SPECTRUM EFFICIENT FREQUENCY ASSIGNMENT FOR CELLULAR RADIO

Thesis by

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to my parents

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Abstract

In this thesis, we first describe some results on the following generalized chromatic number problem that has its origin in cellular radio (the frequency assignment problem with co-channel constraints only): Given a graph G with vertices $V = \{v_1, \ldots, v_n\}$ and an n-vector $M = (m_1, \ldots, m_n)$ of nonnegative integers (the *requirement vector*), find the minimum number of colors, $\chi(G, M)$, required to assign m_i distinct colors to vertex v_i, $i \in \{1, \ldots, n\}$, such that adjacent vertices are assigned disjoint sets of colors. We develop a lower bound on $\chi(G, M)$, which generalizes and strengthens the well-known bound $\chi(G)\alpha(G) \geq n$ for the usual chromatic number. We show that this bound is sharp for a number of interesting graphs (e.g., perfect graphs and odd cycles), but not for all graphs — the Grötzsch graph being a counterexample. We also give examples of the application of this bound to frequency assignment in cellular radio.

In the presence of constraints other than just co-channel constraints (e.g., adjacent channel and co-site constraints), the frequency assignment problem is a further generalization of the graph coloring problem. We describe some heuristic algorithms for frequency assignment in cellular radio that we developed by suitably adapting some of the ideas previously introduced in heuristic graph coloring algorithms. These algorithms have yielded optimal, or near-optimal assignments, in many cases.

We then describe some dynamic channel assignment algorithms for cellular systems that we have developed. In addition to having a considerable advantage over fixed channel assignment in the range of blocking probabilities of interest in current cellular systems (2–4%), these algorithms are feasible for implementation in these systems. Some of these dynamic channel assignment algorithms are also

shown to give good performance under overload (heavy traffic conditions).

Finally, we discuss various methods of computing interference probabilities and the formulation of compatibility constraints on channel assignment based on these calculations. We also formulate the channel assignment problem as one of coloring hypergraphs, instead of graphs, and show that, in the case of dynamic channel assignment, this leads to a considerable increase in the carried traffic for the *same* blocking probability and the *same* maximum probability of interference.

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Chapter 1

Introduction

"Cellular Radio" is the term commonly used to refer to the mobile telephone systems in use today. The service area (city) is divided into a number of *cells*; hence the name "cellular." Each cell is equipped with a base station and these base stations are connected by wire-links to a telephone exchange. The "radio" link is only between the mobile (phone) and the base station of the cell in which the mobile is located.

A bandwidth of 50 MHz is currently allocated in the U.S. for cellular radio operation and this bandwidth is divided equally between two independent operators. Therefore, there are *two* cellular systems in each city, each with its own set of base sites (or base stations) and its own bandwidth. The available bandwidth is divided into 30 KHz frequency channels. These frequency channels come in pairs; one frequency channel is used for communication between the base station and the mobile, and the other for communication between the mobile and the base station. Henceforth, we will use the terms "frequency" and "channel" to mean one such pair of frequency channels. Therefore, there are 416 frequencies available in each cellular system in the U.S. But 21 of these are "control" channels which are used, among other things, for setting up the call, so that only 395 "voice" channels are available*. This extremely limited availability of frequencies means that the same frequency has to be used simultaneously in different cells, in the same way that TV channels are used simultaneously in different regions. But two cells which use the same frequency — more precisely, two cells in which the same frequency is used —

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* These numbers pertain to the U.S. and are by no means universal. For instance, European systems commonly use 25 KHz channels.

must not be located geographically close together, in order to keep the crosstalk (or interference) within acceptable limits.

Current cellular systems use *fixed channel assignment*, i.e., each cell is assigned a fixed subset of the available channels. Assume that the pairs of cells that should not use the same frequency and the distribution of traffic in each cell are known. Then the problem is to find an assignment of (sets of) frequencies to the cells, in such a manner that the number of blocked calls is minimized or equivalently, the carried traffic and (hence the revenue!) is maximized. Consider the following closely related problem: The pairs of cells that cannot use the same frequency and the number of frequencies to be assigned to each cell are given. Find the minimum number of frequencies required. This is a generalization of the usual chromatic number problem in graph theory because: if a graph is drawn where each vertex represents a cell and pairs of cells which are forbidden from using the same frequency are joined by an edge, the problem becomes one of assigning as many distinct colors to each vertex as the number of frequencies required by the correponding cell such that, adjacent vertices are assigned disjoint sets of colors and the total number of colors used is minimized. We study several aspects of this problem in Chapter 2. The main result of Chapter 2 is a generalization of the *independence (lower) bound* on the chromatic number in graph theory. The *clique (lower) bound*, which is commonly used in cellular radio, appears as a special case of this generalization.

But in reality, there are constraints not only on the use of the same frequency but also on the use of nearby frequencies. Consider a mobile located at the boundary of a cell so that the signals it receives from two base stations are of nearly the same strength. If the mobile is engaged in a phone call, only one of these signals is of interest to it (the desired signal) and the other constitutes interference. If the interfering signal is in an adjacent frequency channel, it can cause an unacceptable level of interference because the filtering process is not ideal. Therefore, one may have to prohibit the use of adjacent frequencies in adjacent cells. The signals received by a base station from the mobiles in its cell can have widely differing strengths so that, the interfering signal (which is on a different frequency) can be much stronger than the desired signal; hence, within the same cell, not only the use of adjacent frequency channels, but also the use of frequency channels that have one or more frequency channels between them, may have to be forbidden. This leads to a generalized graph coloring problem that is treated in Chapter 3. In that chapter, we describe some fast heuristic algorithms for finding "good" frequency assignments, which were obtained by suitably modifying heuristics introduced previously for graph coloring. (The lower bounds developed in Chapter 2 for the simpler problem are, however, valid for this more general problem, as well.)

In Chapter 4, we consider the problem of minimizing the blocking probability with the given number of frequencies, using *dynamic channel assignment.* In fixed channel assignment, only a fixed subset of the channels is available in each cell whereas in dynamic channel assignmnent, *all* the frequencies are available in *all* the cells. However, the constraints on the assignment of channels are the same as in fixed channel assignment. We describe some dynamic channel assignment algorithms that, for the same offered traffic, not only result in fewer blocked calls than fixed channel assignment, in the range of blocking probabilities of interest (2–4%), but also are feasible for implementation in current cellular systems. These algorithms are also shown to have good performance under heavy traffic conditions.

The *probability of interference* P_i for a call is the probability that the signal-to-interference ratio falls below a specified level, termed the *protection ratio*, and is a quantitative measure of the amount of interference experienced by that call. The objective in cellular radio operation is to minimize the blocking probability (P_b)

while maintaining an acceptable P_i, i.e., while maintaining $P_i \leq P_i^{\max}$ (specified) for all calls. In Chapter 5, we consider several methods of calculating P_i and the parameters of the graph coloring model. We will then show that a better model for the frequency assignment problem is *hypergraph coloring* rather than graph coloring because, for the same value of P_b and $P_i^{\max}$, we can get an increase in the carried traffic using the hypergraph coloring model, though it is much harder to work with.

The various chapters are more or less independent of one another and each chapter also has an abstract. The references for each chapter are listed at the end of that chapter.

Some "Cellular Jargon"

Constraints on the use of the same channel are termed *co-channel constraints* while those on the use of adjacent channels are termed *adjacent channel constraints.* The term *co-site constraints* refers to the constraints on channels used in the same cell.

Frequently, in the literature, cellular systems are assumed to consist of regular, hexagonal cells, with uniform propagation conditions and traffic. In reality, none of these assumptions are likely to be valid. Though we will use examples from these ideal systems, we will not specifically use the hexagonal geometry to develop methods of frequency assignment; hence, our methods are applicable to *all* cellular systems. But a phrase that is commonly encountered, and which we will also use in our examples, is "number of reuse groups." In the case of infinite (or sufficiently large) regular, hexagonal systems, the phrase "number of reuse groups is M" means that the co-channel constraints on frequency assignment are such that to assign equal numbers of frequencies to all the cells, the total number of frequencies available has to be divided into M groups, with one group being assigned to each cell. This is equivalent to saying that two cells can use the same frequency if, and only if, the distance between their centers is $< \sqrt{M}$ (see [Mac] and [Gam]).

References

[Gam] Gamst, A., "Homogeneous Distribution of Frequencies in a Regular, Hexagonal Cell System," *IEEE Trans. Veh. Tech.*, vol. VT-31, pp. 132–144, 1982.

[Mac] MacDonald, V.H., "The Cellular Concept," *The Bell System Tech. J.*, vol. 58, pp. 15–41, Jan. 1979.

Chapter 2

A Graph Coloring Problem in Cellular Radio

Abstract

This chapter describes some results on the following generalized chromatic number problem that has its origin in cellular radio: Given a graph* G with vertices $V = \{v_1, \ldots, v_n\}$ and an n-vector $M = (m_1, \ldots, m_n)$ of nonnegative integers (the *requirement vector*), find the minimum number of colors, $\chi(G, M)$, required to assign m_i distinct colors to vertex v_i, $i \in \{1, \ldots, n\}$, such that adjacent vertices are assigned disjoint sets of colors. We develop a lower bound on $\chi(G, M)$, which generalizes and strengthens the well-known bound $\chi(G)\alpha(G) \geq n$ for the usual chromatic number. We show that this bound is sharp for a number of interesting graphs (e.g., perfect graphs and odd cycles), but not for all graphs — the Grötzsch graph being a counterexample. We also give examples of the application of this bound to frequency assignment in cellular radio.

2.1. Motivation

In the so-called "(fixed) frequency assignment problem" of cellular radio [Pen], it is necessary to assign to each cell a specified number of frequencies (proportional to the traffic in that cell) subject to the constraint that (in order to minimize interference) certain pairs of cells are forbidden from using common frequencies. If a graph is drawn where each vertex represents a cell, and pairs of vertices corresponding to pairs of cells that are forbidden from using common frequencies are joined by an

* The term "graph" used in this thesis means an undirected graph with no (self-)loops or multiple edges.

edge, the problem becomes one of assigning frequencies (or colors) to the vertices of this graph such that adjacent vertices are assigned disjoint sets of colors. Since frequency spectrum is scarce, this assignment should use as few colors as possible; thus we arrive at the generalized chromatic number problem stated in the abstract. If $M = (1, \ldots, 1)$ we obtain the usual chromatic number problem in graphs, i.e., to find the fewest colors necessary to assign a single color to each vertex so that adjacent vertices are assigned different colors.

2.2. Lower Bounds

Definition. If $G = (V, E)$, $S \subseteq V$ is an *independent set* if $x, y \in S \Rightarrow xy \notin E$. The *independence number* α_i of v_i is the size of the largest independent set containing v_i, i.e.,

$$\alpha_i = \max_{\substack{S: S \subseteq V, \\ \text{independent}, \\ v_i \in S}} |S|.$$

Theorem 1.

$$\chi(G, M) \geq \left\lceil \sum_{i=1}^{n} \frac{m_i}{\alpha_i} \right\rceil.$$

Proof: Consider any coloring that uses $\chi = \chi(G, M)$ colors. This can be represented as an $n \times \chi$ matrix $A = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1, & \text{if color } j \text{ is assigned to vertex } i \\ 0, & \text{otherwise.} \end{cases}$$

Let

$$\delta_j = \min_{i: a_{ij} = 1} \alpha_i. \tag{2.1}$$

By the definition of α_i,

$$\sum_{i=1}^{n} a_{ij} \leq \delta_j. \tag{2.2}$$

Then,

$$\begin{aligned}\sum_{i=1}^{n} \frac{a_{ij}}{\alpha_i} &\leq \sum_{i=1}^{n} \frac{a_{ij}}{\delta_j} \qquad \text{(using (2.1))}\\ &= \frac{1}{\delta_j}\sum_{i=1}^{n} a_{ij}\\ &\leq \frac{1}{\delta_j}\delta_j \qquad \text{(using (2.2))}\\ &= 1. \end{aligned} \tag{2.3}$$

Therefore,

$$\begin{aligned}\sum_{i=1}^{n} \frac{m_i}{\alpha_i} &= \sum_{i=1}^{n} \frac{1}{\alpha_i}\sum_{j=1}^{\chi} a_{ij}\\ &= \sum_{j=1}^{\chi}\left(\sum_{i=1}^{n}\frac{a_{ij}}{\alpha_i}\right)\\ &\leq \sum_{j=1}^{\chi} 1 \qquad \text{(using (2.3))}\\ &= \chi. \end{aligned}$$

∎

Example 2.1. Consider the cellular system shown in Figure 2.1. The cells are located on a regular hexagonal grid and the distance between the centers of adjacent cells is taken to be unity. Let us assume that if the distance between the centers of two cells is ≤ 2, they should not use the same frequency. The number of frequencies to be assigned to each cell is indicated within the cell.

This cellular system can be represented by the graph in Figure 2.2. (The non-edges are indicated by dashed lines.) The frequency requirement m_i at vertex v_i is shown above v_i. The independence numbers of vertices v_4, v_7 and v_8 are unity and

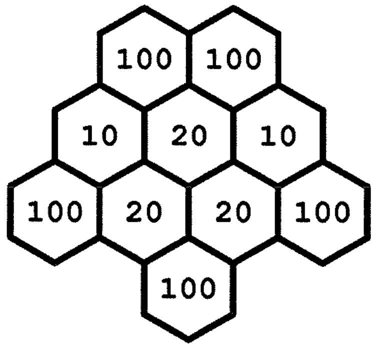

Figure 2.1.

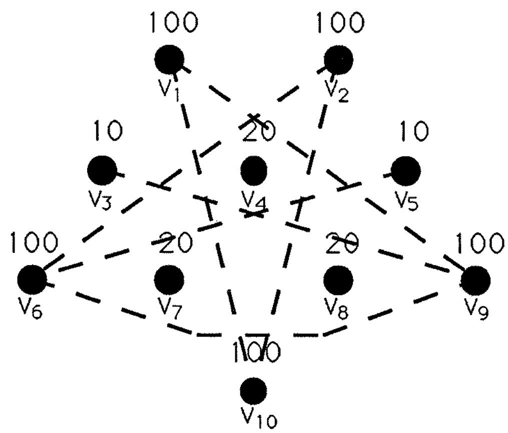

Figure 2.2.

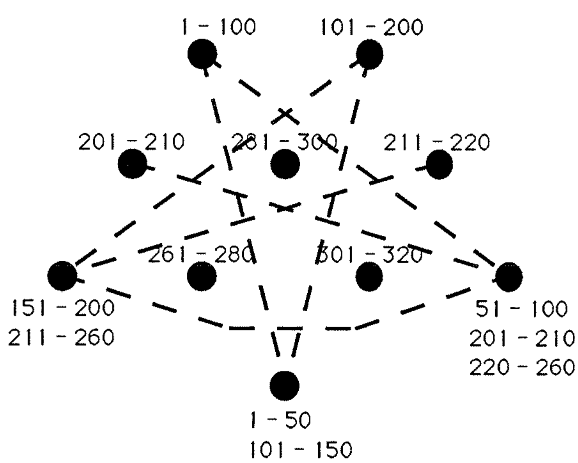

Figure 2.3.

the other vertices have independence number 2. For this problem, from Theorem 1, $\chi(G,M) \geq \lceil 3 \cdot 20 + \frac{1}{2}(5 \cdot 100 + 2 \cdot 10) \rceil = 320$. In this example, 320 colors are also sufficient as demonstrated by the coloring (frequency assignment) in Figure 2.3, where the positive integers are used to denote the various colors. ▮

Let α denote the size of the largest independent set in V (i.e., α is the *independence* or *stability number* of G). Then, from Theorem 1 we have the following

Corollary 1.

$$\chi(G,M) \geq \left\lceil \frac{1}{\alpha} \sum_{i=1}^{n} m_i \right\rceil .$$

If $M = (1, \ldots, 1)$, this is the independence bound on the usual chromatic number ($\chi(G)\alpha(G) \geq n$ [Ber, p. 331]).

Let $V' \subseteq V$ and let $G_{V'}$ be the subgraph* of G induced by V'. (The vertex set of $G_{V'}$ is V' and the edges of $G_{V'}$ are all edges of G both of whose ends are in V'.) Let $M_{V'}$ be the "induced" requirement vector for $G_{V'}$, i.e., if $v_i \in V'$, the requirement at vertex v_i in $G_{V'}$ is m_i. Then, $\chi(G,M) \geq \chi(G_{V'}, M_{V'})$. If $v_i \in V'$, let $\alpha_i^{V'}$ denote the independence number of v_i in $G_{V'}$. Applying Theorem 1 to all such subgraphs, we obtain

Corollary 2.

$$\chi(G,M) \geq \Omega(G,M) \stackrel{\text{def}}{=} \max_{V':V' \subseteq V} \left\lceil \sum_{i:v_i \in V'} \frac{m_i}{\alpha_i^{V'}} \right\rceil .$$

Example 2.2. Consider the cellular system shown in Figure 2.4. The constraints are the same as in Example 2.1. Theorem 1 yields a lower bound of 305 frequencies.

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* The term "subgraph" is always used in this thesis to mean a vertex-induced subgraph.

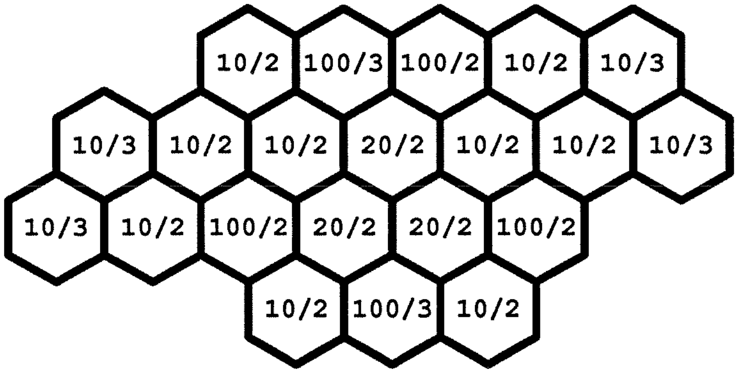

Figure 2.4. Within every cell, the requirement (m_i) and the independence number (α_i) of the vertex v_i representing this cell, are shown in the form m_i/α_i.

However, the graph of Example 2.1 is an induced subgraph of the graph representing this system. Therefore, Corollary 2 yields a lower bound of 320 frequencies. The frequency assignment heuristics described in the next chapter yield an assignment that uses 320 frequencies so that the bound of Corollary 2 is sharp in this example. ∎

If V' is a clique, $\alpha_i^{V'} = 1$ for all $i : v_i \in V'$. Applying Theorem 1 to all cliques, we obtain

Corollary 3. (*The Clique Bound*)

$$\chi(G,M) \geq \omega(G,M) \stackrel{\text{def}}{=} \max_{\text{all cliques } V'} \left\lceil \sum_{i:v_i \in V'} m_i \right\rceil .$$

This is the most commonly used bound in cellular radio [Pen] and appears as a special case of our more general bounds.

Example 2.3. The clique bound only yields a lower bound of 280 in both Examples 2.1 and 2.2 — the largest clique consists of the vertices v_1, v_2, v_3, v_4, v_5, v_7 and

v_8 of Figure 2.2 — so that our more general lower bounds are necessary in these cases. ∎

2.3. Perfect Graphs

For any graph G, let G^M denote the graph obtained by replacing each vertex v_i of G by K_{m_i} (the complete graph on m_i vertices*) and joining each vertex of K_{m_i} to all vertices of K_{m_j} if v_i is adjacent to v_j in G (and to no vertex of K_{m_j} if v_i is not adjacent to v_j in G). Let $V^M = \{v_1^1, \ldots, v_1^{m_1}, \ldots\ldots\ldots, v_n^1, \ldots, v_n^{m_n}\}$ be the vertex set of G^M. Then, clearly $\chi(G, M) = \chi(G^M)$.

Lemma. If $G = (V, E)$, let $\alpha(G)$ be the independence number of G and $\theta(G)$ the minimum number of cliques needed to partition V. Then, for any M, if $m_i \geq 1$ for all i, $\alpha(G^M) = \alpha(G)$ and $\theta(G^M) = \theta(G)$.

Proof: If $\{v_{i_1}, \ldots, v_{i_k}\}$ is an independent set of G then $\{v_{i_1}^1, \ldots, v_{i_k}^1\}$ is an independent set of G^M; hence, $\alpha(G^M) \geq \alpha(G)$. But no independent set of G^M contains two vertices from the same K_{m_i}; hence, $\alpha(G^M) \leq \alpha(G)$. If $\{v_{i_1}, \ldots, v_{i_k}\}$ is a clique in G, then $\{v_{i_1}^1, \ldots, v_{i_1}^{m_{i_1}}, \ldots, v_{i_k}^1, \ldots, v_{i_k}^{m_{i_k}}\}$ is a clique in G^M. Therefore, to every partition of V into cliques, there corresponds a partition of V^M into cliques. Hence, $\theta(G^M) \leq \theta(G)$. On the other hand, given any partition of V^M into cliques, we have a partition of $\{v_1^1, v_2^1, \ldots, v_n^1\}$ into cliques, which corresponds to a partition of V into cliques. Hence, $\theta(G^M) \geq \theta(G)$. ∎

Definition. A graph G is *perfect* if for every (vertex-induced) subgraph G_A of G, $\alpha(G_A) = \theta(G_A)$.

Examples of perfect graphs are complete graphs, bipartite graphs (in particular, paths, trees and even cycles), interval graphs and comparability graphs [Ber, chap. 16].

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* If m_i is zero, this results in the deletion of vertex v_i.

Theorem 2. If G is perfect, G^M is perfect.*

Proof: Every subgraph of G^M is isomorphic to $G_1^{M'}$, for some subgraph G_1 of G and some vector M' with nonzero components. Therefore, applying the above lemma to all subgraphs of G^M, G^M is perfect if G is perfect. ∎

If $\omega(G)$ denotes the size of the largest clique of a graph G, by the Perfect Graph Theorem of Lovász [Ber, chap. 16], a graph G is perfect iff $\chi(G_A) = \omega(G_A)$ for every induced subgraph G_A of G. Together with Theorem 2, this shows that *if G is perfect*, $\chi(G,M) = \omega(G,M)$ $(= \Omega(G,M))$. In particular, $\chi(G,M) = \omega(G,M)$ for complete graphs, bipartite graphs and even cycles.

Theorem 3, which follows, shows that $\chi(G,M) = \Omega(G,M)$ for odd cycles, which are all *imperfect* (not perfect) graphs.

2.4. Imperfect Graphs

Theorem 3. If $C_{2k+1} = (V,E)$ denotes the cycle on $2k+1$ vertices where $E = \{v_1v_2, v_2v_3, \ldots, v_{2k}v_{2k+1}, v_{2k+1}v_1\}$ and

$$L_{2k+1}(M) \overset{\text{def}}{=} \max\left\{ m_1+m_2, m_2+m_3, \ldots, m_{2k+1}+m_1, \left\lceil \frac{m_1+m_2+\ldots+m_{2k+1}}{k} \right\rceil \right\}$$

then, $\chi(C_{2k+1}, M) = L_{2k+1}(M)$.

Proof: From Corollary 2, restricted to the $2k+1$ edges and the entire graph (or by taking the maximum of the bounds obtained from Theorem 1 and Corollary 3), $\chi(C_{2k+1}, M) \geq L_{2k+1}(M)$. To prove the reverse inequality, we use induction on $\sum_i m_i$. If $\sum_i m_i = 1$, exactly one of the m_is is nonzero and $\chi(C_{2k+1}, M) = L_{2k+1}(M) = 1$, so that $\chi(C_{2k+1}, M) \leq L_{2k+1}(M)$. If one of the m_is is 0, the subgraph induced by the other vertices is P_{2k} (the path on $2k$ vertices) which is a perfect graph. Hence, by Theorem 2, $\chi(G,M) = \max\{m_i + m_{i+1}, i = 1, \ldots, 2k+1\} \leq$

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* Note that if $m_i \geq 1$ for all i, the converse is also true since every subgraph of G is then isomorphic to a subgraph of G^M.

$L_{2k+1}(M)$ since the edges are the only (maximal) cliques of C_{2k+1}. (All subscripts in this proof are modulo $2k+1$.) Therefore, we assume that all the m_is are > 0. If $\min_i m_i + m_{i+1} = m_j + m_{j+1}$ assign a single color to each vertex in the independent set $S = \{v_{j+2}, v_{j+4}, \ldots, v_{j-1}\}$. Let

$$m'_i = \begin{cases} m_i - 1, & \text{if } v_i \in S \\ m_i, & \text{otherwise.} \end{cases}$$

and let $M' = (m'_i)$. Since $|S| = k$, $\frac{1}{k}\sum_i m'_i = (\frac{1}{k}\sum_i m_i) - 1$. Therefore, since S contains one vertex from every edge except $v_j v_{j+1}$,

$$L_{2k+1}(M') = L_{2k+1}(M) - 1 \tag{2.4}$$

unless $L_{2k+1}(M) = m_j + m_{j+1}$ and $m_j + m_{j+1} \geq \lceil \frac{1}{k}\sum_{i=1}^{2k+1} m_i \rceil$. But this is impossible because,

$$\begin{aligned} \frac{1}{k}\sum_{i=1}^{2k+1} m_i &> \frac{1}{k}\sum_{i=1}^{k}(m_{2i} + m_{2i+1}) \qquad \text{(since } m_1 > 0\text{)} \\ &\geq m_j + m_{j+1} \qquad \text{(by the minimality of } m_j + m_{j+1}\text{).} \end{aligned}$$

Since only one color has been used to reduce the requirement vector M to M', this yields

$$\begin{aligned} \chi(C_{2k+1}, M) &\leq \chi(C_{2k+1}, M') + 1 \\ &\leq L_{2k+1}(M') + 1 \qquad \text{(by the induction hypothesis)} \\ &= L_{2k+1}(M) \qquad \text{(using (2.4)).} \end{aligned}$$

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It can be verified that the only imperfect graph on five vertices or less is the pentagon; therefore, $\chi(G, M) = \Omega(G, M)$ for all graphs on five or fewer vertices.

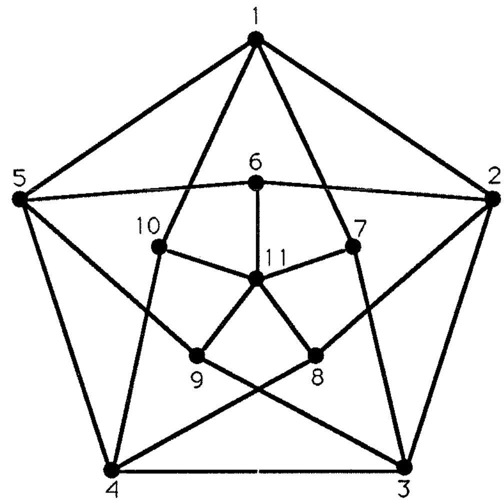

Figure 2.5. The Grötzsch graph.

Using the same method as in the proof of Theorem 3, it can be shown that this holds for all graphs on six vertices and for the complements of the odd cycles. This raises the question: Is $\chi(G,M) = \Omega(G,M)$ for *all* graphs? The answer is *no* as seen from the following example, which is the smallest one we know.

Example 2.4. Consider the graph (called the Grötzsch graph [BCL, p. 241]) shown in Figure 2.5, with $M = (1,\ldots,1)$ so that we have the usual vertex-coloring problem. The chromatic number of this graph is 4 [BCL, pp. 241–242] but the lower bound of Theorem 1 applied to this graph yields only 3. (In Figure 2.5, $\alpha_1 = \cdots = \alpha_5 = 4$, $\alpha_6 = \cdots = \alpha_{10} = 5$ and $\alpha_{11} = 3$. Hence, by Theorem 1, $\chi(G,M) \geq \lceil (5)\frac{1}{4} + (5)\frac{1}{5} + \frac{1}{3} \rceil = 3$.) Moreover, it can be verified — there are only 3 cases to check — that this graph is χ-*critical*, i.e., deletion of any vertex decreases the chromatic number and hence, Corollary 2 also yields a lower bound of only 3. ∎

A class of graphs which is of interest in frequency assignment is the class of *unit disk graphs* [Hal].

Definition. *Unit disk graphs* are graphs whose vertices can be represented by points in the plane such that two vertices are adjacent iff the distance between the corresponding points is ≤ 1. *Hexagonal graphs* are unit disk graphs which satisfy the additional constraint that there exists such a representation where these points form a subset of a regular hexagonal grid in the plane.

Observe that if an induced subgraph of a graph G is not a unit disk graph, then the graph G is also not a unit disk graph. Therefore, the Grötzsch graph, for which the bound of Corollary 2 is not sharp, is not a unit disk graph because the subgraph induced by $\{v_2, v_3, v_4, v_5, v_6, v_8, v_{11}\}$ (shown in Figure 2.6) is not a unit disk graph [Hal, p. 1510].

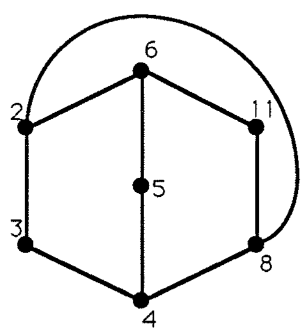

Figure 2.6.

Is the bound of Corollary 2 sharp, i.e., is $\chi(G, M) = \Omega(G, M)$, for all unit disk graphs, or at least for all hexagonal graphs? The answers are again negative as shown by the following example.

Example 2.5. Consider the cellular system shown in Figure 2.7. The constraints are the same as in Example 2.1. Theorem 1 yields $\chi(G, M) \geq \lceil (2)\frac{1}{4} + (3)\frac{1}{5} + (4)\frac{2}{5} + (6)\frac{1}{6} \rceil = 4$. It can be shown that for this problem $\chi(G, M) = 5$. (If there is a 4-coloring of this graph, let color $\{1\}$ be used at v_1, color $\{2\}$ at v_2 and colors $\{3, 4\}$ at v_3. Observe that the colors of vertices $v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}$ and v_{12}

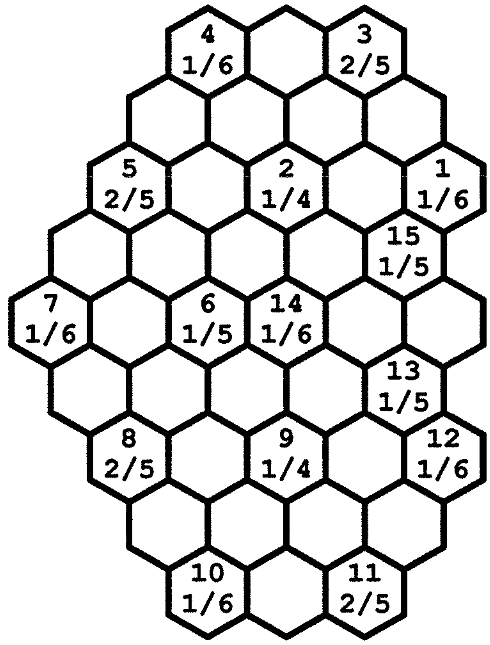

Figure 2.7. Within every cell with a nonzero frequency requirement, this requirement (m_i) and the independence number of the vertex v_i representing this cell (α_i) are shown in the form m_i/α_i, below the cell number i. The blank cells have $m_i = 0$.

are forced to be $\{1\}$, $\{3,4\}$, $\{1\}$, $\{2\}$, $\{3,4\}$, $\{2\}$, $\{1\}$, $\{3,4\}$ and $\{1\}$ respectively. Therefore, none of the vertices $\{v_{13}, v_{14}, v_{15}\}$ can use colors $\{1\}$ and $\{2\}$ but they must use three distinct colors, which is a contradiction.) It can be verified that deleting any vertex decreases the chromatic number, so that Corollary 2 also yields a lower bound of only 4. Hence, the bound of Corollary 2 is not sharp even for all hexagonal graphs. ∎

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Chapter 3

Fixed Channel Assignment

Abstract

In this chapter, we describe some heuristic frequency assignment algorithms for cellular systems that we have developed. These algorithms have yielded optimal, or near-optimal assignments, in many cases. The frequency assignment problem can be viewed as a generalized graph coloring problem, and these algorithms have been developed, in part, by suitably adapting some of the ideas previously introduced in heuristic graph coloring algorithms.

3.1. Introduction and Problem Statement

The frequency assignment problem discussed in the last chapter is a special case of a more general problem, some aspects of which we address in this chapter. The only type of restriction on frequency reuse has hitherto been of the "co-channel" type, i.e., certain pairs of cells are forbidden from using the same frequency. However, in practice, principally because the filtering process is not ideal, it turns out to be necessary to impose restrictions, not only on the use of the same frequency, but also on the use of nearby frequencies. In addition, we wish to find, not only the minimum number of frequencies required, but also an assignment of frequencies that achieves this minimum. This leads to the following more general problem.

Problem Statement

Frequencies are represented by the positive integers 1, 2, 3, ...

Given:

N: the number of cells in the system,

m_i, $1 \le i \le N$: the number of channels required in cell i, and

c_{ij}, $1 \le i, j \le N$: the minimum frequency separation required between a frequency used in cell i and a frequency used in cell j.

Find:

f_{ik}, $1 \le i \le N$, $1 \le k \le m_i$: the frequency assigned to the kth requirement (or call) in the ith cell

such that,

$$\max_{i,k} f_{ik}$$

(i.e., the total number of frequencies required), is a minimum, subject to the separation or compatibility constraints,

$$|f_{ik} - f_{jl}| \ge c_{ij}$$

for all i, j, k, l except for $i = j$, $k = l$.

Example 3.1. The number of cells is $N = 4$. $M = (m_i) = (1, 1, 1, 3)$ is the vector of requirements. The separation or *compatibility matrix* $\mathbf{C} = (c_{ij})$ is

$$\mathbf{C} = \begin{pmatrix} 5 & 4 & 0 & 0 \\ 4 & 5 & 0 & 1 \\ 0 & 0 & 5 & 2 \\ 0 & 1 & 2 & 5 \end{pmatrix}.$$

It is required to find positive integers (frequencies) f_{11}, f_{21}, f_{31}, f_{41}, f_{42} and f_{43}, such that their maximum is a minimum, subject to the separation constraints specified by $\mathbf{C}$. (We will return to this example later.) ∎

This problem is equivalent to the following generalized graph coloring problem. Consider the graph obtained by representing each call by a vertex, with an edge joining two vertices if the corresponding calls cannot use the same frequency. This edge is labelled with the required minimum separation between the frequencies assigned to these calls. The frequency assignment problem is then equivalent to assigning positive integers to the vertices of this graph such that, if two vertices are connected by an edge, the absolute value of the difference of the integers assigned to these vertices, is at least equal to the edge label, and the maximum integer used is as small as possible. If all the c_{ij}'s are 0's and 1's (pure co-channel case), this reduces to the classical graph coloring problem. Since the latter is known to be NP-complete, it follows that the generalized graph coloring problem is also NP-complete [GJ79]. Thus we cannot expect to find an efficient algorithm that solves this problem exactly. However, we have found a number of good heuristic algorithms, which we present in the next section.

3.2. Heuristic Algorithms

The basic idea of all of our algorithms is to list the calls in some order, and use either a *requirement exhaustive strategy* or a *frequency exhaustive strategy* (see [ZB] or [GR]).

Frequency Exhaustive strategy:

1. Starting at the top of the list, assign to each call the *least* possible frequency, consistent with previous assignments, i.e., without violating the separation constraints. (In this strategy, one takes a call and goes through or *exhausts* all the frequencies; hence the name.)

Requirement Exhaustive strategy:

1. Take frequency 1 and assign it to the first call in the list. There may be other calls, further down the list, which can reuse frequency 1. If so, assign frequency

1 again to the first such call in the list. Continue in this manner until there is no call in the list, to which frequency 1 can be assigned.

2. Now take frequency 2, and starting at the top of the list, similarly assign it to all possible calls in the list.
3. Continue in this manner until all the calls have been assigned frequencies. (In this strategy, one takes a frequency and exhausts the requirements (calls); hence the name.)

Remark. If the calls are ordered such that all calls which are assigned frequency 1, in an optimal assignment, are at the top of the list, followed by calls which are assigned frequency 2, and so on, either a frequency exhaustive or a requirement exhaustive strategy will produce an optimal assignment (i.e., using the minimum number (span) of frequencies).

But there are $n!$ possible orderings of n calls! Since the problem is NP-complete, we do not try to find an "optimal ordering" but instead, are content with an ordering that yields a "good" assignment, i.e., an assignment in which the number of frequencies used is close to the minimum. We will, in fact, consider just four different orderings of the calls. These orderings are based on the notion of the *degree* of a call.

The *degree* of cell i is defined as

$$d_i = \left(\sum_{j=1}^{N} m_i c_{ij} \right) - c_{ii}, \qquad 1 \leq i \leq N,$$

which is a heuristic measure of the difficulty of assigning a frequency to a call in that cell. The *degree of a call* is the degree of the cell in which it is contained. In the equivalent graph coloring problem described above, the degree of a call is equal to the sum of the labels on the edges, incident at the vertex corresponding to the call.

Based on this, two different orderings of the cells are considered. They are the *node-color* and *node-degree* orderings considered by Zoellner and Beall in [ZB], except that the *above* definition of the *degree* of a cell is used. In the node-degree ordering, the cells are arranged in decreasing order of their degrees. The node-color ordering is obtained as follows: Of the N cells, the cell with the least degree is placed at the last (Nth) place in the list. This cell is eliminated from the system and the degrees of the remaining cells are recomputed. Now, the cell with the least degree is placed at the $(N-1)$th position in the list, and eliminated from the system. This process is continued until the ordering is complete. These orderings are modifications of the "highest degree first" and "least degree last" heuristics in graph coloring [GJ76, MMI, WP].

Once the *cells* have been ordered, the *calls* can be ordered in two ways. The calls are arranged in an $(N \times m_{\max})$ matrix, where N is the number of cells and $m_{\max}$ is the maximum number of calls in any cell. Each row of the matrix corresponds to the calls in a cell. The rows are arranged in node-color or node-degree order as explained above. The idea is to arrange the calls such that all the columns have nearly the same number of calls. Calls in the first row start at the first column. Calls in the second row start at column $(m_1 + 1)$, if the first row has m_1 calls, and cyclically fill this row. Similarly, calls in the third row start where the second ends and so on.

Example 3.1 (continued). The degrees of the calls are $\mathbf{d} = (4, 7, 6, 13)$. Therefore the node-degree ordering is (cell 4, cell 2, cell 3, cell 1). The matrix of calls corresponding to this is

$$\mathbf{A}_d = \begin{pmatrix} a_{41} & a_{42} & a_{43} \\ a_{21} & & \\ & a_{31} & \\ & & a_{11} \end{pmatrix}.$$

In the node-color ordering, cell number 1 is again the last in the list since it has the least degree. If this cell is eliminated, the degrees of the other cells become $\mathbf{d} = (-, 3, 6, 13)$. Therefore, cell number 2 will be in the third place in the list. The final node-color ordering is (cell 4, cell 3, cell 2, cell 1). The matrix of calls corresponding to this is

$$\mathbf{A}_c = \begin{pmatrix} a_{41} & a_{42} & a_{43} \\ a_{31} & & \\ & a_{21} & \\ & & a_{11} \end{pmatrix}.$$

∎

Once the calls have been so arranged in a matrix, two orderings of the calls are obtained by either listing all the calls in the first row, then the second, and so on (*row-wise ordering*), or listing the calls in the first column, then the second, and so on (*columnwise ordering*). Therefore one obtains *four* ways of ordering the *calls* from *two* ways of ordering the *cells*. Combined with *two* techniques of assigning frequencies, this gives rise to *eight* frequency assignment algorithms.

The assignments obtained using one or the other of these algorithms, in many of the examples we considered, is close to the best lower bound (LB) that we can obtain. In many cases, the best lower bound that we can obtain is the clique bound (see the previous chapter or [Ga86]). In the other cases, we use the lower bounds described in [Ga86]. (If the clique bound is not sharp it is often the case that, in the presence of constraints other than co-channel constraints, especially large c_{ii}, the lower bounds described in [Ga86] are better than the lower bounds of the previous chapter. However, the lower bounds of the previous chapter should prove more useful when the dominant constraints are of the co-channel type.) Some of these examples are described in the next section.

Example 3.1 (continued). Consider the matrix of calls $\mathbf{A}_d$. Ordering the calls row-wise, one obtains (a_{41}, a_{42}, a_{43}, a_{21}, a_{31}, a_{11}) as the list of calls. A frequency exhaustive strategy applied to this list of calls gives the frequency assignment (1, 6, 11, 2, 3, 6). A requirement exhaustive strategy, at the first step, assigns frequency 1 to calls 1 and 7 (a_{41} and a_{11}). The complete assignment using this strategy is (1, 6, 11, 5, 3, 1). The maximum frequency used by both these assignments is 11, which is also the lower bound. This is because, any two of the three calls in cell number 4 require a separation of 5 between them. Ordering the calls columnwise, one gets (a_{41}, a_{21}, a_{42}, a_{31}, a_{43}, a_{11}). The row-wise and columnwise ordering methods applied to the matrix of calls $\mathbf{A}_c$ yield (a_{41}, a_{42}, a_{43}, a_{31}, a_{21}, a_{11}) and (a_{41}, a_{31}, a_{42}, a_{21}, a_{43}, a_{11}), respectively. In all the above cases, in this particular example, a frequency exhaustive strategy gives the assignment (a_{11} : 6, a_{21} : 2, a_{31} : 3, a_{41} : 1, a_{42} : 6, a_{43} : 11) whereas a requirement exhaustive strategy gives the assignment (a_{11} : 1, a_{21} : 5, a_{31} : 3, a_{41} : 1, a_{42} : 6, a_{43} : 11). ∎

The computational complexity of these algorithms is $O(nr)$*, where n is the total number of channel requirements and r is the maximum degree of a call. It is usually the case in cellular sytems, that r is bounded above by a constant, independent of n. In this case, the complexity of these algorithms is only *linear* in n. These algorithms are much faster than the algorithm proposed by Box [Box], which appears to be widely used [Ga86, Ga88]. (Box's algorithm requires a number of

* This assumes that the total number of frequencies used is bounded above by a constant, independent of n. In cellular systems, this is usually the case. Otherwise, the complexity is $O(nr^2)$.

iterations; each iteration has a running time of $O(nr)$ and the number of iterations increases with n.) This feature is particularly important in the case of large cellular systems. This, and the fact that these algorithms are applicable to any cellular system (not necessarily consisting of regular, hexagonal cells), are important when one is trying to choose the optimal locations for the cell sites, by repeated application of a frequency assignment algorithm, since a large number of cases may have to be solved.

3.3. Examples

Example 3.2. The cellular system considered is the 21-cell example found in [Ga86], which is reproduced as Figure 3.1. The frequency requirements (m_i) are shown within each cell. We tabulate the performance of our frequency assignment algorithms for various sets of compatibility constraints in Table 3.1.

Example 3.3. We consider the same system as in the previous example but with a different set of frequency requirements (m_i) which is shown in Figure 3.2. Table 3.2 shows the performance of our algorithms in this case.

Example 3.4. The cellular system, frequency requirements and the compatibility matrix correspond to the example in [Ga88]. The frequency requirements and the compatibility matrix are shown in Table 3.3*. The performance of our algorithms is shown in Table 3.4. It is interesting to note that while the algorithm used in [Ga88] (which is a variant of Box's algorithm [Box]) takes 78 iterations to achieve the lower bound of 222 frequencies, our algorithms result in a frequency assignment that uses only 223 frequencies, which is *nearly optimal*, in the equivalent of 8 iterations (corresponding to trying all 8 algorithms and choosing the best). This is an

* Each of the combiner groups in the example in [Ga88] is treated as a cell for the purposes of frequency assignment.

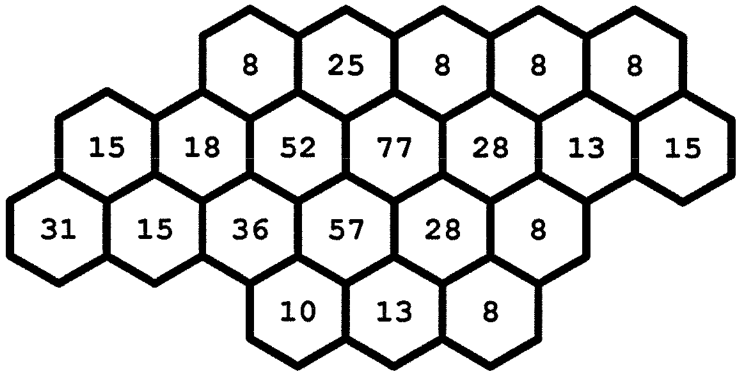

Figure 3.1.

Table 3.1.

N_c	acc	c_{ii}	**LB**	CRF	CRR	CCF	CCR	DRF	DRR	DCF	DCR
12	2	5	**414**	543	464	*460*	476	543	521	475	504
7	2	5	**414**	543	468	451	501	543	466	*447*	495
12	2	7	**533**	*536*	565	546	562	*536*	566	546	565
7	2	7	**533**	536	564	546	559	536	561	*533*	566
12	1	5	**381**	*381*	*381*	*381*	*381*	*381*	*381*	*381*	*381*
7	1	5	**381**	*381*	*381*	*381*	*381*	*381*	*381*	*381*	*381*
12	1	7	**533**	*533*	*533*	*533*	*533*	*533*	*533*	*533*	*533*
7	1	7	**533**	*533*	*533*	*533*	*533*	*533*	*533*	*533*	*533*

N_c denotes the number of reuse groups corresponding to the co-channel constraints. A 2 (resp. 1) in column "acc" implies the presence (resp. absence) of adjacent channel constraints on adjacent cells. The co-site constraint is indicated in column "c_{ii}." **LB** denotes the best lower bound obtained using generalizations of the clique bound described in [Ga86]. A three letter code is used to indicate the algorithms. The first letter is "C" or "D" and denotes "node-Color order" or "node-Degree order" respectively. The second letter is "R" or "C" for "Row-wise" or "Columnwise" ordering. The last letter is "R" or "F" for "Requirement" or "Frequency" exhaustive method of assignment. The entries beneath the acronym for each algorithm are the number of frequencies (span) required by that algorithm. (The numbers closest to the lower bound in each row are in italics.)

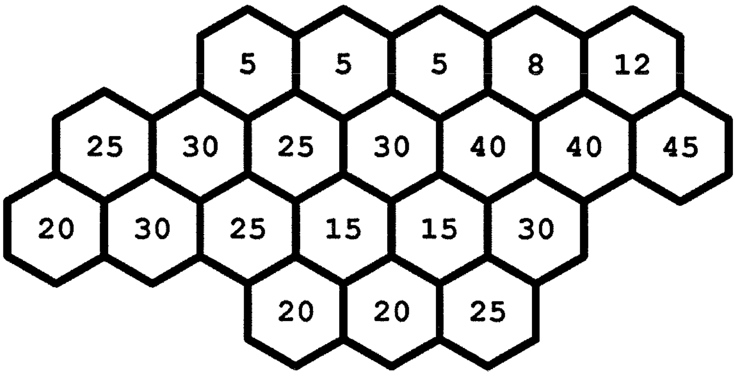

Figure 3.2.

Table 3.2.

N_c	acc	c_{ii}	LB	CRF	CRR	CCF	CCR	DRF	DRR	DCF	DCR
12	2	5	**258**	360	345	296	*283*	346	296	304	297
7	2	5	**229**	347	285	274	272	346	270	280	*269*
12	2	7	**309**	381	325	315	327	384	384	*310*	335
7	2	7	**309**	*310*	319	318	328	358	341	333	338
12	2	12	**529**	*529*	*529*	*529*	*529*	534	530	534	532

N_c denotes the number of reuse groups corresponding to the co-channel constraints. A 2 (resp. 1) in column "acc" implies the presence (resp. absence) of adjacent channel constraints on adjacent cells. The co-site constraint is indicated in column "c_{ii}." **LB** denotes the best lower bound obtained using generalizations of the clique bound described in [Ga86]. A three letter code is used to indicate the algorithms. The first letter is "C" or "D" and denotes "node-Color order" or "node-Degree order" respectively. The second letter is "R" or "C" for "Row-wise" or "Columnwise" ordering. The last letter is "R" or "F" for "Requirement" or "Frequency" exhaustive method of assignment. The entries beneath the acronym for each algorithm are the number of frequencies (span) required by that algorithm. (The numbers closest to the lower bound in each row are in italics.)

Table 3.3.

i	m_i	Compatibility matrix
1	15	5 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
2	14	2 5 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
3	14	2 2 5 2 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
4	14	2 2 2 5 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
5	7	1 1 1 1 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 1 1 0 1 1 1
6	9	1 1 1 1 1 5 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
7	8	1 1 1 1 1 2 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1
8	12	1 1 1 1 1 1 1 5 2 2 1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1
9	12	1 1 1 1 1 1 1 2 5 2 1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1
10	11	1 1 1 1 1 1 1 2 2 5 1 1 1 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 1 1 1 1 1
11	15	1 1 1 1 1 1 1 1 1 1 5 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
12	15	1 1 1 1 1 1 1 1 1 1 2 5 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
13	15	1 1 1 1 1 1 1 1 1 1 2 2 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
14	11	1 1 1 1 1 1 1 0 0 0 1 1 1 5 2 2 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1
15	11	1 1 1 1 1 1 1 0 0 0 1 1 1 2 5 2 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1
16	11	1 1 1 1 1 1 1 0 0 0 1 1 1 2 2 5 1 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 1
17	9	1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 5 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1
18	5	0 0 0 0 1 1 1 0 0 0 1 1 1 1 1 1 1 5 1 0 0 1 0 0 0 1 0 1 1 1 0 0 0 0
19	4	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 5 1 1 1 0 1 0 1 0 1 1 1 0 0 0 0
20	15	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 5 1 1 0 1 0 1 0 1 0 0 0 0 0 0
21	5	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 5 1 1 1 0 1 0 1 0 0 1 0 0 0
22	11	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 5 1 1 1 1 0 1 1 1 1 1 1 1
23	7	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 1 5 1 0 1 0 1 0 0 0 0 0 0
24	7	1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 1 0 1 1 1 1 1 5 1 1 0 1 0 1 1 1 1 1
25	13	1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 5 1 0 0 0 0 0 1 1 0
26	11	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 5 1 1 1 1 1 1 1 1
27	4	0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 5 1 0 0 0 0 0 1
28	8	1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 5 1 1 0 1 1 1
29	5	1 1 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0 1 0 0 0 1 0 1 5 0 0 0 0 0
30	7	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 1 0 1 0 1 0 5 0 0 0 0
31	6	1 1 1 1 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 0 1 0 1 0 0 0 0 5 0 0 0
32	11	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 1 1 0 1 0 0 0 5 2 0
33	10	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 1 1 0 1 0 0 0 2 5 0
34	14	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 0 1 0 1 1 1 0 0 0 0 0 5

Table 3.4.

LB	CRF	CRR	CCF	CCR	DRF	DRR	DCF	DCR
222	267	247	*223*	*223*	250	252	224	224

(See Table 3.1 or 3.2 for expansion of acronyms.)

improvement in speed by an order of magnitude! This advantage in speed can be quite critical in the case of large problems like the next example.

Example 3.5. We applied our frequency assignment algorithms to a typical large cellular system. This system consisted of 58 cells. But many of the cells were divided into 60° sectors. For the purposes of frequency assignment, each sector of a cell can be treated as a cell. When this was done, we had 245 (virtual) cells. All the cells had modest frequency requirements, the maximum number of frequencies required in any cell being 15. The total number of requirements was 2285. The best lower bound that we could obtain on the minimum number of frequencies required for this problem was 298 and this was obtained using one of the lower bounds in [Ga86]. Our algorithms yielded an assignment that used 305 frequencies.

3.4. Comments and Conclusions

If all the nonzero entries in the separation matrix are taken to be unity, the cells ordered using the node-color or node-degree ordering described above, and the calls ordered row-wise, we obtain the node-color and node-degree orderings described in [GR]. The best assignment obtained using our algorithms, in all the examples considered, is better than the assignments obtained by using this ordering of the calls, and a frequency or requirement exhaustive strategy. However, the performance of the frequency assignment algorithms obtained by using this ordering of the cells, and columnwise ordering of calls, is better in some cases. One such case is in Table 3.1, $N_c = 7$, acc $= 2$ and $c_{ii} = 5$. The minimum number of frequencies used by any of the algorithms listed there is 447 but with the node-color ordering of cells described in [GR], *columnwise* ordering of calls and a frequency exhaustive assignment strategy, an assignment which uses only 445 frequencies may be obtained.

Therefore, the two new ideas on channel assignment introduced in this chapter viz., the new definition of the degree of a cell (or call) in the presence of arbitrary constraints (not purely co-channel), and the columnwise ordering of calls, which corresponds essentially to taking a call from each cell in the system in succession (with some modification to accommodate the unequal numbers of calls in each cell), achieve significant savings in the spectrum needed for a frequency assignment problem.

In addition to being NP-complete, graph coloring is one of the most difficult problems to develop approximation algorithms for. It is shown in [GJ76] that the problem of finding a fast (polynomial-time) algorithm that guarantees a coloring using less than twice the minimum number of colors, is itself NP-complete. In the light of these results, the performance of the heuristics we have developed seems good indeed.

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Chapter 4

Dynamic Channel Assignment

Abstract

This chapter describes some dynamic channel assignment algorithms for cellular systems that we have developed. In addition to having a considerable advantage over fixed channel assignment in the range of blocking probabilities of interest in current cellular systems (2–4%), these algorithms are feasible for implementation in these systems. Some of these dynamic channel assignment algorithms are also shown to give good performance under overload (heavy traffic conditions).

4.1. Introduction and Notation

In fixed channel assignment (FCA), only a fixed subset of all the channels can be used in each cell whereas in dynamic channel assignment (DCA), *all* the channels can be used in *all* the cells. A channel is said to be *available in a cell* if, given the existing configuration of calls in progress in the system, this channel can be used for a new call in that cell, without violating any of the compatibility constraints. *A dynamic channel assignment algorithm, or strategy*, is a method for choosing, when more than one channel is available, which of these channels must be assigned to a new call. We will consider both DCA algorithms that do not permit calls already in progress to be reassigned to a different frequency, and those that do. The objective in DCA is to develop a channel assignment strategy which minimizes the total number of blocked calls and is, in addition, feasible for implementation. Of course, we would like to do better than FCA. It may be argued that FCA is a special case of DCA. However, we *assume* that none of the DCA strategies are allowed to block a call, if a channel is available to carry the call (i.e., these algorithms are greedy). This is not true of FCA assignment since, a call may be blocked under FCA even though a channel is available to carry the call, because of the restriction that only a fixed subset of the channels can be used in each cell. Thus FCA is not a special case of DCA, and indeed in certain rare cases may do better [Kel]. We present several DCA algorithms, of increasing levels of complexity of implementation, and compare their performance against that of FCA.

Notation:

N: the number of cells in the system;

c_{ij}, $1 \leq i, j \leq N$: the frequency separation required between a call in cell i and a call in cell j;

n_i, $1 \leq i \leq N$: the number of calls in progress in cell i;

p_i, $1 \leq i \leq N$: the probability that a new call arrives in cell i;

ρ: the total traffic in the system measured in Erlangs*;

$\rho_i = p_i \rho$, $1 \leq i \leq N$: the traffic in cell i;

N_f: the number of (contiguous) frequency channels available. These channels are numbered 1 through N_f.

f_{ik}, $1 \leq i \leq N$, $1 \leq k \leq m_i$: the frequency assigned to the kth call in the ith cell.

Some more notation will be introduced later, as the need arises.

Compatibility Constraints:

$|f_{ik} - f_{jl}| \geq c_{ij}$ for all i, j, k, l except for $i = j$, $k = l$.

---

* If λ is the mean arrival rate of calls, and T is their mean holding time, the traffic is λT Erlangs. The unit "Erlang" is named after Anger Krarup Erlang (1878–1929), a Danish mathematician and a pioneer in the field of traffic engineering [Ino, p. 225].

Assumptions:

- Call arrivals in cell i are independent of all other arrivals and obey a Poisson distribution with parameter ρ_i.
- Call holding times (durations) are exponentially distributed with a mean of 180 seconds.
- There are no calls handed-off between cells.
- Blocked calls are cleared.

The assumption of memoryless arrivals and holding times is standard in wireline telephony [Fel, p. 282, p. 293, pp. 458–459, Ino, p. 225] and it is reasonable to assume that these hold for mobile telephony as well.

Four dynamic channel assignment strategies will be considered. They are:

1. *Simple*: An incoming call is assigned the least available frequency.
2. *Maxavail*: Of all the frequencies that can be assigned to an incoming call, the frequency which maximizes the total number of channels available in the entire system is assigned to an incoming call.
3. *Remax1*: If no frequency is available for assignment to an incoming call (using the Maxavail strategy), one of the calls in progress is permitted to be reassigned to a different frequency. The reassignment is also carried out using the "Maxavail" principle.
4. *Remax2*: If no channel is available for assignment to an incoming call using the Remax1 strategy, one more call is permitted to be reassigned to a different frequency.

Each strategy is more complex (in terms of difficulty of implementation) than the previous one. However, we will see that there is a payoff in terms of performance.

4.2. Details of the Strategies

The details of the simulation and the channel assignment algorithms will now be described. Since the arrival process is assumed to be memoryless, the time between call arrivals is exponentially distributed. The Mean Time Between call Arrivals (MTBA) in the entire system equals $(180/\rho)$ seconds, since the mean call duration is assumed to be 180 seconds. To simulate the process of call arrivals and departures, the following steps are carried out:

Time is discretized to steps of 10 ms. The minimum duration of a call, and the minimum time between call arrivals, are both assumed to be one time step.

1. To start the process, generate an exponentially distributed random variable (ERV) with mean = MTBA.
2. Increment the time by one step and check for a call arrival. If there are no arrivals, go to step 8.
3. Choose a cell for the call with the required distribution (i.e., choose cell i with probability p_i).
4. Generate an ERV with mean = MTBA for the time to the next call arrival.
5. Call the channel assignment routine.
6. If it returns a channel, assign it to the call, and also make the required call rearrangements, if any. If it reports, "call blocked," go to step 8.
7. Generate an ERV with mean 180 seconds for the call duration, and keep track of the channels that are not assignable to future calls in each cell because of this assignment.
8. Check for call departures.
9. If there are any departures, free all the channels that were tied up in each cell because of the frequencies assigned to the departing calls.
10. Go to step 2.

Each of the following channel assignment algorithms requires that the cell in which the call arises, referred to as "callcell" below, be passed to them. They can access all the existing assignments (say, through global variables). They either return a channel that can be assigned to the call, or report, "call blocked." If it is a rearrangement strategy, the required call rearrangements are also returned.

Simple:

1. From the list of channels 1 through N_f in "callcell," return the first channel that is free.
2. If there is no free channel, report "call blocked."

Maxavail:

1. For each available channel in "callcell," compute

$$\text{Systemwide Channel Availability} = \sum_{\text{all cells}} \text{Number of available channels},$$

   assuming that this channel is assigned to the call.
2. Return that channel which maximizes this sum.
3. In the case of a tie, return the least channel.
4. If no channel is available in "callcell," report "call blocked."

Remax1 :

1. Call Maxavail.
2. If Maxavail reports "call blocked," go to step 3. Otherwise, return the channel returned by Maxavail.
3. Make a list of all the channels in "callcell" that are unavailable because of exactly one other call that is in progress.
4. Call Maxavail for each of these calls (interferers).

5. For each of the interferers that are not reported "blocked" by Maxavail, by assuming that the channel returned by Maxavail is assigned to them, and that the corresponding "freed" channel is assigned to the new call, compute "Systemwide Channel Availability."
6. Return that interferer which maximizes this sum, the channel to which it should be reassigned, and the channel that is to be assigned to the new call.
7. If no channel can be freed by reassigning a single call, report "call blocked."

Remax2 :

1. Call Remax1.
2. If Remax1 reports "call blocked," go to step 3. Otherwise, return the channel returned by Remax1.
3. Make a list of all the channels in "callcell" that are unavailable because of exactly one other call that is in progress.
4. Call Remax1 for each of these calls (interferers).
5. For each of the "interferers" that are not reported "blocked" by Remax1, by assuming that the rearrangement required by Remax1 is made, the channel returned by Remax1 is assigned to the "interferer," and that the corresponding "freed" channel is assigned to the new call, compute "Systemwide Channel Availability."
6. Return the two rearrangements, and the freed channel, corresponding to that call which maximizes this sum.
7. If no channel can be freed using this strategy, report "call blocked."

4.3. Performance of the Strategies

The performance of these channel assignment strategies is now investigated. The cellular system that is chosen for this purpose is the $N = 21$ system described in [Gam, p. 13]. This cellular system is reproduced in Figure 4.1. The channel assignment constraints are:

- co-channel constraints corresponding to 12 reuse groups, i.e., two cells can use the same frequency if, and only if, the distance between their centers is $\geq \sqrt{12}$,
- adjacent channel constraints for adjacent cells, i.e., adjacent cells cannot use the same or adjacent frequencies but may use any other frequency, and
- a co-site constraint of 5.

The compatibility matrix $\mathbf{C} = (c_{ij})$ corresponding to these constraints is shown in Table 4.1.

Case 1: Homogeneous Spatial Traffic Distribution

In this case, $p_i = (1/N)$ for all i. The number of channels available is $N_f = 96$. The operation of the cellular system is simulated for a period of 3 hours and the average blocking, which is the ratio of the number of blocked calls and the number of call attempts and is an estimate of the blocking probability, is plotted as a function of the offered traffic, in Figure 4.2, for each of the dynamic channel assignment strategies considered. Because of the constraints chosen, the fixed channel assignment that minimizes the blocking probability* assigns 16 channels each to cells 5, 6, 12, 13 and 14, and 8 channels each to the other cells (see Figure 4.1). The blocking probability for the fixed channel assignment can be calculated using the Erlang B formula and this is also plotted in Figure 4.2. (If, in a telephone exchange (or cell), the arrival process and the holding times are memoryless (as we have assumed), the

* This is for low traffic levels. The optimal fixed channel assignment, i.e., the one that minimizes the blocking probability, is not necessarily independent of the offered traffic. Fixed channel assignments that are optimal for small values of ρ may not be optimal for large values of ρ (i.e., for heavy traffic). This phenomenon requires further investigation.

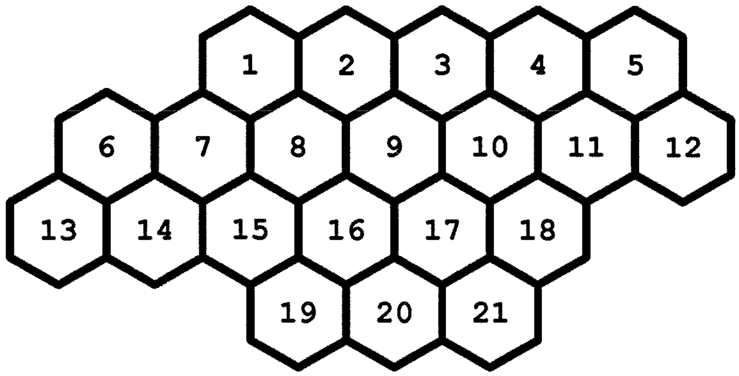

Figure 4.1. The cell number is shown within each cell.

Table 4.1. Compatibility matrix

5	2	1	1	0	1	2	2	1	1	0	0	1	1	1	1	1	0	1	1	0
2	5	2	1	1	1	1	2	2	1	1	0	0	1	1	1	1	1	1	1	1
1	2	5	2	1	0	1	1	2	2	1	1	0	0	1	1	1	1	1	1	1
1	1	2	5	2	0	0	1	1	2	2	1	0	0	0	1	1	1	0	1	1
0	1	1	2	5	0	0	0	1	1	2	2	0	0	0	0	1	1	0	0	1
1	1	0	0	0	5	2	1	1	0	0	0	2	2	1	1	0	0	1	0	0
2	1	1	0	0	2	5	2	1	1	0	0	1	2	2	1	1	0	1	1	0
2	2	1	1	0	1	2	5	2	1	1	0	1	1	2	2	1	1	1	1	1
1	2	2	1	1	1	1	2	5	2	1	1	0	1	1	2	2	1	1	1	1
1	1	2	2	1	0	1	1	2	5	2	1	0	0	1	1	2	2	1	1	1
0	1	1	2	2	0	0	1	1	2	5	2	0	0	0	1	1	2	0	1	1
0	0	1	1	2	0	0	0	1	1	2	5	0	0	0	0	1	1	0	0	1
1	0	0	0	0	2	1	1	0	0	0	0	5	2	1	1	0	0	1	0	0
1	1	0	0	0	2	2	1	1	0	0	0	2	5	2	1	1	0	1	1	0
1	1	1	0	0	1	2	2	1	1	0	0	1	2	5	2	1	1	2	1	1
1	1	1	1	0	1	1	2	2	1	1	0	1	1	2	5	2	1	2	2	1
1	1	1	1	1	0	1	1	2	2	1	1	0	1	1	2	5	2	1	2	2
0	1	1	1	1	0	0	1	1	2	2	1	0	0	1	1	2	5	1	1	2
1	1	1	0	0	1	1	1	1	1	0	0	1	1	2	2	1	1	5	2	1
1	1	1	1	0	0	1	1	1	1	1	0	0	1	1	2	2	1	2	5	2
0	1	1	1	1	0	0	1	1	1	1	1	0	0	1	1	2	2	1	2	5

blocking probability is given by

$$P_b = \frac{\rho^N/N!}{\sum_{i=0}^{N} \rho^i/i!},$$

where ρ is the traffic (measured in Erlangs) and N is the number of trunks (or frequencies) available. This formula is called the Erlang B formula [Ino, pp. 226–227].)

It is readily seen that the dynamic channel assignment strategies can be ranked in decreasing order of performance as follows: Remax2, Remax1, Maxavail and Simple, and that all of them outperform fixed channel assignment in minimizing the blocking probability.

But what about Remax3, Remax4, etc.? To answer this question, we consider an *idealized* DCA strategy that permits everyone to be reassigned in order to accommodate a new call. This is termed the "Maximum Packing" strategy and was first proposed by Everitt and Macfadyen [EMc] (quoted from [EMn]). (In our terminology, this would be Remax∞, but we will adopt the term Maximum Packing.) We would like to compare the performance of our DCA strategies with that of the Maximum Packing strategy. The application of this strategy is equivalent to solving the fixed channel assignment problem every time an incoming call is blocked. This would involve a great deal of computation, even if the fast heuristic algorithms developed in the previous chapter are used. Since, in current analog systems, the process of reassigning a frequency is not likely to be entirely transparent to the user, there would be a considerable decrease in service quality if there are a large number of reassignments during a call. Also, the signalling problem, i.e., the problem of communicating all the necessary changes to the mobiles involved in the reassignment, could become unmanageable. In contrast, Remax2 reassigns *at most* 2 calls to accommodate a new call, and only after attempting to accommodate it without

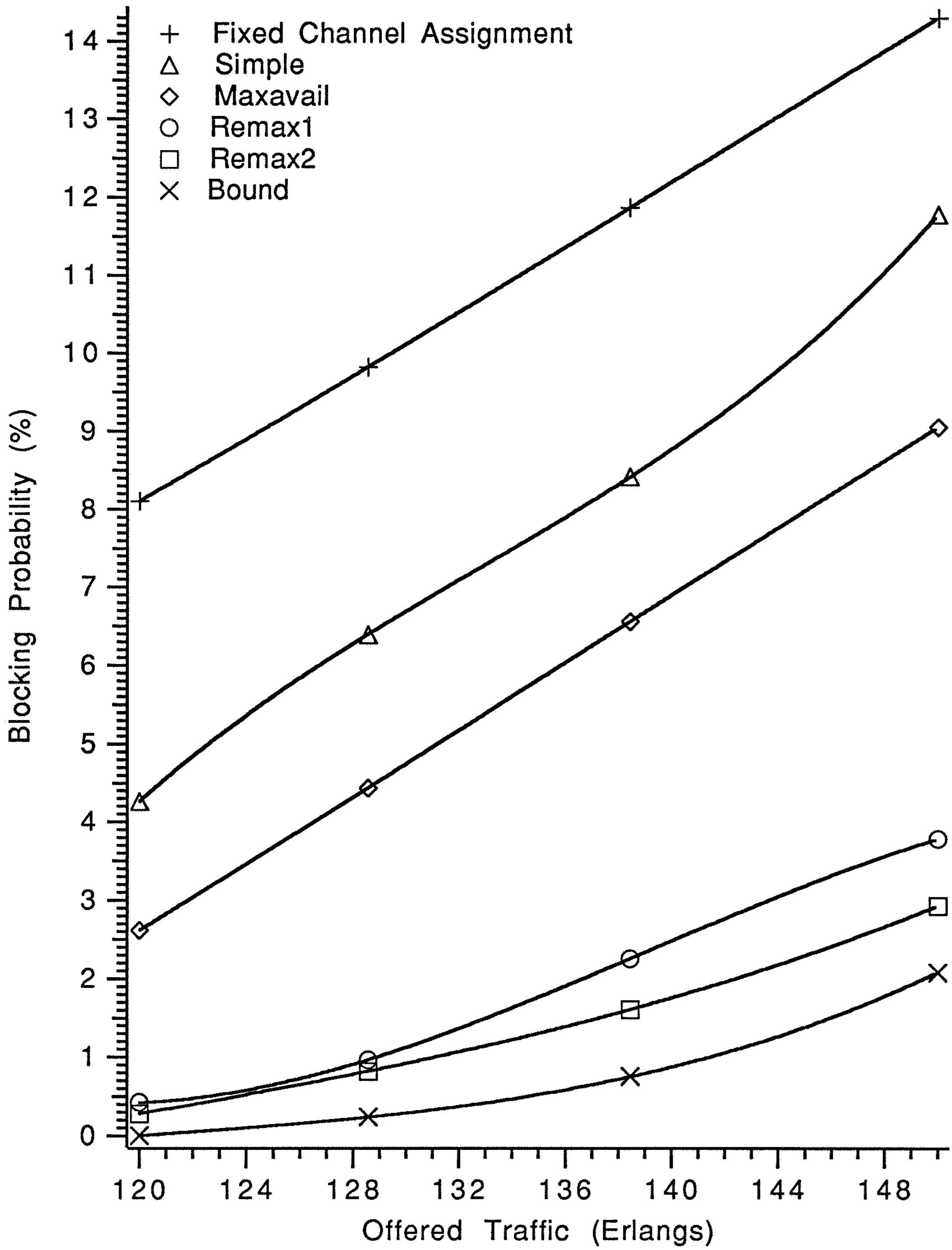

Figure 4.2. Homogeneous traffic case.

reassignments, or with one reassignment only. The natural question to be asked is: "What is the difference in performance between Remax2 and the Maximum Packing strategy?"

The evaluation of the Maximum Packing strategy involves a lot of computation, since the *allowed states* of the system, i.e., the configurations of calls that can be carried by the system, are not easy to find, in general. To determine whether a given state of the system is allowed or not, one has to solve the fixed channel assignment problem which is, in general, difficult. However we can obtain lower bounds for the fixed channel assignment problem which are easy to check. In other words, we can obtain sufficient conditions for a call to be blocked, which are easily checked, though these conditions are by no means necessary. The lower bounds for the fixed channel assignment problem we consider are the clique bound and its generalizations in the presence of arbitrary (not purely co-channel) constraints described in [Gam, lemma 7]. We use these bounds to simulate the performance of an idealized DCA strategy, which we call "Bound." This DCA strategy blocks an incoming call if, and only if, accommodating this call would mean that one of these lower bounds (on the number of channels required to handle this configuration of calls) exceeds N_f (= 96, in this example). This is plotted as "Bound" in Figure 4.2. Note that the complexity of evaluating this strategy will depend on the complexity of computing the lower bounds. The lower bounds that we have chosen [Gam, lemma 7] are easy to compute. However, since we need to resort to simulation in order to evaluate this strategy, the computing time required is nearly the same as that for the Simple strategy (see the last section of this chapter).

We were pleasantly surprised to find that, in this case, the performance of Remax2, in terms of minimizing the average blocking, is *close to that achieved by the Bound strategy.*

Table 4.2. Inhomogeneous spatial distribution of traffic considered

i	p_i
1	0.0166
2	0.0520
3	0.0166
4	0.0166
5	0.0166
6	0.0312
7	0.0374
8	0.1081
9	0.1601
10	0.0582
11	0.0270
12	0.0312
13	0.0644
14	0.0312
15	0.0748
16	0.1185
17	0.0582
18	0.0166
19	0.0208
20	0.0270
21	0.0166

Case 2: Inhomogeneous Spatial Traffic Distribution

The p_i in this case are obtained by treating the channel requirements specified in [Gam] for the example under consideration, as relative traffic densities. These p_i are listed in Table 4.2.

The performance of these algorithms is plotted in Figure 4.3. Once again, of the channel assignment strategies considered, Remax2 is the best; but Remax1 is almost as good. But unlike Case 1, the performance of Remax2 is not extremely close to that of Bound. This difference in performance may be because Remax2 is not a good strategy when the traffic is inhomogeneous. On the other hand, in this case, the performance of no realizable strategy, may come close to that of Bound, which is *not* realizable, even in principle, since the lower bounds used are not always sharp. To shed further light on this question, we need a few preliminaries.

The process of call arrivals and departures is a discrete-state, continuous-time, Markov process, because of the assumption of memoryless arrivals and holding times. The state of the system is specified by the N-dimensional vector $\tilde{n} = (n_i)$ whose components are the number of calls in progress in the N cells in the system. In particular, the Markov process is a birth-death process since only transitions to neighbouring states (in N-dimensional space) are permitted, i.e., the probability of multiple arrivals, or departures, or an arrival and a departure, in a small time interval Δt, is negligible, compared to the probability of a single arrival or departure [Fel, pp. 454–457, Sch, pp. 47–49] (one-dimensional case). The allowed states of the system are all the states $\tilde{n} = (n_i)$, which can possibly exist, with the given number of channels and the constraints on their assignments. Let $\mathcal{N}$ denote the set of all allowed states of the system. From these assumptions it can be shown (by a straightforward generalization of the method in [Sch, pp.47–49] for the one-dimensional case) that the equilibrium probability that the system is in state $\tilde{n}' = (n_i'), 1 \leq i \leq N$, is given by

$$P_{\tilde{n}'} = \frac{\rho_1^{n_1'} \rho_2^{n_2'} \dots \rho_N^{n_N'} / n_1'! n_2'! \dots n_N'!}{\sum_{\tilde{n} \in \mathcal{N}} \rho_1^{n_1} \rho_2^{n_2} \dots \rho_N^{n_N} / n_1! n_2! \dots n_N!}.$$

Now consider the following cellular system consisting of two cells with the separation matrix $\mathbf{C} = (c_{ij})$ given by:

$$\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

Let $N_f = 3$. The allowed states of the system $\mathcal{N}$ are (0,0), (1,0), (0,1), (2,0), (1,1), (0,2), (2,1) and (1,2). It can be shown, by a straightforward calculation using the

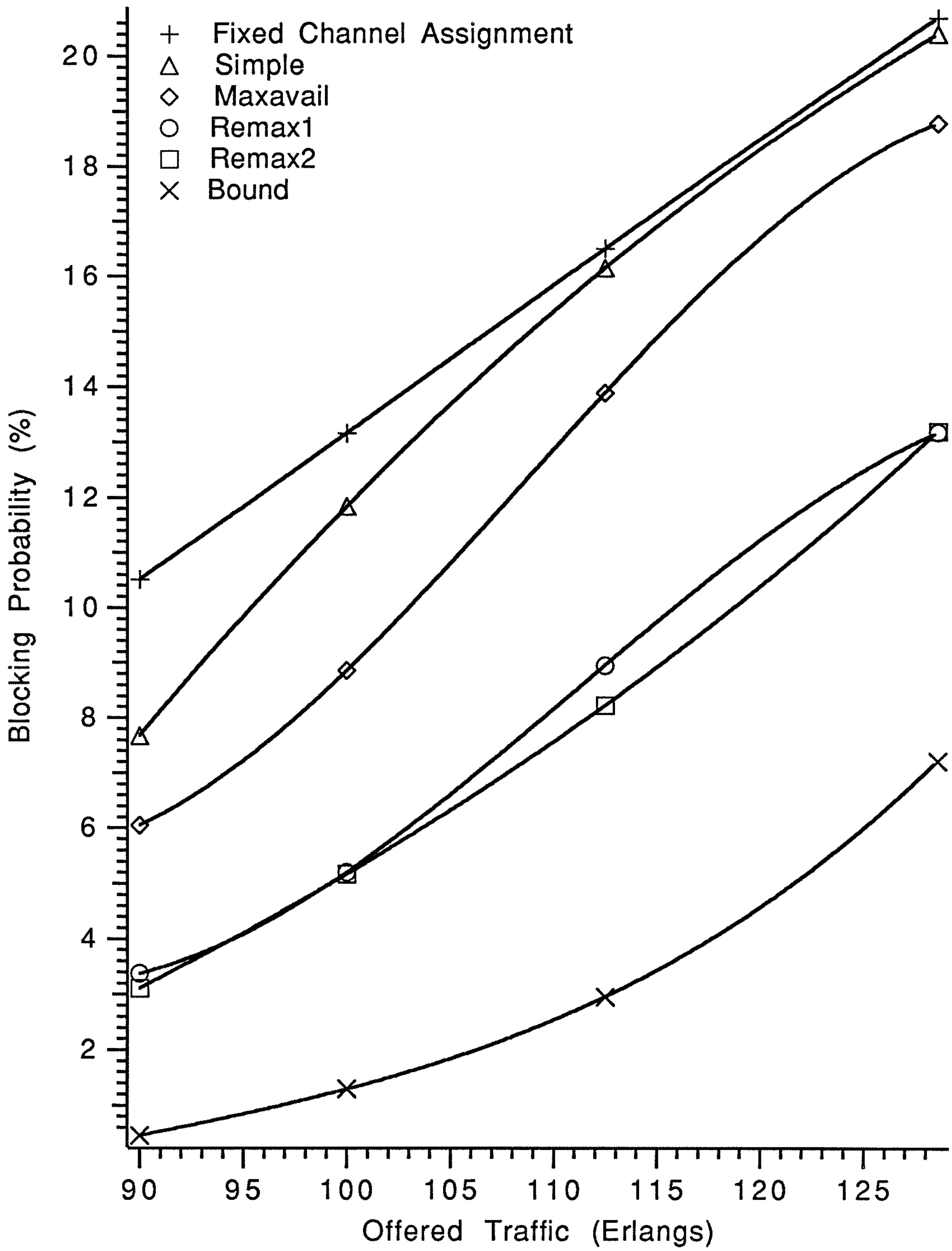

Figure 4.3. Inhomogeneous traffic case.

expression for the equilibrium probabilities for the system states mentioned above that, the probability that an incoming call is blocked under the Maximum Packing strategy is

$$P_b^{\text{MaximumPacking}} = \frac{\rho^2(1 - 3p_1p_2) + p_1p_2\rho^3}{2 + 2\rho + \rho^2 + p_1p_2\rho^3}.$$

(This is because a call is blocked under the Maximum Packing strategy if, and only if, accepting that call would result in the system being in a disallowed state.) The two cells in this system constitute a clique and hence, the total number of calls in the system is a lower bound on the number of frequencies required. If we implement the Bound DCA strategy using only this lower bound, this strategy will block an incoming call if, and only if, 3 calls are already present in this system. The blocking probability is easily calculated using the Erlang B formula to be

$$P_b^{\text{Bound}} = \frac{\rho^3/3!}{1 + \rho + \rho^2/2! + \rho^3/3!}.$$

Let $p = p_1$. Then, $p_1p_2 = p(1 - p)$, is a function only of p, and p serves as a measure of the degree of nonuniformity of the traffic. The above expressions are plotted in Figure 4.4 as a function of the traffic for various degrees of nonuniformity, i.e., for various values of p. (P_b^{Bound} is independent of p, in this example.) The widening of the difference in performance between the two strategies, as the nonuniformity in the traffic increases, is evident from this plot.

Intuitively, one may expect the performance of the Maximum Packing strategy to serve as a lower bound on the performance of any channel assignment strategy since, if the Maximum Packing strategy blocks a call, so must any other channel assignment strategy. Similarly, one may also expect the performance of Bound to serve as a lower bound on the performance of Maximum Packing. Indeed, this is the idea behind the consideration of these idealized strategies. Therefore, the fact that

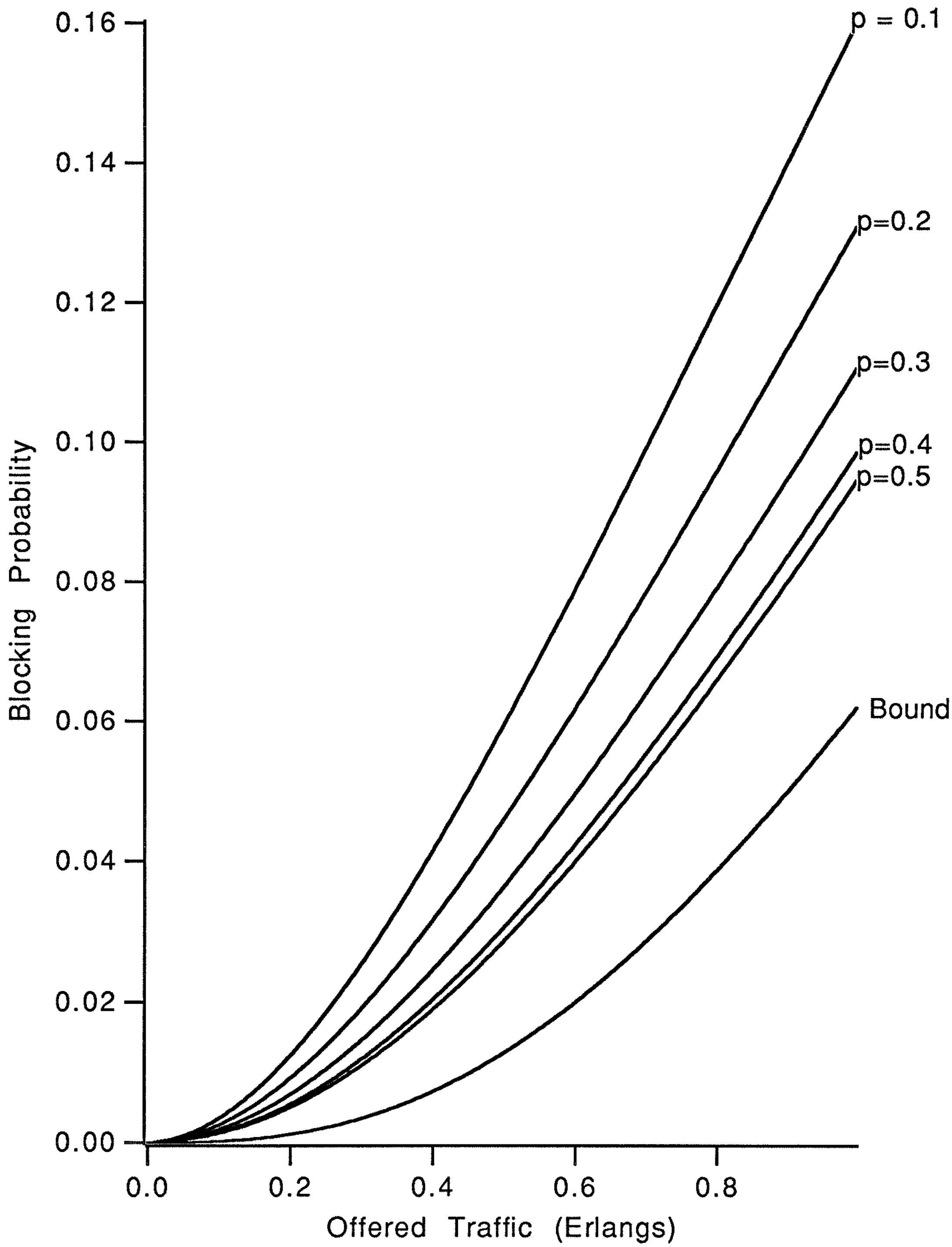

Figure 4.4. Blocking probability for various degrees of inhomogeneity.

the performance of the Maximum Packing strategy diverges from that of the Bound strategy as the traffic becomes more inhomogeneous is encouraging; it suggests that the performance of no realizable strategy may come close to that of Bound and hence, Remax2 may actually be an excellent strategy, in the case of inhomogeneous traffic too.

However, in a remarkable paper Kelly [Kel] showed that under certain conditions, e.g., heavy traffic, the performance of the Maximum Packing strategy may actually be worse than that of FCA. This is because the Maximum Packing strategy is a greedy strategy while FCA is not, i.e., under certain circumstances, blocking a call deliberately enables more calls to be accommodated later.

It is also worth noting that, for the same traffic, the average blocking in this inhomogeneous system is considerably more than that in the homogeneous system. This seems to confirm the widely held belief among cellular system designers that, the traffic in the system should be as homogeneous as possible. ("... matching the spatial density of available channels to the spatial density of demand for channels ...," V. H. MacDonald in [Mac, p. 19]).

4.4. Heavy Traffic and Robustness

In the example in Kelly's paper [Kel] where he demonstrates that the Maximum Packing strategy can actually be worse than FCA, this occurs under heavy traffic conditions. Also, quoted from [EMn, p. 1173]: "... the existing dynamic channel assignment algorithms make the network overload performance no better than that with fixed channel assignment, and can make it worse, depending on the particular algorithm used. ... Much more work is needed to produce dynamic channel assignment algorithms that do work well under overload." How do our algorithms perform under overload (heavy traffic conditions)? We simulated their performance, in the case of homogeneous traffic, for the same system as in Figure 4.1, with $N_f = 96$ and the same compatibility matrix as in Table 4.1, under heavy traffic conditions. The results are shown in Table 4.3.

Table 4.3.

(P_b^S is the blocking probability when strategy S is used.)

Traffic (Erlangs)	P_b^{FCA} (%)	P_b^{Simple} (%)	P_b^{Maxavail} (%)	P_b^{Remax1} (%)	P_b^{Remax2} (%)	P_b^{Bound} (%)
360	48.6	53.0	51.8	47.9	46.6	44.2
720	72.2	74.5	74.0	72.5	71.8	69.9

It can be seen that Remax2 continues to be better than FCA even when the blocking probability gets up to over 70% and that Remax1 is quite good too, although Simple and Maxavail are worse than FCA.

What happens, if for some reason *all* (or most of) the traffic is in only one cell, and for a single cell, this traffic is heavy? (This is a case where we have both heavy and extreme inhomogeneity of traffic.) Because of the co-site constraint of 5, since $N_f = 96$, the maximum number of calls that can be in progress in a single cell is 20. All of our algorithms, from Simple to Remax2, make 20 channels available in the one cell with all the traffic. Therefore, these algorithms can be said to be *robust.*

However, we have only investigated the performance of our algorithms, when the traffic in the system is steady. How do these algorithms perform when the traffic *is increasing* or when all the traffic *is becoming concentrated* in a single cell? More generally, what is the performance of these algorithms in the case of time-varying traffic? We have not investigated this problem but the design of algorithms that do well in the case of time-varying traffic also, is one of several open problems in this area. Some more are listed in the next section.

4.5. Conclusions and Open Problems

Can we implement these strategies in current cellular systems? First, note that the implementation of these strategies would require no changes to the mobile phones. The only changes needed would be at the base stations, which would have to be made *frequency-agile*, i.e., capable of transmitting on all frequencies allocated to cellular operation. (The mobile is already capable of doing this, but the design of frequency-agile base stations is more difficult since base stations transmit at a much higher power than the mobiles. But the author has learned from informed sources that frequency-agile base stations will be possible in the near future.) Reassigning a call to a different frequency is not a problem, since even current systems have to reassign a call to a different frequency during a hand-off, i.e., when the mobile moves from one cell to another. Then, the only question is that of computation. For the examples considered, the running time for Simple is about 10 minutes, while Maxavail, Remax1 and Remax2 take between 20 and 25 minutes, on a MIPS M/120 RISComputer. Considering that the performance of these algorithms is simulated for a period of 3 hours of simulated time, these algorithms can be said to run in "real" time. A cellular system, consisting of 21 cells, is not atypical of existing systems. The number of channels available is typically about 3 to 4 times what had been assumed above. But the running time of these algorithms only increases linearly with increase in the number of cells and quadratically with increase in the number of available channels. Therefore, the deployment of these channel assignment strategies, in current cellular systems, seems well within the scope of today's technology.

Table 4.4. Blocking probabilities for FCA and DCA strategies when $N_f = 400$, and $\rho = 720$.

FCA	Simple	Maxavail	Remax1	Remax2	Bound
10.6	10.1	7.7	3.6	3.4	3.4

On comparing the performance of these algorithms with that of fixed channel assignment, it is found that, in the interesting range of blocking probabilities, viz. 2–4%, an increase in carried traffic of about 70% can be obtained. This can be seen from Figure 4.2 for the uniform traffic case, where the performance of the best fixed channel assignment is also plotted. A similar increase is obtained in the nonuniform traffic case also, where the best fixed channel assignment obtained with the help of the algorithms in the previous chapter was used.

However, it must be noted that the gain in carried traffic over FCA achieved by these DCA algorithms is a function of the number of channels available per cell in FCA (and hence of the total number of channels available in the system). Since the actual number of frequencies available in current cellular sytems in the U.S. is about 400, we compiled the data in Table 4.4 to evaluate the increase that may be achieved in this case. This is again the 21-cell system of Figure 4.1, with homogeneous traffic and the compatibility matrix of Table 4.1.

FCA results in a blocking probability of nearly 3.4% at a traffic of 27.75 Erlangs/cell or 583.75 Erlangs in the system. Therefore, we get an increase of about 22.8% in the carried traffic in going from FCA to DCA with Remax2, when the number of channels per cell is 33–34*. In the earlier case, when the number of channels per cell was 8, and the total number of channels available in the system

* We consider the number of channels in the cells in the center of the system, as we will usually be able to assign more channels to cells at the edge of the system due to fewer compatibility constraints on these cells.

was 96, at a blocking probability of 2.9%, we got an increase of 68% in going from FCA to DCA with Remax2. The actual number of channels per cell (or sector, in the case of sectored cells) will be somewhere between these two cases and so will the increase in carried traffic (or revenue!) in going from FCA to DCA with Remax2.

One of the important open problems in this area is to compute a sharp lower bound on the performance of *any* channel assignment strategy. In the light of the available evidence, this appears to be a difficult problem. (However, the reader is urged to read the next chapter before attacking this problem.)

Furthermore, blocked calls were assumed to be cleared from the system. The carried traffic could be increased further by queuing the blocked calls.

In all of the above, the computation is centralized. It remains to be investigated as to how this may be distributed among the individual cell-sites, or the mobiles, or both.

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Chapter 5

Interference Probabilities and Hypergraphs

Abstract

In this chapter, we discuss various methods of computing interference probabilities and the formulation of compatibility constraints on channel assignment based on these calculations. We also formulate the channel assignment problem as one of coloring hypergraphs, instead of graphs, and show that, in the case of dynamic channel assignment, this leads to a considerable increase in the carried traffic for the *same* blocking probability and the *same* maximum probability of interference.

5.1. Introduction

The primary objective in the design and operation of any cellular telephone system is to maximize the carried traffic while maintaining acceptable call quality. The call quality is usually measured by two parameters, the blocking probability (P_b) and the interference probability (P_i), and is considered acceptable if $P_b \leq P_b^{\max}$ (typically 5%) and $P_i \leq P_i^{\max}$ (typically 10%). Though the number of frequencies available in practice is fixed, it is useful to formulate the problem as one of minimizing the total number of frequencies needed when the number of frequencies required in each cell is specified, especially in the case of fixed channel assignment. As seen in the previous chapters, the frequency (channel) assignment problem in cellular radio is usually formulated as a graph coloring problem with the interference constraints specified by a *compatibility matrix.* In the simplest case, the compatibility matrix specifies, for every pair of cells, whether co-channel use is permissible, or not. The entries in the compatibility matrix are (or should be) determined on the basis of worstcase interference probability calculations so that $P_i \leq P_i^{\max}$. If a graph is drawn, where each vertex represents a cell, and pairs of cells which are forbidden from using the same frequency are connected by an edge, the fixed channel assignment problem is equivalent to assigning as many distinct colors to every vertex of this graph as the number of frequencies required to be assigned to the cell which that vertex represents, while minimizing the total number of colors used. Frequencies (or colors) are represented by the positive integers so that, equivalently, the largest integer used may be minimized.

This formulation is convenient since it enables us to use well-developed graph-theoretical techniques. We presented several results for this formulation of the problem in the previous chapters.

However, a more general, and better, formulation of the channel assignment problem is to forbid all members of certain subsets of cells (not necessarily pairs) from simultaneously using the same channel*. (In the graph-theoretical case described above, these subsets are restricted to be pairs of cells.) As we will demonstrate, this general approach enables us to obtain better results for the same value of $P_b^{\max}$. These constraints can be represented in the form of a *hypergraph* where the vertices of the hypergraph represent the cells and the set of cells which are forbidden from using the same frequency are said to be adjacent, i.e., constitute a (hyper)edge of the hypergraph. We will refer to this formulation of the frequency assignment problem as the hypergraph formulation.

5.2. Interference Probability Calculations

The previous chapters assumed that the restrictions arising out of interference probability considerations could be summarized in the form of a compatibility matrix.

* The first such formulation is due to A. Gamst [Gpc].

How would we actually go about constructing the compatibility matrix? In this section we will provide an answer to this question, and in the process, we will develop an alternative to the compatibility matrix which will give rise to more efficient channel assignments viz., the hypergraph formulation of the problem briefly described above.

The power of the received radio signal in the cellular environment undergoes both random, rapid fading, which is usually modelled by a Rayleigh distribution, and a random, slower variation termed shadowing, which is usually modelled by a log-normal distribution. The rapid fading is due to the multipath nature of the propagation, i.e., because the signal is the result of the addition of a number of components of differing amplitudes and phases. The slower variation is due to the effect of objects like buildings and trees in the propagation path, which results in the signal being attenuated by, essentially, a random amount. By averaging the power of the signal over the rapid fading, i.e., by taking the short-term or "local" average, we obtain what is termed the *local mean power.* The net random attenuation, or variation, of the local mean power is the *product* of a fairly large number of independent, random attenuations, since every time the signal is reflected or diffracted by an object in its path, it is attenuated by a random amount. Therefore, the *logarithm* of the local mean power is the *sum* of independent, random variables and by the central limit theorem, its distribution tends to a normal distribution, when the number of random attenuators becomes large. Hence, the local mean power itself must tend to be log-normally distributed. (If X is log-normal, $\log X$ is normal.) That the distribution of the local mean power is well modelled by the log-normal distribution has been verified by a number of experiments ([VT], p. 46).

We make the following assumptions (as do Nagata and Akaiwa [NA]):

1. The local mean power is log-normally distributed with shadowing parameter

σ. (This is the standard deviation of the logarithm of the local mean power.)

2. The area mean power (mean of the local mean power) is proportional to the inverse fourth power of the distance from the source.
3. Only co-channel interference is significant.
4. The probability of interference is the probability that the ratio of the local mean power of the desired signal and the local mean power of the net interfering signal, which is the sum of all the interfering signals, is less than the specified protection ratio β.
5. If the probability of interference (P_i) for a given call is satisfactory at the base station (i.e., $P_i \leq P_i^{\max}$), it is satisfactory at the mobile.
6. The cellular system is an infinite tiling of the plane by regular, hexagonal cells.
7. The desired mobile is located at the corner of the cell and the interfering mobiles are located at the centers of the cells.

To be able to calculate the probability of interference, we must find the distribution of a sum of (independent) log-normal variables. Consider the case where six interferers are symmetrically located at the six cells which are at distance $\sqrt{13}$ from the cell at the base station of which the interference probability is to be calculated. (This corresponds to calculating the interference probability for the six closest interferers when the number of reuse groups is 13.) For this case we compute the probability of interference for several values of σ and β using three methods:

1. By approximating the sum of log-normal variables by a log-normal variable with the same mean and variance, known as Fenton's method [Fe]. This is computationally the simplest.
2. By the method of Schwartz and Yeh [SY] which is, essentially, approximating the logarithm of a sum of log-normal variables by a normal variable with the

same mean and variance* (except that this approximation is done recursively). We refer to [SY] for the details. This method is computationally more difficult than Fenton's method.

3. Directly. We will briefly describe this method here. This is computationally the most difficult but is useful in determining the accuracy of the above approximate methods.

Let S_i^u denote the local mean power of the ith ($i = 1,\ldots,6$) interfering (undesired) signal and S^d that of the desired signal. Let the probability density function of S_i^u be

$$f_{S_i^u}(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-(\ln x - m_i^u)^2/2\sigma^2}, \quad x \geq 0, \quad 1 \leq i \leq 6$$

and that of S^d be

$$f_{S^d}(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-(\ln x - m^d)^2/2\sigma^2}, \quad x \geq 0.$$

(In all the equations, σ and β are not in dB but in natural units.) Let $X_i = e^\beta S_i^u$. Since $m_i^u = m^u$ for all i, let

$$f_X(x) = f_{X_i}(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-(\ln x - m^u - \beta)^2/2\sigma^2}, \quad x \geq 0, \quad 1 \leq i \leq 6.$$

Then, the probability of interference is

$$P_i = \Pr(S^d < X_1 + \cdots + X_6) = \Pr(S^d - X_1 - \cdots - X_6 < 0).$$

---

* If the sum of independent log-normals were truly log-normal, this should yield the same result as Fenton's method. However, the sum of log-normals is *not* truly log-normal, and as we will see, this yields a better approximation to the true distribution.

Let

$$\phi_X(s) = \int_{-\infty}^{\infty} f_X(x) e^{isx}\, dx$$

denote the characteristic function of the random variable X and

$$F_X(s) = \int_{-\infty}^{s} f_X(x)\, dx$$

its cumulative probability distribution function. Let $Z = S^d - X_1 - \cdots - X_6$. Then,

$$\phi_Z(z) = \left(\int_0^{\infty} e^{-izx} f_X(x)\, dx\right)^6 \int_0^{\infty} e^{izy} f_{S^d}(y)\, dy$$

and

$$\begin{aligned} P_i &= F_Z(0) \\ &= \int_{-\infty}^{0} f_Z(s)\, ds \\ &= \frac{1}{2\pi} \int_{-\infty}^{0} ds \int_{-\infty}^{\infty} \phi_Z(z) e^{-izs}\, dz \\ &= \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} F_{S_d}\left(e^{m^u+\beta}\left(e^{\sigma z_1} + \cdots + e^{\sigma z_6}\right)\right) e^{-(z_1^2+\cdots+z_6^2)/2}\, dz_1 \cdots dz_6. \end{aligned}$$

(Some intermediate steps have been omitted.) If

$$P(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\, dt$$

(the cumulative normal distribution function),

$$F_{S^d}(s) = P\left(\left(\ln s - m^d\right)/\sigma\right).$$

Therefore,

$$P_i = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} P\left(m^u - m^d + \beta + \ln\left(e^{\sigma z_1} + \cdots + e^{\sigma z_6}\right)\right) e^{-(z_1^2+\cdots+z_6^2)/2}\, dz_1 \cdots dz_6.$$

Since the area mean power is assumed to vary inversely as the fourth power of the distance from the source, it is easily shown that $m^u - m^d = \ln(R^4/D^4)$ where R is the distance of the desired mobile from the base station (the radius of the cell) and D is the distance of the undesired mobile from the base station. $D^2/3R^2$ is the reuse ratio, which equals 13, in this case. Therefore, P_i can be calculated for given values of σ and β. The numerical computation is carried out using Gauss-Hermite integration [AS]. The computation of each value of P_i, with an error less than 0.0001, requires about 30 seconds of CPU time on a MIPS M/120 RISComputer.

Table 5.1 lists the interference probabilities calculated by the three methods for several values of σ and β. Prasad and Arnbak published a table of interference probabilities calculated using the two approximate methods for $\sigma = 6$ dB, under the same assumptions [PA]. However, the values tabulated there are incorrect in the case of Fenton's method. Though the conclusion of Prasad and Arnbak in that Fenton's method yields optimistic values for the interference probability is correct for $\sigma \geq 8$ dB, from our table we deduce that Fenton's method is quite accurate for $\sigma \leq 6$ dB.

To determine a set of compatibility constraints that will ensure that $P_i \leq P_i^{\max}$, one can determine the smallest distance $d_{\min}$ such that, if six interfering mobiles are located symmetrically at distance $d_{\min}$ from the center of a given cell v, with the desired mobile being located at the farthest point in the cell v, $P_i \leq P_i^{\max}$. Then all cells within distance d from v are forbidden from using the same frequency as v. If the cellular system is not regular (cells have different sizes), one can make this computation for each cell. One can even make this computation if the propagation exponent (which was assumed to be 4 in the above discussion) and the shadowing parameter σ are different in different regions of the cellular system. In short, these calculations may be carried out with any suitable propagation model

Table 5.1
Probabilities of Interference
(computed by various methods)

σ(dB)	β(dB)	Direct Method	Fenton's Method	Schwartz and Yeh's Method
12.0000	12.0000	0.4147	0.3045	0.4171
12.0000	10.0000	0.3597	0.2621	0.3614
12.0000	8.0000	0.3076	0.2230	0.3086
12.0000	6.0000	0.2593	0.1874	0.2594
12.0000	4.0000	0.2153	0.1555	0.2146
10.0000	12.0000	0.3264	0.2626	0.3275
10.0000	10.0000	0.2666	0.2147	0.2669
10.0000	8.0000	0.2131	0.1723	0.2125
10.0000	6.0000	0.1666	0.1356	0.1652
10.0000	4.0000	0.1273	0.1047	0.1253
8.0000	12.0000	0.2141	0.1946	0.2136
8.0000	10.0000	0.1563	0.1437	0.1550
8.0000	8.0000	0.1099	0.1026	0.1081
8.0000	6.0000	0.0745	0.0708	0.0724
8.0000	4.0000	0.0487	0.0472	0.0465
6.0000	12.0000	0.0897	0.0900	0.0883
6.0000	10.0000	0.0509	0.0518	0.0495
6.0000	8.0000	0.0269	0.0278	0.0257
6.0000	6.0000	0.0133	0.0139	0.0123
6.0000	4.0000	0.0061	0.0064	0.0055
4.0000	12.0000	0.0083	0.0084	0.0080
4.0000	10.0000	0.0023	0.0023	0.0021
4.0000	8.0000	0.0005	0.0005	0.0005
4.0000	6.0000	0.0001	0.0001	0.0001
4.0000	4.0000	0.0000	0.0000	0.0000

and the co-channel restrictions determined. In practice, one needs to introduce adjacent channel and co-site restrictions as well. It will probably be both necessary and sufficient to introduce adjacent channel restrictions on adjacent cells. In current cellular systems, co-site restrictions are determined mainly by the frequency separations required by the combiners and not by interference considerations. Therefore, the entire compatibility matrix may be determined.

5.3. Hypergraphs and Dynamic Channel Assignment

But as the astute reader will have observed at the end of the previous section, it may be possible to reuse a frequency at a distance $d < d_{\text{min}}$, provided that six interferers are not located at distance d. Specifying co-channel restrictions using a compatibility matrix is equivalent to specifying for every pair of cells, whether co-channel use is permitted or not and from the previous section, if two cells are distance d apart, they are forbidden from using the same frequency if and only if $d < d_{\text{min}}$. To determine whether all the cells of any given subset, V', of the cells — not necessarily a pair of cells — may simultaneously use the same frequency while ensuring that $P_i \leq P_i^{\text{max}}$ for all calls, one proceeds as follows:

1. Assuming that all the cells in V' use the same frequency, and these cells are the only cells in the system that use this frequency, calculate the interference probabilities for each of these cells. (By the interference probability for a cell, we mean the interference probability for a call in that cell.)
2. If the interference probability of at least one of the cells is $> P_i^{\text{max}}$, the subset of cells V' is forbidden from using the same frequency, and will be called a forbidden subset; otherwise, these cells may simultaneously use the same frequency.

The determination of all the forbidden subsets of a cellular system seems to be a forbidding task at this time. Examining all the possible subsets of cells is clearly impossible even for small cellular systems. However, it may be possible to determine all the forbidden subsets of size $\leq k$ for moderate values of k. If all the forbidden subsets of a cellular system are determined, one can then attempt to solve the fixed channel assignment problem with these constraints, instead of the compatibility matrix, and achieve greater spectrum efficiency for the same $P_i^{\max}$ (and $P_b^{\max}$).

The forbidden subsets can be represented by a *hypergraph:* each vertex of the hypergraph represents a cell and the forbidden subsets are the edges of this hypergraph. While it has not yet been possible to apply the theory of hypergraphs to the frequency assignment problem with this kind of compatibility restrictions — in a manner similar to the application of graph theory in the case of the compatibility matrix formulation — we hope that we will be able to do so in the future. In anticipation of this, we will refer to this formulation of the problem as the hypergraph formulation.

Even without a knowledge of *all* the forbidden subsets of a cellular system, one can develop dynamic channel assignment schemes based on the hypergraph formulation. In dynamic channel assignment, all the channels are available in all the cells. Calls are to be assigned frequencies in real time, subject to the compatibility restrictions (e.g., subject to the frequency separations specified by a compatibility matrix). For a more detailed description of dynamic channel assignment, the reader is referred to the previous chapter. To determine whether a particular frequency f' is available for assignment to a call in cell v' on arrival, find all the cells in the system that are currently using frequency f'. Let this subset of the cells be denoted by $V_{f'}$. Let $V' = V_{f'} \cup v'$. Frequency f' cannot be assigned to the new call in v' if and only if V' is a forbidden subset. Compared with the compatibility matrix approach, we have to calculate an interference probability instead of looking up an entry in the compatibility matrix. However, we have found that if Fenton's method of approximating the interference probability is used, this approach has almost the same complexity as that of the compatibility matrix approach; the running times for the simulation of both the schemes for a given cellular system and offered traffic are nearly the same.

In a dynamic channel assignment scheme, it is possible to use one of several dynamic channel assignment strategies to assign a frequency to a new call while observing the compatibility constraints. A frequency is said to be available in a cell, if it may be assigned to a call in that cell without violating the compatibility constraints. There are two kinds of strategies: those that do not permit reassignment of calls in progress to different frequencies and those that do. We consider the performance of the following three strategies:

Simple: An incoming call in a cell is assigned the least available frequency in that cell. (Recall that frequencies are identified with the positive integers.)

Maxavail: An incoming call is assigned that frequency which maximizes the total number of channels available in all the cells in the system.

Remax1: An attempt is made to assign a frequency to an incoming call using the Maxavail strategy. If no frequency is available, *one* other call that is currently in progress, is permitted to be reassigned to a different frequency, if it enables the new call to be accommodated.

For more details regarding these strategies, the reader is referred to the previous chapter.

If no channel is available for assignment to an incoming call, the call is said to be blocked. The following assumptions are made.

1. Blocked calls are cleared.

2. Call arrivals in a cell are independent of call arrivals in all other cells and obey a Poisson distribution.
3. The call duration is exponentially distributed with a mean call duration of 3 minutes.
4. The traffic is uniform, i.e., the rate of call arrivals is the same for all the cells in the system.
5. Hand-offs can be neglected.

The first three assumptions are standard in wireline telephony. The cellular system we consider is the same one as in Figure 4.1 and is reproduced in Figure 5.1.

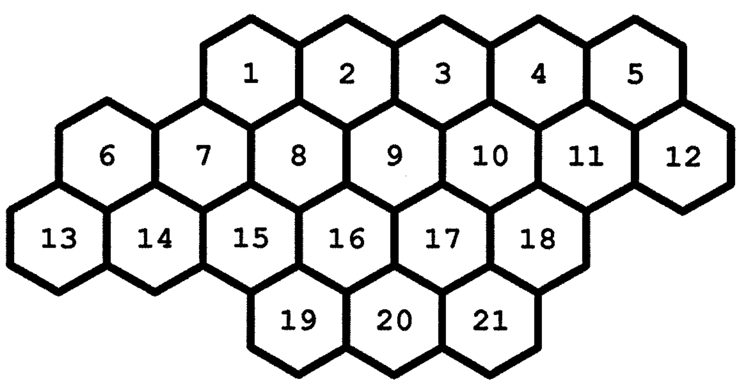

Figure 5.1. The cell number is shown within each cell.

We choose $P_i^{\max} = 0.108$, $\sigma = 6$ dB and $\beta = 12$ dB. With these parameters, $d_{\min} = \sqrt{12}$ so that, with adjacent channel restrictions on adjacent cells and a co-site separation (c_{ii}) of 5, we have the same compatibility matrix, for the graph-theoretical formulation, as in the previous chapter (Table 4.1). The number of channels available in the system is 96. The estimated blocking probability (the ratio of the number of blocked calls to the total number of call attempts) is plotted for both the compatibility matrix and the hypergraph form (indicated by (H)) of the compatibility constraints, in Figure 5.2. It is easily seen that for the same strategy, for blocking probabilities in the range 2–6%, an increase in traffic of 10–15% may be obtained.

If the conventional, fixed channel, regular reuse scheme (which is optimal in the case of an infinite, regular hexagonal system with uniform channel requirements in each cell) is used, since the number of reuse groups is 12 and a total of 96 channels are available, each cell will be assigned 8 channels. However, because there are fewer compatibility restrictions on the cells at the edges of a cellular system, some of these cells can be assigned more channels. The best that we can due to minimize the blocking probability in this example, using fixed channel assignment, is to assign 16 channels to each of cells 5, 6, 12, 13 and 14, and 8 channels each to the other cells (see Figure 5.1). A calculation of blocking probabilities, using the Erlang B formula, shows that an offered traffic of 85.7 erlangs will result in a blocking probability of 2.5%. But from the graph on the following page, we see that our best dynamic channel assignment scheme viz., Remax1(H), handles an offered traffic of 150 erlangs at the same blocking probability.

5.4. Conclusions and the Future

Fenton's method of computing the interference probability is fast but is not accurate for $\sigma \geq 8$ dB. In order to make channel assignment feasible for the hypergraph formulation of the problem, it is necessary to develop approximation techniques for computing the interference probability which are as fast as Fenton's method while being accurate.

In the case of dynamic channel assignment, we have assumed that the traffic is uniform. Nonuniform traffic does not pose any more difficulties in the implementation of the strategies considered. But it is our experience, in the compatibility

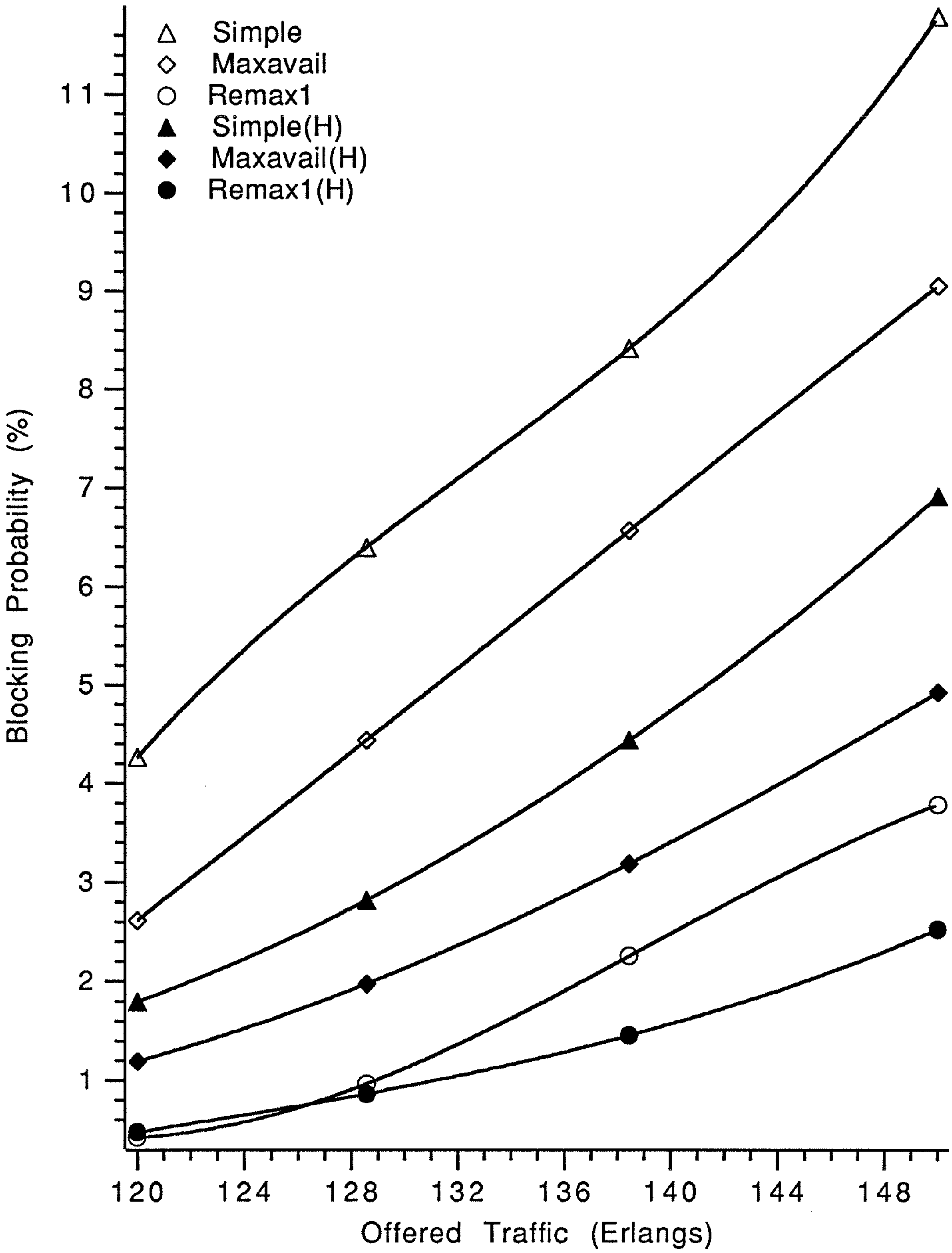
Simple
Maxavail
Remax1
Simple(H)
Maxavail(H)
Remax1(H)
Blocking Probability (%)
1
2
3
4
5
6
7
8
9
10
11
120
124
128
132
136
140
144
148
Offered Traffic (Erlangs)

Figure 5.2.

matrix formulation, that the divergence of the performance of these strategies from that of certain idealized strategies is more in the case of nonuniform traffic. It is likely that the performance of no realizable strategy comes close to that of these idealized strategies in the case of nonuniform traffic; on the other hand, it may be that new channel assignment strategies have to be developed to handle nonuniform traffic in both the compatibility matrix and hypergraph formulations. The effects of hand-offs and time-varying traffic on channel assignment also needs to be investigated.

It is our opinion that the single most important open problem in this area is that of computing the "capacity" of a cellular system analogous to the Shannon capacity of a communication channel. (The word "channel" is not used here in the same sense as it has been used so far in this thesis.) Note that one has to exercise considerable care not to work with a model for the frequency assignment problem, e.g., graph coloring, that does not capture *all* the features of the actual problem. (Any result on the capacity of a cellular system obtained using the graph coloring model, would most likely be invalid for the hypergraph coloring model.) We leave the reader* with the question:

What are the ultimate limits of spectrum efficiency?

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* However, see the Epilogue.

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Epilogue

The work described in this thesis has been principally aimed at improving the spectrum efficieny of current cellular systems which use analog FM frequency channels. If future digital systems use either Frequency Division Multiple Access (FDMA) or Time Division Multiple Access (TDMA) — or a combination of both, which is one of the proposed schemes — the work of this thesis would be directly applicable to them. (If TDMA is used, each time slot is to be treated as a frequency channel.) Recently, Code Division Multiple Access (CDMA) has also been proposed for future digital cellular systems. Since frequency spectrum is scarce (and becoming scarcer) the choice between an FDMA/TDMA system and a CDMA system must be based almost entirely on spectrum efficiency. We hope that this thesis, by enabling the spectrum efficiency of FDMA/TDMA systems to be more accurately ascertained, will contribute significantly to the development of future digital cellular systems.