AXIAL FLOW FAN DESIGN
BY LATTICE THEORY

THESIS

by

Carlos Wood

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CONTENTS

FIGURES and SKETCHES

CONTENTS cont.

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SUMMARY

This thesis takes the much discussed and comparatively little used lattice theory and applies it to the design of fans. The application is made through the use of charts coming out of the theory. It is felt that by the use of the charts that the design of confined fans, that is those where end effects on the blades need not be considered, will be made considerably simpler and easier.

The use of the lattice theory in the design of fairly complicated vane systems is discussed, and an approximate method of analysis is given. This subject requires more study in the near future.

As yet, at the time of writing, the method has not been checked by experiment, and as a result not much may be said concerning the accuracy of the results as actually applied to design. However, the use of the lattice theory gives designs that appear to be reasonable, and it is hoped that the method will prove to be fairly accurate in actual practice. At present a fan and vane installation as shown in Fig. XIII and Fig. XV is being constructed, and it is hoped that it will be possible to present some experimental data on the design in the near future.

INTRODUCTION

During the course of the design of a special wind tunnel model it became necessary to design a small high speed axial flow fan to give a comparatively high pressure rise per stage. Several methods of design were proposed, but they all [redacted] depended on the carrying out of a series of approximations. As a result these methods tended to be very long and somewhat innacurate.

The application of the lattice theory was discussed as a possible method of design, but it was feared that it would be too complicated to handle with any degree of ease. However it was finally found possible to develop design charts that were very simple to use. As a result the design procedure was materially simplified as compared to the other methods proposed.

The lattice theory that was used in the development of the charts was developed several years ago by Dr. Th. v. Karman and Dr. Clark B. Millikan. The theory has not been published, but is described in Reference No.1. A somewhat similar analysis has also been made by S. Kawada, and will be found in Reference No. 2.

REFERENCES

1. Aerodynamic Theory, Durand. Vol. II, Pp. 91 - 96.
2. Proceedings of the 3rd International Congress of Applied Mechanics, Vol. I, Pp. 393 - 402.

DEVELOPMENT OF THE CHARTS

The analysis is one of conformal transformation, and the theory is built up somewhat as follows:

If we have a circle in the ζ plane, it is well known that we may transform it into a straight line in the 'z' plane by subjecting it to the familiar transformation

$$z = \zeta + \frac{c^2}{\zeta} \qquad 1)$$

Now instead of considering the circle as a geometric figure, let it be considered as a streamline. Of course this means that the circle may be considered as a streamline in a flow composed of a doublet in a rectilinear flow. The doublet is in this case placed at the origin, which is at the center of the circle, while the rectilinear flow may be considered as due to a doublet placed at $\pm$ infinity.

Now in the lattice theory we are not concerned with a transformation that will give us only one straight line, but we want one that will give us an infinite family of parallel straight lines. To do this we must of necessity resort to a more complicated transformation than the simple Joukowsky type already mentioned.

Considering the circle as a streamline, we see that in any case for the circle to be a streamline it is only necessary for the sources and sinks inside and outside of the circular streamline to be images of each other. We considered 'c' as the radius of the circle in the first example. Now if we let 'k' be any given real positive value less than unity, we do the following to our original configuration of sources and sinks:

We bring the source at (-) infinity and the sink at (+) infinity in to any given point (-) or (+) c/k respectively. As the internal singular points must be images of the external ones, we see that the internal singular points must be moved to the points (-) or (+) ck respectively.

It may then be shown [redacted] at the contour of the circle that

$$z_c = \frac{Q}{2\pi} \operatorname{Log} \frac{(k + 1/k) + 2 \cos \theta}{(k + 1/k) - 2 \cos \theta} + nQi \qquad 2)$$

where 'n' is any integer, + or —. 'Q' is the strength of any one of the sources or sinks in the system.

(NOTE: 'Log' EQUALS PRINCIPAL VALUE OF $\log_e$.)

From this it may be seen that z_c is a multiple valued function representing an infinite series of straight lines parallel to the 'x' axis, and spaced a distance '[illegible]' apart vertically. However, this arrangement has no stagger, and for the use that we are to make of the lattice we must have stagger.

It is found that by placing vortices of opposite sign at the external singular points and image vortices at the internal singular points that we may obtain stagger in the system. Doing this we find after some calculation and simplification that

$$z = \frac{h}{2\pi}\left[e^{-i\beta}\log\frac{c+k\zeta}{c-k\zeta} + e^{i\beta}\log\frac{\zeta+ck}{\zeta-ck}\right] \qquad 3)$$

where the lattice now has the appearance shown below, and the various geometric properties are defined by their position:

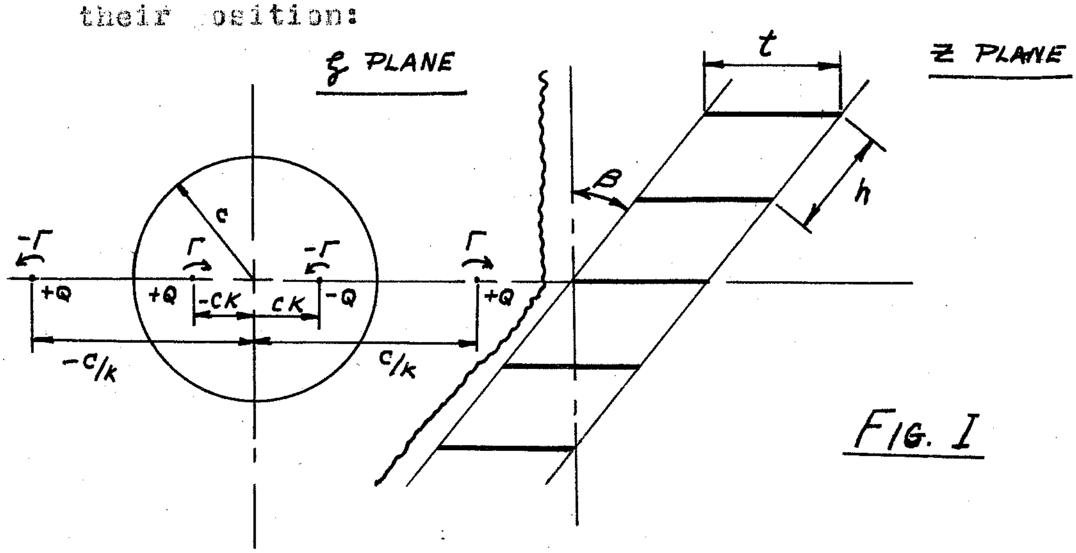


FIG. I

In the design of the fan we are especially interested at this point in the geometric properties of the lattice. It may be shown that if we let

$$R = \sqrt{1 + 2k^2\cos 2\beta + k^4}$$

we will find that

$$t = \frac{2h}{\pi}\left[\cos\beta \cdot \operatorname{Log}\frac{R+2k\cos\beta}{1-k^2} + \sin\beta\tan^{-1}\frac{2k\sin\beta}{R}\right] \qquad 4)$$

where β is the angle of stagger.

h is the spacing of the blades along the line of stagger.

t is the chord of the blade section.

Thus the above equation will determine 't' or 't/h' from 'k'. In any practical problem we do not care about the value of 'k', but we have approximate ideas as to the desired values of 't' and 't/h'. So the above equation is really a transcendental expression to determine 'k'. It is found that the easiest way to use the equation is to assume different values for 'k' and solve for 't' or 't/h'. By this means a family of curves may be built up which will make it possible to determine the value of 'k' from any given value of 't' or 't/h'. (See Fig.V).

Nothing has been said up to this point concerning the introduction of circulation into the system to obtain a lift force on the blades. Remembering that the Kutta condition must be fulfilled, we see that in this case we must as usual have a smooth flow off the trailing edge to avoid infinite velocities. After some calculation the theory shows that

$$\overline{K} = 2K\,\frac{\sin\alpha}{R} \qquad 5)$$

where $\overline{K}$ is the circulation.

α is the angle of attack of the section (from zero lift.)

From this it may be seen that $\overline{K}$ is proportional to '$\sin\alpha$' as in the ordinary airfoil theory.

It is found that for many of the calculations that instead of the values of 'k' versus 't/h' it is more convenient to use values of '$2K/R$' versus 't/h'. This family of curves is presented in Fig.VI.

Due to the action of the infinite series of circulations around the blades in the lattice there will be a change in the angle of attack of the air at the blades (as compared to the angle at which the air would strike the blades if they were isolated in an infinite flow). The infinite series of circulations will also cause a change in the velocity of the air at the blade (as compared to the velocity at which the air would strike the blade if isolated in an infinite flow). The actual angle of attack of the air at the blade is one of the factors entering into the consideration of the 'infinite flow' angle of attack. It is found most convenient to assume a certain angle of attack of the air at the blade and build up a family of curves for this given condition. One of these families is shown in Fig. VII. Likewise in determining the variation of the actual velocity (as compared to the velocity for the isolated airfoil in an infinite flow) it is best to assume an angle of attack at the section and solve for the velocity at the blade (in terms of the velocity for the isolated airfoil). A family of these curves is shown in Fig. VIII.

Using relations developed in the theory it is found that the expression for the angles of attack is

$$\operatorname{Cot} \alpha_1 = \frac{\operatorname{Cot} \alpha + (2k/R)\sin\beta}{1 + (2k/R)\cos\beta} \qquad 6)$$

and that for the velocities is

$$(U/U_1)^2 = \frac{1}{1 + (2k/R)^2\sin^2\alpha + 2(2k/R)\sin\alpha\sin(\alpha+\beta)} \qquad 7)$$

where α_1 is the angle of attack the airfoil would have if isolated in an infinite flow.

α is the actual angle of attack at the blade.

U_1 Is the velocity at the airfoil if isolated in an infinite flow.

U is the actual velocity at the airfoil.

From the characteristics of the NACA airfoil section to be used (2412) it is found that the section will probably be most efficient when the angle of attack $\alpha = 10^\circ$. The figures mentioned above are built up using this value for α where α is measured from zero lift.

We are of course very interested in the forces on the blade elements themselves. From the theory we find that the force on a blade in the lattice may be reduced to the form

$$F = 2\rho U^2 h(2k/R)\sin\alpha \qquad 8)$$

where F is the force on the blade element (in two dimensions).

ρ is the density of the fluid.

But from airfoil theory we know that this may be written in the form

$$F = (\rho/2)U^2 tC_L \qquad 9)$$

Setting these two expressions equal to each other, they may be reduced to the form

$$C_L = 4\frac{h}{t}\frac{2K}{R}\sin\alpha \qquad 10)$$

As '$\sin\alpha$' is practically equal to 'α' in the range commonly encountered, we see that we may write the slope of the lift curve as

$$\frac{dC_L}{d\alpha} = 4\frac{h}{t}\frac{2K}{R} \qquad 11)$$

A family of curves may be built up from this, and is shown in Fig.IX. It is interesting to note that the values of this slope of the lift curve agree very closely to the value shown in the normal airfoil theory up to values of 't/h' of about 0.10, where the values begin to diverge. It is also interesting to note that in many of the cases the slope of the lift curve of an element in the lattice is considerably higher than that of the isolated airfoil.

Now we may write the expression of the force on an isolated airfoil as

$$F_o = \frac{\rho}{2}U_1^2 t\, C_{L_o} \qquad 12)$$

where C_{L_o} is equal to $2\pi\sin\alpha_1$. From this we see that if we take the ratio of the force on the lattice element to the force on an isolated airfoil that we may write the ratio as

$$\frac{F}{F_o} = \frac{2}{\pi}\left[\frac{h}{t}\right]\left[\frac{2K}{R}\right]\left[\frac{U}{U_1}\right]^2\left[\frac{\sin\alpha}{\sin\alpha_1}\right] \qquad 13)$$

From these relations we may now design the fan and calculate the performance we may expect it to have, allowing of course for variations of experimental characteristics from the theoretical ones, somewhat as in the normal airfoil theory. This is simplified if we use a series of ratios to convert from the normal airfoil characteristics to the characteristics of the airfoil in the lattice system. For practical purposes the use of the 't/h' value, or what is called the λ value in the charts, as the contour on the chart is much more satisfactory then the use of the β value that was used in building up the charts mentioned above. Using the value of $\alpha = 10°$, the necessary charts for the design of the fan are replotted in Figs. X and XI.

APPLICATION OF THE CHARTS
TO FAN DESIGN

For any element of the fan we will have a resultant force on the fan blade which we may call 'dF', which we may integrate along the length of the blade in order to get the total resultant force on any given blade. In general in a pressure type fan we will want to keep a constant pressure increase over the disk of the fan, in order to minimize radial flows, and this pressure increase may be called 'δp'. Calling 'P' the total thrust on the fan disk we may say that

$$P = \int \delta p \, dS = 2\pi \int \delta p \, r \, dr \tag{14}$$

This gives us that $dP = 2\pi \, \delta p \, r \, dr$

From which

$$\delta p = \frac{dP}{2\pi r \, dr} \tag{15}$$

But from the sketch below, we see that

$$dP = dF \sin(\alpha + \beta) \tag{16}$$

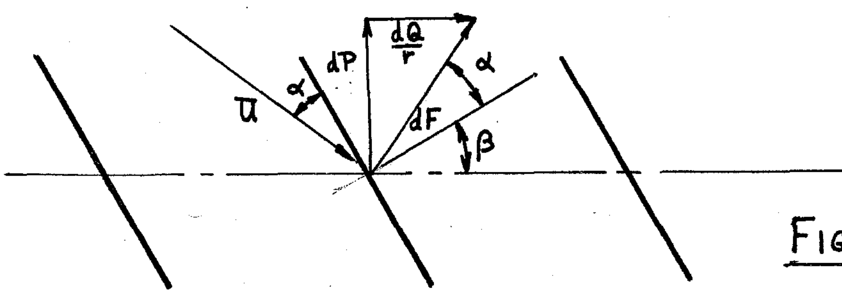


FIG. II

From this, by substitution in the above formula for 'δp' we see that

$$\delta p = \frac{dF \sin(\alpha + \beta)}{2\pi r \, dr} \tag{17}$$

We may say that $dF = dF_0 \frac{F}{F_0}$

and $$dF_0 = C_{L_1} q_1 t \, dr \qquad 18)$$

So for one blade we may write

$$\delta p = \frac{C_{L_1} q_1 t \frac{F}{F_0} \sin(\alpha+\beta)}{2\pi r} \qquad 19)$$

Then if we let 'n' be the number of blades, the total value of 'δp' will be

$$\delta p = \frac{nt}{2\pi r}\left[C_{L_1} q_1 \frac{F}{F_0} \sin(\alpha+\beta)\right] \qquad 20)$$

But $2\pi r$ is the circumference of one of the annular rings of radius 'r', so $\frac{n}{2\pi r} = \frac{1}{h}$

Making this substitution we may then write

$$\delta p = C_{L_1} q_1 \left[\frac{t}{h}\right]\left[\frac{F}{F_0}\right] \sin(\alpha+\beta) \qquad 21)$$

However, in the range in which our values of 'α' generally fall, we may say that C_L is proportional to 'α'. As C_{L_1} is the value of C_L of the airfoil when isolated in an infinite flow at an angle of attack 'α_1', if we then consider that C_L varies directly with the value of 'α' from zero lift, and C_{L_0} is the value that would be obtained from an isolated airfoil at an angle of attack 'α', we may write

$$C_{L_1} = C_{L_0}\left(\frac{\alpha_1}{\alpha}\right)$$

And from this we may write

$$\delta p = C_{L_0} q_1 \left[\frac{\alpha_1}{\alpha}\right]\left[\frac{t}{h}\right]\left[\frac{F}{F_0}\right] \sin(\alpha+\beta) \qquad 22)$$

Where C_{L_0} = lift coefficient of the airfoil, considered isolated in an infinite flow, at the angle of attack 'α'.

q_1 = dynamic pressure of the flow an infinite distance ahead of the lattice.

$\frac{t}{h}$ = ratio of blade chord to distance between the blades parallel to the line of stagger.

$\frac{F}{F_0}$ = ratio of force on the blade in the lattice to that on the blade if isolated in an infinite flow, from Fig.XI.

β = angle of stagger.

$\frac{\alpha_1}{\alpha}$ = ratio of angle of attack of the blade (referred to the flow at infinity ahead of the blade) to the actual angle of attack of the air (at the blade), from Fig.X.

By the use of the above equations we may determine the available thrust and the pressure rise in the fan, and so the output power.

The other portion of the problem necessary to consider is that of finding the torque, which may be called 'Q'. We may define 'Q' as

$$Q = \int dQ \tag{23}$$

From the definition of dQ as shown in Fig.II, we see that we may write

$$dQ = r\left[\cos(\alpha+\beta)dF + dD\right] \tag{24}$$

where the 'dD' term is to take into consideration the effects of the drag of the section on the torque necessary to move the blade. Strictly there should also be a 'dD' term of some sort in the expression for δp, but it was found that in most cases the term was so small that it could be neglected in comparison with the other forces. However, when considering the torque necessary to turn the fan, it was found that some value representing the drag of the airfoil section should be included, or errors of too large a magnitude would occur.

Strictly speaking the 'dD' term should be multiplied by the $\sin(\alpha + \beta)$. But the values of 'dD' are very small, varying somewhat because of the low Reynolds Number at which the section is operating. Also interference effects are a problem about which little is known in a quantitative way. As a result the total value of the 'dD' term is considered as a rough approximation to the actual drag.

Substituting in the equation for 'dQ' we may write

$$dQ = rq_1 t dr\left[C_{L_o}(\alpha_1/\alpha)(F/F_o)\cos(\alpha + \beta) + C_{D_o}\right] \qquad 25)$$

where C_{D_o} = drag coefficient of the airfoil, considered isolated in an infinite flow at an angle of attack α.

The rest of the terms have been defined in Eqn. 22.

DESIGN OF THE FAN

The use of the charts simplifies the design of the fan section to a very short series of calculations. The use of the charts involves the making of an original assumption to the value of β , which will probably be fairly close with care and practice in the use of the charts. However, this approximation is not serious, as the series of approximations converges very fast, and the convergence may all be performed on the chart connecting α_1 and α. This is a very simple procedure, and makes the design of the fan very simple and short.

With such a short procedure for the design of the fan it is possible to conveniently carry through a seried of designs in order to determine the most suitable design for the case under consideration. In the present case a series of sixteen (16) fans was investigated in order to determine the best proportions for the fan to be used.

Generally we are interested in the fan of highest efficiency, or in other words the fan that will give us the highest output power for the power furnished it. In this case it was found that the high efficiency fans would give a very low total thrust per stage, and thus make necessary the use of a very large number of stages in order to obtain the desired overall thrust. Due to the physical limitations on dimensions available for the fan units it was decided to change the criterion for the fan design from one of maximum efficiency to one of maximum thrust per stage for the input power per stage.

As a result a pseudo-efficiency factor connecting the output thrust and the input power was used as the design criterion. As used it was not dimensionless, having the dimensions of a velocity. Instead of obtaining a design in which we had the highest value of thrust output for power input it was found more convenient to use the design that had the lowest value of input power (P_i) for unit thrust output (δpS = thrust output). Of course this is one and the same thing, but the latter ratio was found easier to plot and use as shown in Fig. XII.

All of the investigations were made on the mean radius of the fan, that is halfway between the inner and outer radii of the fan. All of the calculations were made for the critical case for which the fan was to be designed, which was one of a high pressure rise and small flow through the fan disk.

Fans were investigated with values of the inner to outer radius ratio, r_i/r_o, equal to .65, .70, .80, and .90. For each of these cases the values of t/h of .30, .40, .60, and .80 were investigated.

As shown in Fig. XII the best proportions of the fan under the criteria taken above would appear to be where r_i/r_o lies between .65 and .70. As the value of r_i/r_o = .70 will give us higher axial velocities that value was chosen. This seems to be about the best value for all values of t/h. Our pseudo-efficiency factor was then crossplotted at a value of r_i/r_o = .70 for all values of t/h considered, the optimum value of t/h appearing to be about .425.

Using these parameters for the mean section the rest of the fan blade was designed. The characteristics are presented in Table A. A sketch of the fan is given in Fig. XIII together with the variation of the section chord and pitch along the radius in Fig. XIV.

The value of δp for the optimum fan dimensions at the mean radius was determined by means of Eqn. 22. It was desired to have a constant value of δp across the area swept by the fan. Thus by varying the dimensions of the blade at the outer and inn er radii it was possible to pick the chord and the pitch such that we did obtain a constant value of δp across the swept area of the fan. For a sample calculation of δp at the mean radius and a discussion of the methods of determining the fan blade proportions at other radii see appended P. A-1.

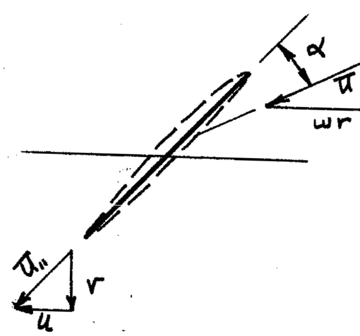
APPLICATION OF THE CHARTS
TO VANE DESIGN

The foregoing discussion, at least as far as that of the pressure rises in the fan section, is built up from the standpoint of perfect fluids. In other words, we would obtain the theoretical pressure rise, or very close to it, if there were no swirl of air aft of the fan section. But there are frictional forces between the blades and the air passing through them, which together with the infinite series of circulations will tend to give a swirl to the air as it passes through the vane sections. As a result we must straighten the air flowing aft of the fan if we are to get the higher pressure rise desired.

The use of vanes is necessary to get this straightening action, and with proper caution the charts that have been built up may be applied to the design of the vanes too. In general the vanes are very close to the trailing edge of the fan. As a result we may assume that the air has a motion along the mean camber line at the trailing edge of the blade, and, at least for a very short distance, will remain flowing in about the same direction.

The equation of continuity must hold in this case all through the fan, and as a result we know the approximate axial velocity of the flow, and from the direction of the flow at the trailing edge of the fan blade we may then obtain the magnitude of the velocity of the air, with respect to the fan blade. But the fan is rotating with a certain definite velocity, while the vanes are being held stationary. As a result we must add the velocity vector representing the rotational speed of the fan to the velocity vector of the air leaving the fan. This then will give us the direction and magnitude of the air as it will tend to enter the vane assembly. The pictures of the velocity vectors will appear somewhat as follows:

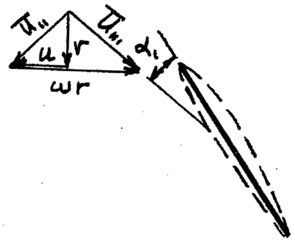
With respect to fan blade.



a)

FIG. III

With respect to vanes.



b)

As we are continuously changing the direction of the flow as it goes through the vanes and the axial component of the flow remains practically constant in order to satisfy the equation of continuity, the magnitude as well as the direction of the velocity vector as we pass through the fan is continually changing. This, combined with the fact that in a small high speed fan like this the amount of straightening is generally too large to be done with one simple vane, makes it necessary to somewhat change the procedure of analysis for the vanes.

If the case is such that each of the vanes is of the same length, and covers a certain percentage of the length of the path that the air has to follow through the vane section, the following approximate method of analysis may be used:

Consider the first vane: Determine the approximate dynamic pressure at both the leading and trailing edges of the first vane in the section. Then determine a mean dynamic pressure that may be assumed to act over the whole vane, and approximate the performance of the vane from this.

A similar method of approximating the dynamic pressures and velocities of the other vanes may be used.

The vanes actually change, or try to change, the dynamic pressure of the tangential component of the flow aft of the fan into static pressure, by the action of stopping the rotation. As a result, we may use pressures instead of kinetic energies to determine the performance of the vanes. Of course this is not exactly rigorous, but the kinetic energies of the motion in any given direction are analagous to the pressure due to the motion in any given direction because of the fact that we are dealing with a disk of constant area. As a result we may say that the vanes will develop a 'bucking pressure' to counteract the tangential flow, and this we may call 'δp_t' . This 'bucking pressure' is a function of one component of the actual force on the vane blade. Thus we may consider this as proportional to the power input to the vane. If there is a power input, there must also be a power output, and this will take the form of a static pressure rise in this case, which may be called a 'decelerating pressure rise', and denoted by 'δp_a' . This 'decelerating pressure rise' will be directly analagous to the 'bucking pressure' developed by the vane, except that it will act from the component of the force on the vane blade that is at right angles to the component that has to do with the 'bucking pressure'.

By analogy to the 'δp' equation developed for the fan blade, and by reference to the sketch which follows, it is seen that we may write

$$\delta p_t = q_1 \left[\frac{t}{h}\right]\left[C_{L_0}\left(\frac{\alpha_1}{\alpha}\right)\frac{F}{F_0}\cos(\beta+\alpha) + C_{D_0}\left(\frac{U}{U_1}\right)^2 \sin(\beta+\alpha)\right] \qquad 26)$$

and

$$\delta p_a = q_1 \left[\frac{t}{h}\right]\left[C_{L_0}\left(\frac{\alpha_1}{\alpha}\right)\frac{F}{F_0}\sin(\beta+\alpha) - C_{D_0}\left(\frac{U}{U_1}\right)^2\cos(\beta+\alpha)\right] \qquad 27)$$

Where all the terms have exactly the same significance as in the formulae for the design of the fan blade.

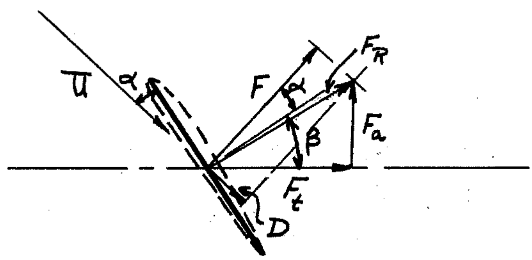


FIG. IV

DESIGN OF THE VANES

As mentioned before, the vane design of this fan was quite a problem. The very high value of the rotational component of velocity in the air after it passed through the fan section made it necessary to bend the air through quite a large angle in order to straighten it so there would be no rotational component of velocity aft of the vane section. As the axial component of the velocity has to remain approximately constant due to the fact that the fan disk acts on an annular ring of approximately constant section this really meant that the vane section acted as a very strong diffuser.

Because of this diffusing action of the vanes it became almost certain that a normal single blade type of vane assembly, that is one with merely a series of single vanes distributed around the circumference of the fan disk, would be conducive to very bad separation characteristics. This applied especially if the vanes were to be kept to a fairly short length. If the single vane type were lengthened the friction losses in the vanes would go up enormously, and the overall efficiencies would tend to drop.

As a result it was decided to use a multiple type of design for the vanes. Several arrangements were considered. The idea of using two vanes arranged so as to give a slot action between them and thus control the separation was discussed and considered quite seriously, but this would tend to large interference drags in the vane system. Systems using two, three and four vanes were considered, but the calculations showed that they would not have enough straightening action.

Finally an arrangement was evolved that seems to be logical, and that appears to have enough straightening action. The only trouble with it is that it has a comparatively large total number of blades, and tends to make the overall length per fan-vane stage longer than was desired. This arrangement is as follows:

There are ten (10) vane sets arranged radially around the periphery of the annular channel aft of the fan section. In each of these sections there are five (5) blades, each of chord of 0.90 inches, [redacted] arranged as shown in Fig. XV. Each of these blades goes completely across the annular channel, and is cast in solidly with the outer channel wall at the outer end, and with the inner wall at the inner end. The inner wall affords support to the shaft bearings that must be placed in the center of the complete assembly to make the fans run true.

When it is said that there are a rather large number of vanes in the assembly, the truth of the saying becomes obvious when it is realized that for each fan stage this assembly actually has fifty (50) blades in the vanes. So for the critical case of the design, in which there are three (3) stages, we have one hundred and fifty (150) vanes need to straighten the flow.

Calculation shows that we have a rotational component of velocity that alone gives us a dynamic pressure of approximately 49.0#/sq.ft. If the vanes are designed to take this rotational component out of the flow and convert the kinetic energy into static pressure rise, the calculations show that we get a static pressure rise of about 39.10#/sq.ft. This gives a vane efficiency of about 80.0%.

In the fan we calculated a pressure rise due to the action of the fan, considering no losses in the vanes, of $\delta p = 92.9$#/sq.ft. It was also found that if the swirl just aft of the fan was taken into account that the actual pressure rise through the fan itself was only 43.9#/sq.ft. The vanes add a pressure rise, like that of a diffuser, of 39.10#/sq.ft. As a result this gives us an overall pressure rise through one fan and vane stage of $\delta p_{tot} = 83.00$#/sq.ft. Calculating the overall efficiencies from this pressure rise we determine that $\eta_{tot} = 73.8\%$.

APPENDED FIGURES, SKETCHES, and TABLES.

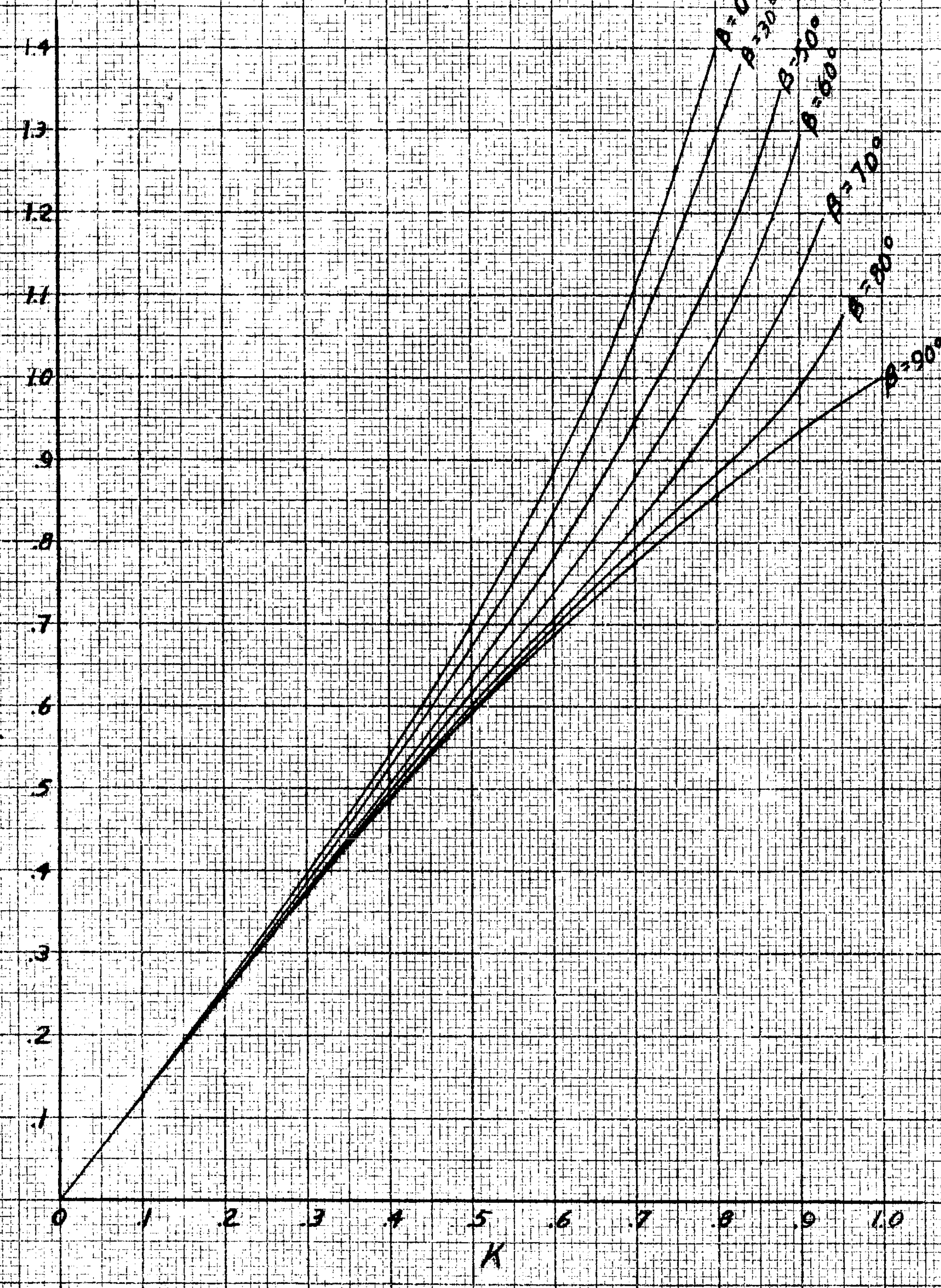


Fig. V .(t/h) AS A FUNCTION OF (k) AND (β).

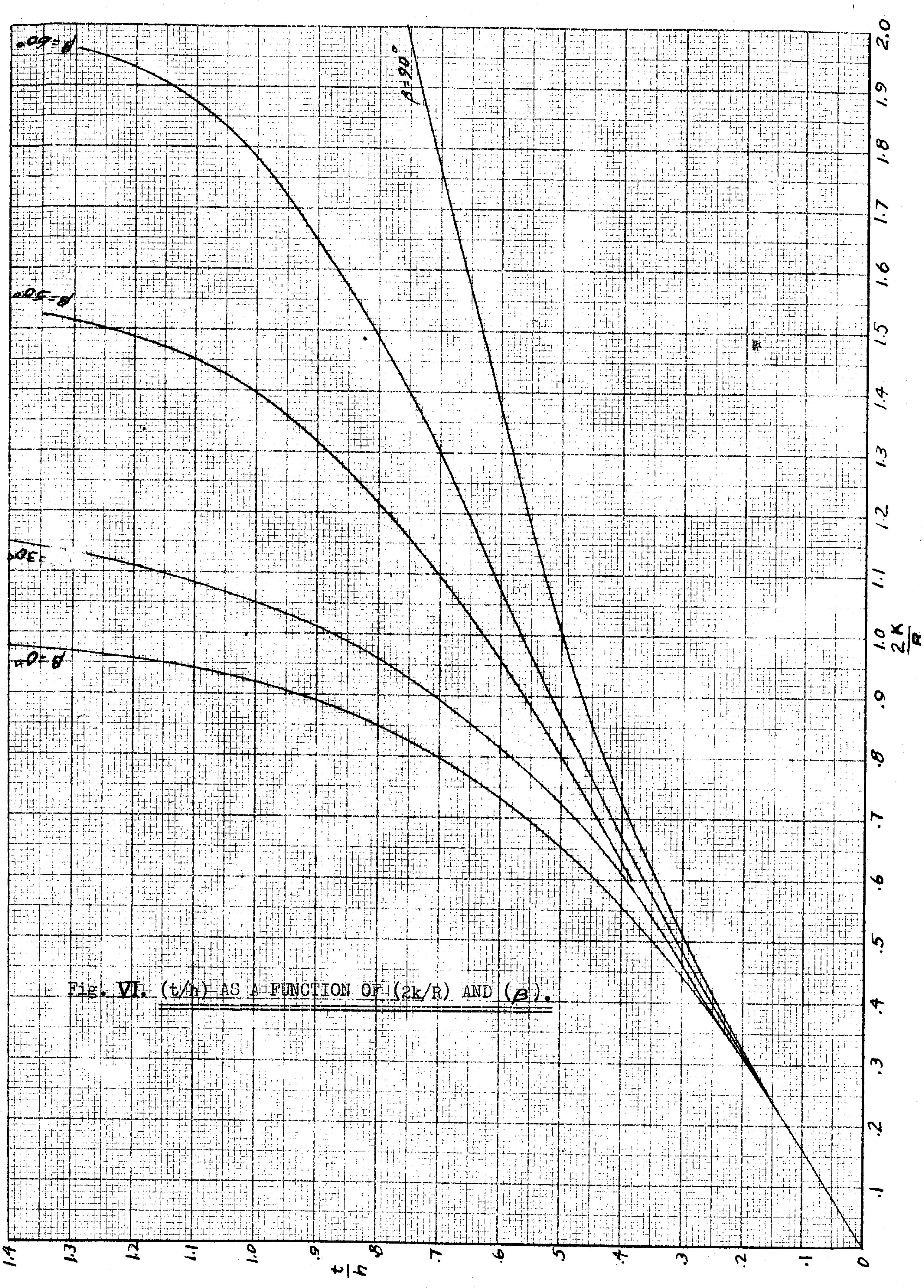


Fig. VI. (t/h) AS A FUNCTION OF (2k/R) AND (β).

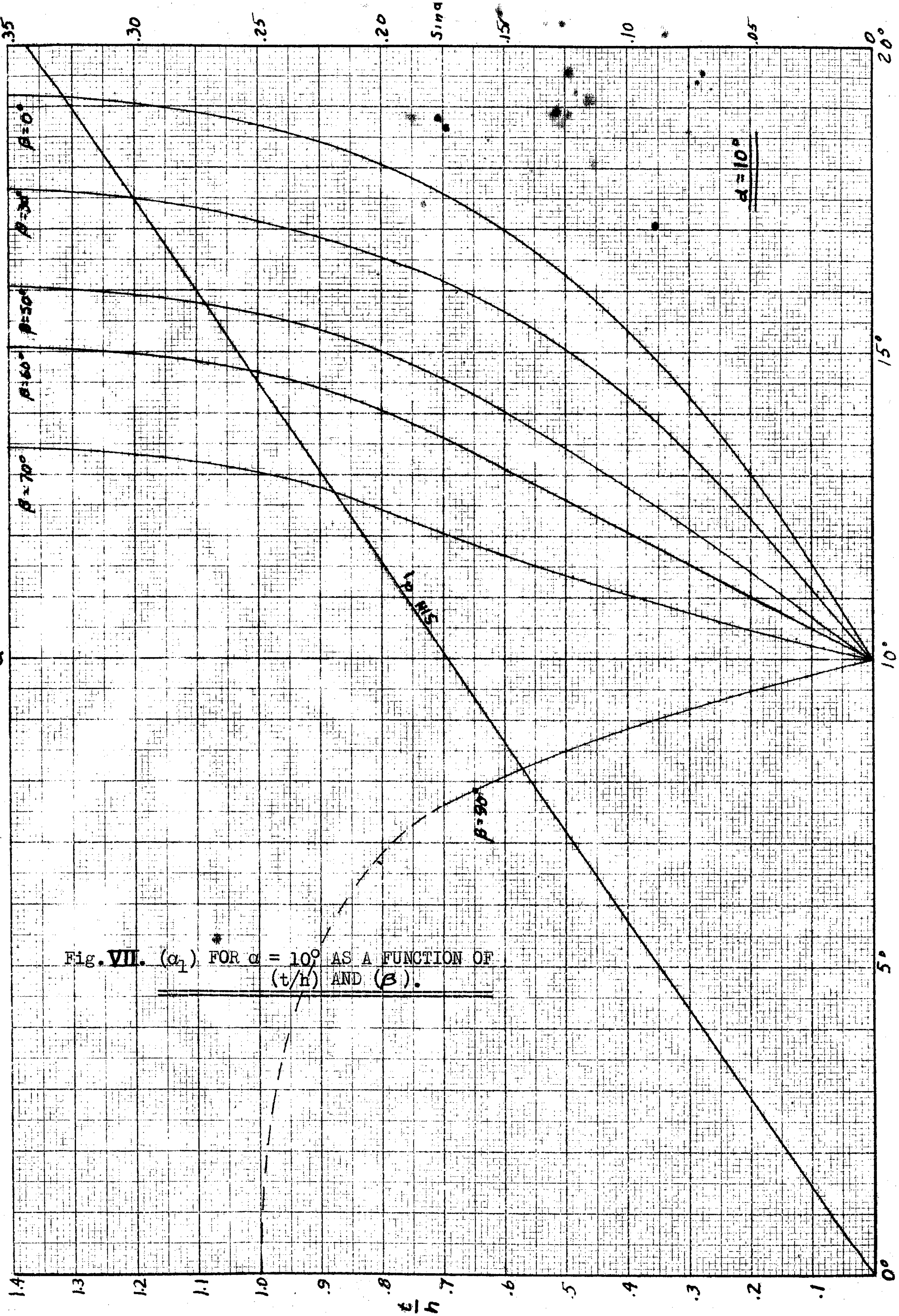


Fig. **VII**. (α_1) FOR α = 10° AS A FUNCTION OF (t/h) AND (β).

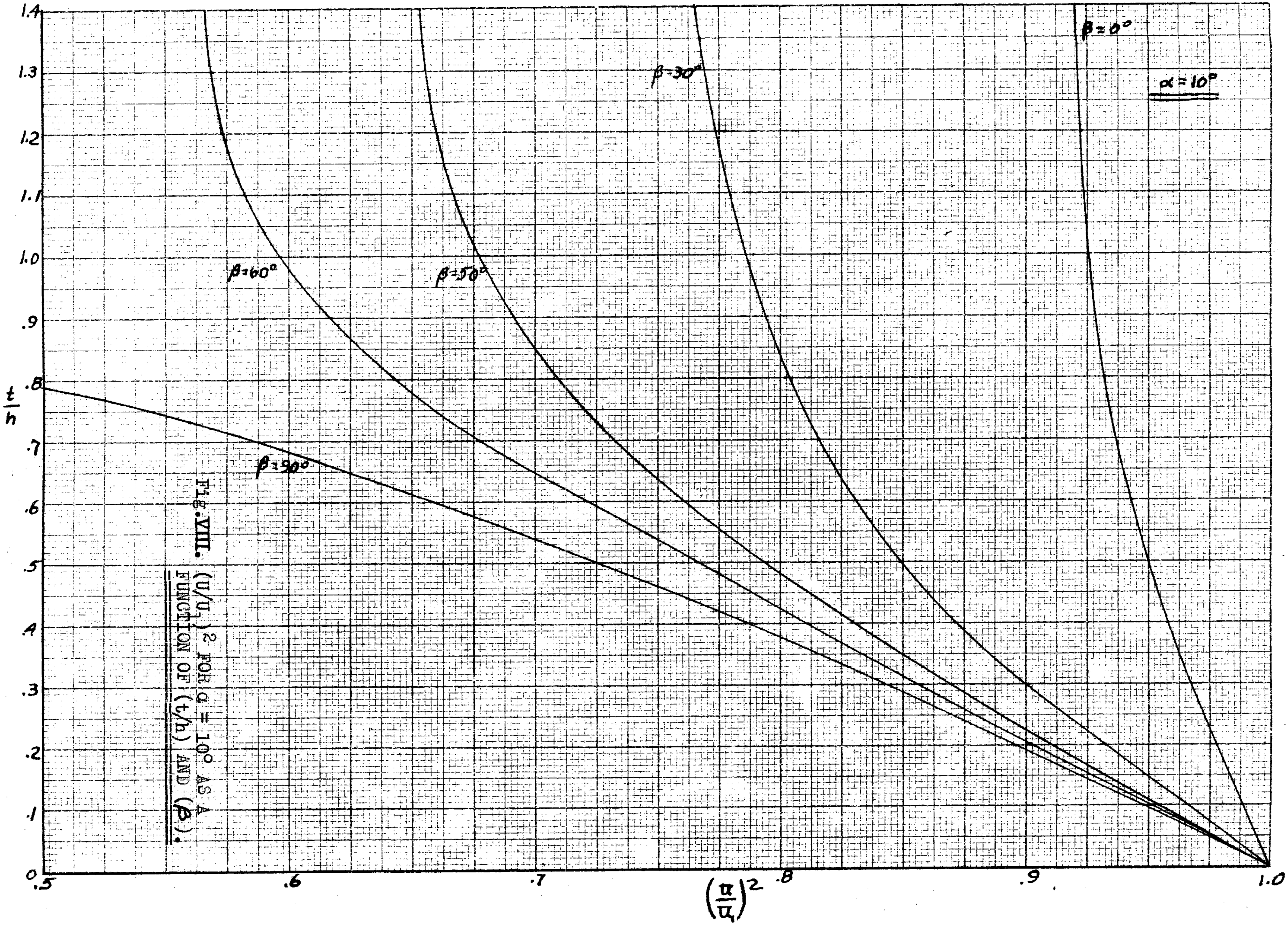


Fig. VIII. $(U/U_1)^2$ FOR $\alpha = 10°$ AS A FUNCTION OF (t/h) AND (β).

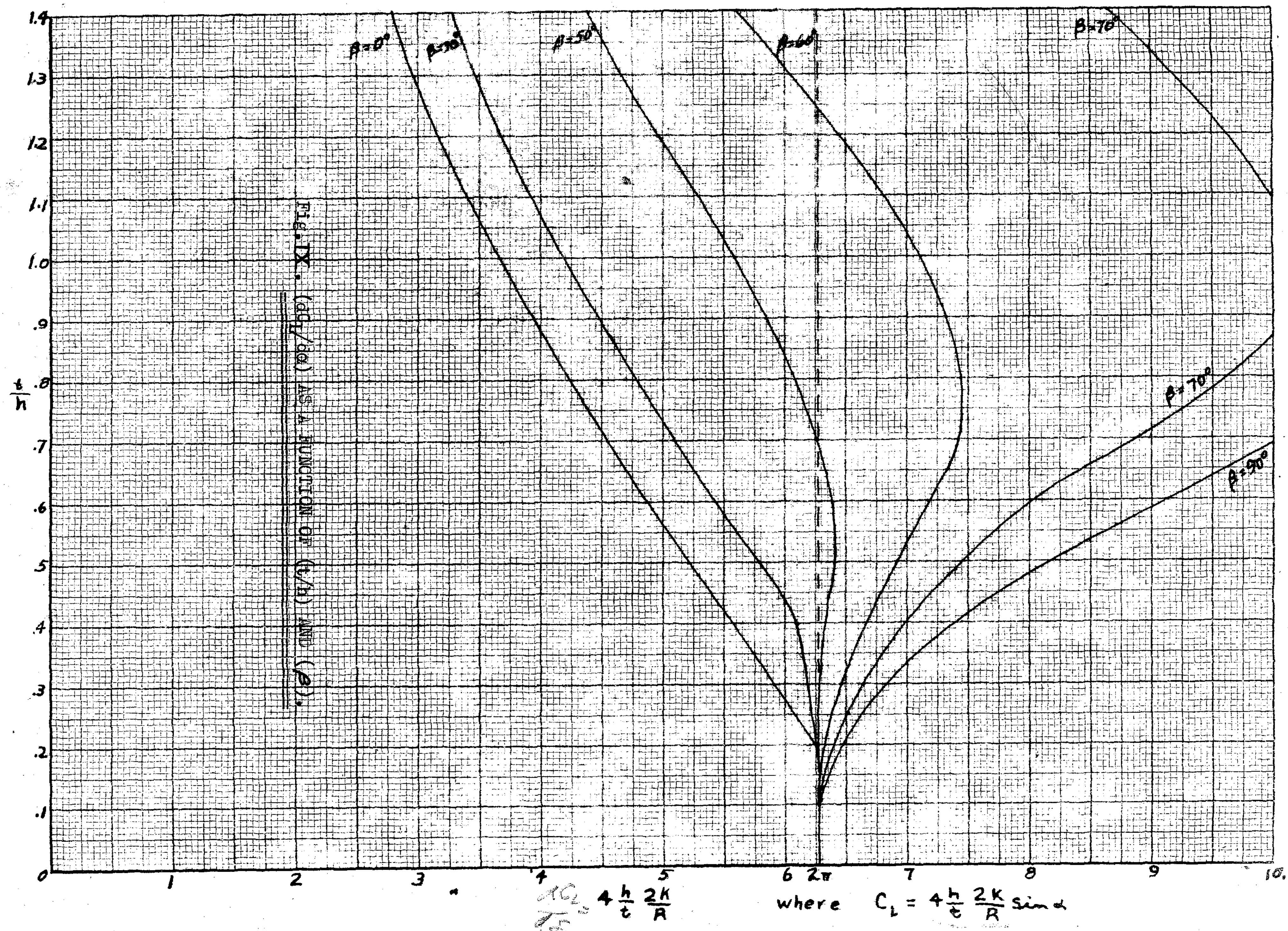
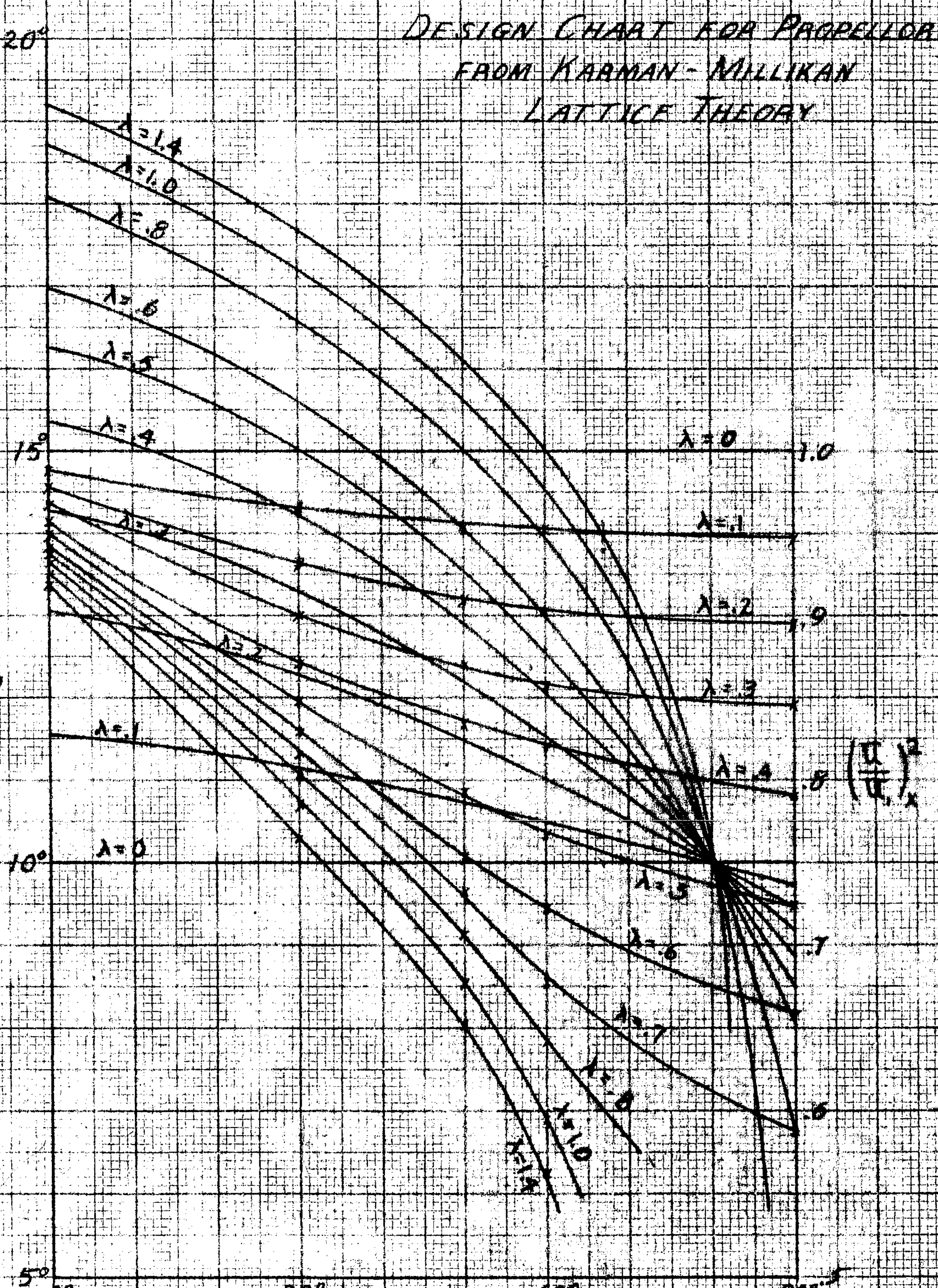


Fig. IX. $(dC_L/d\alpha)$ AS A FUNCTION OF (t/h) AND (β).



DESIGN CHART FOR PROPELLOR
FROM KARMAN - MILLIKAN
LATTICE THEORY
20°
15°
10°
5°
α_1
λ=1.4
λ=1.0
λ=.8
λ=.6
λ=.5
λ=.4
λ=.2
λ=.1
λ=0
λ=.3
λ=.7
1.0
.9
.8
.7
.6
$\left(\frac{U}{U_1}\right)^2_x$
0°
30°
60°
90°
β
$\frac{d}{h}=\lambda$
$\alpha=10°$

Fig. X.

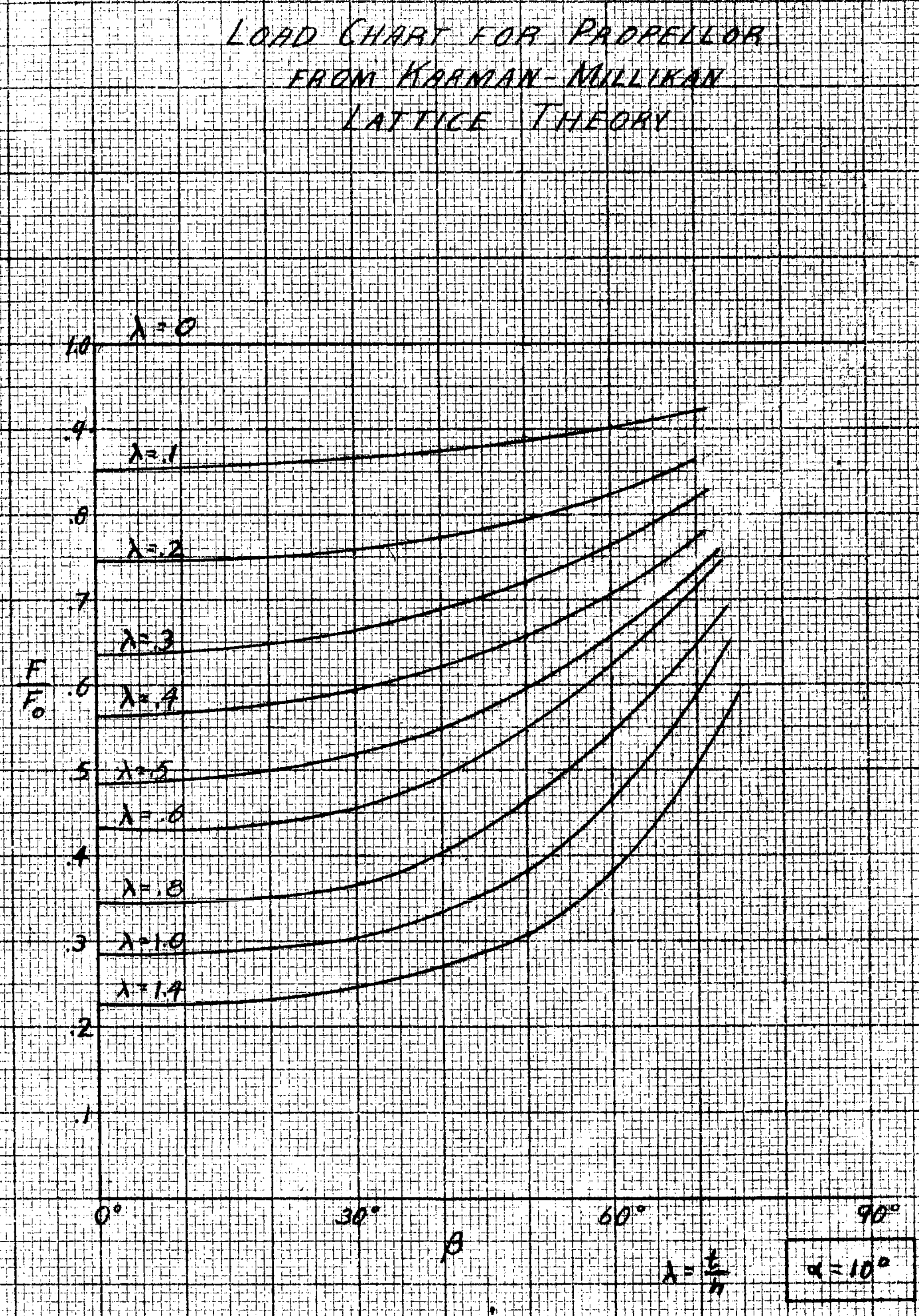
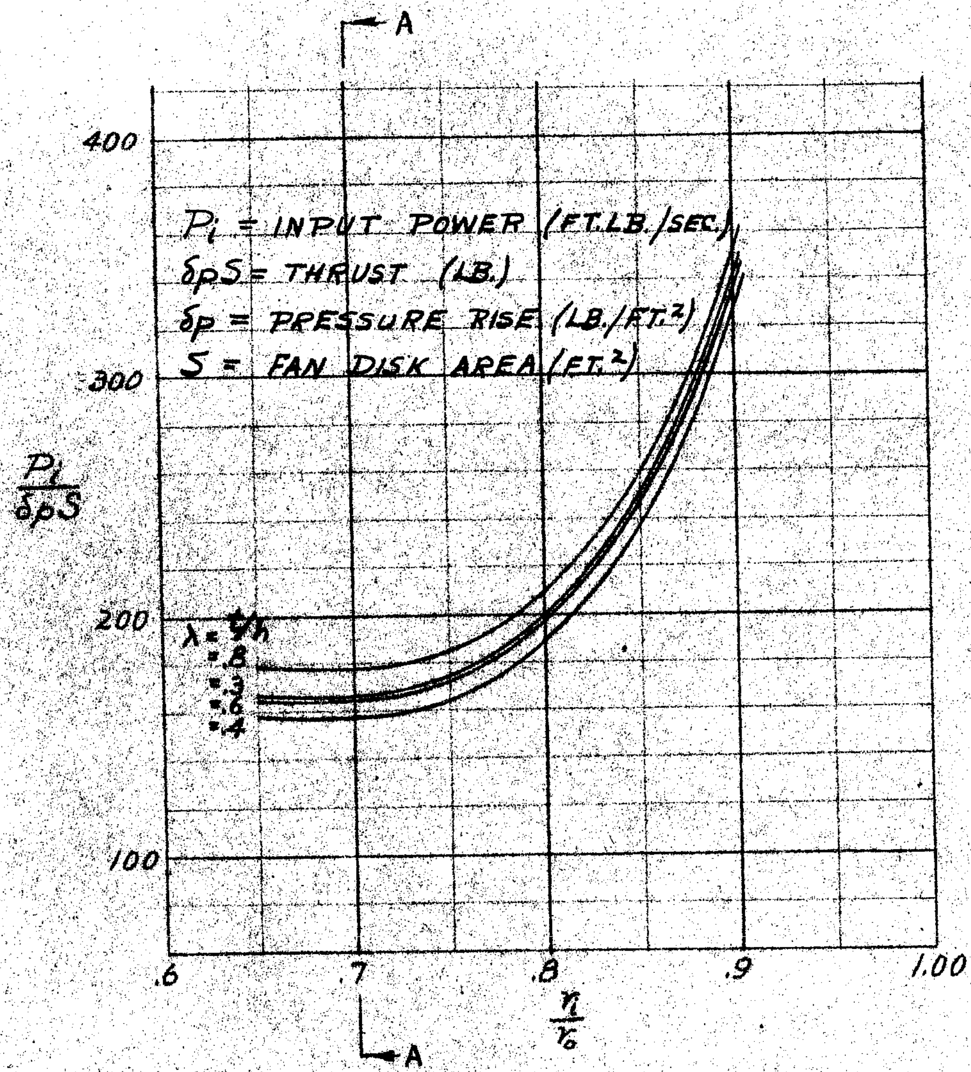
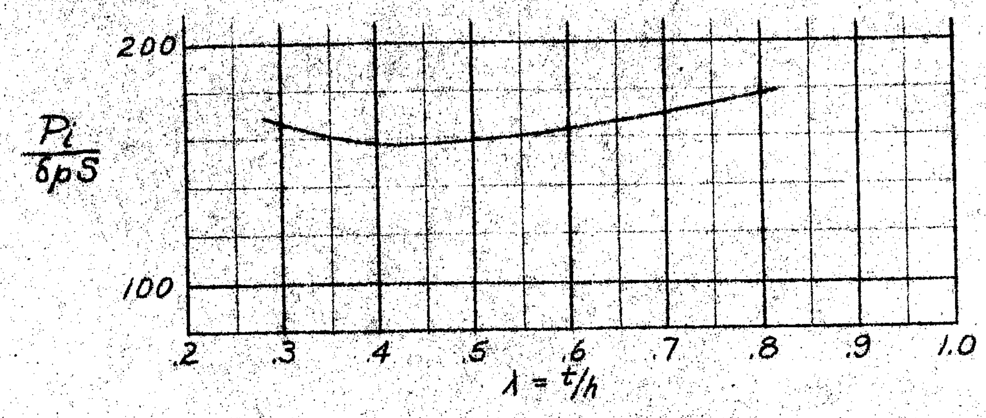

LOAD CHART FOR PROPELLOR
FROM KARMAN-MILLIKAN
LATTICE THEORY
λ=0
λ=.1
λ=.2
λ=.3
λ=.4
λ=.5
λ=.6
λ=.8
λ=1.0
λ=1.4
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
F/F₀
0°
30°
60°
90°
β
λ = t/h
α = 10°

Fig. XI .

FIG. XII. $\left(\frac{P_i}{\delta p S}\right)$ AS A FUNCTION OF (t/h) AND $\left(\frac{r_i}{r_o}\right)$.



SEC. A-A.



PROBLEM ON LATTICE THEORY:

Given a fan of 7" outer ~~radius~~ diameter, rotating at 17,500 RPM, handling 91.0 cubic feet of air per ~~second~~ minute. Take the ratio of the inner to outer radius of the fan as 0.70. Giving the fan 8 blades, let the blade chord at the mean radius, (halfway between the inner and outer radii) be 1". ~~Calculate the approximate optimum value of t/h for the mean radius of the fan (halfway between the inner and outer radius) taking as the criterion the maximum value of the thrust from the fan for the input power.~~ Determine the pressure rise for an element at the mean radius, and determine t/h and the angular setting of the fan blades at both the inner and outer radii to give the same pressure rise as at the mean radius.

Take the lift coefficient at the fan blade as 1.00 (C_{L_0}), and the local angle of attack at the blade as 10°. Then, using this the equation for the pressure rise becomes

$$\delta p = \frac{\alpha_1}{10^\circ} q_1 (t/h)(F/F_o)\sin(10^\circ+\beta)$$

and the equation for the torque becomes

$$dQ = rq_1 t\left[\frac{\alpha_1}{10^\circ}(F/F_o)\cos(10^\circ+\beta) + C_{D_o}\right]dr$$

where α_1 = angle of attack of the blade as referred to the flow at infinity ahead of the blade.

q_1 = dynamic pressure of the flow an infinite distance ahead of the lattice.

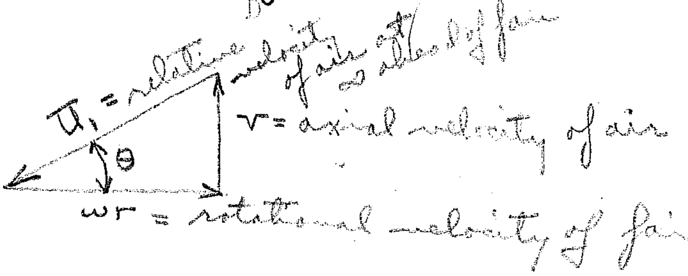
t/h = ratio of the blade chord to distance between the blades parallel to the line of stagger.

F/F_o = ratio of force on the blade in the lattice to that on a similar blade isolated in an infinite flow.

β = angle of stagger

r = radius at the portion of the blade being considered

C_{D_o} = 0.02 approximately for the fan section being considered.



Note that by definition

$\beta + \alpha_1 + \theta = 90^\circ$

So $\beta = 90^\circ - \alpha_1 - \theta$

θ is known, so α_1 becomes the only variable.

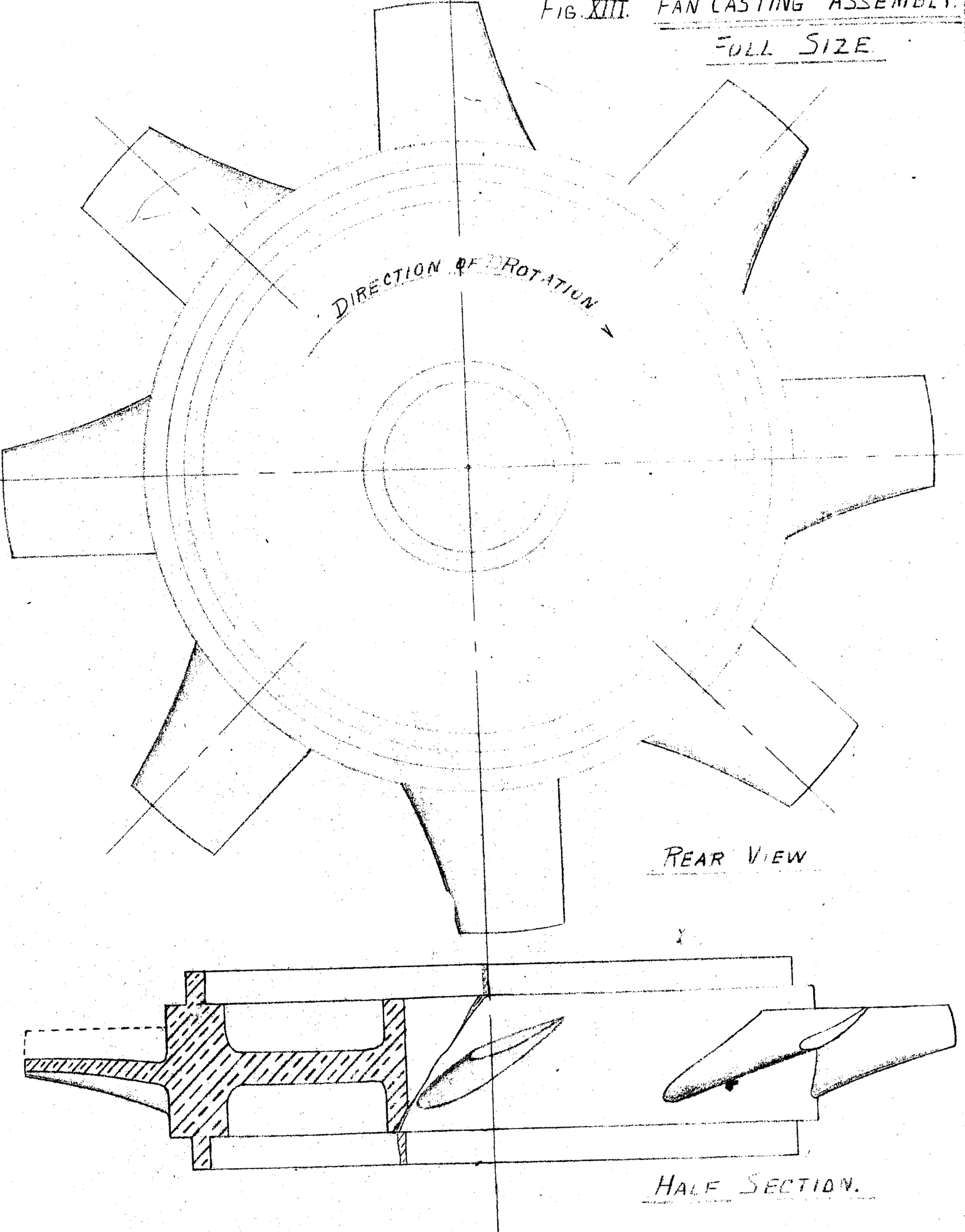
Table A. DESIGN PARAMETERS OF THE FAN

	r_1	r_m	r_o
r (inches)	2.450	2.980	3.500
r (feet)	.204	.248	.292
$90^\circ - (\beta)^\circ$	29.95	25.40	22.60
Setting, to chord line, $(\gamma)^\circ$	28.25	23.70	20.90
(λ) or t/h	.691	.425	.303
h (inches)	1.925	2.340	2.750
t (inches)	1.330	.995	.832

Assuming no swirl aft of fan, δp = 92.9 #/ft^2

Assuming swirl aft of fan, δp = 43.9 #/ft^2

FIG. XIII. FAN CASTING ASSEMBLY.
FULL SIZE



Print to lines only!

FIG. XIV FAN BLADE PROPORTIONS.

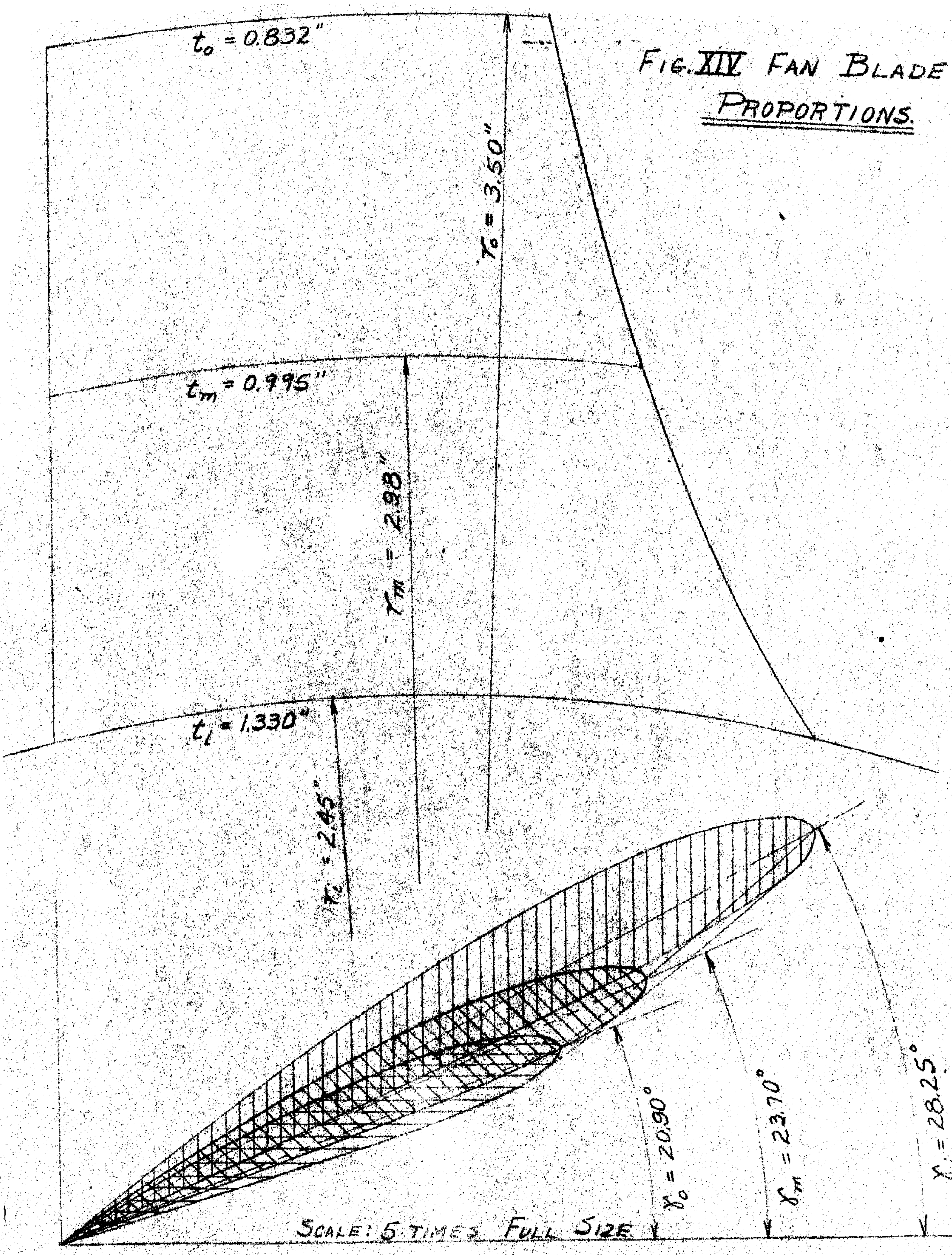
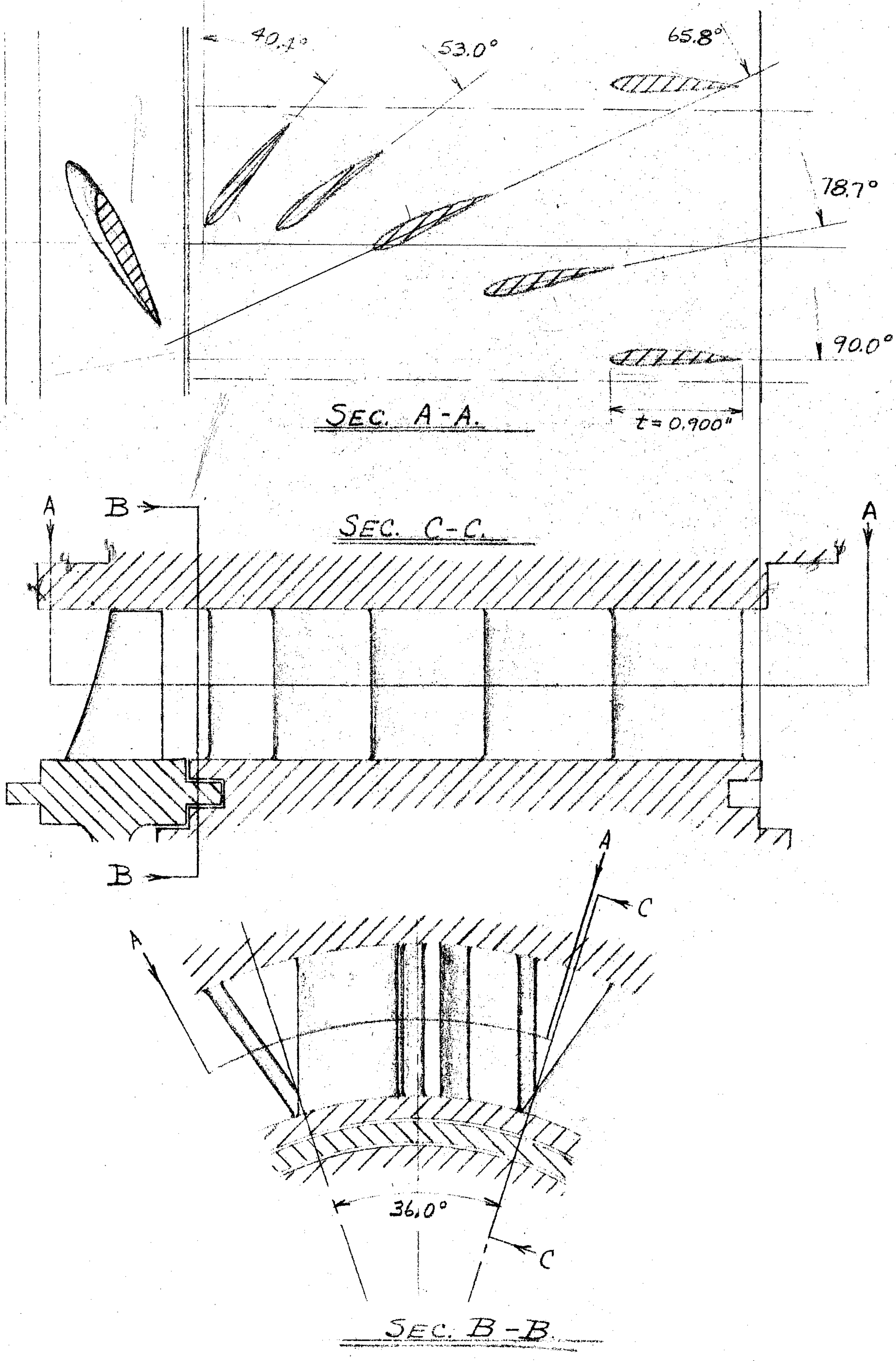


FIG. XV. FAN-VANE CASTINGS, ASSEMBLY
FULL SIZE.



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APPENDIX V

THREE PHASE VARIABLE FREQUENCY POWER SUPPLY