Electoral Institutional Design

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Olga Shvetsova

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ABSTRACT

This dissertation addresses the role played by electoral institutions in the formation of systems of political parties. The introductory chapter contrasts two methodological approaches found in the studies of institutional influence: the approach of cross-country comparisons versus the detailed analysis of specific electoral systems.

In Chapter 2 a set of 25 democratic countries, yielding a total of 53 distinct electoral regimes, is analyzed in order to determine the effect of electoral institutional variables on the number and strengths of political parties. To resolve the argument in the literature of whether institutional influence is systematic, we change the traditional analytical setting and bring into the analysis variables characterizing the internal cohesiveness of the societies under consideration. In particular, we take district magnitude as our institutional variable, and ethnic, religious, and linguistic fragmentation of electorates as characteristics of internal cohesiveness. Our conclusion is that the role of electoral institutions is in mediating the impact of social factors on the formation of political landscapes, and that, therefore, institutional features should enter the analysis interactively with the parameters describing societies, i.e., institutional influence should not be viewed as absolute and independent of social context.

Chapter 3 contains theoretical analysis of a particular electoral system - the Single Non-Transferable Vote (SNTV) system - and establishes some equilibrium properties of SNTV where the entry of new competitors is allowed. Among other things, we show that, in

cohesive electorates, the SNTV system would generate an equilibrium in which there will be either no competitors in excess of the number of seats being filled, or just one extra competitor. Assumptions that lead us to this and other results are: (1) unidimensional policy space, (2) sincere Downsian voters with single-peaked preferences over policies, and (3) strategic candidates who care primarily about getting in office. As our theoretical conclusions depend strongly on these assumptions, we offer in Chapter 4 empirical evidence in support of those conclusions. We analyze Japanese and Taiwanese district-level races (both countries used SNTV for their legislative elections, although Japan is now undergoing an electoral reform), and find that the equilibrium features that we predict theoretically under SNTV are, indeed, systematically present in elections there.

While Chapters 3 and 4 are devoted to candidate competition in multiseat elections, in Chapter 5 we look at the incentives and constraints that electoral rules create for political parties. Our analysis there applies to at-large electoral districts. We find that implicit or explicit thresholds for legislative representation seriously influence the number and strength of the parties that form. We also derive endogenous limits on the number of competing parties. Because the analysis in Chapter 5 proceeds mainly within the same framework as Chapter 3, our resulting conclusions about the candidate-based and party-based electoral competition in the single-member as well as multimember districts are compatible.

ELECTORAL INSTITUTIONAL DESIGN

CHAPTER 1: INTRODUCTION

CHAPTER 2: INSTITUTIONAL AND SOCIAL DETERMINANTS OF PARTY SYSTEMS

CHAPTER 1: INTRODUCTION

1.1 The General Challenge of Institutional Design

In all societies political outcomes are seriously influenced by the rules by which expressed preferences are translated into adopted decisions. In democratic process in particular, the institutional form in which preferences are allowed to be expressed, and the subsequent method of their aggregation, become especially important in that they can alter the directions of political development. Thus, when institutional choices are made for democratic nations, those choices have far-reaching policy consequences that are not always easy to isolate or predict.

Regardless of what social developments follow the adoption of a particular set of democratic rules, evaluation of those rules is necessarily complicated. It is often difficult and sometimes impossible to prove that certain events have an institutional cause. And if agreement on causality is reached, then evaluations of social implications must rely on some normative criteria and thus are prone to controversies. Consequently, while, for example, some scholars believe that representation in legislatures should be as close to proportional as possible, others reasonably point out that proportionality concerns only the representation of political parties in legislatures, whereas parties themselves, formed in order to secure representation, cannot be viewed as an identical reflection of society. Besides, proportionality itself says little about the effectiveness of the government or even about who forms it: as long as government formation is a search for compromises on multiple issues, politics might do better aggregating preferences by engaging only a small number of parties rather than emphasizing variations.

It is unlikely that politicians who conduct, say, electoral reforms would closely consider current academic disputes on the virtues and shortcomings of specific institutional forms (though were there a greater consensus among the scholars, they might). One naturally suspects that politicians are primarily concerned with their own electoral advantage or disadvantage, and also - to a varying degree in different countries - with the popularity of the proposed new rules (e.g., the Italian 1993 choice of a new system of representation, arguably, was a popularly expected rather than politically beneficial decision). Nevertheless, from time to time the design of democratic institutions occurs in a meaningful way. Sometimes decision-makers are both benevolent and have an opportunity to choose on the basis of what they deem best for the country. One possible example is the American state and national constitutional process, about which John Adams wrote in 1776: "You and I, my dear Friend, have been sent into life, at a time when the greatest law-givers of antiquity would have wished to have lived. How few of the human race have ever enjoyed an opportunity of making an election of government more than of air, soil, or climate, for themselves and their children."[1] And even if we take a view that the "Jacksonian movement" that led to a major revision of the party system in the 1830's, and the Civil War a few decades latter were, in institutional terms, the

[1] *Thoughts on Government: Applicable to the Present State of the American Colonies. In a Letter from a Gentleman to his Friend.* in: Robert J. Taylor et al., eds., *The Papers of John Adams*, Cambridge, Harvard University Press, 1977, IV, p. 92.

necessary corrections of the imperfections of an original design by the Founders, such a view still would not contradict the argument that (1) a precedent of design had occurred, and (2) the product of this design allowed needed corrections, which is a virtue that not every institutional system possesses.

Since Adams's time similar opportunities have been enjoyed by many politicians around the globe. The world in the 20th century has seen many cases of what after Montesque can be called "statecraft." Sometimes it proceeded through incremental changes. Sometimes intentions were not entirely benevolent, or were not really aimed at achieving a liberal democracy (e.g., the invention of the Soviet-type state). Nevertheless, particular instances of institutional design are of interest if they are more or less honest attempts to build vital democracies. Several general categories can be distinguished: (1) In the wave of democratic designs that accompanied de-colonization in Africa and Asia we saw the participation of both local political elites and the colonial powers, but the colonial administration played a key role in setting up the preconditions for independence, including institutional ones. (2) More recently, the democratization in the ex-communist countries brought about yet another set of institutional solutions, albeit, in many instances rather unstable ones. (3) The collapse of the "socialist world" deprived many not-so-democratic regimes in unsuccessful post-colonial democracies of an "alternative" political model, and induced the "second wave" of democratization. Finally, (4) like Spain after Franco, post-apartheid South Africa, perhaps Greece, Portugal, Brazil, Argentina, and Chili, some countries do not fall into any of the above categories, but experienced an extensive revision of their political institutions in the direction of democratization.

The idea of practical involvement in democratic design has sometimes disturbed political scientists and left them uncertain about the value of any advice they could offer. As Giuseppe di Palma (1992) recounts the profession's early reaction to the construction of a new democracy in Spain following the death of Francisco Franco: "...Transitions such as Spain's revealed the essential reason for the experts' wavering between noncommittal and pessimistic assessments. In effect, we suffered from blind spots. We were inadequately prepared to prescribe the extent to which innovative political action can contribute to democratic evolution; inadequately prepared, in sum, to entertain and give account of the notion that democracies can be made (and unmade) in the act of making them." The "political action," to which di Palma refers is, of course, a more general concept than the design of institutions.

In this work we deliberately restrict ourselves to the analysis of the latter, which is but one component of "statecraft." Di Palma, for example, differentiates four aspects of "crafting" democracies, although, what we call institutional design occupies the first position. Those aspects are: "(1) the quality of the finished product (the particular democratic rules and institutions that are chosen among the many available); (2) the mode of decision making leading to the selection of rules and institutions (pacts and negotiations versus unilateral action); (3) the type of "craftsmen" involved (the alliances and coalitions forged in the transition); and (4) the timing imposed on the various tasks and stages of the transition." Out of the four components, we believe the design of rules and institutions to be the most suitable for political analysis, since rules and institutions are the least subjective of all noted influences.

Of course, an argument can be made, whenever institutional choice is addressed, that interested parties manipulate the selection of rules to their own advantage. Still, we should not overlook the fact that at least in part actual choices are based on an evaluation of a variety of institutional options, even if the decisions that follow are self-interested. Indeed, because the rules by which democracies are organized affect political outcomes so directly and closely, "democratization is ultimately a matter of political crafting[2]."

In the work that follows we do not analyze the complex, interdependent dilemmas as faced by institutional designers. Instead we look at the *by-element* impact of institutional rules on political outcomes. Moreover, out of the whole body of institutions that we might consider - legislative-executive relations, court jurisdictions, presidential versus parliamentary government - we restrict ourselves to the analysis of electoral institutions only, and more specifically still, to the analysis of rules that govern legislative elections.

1.2 Electoral Systems and Party Systems

Electoral institutions, on the analysis of which we focus in this work, form a substantial, if not a crucial part of democratic institutional arrangements, because they directly influence the basis of democratic government - representation. Accountability of elected

[2] Quoted in di Palma (1992) from: Juan Linz, and Alfred Stepan, "Political Crafting of Democratic Consolidation and Destruction: European and South American Comparisons" (paper delivered at the Conference on Reinforcing Democracy in the Americas, the Carter Presidential Center, Atlanta, Georgia, November 17-18, 1986.

politicians to their constituents can only be defined through the mechanisms by which constituents send their "messages" of satisfaction, dissatisfaction, support, or disagreement. To the voters, on the other hand, the rules may be determine what "messages" to send to politicians. Indeed, intervention of electoral mechanisms of certain types makes sending some signals practically useless, as the system will not generate sufficient response to them, while other rules may put emphasis on these same signals. A simple example is offereded by the ability of single-member district plurality systems in many cases to suppress the salience of fine divisions within electorates, while PR systems, by allowing small groups to obtain representation, inspire voters to stress the identification with these groups (see Chapter 2).

In other words, electoral systems are important because they may be critical in determining the form of political rivalry that takes place in society and the agenda around which this rivalry develops. Political party systems simultaneously reflect both the form and the agenda of political competition. Variation in political party systems, therefore, is the first immediately observable consequence of application of varying electoral rules. Chapter 2 explores this connection in greater depth.

But establishing empirical connections between electoral rules and party systems helps us little in understanding why such connections take a certain form, and what should be changed to bring about some desirable political consequences. In other words, to engage in meaningfull design, one needs to understand in greater detail the causality in institutional influences. It is argued below that approaching the issue from the perspective of cros-country comparisons can provide only very limited results. In fact, we believe that the potential of such comparisons is exhausted by the analysis in Chapter 2. Therefore, in Chapters 3-5 we depart from the methodology of empirical comparative analysis, and study in a theoretical manner more appropriate for designing purposes one particular electoral system - SNTV.

This choice of electoral system to analyze is dictated by the unique properties of SNTV compared to the mechanisms of plurality and proportional representation. If viewed from the position that only individuals can take actions and that only individual goals can be advanced by those actions, electoral systems where political parties (i.e., *groups* of individuals) compete are difficult to analyze, if only because we must simultaneously study the imperatives to form those parties, the actions of individuals, and how both individuals and organizations appeal to voters. However, electoral systems that rely on party competition create strategic environments for political actors that are qualitatively different from the ones generated by the Anglo-American "first-past-the-post" electoral system. The Single Non-Transferable Vote system (SNTV) is an intermediate case between the well-studied Anglo-American plurality in single-member district elections with their emphasis on individual candidates, and PR (proportional representation) electoral mechanisms with their emphasis on parties.

Out of the variety of electoral systems the one best studied theoretically is the Anglo-American plurality in single-member districts. This electoral system, however, is used in only one-fifth of stable and long-lasting democracies (if we count India as such). And of those, New Zealand has now decided to abandon it. Thus, it is viewed by political practitioners as but a special variety, as it competes, implicitely in established democracies, and explicitely in newly democratizing nations, with its counterparts - the party list proportional reprsentation and other multimember district electoral mechanisms. Whatever theory is developed, then, it must be ultimately applicable accross different types of electoral systems, before it can be used to select from existing, or to design new electoral rules. Studying the SNTV system allows us to consider districts of magnitude larger than one, while staying within the familiar framework of individual candidate competition and retaining the exactness of results typical for the the models of single-member district plurality elections.

Of three major types, plurality in single-member districts and party list PR stand at the extremes of the spectrum. The first creates purely individualistic incentives, at least, if viewed at the district level. The latter one in its most developed form of at-large PR is a competition of political parties, where most candidates are but names on party lists. SNTV is located between these extremes in the sense that in those systems individualistic incentives remain, as individual candidates compete for seats, but the larger number of seats per distict promote party multiplication and allow their differentiation in strength within a single district. SNTV, therefore, is of particular analytical interest, and not only because of this "intermediate" position, but also because it promises to provide useful insights in the process of party formation as far as it is influenced by the rationale of individual politicians. In Chapters 3 and 4 we address the Single Non-Transferable Vote system - first, theoretically, and then empirically in Japan and Taiwan.

1.3 Some Methodological Points

There are two principal ways to approach the study of institutional impact on a political system. We can assess it by empirically connecting institutional causes and political consequences. Alternatively, we can try to deduce analytically the logistics with which political system responds to certain institutional incentives, i.e., revert to a "micro"-level, "qualitative" methodology.[3]

A natural way to proceed if we want to learn about the influence of particular electoral rules on the policies that develop under them or on who gets elected to the legislature is by comparing the outcomes resulting from different institutional arrangements. As long as we believe that in some fundamental ways political, institutional, and other (e.g., anthropological, geographic or economic) characteristics of society are interdependent, this approach may lead to the discovery of social laws and some general relationships that are otherwise concealed under the rich diversity of political phenomena. This methodology of broad comparisons is the "classical" one of "comparative" comparative politics (as distinguished from the studies of specific areas and countries). This is the path Duverger (1954) follows to formulate his Law relating the structure of party competition in a country's elections to the number of seats contested in its electoral districts. After ruling out geographically, historically, and culturally based explanations of bipartism versus multipartism, he concludes that it is the type of electoral regime that matters: "the simple-majority single-ballot system favors the two-party

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[3] This separation of the two methodologies somewhat parallels the classification by Stein Rokkan in: Stein Rokkan, "Comparative Cross-National Research: The Context of Current Efforts'" in Richard L. Merritt, and Stein Rokkan, eds., *Comparing Nations: The use of Quantitative Data in Cross-National Research*, New Haven, Yale University Press, 1966.

system" (p. 217), and that "the simple-majority system with the second ballot and proportional representation favor multi-partism" (p. 239). Similarly, it is, in essence, this cross-country empirical approach that characterizes the work of the scholars like Rae (1971), Lijphart (1984, 1991), Sartori (1986), and Taagapera and Shugart (1989). Indeed, this methodological tradition is the one used by the founders of the discipline. Niccolo Machiavelli in *Discourses* suggests to "...turn to those republics that without so many enmities and disturbances have been for a long time free, and.. see of what sort their government was and whether it could have been introduced into Rome... (p. 207)."

But the methodology of general comparisons is necessarily subject to severe practical limitations. To begin with, the sample of countries that can be used as data points is very restricted, and it does not grow fast. Indeed, we would not wish that it grow fast: once established, we want to see democracies safe and uninterrupted in their development, so that each eligible country supplies just one data point. The limited size of the sample is the central cause of many flaws of the cross-case analysis. Only a very broad and general typology of institutional systems can be sustained: incorporation of more than just a few institutional parameters is already sufficient to drain the degrees of freedom and invalidate any inquiry by statistical methods. Meanwhile, political institutions in general, and electoral regimes in particular, are characterized by the same degree of variation as other social phenomena. It is unlikely that we can ever find two completely identical electoral regimes in our sample. And since a pure comparison is complicated if an independent influence is never twice the same, we are left to *ad hoc* hypotheses about the sources of observed differences and similarities in political outcomes.

But institutional differences may be the least complex thing a researcher encounters in case-comparative inquiry. If institutions are the mechanisms that translate what society wants into what it receives, then the side of "social wants" is of no small importance either. But as "social wants" are expressed, they are already institutionally transformed through ballot structures, limited menues of options to choose from, district mapping, awareness of what mechanisms of seat allocation will be applied, and anticipation of coalition possibilities. Thus, the "social" side of the relationship cannot be adequately described through *expressed* preferences. Then, what we must consider are the non-institutional determinants of preferences. But, information about these determinants is the information about the society in question: income, education, age and gender structure, occupational diversity, ethnic, linguistic, religious composition, urban-rural partitioning, the legal system, technological development, positioning in international markets, etc., plus the interdependence of these factors. Without considering everything, we cannot compare the bases of "social wants" in different countries.

As if social and institutional differences were not enough to make a case against relying exclusively on cross-country comparisons, historic factors claim their proper place as well. The sequence in which rules are introduced - the institutional history so to say - matters, even if only because of the environment in which a state's political party system originally forms. The history of enfranchisement and disenfranchisement is important for the same reason. But the most important influence and, perhaps, the most difficult to capture analytically is of history in the conventional meaning of the word - the impact of the epoch when the regime is established, and of the events and people of the time of its introduction and growth. No one doubts that wars and recessions, for example, influence political development. But so do the natural disasters, outstanding political figures, and even some timely written books. And, of course, one country's history is inseparable from the history of the rest of the world. What was the influence of the 17th century revolution in England on the American Revolution, and of the latter on the revolution in France?

To summarize the preceding discussion, the transformation of the "pure" influence of institutions by what we call today the cultural specifics of the "host" countries[4] obscures our ability to learn from those countries' experiences about these systematic institutional influences.

Therefore, although empirical comparisons can often be insightful, what we can infer will always be too general to provide much practical guidance. For example, it may be true that, if plurality in single-member districts is introduced in some hypothetical country, that country is likely to develop a two-party system - but only if a democratic regime survives there. What it needs to survive, or what dangers exist for democratic stability - at best can only be hinted at by the cross-case approach. Addressing issues at this general level may be a first step in the direction of understanding how man-made mechanisms affect political

[4] Defining the cultural specifics that influence the performance of the political institutions in this admittedly all-encompassing way, is, however, in accord with the Gabriel Almond's definition (see Gabriel A. Almond, "The Intellectual History of the Civic Culture Concept", in: Gabriel A. Almond, and Sidney Verba, eds., *The Civic Culture Revisited*, Newbury Park: SAGE Publications, 1989).

development, but from there we still have a long way to go before this knowledge has solid practical application.

What we want, then, is to combine cross-country comparisons with micro-level analysis so as to generate more detailed information about the impact of the tools that people have in their power to apply to the society. The directions that micro-analysis might take are, first, an empirical path, along which we might try to access in some more or less direct form the preference structures in different countries and the differences that institutional changes produce in the political output. Another approach is to assume particular institutional arrangements - electoral regimes, for example - and to deduce their implications. This path may require making assumptions that necessarily depart from the reality - assumptions such as perfect information among voters, perfect issue mobility for candidates, costless entry of new candidates or parties, and so on. If and when these departures from realism do not disrupt causal connections, the implications of our logical constructions can then be applied and tested in those political systems where electoral institutions correspond to the type that we analyze.

Institutionally based micro-level analysis is illustrated by Reed (1990), who demonstrates that some systematically found traits of the Japanese party system correspond to what is to be expected, given the electoral rules in operation there. Similarly, Cox (1994) examines the Japanese system of single non-transferable vote (SNTV) both analytically and empirically in order to place Reed's finding on a firmer theoretical foundation. Other research in the formal field has generated similar results for other electoral rules, where scholars more often emphasize logical conclusions than empirical evidence in teir support (Greenberg and

Weber 1985, Myerson 1993).

1.4 An Outline of What Follows

This work relies in diferent parts on both of the described methodological approaches. Chapter 2 is a cross-country general assessment of the performance of electoral institutions in different social contexts. We begin with the hypothesis that the right way to study the implications of electoral institutions is to evaluate them in conjunction with the non-institutional social factors that affect politics. Or, if we turn it the other way around: it is incorrect to evaluate the extent to which social cleavages influence politics out of the institutional environment of this politics. Indeed, we find that social cleavages influence the number of political parties in a predictable direction, but only if those cleavages are considered in the context of the present institutional constraints - electoral rules.

Chapters 3 and 4 consider a specific electoral system - SNTV. Chapter 3 contains theoretical propositions regarding the number and electoral strength of candidates who compete for seats in a district. Our conclusion is that in cohesive constitutencies either as many candidates will run as there are seats, or there will be just one extra competitor. Moreover, the weakest winner in a district and the strongest loser should have approximately the same vote shares, so that both anticipate a close race prior to election. Chapter 4 deals with electoral data from the countries that use SNTV in their legislative elections (Japan until 1993, and Taiwan) and is aimed to test the validity of theoretical conclusions derived in Chapter 3.

While chapters 3 and 4 are devoted to elections of individual candidates, though in the potentially large districts, Chapter 5, using a similar theoretical methodology considers party list PR elections in which competitors are not candidates, but political parties.

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CHAPTER 2: INSTITUTIONAL AND SOCIAL DETERMINANTS OF PARTY SYSTEMS

2.1 Theoretical and Empirical Studies of Electoral Systems

How political competition as structured by the rules of elections is as yet far from being thoroughly studied. Two classes of electoral systems - plurality in electoral districts returning one legislator each versus other types of plurality and all the variety of proportional representation systems combined until very recently were not treated equally in the theoretical political science literature. Plurality rule in single member districts enjoyed considerably greater attention: most of the models study the behavior of voters and politicians in that framework. Here to mention are the studies on Duverger's Law (Palfrey (1989), Feddersen, Sened and Wright (1990), Feddersen (1991), Cox (1987), as well as the broader and more general class of studies on electoral equilibria under plurality or majority rule (restricted to one single-member district). Neither of the above models is ready to be extended to the proportional representation case.[1]

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Cox (1984,"Electoral Equilibrium in Double member districts" and 1984, Strategic Electoral Choice in Multi-Member Districts: Approval voting in Practice?") extends the model to the approval vote case. His results are controversial and crucially depend on the restrictions imposed on the model. In "Electoral Equilibrium..." Cox shows that if three candidates compete in a two-member district, voters are sincere, and in addition, if they decide to vote, they must

The models of multimember (rather than multicandidate but single-member) electoral districts, though they exist, have not been very successful in answering questions raised by existing literature on proportional representation. For example, Greenberg and Weber (1985) show the existence of an equilibrium in multimember districts, when electoral rules are such that only a full quota (of votes) yields a seat; they do not allow in their model for the distribution of seats by remainders method that is almost universally used in real quota systems, but that threaten to destroy the stability results derived from their analysis. Greenberg and Shepsle (1986) address the issue of the use of remainders for allocation of undistributed seats, and show that equilibrium existence is, indeed, undermined by the introduction of this realistic detail. Strictly speaking, Greenberg and Shepsle's result does not preclude the possibility of equilibria for particular preference configurations, different from the ones used in the proof. A more important deficiency of the party list PR models is that distinction is not stressed enough between candidate competition and party competition; despite the fact that authors explicitly deal with procedures that induce competition among parties, none of them arrives to results predicting fewer competing parties than seats in a district. Parties in multiseat (i.e., where *district magnitude* - number of legislative mandates - exceeds one) constituencies, especially of larger size, can and do seek more than just one seat - something that makes no sense as an objective of individual candidate. Thus, objectives of politicians in a multiseat

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necessarily approve two candidates out of three, then all three candidates in equilibrium converge to the same point in the median interval. In "Strategic Electoral Choice..." Cox assumes that voters behave strategically, but again considers only three-candidate elections in double-member districts by approval voting (approval voting sometimes is also called "plurality in multimember districts"). The results apply to this special case only. The important conclusion of this paper is that voters' subjective beliefs about electoral outcomes play a crucial role in electoral choice.

race often cannot be reduced to the objectives of individual candidates.[2] Recent work by Cox and Shugart (Cox 1993, Cox and Shugart 1994) represents an alternative approach, analyzing the problem of multiseat elections from the perspective of voter rationality. Cox's model of voter response to elections by *plurality* in multimember districts (Cox 1993), when extended to the PR case (Cox and Shugart 1994) fails, like other works in the field, to generate a prediction of the number of competitors (competing parties), location of their policies, and their respective electoral strength as endogenous parameters.

Not only is a satisfactory general model of the multiseat elections needed, but the long-term goal may be to derive as endogenous the institutional choice between "plurality" (i.e., plurality in single-member districts) and proportional representation. Meanwhile, although democratic systems were almost uniformly of a "plurality" type early in this century, many democratic states adopted some form of proportional representation between the 1910's and the end of the World War I, and it has since successfully competed with plurality systems in the world.

Although in theoretical analysis we cannot as yet avoid separating plurality elections (i.e., when candidates compete) and party list PR elections (with political parties as relevant players), in this chapter - preceding theoretical analysis that requires separate treatment of the two cases - we pool all electoral systems together to establish the common empirical traits, the general type of connection between institutional characteristics and structure of competition

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"Closed list" proportional representation is used on the second ballot in Germany and Mexico. "Open list" (when voters have an option in addition to voting for the party to express their preferences within party list) system is used in Belgium and Italy.

in elections. More specifically, we address the impact of electoral rules on the way social diversity translates (via those rules) in the political diversity (latter being represented by the system of political parties).

In contrast to the theoretical literature, which as a rule analyzes single-member district electoral systems separately from all other electoral systems, many contemporary empirical institutional studies view single member districts, operating under plurality rule, as but a special case of a general type. Rae (1967), in his seminal volume "The Political Consequences of Electoral Laws," approaches the numerical analysis by pooling the observations of plurality single-member-district elections together with the proportional representation cases. The same is true for Lijphart's study ("Democracies".., 1984), where all conclusions derive from a sample of 21 stable democracies in which proportional and plurality systems are not separated by any formal criteria. Taagapera and Shugart, in "Seats and Votes.." (1989), discuss this approach specifically and make a connection between Duverger's law and `Duverger's rule', rejecting the hypothesis that number of parties and number of seats in the district should be viewed as dichotomous quantities (two-party versus multiparty systems, and plurality versus proportional representation). In fact, they argue, there exists a continuous relationship between the number of political parties and the estimated number of seats, filled within an average electoral district.

Here to the institutional explanation of political divisions we add the consideration of the impact being exerted by the social divisions which we take to be a crude approximation of what the preference structure might look like. We presume when analyzing political institutions that those institutions mediate between individual preferences and social outcomes such as political stability and the nature and number of political parties. We also know that preferences have, as one source, society's underlying social structure, especially its ethnic structure. So, in learning the influence of institutions on outcomes, we should consider the possibility that similar institutions in different social contexts yield different outcomes. Restated specifically for electoral politics, "[t]he relationship between electoral rules and party systems is not mechanical and automatic: A particular electoral regime does not necessarily produce a particular party system; it merely exerts pressure in the direction of this system ..." (Duverger 1959, p. 40).

This argument, though, is not always made part of our research. The abovementioned Lijphart's (1990) reassessment of Rae's (1971) seminal analysis of electoral laws is a case in point. Despite acknowledging that "there are other important causes of multipartism, particularly the number and depth of cleavages in a society" (1990, p. 488), Lijphart's reanalysis, like Rae's, fails to consider Duverger's argument in its full form. The particular problem is that the "usual suspects" examined in these studies, stable democracies, vary greatly in character: in the degree of social homogeneity, size of electorate (population), economic orientation and level of prosperity, etc. Analyzing the effects of electoral institutions separate from other things ignores the possibility that institutions are intervening structures and that they influence, say, the number of political parties only to the extent that the "more basic" characteristics of a society act through them to increase or decrease this number.

In this chapter, then, we adopt the view of electoral laws as intervening structures and we reconsider Rae and Lijphart's analysis of the institutional parameter - district magnitude - that researchers (c.f. Taagapera and Shugart 1989) regard as the most important characteristic of an electoral system.

In Section 2.2 we reconsider the data and some of the variables that are the focus of earlier studies, and discuss an especially important component of social structure - ethnic heterogeneity - that sets the context for the operation of electoral institutions in general and district magnitude in particular. In Section 2.3 we reanalyze matters using Lijphart's approach, whereas in Section 2.4 we consider Rae's election-by-election method. In Section 2.5 we use both Rae and Lijphart's approaches to assess the extent to which our conclusions about the joint influence of district magnitude and ethnic heterogeneity depend on the inclusion in the data set of single-member district systems, and in Section 2.6 we offer some remarks on the special place that India has among the democratic countries we analyze. Section 2.7 is a conclusion to the chapter.

2.2 Data and Variables

2.2.1 Sample selection and the unit of analysis

Lijphart criticizes Rae's election-by-election approach with the argument that a political system such as the United States, operating under a uniform electoral arrangement throughout this century, ought to be treated as a single observation. Entering all data from the

United States along with data from the three French elections held under d'Hondt in 1945 and 1946 biases the analysis in the direction of the consequences of electoral laws as they appear in the U.S. So, after defining an electoral regime to be a period of time in a country in which electoral rules - the seat allocation formula and the average number of seats to be filled in an electoral district (district magnitude) - are essentially constant, Lijphart takes regimes as the unit of analysis. The values of other variables, such as the number of parties, are then set equal to their average across all elections within the regime.

Lijphart's innovation has at least one theoretically compelling justification - "elections under the same rules are not really independent cases but merely repeated operations of the same electoral system" (p. 482). Specifically, the usual hypotheses about the relationships between the number of parties and electoral laws concern the properties of systems in equilibrium.[3] Indeed, it is here that we find the reason for excluding data from "unstable" democracies. All countries in the sample thus possess a common characteristic; that is, they all can be referred to as "stable" democracies. They all for a prolonged period of time manage to somehow aggregate the expressed preferences of their electorates and are capable of generating on this base outcomes that are stable. Beyond these vague notions there is little that we can say when introducing the common political features of countries in our sample. But even this flexible description has immediate implications. Namely, as we control for the property of political stability and sustainability of democratic regimes, a sample formed on this

[3] We refer here to the spatial modelling literature (c.f. Enelow and Hinich 1984) and to the literature on the equilibrium number of parties, beginning with Duverger (1954) through Cox (1984), Palfrey (1989), and Feddersen et al. (1990).

basis in *not suitable* for testing any sort of hypotheses regarding the *prerequisites of democratic stability*. Such hypotheses, therefore, are not a part of our agenda here. This remark may appear obvious, but it is needed, because it is often tempting to present the similarities found among the countries in thus controlled sample as the desirable conditions (institutional, cultural, economic or other) for the longevity of democratic government. In fact, we find a conclusion of this sort implied in an indirect form in Lijphart's "Democracies" (1984). Thus obtained collections of "preconditions" may or may not be relevant to the sustainability of democracy - while working with the sample biased toward stability, we have no scientific base behind claiming or even implying one or another. What we have the basis for saying is how the specific combinations of inputs (again, social and institutional) influence the form of an output (political) so that the latter remains suitable for stability.

Arguably, Lijphart's approach of taking regimes as observations brings us nearer an ideal of studying systems in their equilibrium states, when in empirical assessment we ought to focus on dependent variables such as "the equilibrium number of parties within a system." However, even if we restrict our attention to stable Western democracies, this approach can be criticized given the nature of the actual data. First, it introduces a bias that is opposite Rae's, because the data includes regimes containing a single election. Thus, Lijphart equates the weight of the first post-war (and presumably out-of-equilibrium) French and German elections to all post-war elections held in, for instance, Canada, the United States, or Australia. Second, although we may predict that a change in electoral law will change the number or strengths of parties, it is not the case that we predict that these changes are instantaneous.

Hence, the results of an election immediately following a change may tell us little about the consequences of that change. Third, averaging values of variables within a regime and taking these averages as the observations introduces a number of econometric problems, including artificially increasing R^2 and t-statistics.

There is no wholly satisfactory methodology to be offered as an alternative since there is no way to learn the "true" equilibrium number of parties in a regime that encompasses, say, two or three elections. Minimally, though, we can do two things. First, we can discard all one-election regimes (Sweden in 1948, Germany in 1949 and 1953, and Israel in 1949). Second, we can set all variables equal to their values in the last election of each regime. But because this approach can be criticized as well (in addition to "wasting" the data from all but the last election of a regime, if changes in electoral laws are endogenous, this last election might actually be an out-of-equilibrium event that signals change), we also consider Rae's original approach and Lijphart's. In this way we assess the extent to which our conclusions depend on our handling of the data.

These adjustments in the treatment of the data bear upon another issue - the endogeneity of electoral laws. First, we should not be surprised to learn that specific institutional arrangements are chosen because they make life more secure for existing parties and political elites. For example, if single-member districting reduces the incentives for parties to fracture in a multi-ethnic environment, then a system that begins somehow with a small number of parties will maintain and even strengthen that system in the face of increasing heterogeneity to the extent that political elites have control over the rules of a game in which they are successful players. Similarly, if a multiparty system experiences some exogenous shock that threatens to disrupt party structures, then whether that shock changes party structures or whether, through the actions of elites, it results in the change of electoral laws in a way that maintains the status quo will depend on things that we cannot specify here (c.f., Shamir 1985).

Ideally, we prefer to test a theory that specifies $[D^*(H),N^*(H)]$, where $D^*(H)$ is the equilibrium district magnitude (or any other parameter of an electoral system under investigation) implied by the equilibrium number of parties; $N^*(H)$, where $N^*(H)$ is a number impled by $D^*(H)$; and where H summarizes the permanent characteristics of a society (e.g., ethnic heterogeneity) that influences the relationship between D^* and N^*. Unfortunately, aside from those models that establish [1,2] and [2,2] as equilibria under plurality rule (c.f. Palfrey 1989, Feddersen et al. 1990, Cox 1984), we can offer only "reasonable arguments" that N^* increases as D^* increases. Thus, our analysis, like Rae's and Lijphart's, implicitly assumes that observed values of D correspond to equilibrium values and that changes in D are due to exogenous factors.

2.2.2 Dependent variable: counting parties

Although he considers several alternatives, Rae's primary dependent variable is party fractionalization, F, based on the Herfindahl-Hirschman concentration index and applied to national election returns for the lower house of parliament or legislature. Lijphart and Taagapera and Shugart (1989) calculate the "effective number of parties" by computing 1/(1-

F).

The use of this and similar indices stems from the belief that the analyst should not give equal weight to parties that receive, say, 60 versus 5 percent of the vote. Historically, attempts to measure "political stratification" generated a set of indices based on the parties' voting (or representation) weights. Most of these indices, contrary to the one used by Lijphart and Taagapera and Shugart, do not adjust for the actual number of parties, but rather attempt to reflect the level of political agreement (or disagreement) in the society:

i) Rice's (1949) index of group cohesion is derived from the split between two alternatives and equals the difference in their "social weight" multiplied by 100:

Rice index score = $\frac{|p_1 - p_2|}{N} * 100\%$, where p_i is the number of votes (or, alternatively, seats) of party *i*; *N* is the total number of votes (or seats).

ii) Anderson and Watts' (1965) index of relative cohesion is simply the weight of the group supporting the most popular alternative.

Index of relative cohesion = $\frac{p_{max}}{N}$.

iii) Jacobson's (1967) Voting concordance measure, defined like Rice's for only two alternatives, also counts "undecided" as a separate group,

Voting concordance =

$$\frac{\textit{Number of pairs abstaining} + \textit{Number of pairs for} + \textit{Number of pairs against}}{\textit{Total number of pairs}}.$$

iv) Lijphart's (1963) measure of pairwise solidarity is similar to Jacobson's index:

Solidarity =

$$\frac{0.5\,(\textit{Number of pairs where one abstains}) + \textit{Number of pairs for} + \textit{Number of pairs against}}{\textit{Total number of pairs}}$$

v) There also has been offered a number of "entropy" measures that in Rae's words "promise, at least, a high level of generality, although there seems to be some confusion about their interpretation." Here we can mention Galtung's (1967) measure of uncertainty, which varies from 0 to infinity and is sensitive to both the number of parties and their relative sizes:

$$\text{Uncertainty} = -\sum_{i=1}^{n}\left[\frac{p_i}{N} * \log_2\left(\frac{p_i}{N}\right)\right].$$

and Kesselman's (1966) multipartism measure (intuition behind which remains unknown):

$$\text{Multipartism} = antilog_e\left[-\sum_{i=1}^{n} \frac{p_i}{N} * \log_e\left(\frac{p_i}{N}\right)\right].$$

More about these early discussions is contained in Rae and Taylor (1970).

The problem with any fractionalization index, though, is that it obscures the motives and actions of voters and political elites so that it becomes difficult or impossible to discern the effect of institutional structure on these separate motives. For example, suppose that whatever theory we possess predicts (for a given institutional structure and distribution of preferences over policies) that four parties will compete and that each will receive, in equilibrium, an approximately equal vote share. Fractionalization indexes would then measure the extent to which some parties are less than "full fledged" owing to differences in, say, organizational talent. On the other hand, suppose, for some other set of issue preferences, we predict that these four parties continue to compete but that they secure unequal vote shares in equilibrium. A fractionalization index, applied in combination with the observed actual number of parties, may then provide the appropriate measure of the extent to which voter choice matches theoretical expectations. But if our theory focuses on the incentives of political elites to form and to maintain parties or if it seeks to uncover the interaction of voter and elite motives, a fractionalization index, if used alone, confuses matters by reporting a number less than four or even three. If we are interested in learning about the variation in voter preferences over a *given* menu of politicians or parties, fractionalization measures will be appropriate. However, unless we assume that all parties gain equal vote shares in equilibrium, application of a fractionalization index can mislead us about the incentives to organize parties.[4]

In addition, then, to employing "effective number of parties" as a dependent variable, we will also simply count the number of formally organized parties that secure more than one percent of the national vote or one or more seats in the lower house of the legislature.[5] A one-percent cutoff is arbitrary, but it does take us part way toward eliminating "parties" that are mere ephemeral protest movements. In addition, we count only those parties who satisfy this cutoff in two or more successive elections, which has the effect of eliminating those candidates

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[4]In chapter 4 we return to the discussion of this measure in a somewhat different context; when it is used to measure the number of "serious" individual candidates in a district based on the vote shares they receive.

[5]A one-percent cutoff reduces the extent to which our analysis depends on potential variations in the "other" category of vote tabulations. Note that this rule sometimes results in the number of parties with seats in parliament exceeding the number of parties receiving more than one percent of the vote.

or parties who may receive a significant proportion of the vote in one election but who do not sustain themselves as a separately organized party.

2.2.3 Time period

Although Rae and Lijphart restrict their attention to post-World War II elections, we should not suppose that the "laws" of electoral competition came into play only after 1945. Elections before World War I may have differed from what followed owing to changes in the franchise; but it is unlikely that anyone would argue that changes in the franchise have deeply altered the voter and politicians' responses to those incentives that rules of elections set for them. We see no reason to suppose that the 1935 elections in Britain, for example, are a less valid observation than, say, the German elections of 1949. Consequently, we add the Continental elections in the interval 1918-39 to our data, along with all elections beginning with 1918 that occurred in those countries that held elections throughout the war.[6] However, when analyzing matters using Lijphart's approach, we assume that 1939-40 (or the pre-war election closest to this date) marked the end of a regime for all countries regardless of the electoral formula that each employed after the war. Even if only because World War II lay in

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The source of electoral data prior to 1990 is Maskie and Rose, *The International Almanac of Electoral History*, 1991. Later electoral information comes from "Notes on Recent Election," *Electoral Studies,* 1988-1994. Sources of Indian electoral data are Butler and Prateek "The Indian Parliamentary Election of 1989," *Electoral Studies* (1990), 9:2, 147-150; Jalan Prateek "The Indian Parliamentary Election of 1991," *Electoral Studies* (1991), 10:4, 353-356; and *Elections Since 1945: A Worldwide Compendium*, 1989. The most recent (1994) electoral data on Sweden and Denmark come from *The Economist*, "The Left Consolidates," September 24, 1994, p. 51-52.

the future is 1918-1939 distinct from the post-World War II period. Finally, we also add post-War election data from Greece, Spain, Portugal, and Japan.

2.2.4 District magnitude

It is by now agreed in the comparative elections literature that the critical institutional variable influencing the formation and maintenance of parties is district magnitude - the number of legislative seats to be filled within an electoral district. The importance of magnitude derives, in part, from its influence on the vote quota a party must secure to ensure representation in parliament. Also, magnitude influences a system's proportionality, which also influences the incentives to form and maintain parties: "[district magnitude] affects the proportionality of PR more than do the various mathematical translation formulas ... [and in] this regard the rule of thumb is that the smaller the district, the lesser the proportionality and, conversely, the larger the district, the greater the proportionality" (Sartori 1986, p. 53).

Unfortunately, characterizing each country by a single measure of magnitude is difficult. Few countries with PR have multiple districts that are of uniform magnitude. Several countries also have at-large or adjustment seats designed to "correct for" the influence of district magnitudes and seat allocation formulas. The most extreme case is Germany, with 249 single-member districts and a single 249-member national "district." Owing to this variability, no single measure captures all relevant aspects of magnitude. Lijphart opts for a simple calculation based on average magnitude. But an average equates a country with N double-member districts to one that has $N/2$ single-member districts plus a single $N/2$-member "adjustment" or at-large district (e.g., Germany). These two situations do not yield equivalent incentives for party formation, because each can yield a different vote share threshold that parties must meet before they secure legislative representation and because each generates different incentives for voters to vote strategically.

In response to this and similar problems, Lijphart, who uses a simple categorical analysis, accommodates adjustment or at-large seats by moving a country with a "significant" number of such seats into the next larger category of average magnitude. Taagapera and Shugart (1989) offer a more extensive analysis of "effective" magnitude and offer a formal adjustment that seeks to accommodate the fact that legal thresholds or adjustment seats can override the strategic imperatives of a simple average. A justification for their approach, though, requires that: (1) all parties are national; (2) overall variations in magnitude within a country are not great; and (3) the number of parties, N, is approximately equal to district magnitude plus one, $D+1$. Assumptions (1) and (2) are also required to justify using average magnitude. Assumption (3), though, reveals that "effective magnitude" is itself an endogenously determined parameter that is function, in part, of a variable we are attempting to predict.

The essential problem, here, of course, is that the incentives to form and maintain parties are a complex function of national and district electoral laws, as well as of parliamentary structure. Hence, we know that except for the simplest systems, no single index or measure can summarize the imperatives of most existing electoral laws. Absent a theory that tells us how to convert a description of an electoral system so that we can enter that description into a statistical analysis, we will focus on Rae-Lijphart's calculation of magnitude based on averages. However, in appreciation of the issues they raise, we also examine Taagapera and Shugart's adjustment. Comparing the performance of these two measures tells us that additional refinements of the calculation of magnitude are likely to be profitable.

Finally, Sartori (1986, p. 67, n 15) argues reasonably that the relationship between proportionality and district magnitude and, by inference, between number or parties and magnitude, is curvilinear. That is, although we might predict that single-member districts imply 2-party systems, and that, say, 15-member districts might allow 4 or 5 parties, it is unreasonable to suppose that 120- or 150-member districts (Israel and the Netherlands) will generate 30 or 40 parties, *ceteris paribus*.[7]

Notice that, since ln(1)=0, letting the measure of social fractionalization enter our model as multiplied by ln(*D*) (as we do in the next section) is equivalent to assuming that heterogeneity is of no consequence in single-member district systems, whereas using *D* or its square root instead allows heterogeneity to "operate" there. Since ln(*D*) provides a considerably better fit, we can tentatively accept the hypothesis that single-member district systems, where successfully applied, suppress and even eliminate the potential divisive effects of ethnic heterogeneity (c.f. Horowitz 1990). Indeed, this finding is strong evidence in support of Taagapera and Shugart's (1989) restatement of Duverger's argument.

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In regressions that we do not report here for the lack of theoretic interest, we note that in choosing between *D*, and two alternative operationalizations of Sartori's argument - ln(*D*) and $D^{1/2}$ - ln(*D*) consistently performs best.

2.2.5 Measures of social diversity

The study of political institutions assumes that outcomes derive from strategic choices based on preferences and taken as responses to institutional constraints. We have discussed constraints (district magnitude) and outcomes (number of political parties). Turning to preferences, we begin by noting that Taagapera and Shugart (1989) summarize Duverger's argument with a conclusion similar to ours, namely that "(1) Plurality rule tends to reduce the number of parties ..., regardless of the number of issue dimensions ... (2) PR rules tend not to reduce the number of parties, if the number of issue dimensions favors the existence of many parties" (p. 65). However, the issue dimensions Taagapera and Shugart count, taken from Lijphart (1984), can be said to be endogenous to the political system. Since different electoral systems give political elites different incentives to seek out new salient issues, we cannot reject the supposition that Taagapera and Shugart's findings are due to the influence of electoral system on issues rather than the effect of issues on outcomes as mediated by electoral system.

What we require, then, is a measure of the exogenous determinants of those preferences that are relevant, *a priori*, to pressures to increase or decrease the number of political parties. In this respect, a key variable that is of evident contemporary concern is a society's ethnic heterogeneity. We need not review the innumerable essays that document the influence of ethnicity on politics. But, keeping in mind those instances in which political engineering must contend with ethnicity and ethnic conflict when attempting to implement stable democratic systems (c.f. Horowitz 1991), focusing on this characteristic of a society should help us ascertain how alternative electoral laws mediate the influence of ethnic heterogeneity.

There are, though, two issues that arise when incorporating ethnic heterogeneity into our analysis: its measurement and the structural form of its incorporation. First, with respect to measurement, we begin by reconsidering our discussion of fractionalization indices. Earlier, we argue against the application of such indices to election returns data, because it confuses the interdependent motives and actions of voters and political elites. But social heterogeneity (with the possible exception of religion) is not a product of individual choice - rather, it is better portrayed as an exogenously determined social state. And an especially convenient characterization of heterogeneity is the probability that two randomly chosen individuals are of the same ethnic group. Hence, if there are valid arguments that such indices measure anything, then they apply to ethnicity.[8] Thus, one indicator of ethnic heterogeneity is simply ethnic fractionalization, F, where F varies between 0 and 1 and denotes the ethnic (linguistic, religious) fractionalization of society (where 1, the upper limit of fractionalization, is approached when every individual is a member of a different group).

Notice that F admits two measures that can be entered into a regression analysis - F itself, and $H = 1/(1-F)$. This second variable, H, measures the "effective number of ethnic groups" in the same way as Lijphart calculates "effective number of parties" from Rae's fractionalization measure. However, in lieu of arguing whether F or H is more theoretically

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See also Lijphart (1977) - especially the discussion and citations in note 10, page 59 - and Rae and Taylor (1970) for additional discussion rationalizing the use of fractionalization indices in this context.

satisfying (we believe that H is the more appropriate calculation for the reasons that Lijphart and Taagapera and Shugart offer), we note simply that in the regressions reported in this essay, H uniformly provides better fits than does F.

Of course, as with most other things, no single index can serve as a wholly satisfactory measure of every aspect of social heterogeneity that we might think is relevant. For example, separate indexes for ethnic, religious, and linguistic heterogeneity might be employed in recognition of the fact that ethnic heterogeneity is but one potential dimension of social cleavage. Although we focus on ethnicity, because we have more confidence in its measurement and the resulting index of fractionalization, our problems do not end even if we employ separate indexes[9] (see discussion below for consideration of religious and linguistic heterogeneity).

First, separate indexes would not tell us whether these cleavages correlate. A society may have two ethnic and two religious groups, but anywhere from two to four distinct ethnic-religious clusters. Second, a fractionalization index cannot measure the salience of these cleavages, which can be endogenous to electoral laws (Lijphart 1977, Rabushka and Shepsle 1972). Finally, a fractionalization index ignores the important matter of territoriality. The particular problem is that ethnic, religious, and linguistic heterogeneity can operate differently

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Ethnicity data for elections and regimes initiated prior to 1971 are taken from *Atlas Narodov Mira*, Moscow, 1962; for regimes and elections after 1970 we use data from *Narodi Mira: Istoriko Etnographitcheskii Spravochnik*, Moscow, 1988 (note that both sets of data are collected by the same institute using the same methodology and thus are comparable; the 1962 data were collected in 12960 whereas the 1988 data were collected in 1985). Religious and linguistic heterogeneity is computed using data from a single source - *Encyclopedia Britannica*, 1980.

when groups are geographically separate than when all groups are mixed (c.f. Horowitz 1985, 1991). Territoriality allows for a heterogeneous society, but homogeneous election districts, and thereby can influence the incentives of parties to compete within a district and nationally. Thus, although Switzerland and the U.S. are both ethnically heterogeneous, the relative absence of territorial considerations in the U.S. as compared to the situation in Switzerland suggests that even if both countries adopted identical electoral laws, those laws would operate differently in each country.

If we were to identify theoretically appropriate measures of all potentially relevant variables and their functional relationships, we would soon exhaust our degrees of freedom. There are many more potential permutations of social and institutional structures than would exist in any data set. But our purpose here is not to ascertain precisely how ethnic heterogeneity influences party systems. Rather, we merely want to determine whether the influence of a single institutional variable, district magnitude, on the number of political parties is better described if we take a simple characterization of society's ethnic structure into account, with the understanding that there is considerable room for additional refinements in the conceptualization and measurement of variables. So suppose that we have an index, H, that we take to measure the effective number of ethnic groups. Our next question is how to enter this variable into the analysis. That is, if the number of political parties, N, is a function of H as well as of the log of district magnitude $\ln(D)$, then we must contemplate alternative functional forms. There are two primary choices. The first choice assumes that heterogeneity and $\ln(D)$ have independent effects modeled by the simple linear relationship

$$N = \alpha + \beta_1 \ln(D) + \beta_2 H. \tag{1}$$

This expression, then, is an implicit assumption of those essays that study electoral laws independent of social context or that study the effect of electoral laws alone (thereby implicitly relegating the influence of H to the error terms of their analyses).

The second possibility, implied by Lijphart's discussion of electoral laws (if not his empirical analysis of them) is the one that is more consonant with the hypothesis that heterogeneity's impact is mediated by electoral structure or, equivalently, that the operation of electoral structure depends on fixed social preconditions:

$$N = \alpha + \beta H \ln(D) \tag{2}$$

This second expression, then, models Taagapera and Shugart's (1989:65) revision of Duverger's hypothesis. However, rather than rely as they do on subjective counts of election issues - the salience of which are almost certainly endogenously determined - we operationalize H as a variable that cannot itself be influenced by electoral laws.

The final possibility, expression (3), combines these two models into one, which we consider for purposes of determining whether H or $\ln(D)$ has any independent effect on the number of parties after we have controlled for $H\ln(D)$.

$$N = \alpha + \beta_1 \ln(D) + \beta_2 H + \beta H \ln(D) \tag{3}$$

The analysis we report here ascertains which of these three functional forms best describes the data that are the focus of Rae and Lijphart's research and whether incorporating ethnic heterogeneity into the analysis contributes anything to our understanding of the consequences of electoral laws.

2.3 Analysis - Lijphart's By-Regime Approach

2.3.1 Reanalysis of Lijphart's data

Beginning with Lijphart's regime data, in Table 1 we present a series of regressions in which the dependent variable is the "effective number of parties" based on each party's share of the vote (*ENPV*) as calculated by Lijphart. Clearly, none of the results this table reports are statistically spectacular (equations 2, 3, and 5 are not statistically significant at a 5 percent level, equation 1 will be accepted at a 5 percent significance level, but will be rejected at 1 percent, and only equation 4 is significant at 1 percent level), but notice first, from regression 1 an additive specification like expression (1) leads to the conclusion that the effective number of ethnic groups has no influence on *ENPV* and that $\ln(D)$ provides whatever explanatory power is available in the two independent variables this study considers.

[Table 2.1 is about here]

The last two regressions in this table, though, show that this conclusion is erroneous. Specifically, the best overall fit is secured by assuming, in accordance with expression (2), that heterogeneity and district magnitude are interactive. Moreover, the comparison of regressions 4 and 5 shows that, at least when *ENPV* is our dependent variable, $H\ln(D)$ wholly absorbs any independent effect that $\ln(D)$ or H alone might have on the effective number of political parties. Moreover, comparing regressions 1 and 3, we see that although the coefficient on ln(D) in 1 is significant and nearly identical to that on Hln(D) in regression 3, multiplying ln(D) in regression 4 by the effective number of ethnic groups increases the fit of our model appreciably. Thus, *although the qualitative conclusion that district magnitude has an important influence on the number of parties does not change with how ln(D) is entered into a regression, the important fact is that its effect is best described by treating it as a variable that intervenes interactively between ethnic heterogeneity and the effective number of parties.*

2.3.2 Countries and time period

What we must now do is ascertain the robustness of this finding against various things, including: (1) the countries and election periods under consideration; (2) the method of counting the number of parties; and (3) alternative measures of district magnitude. First, then, accepting the possibility that averages across regimes admit of too many out-of-equilibrium elections, consider the first three regressions in Tables 2.2a and 2.2b, which parallel regressions 3-5 in Table 2.1, except that now we (1) extend the time period to 1994; (2) add the data from India, Japan, Greece, Spain, and Portugal; (3) delete those regimes that contain only one election; and (4) take only the last election in each regime as the observation corresponding to that regime. Although we are no longer averaging variables within regimes, statistical relationships are a bit stronger - at least with respect to regressions 4 and 5 as compared to 2 and 3.

[Tables 2.2a and 2.2b are about here]

2.3.3 Counting parties

Now let us consider other operationalizations of "number of parties." Table 2.2, then, reports the results of a series of regressions in which the effective number of parties is calculated using their relative share of votes (*ENPV*); here also are the results for dependent variables corresponding to a simple count of the number of parties that receive at least 1 percent of the vote (*NV*) or of the "historical" parties - parties with not less than 1 percent of the vote in two or more successive elections (*HNV*).

The regressions this table reports warrant at least three comments. First, *an interactive relation remains superior to a simple linear additive one regardless of our choice of dependent variable*. Thus, our conclusion about how heterogeneity ought to be entered into the analysis - as a variable that mediates the influence of district magnitude - is invariant with how we count parties. Second, the best overall fit occurs when we use a simple count (*NV* or *HNV*) rather than a calculation of "effective number." Our final comment concerns the explanatory power gained by adding H and $\ln(D)$ to $H\ln(D)$ - regressions run in accordance with expression (3). Although regression pairs (7, 8), (10, 11), (13, 14), and (16, 17), (19, 20), (22, 23) each provide essentially the same goodness of fit, a simple regression using $H\ln(D)$ does in fact perform best in the cases of both simple counts of parties and "historical" counts. An appeal to parsimony and the elimination of variables with insignificant coefficients, then, dictates the choice of the simplest regression, expression (2).[10]

Although we focus on ethnicity, because we have more confidence in its measurement and incorporation into a fractionalization index, other cleavages can have predictive power in party formation as well. As an example, using *ENPV* and *NV* as dependent variables, the following regressions parallel regression sets {15,16} and {18,19} except that language and religious fractionalization are used to calculate heterogeneity - H_l and H_r respectively. First, with respect to linguistic fractionalization,

$$ENPV = \underset{(2.6)}{2.12} + \underset{(2.4)}{.34\ln(D)} + \underset{(1.6)}{1.03H_l}; \qquad \text{adj. } R^2 = .14$$

$$ENPV = \underset{(13.4)}{3.47} + \underset{(2.4)}{.26H_l\ln(D)}; \qquad \text{adj. } R^2 = .12$$

$$NV = \underset{(2.5)}{2.41} + \underset{(6.2)}{1.23\ln(D)} + \underset{(2.2)}{1.64H_l}; \qquad \text{adj } R^2 = .43$$

$$NV = \underset{(12.8)}{4.73} + \underset{(7.5)}{.85H_l\ln(D)}; \qquad \text{adj. } R^2 = .45$$

Thus, linguistic heterogeneity generates results that are nearly equivalent to those generated by ethnic heterogeneity with both R^2 and the magnitude of coefficients being statistically similar. However, if we ignore the issue of whether the differences are statistically significant, ethnicity does provide the better fit. Next, with respect to religious heterogeneity,

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10 Note that when an Indian observation is included, ethnic heterogeneity measure acquires significance in the additive formulation.

$$ENPV = \underset{(7.6)}{3.69} + \underset{(2.4)}{.32\ln(D)} - \underset{(-0.7)}{.09H_r}; \qquad \text{adj. } R^2 = .06$$

$$ENPV = \underset{(14.8)}{3.72} + \underset{(2.1)}{.13H_r\ln(D)}; \qquad \text{adj. } R^2 = .06$$

$$NV = \underset{(7.0)}{5.09} + \underset{(5.5)}{1.18\ln(D)} - \underset{(-1.2)}{0.22H_r}; \qquad \text{adj. } R^2 = .37$$

$$NV = \underset{(13.1)}{5.65} + \underset{(3.0)}{.38H_r\ln(D)}; \qquad \text{adj. } R^2 = .22$$

So religion produces fits that are uniformly inferior to those of language and ethnicity - indeed, in a simple linear model (not reported here), the coefficient on religion has the wrong and insignificant sign. This finding, though, is not surprising if religion is subject to the inherent ambiguity of how people choose to report weak or non-existent affiliations.

2.3.4 District magnitude

Tables 2.2a and 2.2b give us confidence that our conclusion about the superiority of an interactive model is robust to manipulations in the countries and time period considered and to the operationalization of our dependent variable. But we should also consider alternative measures of district magnitude, since no single measure can capture all of the variation in election systems. Let us turn, then, to the measure suggested by Taagapera and Shugart (1989), which seeks to take more explicit account of vote thresholds that parties must exceed before securing legislative or parliamentary representation owing either to legally specified thresholds or to adjustment seats and at-large districts that move a system closer to proportionality.

Tables 2.3a and 2.3b reproduce the regressions in Tables 2.2a and 2.2b, except that now we replace the calculation of *D* based on a simple average with Taagapera and Shugart's calculations of "effective magnitude," D_e[11]. The comparison of Tables 2.2 and 2.3 shows that Taagapera and Shugart's measure does in fact perform slightly better than does *D*, the simple average (equivalent results arise if we compare regressions run using expression (3) as our model). However, since the fit that D_e provides with our primary dependent variables *NV* and *HNV* is very close to that of *D*, and since we are uncertain about theoretical justifications for measuring district magnitude the way Taagapera and Shugart suggest, we continue to use *D* in our reanalysis of Rae's approach in the next section. Nevertheless, Taagapera and Shugart's efforts at devising a measure that involves other institutional features in addition to actual district magnitude warrant closer attention, because additional refinements may generate additional payoff. We want to emphasize, though, that it follows from Tables 2.3a and 2.3b that our conclusion about the superiority of the interactive structure that models district magnitude as an intervening parameter does not depend on how we operationalize district magnitude. Aside from the differences in goodness of fit just noted, the qualitative patterns among our estimated coefficients are identical to those we report in Table 2.2a.

[Tables 2.3a and 2.3b are about here]

2.4 A Brief Reconsideration of Rae's Approach

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11 $D_e = 50\%/T$, where *T* is a legal threshold for legislative representation.

Before putting a "seal of approval" on the interactive model, we must consider two additional matters. First, we should consider Rae's election-by-election data in order to be certain that our conclusions do not depend on definitions of a regime or on regimes that survive for only a few elections. Second, noting that countries with single-member district procedures, on average, are more heterogeneous than are those with PR systems, we want to be certain that it is not the non-PR countries (Australia, the United States, the United Kingdom, New Zealand, Canada, and France for all but one election after 1958) that provide the sole source of explanatory power. Third, we want to resolve the puzzle of whether India's case follows the patterns described above, or whether Riker's (1982) claim that it constitutes an exception from the noted patterns of institutional influence on party systems can be statistically justified.

Turning first to Rae's approach of taking each election outcome as an independent observation, we offer in Tables 2.4a and 2.4b the relevant regressions, and once again a comparison of additive and interactive models for the alternative dependent variables. Perhaps the most important fact to be gleaned from these regressions is that our qualitative conclusion is once again sustained - with one exception in a data set that includes ten Indian observations, the interactive model performs better than a linear additive one. Moreover, neither ln(*D*) nor *H* is universally significant when included with *H*ln(*D*) (regressions 44, 47, 50, 53, 56, and 59), although we must note that whenever India is included in the data, ethnic diversity variable is always significant and always has the right sign.

[Tables 2.4a and 2.4b are about here]

Aside from the fact that R^2's are higher in Table 2.4 than in Table 2.2 (which is to be expected owing to the greater number of observations - 495 and 485 versus 53 and 52), the only difference of note is that in the combined (3) specification coefficients on *H*ln(*D*) are significant everywhere. However, when the interactive model is considered, the similarities between Tables 2.2, 2.3, and 2.4 in the magnitudes of coefficients are more remarkable. Looking at the coefficient on *H*ln(*D*), if we use Lijphart's regime approach and district magnitude averages, we get .34, .91, and .77 for each of the three dependent variables (regressions 7, 10, and 13), with "effective" district magnitudes and the by-regime approach, we get the values of .44, .93, and .82 (regressions 25, 28, and 31), whereas if we use election-by-election data we get .40, .95, and .85 (regressions 43, 46, and 49). Thus, once heterogeneity is appropriately factored into the analysis, there is no reason to modify Rae's original conclusions about the influence of district magnitude or to argue that district magnitude has a different influence on party systems when regimes rather than individual elections are taken as the unit of analysis.

2.5 PR Systems Only

The next issue we want to address concerns the extent to which our results are driven by the fact that the most heterogeneous states on average are those with single-member districts and with the fewest number of parties. Specifically, those countries with single-member district regimes, Australia, Canada, India, France, Great Britain, New Zealand, and the United States, are, on average, more heterogeneous than their proportional-representation counterparts (with an average ethnic homogeneity index of, for example, .52 versus .82 in the 1960 ethnic data and .46 versus .79 in the 1985 data) and are associated, on average, with lower values of *ENPV* (3.07 versus 4.68 for the most recent regime). The particular hypothesis we want to examine, then, is Taagapera and Shugart's (1989, p. 142) assertion that "the decisive question is not whether a particular system is plurality or PR, but what its effective magnitude is."

Focusing on *NV* alone since this is sufficient to establish our conclusions, we report in Table 2.5 the results of a series of regressions using expressions (1), (2), and (3) again as our models. The thing to notice first is that goodness of fit as measured by R^2 between the liner and interactive specifications (expressions (1) and (2)) narrows or disappears altogether so that the apparent advantage of the interactive model over the additive one disappears in PR systems. That is, contrary to Taagapera and Shugart's argument, the superiority of the interactive model appears to derive solely from the fact that single-member district states not only have fewer parties on average, but also are more heterogeneous than their PR counterparts.

[Table 2.5 is about here]

Nevertheless, there are reasons for preferring the hypothesis that district magnitude is best modeled as an intervening variable. Looking first at the intercept terms, if we use the interactive model, then the average absolute difference between the value of this term between the complete and partial data sets is .08 and .05 correspondingly when electoral regimes are considered and in the by-election data set; but if we use the additive model, this difference is 1.63 and 1.26 respectively. Similarly, while the coefficient on ln(*D*) when used in the additive model varies by .16 and .21 in the two data sets, the coefficient on *H*ln(*D*) varies by .02 and .05. In addition, the sign with which ethnicity variable enters additive specification is unstable across the equations. Thus, the interactive model offers estimated coefficients that are far less sensitive to the treatment of the data.

There are some things, however, that do not change when we delete single-member district systems from the sample. First, although we do not report the regressions here, better fits continue to be secured when parties are simply counted rather than computed on the basis of a fractionalization index. Thus, a simple count is not only the more behaviorally meaningful dependent variable, but it is also the more predictable measure of number of parties, regardless of whether we include single-member district systems. Second, the estimated coefficients for *H*ln(*D*) do not change dramatically between the by-regime and by-election data (.89 versus .93 in equations 61 and 64).

Overall, much of the evidence in favor of an interactive model does derive from the character of single-member district states. However, we cannot be altogether indifferent between models even if we restrict our attention to PR systems. Considerations like parsimony, the stability of coefficients, and the theoretical meaningfulness of estimated coefficients lead us to prefer estimations in the form of expression (2) over expressions (1) and (3). And although we can reason that the choice between single- and multi-member district systems is a qualitative one that entails other decisions such as the weight that ought to be given to achieving proportional representation in some form, analyzing the effects of district magnitude can proceed under Taagapera and Shugart's (1989) argument that single-member district systems are quantitatively but not qualitatively different form their multi-member district counterparts.

2.6 Exception from the General Pattern

To our sample of stable democracies India is a problematic addition. When advancing the claim that low district magnitude (*D*=1) neutralizes the impact of social divisions on party formation, we cannot but doubt its applicability in Indian case. India is in fact different from the rest of the countries in a low-magnitude subsample in that it is a much more heterogeneous society (even though, as we have remarked above, the countries employing districts of magnitude 1 tend to be less homogeneous). And, of course, in a full sample this contrast between India and the rest of the countries is even more striking. Along all three lines of divisions that we consider, ethnicity, language, and religion, Indian society is deeply fractionalized. An observer, however, would note that those may not even be the most politically influential divisions; a cast system peculiar to India plays role, too.

Not arguing here that fragmentation of society reaching an extent that it does in India impedes democratic stability, we merely offer with respect to India a hypothesis that there exists an extreme point of social fractionalization, beyond which we may not expect institutional provisions to succeed in blocking it out completely. For the purpose of this chapter it suffices to show that our preferred model does not explain the Indian case: interactive specification in conjunction with the logarithmic transformation of the district magnitude variable conceals the role played by social divisions in determining the Indian party system.

[Table 2.6 is about here]

Table 2.6 presents an indirect test of this proposition. It shows that when India is included into the set of countries with low district magnitude, ethnic variable matters, while once Indian observations are dropped, things change dramatically. This evidence once again points to the need for separate assessment of the democratic processes in extremely divided societies. In recognition of this possibility, all our results above were reported for the samples both with and without Indian observations.

2.7 Conclusion

There are many things this chapter does not consider, such as the influence of seat allocation formulas and ballot structure (Rae 1971, and Lijphart 1990), vote thresholds (Taagapera and Shugart 1989), the influence of presidential versus parliamentary systems (Jones 1992 a, b and Shugart and Carey 1992), and the nature of federal institutions and the territoriality of ethnicity (Horowitz 1991, Lijphart 1977, 1984). And, as we note earlier, we also do not consider district magnitude itself as an endogenously determined parameter chosen to achieve certain ends in the context of a particular environment (Shamir 1985). To the extent, then, that H influences D, our approach probably overstates district magnitude's mediating influence while it understates the role of ethnic heterogeneity.

Sorting out the interdependencies among social structure, electoral laws, and outcomes requires a firmer theoretical footing than is available. An empirical investigation uninformed by rigorously derived theoretical relationships is, with that data at hand, unlikely to yield definitive conclusions. Nevertheless, with this caveat in mind, we can conclude that any general theoretical model should accommodate the fact that if the effective number of ethnic groups is large, political systems become especially sensitive to district magnitude. But if ethnic fractionalization is low, then only especially large average district magnitudes result in any "wholesale" increase in formally organized parties. Finally, if district magnitude equals one, then the party system is relatively "impervious" to ethnic and linguistic heterogeneity (keeping in mind, of course, that this conclusion rests on data from one source: stable, economically prosperous Western democracies).

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Table 2.1: Dependent Variable = *ENPV*, Lijphart's Data (n = 32)*

reg. #	1	2	3	4	5
Const.	3.24 (12.5)	4.10 (13.2)	3.21 (7.29)	3.23 (13.8)	3.86 (7.67)
ln(*D*)	.34 (3.5)	-	.35 (3.31)	-	-0.45 (-1.42)
H	-	-.19 (-.87)	.01 (.07)	-	-0.33 (-1.44)
*H*ln(*D*)	-	-	-	.27 (4.43)	.55 (2.84)
adj. R^2	.17	.001	.14	.23	.22
SEE (mean)	(3.86)		.98	.92	.93

* Throughout this chapter, numbers in parentheses after estimated coefficients are t-statistics; numbers in parenthesis after SEE are means of the sample's dependent variable.

Table 2.2a: Alternative counting of parties (last election only, full data set, with India, n=53)

	ENPV			NV			HNV		
	6	7	8	9	10	11	12	13	14
Intercept	3.06 (8.0)	3.35 (12.9)	3.40 (8.6)	3.95 (6.8)	4.75 (11.5)	4.24 (6.9)	3.58 (7.2)	4.32 (11.7)	3.83 (7.2)
ln(D)	0.40 (2.8)	-	-0.68 (-1.3)	1.31 (5.8)	-	0.37 (0.6)	1.11 (6.0)	-	0.31 (0.5)
H	0.21 (1.9)	-	0.07 (0.9)	0.34 (2.6)	-	0.21 (1.6)	0.32 (2.3)	-	0.60 (1.3)
Hln(D)	-	0.34 (2.8)	0.80 (2.1)	-	0.91 (5.4)	0.70 (1.4)	-	0.77 (5.2)	0.59 (1.3)
adj. R^2	0.08	0.17	0.21	0.37	0.41	0.39	0.34	0.37	0.36
SEE (mean)	1.42 (4.09)	1.35	1.32	2.08 (6.74)	2.02	2.04	1.87 (6.00)	1.82	1.84
F	3.41	11.2	5.44	15.98	36.93	12.32	14.06	31.3	10.89

Table 2.2b: Alternative counting of parties when India is excluded from the sample (last election per regime only, n=52)

	ENPV			NV			HNV		
	15	16	17	18	19	20	21	22	23
Intercept	2.74 (4.1)	3.29 (12.3)	3.75 (6.9)	3.95 (4.5)	4.64 (11.0)	4.95 (6.4)	3.90 (4.9)	4.19 (11.5)	4.90 (8.0)
ln(D)	0.43 (2.7)	-	-0.84 (-1.5)	1.31 (5.5)	-	0.05 (0.1)	1.09 (5.6)	-	-0.16 (-0.3)
H	0.41 (1.1)	-	-0.14 (-0.7)	0.34 (0.8)	-	0.21 (0.8)	0.12 (0.3)	-	-0.41 (-2.0)
Hln(D)	-	0.35 (2.9)	0.90 (2.2)	-	0.94 (5.5)	0.89 (1.8)	-	0.80 (5.4)	0.88 (1.9)
adj. R^2	0.09	0.18	0.21	0.37	0.42	0.40	0.34	0.40	0.38
SEE (mean)	1.42 (4.08)	1.35	1.32	2.10 (6.73)	2.01	2.05	1.89 (5.98)	1.80	1.82
F	3.66	12.5	5.62	15.66	37.72	12.57	14.39	34.8	11.59

Table 2.3a: "Effective" district magnitude as a predictor of a number of parties (n=53)

	ENPV			NV			HNV		
	24	25	26	27	28	29	30	31	32
Intercept	2.58 (7.0)	2.96 (11.6)	2.79 (9.1)	3.57 (5.8)	4.33 (10.5)	3.82 (6.2)	3.01 (5.3)	3.89 (9.7)	3.20 (5.4)
$\ln(D_e)$	0.50 (4.2)	-	-0.54 (-1.5)	1.20 (5.2)	-	-0.08 (-0.1)	1.11 (5.8)	-	0.15 (0.3)
H	0.30 (2.2)	-	0.16 (2.8)	0.43 (3.0)	-	0.25 (1.9)	0.44 (3.3)	-	0.31 (2.1)
$H\ln(D_e)$	-	0.44 (3.8)	0.84 (2.7)	-	0.93 (6.0)	1.03 (2.7)	-	0.82 (5.9)	0.78 (2.2)
adj. R^2	0.15	0.29	0.36	0.32	0.42	0.41	0.35	0.42	0.42
SEE (mean)	1.36 (4.09)	1.25	1.19	2.16 (6.74)	1.99	2.01	1.86 (6.00)	1.76	1.76
F	5.49	21.86	10.4	12.89	38.47	13.36	14.68	38.5	13.36

Table 2.3b: Role of "effective" district magnitude when India is excluded from the sample (n=52)

	ENPV			NV			HNV		
	33	34	35	36	37	38	39	40	41
Intercept	1.96 (2.6)	2.87 (11.2)	2.98 (6.3)	3.24 (3.2)	4.17 (10.3)	4.63 (5.4)	3.01 (3.1)	3.70 (9.7)	4.14 (5.4)
$\ln(D_e)$	0.56 (3.9)	-	-0.61 (-0.2)	1.23 (4.9)	-	-0.35 (-0.6)	1.11 (5.2)	-	-0.17 (0.4)
H	0.65 (1.6)	-	0.05 (0.3)	0.61 (1.3)	-	0.20 (-0.6)	0.44 (0.9)	-	-0.22 (-0.9)
$H\ln(D_e)$	-	0.46 (4.0)	0.88 (2.8)	-	0.97 (6.3)	1.19 (3.0)	-	0.87 (6.4)	0.96 (2.7)
adj. R^2	0.18	0.31	0.36	0.32	0.44	0.42	0.35	0.45	0.43
SEE (mean)	1.35 (4.09)	1.24	1.20	2.18 (6.73)	1.98	2.01	1.88 (5.98)	1.72	1.75
F	6.51	23.53	10.7	13.19	40.91	13.63	14.39	42.6	14.19

Table 2.4a: Election-by-election data, full sample (n=495)

	ENPV			NV			HNV		
	42	43	44	45	46	47	48	49	50
Intercept	2.65 (26.6)	2.96 (49.7)	2.77 (30.2)	2.94 (15.3)	4.13 (32.2)	3.13 (15.6)	3.05 (18.3)	3.91 (35.9)	3.25 (18.7)
ln(D)	0.55 (14.8)	-	0.04 (0.3)	1.41 (18.8)	-	0.64 (3.0)	1.22 (20.3)	-	0.37 (1.8)
H	0.15 (5.7)	-	0.09 (3.6)	0.53 (8.6)	-	0.44 (6.4)	0.39 (6.9)	-	0.29 (4.7)
Hln(D)	-	0.40 (13.3)	0.39 (3.6)	-	0.95 (17.2)	0.58 (3.6)	-	0.85 (18.6)	0.64 (4.1)
adj. R^2	0.30	0.34	0.34	0.46	0.44	0.48	0.42	0.43	0.45
SEE (mean)	1.05 (3.74)	1.02	1.02	1.93 (5.97)	1.96	1.89	1.82 (5.56)	1.80	1.77
F	110.5	254	88.1	209.6	387.3	151.1	178.1	372	133.9

Table 2.4b: Election-by-election data when India is excluded, (n=485)

	ENPV			NV			HNV		
	51	52	53	54	55	56	57	58	59
Intercept	2.67 (17.9)	2.91 (48.6)	3.12 (25.0)	3.33 (12.3)	3.94 (33.5)	4.14 (17.0)	3.30 (13.6)	3.77 (36.7)	4.12 (20.3)
ln(D)	0.55 (13.5)	-	-0.12 (-0.8)	1.37 (17.3)	-	0.20 (0.9)	1.19 (18.7)	-	-0.003 (-0.01)
H	0.14 (1.9)	-	-0.11 (-2.2)	0.30 (2.5)	-	-0.14 (-1.5)	0.25 (2.2)	-	-0.21 (-2.6)
Hln(D)	-	0.41 (13.6)	0.48 (4.4)	-	1.00 (18.6)	0.85 (5.3)	-	0.88 (19.7)	0.87 (5.6)
adj. R^2	0.30	0.36	0.36	0.46	0.49	0.49	0.42	0.46	0.46
SEE (mean)	1.06 (3.73)	1.02	1.02	1.93 (5.92)	1.87	1.86	1.82 (5.53)	1.75	1.75
F	108.3	271.2	90.2	205.3	464.1	160.3	174.5	411	142.2

Table 2.5: PR systems only, Dependent Variable = NV

	Regime data, n=40			Election-by-election data, n=341		
	60	61	62	63	64	65
Intercept	2.32 (1.9)	4.83 (8.0)	-2.10 (-0.6)	2.07 (4.8)	4.19 (19.0)	2.67 (1.6)
ln(D)	1.15 (4.0)	-	3.31 (1.9)	1.20 (11.2)	-	0.90 (1.1)
H	2.00 (2.0)	-	5.51 (1.7)	1.65 (5.2)	-	1.16 (0.8)
Hln(D)	-	0.89 (4.1)	-1.72 (-1.3)	-	0.93 (11.8)	0.25 (0.3)
adj. R^2	0.32	0.32	0.33	0.36	0.35	0.35
SEE (mean)	2.00 (7.40)	2.01	2	1.95 (6.82)	1.96	1.95
F	10.41	18.72	7.35	95.1	190.7	63.2

Table 2.6: "Effective" number of ethnic groups in determining the number of political parties; countries with D=1 only

	ENPV		NV		HNV	
	w/o India, n=144	with India, n=154	n=144	n=154	n=144	n=154
Intercept	2.88 (16.4)	2.52 (22.4)	4.10 (12.3)	2.83 (11.5)	3.88 (13.8)	2.87 (13.3)
H	-0.05 (-0.8)	0.13 (5.0)	-0.14 (-1.2)	0.50 (6.9)	-0.14 (-1.5)	0.36 (5.6)
R^2	2.8×10^{-3}	0.09	4.9×10^{-3}	0.24	8.4×10^{-3}	0.18
SEE (mean)	0.81 (2.78)	0.81 (2.86)	1.62 (3.81)	1.72 (4.10)	1.40 (3.58)	1.48 (3.80)
F	0.4	15	0.7	48	1.2	33.4

APPENDIX: Elections in Twenty-Five Countries

	YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
Australia:					
	1919	0.69	2.43	3	3
	1922	0.69	2.95	4	3
	1925	0.69	2.53	3	3
	1928	0.69	2.74	4	3
	1929	0.69	2.72	3	3
	1931	0.69	3.55	4	4
	1934	0.69	4.11	6	5
	1937	0.69	3.02	4	4
	1940	0.69	3.56	5	3
	1943	0.69	3.09	5	4
	1946	0.69	2.74	6	4
	1949	0.69	2.64	3	3
	1951	0.69	2.49	3	3
	1954	0.69	2.46	4	4
	1955	0.69	2.73	5	5
	1958	0.69	2.97	5	5
	1961	0.69	2.81	5	5
	1963	0.69	2.79	4	4
	1966	0.69	2.98	4	4
	1969	0.69	2.83	4	4
	1972	0.56	2.77	5	5
	1974	0.56	2.66	5	5
	1975	0.56	2.69	4	4
	1977	0.56	3.11	5	5
	1980	0.56	2.81	4	4
	1983	0.56	2.67	4	4
	1984	0.56	2.79	4	4
	1987	0.56	2.90	4	4
	1990	0.56	3.32	5	5
	1993	0.56	2.85	5	5
Austria:					
	1919	0.88	3.03	4	3
	1920	0.88	3.10	5	5
	1923	0.88	2.79	5	4
	1927	0.88	2.41	3	3
	1930	0.88	3.15	5	3
	1945	0.88	2.22	3	3
	1949	0.88	2.78	4	4
	1953	0.88	2.76	4	4
	1956	0.88	2.48	4	4
	1959	0.88	2.48	4	4
	1962	0.88	2.47	4	4
	1966	0.88	2.39	4	3
	1970	0.85	2.29	3	3
	1971	0.85	2.28	4	4

	YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
	1975	0.85	2.27	4	4
	1979	0.85	2.27	3	3
	1983	0.85	2.40	5	3
	1986	0.85	2.72	4	3
	1990	0.85	3.15	4	4
Belgium:					
	1919	0.46	3.33	6	6
	1921	0.46	3.42	6	6
	1925	0.46	3.23	6	6
	1929	0.46	3.48	6	6
	1932	0.46	3.22	5	5
	1936	0.46	4.59	7	6
	1939	0.46	4.17	6	6
	1946	0.46	3.28	6	4
	1949	0.46	3.26	5	4
	1950	0.46	2.76	5	5
	1954	0.46	3.08	6	6
	1958	0.46	2.79	6	6
	1961	0.46	3.08	5	5
	1965	0.46	3.98	6	6
	1968	0.46	4.21	6	6
	1971	0.41	4.60	6	6
	1974	0.41	6.11	9	9
	1977	0.41	5.70	10	9
	1978	0.41	7.53	11	11
	1981	0.41	8.99	14	14
	1985	0.41	8.13	13	13
	1987	0.41	8.12	11	11
	1991	0.41	9.79	13	11
Canada:					
	1921	0.25	3.21	4	4
	1925	0.25	2.60	4	4
	1926	0.25	2.38	4	4
	1930	0.25	2.25	3	3
	1935	0.25	3.27	5	4
	1940	0.25	2.69	4	4
	1945	0.25	3.65	6	4
	1949	0.25	2.83	4	4
	1953	0.25	2.85	5	4
	1957	0.25	2.98	4	4
	1958	0.25	2.44	4	4
	1962	0.25	3.23	4	4
	1963	0.25	3.20	4	4
	1965	0.25	3.31	5	5
	1968	0.25	2.97	4	4
	1972	0.23	3.25	4	4

YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
1974	0.23	2.96	4	4
1979	0.23	3.09	4	4
1980	0.23	2.93	4	4
1984	0.23	2.75	3	3
1988	0.23	3.04	4	3
1993	0.23	3.92	5	3
Denmark:				
1920	0.97	3.75	5	5
1924	0.97	3.75	4	4
1926	0.97	3.65	5	5
1929	0.97	3.40	5	5
1932	0.97	3.47	6	6
1935	0.97	3.49	7	7
1939	0.97	3.85	8	8
1943	0.97	3.50	8	8
1945	0.97	4.56	7	7
1947	0.97	3.80	7	7
1950	0.97	4.01	6	6
1953	0.97	3.92	6	6
1953	0.97	3.80	7	7
1957	0.97	3.90	7	7
1960	0.97	3.81	8	8
1964	0.97	3.75	8	8
1966	0.97	4.22	7	7
1968	0.97	4.56	7	7
1971	0.94	4.52	9	9
1973	0.94	7.11	11	11
1975	0.94	5.60	11	11
1977	0.94	5.23	11	11
1979	0.94	4.99	11	11
1981	0.94	5.75	11	11
1984	0.94	5.25	10	10
1987	0.94	5.83	11	11
1988	0.94	5.83	10	10
1990	0.94	4.84	10	10
1994	0.94	4.76	9	9
Finland:				
1919	0.84	4.18	6	6
1922	0.84	5.46	6	6
1924	0.84	5.14	6	6
1927	0.84	5.08	6	6
1929	0.84	5.02	7	7
1930	0.84	4.18	6	6
1933	0.84	4.23	6	6
1936	0.84	4.28	7	7
1939	0.84	4.05	7	7

	YEAR	ETHNIC HOMOGENEITY	ENPV	NV	HNV
	1945	0.84	5.09	7	7
	1948	0.84	4.90	6	6
	1951	0.84	4.96	6	6
	1954	0.84	4.98	6	6
	1958	0.84	5.19	7	7
	1962	0.84	5.86	8	7
	1966	0.84	5.22	7	7
	1970	0.87	6.17	9	9
	1972	0.87	5.95	8	8
	1975	0.87	5.89	10	9
	1979	0.87	5.74	9	9
	1983	0.87	5.45	8	8
	1987	0.87	6.15	10	8
	1991	0.87	5.90	8	8
France:					
	1945	0.75	4.61	5	5
	1946	0.75	4.51	5	5
	1946	0.75	4.65	6	6
	1951	0.75	5.40	6	6
	1956	0.75	6.08	8	8
	1958	0.75	6.08	9	8
	1962	0.75	4.92	8	8
	1967	0.75	4.55	7	7
	1968	0.75	4.32	7	7
	1973	0.68	5.68	9	7
	1978	0.68	5.08	10	8
	1981	0.68	4.14	7	7
	1986	0.68	4.66	8	8
	1988	0.68	4.38	7	7
	1993	0.68	6.73	9	6
Germany:					
	1919	0.98	4.29	6	6
	1920	0.98	6.73	9	9
	1924	0.98	6.51	10	10
	1924	0.98	7.58	12	11
	1928	0.98	6.66	12	10
	1930	0.98	7.28	11	10
	1932	0.98	4.40	7	7
	1932	0.98	4.94	8	7
	1933	0.98	3.83	7	7
	1949	0.98	5.60	10	9
	1953	0.98	4.20	10	9
	1957	0.98	3.58	6	6
	1961	0.98	3.50	6	5
	1965	0.98	3.15	6	6
	1969	0.98	3.02	5	5

	YEAR	ETHNIC HOMOGENEITY	ENPV	NV	HNV
	1972	0.86	2.86	4	4
	1976	0.86	2.91	4	4
	1980	0.86	3.10	5	5
	1983	0.86	3.21	5	5
	1987	0.86	3.56	5	5
	1990	0.86	3.65	6	5
Greece:					
	1974	0.91	2.74	5	4
	1977	0.91	3.74	7	4
	1981	0.91	2.68	5	4
	1985	0.91	2.59	4	4
	1989	0.91	2.73	3	3
	1989.1	0.91	2.60	3	3
	1990	0.91	2.36	4	3
	1993	0.91	2.63	5	4
Iceland:					
	1916	0.99	4.56	7	6
	1919	0.99	3.05	4	4
	1922	0.99	4.28	5	4
	1923	0.99	2.59	3	3
	1926	0.99	3.58	5	4
	1927	0.99	3.23	4	4
	1930	0.99	2.68	3	3
	1931	0.99	2.88	4	4
	1933	0.99	3.03	4	4
	1934	0.99	3.55	5	5
	1937	0.99	3.58	5	5
	1942	0.99	3.54	5	5
	1942	0.99	3.65	5	5
	1946	0.99	3.58	4	4
	1949	0.99	3.56	4	4
	1953	0.99	4.16	6	5
	1956	0.99	3.62	5	5
	1959	0.99	3.40	5	5
	1959	0.99	3.66	5	5
	1963	0.99	3.37	4	4
	1967	0.99	3.77	6	5
	1971	0.97	4.10	6	5
	1974	0.97	3.47	5	5
	1978	0.97	4.20	5	5
	1979	0.97	3.89	4	4
	1983	0.97	4.26	6	5
	1987	0.97	5.77	9	5
	1991	0.97	4.22	7	7

	YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
Ireland:					
	1922	0.96	3.84	4	3
	1923	0.96	3.77	4	4
	1927	0.96	5.15	6	6
	1927	0.96	3.42	6	5
	1932	0.96	2.94	4	4
	1933	0.96	2.82	4	4
	1937	0.96	2.90	3	3
	1938	0.96	2.55	3	3
	1943	0.96	3.67	4	4
	1944	0.96	3.24	5	5
	1948	0.96	3.99	6	6
	1951	0.96	3.27	5	5
	1954	0.96	3.21	5	5
	1957	0.96	3.12	6	6
	1961	0.96	3.20	6	6
	1965	0.96	2.72	3	3
	1969	0.96	2.82	3	3
	1973	0.97	2.80	4	4
	1977	0.97	2.73	4	4
	1981	0.97	2.85	5	4
	1982	0.97	2.68	4	4
	1982	0.97	2.71	4	4
	1987	0.97	3.46	6	6
	1989	0.97	3.36	7	6
Israel:					
	1949	0.81	5.37	11	8
	1951	0.81	5.10	13	9
	1955	0.81	6.31	10	10
	1959	0.81	5.15	10	10
	1961	0.81	5.49	10	9
	1965	0.81	4.91	12	11
	1969	0.81	3.63	12	10
	1973	0.71	3.83	9	8
	1977	0.71	5.04	13	8
	1981	0.71	3.59	9	8
	1984	0.71	4.28	15	10
	1988	0.71	5.02	15	9
	1992	0.71	4.90	11	9
Italy:					
	1946	0.96	4.67	8	6
	1948	0.96	2.94	7	7
	1953	0.96	4.17	8	8
	1958	0.96	3.87	9	8
	1963	0.96	4.15	8	8
	1968	0.96	3.95	8	7

	YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
	1972	0.89	4.07	8	8
	1976	0.89	3.50	10	9
	1979	0.89	3.90	9	9
	1983	0.89	4.51	10	9
	1987	0.89	4.61	10	9
	1992	0.89	6.56	13	8
Japan:					
	1946	0.98	4.32	5	5
	1947	0.98	4.49	5	5
	1949	0.98	3.88	7	5
	1952	0.98	3.41	6	5
	1953	0.98	4.36	6	5
	1955	0.98	3.99	5	4
	1958	0.98	2.23	3	3
	1960	0.98	2.40	4	4
	1963	0.98	2.55	4	4
	1967	0.98	3.03	5	5
	1969	0.98	3.36	5	5
	1972	0.99	3.40	5	5
	1976	0.99	4.01	6	6
	1979	0.99	3.76	6	6
	1980	0.99	3.44	6	6
	1983	0.99	3.63	6	6
	1986	0.99	3.35	6	6
	1990	0.99	3.39	5	5
	1993	0.99	5.13	8	5
Luxembourg:					
	1919	0.94	3.01	6	0
	1925	0.94	4.17	7	4
	1928	0.94	2.62	4	4
	1931	0.94	3.44	5	4
	1934	0.94	3.48	5	4
	1937	0.94	3.35	7	4
	1945	0.94	3.34	6	5
	1948	0.94	3.24	4	4
	1951	0.94	2.98	4	4
	1954	0.94	3.00	5	4
	1959	0.94	3.26	4	4
	1964	0.94	3.50	5	4
	1968	0.94	3.49	4	4
	1974	0.57	4.26	5	5
	1979	0.57	4.16	7	6
	1984	0.57	3.56	6	6
	1989	0.57	4.65	9	5

	YEAR	ETHNIC HOMOGENEITY	ENPV	NV	HNV
The Netherlands:					
	1918	0.91	5.83	10	9
	1922	0.91	5.80	9	9
	1925	0.91	5.80	10	9
	1929	0.91	5.56	10	8
	1933	0.91	6.18	11	8
	1937	0.91	5.79	10	8
	1946	0.91	4.68	7	7
	1948	0.91	4.98	8	8
	1952	0.91	4.99	8	8
	1956	0.91	4.26	7	7
	1959	0.91	4.46	8	8
	1963	0.91	4.79	9	9
	1967	0.91	6.20	10	10
	1971	0.66	7.09	14	13
	1972	0.66	6.85	13	13
	1977	0.66	3.96	7	7
	1981	0.66	4.56	9	9
	1982	0.66	4.23	9	9
	1986	0.66	3.77	7	7
	1989	0.66	3.90	8	7
New Zealand:					
	1919	0.63	3.54	3	3
	1922	0.63	3.42	3	3
	1925	0.63	2.98	3	3
	1928	0.63	3.52	4	4
	1931	0.63	2.33	3	3
	1935	0.63	2.97	4	3
	1938	0.63	2.10	2	2
	1943	0.63	2.41	3	2
	1946	0.63	2.01	2	2
	1949	0.63	2.03	2	2
	1951	0.63	2.00	2	2
	1954	0.63	2.48	3	3
	1957	0.63	2.31	3	3
	1960	0.63	2.37	3	3
	1963	0.63	2.39	3	3
	1966	0.63	2.61	3	3
	1969	0.63	2.45	3	3
	1972	0.48	2.43	4	4
	1975	0.48	2.55	4	4
	1978	0.48	2.87	4	4
	1981	0.48	2.89	3	3
	1984	0.48	2.99	4	3
	1987	0.48	2.34	3	2
	1990	0.48	2.68	3	3
	1993	0.48	3.52	4	3

	YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
Norway:					
	1921	0.96	4.49	6	6
	1924	0.96	4.90	7	7
	1927	0.96	4.05	7	7
	1930	0.96	4.15	6	6
	1933	0.96	3.98	8	8
	1936	0.96	3.75	8	8
	1945	0.96	4.12	6	6
	1949	0.96	3.76	7	6
	1953	0.96	3.53	6	6
	1957	0.96	3.44	7	7
	1961	0.96	3.58	8	8
	1965	0.96	3.90	8	8
	1969	0.96	3.61	7	7
	1973	0.94	5.17	9	8
	1977	0.94	3.85	8	8
	1981	0.94	3.90	8	8
	1985	0.94	3.63	7	7
	1989	0.94	4.84	7	7
	1993	0.94	4.72	8	7
Portugal:					
	1975	0.99	3.66	7	4
	1976	0.99	4.00	5	5
	1979	0.99	3.01	5	4
	1980	0.99	2.89	5	4
	1983	0.99	3.73	4	4
	1985	0.99	4.77	6	5
	1987	0.99	2.98	5	5
	1991	0.99	2.79	4	4
Spain:					
	1977	0.54	4.29	8	5
	1979	0.54	4.25	9	6
	1982	0.54	3.18	7	7
	1986	0.54	3.59	8	6
	1989	0.54	4.02	7	7
	1993	0.54	3.44	5	5
Sweden:					
	1917	0.93	4.10	6	6
	1920	0.93	4.40	6	6
	1921	0.93	4.04	6	6
	1924	0.93	3.72	7	6
	1928	0.93	3.88	6	6
	1932	0.93	3.81	7	6
	1936	0.93	3.54	6	5

YEAR	ETHNIC HOMOGENEITY	ENPV	NV	HNV
1940	0.93	2.84	5	5
1944	0.93	3.48	5	5
1948	0.93	3.35	5	5
1952	0.93	3.28	5	5
1956	0.93	3.38	5	5
1958	0.93	3.31	5	5
1960	0.93	3.25	5	5
1964	0.93	3.42	7	7
1968	0.93	3.18	7	7
1970	0.86	3.48	6	6
1973	0.86	3.50	6	6
1976	0.86	3.58	6	6
1979	0.86	3.63	6	6
1982	0.86	3.39	7	7
1985	0.86	3.52	6	6
1988	0.86	3.91	7	6
1991	0.86	4.58	8	7
1994	0.86	3.64	8	8
Switzerland:				
1917	0.50	3.41	5	5
1919	0.50	4.79	7	7
1922	0.50	4.83	8	8
1925	0.50	4.68	7	7
1928	0.50	4.49	7	7
1931	0.50	4.44	7	7
1935	0.50	5.20	11	10
1939	0.50	6.00	10	10
1943	0.50	5.14	8	8
1947	0.50	5.33	8	8
1951	0.50	5.09	8	8
1955	0.50	4.99	9	9
1959	0.50	5.03	9	9
1963	0.50	5.00	9	9
1967	0.50	5.54	9	9
1971	0.41	6.07	10	10
1975	0.41	5.79	10	10
1979	0.41	5.50	10	10
1983	0.41	5.99	10	10
1987	0.41	6.80	12	10
1991	0.41	7.03	10	10
United Kingdom:				
1918	0.68	4.07	7	5
1922	0.68	3.54	4	4
1923	0.68	3.06	3	3
1924	0.68	2.75	3	3
1929	0.68	2.95	3	3

YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
1931	0.68	2.51	6	5
1935	0.68	2.62	5	5
1945	0.68	2.67	4	4
1950	0.68	2.44	4	4
1951	0.68	2.13	3	3
1955	0.68	2.16	3	3
1959	0.68	2.28	3	3
1964	0.68	2.53	3	3
1966	0.68	2.42	3	3
1970	0.61	2.46	4	4
1974	0.61	3.13	5	5
1974	0.61	3.15	5	5
1979	0.61	2.87	5	5
1983	0.61	3.46	6	6
1987	0.61	3.33	6	6
1992	0.61	3.06	5	5
United States:				
1920	0.50	2.14	4	4
1922	0.50	2.14	4	4
1924	0.50	2.11	3	3
1926	0.50	2.04	3	3
1928	0.50	2.03	2	2
1930	0.50	2.10	3	2
1932	0.50	2.12	3	3
1934	0.50	2.15	4	4
1936	0.50	2.11	4	3
1938	0.50	2.14	2	2
1940	0.50	2.09	2	2
1942	0.50	2.09	2	2
1944	0.50	2.03	2	2
1946	0.50	2.04	2	2
1948	0.50	2.06	2	2
1950	0.50	2.06	2	2
1952	0.50	2.04	2	2
1954	0.50	2.01	2	2
1956	0.50	2.01	2	2
1958	0.50	1.98	2	2
1960	0.50	2.01	2	2
1962	0.50	2.01	2	2
1964	0.50	1.97	2	2
1966	0.50	2.03	2	2
1968	0.50	2.05	2	2
1970	0.43	2.04	2	2
1972	0.43	2.04	2	2
1974	0.43	2.00	2	2
1976	0.43	2.02	2	2
1978	0.43	2.04	2	2

YEAR	ETHNIC HOMOGE-NEITY	ENPV	NV	HNV
1980	0.43	2.06	2	2
1982	0.43	2.02	2	2
1984	0.43	2.03	2	2
1986	0.43	2.01	2	2
1988	0.43	2.03	2	2
1990	0.43	1.99	2	2
1992	0.43	1.99	2	2

India:

1952	0.11	3.89	9	9
1957	0.11	3.44	6	5
1962	0.11	4.12	8	6
1967	0.11	4.71	10	7
1971	0.11	4.31	10	9
1977	0.11	3.18	7	5
1980	0.11	4.15	9	7
1984	0.11	3.70	10	7
1989	0.11	4.29	8	8
1991	0.11	4.45	7	7

CHAPTER 3: MULTICANDIDATE COMPETITION WITH ENTRY IN PLURALITY ELECTIONS

3.1 General Problem of Electoral Competition with Entry

Political engineering in the form of designing democratic institutions is a growth industry, even among states that have a reasonably long history of democratic government (e.g., Italy, Israel). However, newly introduced forces of democratic competition can exacerbate the internal problems of some states, as when elections give rise to ethnic parties that find it in their interest to raise the salience of divisive issues. But even those who disagree about specifics agree that institutional rules matter and that some rules are less problematical than others, depending on circumstances (see, for example, Lijphart 1977 and 1984, Linz 1990, Riker 1982, Horowitz 1991, Shugart and Carey 1992). And as long as the designing process occurs, we must strive to find those institutional arrangements that have the best chance of preventing conflict from wholly destabilizing a political system.

As we and others argue elsewhere (Shvetsova and Filippov 1993, Horowitz 1985), the period prior to the formation of a coherent political party structure is the most dangerous stage of democratic development, although once formed, the character and function of political parties becomes relatively predictable and stable. But whether a stable democratic equilibrium is achieved at all depends on many things and the chief among them is the state's election laws and ethnic composition (see Chapter 2 here, and also: Ordeshook and

Shvetsova 1993, Horowitz 1991, Mainwaring 1993). Thus, although we cannot model this dynamic process of party formation, the design of an electoral system requires that we understand better how election laws help generate a political party system of a specific type. In particular, of interest is the question of the principal ability of a specific electoral procedure to yield stable and predictable outcomes - in other words, to yield an equilibrium.

To this end, this chapter seeks to understand the nature of election competition fostered by a specific type of election system - the single non-transferable vote (SNTV) - that is used today in parliamentary elections in Taiwan, and had been used in Japan in the legislative elections since 1947. Our interest in this system is motivated by an evident empirical regularity that warrants theoretical explanation. Specifically, the analysis of Japanese elections to the lower House undertaken by Steven Reed (1990) indicated the eventual emergence of $k+1$ "serious" candidates in each of its k-member electoral districts. Cox (1993) identifies Reed's findings as the "extension" of Duverger's Law (1954) to plurality elections in multi-candidate electoral districts. By looking at the district-level electoral results, Reed concludes that

- in equilibrium, only $k+1$ candidates compete in the Japanese SNTV system;
- over time the vote shares of these $k+1$ candidates tend towards uniformity;
- although there is some evidence of strategic voter behavior, the process whereby this equilibrium is achieved appears to depend primarily on the ability of candidate (parties) to coordinate their actions; and
- convergence to this equilibrium is slow, at least in Japan.

The approach we take with respect to Reed's findings are both theoretical and empirical. The theoretical part (the current chapter) is devoted to studying the equilibrium properties of the SNTV system, when only the voters preferences are predetermined, but positions and the number of competing candidates are endogenous. It shows that in addition to k+1 equilibria, there may also exist k-equilibria, depending on the specific form of the distribution of preferences. And for the special case of a uniform distribution of preferences, equilibria exist for any number of candidates greater than or equal to k. Also, we see that, except for the special case of a uniform preference distribution, in equilibrium successful candidates should obtain uneven shares of the total vote. The reanalysis of Reed's data as well as data from Taiwan that we provide in the next part of this work (Chapter 4), moreover, shows that candidates achieve an equilibrium configuration relatively quickly - at least more quickly than Reed suggests - and that if the support of candidates becomes more uniform over time, this is due largely to the actions of voters rather than to the positioning of the candidates on the issues.

Ours is not the first theoretical analysis of SNTV.[1] Cox (1993), motivated specifically by Reed's data, focuses on the analysis of the properties of the equilibria which SNTV yields. He offers a model of SNTV in which any number of candidates greater than k can compete and he approaches the problem from the standpoint of strategic voters rather than

[1]Denzau, Katz and Slutsky (1985) analyze multi-candidate elections under various rules and candidate objectives that admit SNTV as a special case. But, in addition to assuming that preferences are uniformly distributed and that all voters are sincere, they are precluded from addressing Reed's empirical findings by the assumption that the number of candidates is determined exogenously.

strategic candidates. Admittedly, his treatment has the advantage of not imposing any specific assumptions about the distribution of voter preferences (aside from assuming that there is at least one voter of any type) and of not restricting the issue space to any specific number of dimensions. On the other hand, a treatment that does not impose some geometry on the set of alternatives, such as a spatial representation of issues does, cannot allow the candidates any strategic role. Thus, Cox assumes simply that there is a given number of candidates (some number greater than k) with fixed policy positions. Assuming that voters begin with a common knowledge assessment of the electoral prospects of each candidate, he then establishes that the rational expectations equilibrium has the first k candidates receiving equal vote shares and the remaining candidates receiving declining vote shares. In contrast to Cox's, our approach is to take voters as sincere, with single-peaked preferences over a one-dimensional policy space, and to take candidates as strategic players who chose whether to enter the race or not, and if they enter, then at what policy platform.

Our primary objective in analyzing the candidate competition under plurality in the multi-seat districts is to find out whether the very number of competitors in the race can be formalized as an endogenous parameter determined by the preference structure and the institutional arrangements. Another goal being, of course, to determine what the policy stands of these competing candidates are. Also, we would want to know how competitive the race is, in the sense that within the same district some candidates may have "safer" seats than the others, and how the strains vary that competition imposes on different candidates. These multiple tasks would presuppose the consideration of a very complex game with either sequential or simultaneous entry of a potentially large number of candidates. The corresponding problems of coordination that would derive from the static formulation, or the enormously complex belief structure that a dynamic statement of the problem would yield, all threaten to make our multicandidate multiseat competition problem thus defined virtually intractable. And the more potential entrants we allow in the game, the greater the difficulties are. It is intuitive to suppose that the number of equilibria will also be going up at the increasing rate as the mere artifact of our adding more players.

Thus, the game is threatening to become excessively complex. Also the controversy exists whereas the formalization of *endogenous* entry requires first an arbitrary fixing of the number of players so that only a limited number of existing players will actually enter, but then it so happens that the equilibrium number of active participants depends on the total number of potential players that we allowed. By changing the number of players, we might change the set of equilibria of the game, even though the added players may never enter the race. Adding some more, we might be able to upset some of the previously found equilibria. Hence, the equilibrium number of candidates who enter the competition is influenced by a factor which is *badly exogenous*, i.e., is non-institutional and is not determined by the preferences of the voters. Therefore, for as long as we have a finite number of players preset before the game, many equilibria will be but the artifacts of this number, while others may be reasonable and will persist regardless of the number of non-entering players (provided, of course, that equilibria exist at all).

Here we do not aspire to find all possible equilibria of the candidate game of entry for some large number of candidates. Instead, we are looking for the "reasonable" equilibria. And for the sake of convenience and tractability, we decide as "reasonable" the equilibria that occur when there is just one new competitor that can enter the race. The "trick" that we employ in analyzing this problem is that in order to find the endogenously determined number of candidates we choose between the games. We are looking for an equilibrium not within a single game, but across a class of games - for an equilibrium game, so to say.

We analyze a class of games where in every game there are n players already in the race, who now must assume policy positions (either simultaneously, or sequentially, again; but we prefer to think of it in sequential terms, while solving it as a static game). And there also is an $n+1^{st}$ candidate who decides whether to enter the race or not, and if to enter, then at what policy platform. By equilibrium, then, we understand a vector of the policy positions of the n candidates, such that

- all n candidates have a positive probability of winning seats;
- each candidate's position is a best response to the positions of the other candidates (even without the threat of entry, which implies the subgame perfection);
- a new, $n+1^{st}$ candidate does not enter, i.e., by entering he cannot obtain a positive probability of getting a seat.

We, then, are to determine, for which n's (if any) such equilibria exist. Eventually, it can be shown that the equilibria that we find in this context remain ones out of potentially many equilibria to the games with *any* larger number of players. Other parameters for which we are to establish the existence of equilibria are the number of seats in a district (clearly, equilibrium n cannot be below this number), and the structure of voter preferences.

By equilibrium in a broader sense then, we understand an index n of the game Γ_n, where the number of players is $n+1$, and the equilibrium (equilibria) as described above exist, in conjunction with a vector of n candidates' spatial locations that solves Γ_n. This problem as we stated it is, in fact, equivalent to taking a game of sequential entry, where the assumption is made that each new entrant is myopic and enters only if he gets a seat with some positive probability *immediately* after he enters, and cannot rely on the future entrants in providing him with a hope of winning. Such an assumption rules out the entrants who, like Japan's Communist Party, often use the race as an opportunity for political propaganda, rather than to get into the Parliament. When assuming individual electoral gains for the entrant, we also prohibit anyone from entering the race merely to hurt some otherwise successful candidate. In this definition an equilibrium of an election game in a district with k seats to be filled is described by (1) the number of candidates entering the competition, and (2) the vector of spatial positions that those candidates assume. It is in this context that we show first that if more than one candidate can occupy the same policy position, then regardless of district magnitude, k, and the distribution of preferences (as long as there are no mass points), a k-equilibrium never exists, and that when $f(x)$ is strictly quasi-concave there does not exist any equilibrium number of competing candidates.

We then modify this structure by supposing that candidates must be spatially separated by some minimal distance. When this assumption is introduced, we establish the existence of the k and k+1 equilibria. More specifically, for a broad class of electoral distributions, we describe some of the necessary conditions for the existence of an equilibrium when entry is allowed, whereas for unimodal distributions we identify the form of that equilibrium. Briefly,

- for unimodal (quasi-concave) distributions of voter ideal points, if an equilibrium exists, it can only be a k or a k+1 equilibrium;
- a k+1 equilibrium necessarily exists for all symmetric unimodal distributions;
- a k equilibrium exists for all symmetric and convex preference distributions and values of k of 1, 2, and 3;
- uniform distributions yield equilibria with any number of candidates if not less than k; and
- unless voters preferences are distributed uniformly, the vote shares of the k leading candidates do not have to be equal or similar, but the difference must be narrow for the k^{th} and the $k+1^{st}$ candidates.

The rest of this chapter is structured as follows. In Section 2 we offer some basic notation and outline the model's essential structure. In Section 3 we analyze the properties that an equilibrium must satisfy when candidates are allowed to occupy the same spatial position, and we use these properties to show that no such equilibrium is possible. In Section 4 we assume that candidates cannot get spatially closer than some minimal distance δ, whereupon we establish the existence of various equilibria, depending on the distribution of preferences. Finally, in Section 5 we offer some testable propositions about competition in SNTV systems that we test through a reanalysis of Reed's data and data from Taiwan in Chapter 4 of this work.

3.2 The Basic Model

Among the systems of proportional representation, SNTV is one that allows a relatively unambiguous definition of candidate objectives. The goal of a candidate is to win a seat, and for that he must score above the k+1st best competitor. Aside from this, objectives such as maximization of vote share or rank are irrelevant considerations. And since under SNTV there are but three possibilities for a candidate - (1) he receives a seat with certainty; (2) he receives a seat with some positive probability less than one; and (3) his probability of being elected is zero - we assume that candidates maximize the probability of winning a seat.

That we focus on individual candidates rather than political parties is not merely a matter of convenience. In theory, the treatment of parties as strategic players should be different from the treatment of candidates. Some studies show (Balinski and Young 1982) that with many parties, no voting system guarantees that an increase in a party's share of the vote would not lead to a decrease of its share of legislative seats. Thus, a candidate's probability of winning a seat may not always be monotone with his party's vote share. One can still suppose that when elections are conducted in a large (such as national) district, there is no conflict in the objectives of party leaders and party's candidates once the party list has been compiled, and it is reasonable to assume that both maximize vote share, although some constraints may apply, as Chapter 5 will show. However, within a local electoral district it is often the case that the number of seats subject to allocation is comparable to the number of parties represented in the race. Therefore, it might as well be party strategy to promote few individual candidates within the district, even if the system is not SNTV. It may be rational, hence, for parties maximizing the sum of seats won across all districts, to assume the objective function of its candidates running within the districts. Of course, as long as the ballot structure requires voters to choose between parties, not candidates, we can at best look for similarities between the parties' and candidates' incentives within the district, but cannot substitute candidates for parties as active players in the formulation of the model.

Things are different under the plurality system, which SNTV in fact is. We clearly cannot suppose that a party can maximize its share of seats by merely maximizing the summed vote share of its candidates (Cox and Niou 1993). A party's candidates can split the vote in a district in ways that preclude any of them from winning a seat. Whereas if they coordinate their support or if fewer of them run, several of them might secure seats even if their smaller number decreases the party's overall share of the vote. Under SNTV, then, a natural harmony of interests between party leaders and potential candidates puts the interests of political candidates first.

What strengthens the role of individual candidates even more is that SNTV, where used, is an election system that operates exclusively at the local level, without the at-large district or allocation by remainders. There is, then, a strong local component to legislative elections in both Taiwan and Japan in which candidates are the key players and parties are merely the non-binding coordinating entities that negotiate on behalf of the candidates before elections and facilitate legislative structure afterwards. Indeed, in Taiwan, for example, it is not unusual to find members of the Kuomintang running in an election against other KMT members without official party sanction. Thus, our analysis assumes that under SNTV the key strategic players are candidates rather than parties. We emphasize, however, that this focus does not preclude us from discussing the role of parties in SNTV systems. In particular, considerable coordination may be required to preclude entry or to otherwise prevent an unintended supply of candidates. In fact, the model that we suggest here rationalizes the presence of *cooperation* between the "parties" at the district level, that is somewhat typical of the Japanese political system.

Turning now to more formal matters, we begin by assuming that candidates locate themselves somewhere on a one-dimensional policy space, $\mathbb{R}$, and that voters have single-peaked preferences in this space corresponding to the usual Euclidean distance model (Enelow and Hinich 1984) so that the distribution of voters ideal points, $f(x)$, is continuous and contains no mass points. The electoral rule is plurality in k-member districts (SNTV). Thus, the k candidates with the largest vote shares each win a seat, and ties are broken by coin tosses. Insofar as candidate motives are concerned, we assume that each candidate, actual or potential, maximizes his or her probability of winning a seat. In contrast to Cox (1993), we assume that voters vote sincerely for the candidate closest to their ideal (tossing coins if indifferent), but that candidates are strategic in their selection of policy platforms. Entry is allowed, but a new candidate enters only if he can secure a non-zero probability of winning a seat by his/her entry alone. Existing candidates must choose their positions under the threat that new opponents might enter the contest.

Developing this structure further requires some additional notation. Briefly, we let

C a finite set of candidates, where $i, j, ..., v \in C$;

x_i — the position of candidate i in the policy space;

$\mathbf{x}$ — the vector of candidate positions, $(x_1, x_2, ..., x_v)$;

n_i — the number of candidates at the policy position of candidate i;

$l_{x_i} = l_i = \frac{|x_i + x_{i-1}|}{2}$ — the left-most ideal point of voters who most prefer candidate i;

$r_{x_i} = r_i = \frac{|x_i + x_{i+1}|}{2}$ — the right-most ideal point of voters who most prefer candidate i;

$S_{x_i} = \int_{l_i}^{r_i} f(x)\,dx = F(r_i) - F(l_i)$ — the proportion of voters who most prefer x_i. Also, let

$S^L_{X_i}$ — be the proportion of voters to the left of x_i who most prefer x_i and $S^R_{X_i} = S_{x_i} - S^L_{X_i}$ (since $f(x)$ has no mass points, we need not concern ourselves with anyone who might most prefer x_i).

$S_i = \frac{1}{n_i} S_{x_i}$ — candidate i's share of the vote;

$s_i = s_{x_i} = [l_{x_i}; r_{x_i}]$ — the interval of policy positions that are closer to x_i than any other candidate position;

$P_i(\mathbf{x})$ — candidate i's probability of winning a seat, where $P_i = P_j$ if $x_i = x_j$. Note that $P_i > P_j$ only if $S_i > S_j$.

Finally, an equilibrium (**x**,C) to the election game with k seats at stake is a v-element vector **x** and a set C of v candidates such that

a. no new candidate can enter and, ceteris paribus, secure a non-zero probability of winning a seat; and

b. no candidate $i \in C$ can unilaterally alter his position and increase P_i; and

c. $P_i >$ for all candidates in C.

3.3 The Model When Two or More Candidates Can Occupy the Same Policy Platform

We can now prove several lemmata that characterize the properties, in terms of number and location of candidates, that any equilibrium (**x**,C) must possess when two or more candidates may share the same policy platform. We emphasize at the outset, however, that we offer these lemmata to establish that, unless we impose some additional restrictions on the candidate's positions (specifically, unless we preclude the possibility of two or more candidates adopting the same policy platform), no equilibrium exists. Admittedly, there may be easier routes to prove non-existence, but the method we offer here paves the way to establishing existence when candidates are constrained to adopt distinct positions. First, then,

Lemma 1: *If (**x**,C) is an equilibrium, then,*

1. *P_i equals either 1, or 0, or α ($0 < \alpha < 1$) for all i in C (if both P_i and P_j are not equal to 1, but greater than 0, then both are equal to α, where α is some positive number less than one).*

2. *If $n_i \geq 2$, then $P_i < 1$ (a candidate can receive a seat with certainty only if he or she stands alone at a policy position).*

3. *If $P_i > 0$ then $n_i \leq 2$ (not more than two candidates will occupy the same policy position and still have a chance to win).*

4. *If $n_i = 2$ and $P_i > 0$, then $s^L_{x_i} = s^R_{x_i}$.*

5. *$n_i = 2$ only if for all σ such that $\hat{x}_i = x_i + \sigma \in [x_{i-1}; x_{i+1}]$, and $\sigma > 0$*

$$\int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx < 2\int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx,$$

and for $\sigma < 0$

$$\int_{l_{x_i}}^{l_{\hat{x}_i}} f(x)\,dx < 2\int_{x_i+\frac{\sigma}{2}}^{x_i} f(x)\,dx.$$

6. *For all x_i and x_j, if $n_i = n_j = 2$ and P_i, $P_j > 0$, then $s_{x_i} = s_{x_j}$ (paired candidates everywhere have support of the same size).*

7. *If $n_i = 1$ and $P_i < 1$, then for all σ such that $\hat{x}_i = x_i + \sigma \in [x_{i-1}; x_{i+1}]$, and $\sigma >$ 0*

$$\int_{l_{x_i}}^{l_{\hat{x}_i}} f(x)\,dx \geq \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx,$$

and for $\sigma < 0$

$$\int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\,dx \le \int_{r_{\hat{x}_i}}^{r_{x_i}} f(x)\,dx.$$

8. *If* $n_i = 1$ *and* $P_i = 1$, *and if there exists a candidate j in C such that* $P_j = \alpha$, *then*

$$S_j < S_{x_i} < 2S_j \text{ and } S_{x_i}^L < S_j;\ S_{x_i}^R < S_j.$$

9. *No two adjacent policy positions* x_i *and* x_j *in* $\mathbf{x}$ *can be located so that there exists an interval [a, b] in the interval* $[x_i, x_j]$ *such that*

$$|a - b| \le |x_i - x_j|/2$$

and

$$\int_a^b f(x)\,d\hat{x} \ge S_{\min},$$

where S_{min} *is the vote share of a candidate with the lowest positive probability of winning a seat.*

The proof of Lemma 1 is in the Appendix.

At this point it is perhaps useful to say something about ties. Unlike models in which voters are strategic (Cox 1993, Palfrey 1989, Myerson and Weber 1989), a tie here is not a "knife-edged" case involving a single voter. We are not concerned with ties as an electoral outcome but with the candidate's estimates of their electoral prospects. Even if candidates possess perfect knowledge about preferences and candidate positions, a candidate's electoral support remains a random variable subject to determination by such things as variations in turnout (which we do not model) and vote counting errors. In this event a tie in a candidate's calculations becomes a robust possibility.[2]

Our next lemma establishes a restriction on the location of the paired candidates in the case, when more than one seat is allocated through the tie (i.e., five candidates are tied for three seats), and Lemma 3 says that if the distribution of preferences is unimodal, the tie can occur for not more than just one seat out of *k*.

Lemma 2: *If (**x**,C) is an equilibrium, if* $|P_i = 1| < k - 1$ *(if a tie occurs for more than one seat), and if there is an* x_i *in* **x** *such that* $n_i = 2$ *(say candidates i and j), then for all* σ *such that* $\hat{x}_i = x_i + \sigma \in [x_{i-1}; x_{i+1}]$, , *if* $\sigma > 0$

$$\int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx \le \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx,$$

and for $\sigma < 0$

$$\int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\,dx \le \int_{x_i+\frac{\sigma}{2}}^{x_i} f(x)\,dx.$$

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[2] And derived later from the model, this possibility should also be consciously reinforced by the actions of the strategic candidates themselves. Namely, in order to prevent further entry, "safe" candidates are interested in maintaining competition somewhere along the policy space. Because they do not care so much about keeping their excess votes, "safe" candidates can afford to maintain proper distances from "weak" candidates, in order to preserve or generate a tie between them for the sake of a mutually beneficial no-entry balance.

Proof: If more than one seat is allocated by breaking ties, then at least three candidates must be tied. Since, from Lemma 1.2, candidates *i* and *j* win a seat with probability less than 1 and since, from Lemma 1.1, all tied candidates win a seat with the same probability, α, then there exists a third candidate, say *t*, such that $P_i = P_j = P_t$. So the candidate who increases his share of the vote wins a seat for certain. So, by deviating from x_i, candidate *i* secures the vote share

$$S_{\hat{x}_i} = S_{x_i} - \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx + \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx.$$

Hence, if the conditions of the lemma are not satisfied, a deviation increases i's vote share. Although *j*'s share may also increase, *t*'s cannot increase, so *i* wins a seat for certain, and (**x**,*C*) cannot be an equilibrium. QED

Lemma 3: *If the distribution of preferences, f(x), is strictly quasi-concave, then in equilibrium, at most one seat can be allocated through a tie, i.e.,*

$$|P_i = 1| \geq k - 1.$$

Proof: Suppose to the contrary that $|P_i = 1| < k - 1$. Then,

a. from Lemma 2, for all x_i in **x** such that $n_i = 2$, the mode of *f*(*x*), m(f), is in s_{x_i}. Hence, there does not exist an x_j such that $x_j \neq x_i$, $n_j = n_i = 2$. That is, there can be only one set of paired candidates.

b. for all x_j in **x** such that $n_i = 1$ and $0 < P_i < 1$, it follows from the single-peakedness of $f(x)$ and Lemma 1.7 that $m(f) \in s_{x_i}$. Hence, there does not exist an x_j in **x** such that $x_j \neq x_i,\ n_j = n_i = 1,$. That is, there can be at most one candidate in C who does not receive a seat with certainty.

c. From (a) and (b) it follows that

$$\{\exists x_i \in x \ s.t.\ n_i = 1,\ 0 < P_i < 1\} \Rightarrow \{\nexists x_j \in \mathbf{x} \ s.t.\ x_j \neq x_i,\ n_j \leq 2,\ 0 < P_j < 1\}.$$

Hence,

1. there does not exist an $x_i \in$ **x** such that $n_i = 1$, $0 < P_i < 1$,
2. if there is an $i \in C$ such that $0 < P_i < 1$, then there must be two such candidates who are paired at x_i,
3. (1) and (2) together imply that $|P_i = 1| \geq k - 1$, as at most one seat can be allocated through a tie. QED

It follows immediately from this lemma that,

Corollary 1: *If (x,C) is an equilibrium, f(x) is strictly quasi-concave, then* $|C| \leq k+1$, *as by Lemma 3, at most one seat can be allocated through the tie, and at most two candidates can be tied for it.*

Lemmata 1 through 3 now allow us to establish a non-existence result when no restrictions are placed on the candidates spatial positions (i.e., they may share positions).

Proposition 1: *Under the assumptions of the model, for any distribution of voters' ideal points and any k,* $|P_i=1| < k$, *i.e., no* ***k*-*equilibrium exists.***

Proof: Suppose to the contrary that $|P_i=1| = k$. But then for any established candidate $i \in C$, $P_j = \frac{1}{2} > 0$, where $j \notin C$, $x_i = x_j$. Hence entry will occur, and *(**x**, C)* cannot be an equilibrium. QED

Proposition 1, though, does not rule out the possibility of equilibria with more than *k* candidates. To that end, suppose *f(x)* is quasi-concave (unimodal). Then,

Proposition 2: *If the distribution of voters' ideal points is quasi-concave, then under the assumptions of the model no equilibrium (**x**, C) exists.*

Proof: By Proposition 1, $|P_i=1| < k, \Rightarrow \exists i \in C,\ s.t.\ 0 < P_i < 1$. By Lemma 3 - for *f(x)* quasi-concave, $|0 < P_i < 1| = 2$, while both tied candidates are located in the same position. Hence, in equilibrium there cannot be other than *k+1* candidates, *k-1* of which receive seats with certainty, and two others (suppose, candidates *i* and *j*) are tied for a single seat. By Lemma 1.4. $S_{x_i}^L = S_{x_i}^R$. Hence

$$\forall\delta>0,\ \exists\sigma\geq\delta,\ s.t.\ S_{x_i+\sigma}\geq S_{x_i}^{R}-\int_{x_i}^{x_i+\frac{\sigma}{2}}f(x)\,d(x)>\frac{1}{2}\{S_{x_i}^{L}+\int_{x_i}^{x_i+\frac{\sigma}{2}}f(x)\,dx\}.$$

In other words, by hurting at once both candidates *i* and *j*, the entrant receives more than what is left to each of the tied candidates. So such an entrant receives a seat with certainty, which implies that entry occurs and *(x,C)* is not an equilibrium. QED

3.4 The Model When Candidates Must Maintain Some Minimal Separation

The nonexistence result for the unimodal case presented above is driven by the assumption that two or more candidates can run on indistinguishable platforms. Instead, we may want to introduce a minimal distance, δ, that must separate any two candidates in the policy space.[3] This additional assumption can only contribute to the realism of the model, as it merely requires that any two candidates be distinguishable in voters' eyes. All results below hold for any δ, no matter how small. Lemma 4, then, is similar to Lemma 1, in that it lists conditions that must hold in equilibrium, if it exists.

Lemma 4: *If no two candidates can adopt spatial positions that are closer than δ, and if (x,C) is an equilibrium for any small δ, then,*

1. *Lemma 1.1 holds.*

[3] Provided that nowhere in the distribution is $s=2\delta$ associated with positive probability of winning, which for some δ is always true by the continuity of the distribution and finiteness of an integral.

2. *If* $|P_i=1| = k$, *i.e., if all candidates receive seats with certainty, then*

$$\forall i,\ S^L_{x_i} + [F(x_i+\frac{\delta}{2}) - F(x_i)] > S^R_{x_i} - [F(x_i+\frac{\delta}{2}) - F(x_i)]$$

and

$$S^R_{x_i} + [F(x_i) - F(x_i-\frac{\delta}{2})] > S^L_{x_i} - [F(x_i) - F(x_i-\frac{\delta}{2})]\ .$$

That is, no candidate *i* can be located further from the median of his support m_i, than $\frac{\delta}{2}$. (Suppose not. Then new candidate *j* can either win a seat, or tie with candidate *i* for it, if *j* enters at $x_i+\delta$.)

3. *Lemma 1.7 holds, when modified as follows: If there exists candidate* ***i****, such that* $P_i<1$, *then for*

$\forall\sigma\ \ s.t.\ \ \hat{x}_i = x_i+\sigma \in [x_{i-1}+\delta\,;x_{i+1}-\delta]$, *either*

$\int_{l_{x_i}}^{l_{\hat{x}_i}} f(x)\,dx \geq \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx$, *or* $\int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\,dx \leq \int_{r_{\hat{x}_i}}^{r_{x_i}} f(x)\,dx$.

4. *Lemma 1.8 holds.*

5. *Lemma 1.9 holds.*

Note that Lemma 4.2 hints at the possibility of a *k*-equilibrium. We want to explore this possibility further here. No general result with respect to a *k*-equilibrium's existence or non-existence has been established so far. Its existence depends not only on the form of the distribution of voters' ideal points and the size of the district (*k*), but in certain cases - on the size of δ as well (equilibrium may disappear with the decrease of the δ parameter, while existing for its greater values). As the rest of our discussion here is aimed to establishing the results for δ arbitrarily small, we present only one particular case of *k*-equilibrium existence, which fits this requirement:

Proposition 3: *For any concave and symmetric distribution of voters preferences when* ***k****=1, 2, or 3, k-equilibria exist for any small* δ.

Proof: In order to prove the result for any small δ, we need to show that all *k* candidates can be positioned exactly at their respective means, and that none of them gets a twice smaller share of the vote than another (by L.4.2, and L.1.9). Because the cases of *k*=1 and *k*=2 are trivial, consider *k*=3 and locate the middle candidate x_2 at the mode of the distribution. Choose l_2, such that

$$\int_{x_1}^{l_2} f(x)\,dx = \int_0^{x_1} f(x)\,dx, \quad |l_2 - x_1| = |x_2 - l_2|$$

Note that such an l_2 always exists. Now we want to show that $s_1^R > \frac{1}{2} s_2^L$ in order to assure that the middle candidate has less than twice more votes than the candidate on the left. If we show this for the linear slope, it must hold for any other slope of a concave distribution. The vote share of the first candidate on his left by construction is equal to that on his right, i.e.,

$$a*b = \frac{1}{2}(a+\Delta_a)*(b+\Delta_b) \quad i.e. \quad a*b = a*\Delta_a + b*\Delta_b + \Delta_a*\Delta_b,$$

where
$$a = x_1,\ b = f(x_1)$$

$$a+\Delta_a = l_2,\ b+\Delta_b = f(l_2)\ .$$

Thus, as we have to show that $s_1^L < 2s_2^L$, and we know that

$$s_2^L = (a+2\Delta_a)*(b+2\Delta_b) - 2a*b.$$

Hence,

$$s_2^L = a*b + 2\Delta_a*\Delta_b < 2a*b.$$

QED

The following proposition shows that for the special case of a uniform distribution of voter ideal points, for all values of *k*, equilibria exist for any number of candidates greater than or equal to *k*.

Proposition 4: *If* $f(x) \sim U[0,1]$, *then* $\forall k\ \exists$ *equilibria* ***(x**,C) such that* $|C| \geq k$.

Proof: We prove the existence simply by presenting the examples of equilibria for all cases describing $|C| \geq k$. Specifically,

1) for $|C| > 2k$ odd, let $x_i = \frac{2i-1}{2|C|}\ \forall i \in C,\ x_i \in x$;

2) for $|C| \geq 2k$ even, either let $x_i = \frac{2i-1}{2|C|}$,

or let $x_{\{i|odd\}} = \frac{2i-1}{2|C|} - \frac{\delta}{2}$, and $x_{\{i|even\}} = \frac{i}{|C|} + \frac{\delta}{2}$;

3) for $2k > |C| > k$ any combination of certain and tied candidates is possible, including the arrangement described in (1).

Lemma 5: *If (x,C) is an equilibrium, if f(x) is strictly quasi-concave, and if there exists a candidate* $i \in C$, *such that* $0 < P_i = \alpha < 1$, *then* $\exists j \ s.t. \ |x_i - x_j| = \delta, P_j = P_i = \alpha$ *and* $argmax \, f(x) \in \{s_i \cup s_j\}$.

Proof: By Lemma 4.3, if such a candidate *i* exists, it must either be located so that $argmax \, f(x) \in s_{x_i}$, or be blocked from above by another candidate, say *i+1*, such that $|x_{i+1} - x_i| = \delta$. As at least two candidates must have a probability of winning a seat less than one, then at least one such candidate must be "blocked" from above.

If a candidate *i* is blocked from above, he must be blocked by *i+1* such that $0 < P_{i+1} = P_i < 1$. Suppose otherwise. Then $P_{i+1} = 1$. But by definition, $S_{i+1} - S_i = M$, where M is some number strictly greater than 0. Then for any eligible distribution *f(x)*, there exists δ^*, such that

$$\forall \delta < \delta^*, \ \int_{x_i + \frac{\delta}{2}}^{x_{i+1} + \frac{\delta}{2}} f(x) \, dx < M.$$

Note that $|x_{i+1}-x_i| = \delta$. Hence, an entrant at $x_{i+1}+\delta$ receives a seat with certainty, and *(x, C)* is not an equilibrium.

But then $P_{i+1}<1$, and by Lemma 5.3 it must be that $argmax f(x) \in s_{i+1}$, which implies that $|0<P_i<1| = 2$, $|x_{i+1}-x_i|=\delta$, and

$$F(x_i+\frac{\delta}{2})-F(x_i+\frac{\delta-\sigma}{2}) \geq F(l_{x_i})-F(l_{x_i}-\frac{\sigma}{2}),$$

and

$$F(x_{i+1}+\frac{\sigma-\delta}{2})-F(x_{i+1}-\frac{\delta}{2}) \geq F(r_{x_{i+1}}+\frac{\sigma}{2})-F(r_{x_{i+1}}).$$

Hence, as $f(x)$ is strictly quasi-concave, l_{x_i} and $r_{x_{i+1}}$ straddle the mode, i.e., $argmax f(x) \in \{s_i \cup s_{i+1}\}$. QED

Two corollaries follow immediately from Lemma 5:

Corollary 2: *If f(x) is quasi-concave, and if* **(x**, *C) is an equilibrium, then* $|C| \leq k+1$, *and* $|P_i=1| = k-1$.

Corollary 3: *If f(x) is quasi-concave, and if* **(x**, *C) is an equilibrium, and if there exists* ***i****, such that* $0<P_i<1$, *then* $P_i = 1/2$.

We are now positioned to prove the central result of this section of the paper.

Proposition 5:

I. *For any strictly quasi-concave distribution of ideal points, if there are equilibria other than* ***k****-equilibrium existing for all small* $\delta>0$*, they must be of the following form:*

- $|C|=k+1$;
- *two candidates are located maximally close to each other, the union of their constituencies includes the mode of the distribution, and these two candidates are tied for a seat;*
- *the remaining* ***k****-1 candidates each receive a seat with certainty and are located so that Lemma 5 holds.*

II. *For all symmetric single-peaked distributions and for all odd* ***k****'s (and some even), such equilibria exist for any small* $\delta>0$.

Proof: Part I of the proposition follows directly from Lemma 5 and Corollaries 2 and 3; for quasi-concave distributions no other equilibria with $|C|>k$ may exist. Part II is proved by the construction of the corresponding equilibrium.

Since the case of *k=1* is trivial, we start with *k=3*. For δ small enough, set

$$x_i=argmax\,f(x)-\frac{\delta}{2},\quad x_j=argmax\,f(x)+\frac{\delta}{2}.$$

Now on $(\infty;x_i]$ we need to choose l_i to separate the constituencies of candidates *i* and *i-1* so that

$$|l_i-x_{i-1}|=|x_i-l_i|\geq\frac{\delta}{2}.$$

Notice that the choice of l_i uniquely determines x_i. By the strict monotonicity of $f(x)$ on $(\infty;x_i]$,

$$F(x_i)-F(l_i) > F(l_i)-F(x_{i-1}).$$

If in particular we choose l_i such that

$$F(x_i)-F(l_i) = F(x_{i-1}=x_i-2|x_i-l_i|),$$

the conditions of Lemma 5 are satisfied. But by the strict monotonicity of $f(x)$ on $(\infty;x_i]$, such an l always exists: $F(x_i)-F(l_i)$ is continuous, monotonically decreasing from $0.5-\xi(\delta)$ to 0 as l_i goes from 0 to $x_i-\frac{\delta}{2}$, while $F(x_{i-1}=x_i-2|x_i-l|)$ is continuous, monotonically increasing from 0 to $0.5-\xi(\delta)$. Therefore, there exists l_i^* such that $F(x_i)-F(l_i^*)=F(x_{i-1})$. Finally, by symmetry, locate x_{j+1} at $F^{-1}(1-F(x_{i-1}))$, so that $((x_{i-1},x_i,x_j,x_{j+1}),\{1,2,3,4\})$ constitutes a {k+1}-equilibrium for *k=3*.

To construct an equilibrium for *k=5*, choose x_i and x_j as before. From our previous argument we know that the choice of l_{i-1}^* separating the constituencies of candidates *i-1* and *i-2* uniquely determines the choice of l_i^* and, hence, x_{i-1} and x_{i-2} (see Figure 1). And again, for the same reason as in the case of *k=3*, we want the choice of l_{i-1}^* to satisfy

$$F(x_{i-1})-F(l_{i-1}^*) = F(x_{i-2}=x_{i-1}-2|x_{i-1}-l_{i-1}^*|).$$

The existence of such l_{i-1}^* is again asserted by a fixed-point argument. The theorem holds because this method of construction can be extended to any odd *k*[4]. QED

[Figure 3.1 is about here]

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[4] The reason we do not claim the uniqueness of an equilibrium for any particular value of *k* is the arbitrariness of the choice of x_i and x_j, for which the only restrictions are $\mathrm{argmax} f(x)\in\{s_i \cup s_j\}$ and that Lemma 4.3 holds, as well as the flexibility that safe candidates have in *reducing* their vote somewhat below the maximal safe level.

3.5 Implications of the Model that Admit Empirical Testing

That Reed's (1990) work documents the stability of the $k+1$ pattern in Japan is an encouraging fact. Indeed, although our analysis asserts the existence of a $k+1$-equilibrium only for symmetric and unimodal distributions of voters' preferences, there clearly exists in each particular case a large set of equilibrium spatial configurations with the same number of candidates. For this reason we can speculate that symmetry is not a strict requirement. For small variations in a symmetric distribution, there must still exist a subset of configurations that remain equilibria. At the same time, if we relax the restriction that the number of candidates to the right of the mode equal the number to the left, we suspect that for any unimodal distribution (no matter how asymmetric) there exists a district size k for which a $k+1$-equilibrium exists. Thus, the preconditions for testing our analysis are not necessarily as strong as the assumptions vis-a-vis symmetry that we employ in proving our formal results.

The next precondition to empirical testing is to establish that the assumption of a unimodal preference distribution provides a reasonable characterization of individual districts. Fortunately, extremely high ethnic and linguistic homogeneity in both Taiwan and Japan allows us to suppose that constituencies are unlikely to be polarized along these lines. Japan's districts, moreover, are quite small (3-5), as are Taiwan's (with but a few notable exceptions), thereby giving us some confidence in the validity of the unimodality assumption. That those exceptions (i.e., Taipei and Kaoshung) are also likely to be the most heterogeneous districts provides a basis for seeing whether the k+1 rule holds better in homogeneous than heterogeneous election districts.

Insofar as specific hypotheses are concerned, both Reed's empirical and Cox's theoretical analysis agree that the vote shares of the candidates should be approximately the same. Reed asserts this hypothesis for all $k+1$ candidates. Cox, by approaching the problem from the standpoint of voters' rationality, derives that only the top k candidates should all receive identical vote shares, and the $k+1^{st}$ and lower ranked candidates get arbitrarily lower shares of the vote. In contrast, our analysis predicts that candidates in equilibrium do not necessarily secure equal electoral support. Although over time their support may become even as Reed reports, this trend should be the result of strategic voting rather than of any strategic action by the candidates.

An empirical assessment of this hypothesis, though, requires a reconsideration of how we count candidates, the number of "serious" candidates competing in each district. Reed's (1990) approach is not satisfactory for our needs. Reed equates the number of candidates to the number of "effective" candidates, i.e., the inverse of the sum of the candidates' squared shares of the vote. Although the motivation for the use of "effective" number of candidates is to avoid separating "serious" candidates from non-serious ones by some *ad hoc* criterion, the use of this measure involves a number of unsatisfactory assumptions. In particular, testing the hypothesis of $k+1$ candidates running in a district by looking at "effective" number of candidates implies that all such candidates earn identical support from voters in equilibrium. If there are indeed only $k+1$ candidates, but with uneven shares of the vote, the "effective" number will be below $k+1$. Conversely, for "effective" number to be *k+1* when vote shares differ, more than *k*+1 candidates must compete. Indeed, Reed does assert that in equilibrium all candidates should be equally successful at the polls. Looking merely at the "effective" number of candidates makes it impossible to test this part of his hypothesis and confounds the influence of several hypotheses.

In the tables that follow as an illustration in the Addendum, unlike Reed, we will not compute "effective" numbers of candidates, but instead will draw a line rather arbitrarily, cutting off those unsuccessful candidates whose vote shares are "significantly" lower than the candidates' immediately above them. To avoid obvious criticism, the cutoff will be set at a 20% vote decrease, 33%, 50%, and 100%. In other words, we count the number of candidates in the district as *k* plus all those candidates, who gathered not less than 80 (67, 50, or 0) percent of the vote of the competitor immediately above them. For example, if the k^{th} candidate receives 100,000 votes, and

$k+1^{st}$ receives 79,000,

$k+2^{nd}$ receives 52,000,

$k+3^{rd}$ receives 25,000,

and the last one, the $k+4^{th}$, only 10 votes, the number of "serious" candidates in the district will be *k*, *k+1*, *k+2*, and *k+4* correspondingly for the 20, 33, 50 and 100% cutoffs.

Insofar as attempting to see whether a candidate-oriented analysis (our) provides a better explanation of the data than does a voter-oriented explanation (Cox, and Reed), we must look at some additional things. Because Reed predicts that there should be *k+1* "serious" candidates per district and since we predict either *k* or *k+1* such candidates, we cannot use simple counts to discriminate between Reed's analysis and our own. Moreover, Cox does not infer any predictions about the number of candidates. He states only that all victorious candidates should obtain identical shares of the vote. We can, though, get a handle of discriminating between the two alternative approaches by looking at the ratios of the candidates' vote shares within districts. Specifically,

- Cox's hypothesis can be restated as predicting that the ratio of the *vote shares of the k-1st* candidate and the *1st* candidate being close to 1.
- Our analysis predicts that the corresponding ratio of the *k+1st* to the *kth* being close to 1.
- Reed's hypothesis predicts that both ratios should be 1.

In the next chapter we analyze the complete set of Taiwanese elections and reexamine the electoral data from Japan assembled by Reed with these hypotheses in mind. However, in the Addendum to this paper we present a partial set of Taiwanese elections, to provide evidence immediately that our hypothesis is worth consideration.

Finally, we want to offer several comments on the how our analysis might be used to shed light on the potential rational grounds for the party formation under SNTV. One widely held opinion is that for party membership to be individually rational for the candidates, it must be significantly reducing campaigning costs, or otherwise strengthening the candidates positions with the voters (Holler, 1987). Alternatively, Downs's political party does not yield benefits to members on an individual basis, but instead collects them "lump-sum," in the form of the control over the government, which serves as a kind of public good for party members. A political party, then, is but "a team seeking to control the governing apparatus by gaining office in a duly constituted election," "whose members agree on all their goals instead of just part of them" (Downs, 1957 p. 25).

In contrast, our analysis suggests the concept of a party as a coordinating agent among candidates. That is, the particular difficulty candidates confront in an SNTV system is being certain that only the "correct" number of them compete, and that they compete at the "correct" positions on the issue space. Notice now that this perspective admits of the existence of factions within parties and the free use of faction labels in a campaign. The phenomenon of factions, of course, is not foreign to either the Japanese or the Taiwanese political systems.

Specific aspects of intra- and inter-party coordination include, for example, Curtis' (1972) and Reed's (1990) observation in the Japanese system of (for example) "the young entrepreneur... [who] runs as an independent to demonstrate his campaigning ability to the [major party] in the hope that they will reward him with nomination" (Reed 1990, p. 355). Japanese parties also tend to grant their "joint" nomination to independent candidates - something that hints of the attention given by parties to defining the "spatial" location of a candidate. Cases of collusion between the parties in terms of vote transfers are also common.

Notice moreover that our treatment of parties in fact corresponds to Schumpeter's famous definition of party and machine politicians as "simply the response to the fact that the electoral mass is incapable of actions other than a stampede, and they constitute the attempt to regulate political competition exactly similar to the corresponding practice of trade associations" (Schumpeter, 1947, p. 283). Owing to the complexity of the equilibrium our model describes, a complexity that requires the right number of candidates at the right spatial locations, it seems only reasonable to view parties in SNTV systems as both agents of coordination as well as of enforcement.

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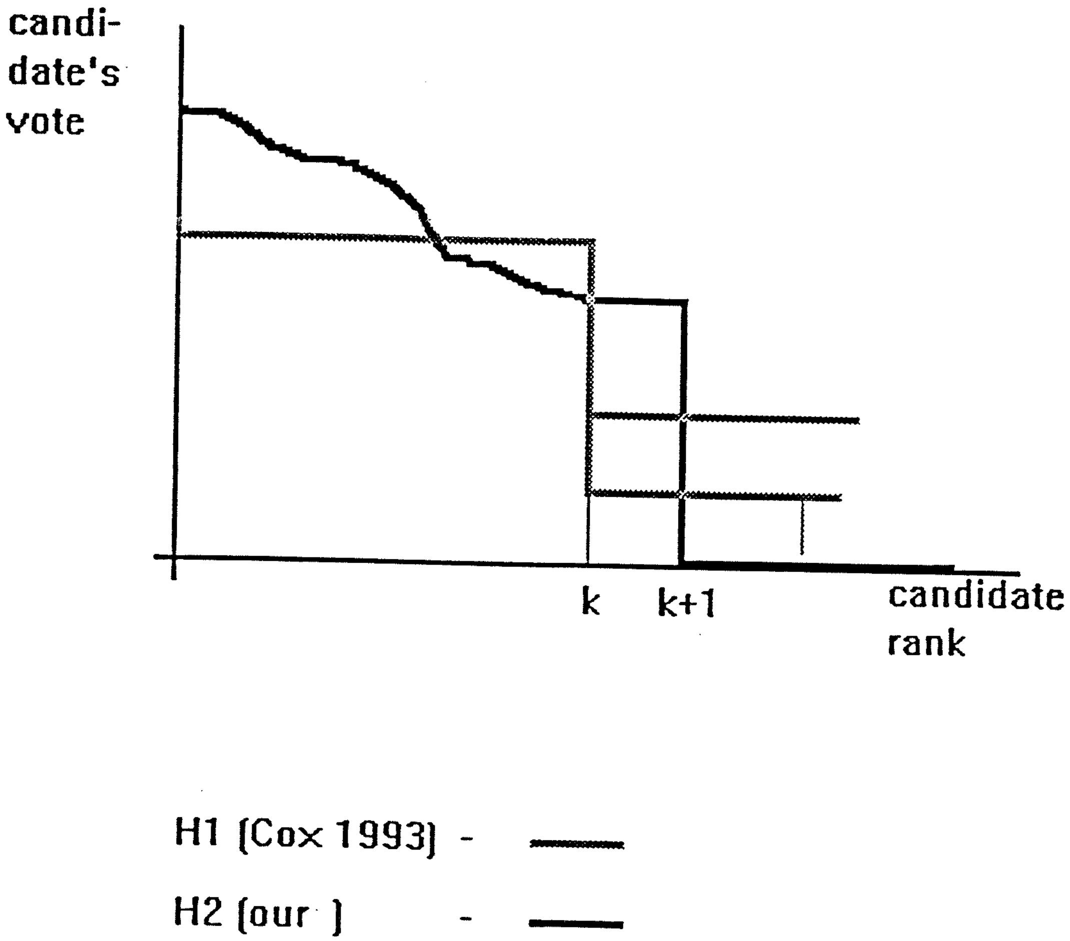
Figure 3.1: **Alternative hypotheses generated by the voter-based (H1 - Cox 1994) and candidate-based (H2 - our) models**



APPENDIX:

Proof of Lemma 1:

1. Follows from the definition of SNTV.

2. Suppose not; suppose $P_i = 1$. Then the share of the vote hold by each of the candidates at $\mathbf{x_i}$ $S_i = S_j = \frac{1}{n_{x_i}} S_{x_i}$ is strictly greater than the share of the vote of all other candidates, except at most $k - n_{x_i}$ of them. Then $\exists\, \sigma$, such that a new candidate **l** can enter at $x_i + \sigma$, and get a share of the vote

$$S_{x_i+\sigma} \geq \frac{1}{2} S_{x_i} - \varepsilon(\sigma)\,, \quad where \;\; \varepsilon(\sigma) = \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\, dx$$

while the candidates at $\mathbf{x_i}$ each receive the vote shares

$$S_{i\,|\,j\; moves\; on\; x_i+\sigma} \leq \frac{1}{n_i - 1}\left(\frac{1}{2} S_{x_i} + \varepsilon(\sigma)\right),$$

for $n_i \geq 2$, Deviation occurs, because the vote share of deviating candidate exceeds the vote shares of all other candidates except at most $k - (n_i + 1)$ of them, and thus $P_j = 1$. Hence, in equilibrium necessarily $0 < P_{\{i\,|\,n_i > 1\}} < 1$.

3. Suppose not; suppose three candidates **i**, **j**, and **l** share policy position x_i. Then by assumption $S_i = S_j = S_l = \frac{1}{3} S_{x_i}$. Consequently, by Lemma 1.2 $0 < P_i = P_j = P_l = P_{x_i} < 1$ - they all are tied for a seat. Hence, anyone with support exceeding $\frac{1}{3} S_{x_i}$ can win a seat with certainty. Then $\exists\, \sigma$, such that by taking a position $\hat{x}_i = x_i + \sigma$, candidate **i** can increase his share of the vote to $S_{\hat{x}_i} \geq \frac{1}{2} S_{x_i} - \varepsilon(\sigma) > \frac{1}{3} S_{x_i}$, and get a seat for certain.

4. Suppose not; suppose $s^L_{x_i} > s^R_{x_i}$. By Lemma 1.2, probability of getting a seat is less than one for both candidates **i** and **j**. Then $\exists\,\sigma$, such that by moving to a position $x_i+\sigma$, candidate **i** can receive

$$S_{\{i\,|\,at\,x_i+\sigma\}} = S^L_{x_i} - \varepsilon(\sigma) > S_{\{i\,|\,at\,x_i\}} > S^R_{x_i} + \varepsilon(\sigma) = S_{\{j\,|\,at\,x_i,\ while\ i\ is\ at\ at\ x_i+\sigma\}}$$

and probability of **i** getting a seat would increase to **1**, hence the above is not an equilibrium.

5. Candidates **i** and **j** at position **a** (and possibly other candidates elsewhere) are tied for a seat (or seats). Then $\exists\sigma$ (w.l.o.g. let σ be positive), such that by taking position $\hat{x}_i = x_i+\sigma$ candidate **i** can obtain the vote share

$$S_{\hat{x}_i} = S_{x_i} - \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx + \int_{r_{x_i}}^{r_{\hat{x}_i}} f(x)\,dx$$

while candidate **j** remaining at $\mathbf{x_j}$ now receives

$$S_{\{j\,|\,i\ is\ at\ \hat{x}_i\}} = \frac{1}{2}S_{x_i} + \int_{x_i}^{x_i+\frac{\sigma}{2}} f(x)\,dx.$$

Suppose the condition does not hold. Then $s_{\hat{x}_i} > s_{j\,|\,i\ is\ at\ \hat{x}_i}$. This implies that **i** now gets a seat with probability one. (If $s_{\{j\,|\,i\ is\ at\ \hat{x}_i\}}$ exceeds the vote share of the weakest candidate who was originally receiving a seat, than **j** also gets a seat for certain; if not, his probability of getting a seat diminishes to 0; if their vote shares coincide, they tie.) Then the former cannot be an equilibrium.

6. Suppose not. Suppose $s_{x_i} > s_{x_j}$. Then $P_{x_i} > P_{x_j} > 0 \Rightarrow P_{x_i} = 1$ (by Lemma 1.1), which contradicts Lemma 1.2.

Alternatively, suppose $s_{x_i} > s_{x_j}$, then $\exists\sigma$, such that a new candidate **l**, if enters at $\hat{x}_l = x_i+\sigma$, can get a vote share

$$S_{\hat{x}_l} = \frac{1}{2} S_{x_i} - \varepsilon(\sigma) > \frac{1}{2} S_{x_j}; \quad where \ \varepsilon(\sigma) = \int_{x_i}^{x_i + \frac{\sigma}{2}} f(x)\, dx.$$

Which yields the entrant positive probability of getting a seat ($P_l = 1$ by L.1.1).

7. Candidate **i**, whose probability of winning a seat is $P_i = \alpha < 1$, is tied for the seat(s) with a number of other candidates. Therefore, if he increases his vote share, he wins a seat for sure. If the stated conditions do not hold for some σ, candidate **i** can increase his vote share by taking a position $\hat{x}_i = x_i + \sigma$. Moreover, the move of candidate **i** can benefit only the candidate(s) at the position immediately adjacent to his, (w.l.o.g. let it be $\mathbf{x}_{i+1}$), whose vote share increases by

$$\frac{1}{n_{x_i+1}} \int_{r_{\hat{x}_i}}^{r_{x_i}} f(x)\, dx \ < \ \int_{l_{\hat{x}_i}}^{l_{x_i}} f(x)\, dx$$

so that they get no advantage over **i**. Hence, candidate **i** has positive incentives to deviate, and the above cannot be an equilibrium.

8. Suppose not. Then entrant can guarantee himself a probability of winning a seat of α or better, as

$$\exists \sigma \ \ s.t. \ \ S^L_{x_i} - \int_{x_i + \frac{\sigma}{2}}^{x_i} f(x)\, dx \geq S_{x_j}.$$

Hence by entering at $x_i + \sigma$ candidate **l** gets positive probability of winning a seat.

9. If not, a new candidate can enter at the point $\frac{|a+b|}{2}$, which yields a positive probability of winning a seat.

ADDENDUM: Elections by SNTV in Taiwan

The table below shows the dynamics of candidate participation in three consecutive Taiwanese national elections - 1980, 1983, and 1986 - prior to the drastic change of electoral laws that occurred in 1989 (26 electoral districts were formed in place of 8, with average district magnitude reduced from 6.9 to 4.6). Columns 4, 5, and 6 of the table show the number of "serious" candidates in excess of k competing in the race within districts for the three cutoff levels specified above. Column 8 gives the ratio of the support for the k-1^{st} leading candidate to that of the first-ranked candidate - statistics that should converge to 1 by both Reed's and Cox's hypotheses. Column 9 reports the ratio of the vote for the strongest loser (the $k+1^{st}$ candidate) to that of a last victor (the k^{th} candidate), which should converge to 1 by both Reed's hypothesis and ours (except for the k-equilibrium cases). The data on this set of elections are interesting in particular because, with opposition to the Kuomintang illegal during this period, there has been no process of party formation during the time covered by the data in this table. This data, then, derives from a period of minimal coordination during a campaign (when, for example, the total number of candidates exceeded the number of seats in the district 2 to 4 times).

Looking at the numbers this table reports, we see that, as predicted by our model (except for the clearly expressed cases of k-candidate competition - the Third district in 1983; the Sixth district in 1980, 1983, and 1986; and the Eighth district in 1980 and 1986), the ratio of the vote for the first loser and the last winner stays close to 1, beginning with the first elections. The average for the year 1980 is 0.89, for the year 1983, 0.94, and for the year 1986, 0.93. At the same time and contrary to Reed and Cox's analyses, the numbers in column 8 stay far from 1 (averaging 0.55 in 1980, 0.76 in 1983, and 0.65 in 1986). Of course, we cannot say whether this ratio would converge to 1 over a longer period (as Reed claims based on his analysis of Japan). But these data do encourage us to examine the Japanese case more closely for support of the hypothesis that the "k+1 rule" is driven more by candidate and party strategies than by individually motivated voters voting strategically.

DISTRICT LEVEL ELECTORAL DYNAMICS IN TAIWANESE GENERAL ELECTIONS, 1980-1986

District	Year	District magni-tude k	Number of "serious" candidates in excess of *k* when cutoff is when the vote reduces by			Total number of candidates	Ratio of the vote	
			20%	33.3%	50%		(k-1st)/1-st	k+1st/k-th
1	2	3	4	5	6	7	8	9
1	1980	8	2	3	3	21	0.6158	0.9736
	1983	9	2	2	2	22	0.5451	0.9776
	1986	9	1	3	3	17	0.5826	0.9976
2	1980	6	0	5	12	18	0.4657	0.7792
	1983	6	2	4	6	16	0.7747	0.9027
	1986	6	2	2	2	11	0.6335	0.9859
3	1980	9	1	1	5	22	0.5559	0.9357
	1983	9	0	0	0	17	0.6829	0.4749
	1986	10	1	1	2	17	0.4668	0.9486
4	1980	8	2	5	14	23	0.4169	0.9172
	1983	8	2	2	2	20	0.6863	0.9998
	1986	9	2	4	4	20	0.6055	0.8387
5	1980	5	1	1	3	11	0.7576	0.856
	1983	5	1	1	2	10	0.6254	0.9904
	1986	5	2	2	2	8	0.8556	0.8094
6	1980	2	0	0	2	6	0.7103	0.5677
	1983	2	0	0	0	5	0.9156	0.4735
	1986	2	0	0	0	2	0.8571	0
7	1980	8	2	2	25	33	0.3371	0.932
	1983	8	2	2	8	25	0.5305	0.9484
	1986	8	2	2	2	16	0.574	0.9885
8	1980	5	0	0	7	16	0.5622	0.5435
	1983	5	2	3	5	17	0.742	0.8473
	1986	6	0	0	0	11	0.5968	0.3742

CHAPTER 4: EMPIRICAL PATTERNS OF MULTICANDIDATE COMPETITION

4.1 Candidates' Strategic Choices

Building on the theoretical results presented in the previous chapter, here we examine Japanese and Taiwanese election data to assess the existence of stable patterns in the number and relative electoral strength of candidates at the district level. At the same time we contrast the empirical support that our predictions receive against that received by the predictions of a "rival" conceptualization - the one offered by Cox (1993), which focuses on voters as strategic actors rather than candidates. Generally, the empirical evidence indicates the stronger impact of the strategic behavior of candidates on patterns of competition. On the other hand, that support is not so strong as to lead us to restrict ourselves to models based exclusively on sincere voting. It is clear that strategic voting must be an important consideration as well.

In Section 2 of this chapter we state the competing hypotheses, which differ both in terms of predicted number of candidates within a district and the distribution votes among them. Sections 3 and 4 are devoted to the analysis of Japanese electoral data. Section 3 looks at the number of candidates, while Section 4 deals with the distribution of votes. Section 5 offers a parallel analysis of Taiwanese elections. Finally, in Section 6 we make a preliminary projection of theoretical findings on the role and operation of political parties.

Few theoretical results have been offered thus far about how electoral systems work when there is more than one legislative seat to be filled per district. This lack of theory, in contrast to the well developed conceptualizations of single-member district elections, is explainable not only by the prevalence of the latter case in American politics, but also by the exploding strategic complexity of multi-seat elections. However, it is the multimember district case that may play a decisive role in improving our general understanding of the true nature of the electoral game. Specifically, in deciding how to approach theorizing about elections, we must often ask one primary question: how much attention should we pay to strategic choice among voters versus among candidates.

Ideally, of course, we would want to consider both forms of strategic behavior. Unfortunately, in most multi-candidate election systems, doing so results in unmanageable analytic complexity. Thus, in examining the implications of a particular election law, it is useful to learn whether that law impacts electoral outcomes primarily through the strategic calculations of voters or that of candidates. If, as in the case here, we are interested in predicting the number of candidates or parties likely to compete as a function of district magnitude or allocation rule, should we focus on the possibility that voters will act strategically by voting for candidates other than those who rank highest on their preference order? Or, are we better served by supposing that voters act sincerely and by focusing instead on candidates who choose policy platforms in anticipation of the fact that competitors enter the contest whenever doing so is profitable?

At least for SNTV multi-member district elections, Reed (1990) and Cox (1993) suggest that the implications of voter and candidate rationality can be separated in terms of the number of candidates competing and the vote shares they obtain. What we require, though, are partial equilibrium models that provide the requisite empirically testable hypotheses that rigorously differentiate between the two assumptions of voter and candidate strategic action. Cox (1993) provides an appropriate analysis when voters are strategic and candidates are mere mannequins. In contrast, the model offered in Part I of this essay lets candidates be strategic (by granting them spatial mobility and by allowing them to choose whether or not to compete) and assumes that they compete for the votes of sincere Downsian voters.[1]

4.2 Discriminating Hypotheses

As Chapter 3 shows, the difference between the predictions of these two models concerns both the number of candidates that compete in equilibrium and the way in which the vote is divided among them. In Cox's model, in its pure form, the difference in the vote for the k winning candidates (where k is district size) should be minimal, i.e., all such candidates should receive equal electoral support. Furthermore, the electoral support of all vote-getting losers must be the same, lower than that of the last winner, and should eventually drop to zero as one moves down the list. Insofar as the number of candidates is concerned, this number should be at least $k+1$. In contrast, our model predicts the equilibrium number of candidates being either k or $k+1$ in a non-polarized constituency (i.e., if the distribution of preferences is unimodal). No prediction is made about the relationship among the vote shares of the winners,

[1] Employing a simple Nash argument in a static context allows us to draw a line between things that can and cannot be if the system is in equilibrium.

but in the $k+1$ case the last winner and the first looser should enjoy approximately the same share of the vote.[2]

Thus, if we indicate the number of votes received by candidate i, $i=1,..k,..n$ as v^i (where k is the number of seats to be filled in a district, and where i is the rank of a candidate within that district), then the two hypotheses we want to test are as follows:

H1 (Cox's): *A1:* $v^i = v^{k+1}$, or $v^i = 0$ for all $i > k$, and

B1: $v^k/v^1 \approx 1$.

H2 (ours): *A2:* k or $k+1$ candidates compete with $v^1 \geq \ldots \geq v^k$, and

B2: In the case of $k+1$ candidate competition, $v^{k+1}/v^k \approx 1$.

[2] It is appropriate here to recall what assumptions were made when our model was developed in Chapter 3, and what assumptions underlie Cox's model. We require that: 1) voters have single-peaked preferences over a one-dimensional policy space and vote sincerely for the candidates nearest their ideal points; 2) each candidate maximizes his/her probability of winning a seat; 3) candidates can adjust their spatial positions, and a new candidate can enter, but only if doing so secures a positive probability, and 4) any two candidates must be spatially separated by a minimal distance $\delta > 0$. In addition, in order to derive the results we require the voter ideal points in the district to be continuously, unimodally and symmetrically distributed. Cox, when developing a voter-based model, assumed that: 1) number of candidates is finite, and their policy platforms are fixed; 2) there is at least one voter for each possible preference configuration (only strict preference ranking is allowed); 3) voters vote strategically, maximizing their expected utility; 4) distribution of voter preferences is common knowledge, and all voters have identical expectations about the number of votes each candidate is going to receive.

Represented graphically, the two hypotheses are compared in Figure 3.1. Cox's hypothesis implies a uniformly distributed vote up to the last winning candidate. After *k*, the vote declines to some v^{k+1}, where it stays until it drops all the way to zero at some $j > k+1$ (the exact location of *j* will depend on the preference structure within the constituency). In contrast, our hypothesis does not preclude the decline of the vote in the set of winners - up to the k^{th} candidate. But for those races where more than *k* candidates compete, our candidate-based model predicts a flat portion in the distribution of the vote between the k^{th} and $k+1^{st}$ candidates, after which the vote should drop to zero, i.e., no other electorally motivated candidates in excess of $k+1$ should exist. Of course, since things other than electoral motivation may activate some candidates, it is sufficient to establish that a "sharp" decline in the vote between the $k+1^{st}$ and $k+2^{nd}$ candidates makes the latter a "non-serious" competitor.

With this structure to our empirical analysis, we can focus on the two most straightforward products of an election: the number of competing candidates and the distribution of votes among them. However, we emphasize that it is not our intention to select any single hypothesis at the expense of another. Rather, it is to study how H1 and H2 interact and whether relying on one or another better serves our purposes. Again, because both sets of results are based on stringent assumptions, we are not in the position to reject the validity of either model. Whenever evidence does not support a hypothesis, an argument can be made that it is due to the specific set of assumptions not being met by the data. With this qualification in mind, we turn first to Japanese electoral data, since Japan over the post-war period has accumulated a large body of internally consistent electoral information (consistent in the sense of district boundaries being largely unaltered and electoral rules largely unchanged).[3]

4.3 The Number of Candidates

4.3.1 Measurement problems

In developing his hypotheses about the number of "serious" candidates that compete in Japanese legislative elections, Reed (1990) counts candidates by computing their "effective" number, where this number corresponds to the inverse of a sum of the candidates' squared shares of the vote:

$$EN = \frac{1}{C} = 1/\sum_{1}^{n} \frac{(vote_i)^2}{(vote\ in\ the\ district)^2}$$

EN, then, is the inverse of a concentration index *C* that, for the same number of candidates, increases as the candidates' vote shares become more uneven. The minimum of *C* for a given number of candidates is attained when their vote shares are identical. At this point *EN* equals the actual number of contestants. But this circumstance corresponds to *EN*'s maximal value, so that for all other distributions of votes, the "effective" number of candidates assumes a

[3] Data is taken from: Reed, Steven, *Japan Election Data: The House of Representatives, 1947-1990*, Ann Arbor, Center for Japanese Studies, 1992. Electoral data on 1993 Japanese elections are courtesy of Professor Gary Cox, University of California at San Diego.

value below the "real" number (with the bias increasing as differences in the candidates' vote shares increase).

Although convenient in some applications, "effective" counting does not help us study the behavior of candidates in the present context. A downward bias may be produced by differences in the vote for the highest ranked winners, whereas our attention is directed at competition down the line - among the lower ranked winners, losers, and potential entrants. In fact, the downward bias of the "effective" measure can be so significant that it can result in counting fewer candidates than there are seats in the district. Consider an example where district magnitude equals four, and five candidates receive votes in a following proportion:

candidate 1: 40%
candidate 2: 20%
candidate 3: 16%
candidate 4: 12% + ε

---

candidate 5: 12% - ε

Despite the fact that candidates *3, 4* and *5* are competing closely for the last seat, the "effective" measure that Reed employs returns the value $EN = 3.93$, which is less than k and which leads to the loss of the very candidate whose behavior interests us most - the $k+1^{st}$ candidate. There is, moreover, nothing extraordinary in the distribution of the vote in this example. Consider Japan's *Niigata* 3^{rd} District in 1972:

Tanaka Kakuei	*182681*
Kobayashi Susumu	*58217*

Miyake Shoichi	*55363*
Murayama Tatsuo	*48329*
Ono Ichiro	*39867*
Furukawa Hisashi	*30747*
Magai Hideji	*18944*

The "effective" number of candidates in this race, *4.22*, is less than the size of the district. Similarly, in the *Kagawa 2nd* District, in 1980, we have for the three seats being filled,

Morita Hajime	*151546*
Kubo Hitoshi	*44027*
Kato Tsunetaro	*34535*
Kubo Fumihiko	*9829*
other candidates	*3016*

EN=2.28 is again below district size.

Not only can *EN* conceal competition, it can indicate competitiveness where it is not present. For instance, Japan's Communist Party has a policy of running a candidate in each district regardless of his or her chances of electoral success. In the majority of cases, these chances are zero. *EN*, though, often counts these candidates despite their electoral hopelessness. For example, in the *Toyama 2nd* District in 1972:

Kataoka Seiichi	*76522*
Watanuki Tamisuke	*64950*
Sano Kenji	*62954*
Yoshida Minoru	*62355*

Arima Shigekazu *9769*

Competition for the last seat clearly occurs here, but the fifth candidate is not part of it. Nevertheless, *EN=4.27*, which indicates more than $k+1$ relevant candidates.

Reed (1990) employs *EN* to avoid an arbitrary definition of "serious" candidates. But because "effective" counts do not carry the information we need, we must find a more satisfactory way to divide candidates into serious and non-serious categories. To that end notice that to the extent that competition among losing candidates focuses on the last seat in the district, we might expect such candidates to win comparable shares of the vote. Hence, if we order all candidates in the descending order of their vote, a sudden "significant" drop in the vote among the losers would indicate the end of the string of "serious" candidates (corresponding to the predicted drop in the distribution of the vote, as in Figure 3.1). The arbitrary thing to decide with this approach, though, is exactly how significant this drop should be. Since we have no theoretical basis for operationalizing "significant" one way or another, we suggest marking this drop after the candidate whose nearest competitor is 20, 33, or 50 percent behind him in terms of vote shares. Of course, that competitor should also be among the losers, as we assume that victorious candidates compete not among themselves, but with those without legislative seats.

Figure 4.1a shows the percentage of races by the year in which a 20-percent drop in the vote occurs immediately after the k^{th} candidate, after the $k+1^{st}$, $k+2^{nd}$, $k+3^{rd}$, or later candidate (Figures 4.1b and 4.1c consider respectively the location of the first 33-percent and 50-percent drop in the vote). A substantial portion of districts supports only as many competitive candidates as there are seats, whereas the dominant number of districts has just one competing candidate in excess of district size.

If we now take the number of candidates before the vote becomes non-competitive (i.e., drops substantially for the first time outside the set of winners) as a measure of the number of "serious" candidates, then Figure 4.1 allows us to make certain preliminary observations supportive of hypothesis H2. For example, we are interested in detecting the presence of a *k*-equilibrium in which the vote drops significantly immediately outside the set of winners. Reed's analysis does not make allowances for this possibility, whereas Cox predicts it will not happen. According to our model, however, it can occur for certain types of voter preference distributions. Hence, if we observe a persistent presence of *k*-candidate races, then we have one argument in favor of a candidate-based model. In Figure 4.1, then, we see that when the 20% criterion is applied, *k*-candidate races take place in 20 to 30 percent of all electoral districts. The combined weight of *k* and *k+1* candidate races by the same criterion lies - in different years - between 65 percent (in 1969) and 87 percent (in 1980, 1986, and 1990).

[Figures 4.1a, 4.1b, and 4.1c are about here]

Naturally, strengthening the threshold of candidate "seriousness" increases the number of counted candidates in the race, so from one diagram to another the shift should be toward the greater weight of the higher count. But the very presence of *k*-candidate elections even with a threshold of 50 percent means that in up to 10 percent of all races the vote drops by more than one half immediately after the last winning candidate (and in some cases the drop in the vote is as significant as 70-80 percent or more). The combined weight of k and $k+1$ races by this stronger criterion is between 33 percent (in 1958) and 67 percent (in 1980).

4.3.2 The pattern of one or fewer extra candidates/district

The average number of "serious" candidates in excess of district size is shown by the years of elections (1958 to 1993) in Figure 4.2.[4] Notice that with the relatively liberal 20 percent criterion, almost exactly $k+1$ candidates compete on average, whereas with the more stringent 50 percent criterion, this number still does not go above $k+2$ in all but two elections.[5]

[Figure 4.2 is about here]

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[4] Taking the average number of "excess" candidates allows us to avoid separating the results for 1, 2, 3, 4, and 5 member districts - something that both Reed and Cox were forced to - being at the same time consistent with our conceptualization.

[5] It is worth mentioning here that our theoretical analysis predicts the equilibrium number of candidates of either k or $k+1$ only if voter preferences are distributed unimodally, i.e., if the constituency is cohesive. This prediction does not have to hold in polarized constituencies, where $k+2$ and higher numbers of candidates can compete (see Appendix). Later we show that in the constituencies where we are not likely to expect polarization of voter preferences, the average number of candidates in the race is indeed much lower than in the districts where polarization is most likely, and basically stays at $k+1$ or below.

Notice also that the decline in the number of candidates across time is much less evident than Reed asserts on the basis of "effective" counts. If learning and adjusting to the peculiarities of SNTV occur, it most probably occurred in the period 1947-1958.

Our way of counting "serious" candidates is, probably, unfavorable to the hypothesis we advocate. Specifically, comparing the vote of each loser to the vote of the candidate immediately ahead of him may result in counting candidates with low vote shares, especially if the descent of the vote among the losers is smooth. Changing the way we count candidates to counting only those losers whose vote is comparable to that of the last winner allows us to better assess the competitiveness of each individual candidate. So drawing the line of "seriousness" when the fraction of the last winner's vote that the loser gathers drops below 80, 67 or 50 percent respectively (same 20, 33 and 50 percent thresholds, but applied in a different context), disregarding now closeness to the immediate front-runner, yields an alternative count of the number of "serious" candidates. As Figure 4.3 shows, the average number of excess candidates moves closer to predicted "one or below" level for stronger thresholds.

[Figure 4.3 is about here]

However, there is no dramatic difference between the graphs generated by this and the previous, less favorable measure (compare Figures 4.2 and 4.3). This fact points at the existence of an actual gap between the candidates in the race, and indicates that steady decline in the vote outside the set of winners does not occur frequently.

4.3.3 Differences in the losers' competitiveness depending on their ranks

To illuminate what happens to candidates' vote shares after all seats are filled, consider Figures 4.4a to 4.4d, which show how the vote for each successive loser (the $k+1^{st}$, $k+2^{nd}$, $k+3^{rd}$, and $k+4^{th}$ candidates) is distributed as a fraction of the last winner's actual vote in that district.

[Figures 4.4a, 4.4b, 4.4c, and 4.4d are about here]

Most $k+1^{st}$ candidates, as Figure 4.4a shows, stay close to the last winner. This is but another way to say that first losers are generally competitive and pose a serious threat to the last winners. At the same time, both the $k+3^{rd}$ and $k+4^{th}$ candidates (Figures 4.4c and 4.4d) concentrate at the lower end of the distribution, being thus out of the competition for seats. Of course, this assumes that the third and fourth losers are present in a district, and in many cases they are not: 592 out of 1632 observations lack the third loser, while 1106 out of 1632 lack the fourth one. We can conclude, then, that neither the third nor the fourth loser is likely to compete seriously for a seat under SNTV.

The second loser is an intermediate case, and his electoral strength is of the greatest interest to us. As Figure 4.4b shows, the $k+2^{nd}$ candidates distribution is generally fairly uniform with some tendency toward bimodality. The left mode of this distribution corresponds to districts with the second loser being effectively out of competition, and constitutes evidence in favor of the prediction of "one or fewer" extra candidates per race. But the right mode of this distribution signals that in a substantial number of races, second losers are competitive and pose a threat to the last winner. Thus, we cannot ignore the fact that there is substantial number of *k+2* candidate races in our data. At this point we can raise a claim that this *k+2* pattern characterizes those constituencies where the assumptions of our model are not met. This is the issue to which we now turn.

4.3.4 Separating constituencies by the type of distribution of ideal points: departing from assumption of preference unimodality

In our theoretical analysis we have not addressed any complexities that may occur in the distributions of electoral preferences within districts, such as there being non-spatial issues (such as personal popularity) or voter preferences spatially bimodally or multimodally distributed. Instead, we have assumed that voter ideal points were unimodally distributed over the one-dimensional policy space. But this assumption may not be met in many electoral districts contributing to our data. In some districts voter preferences may correspond to this assumption closer that in others. The data summarized by Figure 4.4b causes us to address the question: how good are the model's predictions when we have greater confidence in the restrictive assumption about voter preferences? Our assuming unimodality can in fact explain why our hypothesis may not hold for polarized constituencies or constituencies with a broad range of preferences (with a uniform portion at the mode of preference distribution). Even though we are not in the position to give an exhaustive description of what might happen in all of the non-unimodal cases, we can offer examples of equilibria for some preference distributions that may prove generalizable - notably, some bimodal (polarized) distributions and quasi-concave configurations that begin to approximate a uniform distribution. The Appendix to this chapter illustrates some equilibria with more than $k+1$ "serious" contenders in non-cohesive constituencies.

A natural test, then, is to compare two samples: one formed of constituencies that are unlikely to be polarized, and another with more diverse or polarized preferences. Fortunately, demographic type of a constituency is one piece of information about Japanese electoral districts that is part of the data set. Specifically, although Japan is not ethnically or linguistically heterogeneous, we are more likely to find issue polarization in urban districts than in rural ones, if only because of the greater income, class, educational and occupational diversity found there. Hence, it is not unreasonable to suppose that our model will hold better in rural districts than in metropolitan ones.

The prediction that rural districts would follow the pattern of at most $k+1$ "serious" candidates closer than metropolitan ones is borne out by the data.[6] The number of excess candidates competing in Japanese electoral districts with different demographic characteristics is plotted in Figure 4.5.

[Figure 4.5 is about here]

[6] As a null hypothesis we take equal probability of $k+(>1)$ candidate races in both rural and metropolitan districts. This hypothesis is firmly rejected already at the 98% confidence interval. Probability of a $k+(>1)$ candidate race in a metropolitan district exceeds the one in a rural district by at least 20%.

This is the number obtained by the first method described in Section 1, in which "serious" candidates are counted until the vote falls sharply between two adjacent candidates. The average number of excess candidates in rural constituencies is distinctly lower than in metropolitan ones. And surprisingly, the stronger the threshold used for counting "serious" candidates, the larger the gap between rural and metropolitan districts.

And if as an alternative we determine "serious" candidates by comparing their vote shares to the share of the "weakest" winner, then not only does the separation between the number of candidates in rural and metropolitan constituencies persist, but the number of excess candidates in rural constituencies settles firmly at the $k+1$ predicted mark, regardless of the threshold we use (see Figure 4.6).[7]

[Figure 4.6 is about here]

Finally, and in accord with hypothesis H2, if we now analyze the relative strength of the $k+2^{nd}$ candidate (i.e., the second loser) in rural and metropolitan constituencies, we get two distinct distributions. As we expect the rural districts to adhere more closely to our assumptions, we expect the second losers there to be on average much less competitive. Indeed, Figure 4.7 shows in rural districts most such candidates do not pose effective competition for the k^{th} seat (conforming to the hypothesis of at most $k+1$ true competitors in

[7] What the test of hypotheses described above is concerned, its result sustains when alternative measure is used.

the race). In metropolitan constituencies, on the other hand, the $k+2^{nd}$ candidate is generally a competitor for a seat.

[Figures 4.7a and 4.7b are about here]

4.4 Candidate Vote Shares

4.4.1 A direct comparison of H1 and H2

Counting the number of candidates alone is not sufficient to discriminate between H1 and H2. Although k-candidate races are allowed only under hypothesis H2, there is still a pooling effect of both models with respect to the $k+1$-candidate elections. However, some separation is achieved if we consider the relationships between the vote for certain candidates: Cox's model predicts that the vote shares of the 1^{st} and k^{th} candidates will be approximately equal (part *B* of H1), whereas the candidate-based model sets little restriction on the relative vote of winners (part *B* of H2).[8] On the other hand, the candidate-based model requires that the k^{th} and $k+1^{st}$ candidates in the $k+1$-candidate elections be "close."

To assess these predictions Figure 4.8 presents the corresponding ratios of the vote, where the average ratio of the $k+1^{st}$ vote to the k^{th} is computed conditionally on electoral

[8] The only restriction is that their vote should not differ more than twice (for small δ - if δ goes up, so does the allowed variation in the vote).

districts having more than k "serious" candidates in them (ratios are computed for each of the "seriousness" thresholds used in the previous section). But even if we do not control for the k-candidate districts, the vote for the k^{th} and *for the* $k+1^{st}$ candidates stay closer together on average than do the votes within the set of winning candidates.

[Figure 4.8 is about here]

The dynamics of these two vote ratios is meaningful: the first, v^k/v^1, says how prudent voters are in allocating their vote; the other, v^{k+1}/v^k, indicates how precisely the candidates evaluate their chances when deciding about entering the contest. We can suppose that whichever side, voters or candidates, produces the faster convergence to the predicted level is the faster learner and, potentially, the Stackelberg leader in the game. As nothing prevents us from having both candidates and voters being strategic, it then becomes important to know who learns their equilibrium strategies first, and who thereby sets the structure of the system that then persists in future elections.

4.4.2 Accommodating a populist vote

The counter argument to using the described vote ratios as evidence for or against the two competing theories is that the k^{th} and 1^{st} candidates usually are more than one step apart. In other words, unlike the $k+1^{st}$ and the k^{th}, these candidates are not by definition immediately close to each other. Moreover, we should be concerned that the top-ranked candidate assumes that position because of some special, non-spatial appeal. The argument in either case would be that relying on the ratios v^k/v^1 and v^{k+1}/v^k biases our conclusions against Cox's analysis.

Thus, to reduce the potential impact of a heavy "populist" vote for the leading winner, suppose we compare the drop in the vote between v^1 and v^k versus the corresponding drop between v^2 and v^k. Depicted in Figure 4.9, the difference in these two ratios confirms that, indeed, front-runners collect a disproportionate share of the vote. But even recalculated this way, the ratio of the vote within the set of winners remains below the ratio v^{k+1}/v^k.[9]

[Figure 4.9 is about here]

4.5 The Evidence From Taiwan

4.5.1 Number of candidates

[9] Another place to use the per seat decline in the vote is to see if there is a discontinuity in the safety of the *k-1*st and the *k*th candidates that reflects the fact that one is "safe" while the other is under the pressure of the competition. In fact, we find the vote ratio of the *k*th to *the k-1*st vote slightly lower than the ratio of the first loser's to the last winner's vote when *k*-candidate races are excluded from the sample. The null hypothesis that the two ratios are the same can be rejected in favor of the hypothesis that the ratio of the vote for two weakest winners is lower than the one of the vote for the first looser to the vote for the last winner. Taking into account the higher denominator of the first ratio, this translates into substantially greater decline of the vote in absolute terms between the two last winners, than on the edge of the set of winning candidates.

Returning to our original count of "serious" candidates (when we count all candidates before the first substantial drop in the vote among the losers), we now see (Figures 4.10, 4.11) that, although less stable, a pattern similar to the one we observe in Japan arises in Taiwan as well.[10] This regularity in SNTV elections, one extra competitor over the district size, can again be taken as evidence in favor of H2 and a candidate-based model. This is especially so if we recall that every Taiwanese election corresponds essentially to an alteration in electoral regime - for the first three elections it was the gradual change in restrictions on campaigning; for the two later cases, disruption derived from the drastic changes in district boundaries and district magnitudes as well as the rules on allowable campaigns.

[Figures 4.10 and 4.11 are about here]

This regime instability suggests that we cannot expect to infer much here about learning by candidates and voters. However, it seems that the statistics generated by a 20 percent and a 33 percent criterion quickly settle close to the predicted number of candidates, while statistics generated by the 50 percent criterion, although starting high, rapidly approach the other two. This circumstance points in the direction of "quantitative" rather than "qualitative" learning. Namely, the number of "serious" candidates does not change much over time, while the gap in votes between them as a group and the others in the race deepens. Slow

[10] Taiwanese electoral data are courtesy of Professor Emerson Niou, Duke University.

descent of the vote in early Taiwanese elections suggests that the second measure of the number of "serious" competitors - counting those candidates whose vote is comparable to the minimal winning vote in their districts - may produce results more consistent across different thresholds. This expectation is confirmed by Figure 4.12. When we only count losers whose vote is comparable to the minimal winning vote in their district, the number of excess candidates stays mostly at the predicted "one or below" for the 20 percent threshold, and all three thresholds generate comparable numbers.

[Figure 4.12 is about here]

The difference between the two ways of counting candidates, then, is much greater in Taiwan than in Japan. However, an interesting feature of at least the first three Taiwanese elections (1980, 1983, and 1986) is that no institutionalized (party) alternative to the Kuomintang existed, which implied minimal coordination within the opposition with respect to entry and campaigns. Hence, a large number of unaffiliated candidates could run, with each of them likely to either miscalculate his chances or to act in pursuit of non-electoral goals. Indeed, unlike Japanese elections between 1958 and 1993, candidate participation is high in Taiwan. Nevertheless, if one ignores crowds of non-serious candidates, the same pattern emerges as in Japan. The fact that electoral districts were so few at that time (eight, returning to ten candidates each), allows us to look at the by-district dynamics. The 20 percent drop occurs fairly early, and districts display some stability in terms of the number of "serious" excess candidates that is unrelated to the overall number of "actual" candidates in those districts (see Addendum to Chapter 3).

4.5.2 Distribution of votes

As with the number of "serious" candidates, Taiwanese elections generate a pattern similar to the one we observe in Japan in terms of candidates' relative strengths. Figure 4.13 shows the relation between the vote for differently ranked candidates.

[Figure 4.13 is about here]

Recall that to correspond to our analytical predictions, the ratios of the vote should be close to one on the boundary of the set of winners (i.e., the first loser should have nearly as many votes as the last winner), and somewhere below one within the set of winners (the first and last winners can have electoral support of substantially different size). In contrast, the implications of Cox's strategic voter model are met if the ratio of the vote within the set of winners is approximately one, and somewhere (although not necessarily "drastically") below one on the boundary of the set of winners. In fact, the values of various vote ratios stay close to the values we report for Japan, thereby reinforcing our conclusion about the superior performance of a candidate-based model.[11]

[11] Cox's vote ratio within the set of winners equals about .6, which is below the number for Japan. But Taiwanese electoral districts are on average larger, and, therefore, the ranks of the candidates being compared by this ratio are further apart. Elimination of the most popular

4.6 Implications of SNTV for Party Systems

It follows from our theoretical analysis that candidates within a district have considerable incentive to coordinate strategies. Neither a mechanism of enforcement nor *ad hoc* cooperative assumptions are needed to explain the potentially coherent behavior among the winning candidates. The mutual goal of safe winners is to deter entry and, thus, to preserve the status-quo. And, at least in theory, all but one winning candidate are safe winners in equilibrium, while the candidates who are to become the last winner and the first loser cannot improve their situation as they already occupy the best available locations.

The existence of strong national parties then, even if they participate only by nominating or endorsing candidates, obscures the picture of within-district electoral competition. Indeed, the reason why candidates would play cooperatively in a non-cooperative game is that they achieve "satiation" once they have secured seats for themselves, as no single candidate can aspire to more than one seat. The common interest of candidates in a district pushes them to coordinate their actions across party lines, rather than to seek a partisan expansion. This is not true for parties. Parties may have candidate-like incentives only if they are unlikely to place more that one candidate per district into the legislature. Once parties become sufficiently strong to attempt an increase of the number of seats they control in the

candidate in each district (to neutralize any extreme populist vote) does not change the results as sharply as it did in Japan, but one explanation may be the lower number of nationally renown politicians competing in districts. And as in the Japanese case, the ratio of the vote for the first loser to the vote for the last winner stays fairly high, averaging to 0.91 across elections.

district, implicit cooperation among the set of winning candidates is threatened. National parties can introduce challengers in the districts that not only jeopardize the safety of their incumbents, but through the national campaign can disrupt equilibria in "unchallenged" districts as well.

For this reason factionalism in the Japanese party system may have been stabilizing, since factions would normally have not more than one candidate per district, and would resist nominations from their own party that could diminish the security of their candidates. Factions can also distance themselves from a party campaign if that campaign is not to their advantage.

Another feature of Japanese elections is the existence of independent candidates who are endorsed by one or more than one party. Their presence can also be viewed as a manifestation of the gap between district-level and national-level politics. Namely, a structure of voter preferences at the district level at odds with the national picture may require substantial adjustments in party political platforms, so that an independent candidate endorsed by the party would do better. Alternatively, the distribution of voter ideal points may be unusually "thin" where party platforms are spatially concentrated. As a consequence this residual vote has to be taken care of in cooperation with other parties through joint nomination of an independent candidate, so as not to invite organized entry.

Another important relationship - between parties and voters - is modified as well by the electoral imperatives generated by SNTV. SNTV provides strong incentives for parties to coordinate voter strategies within a district. Indeed, as we have seen in our theoretical development (Chapter 3), successful candidates (and their respective local party organizations), once they anticipate enough support to win a seat, should be primarily concerned with preserving a "properly structured" competition within the district. Theoretically at least, electorally-minded candidates may wish to support other competitors in their district with some of their own vote to preserve the overall balance. Alternatively, candidates may find it useful to suppress their own turnout so as to make entry less attractive. Either way, we are talking about candidates manipulating the effective distribution of voter preferences, which can be done if parties can influence voters strategies.

This later possibility, then, that parties might be the instruments of influencing the observable distribution of voter preferences, suggests yet another role for parties. Suppose, for a given district magnitude and preference distribution, that no equilibrium campaign configuration exists. We know, however, that for some derivative preference distribution, after adjustments in turnout, for instance, there may be an equilibrium. Parties then may be a tool for "correcting imperfections" in the distribution of preferences so as to generate a favorable equilibrium, one that maintains the viability of almost all incumbent politicians.

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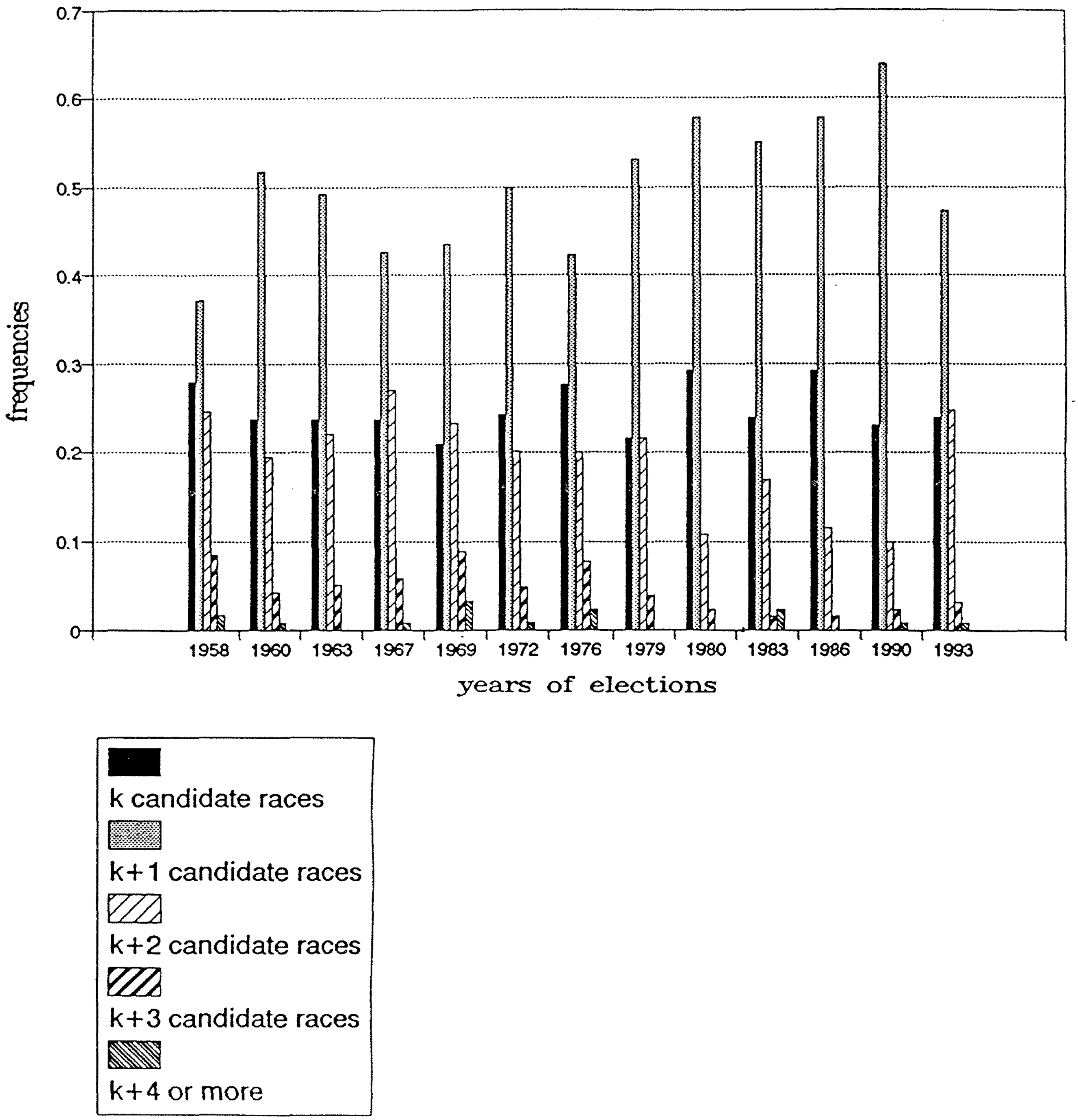
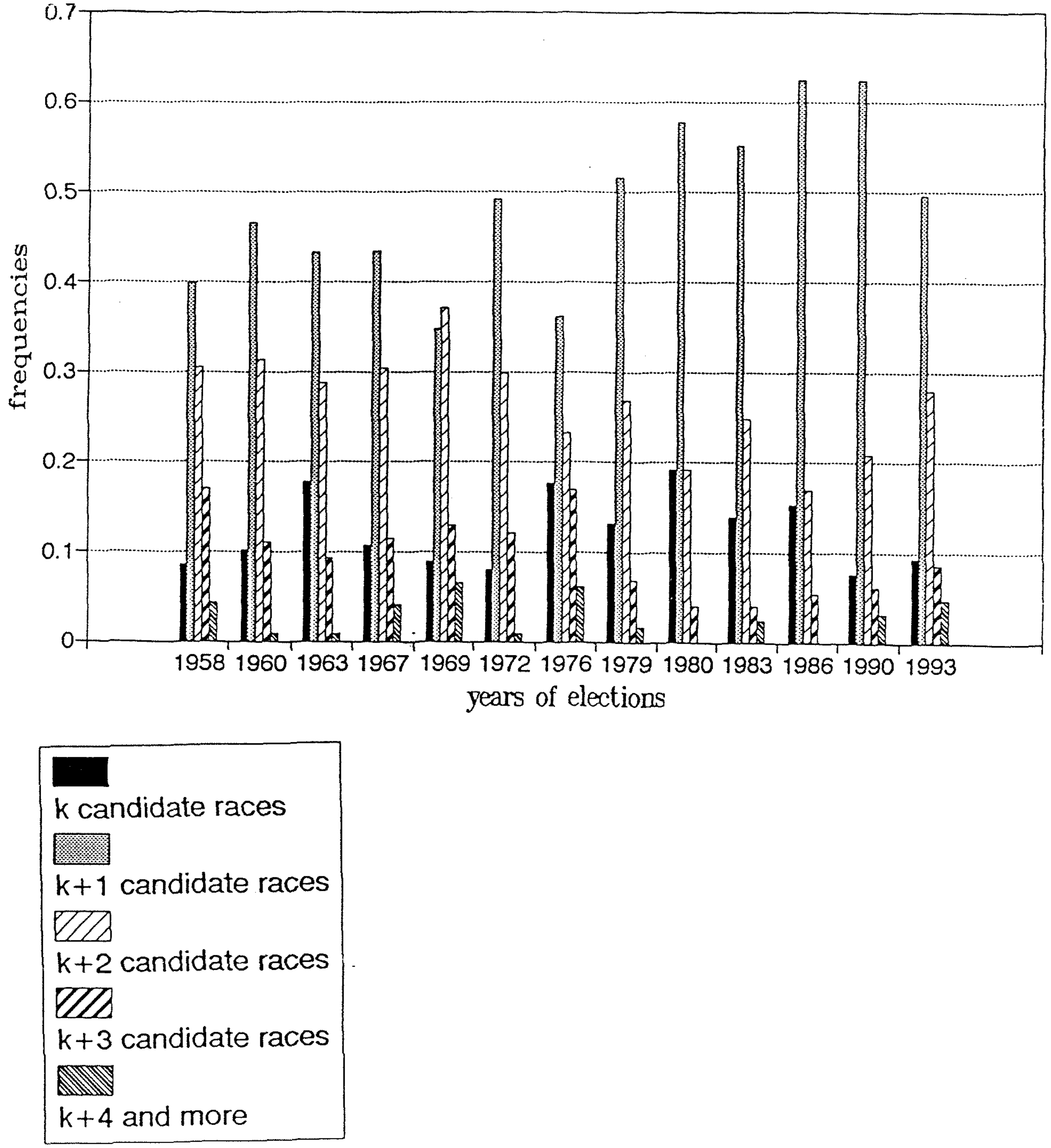
Figure 4.1 Location of the First Substantial Decline in Vote Outside the Set of Winners

4.1a: Considered decline is 20% or more



4.1b: Considered decline is 33% or more



4.1c: Considered decline is 50% or more

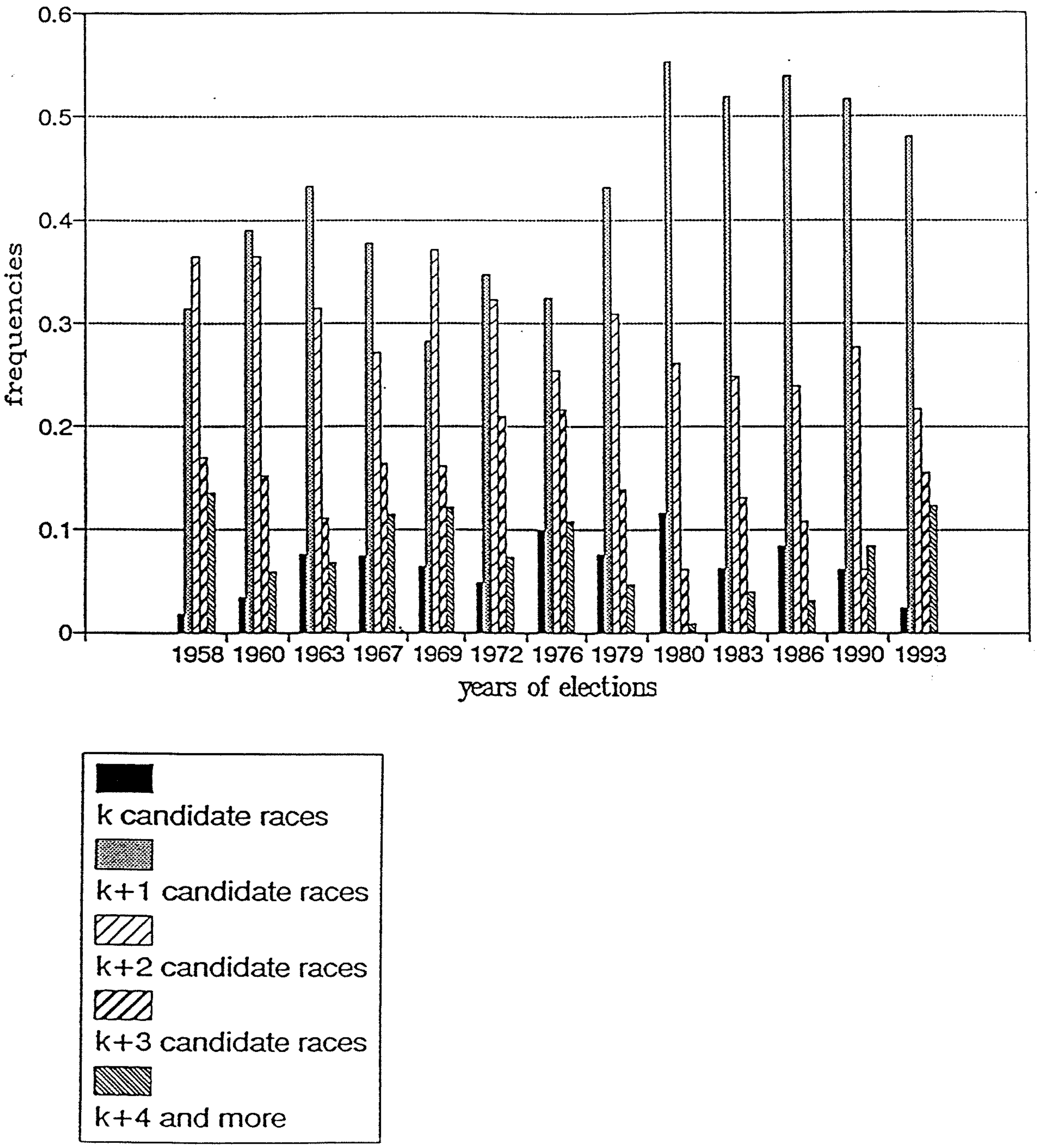
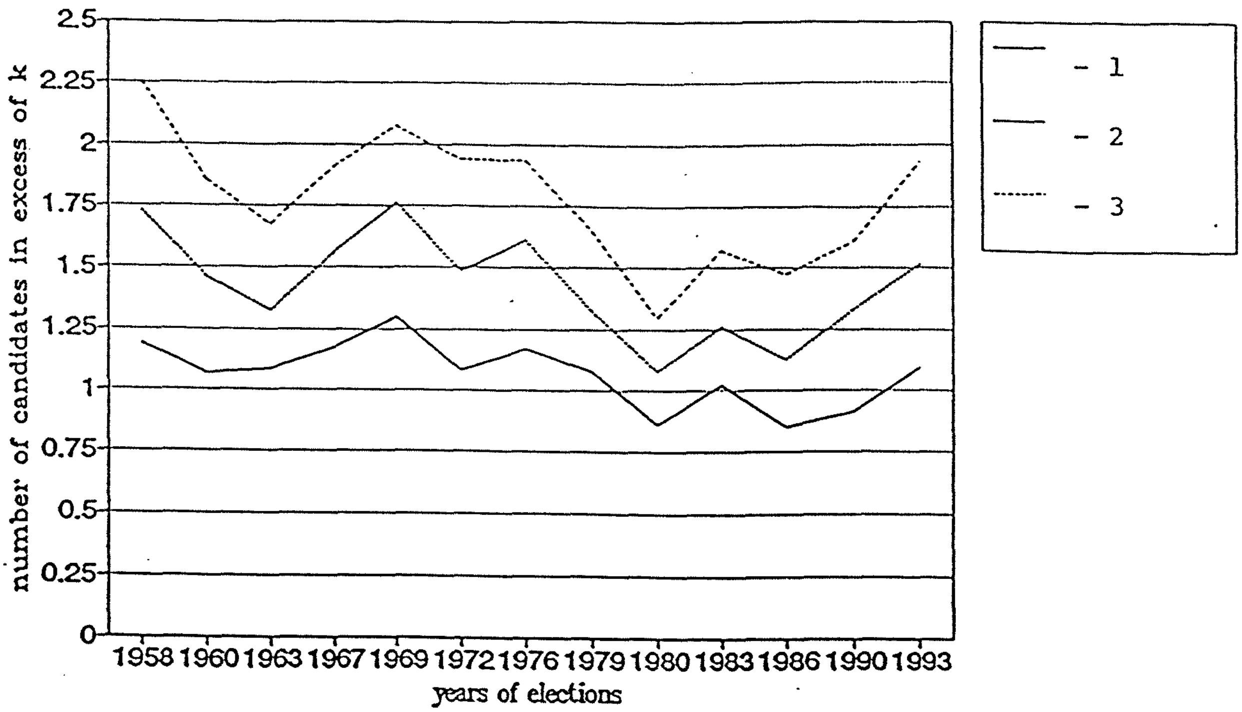
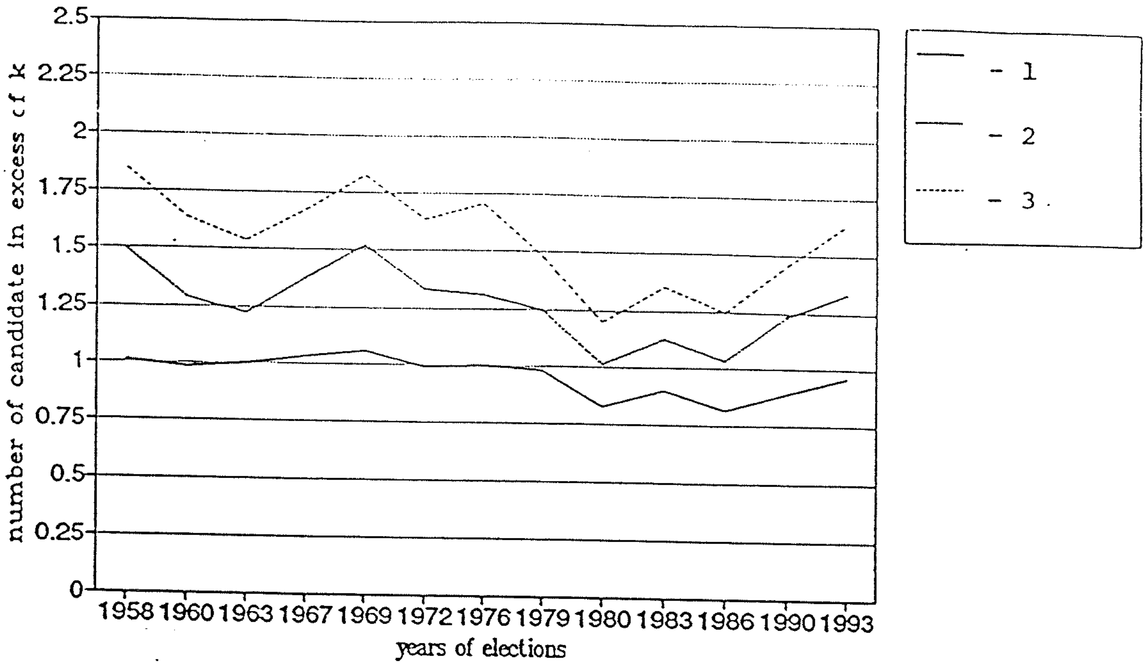


Figure 4.2. **Average Number of "Serious" Candidates per Race in Excess of the District Size**



1 - **before the first 20% drop in vote;**
2 - **before the first 33% drop in vote;**
3 - **before the first 50% drop in vote.**

Figure 4.3. **Average Number of "Competitive" Candidates per Race**



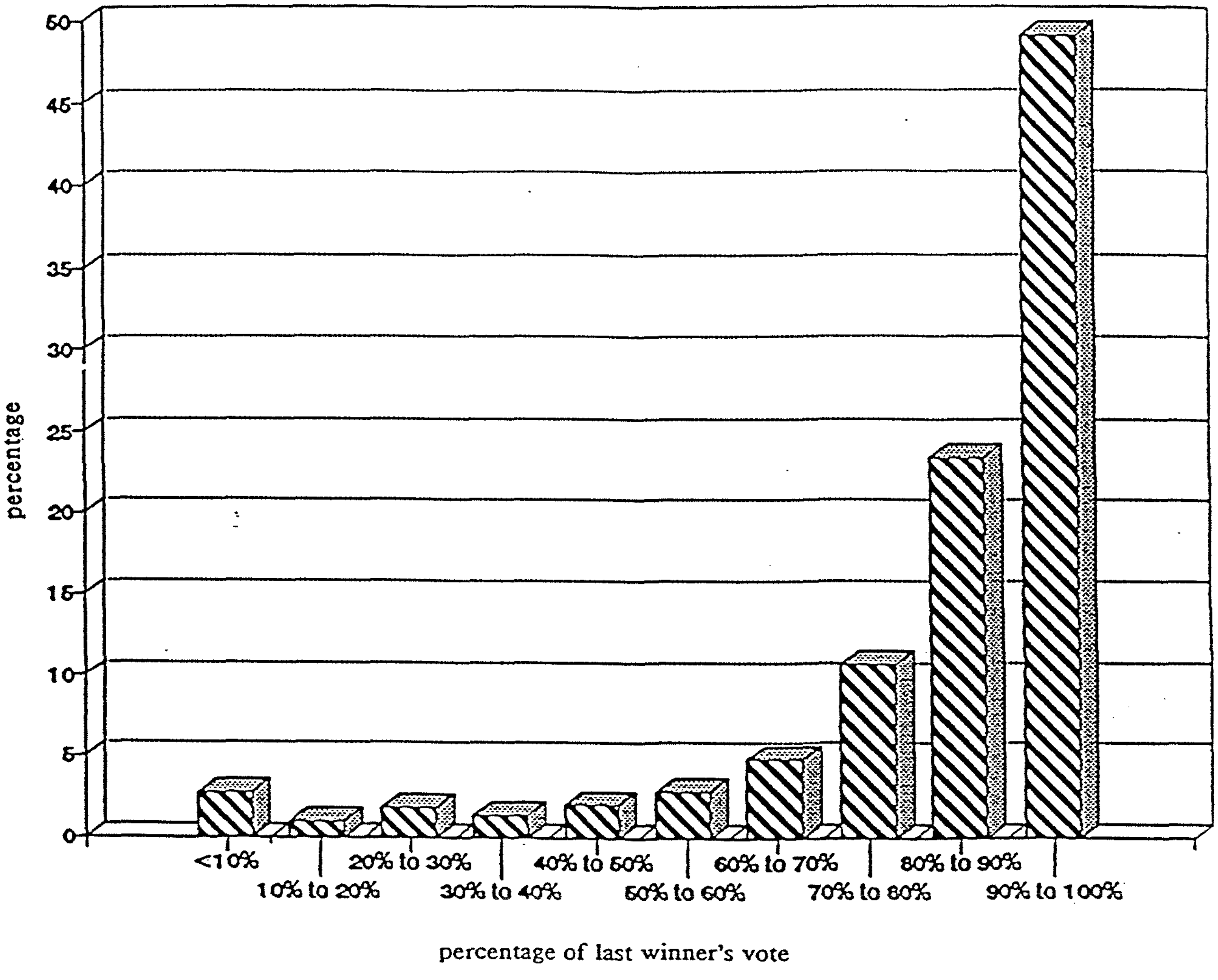
1 - **considering losers with as little as 50% of the minimal winning vote in their districts;**

2 - **with 67 or more percent;**

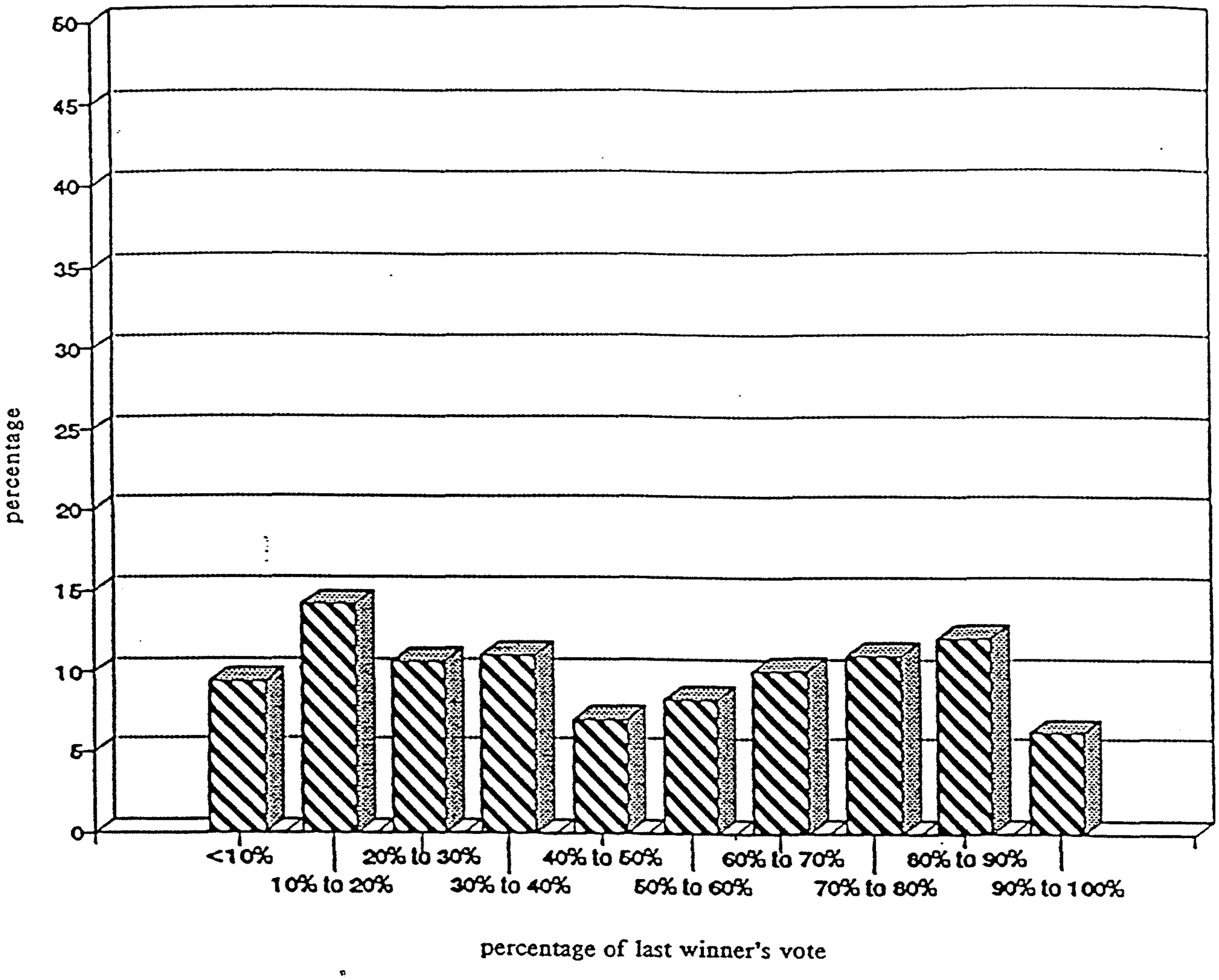
3 - **with 80% or more.**

Figure 4.4. **Performance of Differently Ranked Losers Relatively to Their Districts' Last Winner Vote - Japan**

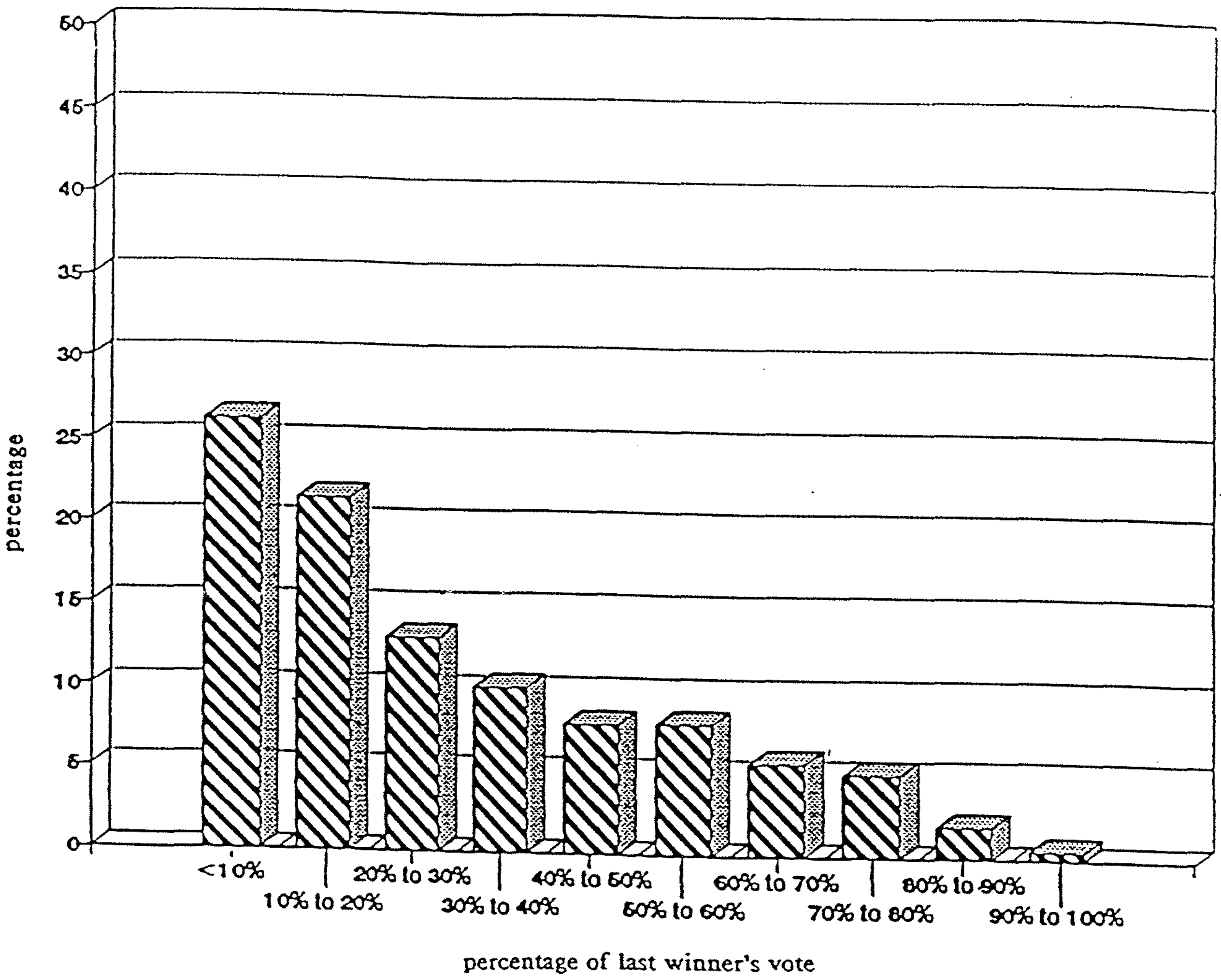
4.4a. First loser's vote as a percentage of last winner's vote



4.4b. Second loser's vote as a percentage of a last winner's vote - Japan



4.4c. Third loser's vote as a percentage of last winner's vote



4.4d. Fourth loser's vote as a percentage of last winner's vote

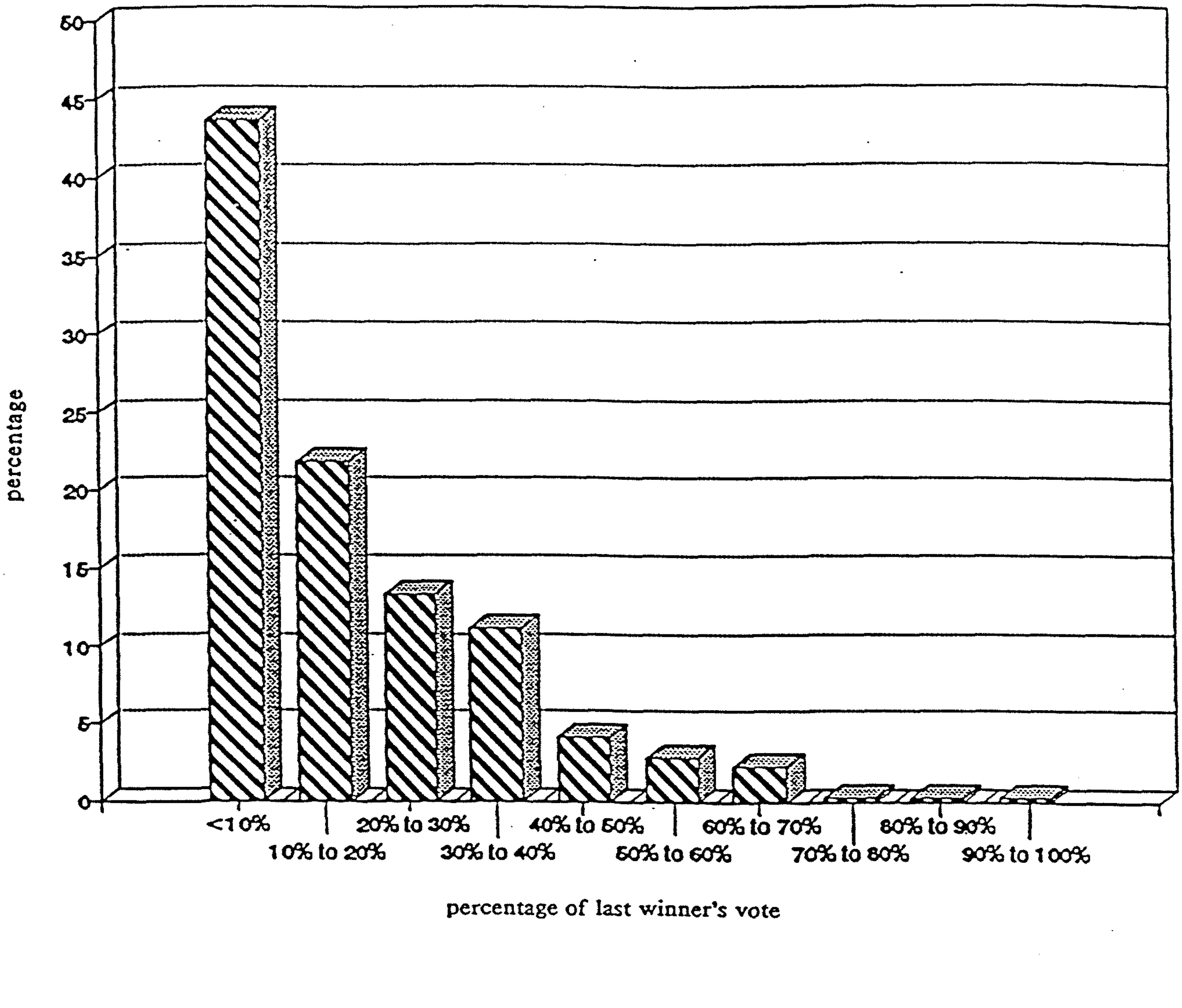
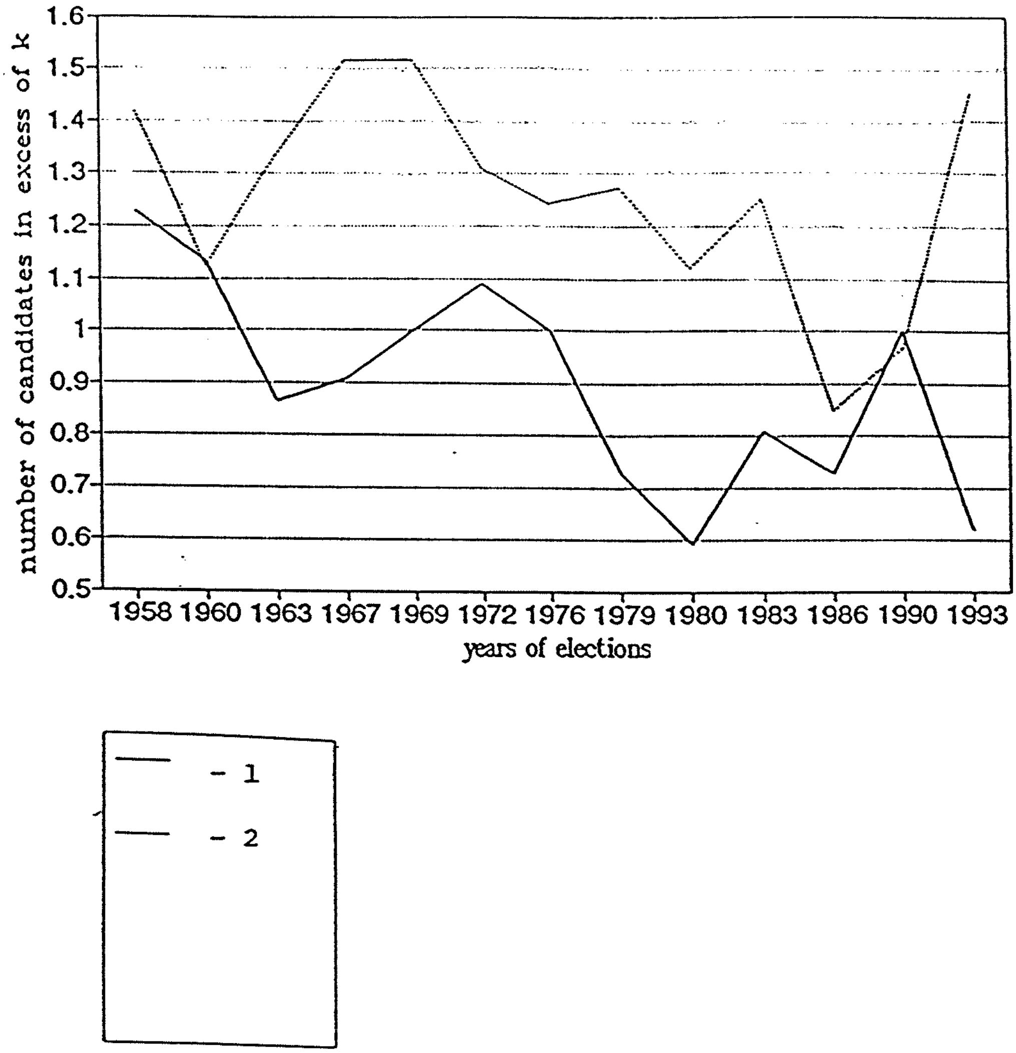


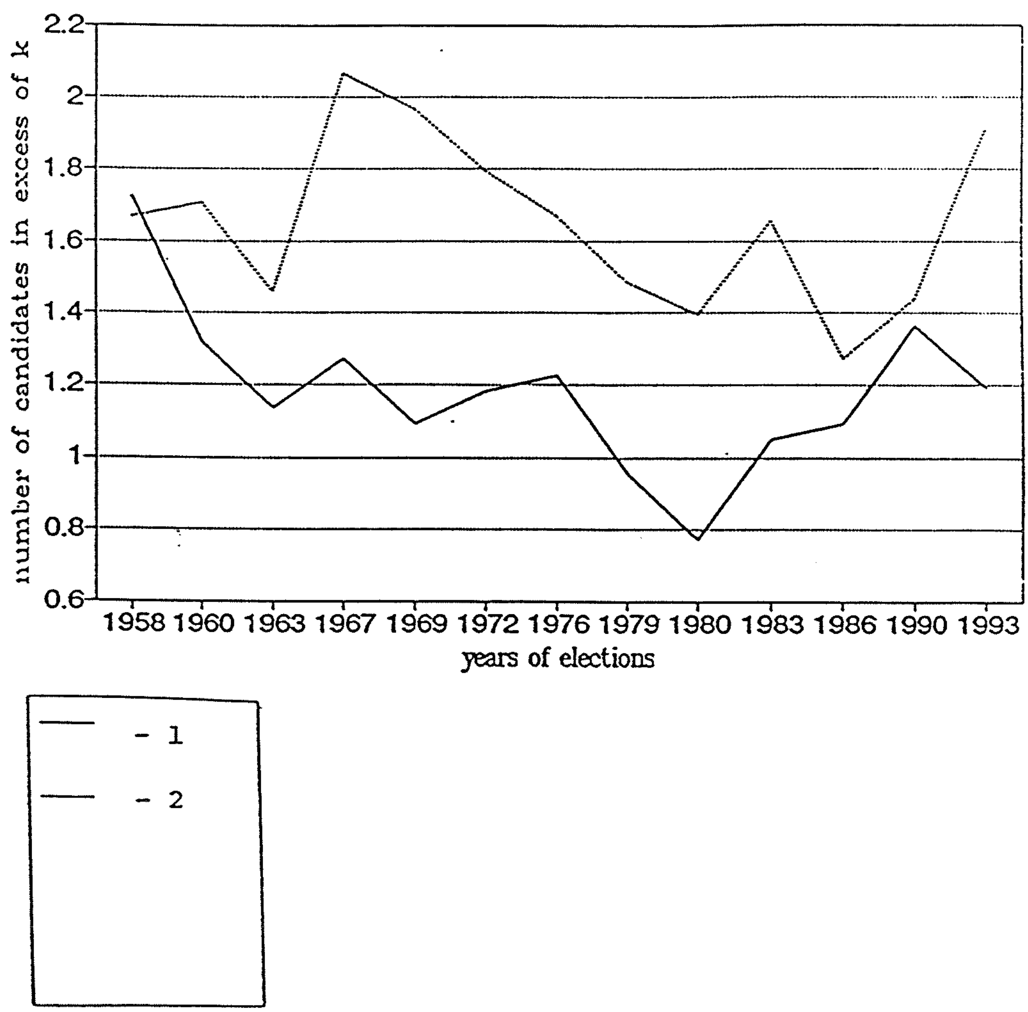
Figure 4.5. **Average Number of "Serious" Excess Candidates before the First Substantial Drop in the Vote among Losers in Constituencies of Different Demographic Types - Japan**

4.5a: Considering a drop of 20% or more



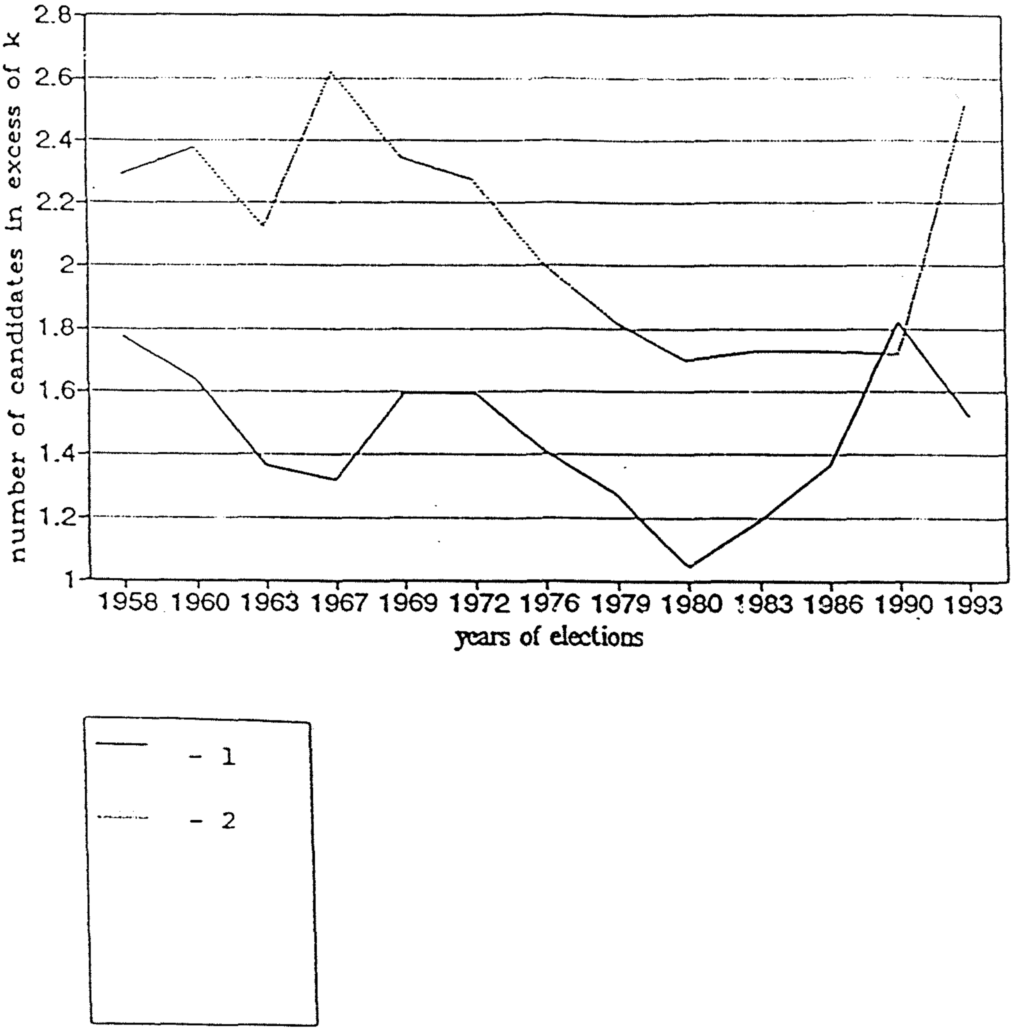
1 - in rural districts; **2 - in metropolitan districts**

4.5b: Considering a drop of 33% or more



1 - in rural districts; **2 - in metropolitan districts**

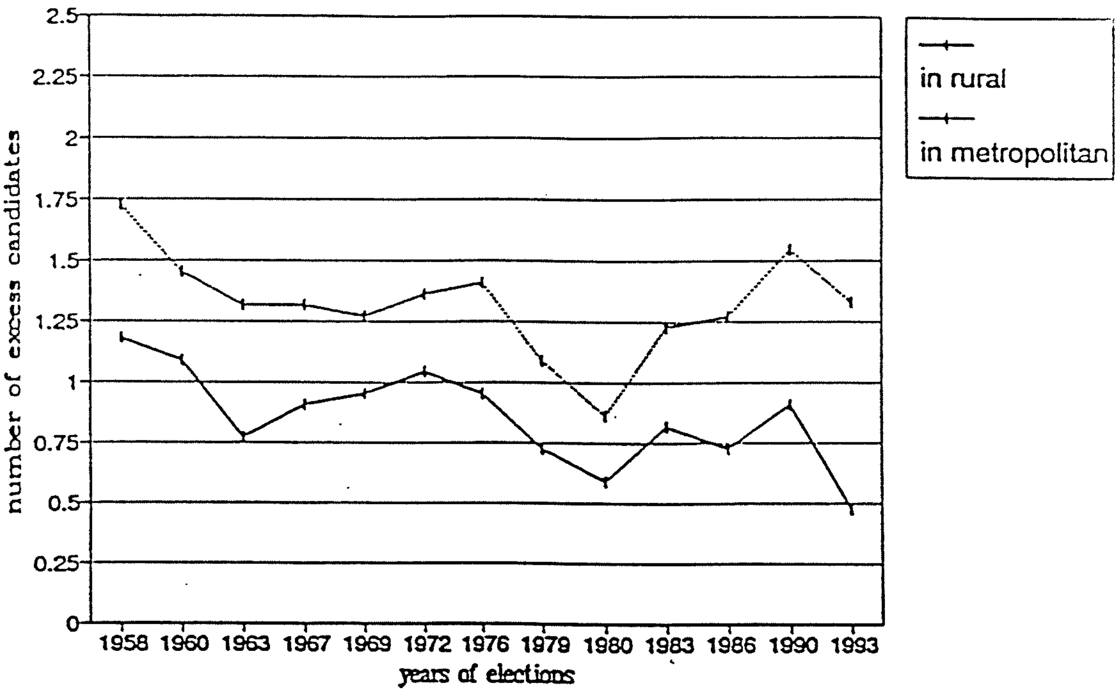
4.5c: Considering a drop of 50% or more



1 - in rural districts; 2 - in metropolitan districts

Figure 4.6. **Average Number of "Competitive" Losers per Race in Constituencies of Different Demographic Types - Japan**

4.6a: Considering candidates with 80% of their districts' minimal winning vote



4.6b: Considering candidates with at least 50% of their districts' minimal winning vote

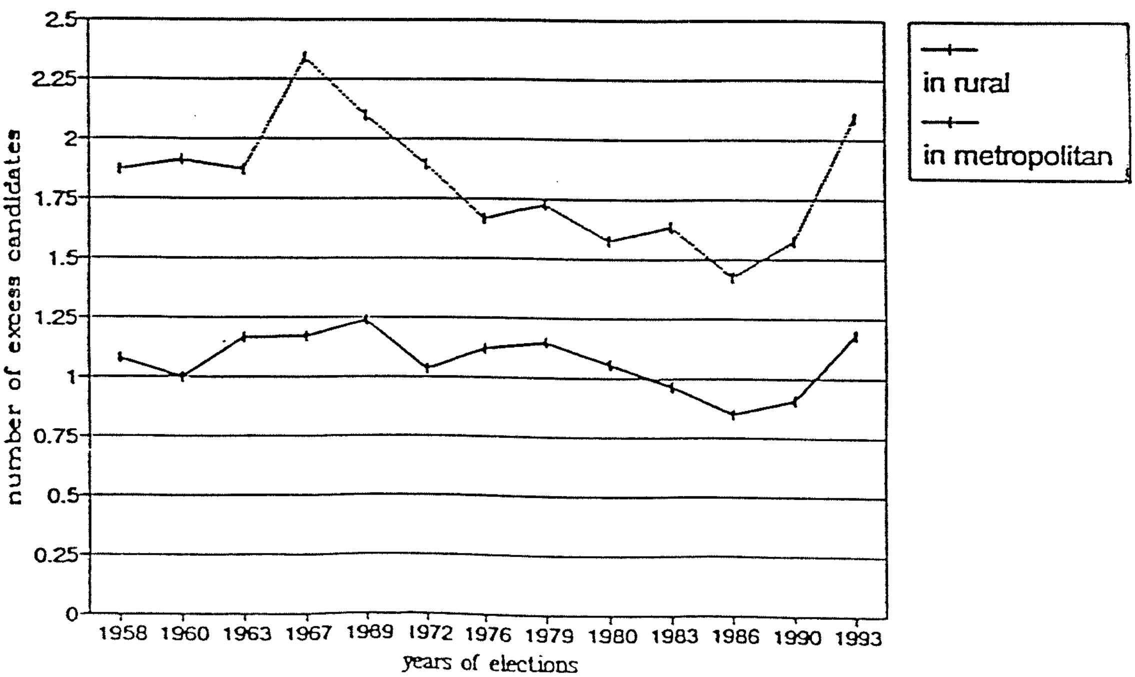
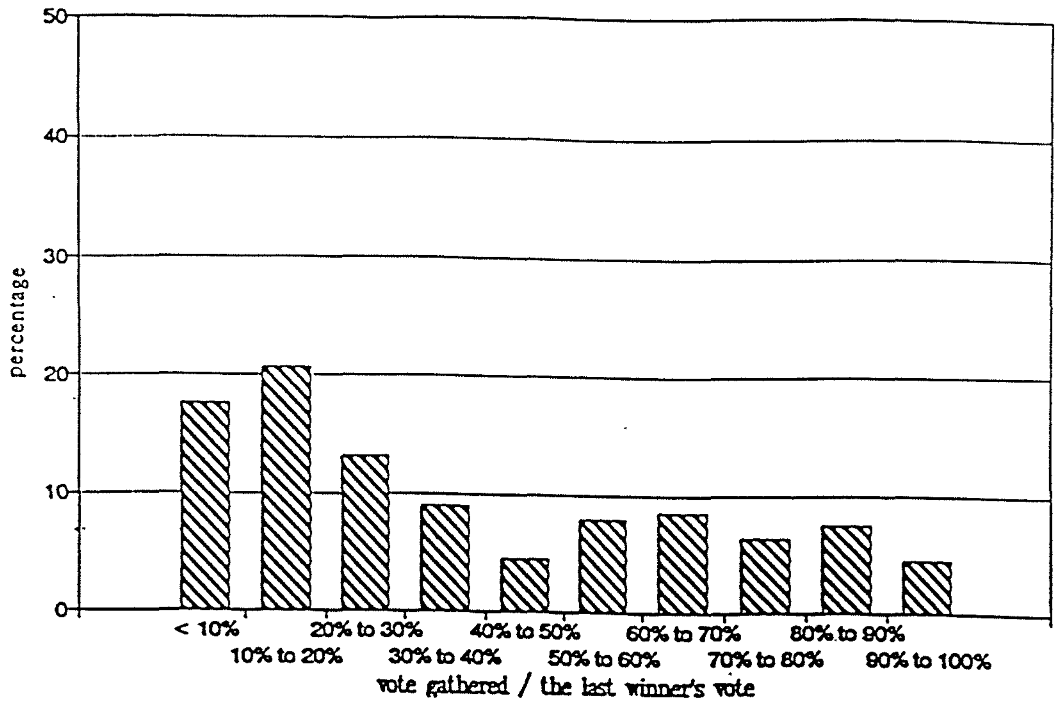


Figure 4.7. **Second Loser's Vote as a Percentage of the Last Winner's Vote - Japan**

4.7a. Rural constituencies



4.7b. Metropolitan Constituencies

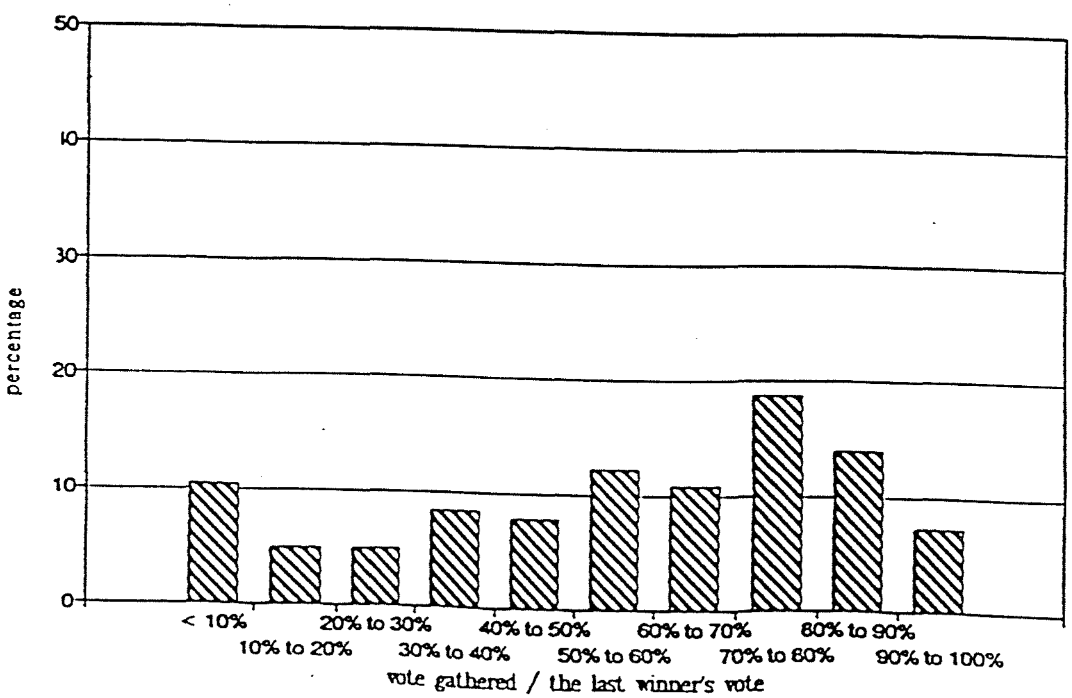
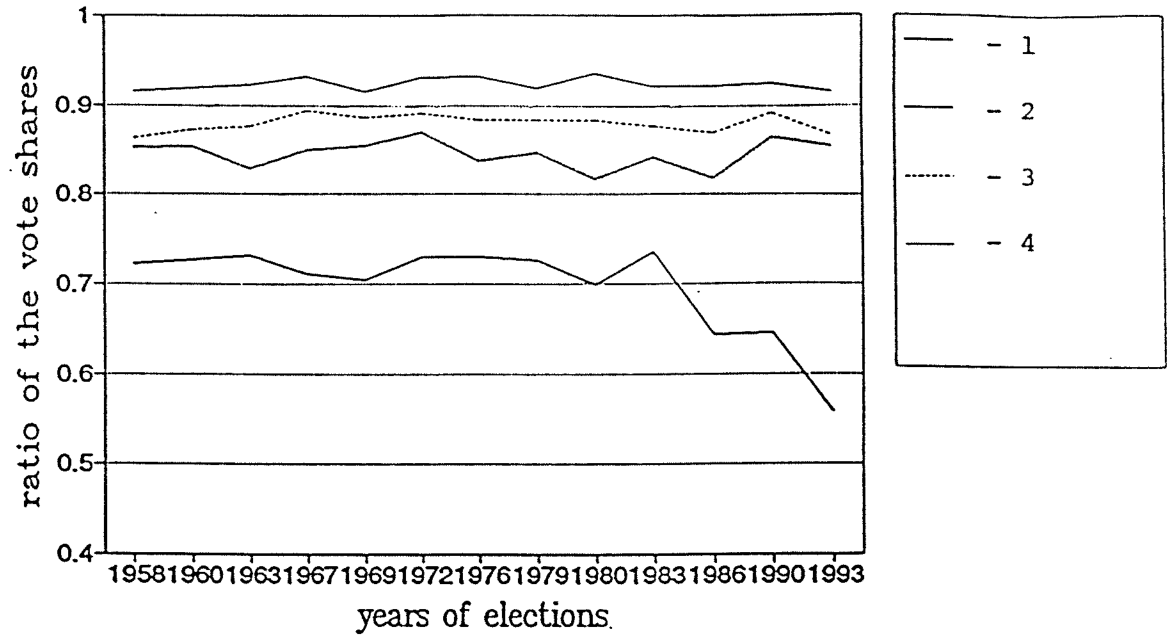


Figure 4.8. **Comparison of H1 and H2 in Terms of Vote Dynamics**



1 - **ratio of the vote within the set of winners - Cox 1993 (H1: $k^{th}/first=1$)**

ratio of the vote for the first loser to the vote for the last winner:

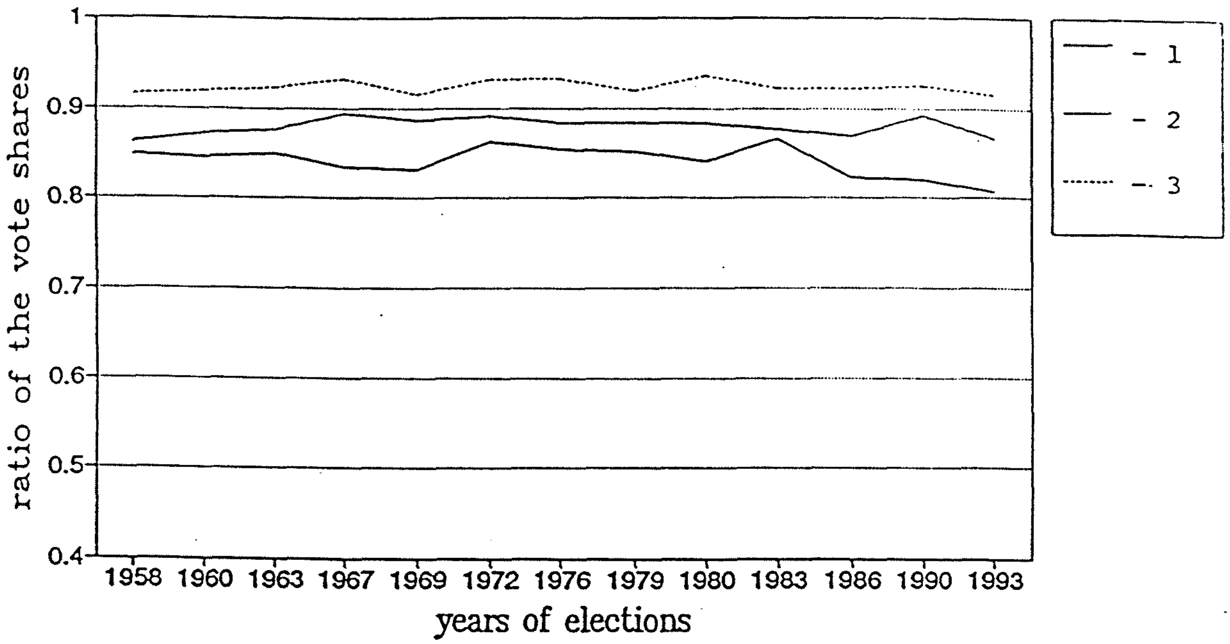
2 - **in all races;**

in competitive races

3 - **when the first loser is not more than 20% behind the last winner;**

4 - **when the first loser is not more than 50% behind the last winner.**

Figure 4.9. **Comparison of H1 and H2 Controlled for the Populist vote - Japan**



1 - **ratio of the vote for the last winner to the vote for the second winner;**

ratio of the vote for the first loser to the vote for the last winner in competitive races:

2 - **when the first loser is not more than 50% behind the last winner;**

3 - **when the first loser is not more than 20% behind the last winner.**

Figure 4.10. **Location of the First 33% Decline in the Vote Outside the Set of Winners - Taiwan**

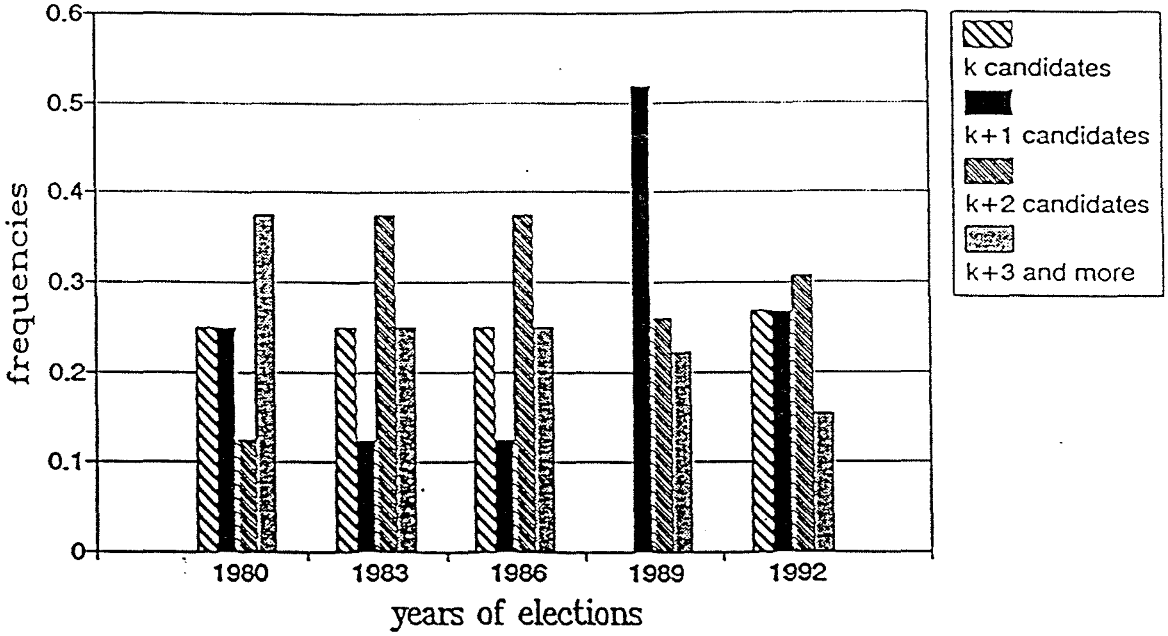
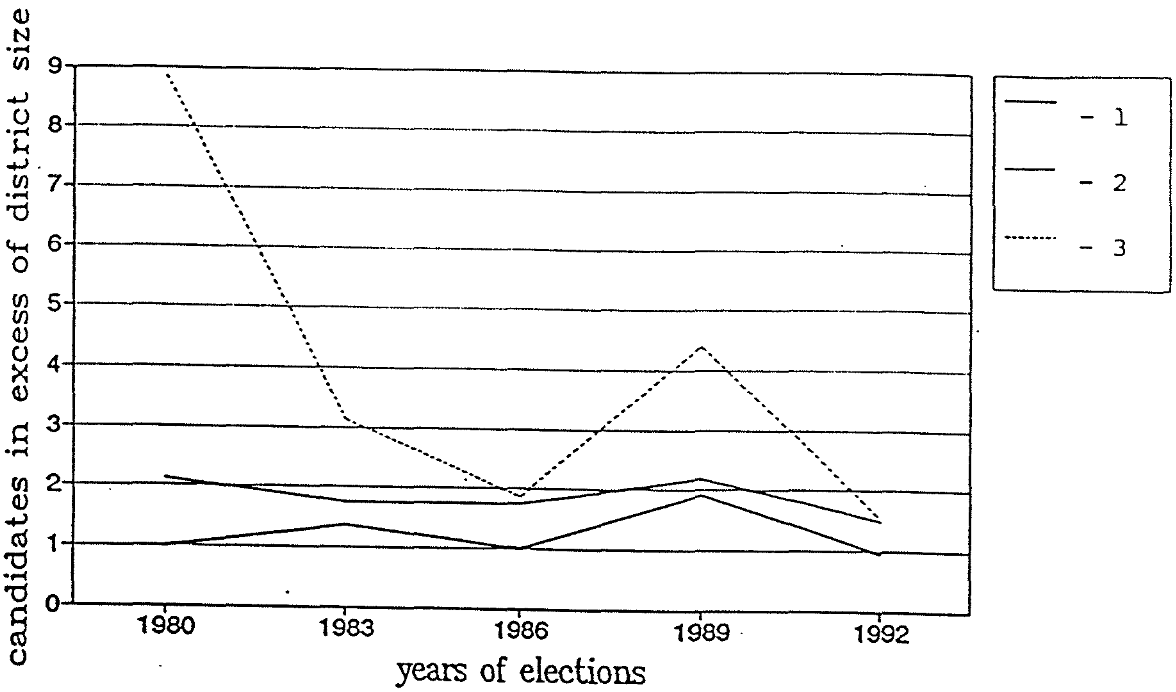
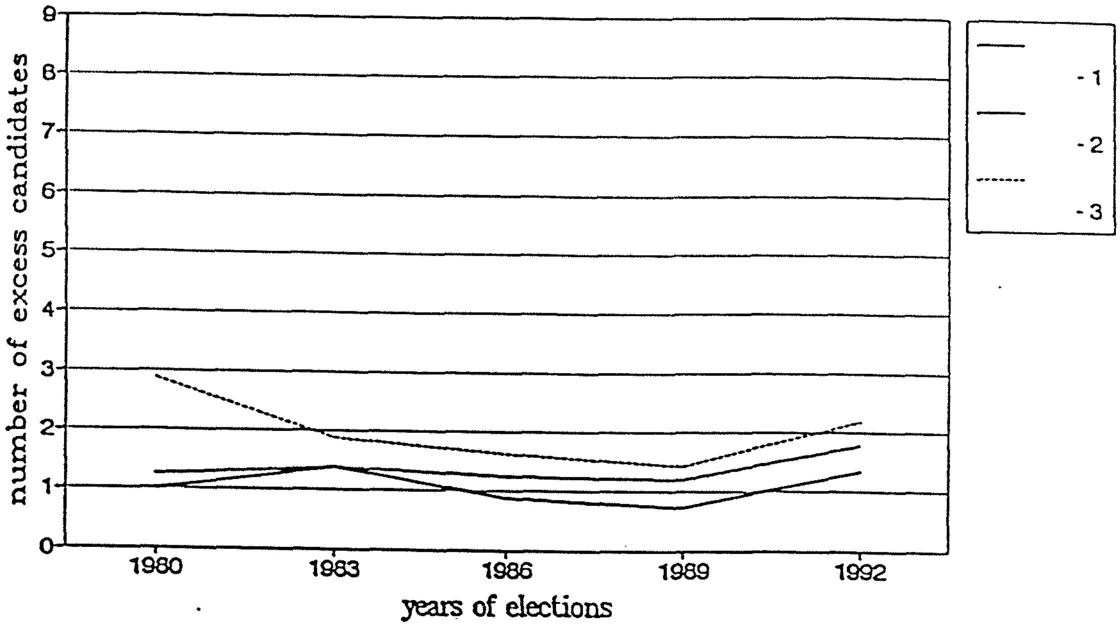


Figure 4.11. **Average Number of "Serious" Candidates per Race in Excess of the District Size -Taiwan**



1 - **before the first 20% drop in the vote outside the set of winners;**
2 - **before the first 33% drop in the vote outside the set of winners;**
3 - **before the first 50% drop in the vote outside the set of winners.**

Figure 4.12. **Average Number of "Competitive" Losers per Race - Taiwan**

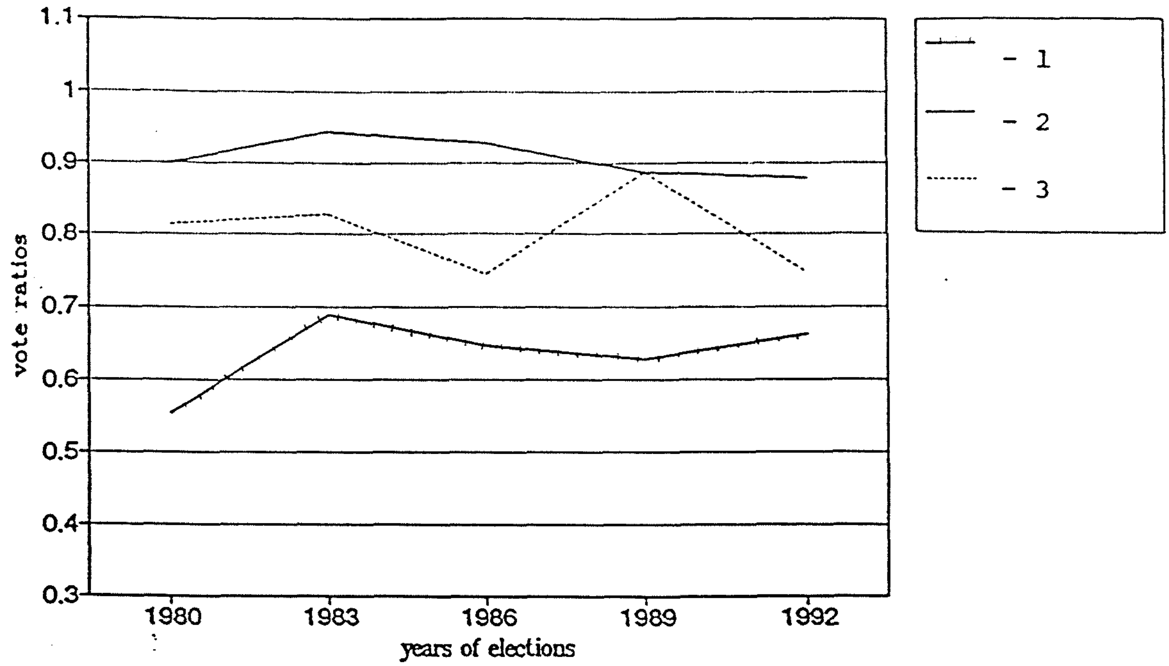


1 - **considering candidates with at least 80% of their districts' minimal winning vote;**

2 - **considering candidates with at least 67% of their districts' minimal winning vote;**

3 - **considering candidates with at least 50% of their districts' minimal winning vote;**

Figure 4.13. **Comparison of H1 and H2 (Vote Ratios) - Taiwan Data**



1 - ratio of the vote within the set of winners - Cox 1993 (H1: k^{th}/*first=1*)

ratio of the vote for the first loser to the vote for the last winner:

2 - in all races;

3 - in competitive races when the first loser is not more than 20% behind the last winner.

APPENDIX: DEPARTING FROM SOME ASSUMPTIONS: POLARIZED PREFERENCES

Analysis in Chapter 3 of this work does not apply to the cases of complex electoral preferences within districts, such as non-spatial issues or bimodally or multimodally distributed spatial preferences. Instead we assume that voter ideal points are unimodally distributed over the single-dimensional policy. But this assumption may not be met in many electoral districts included in our data. Voter preferences in some districts may correspond to this assumption closer than in the others. As was mentioned earlier, the structure of competition appears significantly different in rural and metropolitan constituencies. We believe that rural constituencies correspond better to our assumptions, and in fact the predictions of our model are met closer in those districts.

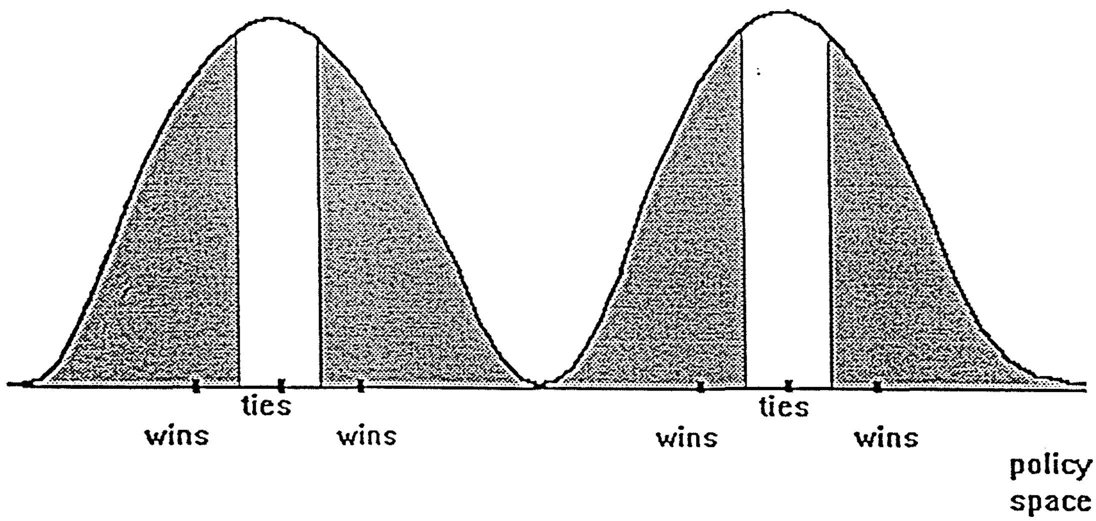
Even though we are not in the position to give an exhaustive description of what might happen in the non-unimodal cases, we can offer examples of equilibria for certain special distributions, that may prove generalizable - notably, some bimodal (polarized) distributions and quasi-concave configurations that begin to approximate a uniform distribution.

In Chapter 3 we establish the existence of $k+1$ equilibria (k odd) for all symmetric unimodal distributions and for distributions that approximate them "sufficiently closely". And although we do not know whether equilibria exist for other unimodal distributions, we know that if they exist, they cannot be but of the same form - either k, or $k+1$ with two tied candidates at the mode.

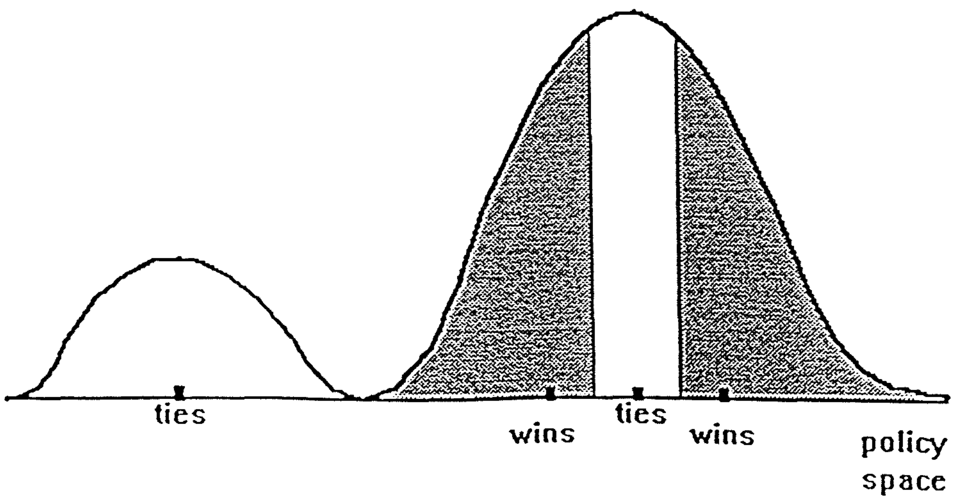
Now there are certain things that we can say about the number of candidates in polarized (bimodal) constituencies. First, building on the above mentioned result, if the distribution of voter preferences is bimodal and symmetric, as well as symmetric around each mode, then at least for $k=4a+1$ (where a is a positive integer) a $k+1$ equilibrium exists (see Figure A1). Without symmetry around each mode, a $k+1$ equilibrium may exist that looks like the configuration in Figure A2. Figures A3 and A4 illustrate $k+2$ equilibria. Note, that the configuration in Figure A4 is simply a combination of two $k+1$ equilibria for unimodal distributions. Note also that the same configuration is an equilibrium if district magnitude is five, six or seven.

The intermediate case between a strict unimodal distribution and a polarized (bi-modal) constituency occurs when the preference distributions has a flat region for a mode. In this case the number of candidates in excess of k that compete in equilibrium depends on the size of the district and the specific form of the distribution, because as we know from Chapter 3, a uniform preference distribution can support a great variety of equilibria configurations. The existence of finite slopes in $f(x)$ and how candidates respond to them restrict this variety. Figures A5 and A6 illustrate the $k+1$ and $k+3$ equilibria.

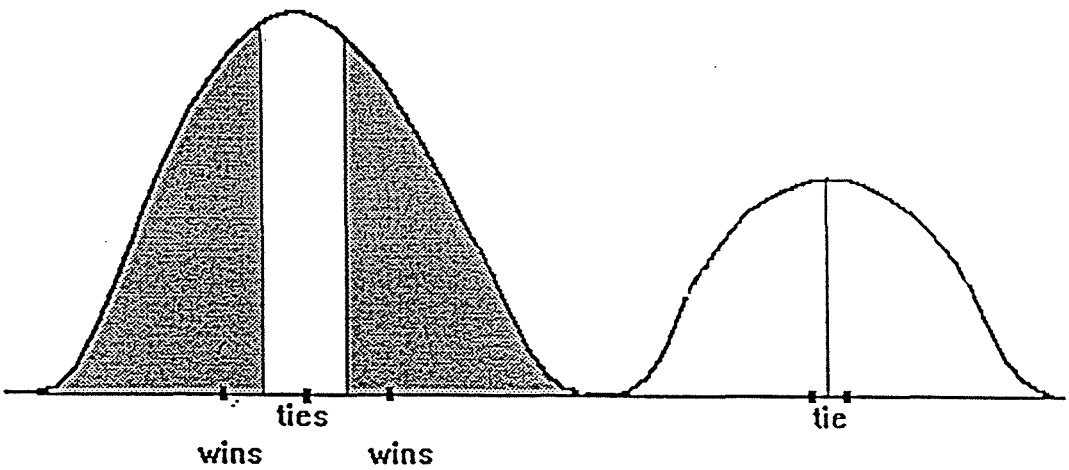
A1: Example of a k+1 equilibrium (k=5)



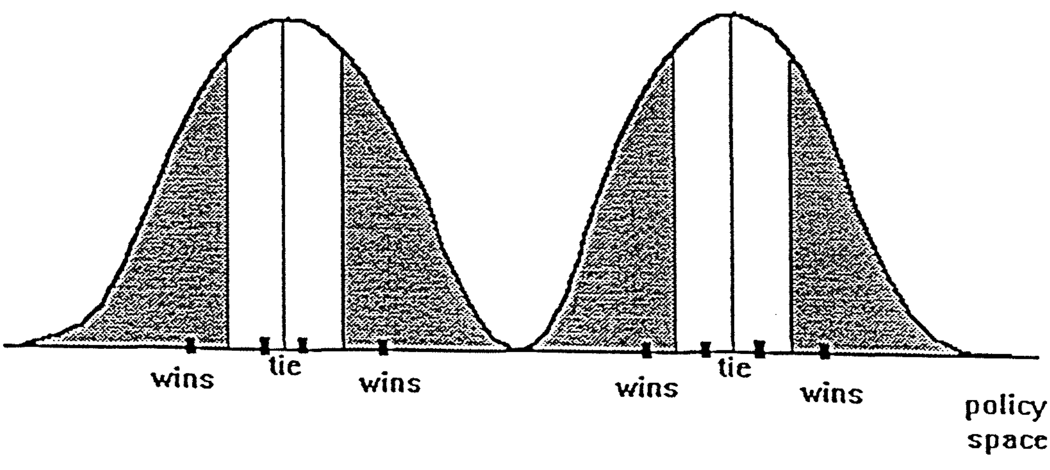
A2: Example of an asymmetric k+1 equilibrium (k=3)



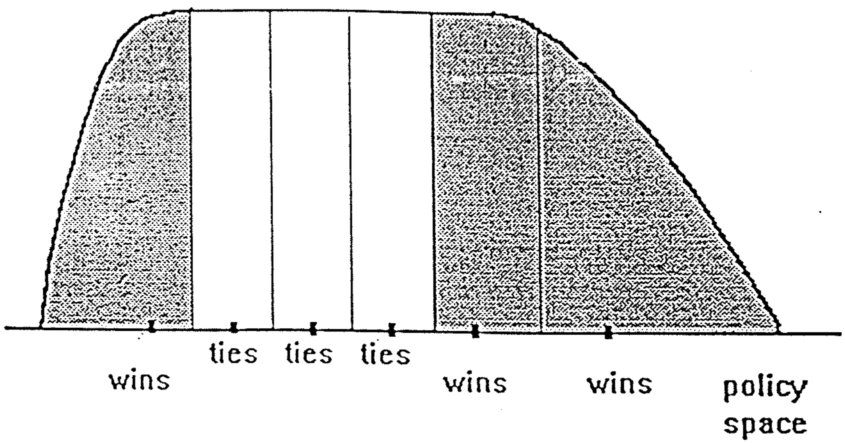
A3: Example of a k+2 equilibrium (k=3)



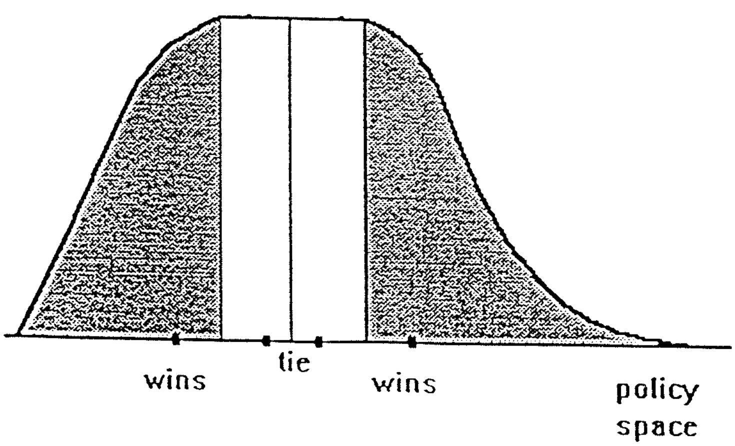
A4: k+2 equilibrium as a combination of two k+1 equilibria (k=5, 6, or 7)



A6: Possibility of a k+2 equilibrium when distribution of preferences has a "flat" portion (k=4)



A5: Possibility of a k+1 equilibrium when distribution of preferences has a "flat" portion (k=3)



CHAPTER 5: MODELING PARTY COMPETITION WITH ENTRY

5.1 Parties as the Key Players

The object of study in this chapter is electoral competition under proportional representation. Because the concept of proportional representation is applied to parties, our analysis is necessarily confined to competition among political parties. Proportionality in representation as conventionally understood deals with how well in proportion to their electoral support (either regional or nationwide) *existing parties* are represented in the legislature. Thus, "proportionality" has only an indirect connection to the proportion with which "interests," groups, or opinions receive recognition in the government. The difference between proportionality in the electoral and in this "social" sense derives from the fact that the vote received by political parties is not an "objective" description of social preferences, but can itself be biased by the rules of election. Indeed, one can even argue that what political parties emerge and where they stand in relation to the electorate is derivative of the politicians' and voters' awareness of the rules, i.e., that parties themselves may be implied by the electoral system, as opposed to just constituency preferences.

As we note earlier (Chapter 1), the choice of what electoral rules to adopt itself may reflect current needs of parties in power (see, for example, Bawn 1993). However, we do not address this direction of causality. Instead, in this as in earlier chapters, we look at how electoral rules influence the party system through the rational responses of politicians to those rules. The difference between this and earlier chapters is that now we view politicians as representing the interests of political parties who wish to receive as many legislative seats as possible, rather than individual candidates who pursue one seat.

Our more specific objectives are to establish some of the effects of proportional electoral rules on party formation, and on the policy platforms of parties that decide to compete. To this end we view parties like individual candidates in that a party's policy platform is a point in the policy space. In other words, we view parties not as coalitions of individual politicians each with a distinct policy stand, but rather as a single position introduced to voters, perhaps by one noticeable figure (say, the party chairman) whose views are taken as the party's platform. This simplified description deprives parties of the many organizational and political features they might use to increase their competitiveness and survivability. But this simplification does allow us to determine the "raw" electoral institutional influence on the number, locations, and relative strengths of the parties.

Insofar as previous research is concerned, Cox and Shugart (1994) show that once voters are presumed to be strategic, the excess votes a party receives on top of the seat-yielding quotas may depend on institutional rules of seat allocation. In their model strategic voters deprive a party of excess votes if the next seat for that party is unattainable. However, except for the last seat, that model leaves unexplained all other seats a party receives. It also does not tell us why no other party or candidate would attempt to "steal" those seats, and the model generates no predictions about the number and relative strengths of parties. Thus, the model cannot move us in the direction of answering the key question that should concern voters in party list PR elections: Who forms the government.

Schofield (1994) suggests approaching the voters' calculus from a different perspective. In Cox and Shugart's model, voters seek to elect as many most preferred MPs as possible, but when their vote cannot help their first-best party, they spend it elsewhere. If instead voters are required to think strategically about the final outcome in terms of the platform of the government, we may find a trade-off between voting for a most-liked policy program and supporting the most suitable *strong* party. Voters might then forego the opportunity to elect one or two additional small party MPs, and help instead a stronger party. Schofield's approach, though, leaves unanswered any questions about the primary subject of this study - the entry of new parties.

To explore party entry, we treat voters as sincere since we are uncertain how to model their strategic calculations, and because, empirically at least, voters do vote for smaller parties.[1] More importantly, were we to restrict ourselves to considering only the rationality of voters, we would lose the important influence of political entrepreneurship on the structure of the party system. Even if one decides to ascribe rational behavior to both relevant sets of players - voters and political - the decision on how to subordinate their respective decision-making (which side is "leading" and in what sense, or are they moving "simultaneously") ought not to be made on an *ad hoc* basis, without attempting first to derive some insights by considering the one-sided cases. Thus, in this chapter we assume parties (politicians) to be

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1 Even the fairly "strategic" electorates do so, and the British Liberals' vote can serve as an example here.

strategic players.

5.2 The Model

As before we assume that voter have single-peaked preferences over a unidimensional policy space, and that they vote for parties whose electoral platforms are nearest their policy ideal points. Parties are viewed as individual actors in the sense that we do not consider the complexities of decision-making within them. We consider parties to be strategic seat-maximizers; thus we do not deal with the parties' preferences over policies. And as long as they have a chance of getting representation in the legislature, new parties are allowed to enter the competition. At some point, of course, this new entry must stop, but there is no exogenous constraint on the number of potential competitors. There is, however, a precondition for getting seats. Namely, a party's vote share must exceed some number T - a legal threshold for legislative representation. Thus, each party's objective is a compound one: it pursues additional seats only after it secures representation in the legislature.

To establish whether there exists an equilibrium party configuration in such an environment, imagine a sequential entry game in which each player has complete information about the locations of previous entrants and knows that entry can only stop endogenously. Each player i's strategy space is a set of all possible locations for entry, plus an option of not entering. For a game as specified so far, those strategies are

$$x_i \in \{\mathbb{R}; \varnothing\}.$$

Players - political parties - are seat maximizers, but only conditional on obtaining a threshold for legislative representation. Their utility increases in the number of seats, and for our purposes its functional form is of little importance.[2] We assume that the utility of a party that does not receive a threshold is 0, since it receives no seats, and that the utility of not entering the race is 0 as well. Since the latter state is the status-quo, we assume that whenever the choice is between these two zero-utility options, the latter (of not entering) is chosen.

Now if we define an equilibrium in a standard way, as a vector of strategies

$$S^* = (s_1^*, \ldots s_n^*, s_{n+1}^*) \ s.t\ \forall i=1, \ldots (n+1) \ \ u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*),$$

we loose the notion of the endogenously controlled entry. If that is taken into account, an equilibrium vector of party strategies must be

$$S^* = (x_1^*, \ldots x_n^*, s_{n+1}^* = \emptyset),$$

where the number of position-taking parties n is endogenously determined in equilibrium. Therefore, we modify the usual definition of an equilibrium, and are interested in an equilibrium vector $(C, \mathbf{x})$, where C is a set of position-taking parties of size n^*, and $\mathbf{x}$ is a vector of their respective spatial locations. The $n+1^{st}$ party, whose equilibrium strategy is to not enter,

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2

However, although it does not influence the results, it is worth noting that if we suppose that the welfare of a party is a sum of the welfare of individual candidates on its electoral list (who value primarily their own legislative seats), then, as candidates ranking on the list is likely to correspond to their weights in the party, party's incentives for securing seats for candidates are the smaller, the lower those candidates stand on its list. Of course, this reasoning does not take into account the utility gain the party might be receiving from its improved chances of controlling the government (which, other things being equal, are positively associated with legislative strength), and its coalition worth (the dynamics of which might be other than monotone in seats, depending on the overall balance in the party system).

is implied, but omitted from the definition of equilibrium.

Thus, by equilibrium we mean a party system, in which

- all *n* parties currently in the race have positive probability of passing the threshold and securing legislative representation,
- no party can improve its seat share by choosing another policy platform, and
- no new party wants to enter, because no new entrant can secure legislative representation.

Actually, if we constrain new entry by requiring that a party enters the race only if by doing so it can get representation immediately, before any further adjustments occur, we can relax even further the assumptions about what the parties already in the race can and cannot do. We may allow parties already in the race to adjust their spatial locations before the potential entrant's move, for as long as they understand the possibility of new entry. A party does not get a higher utility if it invites excessive entry and the vote is now shared $n+1$ rather than n ways.

One way to picture the restraint on a party in pursuing its seat-maximization objective is to consider the party's internal composition. Assuming that its list of candidates ranks individuals according to their intraparty importance, a marginal candidate's attempt to shift a party's platform in order to increase the remainder to a level that yields him a seat, might not be well received by people higher on the list. If new entry is a likely consequence of a positional change, and if entry reduces the party's vote sufficiently (below the threshold) so that senior candidates lose their seats, the change will not occur. This is something we cannot impose on the candidate-based model, where a single seat is under consideration. But in the context of party position-taking, the first *n* candidates on the party list who are already entitled to seats may firmly oppose the $n+1^{st}$ candidate's attempts to move the party. This is not an exhaustive explanation of why a party may stick with its platform. But when we describe equilibria, we see that this explanation corresponds to the circumstances in which parties find themselves.

Notation and definitions:

D - the number of seats to be filled in the electoral district under consideration (district magnitude);

T - the electoral threshold for parties to get legislative representation, measured as a percentage or a fraction of the total vote (we disregard turnout here, but technically speaking *T* can be defined as a fraction of the total population of eligible voters, as it is, for example, in France).

Q - the quota for the seat allocation. To get a generic interpretation of a party list PR system, let $Q=\frac{1}{D}$. But all results here unless otherwise noted also hold for

$Q=\frac{\sum_{V_i \geq T} V_i}{D}$, where V_i is the vote share of the party *i*, where the actual share of the vote that yields a seat goes down, as some votes are wasted on the parties that fail to meet a threshold for legislative representation *T*.

F(x) - a cumulative distribution function of voter ideal points.

f(x) - the density function, $f(x) = F'(x) > 0$ for $x \in [F^{-1}(T); F^{-1}(1-T)]$ and continuous everywhere on $\mathbb{R}$.

K - a finite set of political parties participating in the race, where $i, j, \ldots, v \in K$. Later we also use *K* to denote the number of parties in the race. When we do so, it is clear from the context.

$x_i \in \mathbb{R}$ - the position of party *i* in the policy space.

x - the vector of party policy platforms, $\mathbf{x} = (x_1, x_2, \ldots, x_K)$.

l_i - the left-most ideal point of voters who most prefer party *i*, $l_i = \frac{|x_i + x_{i-1}|}{2}$ (for the first candidate an equivalent of l_1 is $-\infty$).

r_i - the right-most ideal point of voters who most prefer party *i*, $r_i = \frac{|x_i + x_{i+1}|}{2}$ (for the last candidate to the right an equivalent of r_K is $=\infty$).

V_i - the fraction of the total vote secured by party *i*.

Seats are allocated by the number of full quotas a party receives and undistributed seats are allocated by the "largest remainders" principle.

To summarize, we make the following assumptions:

1. Sincere Downsian voters in a unidimensional policy space, whose ideal points are distributed according to some continuous distribution, *F(x)*.

2. Parties' policy positions are points in the policy space.

3. Parties maximize the number of seats they win.

4. Unrestricted competition: new entry is not precluded if an additional party can choose a spatial position so as to have a chance of winning at least one seat (effectively, for $T \geq Q$, - of passing the threshold).

5. Parties can share policy platforms. Two or more partes can assume the same position on policy.

In this framework we now establish the equilibrium properties of an electoral system where the members of parliament are elected by the party lists in a single electoral district, and when undistributed seats are allocated by the "largest remainders" rule. Our first lemma states that in equilibrium no party can have more than T of its vote on either side of its policy position, since entry will occur if there are enough votes to be won by a new party that closely (but not too closely) imitates the overgrown, overly-successful party. Formally,

Lemma 1: *In equilibrium,* $\nexists i, \; s.t. \; \int_{l_i}^{x_i} f(x)\,dx > T$, *or* $\int_{x_i}^{r_i} f(x)\,dx > T$.

Proof: Suppose not, suppose

$$\exists i, \text{ s.t. } \int_{l_i}^{x_i} f(x)\,dx > T.$$

Then

$$\exists \varepsilon > 0, \text{ s.t. } \sigma(\varepsilon) = \int_{x_i - \varepsilon}^{x_i} f(x)\,dx < M,$$

where

$$M=\int_{l_i}^{x_i} f(x)\,dx - T > 0.$$

But then any new party that enters at $x_{new} \in [x_i - 2\varepsilon; x_i)$ passes the threshold and wins at least one seat. QED

The reason why a party, say, *i*, may be careful to avoid such entry can be found in the nature of its electoral rewards. Currently, with no more than a full threshold on either side of its support, *i*'s vote share entitles it to several seats, so that several candidates on the party's list receive seats with certainty. Suppose we have *n* candidates with "secure" (non-probabilistic) seats. Because these people lead party *i*'s list of candidates, we can suppose that these are people whose voice in the matters of party policy is decisive (at least in the short run). If the party makes itself vulnerable to entry, the immediate consequence is the loss of the threshold and, therefore, of the right to have *any* representation in parliament. In other words, if an attempt is made to win one more seat (which will be filled by the $n+1^{st}$ name on its list), the top *n* safe seats will be lost. Thus, the "incumbents" within the party will be careful not to give in to the policy demands of the party's marginal candidates.

This argument leads to several conclusions about the parties' strength and location. First, in equilibrium, no party can have a vote share that exceeds more than twice the threshold, because this would imply that there is room for a successful entry. Thus,

Corollary 1: $V_i \leq 2T$.

Moreover, if two or more parties assume identical policy platforms, they will have to share the same maximum of *2T* votes, which can be enough to support *at most* two parties. Thus,

Corollary 2: *If* K *is an equilibrium number of parties, if* $x_i < x_j \Rightarrow i < j$, *and if the threshold* T$\geqq$Q, *then* $x_1 \leq F^{-1}(T)$, *and* $x_K \geq F^{-1}(1-T)$.

This corollary says in effect that in equilibrium, the "extremist" parties' platforms cannot be located so that a full threshold of the vote lies yet further to either extreme; otherwise new entry will occur.

Corollary 3: *In equilibrium it is never the case that* $x_i = x_j = x_t,\ s.t.\ i \neq j \neq t.$

Moreover, it takes rather exceptional circumstances for two parties in equilibrium to share a policy platform and to pass the threshold for representation:

Corollary 4: *In equilibrium, if* $x_i = x_j,\ s.t.\ i \neq j,$ *then* $V_i = V_j = T$ *and*

$$\int_{l_{i(j)}}^{x_{i(j)}} f(x)\, d(x) = \int_{x_{i(j)}}^{r_{i(j)}} f(x)\, dx = T.$$

One may doubt whether the conditions of Corollary 4 are enough for two parties to share the policy point. Indeed, this is a necessary, not a sufficient condition. Corollary 4 says only that the vote for the paired candidates must come in equal proportion from both sides of their platform. It does not say anything, however, about the incentives for the paired parties to stay together at this position.

Our next lemma, lemma 2 gives the more detailed requirements for two parties to be able to share the same policy when the distribution of constituency preferences is strictly quasi-concave: the parties' platform must be the one nearest the mode of the distribution of voter ideal points, and such that the supporters of the two parties, who stand the furthest from the parties' position, are less numerous then those whose views coincide with the party position.

Lemma 2: *If f(x) is strictly quasi-concave, then if in equilibrium* $x_i = x_j,\ s.t.\ i \neq j,$ *then*

(1) $m \in [l_{i(j)}; r_{i(j)}]$, *and*

(2) $\max\{f(l_{i(j)}); f(r_{i(j)})\} < f(x_{i(j)})$.

Proof: Note that (1) follows immediately from (2) and the strict quasi-concavity of *f(x)*. To establish (2), note that in equilibrium, it must be the case that $V_i = V_j \geq T$, which from Corollary 3, implies

$$\int_{l_{i(j)}}^{x_{i(j)}} f(x)\, d(x) = \int_{x_{i(j)}}^{r_{i(j)}} f(x)\, dx = T.$$

Now suppose that the conditions of the Lemma do not hold and

$$f(l_i) = \max\{f(l_i); f(r_i)\} > f(x_i).$$

Then

$$\exists \varepsilon \ \ s.t. \ \ \sigma(\varepsilon) = \int_{l_i-\varepsilon}^{l_i} f(x)\, dx > \psi(\varepsilon),$$

where

$$\psi(\varepsilon) = \int_{x_i-\varepsilon}^{x_i} f(x)\, dx.$$

Hence, a new party that enters at $x_{new} = x_i - 2\varepsilon$ passes the threshold and wins at least one seat. QED

We are now in a position to confront the central task of this chapter - to analyze the equilibrium properties of electoral competition among parties. The following proposition does two things: it (1) asserts the existence of an equilibrium with entry for all unimodally distributed constituency preferences, and (2) contains in its proof a description of the structure of this equilibrium in terms of the number of competing parties and their respective policies.

Proposition 1: *For any strictly quasi-concave distribution of voter ideal points and $T \geq Q$, there always exists an equilibrium number of parties K and a* *vector of their respective locations in the policy space* **x** *so that no further entry or adjustment occurs.*

Proof: We prove existence by construction; that no other equilibria exist is guaranteed by the preceding lemmas and their corollaries. For any strictly quasi-concave preference distribution, construct an equilibrium the following way:

1) Consider separately the intervals $(-\infty; m]$ (the LHS) and $[m; +\infty)$ (the RHS).

2) On the LHS choose x_1^l $s.t.$ $x_1^l = F^{-1}(T)$.

3) Chose x_i^l where *i=2,3,..* so that

(i) $x_i^l \in [x_{i-1}^l; m]$, and

(ii) $s^i(x_i^l) = T$, where $s^i(x_i^l) = \int_{\frac{x_i^l + x_{i-1}^l}{2}}^{x_i^l} f(x)\,dx$.

Note that $s^i(x_i^l)$ is strictly increasing from *0* to $s^i(m)$ as x_i^l increases from x_{i-1}^l to *m*. If $s^i(m) \geq T$, then $\exists x_i^l$ $s.t.$ $s^i(x_i^l) = T$. If $s^i(m) < T$, set $L = i-1$, $x_{i-1}^l = x_L^l$ will be the last one on the LHS of the distribution.

4) On the RHS choose x_1^r s.t. $x_1^r = F^{-1}(1-T)$.

5) Choose x_i^r, *i=2,3,...,* so that

(i) $x_i^r \in [m; x_{i-1}^r]$, and

(ii) $\tau^i(x_i^r) = T$, where $\tau^i(x_i^r) = \int_{x_i^r}^{\frac{x_{i-1}^r + x_i^r}{2}} f(x)\,dx$. On the RHS, as on the LHS, we index the candidates from the tail toward the center of the distribution. Again, $\tau^i(x_i^r)$ is strictly increasing from *0* to $\tau^i(m)$ as x_i^r decreases from x_{i-1}^r to *m*. If $\tau^i(m) \geq T$, then $\exists x_i^r$ $s.t.$ $\tau^i(x_i^r) = T$. If $\tau^i(x_i^r) < T$, set $R = i-1$; $x_{i-1}^r = x_R^r$ will be the last one on the RHS of the distribution.

6) If either $x_1^l = m$ or $x_R^r = m$ or both, then *K=L+R*. If both $x_1^l = m$ and $x_R^r = m$, then two parties, *L* and *R*, share the platform at the mode receiving exactly one threshold each. If not, if no party is placed at the mode, then:

(i) if $s^{L+1}(x_R^r) \geq T$, or $\tau^{R+1}(x_L^l) \geq T$, a new party must be added between the points x_{L+1}^l (obtained as in (3)) and x_{R+1}^r (obtained as in (5)), *K=L+R+1*;

(ii) if neither holds, *K=L+R*.

The resulting vector *(K;***x***)* is an equilibrium.

QED

Corollary 5: *K as in Proposition 1 is the minimal number of parties competing in equilibrium.*

Proof: For the number of parties **C** to be below *K*, at least one competing party's vote share must increase. W.l.g., suppose this is party x_i^r. There are three ways for party x_i^r's vote to get increased. (1) x_i^r moves nearer x_{i+1}^r; (2) x_{i+1}^r moves away from x_i^r (to the left); or (3) x_{i-1}^r moves away from x_i^r (to the right). By Lemma 1 none can be an equilibrium. QED

Thus, Proposition 1 supplies us with the equilibrium number of parties, *K*, and the vector of those parties' spatial locations, **x** (or **x**= $(x_1^l, \dots x_L^l, x^c, x_R^r, \dots x_1^r)$). This equilibrium exhausts the set of possible equilibrium configurations, except for the flexibility that parties may have in "keeping" less than a full threshold of votes on the "thinner" side of their support. Thus, *K* is the *minimal* equilibrium number of parties. But the parties' flexibility in this matter is severely restricted by their seat-maximization objective, when no full quota of votes and no meaningful remainder should be foregone. Even though a slight reduction in the party's vote would not threaten its passing a threshold, it may, however, cause this party to lose its marginal seat. This is especially so in view of the fact that an increase in the equilibrium number of parties implies at least one extra party getting representation.

5.3 Examples and The Model's Relevance in Large Electoral Districts

The structure of an equilibrium described in Proposition 1 highlights the important role of legal thresholds for legislative representation in a party's decision of where and when to enter. We see that when this institutional constraint is combined with the "social" constraint of voter preferences, one can establish the existence of equilibria for a large class of preference distributions. This, in view of our definition of equilibrium, implies stability in number and platforms of competing political parties. The electoral threshold, when preferences are known, also allows us to place upper and lower bounds on the number of political parties. Considerations like the magnitude of electoral districts then, will in many cases return unique predictions about the number of competitors. Proposition 1 also makes it possible to assess theoretically (although with less precision) the relative strengths of parties in terms of their seat and vote shares.

It may be difficult, though, to believe that a single characteristic - legal legislative thresholds - plays such a crucial role in influencing the number of political parties. Here we need to recall that our model in effect equates the at-large electorate with an electoral district. This approach is only valid for a few real world electoral systems: the Netherlands, Israel, Germany (for the party list contest), Hungary, and Russia. More generally, proportional representation formulas are applied within electoral districts of fairly low magnitude: from 4 (Ireland) to 20 (Italy 1946-76, Austria 1971-79) seats per district. In this more general case, the impact of thresholds and of competition within districts should be somewhat separate. The smaller the size of the district, the closer party district-level competition will resemble the candidate competition under plurality rule (as in Japan and Taiwan, which use plurality in multimember districts), unless a very high threshold applies within a district. However, the importance of the threshold-like constraints that operate at the district level is now being stressed in the literature independently of its analytical usefulness. Lijphart (1994) designs an "effective" threshold variable - a small-district substitute of a legal threshold calculated on the basis of the magnitude of a district. Alongside legal thresholds, this single measure, in Lijphart's opinion, allows for the greatest comparability across the variety of electoral systems.

But even if we limit our inquiry to at-large electoral districts, there are still factors that, although not affecting the general logic of Proposition 1, can influence (downward) the number of predicted competitors. First, we have thus far allowed parties to assume identical policy platforms, but even when we deal with individual candidates, the assumption that two or more candidates can be indistinguishable does not seem realistic. When the subject of our analysis are political parties, whose success or failure depends largely on the appeal and comprehensiveness of their programs, this assumption is even less tenable.

Prohibiting parties from taking the same platform is equivalent to requiring that there be some minimal distance between the policy positions of any two parties. Although we have no basis for saying how large this parameter should be, requiring even a small separation can lead to variations in electoral strengths if the distribution of preferences is steep (i.e., if electorate is cohesive). And it can reduce the predicted number of parties competing in the race. For example, if the electorate is described by a normal distribution of preferences, and if the threshold for legislative representation is 5% of the vote, then by the logic of Proposition 1, 11 parties will compete in equilibrium. Party platforms will be located at

$F^{-1}(.050)$, $F^{-1}(.136)$, $F^{-1}(.230)$, $F^{-1}(.326)$, $F^{-1}(.425)$, $F^{-1}(.500)$,

and symmetrically at

$F^{-1}(.575)$, $F^{-1}(.674)$, $F^{-1}(.770)$, $F^{-1}(.864)$, $F^{-1}(.950)$

(see Figure 5.1 for the relative electoral strength of the competing parties).

[Figure 5.1 about here]

If, however, we introduce a requirement that party platforms be separated by some minimal distance δ, which can be expressed as a function of the standard deviation of preferences, then we witness a decline in the number of parties. For $\delta = 0.2\sigma$, for example, in equilibrium there will be ten parties located at

$F^{-1}(.061)$, $F^{-1}(.159)$, $F^{-1}(.261)$, $F^{-1}(.363)$, $F^{-1}(.464)$,

and symmetrically at

$F^{-1}(.536)$, $F^{-1}(.637)$, $F^{-1}(.739)$, $F^{-1}(.841)$, $F^{-1}(.939)$

(see Figure 5.2 for the parties' vote shares, and Appendix for technical details of our example).

[Figure 5.2 is about here]

Another reduction in the number of parties may be produced by extending the model's framework to incorporate broader objectives than mere representation, namely, the ability to form or enter the government or to influence policy outcomes in the legislature. In this extended setting what we have considered thus far would be a process of "proto-party" formation. Then two developments are possible. The first possibility is that these proto-parties form and then enter coalitional arrangements with each other, broadening their goal along the way from the representation in the legislature to the representation in the government. Another possibility is for proto-parties to remain an ideal type, to never materialize in reality while remaining a potential threat, but where the threat of entry forces existing party organizations to offer the leaders-to-be of potential rival parties slots on their electoral lists high enough to yield seats. The latter scenario of existing parties seeking to "accommodate" their potential rivals opens new opportunities for analyzing the development of party structures. Indeed, by combining politicians of different political profiles, parties would no longer need to worry about obtaining the threshold for representation: a substantial economy of scale can be achieved by combining efforts. And by holding more than one platform on policy at a time, a party can make whole areas of the policy space safe from entry.

An alternative line of future inquiry would be to consider competition in districts of smaller size. When the size of a district is small, parties may not be able to successfully promote more than one candidate per district; the number of parties is bounded from above by the number of potentially successful individual candidates. Candidates, in turn, may find it in their interest to form coalitions - within districts as well as across districts. Pursuing this direction of research, one could establish the equilibrium number of parties resulting from the coalition-making strategies of individual candidates within a district.

For most electoral systems, though, the truth lies somewhere between the large district and small district cases: as most districts are of intermediate size, party competition will be constrained by, but cannot be reduced to the candidates' district-level ambitions. This intermediate case is, for now, beyond our analytical reach. However, when electoral thresholds can be meaningfully defined at the district level, then even in a district of low magnitude the logistics of "party entry" as analyzed above would apply. France offers an example of such high district-level threshold: in order to have a chance of winning a seat in a French electoral district, a party must gather on the first ballot a vote of not less than *12.5* percent of the constituency. Given turnout variations, this may translate into *17* to *18* percent of the actual vote. Without this provision, the French electoral system (although only one seat is usually filled per district) would greatly encourage party proliferation. However, France has a modest number of strong political parties. If we hypothesize that preferences of voters in a representative French constituency are distributed according to a one-dimensional standard normal distribution, then, as in the illustrations above, we can find the number of parties that will be willing to compete in such a district. Note that in this particular case it is irrelevant that a threshold does not guarantee a seat, as (1) seats are likely to be allocated only in the second round of voting, and (2) given the ability of French parties to enter agreements with each other, good electoral performance yields expected seats once all constituencies are considered.

If the turnout is 100 percent in our hypothetical constituency, there will be just five parties, located at

$$F^{-1}(.125),\ F^{-1}(.345),\ F^{-1}(.500),\ F^{-1}(.655),\ F^{-1}(.875)$$

so that the vector of the parties' respective vote shares will be

$$v=(0.219,\ 0.201,\ 0.159,\ 0.201,\ 0.219)\ .$$

In other words, in the first round of elections the five parties in the race will gather between 18 and 24 percent of the total vote each.

Now suppose only 65 percent of those eligible to vote show up at the polls. Suppose also that the preferences of those who have shown up to vote are again distributed according to the standard normal distribution. The threshold parties must now meet is *18* percent of the total vote. If parties know that turnout is going to be low (i.e., the actual threshold is going to be correspondingly high), only *three* will enter the competition. The vector of their locations

$$\mathbf{x}=(F^{-1}(.180),\ F^{-1}(.500),\ F^{-1}(.820))\ ,$$

which implies the vote shares of these parties of 32.3, 35.4 and 32.3 percent respectively. Of course, for other than the standard normal distribution of preferences, the vote shares would be different, while the predicted number of parties for small variations in distribution will remain the same.

The above example should not be interpreted as an attempt to explain the actual number of French parties, although the predictions fit reality. First, we arbitrarily hypothesized about the form of the preference distribution. Second, we should not lose sight of the historical fact that thresholds were strengthened *after* the party system had formed. However, the choice of the exact level of the threshold could have been dictated by the needs of the major parties, as it fits them so well in restricting further entry.

Unlike the French case, most small-district electoral systems do not use prohibitive formal district-level thresholds, so it makes sense to run candidates in a district even if only one seat can be won with positive probability. Future research may lead us to describe the endogenous party formation as a two-stage process. First candidates enter *as if* the electoral rule were simple plurality. Then, inspired by the peculiarities of the actual electoral rule, parties form endogenously as coalitions of individual candidates.

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Figure 5.1. Positions of political parties in elections with a 5% threshold (no-entry equilibrium is reached at 11 parties if the distribution of preferences is assumed to be standard normal)

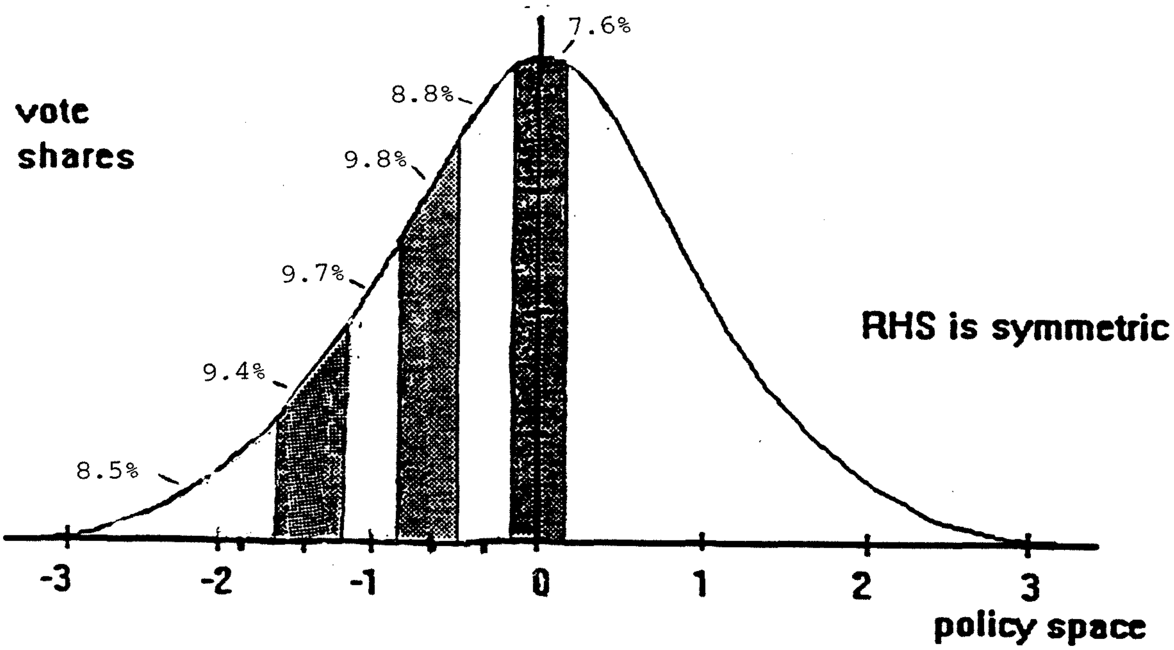
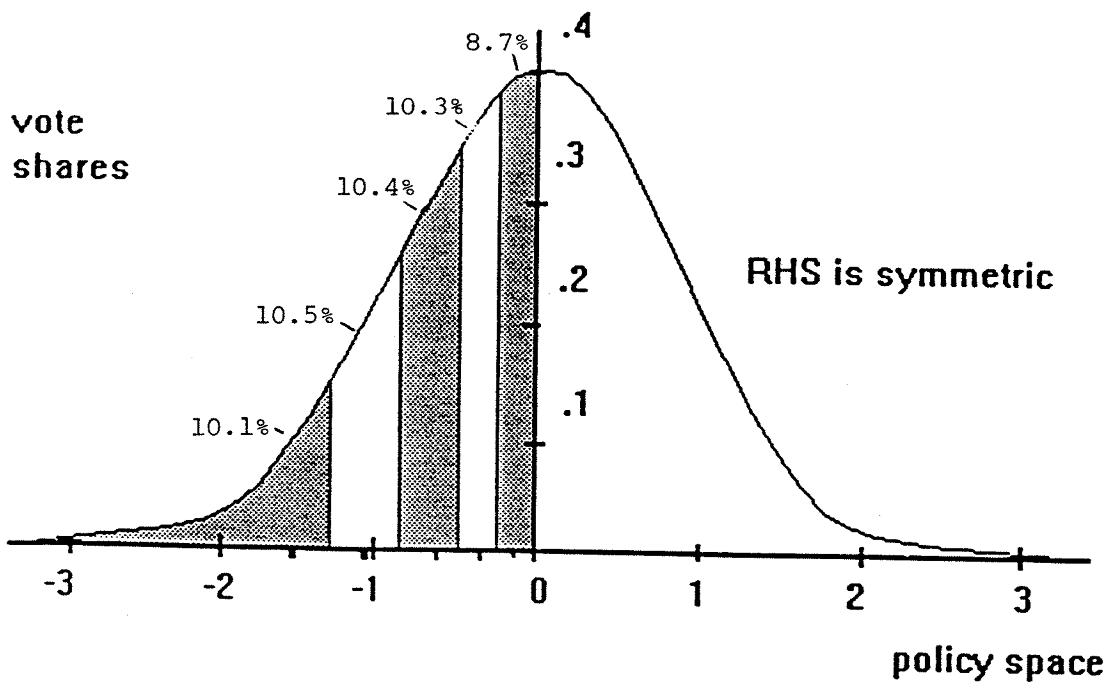


Figure 5.2. Positions of the parties when T=5%, distribution of preferences is assumed to be standard normal, and delta is 20% of the variance (equilibrium is reached at 10 parties)



APPENDIX: PARTY PLATFORMS WHEN MINIMAL SEPARATION IS REQUIRED

The density function of a standard normal distribution is given by

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-0.5x^2}.$$

Based on this, probability of a random variable taking a value within some interval $[\alpha; \beta]$ can be expressed as

$$P(\alpha < X < \beta) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\beta} e^{-0.5x^2} dx .$$

Now if we substitute t^2 for $0.5x^2$ ($t = x\sqrt{0.5}$), this expression becomes

$$\frac{1}{\sqrt{\pi}} \int_{\alpha\sqrt{.5}}^{\beta\sqrt{.5}} e^{-t^2} dt .$$

Note that we can use a convenient tabulation that exists for the values of *the Laplas function*

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt .$$

We can now express $P(\alpha < X < \beta)$ through the Laplas function:

$$P(\alpha < X < \beta) = \frac{1}{2} \{ \frac{2}{\sqrt{\pi}} \int_0^{\beta\sqrt{.5}} e^{-t^2} dt - \frac{2}{\sqrt{\pi}} \int_0^{\beta\sqrt{.5}} e^{-t^2} dt \}$$

$$= \frac{1}{2} \{\Phi(\beta\sqrt{.5}) - \Phi(\alpha\sqrt{.5})\}.$$

In order to find the location x_1^* of the leftmost party, so that party *1* gets the largest vote possible, we need to make sure that no new party can enter to the left of it. We know that no entry occurs unless the entrant can secure at least a vote of size *T*. With the minimal distance δ required between the parties, the vote for this entrant is at most $P(-\infty < X < x_1^* - \delta/2)$. The condition for the proper location of party *1* then is that

$$x_1^* = -\max\{x_1 \mid x_1 \in (-\infty, +\infty);\ \frac{1}{2}[\Phi(-\infty) - \Phi(x_1\sqrt{.5})] < T\} + \frac{\delta}{2}.$$

As $\Phi(-\infty) = -1$, the above simplifies to

$$x_1^* = -\max\{x_1 \mid \Phi(x_1\sqrt{.5}) > 1 - 2T\} + \frac{\delta}{2} = -\sqrt{2}\Phi^{-1}(1-2T) + \frac{\delta}{2}.$$

In our case $T = 0.05$ and $\delta = 0.2\sigma = 0.2$. This yields $x_1^* \approx -1.545$.

To find the location of other parties on the left-hand side of the distribution, denote its mode (which, by symmetry, is also its mean and median) as $m = 0$. Then the platform of party *i>1* must satisfy

$$x_i^* = -\max\{x_i \mid x_i \in (x_{i-1}; m];\ \Phi(\frac{2x_i - \delta}{2\sqrt{2}}) - \Phi(\frac{x_i + x_{i-1} - \delta}{2\sqrt{2}}) < 2T\}.$$

Party platforms on the right slope are symmetric to those on the left one, and can be found by taking the absolute values of the latter.

If we allow parties to share policy platforms, this is equivalent to taking the parameter $\delta = 0$. Then proceed as in Proposition 1.