

# The Formation of Teams Under Incomplete Information

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**Dedication**

*To Andrei, my parents and family*

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## ABSTRACT

Organizational forms such as task-oriented teams have often been proposed as a method to enhance the efficiency of a firm. Under asymmetric information, however, the costs of acquiring the information needed to improve efficiency may outweigh the efficiency gain and lead to lower profits. This dissertation analyzes profitability-efficiency trade-offs faced by a profit-maximizing principal who wants to select teams from a given group of heterogeneous agents to work on a number of projects, given that the principal has incomplete information about the agents' abilities.

The dissertation consists of two main chapters. In chapter one, we take a theoretical mechanism design approach to analyze the problem of the formation of multiple teams under different information structures and behavioral assumptions. We study feasible incentive-compatible (truth-revealing) individually rational mechanisms under both the dominant strategy and Bayesian Nash behavioral assumptions. Some attention is also paid to Nash equilibrium mechanisms. The chapter covers derivation of optimal mechanisms, efficiency analysis, and analysis of the principal's expected profit as a function of different types of environment and information structures. We find that if the principal has little or no information about the agents' private characteristics and the agents follow dominant strategy behavior, the principal may often run into losses in an attempt to discover the hidden information. Paradoxically, the loss occurs when the efficiency gains from team production are high and the competition among the agents is low. If the hidden information about each agent can be summarized as a one-dimensional type parameter, and if a prior distribution function of the agents' types is common knowledge among the agents and the principal, an expected-profit maximizing Bayesian equilibrium mechanism exists and is of the optimal

auction form (Myerson, 1981). Moreover, the mechanism can be equivalently implemented in dominant strategies with no expected profit loss for the principal. Yet, the principal's profit often decreases with an increase in the number of projects. The findings suggest that, in profit-maximizing firms with low competition among the employees, efficient organizational forms may often be foregone in favor of profits.

In chapter 2 we consider, theoretically and experimentally, one specific type of the team-formation mechanisms, a wage-demand mechanism, first suggested by Bolle (1991). Under these mechanisms, potential team members submit their wage demands to the principal and the principal chooses a team which gives her the highest profit – defined as the output of the team net of wages demanded by the team-members, and then pays all the employed agents their demanded wages. Bolle found that the principal's ability to detect and choose efficient teams among the profit-maximizing teams is essential for the existence of pure strategy Nash equilibria of the wage-demand games. We consider wage-demand mechanisms in a framework when the principal might have incomplete information about the agents' characteristics. In this case, the pure strategy Nash equilibria of the wage-demand game do not exist. However, there are  $\epsilon$ -Nash equilibria, which are close in efficiency and profitability to the Nash equilibria of the complete information game.

We present the results of experimental tests of the Nash and  $\epsilon$ -Nash behavioral hypothesis for the team-selection wage-demand games corresponding to complete and incomplete information cases. If the agents do follow the Nash equilibrium behavior, then the principal's information should not significantly affect the outcomes of the games regarding team's profitabilities and efficiencies. In his experimental investigation of the wage-demand games, Bolle found that the subjects often do not follow the competitive Nash equilibrium behavior,

but engage in “tacit collusion.” We test the robustness of Bolle’s findings by introducing asymmetry into agent’s productivity characteristics. We find that although some collusive tendencies are present in the subjects behavior, they are not sustainable; with repetition, the outcomes of the wage-demand games converge to the Nash equilibrium outcomes. However, we find that the two experimental treatments corresponding to the complete and incomplete information on the principal’s part are not equivalent in the degrees of agents’ competition and cooperation. In our experiments the agents were significantly more collusive when the principal had incomplete information, and the outcomes were less profitable for the principal. Thus, we once again confirm that information does matter for the profit-maximizing principal.

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## **Chapter 1**

# **A Mechanism Design Perspective on the Formation of Teams**

### **1.1 Introduction**

The notion of a good organization as a robustly structured hierarchical unity has been recently changing in favor of the idea of more flexible organizational forms. With the growing diversity and complexity of tasks faced by a firm, one may expect high efficiency gains from the more flexible and responsive organizational structures. In particular, the notion of a “virtual corporation,” which can be thought of as a flexible task-oriented partnership, has become quite popular in the literature (Byrne, Brandt, Port (1993)). One might ask whether flexible organizations should necessarily take the form of partnerships, or whether owners of private firms can also gain from internal organizational flexibility. To answer this, one needs to consider the possible implications of flexible organizational forms, both for production efficiency and for owners’ profit, as well as the informational requirements

imposed on the management by the organization structure. In this study, we address the above question in the principal-agent framework and focus on one specific problem that a principal is likely to face in a flexible multi-project organization – the problem of forming temporary task-oriented teams from a given set of agents. Assuming the principal has incomplete information about the agents' characteristics, we consider whether she can gain from teams-based production. Curiously, we learn that rather often, when the potential efficiency gains from flexible organizations are high, it becomes too costly for the principal to run such organizations.

We consider an adverse selection model with a principal who has a number of (possibly profitable for her) projects to carry out and needs to hire the agents to work on these projects – each person for at most one project – from a given group of people. Each agent is characterized by his preference over working on projects (which might represent his personal taste and/or the difficulty of each job for him), and each subgroup of agents – by its productivity of working on each project. The principal needs to choose an allocation of the agents among projects that would maximize the principal's profit, which is the share of the revenue from all the projects net of the payments to the agents necessary to induce them to behave in the principal's interest. A difficulty arises when the principal does not have complete information about the agents' characteristics and hence cannot impose her most preferred outcome without the costly creation of the “right” incentives for the agents.

The recent literature in the economics of incomplete information pays a lot of attention to both the issue of inducing the “right” incentives for agents working in teams (Groves (1973), Holmstrom (1982), McAfee and McMillan (1991)), and the one of choosing the right agent for a job (Laffont and Tirole (1987), McAfee and McMillan (1987)). Both the moral

hazard and the adverse selection aspects of the problem have been studied. Yet, the problem of optimally selecting teams for a given set of projects has been hardly addressed. Bolle's "Team Selection" paper (1991) is a rare study that addresses the issue of team formation in the Nash equilibrium framework with complete information. In our study, we use Bolle's approach to model the team production process, but consider incomplete information environments and seek to find optimal incentive-compatible team selection mechanisms under various behavioral assumptions. We use a simple model of team production where the moral hazard problem is absent. The social surplus produced by a team of agents on a project is a deterministic function of the agents' joint productivity parameter, which is assumed to be common knowledge, and the agents' private characteristics, or cost types. The agents' private characteristics are assumed to be independent.

We mainly focus on two information structures and two behavioral assumptions. In section 1.2 we assume a "complete ignorance" information structure, where there is no well-defined probability assessment over the agents' (possibly multi-dimensional) private characteristics that is common knowledge among the agents and the principal. Therefore, the agents are assumed to follow dominant strategy behavior. We consider feasible dominant strategy incentive compatible (DSIC) and individually rational (IR) mechanisms. Under these assumptions, the social efficiency maximizing information revelation mechanisms have been broadly studied in the literature, and truth is found to be a dominant strategy if and only if the mechanism is Groves-Clark (Groves (1973), Clark (1971), Green and Laffont (1977)). However, it is generically not budget-balancing (Green and Laffont (1977), Walker (1980)), and hence the "social planner" (the principal) may run into losses. The inconsistency of efficiency maximization with the principal's profit maximization is recognized by a



number of authors (Groves and Loeb (1975, 1979), J. Miller and P. Murrell (1981), G. Miller (1992)), but the problem of profit-maximization constrained by dominant strategy incentive compatibility and individual rationality of the agents has not been explicitly studied. We address this problem and find that under the “complete ignorance” assumption, there does not exist a uniform strongly optimal mechanism for a principal – a mechanism that in any environment<sup>1</sup> produces a higher level of profit than any other DSIC IR mechanism. The mechanisms that are optimal under an alternative, weaker criterion of optimality (Arrow and Hurwicz, 1972) exist, but they can be “ad hoc,” very inefficient and hardly sensitive to the environment. Among the efficiency maximizing mechanisms, any DSIC mechanism that is individually rational for the agents is not individually rational for the principal. The result indicates that, in contrast to the efficiency-maximization case, in generic environments a principal cannot successfully organize production without having substantial information about agents’ characteristics.

We further analyze specific types of environments that can be of interest and find that the more competitive the environment is, the higher the principal’s profit is. In this part, our results are very similar to Makowski and Ostroy (1987), who establish a close connection between perfect competition and efficient dominant strategy incentive compatible mechanisms. We find that in the perfectly competitive environments, where each agent is dispensable, the principal can use efficient DSIC mechanism to acquire the whole social surplus for herself. On the other hand, perhaps surprisingly, we establish that the higher the social gains from the teams’ production are, the lower is the principal’s profit. Thus,

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<sup>1</sup>By an environment we mean the teams’ productivity parameters and the agents’ private cost characteristics.

pursuing efficiency gains from a flexible organization form appears to be completely at odds with the principal's self-interest.

In section 1.3 we move to the Bayesian Nash equilibrium framework, where the information incompleteness on the principal's part is less extreme. The full consideration of this problem would require us to address a multi-dimensional adverse selection issue, allowing agents' costs to be independent (or stochastically correlated) across project. While a number of recent studies approach the latter problem (Rochet (1985), McAfee and McMillan (1988), Wilson (1993), Armstrong (1993a, 1993b)), all of them indicate that explicit characteristics of the solutions are difficult to obtain even for a single-agent case. In characterizing incentive-compatible mechanisms in a multi-dimensional setting, McAfee and McMillan (1988) derive a generalized single-crossing property which requires, essentially, the agents' types to line up. In this study, we reduce the problem to a one-dimensional case by assuming that agents' costs on different projects are deterministic functions of one-dimensional types. The types' probability distribution is known to the principal and the agents. Both the principal and agents are risk-neutral. Under these assumptions, we derive Bayesian incentive compatible (BIC) IR mechanisms that maximize the principal's expected profit, and present necessary and sufficient conditions for such mechanisms to exist. We show that an optimal BIC IR mechanism is similar to an optimal auction with risk-neutrality and independent private values a la Myerson (1981). Furthermore, following a technique presented by Mookherjee and Reichelstein (1992), we show that an optimal mechanism can be equivalently implemented in dominant strategies with no expected profit loss to the principal. In this way, the problem of finding an optimal DSIC IR mechanism is resolved for this type of information structure. However, we find that the larger the number of projects,

i.e., the more complicated the allocation problem is, the stronger the restrictions are on the mechanism needed to satisfy the agents' incentive compatibility. The principal is often bound to treat agents of different types alike and hence faces losses in her expected profits.

In section 1.4 we briefly explore the levels of profit achievable by the principal if she is “completely ignorant” (as in section 1.2), but the agents have complete information about each other's characteristics and follow Nash equilibrium behavior. We present our main conclusions and discuss their implications for issues about flexible organizational forms in section 1.5. Section 1.6 contains proofs of the propositions.

## 1.2 Dominant strategy mechanisms with “complete ignorance”

### 1.2.1 The model

Consider a simple case of the multiple team formation problem with pure adverse selection, where nature-induced uncertainty and moral hazard are absent. We are given the set of agents  $N = \{1, \dots, n\}$ ,  $n \geq 1$ , and the set of projects  $K = \{1, \dots, k\}$ ,  $k \geq 1$ , among which the agents are to be allocated. Each agent  $i$  is characterized by a vector of costs (disutility levels)  $c_i = \{c_{ij}\}$ , with each  $c_{ij}$  denoting  $i$ 's disutility of being assigned to the project  $j$ . For every  $i$ , let  $c_i \in \mathcal{C}_i$ , where  $\mathcal{C}_i$  is a convex bounded subset of  $R^k$  with a non-empty interior. We may interpret these disutilities as exogenously given costs of an agent's effort which vary depending on the project to which he is assigned. Let  $C = (c_1, \dots, c_n)$  denote the matrix of all agents' costs, and  $C_{-i}$  – the matrix of costs of agents other than  $i$ , for every  $i \in N$ . Then let  $\mathcal{C} = \times_i \mathcal{C}_i$  denote the set of all possible disutility profiles. Assume that the costs  $c_{ij}$

are expressed in monetary terms and are the private information of each respective agent.

A team of agents  $T \subseteq N$  assigned to a project  $j \in K$  is represented by an  $n$ -dimensional vector  $x_j = (x_{1j}, \dots, x_{nj})$ , where for all  $i \in N$   $x_{ij} = 1$  if agent  $i$  is assigned to the project  $j$ , and  $x_{ij} = 0$  otherwise. Then an  $n \times k$  matrix  $X = (x_1, \dots, x_k)$  denotes a particular allocation, or assignment, of agents across the projects. An allocation  $X$  is *feasible* if for all  $i \in N$ ,  $\sum_{j \in K} x_{ij} \leq 1$  and for all  $i \in N$ ,  $j \in K$ ,  $x_{ij} \in \{0, 1\}$ <sup>2</sup>. Let  $\mathcal{X}$  denote the set of all feasible allocations.

Suppose each team  $x_j$ ,  $x_{ij} \in \{0, 1\}$  for all  $i \in N$  is characterized by its potential productivity on a project  $j$ ,  $F_j(x_j)$ ,  $|F_j(x_j)| < \infty$ , expressed in monetary terms. Assume  $F_j(0, \dots, 0) = 0$  for all  $j$ . For each  $x_j$  and each  $j \in K$ , let  $\mathcal{F}_j(x_j) \subset \mathcal{R}$  be the set of all possible productivity parameters. Assume that the projects have no external productivity effects on each other, and that each agent can be assigned to at most one project. Then for any feasible allocation  $X$  of agents among the projects, the total gross productivity of the allocation equals the sum of the teams' productivities over projects:

$$F(X) = \sum_{j \in K} F_j(x_j) .$$

Note that  $F(0) = 0$ . For future convenience, for every agent  $i \in N$ , let  $x_i$  denote a  $k$ -dimensional vector of  $i$ 's assignment, and  $X_{-i}$  a matrix of allocations of agents other than  $i$ . Let  $\mathcal{F} \subset \times_{X \in \mathcal{X}} \times_{j \in K} \mathcal{F}_j(x_j) \subset \mathcal{R}^k$  denote the set of all possible productivity profiles.

We denote an *environment* as  $(F, C)$ , a set of parameters characterizing the teams'

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<sup>2</sup>Alternatively, we could assume that an agent's contribution can be distributed among several projects and, therefore,  $x_{ij}$  is a continuous variable,  $x_{ij} \in [0, 1]$ . Most of the results presented in this chapter are still valid in the continuous case. We will indicate which parts of the analysis hold for  $x_{ij} \in \{0, 1\}$  case only.

productivities and the agents' private costs on each project. Let  $(\mathcal{F}, \mathcal{C})$  denote the set of possible environments.

Given an environment  $(F, C)$ , the net productivity, or the *social surplus*, of a feasible allocation  $X$  equals the difference between the gross productivity and the agents' costs:

$$S(X) \equiv F(X) - \sum_{i=1}^n \sum_{j=1}^k c_{ij} x_{ij} . \quad (1.1)$$

A feasible allocation  $X^* \in \mathcal{X}$  is called *efficient* if it maximizes the social surplus among all the feasible allocations. We assume that  $(\mathcal{F}, \mathcal{C})$  is such that for every  $F \in \mathcal{F}$  there exist  $C, C' \in \mathcal{C}$  such that  $S(X; C) < 0$  for every  $X \in \mathcal{X} \setminus 0$  and  $S(X; C') > 0$  for some  $X \in \mathcal{X}$ , i.e., there exist environments where production is efficient and other environments where the only efficient allocative option is “no production”  $X = 0$ .

Assume that the teams' productivities on each project are common knowledge, whereas the agents' costs are the agents' private information<sup>3</sup>. Suppose that the principal has no specific probability assessment about the distributions of the agents' costs and assumes that the agents have no common priors. Therefore, the principal is restricted to consideration of dominant strategy mechanisms<sup>4</sup>.

The principal's problem is to offer the agents an allocation rule  $X(\cdot)$  and a menu of wages  $W(\cdot)$ , with  $w_{ij}$  denoting the wage paid to agent  $i$  if he is employed on the project  $j$ , so that

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<sup>3</sup>Almost equivalently, we can assume that the team productivities ex-ante are only known to members of the teams, but the output of each team is ex-post observable. Then, with no uncertainty involved, a simple forcing contract (as in Holmstrom (1982)) could enforce truthful revelation of productivities as a Nash equilibrium. A coordination problem, however, prevents making the truthful productivity reports dominant strategies.

<sup>4</sup>If the principal were uninformed about the distribution of the agents' costs but the distribution was the common knowledge among the agents, there might exist extended revelation mechanisms in which the agents first report their common priors to the principal, and then the actual costs are revealed. In this case Bayesian equilibrium mechanisms could be considered by the principal.

it will be always in the agents' self-interest to submit to the principal the information which will allow the latter to choose her most desirable allocation, i.e., dominant strategy incentive compatibility will be sustained. The principal seeks to maximize her profit, defined as her share of surplus net of the payments to the agents, which is

$$\pi(X) = F(X) - \sum_{i=1}^n \sum_{j=1}^k w_{ij} x_{ij} . \quad (1.2)$$

Suppose that the agents are indifferent to the outcome of production per se and care only about their own costs and wages. Specifically, we assume each agent is characterized by a quasi-linear utility function

$$u_i(x_i, w_i; c_i) = \sum_{j=1}^k (w_{ij} - c_{ij}) x_{ij} . \quad (1.3)$$

Thus, each agent is maximizing his payoff from employment, which, given his assignment, equals the difference between his wage and cost.

By the revelation principle (Dagusta, Hammond and Maskin (1979)), without loss of generality, we can restrict our attention to direct revelation mechanisms, where the agents report their cost vectors to the principal. For each  $i \in N$ , let  $\tilde{c}_i$  denote the reported costs as opposed to the true costs  $c_i$ . Given the reported costs  $\tilde{C}$ , the principal chooses allocation and wage matrices according to a prespecified rule  $g(\tilde{C}) = (X(\tilde{C}), W(\tilde{C}))^5$ . The principal's task, then, is to choose a dominant strategy incentive compatible decision rule  $g(\tilde{C})$  that will maximize her objective function 1.2. We also assume that the agents cannot be forced to

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<sup>5</sup>Since the productivity environment  $F$  is observable to the principal, the rule may and will, in general, depend on  $F$  as well as  $\tilde{C}$ :  $g = g_F(\tilde{C})$ . The dependence of mechanisms on the observable elements of the environment is omitted, where possible, to simplify exposition.

participate in the project, and have a reservation utility level of 0 if they do not participate. Hence the principal has to observe the individual rationality constraints for every agent to be able to employ him. Given the above assumptions, we can present the principal's problem as follows:

$$\max_{X(\tilde{C}), W(\tilde{C})} F(X(\tilde{C})) - \sum_{i=1}^n \sum_{j=1}^k w_{ij}(\tilde{C}) x_{ij}(\tilde{C}) \quad (1.4)$$

subject to:

$$x_{ij}(\tilde{C}) \in \{0, 1\} \quad \text{for any } i \in N, j \in K \quad (1.5)$$

$$\sum_{j=1}^k x_{ij}(\tilde{C}) \leq 1 \quad \text{for every } i \in N \quad (1.6)$$

$$\sum_{j=1}^k (w_{ij}(\tilde{C}_{-i}, c_i) - c_{ij}) x_{ij}(\tilde{C}_{-i}, c_i) \geq \sum_{j=1}^k (w_{ij}(\tilde{C}) - c_{ij}) \tilde{x}_{ij}(\tilde{C})$$

for every  $i \in N$ , any  $c_i$ , any  $\tilde{C}$  (1.7)

$$\sum_{j=1}^k (w_{ij}(\tilde{C}_{-i}, c_i) - c_{ij}) x_{ij}(\tilde{C}_{-i}, c_i) \geq 0$$

for every  $i \in N$ , any  $\tilde{C}_{-i}$  (1.8)

In the above formulation, 1.5 and 1.6 are feasibility constraints, 1.7 is the incentive compatibility constraint which guarantees that each agent cannot gain from a non-truthful report no matter what the others' reports are, and 1.8 is the individual rationality, or the voluntary participation, constraint.

The problem would be a variant of a traditional resource allocation problem if the principal were a social surplus (expression 1.1) maximizer; it becomes quite different when the principal's objective function is to maximize her own share of surplus (expression 1.2). Be-

low, we consider the mechanisms that are feasible and optimal for a self-interested principal under various productivity-cost environments.

### 1.2.2 Complete information solution

We start with the complete information solution. Consider the payoff that would be available to the principal under complete information (principal's first best). If each agent's type were observable to the principal, she would choose the allocation and payment schedule  $(X^*, W^*)$  such that

$$\{X^*\} \text{ maximizes } F(X) - \sum_{i=1}^n \sum_{j=1}^k c_{ij} x_{ij} , \quad (1.9)$$

$$w_{ij}^* = c_{ij} x_{ij}^* \text{ for all } i \in N, j \in K . \quad (1.10)$$

Thus, under complete information, the principal chooses an efficient allocation – one that maximizes the social surplus. Each employed agent is compensated for the cost he bears at the project he is assigned to, and hence the individual rationality constraint is satisfied. However, the agents get a zero share of the social surplus, which goes exclusively to the principal. Note also that none of the agents can gain from changing his assignment to a different project under the suggested payment scheme since he does not get any compensation for his costs elsewhere.

In what follows, we compare the principal's payoffs under “complete ignorance” to her first best payoff and determine the information rents that the agents are able to extract from the principal.



### 1.2.3 The principal's choice of mechanisms under complete ignorance

We begin our consideration with the direct revelation dominant strategy incentive compatible (DSIC) individually rational (IR) mechanisms that are not environment-specific. Suppose the principal ex-ante has no well-defined beliefs about the distribution of the agents' costs, except, perhaps, it is known that each particular  $C \in \mathcal{C}$  occurs with probability zero. We will call this a "complete ignorance" situation, keeping in mind the inaccuracy of the term. A number of possible optimality criteria can be used for comparison of various dominant strategy mechanisms under complete ignorance. For a broad class of problems with a social surplus maximizing principal, it has been shown (Groves (1973), Green and Laffont (1977), Walker (1980)) that there are DSIC mechanisms that are ex-post efficient even under the complete ignorance assumption. For any environment, they guarantee a level of social surplus no less than any other DSIC mechanisms. In the analogy with the efficiency-maximization case, we first consider the strongest possible criterion of optimality for a self-interested principal – ex-post profitability. We then discuss an alternative – and much weaker – optimality criterion.

Given the agents' cost reports  $\tilde{C}$ , let  $(F, \tilde{C})$  denote the *reported environment*.

**Definition 1** *Within the class of direct revelation DSIC IR mechanisms, a mechanism  $g(C) = (X(C), W(C))$ <sup>6</sup> is called strongly optimal if for any other DSIC IR mechanism  $\tilde{g}(C) = (\tilde{X}(C), \tilde{W}(C))$ , for every environment  $(F, C)$*

$$\pi(g(C)) \geq \pi(\tilde{g}(C)) .$$

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<sup>6</sup>Hereafter, we will denote DSIC mechanisms by  $g(C)$  instead of  $g(\tilde{C})$  so as to not cause confusion.

The following proposition shows that this optimality criterion is too strong a requirement for any incomplete information structure.

**Proposition 1** *If a principal has incomplete information about the environment, there does not exist a strongly optimal DSIC IR mechanism.*

We prove the above statement with the help of several lemmas<sup>7</sup>.

**Lemma 1** *A DSIC IR mechanism is strongly optimal only if in any environment it guarantees the first best, i.e., the complete information level of profit to the principal.*

**Corollary 1** *A mechanism  $G(F, C)$  is strongly optimal only if it is social surplus maximizing.*

Therefore, we need to consider a class of DSIC mechanisms that maximize social efficiency. For this class of mechanisms, Green and Laffont (1977) have shown that the only truth-dominant direct revelation mechanisms are Groves mechanisms, and, moreover, these mechanisms are not generically budget balancing (Walker (1980)). We now define a class of Groves mechanisms corresponding to the team formation problem.

**Definition 2** *A mechanism is called a Modified Groves mechanism if, given a reported environment  $(F, \tilde{C})$ , it chooses an allocation  $X^*(F, \tilde{C})$  such that*

$$X^* \text{ maximizes } F(X) - \sum_{i=1}^n \sum_{j=1}^k \tilde{c}_{ij} x_{ij} \quad (1.11)$$

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<sup>7</sup>The proofs for the statements, if not presented in the text, are given in section 1.6.

and a set of transfers  $W^*(F, \tilde{C})$  defined by

$$\sum_j w_{ij}^* x_{ij}^* = F(X^*) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj}^*(\tilde{c}_i, \tilde{C}_{-i}) + h(\tilde{C}_{-i}) , \quad (1.12)$$

where  $h(\tilde{C}_{-i})$  is an arbitrary deterministic function of the other agents' cost reports.

**Lemma 2** *The Modified Groves mechanisms in the problem with observable production have the same properties as the standard Groves mechanisms in the allocation problem without production. That is, the Modified Groves mechanisms are the only DSIC efficient mechanisms, and they are not generically budget-balancing.*

**Corollary 2** *There does not exist an efficient DSIC IR mechanism which in every environment allocates the whole social surplus to the principal.*

*Proof* Follows from the fact that the Groves mechanisms are generically not budget-balancing (Walker (1980)).  $\square$

Combining the results of lemmas 1,2 and corollaries 1,2 concludes the proof of proposition 1.

Proposition 1 shows that if a self-interested principal has no (or incomplete) information about the agents' cost types, she cannot choose a mechanism that will perform better for her in any environment compared to other DSIC mechanisms. This conclusion contrasts with the results obtained for a social surplus maximizing principal: in the latter case, the principal ex-ante need not have any information about the agents' costs to implement a socially efficient outcome. The difference in the results apparently emerges from the fact that with social efficiency maximization there is "enough" coincidence of interests between

the social planner and the agents; the Groves-type transfer rules compensate individuals for the differences in the social and individual objective functions. On the contrary, with profit maximization, the principal and the agents have opposing interests with respect to the social surplus division, which makes the principal's first best not implementable in dominant strategies<sup>8</sup>.

The above results rest heavily on the optimality criterion used and the complete ignorance assumption. In section 1.2.5 below we consider a much weaker optimality criterion suggested by Arrow and Hurwicz (1972) for decision-making under complete ignorance; we find that under this criterion optimal mechanisms often exist. Then in section 1.3 we show that if the principal has a prior over the distribution of the agents' cost types, then the ex-ante optimal mechanism that maximizes the principal's expected profit subject to dominant strategy incentive compatibility is well defined. Before turning to these issues, however, we characterize certain suboptimal feasible mechanisms. In the next section, we consider efficiency-maximizing mechanisms and analyze the range of payoffs (or the share of social surplus) that the principal can guarantee for herself under these mechanisms depending on the type of environment she operates in.

#### 1.2.4 Efficient dominant strategy mechanisms

Makowski and Ostroy (1987) consider the connection between the properties of efficiency-maximizing DSIC mechanisms and the competitive characteristics of an economy for a

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<sup>8</sup>Roberts (1979) presents a complete characterization of the social choice functions that are implementable in dominant strategies for the class of quasi-linear utility functions. He shows that such social choice functions maximize the weighted sum of individual utilities of an allocation plus a function that does not depend on individuals' preferences. Hence, coincidence of interests between the social choice function and individual preferences is a necessary condition for implementation in dominant strategies.

general class of incentive problems with incomplete information. They establish a direct connection between the DSIC property of mechanisms in a mechanism design framework and the notion of perfect competition in Walrasian equilibrium theory. They find that a perfectly competitive economy in which no individual can change equilibrium prices is equivalent to the special kind of DSIC IR mechanism – the marginal product mechanism under which each agent is rewarded with the level of utility exactly equal to the value of his marginal product, when agents’ characteristics exhibit no complementarity with each other. Our findings presented in this section are remarkably coherent with the Makowski and Ostroy results, although we approach the problem from a different perspective. We find that the “best” profit-maximizing mechanism for the principal restricted to the use of ex-post efficient DSIC IR mechanisms is the marginal product mechanism, and then investigate under what classes of environments the principal can, using this mechanism, extract all the social surplus from the agents.

Suppose the principal – for some reason – can only use DSIC mechanisms that are social surplus maximizing. Consider the implications of this restriction for the principal’s profit. From the previous section we know (lemma 2) that the principal in this case is restricted to the class of Modified Groves mechanisms. Within this class, and taking into account that a mechanism should satisfy individual rationality, define the principal’s preference over mechanisms by

**Definition 3** *Within the class of Modified Groves individually rational mechanisms, a mechanism  $g(C) = (X(C), W(C))$  is preferred to a mechanism  $\tilde{g}(C) = (\tilde{X}(C), \tilde{W}(C))$  if for all environments  $(F, C)$*

$$\pi(g(C)) \geq \pi(\tilde{g}(C)) .$$

A mechanism is called dominant if it is preferred to every other Modified Groves IR mechanisms.

**Corollary 3** A Modified Groves IR mechanism  $g(C) = (X(C), W(C))$  is preferred to a Modified Groves IR mechanism  $\tilde{g}(C) = (\tilde{X}(C), \tilde{W}(C))$  if and only if for every  $(F, C)$

$$\sum_{i \in N} h_i(C_{-i}) * \left( \sum_j x_{ij}^* \right) \leq \sum_{i \in N} \tilde{h}_i(C_{-i}) * \left( \sum_j x_{ij}^* \right), \quad (1.13)$$

where  $h_i(C_{-i})$ ,  $\tilde{h}_i(C_{-i})$  are arbitrarily components of transfers in  $g(C)$  and  $\tilde{g}(C)$ , respectively, as given by 1.12.

In the above definition the preference relation is not strict: There may exist more than one dominant Modified Groves IR mechanism. What matters, however, is that all dominant mechanisms are ex-post profit-equivalent, i.e., they provide the principal with an equal amount of profit for every environment. Therefore, it is sufficient to find just one dominant mechanism. We now introduce a Modified Groves individually rational mechanism that satisfies the desired dominance property.

**Definition 4** A direct revelation mechanism  $g^*(F, \tilde{C})$  is called the Marginal Product Wage (MPW) mechanism if, given a reported environment  $(F, \tilde{C})$ , it chooses  $(X^*(F, \tilde{C}), W^*(F, \tilde{C}))$  such that

$$X^* \text{ maximizes } F(X) - \sum_{i=1}^n \sum_{j=1}^k \tilde{c}_{ij} x_{ij}; \quad (1.14)$$

$W^*$  such that for each  $i$

$$w_{ij}^* = \begin{cases} [F(X) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj}^*] - [F(\tilde{X}_{-i}) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} \tilde{x}_{lj}] & \text{if } x_{ij}^* = 1 \\ 0 & \text{if } x_{ij}^* = 0, \end{cases} \quad (1.15)$$

where  $\tilde{X}_{-i}$  is an  $(n-1) \times k$  allocation matrix that maximizes

$$F(X_{-i}) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj} \equiv S(X_{-i}).$$

**Proposition 2** *The Marginal Product Wage mechanism is efficient, dominant strategy incentive compatible and individually rational.*

It can be easily seen that the MPW mechanism pays every employed agent his “raw marginal product” – the net marginal product that the agent produces in the most efficient allocation and his cost compensation. Let  $S^* \equiv S(X^*)$  denote the social surplus produced in the efficient allocation, and  $\tilde{S}_{-i} \equiv S(\tilde{X}_{-i})$  denote the social surplus produced in the efficient allocation without agent  $i$ . Then

$$\sum_j w_{ij}^* x_{ij}^* = S^* - \tilde{S}_{-i} + \sum_j \tilde{c}_{ij} x_{ij}^*{}^9.$$

Two important properties of the mechanism (DSIC and IR) follow: first, the agents can only gain from truthful revelation since they are rewarded with the value of the whole social surplus minus a lump sum transfer. Second, since only the agents who produce non-negative marginal social surplus are employed, each agent is guaranteed to have a non-negative level

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<sup>9</sup>It follows that each agent’s utility equals the value of his (net) marginal product, and therefore this mechanism is indeed the Makowski-Ostroy marginal product mechanism. So proposition 2 directly follows from Makowski-Ostroy (1987).

of utility. We also note that this mechanism is envy-free, i.e., no agent could gain from changing his employment given the wages he is offered<sup>10</sup>. The next proposition shows that the MPW mechanism is indeed dominant.

**Proposition 3** *The Marginal Product Wage mechanism is dominant in the class of Modified Groves individually rational mechanisms.*

Knowing that the Marginal Product Wage mechanism is “the best” for the principal in the class of efficient DSIC IR mechanisms, we turn to the question of how profitable this mechanism could be. Unfortunately, as the next proposition shows, the principal is not guaranteed against losses under this mechanism.

**Proposition 4** *The principal cannot guarantee herself a non-negative profit for every environment under the Marginal Product Wage mechanism.*

*Proof* It is sufficient to present an example of an environment in which the principal gets a negative payoff. Let  $n = 2$ ,  $k = 2$ ,  $F_1(\{1\}) = 5$ ,  $F_1(\{2\}) = 7$ ,  $F_1(\{1, 2\}) = 15$ ,  $F_2(\{1\}) = 0$ ,  $F_2(\{2\}) = 3$ ,  $F_2(\{1, 2\}) = 4$ ,  $c_{ij} = 2$  for all  $i, j$ . Then the efficient allocation is  $x_{11}^* = 1$ ,  $x_{12}^* = 0$ ,  $x_{21}^* = 1$ ,  $x_{22}^* = 0$ , with  $F(X^*) = 15$ . The MPW mechanism wages are  $w_{11} = (15 - 2) - (7 - 2) = 8$ ,  $w_{21} = (15 - 2) - (5 - 2) = 10$ ,  $w_{12} = w_{22} = 0$ . Then the principal's payoff is  $\pi(X^*) = 15 - 8 - 10 = -3 < 0$ .  $\square$

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<sup>10</sup>Formally, we call a mechanism  $(X^*(\vec{C}), W^*(\vec{C}))$  envy-free if for every  $i \in N$ ,

$$\sum_{j \in K} (w_{ij}^* - c_{ij}) x_{ij}^* \geq \sum_{j \in K} (w_{ij}^* - c_{ij}) x_{ij}$$

for any feasible  $x_i$ . Since under the MPW mechanism an agent, if employed on a project  $j$ , can only gain from employment, and is offered zero wages at the projects other than  $j$ , the envy-free requirement is satisfied.



Under specific types of environments, however, the principal is able to extract a non-negative surplus from the agents. These are highly competitive environments, in which the agents' marginal contributions to the total surplus produced are "low enough." We now characterize these environments.

**Corollary 4** *If the agents' net marginal products  $(S^* - \tilde{S}_{-i})$  are "low enough" in the sense that the following inequality holds:*

$$(n - 1)S^* \leq \sum_{i=1}^n \tilde{S}_{-i} , \quad (1.16)$$

*then the principal gets a non-negative payoff in the MPW mechanism.*

The above conditions may correspond to different types of economic situations: in one type, an efficient allocation does not employ every available agent, but for many employed agents there are unemployed ones that closely match them in productivity and cost characteristics. In this case there exists a high degree of substitution among some agents, and an environment can be called highly competitive. The other possible situation is when most agents are employed but the teams' net productivities are characterized by "decreasing returns to scale." We start with analysis of the latter case. Consider an environment in which it is efficient to employ every available agent under the MPW mechanism. We call it a *full employment environment*.

**Definition 5** *A full employment environment is one in which every available agent is employed under the MPW mechanism. Formally, given the set of agents  $N$  and any  $N_h \subseteq N$ , let  $X^*(N_h)$  be the matrix of efficient assignments when the subset  $N_h$  of agents is available.*

Then the environment is called full employment if

$$\sum_{i \in N_h} \sum_j x_{ij}^*(N_h) = |N_h| \text{ for all } N_h \subseteq N, \quad (1.17)$$

where  $|N_h|$  denotes the number of elements in  $N_h$ .

Next, we introduce the notion of decreasing returns to scale. For our model it is easier to define decreasing returns to scale in terms of average per person surplus (net product) produced under the MPW mechanism. Let  $s^*(N) = S^*/(\sum_i \sum_j x_{ij}^*)$  denote the average per person surplus produced when all the agents are available for employment, and  $\tilde{s}_{-i} = \tilde{S}(N_{-i})/(\sum_{l \neq i} \sum_j \tilde{x}_{lj})$  – average per person surplus when all the agents but  $i$  are available. Finally, let  $\tilde{s}(N_{-i}) = (\sum_{i=1}^n \tilde{s}_{-i})/n$  denote the average per person surplus produced when an “average” agent is excluded from possible employment.

**Definition 6** *A production environment is characterized by decreasing net returns to scale if an average agent, efficiently employed under MPW mechanism, decreases the average per person social surplus produced, as compared to the efficient employment without this agent. That is,*

$$s^*(N) \leq \tilde{s}(N_{-i}). \quad (1.18)$$

**Corollary 5** *If an environment is full employment and characterized by decreasing net returns to scale, the principal can guarantee herself a non-negative payoff under the MPW mechanism.*

We now turn to another type of environment which is extremely favorable to the principal – a perfectly competitive environment, where no agent is indispensable. In such

environments, each agent faces the competition of at least one other agent who is identical to him. With a continuum of possible types of agents and independence of types, such environments occur with probability zero, but if the number of agents available for employment is large, there might exist some agents who closely match each other in productivity-costs characteristics. Then each agent's marginal contribution, and, consequently, his share of the social surplus will be small, therefore increasing the principal's share of the surplus. The analysis of the extreme, perfect competition case indicates that, in general, competition serves the interests of the principal.

**Definition 7** *An environment  $(F, C)$  is called perfectly competitive if for any agent  $i \in N$  who is employed under MPW mechanism*

$$S(X^*(F, C)) = S(\tilde{X}_{-i}(F, C_{-i}))^{11} .$$

**Proposition 5** *If an environment is perfectly competitive, then under the MPW mechanism the principal achieves her first best level of profit, i.e., she captures the whole social surplus of production.*

This statement follows from the definitions of the MPW mechanism and the perfectly competitive environment.

To summarize, we find that efficient dominant strategy individually rational mechanisms do not always leave the principal with a non-negative profit. The environments in which the principal can guarantee herself a non-negative profit are rather restrictive and look a

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<sup>11</sup>Makowski and Ostroy define perfect competition as a situation in which no individual can change equilibrium prices. They further show that this is equivalent to an environment where each agent's marginal product equals zero.

lot like traditional labor markets with homogeneous workers; competition among agents serves to the principal's advantage. On the other hand, if the agents have complementary characteristics and their joint production in teams produces increasing social surplus, the principal is doomed to run into losses. The last observation is curious since from a production efficiency viewpoint, the teams should be formed exactly when the efficiency gain from the joint production is high. Our analysis indicates, however, that a self-interested principal who is forced to implement efficient allocations only loses from high efficiency.

### **1.2.5 An alternative optimality criterion**

The above analysis does not imply, of course, that any DSIC IR mechanism would necessarily make a self-interested principal bear losses under certain environments. It only shows that no-loss DSIC mechanisms cannot have socially desirable properties such as economic efficiency. From the profit-maximizing principal's perspective, however, efficiency is not nearly as important as the profits that a mechanism produces in every possible environment. In section 1.2.3 above we have found that there is no DSIC IR mechanism that is ex-post profit maximizing for all environments. With this result in mind, the principal may prefer any mechanism which insures her against losses and provides high profits at least in some environments. This reasoning corresponds to the criterion of optimality suggested by Arrow and Hurwicz (1972) for decision-making under ignorance. In their terms, an action is called optimal if the minimal and maximal possible payoffs from this action are not lower than the respective payoffs from any other action. In application to our problem, this criterion implies that a mechanism is optimal if it always provides a non-negative profit and in the

environment with minimal costs guarantees the principal her first best<sup>12</sup>:

**Definition 8** *Within the class of direct revelation DSIC IR mechanisms, a mechanism  $g(C) = (X(C), W(C))$  is called weakly optimal if for any  $C \in \mathcal{C}$*

$$\pi(g(C)) \geq 0$$

*and there exists  $C^* \in \mathcal{C}$  such that for any  $C \in \mathcal{C}$ , any  $X \in \mathcal{X}$*

$$\pi(g(C^*)) \geq S(X; C) .$$

From the above discussion, it immediately follows that if  $\mathcal{C}$  is closed from below, then a weakly optimal mechanism exists.

**Proposition 6** *Suppose that  $C^* \in \mathcal{C}$ , where  $C^*$  is defined by  $c_{ij}^* \equiv \inf\{c_{ij} | c_{ij} \in C_{ij}\}$  for all  $(i, j) \in N \times K$ . Then there exists a weakly optimal mechanism.*

*Proof* Suppose that  $C^* \in \mathcal{C}$ , where  $C^*$  is defined as above. Consider a mechanism  $g^*(C)$  which chooses an efficient allocation  $X^*$  when  $C = C^*$  is reported, and no production  $X^* = 0$  otherwise. Let  $w_{ij}^* = c_{ij} x_{ij}^*$ . Obviously,  $g^*$  is weakly optimal.  $\square$

Unfortunately for the principal, weakly optimal mechanisms may always result in zero profit except for a single lowest cost environment. We now suggest an alternative suboptimal

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<sup>12</sup>Note that, first, there do not exist individually rational mechanisms that guarantee strictly positive profits in all environments, since we have assumed (section 1.2.1) that there is  $C \in \mathcal{C}$  such that for any feasible  $X$ ,  $S(X) \leq 0$ . For the same reason, any mechanism that gives the principal a profit higher than the corresponding level of the social surplus cannot be individually rational.

class of “no-loss” mechanisms which allow the possibility of positive profits for uncountable sets of environments.

Consider the following direct revelation mechanism. Given the observable productivity parameters  $F(\cdot)$ , choose an arbitrary  $n \times m$  matrix of constants  $B$ ,  $B \in \mathcal{C}$ , such that

$$\max_{X \in \mathcal{X}} (F(X) - \sum_{i \in N} \sum_{j \in K} b_{ij} x_{ij}) > 0 .$$

Let  $X^*(B)$  denote an allocation where the maximum is achieved. Next, given the cost reports  $\tilde{C}$ , form the matrix  $\tilde{X}(F, \tilde{C})$  using the following rule:

$$\tilde{x}_{ij} = \begin{cases} 1 & \text{if } x_{ij}^*(B) = 1 \text{ and } \tilde{c}_{ij} \leq b_{ij} \\ 0 & \text{otherwise .} \end{cases}$$

Now consider the mechanism  $\hat{g}(\tilde{C}) = (\hat{X}(\tilde{C}), \hat{W}(\tilde{C}))$  such that  $\hat{X}$  solves

$$\max_X (F(X) - \sum_{i \in N} \sum_{j \in K} b_{ij} x_{ij})$$

subject to:

$X$  is feasible ,

$$F(X) - \sum_{i \in N} \sum_{j \in K} b_{ij} x_{ij} \geq 0$$

if  $\tilde{x}_{ij} = 0$  then  $\hat{x}_{ij} = 0$  ,

and  $\hat{W}$  is defined by

$$\hat{w}_{ij} = \begin{cases} b_{ij} & \text{if } \hat{x}_{ij} = 1 \\ 0 & \text{otherwise .} \end{cases}$$

This mechanism is DSIC, IR and never gives the principal a negative profit. For  $C^* = B$ , the mechanism provides the principal her first-best outcome; for any  $C \in \mathcal{C}$  such that  $c_{ij} \leq b_{ij}$  for all  $(i, j) \in N \times K$ , the principal gets a positive profit. In this respect, such suboptimal mechanisms may be more reasonable than the weakly optimal ones. Yet, these mechanisms are still generically inefficient and not profit-maximizing, with the resulting allocations being chosen almost ad hoc; the only purpose of the agent's cost reports may be to insure individual rationality. The "constant rule" weakly optimal mechanisms which always assign the same allocation and wages, unless vetoed by the agents, are even less profitable and sensitive to the environment than the mechanism suggested above. Unfortunately, it is hard to find a no-loss dominant strategy IR mechanism that produces an allocation which is responsive to the cost reports without disturbing the agents' incentives to report the truth. For example, suppose the principal adapts an MPW mechanism truncated at zero level of profit: given the reported environment, she uses the MPW mechanism if it gives her non-negative profit, and chooses not to engage the production process otherwise. Then the agents might be tempted to misrepresent their costs in favor of less efficient allocations for fear of having no production in the case of truthful reports.

To conclude, we find that if the principal has no information about the agents' costs, in the sense specified earlier, the dominant strategy incentive compatible mechanisms that she might use are either almost ad hoc and not sensitive to the agent's private information, or make the principal bear losses in some environments. There is no strongly optimal mechanism for the principal. The result is not surprising in view of the principal's lack of information and a strong dominant strategy requirement imposed on the agent's behavior. We next consider how our conclusions change if we move away from the principal's complete

ignorance assumption.

In the next section, we consider the Bayesian Nash equilibrium framework and characterize optimal Bayesian mechanisms for the expected profit-maximizing principal. We then return to the question of their dominant strategy implementation.

### **1.3 Expected profit maximizing mechanisms**

The results obtained for dominant strategy mechanisms under complete ignorance do not require any assumptions on the consistency of agents' costs characteristics across projects. In this section we introduce such assumption to be able to reduce the general problem to a special single-dimensional case. Perhaps surprisingly, we find that the conditions necessary for Bayesian incentive compatibility of team-formation mechanisms can be quite restrictive even under this strong assumption.

#### **1.3.1 The model**

Suppose now that the principal, when hiring an agent, can observe his profession, but cannot recognize the quality of the training, or the agent's type. In the other words, the principal can tell an engineer from a carpenter, but does not know how qualified each of them is. In general, an agent's qualification (type) may affect both the team's output and the agent's personal costs. Yet in what follows we assume that the team's output is a deterministic function of the agents' professions and is uninformative about their quality<sup>13</sup>. The role of agents' types is to effect their personal costs of performing any task in a consistent way. The high-quality agents bear lower costs compared to the low-quality agents of the same

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<sup>13</sup>For example, we can imagine that a forcing contract makes each team produce an assigned task.



profession. The problem of type revelation exists because of the presence of individual rationality constraints.

Formally, assume that for every agent  $i$ ,  $i \in N$ , his cost of working on each project  $j$ ,  $j \in K$ , is a deterministic twice continuously differentiable non-increasing function of his single-dimensional type  $t_i$ ,  $c_{ij} = c_{ij}(t_i)$ ,  $c'_{ij}(t_i) \leq 0$ ,  $c''_{ij}(t_i) \geq 0$ . The agents' types are stochastically independent random variables, distributed over the supports  $T_i = [0, \bar{t}_i] \subset R$  according to the probability distributions  $H_i(t_i)$ . Each  $H_i(t_i)$  is twice continuously differentiable, with corresponding density  $h_i(t_i)$ ,  $i \in N$ . The distribution functions conform to the Monotone Hazard Rate property, i.e., for every  $i \in N$ ,  $(1 - H_i(t_i))/h_i(t_i)$  is non-increasing in  $t_i$ <sup>14</sup>. Let  $H(t)$  and  $h(t)$  denote the cumulative distribution and density functions of vectors of types  $t = (t_1, \dots, t_n)$  over the support  $T = \times_i T_i$ , and  $H_{-i}(t_{-i})$ ,  $h_{-i}(t_{-i})$  denote the corresponding distribution functions of vectors  $t_{-i} = (t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_n)$ . For all  $i \in N$ , the support  $T_i$ , the probability distribution function  $H_i(t_i)$ , and the cost functions  $c_i(\cdot) : R \rightarrow R^k$  are common knowledge. Each agent's cost type is his private information. Both the principal and the agents are risk-neutral. The agents follow Bayesian Nash Equilibrium behavior. The principal's purpose is to construct a Bayesian incentive compatible (BIC) IR mechanism that maximizes her expected profit. Using the revelation principle<sup>15</sup>, we again restrict our attention to the direct revelation mechanisms. Consider mechanisms that determine probabilities of matrix allocations and wages for the agents as a function of their reported types  $\tilde{t}$ .

<sup>14</sup>The Monotone Hazard Rate property together with  $c''_{ij}(t_i) \geq 0$  are the standard assumptions made in the literature to guarantee that monotonicity constraints (see below) are not binding in the optimal allocation problems with single-dimensional decision space (see, for example, Fudenberg and Tirole (1992)). As we demonstrate in what follows, the assumptions remain important in the analysis of multi-dimensional allocation problems as well.

<sup>15</sup>See, for example, Myerson (1979).

Let  $\mathcal{X} = \{X|X \text{ is feasible}\}$  denote the set of all feasible matrix allocations. We can arbitrarily order the elements in  $\mathcal{X}$  so that  $\mathcal{X} = \{X_1, \dots, X_l, \dots, X_{(k+1)^n}\}$ . Let  $L = \{1, \dots, l, \dots, (k+1)^n\}$  be the set of corresponding indices, and for every  $l \in L$  let  $F_l \equiv F(X_l)$ . Then we can consider a  $(k+1)^n$ -dimensional probability vector  $P = \{p_1, \dots, p_l, \dots, p_{(k+1)^n}\}$ , where  $p_l$  represents the probability of  $l$ -th feasible matrix allocation  $X_l$ . This implies the new feasibility conditions:

$$p_l \geq 0 \quad \text{for all } l \in L$$

$$\sum_{l \in L} p_l = 1 .$$

Equality in the second condition above indicates that  $X = 0$  is a feasible allocation.

Further, let  $W = (w_1, \dots, w_i, \dots, w_n)$  denote the vector of agents' wages. A direct revelation mechanism then is a function from the reported types  $\tilde{t} \in T$  into the probability vector  $P \in R^{(k+1)^n}$  and the wage vector  $W \in R^n$ :  $g(\tilde{t}) = (P, W)$ . We restrict our attention to the mechanisms such that  $P(t)$  is piecewise continuously differentiable.

Some additional notation is needed for further analysis. For a given probability vector  $P$ , for every  $i \in N$ ,  $j \in K$ , let  $L_{ij}(P) \subset L$  denote the set of indices whose corresponding allocations assign agent  $i$  to project  $j$ . Then

$$q_{ij}(P) = \sum_{l \in L_{ij}(P)} p_l$$

denotes the probability of agent  $i$  being assigned to project  $j$ . Given the profile of reported

strategies  $\tilde{t}(t)$ , agent  $i$ 's utility under the mechanism  $(P(\tilde{t}), W(\tilde{t}))$  is:

$$u_i(P, W, \tilde{t}, t) = w_i(\tilde{t}) - \sum_{j \in K} c_{ij}(t_i) * \sum_{l \in L_{ij}(P)} p_l(\tilde{t}). \quad (1.19)$$

Similarly,  $i$ 's expected probability of being assigned to a project  $j$  is

$$Q_{ij}(\tilde{t}, t_i) = \int_{T_{-i}} \sum_{l \in L_{ij}(P)} p_l(\tilde{t}(t_{-i}, t_i)) h_{-i}(t_{-i}) dt_{-i}, \quad (1.20)$$

and  $i$ 's expected utility is, correspondingly,

$$\begin{aligned} U_i(P, W, \tilde{t}, t_i) &= \\ &= \int_{T_{-i}} [w_i(\tilde{t}(t_{-i}, t_i)) - \sum_{j \in K} c_{ij}(t_i) * \sum_{l \in L_{ij}(P)} p_l(\tilde{t}(t_{-i}, t_i))] h_{-i}(t_{-i}) dt_{-i}. \end{aligned} \quad (1.21)$$

Given that the agents' type is  $t$  and their reported strategy is  $\tilde{t}(\cdot)$ , the principal gains the profit

$$\pi(P, W, \tilde{t}, t) = \sum_{l \in L} F_l p_l(\tilde{t}(t)) - \sum_{i \in N} w_i(\tilde{t}(t)) \quad (1.22)$$

and his expected profit is

$$\Pi(P, W, \tilde{t}) = \int_T (\sum_{l \in L} F_l p_l(\tilde{t}(t)) - \sum_{i \in N} w_i(\tilde{t}(t))) h(t) dt. \quad (1.23)$$

Let  $u_i(P, W, t)$  and  $U_i(P, W, t_i)$  denote agent's  $i$  utility and expected utility when the agents report their true types, i.e.,  $\tilde{t}(t) = t$ . Then the principal's problem can be stated as

follows:

$$\max_{P(t), W(t)} \int_T (\sum_{l \in L} F_l p_l(t) - \sum_{i \in N} w_i(t)) h(t) dt \quad (1.24)$$

subject to:

$$p_l(t) \geq 0 \quad \text{for any } l \in L \quad (1.25)$$

$$\sum_{l \in L} p_l(t) = 1 \quad (1.26)$$

$$\begin{aligned} U_i(P, W, t_i) &\geq \\ &\geq \int_{T-i} (w_i(t_{-i}, \tilde{t}_i) - \sum_{j \in K} c_{ij}(t_i) * \sum_{l \in L_{ij}(P)} p_l(t_{-i}, \tilde{t}_i)) h_{-i}(t_{-i}) dt_{-i} \\ &\quad \text{for every } i \in N, \text{ any } t_i, \text{ any } \tilde{t}_i \end{aligned} \quad (1.27)$$

$$U_i(P, W, t_i) \geq 0 \quad \text{for every } i \in N, \text{ any } t_i \quad (1.28)$$

Here the feasibility constraints take the form of 1.25-1.26, and the incentive compatibility (BIC) 1.27 and individual rationality 1.28 constraints are written assuming Bayesian equilibrium behavior.

A direct revelation mechanism  $(P(t), W(t))$  is *optimal* if it solves the problem 1.24-1.28.

### 1.3.2 Optimal Bayesian equilibrium mechanisms

Note that the assumption  $c'_i(t_i) \leq 0$  insures that the single-crossing property, or the constant sign condition (Guesnerie, Laffont (1984)) holds; i.e., the agents' costs on each project change with their types in a consistent manner. This allows us to use the standard techniques developed for the analysis of mechanism design problems in a Bayesian framework<sup>16</sup>.

<sup>16</sup>See Fudenberg and Tirole (1992), chapter 7, for an overview.

Consider the necessary conditions for Bayesian incentive compatibility<sup>17</sup>.

**Proposition 7** (*Interim monotonicity*) *If a feasible direct revelation mechanism  $(P(t), W(t))$  is Bayesian incentive compatible, then for any  $i \in N$ , any  $s_i, t_i \in T_i$  the following is true:*

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(Q_{ij}(P, t_i) - Q_{ij}(P, s_i)) \leq 0. \quad (1.29)$$

We call the above condition “monotonicity” in analogy to the one-project case, where the condition reduces to the requirement that the expected probability of employment is monotonic in an agent’s type. For the multi-project case the condition becomes more demanding. First, as in a one-project case, it requires that an agent of a higher type should be hired with higher expected probability than an agent of a lower type. Second, with respect to shifting an agent’s employment probabilities among the projects, it requires that an agent should be more likely to be assigned to the project where his cost decrease is the fastest among the projects. We first solve for the optimal mechanism assuming that the necessary conditions for BIC hold, and then characterize the conditions under which this assumption holds.

In the spirit of Myerson’s (1981) analysis, for every  $i \in N, j \in K$ , let

$$J_{ij}(t_i) \equiv c_{ij}(t_i) - c'_{ij}(t_i) \frac{1 - H_i(t_i)}{h_i(t_i)} \quad (1.30)$$

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<sup>17</sup>The proofs for the propositions in this section are given in section 1.6.

denote agent's  $i$  virtual cost of working on a project  $j$ . Similarly, define a virtual surplus  $\tilde{S}(X)$  of an allocation  $X$  by

$$\tilde{S}(X; t) = F(X) - \sum_{i \in N} \sum_{j \in K} J_{ij}(t_i) x_{ij} . \quad (1.31)$$

**Proposition 8** *A direct revelation mechanism  $(P(t), W(t))$  of the form*

$$p_l = \begin{cases} 1 & \text{if } X_l \text{ maximizes } \tilde{S}(X; t) \text{ subject to} \\ & \text{interim monotonicity constraint 1.29;} \\ 0 & \text{otherwise ;} \end{cases} \quad (1.32)$$

$$w_i(t_i) = \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau_i) Q_{ij}(\tau_i) d\tau_i$$

for all  $i \in N$ , all  $t_i \in T_i$  (1.33)

is optimal for the principal<sup>18</sup>.

Hence, an optimal mechanism, within the constraint imposed by monotonicity, chooses an allocation that maximizes virtual social surplus and offers each agent an expected payment which is never lower than his expected costs of employment (the latter follows from the assumption that  $c'_i(t_i) \leq 0$ ). Similar to the optimal auctions results (Myerson (1981)), we find that, first, the optimal team-formation mechanisms are generically inefficient, since the principal trades off some efficiency for higher expected profit. Second, there exists the whole class of equivalent optimal BIC mechanisms which differ from each other in the form

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<sup>18</sup>In the above statement we ignore the possibility that there may exist more than one allocation  $X$  that maximizes the virtual surplus. Generically, these cases occur with probability zero. However, if such a situation emerges, an optimal mechanism, equivalent to 1.32-1.33, randomly chooses one of the efficient allocations.

of actual wages paid to the agents; expected wages are given by 1.33. Also note that, similarly to the optimal auction, an agent's probability of being hired under an optimal mechanism is a non-decreasing function of his type: Provided that the Monotone Hazard rate condition and the assumption  $c''_{ij}(t_i) \geq 0$  hold for all  $i, j$ , we get that  $J'_{ij}(t_i) \leq 0$  for any  $i, j$ . This important observation is useful for the further analysis; we state it as the following lemma:

**Lemma 3** *Under an optimal mechanism of the form 1.32-1.33, for any  $t \in T$ , any  $i \in N$ , agent  $i$ 's probability of employment  $\sum_{j \in K} x_{ij}(t_{-i}, t_i)$  is a non-decreasing function of his type  $t_i$ .*

The optimal BIC mechanisms differ depending on whether the interim monotonicity constraints 1.29 are ever binding. If  $H(t)$  and  $c_i$ 's are such that the constraints are not binding, then the optimal mechanism is explicitly given in Proposition 7. Otherwise, bunching is optimal over the ranges of types where the monotonicity is binding<sup>19</sup>. We now proceed with the analysis of the restrictiveness of the monotonicity constraints 1.29.

### 1.3.3 Analysis of monotonicity requirements

To see that the constraints 1.29 can be indeed rather restrictive and often binding, consider the following example.

**Example** (*Restrictiveness of the monotonicity constraint*) Consider the problem of hiring one agent on two alternative projects, i.e., let  $n = 1, k = 2$ . Let  $F_1 = 100, F_2 = 98$ . Suppose

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<sup>19</sup>See Guesnerie and Laffont (1984) for an exposition of bunching technique.

that the agent's cost type  $t$  is uniformly distributed on  $[0, 5]$ , and let  $c_1(t) = 3/(t+0.1) + 10$ ,  $c_2(t) = 20 - 4t$ . Note that the uniform distribution conforms to the Monotone Hazard Rate property and the cost functions are decreasing and convex, as required by the initial assumptions. We demonstrate that the monotonicity condition 1.29, which in this case takes the form of ex-post monotonicity,

$$\sum_{j \in K} (c_j(t) - c_j(s))(x_j(P, t) - x_j(P, s)) \leq 0, \quad (1.34)$$

is not trivially satisfied. Take  $s = 0.23$  and  $t = 2.5$  and let us compute the virtual surplus maximizing allocations  $X(s)$ ,  $X(t)$ . Since  $c_1(s) = 19.08$ ,  $c_2(s) = 19.08$ ,  $c_1(t) = 11.15$ ,  $c_2(t) = 10$ ;  $(1 - H(s))/h(s) = 0.19$ ,  $(1 - H(t))/h(t) = 0.1$ ;  $c'_1(s) = -27.59$ ,  $c'_2(s) = -4$ ,  $c'_1(t) = -0.44$ ,  $c'_2(t) = -4$ , we obtain  $J_1(s) = 24.7$ ,  $J_2(s) = 19.84$ ,  $J_1(t) = 11.55$ ,  $J_2(t) = 10.04$ . Therefore,

$$\begin{aligned} \tilde{S}_1(s) &= 75.3 & \tilde{S}_2(s) &= 78.2 & \tilde{S}_1(t) &= 88.45 & \tilde{S}_2(t) &= 87.96 \\ x_1(s) &= 0 & x_2(s) &= 1 & x_1(t) &= 1 & x_2(t) &= 0. \end{aligned}$$

However, the above allocations do not conform to the monotonicity constraint 1.34 and therefore cannot be chosen:

$$\begin{aligned} (c_1(t) - c_1(s))(x_1(t) - x_1(s)) + (c_2(t) - c_2(s))(x_2(t) - x_2(s)) &= \\ = (11.15 - 19.08)(1 - 0) + (10 - 19.08)(0 - 1) &= 1.15 > 0. \end{aligned}$$

Figure 1.1 gives a graphical illustration of the above example. Note that the graphs



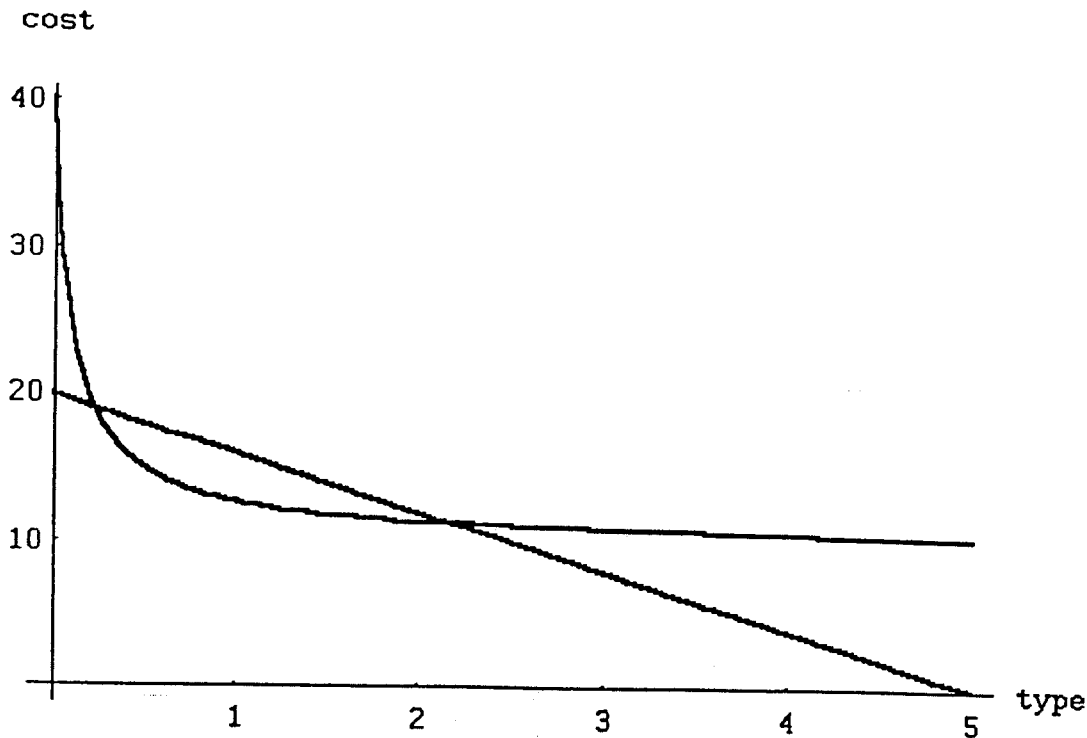


Figure 1.1: An example of cost functions violating sufficient conditions for monotonicity:  $c_j(t) = 3/(t + 0.1) + 10$ ,  $c_k(t) = 20 - 4t$ ,  $t \in [0, 5]$ .

of the cost functions for the two projects intersect more than once, which indicates that there is no consistency in the cost change on one project relative to the other. Specifically, there is a switch in the relative rate of cost decrease between the projects:  $c'_1(s) < c'_2(s)$ , but  $c'_1(t) > c'_2(t)$ . As a consequence it is possible that with the change in type the true costs change in favor of one project, whereas the virtual costs change in favor of the other. Under these rather typical circumstances, the monotonicity constraint can be violated quite easily, as the example above shows. Below, we present sufficient conditions for monotonicity, which guarantee that the above situation is never the case<sup>20</sup>. The condition states that the difference in the rates of cost decreases between any two projects does not change “too much” compared to the difference in the change in the absolute costs between the projects.

<sup>20</sup>The sufficient conditions for monotonicity presented in propositions 9 and 10 are specific for  $x_{ij} \in \{0, 1\}$  case.

**Proposition 9** (*Sufficient conditions for monotonicity*) *The monotonicity conditions 1.29 are satisfied if the following is true:*

*For every  $i \in N$ , any  $s_i, t_i \in T_i$ , any  $j, k \in K$ ,  $j \neq k$ , if*

$$(c_{ij}(s_i) - c_{ij}(t_i)) - (c_{ik}(s_i) - c_{ik}(t_i)) > 0, \quad (1.35)$$

*then*

$$\begin{aligned} (c_{ij}(s_i) - c_{ij}(t_i)) - (c_{ik}(s_i) - c_{ik}(t_i)) &> \left(\frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ij}(s_i) - \frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ij}(t_i)\right) - \\ & - \left(\frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ik}(s_i) - \frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ik}(t_i)\right). \end{aligned} \quad (1.36)$$

*Moreover, the above condition is also sufficient to guarantee that the ex-post monotonicity requirement is satisfied: if for any  $i \in N$ , any  $s_i, t_i \in T_i$  1.35 implies 1.36, then for all  $t_{-i} \in T_{-i}$  the following inequality holds:*

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t) - x_{ij}(s_i, t_{-i})) \leq 0. \quad (1.37)$$

*Outline of the proof* The complete proof is given in the section 1.6; here we present the outline to show that the above conditions guarantee not only interim, but also ex-post monotonicity.

We show that virtual surplus maximization

$$\max_X \{F(X) - \sum_{i \in N} \sum_{j \in K} J_{ij}(t_i) x_{ij}\} \quad (1.38)$$

implies that for any  $i \in N$ , any  $s_i, t_i$

$$\sum_{j \in K} (J_{ij}(t_i) - J_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0, \quad (1.39)$$

which, under the conditions stated in the proposition, in turn implies

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0. \quad (1.40)$$

The latter inequality is ex-post monotonicity, which is clearly stronger than interim monotonicity and implies it.  $\square$

The form of the sufficient conditions together with the earlier example suggest that the conditions that guarantee monotonicity are not trivial and do not generically hold. Rather, they are satisfied for certain groups of type distributions and cost functions. In particular, the monotonicity conditions hold if each agent's costs change in a consistent manner not only with types, but also from project to project. We state this case in the following proposition.

**Proposition 10** *Suppose that for every  $i \in N$ , for every pair of projects  $j, k \in K$ , either*

$$(i) \quad c'_{ij}(t_i) < c'_{ik}(t_i) \text{ and } c''_{ij}(t_i) \geq c''_{ik}(t_i) \text{ for all } t_i \in T_i, \quad (1.41)$$

or

$$(ii) \quad c'_{ij}(t_i) \geq c'_{ik}(t_i) \text{ and } c''_{ij}(t_i) \leq c''_{ik}(t_i) \text{ for all } t_i \in T_i. \quad (1.42)$$

*Then the monotonicity conditions 1.29, as well as 1.37, are satisfied.*

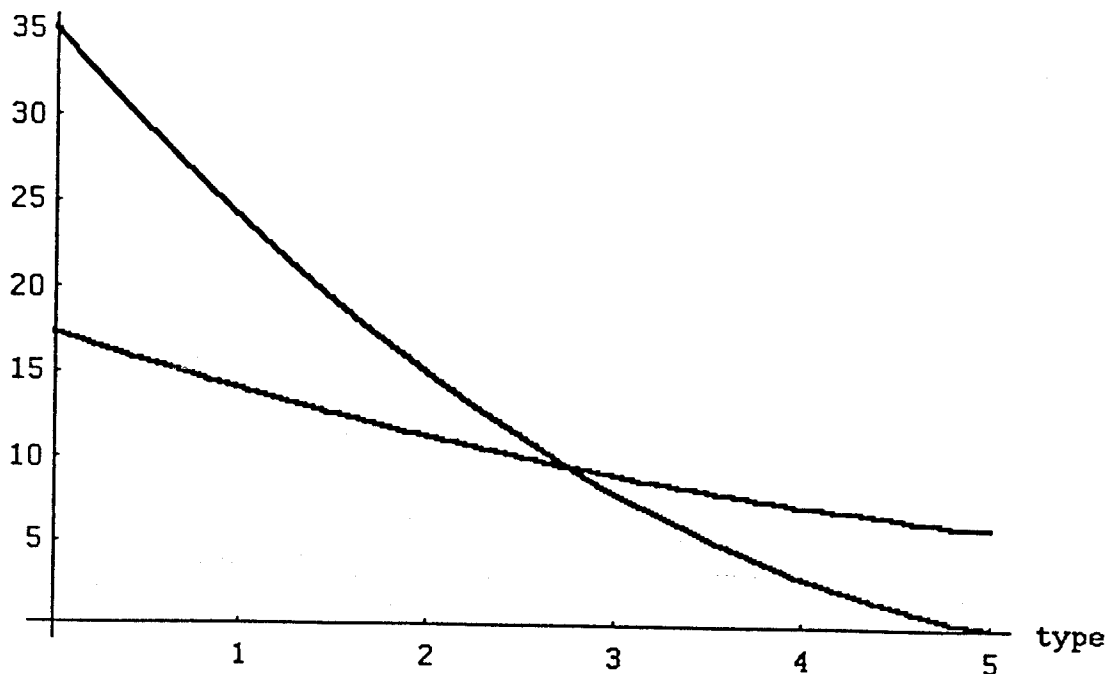


Figure 1.2: An example of cost functions satisfying sufficient conditions for monotonicity:  $c_j(t) = (x - 6)^2 - 1$ ,  $c_k(t) = (x - 7)^2/4 + 5$ ,  $t \in [0, 5]$ .

The set of assumptions presented in the proposition 10 implies that for each agent, the projects are ranked with respect to the rates in cost changes; this ranking does not change with types. This can be interpreted as a single-crossing property for the cost functions of each agent: under the given conditions, an agent's cost functions for any two projects  $j$  and  $k$  can intersect at most once (compare this to the cost functions in the example above). Figure 1.2 illustrates the idea.

The above versions of sufficient conditions for the monotonicity, together with the standard single-crossing property guaranteed by the assumption  $c'_{ij} \leq 0$  for any  $i, j$ , present a set of very restrictive regularity requirements. Although the necessary and sufficient conditions<sup>21</sup> for the interim monotonicity may be less restrictive, the example presented

<sup>21</sup>The necessary and sufficient conditions are not considered here for the reason that they differ depending on the values of the observed productivity parameters; obtaining sensible necessary conditions requires imposition of certain regularity requirements on productivities, which would narrow the scope of the analysis.

in this section demonstrates that some consistency in each agent's cost functions for different projects may still be required. The above analysis indicates that the monotonicity conditions become much harder to satisfy once we move from one-dimensional to multi-dimensional decision spaces. Therefore, under a broad range of circumstances, bunching will be optimal over wide ranges of an agent's type. This suggests that multidimensionality of decision variables, through making monotonicity conditions more restrictive, may substantially decrease the principal's expected profit compared to a one-dimensional (one-project) allocation problem.

For the rest of the section, we restrict our attention to the cases when the necessary monotonicity conditions are not binding, and therefore the optimal BIC mechanism is of the form explicitly presented in proposition 7.

#### 1.3.4 Implementation in dominant strategies

Following the results of Mookherjee and Reichelstein (1992), we now show that an optimal BIC mechanism can be equivalently implemented in dominant strategies with no expected loss to the principal. This brings us back to the problem initially stated in section 1.2 – the one of finding an optimal profit-maximizing dominant strategy incentive compatible mechanism. The following result shows that if the principal knows a prior distribution of the agents' cost types there exists an optimal, in the sense of expected profit maximization, DSIC IR mechanism.

**Proposition 11** *Suppose that the cost functions  $c_i(t_i)$  and the distribution functions  $H_i(t_i)$ ,  $i \in N$ , are such that the ex-post monotonicity conditions 1.37 are satisfied. Then the direct*

revelation mechanism  $(X(t), W(t))$ , given by

$$X^*(t) = X \text{ that maximizes } \tilde{S}(X; t) , \quad (1.43)$$

$$w_i(t) = \sum_{j \in K} c_{ij}(t_i) x_{ij}^*(t) - \int_0^{t_i} \sum_{j \in K} c_{ij}'(\tau) x_{ij}^*(t_{-i}, \tau) d\tau$$

for all  $i \in N$ ,  $t_i \in T_i$ ,  $t_{-i} \in T_{-i}$  ,

(1.44)

is a dominant strategy incentive compatible and ex-post individually rational mechanism which yields the same expected profit as the optimal Bayesian incentive compatible mechanism 1.32-1.33.

The above proposition says that we can replace the optimal BIC mechanism with an equivalent DSIC mechanism<sup>22</sup>. Moreover, since the DSIC constraints are more restrictive than the BIC constraints, we have also established

**Corollary 6** *If the cost functions  $c_i(t_i)$  and the distribution functions  $H_i(t_i)$ ,  $i \in N$ , are such that the sufficient conditions for monotonicity (proposition 9) are satisfied, then 1.43-1.44 is an optimal expected profit maximizing dominant strategy incentive compatible individually rational mechanism.*

To draw a parallel with the section 1.2 results, note that the above mechanism is not an efficiency-maximizing Groves mechanism, but it does have certain incentive properties in common with it. First observe that an agent's type report affects his wage only through the allocation decision; within the same allocation of an agent, his wage is constant in his own report. Indeed, for any  $i \in N$  and any  $t_{-i} \in T_{-i}$ , suppose  $x_i^*(t_{-i}, t_i) = x_i^*(t_{-i}, s_i) \equiv x_i^*$

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<sup>22</sup>The finding still holds if the  $x_{ij}$ 's are continuous variables.

for some  $t_i \neq s_i$ . Let  $t_i > s_i$ . Then, by 1.44,

$$\begin{aligned}
 w_i(t_{-i}, t_i) - w_i(t_{-i}, s_i) &= \\
 &= \sum_{j \in K} c_{ij}(t_i) x_{ij}^* - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau) x_{ij}^*(t_{-i}, \tau) d\tau - \\
 &\quad - \sum_{j \in K} c_{ij}(s_i) x_{ij}^* + \int_0^{s_i} \sum_{j \in K} c'_{ij}(\tau) x_{ij}^*(t_{-i}, \tau) d\tau = \\
 &= \sum_{j \in K} c_{ij}(t_i) x_{ij}^* - \sum_{j \in K} c_{ij}(s_i) x_{ij}^* - \int_{s_i}^{t_i} \sum_{j \in K} c'_{ij}(\tau) x_{ij}^*(t_{-i}, \tau) d\tau = 0. \quad (1.45)
 \end{aligned}$$

Second, sufficient conditions for monotonicity 1.35-1.36 guarantee that, given the allocation rule 1.43, an agent's true cost report maximizes his own utility as well as the principal's objective function<sup>23</sup>. Comparing these findings with the properties of the DSIC IR mechanisms under complete ignorance, we find that the principal's knowledge of the probability distributions of the agents' cost types – at least in the special case when these types are one-dimensional – is decisive in determining the employment rules which insure that (i) individual rationality holds and (ii) the expected profit is nonnegative<sup>24</sup> and is maximized. In contrast, under complete ignorance nearly the only way to satisfy individual rationality in DSIC mechanisms was to sacrifice the profit maximization motive and either choose random allocations or pay unreasonably high wages to the agents. One might conclude that no matter how well or poorly informed the agents are about each other's costs, the principal's

<sup>23</sup>In fact, given any  $t_{-i} \in T_{-i}$ ,  $i$ 's assignment and, respectively, his wage are step functions of his type report  $t_i$ . This follows from the form of the allocation rule 1.43 and continuity of  $J_{ij}(t_i)$  in  $t_i$ . Thus, for every  $i \in N$  we can identify a collection of threshold types  $\{s_{i0}(t_{-i}) = 0, s_{i1}(t_{-i}), \dots, s_{iL}(t_{-i}) = \bar{t}_i\}$ ,  $L < \infty$ , such that  $x_i^*(t_{-i}, t_i)$  and, consequently,  $w_i^*(t_{-i}, t_i)$  are constant within each interval  $(s_{i,l-1}(t_{-i}), s_{i,l}(t_{-i}))$ ,  $l = 1, \dots, L$ . Note that since  $i$ 's probability of employment is non-decreasing in his type (lemma 3),  $x_i^*(t_{-i}, t_i) = 0$  if  $t_i \in [0, s_{i1}(t_{-i}))$  and  $\sum_{j \in K} x_{ij}^*(t_{-i}, t_i) = 1$  otherwise. Then one can easily show that  $i$ 's wage at each allocation is a function of his costs at the threshold types only, and does not directly depend on his own type report.

<sup>24</sup>By construction of the optimal mechanism, an allocation  $X = 0$  is always an option, which guarantees that the expected profit is nonnegative.

possession of information is crucial for her profit maximization.

## **1.4 A note on Nash equilibrium mechanisms**

In the above study, we ignored the case of the extreme informational asymmetry – that is, when the principal is “completely ignorant” (as defined in section 1.2) but the agents themselves are well-informed about each other’s characteristics. Assuming that under this information structure the agents follow Nash equilibrium behavior, we present two notes on the Nash equilibrium mechanisms. First, we find that if the agents follow Nash equilibrium behavior, the principal can design mechanisms that will always secure her a non-negative profit, but generically cannot guarantee her most preferred outcome. However, we further demonstrate that if the agents are sequentially rational, there exist sequential mechanisms that allow the principal to acquire, almost costlessly, all the hidden information and obtain an outcome which is arbitrarily close to her most preferred alternative. In other words, under certain circumstances the principal can use the agents’ self-interest to accumulate the hidden information at a low cost.

### **1.4.1 Nash implementation and the first best**

Assume that the agents have complete information about each other’s types and follow Nash equilibrium behavior. However, the principal has no information about the agents’ costs characteristics and can pursue her own interests only by setting the “rules of the game”, or the mechanism under which the agents are employed and paid for their jobs. Without focusing on any specific Nash equilibrium mechanism, we use the Nash implementation theory framework to consider what levels of principal’s profit are implementable in Nash



equilibrium.

As in section 1.2, let  $(F, C)$  characterize an environment, and let  $(\mathcal{F}, \mathcal{C})$  be the set of all possible environments, as given in section 1.2.1. A feasible alternative  $a = (X, W)$  is a feasible allocation  $X$  and a wage matrix  $W$  such that  $|\sum_{i \in N} \sum_{j \in K} w_{ij}| \leq M$ , where  $M$  is a big enough real number<sup>25</sup>. Denote by  $A$  the set of all feasible alternatives<sup>26</sup>. The principal's profit corresponding to an alternative  $a$  in an environment  $(F, C)$ ,  $\pi(a; (F, C))$ , is given by 1.2. With the agents' utility functions given by equation 1.3, the agents' cost parameters bear sufficient information about their utility functions. Since  $F$  is observable for the principal, given any  $F \in \mathcal{F}$  we can present a choice rule as a correspondence  $G_F: \mathcal{C} \rightarrow A$ ; denote by  $G_F(C)$  the resulting choice set. Then  $G_F$  is implementable in Nash equilibrium if there exists a game with the set of Nash equilibria coinciding with the choice set  $G_F(C)$ .

Consider whether the principal's most preferred choice set is implementable in Nash equilibrium. Since we assume that the agents always have an option not to participate in the game proposed by the principal, a chosen allocation and wage have to be individually rational for every agent. Therefore, in any environment the principal prefers the choice rule that gives her first best, or the complete information outcome given by 1.9-1.10. Given

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<sup>25</sup>For example, we can choose  $M = \sum_i \sum_j \bar{c}_{ij} \bar{x}_{ij}$ , where  $\bar{c}_i = \sup\{c_i | c_i \in \mathcal{C}_i\}$ , and  $\bar{X}$  maximizes  $F(X) - \sum_i \sum_j \bar{c}_{ij} x_{ij}$ . Restrictions on the range of possible wages are imposed for the tractability of analysis; otherwise the agents might unanimously prefer infinitely large wages.

<sup>26</sup>Note that the set of feasible alternatives is not constrained to the set of individually rational alternatives and therefore stays the same for every environment.

$F \in \mathcal{F}$ , the principal's first best is the following choice rule:

$$G_F(C) = \begin{cases} X^* & = X \text{ that maximizes } S(X; (F, C)) \\ w_{ij}^* & = c_{ij}x_{ij}^* \text{ for all } i, j, \end{cases} \quad (1.46)$$

where, as before,  $S(X; (F, C))$  denotes the social surplus of an allocation. We can show that for almost all  $(\mathcal{F}, \mathcal{C})$  this rule violates the monotonicity property which is necessary for implementability in Nash equilibrium (Maskin, 1979), and thus establish

**Proposition 12** *Suppose the set of possible cost profiles  $\mathcal{C}$  is such that for every  $F \in \mathcal{F}$  there exists  $C \in \mathcal{C}$  which satisfies the following conditions: for every surplus-maximizing allocation  $X^*(F, C)$  there exists at least one pair  $(i, j)$ ,  $i \in N$ ,  $j \in K$  for which  $x_{ij}^*(F, C) = 1$  and  $c_{ij} > \inf\{c_{ij} | c_{ij} \in \mathcal{C}_{ij}\}$ . Then the principal's first best is not implementable in Nash Equilibrium.*

We conclude that under the Nash equilibrium behavior hypothesis, the principal cannot always get her most preferred alternative if she has no information about the environment<sup>27</sup>. The proof of the proposition 12<sup>28</sup> also shows that no matter what Nash equilibrium mechanism the principal uses, under many cost profiles she has to give to the agents substantial shares of social surplus in order to sustain monotonicity. Note, however, that the principal can use her power as mechanism designer to obtain the outcomes which are always individually rational for herself. We introduce the notion of an *acceptable* alternative, which corresponds to the notion of feasibility for the game of surplus redistribution without a principal.

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<sup>27</sup>The result depends crucially on the assumption that  $x_{ij} \in \{0, 1\}$ , as the proof in section 1.6 shows.

<sup>28</sup>The proof is given in section 1.6.

**Definition 9** An alternative  $a = (X, W) \in A$  is called acceptable if

$$F(X) - \sum_i \sum_j w_{ij} x_{ij} \geq 0 .$$

**Proposition 13** For any environment, the principal can guarantee an outcome from the set of acceptable alternatives.

*Proof* Consider any mechanism that includes the following element. Given observable productivities  $F$ , for any  $(X, W)$ , determined according to some choice rule, let

$$(X^*, W^*) = \begin{cases} (X, W) & \text{if } F(X) - \sum_{i \in N} \sum_{j \in K} w_{ij} x_{ij} \geq 0 \\ (0, 0) & \text{otherwise .} \end{cases} \quad (1.47)$$

Thus, the principal can “veto” any outcome that gives her a negative payoff by choosing not to employ anybody.  $\square$

We summarize the above findings in the following corollary.

**Corollary 7** If the agents follow Nash equilibrium behavior, the principal can guarantee herself a non-negative profit, but cannot guarantee her first best.

### 1.4.2 Sequential mechanisms

Surprisingly, the situation drastically changes in favor of the principal if we assume that the agents follow subgame perfect Nash equilibrium behavior. In this case, the principal can use simple sequential mechanisms to implement the outcomes that in any environment are arbitrarily close to her first best alternative. Indeed, we prove the following proposition.

**Proposition 14** *If the agents follow subgame perfect Nash equilibrium behavior, for any arbitrarily small  $\epsilon > 0$  there exists an individually rational mechanism  $G_\epsilon(F, C) = (X, W)$  such that for any environment  $(F, C)$*

$$\pi(G_\epsilon(F, C)) \geq (1 - \epsilon)\pi^*(F, C) ,$$

where  $\pi^*(F, C)$  is the principal's complete information profit level.

*Proof* Consider the following sequential mechanism<sup>29</sup>. For an arbitrary  $\epsilon > 0$ , choose  $\epsilon_1 > 0$ ,  $\epsilon_2 > 0$  so that  $\epsilon_1 + \epsilon_2 \leq \epsilon$ . Pick randomly two agents  $m, l \in N$ . Let each stage of the mechanism be observable to the agents. At stage 1, let agent  $l$  choose a  $k$ -dimensional vector  $x_m$  and a scalar  $s_m$ , such that  $x_m$  constitutes a feasible allocation of agent  $m$ . At stage 2, let agent  $m$  choose an  $(n - 1) \times k$  matrix  $X_{-m}$  and a  $(n - 1)$ -dimensional vector  $S_{-m}$ , where, again,  $X_{-m}$  is a feasible allocation of agents other than  $i$ . At stage 3, allow any agent to veto  $m$ 's or  $l$ 's choices by reporting "no." Define the following "profit" function:

$$p(X, S) \equiv F(X) - \sum_{i \in N} s_i . \quad (1.48)$$

Finally, let the mechanism choose the resulting outcome  $(X^*, W^*)$  by the following rule.

For every  $i \in N$ , every  $j \in K$ , let

$$x_{ij}^* = \begin{cases} 0 & \text{if "no" has been reported by any agent} \\ x_{ij} & \text{otherwise ;} \end{cases} \quad (1.49)$$

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<sup>29</sup>I am grateful to John Duggan for suggesting the idea of this mechanism to me.

$$w_{ij}^* = \begin{cases} s_i & \text{if } x_{ij}^* = 1 \text{ and } i \neq l, i \neq m \\ s_l + \epsilon_1 p(X, S) & \text{if } x_{ij}^* = 1 \text{ and } i = l \\ s_m + \epsilon_2 p(X, S) & \text{if } x_{ij}^* = 1 \text{ and } i = m \\ 0 & \text{otherwise .} \end{cases} \quad (1.50)$$

Using backwards induction reasoning, we now show that any two chosen agents will pick an efficient allocation  $X$  and the “base wage” vector  $S$  with  $s_i = \sum_{j \in K} c_{ij} x_{ij}$  for every  $i \in N$ . From stage 3, agents  $l$  and  $m$  cannot be better-off by choosing  $s_i < \sum_{j \in K} c_{ij} x_{ij}$  for any  $i$  since then their choice will be vetoed and their gain will be identically zero. This guarantees individual rationality of the mechanism. Next, on stage 2 agent  $m$ , with his assignment and “base wage”  $s_m$  already given, can only maximize his utility by maximizing the principal’s profit. This implies that he will choose a surplus-maximizing allocation, constrained to his own allocation, and “base wages”  $s_i \leq \sum_{j \in K} c_{ij} x_{ij}$ , i.e.,  $s_i = \sum_{j \in K} c_{ij} x_{ij}$  for every agent including  $l$ . Hence at stage 1 agent  $l$  knows that no matter what choices he makes, the difference between his “base wage” and cost at his assignment will be identically zero. Therefore, agent  $l$  also can increase his utility only through maximizing the principal’s profit. It follows that agent  $l$  will pick an assignment for  $m$  that is consistent with an efficient allocation, and choose  $s_m = \sum_{j \in K} c_{mj} x_{mj}$ .  $\square$

The above analysis indicates that, assuming sequential rationality, a completely uninformed principal can almost costlessly acquire all the information he needs to implement his most preferred outcome. All she needs to do is to hire two informed agents on a profit-sharing basis<sup>30</sup>. One might conclude that the agents do not always gain from having more

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<sup>30</sup>Think of foremen who are put in authority over groups of workers or particular operations in a plant.

information: As our results on dominant strategy mechanisms show, if the agents themselves are ignorant of the other agents' cost types, the principal often needs to pay huge information rents to accumulate the dispersed private information. Yet, the agents' complete information case more readily applies to situations where the agents have a past experience of working together than to newly formed teams in flexible organizations.

## **1.5 Conclusion**

In considering the multiple teams formation problem, we were able to demonstrate several points. First, most generally, the principal's knowledge of the information structure of the agents' characteristics is crucial for her profit-maximization motive – as opposed to efficiency maximization, where no information on the principal's part is required to make an efficient decision. If a principal starts a new project (or a new firm) with no idea how costly this project might be for her, then, even with no nature-induced uncertainty and the information dispersed among the agents, she is likely to run into losses in an attempt to have agents truthfully reveal this information. Yet, if the agents themselves are well-informed about each others' characteristics, the principal can use their self-interest to accumulate this information at a low cost. If the principal is aware of the distribution of the agents' cost characteristics, there exists a well-defined optimal mechanism that maximizes her expected profit. However, when the decision space becomes more complicated, as in the multiproject case, the incentive compatibility constraints are more likely to become binding and thus reduce the principal's profit by often making her treat "good" and "bad" agents equally.

Competition among agents might substitute for the information needed by the principal.

We learn that in perfectly competitive environments the principal may collect all the social surplus without having any information on the distribution of the agent's costs. This is also the case when efficiency gains from team production are low. On the other hand, when each agent is indispensable and the efficiency gains from team production are high, the principal is very likely to bear losses. This suggests that a principal might prefer to run a robustly-structured enterprise with homogeneous labor factors and standard tasks rather than a flexible corporation with highly innovative tasks and indispensable agents. Changing to the latter requires acquisition of new information about the agents' characteristics, which might turn out to be very costly for the principal in a hierarchy.

Turning back to our initial question, we find that the appeal for partnerships in the context of flexible organizational forms has theoretical explanations. The agents might want to organize themselves as partners and share efficiency gains from their joint activities when it cannot be profitably done by an outside principal. Yet, incentive compatibility needs to be sustained within a partnership, as well as a principal-run firm, if the agents have incomplete information about each other. This important problem has to be addressed before we can argue in favor of partnerships. Still, when the agents are well-informed about each others' characteristics, partnerships appear to be a feasible way to achieve higher efficiency by means of flexible organizational forms. Small consulting firms working on a variety of different tasks is the most obvious example.

As the other side of the same conclusion, we might expect large hierarchical structures to have major incentive problems, either on the principals' or on the agents' side, in attempts to reorganize towards more flexible internal structures. A possible solution might be in reducing informational asymmetries or, possibly, changing the ownership structure towards

partnerships.

## 1.6 Proofs of the statements

**Proofs for section 1.2** *Proof of Lemma 1* Let  $\pi^*(F, C)$  denote the profit that the principal would be able to get in an environment  $(F, C)$  if she had complete information. Suppose there exists a strongly optimal DSIC IR mechanism  $g(F, C)$  such that  $\pi(g(\hat{F}, \hat{C})) < \pi^*(\hat{F}, \hat{C})$  for some environment  $(\hat{F}, \hat{C})$ . Then consider the following degenerate direct revelation mechanism  $\tilde{g}(F, C)$ : Denote by  $X^*(F)$  an allocation that maximizes

$$F(X) - \sum_{i=1}^n \sum_{j=1}^k \hat{c}_{ij} x_{ij} .$$

Then for any  $(F, \tilde{C})$  choose  $(\tilde{X}(\tilde{C}), \tilde{W}(\tilde{C}))$  such that

$$\tilde{x}_{ij} = \begin{cases} 1 & \text{if } x_{ij}^*(F) = 1 \text{ and } \tilde{c}_{ij} \leq \hat{c}_{ij} \\ 0 & \text{otherwise} \end{cases} ;$$

$$\tilde{w}_{ij} = \begin{cases} \hat{c}_{ij} & \text{if } \tilde{x}_{ij} = 1 \\ 0 & \text{otherwise} \end{cases} .$$

Note that, first,  $\tilde{g}(F, C)$  is DSIC and IR for any environment and, second, for the environment  $(\hat{F}, \hat{C})$  it provides the principal the level of profit

$$\pi(\tilde{g}(\hat{F}, \hat{C})) = \pi^*(\hat{F}, \hat{C}) > \pi(g(\hat{F}, \hat{C})) .$$

This contradicts our initial supposition that  $g(F, C)$  is strongly optimal.  $\square$



*Proof of Lemma 2* For the class of problems with no production, any efficiency-maximizing DSIC mechanism has to be a Groves mechanism, with the transfers (wages) given in the form (Green and Laffont, 1977):

$$\sum_j w_{ij} x_{ij}^* = - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj}^*(\tilde{c}_i, \tilde{C}_{-i}) + h(\tilde{C}_{-i}) .$$

Introduction of production, however, changes the social efficiency criterion and correspondingly modifies the form of the transfer function. Consider the problem of choosing a socially efficient allocation in the variant with observable production. One can easily show that a direct revelation mechanism is DSIC if and only if it satisfies the following properties (this is a modified “Property A” (Green and Laffont, 1977)):

(i). For all  $i$ ,  $w_i$  is independent of  $\tilde{c}_i$  at  $X^*$ ; i.e., for any  $F$ ,  $\tilde{C}_{-i}$ ,  $\tilde{c}_i$ ,  $\tilde{c}'_i$ , if  $X^*(\tilde{c}_i, \tilde{C}_{-i}, F) = X^*(\tilde{c}'_i, \tilde{C}_{-i}, F)$ , then  $w_i(\tilde{c}_i, \tilde{C}_{-i}, F) = w_i(\tilde{c}'_i, \tilde{C}_{-i}, F)$ .

$$\begin{aligned} (ii). \quad & w_i(\tilde{c}_i, \tilde{C}_{-i}, F) - w_i(\tilde{c}'_i, \tilde{C}_{-i}, F) = \\ & = [F(X^*) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj}^*(\tilde{c}_i, \tilde{C}_{-i}, F)] - [F(\{\tilde{x}_{ij}\}) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} \tilde{x}_{lj}(\tilde{c}'_i, \tilde{C}_{-i}, F)] , \end{aligned}$$

where  $X^*$  maximizes  $F(X) - \sum_i \sum_j \tilde{c}_{ij} x_{ij}(\tilde{c}_i, \tilde{C}_{-i})$ , and  $\tilde{X}$  maximizes  $F(X) - \sum_i \sum_j \tilde{c}_{ij} x_{ij}(\tilde{c}'_i, \tilde{C}_{-i})$ .

Next we can show that the only mechanisms that satisfy these properties are the Modified Groves (as defined above). Moreover, all the properties of the standard Groves mechanism hold for the modified version<sup>31</sup>. Therefore, the results regarding the standard Groves

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<sup>31</sup>The proofs go exactly as they would for the Groves mechanisms and hence do not present anything new of interest; see Green and Laffont (1977) for the original proofs.

mechanisms apply.  $\square$

*Proof of Proposition 2* By construction, the MPW mechanism is Modified Groves, which implies that it is DSIC and efficient. It is left to show that it is individually rational. Note that if an agent  $i$ ,  $i \in N$ , reports the truth, then

$$\begin{aligned} \sum_{j=1}^k w_{ij}^* x_{ij}^* &= \max_X [F(X) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj} - \sum_j c_{ij} x_{ij}] - \\ &\quad - \max_{X_{-i}} [F(\{x_{ij}\}_{-i}) - \sum_{l \neq i} \sum_j \tilde{c}_{lj} x_{lj}] + \sum_j c_{ij} x_{ij}^* = \\ &= S(X^*(\tilde{C}_{-i}, c_i)) - S(\tilde{X}_{-i}(\tilde{C}_{-i})) + \sum_j c_{ij} x_{ij}^* \geq \sum_j c_{ij} x_{ij}^* \end{aligned} \quad (1.51)$$

since

$$S(X^*(\tilde{C}_{-i}, c_i)) \geq S(\tilde{X}_{-i}(\tilde{C}_{-i})) .$$

Therefore,

$$\sum_{j=1}^k (w_{ij}^* - c_{ij}) x_{ij}^* \geq 0 . \quad (1.52)$$

$\square$

*Proof of Proposition 3* As before, let  $X^*(F, C)$  denote an efficient allocation of the set  $N$  of agent, and  $\tilde{X}_{-i}(F, C_{-i})$  – an efficient allocation of the set  $\{N \setminus i\}$  agents. Besides, let  $N^*(F, C)$  be the set of employed agents:  $N^*(F, C) = \{i \in N \mid \sum_j x_{ij}^* = 1\}$ . By corollary 3, it is sufficient to show that for every  $(F, C)$

$$- \sum_{i \in N^*} S(\tilde{X}_{-i}(F, C_{-i})) \leq \sum_{i \in N^*} \tilde{h}_i(C_{-i}) , \quad (1.53)$$

for any  $\tilde{h}(C) = (\tilde{h}_1(C_{-1}), \dots, \tilde{h}_n(C_{-n}))$  such that the corresponding Modified Groves mech-

anism is individually rational for any  $(F, C)$ . Suppose this is not the case. Then there exists an environment  $(F, C)$  and a vector-function  $\tilde{h}(C) = (\tilde{h}_1(C_{-1}), \dots, \tilde{h}_n(C_{-n}))$ , with the corresponding individually rational Modified Groves mechanism  $\tilde{g}(F, C)$ , such that

$$- \sum_{i \in N^*} S(\tilde{X}_{-i}(F, C_{-i})) > \sum_{i \in N^*} \tilde{h}_i(C_{-i}). \quad (1.54)$$

This in turn implies that

$$- S(\tilde{X}_{-i}(F, C_{-i})) > \tilde{h}_i(C_{-i}) \text{ for some } i \in N^*. \quad (1.55)$$

We now show that this leads to the violation of individual rationality in certain environments. Note that since any Modified Groves mechanism is social surplus maximizing, we have

$$S(X^*(F, C)) - S(\tilde{X}_{-i}(F, C_{-i})) \geq 0 \text{ iff } i \in N^*.$$

Two cases are possible:

(i)  $S(X^*(F, C)) = S(\tilde{X}_{-i}(F, C_{-i}))$ . If condition 1.55 holds, then individual rationality for  $i$  is violated. Hence, this cannot be the case.

(ii)  $S(X^*(F, C)) > S(\tilde{X}_{-i}(F, C_{-i}))$ . Then suppose  $S(X^*) - S(\tilde{X}_{-i}) = a > 0$ . In accordance with 1.55, let  $\tilde{h}_i(C_{-i}) = -S(\tilde{X}_{-i}) - \epsilon$  for some  $\epsilon > 0$ . Then

$$\begin{aligned} \sum_j w_{ij}^* x_{ij}^* &= F(X^*) - \sum_{l \neq i} \sum_j c_{lj} x_{lj}^* + \tilde{h}(C_{-i}) = \\ &= S(X^*) + \sum_j c_{ij} x_{ij}^* - S(\tilde{X}_{-i}) - \epsilon = \sum_j c_{ij} x_{ij}^* + a - \epsilon \end{aligned}$$

and

$$u_i = \sum_j (w_{ij}^* - c_{ij}) x_{ij}^* = a - \epsilon .$$

Now consider a different environment  $(F, \hat{C})$ , such that  $\hat{c}_{ij} = c_{ij} + a - \epsilon/2$  for all  $j$ ,  $\hat{c}_{lj} = c_{lj}$  for all  $l \neq i$ , all  $j$ . Then

$$S(X^*(F, \hat{C})) - S(\tilde{X}_{-i}(F, \hat{C}_i)) = \epsilon/2 > 0 ,$$

$i$  is still chosen and the whole allocation does not change:  $\hat{X} \equiv X^*(F, \hat{C}) = X^*(F, C)$ .

Further,  $\hat{w}_i = w_i^*$ , where  $\hat{w}_i \equiv w_i^*(F, \hat{C})$ ,  $w_i^* \equiv w_i^*(F, C)$ . Then

$$\begin{aligned} u_i(F, \hat{C}) &= \sum_j (\hat{w}_{ij} - \hat{c}_{ij}) \hat{x}_{ij} = \\ &= \sum_j (w_{ij}^* - c_{ij} - a + \epsilon/2) x_{ij}^* = (a - \epsilon) - a + \epsilon/2 = -\epsilon/2 < 0 , \end{aligned} \quad (1.56)$$

which contradicts individual rationality.  $\square$

*Proof of Corollary 4* The principal gets a non-negative payoff if  $\pi(X^*) = F(X^*) - \sum_i \sum_j w_{ij}^* x_{ij}^* \geq 0$ . But

$$\begin{aligned} F(X^*) - \sum_i \sum_j w_{ij}^* x_{ij}^* &= \\ &= F(X^*) - \sum_i S^* - \sum_i \sum_j c_{ij} x_{ij}^* + \sum_i \tilde{S}_{-i} = \\ &= (1 - n)S^* + \sum_i \tilde{S}_{-i} , \end{aligned} \quad (1.57)$$

which is non-negative only if 1.16 holds.  $\square$

*Proof of Corollary 5* In this case,

$$\begin{aligned}
 \pi(X^*) &= F(X^*) - \sum_i \sum_j (w_{ij}^* - c_{ij}) x_{ij}^* = \\
 &= \sum_i \tilde{S}_{-i} - (n-1)S^* = n(n-1)\tilde{s}(N_{-i}) - (n-1)ns^*(N) = \\
 &= n(n-1)[\tilde{s}(N_{-i}) - s^*(N)] \geq 0
 \end{aligned} \tag{1.58}$$

since  $n \geq 1$  and by definition 2.  $\square$

**Proofs for section 1.3** *Proof of Proposition 7* Let  $U(P, W, s_i | t_i)$  denote  $i$ 's utility of reporting  $s_i$  when his true type is  $t_i$ , given the mechanism  $(P, W)$ . Then one can easily show that

$$U(P, W, s_i | t_i) = U(P, W, s_i) - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(P, s_i) .$$

If the mechanism  $(P, W)$  is BIC, then for any  $s_i, t_i \in T_i$

$$U(P, W, t_i) \geq U(P, W, s_i | t_i)$$

and

$$U(P, W, s_i) \geq U(P, W, t_i | s_i)$$

or, equivalently,

$$U(P, W, t_i) \geq U(P, W, s_i) - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(P, s_i) , \tag{1.59}$$

$$U(P, W, s_i) \geq U(P, W, t_i) - \sum_{j \in K} (c_{ij}(s_i) - c_{ij}(t_i)) Q_{ij}(P, t_i) . \tag{1.60}$$

It follows that

$$\begin{aligned} \sum_{j \in K} (c_{ij}(s_i) - c_{ij}(t_i)) Q_{ij}(P, s_i) &\leq \\ &\leq U(P, W, t_i) - U(P, W, s_i) \leq \sum_{j \in K} (c_{ij}(s_i) - c_{ij}(t_i)) Q_{ij}(P, t_i) \end{aligned} \quad (1.61)$$

which yields 1.29.  $\square$

*Proof of Proposition 8* For simplicity of notation, let  $U(P, W, s_i | t_i) \equiv U(s_i | t_i)$ ,  $U(P, W, t_i) \equiv U(t_i)$ . Incentive compatibility means that

$$U(t_i) = \max_{s_i} U(s_i | t_i) .$$

From the Envelope theorem, if the mechanism is incentive compatible, then

$$U'_i(t_i) = \frac{\partial U(s_i | t_i)}{\partial t_i} ,$$

or

$$U'_i(t_i) = - \sum_{j \in K} c'_{ij}(t_i) Q_{ij}(t_i) \quad (1.62)$$

for all  $i$ , all  $t_i \in T_i$ . Integrating both sides of the equation and letting  $U_i(0) = 0$ , we obtain:

$$U_i(t_i) = - \int_0^{t_i} \left( \sum_{j \in K} c'_{ij}(\tau_i) Q_{ij}(\tau_i) \right) d\tau_i . \quad (1.63)$$

Individual rationality is then guaranteed for all  $i$ , all  $t_i \in T_i$  since  $c'_i(t_i) \leq 0$  by assump-

tion. Let  $W_i(t_i)$  denote  $i$ 's expected wage. Then, by definition,

$$U_i(t_i) = W_i(t_i) - \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) ,$$

and therefore the expected wage is

$$W_i(t_i) = \sum_{j \in K} c_{ij}(t_i) Q_{ij}(t_i) - \int_0^{t_i} \left( \sum_{j \in K} c'_{ij}(\tau_i) Q_{ij}(\tau_i) \right) d\tau_i . \quad (1.64)$$

We now show that 1.29 and 1.63 together imply incentive compatibility, i.e.,

$$U(t_i) \geq U(s_i | t_i) .$$

for any  $t_i, s_i$ . From 1.63 and 1.64,

$$U(s_i | t_i) = U(s_i) - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(s_i) ,$$

and, therefore, it is sufficient to show that for any  $s_i < t_i$

$$- \int_{s_i}^{t_i} \sum_{j \in K} c'_{ij}(\tau) Q_{ij}(\tau) d\tau \geq - \sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i)) Q_{ij}(s_i) ,$$

or

$$- \int_{s_i}^{t_i} \sum_{j \in K} c'_{ij}(\tau) Q_{ij}(\tau) d\tau \geq - \int_{s_i}^{t_i} \sum_{j \in K} c'_{ij}(\tau) Q_{ij}(s_i) d\tau .$$

Observe that the above always holds since the necessary condition for monotonicity 1.29

implies that

$$\sum_{j \in K} c'_{ij}(\tau)(Q_{ij}(\tau) - Q_{ij}(s_i)) \leq 0$$

for all  $\tau \geq s_i$ . The case  $s_i > t_i$  is established by reversing the inequality signs twice.

Incentive compatibility and individual rationality are therefore established. Finally, since

$$\Pi(P) = E_t \left\{ \sum_{l \in L} F_l p_l(t) - \sum_{i \in N} \sum_{j \in K} c_{ij}(t) Q_{ij}(t) - \sum_{i \in N} U_i(t_i) \right\},$$

where  $E_t$  denotes expected value over the domain of  $t$ , substitution of the expression 1.63 into the principal's objective function, after standard transformations, yields:

$$\begin{aligned} \Pi(P) &= E_t \left\{ \sum_{l \in L} F_l p_l(t) - \sum_{i \in N} \sum_{j \in K} \left( c_{ij}(t_i) - c'_{ij}(t_i) \frac{1 - H_i(t_i)}{h_i(t_i)} \right) \left( \sum_{l \in L_{ij}} p_l \right) \right\} = \\ &= E_t \left\{ \sum_{l \in L} F_l p_l(t) - \sum_{i \in N} \sum_{j \in K} J_{ij}(t_i) \left( \sum_{l \in L_{ij}} p_l \right) \right\}. \end{aligned} \quad (1.65)$$

Since both the principal's and the agents' utility functions are linear in allocation  $X$ , there cannot be any gain from randomization over  $X$ . The optimal choice of  $P(t)$  follows.

□

*Proof of Proposition 9* By proposition 8, the optimal mechanism chooses an allocation  $X(t)$  to maximize, subject to the monotonicity constraint,

$$\tilde{S}(X; t) = F(X) - \sum_{i \in N} \sum_{j \in K} J_{ij}(t_i) x_{ij}. \quad (1.66)$$



We first show that this implies that for any  $i \in N$ , any  $s_i, t_i$

$$\sum_{j \in K} (J_{ij}(t_i) - J_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0 . \quad (1.67)$$

Given the form of the optimal employment rule an agent's probability of being hired is non-decreasing in his type (lemma 3). Thus, we can restrict our attention to the case of an agent being moved from one project to another, within the range of the types where the agent is employed. In the latter case, for an arbitrary  $t_{-i} \in T_{-i}$ , let  $X^*$  be the allocation that maximizes 1.66 given  $(t_{-i}, s_i)$ , and  $\tilde{X}$  – an allocation that maximizes 1.66 given  $(t_{-i}, t_i)$ . Suppose that agent  $i$  is optimally employed at project  $j$  being of type  $s_i$ , but is optimally moved to the project  $k$  when his type changes to  $t_i$ :  $x_{ij}^* = 1, x_{ik}^* = 0$  for all  $k \neq j$ , and  $\tilde{x}_{ik} = 1, \tilde{x}_{ij} = 0$  for all  $j \neq k$ . Then

$$\tilde{S}(X^*; t_{-i}, s_i) = F(X^*) - \sum_{l \neq i} \sum_{j \in K} J_{ij}(t_i) x_{ij}^* - J_{ij}(s_i) \geq \tilde{S}(X; t_{-i}, s_i)$$

for any feasible  $X$  ;

$$\tilde{S}(\tilde{X}; t_{-i}, t_i) = F(\tilde{X}) - \sum_{l \neq i} \sum_{j \in K} J_{ij}(t_i) \tilde{x}_{ij} - J_{ik}(t_i) \geq \tilde{S}(X; t_{-i}, t_i)$$

for any feasible  $X$  .

In particular,

$$F(X^*) - \sum_{l \neq i} \sum_{j \in K} J_{ij}(t_i) x_{ij}^* - J_{ij}(s_i) \geq F(\tilde{X}) - \sum_{l \neq i} \sum_{j \in K} J_{lj}(t_i) \tilde{x}_{lj} - J_{ik}(s_i) \quad (1.68)$$

and

$$F(\tilde{X}) - \sum_{l \neq i} \sum_{j \in K} J_{lj}(t_l) \tilde{x}_{lj} - J_{ik}(t_i) \geq F(X^*) - \sum_{l \neq i} \sum_{j \in K} J_{lj}(t_l) x_{lj}^* - J_{ij}(t_i) . \quad (1.69)$$

Combining the two above inequalities, we obtain

$$J_{ij}(t_i) - J_{ij}(s_i) \geq J_{ik}(t_i) - J_{ik}(s_i)$$

or

$$\sum_{j \in K} (J_{ij}(t_i) - J_{ij}(s_i))(x_{ij}(t_i, t_{-i}) - x_{ij}(s_i, t_{-i})) \leq 0 . \quad (1.70)$$

Note that interim monotonicity certainly holds if ex-post monotonicity holds, i.e.,

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t_i, t_{-i}) - x_{ij}(s_i, t_{-i})) \leq 0 , \quad (1.71)$$

which is the case when

$$c_{ij}(t_i) - c_{ij}(s_i) \geq c_{ik}(t_i) - c_{ik}(s_i)$$

whenever

$$J_{ij}(t_i) - J_{ij}(s_i) \geq J_{ik}(t_i) - J_{ik}(s_i) .$$

Equivalently, monotonicity holds if

$$c_{ij}(s_i) - c_{ij}(t_i) > c_{ik}(s_i) - c_{ik}(t_i) \quad (1.72)$$

implies

$$J_{ij}(t_i) - J_{ij}(s_i) < J_{ik}(t_i) - J_{ik}(s_i) . \quad (1.73)$$

But, using the definition of  $J_{ij}$ , the latter is always the case if the conditions stated in the proposition hold.  $\square$

*Proof of Proposition 10* For an arbitrary  $i \in N$ , take any two  $j, k \in K$  and, without loss of generality, suppose (i) is the case. Then for any  $s_i, t_i \in T_i$ , such that  $s_i < t_i$ , we have

$$\int_{s_i}^{t_i} c'_{ij}(\tau) d\tau < \int_{s_i}^{t_i} c'_{ik}(\tau) d\tau ,$$

which implies

$$c_{ij}(s_i) - c_{ij}(t_i) > c_{ik}(s_i) - c_{ik}(t_i) .$$

Therefore, by proposition 9 it is sufficient to show that for any  $s_i, t_i \in T_i$  such that  $s_i < t_i$ , the following inequality holds:

$$\frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ij}(t_i) - \frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ij}(s_i) \geq \frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ik}(t_i) - \frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ik}(s_i) . \quad (1.74)$$

Note that

$$\frac{1 - H_i(t_i)}{h_i(t_i)} c'_{ij}(t_i) - \frac{1 - H_i(s_i)}{h_i(s_i)} c'_{ij}(s_i) = \int_{s_i}^{t_i} \left( \frac{1 - H_i(\tau)}{h_i(\tau)} c''_{ij}(\tau) + \frac{d}{d\tau} \left( \frac{1 - H_i(\tau)}{h_i(\tau)} \right) c'_{ij}(\tau) \right) d\tau .$$

Hence it is sufficient to show that for any  $\tau \in [s_i, t_i]$

$$(c''_{ij}(\tau) - c''_{ik}(\tau)) \frac{1 - H_i(\tau)}{h_i(\tau)} + (c'_{ij}(\tau) - c'_{ik}(\tau)) \frac{d}{d\tau} \left( \frac{1 - H_i(\tau)}{h_i(\tau)} \right) \geq 0 . \quad (1.75)$$

Since  $(1 - H_i(\tau))/h_i(\tau) \geq 0$  and the Monotone Hazard Rate condition holds, the above inequality follows directly from the assumptions stated in the proposition.  $\square$

*Proof of Proposition 11* First note that, given that the sufficient conditions for monotonicity hold, the mechanism belongs to the class of optimal BIC mechanisms as defined in proposition 8; hence it is expected profit maximizing. Applying the theorem of Laffont and Maskin (1982) to the team-formation problem, we obtain that an allocation rule  $X(t)$  is implementable in dominant strategies if and only if the following conditions are satisfied:

$$\sum_{j \in K} (c_{ij}(t_i) - c_{ij}(s_i))(x_{ij}(t_i) - x_{ij}(s_i)) \leq 0, \quad (1.76)$$

$$w_i(t_i) = \sum_{j \in K} c_{ij}(t_i)x_{ij}^*(t) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau)x_{ij}^*(t_{-i}, \tau) d\tau + e_i(t_{-i})$$

for all  $i \in N$ , all  $t_{-i} \in T_{-i}$ ,

(1.77)

where  $e_i(t_{-i})$  is an arbitrary function that does not depend on  $t_i$ .

The first condition above is dominant strategy monotonicity; by construction of the sufficient condition for interim monotonicity, it holds whenever the sufficient condition for interim monotonicity holds. In the wage function, using  $e_i(t_{-i}) = 0$  guarantees that

$$E_{t_{-i}}(w_i(t_{-i}, t_i)) = \sum_{j \in K} c_{ij}(t_i)Q_{ij}(t_i) - \int_0^{t_i} \sum_{j \in K} c'_{ij}(\tau)Q_{ij}(\tau) d\tau$$

for any  $i$ , any  $t_i \in T_i$ , and therefore the principal's expected profit is preserved at the BIC optimal level. Finally, ex-post individual rationality follows from the property that  $c'_{ij}(t_i) \leq 0$  for all  $i, j$ .  $\square$

**Proofs for section 1.4** *Proof of Proposition 12* The principal's role is restricted to the one of mechanism designer, and we only need to prove that the choice function given by 1.46 is not Nash implementable among the agents. Since monotonicity of a social choice function is a necessary condition for implementability (Maskin, 1979), it is sufficient to show that for any  $F \in \mathcal{F}$   $G_F(C)$  given by 1.46 is non-monotonic. Consider the choice rule given by 1.46 for an arbitrary  $F \in \mathcal{F}$ . If  $G_F(C)$  is monotonic, then for any  $C, \tilde{C}$ , if  $a \in G_F(C)$  and for any  $b \in A$ , for any  $i$ ,  $u_i(a; C) \geq u_i(b; C)$  implies  $u_i(a; \tilde{C}) \geq u_i(b; \tilde{C})$ , then  $a \in G_F(\tilde{C})$ . Take an arbitrary cost profile  $C \in \mathcal{C}$  such that for every surplus-maximizing allocation  $X^*(F, C)$  there exists at least one pair  $(i, j)$ ,  $i \in N$ ,  $j \in K$  for which  $x_{ij}^*(F, C) = 1$  and  $c_{ij} > \inf\{c_{ij} | c_{ij} \in \mathcal{C}_{ij}\}$  (under the conditions specified in the statement above, such  $C$  always exists). Let an alternative  $a = (X^*, W^*)$ , as defined in 1.46 be in the choice set,  $(X^*, W^*) \in G_F(C)$ . Let  $N_1 \subseteq N$  denote the set of agents  $i$  employed in  $X^*$  for which there exist a project  $j(i)$  such that  $x_{ij}^*(F, C) = 1$  and  $c_{ij} > \inf\{c_{ij} | c_{ij} \in \mathcal{C}_{ij}\}$ . Note that  $N_1$  is always non-empty by the assumption of the proposition. Next, consider another environment  $\tilde{C}$  such that  $\tilde{c}_{ij} = c_{ij} - \epsilon$  if  $i \in N_1$  and  $x_{ij}^* = 1$ , for some  $\epsilon > 0$  small enough to ensure that  $\tilde{c}_{ij} \in \mathcal{C}_{ij}$ , and  $\tilde{c}_{ij} = c_{ij}$  otherwise. Then for any  $b \in A$ ,  $u_i(b; \tilde{C}) = u_i(b; C) + \epsilon \sum_j x_{ij}(b)$  if  $i \in N_1$  and  $x_{ij}(b) = x_{ij}^*$ , and  $u_i(b; \tilde{C}) = u_i(b; C)$  otherwise; note also that  $x_{ij} \in \{0, 1\}$  for all  $i, j$ . Therefore, for any  $b \in A$ , any  $i \in N$ ,  $u_i(a; C) \geq u_i(b; C)$  implies  $u_i(a; \tilde{C}) \geq u_i(b; \tilde{C})$ ; then monotonicity requires that  $a \in G_F(\tilde{C})$ <sup>32</sup>. However, 1.46 requires that  $u_i(a; \tilde{C}) = 0$  if  $a \in G_F(\tilde{C})$  which obviously does not hold. Therefore,  $a \notin G_F(\tilde{C})$  and the necessary

<sup>32</sup>Notice that the assumption that  $x_{ij} \in \{0, 1\}$  is crucial for the analysis: Suppose that  $0 < x_{ij}(a) < 1$  for some  $(i, j)$ . Then consider an alternative  $b$  such that  $x_{ij}(b) = 1$  for this  $(i, j)$ , and  $w_i(b) = c_{ij} - \epsilon_1$ , where  $0 < \epsilon_1 < \epsilon$ . Then  $u_i(a; C) = 0 > u_i(b; C)$ . However,  $u_i(a; \tilde{C}) = \epsilon * x_{ij}(a)$ , whereas  $u_i(b; \tilde{C}) = \epsilon_1$ . Thus, if  $\epsilon_1 > \epsilon * x_{ij}(a)$ , we obtain that  $u_i(a; \tilde{C}) < u_i(b; \tilde{C})$ .

monotonicity condition is violated under 1.46.  $\square$

## Chapter 2

# Wage-Demand Mechanisms for the Formation of Teams

### 2.1 Introduction: the purpose of the study

In this chapter, we proceed with the analysis of the problem of the profit-maximizing principal who needs to assign agents to work on a given project or projects when agents differ in their contributions to the production process. The chapter includes some theoretical analysis of the possible outcomes of specific team-formation mechanisms, and the results of an experimental investigation. In the theoretical part, we assume that the agents have complete information about each other's characteristics, but the teams' efficiencies might not be fully observable to the principal. As it has been shown in chapter 1, if the principal is uninformed about the agents' costs, then her first best outcome, or the level of profit that she would get under complete information, is not implementable in Nash equilibrium by the agents. Below, we examine a specific  $\epsilon$ -Nash equilibrium mechanism that always

provides the principal with a non-negative profit, and under some environments guarantees the principal her first best outcome. We consider the one-team case, but the theoretical results are fully extendible to multiple teams.

Bolle (1991) first considered, theoretically and experimentally, complete information team-selection **wage-demand** games. In these games, each potential team is characterized by a productivity, or output, parameter. Potential team members submit their individual wage demands to the principal, the principal selects a team which gives her the highest profit – defined as the output of the team net of wages demanded by the team-members, – and then pays all the employed agents their demanded wages. Bolle found that the principal’s ability to detect and choose efficient teams among all profit-maximizing teams is essential for the existence of pure strategy Nash equilibria of the game. The reason is that an equilibrium, if it exists, is always characterized by more than one profit-maximizing team; if the principal always chooses the most efficient, among all profit-maximizing teams, then all agents’ equilibrium strategies are well defined. The members of efficient teams choose their wage-demands to provide the principal with the level of profit which makes her indifferent between the efficient and some other, inefficient, team; i.e., no extra surplus is “given away” to the principal. If, instead, the principal chooses any profit-maximizing team, which is the case when she cannot observe teams’ efficiencies, then the members of efficient teams are faced with a no-best-response problem. It is sufficient that each “efficient” agent decreases his wage demand by an infinitely small amount to get selected<sup>1</sup>; however, since there does not exist a smallest positive number, the Nash equilibrium collapses. The situation is quite

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<sup>1</sup>Hereafter, we will use terms “select,” “choose,” “hire” and “employ” to denote the principal’s decision on the team formation.



similar to other games with no best response properties; for example, in the first price auction with complete information, the bidder with the highest value needs to bid just above the second highest value to get selected, but his Nash equilibrium bid is undefined<sup>2</sup>.

Therefore, when the team efficiencies are not fully observable to the principal, and this is the case we are interested in, the pure strategy equilibria of the wage-demand game do not exist. However, we find that there exist “reasonable”  $\epsilon$ -Nash equilibria of the incomplete information game which, in all important respects, such as efficiency and profitability of outcomes, are “almost” identical to the Nash equilibria of the complete information game. Using again the first price auction analogy, we can postulate the agents’ behavior in this game, even though the pure strategy equilibrium of the game does not exist.

The main purpose of the experimental investigation in this chapter is to test the predicted robustness of the outcomes of the wage-demand games with respect to the principal’s information. Our experiments investigate two kinds of wage-demand games corresponding to complete and incomplete information on the principal’s part: the one in which only the most efficient profit-maximizing teams are chosen, and the other in which any profit-maximizing team can be selected. We analyze and compare the outcomes of these two games with respect to their profitability, efficiency and employment and wage structures. If we find that the outcomes are not substantially different, then we can conclude that the incomplete information on the part of the principal in the wage-demand games is not a serious problem, since all the important characteristics of outcomes stay the same under either regime.

There is also another, related aspect of our experimental study. Bolle (1991) conducted

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<sup>2</sup>The problem vanishes if only discrete increments of, say, \$.01 are allowed.

a set of complete information experiments with two symmetric agents and a principal to test the Nash equilibrium behavioral hypothesis in the team-selection games. He found that the agents did not always follow Nash equilibrium behavior in his experiments and exhibited “tacit collusion.” Bolle argued that his results demonstrate that fairness and cooperation often play an important role in agents’ behavior. Correspondingly, the second objective of our investigation is to test whether collusive behavior found by Bolle is robust to changes in team size and payoff symmetry. To investigate this issue, we consider three-agent experiments, in which agents have asymmetric contributions to team productivity. We analyze whether the agents in our experiments follow competitive Nash equilibrium behavior or they sustain cooperative behavior as in Bolle’s experiments.

The experimental study of wage-demand games has relevance to at least two different bodies of existing literature. First, to the extent that we are testing two different wage-demand mechanisms for team formation with respect to their efficiency and profitability, we are concerned with mechanism design issues. Earlier experimental research in mechanism design (Banks, Ledyard, Porter, 1989, Olson and Porter 1994) proved to be very useful to evaluate and discriminate among different mechanisms for such problems as resource allocation and assignment. Our objective is to test whether the complete information variant of the team-formation mechanism produces substantially higher gains in profit and efficiency for the principal.

Second, since we are considering the competitiveness of agents’ behavior in wage-demand mechanisms, earlier experiments that test the Nash equilibrium behavioral hypothesis against cooperative, or “collusive” behavioral theories are relevant. The most important ones include the studies of imperfect competition/collusion (see, for example, Plott’s review of

experimental studies in industrial organization, 1989), and collective actions/public goods experiment (see Ledyard's survey, 1993). The results indicate that in small groups collusive behavior, even if it does not constitute a Nash equilibrium, often occurs if it is mutually beneficial to the subjects compared to the Nash equilibrium behavior. However, collusion is generally sensitive to such factors as repeatedness of the game, asymmetry of payoffs, players' information about other players' payoffs, size of the group and the presence or absence of communication (Plott, 1989, Ledyard, 1993). Cooperation is more likely to occur in small groups of symmetric agents, when each agent is informed about others' payoffs, the games are repeated and communication is allowed. In this respect, Bolle's results on the presence of collusion in wage-demand games with two symmetric perfectly informed agents is not surprising. On the other hand, the studies indicate (see the same reviews) that a one-shot game, asymmetric payoffs, subjects' uncertainty about each other's payoffs, large group size tend to prevent cooperation and cause more competition among the agents even if the cooperative outcome is strictly preferred by all subjects. Therefore, we might expect to find that in settings with asymmetric agents and bigger groups, the cooperation easily breaks down in wage-demand games.

Finally, the studies of bargaining (Rubinstein, 1982) are remotely relevant, since they concern the issue of dividing a common good among several individuals. Yet, the games we consider have the principal restricted to the role of mechanism designer and the agents acting simultaneously, which substantially constrains the bargaining freedom of the players.

To summarize, the current study considers two related research questions. First, do team-selection mechanisms, and, in particular, two variants of the wage-demand mechanisms that correspond to the complete and incomplete information on the part of the

principal, substantially affect efficiency and profitability of the team-formation process for the principal? Second, is collusive behavior among agent characteristic to the wide range of wage-demand games or do competitive tendencies prevail when the features of experimental design are altered? These questions are related since the mechanisms themselves might to some extent influence – induce or reduce – competitive or cooperative tendencies in agents' behavior. We proceed with the investigation as follows.

In section 2.2 below, we present the theoretical solutions to the team-formation wage-demand games. The Nash equilibrium and cooperative solutions are considered. According to the Nash equilibrium theory, the outcomes of the complete and the incomplete information variants of the mechanism should be almost identical. However, the set of cooperative solutions is different under the two mechanisms. In view of these theoretical results, in section 2.3 we restate the main questions addressed by the experimental study and discuss experimental design. The preliminary experimental results are reviewed in section 2.4. In section 2.5 we analyze the subjects' individual behavior and present the answers to the questions above. We conclude in section 2.6. Proofs of theoretical statements are given in section 2.7.

## 2.2 Theoretical findings

The wage-demand games considered below and the findings on the Nash equilibria of these games are due to Bolle (1991). We extend his results by considering the  $\epsilon$ -Nash equilibria of the games and possible cooperative outcomes. The framework given below and the results are presented for the one-team case for simplicity of exposition, but they are easily

extendible to the multiple team case. Likewise, the agents' private costs of working on the projects, introduced in chapter 1, are not explicitly present here. They are implicitly introduced with the assumption that the teams' efficiencies are not fully observable to the principal (see footnote 3 below).

### 2.2.1 The formal model

Suppose there is a principal who has to hire a team  $T$  of workers from a given set of agents  $N = \{1, \dots, n\}$ ,  $n \geq 1$ ,  $T \subseteq N$ , to accomplish a task. The principal is constrained to a set of mechanisms in which each agent submits his wage demand  $v_i \in R$  to the principal, and the principal has to meet this demand if she decides to hire the agent. Each potential team is characterized by its productivity  $F(T) \in R$ , expressed in monetary terms; assume  $F(\emptyset) = 0$ . We will assume that the teams' productivities are known to all agents, but may be not known to the principal<sup>3</sup>. A team  $T$  is called **efficient** if it has the highest productivity among all possible teams. We will call an agent  $i \in N$  **efficient** if he belongs to every efficient team. Denote by  $I^* \subseteq N$  the set of efficient agents. If an agent is not efficient, he is **inefficient**<sup>4</sup>.

For a given vector of agents' wage-demands  $v = (v_1, \dots, v_n) \in R^N$ , a team's **profit** for

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<sup>3</sup>This assumption may seem unnatural since we will further assume that the principal can observe each team's profitability, which is a deterministic function of the productivity and agent's wage demands, and the latter are known to the principal. However, in more general situations, where the agents may bear some private costs of working on a task, it might be the case that the team's profitability – defined as the difference between the output and the wages – is observable to the principal, while the team's productivity – the difference between the output and the agents' costs – is not; this is the case when the team's outputs alone are known. In this chapter, the agents' costs are not included for simplicity of the exposition.

<sup>4</sup>Note that there may be efficient agents who belong to inefficient teams and inefficient agents who belong to efficient teams.

the principal is defined by

$$\pi(T, v) = F(T) - \sum_{i \in T} v_i . \quad (2.1)$$

We assume that, given the agents' wage-demands, the profit of each potential team is known to the principal. A team is called **profit-maximizing** if, given  $v$ , it offers the principal the highest profit among all other teams. We assume that the principal is a profit-maximizer, and each agent maximizes his expected payoff from employment, which depends on his wage  $w_i$  and the probability of being employed  $p_i$ :

$$U_i = p_i w_i . \quad (2.2)$$

We therefore assume that agents are risk-neutral<sup>5</sup>.

We consider the following class of wage-demand mechanisms, first suggested by Bolle (1991). Let each agent  $i$ 's message  $v_i$  be the agent's demanded wage.

**Definition 10** *A wage-demand mechanism is a rule that maps the agents' wage demands  $v = (v_1, \dots, v_n)$  into the probability distribution  $Q = (Q_1, \dots, Q_{2^N})$  over possible teams  $T \subseteq N$ , and wages  $w = (w_1, \dots, w_n)$  for the agent, such that*

- $Q(T) > 0$  only if  $T$  is profit-maximizing;
- For all  $i \in N$ ,

$$w_i = \begin{cases} v_i & \text{if } i \text{ is employed;} \\ 0 & \text{otherwise.} \end{cases}$$

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<sup>5</sup>For the analysis of pure strategy Nash equilibria of the wage-demand games, the agents' attitudes towards risk do not matter as will be demonstrated below, and the assumption therefore is not restrictive. However, agents' attitudes towards risk affect the cooperative solutions of the games. We assume risk-neutrality for simplicity of the analysis.

Given  $v$ , a team  $T$  is called **selectable** by a wage-demand mechanism if  $Q(T) > 0$ . An agent  $i$  is called *selectable* if he belongs to a selectable team.

Let  $p = (p_1, \dots, p_n)$  denote the vector of agents' probabilities of being hired under a wage-demand mechanism. Then for each agent  $i$ ,  $p_i = \sum_{T|i \in T} Q(T)$ .

Note that no production (an empty team) can be selected under the wage-demand mechanism if all other teams impose losses on the principal.

### 2.2.2 Characterization of the Nash equilibria

In this section we consider the pure strategy Nash equilibria<sup>6</sup> of the games induced by wage-demand mechanisms. Denote the latter games as the wage-demand games.

The class of wage-demand mechanisms can include many different mechanisms which differ from each other by the choice rule they assign in case a profit-maximizing team is not unique.

Bolle (1991) finds that Nash equilibria restricted by an equilibrium selection assumption of a wage-demand game exist only if only the most efficient, among profit-maximizing teams, are chosen with positive probability. He assumes that the agents who are not employed with certainty demand zero wages.

**Assumption 1** (*Equilibrium selection, Bolle, 1991*) *If an equilibrium exists in a wage-demand game, then, if for some  $i \in N$   $p_i = 0$  then  $v_i = 0$ .*

Since, further,  $v_i \geq 0$  if  $p_i > 0$  by the utility-maximization assumption, without loss of generality we can restrict our attention to the wage-demand games where only non-negative

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<sup>6</sup>Mixed strategy equilibria are not considered.

wage demands are allowed. We now define two kinds of the wage-demand mechanisms, which correspond to complete and incomplete information on the part of the principal.

**Definition 11** *Given the agents' wage-demands  $v = (v_1, \dots, v_n)$ , the Bolle Mechanism is a wage demand-mechanism in which only the most efficient, among profit-maximizing teams, are selectable<sup>7</sup>*

**Definition 12** *Given the agents' wage-demands  $v = (v_1, \dots, v_n)$ , the Generalized Wage-Demand Mechanism is a wage demand-mechanism in which every profit-maximizing team is selectable with equal probability.*

We will denote the corresponding games, the Bolle game and the Generalized Wage-Demand Game, by BG and GWDG, respectively. With the above assumption, we obtain the following:

**Proposition 15** *(Bolle, 1991) A pure strategy Nash equilibrium of a wage-demand game exists if and only if the game is induced by the Bolle Mechanism. Therefore, there are no pure strategy Nash equilibria in the Generalized Wage-Demand Game.*

This proposition points out the restrictiveness of the conditions under which the pure strategy Nash equilibria of the wage-demand game exist. In particular, the principal should know (or be able to detect) which team is the most efficient. Yet, we are interested in considering situations where the principal is uninformed about the productivities, and therefore cannot use the Bolle mechanism. To get an idea about what may happen in the wage-demand game where the principal cannot observe the productivities, it is useful to

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<sup>7</sup>It follows that the Bolle mechanism maps not only agents' wage-demands, but also teams' productivities, into selection probabilities and wages.



first consider properties of pure strategy Nash equilibria of wage-demand games when the equilibria do exist.

**Proposition 16** (*Bolle, 1991*) *If a pure strategy Nash equilibrium of a wage-demand game exists, then, under assumption 1, it is characterized by the following properties:*

1. *The set of selectable teams equals the set of efficient teams.*
2. *Only the agents who are employed with certainty can gain from employment; i.e.,  $v_i > 0$  only if  $p_i = 1$ ;  $v_i = 0$  otherwise.*
3. *An agent is employed with certainty if and only if he is efficient.*
4. *Given  $v$ , for every selectable team  $T^*$  and for any agent  $i$ ,  $i \in T^*$ , there is at least one team  $T_{-i}$  with  $i \notin T_{-i}$  and such that*

$$\pi(T^*, v) = \pi(T_{-i}, v) .$$

5. *An agent's equilibrium wage demanded is never higher than his marginal product: for all  $i \in N$ ,*

$$v_i \leq F(T^*) - F(\tilde{T}_{-i}) ,$$

*where  $T^*$  is a selectable team from the set  $N$  of agents, and  $\tilde{T}_{-i}$  – a selectable team from the set  $(N \setminus i)$  of agents.*

The above properties explain why the ability of the mechanism to recognize and choose efficient teams is essential for the existence of equilibria. The problem is that competition

makes the efficient agents decrease their wage demands down to the point where efficient teams provide the same level of profit to the principal as some other team, but not below that, since there does not exist a minimal positive number (which would allow efficient agents to lose a minimal amount of wage and get selected with certainty under any wage-demand mechanism). The marginal product produced by an agent gives an upper limit on his wage demand. Curiously, we are able to make the following observation: first, the equilibria of the wage-demand game exist only if the mechanism is able to recognize efficient teams; second, no agent can be better-off if an inefficient team is chosen (efficient agents can only lose from not being employed, and inefficient agents do not gain from employment anyway). Yet, if the productivities are not observable to the principal, she will have to use GWD mechanisms<sup>8</sup>, and therefore cannot guarantee to always choose an efficient team. Hence the only way that the efficient agents may signal about the teams being efficient may be by decreasing their wage demands by some arbitrarily small amounts. Although this behavior does not constitute equilibrium behavior in its strict sense, we can consider the existence of  $\epsilon$ -equilibria in the game with incomplete information, where each agent's  $\epsilon$ -equilibrium strategy "almost" maximizes his payoff from the game.

**Definition 13** *For any given  $\epsilon > 0$ , an  $\epsilon$ -equilibrium of the wage-demand game is a set of wage demands  $v = (v_1, \dots, v_n)$  such that for any  $i$*

$$U_i(v_i|v_{-i}) \geq U_i(\tilde{v}_i|v_{-i}) - \epsilon \quad \text{for any } \tilde{v}_i . \quad (2.3)$$

---

<sup>8</sup>There might be extended wage-demand mechanisms that implement the selection of efficient teams. In this study, we restrict our attention to a class of "direct" wage-demand mechanisms, where the agents' messages are restricted to their wage-demands; other mechanisms are not considered.

We can obtain the following desirable existence property of  $\epsilon$ -equilibria of the wage-demand games.

**Proposition 17** *In any wage-demand game, for any  $\epsilon > 0$ , there exists an  $\epsilon$ -equilibrium for which the set of profit-maximizing teams equals the set of efficient teams. This  $\epsilon$ -equilibrium differs by at most  $\epsilon$  from the corresponding pure strategy Nash equilibrium of the Bolle game with respect to each agent's equilibrium wage demand and the profitability of selectable teams<sup>9</sup>.*

In this  $\epsilon$ -equilibrium – which we will denote by  $\epsilon^*$ -**equilibrium** – inefficient agents demand zero wages, and efficient agents decrease their Bolle game equilibrium wage demands, if they were positive, by arbitrarily small numbers to “signal” efficient teams. As a result, only efficient teams become profit-maximizing, the efficient agents are employed with certainty and only efficient teams get selected. Since in such  $\epsilon^*$ -equilibrium the mechanism that chooses any profit-maximizing team is guaranteed to select an efficient one, the problem with non-existence of equilibrium does not arise. From the description of the wage-demands it is also clear that such  $\epsilon^*$ -equilibrium of the incomplete information game (GWDG) can get arbitrarily close to the Nash equilibrium of the Bolle game with respect to wage demands and profits. We can conclude:

**Corollary 8** *If the agents follow  $\epsilon^*$ -equilibrium behavior, then, as  $\epsilon \rightarrow 0$ , the set of possible outcomes of the GWDG becomes arbitrarily close to the set of possible outcomes of the BG with respect to the agent's wage-demands, efficiency and profitability of the selectable teams.*

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<sup>9</sup>The proof of this proposition is given in the section 2.7.

We next characterize the properties of the outcomes of the wage-demand games if the agents follow cooperative behavior.

### 2.2.3 Cooperative solutions

Let us now consider the possible outcomes of the wage-demand games assuming the agents follow collusive behavior if it is mutually beneficial to them compared to Nash equilibrium behavior<sup>10</sup>. Below we define cooperative and collusive outcomes and characterize their properties.

Since a wage-demand vector  $v = (v_1, \dots, v_n)$  uniquely determines the outcome  $(p(v), w(v))$  of a given wage-demand game (either BG or GWDG), by the abuse of notation we will often identify a wage-demand vector  $v$  with the outcome;  $\pi(v)$  will denote the corresponding level of profit of a selectable team. We will denote Nash equilibrium outcomes by  $v^N$ , and cooperative and collusive outcomes (to be defined below) by  $v^c$  and  $v^C$ , respectively.

**Definition 14** *An outcome  $v^c = (v_1^c, \dots, v_n^c)$  is called **cooperative** if there exists a pure strategy Nash equilibrium outcome  $v^N$  such that*

$$U_i(v^c) \geq v_i^N \quad \text{for all } i = 1, \dots, n, \text{ and either}$$

$$(i) \quad U_i(v^c) > v_i^N \quad \text{for some } i \in N ; \text{ or}$$

$$(ii) \quad v^c = v^N \text{ and } \pi(v^N) = 0 .$$

*An outcome is called **fully cooperative** if, in addition to the above,  $\pi(v^c) = 0$ .*

---

<sup>10</sup>Hereafter, when mentioning the Nash equilibria of the wage-demand games, we will be referring to the pure strategy Nash equilibria of the BG and the corresponding efficient  $\epsilon^*$ -equilibria of the GWDG.

In the above definition, we use the result that in a Nash equilibrium for any  $i \in N$ ,  $U_i(v^N) = v_i^N$ .

Thus, an outcome of the wage-demand game is cooperative if either (i) no agent is worse-off compared to some Nash equilibrium and at least one agent is strictly better-off or (ii) it coincides with a Nash equilibrium in which all the output is divided among the agents, and therefore the corresponding principal's profit equals zero. The cooperative outcomes defined in this way may not be stable in the sense that the agents might gain from deviations. However, these outcomes are Pareto-superior to Nash equilibrium outcomes from the agents' viewpoint. We can make the following observations about the properties of cooperative solutions of wage-demand games.

**Proposition 18** (*Characterization of cooperative outcomes*)<sup>11</sup>

1. *Any inefficient agent is always weakly better-off in any outcome other than a Nash equilibrium as long as he demands a non-negative wage: for any  $i \notin I^*$ , for any  $v^N$  and any  $v$  such that  $v_i \geq 0$ ,  $U_i(v) \geq U_i(v^N)$ . Therefore, the set of cooperative solutions of a wage-demand game is determined solely by the payoffs of the efficient agents.*
2. *In any outcome  $v^c$  that is cooperative relative to a Nash equilibrium outcome  $v^N$ , any efficient agent  $i \in I^*$  such that  $v_i^N > 0$  is chosen with positive probability  $p_i(v^c) > 0$ , and  $v_i^c \geq (v_i^N / p_i(v^c)) \geq v_i^N$ .*
3. *There may exist cooperative solutions in which efficient agents are not selected with certainty.*

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<sup>11</sup>The proof is omitted for the reason of simplicity.

4. *The principal's profits are never higher under a cooperative outcome  $v^c$  than under the corresponding Nash equilibrium  $v^N$ :*

$$\pi(v^N) \geq \pi(v^c)$$

*Moreover,  $\pi(v^N) = \pi(v^c)$  if and only if the Nash equilibrium itself is cooperative.*

*Otherwise,  $\pi(v^N) > \pi(v^c)$ .*

The above observations rest on the properties of the Nash equilibria of the wage-demand games (proposition 16): inefficient agents always demand zero wages in any Nash equilibrium of a wage-demand game, and the efficient agents are selected with certainty. We note that there is a class of cooperative solutions in which the efficient agents trade off their certainty of being selected in Nash (or  $\epsilon^*$ -Nash) equilibrium for higher expected wages.

**Example 1** Suppose  $F(1, 2) = 100$ ,  $F(1) = F(2) = 90$ . The Nash equilibrium solution is  $v^N = (10, 10)$ . The principal's corresponding profit is  $\pi^N = 80$ . The set of the cooperative solutions is defined by  $v^c = (x, x)$ , where  $x \in [20, 90]$ . In the cooperative outcome each agent is selected with probability  $p_i = 1/2$  and gains  $U_i = x/2$  in expected utility.

Let us now define collusive outcomes as outcomes in which every agent who can affect the payoffs of other agents in the game by altering his wage demand strictly gains from cooperation compared to some Nash equilibrium outcome. Basically, we want to exclude from the set of the cooperative outcomes the ones in which some inefficient agent or agents altruistically keep their wage demands high enough to allow other agents to gain from

cooperation, whereas their own probability of being selected is zero.

In general, if any wage-demands are allowed, almost always any agent can affect other agent's payoffs by demanding  $v_i = -\infty$ . To make the analysis interesting, we would like to restrict our attention to the set of non-negative wage-demands:  $v \in \mathbb{R}_+^n$ . Since the agents are utility-maximizers, if  $p_i > 0$ , then  $v_i \geq 0$ . Then, by adding a weaker analog of assumption 1 from section 2.2.2,

**Assumption 2** *If  $p_i = 0$ , then  $v_i \geq 0$ ,*

we obtain that only non-negative wage-demands are possible. We can then define the following.

**Definition 15** *Given the agents' wage demands  $v$ , an agent  $i \in N$  is called **offensive** if there exists some  $v'_i \neq v_i$ ,  $v'_i \geq 0$ , and some  $j \in (N \setminus i)$  such that  $p_j(v_{-i}, v'_i) \neq p_j(v)$ .*

**Definition 16** *An outcome  $v^C$  is called **collusive** if it is cooperative and every offensive agent  $i$  strictly gains from cooperation compared to the Nash equilibrium outcome:*

$$U_i^C(v) > v_i^N$$

for every offensive  $i \in N$ .

The collusive outcomes are characterized by the following properties.

**Proposition 19** *(Characterization of collusive outcomes)<sup>12</sup> In any outcome  $v$  which is collusive relative to the Nash equilibrium  $v^N$ , the following is true:*

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<sup>12</sup>The proof is presented in section 2.7.

1. All agents who are selected with positive probability are strictly better-off than in the Nash equilibrium: for all  $i \in N$ , if  $p_i(v) > 0$ , then  $U_i(v) > v_i^N$ .
2. Only the agents who are not offensive may be kept at their Nash equilibrium utility levels. Therefore, for all  $i \in N$ , if  $p_i(v) = 0$ , then  $p_i(v_{-i}, v'_i) = 0$  for any  $v'_i \geq 0$ .
3. If  $i \in N$  belongs to at least one of the profit-maximizing teams, then either (i)  $p_i > 0$  and  $v_i > 0$ , and  $i$  is offensive; or (ii)  $p_i = 0$  and  $v_i = 0$ , and  $i$  is not offensive.
4. Every efficient agent whose Nash equilibrium wage-demand is positive is strictly better-off than in the Nash equilibrium: for any  $i \in I^*$  such that  $v_i^N > 0$ ,  $U_i(v) > v_i^N$ .
5. The principal's profits are strictly lower under a collusive outcome than under the corresponding Nash equilibrium:

$$\pi(v^N) > \pi(v^C)$$

We now state some properties of the cooperative outcomes in the BG and then compare them to the cooperative outcomes of the GWDG. First consider the possible types of cooperative outcomes. We divide them into two types with respect to efficiency of selectable teams.

**Definition 17** *A cooperative outcome of a wage-demand game is called a **type 1** outcome if at least one efficient team is selected with positive probability. An outcome is called **type 2** outcome if none of the efficient teams is selectable.*



Notice that a type 1 cooperative outcome can be obtained from a Nash equilibrium outcome by increasing the agents' wage-demands in such a way that the set of profit-maximizing teams does not change, but the level of profits decreases compared to the Nash equilibrium. In this way, as we will demonstrate below, in the BG, type 1 outcomes preserve some properties of the Nash equilibria: the outcomes are always efficient and the efficient agents are selected with certainty.

**Example 2: a type 1 outcome** Let  $F(1, 2) = 110$ ,  $F(1, 3) = F(2, 3) = 100$ . Then the Nash equilibrium outcome of the BG is  $v^N = (10, 10, 0)$ , and a possible type 1 cooperative solution is  $v^c = (x, x, x - 10)$ , with  $a < x \leq 55$ ,  $a = 10$  for the BG and  $a = 15$  for the GWDG. Under the BG, only efficient team  $\{1, 2\}$  is selectable; under the GWDG, any of three teams are.

Type 2 cooperative solutions occur, in particular, when efficient agents trade-off their certain (but low) wages under Nash equilibria for higher, but risky, expected payoffs under cooperation. This may happen when the agents' marginal contributions to the efficient teams are low, but every efficient agent can be a member of an alternative – inefficient team – which produces high enough output to allow the agent to demand high wages if this team is selected.

**Example 3: a type 2 outcome** Let  $F(1, 2) = 110$ ,  $F(1, 3) = F(2, 3) = 100$ . The Nash equilibrium solution is  $v^N = (10, 10, 0) = U$ , where  $U$  denotes the vector of agents' expected payoffs. A type 2 cooperative outcome is  $v^c = (x, x, 100 - x)$ , where  $55 < x \leq 100$ , with corresponding expected payoffs  $U = (x/2, x/2, 100 - x)$ . The solution is (fully) cooperative (and collusive) under both BG and GWDG.

The following properties of type 1 cooperative outcomes are presented to show that the selection rule, or the type of the wage-demand game, significantly affects the set of possible cooperative (and collusive) outcomes of the game.

**Proposition 20** (*Type 1 cooperative outcomes*)<sup>13</sup>

1. *In type 1 cooperative outcomes of the BG, all efficient agents are selected with certainty, and only efficient teams are selected. This is not generally true for the GWDG.*
2. *If, in the wage-demand game, a type 1 cooperative solution is collusive, then*
  - *every efficient agent is strictly better-off than in the Nash equilibrium;*
  - *if the game is BG, every inefficient agent either belongs to at least one efficient profit-maximizing team and demands  $v_i > 0$ , or is inoffensive;*
  - *if the game is GWDG, every inefficient agent either belongs to at least one profit-maximizing team and demands  $v_i > 0$ , or is inoffensive.*
3. *Suppose that every type 1 cooperative solution of a wage-demand game contains at least one inefficient offensive agent  $i \in (N \setminus I^*)$  with  $v_i > 0$ ,  $p_i = 0$ . Then the only collusive outcomes that are possible are type 2.*

The findings indicate that quite often, unless all inefficient offensive agents belong to some efficient allocations, the set of type 1 collusive outcomes of the BG is empty. Therefore, if agents follow collusive behavior under BG, efficient outcomes will rarely occur. In contrast, in the GWDG, type 1 collusive outcomes do not require inclusion of every inefficient agent

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<sup>13</sup>The proof is presented in section 2.7.

into some efficient team; inefficient agents can secure positive chances to be selected by belonging to any – not necessarily efficient – profit-maximizing team. Therefore, the efficient outcomes may occur along with the inefficient ones in GWDG.

We now offer a general observation concerning the non-equivalence of the sets of cooperative and collusive outcomes of the BG and the GWDG.

**Proposition 21** *1. The set of cooperative (collusive) solutions of the BG is not equivalent to the set of cooperative (collusive) solutions of the GWDG for every wage-demand game. There may be solutions which are cooperative (collusive) under the BG but not under the GWDG, and solutions that are cooperative (collusive) under the GWDG but not under the BG.*

*2. If the agents follow collusive behavior and there are no type 1 collusive outcomes in the BG, then the efficient outcomes never occur in the BG, but may occur in the corresponding GWDG.*

Part (2) of proposition 21 follows from proposition 20; we illustrate part (1) by means of the following examples.

**Example 4: an outcome which is collusive under the BG but not under the GWDG** Let  $F(1,2) = F(1,3) = 100$ ,  $F(2,3) = 80$ . The Nash equilibrium solution is  $v^N = (20, 0, 0)$ . Then the outcome  $v = (25, 5, 5)$  is collusive under the BG with  $p(v|BG) = (1, 1/2, 1/2)$ , and  $U_1(v|BG) = 25 > 20$ ,  $U_2(v|BG) = U_3(v|BG) = 5/2 > 0$ . Yet it is not collusive under the GWDG, since agent 1's expected payoff is now lower than his Nash equilibrium payoff:  $p(v|GWDG) = (2/3, 2/3, 2/3)$ , and  $U_1(v|GWDG) = 50/3 < 20$ .

In this example, the efficient agent 1 lost in the probability of being selected when the game changed from the BG to the GWDG, and therefore his expected payoff decreased. We may expect that rather often the probability of being selected for the agents-members of the efficient profit-maximizing teams will decrease in the GWDG as compared to the BG, therefore eliminating some outcomes from the set of collusive outcomes.

**Example 5: an outcome which is collusive under the GWDG but not under the BG** Let  $F(1, 2) = 110$ ,  $F(1, 3) = F(2, 3) = 100$ . The Nash equilibrium solution is  $v^N = (10, 10, 0)$ . An outcome of the form  $v = (x, x, x - 10)$  with  $15 < x \leq 55$  is collusive under the GWDG with  $p(v|GWDG) = (2/3, 2/3, 2/3)$ , and  $U_1(v|GWDG) = U_2(v|GWDG) = 2x/3 > 10$ ,  $U_3(v|GWDG) = 2/3(x - 10) > 0$ . Yet it is not collusive under the BG, since agent 3's probability of being selected decreases to zero:  $p(v|BG) = (1, 1, 0)$ , and  $U_3(v|BG) = 0$ .

This example demonstrates that a lot of outcomes that are collusive under the GWDG are no longer collusive under the BG because inefficient profit-maximizing teams are no longer selectable in the BG. By the same reason, in the latter example, the set of collusive outcomes that allow selection of the efficient team  $\{1, 2\}$  is empty in the BG but not in the GWDG, which illustrates the second statement in the above proposition.

To conclude, the  $\epsilon$ -Nash and the collusive behavioral hypothesis give different predictions about the outcomes of the BG as compared to the GWDG: the former suggests the outcomes should be almost identical, whereas the latter claims they might be quite different. In our experimental investigation, we test which theory gives better predictions of the actual outcomes of the wage-demand games.

## 2.3 Experimental design

### 2.3.1 Bolle's experiments and research questions restated

Bolle (1991) conducted a set of experiments to test his theoretical findings and the Nash equilibrium behavioral hypothesis in the team selection games. He considered three-person experiments with a principal (being a subject) and two symmetric agents. The agents could produce a value of 100 as a team, and each agent was characterized by a value  $F_i$  that he was able to produce alone; in all experiments this “marginal productivity” value was equal for two agents and it ranged from 0 to 100 in different experiments. Bolle found that the principals did use profit-maximization as the only team selection criterion, but on the part of the agents a pattern of behavior quite different from the Nash equilibrium behavior was observed. The Nash equilibrium hypothesis predicted that the wage demands are  $v = (100 - F_i, 100 - F_i)$  for  $F_i \in [50, 100]$ , and  $v = (v_1, 100 - v_1)$ ,  $v_1 \in [0, 100]$  for  $F_i < 50$ . However, Bolle reports that for different (symmetric) values of agent's marginal productivity parameters, the most common agent's demands were of about 40 each, inducing a split pattern (35,35,30) of total output of 100 among two agents and the principal, or a split pattern of (35,  $F_i - 35$ ) between one of the agents and the principal, respectively. Bolle argues that this result supports his hypothesis that fairness, in the sense of equal split, and collusion considerations often play an important role in agents' behavior.

Faced with the theoretical predictions and the results of Bolle's experiments, we can now restate the purpose of our experimental investigation. We want to consider the outcomes – profitability and efficiency – and the agents' behavioral patterns in two variants of wage-demand games – the Bolle Game and the Generalized Wage-Demand Game. Are the

outcomes of the GWDG distinguishable from the outcomes of the BG? Do the agents follow competitive Nash-equilibrium-type behavior in these wage-demand games or do they always collude? If the agents are competitive in the BG, do they follow  $\epsilon^*$ -equilibrium behavior in the corresponding GWDG? If the agents are collusive, in what way can the outcomes of the BG differ from the outcomes of the GWDG? Is the BG inefficient under a broad range of circumstances, as the collusive theory predicts?

Below, we state two alternative sets of conjectures which contain possible answers to these questions stated from the competitive and cooperative perspectives. We further offer an experimental design in view of the possibilities to test these conjectures.

### 2.3.2 Two alternative sets of conjectures

The first two conjectures below contain the answers to the questions presented in the previous section given from the competitive perspective. The first conjecture rests on the Nash equilibrium theoretical findings presented in section 2.2, and the second one, on the results on non-robustness of cooperation reported in the studies reviewed in the introduction.

**Conjecture 1** *The Bolle Game and the GWD Game are essentially equivalent with respect to the structure of agents' wage demands, efficiency and profitability.*

**Conjecture 2** *Collusion is not robust in the wage-demand games. It is sufficient to introduce asymmetry in the agents' roles, and the presence of inefficient agents to make the outcomes of the wage-demand games closer to the competitive (Nash) than to the cooperative solution.*

Note that the first conjecture can be true only if conjecture 2 is true. Alternatively, we can expect collusion to be a rather common pattern of behavior within small groups of agents in the wage-demand games, if cooperation is mutually beneficial. Then the results of section 2.2.3 applies: the sets of collusive outcomes of the BG and the GWDG are not generally equivalent. Therefore, we might make different predictions about the outcomes of the BG as compared to the outcomes of the GWDG. The obvious difference among the two games is that the latter (GWDG) treats all profit-maximizing teams – and therefore all agents who are the members of all profit-maximizing teams – equally, whereas the former (BG) discriminates among the teams in favor of more efficient ones, and therefore discriminates among the agents. Hence, we may suppose that the BG will trigger more competition on the part of the members of profit-maximizing inefficient teams, who might fight fiercely against the more efficient teams in an attempt to get selected with positive probability. On the other hand, the members of efficient teams are strongly interested in cooperation on the part of the inefficient agents in the BG, and therefore might not initiate attempts to drive the wage-demands down. In the GWDG, the inefficient agents that were selected with zero probability under the BG now have a positive chance to get selected, and therefore may be interested in cooperating and not driving the wage-demands down. Yet, the efficient agents in the GWDG might try to restore their high probability (and often certainty) of being selected and in this way trigger competition. Thus, we can expect more or less cooperation in the BG relative to the GWDG depending on which tendency is predominant in a given game.

We now state the set of conjectures based on the collusive behavioral assumptions and the corresponding theoretical findings:

**Conjecture 3** *If the number of agents in a wage-demand game is small and there are substantial gains from cooperation, then collusive behavior prevails. Consequently, the level of the principal's profit in the wage-demand games is sustained at a level lower than that predicted by Nash equilibrium behavior.*

**Conjecture 4** *The outcomes of the the BG and the GWDG differ if the sets of collusive solutions of the two wage-demand games are not identical.*

**Conjecture 5** *If a wage-demand game is such that the set of cooperative solutions is non-empty and there are inefficient agents who do not belong to efficient teams, then the BG is less efficient than the GWDG in the sense that efficient teams are never selected under the BG.*

### 2.3.3 Experimental design: parameter choices

To test which set of conjectures concerning the comparison of the two wage-demand mechanisms gives better predictions for the outcomes of the wage-demand games, we consider a three-agent game with asymmetric productivity (output) values for teams. The productivity values are given in table 2.1, where the agents are denoted by roles 1, 2 and 3. As it is apparent from the table, the design is competitive in the sense that all three agents cannot be selected; only two-person teams –  $\{1, 2\}$ ,  $\{1, 3\}$  and  $\{2, 3\}$  – are selectable. Moreover, team  $\{1, 2\}$  is efficient, while the other two are not. This induces asymmetry in the agents' roles: agents 1 and 2 are efficient, whereas agent 3 is inefficient. The game is characterized by the unique Nash equilibrium of the BG and a wide range of possible cooperative solutions. We first present competitive and cooperative solutions of the above game and then



	Team						
	{1}	{2}	{3}	{1,2}	{1,3}	{2,3}	{1,2,3}
Productivity	-1	-1	-1	1100	1000	1000	0

Table 2.1: Experimental parameters: output values for teams

discuss the parameter choices in more detail.

**Nash and  $\epsilon^*$ -Nash Equilibrium Solutions** The Bolle game has a unique Nash equilibrium with agents' wage-demand vector  $v^N = (100, 100, 0)$  (the solution is obtained using the properties of Nash equilibria stated in proposition 16). The principal's profits are  $\pi(1, 2) = \pi(1, 3) = \pi(2, 3) = 900$ , and team  $\{1, 2\}$  is selected with certainty. The corresponding  $\epsilon^*$ -equilibrium of the GWDG is  $v^\epsilon = (100 - \epsilon_1, 100 - \epsilon_2, 0)$ , with the principal's profits  $\pi(1, 2) = 900 + \epsilon_1 + \epsilon_2$ ,  $\pi(1, 3) = 900 + \epsilon_1$ ,  $\pi(2, 3) = 900 + \epsilon_2$ , and team  $\{1, 2\}$  is still selected with certainty.

**Cooperative Solutions** The sets of cooperative and collusive solutions differ under the BG and the GWDG<sup>14</sup>. Under the BG, two types of cooperative solutions are possible: either  $\{1, 2\}$  is the only selectable team (type 1 cooperative solutions) or (only) teams  $\{1, 3\}$  and  $\{2, 3\}$  are profit-maximizing and are selectable with equal probability 1/2 each (type 2 cooperative solution). Type 1 cooperative solutions are not collusive since they exclude agent 3 from the set of selectable agents; most of the type 2 solutions (except on the boundary) are collusive. Note that the efficient agents' asks are never lower than 200 under any collusive solution of the BG (assuming risk-neutrality).

The set of cooperative solutions of the GWDG is of a wider variety and includes the cases where (i) only team  $\{1, 2\}$ ; (ii) any two of the two-person teams; (iii) all three two-person

<sup>14</sup>The full description of the set of cooperative solutions is given in the appendix A.

teams are selectable. The set of collusive outcomes excludes case (i) (as in the BG) but otherwise differs from the cooperative outcomes by the exclusion of certain boundaries. The efficient agents' asks are never lower than 150 under any collusive solution of the GWDG (assuming risk-neutrality).

Among the fully cooperative solutions, where the principal gets a zero profit, we should mention the unique fair solution of the GWDG  $v = (550, 550, 450)$ . Under this outcome the selected agents share all the output among themselves; all three two-person teams are equally selectable, and therefore each agent is hired with equal probability  $2/3$ ; and the agents' wage demands are close to each other in absolute value. Thus, the proximity of the actual outcomes of the GWDG to this solution may indicate how important collusion and fairness considerations are to the agents.

Most importantly, we can state the following<sup>15</sup>.

**Proposition 22** *Let  $BG^C$  and  $GWDG^C$  denote the sets of collusive solutions of the BG and the GWDG, correspondingly, where the teams' productivities are as given in table 2.1.*

*Then*

$$BG^C \subset GWDG^C .$$

*That is, the set of collusive solution of this GWDG is wider than the set of collusive solutions of the corresponding BG.*

We now comment on the design features of the above wage-demand game in view of our research objectives. First, we are able to compare the outcomes of two variants of the wage-demand games, the BG and the GWDG. Both games have a unique Nash (respectively,

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<sup>15</sup>The statement follows directly from the analysis of the collusive solutions given in appendix A.

	Bolle	Our design
Is principal a subject?	yes	no
Number of agents in a group	2	3
Symmetric productivities?	yes	no
Varying productivity values?	yes	no
Unique Nash equilibrium?	varies	yes
Varying selection rule?	no	yes

Table 2.2: Features of experimental design as compared to Bolle's

$\epsilon^*$ -Nash) equilibrium; this gives us a well-defined prediction of the competitive outcomes, which are practically the same for the BG and the GWDG. On the other hand, the BG has a wider set of collusive outcomes than the GWDG. Thus we can test whether the mechanisms matter for the outcomes of these wage-demand game, as well as consider which conjecture, competitive (conjecture 1) or cooperative (conjecture 4), gives a better prediction for the outcomes of the games. Besides, the design satisfies the assumptions of conjecture 5: the inefficient agent 3 does not belong to the efficient team  $\{1, 2\}$ . As a consequence, there are no collusive outcomes that allow the choice of  $\{1, 2\}$  in the BG, and the efficient team can never be chosen under the collusive hypothesis. Yet, collusive solutions of the GWDG allow the choice of the efficient team  $\{1, 2\}$  as long as some other team that includes agent 3 can also be selected. In this way we can test the prediction of inefficiency of the BG relative to the GWDG if the agents are collusive.

To consider the robustness of collusive tendencies in agents' behavior found by Bolle (1991), we introduce the features that were found elsewhere (Ledyard, 1993, Plott, 1989) to induce competition among agents. Table 2.2 summarizes the differences of our experimental design as compared to Bolle's. In our design there are asymmetric productivity values for

teams<sup>16</sup>, and, as a consequence, efficient (roles 1 and 2) and inefficient (role 3) agents. The inefficient agent 3 does not belong to any of the efficient teams, and therefore we can expect very competitive behavior on his part in the BG if agents 1 and 2 try to form an efficient profit-maximizing team and get selected with certainty (note that such an outcome cannot be collusive). In this way we can test whether the agents' asymmetry of roles, at least in the form of presence of inefficient agents, can be sufficient to destroy cooperation (conjectures 2 and 3). A three-agent (instead of Bolle's two-agent) design is suggested to create extra competition – efficient agents 1 and 2 may be competing for employment with each other as well as with agent 3; it is also the smallest number of agents which allows us to combine gains from team production with a competitive environment<sup>17</sup>. In particular, in our design only two-agent teams can be selectable. In other respects, such as a small group setting and complete information among the agents, we choose, as in Bolle's experiments, an environment which may enhance cooperation. The team productivity values are chosen in way a that the set of collusive outcomes is non-empty under both BG and GWDG, i.e., cooperation is mutually beneficial for every agent (see above).

### 2.3.4 Experimental procedures

We conducted two sets of computerized experiments. In the first experiment, the participating subjects were involved in the Bolle Game (BG): given the table of parameters describing

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<sup>16</sup>Asymmetry is also necessary to distinguish between the BG and the GWDG. In our setting, if there is a tie in profit among certain teams (teams  $\{1, 2\}$  and  $\{1, 3\}$  or  $\{1, 2\}$  and  $\{2, 3\}$ ), different sets of teams are selected with positive probability under the BG and the GWDG. (The requirement that we be able to distinguish between the BG and the GWDG excludes symmetric designs such as  $F(1) = 10$ ,  $F(2) = 10$ ,  $F(1, 2) = 0$ .) For this purpose, inefficient teams  $\{1, 3\}$  and  $\{2, 3\}$  that can be profit-maximizing are present.

<sup>17</sup>In a two-agent game, such as studied by Bolle, it is always the case that either agents gain more from the joint production and therefore do not compete with each other for employment, or each agent expects to gain more – in expected utility – if he works alone, in which case there can be no gains from team production for the agents.

the output values (productivities) of potential teams, they submitted their wage demands to the computer, which then selected the most efficient, among profit-maximizing, team; ties were broken by the computer using a fair lottery. The second set of experiments, the GWDG, was identical to the first one, except in the latter all profit-maximizing teams were chosen with equal positive probability. The principal was not a subject and the team selection process was fully determined by the mechanism and realized by the computer. Thus, the experiments were considered from the mechanism designer's point of view and focused on the effects of the different selection rules on the agents' behavior and the outcomes of wage-demand games.

A total of seven experiments were conducted, with four experiments – #1, 3, 5 and 7 being the Bolle Game (BG) experiments, and three experiments – #2, 4 and 6 – GWDG experiments. All the subjects were from Caltech community, mostly undergraduates. Except for one case<sup>18</sup>, each subject was used in only one of the experiments. Twelve subjects divided into four three-agent groups participated in each of six experiments; nine subjects divided into three three-agent groups participated in experiment #7. Parameters for all seven experiments were identical, as given in table 2.1. The design characteristics of all experiments are summarized in table 2.3. The subjects were provided with the set of instructions for the experiment and with the table of teams' productivity parameters (see appendix B).

At the beginning of each experiment, eight subjects (six subjects in experiment #7) were chosen by the computer to be of the efficient type (roles 1 and 2), and four subjects

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<sup>18</sup>One subject participated in experiments #1 and #4.

Exp. #	Selection rule	# of periods	# of subjects			# of groups p/per.	Total # of trials
			total	eff.	ineff.		
1	BG	28	12	8	4	4	112
2	GWDG	29	12	8	4	4	116
3	BG	29	12	8	4	4	116
4	GWDG	29	12	8	4	4	116
5	BG	29	12	8	4	4	116
6	GWDG	29	12	8	4	4	116
7	BG	31	9	6	3	3	93

Table 2.3: Experimental design (practice periods excluded)

(three subjects in experiment #7) – of the inefficient type (role 3)<sup>19</sup>. The types stayed the same for the whole experiment to decrease possible repeated game reciprocity effects, where agents, being in the inefficient role in the BG, cooperate to induce the same type of behavior from the other agents when it is their turn to take this role. Subjects were informed that their roles could be either only 3 or change between 1 and 2 during the experiment.

Each experiment included about 30 repetitions (periods), preceded by two practice periods. The number of periods was unknown to the subjects, but the last period was announced. At the beginning of each period, the subjects were divided by the computer into four three-subject groups, with two efficient and one inefficient agents in each. The efficient agents were assigned roles 1 or 2 within the group, and the inefficient agent was in role 3. The subjects did not know who the other subjects in their group were. The group and role reassignment was computerized and occurred at the beginning of each period to eliminate the repeated game effects.

Each period was organized as a sealed bid, independent for each group<sup>20</sup>. The subjects

<sup>19</sup>Letters A, B and C were used in the experiments to denote the roles.

<sup>20</sup>A curious observation is that an English auction cannot be used to test one-shot wage-demand games, since the English auction which allows ties among teams induces a quite different sequential game among

submitted their wage demands (any non-negative real numbers between 0 and 10000) to the computer. Once all the agents submitted their demands, the team selection was made according to the BG or GWDG selection rules, depending on the experiment. The subjects were informed about the selection rule used in their experiment. At the end of the period, the subjects are informed about the wages submitted, the corresponding profits of teams, the team selected and the wages paid to the subjects in their group.

The information table displaying outcomes of the game for the previous period for the groups in which each subject participated was available to this subject at any time during the current period.

At the end of the experimental session, the subjects were paid in accordance with their accumulated payoffs in the experiment using the exchange rate \$0.005 per franc. The participation fee of \$8 (the value of which was private information) was added for the payoff of inefficient agents to compensate for the potential differences in earnings.

## 2.4 Experimental results: summary of the data

The data for all experiments pooled by selection rules are summarized in tables 2.4-2.7 and figures 2.1-2.2. The detailed data for each experiment is presented in tables D.1-D.10 in appendix D and figures C.1-C.7 in appendix C. We have structured the data into two categories: group data, which includes descriptive statistics regarding level of profits and efficiency of the teams, and individual data, which concerns the individuals' asks<sup>21</sup>. For

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the agents. Moreover, a large number of cooperative solutions can be supported as Nash equilibria in an English auction version of this game.

<sup>21</sup>The term "ask" was used to denote a wage-demand in the experiments. Hereafter, we will use both terms when referring to wage-demands.

convenience of the analysis, we have also divided the data for each experiment into three time intervals: The first 9 periods of an experiment are referred to as “time 1,” the second 10 periods, as “time 2,” and all the following periods, as “time 3.” Further, we have divided the data into two “regions” of outcomes according to the proximity to the Nash equilibrium prediction: the region when profits of selected teams are below 800 is referred to as “far from Nash equilibrium,” and the region when profits are 800 or above is referred to as “close to Nash equilibrium” (for the remainder, the Nash equilibrium level of profit of a selected team is 900). The cutpoint of 800 is chosen rather arbitrarily with the only purpose to test whether the experimental data differs significantly depending on the proximity of the outcomes to the Nash equilibrium prediction<sup>22</sup>.

The first, major result is immediately apparent from the data.

**Result 1** *In both BG and GWDG, most of the time the agents followed neither fully collusive (cooperative) nor Nash ( $\epsilon$ -Nash) equilibrium behavior.*

**Support:** Tables 2.4, 2.5; figures 2.1, 2.2. For all experiments and all time intervals, the average per period asks were above the Nash equilibrium level of 100 or 0 francs for efficient and inefficient agents, respectively, but below the fully cooperative level of 550 and 450 francs. (See also tables D.7, D.8 in appendix D.) Consequently, in all experiments and all time intervals, the average per period profits of the selected teams were significantly above zero, the fully cooperative level. In fact, the minimal level of profit of the selected team that was ever observed was 190. On the other hand, in 58.9% of the observed data, the level of profit of the selected teams was below 800, with 900 being the Nash equilibrium

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<sup>22</sup>In particular, the difference should occur if we observe converges of wage-demands to the Nash equilibrium. This issue is treated in more detail in the next section.



Mean St. deviation	Period 1	time 1	time 2	time 3	Total
Average profit, all data	567.6 180.1	626.2 150.6	723.9 122.2	793.6 91.8	717.6 141.5
-BG pooled	630.2 175.9	714.9 103.6	809.0 49.2	848.4 60.2	793.6 91.8
-GWDG pooled	489.3 159.1	515.4 124.6	617.6 101.1	724.5 90.9	622.7 135.3
Profit change, all data	— —	17.91 87.35	10.09 38.61	3.86 56.78	10.32 63.21
-BG pooled	— —	19.09 80.84	9.59 35.73	-3.06 67.81	8.02 63.99
-GWDG pooled	— —	16.52 95.02	10.73 42.08	12.64 37.03	13.18 62.18
% Efficiency, all data	—	43.6	49.6	48.5	47.4
-BG pooled	—	43.0	47.3	57.9	49.7
-GWDG pooled	—	44.4	52.5	36.7	44.5

Table 2.4: Experimental results, group data. Profit and profit changes are the averages per group, per period, in francs. The numbers below are the standard deviations, in francs. Percentage efficiency shows the percentage of efficient teams selected, in %.

Mean St. deviation	Period 1	time 1	time 2	time 3	Total
All data	Efficient agents				
	438.5 790.2	285.3 397.7	194.2 65.1	163.4 64.1	212.4 234.9
BG pooled	256.4 121.6	208.4 76.6	151.8 33.6	136.2 61.6	164.6 66.9
	GWDG pooled	666.2 115.0	384.4 581.0	247.3 55.2	196.9 49.7
All data	Inefficient agents				
	230.9 119.5	182.9 99.8	170.9 678.3	122.4 160.7	158.1 412.6
BG pooled	202.7 118.1	143.4 99.0	83.6 124.7	121.0 201.5	115.3 150.7
	GWDG pooled	266.3 116.3	233.7 74.8	280.0 999.4	124.3 88.0

Table 2.5: Average per period per person asks, francs. Standard deviations are listed below the means, in francs.

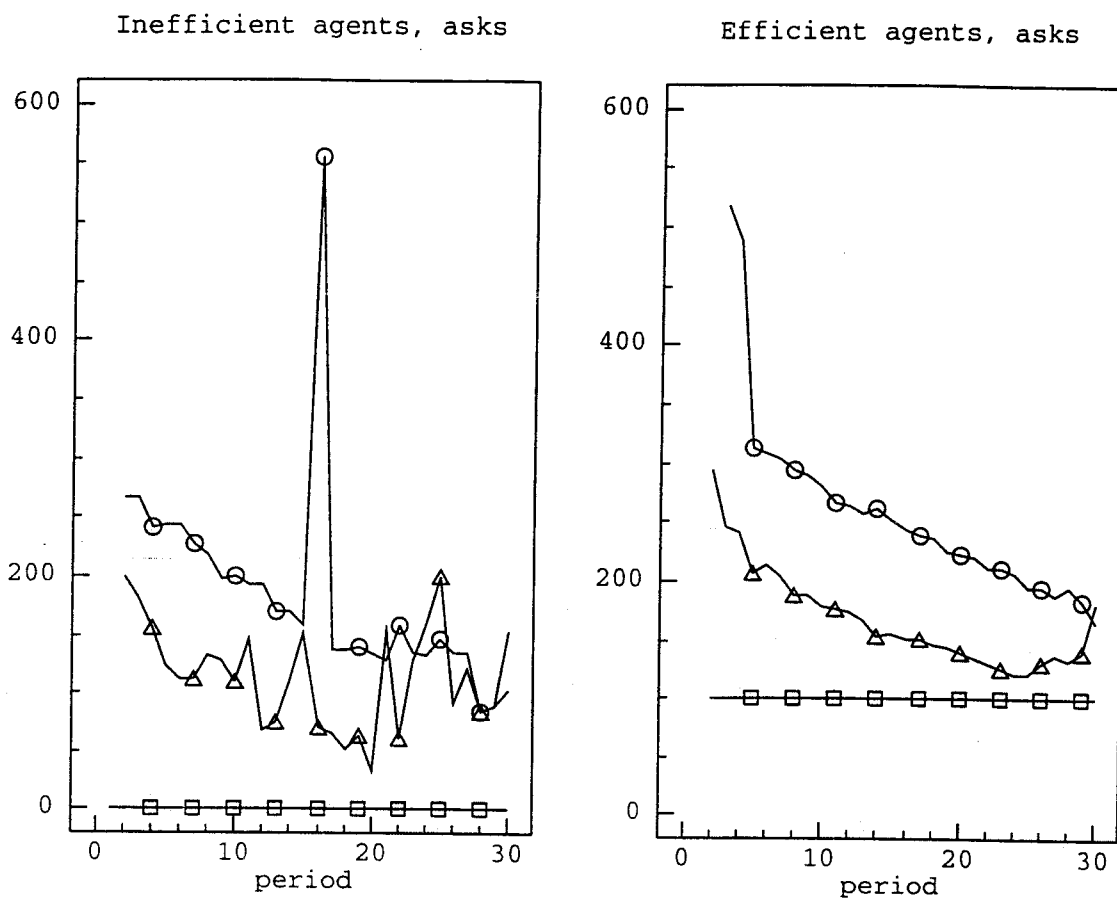


Figure 2.1: Average per period asks in BG and GWDG experiments. The graphs on the left present average asks of inefficient agents; the graphs on the right - of efficient agents.  $\Delta$  - average asks in BG;  $\odot$  - average asks in GWDG;  $\square$  - Nash equilibrium asks.

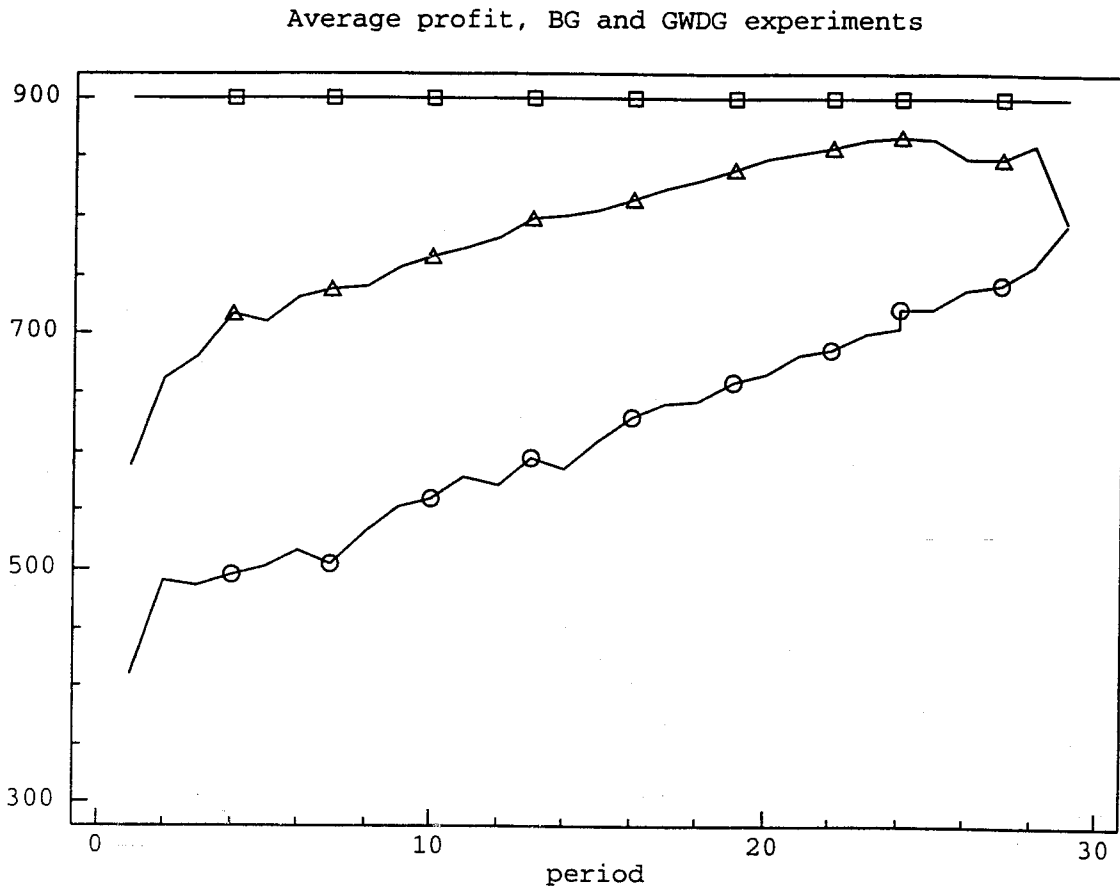


Figure 2.2: Average per period profits of selected teams in BG and GWDG experiments.  $\triangle$  - average profits in BG;  $\odot$  - average profits in GWDG;  $\square$  - Nash equilibrium profit.

level.

The percentage of efficient teams selected was 49.7% in BG and 44.5% in the GWDG, which is far below fully the competitive level of 100%, but is, on the other hand, above the “fair” cooperative level of 33%<sup>23</sup>. □

We can further make some conclusions about the presence and characteristics of competitive and cooperative tendencies in the subjects’ behavior.

**Result 2** *Cooperative tendencies, if they were present, were not sustainable in either the BG or the GWDG experiments: the average individual asks were decreasing from period to period, and the principal’s profits from the selected teams were increasing.*

**Support:** Tables 2.4, 2.5, 2.7; figures 2.1, 2.2. Average per period asks of efficient agents were decreasing in time in all experiments; asks of inefficient agents were decreasing rather consistently until the outcomes were close to Nash equilibrium (Profit >800). Correspondingly, the average profits of the selected teams were increasing until they reached the level close to the Nash equilibrium. (See also tables in appendix D.) □

To identify the presence of competitive tendencies, the notion of competition needs to be reconsidered. As result 1 indicates, the Nash equilibrium behavioral hypothesis does not explain all the data, although 63.6% of the outcomes of the BG experiments and 12.9% of the outcomes of the GWDG experiments are close to the Nash equilibrium in the level of profit. Still, the dynamics of the data – the changes in individual asks and profits of the teams

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<sup>23</sup> Assuming each team is selected, on average, with equal probability, as in the “fair” fully cooperative solution  $v = (550, 550, 450)$ . Trembles can insure that this solution is fair and collusive under both BG and GWDG.

	BG pooled		GWDG pooled		All data	
	yes	no	yes	no	yes	no
Selected last period?						
% of asks increased	34.4	9.9	40.4	5.2	37.1	7.8
% of asks unchanged	33.5	13.9	26.3	7.8	30.3	11.1
% of asks decreased	32.1	76.2	33.3	87.1	32.7	81.0

Table 2.6: Last period selections and directions of individual ask changes, %.

over periods (see also figures 6-12) – indicates that both BG and GWDG experiments were similar to market oral auction and sealed bid experiments (Plott, 1989) with a definite trend of convergence towards the Nash (competitive) equilibrium. Although the convergence in our experiments – especially in the GWDG – was slow and noisy, the trend is apparent from tables 2.4 and 2.5 and figures 2.1 and 2.2. The same arguments which were used to show the instability of cooperative tendencies can be used to support the competitive tendencies: the individual asks were decreasing consistently in time until the outcomes were close to the Nash equilibrium, and the profits from the selected teams were increasing consistently. In addition, we have computed the frequencies of directions of individual wage demand changes. Table 2.6 displays how often the agents increased, decreased or did not change their wage demands depending on whether they were selected or not in the previous period.

The data show that 81% of the time subjects decreased their asks, i.e., acted in the direction of the best response to the last period asks following their non-selection. Moreover, 32.7% of the time the subjects decreased their asks even if they were selected in the previous period. Viewing competition in the context of the wage-demand games as a desire to get selected with certainty by means of the wage-demand decreases, we state

**Result 3** *Competitive tendencies were present and rather persistent in both BG and GWDG*

*experiments: asks were decreasing from period to period, and profits of selected teams were increasing.*

**Support:** The above, support for result 2. □

The next group of results regards the differences between the BG and the GWDG experiments.

**Result 4** *On average, the BG experiments were closer to the Nash equilibrium prediction than the GWDG experiments: the average per period asks were much lower in the BG than in the GWDG experiments.*

**Support:** Tables 2.4, 2.5, 2.7; figures 2.1, 2.2. On average, the initial – first period level of asks was higher in the GWDG than in the BG. Average per period asks were much higher under GWDG than under the BG. Moreover, under the BG an efficient agent's average ask was 164.4 francs, which is below the lower bound of the collusive solutions (200 francs), whereas under the GWDG an efficient agent's average ask was 272.5 francs, which is above both 150 and 200, the two lower bounds of collusive solutions possible under GWDG. In the region of outcomes far from the Nash Equilibrium level (Profit < 800), the average per period speed of ask decrease was much higher under the BG than under the GWDG. (See also tables D.9-D.10 in appendix D.)

Consequently, the average per period profits of the selected teams were much lower under the GWDG than under the BG. The GWDG experiments have less than 1/7 of total number of outcomes in the region close to Nash equilibrium (Profit  $\geq$  800); the BG experiments have over 60% of observations in this region. In fact, except for experiment #6, the GWDG

	Profit < 800		Profit ≥ 800		Total	
	Ask change	% of obs.	Ask change	% of obs.	Ask change	% of obs.
All data	-22.12	60.9	0.23	39.1	-13.99	100
-efficient agents	-11.93	60.9	-0.85	39.1	-7.06	100
-inefficient agents	-42.49	60.9	2.38	39.1	-24.97	100
BG pooled	-30.90	38.3	0.34	61.7	-11.64	100
-efficient agents	-9.07	38.3	-1.18	61.7	-4.20	100
-inefficient agents	-74.57	38.3	3.37	61.7	-26.51	100
GWDG pooled	-17.420	89.1	-0.54	10.9	-15.57	100
-efficient agents	-13.467	89.1	1.46	10.9	-11.87	100
-inefficient agents	-25.326	89.1	-4.52	10.9	-23.05	100

Table 2.7: Average per period individual ask changes far and close to Nash equilibrium, francs.

experiments have no outcomes close to the Nash Equilibrium level; all of BG experiments had more than 45% of outcomes with the level of profit above 800.  $\square$

Note that in the region of outcomes close to Nash equilibrium, subjects in the BG experiments attempted to be more cooperative in the sense of increasing their wage demands. It is difficult to compare the subjects' behavior across the treatments (selection rules) because of the absence of observations in this region for two of three GWDG experiments. The data from the only GWDG experiment (#6) that has outcomes close to Nash equilibrium indicates that the behavior of subjects in this region was not significantly different from the BG experiments (table D.10 in appendix D).

To get the idea why the BG experiments could be more competitive than the GWDG experiments, we consider the differences in the behavior of efficient and inefficient agents.

**Result 5** *Both in BG and GWDG experiments, inefficient agents were, overall, more competitive than efficient agents: their average per period ask decrease was higher. The ineffi-*

*cient agents became less competitive only when the outcomes approached the Nash equilibrium level in profits.*

**Support:** Table 2.7. The table indicates that the average asks of the inefficient agents increased in the region of outcomes close to Nash equilibrium. However, this tendency was unstable and did not occur in all experiments (see table D.10 in appendix D). □

**Result 6** • *Overall, inefficient agents were more competitive in the BG than in the GWDG: their average ask decreases were higher in the BG experiments.*

- *Overall, efficient agents were more competitive in the GWDG than in the BG: their average ask decreases were higher in the GWDG experiments.*

**Support:** Table 2.7. Results 5 and 6 become even stronger if we consider rates of subjects' wage demand decreases in the region of outcomes far from Nash equilibrium. □

The above observations allow us to make the following conclusions. The data indicates that both the BG and GWDG experiments exhibited substantial competitive tendencies in the subjects' behavior, although the subjects did not strictly follow the Nash equilibrium behavior. The subjects' behavior was substantially different from the behavior that Bolle observed in his experiments. In the BG experiments, the competitive tendencies were much stronger than in the GWDG experiments, and the outcomes converged to the region close to the Nash equilibrium much faster under the BG treatment. The reason was that the inefficient agents were much more competitive in the BG than in the GWDG experiments, therefore driving the outcomes to the levels close to Nash equilibrium faster. This tendency was consistent over most experiments and, therefore, can be attributed to the differences



in the selection rules rather than subject pool effects. In particular, it might have been the case that the subjects behaved more competitively in the BG experiments since the BG allowed fewer collusive possibilities than the GWDG. We can draw a conclusion: the experiments showed that the mechanisms do matter for the outcomes of wage-demand games. The behavior of the inefficient, “marginal” agents in the wage-demand games was driving the process, just as in traditional competitive market experiments (Plott, 1989). When getting closer the Nash equilibrium outcomes, the agents, especially the inefficient agents who were gaining almost nothing in this region even if they were selected, in several cases tried to induce cooperation by increasing their wage demands, but the tendency was weak and unstable; the outcomes stayed in the region close to the Nash equilibrium (see figures in appendix C).

In the following sections, we consider whether the above differences between the outcomes of the BG and the GWDG experiments can be explained by the differences in subjects’ behavior on individual level.

## 2.5 Classification of individual behavior

The results presented in the previous section indicate that neither the Nash equilibrium solution nor any one of the cooperative solutions describe the behavior of the individuals in the wage-demand experiments very well. The asks were above the Nash equilibrium level most of the time<sup>24</sup>, which can be attributed to cooperative trends in the behavior. Yet there was an apparent tendency for the subjects’ asks to decrease through the duration of each

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<sup>24</sup>While the inefficient agents could not submit their asks below the Nash equilibrium level of 0, the efficient agents could: their Nash equilibrium ask was 100.

experiment, clearly indicating the presence of competitiveness. In the BG experiments, the asks were decreasing faster and were closer to the Nash equilibrium level than in the GWDG experiments. In this section, we take a closer look at the subjects' individual behavior in order to establish whether the difference between the experiments could be attributed to the differences in subjects' behavior. We assume that the subjects' wage demands were history-dependent, and then estimate whether they were converging to the Nash equilibrium. We further classify the individual behavior by the degree of competitiveness and distinguish the competitive-type subjects from the others. Since such competitive types are the only ones who systematically initiate the decrease in the level wage demands, the share of these types among all subjects affects the speed of convergence of each experiment to the Nash equilibrium outcome: the higher the number of competitive types, the faster the experiment converges. We compare the number of competitive subjects across the experiments and find that there were more competitive subjects in the BG than in the GWDG experiments; this explains why the outcomes of the BG were closer to the Nash equilibrium prediction. Moreover, since the difference is persistent across the two types experiments, it can be attributed to the difference between mechanisms.

We now turn to the detailed consideration of subjects' behavior.

### **2.5.1 History-dependence hypothesis**

There may be many reasons why competitive behavioral tendencies do not immediately lead to Nash equilibrium outcomes: for example, "competitive" agents may be counting on the presence of cooperative – altruistic or not – agents in the subject pool (as in McKelvey and Palfrey, 1992); or, some of the agents may be "more competitive" than the others, thus

allowing agents to demand wages above the Nash equilibrium level and still be selected. More generally, the Nash equilibrium behavior is not a dominant strategy and, therefore, if some agents do not follow the Nash equilibrium behavior, the others could also gain from deviations from Nash equilibrium. In particular, competitive agents could follow the Nash best response behavior: depending on what wage demands an agent expects his opponents to submit, the agent could submit an ask just below his opponent's asks in order to get selected with certainty and maximize the wage. There may be other behavioral rules that are consistent with competitiveness; however, all these rules depend strongly on an agent's expectations about other agents' asks.

We employ a simple variant of the history-dependence hypothesis to specify the agents' expectations about their opponents' asks. We assume that the agents take the asks observed in the previous period as a signal about the current level of asks. The similar approach is applied to the Nash-Cournot best response model which has been proposed and used before to analyze subjects' behavior in the context of public good experiments (Ledyard, 1978, Y. Chen and Plott, 1993). In our situation, the use of the Cournot model might be criticized on the grounds that the subjects are most probably matched with different opponents each period, and therefore the wage demands observed in the last period might be a very inaccurate prediction of the asks in this period. We use econometric analysis to test the validity of the one-period history-dependence hypothesis for our situation.

We have performed a least squares regression analysis to consider whether the information observed by individuals in the previous periods affected their current wage demands. Table 2.8 presents the results of the regression of individual period-to-period ask changes on the factors that could affect the agents' decisions on wage demands, with the observations

pooled for all experiments. The independent variables include a constant as well as dummy variables for the selection rule, subject's role (efficient or inefficient), three time intervals, and their cross-products. To account for past history, the set of independent variables includes the subjects' own wage demands, a dummy for whether the subject was selected or not, and the subjects' best response  $ask^{25}$  in the previous three periods. We have also included independent variables to differentiate for the influence of this factor depending on subjects' roles and whether they were selected or not in each of the three previous periods. Finally, in case a subject was selected in the previous period, we tested whether he was trying to match the ask of the other selected subject in his group: a "team-partner's" ask is included as an independent variable also<sup>26</sup>.

The results indicate (see table 2.8,  $t$ -statistics for the respective variables) that none of the variables concerning the history beyond one period back is significant at the 5% level. On the contrary, the individual ask, the best response ask, the selection outcome and the team-partner's ask from the previous period are all significant at the 1% level.

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<sup>25</sup>Best response asks for each agent are calculated given their opponents' asks in the previous period. (The exact formula is given below.) If the agents follow the Cournot best response behavior, then an individual's ask change in a current period should equal to the difference of his best response ask and his actual ask last period, and no other factors should matter. If other factors matter but the agents take into account how much they "missed" from the the best response in the previous periods, then the latter factor should still appear significant in the results of the regression.

<sup>26</sup>Here is the full list of independent variables: **one** - a constant; **srule** - a dummy for the selection rule: = 1 if GWDG, = 0 if BG; **role1** - a dummy for role: = 1 if efficient, = 0 if inefficient; **time2**, **time3** - dummies for the second and the third time intervals; **asklag**, **asklag2**, **asklag3** - a subject's asks previous period and 2 and 3 periods before; **askBRlag**, **askBRla2**, **askBRla3** - a subject's best response ask previous period and 2 and 3 periods before; **Sidlag** - a selection dummy: = 1 if a subject was selected in the previous period, = 0 otherwise; **Sidlag2**, **Sidlag3** - same as Sidlag for 2 and 3 periods before; **askopS1** - in case a person was selected in the previous period, this is the adjusted ask of the other selected person (equals the actual ask if both subjects were of the same role, and is adjusted by +100 or -100 francs otherwise), = 0 if a person was not selected; **askopS2**, **askopS3** - same as askopS1 for 2 and 3 periods before; **timerul1** -  $time1*srule$ ; **timerul2** -  $time2*srule$ ; **timerul3** -  $time3*srule$ ; **timrol11** -  $time1*role1$ ; **timrol21** -  $time2*role1$ ; **timrol31** -  $time3*role1$ ; **rolrul1** -  $role1*srule$ ; **Sidask1** -  $Sidlag*asklag$ ; **Sidask12** -  $Sidlag2*asklag2$ ; **Sidask13** -  $Sidlag3*asklag3$ ; **asklagr1** -  $asklag*role1$ ; **asklagr2** -  $asklag2*role1$ ; **asklagr3** -  $asklag3*role1$ ; **asklr1t2** -  $asklag*role1*time2$ ; **asklr1t3** -  $asklag*role1*time3$ .

Independent Variable	3 periods back		1 period back	
	Estimated Coefficient	t-Statistic	Estimated Coefficient	t-statistic
one	109.27***	3.19	118.37***	5.85
srule	34.40**	2.41	53.40***	5.07
role1	-147.68***	-2.87	-120.20*	-2.31
rolrul1	-45.00	-1.29	-48.70	-1.53
time2	1.92	0.06	-5.36	-0.19
time3	7.47	0.29	-4.92	-0.25
asklag	-0.98***	-72.04	-0.98***	-76.81
asklag2	-0.0007	-0.12	—	—
asklag3	-0.01	-1.89	—	—
askBRlag	0.16***	2.61	0.21**	2.52
askBRla2	0.06	1.14	—	—
askBRla3	0.08	1.49	—	—
Sidlag	-46.08***	-3.71	-62.24***	-5.05
Sidlag2	-15.69	-1.47	—	—
Sidlag3	-16.28	-1.47	—	—
askopS1	0.26***	4.01	0.35***	4.23
askopS2	0.14	1.84	—	—
askopS3	0.03	0.43	—	—
asklagr1	0.58***	2.80	0.64***	2.96
asklagr2	0.17	0.84	—	—
asklagr3	-0.08	-1.03	—	—
asklr1t2	-0.60**	-2.01	-0.48	-1.71
asklr1t3	-0.16	-0.87	-0.08	-0.38
timerul2	65.05	1.34	55.09	1.19
timerul3	-18.87	-1.34	-29.08	-1.83
timrol21	115.09**	2.41	86.22	1.72
timrol31	62.93	1.20	45.02	0.82
Corrected $R^2$	0.602	—	0.600	—
# of observations	2343	—	2343	—
DW statistic	1.987	—	2.035	—

Table 2.8: Ordinary least squares estimation of per period individual ask changes, all data.  
 \* – significant at 5% level; \*\* – 2% level; \*\*\* – 1% level.

In the regression performed with the “past one period back” variables excluded, the fit of the regression (see the corresponding values of  $R$ -squared) did not change much, and the significance of the previous period variables was confirmed. On this basis hereafter we adopt the assumption of previous period history-dependence and assume that history beyond one period did not significantly affect agents’ current decisions.

### 2.5.2 The approach to classification of individual behavior

Our purpose is to consider, on individual level, whether the experiments were competitive in the sense that the agents’ asks were converging to the Nash equilibrium, and whether the BG experiments were more competitive than the GWDG experiments on the individual level. We take the following approach. Given the history-dependence assumption, we first statistically estimate individual ask-adjustment rules and then consider whether, according to these rules, individual asks were converging to the Nash equilibrium. Next, we distinguish the individuals who were actively inducing competition from those who just followed the trend initiated by others, and thus classify the individual behavior by the degree of competitiveness. Finally, we compare the number of competitive subjects across the BG and the GWDG experiments.

A short comment on statistical procedures used in this section is necessary. Below, we deal with the analysis of each subject’s behavior separately, and therefore the number of data points per individual is relatively low (about 30 observations). Hence, we cannot expect the statistical analysis to produce very accurate estimations. We use this analysis in order to highlight the most interesting qualitative features in individual behavior, such as the decreasing trend in asks, rather than to obtain accurate quantitative estimates of

individual behavioral rules. We proceed with this comment in mind.

**Ask-adjustment rules** First consider what rules individuals may use to determine their current asks given the asks observed in the previous period. If agents follow the Cournot-Nash **best response** behavior, then, assuming that the opponents' asks this period are equal to the opponents' asks in the previous period, the agent should submit an ask equal to (or just below, depending on the role and the selection rule) the highest of two of his opponents' adjusted asks<sup>27</sup>. This is equivalent to maximizing the wage conditional on being employed. Formally, given the specific parameters of the game (table 2.1) and the other agents' wage demands in the group, each agent's best response, depending on the role, is

$$\begin{aligned}
 v_1 BR &= \begin{cases} \max\{v_2, 100 + v_3\} & \text{if } 1000 - v_2 - v_3 \geq 0 ; \\ \max\{1100 - a_2, 1000 - a_3, 0\} & \text{otherwise;} \end{cases} \\
 v_2 BR &= \begin{cases} \max\{v_1, 100 + v_3\} & \text{if } 1000 - v_1 - v_3 \geq 0 ; \\ \max\{1100 - v_1, 1000 - v_3, 0\} & \text{otherwise;} \end{cases} \\
 v_3 BR &= \begin{cases} \max\{v_1, v_2\} - 100 & \text{if } 1100 - v_2 - v_3 \geq 0 ; \\ \max\{1000 - v_1, 1000 - v_2, 0\} & \text{otherwise.} \end{cases}
 \end{aligned}$$

The best response ask from the previous period could have been, for some agents, the most important factor that determined the current ask. However, the data overview above (tables 2.6, D.6) indicates that all agents did not precisely follow the Nash best response behavior and about 30% of the time changed their asks in the direction opposite from the

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<sup>27</sup>Hereafter, we will consider the asks adjusted for the role differences to make the agents symmetric; this can be done by decreasing the efficient agents' asks by 100 for inefficient agents or increasing the inefficient agent's ask by 100 for efficient agents.

best response. Therefore, we also consider some alternative rules.

Another possibility is that the agents use the highest winning (i.e., resulting in employment) ask from the previous period as a basis to determine their current asks. Observe that given the three asks submitted by the agents in a group, the highest winning ask is the median among the (adjusted) asks; we will refer to it as the **median ask**. Matching the previous period average ask rather than submitting the previous period best response ask can be explained by the following bounded rationality argument. If the agents follow a Cournot best response, then both of the agents that were selected in the previous period should match the “losing” (not selected) agent’s ask from the previous period; in the meanwhile, the agent who was not selected in the previous period will decrease his ask to match the highest among the “winning” asks, the median ask. If the previously selected agents anticipate this action of the non-selected agent, they might also submit their asks close to the average ask (again, assuming that the asks from the last period are a perfect signal for the asks in this period). Note that the “median ask behavior” is also fair – at least, in the GWDG – in the sense of giving all the agents equal chances of employment if they all match the median ask.

Finally, agents could use simple rules based only on their **selection** (or non-selection) in the previous period. In particular, an agent’s ask could stay equal to his previous period ask if the agent were selected, and be decreased by some increment if the agent were not selected. Besides, in all three cases, whether the agents used the best response ask, the median ask, or the simple selection response rule, we assume the possibility that an agent’s behavior includes the element of inertia, i.e., his own previous period ask affects his current ask.



Let  $v_i$  denote agent  $i$ 's current ask;  $v_iL$ ,  $i$ 's previous period (lagged) ask;  $v_iBR$ ,  $i$ 's best response ask given the previous period asks;  $v_iM$ , a previous period median ask; and  $S$ , an indicator for the previous period selection shock:  $S = 1$  if the agent was selected, and  $S = 0$  otherwise. We can formalize three types of simple rules discussed above that could determine the agents' current asks.

1. Selection response (SR) rule:

$$v_i = \alpha_0 + \alpha_1 * v_iL + \alpha_2 * S \quad (2.4)$$

2. Median ask (MA) rule:

$$v_i = \beta_0 + \beta_1 * v_iL + \beta_2 * v_iM \quad (2.5)$$

3. Cournot best response (BR) rule:

$$v_i = \gamma_0 + \gamma_1 * v_iL + \gamma_2 * v_iBR \quad (2.6)$$

For each individual, we can use least squares estimations to evaluate the coefficients of each equation, and then select one of the three rules as a model of an individual's behavior. However, since each of the models determines a whole family of rules depending on the coefficients, the type of rule itself does not determine whether an individual using a particular rule is cooperative or competitive. Our next objective is to determine a criterion to distinguish competitive behavior from cooperative.

**Competitive or cooperative behavior?** We use the following idea to evaluate the competitiveness of the rule. First, intuitively, we may consider the rule competitive if it results in a decreasing trend in asks from period to period; we may consider the rule cooperative if the asks are increasing or are sustained at some cooperative level. For this purpose, we will consider whether the rule produces an increasing or decreasing sequence of asks if all the agents were identical and submit equal (adjusted) asks every period, and each agent had an equal probability of being selected. Second, we will test the significance of an increasing or decreasing trend imposed by the rule. If the trend is insignificant in the sense that we cannot reject the hypothesis that the rule is stationary, then we classify the behavior as marginal between cooperative and competitive rather than either of those. Interestingly, at this point we can relax the assumption of identical previous period asks and interpret the rule as marginal if it conforms to the hypothesis that the current period ask is a convex combination of the asks observed in the previous period. In this interpretation, the marginal rule may follow a decreasing or increasing trend in asks if it is initiated by other agents.

We now discuss the above idea in more detail.

### Decreasing and increasing regions and stationary points

**Definition 18** *A rule of the form*

$$v_{it} = f(v_{t-1}) ,$$

where  $v_{it}$  is agent  $i$ 's current ask and  $v_{it-1}$  is the vector of asks observed by  $i$  in the previous period, is called **decreasing** in the region of asks  $V \subseteq R_+$  if, given that all the agents submitted equal adjusted asks in the previous period, an agent's ask determined by this rule decreases in the next period:

$$v_{it} < v_{it-1} \text{ for } v_{it-1} \in V .$$

Similarly, a rule is called **increasing** in the region of asks  $V \subseteq R_+$  if the ask is increasing:

$$v_{it} > v_{it-1} \text{ for } v_{it-1} \in V .$$

Finally, the rule is called **stationary** in the region of asks  $V \subseteq R_+$  if the ask does not change:

$$v_{it} = v_{it-1} \text{ for } v_{it-1} \in V .$$

Using the above definition, we can compute increasing, stationary and decreasing regions for the MA and BR type rules using the substitution  $v_i L = v_i M = v_i BR$ . We should keep in mind, however, that if an agent's ask in the previous period was different from the median or best response ask for this period, than the agent's asks could increase in the decreasing region or vice versa. To get an idea how often it could have been the case, we graphically present in figure 2.3 deviations of individual asks from the same period median and best response asks, across all individual observations. In both cases, the mean deviations are small, but different from zero: the mean deviation from the median was 21.4 francs, and from the best response ask was -16.3 francs. Therefore, we may expect to get slightly biased

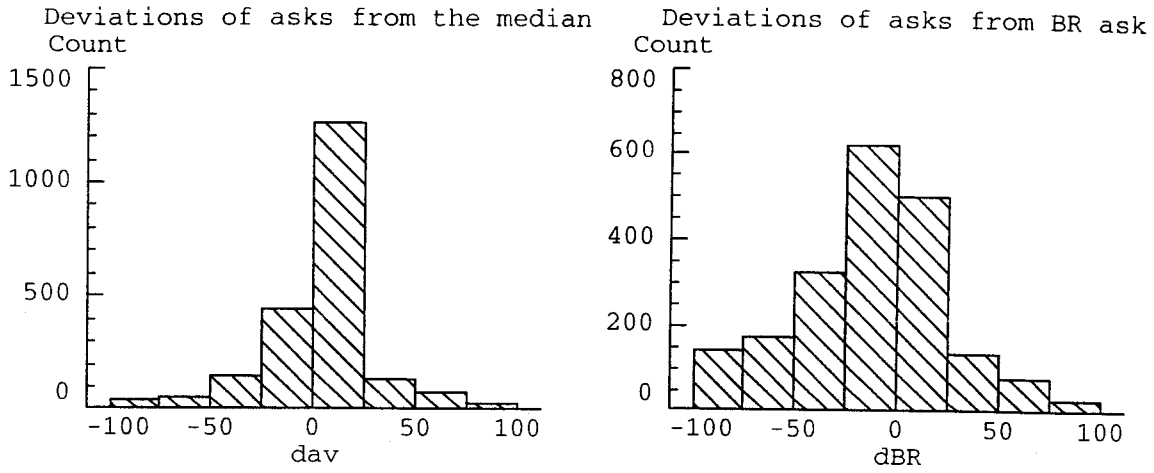


Figure 2.3: Distribution of deviations of individual asks from the current period median and best response asks, in francs, all data.

estimates of the regions where the asks were actually increasing or decreasing; however, our purpose is to estimate the regions that would be increasing or decreasing due to this particular rule. The confidence intervals for the regions will be evaluated below.

In estimating the increasing and decreasing regions for the selection response rules, we encounter a problem with the selection shocks that enter the ask adjustment rule as a discrete variable. To get a simple estimate of the regions, we observe that an agent using rule 2.4 (SR) is, in fact, switching between two models,

$$v_i = (\alpha_0 + \alpha_2) + \alpha_1 * v_i L ,$$

and

$$v_i = \alpha_0 + \alpha_1 * v_i L ,$$

depending on the previous period selection outcome. Therefore, decreasing and increasing

regions for the rule 2.4 will oscillate between the respective regions of the two above rules. Since our main purpose is to evaluate competitiveness of an agent's behavior overall, we use the observation that, on average,

$$\alpha_2 * S = \alpha_2 * r ,$$

where  $r$  is an individual selection rate, or the share of the times when a given agent was selected. Therefore we can substitute the selection shock variable  $S$  in the rule 2.4 with a constant  $r$  to get an approximate estimate of the decreasing and increasing regions<sup>28</sup>.

With these modifications, we can evaluate the decreasing and increasing regions for each rule. A selection response rule is decreasing in the region  $V^d \subseteq R_+$  if for all  $v_i \in V^d$

$$(1 - \alpha_1)v_i > \alpha_0 + \alpha_2 * r ; \tag{2.7}$$

likewise, it is increasing in the region  $V^{in} \subseteq R_+$  if for all  $v_i \in V^{in}$  the above inequality is reversed. It is stationary if

$$(1 - \alpha_1)v_i = \alpha_0 + \alpha_2 * r . \tag{2.8}$$

Similarly, a median ask rule is decreasing in the region  $V^d \subseteq R_+$  if for all  $v_i \in V^d$

$$(1 - \beta_1 - \beta_2)v_i > \beta_0 ; \tag{2.9}$$

it is increasing in the region  $V^{in} \subseteq R_+$  if for all  $v_i \in V^{in}$  the above inequality is reversed.

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<sup>28</sup>Confidence intervals for the estimates may and will be further tested.

It is stationary if

$$(1 - \beta_1 - \beta_2)v_i = \beta_0 . \quad (2.10)$$

Finally, a best response rule is decreasing in the region  $V^d \subseteq R_+$  if for all  $v_i \in V^d$

$$(1 - \gamma_1 - \gamma_2)v_i > \gamma_0 ; \quad (2.11)$$

it is increasing in the region  $V^{in} \subseteq R_+$  if for all  $v_i \in V^{in}$  the above inequality is reversed.

It is stationary if

$$(1 - \gamma_1 - \gamma_2)v_i = \gamma_0 . \quad (2.12)$$

It is easily seen that there may be only a finite number of possibilities regarding decreasing and increasing regions for each rule<sup>29</sup>.

**Proposition 23** *If the ask adjustment rules are of the form 2.4, 2.5 or 2.6, and the decreasing/increasing regions are evaluated using the expressions 2.7-2.12, then there are only five possible cases:*

1. *The rule is increasing for all  $v_i > 0$ ;*
2. *The rule is decreasing for all  $v_i > 0$ ;*
3. *The rule is stationary for all  $v_i \geq 0$ ;*
4. *The rule is decreasing for all  $v_i > v_i^*$ , stationary at  $v_i = v_i^* > 0$ , and increasing for*

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<sup>29</sup>The proof for the following proposition is sketched in section 2.7.

all  $v_i < v_i^*$ ; in this case  $v_i^*$  is the asymptote of the asks:

$$v_i \rightarrow v_i^* \text{ as } t \rightarrow \infty$$

where  $t$  is the period index.

5. The rule is increasing for all  $v_i > v_i^{**} > 0$ , stationary for  $v_i = v_i^{**}$  and decreasing for all  $v_i < v_i^{**}$ .

Observe that case (1) is equivalent to case (5) with  $v_i^{**} = 0$ ; case (2) is equivalent to case (4) with  $v_i^* = 0$ . We will further refer to  $v_i^*$  as **type 1 stationary point**, or the **asymptote**, and to  $v_i^{**}$  as **type 2 stationary point**.

Some interpretation of the possibilities presented above is useful. In case (1) we may call an agent following the corresponding rule cooperative, and in case (2), competitive. Case (3) would be the case, for example, if agents followed the Nash equilibrium behavior, or sustained one of the cooperative outcomes. Case (4) is the case of convergence: no matter at what level of asks the agents start, they end up at  $v_i^*$ . In particular, if  $v_i^* = v_i^N$ , then the process converges to the Nash equilibrium; the agents who follow this rule are, therefore, competitive. In our statistical estimation for the individual rules which fall under case (4), we will test whether the asymptote  $v_i^*$  is significantly different from the Nash equilibrium ask. If the difference is insignificant, then we get an indication of the Nash equilibrium behavior. Finally, case (5) may occur if, for example, efficient agents find it to be worthwhile sustaining cooperation above some level of asks (for example, the lower bound of cooperative solutions), but act competitively if the level of asks is already low.

**Marginal behavior** After an initial evaluation of the decreasing and increasing regions for each rule is completed, the significance of an increasing or decreasing trend in a given rule should be tested. We should test the hypothesis  $H_0$  of stationarity of the rule in the case when all agents submitted identical adjusted asks in the previous period:

$$H_0 : v_{it} = v_{it-1} .$$

If we find that the hypothesis cannot be rejected, then the trend is insignificant and therefore the rule should be considered **marginal** rather than competitive or cooperative.

Marginal behavior has an alternative interesting interpretation. If we relax the assumption of identical previous period ask, then we could interpret the behavior as marginal if it does not induce substantial changes in the level of asks by itself but follows the changes induced by other agents. In this respect, we note the following:

**Observation 1** *Suppose a set  $N$  of individuals is involved in a wage-demand experiment. If each individual's  $i \in N$  ask-adjustment rule is of the form*

$$v_{it} = f(v_{t-1}) ,$$

*and is such that for any  $v_{t-1}$*

$$\min_{j \in N_{t-1}^i} v_{jt-1} \leq v_{it} \leq \max_{j \in N_{t-1}^i} v_{jt-1} , \quad (2.13)$$

*where  $N_j^i$  is a subset of agents whose asks  $i$  observes in the period  $(t-1)$ , then in any period*



$t$

$$\min_{i \in N} v_{i0} \leq \min_{i \in N} v_{it} , \quad (2.14)$$

$$\max_{i \in N} v_{it} \leq \max_{i \in N} v_{i0} , \quad (2.15)$$

where  $t = 0$  refers to the initial period.

In particular, if every agent chooses his next period ask as a convex combination of the asks observed in the previous period, then the asks submitted by the agents in any period will stay within the range of the initial period asks. The asks will neither significantly increase nor decrease. Therefore, if an ask-adjustment rule is of the form:

$$v_{it} = \sum_{j \in N_{t-1}^i} \delta_j v_{jt-1} ,$$

where  $\delta_j \geq 0$  and  $\sum_{j \in N_{t-1}^i} \delta_j = 1$ , then we can classify the rule as marginal; it does not induce significant changes in the level of asks by itself, but follows the change if it is induced externally.

Depending on the type of the rule, we adopt the following variants of the hypothesis of marginal behavior:

- For the selection response rule:

$$H_0^S : (\alpha_1 = 0) \quad \text{or} \quad (\alpha_0 + r * \alpha_2 = 0, \alpha_1 = 1) ; \quad (2.16)$$

- For the median ask rule:

$$H_0^M : (\beta_0 = 0, \beta_1 + \beta_2 = 1) ; \quad (2.17)$$

- For the best response rule:

$$H_0^{BR} : (\gamma_0 = 0, \gamma_1 + \gamma_2 = 1) . \quad (2.18)$$

We now summarize our approach to testing the competitiveness of individual behavior. We first approximate each individual ask-adjustment process by a simple linear model, assuming the the current individual decision depends on the observed outcomes of the previous period. For a given model, we then evaluate the regions of increasing and decreasing asks; increasing asks indicate on the presence of cooperative tendencies in an agent's behavior; likewise, decreasing tendencies indicate competitive behavior. We further test the hypothesis that the behavior was marginal between cooperative and competitive; if the latter hypothesis is rejected, then we can further test if the region of competitive behavior (i.e., decreasing asks) coincides with the region of asks above the Nash equilibrium level. If this is the case, then we can classify the behavior as competitive. If we find that an agent's asks exhibit increasing or stationary trend at the level significantly above the Nash equilibrium asks, then we can classify the behavior as cooperative. Finally, we can compare our results on the competitiveness of behavior between the BG and the GWDG experiments and find out the differences in the behavior that could have been induced by the differences in the selection rules.

We next describe the statistical procedures used to implement the above approach and the resulting classification of individual behavior.

### 2.5.3 Statistical procedures and classification criteria

**Selection of individual rules** We performed the least squares estimations of individual behavioral rules using three alternative models 2.4 (SR), 2.5 (MA) and 2.6 (BR) for each of 81 subjects who participated in the experiments<sup>30</sup>. The *J*-test by Davidson and McKinnon (as described in Green, 1990, p. 231) for comparison of non-nested linear models<sup>31</sup> as well as *F*-statistics for the significance of regressions (in case the results of the *J*-test were inconclusive) were used to select one of the three rules as a model of behavior for each individual. We considered the individual behavior unclassified if none of the three regression models had a value of *F*-statistic for the significance of the regression above the 5% significance level.

**Statistical procedures for classification by competitiveness** Given the estimated coefficients of the selected models, we computed the increasing and decreasing regions and stationary points for each individual rule using inequalities 2.7-2.12. To evaluate standard errors on the estimates of stationary points, we used Taylor series approximations of the stationary points as functions of the estimated coefficients of the linear regression models, as described in Green (1990, pp. 228-230). The values of the test statistic – we will refer to it

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<sup>30</sup>Since the regression models contain lagged dependent variables, we performed the *h*-test for the presence of autocorrelation as described in Green, 1990, p. 454. The results (tables E.1-E.14 in appendix E) indicate that in most cases the value of the *h*-statistic was not significantly different from zero and therefore the hypothesis of no autocorrelation was sustained.

<sup>31</sup>The test is valid asymptotically.

as a  $z$ -statistic<sup>32</sup> – was then calculated to test whether the estimated stationary points were significantly different from the Nash equilibrium asks. Finally, we tested the hypotheses of the marginal behavior represented as linear restrictions 2.16, 2.17 or 2.18 for SR, MA and BR rules, respectively; the  $t$ -test was used to test if  $\alpha_1 = 0$ , and the  $F$ -statistics for testing the set of linear restrictions were calculated for the rest of the cases.

**Criteria for classification of rules by competitiveness** We conducted a classification of rules into competitive, cooperative and marginal types as follows. If the rule had an asymptote (type 1 stationary point), it was classified as **competitive** if the hypothesis  $H_0$  that the rule was marginal was rejected (i.e., the value of  $F$ -statistics for testing the marginality restrictions on the model was significant), and the asymptote was not significantly different from the Nash equilibrium ask (the value of  $z$ -statistic was below the 5% significance level). The behavior was also classified as competitive if it had a type 2 stationary point, the hypothesis of marginal behavior was rejected, and the hypothesis that the stationary point was above the maximal ask submitted by the given subject was not rejected (the latter implied that the agent's asks were in the decreasing region). The behavior was classified as **marginal** whenever the hypothesis that the rule was marginal was not rejected (the value of the  $F$ -statistic for marginality restrictions was below the 5% significance level), except for one case described below. The behavior was classified as **cooperative** if it was marginal and had the asymptote significantly above the Nash equilibrium ask, or it had a type 2 stationary point significantly below the maximal ask and was not marginal. Finally, if the rule had an asymptote, was not marginal but the asymptote was significantly above

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<sup>32</sup>Since the technique relies on asymptotic properties of the least squares, the standard normal distribution was used instead of the  $t$ -distribution for the test statistic.

Type of stat. point	If type 1, $\approx$ ask Nash?	If type 2, $\approx$ ask max?	Marginal?	Classification
type 1	yes	—	no	competitive
type 1	yes	—	yes	marginal
type 1	higher	—	yes	cooperative
type 1	higher	—	no	switch
type 2	—	yes	no	competitive
type 2	—	lower	no	cooperative
type 2	—	yes	yes	marginal

Table 2.9: Criteria for classification of individual behavior

the Nash equilibrium ask, we classified it as a rule with a **switch**: apparently, at the high level of asks, the rule was competitive (decreasing), but then stabilizes at some level above the Nash equilibrium and sustains cooperation. Individuals whose wage-demand rules were initially unclassified, stayed unclassified. Table 2.9 summarizes the criteria for classification of the rules.

#### 2.5.4 Results

The results of our statistical estimations of individual behavior are presented in tables 2.10-2.12 and figure 2.4. The detailed results of the estimations for each individual are given in tables E.1-E.14 in appendix E. We start our review of the results with classification of individual behavior by the types of rules.

**Individual behavioral rules** Table 2.10 shows how often each of the three rules – SR, MA and BR – was used by the individuals. The results indicate that the median ask rule was used by about 50% of the subjects, i.e., more often than either the selection response rule (22% of the subjects) or the best response rule (15% of the subjects). There

%	Selection (SR)	Med. ask (MA)	BR ask (BR)	Unclassified	Total
All data	22	49	15	14	100
Efficient agents	22	52	13	13	100
Inefficient agents	29	44	19	8	100
BG pooled	24	44	18	13	100
Experiment 1	50	33	17	—	100
Experiment 3	17	41	17	25	100
Experiment 5	8	59	25	8	100
Experiment 7	22	45	11	22	100
GWDG pooled	25	56	14	5	100
Experiment 2	33	33	25	8	100
Experiment 4	17	67	8	8	100
Experiment 6	25	67	8	—	100

Table 2.10: Frequences of individual behavioral rules, %

were no substantial differences in the frequencies of use of the rules either between efficient and inefficient agents, nor between BG and GWDG experiments. We next considered the competitiveness of the selected rules.

**Types of stationary points** Let us overview the results on the types of competitive and cooperative regions and the corresponding stationary points that prevailed in our estimations. Table 2.11 presents the number of cases in which individual rules have type 1 and type 2 stationary points<sup>33</sup>. For the rules with type 1 stationary points, or the asymptotes, the table indicates in what number of cases the asymptotes were significantly different from the the Nash equilibrium asks<sup>34</sup>. As table 2.11 indicates, the overwhelming majority of the rules in both BG and GWDG experiments have type 1 stationary points, or the asymptotes:

<sup>33</sup>Since the subjects in the experiments could submit only non-negative asks, the estimates of the stationary points were truncated at zero: negative stationary points were assigned values of 0.

<sup>34</sup>The values of  $z$ -statistics used to evaluate the significance of the differences are presented in tables E.1-E.14 in appendix E.

	Type 1 stationary point			Type 2 st.point	Unclassi- fied	Total
	$\approx$ ask Nash	$>$ ask Nash	Total			
Exp. 1	10	2	12	—	—	12
Exp. 2	8	2	10	1	1	12
Exp. 3	6	3	9	—	3	12
Exp. 4	8	—	8	3	1	12
Exp. 5	10	—	10	1	1	12
Exp. 6	8	1	9	3	—	12
Exp. 7	7	—	7	—	2	9
All data	57	8	65	8	8	81

Table 2.11: Classification of individuals by types of stationary points, counts

the rules are decreasing in the region of asks above  $v_i^*$ , increasing in the region below  $v_i^*$ , and converge to  $v_i^*$  in time. Moreover, in the majority of cases the asymptotes are not significantly different from the Nash equilibrium asks, which is the case in both BG and GWDG experiments. Therefore, we cannot reject the hypothesis that the majority of the rules followed convergence to the Nash equilibrium under both experimental treatments.

**Distribution of asymptotes** To get an idea whether the decreasing and increasing regions of individual rules were any different under the BG than under the GWDG experiments, in figure 2.4 we plotted the distribution of asymptotes for the two types of experiments<sup>35</sup>. This representation should be treated with caution since, as table 2.11 indicates, in most cases the estimates of asymptotes were not significantly different from zero; however, it may give some indication of the differences among the two types of experiments regarding the levels of asks and its dynamics. Overall, we can see that the asymptotes in the GWDG experiments are, on average, higher than in the BG experiment both for efficient

<sup>35</sup>As table 2.11 indicates, the stationary points were type 1, or the asymptotes, in most cases; therefore, their distribution may be informative of characteristics of the experiments.

and inefficient agents. In the BG, the highest number of the asymptotes are concentrated around the Nash equilibrium level of asks; in the GWDG experiments, the majority of the asymptotes are about 100 francs or more higher than the Nash equilibrium level. This indicates that in the GWDG experiments the process of ask decrease was slowing down at a higher level of asks than in the BG. In this respect, BG experiments were more competitive than the GWDG experiments.

**Classification of individual behavior** We now present the central results of our analysis. Table 2.12 contains the results of classification of subjects' behavior into competitiveness types. We can immediately make several interesting observations. First, the share of competitive agents was higher among both efficient and inefficient agents in the BG experiments than in the GWDG experiments. Therefore, our earlier observations that the BG experiments were more competitive than the GWDG experiments are confirmed.

Next, surprisingly, in both experiments there was a lower share of competitive behavior among inefficient than among efficient agents. This finding may seem at odds with our earlier observation (result 5) that inefficient agents were more competitive than efficient agents, in the sense that their average per period ask decrease was lower. However, the situation changes if we take into account the agents who had a switch in their behavior, i.e., started acting competitively and then switched to cooperative behavior. Since in most cases the switch occurred close to the Nash equilibrium level of asks (the asymptotes of the inefficient switch-type agents were often insignificantly higher than 40-60 francs; see tables E.1-E.14 in appendix E), in the early periods of experiments, the ones that were particularly gainful for subjects, the switch-type agents were acting competitively. Together



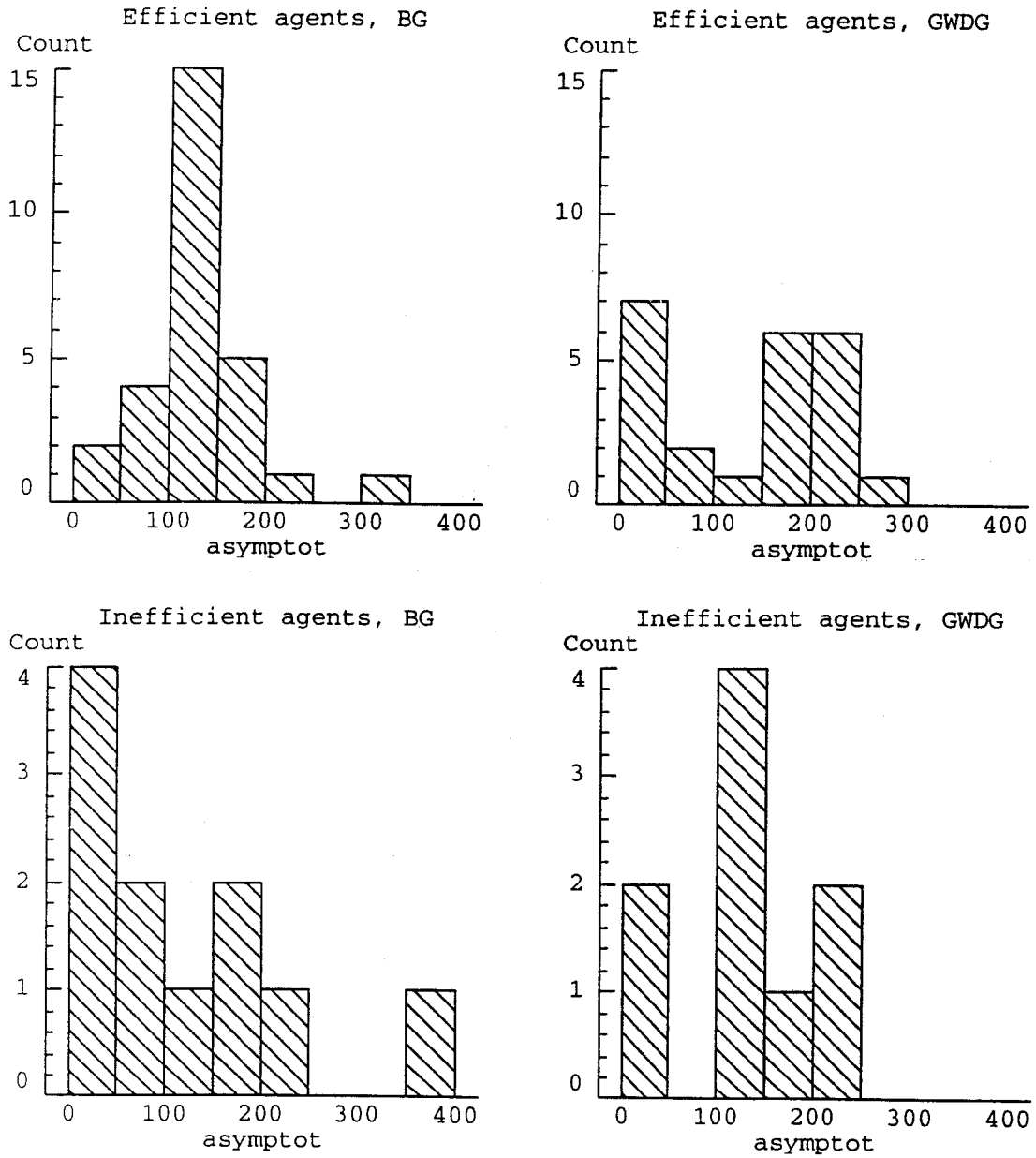


Figure 2.4: Distribution of asymptotes in the BG and GWDG experiments

%	Agent type	Comp.	Marg.	Coop.	Switch	Unclass.
All data	efficient	48	41	2	4	5
	inefficient	19	41	—	22	18
BG pooled	efficient	53	37	—	3	7
	inefficient	27	20	—	27	26
Exper. 1	efficient	37.5	50	—	12.5	—
	inefficient	50	25	—	25	—
Exper. 3	efficient	62.5	12.5	—	—	25
	inefficient	—	—	—	75	25
Exper. 5	efficient	62.5	37.5	—	—	—
	inefficient	25	50	—	—	25
Exper. 7	efficient	50	50	—	—	—
	inefficient	33	—	—	—	66
GWDG pooled	efficient	42	46	4	4	4
	inefficient	8	67	—	17	8
Exper. 2	efficient	37.5	37.5	—	12.5	12.5
	inefficient	—	75	—	25	—
Exper. 4	efficient	62.5	37.5	—	—	—
	inefficient	25	50	—	—	25
Exper. 6	efficient	25	62.5	12.5	—	—
	inefficient	—	75	—	25	—

Table 2.12: Percentages of individual behavior by competitive categories, %

with the switch-type agents, the share of initially competitive behavioral types in the BG was close between efficient and inefficient agents and in both cases exceeded 50%. On the contrary, in the GWDG experiments, even together with the switch-type agents, the share of the competitive agents among inefficient was only 25%, which was almost twice as low as among efficient agents. Apparently, this made a big difference in the speed of convergence of the experiments to the low level of asks; with the higher percentage of competitive types among the subjects, the level of asks decreased faster in the BG than in the GWDG.

The difference in the shares of marginal types among the subjects in the BG and the GWDG experiments is also apparent. There were fewer marginal agents among efficient and, especially, inefficient agents in the BG as compared to the GWDG. Observe that the marginal types do not themselves initiate the decrease in the level of asks, although they follow it when it is induced by the competitive types. Therefore, the higher the share of marginal types as compared to competitive types among the subjects, the slower the asks converge to the competitive stationary point, i.e., the Nash equilibrium.

The next observation regards the presence of cooperation. As the table shows, we have detected practically no cooperative types among the subjects according to our classification. Cooperation efforts were mostly either “passive” (marginal behavior) or late in the experiment (switching behavior); the asks almost converged to the Nash equilibrium level before the agents – especially, inefficient agents – exhibited some cooperation attempts. However, the marginal behavior does not contradict cooperative theories discussed in section 2.2.3. The crucial thing to notice is that the cooperative solutions for the wage demand game induced by our design were unstable in two ways: first, each agent had an incentive to deviate unilaterally; second, cooperation could not be sustained if any one subject devi-

ated. Apparently, in both BG and GWDG experiments, the presence of some number of competitive subjects resulted in the tendency of asks among all subjects to decrease until they approached to the Nash equilibrium level.

We can now formulate our conclusions regarding the BG and the GWDG experiments based on the analysis of individual behavior of the subjects.

**Result 7** *The following conclusions can be drawn from the analysis of individual behavior in the wage-demand experiments.*

1. *Competitive tendencies were present in the behavior of subjects in both the BG and the GWDG experiments: the share of competitive types was significant in all experiments.*
2. *The stationary points of the ask-adjustment rules for the majority of subjects in each experiment were of the asymptote-type and not significantly different from the Nash equilibrium level. That is, the outcomes of every experiment were converging to the Nash equilibrium outcomes.*
3. *The share of competitive behavior among the subjects was higher in the BG than in the GWDG experiments. Especially, the inefficient agents were much more competitive in the BG than in the GWDG. Consequently, the BG experiments were converging to the Nash equilibrium faster than the GWDG experiments, which resulted in lower payoffs to the agents in the BG as compared to the GWDG experiments.*
4. *Since the differences in individual behavior between the BG and the GWDG were persistent across the experiments, they can be attributed to the differences in the selection mechanisms of the BG and GWDG.*

It is appropriate now to look back at the two alternative sets of conjectures (section 2.3.2) that were stated from the competitive and cooperative perspectives on the agents' behavior. As our findings show, none of the two sets gave the correct predictions of the experimental outcomes. The competitive prediction that the BG and the GWDG games are essentially equivalent did not prove to be right, but neither did the cooperative prediction that the outcomes should stabilize at some cooperative level of asks. However, we have observed that from period to period the outcomes of both the GWDG and the BG experiments were getting closer to the competitive prediction, although under GWDG it happened much slower than under the BG. Thus, it is quite possible that, allowing enough repetition, the competitive conjectures would give a better prediction of the outcomes of the wage-demand games than the cooperative conjectures.

## **2.6 Conclusion**

Our experimental investigation of the wage-demand games for the formation of teams has revealed several interesting points. First of all, we have discovered that although the agents did not strictly follow Nash equilibrium behavior, competitive tendencies were always present in the behavior of some individuals and therefore the cooperative outcomes of the games were not sustainable. Our results are, therefore, at odds with Bolle's (1991) experimental findings; Bolle reported that cooperation was persistent in his wage-demand games with two symmetric agents. We have thus demonstrated that cooperation is non-robust to such factors as asymmetries of the agents' roles and small increases in the size of the teams.

The main research question of our investigation was to consider whether the type of

the team-selection rule used in the wage-demand mechanism affected the degree of competitiveness of the agents' behavior and, through the latter, the principal's profit from the team formation. We have discovered that, at least in the short run, the type of the mechanism mattered significantly: the share of competitive agents was much lower and the agents' wage-demands were much higher in the GWDG experiments than in the BG experiments. It is reasonable to assume that extra competition in the BG experiments was caused by an additional asymmetry among the agents induced by the mechanism. In the BG, only efficient teams among the profit-maximizing were selected, whereas in the GWDG any profit-maximizing team was selected with equal probability. This reduced the set of collusive outcomes in the BG compared to the GWDG and caused extra competition on the part of the inefficient agents in the BG. As a consequence, the outcomes of the BG converged faster to the Nash equilibrium than the outcomes of the GWDG experiments, which resulted in lower payoffs to the agents and higher profits to the principal. This allows us to conclude that the BG mechanism, which can be used by the principal only if she is informed about the teams' efficiencies – as opposed to profits – may be more profitable to the principal than the GWDG mechanism, which the principal can use under incomplete information. That is, we find once again that the principal's information matters for the amount of profits she can extract from the agents.

Yet, both mechanism, GWDG as well as BG, proved to be rather competitive and, therefore, profitable for the principal. The presence of asymmetric agents (which is often likely to be the case) and even a small increase in the number of potential team-members resulted in considerable competitive tendencies in agent's behavior. As a bottom line, we conclude that in the team-formation mechanisms, as in many other economic situations,

the extent of a principal's information is important, but markets, if they can be used, often work for the principal even if she has incomplete information.

## 2.7 Proofs of the statements

**Proof of proposition 17** We will use results from the propositions 15, 16 and the following lemma and its corollary in the proof of the proposition 17.

**Lemma 4** *In any pure strategy equilibrium of a wage-demand game, the sum of the wages demanded by the agents of an efficient team  $T^*$  is necessarily higher than the sum of the wages demanded by the agents of any inefficient profit-maximizing team  $\tilde{T}$ . That is,*

$$\sum_{i \in T^*} v_i > \sum_{i \in \tilde{T}} v_i . \quad (2.19)$$

*Proof* By proposition 16(1), in equilibrium any efficient team is profit-maximizing. Now let  $\pi(\tilde{T}) = \pi(T^*)$ , but  $F(\tilde{T}) < F(T^*)$ . Then, since

$$\pi(\tilde{T}) = F(\tilde{T}) - \sum_{i \in \tilde{T}} v_i$$

and

$$\pi(T^*) = F(T^*) - \sum_{i \in T^*} v_i ,$$

we get

$$\sum_{i \in \tilde{T}} v_i < \sum_{i \in T^*} v_i .$$

□

**Corollary 9** *Let  $I^+(T) \equiv \{i \in N \mid i \in T, v_i > 0\}$  denote the set of agents in a team  $T$  who demand positive wages. Then in a Nash equilibrium of a wage-demand game,*

$$I^+(\tilde{T}) \subset I^+(T^*) \subseteq I^* ,$$

where  $\tilde{T}$  is an arbitrary inefficient profit-maximizing team and  $T^*$  is an arbitrary efficient team<sup>36</sup>.

*Proof* Follows from the definition of efficient agents, proposition 16(2,3) and the lemma above.  $\square$

*Proof of proposition 17* By proposition 15, there always exists a pure strategy Nash equilibrium of the corresponding Bolle Game; denote by  $v^* = (v_1^*, \dots, v_i^*, \dots, v_n^*)$  the equilibrium wage demands. Let  $I^+$  denote the set of agents who demand positive wages in this equilibrium:  $I^+ \equiv \{i \in N \mid v_i^* > 0\}$ . By proposition 16(2,3),  $I^+ \subseteq I^*$ . Take an  $\epsilon > 0$  and consider the outcome of an arbitrary wage-demand game in which the agents use the following vector of strategies  $v$ : for every  $i \in N$ ,  $v_i = v_i^* - \epsilon_i$ , where  $\epsilon_i = 0$  if  $i \notin I^+$ , and  $0 < \epsilon_i < \min\{v_i^*, \epsilon/k\}$  for  $i \in I^+$ , where  $k$  is the number of agents who demand positive wages:  $k = |I^+|$ . We now show that given such  $v$ , a profit-maximizing team is efficient if and only if it is profit-maximizing. Let  $T^*$  denote an arbitrary efficient team. Compare the profits of every team under the the wage-demands  $v^*$  and  $v$ . By construction and by

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<sup>36</sup>For the reminder,  $I^*$  denotes the set of efficient agents.



proposition 16(1,2,3), for any  $T \subseteq N$ ,

$$\pi(T, v) \leq \pi(T, v^*) + \sum_{i \in I^+} \epsilon_i$$

and

$$\pi(T^*, v) = \pi(T^*, v^*) + \sum_{i \in I^+} \epsilon_i$$

for any efficient  $T^*$ . Therefore, since all efficient teams are profit-maximizing given  $v^*$  (proposition 16(1)), they all are profit-maximizing given  $v$ . By the same reason, for any  $T$  – inefficient and  $T^*$  – efficient,

$$\text{if } \pi(T^*, v) > \pi(T, v) \text{ then } \pi(T^*, v^*) > \pi(T, v^*).$$

Next, consider an inefficient team  $\tilde{T}$  such that  $\pi(\tilde{T}, v^*) = \pi(T^*, v^*)$ , i.e., it is profit-maximizing given  $v^*$ . By corollary 9,

$$\sum_{i \in \tilde{T}} \epsilon_i < \sum_{i \in T^*} \epsilon_i$$

and therefore

$$\pi(T^*, v) > \pi(T, v).$$

Therefore, the set of profit-maximizing teams equals the set of efficient teams given  $v$ . Next we show that the strategy vector  $v$  constitutes an  $\epsilon$ -equilibrium. If an agent is inefficient, he cannot gain from increasing his wage demand, since, by the definition of an inefficient agent, there exists  $T^*$  – efficient such that  $i \notin T^*$ ; therefore, increasing the wage

is equivalent to not getting selected with certainty. On the other hand, any  $i \notin I^*$  can only lose from decreasing his wage demand from  $v_i = 0$ . An efficient agent  $i \in I^*$  may gain from increasing his wage-demand from  $v_i$  by capturing the difference  $(\pi(T^*, v) - \pi(\tilde{T}, v))$ , where  $\tilde{T}$  is the team with the highest profit among inefficient teams such that  $i \notin \tilde{T}$ . However, by construction and by proposition 16(4),

$$\pi(T^*, v) - \pi(\tilde{T}, v) < \epsilon .$$

Hence, the gain is always below  $\epsilon$ . Since  $v$  guarantees every  $i \in I^*$  employment with certainty with the wage  $w_i = v_i$ , the efficient agents can only lose from decreasing their wage-demands from  $v_i$ . Therefore,  $v_i = v_i^* - \epsilon_i$  is an  $\epsilon$ -equilibrium strategy for every  $i \in N$  and  $v$  is an  $\epsilon$ -equilibrium wage-demand vector. Finally, by construction such  $\epsilon$ -equilibrium exists for every  $\epsilon > 0$ .  $\square$

**Proof of propositions in section 2.2.3** *Proof of proposition 19* (1). Such an agent can always change (at least increase) his wage demand and effect the other agents' probability of being selected; therefore, he is offensive. (2). An agent can affect other agents' payoffs only if he can change his own probability of being selected. (3). Suppose  $p_i > 0$ . Then by (1) above,  $i$  is offensive, and therefore  $v_i > 0$ . Now suppose  $p_i = 0$ . If  $v_i > 0$ , then  $i$  can decrease his wage demand and get selected. Therefore,  $i$  is offensive and the collusive outcome requires  $p_i > 0$ , which is a contradiction. It follows that  $v_i = 0$ . Since  $p_i = 0$ , the profit-maximizing team to which  $i$  belongs is selected with zero probability. Since  $i$  can only increase his wage-demand, from (2) above, he is not offensive. (4) Follows from

the proposition 18 that any efficient agent such that  $v_i^N > 0$  is selected with positive probability in the corresponding cooperative outcome, and from (1) above. (5) Observe that, by definition, if a collusive outcome  $v_C$  exists, then  $v_i^C \geq v_i^N$  for all  $i \in N$ , and  $v_i^C > v_i^N$  at least for one  $i$ .  $\square$

*Proof of Proposition 20* (1) Suppose an efficient team is selectable in an outcome of the BG; therefore, first, it is profit-maximizing, and, second, only efficient profit-maximizing teams can be selected. By definition, an efficient agent  $i \in I^*$  belongs to every efficient team, and therefore every efficient agent belongs to every selectable team; i.e., every  $i \in I^*$  is selected with certainty. For the GWDG, since any profit-maximizing team is selectable, the above logic does not apply (consider also example 2 in section 2.2.3). (2) Follows directly from propositions 19(1,2) and (1) above. (3) Follows directly from propositions 19(2).  $\square$

**Proof of proposition 23** The proposition follows straightforwardly from the analysis of inequalities 2.7-2.12. We illustrate here the proof for the case (4), assuming an agent follows an MA rule. Observe that for the purpose of determining the regions we use  $v_i L = v_i M$ , and the rule 2.5 can be rewritten as:

$$v_{it} = \beta_0 \sum_{\tau=0}^{t-1} (\beta_1 + \beta_2)^\tau + (\beta_1 + \beta_2)^t * v_{i0} ,$$

where  $v_{i0}$  is the first period ask. Therefore, if  $|\beta_1 + \beta_2| \geq 1$ , the sequence  $\{v_{it}\}$  diverges; if  $|\beta_1 + \beta_2| < 1$  then the sequence converges and

$$\lim_{t \rightarrow \infty} v_{it} = \frac{\beta_0}{1 - \beta_1 + \beta_2} \equiv v_i^* .$$

Since  $v_{it}$  is non-negative for any  $t$ , so is  $v_i^*$ ; therefore, only two cases are possible for the limit to exist and be non-negative: (1)  $\alpha_0 < 0$  and  $-1 < (\beta_1 + \beta_2) < 0$ ; however, in this case  $v_{it}$  is negative whenever  $v_{it-1}$  is positive, and hence this case is not feasible. (2)  $\beta_0 \geq 0$  and  $0 < (\beta_1 + \beta_2) < 1$ ; this is the only feasible case when the limit exists. If  $\beta_0 = 0$  then  $v_i^* = 0$ , and the rule is decreasing for all  $v_{it} > 0$ . If  $\beta_0 > 0$ , then  $v_i^* > 0$ . Consider this case. Since  $\beta_0 > 0$  and  $0 < (\beta_1 + \beta_2) < 1$ , from inequality 2.9 we derive that

$$v_{it+1} > v_{it} \quad \text{if} \quad v_{it} < v_i^*$$

and

$$v_{it+1} < v_{it} \quad \text{if} \quad v_{it} > v_i^* .$$

□

## Appendix A

# Cooperative Solutions

**Cooperative solutions** From section 2.2.3, in any cooperative solution  $v$  of the wage-demand game (either BG or GWDG) with teams' productivities give in table 2.1,

$$p_1(v)u_1(v_1) \geq 100$$

$$p_2(v)u_2(v_2) \geq 100$$

$$p_3(v)u_3(v_3) \geq 0$$

and each inequality is strict if the solution is collusive<sup>1</sup>.

*Cooperative solutions of the BG.* Two types of cooperative solutions are possible under the BG: either  $\{1, 2\}$  is among the efficient profit-maximizing teams and therefore is selected with certainty (type 1 cooperative solutions) or (only) teams  $\{1, 3\}$  and  $\{2, 3\}$  are profit-

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<sup>1</sup>As a reminder, we suppose that all the agents are risk-neutral and therefore consider beneficial any lottery which gives them higher expected payoff than in the Nash equilibrium. Therefore, agents 1 and 2 should be chosen with positive probability and have an expected payoff of at least 100 (the Nash equilibrium payoff) each in any cooperative solution. If the agents were risk-averse, than the lowest expected payoff which they considered equivalent to their Nash equilibrium payoff should have been higher and therefore the set of cooperative solutions would be shrunk compared to the risk-neutral case.

maximizing and are selected with equal probability 1/2 each (type 2 cooperative solution). Note that type 1 cooperative solutions exclude agent 3 from the set of selectable agents and therefore none of them is collusive.

Type 1 cooperative solutions are defined by  $v \in B_1 \subset R_+^3$  where

$$B_1 \equiv \{(v_1, v_2, v_3) | v_1 \geq 100, v_2 \geq 100, v_3 \geq 0, v_1 + v_2 \leq 1100, v_1 - v_3 \leq 100, v_2 - v_3 \leq 100\}.$$

Type 2 cooperative solutions require that  $v_1 = v_2 \geq 200$  and are defined by  $v \in B_2 \subset R_+^3$  where

$$B_2 \equiv \{(v_1, v_2, v_3) | v_1 = v_2 \geq 200, v_1 + v_3 \leq 1000, v_1 - v_3 > 100\}.$$

Therefore, the set of cooperative solutions of the BG is defined by

$$B = B_1 \cup B_2.$$

The set of collusive solutions  $B^C$  of the BG is the subset of  $B_2$ :

$$B^C \equiv \{(v_1, v_2, v_3) | v_1 = v_2 > 200, v_3 > 0, v_1 + v_3 \leq 1000, v_1 - v_3 > 100\}.$$

*Cooperative solutions of the GWDG* Under the GWDG, there is a bigger variety of possible cooperative solutions, which we can group by the teams that are profit-maximizing, and therefore, selectable under each type. Let us denote  $i$ -th group (set) of cooperative solution by  $G_i$ , and the corresponding set of collusive solutions – by  $G_i^C$ . Then we can define the set of solutions as follows:

1.  $G_1$  – the set of solutions where team  $\{1, 2\}$  only is selectable. This set differs from  $B_1$  only by the boundary:

$$G_1 \equiv \{(v_1, v_2, v_3) | v_1 \geq 100, v_2 \geq 100, v_3 \geq 0, v_1 + v_2 \leq 1100, v_1 - v_3 < 100, v_2 - v_3 < 100\}.$$

As in the BG, the set of corresponding collusive solutions is empty.

2.  $G_2$  – the set of solutions where teams  $\{1, 3\}$  and  $\{2, 3\}$  are selected with probability  $1/2$  each. This set coincides with the set  $B_2$  of the BG:

$$G_2 \equiv \{(v_1, v_2, v_3) | v_1 = v_2 \geq 200, v_3 \geq 0, v_1 + v_3 \leq 1000, v_1 - v_3 > 100\}.$$

Its subset  $G_2^C$  of collusive solutions is

$$G_2^C \equiv \{(v_1, v_2, v_3) | v_1 = v_2 > 200, v_3 > 0, v_1 + v_3 \leq 1000, v_1 - v_3 > 100\}.$$

3.  $G_3$  – the set of solutions where teams  $\{1, 2\}$ ,  $\{1, 3\}$  and  $\{2, 3\}$  are selected with probability  $1/3$  each. It is defined by:

$$G_3 \equiv \{(v_1, v_2, v_3) | v_1 = v_2 \geq 150, v_1 = v_3 + 100, v_1 + v_3 \leq 1000\}.$$

The corresponding subset of collusive solutions  $G_3^C$  excludes from  $G_3$  the single point  $(150, 150, 50)$ :

$$G_3^C \equiv \{(v_1, v_2, v_3) | v_1 = v_2 > 150, v_1 = v_3 + 100, v_1 + v_3 \leq 1000\}.$$

4.  $G_4$  – the set of solutions where teams  $\{1, 2\}$  and  $\{1, 3\}$  are selected with probability  $1/2$  each. It is defined by:

$$G_4 \equiv \{(v_1, v_2, v_3) | v_1 \geq 100, v_2 \geq 200, v_3 \geq 0, v_2 > v_1, v_2 - v_3 = 100, v_1 + v_3 \leq 1000\}.$$

The corresponding subset of collusive solutions  $G_4^C$  excludes from  $G_4$  three boundaries:

$$G_4^C \equiv \{(v_1, v_2, v_3) | v_1 > 100, v_2 > 200, v_3 > 0, v_2 > v_1, v_2 - v_3 = 100, v_1 + v_3 \leq 1000\}.$$

5.  $G_5$  – the set of solutions where teams  $\{1, 2\}$  and  $\{2, 3\}$  are selected with probability  $1/2$  each. It is symmetric to the set  $G_4$  and is defined by:

$$G_5 \equiv \{(v_1, v_2, v_3) | v_1 \geq 200, v_2 \geq 100, v_3 \geq 0, v_1 > v_2, v_1 - v_3 = 100, v_2 + v_3 \leq 1000\}.$$

The corresponding subset of collusive solutions  $G_5^C$  excludes from  $G_5$  three boundaries:

$$G_5^C \equiv \{(v_1, v_2, v_3) | v_1 > 200, v_2 > 100, v_3 > 0, v_1 > v_2, v_1 - v_3 = 100, v_2 + v_3 \leq 1000\}.$$

The entire set of cooperative solutions is then defined by

$$G = \bigcup_{i=1}^5 G_i,$$



and the set of collusive solutions is its subset:

$$G^C = \bigcup_{i=2}^5 G_i^C.$$

## Appendix B

# Experimental Instructions

### B.1 Instructions

This is an experiment in decision-making. The instructions are simple and if you follow them carefully, you may earn a considerable amount of money that will be paid to you IN CASH at the end of the experiment. During the experiment all units of account will be in francs. Upon concluding the experiment the amount of francs you earned will be converted into dollars at a conversion rate of ..... dollars per franc. Your earnings plus a lump sum amount of ..... dollars will be paid to you in private.

Do not communicate with the other participants except according to the specific rules of the experiment. If you have a question, feel free to raise your hand. An experiment monitor will come over to where you are sitting and answer your question in private.

This experiment will last several PERIODS. At the beginning of each period, you will be

assigned to a UNIT with two other participants. Each person within a unit will be given a role denoted by a A, B or C. You will not be told which of the other participants are in your unit. WHAT HAPPENS IN YOUR UNIT HAS NO EFFECT ON THE PARTICIPANTS THAT ARE NOT IN YOUR UNIT AND VICE VERSA.

### **Market organization and payoffs**

In each period of this session you are going to participate in a GROUP SELECTION process. During the period, you will submit to the market your ASK, which is the amount of francs you want to be paid in case you are in the selected group. Only non-negative asks between 0 and 10000 francs are allowed. During a period, you can submit only ONE ASK. You will not be informed about the asks submitted by the other participants in your unit until the end of the period.

The period ends when all participants submit their asks. Once the period is over, a mechanism, which will be specified below, will select a group of participants on the basis of their asks.

In each period, your EARNINGS are equal to your ask if a group containing yourself is selected; you earn zero otherwise. A group containing yourself cannot be selected if you do not submit an ask during the period.

**Example 1** Suppose in period 1 you are in role A, you submit an ask of 15, and in your unit the group {B,C} is selected. Then you earn zero in period 1. Suppose in period 2 you are in role B, your ask is 15, and the group {A,B} in your unit is selected. Then you earn

15 in period 2. Your earnings table for this case is given below.

EARNINGS TABLE (in francs)

Period #	Your role	Your ask	Group selected	Your earnings
1	A	15	{B,C}	0
2	B	15	{A,B}	15
Total				15

At the end of each period you are required to enter your ask, the group selected and your earnings for the period in the enclosed record sheet (table 2).

### The group selection mechanism

Each group of participants in a unit is characterized by a VALUE which can be realized only if the group is selected by the mechanism. Table 1 (enclosed) contains the values for every possible group of participants in a unit. These values are the same for every unit and stay unchanged in every period of the experiment. The table below provides a hypothetical example of group values for a unit.

	Group						
	{A}	{B}	{C}	{A,B}	{A,C}	{B,C}	{A,B,C}
Group value	0	0	0	22	30	13	-5

For example, group {B} has a value of 0, group {A,C} has a value of 30, and group

{A,B,C} has a value of  $-5$ .

The key to the group selection mechanism is the RESIDUAL of a group. Given the asks of the participants in a unit, the residual of a group is as follows:

RESIDUAL of a group = VALUE of the group – ASKS submitted by the group members

Suppose, for example, that participant's A, B and C asks are 5, 7 and 15, respectively. Then, if the value of group {A,B} is 22, its residual equals  $22 - (5 + 7) = 10$ ; if the value of group {A,B,C} is  $-5$ , its residual equals  $-5 - (5 + 7 + 15) = -32$ .

At the end of each period, in each unit THE MECHANISM WILL SELECT A GROUP WITH THE HIGHEST NON-NEGATIVE RESIDUAL. No more than one group in a unit can be selected. If there are several groups with the highest non-negative residual, the mechanism will select among them the group with the highest value. If there are several groups with the highest non-negative residual and the highest value, a fair lottery will determine which of these groups is selected. If all the groups have negative residuals, no one will be selected<sup>1</sup>.

After the group selection is made, your computer terminal will display the asks submitted by the participants in your unit, the residuals of the groups and the group selected. You

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<sup>1</sup>The paragraph above was used in the instructions for the BG experiments. In the GWDG experiments, it was substituted by the following:

At the end of each period, in each unit THE MECHANISM WILL SELECT A GROUP WITH THE HIGHEST NON-NEGATIVE RESIDUAL. No more than one group in a unit can be selected. If there are several groups with the highest non-negative residual, a fair lottery will determine which of these groups is selected. If all the groups have negative residuals, no one will be selected.

will calculate your earnings for the period as described above.

You will be given a residual accounting sheet (table 3) to help you record the participants' asks and the groups' residuals for every period.

This will continue for a fixed number of periods. Your unit and role assignment will change from period to period except for the following. The participants who are assigned the role C in the first period will stay in this role for the whole experiment. The participants who are assigned the roles A or B in the first period will never be assigned the role C in later periods. At the end of the experiment, you will be asked to calculate your total earnings, which is the sum of your earnings over periods.

**Exercise 1** Suppose the group values are as given in the table above (page 2 of the instructions), and the following asks are submitted by the participants in a unit:

Participant role	Ask submitted
A	15
B	2
C	10

Calculate the group residuals corresponding to these asks and enter them in the practice accounting sheet below.

## GROUP RESIDUAL ACCOUNTING SHEET

	Group						
	{A}	{B}	{C}	{A,B}	{A,C}	{B,C}	{A,B,C}
Group value (V)	0	0	0	22	30	13	-5
Asks (a)							
Residual (R)=(V)-(a)							

Which group will be selected by the group selection mechanism?

Selected group: .....

Enter the corresponding earnings of the participants into the table below:

Participant role #	Earning
A	
B	
C	

**Exercise 2** Given the information below, determine the residuals, the group selected and the participants' earnings, and record them in the corresponding tables below.

Participant role	Ask submitted
A	10
B	30
C	30

GROUP RESIDUAL ACCOUNTING SHEET

	Group						
	{A}	{B}	{C}	{A,B}	{A,C}	{B,C}	{A,B,C}
Group value (V)	-2	-2	-2	40	40	50	0
Asks (a)							
Residual (R)=(V)-(a)							

Selected group: .....

Participant role #	Earning
A	
B	
C	

ARE THERE ANY QUESTIONS?



## B.2 Tables used by experimental subjects

Table 1. GROUP VALUES

	Group						
	{A}	{B}	{C}	{A,B}	{A,C}	{B,C}	{A,B,C}
Group value	-1	-1	-1	1100	1000	1000	0

Table 2. EARNINGS TABLE (in francs)

Period #	Your role	Your ask	Group selected	Your earnings
Total				

Table 3. RESIDUAL ACCOUNTING SHEET

Period #		Group						Group Selected
		{A}	{B}	{C}	{A,B}	{A,C}	{B,C}	
	Group value (V)	-1	-1	-1	1100	1000	1000	
	Asks (a)							
	Residual (R)=(V)-(a)							
	Asks (a)							
	Residual (R)=(V)-(a)							
	Asks (a)							
	Residual (R)=(V)-(a)							

## **Appendix C**

# **Experimental Data, Figures**

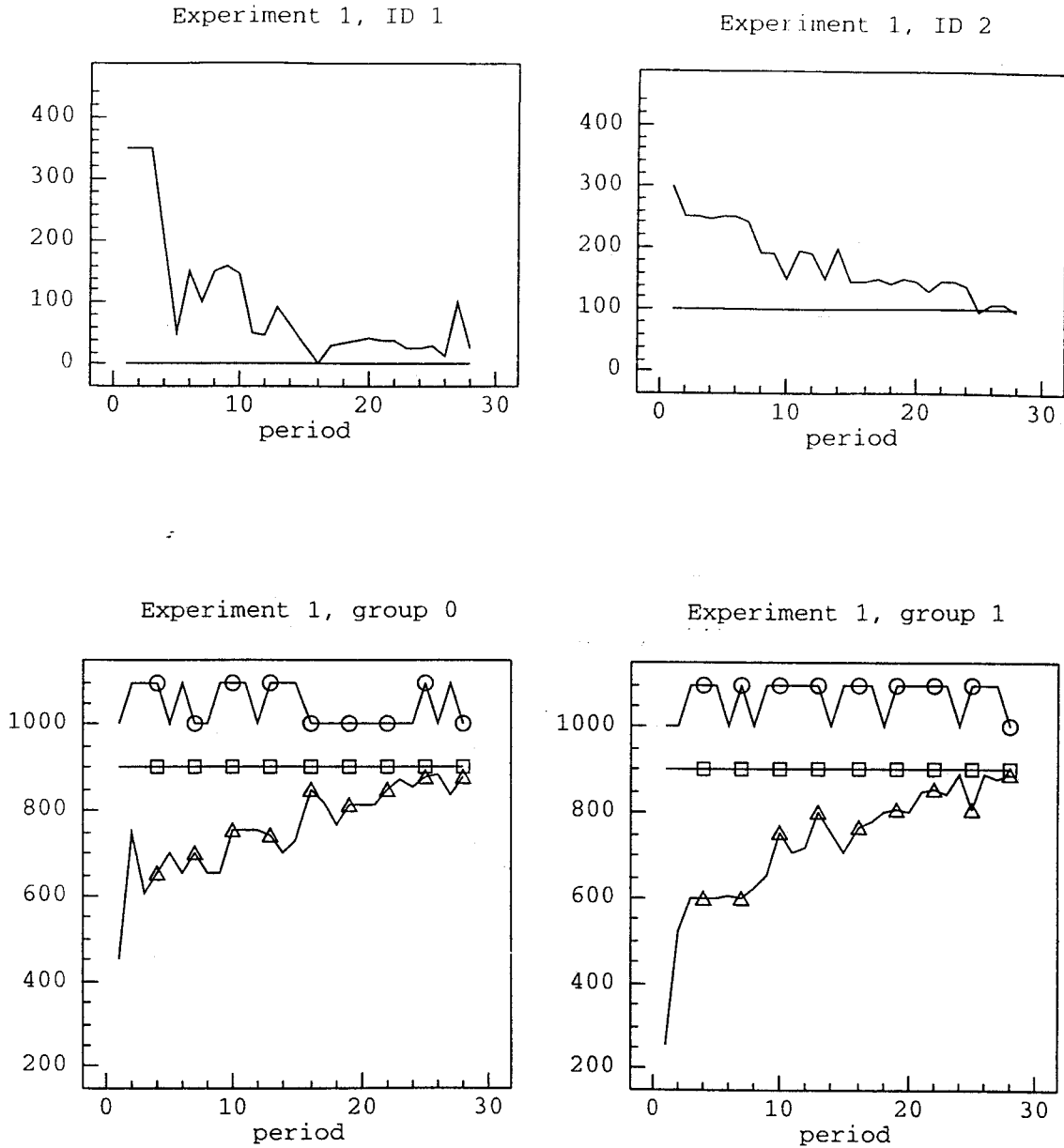


Figure C.1: Representative dynamics for Experiment 1 (BG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\triangle$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).

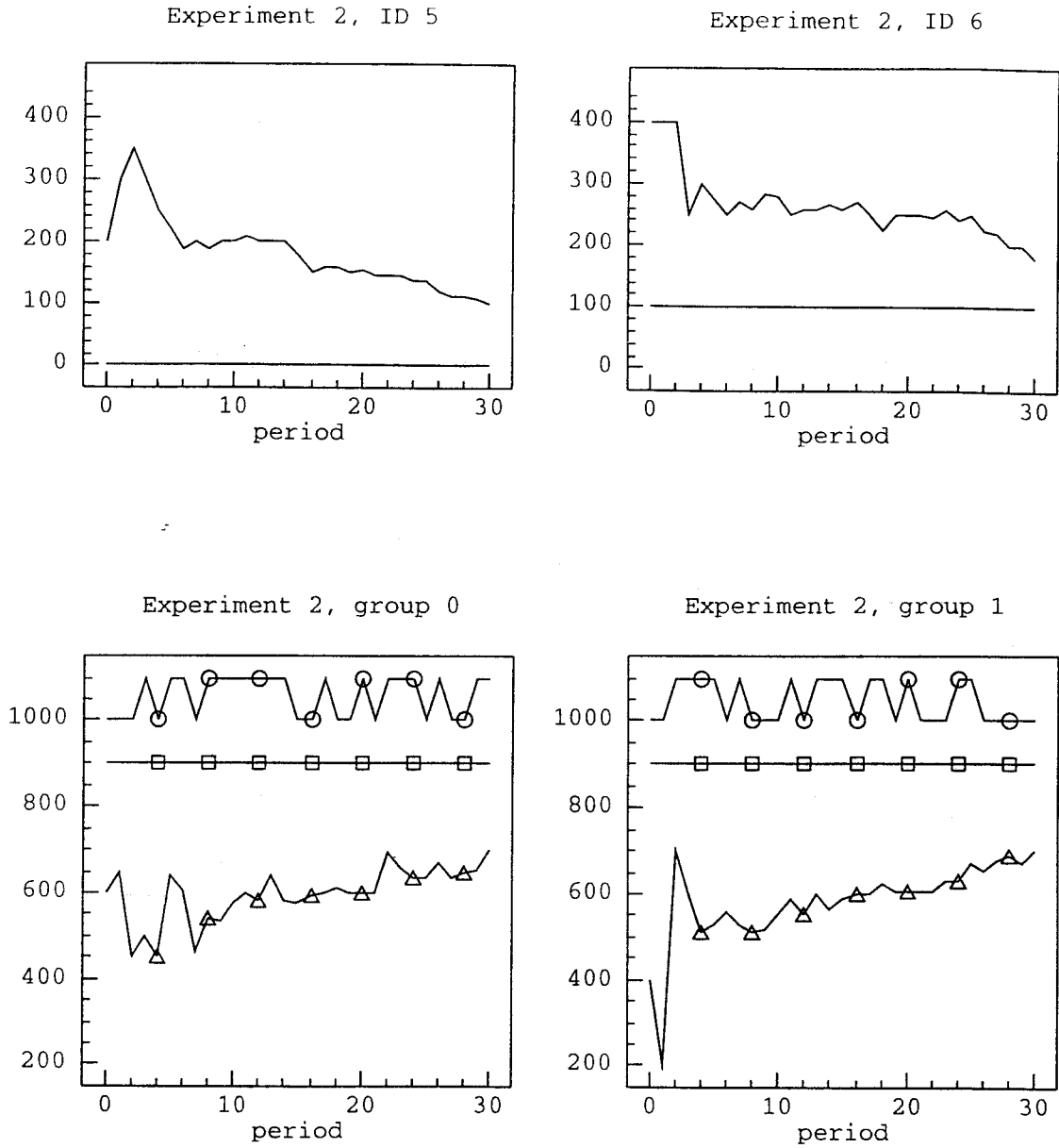


Figure C.2: Representative dynamics for Experiment 2 (GWDG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\Delta$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).

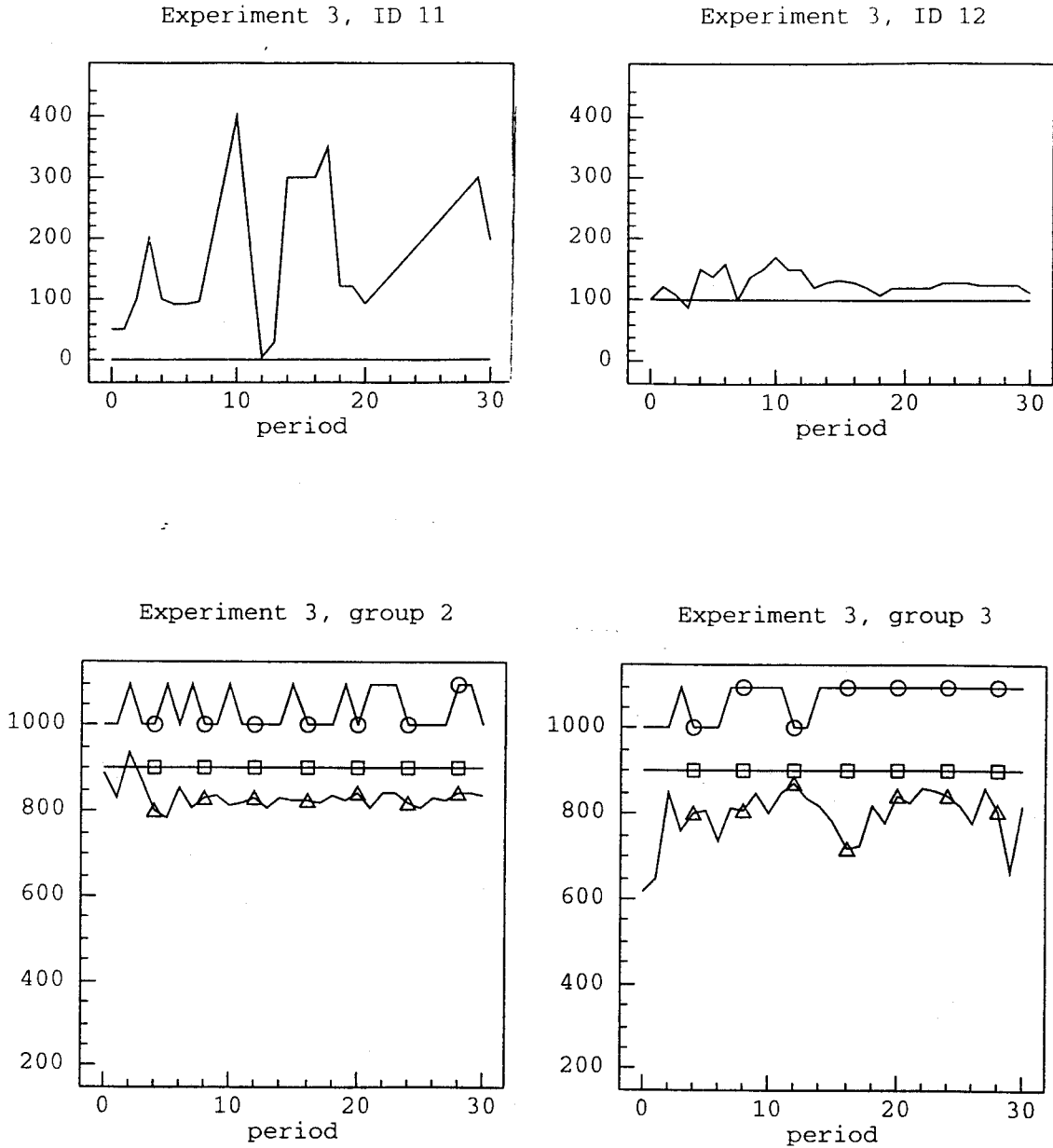


Figure C.3: Representative dynamics for Experiment 3 (BG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\Delta$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).

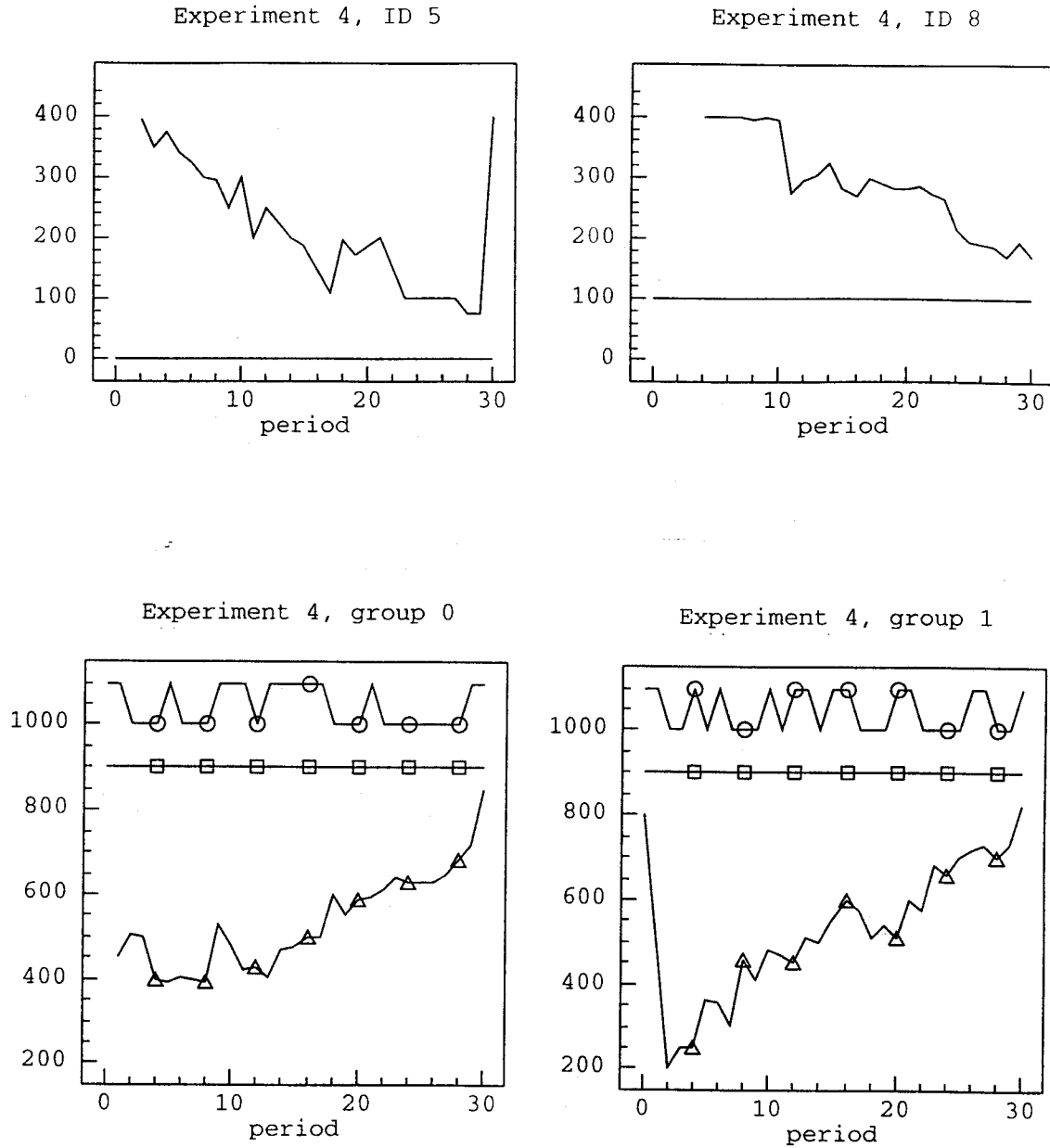


Figure C.4: Representative dynamics for Experiment 4 (GWDG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\Delta$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).

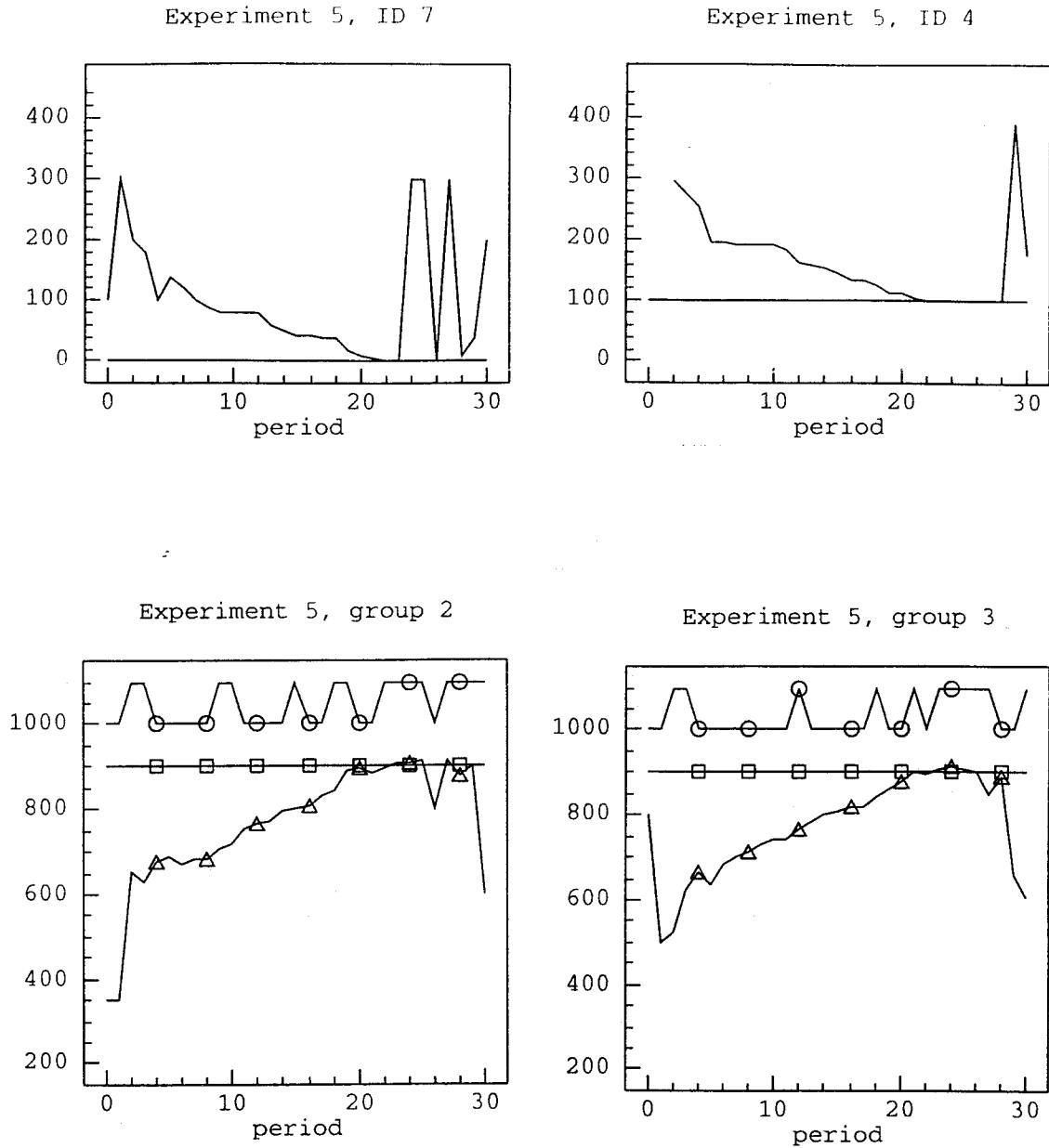


Figure C.5: Representative dynamics for Experiment 5 (BG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\Delta$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).

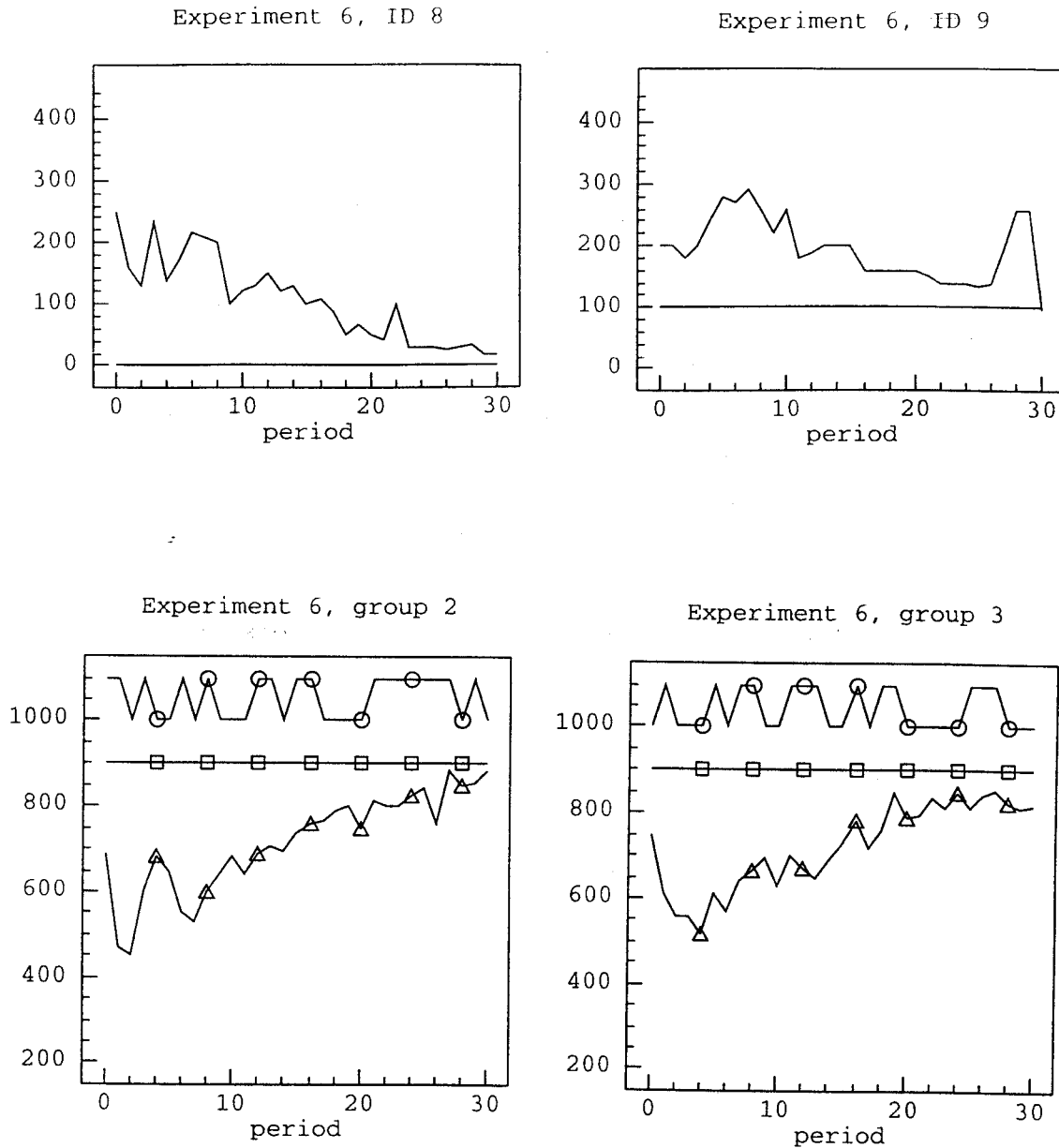


Figure C.6: Representative dynamics for Experiment 6 (GWDG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\Delta$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).



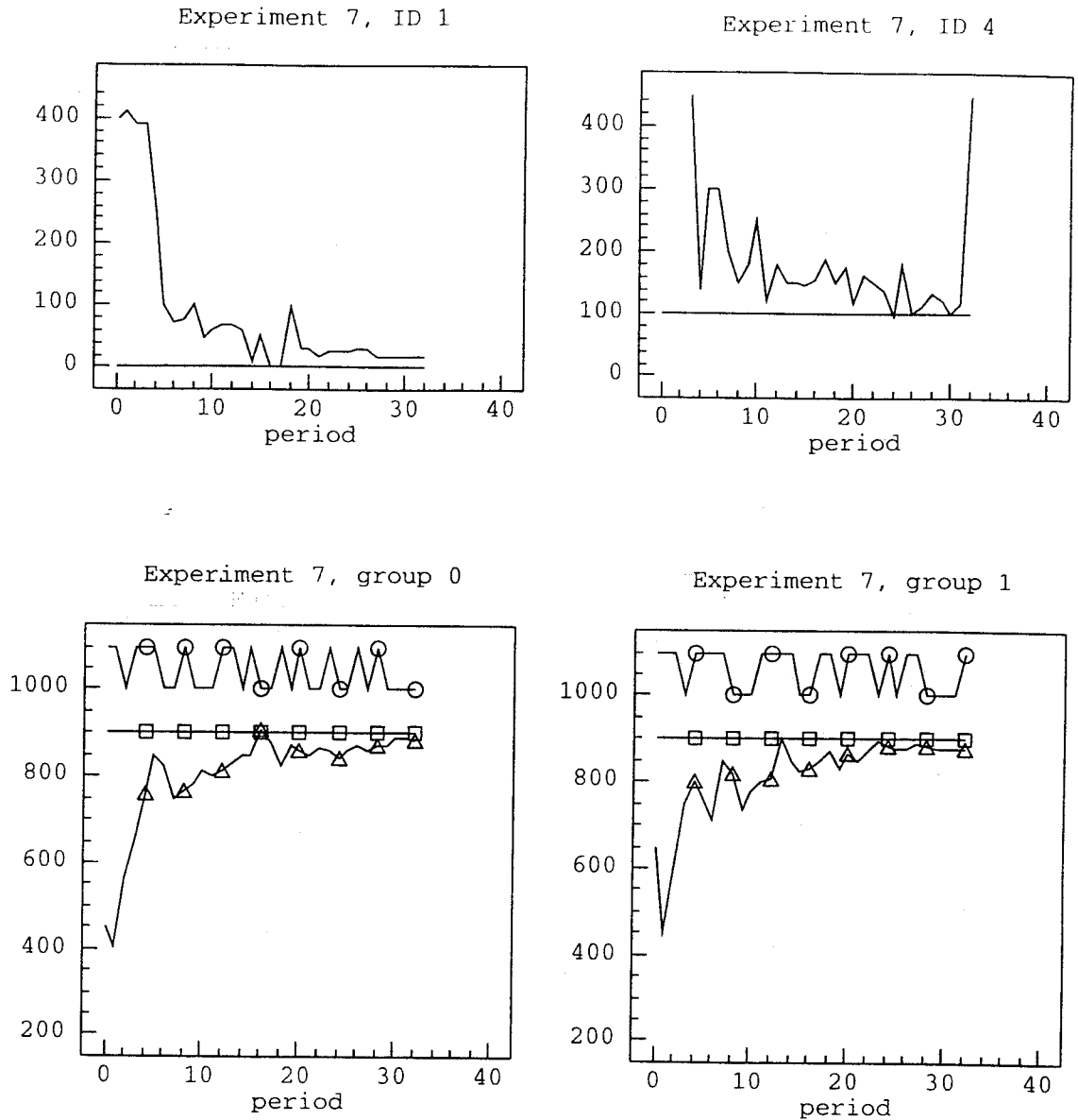


Figure C.7: Representative dynamics for Experiment 7 (BG). The graphs above present individual asks of representative subjects, in francs, for each period: an inefficient subject (left) and an efficient subject (right). The horizontal lines indicate the Nash equilibrium level of asks (0 and 100 francs, respectively). The graphs below display output-sharing in two of the four groups, in francs. (Each group was identified with a given inefficient agent, with efficient agents rotating from period to period.)  $\odot$  - output of the selected team;  $\triangle$  - profit of a selected team;  $\square$  - the Nash equilibrium level of profit (900 francs).

## Appendix D

# Experimental Data, Tables

	period 1	time 1	time 2	time 3	Total
All data	27	243	270	272	785
BG pooled	15	135	150	152	437
Experiment 1	4	36	40	36	112
Experiment 3	4	36	40	40	116
Experiment 5	4	36	40	40	116
Experiment 7	3	27	30	36	93
GWDG pooled	12	108	120	120	348
Experiment 2	4	36	40	40	116
Experiment 4	4	36	40	40	116
Experiment 6	4	36	40	40	116

Table D.1: Number of observations, group data (efficiency and profit)

%	time 1	time 2	time 3	Total
All data	43.6	49.6	48.5	47.4
BG pooled	43.0	47.3	57.9	49.7
Experiment 1	44.4	45.0	44.4	44.6
Experiment 3	41.7	47.5	55.0	48.3
Experiment 5	44.4	37.5	82.5	55.2
Experiment 7	40.7	63.3	47.2	50.5
GWDG pooled	44.4	52.5	36.7	44.5
Experiment 2	50.0	47.5	40.0	45.7
Experiment 4	38.9	57.5	30.0	42.2
Experiment 6	44.4	52.5	40.0	45.7

Table D.2: Frequencies of efficient teams selected, %

Mean St. deviation	Period 1	time 1	time 2	time 3	Total
All data	567.6 180.1	626.2 150.6	723.9 122.2	793.6 91.8	717.6 141.5
BG pooled	630.2 175.9	714.9 103.6	809.0 49.2	848.4 60.2	793.6 91.8
Experiment 1	499.2 190.0	626.0 100.3	763.7 47.3	851.6 31.0	747.7 112.5
Experiment 3	783.4 138.1	802.5 56.2	826.0 34.2	829.3 32.7	819.8 43.2
Experiment 5	574.8 107.3	675.1 59.0	813.0 44.2	842.4 104.3	780.3 102.9
Experiment 7	674.7 161.5	769.6 77.0	841.5 29.8	873.2 16.2	832.9 62.4
GWDG pooled	489.3 159.1	515.4 124.6	617.6 101.1	724.5 90.9	622.7 135.3
Experiment 2	570.5 139.4	557.5 61.2	598.2 21.3	660.3 28.5	607.0 57.9
Experiment 4	334.5 124.8	376.5 91.0	520.0 54.7	681.8 76.7	531.3 144.8
Experiment 6	563.0 98.9	612.8 65.3	734.5 59.7	831.3 26.5	729.9 103.3

Table D.3: Average profits of selected teams, francs

Mean St. deviation	time 1	time 2	time 3	Total
All data	17.91 87.35	10.09 38.61	3.86 56.78	10.32 63.21
BG pooled	19.09 80.84	9.59 35.73	-3.06 67.81	8.02 63.99
Experiment 1	24.13 101.00	12.13 48.57	8.03 31.06	14.31 64.61
Experiment 3	7.45 78.68	3.15 35.60	-1.38 43.53	2.92 54.49
Experiment 5	24.75 61.42	15.18 9.72	-17.55 119.86	6.86 79.90
Experiment 7	21.11 82.66	7.33 37.98	0.06 19.29	8.52 51.01
GWDG pooled	16.52 95.02	10.73 42.08	12.64 37.03	13.18 62.18
Experiment 2	11.08 114.63	5.27 24.51	8.57 29.18	8.21 67.06
Experiment 4	25.52 94.67	11.20 49.31	25.70 40.38	20.64 64.44
Experiment 6	12.94 73.27	15.70 48.10	3.65 37.77	10.69 54.13

Table D.4: Average per period profit change in selected teams, francs

	period 1	time 1	time 2	time 3	Total
All data	81	729	810	804	2343
-efficient agents	54	486	540	536	1562
-inefficient agents	27	243	270	268	781
BG pooled	45	405	450	444	1299
-efficient agents	30	270	300	296	866
-inefficient agents	15	135	150	148	433
Experiment 1	12	108	120	96	324
-efficient agents	8	72	80	64	216
-inefficient agents	4	36	40	32	108
Experiment 3	12	108	120	120	348
-efficient agents	8	72	80	80	232
-inefficient agents	4	36	40	40	116
Experiment 5	12	108	120	120	348
-efficient agents	8	72	80	80	232
-inefficient agents	4	36	40	40	116
Experiment 7	9	81	90	108	279
-efficient agents	6	54	60	72	186
-inefficient agents	3	27	30	36	93
GWDG pooled	36	324	360	360	1044
-efficient agents	24	216	240	240	696
-inefficient agents	12	108	120	120	348
Experiment 2	12	108	120	120	348
-efficient agents	8	72	80	80	232
-inefficient agents	4	36	40	40	116
Experiment 4	12	108	120	120	348
-efficient agents	8	72	80	80	232
-inefficient agents	4	36	40	40	116
Experiment 6	12	108	120	120	348
-efficient agents	8	72	80	80	232
-inefficient agents	4	36	40	40	116

Table D.5: Number of observations, individual data (asks)

%	time 1	time 2	time 3	Total
All data	80.5	81.2	78.2	80.0
-efficient agents	83.5	87.2	85.1	85.3
-inefficient agents	74.5	69.3	64.6	69.3
BG pooled	81.5	84.2	75.7	80.4
-efficient agents	84.1	88.0	84.1	85.5
-inefficient agents	76.3	76.7	58.8	70.4
Experiment 1	81.5	80.8	84.4	82.1
-efficient agents	83.3	86.3	85.9	85.2
-inefficient agents	77.8	70.0	81.2	75.9
Experiment 3	78.7	80.8	72.5	77.3
-efficient agents	81.9	85.0	82.5	83.2
-inefficient agents	72.2	72.5	52.5	65.5
Experiment 5	84.3	94.2	74.2	84.2
-efficient agents	86.1	93.8	91.3	90.5
-inefficient agents	80.6	95.0	40.0	71.6
Experiment 7	81.5	80.0	73.1	77.8
-efficient agents	85.2	86.7	76.4	82.3
-inefficient agents	74.1	66.7	66.7	68.8
GWDG pooled	79.3	77.5	81.4	79.4
-efficient agents	82.9	86.3	86.3	85.2
-inefficient agents	72.2	60.0	71.7	67.8
Experiment 2	73.1	73.3	75.8	74.1
-efficient agents	79.2	78.8	82.5	80.2
-inefficient agents	61.1	62.5	62.5	62.1
Experiment 4	81.5	77.5	87.5	82.2
-efficient agents	79.2	90.0	90.0	86.6
-inefficient agents	86.1	52.5	82.5	73.3
Experiment 6	83.3	81.7	80.8	81.9
-efficient agents	90.3	90.0	86.3	88.8
-inefficient agents	69.4	65.0	70.0	68.1

Table D.6: Percentage of asks below the level of last period best response, %

Mean St. deviation	Period 1	time 1	time 2	time 3	Total
All data	438.5	285.3	194.2	163.4	212.4
	790.2	397.7	65.1	64.1	234.9
BG pooled	256.4	208.4	151.8	136.2	164.6
	121.6	76.6	33.6	61.6	66.9
Experiment 1	317.8	250.9	172.9	132.0	189.1
	127.0	80.5	33.6	52.1	76.6
Experiment 3	184.6	163.8	147.9	145.9	152.1
	108.6	59.3	36.8	40.1	46.4
Experiment 5	272.1	221.0	146.8	142.5	168.3
	52.4	40.4	23.6	91.5	69.4
Experiment 7	249.3	188.2	135.5	122.3	145.7
	167.8	89.8	27.0	43.8	63.7
GWDG pooled	666.2	384.4	247.3	196.9	272.5
	115.0	581.0	55.2	49.7	335.3
Experiment 2	296.0	285.4	256.4	224.5	254.4
	133.9	65.5	19.4	19.1	46.7
Experiment 4	942.5	553.6	298.4	221.2	351.0
	1463.1	868.7	32.0	44.0	502.6
Experiment 6	760.0	314.1	186.9	145.2	212.0
	1390.9	468.2	37.3	32.9	270.6

Table D.7: Average per period asks of efficient agents, francs



Mean St. deviation	Period 1	time 1	time 2	time 3	Total
All data	230.9 119.5	182.9 99.8	170.9 678.3	122.4 160.7	158.1 412.6
BG pooled	202.7 118.1	143.4 99.0	83.6 124.7	121.0 201.5	115.3 150.7
Experiment 1	277.5 114.1	187.2 82.9	121.7 143.7	56.5 80.8	126.5 119.3
Experiment 3	94.8 45.1	117.9 134.3	103.1 183.7	145.5 186.0	122.3 170.2
Experiment 5	218.0 75.1	137.0 55.1	46.0 21.2	200.5 272.4	127.5 174.5
Experiment 7	226.3 175.7	121.3 94.7	56.7 30.5	62.6 168.1	77.8 120.0
GWDG pooled	266.3 116.3	233.7 74.8	280.0 999.4	124.3 88.0	211.9 592.7
Experiment 2	262.5 62.9	217.0 49.7	163.7 21.3	142.4 44.7	172.9 50.4
Experiment 4	319.3 182.0	302.4 64.2	573.1 1706.5	129.3 60.7	336.0 1012.4
Experiment 6	217.0 74.2	181.8 51.3	103.2 31.5	101.1 130.6	126.9 91.1

Table D.8: Average per period asks of inefficient agents, francs

	time 1	time 2	time 3	Total
All data	-37.57	6.38	-11.4	-13.39
-efficient agents	-18.40	-4.92	-0.53	-7.06
-inefficient agents	-75.91	28.99	-33.14	-24.97
BG pooled	-37.22	-5.38	5.35	-11.64
-efficient agents	-12.89	-4.28	3.79	-4.20
-inefficient agents	-85.88	-7.57	8.46	-26.51
Experiment 1	-14.37	-5.62	-0.84	-7.12
-efficient agents	-12.96	-4.70	0.48	-5.91
-inefficient agents	-17.22	-7.45	-3.50	-9.53
Experiment 3	-97.71	-2.97	2.26	-30.57
-efficient agents	-10.66	0.52	0.87	-2.82
-inefficient agents	-271.81	-9.95	5.06	-86.04
Experiment 5	-17.81	-7.79	15.63	-2.82
-efficient agents	-18.44	-7.63	8.58	-5.40
-inefficient agents	-16.53	-8.10	29.73	2.33
Experiment 7	-12.91	-5.04	2.85	-4.26
-efficient agents	-8.37	-5.63	4.66	-2.44
-inefficient agents	-22.00	-3.85	-0.76	-7.92
GWDG pooled	-38.00	21.01	-32.05	-15.57
-efficient agents	-25.29	-5.72	-5.85	-11.84
-inefficient agents	-63.43	74.70	-84.47	-23.05
Experiment 2	-10.82	-3.65	-3.67	-5.88
-efficient agents	-14.15	-3.54	-4.14	-7.04
-inefficient agents	-4.17	-3.88	-2.75	-3.58
Experiment 4	-86.56	74.56	-90.49	-32.36
-efficient agents	-53.87	-5.56	-11.90	-22.74
-inefficient agents	-151.94	234.83	-247.67	-51.58
Experiment 6	-16.62	-7.65	-2.01	-8.49
-efficient agents	-7.83	-8.05	-1.53	-5.73
-inefficient agents	-34.19	-6.85	-2.98	-14.00

Table D.9: Average per period ask change, francs

	Profit < 800		Profit ≥ 800		Total	
	Ask change	# of obs.	Ask change	# of obs.	Ask change	# of obs.
All data	-22.12	1428	0.23	915	-13.99	2343
-efficient agents	-11.93	952	-0.85	610	-7.06	1562
-inefficient agents	-42.49	476	2.38	305	-24.97	781
BG pooled	-30.90	498	0.34	801	-11.64	1299
-efficient agents	-9.07	332	-1.18	534	-4.20	866
-inefficient agents	-74.57	166	3.37	267	-26.51	433
Experiment 1	-10.99	186	-1.90	138	-7.12	324
-efficient agents	-10.24	124	-0.08	92	-5.91	216
-inefficient agents	-12.50	62	-5.54	46	-9.53	108
Experiment 3	-155.15	66	-1.41	282	-30.57	348
-efficient agents	4.05	44	-4.43	188	-2.82	232
-inefficient agents	-473.55	22	4.65	94	-86.04	116
Experiment 5	-13.42	186	9.33	162	-2.82	348
-efficient agents	-15.57	124	6.27	108	-5.40	232
-inefficient agents	-9.11	62	15.46	54	2.33	116
Experiment 7	-10.13	60	-2.66	219	-4.26	279
-efficient agents	0.30	40	-3.19	146	-2.44	186
-inefficient agents	-30.99	20	-1.61	73	-7.92	93
GWDG pooled	-17.420	930	-0.54	114	-15.57	1044
-efficient agents	-13.467	620	1.46	76	-11.87	696
-inefficient agents	-25.326	310	-4.52	38	-23.05	348
Experiment 2	-5.89	348	—	0	-5.88	348
-efficient agents	-7.03	232	—	0	-7.04	232
-inefficient agents	-3.58	116	—	0	-3.58	116
Experiment 4	-32.39	345	-28.27	3	-32.36	348
-efficient agents	-22.59	230	-40.00	2	-22.74	232
-inefficient agents	-51.99	115	-4.80	1	-51.58	116
Experiment 6	-12.56	237	0.21	111	-8.49	348
-efficient agents	-9.62	158	2.57	74	-5.73	232
-inefficient agents	-18.44	79	-4.51	37	-14.00	116

Table D.10: Average per period ask change far and close to Nash equilibrium, francs

## Appendix E

# Statistical Estimations, Tables

**How to read tables E.1-E.14** Tables E.1-E.14 present the results of statistical estimations of individual behavior; the data is grouped by experiments. The entries regarding subjects in the efficient role (1(A) and 2(B)) are grouped at the top of the tables, and subjects in the inefficient roles (3, or C), at the bottom. \*\* next to subject ID (**ID**) marks the inefficient role. **Rule:** SR – selection response rule, MA – median ask rule, BR – Cournot best response rule, ??? - unclassified. **Estimated coefficients:** coefficients that are significant at 5% significance level according to on the *t*-test are marked with \*. Independent variables: **one** – constant; **asklag** – subject’s previous period ask; **Select** – a dummy for subject’s selection in the previous period: = 1 if ID was selected, = 0 otherwise; **Med.ask** – previous period median ask observed by the subject; **BR ask** – subject’s previous period best response ask. **R<sup>2</sup>** – *R*-squared corrected for the degrees of freedom. **h-stat.** – a statistic for presence of autocorrelation in the regressions with lagged dependent variable; the values that are different from zero at 5% significance level are marked with \*; significance indicates the presence of autocorrelation. **Askmin**, **Askmax** – minimal and

maximal asks submitted by the subject, in francs. **F-stat. marg.** –  $F$ -statistic for testing the set of restrictions corresponding to marginal behavior. The values that are significant at 5% level are marked with \*. Significance indicates that the hypothesis of marginal behavior should be rejected. **Asymptote** – estimated value of the stationary point; most of the points were type 1 (asymptotes); type 2 stationary points are marked with #. **Stat. for diff. from Zero/Nash ask** –  $z$ -statistic for testing the hypotheses (Asymptote= 0) and (Asymptote=Nash equilibrium ask), respectively. The values that are significant at 5% level are marked with \*. Significance indicates that the corresponding hypotheses should be rejected. **Classification** – competitive, cooperative, marginal, or switch type.

ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
2	BR	24.06	0.64*	—	—	0.13*	0.797	-0.55
3	MA	36.72	0.22	—	0.48*	—	0.682	-1.52
4	MA	148.83*	-0.72*	—	0.94*	—	0.724	0.77
5	SR	-57.08*	0.91*	71.70*	—	—	0.950	0.23
7	SR	29.50	0.68*	59.55	—	—	0.229	—
8	MA	-1.94	0.33*	—	0.70*	—	0.952	-0.52
9	SR	-7.48	0.84*	41.39*	—	—	0.558	-0.83
12	SR	-2.16	0.84*	28.78	—	—	0.635	-1.11
1**	MA	28.24	0.33	—	0.59	—	0.187	—
6**	SR	142.42*	-0.12	168.09*	—	—	0.406	-0.29
10**	BR	4.00	0.52*	—	—	0.28*	0.879	-0.64
11**	SR	-10.81	0.87*	24.86*	—	—	0.911	-1.02

Table E.1: Experiment 1: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
2	BR	99	299	6.64*	110.09	1.60	0.15	comp.
3	MA	100	250	7.83*	122.88	2.01*	0.37	comp.
4	MA	140	500	65.53*	191.41	9.72*	4.64*	switch
5	SR	99	444	9.64*	49.57	0.55	-0.56	comp.
7	SR	99	500	1.09	219.01	1.49	0.80	marg.
8	MA	101	499	0.74	55.14	0.15	-0.12	marg.
9	SR	115	300	0.62	162.78	0.97	0.38	marg.
12	SR	100	207	1.02	136.89	1.09	0.29	marg.
1**	MA	2	750	0.25	390.21	0.61	0.61	marg.
6**	SR	8	400	26.59*	169.58	5.97*	5.97*	switch
10**	BR	7	170	6.54*	20.37	0.62	0.62	comp.
11**	SR	40	350	6.48*	26.38	0.44	0.44	comp.

Table E.2: Experiment 1: classification of individual behavior

ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
1	SR	45.46	0.72*	25.75	—	—	0.479	-1.39
2	?	—	—	—	—	—	—	—
4	SR	36.84	0.77*	23.36*	—	—	0.700	-1.57
6	MA	83.51*	-0.19	—	0.84*	—	0.812	0.14
8	SR	9.70	0.87*	29.68*	—	—	0.909	-0.02
9	BR	49.52	0.27*	—	—	0.50*	0.564	1.38
10	MA	34.70	0.29*	—	0.53*	—	0.875	-1.44
11	SR	-3.54	0.97*	14.17*	—	—	0.942	-0.53
3**	BR	3.20	0.66*	—	—	0.31*	0.924	-2.87*
5**	MA	-10.43	0.61*	—	0.43*	—	0.965	-0.63
7**	BR	72.96*	0.33	—	—	0.17*	0.185	3.79*
12**	MA	49.23	-0.19	—	0.95*	—	0.251	—

Table E.3: Experiment 2: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
1	SR	120	290	2.01	238.05	2.04*	1.18	marg.
2	???	98	500	—	—	—	—	???
4	SR	199	300	2.88	241.84	2.06*	1.21	marg.
6	MA	180	500	24.31*	235.03	5.29*	3.04*	switch
8	SR	200	500	6.58*	199.06	1.70	0.85	comp.
9	BR	200	500	4.06*	218.87	1.39	0.75	comp.
10	MA	200	500	16.18*	198.87	2.06*	1.03	comp.
11	SR	219	325	1.72	184.52	0.54	0.25	marg.
3**	BR	135	300	0.35	122.27	0.38	0.38	marg.
5**	MA	100	350	1.26	215.87#	1.34	1.34	marg.
7**	BR	100	250	4.01*	148.34	2.28*	2.28*	switch
12**	MA	97	300	1.09	201.61	1.26	1.26	marg.

Table E.4: Experiment 2: classification of individual behavior

ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
1	MA	113.31*	-0.21	—	0.43*	—	0.256	-1.74
2	???	—	—	—	—	—	—	—
6	MA	8.78	0.24*	—	0.66*	—	0.963	-0.05
7	SR	8.37	0.86*	22.78*	—	—	0.873	0.84
8	SR	-13.22	0.90*	32.97*	—	—	0.733	-2.06*
9	BR	76.78*	0.28	—	—	0.31*	0.509	1.77
10	???	—	—	—	—	—	—	—
12	MA	54.97	0.35	—	0.19*	—	0.145	-0.42
3**	MA	11.50*	0.27*	—	0.40*	—	0.817	0.33
4**	MA	36.61*	-1.07	—	0.31*	—	0.196	1.07
5**	???	—	—	—	—	—	—	—
11**	BR	490.39*	0.05	—	—	-2.01*	0.212	—

Table E.5: Experiment 3: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
1	MA	90	202	7.70*	145.88	3.46*	1.09	comp.
2	???	50	200	—	—	—	—	???
6	MA	130	330	11.67*	94.37	1.54	-0.09	comp.
7	SR	130	300	4.39*	139.28	1.71	0.48	comp.
8	SR	117	250	0.69	122.94	0.78	0.15	marg.
9	BR	100	349	4.94*	184.56	2.36*	1.08	comp.
10	???	50	500	—	—	—	—	???
12	MA	90	170	3.60*	121.26	1.90	0.33	comp.
3**	MA	2	250	13.04*	35.95	3.03*	3.03*	switch
4**	MA	29	9999	14.61*	53.62	4.59*	4.59*	switch
5**	???	25	200	—	—	—	—	???
11**	BR	500	1101	8.87*	165.19	4.14*	4.14*	switch

Table E.6: Experiment 3: classification of individual behavior



ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
1	MA	-19.39	0.56*	—	0.45*	—	0.941	1.14
2	MA	104.96*	-0.11	—	0.69*	—	0.534	4.42*
3	MA	8.36	0.39*	—	0.55*	—	0.861	-2.24*
4	BR	89.82*	0.34*	—	—	0.31*	0.587	0.52
6	MA	-1240	0.56*	—	4.67*	—	0.728	2.25*
7	MA	-14.45	-0.05	—	1.04*	—	0.925	—
8	SR	-41.74	0.98*	50.71*	—	—	0.925	-1.38
9	MA	-29.61	0.90*	—	0.15*	—	0.897	-1.09
5**	???	—	—	—	—	—	—	—
10**	MA	28.13	-0.36	—	1.21*	—	0.842	—
11**	SR	-13.69*	0.99*	5.66	—	—	0.990	1.28
12**	MA	47.65	-0.06*	—	0.83*	—	0.637	1.21

Table E.7: Experiment 4: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
1	MA	99	498	6.34*	1489.03 <sup>#</sup>	1.00	0.93	comp.
2	MA	99	750	16.84*	246.51	3.18*	0.60	comp.
3	MA	178	450	1.00	176.72	0.37	0.16	marg.
4	BR	200	450	6.24*	257.83	2.32*	1.42	comp.
6	MA	199	5001	2.06	292.60 <sup>#</sup>	1.96*	1.29	marg.
7	MA	150	500	5.49*	0	-1.48	-0.06	comp.
8	SR	173	549	2.52	0	-0.31	-0.13	marg.
9	MA	100	350	5.37*	488.63 <sup>#</sup>	1.32	1.05	comp.
5**	???	75	10000	—	—	—	—	???
10**	MA	50	500	2.82	184.83	1.67	1.67	marg.
11**	SR	45	400	22.14*	0	-3.92*	-3.92*	comp.
12**	MA	50	5000	1.63	213.43	1.87	1.87	marg.

Table E.8: Experiment 4: classification of individual behavior

ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
1	BR	63.15*	0.23*	—	—	0.34	.694	-1.88
2	MA	-7.36	-0.18*	—	1.21*	—	.959	2.28*
4	MA	77.44*	-0.01	—	0.51	—	.292	-0.97
8	MA	45.51	0.03	—	0.71	—	.141	-0.59
9	MA	28.32	0.37*	—	0.40*	—	.805	1.39
10	MA	8.87	0.69*	—	0.21*	—	.878	-1.47
11	MA	12.33*	0.18	—	0.68*	—	.961	2.76*
12	BR	42.63	0.51*	—	—	0.19*	.492	-0.74
3**	BR	14.07	0.47*	—	—	0.33*	0.656	-2.62*
5**	SR	151.35	0.52*	-108.32	—	—	.312	2.24*
6**	MA	23.05	0.98*	—	-0.17	—	.784	0.54
7**	???	—	—	—	—	—	—	—

Table E.9: Experiment 5: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
1	BR	99	500	16.3*	149.6	3.62*	1.20	comp.
2	MA	85	400	1.22	426#	0.74	0.572	marg.
4	MA	100	530	5.26*	155.3	2.67*	0.95	comp.
8	MA	100	600	0.36	177.2	0.84	0.37	marg.
9	MA	91	250	6.08*	123.2	2.03*	0.38	comp.
10	MA	85	200	2.71	92.7	0.903	-0.07	marg.
11	MA	99.9	350	18.87*	92.1	2.23*	-0.19	comp.
12	BR	100	400	4.10*	141.9	1.53	0.42	comp.
3**	3	12	300	1.47	76.7	1.07	1.07	marg.
5**	1	5	1000	3.96*	202.4	1.75	1.75	comp.
6**	2	0	350	0.87	118.7	1.32	1.32	marg.
7**	???	MA	300	—	—	—	—	???

Table E.10: Experiment 5: classification of individual behavior

ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
1	BR	-651.95*	-0.06	—	—	3.66*	.523	2.09*
3	MA	20.41	-0.30	—	1.16*	—	.882	—
4	SR	-83.50*	0.89*	104.27*	—	—	.819	-0.60
5	MA	-1.12	0.24*	—	0.70*	—	.959	-0.12
6	SR	17.95	0.68*	84.16*	—	—	.418	-1.51
9	MA	27.98	0.42*	—	0.41*	—	.512	2.77*
11	MA	0.94	0.01	—	0.97*	—	.926	-0.26
12	SR	-13.97	0.94*	26.86*	—	—	.926	-0.21
2**	MA	4.42	0.23*	—	0.77*	—	.894	-0.74
7**	MA	110.94*	0.53*	—	-0.34	—	.381	1.30
8**	MA	-13.23	-0.17	—	1.30*	—	.751	-1.73
10**	MA	1.37	0	—	1.01*	—	.956	-0.15

Table E.11: Experiment 6: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
1	BR	75	4200	8.63*	251.1 <sup>#</sup>	3.28*	1.98*	coop.
3	MA	140	325	4.36*	151.8	1.53	0.52	comp.
4	SR	100	350	1.66	130.4	0.96	0.22	marg.
5	MA	107	500	14.32*	0	—	-0.70	comp.
6	SR	30	475	2.07	200.3	1.82	0.91	marg.
9	MA	99	290	1.19	160.2	0.94	0.35	marg.
11	MA	120	300	0.31	61.8	0.11	-0.07	marg.
12	SR	120	300	3.19	82.1	0.49	-0.11	marg.
2**	2	30	400	0.61	0 <sup>#</sup>	—	—	marg.
7**	2	10	800	4.91*	135.9	2.91*	2.91*	switch
8**	2	17	234	0.51	105.7	0.91	0.91	marg.
10**	2	34	399	0.69	0 <sup>#</sup>	—	—	marg.

Table E.12: Experiment 6: classification of individual behavior

ID	Rule	Estimated coefficients					$R^2$	h-stat.
		one	asklag	Select	Med.ask	BR ask		
2	SR	-34.13	0.70*	67.67*	—	—	0.403	0.61
3	MA	48.91*	-0.06	—	0.66*	—	0.799	—
4	MA	50.29	-0.25	—	1.10*	—	0.558	—
5	MA	72.59*	-0.28	—	0.61*	—	0.455	0.91
6	SR	40.65	0.61*	27.80	—	—	0.508	-2.08*
9	MA	24.66	0.69*	—	0.13	—	0.723	-1.18
1**	BR	-11.22	0.39*	—	—	0.58*	0.942	-2.60*
7**	???	—	—	—	—	—	—	—
8**	???	—	—	—	—	—	—	—

Table E.13: Experiment 7: individual behavioral rules, least squares estimation

ID	Rule	Ask min	Ask max	F-stat. marg.	Asymp-tote	Stat. for diff. from		Classification
						Zero	Nash ask	
2	SR	47	200	2.00	106.76	2.05*	0.11	marg.
3	MA	109	300	14.26*	123.57	3.96*	0.75	comp.
4	MA	99	499	2.41	348.05	1.93	1.38	marg.
5	MA	100	250	16.01*	107.51	3.66*	0.26	comp.
6	SR	100	400	7.16*	146.09	2.92*	0.92	comp.
9	MA	109	249	2.06	136.21	1.68	0.45	marg.
1**	BR	1	410	5.33*	0	-2.40*	-2.40*	comp.
7**	???	0	999	—	—	—	—	???
8**	???	15	300	—	—	—	—	???

Table E.14: Experiment 7: classification of individual behavior

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