Applications of Heavy Quark Symmetry and Long Distance Contributions to Weak Decays

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Abstract

Heavy quark chiral perturbation theory is used to predict the form factors for $B_{\ell 4}$ and $D_{\ell 4}$ decays. We also look at the long distance contributions to some hyperon and kaon weak decays which are important for CP violation or extracting information of CKM matrix.

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Chapter 1. Heavy Quark Symmetry and Applications

1.1 Introduction

Effective field theory is a very useful framework to understand particle physics. It is conventional wisdom that physics at low energies is not sensitive to physics at much higher energy scales (the "decoupling theorem" [1]). The idea of the effective field theory is to integrate heavy particles out and arrive at an almost renormalizable low energy theory, with the coefficients in front of the nonrenormalizable terms suppressed by inverse powers of heavy particle mass. For example, suppose we want to integrate out a heavy particle of mass M. At a subtraction point $\mu = M$ we match the Green's functions of the full theory with those of the effective theory without the heavy particle. Of course the effective theory has all kinds of nonrenormalizable terms. Then we can use the renormalization group to run the theory down to some lower scale $\mu \ll M$. The couplings in the effective theory scale with μ, and can be calculated using the renormalization group equations. The contributions from a typical nonrenormalizable term to some matrix elements would be proportional $(\frac{\mu}{M})^\delta$, with $\delta > 0$. Hence at low energy ($\mu \ll M$), we are left with an "effective" theory without the heavy particle, whose major effect is to give rise to many nonrenormalizable terms with small coefficients [2]. In practice the effective field theory approach is very useful in situations involving different energy scales. Two examples are the heavy quark effective theory and chiral perturbation theory.

In the limit of the heavy quark mass $m_Q \to \infty$, a new symmetry of QCD arises [3]. The strong QCD physics of most hadronic system is determined by momentum of order Λ_{QCD} (about several hundred MeV.) In order to change the heavy quark from velocity v to v', it has to emit/absorb a gluon with the coupling $\alpha_s(m_Q(v \cdot v' - 1))$. As $m_Q \to \infty$, $\alpha_s(m_Q(v \cdot v' - 1)) \to 0$ [4] for $v \neq v'$, therefore hard gluon emission and absorption is suppressed. The heavy quark velocity is a conserved quantity, i.e., the heavy quark moves along a straight worldline, and the QCD interaction can't tell its flavor. Furthermore, the spin-flipping magnetic moment type operator is of order $\mathcal{O}(\frac{1}{m_Q})$, and vanishes as $m_Q \to \infty$. Hence the heavy quark spin also decouples from QCD interactions. For N_f flavors of heavy quarks moving with the same four velocity, their QCD interactions respect a new $SU(2N_f)$ spin-flavor symmetry. (Notice this symmetry exists for hadrons containing only one heavy quark. For hadrons containing two or more heavy quarks, the important interactions between heavy quarks are not of order Λ_{QCD} and the above arguments don't apply.) In the real world of six quarks, the t, b and c quarks have masses much greater than Λ_{QCD}. The top quark is very heavy [5] and decays quickly before hadronization. Consequently, we are left with a $SU(4)$ symmetry for hadrons containing a bottom or a charm quark. There are many phenomenological applications of this symmetry developed over the last five years, among them are hadron spectroscopy, and exclusive and inclusive semileptonic decays of hadrons containing a bottom or a charm [6]. The largest uncertainty of the symmetry predictions comes from the fact that the charm mass is not much greater than Λ_{QCD}. The ultimate usefulness of heavy quark symmetry depends on how large the $\mathcal{O}(\frac{\Lambda_{QCD}}{m_Q})$ corrections are, and they have to be examined on a case by case basis.

For practical applications of the heavy quark symmetry, it is very convenient to use the heavy quark effective theory [7]. This is different from the conventional effective theory, in which one integrates out heavy particles completely. Instead, we only integrate out some degrees of freedom from a heavy quark, and go to an effective theory in which the heavy quark has a fixed velocity. Explicitly, for a heavy quark of mass m_Q, the QCD Lagrangian is

$$\mathcal{L}(x) = \bar{Q}(x)(i\not{D} - m_Q)Q(x) \ , \tag{1.1}$$

where $\not{D} = \gamma^\mu D_\mu = \gamma^\mu(\partial_\mu + igA^a_\mu T^a)$, T^a's are generators for color $SU(3)$. For a hadron containing one Q and moving with velocity v, the momentum of the heavy quark is $p^\mu_Q = m_Q v^\mu + k^\mu$, where k^μ is of the order Λ_{QCD}. As $m_Q \to \infty$, the QCD interaction can not change the four-velocity v, and we can factor out m_Q dependence by redefining $Q(x) = e^{-im_Q v\cdot x} h_v(x)$. The equation of motion becomes

$$0 = (i\not{D} - m_Q)Q(x) = e^{-im_Q v\cdot x}(m_Q(\not{v} - 1) + i\not{D})h_v(x) \ , \tag{1.2}$$

where D_μ acting on $h_v(x)$ only brings down residual momentum k_μ. In the limit $m_Q \to \infty$, this gives

$$\not{v} h_v(x) = h_v(x) \ , \tag{1.3}$$

which is an operator identity. In terms of the velocity-dependent field, the Lagrangian becomes

$$\mathcal{L}_v(x) = \bar{h}_v(x) iv \cdot D h_v(x) \; . \tag{1.4}$$

The whole QCD Lagrangian becomes $\mathcal{L} = \sum_v \mathcal{L}_v(x)$ plus similar velocity-dependent terms for anti-heavy quark fields. Since $\mathcal{L}_v$ doesn't have explicit m_Q dependence and γ matrix structure, the spin-flavor symmetry is manifest.

The above discussion is valid for the limit $m_Q \to \infty$. To incorporate $\mathcal{O}(\frac{1}{m_Q})$ corrections, the field redefinition should be taken as

$$Q(x) = e^{-im_Q v\cdot x}(h_v(x) + \chi_v(x)) \; , \tag{1.5}$$

with $\not{v} h_v(x) = h_v(x)$, $\not{v}\chi_v(x) = -\chi_v(x)$. Since heavy quark field has only two degrees of freedom (two spin states), one can express $\chi_v(x)$ in terms of $h_v(x)$ field by using the equation of motion

$$0 = (i\not{D} - m_Q)Q(x) = (i\not{D} - m_Q)[e^{-im_Q v\cdot x}(h_v(x) + \chi_v(x))] \; , \tag{1.6}$$

which gives

$$\chi_v(x) = \frac{1-\not{v}}{2}\frac{1}{2m_Q} i\not{D}(h_v(x) + \chi_v(x)) = \sum_{n=1}^{\infty}\left(\frac{1-\not{v}}{2}\frac{i\not{D}}{2m_Q}\right)^n h_v(x) \; . \tag{1.7}$$

The field redefinition (equation 1.5) becomes

$$Q(x) = e^{-im_Q v\cdot x}\sum_{n=0}^{\infty}\left(\frac{1-\not{v}}{2}\frac{i\not{D}}{2m_Q}\right)^n h_v(x) \; . \tag{1.8}$$

Substituting this into the QCD Lagrangian (equation 1.1) gives

$$\mathcal{L}_v(x) = \bar{h}_v(x) iv\cdot D h_v(x) + \frac{1}{2m_Q}\bar{h}_v(x)[-(iv\cdot D)^2 + (iD)^2 - \frac{1}{2} g\sigma^{\mu\nu} G^a_{\mu\nu} T^a] h_v(x)$$

$$+\mathcal{O}(\frac{1}{m_Q^2}) \; . \tag{1.9}$$

This is the effective Lagrangian at scale $\mu = m_Q$ with the matching performed at tree level. At scale $\mu = m_Q$, there are large logarithms $\log(\frac{p}{m_Q})$ present in matrix elements of the higher dimensional operators in $\mathcal{L}_v$ (here p is a typical momentum).

These large logarithms can be summed using the renormalization group equation, and factored out as the coefficients of the respective operators by scaling down to a scale $\mu \simeq p$

$$\mathcal{L}_v(x) = \bar{h}_v(x) iv \cdot D h_v(x)$$

$$+\frac{1}{m_Q}\bar{h}_v(x)\left[C_1(\mu)(iv\cdot D)^2 + C_2(\mu)(iD)^2 + C_3(\mu)\frac{g}{2}\sigma^{\mu\nu}G^a_{\mu\nu}T^a\right]h_v(x) + \mathcal{O}(\frac{1}{m_Q^2}) \ . \tag{1.10}$$

The coefficients $C_i(\mu)$'s were calculated in leading logarithmic approximation [8](in which all terms of powers $(\alpha_s \log \frac{m_Q}{\mu})^n$ are summed up). The results are

$$\begin{aligned} C_1(\mu) &= 1 - \frac{3}{2}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{8}{25}} \ , \\ C_2(\mu) &= \frac{1}{2} \ , \\ C_3(\mu) &= -\frac{1}{2}\left(\frac{\alpha_s(\mu)}{\alpha_s(m_Q)}\right)^{-\frac{9}{25}} \ . \end{aligned} \tag{1.11}$$

(The μ-independence of C_2 can be understood from the reparametrization invariance [9].) These higher dimension operators are important when one considers $\mathcal{O}(\frac{1}{m_Q})$ corrections.

Now let's get back to the limit $m_Q \to \infty$. In this limit the heavy quark spin $\vec{S}_Q$ completely decouples from QCD interaction, and is a good quantum number. The total spin for a hadron $\vec{S} = \vec{S}_Q + \vec{S}_\ell$ (where $\vec{S}_\ell$ is the spin for the light degrees of freedom) is also a conserved quantity. This tells us that s_ℓ is also a good quantum number. Consequently for each s_ℓ, there are two degenerate states with spin $s = s_\ell \pm \frac{1}{2}$, i.e., hadrons containing one heavy quark come in doublets (except for the case $s_\ell = 0$). The splitting inside a doublet is an $\mathcal{O}(\frac{1}{m_Q})$ effect. For example, the lowest-lying meson doublet containing a heavy quark Q has $s_\ell^{P_\ell} = \frac{1}{2}^-$ (P_ℓ is the parity of light degrees of freedom). This gives two states (P_Q, P_Q^*) with quantum numbers $J^P = (0^-, 1^-)$. (In the case of b quark, these states are B and B^*, whose mass difference is only about 50 MeV, much smaller than Λ_{QCD}.) Just as the octet of pions, kaons and eta fields can be put in one single representation of $SU(3)$, we may put P_Q and P_Q^* fields together into a superfield $H_a(x)$ [10]

$$H_a = \frac{1+\not{v}}{2}(P^*_{a\mu}\gamma^\mu - P_a\gamma_5) \ . \tag{1.12}$$

Here a is the ordinary flavor $SU(3)$ index, $P^*_{a\mu}$ satisfies $P^*_{a\mu}v^\mu = 0$. (P_a, P^*_a) annihilate respective mesons with velocity v. The H_a satisfies $\not{v}H_a = H_a$. Under heavy quark spin transformation, $H_a \to SH_a$, where S is a rotation matrix in the four component spinor representation. Under Lorentz transformation, $H_a(x) \to D(\Lambda)^{-1}H_a(\Lambda x)D(\Lambda)$, where $D(\Lambda)$ belongs to 4×4 matrix representation of the Lorentz group. We can also introduce $\bar{H}_a = \gamma^0 H^\dagger_a \gamma^0$. Explicitly,

$$\bar{H}_a = (P^{*\dagger}_{a\mu}\gamma^\mu + P^\dagger_a\gamma_5)\frac{1+\not{v}}{2} \ . \tag{1.13}$$

The $(P^{*\dagger}_{a\mu}, P^\dagger_a)$ create a respective meson state with velocity v.

As an application of the above formalism, let's look at the semileptonic decays of B with velocity v into (D, D^*) with velocity v'. The weak current mediating this transition is $\mathcal{J}_{HQET} = \bar{c}_{v'}\Gamma b_v$. The most general form of the relevant matrix element is

$$< \mathcal{D}, v'|\bar{c}_{v'}\Gamma b_v|\mathcal{B}, v >= -\xi(v\cdot v')Tr(\bar{\mathcal{D}}(v')\Gamma\mathcal{B}(v))$$

$$= -\xi(v\cdot v')Tr((D^{*\dagger}_\alpha\gamma^\alpha + D^\dagger\gamma_5)\frac{1+\not{v}'}{2}\Gamma\frac{1+\not{v}}{2}(B^*_\beta\gamma^\beta - B\gamma_5)) \ . \tag{1.14}$$

For the weak transition, $\Gamma = \gamma_\mu$ and $\gamma_\mu\gamma_5$. The explicit form of equation (1.14) is

$$\begin{aligned} < D, v'|\bar{c}_{v'}\gamma_\mu b_v|B, v > &= \xi(v\cdot v')\sqrt{m_D m_B}(v+v')_\mu \ , \\ < D^*, \epsilon', v'|\bar{c}_{v'}\gamma_\mu b_v|B, v > &= i\xi(v\cdot v')\sqrt{m^*_D m_B}\epsilon_{\mu\nu\alpha\beta}\epsilon'^{\nu}{}^*v^\alpha v'^\beta \ , \\ < D^*, \epsilon', v'|\bar{c}_{v'}\gamma_\mu\gamma_5 b_v|B, v > &= \xi(v\cdot v')\sqrt{m^*_D m_B}[(1+v\cdot v')\epsilon'^*{}_\mu - \epsilon'^*\cdot v v'_\mu] \ . \end{aligned} \tag{1.15}$$

Therefore, in the heavy quark limit, the semileptonic decays of B into (D, D^*) depend on only one universal form factor, the Isgur-Wise function $\xi(v\cdot v')$. Furthermore, the heavy quark flavor symmetry determines the normalization of $\xi(v\cdot v')$ at zero recoil point $v = v'$ to be $\xi(1) = 1$ [3]. To complete the story, the current $\mathcal{J}_{HQET}$ in the effective theory has to be related to the current $\mathcal{J}_{QCD}$ in the full QCD. The calculation from the matching and the renormalization group running gives

$$\mathcal{J}_{QCD} = C_{cb}\mathcal{J}_{HQET} \; . \tag{1.16}$$

In the leading logarithmic approximation,

$$C_{cb}(\mu) = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-\frac{6}{25}} \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right)^{a_L(v\cdot v')} \; , \tag{1.17}$$

where

$$a_L(v\cdot v') = \frac{8}{25}[v\cdot v' r(v\cdot v') - 1] \; , \tag{1.18}$$

and

$$r(v\cdot v') = \frac{1}{\sqrt{(v\cdot v')^2 - 1}} \log\left(v\cdot v' + \sqrt{(v\cdot v')^2 - 1}\right) \; . \tag{1.19}$$

Factors of C_{cb} are to be inserted into equation (1.15) for the full prediction of the heavy quark effective theory in the heavy quark limit. Notice that $a_L(0) = 1$ as required by the heavy quark symmetry. Therefore, we can use the absolute prediction at zero recoil of B semileptonic decays to (D, D^*) to measure $|V_{cb}|$ [11]. This has been the most important phenomenological application of the heavy quark effective theory. Another significant aspect of the heavy quark effective theory is that it supplies us with a systematic tool to take into account the symmetry breaking effects.

In the light quark sector (u, d, s), the current quark masses (m_u, m_d, m_s) are believed to be small compared with the typical hadronic scale Λ_{QCD}. If we take the limit of zero current quark masses, QCD would possess a chiral symmetry $SU(3)_L \times SU(3)_R$. Since the hadrons are well classified according to the representation of $SU(3)$, this chiral $SU(3)_L \times SU(3)_R$ symmetry must be spontaneously broken, which gives rise to eight massless odd-parity Goldstone bosons. Because light quark masses don't exactly vanish, these (pseudo)Goldstone bosons get small masses. They are identified with the three pions, four kaons and the eta. In fact, this scenario gives a natural explanation of why pions (and kaons) are so much lighter than other hadronic resonances. It is generally regarded that the chiral symmetry is broken by the strong color forces of $SU(3)$ at long distance. The specific symmetry breaking mechanism is an unsolved dynamical problem. Nevertheless, the low energy interaction between Goldstone bosons is very much constrained by the ansatz of the chiral symmetry breaking. One can build a chiral Lagrangian [12] of the Goldstone boson fields which represent the excitations along the directions of the broken generators of the symmetry breaking $SU(3)_L \times SU(3)_R \to SU(3)_V$. Explicitly, the Goldstone boson fields can be included in a nonlinear field $\Sigma(x)$

$$\Sigma = \exp\left(\frac{2iM}{f}\right) , \tag{1.20}$$

where

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} . \tag{1.21}$$

Under chiral $SU(3)_L \times SU(3)_R$ transformation, $\Sigma \to L\Sigma R^\dagger$, where $L \in SU(3)_L$ and $R \in SU(3)_R$. Under an unbroken $SU(3)_V$ transformation $V = L = R$, $\Sigma \to V\Sigma V^\dagger$, which gives $M \to VMV^\dagger$, i.e., the Goldstone bosons transform as an octet under the unbroken $SU(3)_V$. The $SU(3)_L \times SU(3)_R$ invariant kinetic term is given by

$$\mathcal{L}_{kin} = \frac{f^2}{8} Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) . \tag{1.22}$$

The chiral symmetry is broken by the quark mass term, which transforms as $(3_L, \bar{3}_R)+(\bar{3}_L, 3_R)$ under chiral $SU(3)_L \times SU(3)_R$. Including the symmetry breaking terms and higher dimensional terms, the chiral Lagrangian reads

$$\mathcal{L} = \frac{f^2}{8} Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \lambda_0 Tr(m_q \Sigma + m_q \Sigma^\dagger) + ... \tag{1.23}$$

where

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{1.24}$$

is the quark mass matrix, $f \simeq 132$ MeV is the pion decay constant, and the ellipsis denotes terms with more than two derivatives and more insertions of the quark mass matrix. Neglecting effects from these extra terms, a fit to the masses of pions, kaons and eta gives $m_u/m_d = 0.55$, $m_s/m_d = 20$ [13]. By dimensional analysis, the contributions from the higher dimensional terms in the chiral Lagrangian are suppressed by $\mathcal{O}(p^2/\Lambda_{\chi SB}^2)$ or higher, where $\Lambda_{\chi SB} \simeq 1GeV$ is the chiral symmetry breaking scale, p is the typical momentum of a particular scattering process. Therefore these contributions are less important than those from leading terms in the chiral Lagrangian. The most dubious feature of this approach is that the mass of the strange quark may be too large for low momentum expansion to be valid. But this approach has had some success in describing kaon physics.

It is possible to construct chiral Lagrangians for matter fields which give their interactions with the Goldstone boson fields [14]. For example, baryon chiral perturbation theory describes interactions of baryon octet and decuplet with the meson octet [15]. There is also the heavy quark chiral perturbation theory [16], which incorporates the interactions between the heavy hadron multiplets with soft pions. Notice that the chiral Lagrangian description fails if the momenta of the external pions are large compared with $\Lambda_{\chi SB}$. This approach only applies to the processes with soft external pions (and kaons).

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1.2 $B_{\ell 4}$ and $D_{\ell 4}$ Decay

Heavy quark symmetry and chiral symmetry put constraints on $B_{\ell 4}$ and $D_{\ell 4}$ semileptonic weak decay amplitudes[1,2,3]. In this work we explicitly display the implications of these symmetries for $D \to K\pi\bar{\ell}\nu_\ell$, $D \to \pi\pi\bar{\ell}\nu_\ell$, $B \to \pi\pi\ell\bar{\nu}_\ell$ and $B \to D\pi\ell\bar{\nu}_\ell$ decays.

The strong interactions of the lowest lying mesons containing a heavy quark Q with the pseudo Goldstone bosons π, K, η are determined by the chiral Lagrangian density

$$\mathcal{L} = \frac{f^2}{8} Tr(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + \lambda_0 Tr[m_q \Sigma + m_q \Sigma^\dagger]$$

$$-iTr\bar{H}_a v_\mu \partial^\mu H_a + \frac{i}{2} Tr\bar{H}_a H_b v^\mu [\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba}$$

$$+\frac{ig}{2} Tr\bar{H}_a H_b \gamma^\mu \gamma_5 [\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger]_{ba} + \lambda_1 Tr\bar{H}_a H_b$$

$$\cdot[\xi m_q \xi + \xi^\dagger m_q \xi^\dagger]_{ba} + \lambda_1' Tr\bar{H}_a H_a [m_q \Sigma + m_q \Sigma^\dagger]_{bb}$$

$$+\frac{\lambda_2}{m_Q} Tr\bar{H}_a \sigma_{\mu\nu} H_a \sigma^{\mu\nu} \quad + \ldots \tag{2.1}$$

where the ellipsis denotes terms with additional derivatives, factors of the light quark mass matrix

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{2.2}$$

associated with explicit violation of $SU(3)_L \times SU(3)_R$ chiral symmetry, or factors of $1/m_Q$ associated with violation of heavy quark spin-flavor symmetry. The Lagrangian is written in terms of a 4×4 matrix H_a which is defined in section 1.1. For completeness, let's display the form of H_a here[4,5]

$$H_a = \frac{(1 + \not{v})}{2} [P^*_{a\mu} \gamma^\mu - P_a \gamma_5] \, , \tag{2.3a}$$

$$\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0 \, . \tag{2.3b}$$

In cases where the type of heavy quark Q and its four-velocity v are important the 4×4 matrix is denoted by $H_a^{(Q)}(v)$.

In the Lagrangian density (2.1) the light quark flavor indices a, b go over $1, 2, 3$ and repeated indices are summed. For $Q = c, (P_1, P_2, P_3) = (D^0, D^+, D_s)$ and $(P_1^*, P_2^*, P_3^*) = (D^{0*}, D^{+*}, D_s^*)$ while for $Q = b$, $(P_1, P_2, P_3) = (B^-, \bar{B}^0, \bar{B}_s)$ and $(P_1^*, P_2^*, P_3^*) = (B^{-*}, \bar{B}^{0*}, \bar{B}_s^*)$. Factors of $\sqrt{m_P}$ and $\sqrt{m_{P^*}}$ have been absorbed into the P and P^* fields. Consequently they have dimension 3/2.

The field H_a is a doublet under heavy quark spin symmetry $SU(2)_v$ and a $\bar{3}$ under the unbroken $SU(3)_V$ light quark flavor symmetry. Under $SU(2)_v$ and $SU(3)_L \times SU(3)_R$ it transforms as

$$H_a \to S(HU^\dagger)_a \ , \tag{2.4}$$

where $S \epsilon\ SU(2)_v$ and U is the usual space-time dependent 3×3 unitary matrix that is introduced to transform matter fields in a chiral Lagrangian.

The low energy strong interactions of the pseudo-Goldstone bosons are described by the chiral Lagrangian in eq. (1.23). In heavy quark chiral perturbation theory (described by the Lagrangian in eq. (2.1)), it is convenient to use

$$\xi = \exp(iM/f) \ , \tag{2.5}$$

and

$$\xi^2 = \Sigma = \exp(2iM/f) \ . \tag{2.6}$$

Here M is the usual meson octet (eq. (1.21)).

Under $SU(3)_L \times SU(3)_R$ chiral symmetry, $\Sigma \to L\Sigma R^\dagger$, implying that

$$\xi \to L\xi U^\dagger = U\xi R^\dagger \ , \tag{2.7}$$

where $L \epsilon SU(3)_L, R \epsilon SU(3)_R$ and U is a function of L, R and the meson fields. Typically U is space-time dependent. However, for $SU(3)_V$ transformations, $V = L = R$, U is equal to V.

Heavy quark flavor symmetry implies that, to leading order in $\Lambda_{QCD}/m_Q, g$ is independent of heavy quark flavor. For $Q = c$ the $D^* \to D\pi$ decay width is determined by g

$$\Gamma(D^{+*} \to D^0\pi^+) = \left(\frac{1}{6\pi}\right)\frac{g^2}{f^2}|\vec{p}_\pi|^3 \ . \tag{2.8}$$

The present experimental limit[6] on this width ($\Gamma(D^{+*} \to D^0\pi^+) < 72KeV$) implies that $g^2 < 0.4$. Applying the Noether procedure, the Lagrangian density (2.1) gives the following expression for the axial current,

$$\bar{q}_a T_{ab}\gamma_\nu\gamma_5 q_b = -gTr\bar{H}_a H_b \gamma_\nu\gamma_5 T_{ba} + \ldots \ . \tag{2.9}$$

In eq. (2.9) the ellipses represent terms containing the pseudo-Goldstone boson fields and T is a flavor SU(3) generator. Treating the quark fields in eq. (2.9) as constituent quarks and using the non-relativistic quark model (i.e., static SU(6)) to estimate the D^* matrix element of the l.h.s. of eq. (2.9) gives[3] $g = 1$. (A similar estimate for the pion-nucleon coupling gives $g_A = 5/3$.) In the chiral quark model[7] there is a constituent-quark pion coupling. Using the measured pion nucleon coupling to determine the constituent-quark pion coupling gives that $g \simeq 0.75$. The decay $B^* \to B\pi$ is kinematically forbidden and so it will not be possible to use it to test the heavy quark flavor independence of g. The amplitude for the semileptonic decay $B \to D\pi\ell\bar{\nu}_\ell$, in the kinematic region where the pion has low momentum (and the $D\pi$ mass is greater than that of the D^*), can be predicted using chiral perturbation theory. In principle, experimental study of this decay can give information on the flavor dependence of g.

In the next section we discuss the kinematics of weak semileptonic $D_{\ell 4}$ and $B_{\ell 4}$ decay. The fully differential decay rates are expressed in terms of form factors. The results of the next section are a slight modification of the kinematics of $K_{\ell 4}$ decay to the situation where the two hadrons in the final state have different masses. The generalization of $K_{\ell 4}$ decay kinematics to $D \to K\pi\bar{\ell}\nu_\ell$ decay was previously discussed by Kane, et al.[7] We have included a short review of the kinematics for completeness. The two following sections give the predictions of chiral perturbation theory for $D \to K\pi\bar{\ell}\nu_\ell$, $D \to \pi\pi\bar{\ell}\nu_\ell$ and $B \to \pi\pi\ell\bar{\nu}_\ell$ decay form factors, and for $B \to D\pi\ell\bar{\nu}_\ell$. After that there is a brief discussion of the expected kinematic range where chiral perturbation theory for $B \to D\pi\ell\bar{\nu}_\ell$ is applicable. Concluding remarks are made in the end.

For $B_{\ell 4}$ and $D_{\ell 4}$ decay the kinematic region where chiral perturbation theory is applicable is small. In the kinematic region where chiral perturbation theory is applicable $Br(B \rightarrow D\pi\ell\bar{\nu}_\ell) \sim (1/16\pi^2)Br(B \rightarrow D\ell\bar{\nu}_\ell) \sim 10^{-4}$. The situation is worse for the modes with two pseudo-Goldstone bosons in the final state. For example we expect that $Br(D \rightarrow \pi\pi\bar{\ell}\nu_\ell) \sim (1/16\pi^2)\sin^2\theta_c(f_D/m_D)^2 Br(D \rightarrow X_s\bar{\ell}\nu_\ell)$, where f_D is the decay constant for the D-meson. For $f_D \sim 200 MeV$ this crude order of magnitude estimate gives $Br(D \rightarrow \pi\pi\bar{\ell}\nu_\ell) \sim 10^{-6}$. The factor of $\sin^2\theta_c$ is absent for the Cabibbo allowed decay $D \rightarrow K\pi\bar{\ell}\nu_\ell$, but the fact that the kaon mass is not very small makes the validity of lowest order chiral perturbation theory dubious. It will be very difficult, in the kinematic region where chiral perturbation theory applies, to observe $B_{\ell 4}$ and $D_{\ell 4}$ decay to two pseudo-Goldstone bosons. However, the results of this work may still prove useful for these decays. Phenomenological models that predict the form factors over the whole phase space should be constrained to agree with chiral perturbation theory in the kinematic region where it applies.

Review of the Kinematics

Consider for definiteness the decay $D \to K\pi\bar{\ell}\nu_\ell$. At the end of this section we show how to modify the formulae so they apply to the other decays we are considering. It is convenient, following the analysis of $K_{\ell 4}$ decay by Pais and Treiman[9], to form the following combinations of four-momenta

$$P = p_K + p_\pi, \quad Q = p_K - p_\pi, \quad L = p_\ell + p_{\nu_\ell}, \quad N = p_\ell - p_{\nu_\ell} \ . \tag{2.10}$$

Like $K_{\ell 4}$ decay, $D_{\ell 4}$ decay is kinematically parametrized by five variables. For two of these we take the $K\pi$ and $\bar{\ell}\nu_\ell$ squared masses

$$s_{K\pi} = P^2, \quad s_{\ell\nu} = L^2 \ . \tag{2.11}$$

For the remaining three variables we choose: θ_K, the angle formed by the kaon three-momentum in the $K\pi$ rest frame and the line of flight of the $K\pi$ in the D rest frame; θ_ℓ, the angle formed by the $\bar{\ell}$ three-momentum in the $\bar{\ell}\nu_\ell$ rest frame and the line of flight of the $\bar{\ell}\nu_\ell$ in the D rest frame; ϕ, the angle between the normals to the planes defined in the D rest frame by the $K\pi$ pair and the $\bar{\ell}\nu_\ell$ pair. (The sense of the angle is from the normal to the $K\pi$ plane to the normal to the $\bar{\ell}\nu_\ell$ plane.)

Over most of the available phase space (including the kinematic regime where chiral perturbation theory can be applied) the mass of the lepton can be neglected (i.e., $m_\ell^2/s_{\ell\nu} \ll 1$) and we find that with $m_\ell = 0$;

$$P \cdot L = \frac{m_D^2 - s_{K\pi} - s_{\ell\nu}}{2} \ , \tag{2.12a}$$

$$L \cdot N = 0, \quad P \cdot Q = m_K^2 - m_\pi^2 \ , \tag{2.12b}$$

$$Q^2 = 2(m_K^2 + m_\pi^2) - s_{K\pi}, \quad N^2 = -s_{\ell\nu} \ , \tag{2.12c}$$

$$L \cdot Q = \left(\frac{m_K^2 - m_\pi^2}{s_{K\pi}}\right) P \cdot L + \beta X \cos\theta_K \ , \tag{2.12d}$$

$$P \cdot N = X \cos\theta_\ell \tag{2.12e}$$

$$Q \cdot N = \left(\frac{m_K^2 - m_\pi^2}{s_{K\pi}}\right) X \cos\theta_\ell + \beta P \cdot L \cos\theta_K \cos\theta_\ell - \beta (s_{\ell\nu} s_{K\pi})^{1/2} \sin\theta_K \sin\theta_\ell \cos\phi \ , \tag{2.12f}$$

$$\epsilon_{\mu\nu\rho\sigma} Q^\mu P^\nu N^\rho L^\sigma = -\beta X (s_{\ell\nu} s_{K\pi})^{1/2} \sin\theta_K \sin\theta_\ell \sin\phi \ . \tag{2.12g}$$

In eqs. (2.12)

$$X = [(P \cdot L)^2 - s_{K\pi} s_{\ell\nu}]^{1/2} \ , \tag{2.13}$$

and β is $(2/\sqrt{s_{K\pi}})$ times the magnitude of the kaon three-momentum in the $K\pi$ rest frame,

$$\beta = [s_{K\pi}^2 + m_\pi^4 + m_K^4 - 2m_K^2 m_\pi^2 - 2s_{K\pi} m_K^2 - 2s_{K\pi} m_\pi^2]^{1/2} / s_{K\pi} \ . \tag{2.14}$$

Taking the limit, $m_K = m_\pi$, eqs. (2.12) agree with the results of Pais and Treiman for $K_{\ell 4}$ decay.

The invariant matrix element for $D \to K\pi\bar{\ell}\nu_\ell$ semileptonic decay is

$$M_{fi} = \frac{G_F}{\sqrt{2}} V_{cs} < \pi(p_\pi) K(p_K) | \bar{s}\gamma_\mu (1 - \gamma_5) c | D(p_D) >$$

$$\bar{u}(p_\nu) \gamma^\mu (1 - \gamma_5) v(p_{\bar{\ell}}) \ , \tag{2.15}$$

where V_{cs} is the $c \to s$ element of the Cabibbo-Kobayashi-Maskawa matrix and G_F is the Fermi constant. The hadronic matrix element can be written in terms of four form factors $w_\pm, r$ and h that are defined by

$$< \pi(p_\pi) K(p_K) | \bar{s}\gamma_\mu (1 - \gamma_5) c | D(p_D) > = [iw_+ P_\mu + iw_- Q_\mu$$

$$+ ir(p_D - P)_\mu + h\epsilon_{\mu\alpha\beta\gamma} p_D^\alpha P^\beta Q^\gamma] \ . \tag{2.16}$$

The form factors $w_\pm$, r and h are function of $s_{\ell\nu}, s_{K\pi}$ and $\cos\theta_K$. Summing over the lepton polarizations the absolute value of the square of the matrix element is

$$\sum_{spins} |M_{fi}|^2 = 4G_F^2 |V_{cs}|^2 H_{\mu\nu} L^{\mu\nu} \ , \tag{2.17}$$

where

$$H_{\mu\nu} = < \pi(p_\pi) K(p_K) | \bar{s}\gamma_\mu (1 - \gamma_5) c | D(p_D) >$$

$$\cdot < \pi(p_\pi) K(p_K) | \bar{s}\gamma_\nu (1 - \gamma_5) c | D(p_D) >^* \tag{2.18a}$$

$$L^{\mu\nu} = \frac{1}{2} [L^\mu L^\nu - N^\mu N^\nu - s_{\ell\nu} g^{\mu\nu} - i\epsilon^{\alpha\mu\gamma\nu} L_\alpha N_\gamma] \ . \tag{2.18b}$$

The differential decay rate takes the form

$$d^5\Gamma = \frac{G_F^2 |V_{cs}|^2}{(4\pi)^6 m_D^3} X\beta I(s_{K\pi}, s_{\ell\nu}, \theta_K, \theta_\ell, \phi) ds_{\ell\nu} ds_{K\pi} \cdot d\cos\theta_K d\cos\theta_\ell d\phi \ . \qquad (2.19)$$

The dependence of I on θ_ℓ and ϕ is given by

$$\begin{aligned} I = I_1 + I_2 \cos 2\theta_\ell + I_3 \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_\ell \cos\phi \\ + I_5 \sin\theta_\ell \cos\phi + I_6 \cos\theta_\ell + I_7 \sin\theta_\ell \sin\phi \\ + I_8 \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_\ell \sin 2\phi \ , \end{aligned} \qquad (2.20)$$

where $I_1, ..., I_9$ depend on $s_{K\pi}, s_{\ell\nu}$ and θ_K.

To display $I_1, ..., I_9$ in as compact form as possible it is convenient to introduce the following combinations of kinematic factors and form factors

$$F_1 = Xw_+ + [\beta P \cdot L \cos\theta_K + \left(\frac{m_K^2 - m_\pi^2}{s_{K\pi}}\right) X] w_- \qquad (2.21a)$$

$$F_2 = \beta (s_{\ell\nu} s_{K\pi})^{1/2} w_- \qquad (2.21b)$$

$$F_3 = \beta X (s_{\ell\nu} s_{K\pi})^{1/2} h \ . \qquad (2.21c)$$

In terms of these combinations of form factors

$$I_1 = \frac{1}{4}\left\{|F_1|^2 + \frac{3}{2}\sin^2\theta_K (|F_2|^2 + |F_3|^2)\right\} \qquad (2.22a)$$

$$I_2 = -\frac{1}{4}\left\{|F_1|^2 - \frac{1}{2}\sin^2\theta_K (|F_2|^2 + |F_3|^2)\right\} \qquad (2.22b)$$

$$I_3 = -\frac{1}{4}[|F_2|^2 - |F_3|^2] \sin^2\theta_K \qquad (2.22c)$$

$$I_4 = \frac{1}{2} Re(F_1^* F_2) \sin\theta_K \qquad (2.22d)$$

$$I_5 = Re(F_1^* F_3) \sin\theta_K \qquad (2.22e)$$

$$I_6 = Re(F_2^* F_3) \sin^2\theta_K \qquad (2.22f)$$

$$I_7 = Im(F_1 F_2^*) \sin\theta_K \qquad (2.22g)$$

$$I_8 = \frac{1}{2} Im(F_1 F_3^*) \sin\theta_K \qquad (2.22h)$$

$$I_9 = -\frac{1}{2} Im(F_2 F_3^*) \sin^2\theta_K \ . \qquad (2.22i)$$

Eqs. (2.20) and (2.22) are the same as eqs. (11) of Pais and Treiman. However, the definitions of F_1, F_2 and F_3 are slightly different because $m_K \neq m_\pi$.

It is evident from eqs. (2.22) that the partial wave expansions for the form factors F_1, F_2 and F_3 are

$$F_1(s_{K\pi}, s_{\ell\nu}, \cos\theta_K) = \sum_{\ell=0}^{\infty} \tilde{F}_{1,\ell}(s_{K\pi}, s_{\ell\nu}) P_\ell(\cos\theta_K) \tag{2.23a}$$

$$F_2(s_{K\pi}, s_{\ell\nu}, \cos\theta_K) = \sum_{\ell=1}^{\infty} \frac{1}{[\ell(\ell+1)]^{1/2}} \tilde{F}_{2,\ell}(s_{K\pi}, s_{\ell\nu}) \frac{d}{d\cos\theta_K} P_\ell(\cos\theta_K) \tag{2.23b}$$

$$F_3(s_{K\pi}, s_{\ell\nu}, \cos\theta_K) = \sum_{\ell=1}^{\infty} \frac{1}{[\ell(\ell+1)]^{1/2}} \tilde{F}_{3,\ell}(s_{K\pi}, s_{\ell\nu}) \frac{d}{d\cos\theta_K} P_\ell(\cos\theta_K) \tag{2.23c}$$

Integrating over the angles gives

$$d^2\Gamma = \frac{G_F^2 |V_{cs}|^2}{3(4\pi)^5 m_D^3} X\beta \sum_\ell \frac{2}{(2\ell+1)} \Big[|\tilde{F}_{1,\ell}|^2 + |\tilde{F}_{2,\ell}|^2 + |\tilde{F}_{3,\ell}|^2 \Big] ds_{\ell\nu} ds_{K\pi} \ , \tag{2.24}$$

and the total decay rate is

$$\Gamma = \int_{(m_K+m_\pi)^2}^{m_D^2} ds_{K\pi} \int_0^{(m_D - s_{K\pi}^{1/2})^2} ds_{\ell\nu} \left(\frac{d^2\Gamma}{ds_{\ell\nu} ds_{K\pi}} \right) . \tag{2.25}$$

One advantage of the variables $\theta_K, \theta_\ell, \phi, s_{\ell\nu}$ and $s_{K\pi}$ is that in terms of these variables the region of phase space integration is quite simple. The angles are unrestricted and eq. (2.25) gives the region for $s_{K\pi}$ and $s_{\ell\nu}$.

Although we have focused on $D \to K\pi\bar{\ell}\nu_\ell$ decay the results presented above can be straightforwardly altered to apply to the other decays we discuss in this work. For $D \to \pi\pi\bar{\ell}\nu_\ell$ decay one simply changes $V_{cs} \to V_{cd}$ and $m_K \to m_\pi$. For $B \to \pi\pi\ell\bar{\nu}_\ell$ decay one changes $V_{cs} \to V_{ub}^*, m_D \to m_B$ and $m_K \to m_\pi$. Also, in eq. (2.15) $p_{\bar{\ell}}$ and p_ν are switched. Consequently the term proportional to the alternating tensor in eq. (2.18b) and the expressions for I_5, I_6 and I_7 in eqs. (2.22e), (2.22f) and (2.22g) change sign. Finally, for $B \to D\pi\ell\bar{\nu}_\ell$ decay the changes $V_{cs} \to V_{cb}^*, m_D \to m_B, m_K \to m_D$ and the same sign changes as for $B \to \pi\pi\ell\bar{\nu}_\ell$ decay are made.

Decays to Two Pseudo-Goldstone Bosons

The semileptonic decays $D \to K\pi\bar{\ell}\nu_\ell$ $D \to \pi\pi\bar{\ell}\nu_\ell$ and $B \to \pi\pi\ell\bar{\nu}_\ell$ are determined by matrix elements of the left-handed current

$$L_{\nu a} = \bar{q}^a \gamma_\nu (1-\gamma_5) Q \ . \tag{2.26}$$

This operator transforms under chiral $SU(3)_L \times SU(3)_R$ as $(\bar{3}_L, 1_R)$. In chiral perturbation theory its matrix elements are given by those of

$$L_{\nu a} = \left(\frac{i\alpha}{2}\right) Tr\gamma_\nu(1-\gamma_5) H_b \xi^\dagger_{ba} + \dots \ , \tag{2.27}$$

where the ellipsis denotes terms with derivatives, factors of the light quark mass matrix m_q or factors of $1/m_Q$. The constant α is related to the decay constant of the heavy meson,

$$< 0|\bar{q}^a \gamma^\nu \gamma_5 Q|P_a^{(Q)}(v)> = i f_{P_a^{(Q)}} m_{P_a^{(Q)}} v^\nu \ . \tag{2.28}$$

Taking the $P_a^{(Q)}$ to vacuum matrix element of eq. (2.27) (for this matrix element $\xi^\dagger$ can be replaced by unity) gives

$$\alpha = f_{P_a^{(Q)}} \sqrt{m_{P_a^{(Q)}}} \ . \tag{2.29}$$

The parameter α has a calculable logarithmic dependence on the heavy quark[10,11] mass from perturbative QCD.

For $D_{\ell 4}$ and $B_{\ell 4}$ decay to two pseudo-Goldstone bosons the Feynman diagrams in Fig. 1 determine the required matrix element. In Fig. 1 a solid line represents a heavy meson and a dashed line represents a pseudo-Goldstone boson. The shaded square denotes an insertion of the left handed current. The form factors $w_\pm, r$ and h that follow from calculation of these Feynman diagrams are given below.

(i) $D \to K\pi\bar{\ell}\nu_\ell$

$D \to K\pi\bar{\ell}\nu_\ell$ decays are determined by $Q = c$ matrix elements of $L_{\nu 3}$. For the decay $D^+ \to K^-\pi^+\bar{\ell}\nu_\ell$ computation of the Feynman diagrams in Fig. 1 gives

$$w_- = -\left(\frac{f_D m_D g}{2f^2}\right) \frac{1}{[v\cdot p_\pi + \Delta_c]} \quad w_+ = -w_- + r \tag{2.30a}$$

$$r = \left(\frac{f_D}{f^2}\right)\left\{\frac{1}{2} - \frac{1}{2}\frac{(v \cdot p_K - v \cdot p_\pi)}{[v \cdot (p_K + p_\pi) + \mu]} - \frac{g(v \cdot p_\pi)}{[v \cdot p_\pi + \Delta_c]}\right.$$
$$\left. - g^2 \frac{[p_\pi \cdot p_K - v \cdot p_K v \cdot p_\pi]}{[v \cdot (p_K + p_\pi) + \mu][v \cdot p_\pi + \Delta_c]}\right\} \tag{2.30b}$$
$$h = \left(\frac{f_D g^2}{2f^2}\right) \frac{1}{[v \cdot (p_\pi + p_K) + \Delta_c + \mu]} \frac{1}{[v \cdot p_\pi + \Delta_c]} . \tag{2.30c}$$

In eqs. (2.30)

$$\Delta_c = m_{D^*} - m_D \ , \tag{2.31a}$$

$$\mu = m_{D_s} - m_D \ , \tag{2.31b}$$

and v^μ is the four velocity of the D–meson, i.e., $p_D^\mu = m_D v^\mu$. Isospin symmetry implies that the form factors for $D^0 \to K^- \pi^0 \bar{\ell} \nu_\ell$ are $1/\sqrt{2}$ times those above, the form factors for $D^+ \to \bar{K}^0 \pi^0 \bar{\ell} \nu_\ell$ are $-1/\sqrt{2}$ times those above, and the form factors for $D^0 \to \bar{K}^0 \pi^- \bar{\ell} \nu_\ell$ are equal to those above. It is straightforward using eqs. (2.11) and (2.12) to express these form factors in terms of $\theta_K, s_{K\pi}$ and $s_{\ell\nu}$.

(ii) $D^+ \to \pi^+ \pi^- \bar{\ell} \nu_\ell$

For this decay a $Q = c$ matrix element of $L_{\nu 2}$ is needed. It is straightforward to see that the form factors in this case are given by those in eqs. (2.30) if the changes $p_K \to p_{\pi^-}$ and $p_\pi \to p_{\pi^+}$ are made and μ is set to zero. Again using eqs. (2.11) and (e.12) these form factors can be expressed in terms of $\theta_{\pi^-}, s_{\pi\pi}$ and $s_{\ell\nu}$.

(iii) $B^- \to \pi^+ \pi^- \ell \bar{\nu}_\ell$

In this case a $Q = b$ matrix element of $L_{\nu 1}$ is required. The form factors are given by those in eqs. (2.30) if the changes $f_D \to f_B, m_D \to m_B, \Delta_c \to \Delta_b, p_K \to p_{\pi^+}$ and $p_\pi \to p_{\pi^-}$ are made and μ is set to zero. Using eqs. (2.11) and (2.12) these form factors can be expressed in terms of $\theta_{\pi^+}, s_{\pi\pi}$ and $s_{\ell\nu}$.

(iv) $D^0 \to \pi^- \pi^0 \bar{\ell} \nu_\ell$

In this case the $Q = c$ matrix element of $L_{\nu 2}$ is required. Computation of the Feynman diagrams in Fig. 1 gives that the form factors are

$$w_- = \left(\frac{g f_D m_D}{2\sqrt{2} f^2}\right)\left\{\frac{1}{[v \cdot p_{\pi^-} + \Delta_c]} + \frac{1}{[v \cdot p_{\pi^0} + \Delta_c]}\right\} \tag{2.32a}$$

$$w_+ = \left(\frac{g f_D m_D}{2\sqrt{2} f^2}\right)\left\{\frac{1}{[v\cdot p_{\pi^-}+\Delta_c]}-\frac{1}{[v\cdot p_{\pi^0}+\Delta_c]}\right\}+r \tag{2.32b}$$

$$\begin{aligned} r = &\left(\frac{f_D}{\sqrt{2} f^2}\right)\left\{\frac{v\cdot(p_{\pi^-}-p_{\pi^0})}{v\cdot(p_{\pi^-}+p_{\pi^0})}+g\frac{(v\cdot p_{\pi^0})}{[v\cdot p_{\pi^0}+\Delta_c]}\right.\\ &-g\frac{(v\cdot p_{\pi^-})}{[v\cdot p_{\pi^-}+\Delta_c]}-g^2\frac{(p_{\pi^-}\cdot p_{\pi^0}-(v\cdot p_{\pi^-})(v\cdot p_{\pi^0}))}{[v\cdot(p_{\pi^-}+p_{\pi^0})]}\\ &\left.\cdot\left(\frac{1}{[v\cdot p_{\pi^-}+\Delta_c]}-\frac{1}{[v\cdot p_{\pi^0}+\Delta_c]}\right)\right\} \end{aligned} \tag{2.32c}$$

$$h = -\left(\frac{f_D g^2}{2\sqrt{2} f^2}\right)\frac{1}{[v\cdot(p_{\pi^-}+p_{\pi^0})+\Delta_c]}\left\{\frac{1}{[v\cdot p_{\pi^-}+\Delta_c]}+\frac{1}{[v\cdot p_{\pi^0}+\Delta_c]}\right\}\ . \tag{2.32d}$$

It is straightforward using eqs. (2.11) and (2.12) to express these form factors in terms of $\theta_{\pi^-}, s_{\pi\pi}$ and $s_{\ell\nu}$. (Here the difference of four-momenta $Q^\mu = p^\mu_{\pi^-} - p^\mu_{\pi^0}$.)

(v) $\bar{B}^0 \to \pi^+\pi^0\ell\bar{\nu}_\ell$

In this case the $Q = b$ matrix element of $L_{\nu 1}$ is needed. The form factors are given by those in eqs. (2.32) if the following changes are made: $f_D \to f_B, m_D \to m_B, \Delta_c \to \Delta_b$, and $p_{\pi^-} \to p_{\pi^+}$. Using eqs. (2.11) and (2.12) the form factors can be expressed in terms of $\theta_{\pi^+}, s_{\pi\pi}$ and $s_{\ell\nu}$

$\underline{B \to D\pi\ell\bar{\nu}_\ell}$

In this case matrix elements of the operator $\bar{c}\gamma_\mu(1-\gamma_5)b$ are needed. This operator is a singlet under chiral $SU(3)_L \times SU(3)_R$ and in chiral perturbation theory its matrix elements are equal to those of

$$\bar{c}\gamma_\mu(1-\gamma_5)b = -\eta(v \cdot v')Tr\bar{H}_a^{(c)}(v')\gamma_\mu(1-\gamma_5)H_a^{(b)}(v) + \dots \,. \tag{2.33}$$

The ellipsis in eq. (2.33) denotes terms with derivatives, insertions of the light quark mass matrix or factors of $1/m_Q$. The $B \to D$ and $B \to D^*$ matrix elements of this current are[10]

$$< D(v')|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v) >= \sqrt{m_B m_D}\eta(v \cdot v')[v + v']_\mu \tag{2.34a}$$

$$< D^*(v',\epsilon)|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v) >= \sqrt{m_B m_{D^*}}\eta(v \cdot v')[-\epsilon_\mu^*(1 + v \cdot v')$$

$$+(\epsilon^* \cdot v)v'_\mu + i\epsilon_{\alpha\lambda\mu\sigma}\epsilon^{*\alpha}v'^\lambda v^\sigma] \,. \tag{2.34b}$$

The normalization of η at zero recoil, i.e., $v{\cdot}v' = 1$, is determined by heavy quark flavor symmetry and by high momentum strong interaction effects that are computable using perturbative QCD methods,[10–14]

$$\eta(1) \simeq \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} \quad .$$

Since the operator in eq. (2.33) doesn't involve the pseudo-Goldstone boson fields, in the leading order of chiral perturbation theory $B \to D\pi$ matrix elements of the current are determined by the pole-type Feynman diagrams in Fig. 2. They give for a charged pion

$$w_+ - w_- = \frac{g}{f}\sqrt{m_B m_D}\eta[v \cdot v' + 1]\left\{\frac{1}{(v' \cdot p_\pi - \Delta_c)} - \frac{1}{(v \cdot p_\pi + \Delta_b)}\right\} + r \tag{2.35a}$$

$$w_+ + w_- = \frac{-g}{f}\sqrt{\frac{m_B}{m_D}}\eta\left\{\frac{p_\pi \cdot (v + v')}{(v' \cdot p_\pi - \Delta_c)}\right\} + r \tag{2.35b}$$

$$r = \frac{g}{f}\sqrt{\frac{m_D}{m_B}}\eta\left\{\frac{p_\pi \cdot (v + v')}{(v \cdot p_\pi + \Delta_b)}\right\} \tag{2.35c}$$

$$h = \frac{g}{2f}\frac{\eta}{\sqrt{m_B m_D}}\left\{\frac{1}{(v'\cdot p_\pi - \Delta_c)} - \frac{1}{(v\cdot p_\pi + \Delta_b)}\right\} . \qquad (2.35d)$$

In eqs. (2.35)

$$\Delta_c = m_{D^*} - m_D \simeq 140 MeV \qquad (2.36a)$$

$$\Delta_b = m_{B^*} - m_B \simeq 50 MeV . \qquad (2.36b)$$

The form factors for a neutral pion are obtained from the above by multiplying by $\pm 1/\sqrt{2}$.

We have assumed in writing eqs. (2.35) that the kinematic region is chosen so that $v' \cdot p_\pi$ is not too close to Δ_c. For use of the effective theory propagator to be appropriate it is necessary that

$$v'\cdot p_\pi - \Delta_c \gg m_\pi(m_\pi/2m_D) \simeq 5MeV . \qquad (2.37)$$

This also ensures that the D^* width can be neglected in the propagator (it is expected to be only about a hundred KeV).

It is convenient to reexpress some of the formulae of "review of kinematics" in a way that makes the dependence on the heavy meson masses explicit and neglects terms suppressed by m_π/m_D or m_π/m_B. Introducing the pions four-velocity $v_\pi^\mu = p_\pi^\mu/m_\pi$ we change integration variables from $s_{D\pi}$ and $s_{\ell\nu}$ to $v'\cdot v_\pi$ and $v \cdot v'$ using

$$ds_{D\pi} ds_{\ell\nu} \simeq 4 m_B m_\pi m_D^2 d(v'\cdot v_\pi) d(v\cdot v') . \qquad (2.38)$$

The form factors F_j are conveniently written in terms of dimensionless quantities $\hat{F}_j$,

$$F_j = \frac{m_B^{3/2} m_D^{1/2}}{f} g\eta(v\cdot v')\hat{F}_j . \qquad (2.39)$$

Using $\beta \simeq (2m_\pi/m_D)[(v'\cdot v_\pi)^2 - 1]^{1/2}$ and $X \simeq m_B m_D[(v\cdot v')^2-1]^{1/2}$ the differential rate (after integrating over θ_ℓ and ϕ) becomes

$$d^3\Gamma = \frac{8G_F^2 m_B^2 m_D^3 |V_{cb}|^2}{3(4\pi)^5}\left(\frac{m_\pi}{f}\right)^2 g^2\eta^2[(v'\cdot v_\pi)^2-1]^{1/2}$$

$$\cdot[(v\cdot v')^2-1]^{1/2}\{|\hat{F}_1|^2 + \sin^2\theta_D(|\hat{F}_2|^2 + |\hat{F}_3|^2)\} d(v'\cdot v_\pi) d(v'\cdot v) d\cos\theta_D . \qquad (2.40)$$

Combining eqs. (2.39), (2.35) and (2.21) the dimensionless form factors $\hat{F}_j$ are found to be:

$$\hat{F}_1 = [(v\cdot v')^2-1]^{1/2}(v+v')\cdot v_\pi \left\{\left(\frac{m_D}{m_B}\right)\frac{1}{(v\cdot v_\pi+\hat{\Delta}_b)}-\frac{1}{(v'\cdot v_\pi-\hat{\Delta}_c)}\right\}$$

$$-v'\cdot v_\pi[v\cdot v'+1][(v\cdot v')^2-1]^{1/2}\left\{\frac{1}{(v\cdot v_\pi+\hat{\Delta}_b)}-\frac{1}{(v'\cdot v_\pi-\hat{\Delta}_c)}\right\}$$

$$+\cos\theta_D[(v'\cdot v_\pi)^2-1]^{1/2}[v\cdot v'+1][v\cdot v'-m_D/m_B]\left\{\frac{1}{(v\cdot v_\pi+\hat{\Delta}_b)}-\frac{1}{(v'\cdot v_\pi-\hat{\Delta}_c)}\right\} \tag{2.41}$$

$$\hat{F}_2 = [(v'\cdot v_\pi)^2-1]^{1/2}[v\cdot v'+1][1+(m_D/m_B)^2$$

$$-2(m_D/m_B)v\cdot v']^{1/2}\left\{\frac{1}{(v\cdot v_\pi+\hat{\Delta}_b)}-\frac{1}{(v'\cdot v_\pi-\hat{\Delta}_c)}\right\} \tag{2.42}$$

$$\hat{F}_3 = -[(v'\cdot v_\pi)^2-1]^{1/2}[(v\cdot v')^2-1]^{1/2}[1+(m_D/m_B)^2-2(m_D/m_B)v\cdot v']^{1/2}$$

$$\cdot\left\{\frac{1}{(v\cdot v_\pi+\hat{\Delta}_b)}-\frac{1}{(v'\cdot v_\pi-\hat{\Delta}_c)}\right\}\ . \tag{2.43}$$

In eqs. (2.41) to (2.43)

$$v\cdot v_\pi = (v'\cdot v_\pi)(v\cdot v')-[(v'\cdot v_\pi)^2-1]^{1/2}[(v\cdot v')^2-1]^{1/2}\cos\theta_D \tag{2.44}$$

and

$$\hat{\Delta}_c = (m_{D^*}-m_D)/m_\pi,\quad \hat{\Delta}_b = (m_{B^*}-m_B)/m_\pi\ . \tag{2.45}$$

Chiral perturbation theory should be valid for $v\cdot v_\pi$ and $v'\cdot v_\pi$ not too much greater than unity. From eq. (2.44) it is clear that the kinematic region where $\cos\theta_D$ is positive yields (for given $v'\cdot v_\pi$ and $v\cdot v'$)a smaller value for $v\cdot v_\pi$. Note that because m_π and f are comparable, the rate for $B\to D\pi\ell\bar{\nu}_\ell$ is not suppressed by factors of m_π/m_D or m_π/m_B. In fact the above formulas indicate that there is a significant rate for $B\to D\pi\ell\bar{\nu}_\ell$ in the kinematic region where chiral perturbation theory is expected to be applicable (and the $D\pi$ mass is large enough to neglect the width in the virtual D^* propagator). To illustrate this we write,

$$d^3\Gamma = \frac{G_F^2 m_B^5}{192\pi^3} |V_{cb}|^2 g^2 \eta^2 d^3\hat{\Gamma} \ . \tag{2.46}$$

In Table 1 we give $d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$ for various values of $(v \cdot v')$ and $(v' \cdot v_\pi)$. Provided η does not fall off very rapidly as $v' \cdot v$ increases, the rate for $B \to D\pi\ell\bar{\nu}_\ell$, in the region where chiral perturbation theory is expected to be applicable (i.e., $v \cdot v_\pi$ and $v' \cdot v_\pi$ around unity) is comparable with what was estimated in the introduction. In Table 1 we used $\hat{\Delta}_c = 1$. The rate in the kinematic region where $v_\pi \cdot v'$ is near one is quite sensitive to the value of $\hat{\Delta}_c$. For $B^+ \to D^+\pi^-\ell\bar{\nu}_\ell$ decay $\hat{\Delta}_c = 1$ is consistent with the measured masses, but for $B^o \to D^o\pi^+\ell\bar{\nu}_\ell$ decay $\hat{\Delta}_c = 1$ is slightly less than the experimental value.

$d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$	$(v \cdot v')$	$(v' \cdot v_\pi)$
0.030	1.2	1.2
0.042	1.4	1.2
0.024	1.2	1.3
0.034	1.4	1.3
0.021	1.2	1.4
0.030	1.4	1.4
0.018	1.2	1.5
0.027	1.4	1.5

Table 1: $d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$ for various values of $(v \cdot v')$ and $(v' \cdot v_\pi)$

Validity of Chiral Perturbation Theory

Chiral perturbation theory is an expansion in momenta so our results are expected to be valid for only a limited kinematic range. For $B \to D\pi\ell\bar{\nu}_\ell$ naive dimensional analysis suggests that the expansion parameters are $(v \cdot p_\pi)/\Lambda$ and $(v' \cdot p_\pi)/\Lambda$, where Λ is a nonperturbative strong interaction scale around 1 GeV. However, it is far from clear precisely how small these quantities must be for the $B \to D\pi\ell\bar{\nu}_\ell$ differential decay rate given in eqs. (2.40) - (2.45) to be a good approximation. We do have some experience from comparisons of the predictions of chiral perturbation theory for $\pi\pi$ scattering, weak kaon decays etc., with experiment. As we shall see shortly, the situation in $B \to D\pi\ell\bar{\nu}_\ell$ decay is somewhat different.

For $B \to D\pi\ell\bar{\nu}_\ell$ the leading contribution is of order unity. One factor of p_π from the $D^*D\pi$(or $B^*B\pi$) vertex is canceled by a factor of $1/p_\pi$ from the D^*(or B^*) propagator. At the next order of chiral perturbation theory, corrections come from two sources: (i) operators in the chiral Lagrangian for strong D^* and D (or B^* and B) interactions with pions containing two derivatives or one factor of the light quark mass matrix; (ii) operators representing the weak current $\bar{c}\gamma_\mu(1-\gamma_5)b$ that contain one derivative.

For example, one term in the ellipsis of eq. (2.33) is

$$\frac{i\tilde{\eta}(v \cdot v')}{\Lambda} Tr\left(\bar{H}_a^{(c)}(v')\gamma_\mu(1-\gamma_5)H_d^{(b)}(v)\gamma^\lambda\gamma_5\right)\left[\xi^\dagger\partial_\lambda\xi - \xi\partial_\lambda\xi^\dagger\right]_{da}, \tag{2.47}$$

where $\tilde{\eta}(v \cdot v')$ is a new universal function of $v \cdot v'$. This "higher order" contribution to the current $\bar{c}\gamma_\mu(1-\gamma_5)b$ gives rise to the following changes in the form factors $w_\pm r$ and h

$$\delta(w_+ - w_-) = \frac{2}{\Lambda f}\sqrt{m_B m_D}\tilde{\eta}[v \cdot v' + 1] + \delta r \tag{2.48a}$$

$$\delta(w_+ + w_-) = \frac{2}{\Lambda f}\sqrt{\frac{m_B}{m_D}}\ \tilde{\eta}[p_\pi \cdot v] + \delta r \tag{2.48b}$$

$$\delta r = \frac{2}{\Lambda f}\sqrt{\frac{m_D}{m_B}}\ \tilde{\eta}[p_\pi \cdot v'] \tag{2.48c}$$

$$\delta h = \frac{-1}{\Lambda f}\frac{\tilde{\eta}}{\sqrt{m_B m_D}}. \tag{2.48d}$$

For the $\pi\pi$ phase shifts, the first corrections to the leading predictions of chiral perturbation theory are suppressed by s/Λ^2 and come from operators in the chiral

Lagrangian with four derivatives and from one-loop diagrams. However, for $B \to D\pi\ell\bar{\nu}_\ell$ loops do not contribute to the leading correction which is only suppressed by $v \cdot p_\pi/\Lambda$ or $v' \cdot p_\pi/\Lambda$.

There are too many higher dimension operators with unknown coefficients to make any predictions for the next order contribution to the form factors for $B \to D\pi\ell\bar{\nu}_\ell$. However, it is certainly possible that our leading prediction for the $B \to D\pi\ell\bar{\nu}_\ell$ differential decay rate is valid at the 30% level over the kinematic range displayed in Table 1. Eventually the range of validity of lowest order chiral perturbation theory for $B \to D\pi\ell\bar{\nu}_\ell$ may be determined by experiment.

Concluding Remarks

In this section the semileptonic B and D meson decays, $D \to K\pi\bar{\ell}\nu_\ell$, $D \to \pi\pi\bar{\ell}\nu_\ell$, $B \to \pi\pi\ell\bar{\nu}_\ell$ and $B \to D\pi\ell\bar{\nu}_\ell$ were considered. Chiral symmetry and heavy quark symmetry were combined to deduce the decay amplitudes in the kinematic region where the pseudo-Goldstone bosons are soft. There was earlier work on these decays that considered the implications of chiral symmetry but it did not implement heavy quark symmetry in a model independent fashion.[15]

For $B \to D\pi\ell\bar{\nu}_\ell$ decay the rate is large enough that detailed experimental study of the decay (in the kinematic regime where chiral perturbation theory is expected to be applicable) may be possible at a B factory. Table 1 gives $d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$ for various values of $v \cdot v'$ and $v' \cdot v_\pi$ (see eq. (2.46)). These indicate that the branching ratio for semileptonic $B_{\ell 4}$ decay to nonresonant $D\pi$ (in the kinematic regime where the pion is soft, i.e., $v \cdot v_\pi$ and $v' \cdot v_\pi$ around unity), is about 10^{-4}.

The results of this paper rely on heavy quark spin and flavor symmetry. There is experimental evidence from semileptonic B decay[16] and from the decays of excited charm mesons[17] that (at least in some cases) the charm quark is heavy enough for heavy quark symmetry to be applicable. However, several theoretical analyses suggest that there are large Λ_{QCD}/m_c corrections to the prediction of heavy quark symmetry for the relation between B and D meson decay constants.[18,19,20] If this is an isolated case, where the Λ_{QCD}/m_c corrections that break the flavor symmetry are anomalously large, then the results of this paper can still be used (with f_B and f_D treated as independent constants).

Semileptonic $B \to D\ell\bar{\nu}_\ell$ and $B \to D^*\ell\bar{\nu}_\ell$ decay can be utilized to check that there are not large Λ_{QCD}/m_c corrections to the expression for the $b \to c$ transition current in eq. (2.33). However, our predictions for $B \to D\pi\ell\bar{\nu}_\ell$ decay still depend on the validity of heavy quark spin-flavor symmetry for the chiral Lagrangian in eq. (2.1). The dependence on the flavor symmetry arises from the equality of the $B^*B\pi$ and $D^*D\pi$ couplings. If heavy quark flavor symmetry is *not* used then the form factors for $B \to D\pi\ell\bar{\nu}_\ell$ decay given in eq. (2.35) become

$$w_+ - w_- = \frac{\sqrt{m_B m_D}}{f}(v \cdot v' + 1)\eta \left\{ \frac{g_c}{(v' \cdot p_\pi - \Delta_c)} - \frac{g_b}{(v \cdot p_\pi + \Delta_b)} \right\}$$

$$+r\,, \tag{2.49a}$$

$$w_+ + w_- = -\frac{g_c}{f}\sqrt{\frac{m_B}{m_D}}\eta\left\{\frac{p_\pi\cdot(v+v')}{(v'\cdot p_\pi-\Delta_c)}\right\} \quad +r\,, \tag{2.49b}$$

$$r = \frac{g_b}{f}\sqrt{\frac{m_D}{m_B}}\eta\left\{\frac{p_\pi\cdot(v+v')}{(v\cdot p_\pi+\Delta_b)}\right\}\,, \tag{2.49c}$$

$$h = \frac{1}{2f}\,\frac{\eta}{\sqrt{m_B m_D}}\left\{\frac{g_c}{(v'\cdot p_\pi-\Delta_c)} - \frac{g_b}{(v\cdot p_\pi+\Delta_b)}\right\}\,. \tag{2.49d}$$

It would be interesting to use $B \to D\pi\ell\bar{\nu}_\ell$ decay to test the heavy quark flavor symmetry prediction, $g_b = g_c$.

It is not known precisely for what range of $v \cdot p_\pi$ and $v' \cdot p_\pi$ chiral perturbation theory will be valid. Our experience with light hadrons suggests that the relevant expansion paraments are roughly $v \cdot p_\pi/1GeV$ and $v' \cdot p_\pi/1GeV$. It may be possible in $B \to D\pi\ell\bar{\nu}_\ell$ to study the range of validity of chiral perturbation theory for heavy-meson pion interactions.

A number of extensions and improvements on this work are possible. The decay $B \to D^*\pi\ell\bar{\nu}_\ell$ can be considered.[3] It is interesting to explore to what extent it can also be used to fix g and to test the heavy quark flavor symmetry prediction $g = g_b = g_c$. There are computable $\alpha_s(m_b)$ and $\alpha_s(m_c)$ corrections to the form factors for the decays discussed in this paper[4,21,22] and it is worth examining their influence on the rates for $B_{\ell 4}$ and $D_{\ell 4}$ decays.

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Chapter 2. Long Distance Contribution to Some Weak Decays

2.1 Introduction

The Standard Model [1] based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ gives successful description of particle physics up to several hundred GeV's. One of its prominent features is the spontaneous breaking of electroweak symmetry down to electromagnetism $SU(2)_L \times U(1)_Y \to U(1)_{em}$ [2]. This gives the coupling of three families of quarks to the charged W-bosons through

$$\mathcal{L}_{int} = -\frac{g_2}{2\sqrt{2}}\bar{u}^i\gamma_\mu(1-\gamma_5)V^{ij}d^jW^\mu + h.c. \; . \tag{1.1}$$

Here g_2 is the $SU(2)_L$ coupling, W^μ the charged W-boson field, u^i the three up-type quarks (u, c, t), d^j the three down-type quarks (d, s, b), and V the Cabibbo-Kobayashi-Maskawa matrix. V is a 3×3 unitary matrix, arising from rotating the quark weak eigenstates to mass eigenstates. The number of degrees of freedom for V is $N_f^2 - (2N_f - 1) = (N_f - 1)^2$, where the factor $(2N_f - 1)$ comes from redefinition of the phases of quark fields (N_f = number of families of quarks.) For the standard six-quark model, there are $(3-1)^2 = 4$ degrees of freedom in the CKM matrix V. The standard parametrization uses four angles $\theta_1, \theta_2, \theta_3$ and δ. Explicitly [3]

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \tag{1.2}$$

where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$. It's possible to choose the θ_i's to lie in the first quadrant by redefining the phases of the quark fields. A phase δ not equal to zero or π gives rise to CP violation.

There is no satisfactory understanding of why the CKM angles take their values. Nevertheless, the CKM matrix plays an important role in particle physics and it is useful to know its parameters very accurately. A convenient parametrization due to

Wolfenstein [4] is

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} . \tag{1.3}$$

(Note that in this parametrization some higher order terms in powers of λ are neglected.) The upper-left 2×2 corner of V is essentially determined by the Cabibbo angle [5]. From data of nuclear β decay and semileptonic decays of kaons it is determined to be $|V_{us}| = 0.22$ [6] with the error at one percent level.

Semileptonic B decays give information on $|V_{cb}|$ and $|V_{ub}|$. At present the best information on $|V_{cb}|$ comes from comparing the data of exclusive decay $B \rightarrow D^* l\nu$ near zero recoil point (at which D^* is at rest in the B rest frame) with the prediction of the heavy quark effective theory [7]. A recent fit gives $|V_{cb}| = 0.040 \pm 0.003$ [8]. The value of $|V_{ub}|$ is determined by comparing data on the endpoint of the electron spectrum in B semileptonic decays with phenomenological models [9]. This gives $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ [6]. The error ought to be taken as only a rough measure of the large uncertainties of this method.

Information on V_{td} comes from $B\bar{B}$ mixing, $K^+ \rightarrow \pi^+\nu\bar{\nu}$ [10], $K^+ \rightarrow \pi^+\mu^+\mu^-$ [11], etc. Measurement of $\frac{\Delta M}{\Gamma} = 0.71 \pm 0.06$ [6] for $B^0 - \bar{B}^0$ mixing gives $|V_{td}|$ (after m_t is determined). But this method has large uncertainties due to unknown value of $B_B f_B^2$. The best way to constrain V_{td} perhaps is the measurement of the rate $\Gamma(K^+ \rightarrow \pi^+\nu\bar{\nu})$, for which there is a fairly clean theoretical prediction. Of course the experiment is very difficult. One may also look at the asymmetry of the decay rate for $K^+ \rightarrow \pi^+\mu^+\mu^-$ to the right-handed μ^+ versus the left-handed μ^+ (which constrains $Re(V_{td}V_{ts}^*/V_{ud}V_{us}^*) = Re(\rho - 1 + i\eta)|V_{cb}|^2$ in the Wolfenstein parametrization).

The CKM matrix also explains why there is CP violation. It was observed more than thirty years ago that the discrete symmetry of combined charge conjugation and parity is violated in neutral $K^0 - \bar{K}^0$ systems [12]. It has been a mystery why CP is violated and why the violation is so small (at 2×10^{-3} level). Since the CKM matrix is in general complex, CP violation occurs naturally. (Of course at some level the CKM matrix is merely a phenomenological description of the weak interactions among quarks. One still should try to understand the underlying physical principles which give rise to a complex CKM matrix or some other alternative CP-violating mechanism.) However, since CP violation is only observed in $K^0 - \bar{K}^0$ system, there are many beyond-the-Standard-Model CP-violating mechanisms which are not ruled out by experiment. For example, it's possible that low energy physics (described by the Standard Model) is CP-conserving, and the CP violation arises due to some new physics at very high energy scales (the superweak models [13]). So it is important to observe other independent CP-violating effects. At present the most promising experiment is to observe CP-violating asymmetries in neutral B mesons decaying into CP-eigenstates at the B-factory. This should give a clean test of the Standard Model explanation of CP violation. There are many other channels to observe CP violation in rare kaon decays, and also hyperon decays [14]. Unfortunately, for these decays the theoretical uncertainties are much larger.

The rest of this chapter is organized as follows. In the next section we discuss the contribution of the final state interaction for CP violation in weak $\Xi \to \Lambda\pi$ [15] decay. The result is that this effect is relatively small and the CP violation in the hyperon decay chain $\Xi \to \Lambda\pi \to p\pi\pi$ is likely to be dominated by the CP violation in the $\Lambda \to p\pi$ part. In section 2.3 we estimate the long distance contribution to the $K^+ \to \pi^+\nu\bar{\nu}$ decay [16]. This contribution is also small, which makes this a promising channel to provide information on V_{td}. In section 2.4 we look at the two-photon contribution to the polarization in $K^+ \to \pi^+\mu^+\mu^-$ decay.

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2.2 Strong $\Lambda\pi$ Phase Shifts for CP Violation in Weak $\Xi \to \Lambda\pi$ Decay

In the standard six-quark model CP violation arises from the phase δ in the Cabibbo-Kobayashi-Maskawa matrix. So far CP violation has only been observed in second order weak $K^0 - \bar{K}^0$ mixing and it is not known if this arises from the phase δ or from some new interaction associated with a very large mass scale. The latter possibility leads to a superweak scenerio for CP violation [1]. Observation of CP violation in a first order weak decay amplitude (sometimes referred to as direct CP violation) would rule out the superweak model as its sole origin. Avenues for detecting CP violation in first order weak decay amplitudes are the measurement of a nonzero value for the parameter ϵ' and the measurement of asymmetries in B decay. Recently it has been proposed to measure direct CP violation in the nonleptonic hyperon decay chain $\Xi \to \Lambda\pi \to p\pi\pi$ at Fermilab [2]. To observe CP violation in this case requires not only a CP violating phase in the weak hyperon decay amplitudes but also a phase from final state interactions. In this letter we calculate the strong $\Lambda\pi$ phase shifts that are important for observing CP violation in Ξ nonleptonic weak decay using baryon chiral perturbation theory. An interesting aspect of our work is that we are able to make predictions using only chiral $SU(2)_L \times SU(2)_R$ symmetry by utilizing the measured $\Sigma^- \to \Lambda e\bar{\nu}_e$ decay rate to determine the magnitude of the $\Sigma\Lambda\pi$ coupling. Strong phase shifts relevant for weak $\Xi \to \Lambda\pi$ decay were first calculated about 30 years ago and we compare our results with these earlier calculations [3-5]. One important difference between our approach and the previous work is that we don't rely on $SU(3)$ symmetry. We find that the S-wave phase shift vanishes at leading order in chiral perturbation theory and that the P-wave $J = \frac{1}{2}$ phase shift is only -1.7°. This suggests that CP violation in the recently proposed hyperon decay experiment at Fermilab will be dominated by the $\Lambda \to p\pi$ part of the decay chain.

Nonleptonic $\Xi \to \Lambda\pi$ decay is characterized by S-wave and P-wave amplitudes which we denote respectively by S and P. The differential decay rate (in the Ξ rest frame) has the form

$$\begin{aligned}\frac{d\Gamma}{d\Omega} \;\propto\; 1 \;-\; \alpha\,(\hat{s}_\Xi \cdot \hat{p}_\pi \;+\; \hat{s}_\Lambda \cdot \hat{p}_\pi) \;+\; \beta\,\hat{p}_\pi \cdot (\hat{s}_\Xi \times \hat{s}_\Lambda) \\ +\; \gamma\,\hat{s}_\Xi \cdot \hat{s}_\Lambda \;+\; (1-\gamma)\,(\hat{s}_\Lambda \cdot \hat{p}\pi)\,(\hat{s}_\Xi \cdot \hat{p}_\pi) \quad ,\end{aligned} \tag{2.1}$$

where $\hat{s}_\Xi$ and $\hat{s}_\Lambda$ are unit vectors along the direction of the Ξ and Λ spins and $\hat{p}_\pi$ is a unit vector along the direction of the pion momentum. The parameters α, β and γ which characterize the decay distribution are expressed in terms of the S-wave and P-wave amplitudes as follows;

$$\alpha = \frac{2\,\mathrm{Re}\,(S^*P)}{|S|^2+|P|^2}\,, \tag{2.2}$$

$$\beta = \frac{2\,\mathrm{Im}\,(S^*P)}{|S|^2+|P|^2}\,, \tag{2.3}$$

$$\gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}\,. \tag{2.4}$$

The S-wave and P-wave amplitudes are complex numbers

$$S = |S|e^{i\delta_S} \quad , \quad P = |P|e^{i\delta_P}\,. \tag{2.5}$$

In terms of the modulus and phase of S and P the parameters α and β are

$$\alpha = 2\frac{|S||P|}{|S|^2+|P|^2}\cos(\delta_S-\delta_P)\,, \tag{2.6}$$

$$\beta = -2\frac{|S||P|}{|S|^2+|P|^2}\sin(\delta_S-\delta_P)\,. \tag{2.7}$$

Isospin symmetry ensures that S and P for the decays $\Xi^- \to \Lambda\pi^-$ and $\Xi^0 \to \Lambda\pi^0$ are related by a factor of $\sqrt{2}$. The quantities δ_S and δ_P are respectively equal (up to a factor of π) to the strong interaction S-wave and $J=\frac{1}{2}$ P-wave $\Lambda\pi$ phase shifts plus small but important contributions from direct weak interaction CP violation [6].

The decay distribution for $\bar{\Xi} \to \bar{\Lambda}\pi$ is also given by eq.(2.1) with parameters $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ whose expression in terms of the S-wave and P-wave amplitudes $\bar{S}$ and $\bar{P}$ are similar to eqs.(2.2), (2.3) and (2.4). The only difference is that the analog of eqs.(2.2) and (2.3) have a minus sign. The Λ's produced in the decay of unpolarised Ξ's have a polarisation α (as seen in eq.(2.1)) and an important measure of CP violation is the asymmetry

$$\mathcal{A} = \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} . \tag{2.8}$$

In terms of the phases of the S-wave and P-wave amplitudes the above becomes

$$\mathcal{A} = \frac{\cos(\delta_S - \delta_P) - \cos(\delta_{\bar{S}} - \delta_{\bar{P}})}{\cos(\delta_S - \delta_P) + \cos(\delta_{\bar{S}} - \delta_{\bar{P}})} . \tag{2.9}$$

We denote the $J = \frac{1}{2}$ S-wave and P-wave $\Lambda\pi$ phase shifts by δ_0 and δ_1 respectively and write

$$\delta_S - \delta_P = \delta_0 - \delta_1 + \phi_{CP} + \pi , \tag{2.10a}$$

$$\delta_{\bar{S}} - \delta_{\bar{P}} = \delta_0 - \delta_1 - \phi_{CP} + \pi , \tag{2.10b}$$

with ϕ_{CP} the phase that results from direct weak interaction CP violation. Data from $\Xi^- \to \Lambda\pi^-$ decay and $\Xi^0 \to \Lambda\pi^0$ decay give (neglecting CP violation) $\delta_0 - \delta_1 = (8 \pm 8)^\circ$ and $\delta_0 - \delta_1 = (38^{+12}_{-19})^\circ$ respectively. Putting eqs.(2.10) into eq.(2.9) yields

$$\mathcal{A} = -\tan\phi_{CP} \cdot \tan(\delta_0 - \delta_1) . \tag{2.11}$$

Eq.(2.11) indicates that the CP violating observable $\mathcal{A}$ is small if the difference of phase shifts $\delta_0 - \delta_1$ is small. In this letter we calculate the strong interaction $\Lambda\pi$ phase shifts δ_0 and δ_1 using chiral perturbation theory.

In chiral perturbation theory the low energy strong interactions of pions are described by the chiral Lagrangian. (See Chapter 1 for a detailed description.) For the following calculation all is needed is the chiral $SU(2)_L \times SU(2)_R$ symmetry. It is convenient to introduce the following combination of meson fields

$$(A^\mu)^b_a = \frac{i}{2}(\xi \partial^\mu \xi^\dagger - \xi^\dagger \partial^\mu \xi)^b_a , \tag{2.12a}$$

$$(V^\mu)^b_a = \frac{i}{2}(\xi \partial^\mu \xi^\dagger + \xi^\dagger \partial^\mu \xi)^b_a , \tag{2.12b}$$

where $\xi^2 = \Sigma = \exp(2iM/f)$. Note that V^μ contains terms with an even number of pion fields and A^μ contains terms with an odd number of pion fields.

The baryon fields we use are the spin $\frac{1}{2}$ isosinglet Λ, the spin $\frac{1}{2}$ isotriplet Σ_{ab} ($\Sigma_{11} = \Sigma^+$, $\Sigma_{12} = \Sigma_{21} = \frac{1}{\sqrt{2}}\Sigma^0$ and $\Sigma_{22} = \Sigma^-$) and the spin $\frac{3}{2}$ isotriplet $\Sigma^{*\mu}_{ab}$ (the assignment of the Σ^*'s to $\Sigma^{*\mu}_{ab}$ is analogous to the assignment of the Σ's to Σ_{ab}). Under chiral $SU(2)_L \times SU(2)_R$ these fields transform as

$$\Lambda \to \Lambda \ , \tag{2.13a}$$

$$\Sigma^{(*\mu)}_{ab} \to U_{ac}U_{bd}\Sigma^{(*\mu)}_{cd} \ , \tag{2.13b}$$

where repeated roman indices a,b,... are summed over 1,2. Strong interactions of these baryons with pions are described by a chiral Lagrangian that is invariant under parity and chiral $SU(2)_L \times SU(2)_R$ symmetry. Expanding in derivatives this chiral Lagrangian density is [7,8],

$$\mathcal{L} = \mathcal{L}_\Lambda + \mathcal{L}_\Sigma + \mathcal{L}_{\Sigma^*} + \mathcal{L}_{int} \ , \tag{2.14}$$

where

$$\mathcal{L}_\Lambda = \bar{\Lambda}\ iv \cdot \partial\ \Lambda, \tag{2.15a}$$

$$\mathcal{L}_\Sigma = \bar{\Sigma}^{ab}\ iv \cdot \partial\ \Sigma_{ab} + 2\bar{\Sigma}^{ab}\ v \cdot V^c_a\ \Sigma_{cb} + (m_\Lambda - m_\Sigma)\bar{\Sigma}^{ab}\ \Sigma_{ab} \ , \tag{2.15b}$$

and

$$\mathcal{L}_{int} = g_{\Sigma\Lambda}\ \bar{\Lambda}\ S \cdot A^b_a\ \Sigma_{cb}\ \epsilon^{ac} + g_{\Sigma^*\Lambda}\ \bar{\Lambda}\ A^b_a \cdot\ \Sigma^*_{bc}\ \epsilon^{ac} + h.c. \ . \tag{2.15c}$$

The expression for $\mathcal{L}_{\Sigma^*}$ is similar to eq.(2.15b). In eqs.(2.15) S is the spin operator four-vector, ϵ^{ac} is the antisymmetric tensor, $\epsilon^{11} = \epsilon^{22} = 0$, $\epsilon^{12} = -\epsilon^{21} = 1$, and v is the baryon four-velocity. There are also interaction terms with one derivative involving two $\Sigma^{(*)}$ fields and an odd number of pions. However these interactions are not needed for our computation. We treat $m_\Sigma - m_\Lambda$ and $m_{\Sigma^*} - m_\Lambda$ as small quantities. For power counting purposes these mass differences are considered to be the same order as a single derivative.

The magnitude of the couplings $g_{\Sigma^*\Lambda}$ and $g_{\Sigma\Lambda}$ can be determined from experiment. Comparison of the measured $\Sigma^{*+} \to \Lambda\pi^+$ decay width with

$$\Gamma(\Sigma^{*+} \to \Lambda\pi^+) = g^2_{\Sigma^*\Lambda}\ \frac{1}{6\pi}\frac{|\vec{p}_\pi|^3}{f^2}\frac{m_\Lambda}{m_{\Sigma^*}} \ , \tag{2.16}$$

gives $g^2_{\Sigma^*\Lambda} \simeq 1.49$. There is a Goldberger-Treiman type relation that relates matrix elements of the axial current to the $\Sigma\Lambda\pi$ coupling $g_{\Sigma\Lambda}$. Using the Noether procedure we find that in chiral perturbation theory matrix elements of the left-handed current are given by

$$\bar{u}\gamma^\mu(1-\gamma_5)d \;= g_{\Sigma\Lambda}\;\bar{\Lambda}\;S^\mu\;\Sigma^-\;+\ldots\quad . \tag{2.17}$$

In eq.(2.17) the ellipses denote pieces involving other baryon fields, the pion fields and terms with derivatives. The resulting $\Sigma^- \to \Lambda e\bar{\nu}_e$ decay rate is

$$\Gamma(\Sigma^- \to \Lambda e\bar{\nu}_e) \;=\; \frac{G_F^2}{80\pi^3}|V_{ud}|^2 g_{\Sigma\Lambda}^2\,(m_\Sigma - m_\Lambda)^5\;. \tag{2.18}$$

Comparing with the measured $\Sigma^- \to \Lambda e\bar{\nu}_e$ decay rate yields $g_{\Sigma\Lambda}^2 \;\simeq\; 1.44$.

The Feynman diagrams in Fig.3 determine the S-wave and $J = \frac{1}{2}$ P-wave $\Lambda\pi$ phase shifts at the leading order of chiral perturbation theory. As a function of the pion energy in the center of mass frame,* E_π, we find the S-wave phase shift to be

$$\delta_0(E_\pi) \;=\; 0\;, \tag{2.19}$$

and the $J = \frac{1}{2}$ P-wave phase shift to be

$$\delta_1(E_\pi) \;=\; -\frac{(E_\pi^2 - m_\pi^2)^{3/2}}{12\pi f^2}\cdot\Big[\frac{1}{4}\frac{g_{\Sigma\Lambda}^2}{E_\pi + m_\Sigma - m_\Lambda} + \frac{3}{4}\frac{g_{\Sigma\Lambda}^2}{E_\pi + m_\Lambda - m_\Sigma} - \frac{4}{3}\frac{g_{\Sigma^*\Lambda}^2}{E_\pi + m_{\Sigma^*} - m_\Lambda}\Big]\;. \tag{2.20}$$

The expression inside the bracket for $\delta_1(E_\pi)$ is singular at the unphysical pion energy $E_\pi = m_\Sigma - m_\Lambda$ because of the Σ pole. (When the energy is near this value other terms we have neglected become important and tame the singularity.) Note that there is no singularity at $E_\pi = m_{\Sigma^*} - m_\Lambda$ as there is no Σ^* pole in the $J = \frac{1}{2}$ channel. The S-wave phase shift vanished because Λ is an isospin-zero baryon (hence there is no coupling to two pions in $\mathcal{L}_\Lambda$) and because the $\Sigma\Lambda\pi$ and $\Sigma^*\Lambda\pi$ interactions have the pions derivatively coupled. At higher order in chiral perturbation theory we expect an S-wave phase shift suppressed by a factor of order E_π/Λ_χ (where Λ_χ

⋆ In heavy baryon chiral perturbation theory the centre of mass frame and the baryon rest frame coincide.

is the chiral symmetry breaking scale) compared, for example, with the S-wave pion-nucleon phase shifts. At $E_\pi \sim 200$MeV the pion-nucleon S-wave phase shifts are several degrees. A contribution to the S-wave $\Lambda\pi$ phase shift suppressed by E_π/Λ_χ arises from higher derivative terms, e.g.,

$$\mathcal{L}_{\text{higher}} = \frac{c}{\Lambda_\chi} \overline{\Lambda}\Lambda \; A^a_b \cdot A^b_a \quad . \tag{2.21}$$

The coefficient c is expected to be of order unity, but as it is an unknown quantity the S-wave phase shift at this order is not calculable. Previous calculations [5] did not find a small S-wave $\Lambda\pi$ phase shift. Fig.4 contains a plot of the $J = \frac{1}{2}$ P-wave phase shift δ_1 as a function of E_π. For the hyperon decay $\Xi \to \Lambda\pi$ we need the phase shift evaluated at $E_\pi \simeq m_\Xi - m_\Lambda = 206$MeV. At this energy the $J = \frac{1}{2}$ P-wave phase shift is $\delta_1 = -1.7^\circ$. This is within a factor of two of the value for δ_1 obtained in previous calculations [3,5].

Our predictions for the phase shifts do not make use of chiral $SU(3)_L \times SU(3)_R$ symmetry. However, chiral perturbation theory is an expansion in E_π and our result for $\delta_1(m_\Xi - m_\Lambda)$ relies on $m_\Xi - m_\Lambda$ and hence the strange quark mass being small compared with the chiral symmetry breaking scale.

The smallness of the $J = \frac{1}{2}$ P-wave phase shift $\delta_1(E_\pi)$ is partly the result of a cancellation between the Feynman diagrams involving the Σ and Σ^*. This cancellation becomes exact in the large N_c limit [8,9] where $m_\Sigma = m_{\Sigma^*} = m_\Lambda$ and $g^2_{\Sigma^*\Lambda} = \frac{3}{4} g^2_{\Sigma\Lambda}$.

Our calculations indicate that for the weak decay $\Xi \to \Lambda\pi$ the difference between the S- and $J = \frac{1}{2}$ P-wave phase shifts $\delta_0 - \delta_1$ and consequently the CP violating asymmetry $\mathcal{A}$ are small. Therefore, it is likely that any CP violation observed in the recently proposed Fermilab experiment will be dominated by CP violation in the $\Lambda \to p\pi$ part of the $\Xi \to \Lambda\pi \to p\pi\pi$ decay chain.

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2.3 Long Distance Contribution to $K^+ \to \pi^+\nu\bar{\nu}$

The decay $K^+ \to \pi^+\nu\bar{\nu}$ is likely to be observed in the near future. An accurate measurement of its branching ratio can provide a precise determination of the weak mixing angle V_{td} (once the t-quark mass is known). The general form for the $K^+ \to \pi^+\nu\bar{\nu}$ invariant matrix element is

$$\mathcal{M}(K^+ \to \pi^+\nu\bar{\nu}) = \frac{G_F}{\sqrt{2}}\frac{\alpha f_+}{2\pi\sin^2\theta_W}\Bigg[V_{ts}^* V_{td}\xi_t(m_t^2/M_W^2)$$

$$V_{cs}^* V_{cd}\xi_c(m_c^2/M_W^2) + V_{us}^* V_{ud}\xi_{LD}\Bigg](p_K + p_\pi)^\mu \bar{u}(p_\nu)\gamma_\mu(1-\gamma_5)v(p_{\bar{\nu}}) \ . \qquad (3.1)$$

In eq. (3.1) G_F is the Fermi constant, θ_W is the weak mixing angle, α is the fine structure constant, f_+ is the form factor in $\bar{K}^0 \to \pi^+ e\bar{\nu}_e$ decay and V_{ab} denotes the $a \to b$ element of the Cabibbo-Kobayashi-Maskawa matrix. The factors ξ_t and ξ_c arise from the short distance contribution to $K^+ \to \pi^+\nu\bar{\nu}$ and ξ_{LD} from the long distance contribution. Neglecting perturbative strong interaction corrections [1]

$$\xi_t(x) = \frac{x}{8}\left[-\frac{2+x}{1-x} + \frac{3x-6}{(1-x)^2}\ln x\right] \ , \qquad (3.2a)$$

$$\xi_c(x) \simeq \frac{x}{8}\left[-6\ln x - 2\right] \ . \qquad (3.2b)$$

Recently QCD corrections to $\xi_t(x)$ of order $\alpha_s(m_t)$ have been calculated [2] and the QCD corrections to $\xi_c(x)$ have been summed in the next to leading logarithmic approximation [2,3]. The value of ξ_c is about 10^{-3} . (This is for ν_e and ν_μ neutrinos. A somewhat smaller value is obtained for ν_τ because the τ lepton mass cannot be neglected in the W-box diagram.) With the next to leading logarithms included the largest uncertainty in ξ_c comes from our imprecise knowledge of the charm quark mass and Λ_{QCD} .

This paper contains an estimate of ξ_{LD} using chiral perturbation theory. Previous estimates of this type were made by Rein and Sehgal [4] and Hagelin and Littenberg [5]. Our work is similar to theirs in approach and conclusions, however, some of the details are different.

One of the most prominent features of the pattern of kaon (and hyperon) decays is the $\Delta I = 1/2$ rule. Nonleptonic kaon decay amplitudes that arise from the $I = 1/2$ part of the $\Delta S = 1$ effective weak Hamiltonian are enhanced by a factor of twenty over those that arise from the $I = 3/2$ part. In this letter we focus primarily on the part of ξ_{LD} that arises from the time ordered product of the weak $\Delta S = 1$ effective Hamiltonian with the Z^0 neutral current, since it receives a $\Delta I = 1/2$ enhancement. The Z^0 coupling to light u, d, and s quarks is given by

$$\mathcal{L}_{int} = \frac{\sqrt{g_1^2+g_2^2}}{2} Z^{0\mu}\ [J_\mu^{(L)} - 2\ \sin^2\theta_{\rm W} J_\mu^{(e.m.)}]\ , \tag{3.3}$$

where $J_\mu^{(L)}$ is the left-handed current

$$J_\mu^{(L)} = \bar{u}_L\gamma_\mu u_L - \bar{d}_L\gamma_\mu d_L - \bar{s}_L\gamma_\mu s_L\ , \tag{3.4}$$

(the part of $J_\mu^{(L)}$ involving the strange quark was neglected in Ref. [4]) and $J_\mu^{(e.m.)}$ is the electromagnetic current

$$J_\mu^{(e.m.)} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s\ . \tag{3.5}$$

The left-handed current $J_\mu^{(L)}$ can be written as the sum of a piece that transforms as $(8_L, 1_R)$ and a piece that transforms as $(1_L, 1_R)$ with respect to the chiral symmetry group $SU(3)_L \times SU(3)_R$.

$$J_\mu^{(L)} = J_{8\mu}^{(L)} + J_{1\mu}^{(L)}\ , \tag{3.6}$$

where

$$J_{8\mu}^{(L)} = \frac{4}{3}\bar{u}_L\gamma_\mu u_L - \frac{2}{3}\bar{d}_L\gamma_\mu d_L - \frac{2}{3}\bar{s}_L\gamma_\mu s_L\ , \tag{3.7}$$

$$J_{1\mu}^{(L)} = -\frac{1}{3}\bar{u}_L\gamma_\mu u_L - \frac{1}{3}\bar{d}_L\gamma_\mu d_L - \frac{1}{3}\bar{s}_L\gamma_\mu s_L\ . \tag{3.8}$$

The electromangnetic current transforms as $(8_L, 1_R) + (1_L, 8_R)$ under $SU(3)_L \times SU(3)_R$.

The interactions of kaons, pions, and the eta are constrained by chiral symmetry. At low momentum they are described by an effective chiral Lagrangian, which was given by eq. (1.23) in chapter 1. The enhanced part of the effective Hamiltonian for weak $\Delta S = 1$ nonleptonic kaon decays transforms as $(8_L, 1_R)$ under chiral symmetry and is given by

$$\mathcal{H}^{|\Delta S=1|} = \frac{g_8 G_F f^4}{4\sqrt{2}} V_{us}^* V_{ud} \mathrm{Tr} O_W \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + h.c. \;, \tag{3.9}$$

where

$$O_W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \;, \tag{3.10}$$

projects out the correct part of the octet and the measured $K_S \to \pi^+\pi^-$ decay amplitude determines that $|g_8| \simeq 5.1$.

$J_{8\mu}^{(L)}$ and $J_\mu^{(e.m.)}$ are currents associated with generators of chiral symmetry. At leading order in chiral perturbation theory the $K^+ \to \pi^+ Z^0$ vertex that arises from the Z^0 coupling to these currents is obtained by gauging the chiral Lagrangian density (eq. (1.23) in chapter 1) and the Hamiltonian density (3.9) via the replacement

$$\partial_\mu \Sigma \to D_\mu \Sigma = \partial_\mu \Sigma + i\sqrt{g_1^2 + g_2^2} Z_\mu^0 \left(Q\Sigma - \sin^2\theta_\mathrm{W}[Q, \Sigma]\right) \;, \tag{3.11}$$

where

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \tag{3.12}$$

is the electromagnetic charge matrix, and computing the tree level Feynman diagrams in Fig. 5 (using the interactions that follow from eqs. (3.9-12) and (1.23) in chapter 1). In Fig. 5 the incoming dashed line denotes the K^+, the outgoing dashed line denotes the π^+ and the wiggly line denotes the Z^0. (Only some of the diagrams in Fig. 5 were considered by Ref. [5].) We are interested in the part of the $K^+ \to \pi^+ Z^0$ vertex proportional to $(p_K + p_\pi)^\mu$. The part proportional to $(p_K - p_\pi)^\mu$ doesn't contribute to $K^+ \to \pi^+ \nu\bar{\nu}$ since the neutrinos are massless. The Z^0 coupling to $J_\mu^{(e.m.)}$ doesn't give rise to a $K^+ \to \pi^+ Z^0$ vertex at the leading order of chiral perturbation theory (the three diagrams in Fig. 5 cancel [6]). The Z^0 coupling to $J_{8\mu}^{(L)}$ gives

$$\frac{G_F}{2\sqrt{2}}\sqrt{g_1^2+g_2^2}\; g_8 V_{us}^* V_{ud} f^2 \left[0+1-\frac{2}{3}\right] (p_K+p_\pi)^\mu \;, \tag{3.13}$$

for the $K^+ \to \pi^+ Z^0$ vertex (m_u and m_d are neglected here). The three terms 0, 1 and $-2/3$ in the square brackets of eq. (3.13) arise from Figs. (5a), (5b) and (5c) respectively. The long distance contribution to $K^+ \to \pi^+ \nu\bar{\nu}$ that results from the vertex in eq. (3.13) is

$$\xi_{LD}^{(8)} = \frac{g_8 \pi^2}{3} (f/M_W)^2 \;. \tag{3.14}$$

Numerically eq. (3.14) is about 5×10^{-5} which is only 5% of the charm quark short distance contribution, ξ_c.

The $K^+ \to \pi^+ Z^0$ vertex arising form the Z^0 coupling to $J_{1\mu}^{(L)}$ cannot be calculated using chiral perturbation theory alone because through the anomaly instanton field configurations in the QCD path integral spoil the axial $U(1)$ symmetry. (It is expected to have the same order of magnitude as eq. (3.13).) However, in the large N_c limit [7] effects of the anomaly are suppressed and the axial $U(1)$ is a good symmetry [8]. Then the Z^0 coupling to $J_{1\mu}^{(L)}$ is taken into account by adding to the covariant derivative in eq. (3.11) the term

$$i\sqrt{g_1^2+g_2^2} Z_\mu^0 \; (-\frac{1}{6}\Sigma) \;. \tag{3.15}$$

In the large N_c limit the coupling of the Z^0 to $J_{1\mu}^{(L)}$ gives rise to the $K^+ \to \pi^+ Z^0$ vertex

$$\frac{G_F}{2\sqrt{2}}\sqrt{g_1^2+g_2^2}\; g_8 V_{us}^* V_{ud} f^2 \left[0+0-\frac{1}{3}\right] (p_K+p_\pi)^\mu \;. \tag{3.16}$$

The three terms 0, 0 and $-1/3$ in the square brackets of eq. (3.16) come respectively from Figs. (5a), (5b) and (5c). This implies that the Z^0 coupling to $J_{1\mu}^{(L)}$ gives a contribution to ξ_{LD} that satisfies

$$\lim_{N_c \to \infty} \xi_{LD}^{(1)} = -\frac{g_8 \pi^2}{3} (f/M_W)^2 \;. \tag{3.17}$$

This cancels the contribution in eq. (3.14).

In the large N_c limit the leading (in chiral perturbation theory) long distance contribution to $K^+ \to \pi^+\nu\bar{\nu}$ from Feynman diagrams with a W boson and a Z^0 boson vanishes. However, in the large N_c limit the η' is a pseudo-Goldstone boson ($m_{\eta'}^2$ is of order $1/N_c$). It's large mass, 958 MeV, is an indication that this limit is not trustworthy [9]. Nonetheless, it seems likely that some remnant of the cancellation that occurs as $N_c \to \infty$ survives in the physical case of three colors. We do not expect the long distance contribution to $K^+ \to \pi^+\nu\bar{\nu}$ from Feynman diagrams with a W and Z^0 to exceed eq. (3.14). Contributions from Feynman diagrams with two W bosons are expected to be even smaller. They are not enhanced by the factor g_8 which reflects the $\Delta I = 1/2$ rule.

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2.4 Two-Photon Contribution to Polarization in $K^+ \to \pi^+\mu^+\mu^-$

In the minimal standard model the unitarity of the Cabibbo-Kobayashi-Maskawa matrix V gives that

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \ . \tag{4.1}$$

We can think of each of the three complex numbers ($V_{ud}V_{ub}^*$, etc.) on the l.h.s. of eq. (4.1) as vectors in the complex plane. These vectors add to zero and so by translating them they form the sides of a triangle that is often called the unitarity triangle. With the parametrization of the Cabibbo-Kobayashi-Maskawa matrix in eq. (1.2) we have

$$V_{ud}V_{ub}^* \simeq -s_1 s_3 \tag{4.2a}$$

$$V_{td}V_{tb}^* \simeq -s_1 s_2 e^{-i\delta} \tag{4.2b}$$

$$V_{cd}V_{cb}^* \simeq s_1(s_3 + s_2 e^{-i\delta}) \ . \tag{4.2c}$$

The unitarity triangle specifies the angles θ_2, θ_3 and δ. From eqs. (4.2) it is clear that the length of two sides gives θ_2 and θ_3 while the angle between two of the sides is $\pi - \delta$.

The orientation of the unitarity triangle in the complex plane depends on the phase convention in the Cabibbo-Kobayashi-Maskawa matrix. The length of the sides and the angles at each vertex α, β, γ are independent of the phase convention. When there is no CP violation the unitarity triangle collapses to a line. One common orientation for the triangle has $V_{cd}V_{cb}^*$ lying along the real axis. It is conventional to rescale the side on the real axis to unit length and locate one vertex at the origin of the complex plane. This is shown in Fig. 6. With this convention the unitarity triangle is specified by the coordinates in the complex plane, $\rho + i\eta$, of the vertex associated with the angle α.

It is important to determine the unitary triangle by measuring quantities that do not violate CP. The resulting values of the weak mixing angles can then be used to predict the expected values of CP violating quantities. In this way the standard six-quark model for CP violation can be tested. At the present time it is not known if the CP violation observed in kaon decays is due to the phase in the Cabibbo-Kobayashi-Maskawa matrix or from new physics, beyond that in the minimal standard model, or both.

B-meson decays give valuable information on the unitarity triangle. However, rare kaon decays where a virtual top quark plays an important role can also be useful. For example, an accurate measurement of the branching ratio for $K^+ \to \pi^+\nu\bar{\nu}$ would restrict the α corner of the unitarity triangle to lie on a circle.

In Ref. [3] it was pointed out that the measurement of polarization in $K^+ \to \pi^+\mu^+\mu^-$ decay can also lead to valuable information on the weak mixing angles. The dominant contribution to the $K^+ \to \pi^+\mu^+\mu^-$ decay amplitude comes from Feynman diagrams where a single photon produces the $\mu^+\mu^-$ pair. Even though the weak interactions violate parity maximally the one-photon part of the decay amplitude is necessarily parity conserving and doesn't contribute to the parity violating asymmetry $\Delta_{LR} = (\Gamma_R - \Gamma_L)/(\Gamma_R + \Gamma_L)$, where Γ_R and Γ_L are the rates to produce right and left-handed μ^+ respectively.* This parity violating asymmetry arises predominantly from two sources:

(i) the interference of W-box and Z-penguin Feynman diagrams (see Figure 7) with the one-photon piece.

(ii) the interference of Feynman diagrams where two photons create the $\mu^+\mu^-$ pair with the one-photon piece.

If the short distance W-box and Z-penguin part dominates the asymmetry then its measurement can lead to important information on the unitarity triangle. The main purpose of this paper is to examine the long-distance two-photon contribution to the $K^+ \to \pi^+\mu^+\mu^-$ decay amplitude and in particular its influence on the parity violating asymmetry Δ_{LR}. Ref. [3] also noted that there are T-odd asymmetries which involve both the μ^+ and μ^- polarizations and can arise from the interference of the Z-penguin and W-box Feynman diagrams with the one photon piece. Detailed predictions for the short distance contribution to these T-odd asymmetries were made in Ref. [4]. Here we stress that the T-odd asymmetries also receive a contribution from the interference of the absorptive part of the parity violating the two-photon contribution with the one-photon piece.

$\star$ By right- (or left-) handed we mean that the spin is directed along (or opposite) the direction of motion, i.e., helicity $+1/2$ (or $-1/2$).

Kinematics

The dominant part of the $K^+ \to \pi^+\mu^+\mu^-$ decay amplitude comes from Feynman diagrams where a single photon produces the $\mu^+\mu^-$ pair. The one-photon contribution to the invariant matrix element has the form

$$\mathcal{M}^{(pc)} = \frac{s_1 G_F}{\sqrt{2}} \alpha f(s)(p_K + p_\pi)^\mu \bar{u}(p_-, s_-)\gamma_\mu v(p_+, s_+) \ , \tag{4.3}$$

where p_K and p_π are the four-momentum of the kaon and pion and $p_\pm$ are the four-momenta of the $\mu^\pm$. In eq. (4.3) $s_\pm$ are the spin vectors for the $\mu^\pm$ while $\sqrt{s}$ is the invariant mass of the $\mu^+\mu^-$ pair

$$s = (p_+ + p_-)^2 \ . \tag{4.4}$$

We shall parametrize the differential decay rate in terms of s and θ the angle between the three-momentum of the kaon and the three-momentum of the μ^- in the $\mu^+\mu^-$ pair rest frame. In terms of these variables the inner products of four-momenta are

$$p_- \cdot p_+ = s/2 - m_\mu^2 \ , \tag{4.5a}$$

$$(p_K + p_\pi)^2 = 2(m_K^2 + m_\pi^2) - s \ , \tag{4.5b}$$

$$2p_+ \cdot (p_K + p_\pi) = (m_K^2 - m_\pi^2) + \sqrt{1 - \frac{4m_\mu^2}{s}}[(s + m_K^2 - m_\pi^2)^2 - 4sm_K^2)]^{1/2} \cos\theta \ . \tag{4.5c}$$

For a right or left-handed μ^+ the dot products of the polarization four-vector s_+^μ with the μ^- and kaon four-momenta are

$$s_+^{(R)} \cdot p_- = -s_+^{(L)} \cdot p_- = \frac{s}{2m_\mu}\sqrt{1 - \frac{4m_\mu^2}{s}} \ , \tag{4.6a}$$

$$s_+^{(R)} \cdot p_K = -s_+^{(L)} \cdot p_K = \frac{1}{4m_\mu}\left\{\sqrt{1 - \frac{4m_\mu^2}{s}}(s + m_K^2 - m_\pi^2) + [(s + m_K^2 - m_\pi^2)^2 - 4sm_K^2]^{1/2} \cos\theta\right\} \ . \tag{4.6b}$$

The total differential decay rate is dominated by the one photon piece and the invariant amplitude in eq. (4.3) gives

$$d(\Gamma_R + \Gamma_L)/d\cos\theta ds = \frac{s_1^2 G_F^2 \alpha^2 |f(s)|^2}{2^9 m_K^3 \pi^3}\sqrt{1-\frac{4m_\mu^2}{s}}[(m_K^2 - m_\pi^2 + s)^2 - 4sm_K^2]^{3/2}$$

$$\left[1-\left(1-\frac{4m_\mu^2}{s}\right)\cos^2\theta\right] . \tag{4.7}$$

The parity violating part of the decay amplitude has the form

$$\mathcal{M}^{(pv)} = \frac{s_1 G_F \alpha}{\sqrt{2}}[B(p_K + p_\pi)^\mu + C(p_K - p_\pi)^\mu]$$

$$\cdot\bar{u}(p_-, s_-)\gamma_\mu\gamma_5 v(p_+, s_+) . \tag{4.8}$$

The parameters B and C in eq. (4.8) get contributions from the Z-penguin and W-box Feynman diagrams as well as from Feynman diagrams with two photons.

The difference in decay amplitudes for right and left-handed μ^+ arises from the interference of the parity conserving part of the decay amplitude in eq. (4.3) with the parity violating part of the decay amplitude in eq. (4.8). This gives

$$d(\Gamma_R - \Gamma_L)/d\cos\theta ds = \frac{-s_1^2 G_F^2 \alpha^2}{2^8 m_K^3 \pi^3}\sqrt{1-\frac{4m_\mu^2}{s}}[(s + m_K^2 - m_\pi^2)^2 - 4sm_K^2]$$

$$\left\{ Re(f^*(s)B)\sqrt{1-\frac{4m_\mu^2}{s}}[(s+m_K^2-m_\pi^2) - 4sm_K^2]^{1/2}\sin^2\theta \right.$$

$$\left. +4\left[Re(f^*(s)B)\left(\frac{m_K^2 - m_\pi^2}{s}\right) + Re(f^*(s)C)\right] m_\mu^2\cos\theta \right\} . \tag{4.9}$$

Note that in eq. (4.9) the contribution of C vanishes when the difference of decay rates is integrated over θ.

The Parity Conserving Amplitude

The parity conserving amplitude arises predominantly from Feynman diagrams where a single photon produces the $\mu^+\mu^-$ pair. It is characterized by the function $f(s)$ introduced in eq. (4.3). The absolute value of $f(s)$ has been determined by experimental data on $K^+ \to \pi^+ e^+ e^-$. A good fit to the differential decay rate is obtained from[5]

$$|f(s)| = |f(0)|(1 + \lambda s/m_\pi^2) , \tag{4.10}$$

with $\lambda = 0.11$ and $|f(0)| = 0.31$.

Using chiral perturbation theory, the imaginary part of $f(s)$, arises from the Feynman diagrams in Fig. 8, with the pions in the loop on their mass shell. The strong interactions of the pseudo-Goldstone bosons π, K and η are described by the effective chiral Lagrangian

$$\mathcal{L} = \frac{f^2}{8} Tr \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + v Tr(m_q \Sigma + \Sigma^\dagger m_q) + \dots . \tag{4.11}$$

(For explanation of various terms in eq. (4.11), see eqs. (1.20-24) in chapter 1.) In Fig. 8 a shaded circle denotes an interaction vertex arising from the strong interaction effective Lagrangian density in eq. (4.11). The effective Lagrangian for $\Delta s = 1$ weak nonleptonic decays transforms under chiral $SU(3)_L \times SU(3)_R$ as $(8_L, 1_R) + (27_L, 1_R)$. In terms of Σ the $(8_L, 1_R)$ part of the effective Lagrangian density* for weak nonleptonic kaon decays is

$$\mathcal{L} = g_8 \frac{G_F}{4\sqrt{2}} s_1 f^4 Tr O_W \partial^\mu \Sigma \partial_\mu \Sigma^\dagger + \dots , \tag{4.12}$$

where

$$O_W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} . \tag{4.13}$$

The measured $K_S \to \pi^+\pi^-$ decay rate implies that[6] $|g_8| \simeq 5.1$. In Fig. 8 a shaded square denotes an interaction vertex from the effective Lagrangian in eq. (4.12). The

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⋆ It dominates over the $(27_L, 1_R)$ part of the Lagrangian.

Feynman diagrams in Fig. 8 give[6]

$$Imf(s) = -(g_8/24)(1 - 4m_\pi^2/s)^{3/2}\theta(s - 4m_\pi^2) \ . \tag{4.14}$$

The imaginary part of $f(s)$ is largest at the maximum value of s, $s_{max} = (m_K - m_\pi)^2$. Eq. (4.14) implies that (up to a sign) $Imf(s_{max}) \simeq 0.05$ and so the imaginary part of $f(s)$ is expected to be much smaller than its real part.

Chiral perturbation theory also predicts $Ref(s)$ up to a s independent constant that is determined by the total decay rate.[6] The measured s dependence given in eq. (4.10) is somewhat greater than what chiral perturbation theory gives but the experimental error is still quite large, i.e., $\lambda = 0.105 \pm 0.035 \pm 0.015$.

Short Distance Contribution to the Parity Violating Amplitude

The Z-penguin and W-box Feynman diagrams contribute to both B and C of the parity violating amplitude in eq. (11). Explicitly

$$B = f_+(s)\xi \qquad\qquad C = f_-(s)\xi \ , \tag{4.15}$$

where $f_+(s)$ and $f_-(s)$ are the form factors for $K_{\ell 3}$ semileptonic decay. Conventionally their s-dependence is parametrized by $f_\pm(s) = f_\pm(0)(1+\lambda_\pm s/m_\pi^2)$. We use[2] $f_+(0) = 1.02, \lambda_+ = 0.03, f_-(0) = -0.17$ and $\lambda_- = 0$. ξ is a quantity that, apart from mixing angles, is essentially the same as occurs in $B \to X_s e^+e^-$. As noted in Ref. [3] it is given by

$$\xi \simeq -\tilde{\xi}_c + \left(\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}\right)\tilde{\xi}_t \ , \tag{4.16}$$

where

$$\tilde{\xi}_q = \tilde{\xi}_q^{(Z)} + \tilde{\xi}_q^{(W)} \ , \tag{4.17}$$

is the sum of the contributions of the Z-penguin (superscript Z) and W-box (superscript W). In eq. (4.17)

$$\tilde{\xi}_t^{(Z)} = \frac{x}{\sin^2\theta_W}\,\frac{1}{16\pi}\left[\frac{(x-6)(x-1)+(3x+2)\ln x}{(x-1)^2}\right] \tag{4.18a}$$

$$\tilde{\xi}_t^{(W)} = \frac{x}{\sin^2\theta_W}\,\frac{1}{8\pi}\left[\frac{x-1-\ln x}{(x-1)^2}\right] \ , \tag{4.18b}$$

with $x = m_t^2/M_W^2$ and

$$\tilde{\xi}_c^{(Z)} \simeq \frac{\eta^{(Z)}}{\sin^2\theta_W}\,\frac{1}{8\pi}\left(\frac{m_c^2}{M_W^2}\right)\ln(m_c^2/M_W^2) \tag{4.19a}$$

$$\tilde{\xi}_c^{(W)} \simeq -\frac{\eta^{(W)}}{\sin^2\theta_W}\,\frac{1}{8\pi}\left(\frac{m_c^2}{M_W^2}\right)\ln(m_c^2/M_W^2) \ . \tag{4.19b}$$

The QCD correction factors $\eta^{(Z)}$ and $\eta^{(W)}$ have been computed in the leading logarithmic approximation[7] and they have the values $\eta^{(Z)} \simeq 0.3$ and $\eta^{(W)} \simeq 0.6$. Using $m_c = 1.5$ GeV and $M_W = 81$ GeV and $\sin^2\theta_W = 0.23$ equations (4.19a) and (4.19b) imply that $\tilde{\xi}_c = 1.4 \times 10^{-4}$. The value of $\tilde{\xi}_t$ depends sensitively on the top quark mass. For $m_t = 140$ GeV, $\tilde{\xi}_t \simeq 0.51$ and for $m_t = 200$ GeV, $\tilde{\xi}_t \simeq 0.89$. Notice that ξ depends on the weak mixing angles $V_{ts}^* V_{td}/V_{us}^* V_{ud} = (\rho - 1 + i\eta)|V_{cb}|^2$. Therefore, the information on ξ would yield information on the weak mixing angles.

Since $Imf(s)$ is small (provided the two-photon contribution to the parity violating amplitude is negligible) measurement of the polarization asymmetry Δ_{LR} determines the value of ρ restricting the α vertex of the unitary triangle to lie on a vertical line in the $\rho - \eta$ plane. Integrating over the whole available phase space we find that the interference of the short distance contribution to the parity violating amplitude with the parity conserving part implies that⋆

$$|\Delta_{LR}| = |2.3 Re\xi| \ . \tag{4.20}$$

For $m_t = 140$ GeV and $\rho = -0.51$ this gives $|\Delta_{LR}| = 3.7 \times 10^{-3}$ while for $m_t = 200$ GeV and $\rho = -0.12$ this gives $|\Delta_{LR}| = 4.7 \times 10^{-3}$.

The magnitude of the asymmetry Δ_{LR} is larger for $\cos\theta$ positive than for $\cos\theta$ negative as eq. (4.9) indicates. Hence, the asymmetry can be increased by a cut on $\cos\theta$. If $\cos\theta$ is restricted to lie in the region

$$-0.5 < \cos\theta < 1.0 \ , \tag{4.21}$$

the asymmetry arising from the interference of the short distance parity violating amplitude with the parity conserving part is

$$|\Delta_{LR}| = |4.1 Re\xi| \ . \tag{4.22}$$

For $m_t = 140$ GeV, and $\rho = -0.51$ this gives $|\Delta_{LR}| = 6.5 \times 10^{-3}$ while for $m_t = 200$ GeV and $\rho = -0.12$, eq. (4.22) implies that $|\Delta_{LR}| = 8.3 \times 10^{-3}$. This cut increases the magnitude of the asymmetry by almost a factor of two and reduces the number events by only a factor of 0.77. In Fig. 9 we show the constraint on ρ extracted from a Δ_{LR} measurement (with the cut in eq. (4.21)) for some values of the top quark mass

⋆ This differs slightly from the result of Ref. [3] because in this paper the measured s-dependence of $f(s)$ has been used.

and asymmetry. The values of the asymmetry and top quark mass are chosen to be compatible with the measured value for $B^0 - \bar{B}^0$ mixing when $\sqrt{B_B} f_B$ lies between 120 MeV and 250 MeV. ξ is dominated by the top quark loop for the values of the asymmetry shown in Fig. 9.

Two-Photon Contribution to the Parity Violating Amplitude

In this section we use chiral perturbation theory to examine the two-photon contribution to the parity violating form factors B and C. There are local operators that can contribute to the parity violating $K^+ \to \pi^+\mu^+\mu^-$ amplitude. At the leading order of chiral perturbation theory they are included in the effective Lagrange density

$$\mathcal{L} = \frac{iG_F\alpha s_1}{\sqrt{2}}\bar{\mu}\gamma_\mu\gamma_5\mu\big[\gamma_1 Tr(O_W Q^2\Sigma\partial^\mu\Sigma^\dagger)$$

$$+\gamma_2 Tr(O_W\partial^\mu\Sigma Q^2\Sigma^\dagger - O_W\Sigma Q^2\partial^\mu\Sigma^\dagger)$$

$$+\gamma_3 Tr(O_W\partial^\mu\Sigma Q\Sigma^\dagger Q - O_W\Sigma Q\partial^\mu\Sigma^\dagger Q)\big]\ . \tag{4.23}$$

In eq. (4.23) Q is the electromagnetic charge matrix

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix}\ . \tag{4.24}$$

Each term contains two factors of Q because the Lagrange density in eq. (30) arises from Feynman diagrams with two photons. When the photons (and other virtual particles) are off-shell by an amount that is large compared with the pseudo-Goldstone boson masses their effects are reproduced by those of the local operators in eq. (4.23). CPS symmetry[18] has been used to reduce the effective Lagrangian to the form in eq. (4.23). Under a CPS transformation

$$\Sigma(\vec{x},t) \to S\Sigma^*(-\vec{x},t)S\ , \tag{4.25}$$

where S is the matrix that switches strange and down quarks

$$S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\ . \tag{4.26}$$

It is CPS symmetry that forces the two terms in the last two traces of eq. (4.23) to occur with a relative minus sign (the linear combination with a relative plus sign is not invariant under CPS). Expanding out the Σ matrices in terms of the pseudo-Goldstone boson fields it is easy to see that the effective Lagrange density in eq. (4.23) gives a contribution to B proportional to $\gamma_1 - 8\gamma_2 - 4\gamma_3$, but gives *no* contribution to C. We shall not be able to predict B using chiral perturbation theory as γ_1, γ_2 and γ_3 are not known.

CPS symmetry forces the contribution to C from local operators (without factors of m_q) to vanish. This symmetry is broken by the difference between strange and down quark masses. In the pole type graphs of Fig. 10 the quark masses cannot be neglected and it is these diagrams that (in chiral perturbation theory) give the dominant contribution to C. In Fig. 10 the shaded square is an interaction vertex from the weak $\Delta s = 1$ Lagrangian in eq. (4.12), the shaded circle is a $\eta\gamma\gamma$ or $\pi^0\gamma\gamma$ vertex from the Wess-Zumino term[19]

$$\mathcal{L}_{WZ} = \frac{\alpha}{4\pi f}\epsilon_{\mu\nu\lambda\sigma}F^{\mu\nu}F^{\lambda\sigma}(\pi^0/\sqrt{2} + \eta/\sqrt{6}) + \dots \; . \tag{4.27}$$

The cross denotes a $\eta\mu^+\mu^-$ or $\pi^0\mu^+\mu^-$ vertex that arises from the local terms in the effective Lagrange density for strong and electromagnetic interactions

$$\mathcal{L} = \frac{i\alpha^2}{4\pi^2}\bar{\mu}\gamma^\mu\gamma_5\mu\big[\chi_1 Tr(Q^2\Sigma^\dagger\partial_\mu\Sigma - Q^2\partial_\mu\Sigma^\dagger\Sigma)$$

$$+\chi_2 Tr(Q\Sigma^\dagger Q\partial_\mu\Sigma - Q\partial_\mu\Sigma^\dagger Q\Sigma)\big] \; , \tag{4.28}$$

that couple a π^0 or η to a $\mu^+\mu^-$ pair.

In the Feynman diagrams of Fig. 10 the "infinite part" of the loop integrals is cancelled by the terms from eq. (4.28) yielding the following prediction for C

$$C = \frac{g_8}{12}\left\{\frac{3m_\eta^2 - m_\pi^2 - 2m_{K^+}^2}{s - m_\eta^2}\right\}\mathcal{A}(s) \; , \tag{4.29}$$

where

$$Re\mathcal{A}(s) = \frac{\alpha}{4\pi^2}\bigg\{w + \frac{1}{2}(s/m_\mu^2) - \frac{1}{4}(s/m_\mu^2)^2$$

$$+(s/m_\mu^2)ln(s/m_\mu^2) + \frac{1}{2}(s/m_\mu^2)^2 ln(s/m_\mu^2)$$

$$
-\int_0^1 dx \left[3 + \frac{2[(s/4m_\mu^2) - 1]\sqrt{x}}{\sqrt{x + (4m_\mu^2/s)(1-x)}}\right] \lambda_+^2 ln|\lambda_+/2|
$$

$$
-\int_0^1 dx \left[3 - \frac{2[(s/4m_\mu^2) - 1]\sqrt{x}}{\sqrt{x + (4m_\mu^2/s)(1-x)}}\right] \lambda_-^2 ln|\lambda_-/2| \Bigg\} , \tag{4.30a}
$$

and*

$$
Im\mathcal{A}(s) = \frac{\alpha}{\pi} \frac{1}{\sqrt{1-(4m_\mu^2/s)}} ln \left(\frac{1 + \sqrt{1-(4m_\mu^2/s)}}{2m_\mu/\sqrt{s}} \right) . \tag{4.30b}
$$

The Feynman diagrams in Fig. 10 give no contribution to B. In eq. (4.30a) w is a constant independent of s and

$$
\lambda_\pm = \sqrt{x(s/m_\mu^2)} \pm \sqrt{x(s/m_\mu^2) + 4(1-x)} . \tag{4.31}
$$

The constant w gets contributions both from the one-loop diagrams and from the tree diagrams in Fig. 10. It can be determined from the relative strength of the decays $\eta \to \gamma\gamma$ and $\eta \to \mu^+\mu^-$. At the leading order of chiral perturbation theory

$$
\Gamma(\eta \to \gamma\gamma) = \frac{\alpha^2 m_\eta^3}{96\pi^3 f^2} , \tag{4.32}
$$

and

$$
\Gamma(\eta \to \mu^+\mu^-) = \frac{|\alpha\mathcal{A}(m_\eta^2)|^2}{48\pi} \left(\frac{m_\mu}{f}\right)^2 \sqrt{m_\eta^2 - 4m_\mu^2} . \tag{4.33}
$$

The recent measurement[21] of the branching ratio for $\eta \to \mu^+\mu^-, Br(\eta \to \mu^+\mu^-) = (5 \pm 1) \times 10^{-6}$, is within 1σ of the unitarity limit which is 4.3×10^{-6} (arising from an on shell two-photon intermediate state.) The measured branching ratio for $\eta \to \mu^+\mu^-$ implies that $|ReA(m_\eta^2)| < 2.5 \times 10^{-3}$ which gives $-2 < w < 25$. Using the cut on $cos\theta$, given in eq. (4.21), we find that the two-photon contribution of the parity violating form factor C, to the asymmetry satisfies, $|\Delta_{LR}| < 1.2 \times 10^{-3}$. Improving the measurement of the branching ratio for $\eta \to \mu^+\mu^-$ would reduce the uncertainty in w and consequently improve our knowledge of the two-photon contribution to C.

⋆ The imaginary part is related to the unitarity limit for $\eta \to \mu^+\mu^-$. This was computed in Ref. [20]. The real part of the $\eta \to \mu^+\mu^-$ amplitude was also computed in Ref. [20] using a phenomenological model for the form factor associated with the $\eta \to \gamma\gamma$ vertex.

If the short distance contribution to the asymmetry Δ_{LR} (with the cut on $\cos\theta$ given in eq. (4.21)) is at the $\frac{1}{2}\%$ level then it is likely that the two-photon contribution to C can be neglected. (Of course, if the full range of $\cos\theta$ is used then the contribution of C to the asymmetry vanishes.) We have not been able to predict using chiral perturbation theory, the two-photon contribution to the parity violating form factor B. However, we do not expect its influence on Δ_{LR} (with the cut on $\cos\theta$ given in eq. (4.21)) to be larger than that of C. (Our naive expectation is that it gives $|\Delta_{LR}| \sim O(\alpha/\pi) \sim 2\times 10^{-3}$.) It would be interesting to try to estimate the two photon contribution to B using phenomenological models. Experimental information on the decay $K^+ \to \pi^+\gamma\gamma$ may also prove useful.

There are T-odd asymmetries that involve both the μ^+ and μ^- polarizations. They will be much more difficult to measure than the parity violating asymmetry we have been discussing. The T-odd asymmetries also violate parity and are determined by $ImBf^*(s)$ and $ImCf^*(s)$. They get a contribution from the interference of the two photon contribution to the imaginary part of C, given in eqs. (4.29) and (4.30),with the real part of the parity conserving amplitude (as well as from short distance physics).

Concluding Remarks

We have calculated the two-photon contribution to the parity violating $K^+ \to \pi^+\mu^+\mu^-$ decay amplitude arising from the diagrams in Fig. 10. They give rise to an invariant matrix element with Lorentz structure $(p_K - p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$ and do not contribute to the other possible form for the parity violating amplitude, $(p_K + p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$. CPS symmetry of the chiral Lagrangian forces the contact terms (that arise from Feynman diagrams where the virtual particles have large momentum) to have the structure $(p_K + p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$. Therefore, the diagrams in Fig. 10 give the leading value for the coefficient of $(p_K - p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$ in chiral perturbation theory. The prediction of chiral perturbation theory contains an s-independent constant that is fixed by the measured $\eta \to \mu^+\mu^-$ decay rate. Improving the experimental value for the $\eta \to \mu^+\mu^-$ branching ratio would reduce the uncertainty in this constant and hence improve our prediction for the coefficient of $(p_K - p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$. Unfortunately we cannot compute the coefficient of $(p_K + p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$ using chiral perturbation theory since there are several local contact terms that contribute to it which we cannot fix experimentally. These contact terms also contribute to the $K_L \to \mu^+\mu^-$ decay amplitude, but for this amplitude they enter in a different linear combination than for the $K^+ \to \pi^+\mu^+\mu^-$ matrix element and furthermore the measured $K_L \to \mu^+\mu^-$ branching ratio is not accurate enough to provide a significant constraint.

If all the available phase space is integrated over then the $(p_K - p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$ piece of the parity violating decay amplitude does not contribute to the parity violating asymmetry $\Delta_{LR} \equiv (\Gamma_R - \Gamma_L)/(\Gamma_R + \Gamma_L)$. However, it is advantageous to make the cut, $-0.5 < \cos\theta < 1$, since it increases the short distance contribution to the asymmetry by almost a factor of two and diminishes the number of events by only a factor of 0.77. With this cut the measured $\eta \to \mu^+\mu^-$ branching ratio implies that the two photon contribution to Δ_{LR} from the diagrams in Fig. 10 satisfies $|\Delta_{LR}| < 1.2 \times 10^{-3}$. This asymmetry is much less than the asymmetry arising from short distance physics involving virtual top and charm quarks, provided that ρ is negative. For ρ positive, the Feynman diagrams in Fig. 10 may contribute a non-negligable portion of the asymmetry. It seems likely to us that the asymmetry coming from the two photon contribution to the part of the $K^+ \to \pi^+\mu^+\mu^-$ decay amplitude of the form $(p_K + p_\pi)^\mu \bar{u}\gamma_\mu\gamma_5 v$ is not much larger than that arising from the diagrams in Fig. 10. Our naive expectation is that it gives rise to an asymmetry $|\Delta_{LR}| \sim O(\alpha/\pi) \sim 2 \times 10^{-3}$. It would be interesting to estimate this part of the parity violating $K^+ \to \pi^+\mu^+\mu^-$ decay amplitude using phenomenological models. (Such calculations may reveal a further physical suppression of this amplitude.) Experimental information on the decay $K^+ \to \pi^+\gamma\gamma$ could also be valuable.

The asymmetry Δ_{LR} can provide information on the unitarity triangle. Even an experimental limit at the percent level would provide interesting information on ρ. This may be within the reach of a dedicated experiment at existing facilities.[22]

Short distance physics contributes to T-odd (and P-odd) correlations involving both the μ^+ and μ^- polarizations. We have found that the imaginary part of the Feynman diagrams in Fig. 5 (that arises from on shell photons) also contributes. A crude measure of the importance of this long-distance physics contribution is the ratio, $r(s) = ImC(s)/|f(s)|$. Using eqs. (4.10), (4.29) and (4.30b) we find, for example, that $r(4m_\pi^2) \simeq -5 \times 10^{-3}$. The effect of the long distance contribution to $ImC(s)$ should be included in analysis of the implications of measuring these T-odd correlations.

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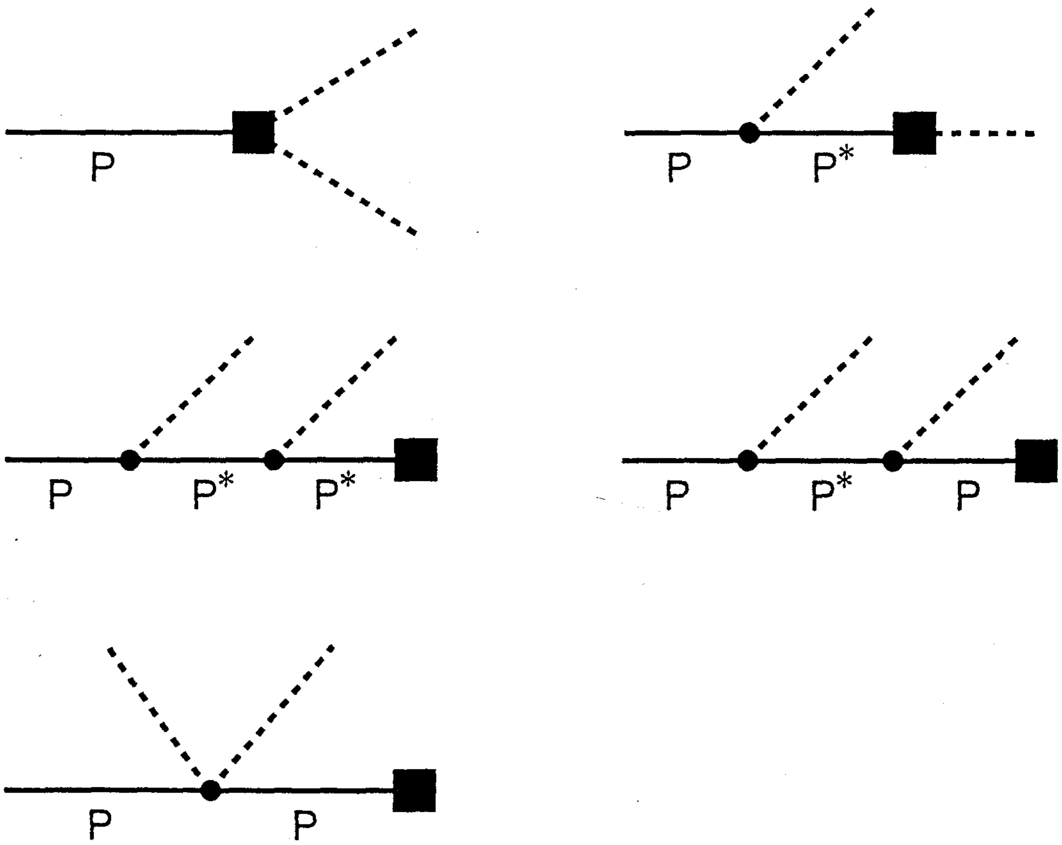

Figure 1. Feynman diagrams for $D \to K\pi, D \to \pi\pi$ and $B \to \pi\pi$ matrix elements of the current $L_{\nu a}$. The shaded square denotes an insertion of the current in eq. (2.27) of section 1.2. Dashed lines denote pseudo-Goldstone bosons.

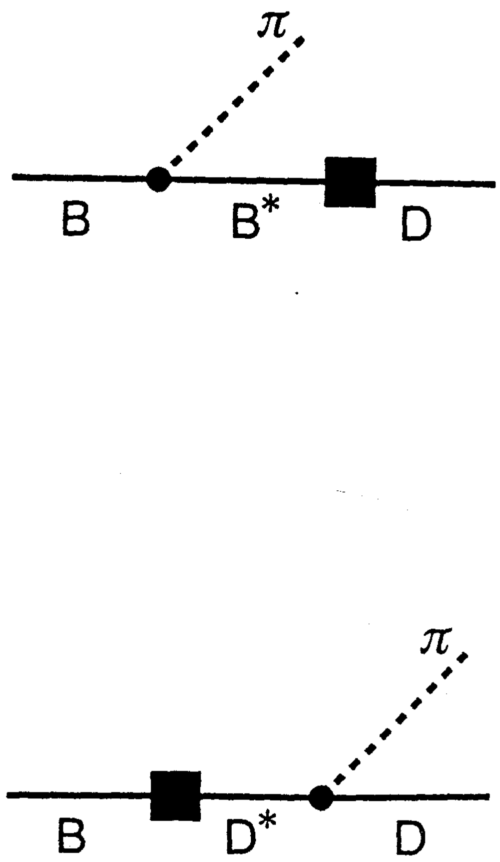

Figure 2. Feynman diagrams for $B \to D\pi$ matrix element of $\bar{c}\gamma_\nu(1-\gamma_5)b$. The shaded square denotes an insertion of the current in eq. (2.33) of section 1.2.

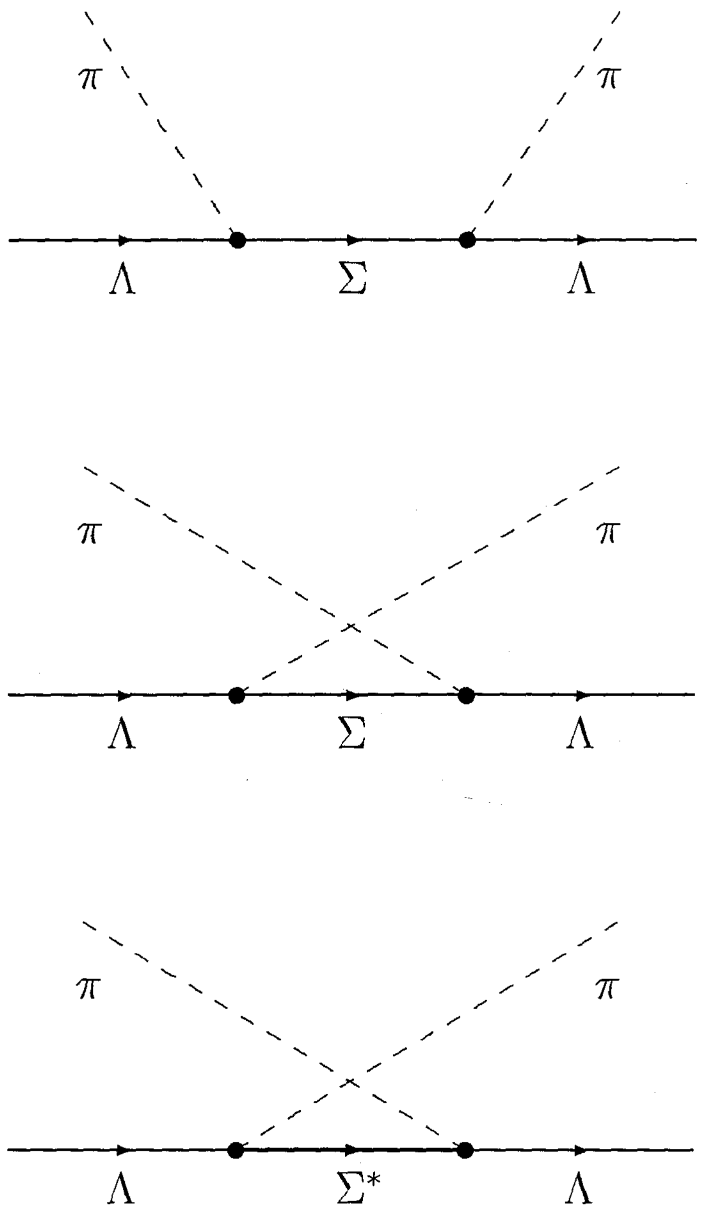

Figure 3. Feynman diagrams contributing to the $J = \frac{1}{2}$ P-wave $\Lambda\pi$ phase shift δ_1. There is no contribution to the S-wave phase shift at leading order in chiral perturbation theory.

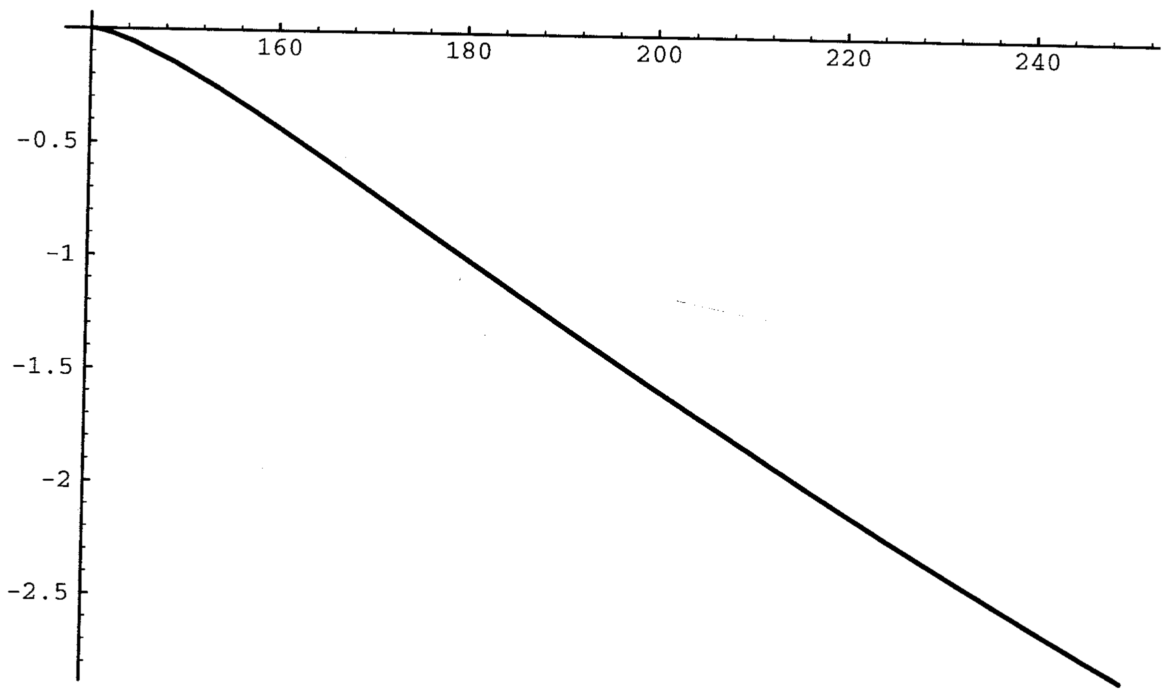

Figure 4. The $J = \frac{1}{2}$ P-wave phase shift δ_1 (in degrees) as a function of pion energy E_π (in MeV).

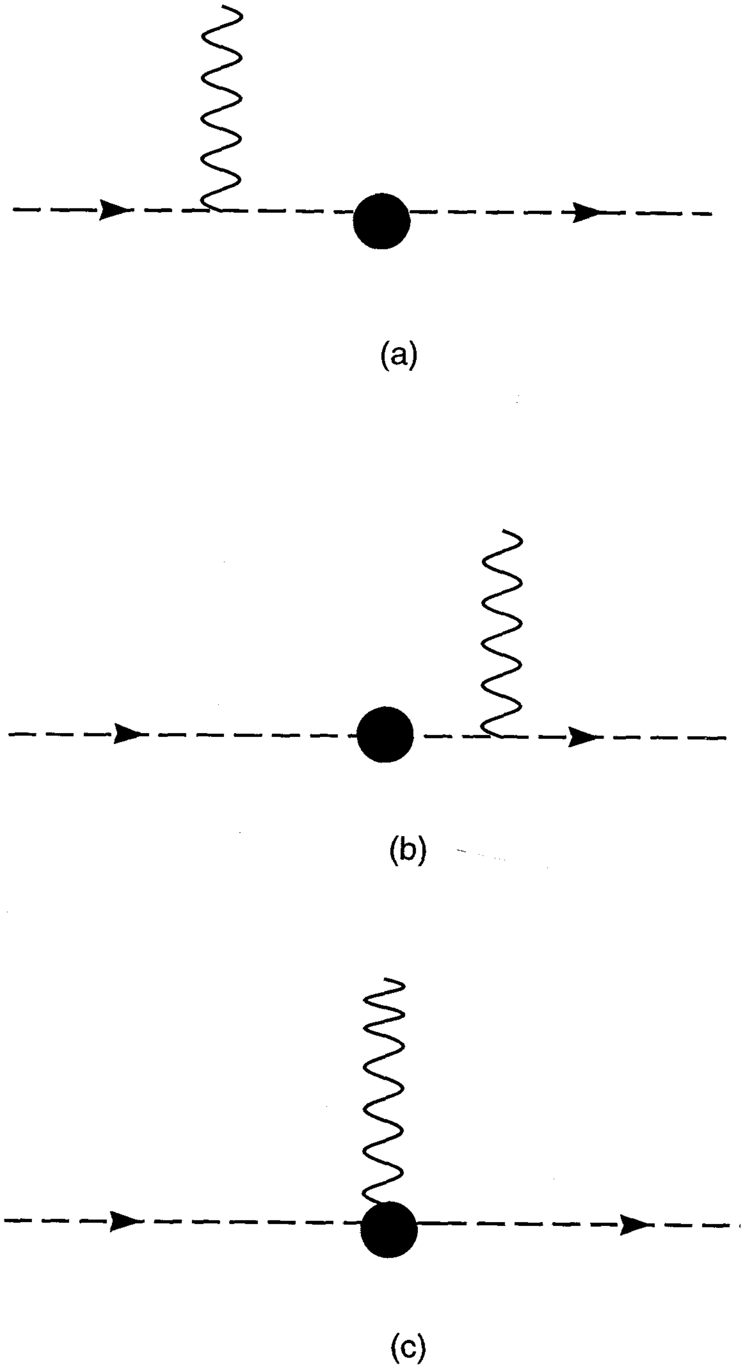

Figure 5. Feynman diagrams that dominate the $K^+ \to \pi^+ Z^0$ vertex in chiral perturbation theory. The shaded circle denotes an insertion of the weak $\Delta S = 1$ nonleptonic Hamiltonian in eq. (3.11) of chapter 2.

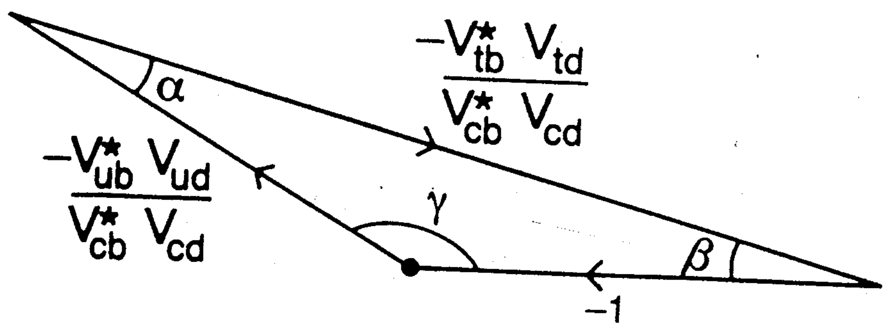

Figure 6. The unitary triangle.

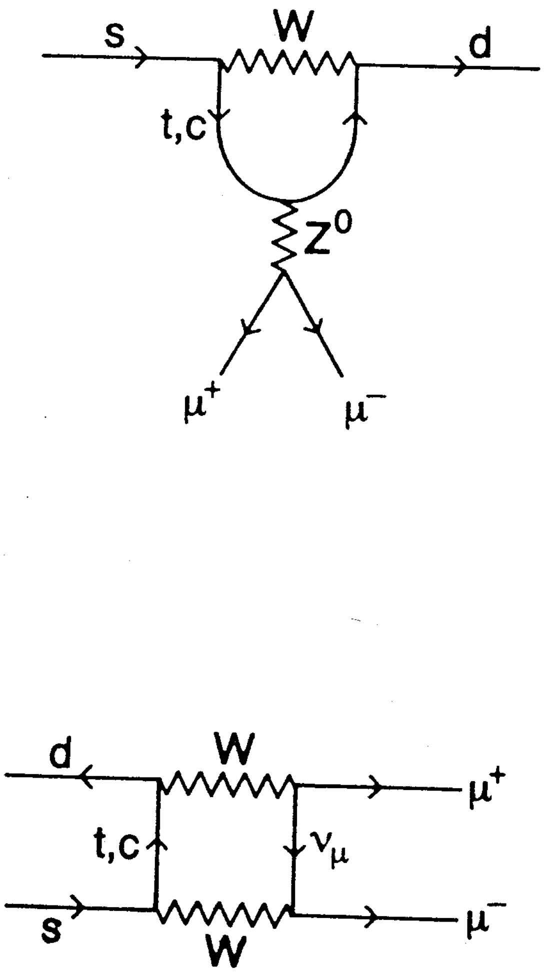

Figure 7. Z-penguin and W-box Feynman diagrams that contribute to the $K^+ \rightarrow \pi^+\mu^+\mu^-$ decay amplitude.

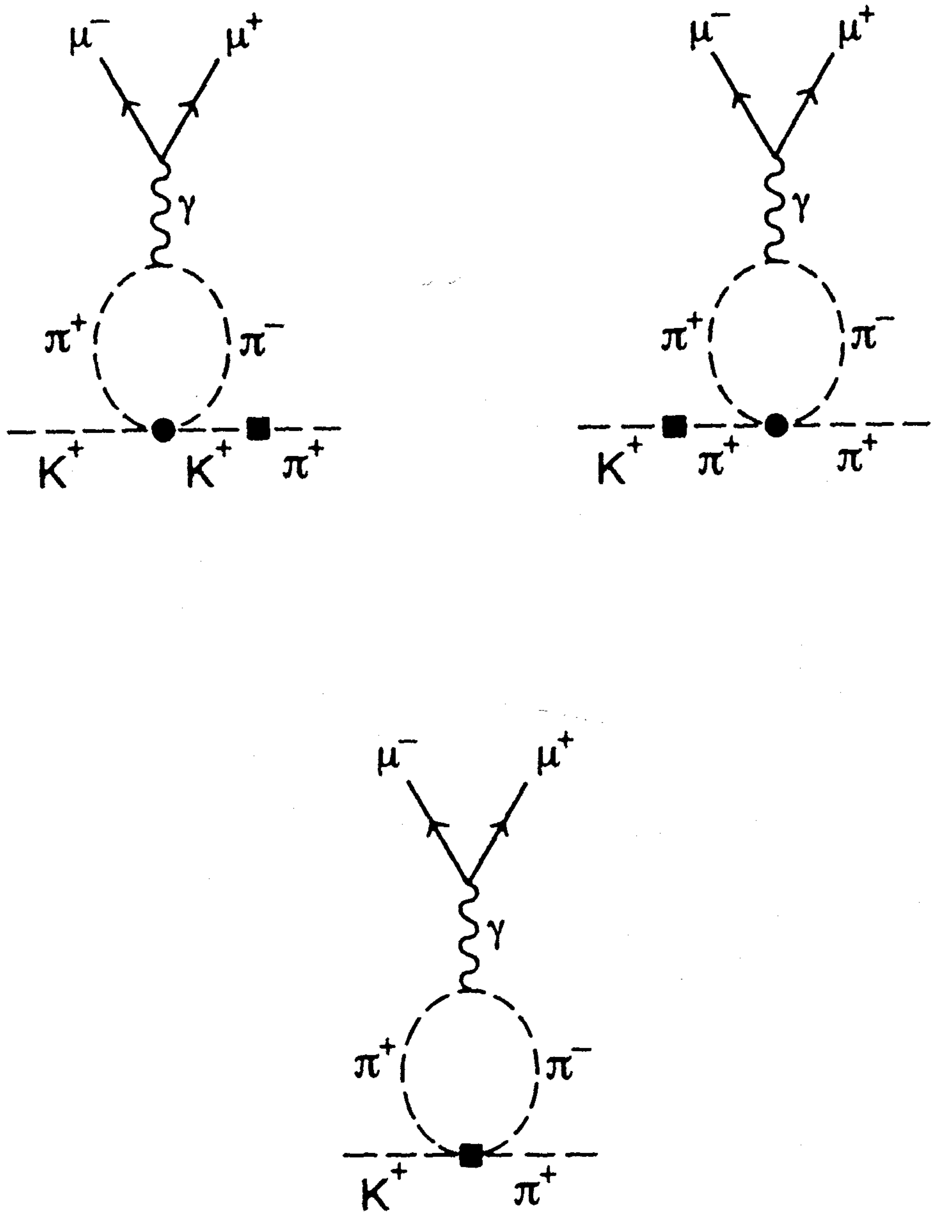

Figure 8. Feynman diagrams that give the leading contribution to $Imf(s)$ in chiral perturbation theory.

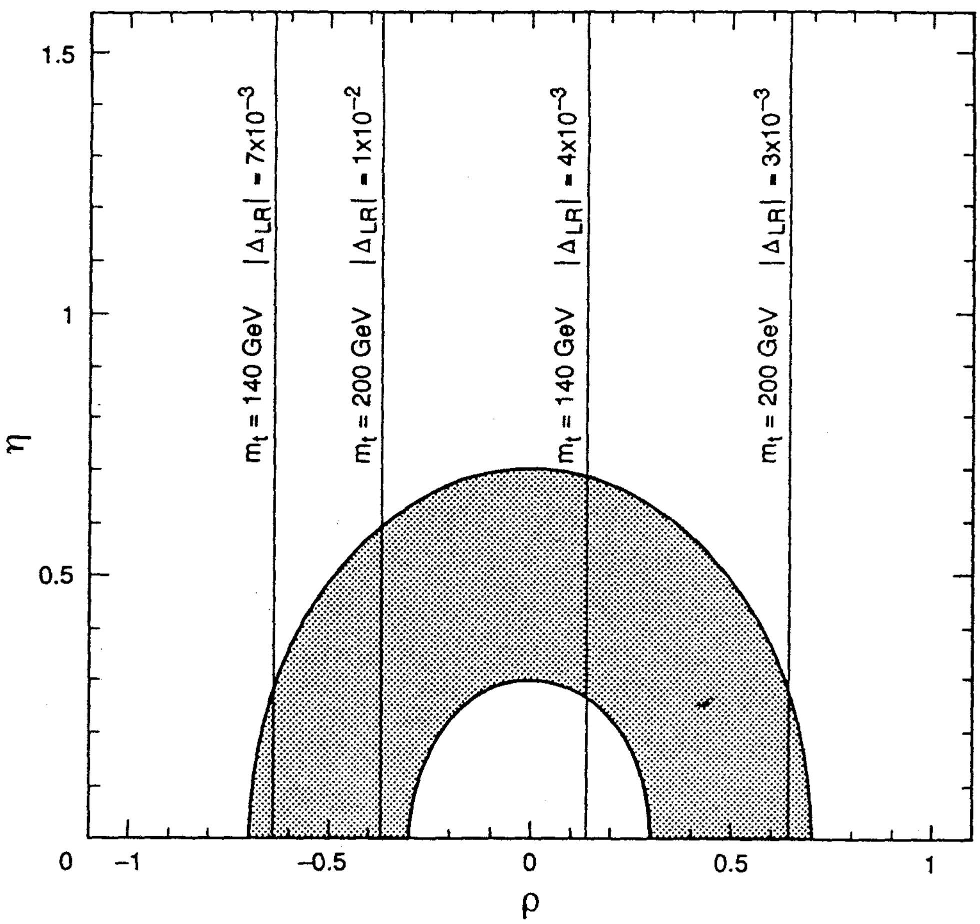

Figure 9. Implications of measurement of the asymmetry Δ_{LR} for the location of the α vertex of the unitarity triangle.

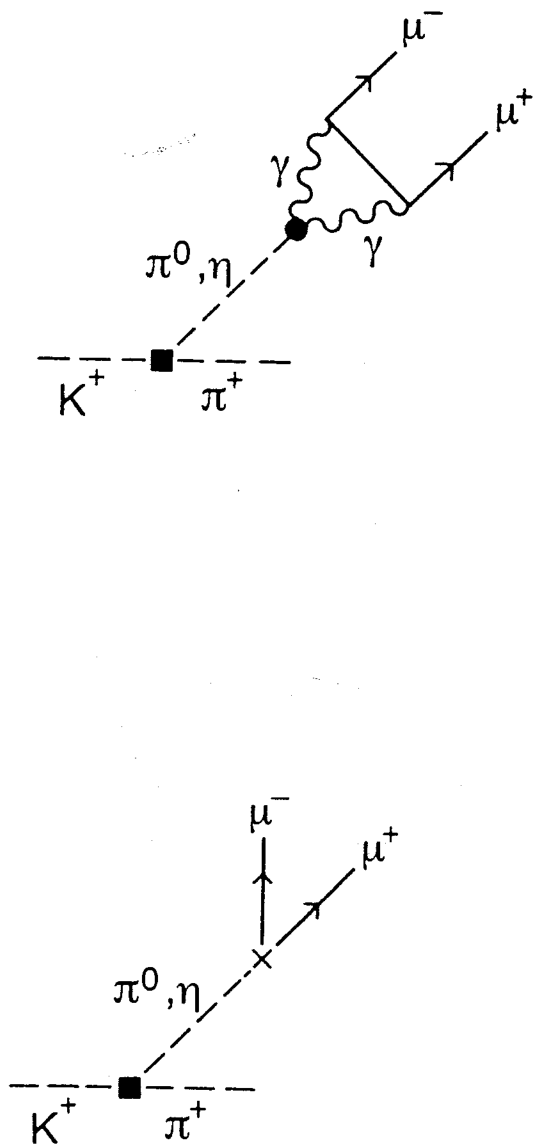

Figure 10. Feynman diagrams that give the dominant two photon contribution to C in chiral perturbation theory.