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THESIS

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SPACE-DISTRIBUTION OF X-RAY PHOTOELECTRONS EJECTED FROM THE K AND L ATOMIC ENERGY-LEVELS

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Presented

by

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In partial fulfillment of the Requirements

for the degree of Doctor of Philosophy

California Institute of Technology

Pasadena, California

1930.

A B S T R A C T

A C. T. R. Wilson expansion-chamber was used to study the space-distribution of photoelectrons ejected from a gas by monochromatic x-rays. Distribution curves were obtained for electrons ejected from both the K and the L atomic energy-levels. A more isotropic space-distribution was found for electrons ejected from the L energy-levels than for those from the K energy-level. The distribution of the electrons from the L energy-levels became less isotropic with an increase in the frequency of the incident radiation. For a given radiation, the average forward momentum of the electrons from the K energy-level was found to decrease with an increase in the binding energy of the parent atom. Within experimental error, however, for electrons from the K energy-level, even for different binding energies, the average forward momentum remained the same for a given velocity of ejection of the electron. The average forward momentum of electrons ejected from the L energy-levels was greater than that for electrons from the K energy-level for a given velocity of ejection. The average forward momentum of the electrons ejected from the K shell was in fair accord with recent results of quantum mechanics. The observed values of the average forward momentum of the electrons ejected from the L energy-levels were slightly greater than those given by the theory.

The longitudinal space-distribution of photoelectrons ejected from a gas by x-rays has been extensively studied by means of the C. T. R. Wilson expansion-chamber by several investigators. The general shape of the distribution curve for electrons ejected from the K energy-level and the dependence on the frequency of the incident radiation has been determined, and are in approximate agreement with the recent quantum mechanical expressions. Results so far obtained for the space-distribution of electrons ejected from the L energy-levels are rather meager. It has been shown, however, that a more isotropic space-distribution exists in this case. In the present work, the distribution of electrons ejected from the L energy-levels is found to become less isotropic as the frequency of the incident radiation is increased.

The C. T. R. Wilson expansion-chamber employed in this investigation was essentially that described by Simon and Loughridge. Only minor refinements were effected to insure greater accuracy in the data obtained.

Simple filtering of the general radiation from an x-ray tube was found to produce radiation not sufficiently monochromatic. Measurements made of the lengths of photoelectron tracks formed by simple filtered radiation showed the track-lengths to vary by a factor of ten, indicating a variation in the initial kinetic energy of the photoelectrons

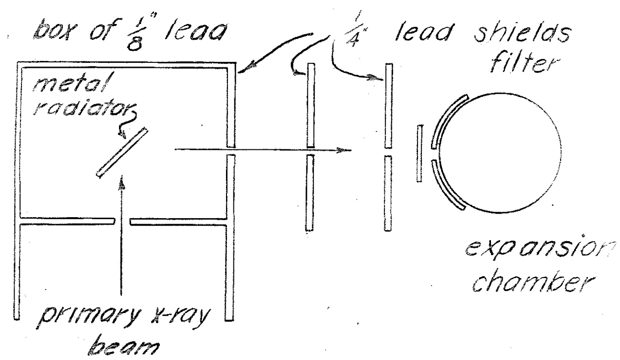

Fig. I. Fluorescence radiator as source of monochromatic x-rays.

of about 300%. It was necessary therefore to employ other means of monochromatizing the x-rays. Monochromatism was insured, in one instance, by the selection of the K line of molybdenum by means of a calcite crystal spectrometer. For the other frequencies, the secondary fluorescence radiation from a metal plate, irradiated by primary x-rays from a tungsten-target Coolidge type tube operated at a potential just insufficient to excite the tungsten K lines, was collimated to a narrow beam and passed into the expansion-chamber (Fig. I). A. H. Compton has shown the fluorescence radiation obtained in this manner to be very homogeneous, having 99 per cent of its energy in the characteristic K line-radiation of the metal radiator. Since the radiation obtained in this manner is of very low intensity, it was necessary to operate the tube at a space current of 35-40 milliamperes. It was possible then to obtain 25 to 30 photo-electron tracks per expansion. Upon removing the metal radiator, but otherwise leaving conditions unchanged, only one to two tracks were obtained per expansion. The effect therefore of stray, scattered radiation was small. Both of the means of monochromatizing the x-rays described above gave tracks uniform in length, variations seldom exceeding 30%.

The wave-lengths of the x-rays used in this investigation are given in Table I together with the metal radiator employed in three cases.

TABLE I

Source	Monochromatizer	λ
Mo target tube	Calcite spectrometer	.71 A
W " "	Silver radiator	.56 A
W " "	Palladium radiator	.59 A
W " "	Tin radiator	.49 A

The relatively faint K β lines of palladium were filtered out by means of a ruthenium filter. The filter consisted of a 3 mm. thickness of crystalized ruthenium nitrate supported between two vertical pieces of thin cardboard. The presence of the K β lines in the other cases was not objectionable.

In the interests of increased accuracy in the measurements of the tracks some changes in the stereo-comparator previously described were found advantageous. The scale and vernier for measuring displacements of the carriage containing the cross-hairs were replaced by a micrometer screw and scale calibrated to read to .001 cm. Very much finer cross-hairs and higher power lenses were employed throughout, and a frame and cross-hair were substituted for the simple pointer previously employed. Careful alignment of the two photographic plates in the comparator before attempting measurements was found to be an essential step in the procedure of measurement. Usually either the surface of the piston in the expansion-chamber or the glass plate covering the chamber will appear on the photographs due to the scattering of light by various small projecting points on these surfaces. It is then imperative that one rotate slightly one of the photographic plates in the comparator so that the cross-hairs as seen stereoscopically lie parallel to the plane of these surfaces. If this apparently trivial adjustment is not made large errors in measurement will result from only a very slight displacement of one of the two photographic plates. After these precautions were taken random errors were reduced five fold and track measurements could be repeated within limits of 3°.

A brief discussion of the errors in measurement may not be out of place at this point. They may be classified into three groups, random errors, systematic errors and statistical fluctuations.

The random errors include (1) the natural variations which occur in setting a cross-hair upon a point, and (2) the effect of scattering by nuclei and electrons of the gas in changing the initial direction of track, as well as producing deflections along the path. While the effect of these random errors will cancel out to some extent for a large number of measurements, their final effect will, in general, be to broaden the distribution curve and decrease its asymmetry. Also of course any fine structure, minor maxima and minima will tend to be ironed out.

Systematic errors are more injurious to the final result. A systematic error inherent to the stereoscopic method employed for measurement arises due to the fact that for the depth measurements one cross-hair is moved and the other remains fixed to the frame of the carriage. This has the effect, for instance, considering a track in the expansion-chamber directed upward toward the camera, of decreasing its forward component if it occurs in one quarter-part of the chamber, and increasing its forward component if it occurs in the remaining three-quarter-part of the chamber. The final effect then is to give a forward bias to the average forward component of a number of tracks. This error may be corrected by taking two measurements on each track, reversing the photographic plates in the comparator for the second measurement. If, however, approximately an equal number of tracks are directed upward as downward in the expansion-chamber, this error will balance out leaving only a very small second order effect. By a careful alignment of the x-ray beam this condition can readily be brought about making only a single measurement necessary.

Statistical variations are always present but may be reduced to any degree if a sufficiently large number of measurements are made.

The ultimate accuracy of the data obtained is rather hard to estimate. It is to be noted that the average forward component of the forward momentum as evaluated later is particularly sensitive to error in measurement. An error in measurement of only 2^{o} in the region where the majority of tracks occur, i.e. $\cos\theta = 80^{o}$, will mean an error of 20% in cos θ . For this reason the value of cos θ (obs.) listed below may be in error by 20%. The other data, however, should be considerably more accurate.

A comparison of the relative merits of the stereoscopic method as compared to that in which two cameras are employed having their focal axes perpendicular to one another, is hard to make since the care and precision used in the construction of the apparatus is the determining factor. The latter method, however, requires fewer readings to be taken for each track and can be used therefore in measuring readily a large number of tracks.

EXPERIMENTAL RESULTS

The longitudinal space-distribution curves, representing the density of emission per unit angle of the photoelectrons as a function of the angle between the direction of ejection and the forward direction of the x-ray beam, were plotted in a number of cases to show the

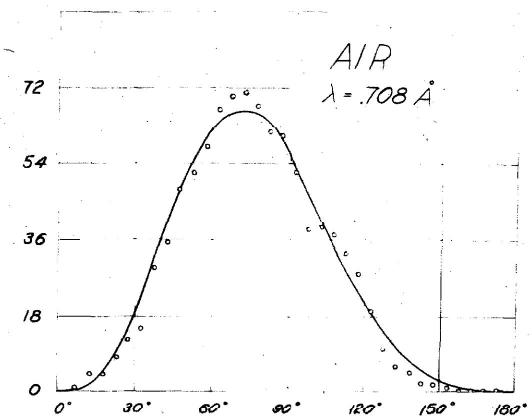

Fig. II

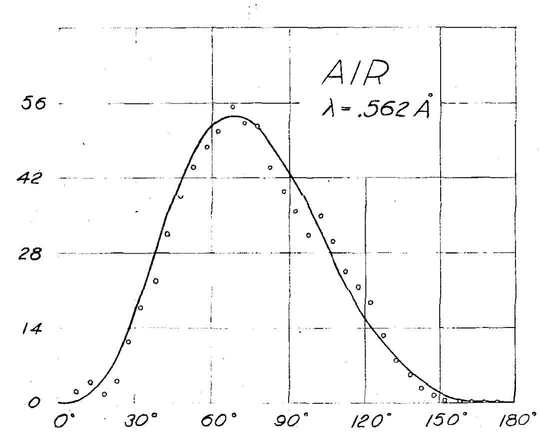

Fig. III

effect of the energy level from which the electron is taken, and also the effect of a change in the frequency of the incident radiation.

In Figs. II and III are plotted the results obtained respectively from measurements on 272 tracks in air produced by radiation of .71 A and on 200 tracks in air produced by radiation of .56 A. Each small circle represents the number of electrons ejected in a 15° interval the point being plotted at an angle corresponding to the center of the interval. Points are plotted every 5° and therefore represent overlapping intervals. In order to study the distribution where the binding energy was of appreciable magnitude, C_2H_5Br was introduced into the expansion-chamber in an atmosphere of hydrogen. The photoelectrons were produced by radiation of .59 A, most of them being ejected from the K shell of the bromine atom. About 64 per cent of the energy of the incident radiation was required to remove the electron from the atom, the remaining 36 per cent appearing as kinetic energy. The secondary and tertiary photoelectrons could be easily distinguished from one another due to the difference in path length. Photoelectrons ejected from levels other than the K level of bromine or from light atoms could be distinguished by their long path length and hence omitted

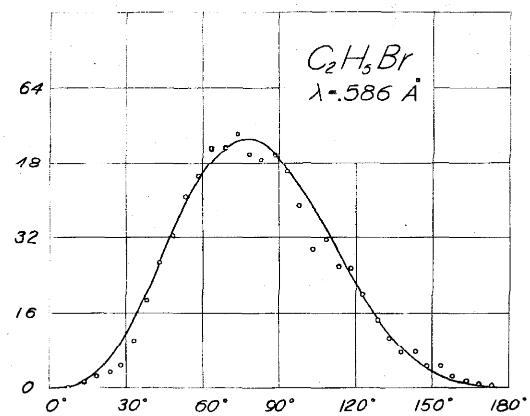

Fig. IV

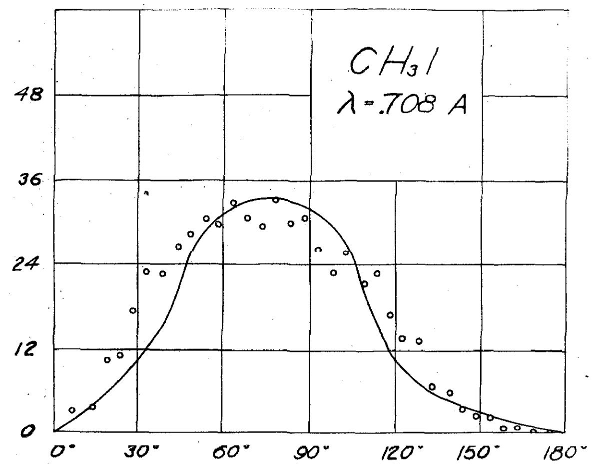
CH₃ I
λ=.708 A
48
36
24
12
0
0°
30°
60°
90°
120°
150°
180°

Fig. V

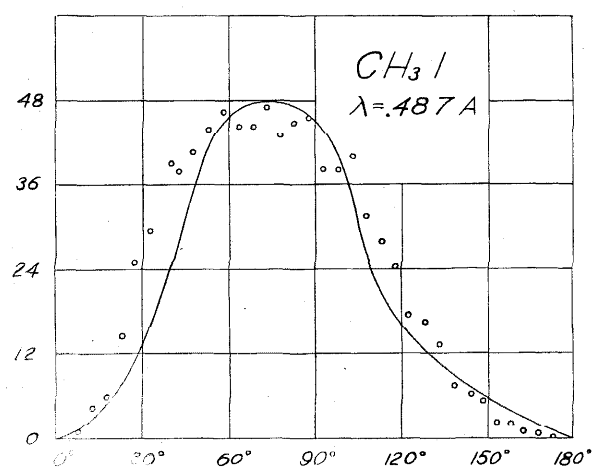
CH₃ I
λ=.487 A
48
36
24
12
0
0°
30°
60°
90°
120°
150°
180°

Fig. VI

in the measurements*. Thus, only the photoelectrons having their origin in the K shell of bromine were included. Fig. IV represents the distribution curve plotted as before for 233 tracks of electrons ejected from the K level of bromine by radiation of .59 A.

For the study of the distribution of electrons ejected from the L energy levels, CH_3I was introduced into the chamber in an atmosphere of hydrogen and photoelectrons produced by radiation of .71 A and .49 A emission being from the L levels of iodine. The results of measurements on 200 tracks formed by radiation of .71 A are shown in Fig. V. In agreement with the work of Auger the curve is broader than that found for the K electrons indicating a more isotropic distribution. The small circles as before represent the experimental points. Fig. VI represents the distribution of 264 photoelectrons ejected from the L levels of iodine by radiation of .49 A. The curve here is narrower than in Fig. V, indicating a decrease in the isotropy of the distribution with an increase in the frequency of the incident radiation. Watson and Van den Akker have shown that photoelectrons ejected from the L_{11} and L_{111} energy levels have a space-distribution more isotropic than those from the K and L_1 energy levels and Robinson and Cassie have found the relative

* The ratio of the number of secondary to tertiary photoelectrons was found to be 2.5.

number of L_1 electrons ejected to increase with an increase in the frequency of the incident radiation. The narrower distribution curve of Fig. VI may then be explained by the presence of a greater proportion of L_1 electrons than is the case for the curve of Fig. V.

A theoretical expression derived recently by Wentzel on the basis of quantum mechanics,

$$P(\theta) \propto \frac{\sin^3\theta}{\left[1 - \frac{v}{c}\cos\theta + \frac{h\nu}{2mc^2}\right]^4} \qquad (1)$$

gives the probability of ejection per unit angle of a photoelectron from the K energy level as a function of the angle, θ, between the direction of ejection and the forward direction of the x-ray beam, where v represents the velocity of ejection, ν the frequency of the incident radiation, and h, m and c are the customary physical constants.

The solid-line curves of Figs. II, III and IV represent $P(\theta)$ with the proper values of v and ν inserted, the curves being plotted on a scale to conform to the experimental points.

The observed asymmetry of the distribution about a plane normal to the x-ray beam may be compared with the theory in several ways. The value of $\cos\theta$ averaged over all the photoelectron tracks, a quantity proportional to the average forward momentum of the photoelectrons may be computed (see Appendix A) giving

$$\overline{\cos\theta} = \frac{4}{5}\frac{v}{c} + \cdots \qquad (2)$$

if only first order terms in $\frac{v}{c}$ are retained. Table II gives the values of $\overline{\cos\theta}$ (obs.) and $\overline{\cos\theta}$ (calc.).

TABLE II

Energy Level	Gas	λ	$\overline{\cos\theta}$ (obs.)	$\overline{\cos\theta}$ (calc.)
K	Air	.71 A	.182	.210
K	Air	.56 A	.210	.235
K	C_2H_5Br	.59 A	.133	.138
L	CH_3I	.71 A	.230	
L	CH_3I	.49 A	.255	

The average forward momentum of the K electrons for a given radiation decreases with an increase of the binding energy of the parent atom. Within experimental error, however, K electrons of the same initial velocity have the same average forward momentum. It is to be noted, moreover, that in accord with the results of Auger, and Watson and Van den Akker for a given velocity of ejection, the L electrons have an average forward momentum greater than that of the K electrons. The difference in behavior of the K and L electrons is more marked for the lower frequencies.

The bi-partition angle, θ_b, the half-angle at the apex of a cone which divides the photoelectrons into two groups of equal numbers, is defined by Eq. (3). Calculation of cos θ_b shows it to be equal to $\frac{v}{c}$ to a first approximation (Appendix A).

$$\int_0^{\theta_b} P(\theta)\,d\theta = \int_{\theta_b}^{\pi} P(\theta)\,d\theta \qquad (3)$$

Table III gives the values of cos θ_b(obs.) and cos θ_b(calc.).

TABLE III

Energy Level	Gas	λ	cos θ_b(obs.)	cos θ_b(calc.)
K	Air	.71 A	.242	.262
K	Air	.56 A	.292	.294
K	C_2H_5Br	.59 A	.191	.173
L	CH_3I	.71 A	.174	
L	CH_3I	.49 A	.242	

For a given value of $\frac{v}{c}$ the bi-partition angle for electrons ejected from the L energy-levels seems to occur nearer 90° than for those from the K energy-level, in agreement with the conclusions of Auger.

The ratio of the number of electrons ejected forward of the plane normal to the x-ray beam, to the number ejected backward, ρ , (Appendix A) is given by Eq. (4):

$$\rho = \frac{2 + 3\frac{v}{c}}{2 - 3\frac{v}{c}} + \cdots \tag{4}$$

The values of ρ (obs.) and ρ (calc.) are given in Table IV.

TABLE IV

Energy Level	Gas	λ	ρ (obs.)	ρ (calc.)
K	Air	.71 A	2.17	2.30
K	Air	.56 A	2.39	2.58
K	C_2H_5Br	.59 A	1.74	1.70
L	CH_3I	.71 A	2.00	
L	CH_3I	.49 A	2.30	

It is to be noted that no marked difference was found in the behavior of ρ for the K and L electrons.

Calculations made recently by G. Schur on the basis of quantum mechanics gives the following expression for the space-distribution of photoelectrons ejected from the L energy-levels of an atom:

$$Q(\theta) \propto \sin^3\theta + \frac{4v}{c}\sin^3\theta\cos\theta\left(1 - \frac{I_L}{h\nu}\right) + \frac{I_L \sin\theta}{h\nu + 3I_L}\left\{1 + \frac{8I_L}{h\nu}\sin^2\theta + 2\frac{v}{c}\cos\theta\left(1 + 2\sin^2\theta\left(1 + \frac{11 I_L}{h\nu}\right)\right)\right\}$$

where I_L represents the mean value of the binding energy of the L energy-levels, and the other quantities remain as before.

The solid-line curves of Figs. V and VI represent $Q(\theta)$ with the proper values of v, ν, and I_L inserted, the curves being plotted on a scale to conform to the observed points.

Calculation of the average value of cos θ for L electrons (Appendix B), in the manner carried out for K electrons, leads to the following results,

TABLE V

Energy Level	Gas	λ	$\overline{\cos\theta}$ (obs.)	$\overline{\cos\theta}$ (calc.)
L	CH_3I	0.71 A	.23	.17
L	CH_3I	0.49 A	.25	.22

The agreement here is not satisfactory, the observations seeming to indicate a greater average forward momentum of the photoelectrons than the theory.

Calculation of ρ, defined as above, leads to the following results,

TABLE VI

Energy Level	Gas	λ	ρ (obs.)	ρ (calc.)
L	CH_3I	0.71 A	2.0	1.9
L	CH_3I	0.49 A	2.3	2.4

With regard to ρ, experiment and theory are in fair accord.

A decrease in the isotropy of the longitudinal space-distribution curve for L electrons, with an increase in the frequency of the incident radiation, as was found above, is also to be expected from the theory. The ratio of the number of L_{11} and L_{111} electrons to the L_1 electrons is given by

$$\frac{L_{II} + L_{III}}{L_I} = \frac{I_L}{h\nu + 3I_L}\left(3 + 8\frac{I_L}{h\nu}\right)$$

which for this case, leads to

TABLE VII

Energy Level	Gas	λ	Relative proportion of L_1 Electrons	L_{11} and L_{111} Electrons
L	CH_3I	0.71 A	67 %	33 %
L	CH_3I	0.49 A	72 %	28 %

For the harder radiation, due to the greater proportion of L_1 electrons ejected, a slightly narrower space-distribution curve is to be expected.

APPENDIX A

SPACE DISTRIBUTION OF K ELECTRONS

On the basis of quantum mechanics several expressions have been derived recently for the space-distribution of K photoelectrons as a function of the frequency of the incident radiation and the binding energy of the parent atom. For electrons of not too great a velocity the expressions given by several authors agree to first order terms in $\frac{v}{c}$ but may differ in the higher order terms. We shall consider that relation derived by G. Wentzel, and communicated in a series of lectures at the Norman Bridge Laboratory in January 1930.

He finds for the probability of ejection of a photoelectron from the K energy-level of an atom

$$P(\theta,\phi) \propto \frac{\sin^2\theta \cos^2\phi}{\left[1 - \frac{v}{c}\cos\theta + \frac{h\nu}{2mc^2}\right]^4} \qquad (1)$$

where ϕ is the angle measured between direction of ejection of the electron and the electric vector of the incident radiation, θ the angle between the direction of ejection and the forward direction of the x-ray beam, v the velocity of ejection, ν the frequency of the incident radiation, and h, m and c are the customary physical constants.

If the incident radiation is unpolarized, and if we consider the longitudinal space distribution, the probability of ejection per unit angle is given by,

$$P(\theta) \propto \frac{\sin^3\theta}{\left[1 - \frac{v}{c}\cos\theta + \frac{h\nu}{2mc^2}\right]^4} \qquad (2)$$

MEAN VALUE OF COS θ

The mean value of cos θ averaged over all the photoelectrons ejected, a quantity proportional to the average momentum of the photoelectrons in the direction of the propogation of the radiation, may be calculated as follows:

$$\overline{\cos\theta} = \frac{\int_0^\pi P(\theta)\cos\theta\, d\theta}{\int_0^\pi P(\theta)\, d\theta} = \frac{I_1}{I_2} \qquad (3)$$

Consider first the integral in the numerator:

$$I_1 = \int_0^\pi \frac{\sin^3\theta\cos\theta\, d\theta}{(a - b\cos\theta)^4}$$

where $$a = 1 + \frac{h\nu}{2mc^2}$$

and $$b = \frac{v}{c}$$

Making the substitution,

$$u = \cos\theta$$

$$I_1 = \int \frac{(1-u^2)u}{(a-bu)^4}\,du$$

substituting $z = a - bu$,

$$= \frac{1}{b^4}\int_{a-b}^{a+b} \frac{(b^2-a^2+2az-z^2)(a-z)}{z^4}\,dz$$

$$= \frac{1}{b^4}\left\{(ab^2-a^3)\int\frac{dz}{z^4} + (3a^2-b^2)\int\frac{dz}{z^3} - 3a\int\frac{dz}{z^2} + \int\frac{dz}{z}\right\}$$

$$= \frac{1}{b^4}\left\{\frac{a^3-ab^2}{3z^3} + \frac{b^2-3a^2}{2z^2} + \frac{3a}{z} + \log z\right\}\Bigg|_{a-b}^{a+b}$$

substituting the limits of integration and simplifying,

$$I_1 = \left\{\frac{-6a^3+10ab^2}{3b^3(a+b)^2(a-b)^2} + \frac{1}{b^4}\log\frac{a+b}{a-b}\right\}$$

Consider now the integral in the denominator of Eq. (3):

$$I_2 = \int_0^{\pi} \frac{\sin^3\theta}{(a-b\cos\theta)^4}\,d\theta$$

substituting $u = \cos\theta$,

$$I_2 = \int \frac{(1-u^2)}{(a-bu)^4}\,du$$

substituting $a - bu = z$,

$$I_2 = \frac{1}{b^3}\int_{a-b}^{a+b} \frac{b^2-a^2+2az-z^2}{z^4}\,dz$$

$$= \frac{1}{b^3}\left\{(b^2-a^2)\int\frac{dz}{z^4} + 2a\int\frac{dz}{z^3} - \int\frac{dz}{z^2}\right\}$$

$$= \frac{1}{b^3}\left\{\frac{a^2-b^2}{3z^3} - \frac{a}{z^2} + \frac{1}{z}\right\}\Bigg|_{a-b}^{a+b}$$

$$= \frac{4}{3(a+b)^2(a-b)^2}$$

Hence

$$\overline{\cos\theta} = \frac{I_1}{I_2} = \frac{-6a^2b + 10ab^3 + 3(a^2-b^2)^2 \log\frac{a+b}{a-b}}{4b^4}.$$

where $a = 1 + \frac{h\nu}{2mc^2}$ and $b = \frac{v}{c}$

Since in the derivation of expression (1), electron spin and relativity terms were not considered and various other approximations made the expression above is significant only to first order terms in $\frac{v}{c}$. Expanding in terms of $\frac{v}{c}$, we find

$$\overline{\cos\theta} = \frac{4}{5}\frac{v}{c} + \frac{4}{35}\left(\frac{v}{c}\right)^3 + \cdots$$

Neglecting the third and higher order terms,

$$\overline{\cos\theta} = \frac{4}{5}\frac{v}{c} \tag{4}$$

BI-PARTITION ANGLE

The bi-partition angle θ_b, the half-angle at the apex of a cone which divides the photoelectrons into two groups of equal number is defined by Eq. (5)

$$\int_0^{\theta_b} P(\theta)\, d\theta = \int_{\theta_b}^{\pi} P(\theta)\, d\theta \tag{5}$$

As we saw previously

$$P(\theta)\,d\theta = \frac{1}{b^3}\left\{ \frac{a^2-b^2}{3Z^3} - \frac{a}{Z^2} + \frac{1}{Z} \right\}$$

where $Z = a - b\cos\theta$,

Hence

$$\int_0^{\theta_b} P(\theta)\,d\theta = \frac{a^2-b^2-3ab\cos\theta+3b^2\cos^2\theta}{3b^3(a-b\cos\theta)^3}\Bigg|_0^{\theta_b}$$

and

$$\int_{\theta_b}^{\pi} P(\theta)\,d\theta = \frac{a^2-b^2-3ab\cos\theta+3b^2\cos^2\theta}{3b^3(a-b\cos\theta)^3}\Bigg|_{\theta_b}^{\pi}$$

Eq. (3) then becomes:

$$\frac{a^2-b^2-3ab\cos\theta_b+3b^2\cos^2\theta_b}{(a-b\cos\theta_b)^3} - \frac{a^2-b^2-3ab+3b^2}{(a-b)^3} =$$

$$= \frac{a^2-b^2+3ab+3b^2}{(a+b)^3} - \frac{a^2-b^2-3ab\cos\theta_b+3b^2\cos^2\theta_b}{(a-b\cos\theta_b)^3}$$

Simplifying and neglecting third and higher order terms in $\frac{v}{c}$:

$$3a^2 b + 3a^3 \cos\theta_b = 0$$

where $a = 1 + \frac{h\nu}{2mc^2}$ and $b = \frac{v}{c}$

Hence
$$\cos\theta_b = \frac{v}{c} + \cdots \qquad (6)$$

RATIO OF NUMBER OF PHOTOELECTRONS
EJECTED FORWARD OF THE PLANE NORMAL TO THE
X-RAY BEAM TO THE NUMBER EJECTED BACKWARD.

The ratio of the number of electrons ejected forward of the plane normal to the x-ray beam to the number ejected backward, ρ , is given by Eq. (7),

$$\rho = \frac{\int_0^{\frac{\pi}{2}} P(\theta)\, d\theta}{\int_{\frac{\pi}{2}}^{\pi} P(\theta)\, d\theta} \qquad (7)$$

As shown in previous section,

$$\int P(\theta)\, d\theta = \frac{1}{b^3}\left\{ \frac{a^2-b^2}{3Z^3} - \frac{a}{Z^2} + \frac{1}{Z} \right\}$$

where

$$Z = a - b\cos\theta$$

Hence

$$\int_0^{\frac{\pi}{2}} P(\theta)\, d\theta = \frac{a^2-b^2}{a^3} - \frac{a^2-b^2-3ab+3b^2}{(a-b)^3}$$

and

$$\int_{\frac{\pi}{2}}^{\pi} P(\theta)\, d\theta = \frac{a^2-b^2+3ab+3b^2}{(a+b)^3} - \frac{a^2-b^2}{a^3}$$

After simplification:

$$\rho = \frac{2a^3+3a^2b-b^3}{2a^3-3a^2b-b^3}$$

Substituting for a and b and neglecting third and higher order terms in $\frac{v}{c}$;

$$\rho = \frac{2+3\frac{v}{c}}{2-3\frac{v}{c}} \qquad (8)$$

DIRECTION OF GREATEST EMISSION

The position of the maximum of the longitudinal space-distribution curve is given by

$$\left[\frac{d\,P(\theta)}{d\theta}\right]_{\theta=\theta_m} = 0 \qquad (9)$$

$$\frac{d}{d\theta}\left[\frac{\sin^3\theta}{(a-b\cos\theta)^4}\right] = 3\sin\theta_m\cos\theta_m\,(a-b\cos\theta_m)^4 -$$

$$4b\sin^4\theta_m\,(a-b\cos\theta_m)^3 = 0$$

$$b\cos^2\theta_m + 3a\cos\theta_m - 4b = 0$$

$$\cos\theta_m = \frac{-3a \pm \sqrt{9a^2+16b^2}}{2b}$$

APPENDIX B

SPACE DISTRIBUTION OF L ELECTRONS

G. Schur has recently published a theoretical expression for the space-distribution of photoelectrons ejected from the L energy-levels of an atom. He finds for the probability of ejection of a photoelectron from the L energy-levels:

$$Q(\theta,\phi) \propto \sin^2\theta\cos^2\phi + \frac{4v}{c}\sin^2\theta\cos^2\phi\cos\theta\left(1-\frac{I_L}{h\nu}\right)$$

$$+\frac{I_L}{h\nu+3I_L}\left\{1+\frac{8I_L}{h\nu}\sin^2\theta\cos^2\phi+\frac{2v}{c}\cos\theta\right.$$

$$\left.\left(1+2\sin^2\theta\cos^2\phi\left(1+\frac{11I_L}{h\nu}\right)\right)\right\}$$

where I_L represents the mean value of the binding energy of the L electron-levels, and the other quantities remained as previously defined.

If the incident radiation is unpolarized, and if we consider the longitudinal space-distribution, the probability of ejection per unit angle is given by,

$$Q(\theta) \quad \propto \quad \sin^3\theta + \frac{4v}{c}\sin^3\theta\cos\theta\left(1 - \frac{I_L}{h\nu}\right)$$

$$+ \frac{I_L \sin\theta}{h\nu + 3I_L}\left\{ 1 + \frac{8I_L}{h\nu}\sin^2\theta + \frac{2v}{c}\cos\theta \right.$$

$$\left. \left(1 + 2\sin^2\theta\left(1 + \frac{11 I_L}{h\nu}\right)\right)\right\} \qquad (11)$$

MEAN VALUE OF COS θ

The mean value of cos θ, for L electrons, is given by,

$$\overline{\cos\theta} = \frac{\int_0^\pi Q(\theta)\cos\theta\, d\theta}{\int_0^\pi Q(\theta)\, d\theta} = \frac{J_1}{J_2} \qquad (12)$$

Let

$$\left.\begin{aligned} a &= 4\frac{v}{c} \\ b &= 1 - \frac{I_L}{h\nu} \\ c &= \frac{I_L}{h\nu + 3 I_L} \\ d &= \frac{8 I_L}{h\nu} \\ e &= 1 + \frac{11 I_L}{h\nu} \end{aligned}\right\} \qquad (13)$$

Then,

$$J_1 = \int_0^\pi \sin^3\theta \cos\theta \, d\theta + ab\int_0^\pi \sin^3\theta \cos^2\theta \, d\theta$$

$$+ c\left\{ \int_0^\pi \sin\theta \cos\theta \, d\theta + d\int_0^\pi \sin^3\theta \cos\theta \, d\theta \right.$$

$$\left. + \frac{a}{2}\int_0^\pi \sin\theta \cos^2\theta \, d\theta + ae\int_0^\pi \sin^3\theta \cos^2\theta \, d\theta \right\}$$

$$J_1 = \frac{4}{15} ab + \frac{1}{3} ac + \frac{4}{15} aec$$

And,

$$J_2 = \int_0^\pi \sin^3\theta \, d\theta + ab \int_0^\pi \sin^3\theta \cos\theta \, d\theta$$

$$+ c \left\{ \int_0^\pi \sin\theta \, d\theta + d \int_0^\pi \sin^3\theta \, d\theta \right.$$

$$\left. + \frac{a}{2} \int_0^\pi \sin\theta \cos\theta \, d\theta + ae \int_0^\pi \sin^3\theta \cos\theta \, d\theta \right\}$$

$$J_2 = \frac{4}{3} + 2c + \frac{4}{3} cd$$

Therefore,

$$\overline{\cos\theta} = \frac{J_1}{J_2} = \frac{a\left(\frac{4}{15} b + \frac{c}{3} + \frac{4}{15} ec\right)}{\frac{4}{3} + 2c + \frac{4}{3} cd} \qquad (14)$$

where a, b, c, d, and e are defined by Eqs. (13).

RATIO OF NUMBER OF PHOTOELECTRONS

EJECTED FORWARD OF THE PLANE NORMAL TO THE

X-RAY BEAM TO THE NUMBER EJECTED BACKWARD

The ratio of the number of electrons ejected forward of the plane normal to the x-ray beam to the number ejected backward, ρ , is given by Eq. (15).

$$\rho = \frac{\int_0^{\frac{\pi}{2}} Q(\theta)\, d\theta}{\int_{\frac{\pi}{2}}^{\pi} Q(\theta)\, d\theta} \tag{15}$$

For L electrons, as previously shown

$$\int Q(\theta)\, d\theta = \int \sin^3\theta\, d\theta + ab \int \sin^3\theta \cos\theta\, d\theta$$

$$+ \left\{ \int \sin\theta\, d\theta + d \int \sin^3\theta\, d\theta \right.$$

$$\left. + \frac{a}{2} \int \sin\theta \cos\theta\, d\theta + ae \int \sin^3\theta \cos\theta\, d\theta \right\}$$

Therefore,

$$\int_0^{\frac{\pi}{2}} Q(\theta)\, d\theta = \frac{2}{3} + \frac{ab}{4} + c + \frac{2}{3}cd + \frac{ac}{4} + \frac{aec}{4}$$

and

$$\int_{\frac{\pi}{2}}^{\pi} Q(\theta)\, d\theta = \frac{2}{3} - \frac{ab}{4} + c + \frac{2}{3}cd - \frac{ac}{4} - \frac{aec}{4}$$

Hence

$$\rho = \frac{\frac{2}{3} + \frac{ab}{4} + c + \frac{2}{3}cd + \frac{ac}{4} + \frac{aec}{4}}{\frac{2}{3} - \frac{ab}{4} + c + \frac{2}{3}cd - \frac{ac}{4} - \frac{aec}{4}} \tag{16}$$

where a, b, c, d, and e are defined by Eqs. (13).

APPENDIX C

NUMERICAL TABLES

Values of ν/R for Radiation Employed

Radiation	ν/R
0.71 A	1287
0.56 A	1621
0.59 A	1555
0.49 A	1871

Binding Energies in Terms of ν/R

Atom	ν/R
O, N (Air)	Small
Br - K level	993
I L_{1} "	336
I L_{11} "	358
I L_{111} "	393

Velocities of Ejection of Electrons

Radiation	Atom	Energy-level	Velocity
0.71 A	O, N (Air)		.262 c
0.56 A	O, N (Air)		.294 c
0.59 A	Br	K	.173 c
0.71 A	I	L	.221 c
0.49 A	I	L	.282 c

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