A STUDY OF THE EFFECT OF VERTICAL SAND DRAINS AS A MEANS FOR THE RAPID CONSOLIDATION OF SOILS OF LOW PERMEABILITY

Thesis by

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NOTATION

Dimensions are given in terms of Force (F), Length (L), and Time (T)

a --(L) One half spacing of soil drains

c --(L^2F^{-1}) Modulus of compressibility or slope of void ratio pressure curve

e -- Void ratio, volume of voids per unit volume of solid soil particles.

e -- Base of Naperian Logarithms, differentiated from void ratio by context

γ --(FL^{-3}) Unit weight of water

H_w --(L) Hydraulic head

H --(L) Depth of soil layer

h_t --(L) Total deformation or settlement at time t

$h_t\%$ -- Degree of consolidation

k --(LT^{-1}) Coefficient of permeability

m --(L^2F^{-1}) Modulus of volume change

M --(L^2T^{-1}) Modulus of consolidation

p --(FL^{-2}) Pore water pressure - pressure in excess of hydrostatic pressure

s -- Hydraulic gradient

σ_s --(FL^{-2}) Unit soil pressure causing consolidation

σ_z --(FL^{-2}) Unit imposed load pressure on soil layers

T -- Dimensionless, time factor of consolidation

t --(T) Unit time

INTRODUCTION

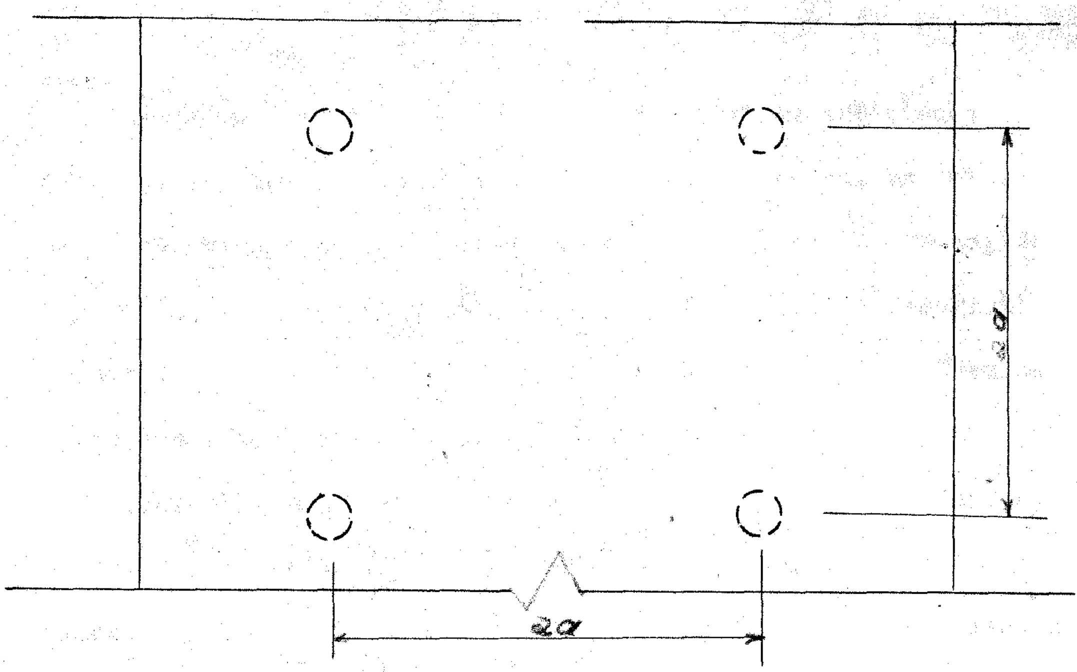
The phenomenon of consolidation or the settlement of soils under load is well known to civil engineering practice in all works dealing with earth movement and foundations. Unlike true elastic materials, this deformation takes place at a variable rate over an extended period of time and is especially apparent in clays saturated with water. This phenomenon was first explained by K. Terzaghi[1] who assumed the soil mass to be an elastic porous medium with voids filled with water or the concept of a saturated rubber sponge. The deformation of such a mass upon application of a load would then be variable, depending upon the rate at which water was forced from the voids. The application of mathematical analysis to this concept led to the complete solution by Terzaghi for the one dimensional process to be followed by the studies of M. A. Biot[2], L. Rendulic[3] and N. Carillo[4] dealing with the three dimensional case.

Within recent years, the construction of earthworks of considerable size upon foundations of saturated clay has stimulated interest in the practical aspects of the three dimensional process, in particular to the method of using vertical sand drains to accelerate the consolidating process and thus effect rapid stabilization of deep layers. Such stabilization becomes of considerable import for foundations composed of saturated clay masses in which, with increasing loads, the shearing resistance soon becomes less than the shearing stress with consequent failure through plastic flow. This decreased shearing resistance is due, in large part, to the entrapped pore water, the rapid draining of which would allow more rapid construction of a stable fill. It is apparent that this end can be accomplished by reducing the length of flow path through the provision of drainage surface in three planes. Figure 1 and 2 illustrate in plan and front elevation respectively, the usual scheme of vertical sand drains to accomplish this purpose.

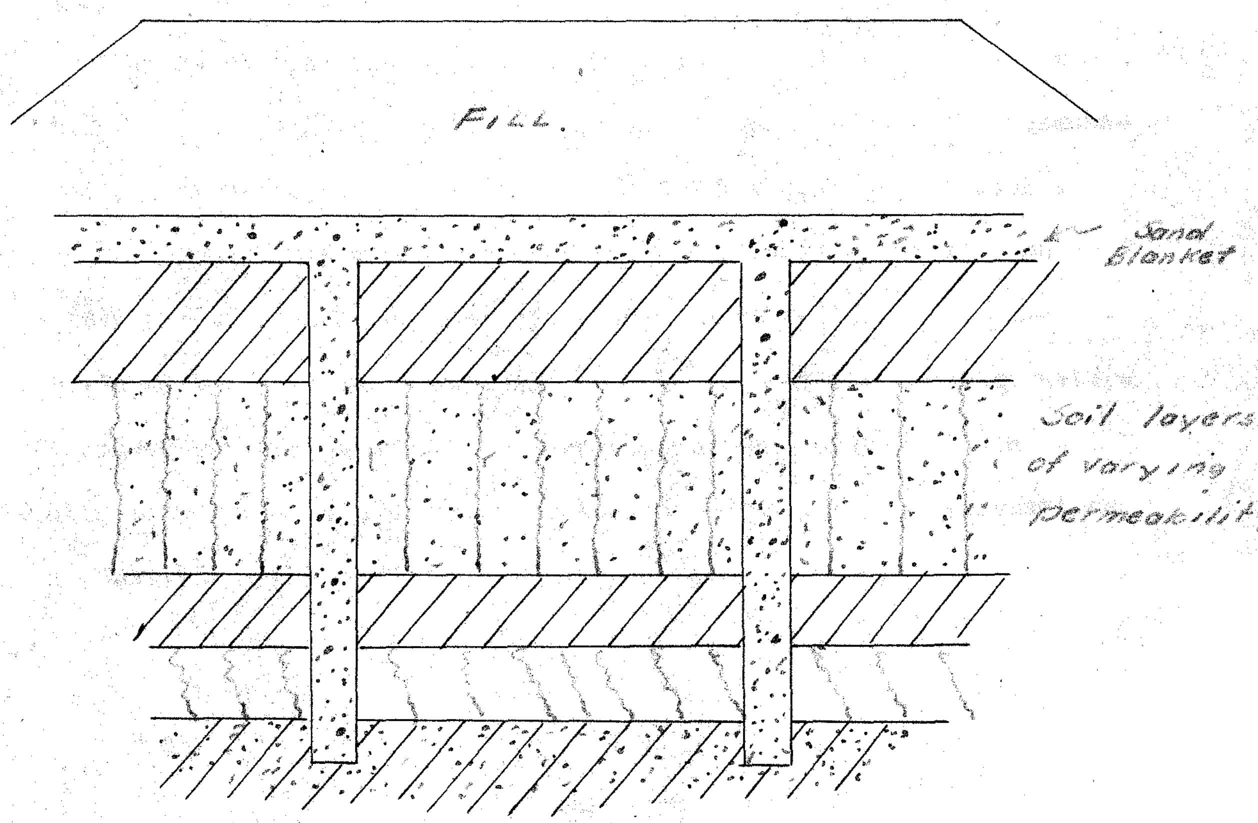
The use of vertical sand drains to effect deep soil stabilization probably found first application on the European continent, there being records of its use, with good success, in the construction of the Canal du Nord[5] in the Somme Valley. In this case the soil formation consisted in main of peat marsh and drains were spaced approximately sixteen feet apart, apparently an arbitrary estimate for spacing.

The California State Division of Highways has been foremost in American practice in the use of sand drains, principally for highway construction. Such uses have been reported by O. J. Potter[6] and G. R. Halton[7], both of the California Division of Highways. Halton lists the most important effect as that due to the generally greater permeability of soil masses along horizons or bedding planes made available by the penetration of less pervious beds by the sand drain. This is especially applicable to most California alluvial fan formations which have been laid down in alternating beds of clay and silty sand. He offers criteria for spacing as a function of height of fill, listed as follows:

Height of Fill	Surface Area of Foundation in SQ.FT. per Drain
Less than 12 feet	No drains
12 feet to 20 feet	200 square feet
Greater than 20 feet	150 square feet
Less than 12 feet at abutments	100 square feet



Plan



Elevation

Figures 1 & 2

Though no claims are made as to the relative efficiency of this criteria, it should be noted that it is quite general in nature in that properties of the soil and relative time lengths are neglected. However, his article is most valuable in its detailed description of the method of construction of drains for highways in the Terminal Island area, Los Angeles, California

Apparently the first and only published exact criteria for vertical sand drain spacing has been derived by Terzaghi[3] in collaboration with L. Rendulic. The end product of this analysis is presented in the form of two curves fixing what can be considered as the extreme upper and lower limits of spacing but leaving the region between these limits subject to estimate through interpolation.

The results of Terzaghi and Rendulic are no doubt based on a rigorous mathematical treatment as such is indicated by the presentation of the basic differential equation in Terzaghi's text book[8].

Because of the general paucity of analytical information on this subject, it will be the object of this study to outline the development of the exact analysis and, because of the unwieldy nature of consequent evaluation, to present approximate criteria with laboratory tests to determine physically the validity of assumptions and analysis.

RESULTS

A. Analytical

Appendix A presents the development of exact criteria which was not carried to conclusion because the time element required for evaluation was not within the scope of this study.

Appendix B presents the derivation of approximate criteria based on a simplifying assumption, the results of which are presented in Figure 3.

By considering the consolidation of an infinitely long, right circular cylinder, draining at all surfaces, Carillo[3] has shown that the following equation holds for the pore water pressure:

$$\frac{P}{P_0} = \frac{P_1}{P_0} \times \frac{P_2}{P_0}$$

Where P_0 equals the initial pore pressure, P equals the resultant pore pressure and P_1 and P_2 equal the pore pressures due to three dimensional flow.

In terms of degree of consolidation, this relation becomes

$$100 - \mu_t\,\% = \frac{1}{100}\left(100 - \mu_{tz}\,\%\right)\left(100 - \mu_{txy}\,\%\right)$$

Where $\mu_t\,\%$ equals the total degree of consolidation due to combined linear and radial drainage and $\mu_{tz}\,\%$ and $\mu_{txy}\,\%$ equals the degree of consolidation due to vertical linear drainage and plane radial drainage respectively

With the above relation, the influence of vertical sand drains on the rate of settlement can be computed, using the results of Figure 3 and those for vertical drainage as found in Figure 98, Krynine's textbook[9].

To illustrate, assume the following data: depth of clay layer H equal to 15 feet, spacing of sand drains equal to 15 feet, (a equals 7.5 feet) and that the formation is isotropic or a uniform modulus of consolidation, M. It is desired to determine what the total degree of consolidation will be when the consolidation without sand drains would be 40 per cent.

From Figure 98, above reference, for rectangular pressure area and $h_{zt}\% = 40$, T is found equal to 0.127

Then

$$T = \frac{M}{H^2} t = 0.127$$

$$t = \frac{0.127 H^2}{M}$$

Substituting this value of t in the time factor, T, for plane radial drainage, Figure 3

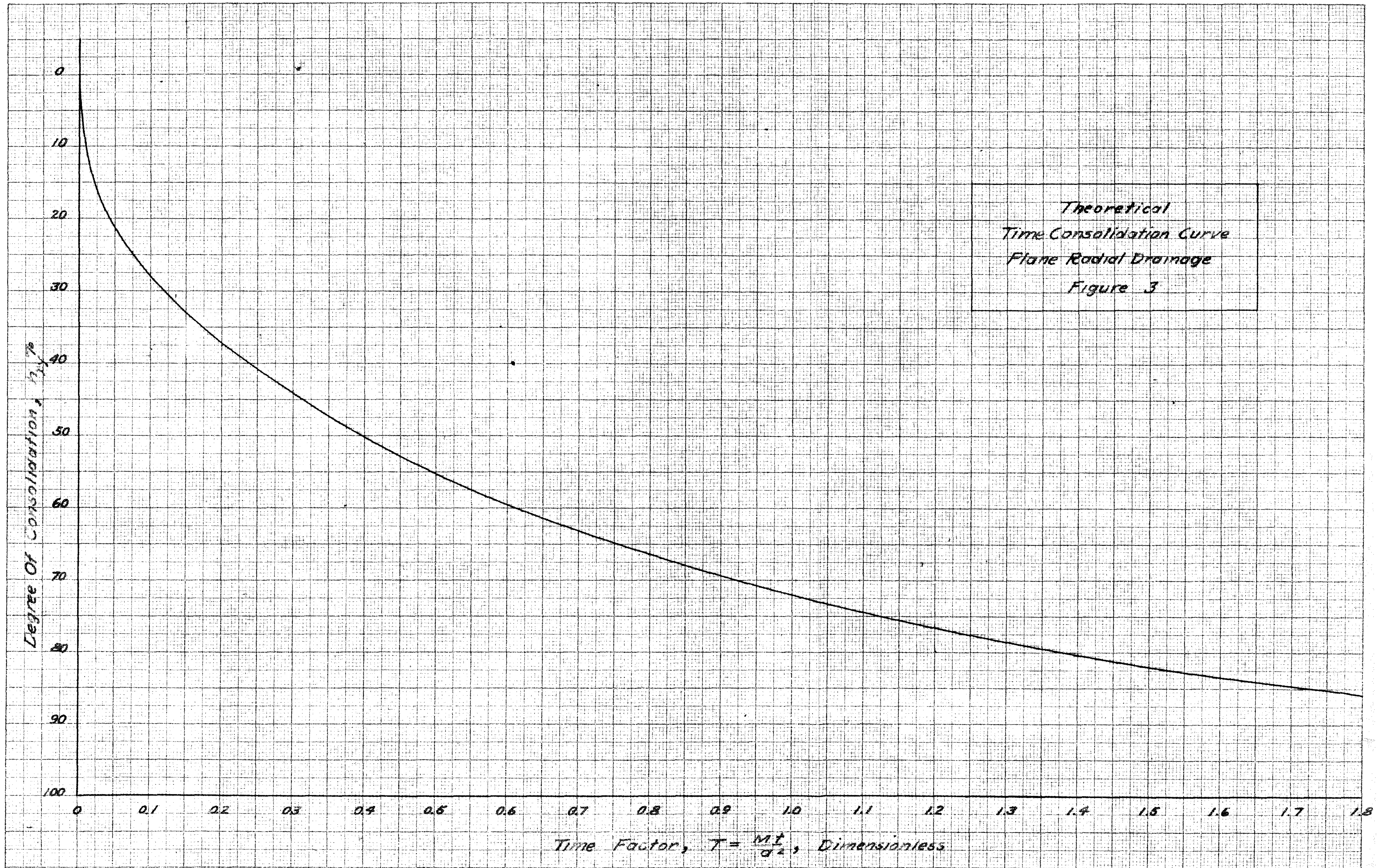
$$T = \frac{M}{a^2} t = \frac{M}{a^2} \frac{0.127 H^2}{M} = 0.508$$

Entering Figure 3 with this value, $h_{txy}\%$ is obtained as 56 per cent.

Evaluating for the total consolidation

$$100 - h_t\% = \frac{1}{100} \times 60 \times 44 = 26$$

$$h_t\% = 74\%$$



Theoretical
Time Consolidation Curve
Plane Radial Drainage
Figure 3
Degree Of Consolidation, U_r, %
0
10
20
30
40
50
60
70
80
90
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
Time Factor, $T = \frac{Mt}{a^2}$, Dimensionless

or the presence of drains increases the degree of consolidation at time t from 40 to 74 per cent.

If greater permeability occurs along horizontal bedding planes, say $K_{xy} = 2K_z$ then $M_{xy} = 2M_z$ and the time factor for radial flow is

$$T = \frac{M_{xy}}{a^2}t = \frac{2M_z}{a^2} \times \frac{0.127H^2}{M_z} = 1.02$$

or a resulting value of 84 per cent for the total degree of consolidation.

It is thus evident that sand drains are more effective when permeability of the soil mass is greater in a horizontal than in a vertical direction, a condition that usually exists because of the deposition process of formation of sedimentary silts and clays.

B. Experimental

The results of laboratory investigations, as a verification of Figure 3, are given in Appendix 3. The comparison between theoretical and actual values as illustrated by Figures 9, 10 and 11 result in reasonable agreement. The assumption on which the results of Figure 3 are based could only, at best, result in an approximation to the mean for all values of drain spacing. The experimental results point to this fact in that both curve shape and absolute values trend to closer agreement with increasing values of drain spacing. Thus, it is reasonable to assume that a point of maximum accuracy could be obtained followed by increasing inaccuracy for larger values of 2a. Further laboratory work is necessary to delineate this spectrum.

Because of experimental difficulties in obtaining consistent values for K, the coefficient of permeability, it is desirable to point out that the intrinsic accuracy of any analytical criteria is dependent upon M, the modulus of consolidation, which is in turn a direct function of K. This fact should be considered in the evaluation of accuracy of any method.

CONCLUSIONS

1. Vertical sand drains are an effective aid in accomplishing the rapid consolidation of foundations composed of saturated, loosely consolidated soil masses of low permeability.

2. The effectiveness of sand drains is due to the following major factors:

(a) Provision of drainage area in a vertical plane with a reduction in the length of flow path of the pore water.

(b) Making available the generally greater permeability in the horizontal direction or along bedding planes by puncturing impervious horizontal layers that serve to check flow in a vertical direction.

3. The theoretical effect of sand drains agrees closely with experimental results.

4. The criteria presented within this study offers a means, within limits of design accuracy, for computing the optimum spacing of drains when the soil characteristics and desired rate of loading are known.

5. Through the savings of yardage lost through fill slumping and the gain in time required for the construction of stable fills, the use of sand drains is economically justifiable from the standpoint of both time and expense. They should require ready application in military engineering work where strategic and tactical requirements militate construction of roads and air fields across marsh lands.

APPENDIX A

Development of Exact Criteria

The object of this analysis is to develop a relation between the rate of consolidation of clay layers as a function of time, soil characteristics and linear dimensions when flow of pore water is into vertical central well or drain.

The limitations of the following assumptions concerning the basic properties of the soil mass will apply:

(1) That Hooks law applies

(2) That the material is isotropic.

(3) That Darcy's law is valid for the flow of pore water.

(4) That the pore water is incompressible.

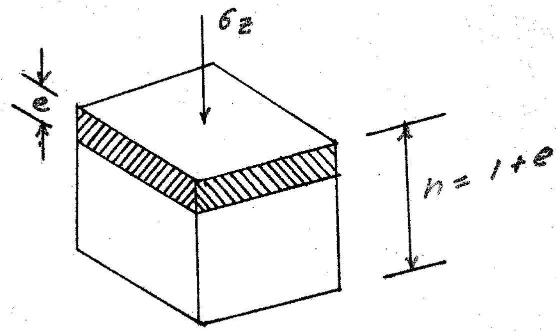
(5) That the distribution of pressure is symmetrical with respect to the coordinate axis or $\frac{\partial p}{\partial \phi} = 0$

(6) That the distribution of pressure is linear along dimensions parallel to the plane of consolidation or $\frac{\partial p}{\partial z} = K$

(7) That the initial stress is first taken by the pore water and subsequently released to the soil particles.

Assumptions (1) and (6) are most open to debate, but for brevity, discussion is omitted because of previous detailed argument by investigators such as Terzaghi and Biot.

Consider first a cube of saturated earth material, confined on all sides and the bottom by a retaining impervious material and subjected to a unit vertical pressure, σ_z, causing a unit vertical pressure within the soil particles equal to σ_s.



Assuming Hooks law applies $\quad E = \dfrac{\sigma_s}{\epsilon} = \dfrac{\sigma_s h}{\delta}$

δ total deformation

ϵ unit deformation $= \dfrac{\delta}{h}$

Applying the decrease in void ratio, e, as a measure of deformation and e_f as the final void ratio after deformation, then

$$\delta = e - e_f \qquad\qquad \epsilon = \frac{\delta}{h} = \frac{e - e_f}{1+e}$$

Set c = a constant for the range of deformation $= \dfrac{e - e_f}{\sigma_s}$

Then $\quad E = \sigma_s \dfrac{1+e}{e - e_f} = \dfrac{1+e}{c} = \dfrac{\sigma_s}{\epsilon} \qquad \epsilon = \sigma_s \dfrac{c}{1+e}$

Setting $\dfrac{c}{1+e} = m$ a constant for the range of deformation

Then $\quad \epsilon = m\,\sigma_s$

Next consider a section, a -- a , cut through the cube of figure 3.

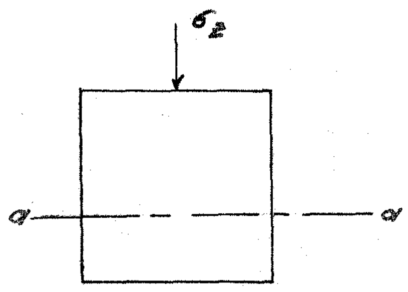


Figure 4

Because of the low permeability of the saturated mass the entrapped water cannot escape quickly and therefore, from statics, along plane, a -- a, the unit water pressure, p, plus the unit soil pressure, σ_s, must equal σ_z

$$\sigma_s + p = \sigma_z$$
$$\sigma_s = \sigma_z - p$$

The force causing consolidation is σ_s while p is the water pressure within the soil voids in excess of hydrostatic pressure, defined as the pore pressure.

Thus, consolidation progresses only when p becomes smaller or as the quantity of water is reduced by being squeezed out.

In being forced from the soil mass, the pore water passes through the many stream lines formed by the soil voids and it thus becomes necessary to accept the cross section of flow as that determined by the impervious boundaries.

Since the resulting velocity will be very small, the velocity head may be neglected and the total energy becomes the sum of pressure head and elevation.

$$E_w = \frac{p}{\gamma} + Z = H_w$$

Because the flow is laminar the effective velocity of flow will be directly proportional to the rate of energy loss in the direction of flow. Introducing k as a coefficient of proportionality

$$V = K \frac{H_{W2} - H_{W1}}{L} = KS$$

where s is the slope of the energy line and k is termed the coefficient of permeability in units of LT^{-1}. Although k is a function of both the soil characteristics and dynamic viscosity of water, there is no known functional relationship between the two. Since the temperature of the soil water remains within a small range, the value of k obtained experimentally for a specified fluid and medium may be used with negligible error.

The simplification is, of course, the well known Darcy law.

For a differential depth Δz the total differential settlement must equal unit deformation by length.

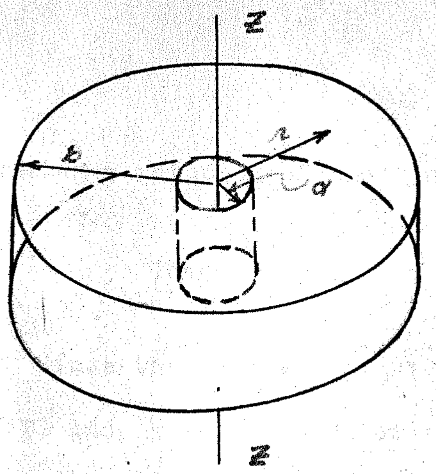
$$\Delta h = e \Delta z = m(\sigma_z - p)\Delta z$$

At the application of σ_z or time zero, no deformation takes place because no water is squeezed from the voids or $p \simeq \sigma_z$. However, at time $t > 0$ an amount of pore water has been squeezed out with a decrease in p and an increase in $\bar{\sigma}_z$ and a commencement of settlement. Then at time t the following relation exists $\Delta h_t = m \Delta z (\sigma_z - p(tz))$

$$h_t = m \int_0^H [\sigma_z - p(tz)]\, dz \qquad (1\text{-}a)$$

where h_t settlement of a layer, H, in depth at time t.

$p(tz)$ must now be determined. Using the convenience of cylindrical coordinates, consider a cylinder of the mass, Z in depth as illustrated by Figure 5.



a = radius of the central well or drain

b = radius of the consolidating earth mass

r = a variable radius

$b > r > a$

Figure 5

Since the voids are saturated the quantity squeezed out of a slice Z in depth is $Q = -2\pi Z \int_a^b \epsilon\, r\, dr$

$$\text{(a)} \qquad -\frac{\partial Q}{\partial r} = 2\pi Z r m \sigma_s$$

From Darcy's law $V = SK = \frac{K}{\gamma} \frac{\partial p}{\partial r}$

and the total Q flowing through a section at radius r is

$$\text{(b)} \qquad \frac{\partial Q}{\partial t} = AV = \frac{K}{\gamma} 2\pi r Z \frac{\partial p}{\partial r}$$

Differentiating (a) with respect to t and using the relation $\frac{\partial \sigma_s}{\partial t} = -\frac{\partial p}{\partial t}$

$$\frac{\partial Q}{\partial r \partial t} = 2\pi Z m r \frac{\partial p}{\partial t}$$

Differentiating (b) with respect to r

$$\frac{\partial Q}{\partial r \partial t} = 2\pi Z \frac{K}{\gamma} \left(r^2 \frac{\partial^2 p}{\partial r^2} + \frac{\partial p}{\partial r} \right)$$

Subtracting

$$m r \frac{\partial p}{\partial t} = \frac{K}{\gamma} \left(r \frac{\partial^2 p}{\partial r^2} + \frac{\partial p}{\partial r} \right)$$

Setting $\frac{k}{m\gamma} = M$, a constant for the range of deformation and termed the modulus of consolidation

$$\frac{\partial p}{\partial t} = M \left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right)$$

Assuming a solution of the type, $p(r,t) = R(r) \cdot T(t)$

Then
$$\frac{T'}{MT} = \frac{R''}{R} + \frac{R'}{rR}$$

Since the left and right sides of the equation are functions of T and R alone respectively, they both must equal a constant, $-\beta^2$, which must be negative in sign to obtain physical compatibility.

Then
$$T' = -M\beta^2 T$$
$$T = A\, e^{-M t \beta^2}$$

Where A and β^2 are arbitrary constants to be determined

The equation for R becomes $\dfrac{d^2R}{dr^2} + \dfrac{dR}{r\,dr} + \beta^2 R = 0$

By letting $r = \gamma Z$ where γ is a constant chosen equal to $\frac{1}{\beta}$ the above equation is identified as Bessel's equation of order zero

or
$$\frac{d^2R}{dZ^2} + \frac{dR}{Z\,dZ} + Z = 0$$

and
$$R = J_0(Z) = J_0(\beta, r)$$

Because of the discontinuity at the inner radius a, Bessel functions of the second kind will appear in the evaluation and the general solution for R becomes

$$R(\beta r) = C_1 J_0(\beta r) + C_2 Y_0(\beta r)$$

The following boundary conditions apply

(1) The pore water pressure at the inner surface is zero for all time $p(at) = 0$

(2) The pore water pressure throughout the cylinder is zero at time infinity $p(r, \infty) = 0$.

(3) The flow of pore water across the outer surface is zero for all time $\frac{dp}{dr}(b, t) = 0$

One initial condition applies, the pore water pressure between the two cylinder surfaces is assumed constant throughout the mass at time zero $p(r, 0) = \sigma_z$

Applying the above conditions to the general solution

$$(1) \quad p(a, t) = Ae^{-M\beta^2 t}\left[C_1 J_0(\beta a) + C_2 Y_0(\beta a)\right] = 0$$

$$C_1 J_0(\beta a) + C_2 Y_0(\beta a) = 0$$

$$(2) \quad \frac{dp}{dr}(r t) = Ae^{-M\beta^2 t}\left[C_1 J_0'(\beta b) + C_2 Y_0'(\beta b)\right] = 0$$

$$C_1 J_0'(\beta b) + C_2 Y_0'(\beta b) = -C_1 J_1(\beta b) - C_2 Y_1(\beta b) = 0$$

$$C_1 J_1(\beta b) + C_2 Y_1(\beta b) = 0$$

(3) Condition three has been satisfied by the selection of a negative sign for β^2

$$(4) \quad \sigma_z = \sum_{n=0}^{\infty} A_n\left[C_1 J_0(\beta_n r) + C_2 Y_0(\beta_n r)\right]$$

Then from (1) and (2) above

$$C_1 = -C_2 \frac{Y_0(\beta a)}{J_0(\beta a)} = -C_2 \frac{Y_1(\beta b)}{J_1(\beta b)}$$

and

$$\frac{Y_0(\beta a)}{J_0(\beta a)} = \frac{Y_1(\beta b)}{J_1(\beta b)}$$

This equation is satisfied by an infinite number of Betas or by the constant C_2 equal to zero. But the stated boundary conditions require that C_2 not be zero.

Selecting a dimensionless parameter $K = \frac{b}{a}$ with the inner radius a as unity and further, taking as a new variable $x_K = \beta_K a$ the equation above becomes

$$J_0(x_K)Y_1(Kx_K) - J_1(Kx_K)Y_0(x_K) = 0$$

The problem is now resolved into the task of determining by trial a number of roots x_K, corresponding to an arbitrarily selected K sufficient to allow the expansion and summation of the series represented by the initial condition to the desired degree of approximation. Complete tabulated values of J_n and Y_n corresponding to the selected argument of x_K and Kx_K are found in "Theory of Bessel Functions" by G. N. Watson. This task, however involves a lengthy computation period which includes the expansion and summation of the resulting series. Such length of time is not available within the scope of this study.

For the discussion of a somewhat similar problem but dealing with heat flow, readers are referred to the paper, "Temperature and Stress Distribution in Hollow Cylinders" by O. G. C. Dahl, Proceedings of the American Society of Mechanical Engineers, 1924.

For the purposes of this study, recourse will be made to a less rigorous solution but one capable of more rapid evaluation. Such solution is presented in Appendix B herewith.

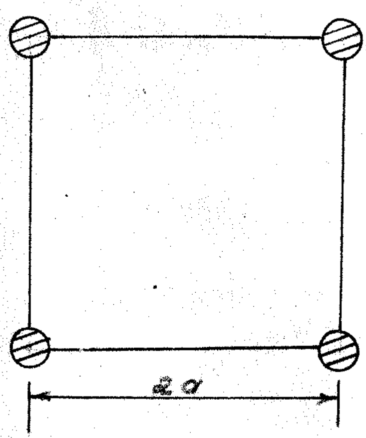
APPENDIX B

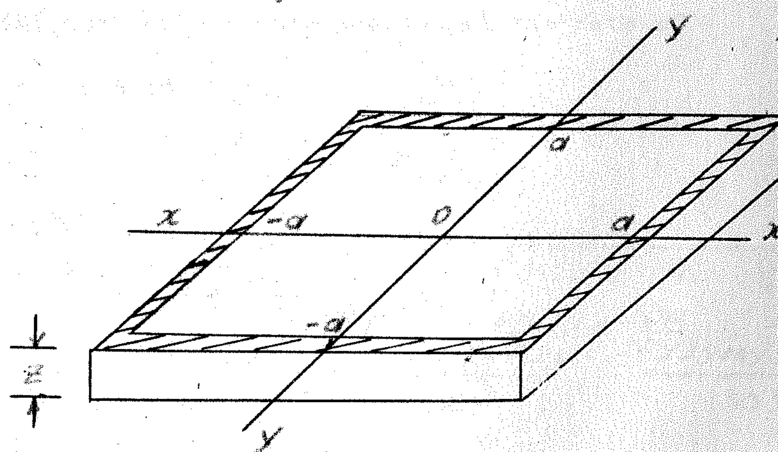
Derivation of Approximate Criteria

The object of this analysis is similar to that of Appendix A, to determine, in an approximate manner, the rate of settlement of clay layers as a function of time, soil characteristics, and coordinate space when flow of pore water is into central wells or drains. The basic assumptions of Appendix A will apply.

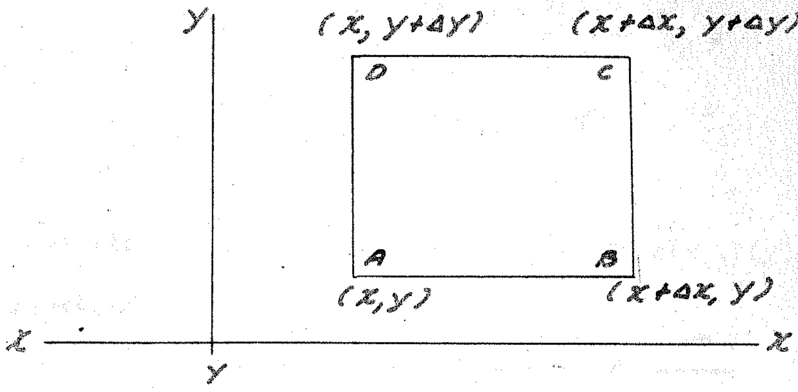
A simplifying approximation will be made, that the drains or wells at each corner of the consolidating mass may be replaced by equivalent line drains surrounding the perimeter of each block whose side, in a horizontal plane, is taken equal to 2a or the spacing of the drains. The error of this simplification, of course, increases with increasing size of spacing and is due to the resulting decreased length of flow path. However, an adjustment to the length of spacing will partially correct for this discrepancy.

Figure a below illustrates the normal plan of sand drains with spacing for this analysis taken equal to 2a. Figure 2a illustrates the above assumption with the coordinate system chosen.





Next consider a differential solid Z in thickness with coordinates as shown .



The quantity entering AB in unit time is

$$\Delta Q = \frac{KZ}{\gamma} \Delta x \left.\frac{\partial p}{\partial y}\right]_{y}$$

and the quantity leaving CD must equal $-\frac{K}{\gamma} Z \Delta x \left.\frac{\partial p}{\partial y}\right]_{y+\Delta y}$

Then the rate of loss of water due to these two boundaries is approximately $\frac{K}{\gamma} Z \Delta x \left(\left.\frac{\partial p}{\partial y}\right]_{y+\Delta y} - \left.\frac{\partial p}{\partial y}\right]_{y} \right)$

In like manner, the approximate rate of loss of moisture due to boundaries AD and BC is $\frac{K}{\gamma} Z \Delta y \left(\left.\frac{\partial p}{\partial x}\right]_{x+\Delta x} - \left.\frac{\partial p}{\partial x}\right]_{x} \right)$

Further, this total loss of moisture must equal the rate of loss in void space due to consolidation or

$$\Delta x \Delta y \epsilon Z = m Z \Delta x \Delta y \frac{\partial \sigma_s}{\partial t}$$

and

$$m Z \Delta x \Delta y \frac{\partial \sigma_s}{\partial t} = \frac{KZ}{\gamma} \Delta x \Delta y \left(\frac{\left.\frac{\partial p}{\partial x}\right]_{x+\Delta x} - \left.\frac{\partial p}{\partial x}\right]_{x}}{\Delta x} + \frac{\left.\frac{\partial p}{\partial y}\right]_{y+\Delta y} - \left.\frac{\partial p}{\partial y}\right]_{y}}{\Delta y} \right)$$

Using the relation, $\frac{\partial \sigma_s}{\partial t} = \frac{\partial p}{\partial t}$ and passing to the limit as Δx & Δy approach zero

$$\frac{\partial p}{\partial t} = \frac{K}{\delta m}\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right)$$

or

$$\frac{\partial p}{\partial t} = M\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right)$$

Again assuming a solution of the form $P = X(x) \cdot Y(y) \cdot T(t)$ the above equation becomes

$$\frac{T'}{M\,T} = \frac{X''}{X} + \frac{Y''}{Y}$$

For the above relation to be true, each of the three terms must equal a constant and in order that the function $T(t)$ be physically compatible, each member must be a negative constant. Then

$$\frac{X''}{X} = -\alpha^2, \quad \frac{Y''}{Y} = -\mu^2 \quad \text{and} \quad \frac{T'}{T} = -M(\alpha^2+\mu^2)$$

The solution for T becomes $T = e^{-M(\alpha^2+\mu^2)t}$

and for X and Y

$$X = A\cos\alpha x + B\sin\alpha x$$

$$Y = C\cos\mu y + D\sin\mu y$$

The following boundary conditions apply:

(1) The pore pressure at all edges is zero

$$p(-a, a; -a, a; t) = 0$$

(2) The pore pressure at time infinity is zero throughout the mass

$$p(x, y, \infty) = 0$$

One initial condition maintains; The pore pressure at time zero is a constant throughout the mass $p(x, y, 0) = \sigma_E$

Because of symmetry, the terms possessing sines in X and Y will vanish leaving as a solution

$$p = \left[A_{mn}\cos\alpha x\cos\mu y\right] e^{-M(\alpha^2+\mu^2)t}$$

Evaluating at the limits for condition (1), the constants α and μ are determined as

$$\alpha = (m+\tfrac{1}{2})\frac{\pi}{a}$$

$$\mu = (n+\tfrac{1}{2})\frac{\pi}{a}$$

where m and n are integers

Then to obtain the complete solution it is necessary to form the double series

$$p(xyt) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}\, e^{-\frac{K_{mn} M t}{a^2}} \cos(m+\tfrac{1}{2})\pi\frac{x}{a} \cos(n+\tfrac{1}{2})\frac{\pi y}{a}$$

where $K_{mn} = \pi^2[(m+\tfrac{1}{2})^2 + (n+\tfrac{1}{2})^2]$ and A_{mn} is arbitrary

This equation will satisfy all stated conditions if the initial condition $p(x,y,0) = \sigma_z$ can be represented by the double series

$$\sigma_z = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} A_{mn} \cos(n+\tfrac{1}{2})\frac{\pi x}{a} \cos(m+\tfrac{1}{2})\frac{\pi y}{a}$$

Multiplying both sides by $\cos(m+\tfrac{1}{2})\frac{\pi x}{a}dx \cos(n+\tfrac{1}{2})\frac{\pi y}{a}dy$ and integrating

$$4\sigma_z\left(\frac{(-1)^m}{(m+\tfrac{1}{2})\frac{\pi}{a}} \times \frac{(-1)^n}{(n+\tfrac{1}{2})\frac{\pi}{a}}\right) = a^2 A_{mn}$$

or

$$A_{mn} = 4\sigma_z \frac{(-1)^{m+n}}{(m+\tfrac{1}{2})(n+\tfrac{1}{2})\pi^2}$$

But the average pressure over the mass is desired or $P_{av} = \int \frac{p\,dA}{A}$

and

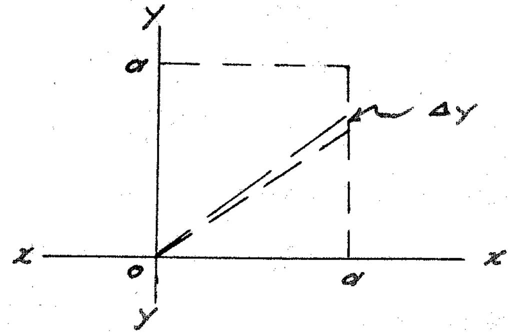
$$P_{av} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{1}{a^2}\left[A_{mn}\, e^{-\frac{K_{mn}}{a^2}Mt} \int_0^a \cos(m+\tfrac{1}{2})\frac{\pi x}{a}dx \int_0^a \cos(n+\tfrac{1}{2})\frac{\pi y}{a}dy\right.$$

$$\int_0^a \cos(m+\tfrac{1}{2})\frac{\pi x}{a}\,dx \int_0^a \cos(m+\tfrac{1}{2})\frac{\pi y}{a}\,dx = \left[\frac{(-1)^m}{(m+\frac{1}{2})\frac{\pi}{a}} \times \frac{(-1)^n}{(n+\frac{1}{2})\frac{\pi}{a}}\right]$$

Then $$p_{av} = \frac{4 6_z}{\pi^4} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{(m+\frac{1}{2})^2(n+\frac{1}{2})^2} e^{-K_{mn}\frac{Mt}{a^2}}$$

Because of the approximate nature of this analysis, an adjustment to the value of the spacing, a , should be applied to correct for the actual length of flow path.

consider the mean of the actual flow path, l_m



$$l_m = \frac{1}{a}\int_0^a l\,dy = \frac{1}{a}\int_0^a \sqrt{y^2+a^2}\,dy$$

$$= \frac{a}{2}\left[\sqrt{2} + \ln(1+\sqrt{2})\right) = 1.2\,a$$

Likewise the mean of the assumed flow path, l_{ma}

$$l_{ma} = \frac{1}{a}\int_0^a x\,dx \tan\frac{\pi}{4} = \frac{a}{2}$$

Then the factor for the corrected value of a to be used in computation would be the ratio of the two means.

$$a_c = \frac{1.2a}{0.5} = 2.4a$$

Referring now to formula (1-a) of Appendix A or

$$h_t = m\int_0^H \sigma_z - p(xyt)\,dz$$

which now becomes.

$$h_t = m\sigma_z H\left[1 - \frac{4}{\pi^4}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{e^{-K_{mn}T}}{(m+\frac{1}{2})^2(n+\frac{1}{2})^2}\right.$$

where $T = \frac{Mt}{a^2}$, a variable, depending upon the soil characteristics, type of loading and time. T, termed the time factor of consolidation is a dimensionless number and for computation, care must be taken to use consistent units of force, length and time.

Noting that the total ultimate settlement is represented by $h = m\sigma_z H$ the percent total settlement at time t becomes,

$$h_t\,\% = 100\,\frac{h_t}{h}$$

It follows that the percent total settlement depends only on the type of load, soil characteristics, drain spacing and time and not upon the unit pressure or depth of the layer.

The problem is now resolved into the task of computing the values for the following expression for varying values of T and plotting such values as ordinates with the variable T as abscissa;

$$h_t\,\% = 100\left[1 - \frac{4}{\pi^4}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{e^{-K_{mn}T}}{(m+\frac{1}{2})^2(n+\frac{1}{2})^2}\right.$$

The method of computation is illustrated by tables one and two below for values of T equal to zero and one tenth. Table three consists of values for T varying from zero to two, from which the curve of degree of consolidation has been drawn as illustrated by figure 3.

TABE 1
($T=0$)

n \ m	0	1	2	3	4	5	6	7
0	16.000	1.778	0.641	0.327	0.198	0.133	0.095	0.081
1	1.778	0.198	0.071	0.036	0.022	0.015	0.011	0.008
2	0.641	0.071	0.026	0.013	0.008	0.005	0.004	0.003
3	0.327	0.036	0.013	0.007	0.004	0.003	0.002	0.001
4	0.198	0.022	0.008	0.004	0.002	0.001	0.001	
5	0.133	0.015	0.005	0.003	0.001	0.001		
6	0.095	0.011	0.004	0.002	0.001			
7	0.081	0.008	0.003	0.001				
Σ	19.253	2.139	0.771	0.393	0.236	0.158	0.113	0.093

$\Sigma\Sigma = 23.16$

$\frac{\pi^4}{4} = 24.30$ 5% error at 7 terms

TABLE 2
($T = 0.1$)

n \ m	0	1	2	3
0	14.70	1.14	0.21	0.04
1	1.14	0.09	0.02	
2	0.21	0.02		
3	0.04			
Σ	16.09	1.25	0.23	0.04

$\Sigma\Sigma = 17.61$

$\frac{17.61}{24.30} = 0.725$

$100(1-0.725) = 27.5\%$

TABLE 3

T	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.80	1.0	1.2
U %	21.7	27.5	37.6	43.8	50.0	55.4	59.6	66.4	72.0	76.4

T	1.4	1.6	1.8	2.0	
U %	80.2	83.3	85.9	88.1	

APPENDIX C

Laboratory Investigations

The object of laboratory tests was to determine the agreement between computed and actual time rates of settlement for a soil sample with varying values of sand drain spacing.

The procedure consisted of selecting a soil sample, and, for a uniform moisture content and void ratio, determining the modulus of consolidation using standard laboratory practice and apparatus. Experimental time settlement curves for a model soil sample with prototype drain spacings of 5.5, 7.5 and 9.5 feet were then determined, adjusted and plotted. The models were to a scale 1/4 inch to one foot with the drains 1/4 inch in diameter and backfilled with standard Ottawa sand. The computed time settlement curve for each of the three spacings was obtained by substituting the respective values of a in the time factor of Figure 3, using the determined value of M.

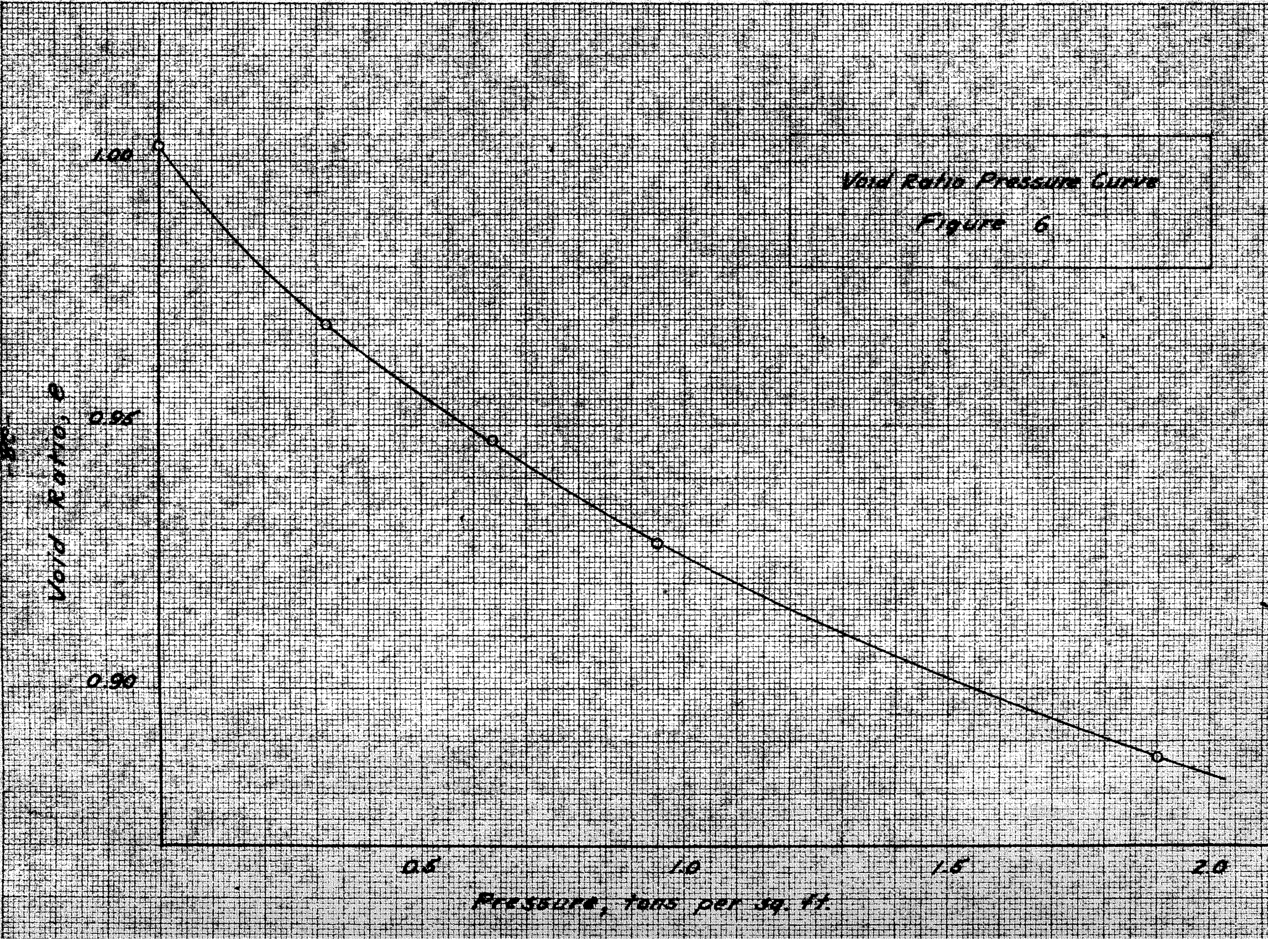
The apparatus was standard except for the retaining rings of the consolidometer. These were made from standard brass pipe sections one inch long and machined to inside diameters of 2-3/4, 3-3/4 and 4-3/4 inches. Each ring was equipped with a brass base against which the ring was clamped, and in addition was fitted with a brass cover plate machined so as to fit within the ring with 0.01" tolerance. A 1/4 inch hole was drilled in the center of each coverplate to provide an outlet for the sand drain. Figure 12 illustrates these rings and the test specimens.

The soil sample was a remolded silty clay, specific gravity 2.67, mass specific gravity 1.76, maintained at a uniform void ratio of 1.03 and moisture content of 34%.

Figures 6 and 7 illustrate the void ratio pressure curve and K as a function of void ratio, respectively. Mean values were taken for K because of the variance caused apparently by continuous soil structure readjustment.

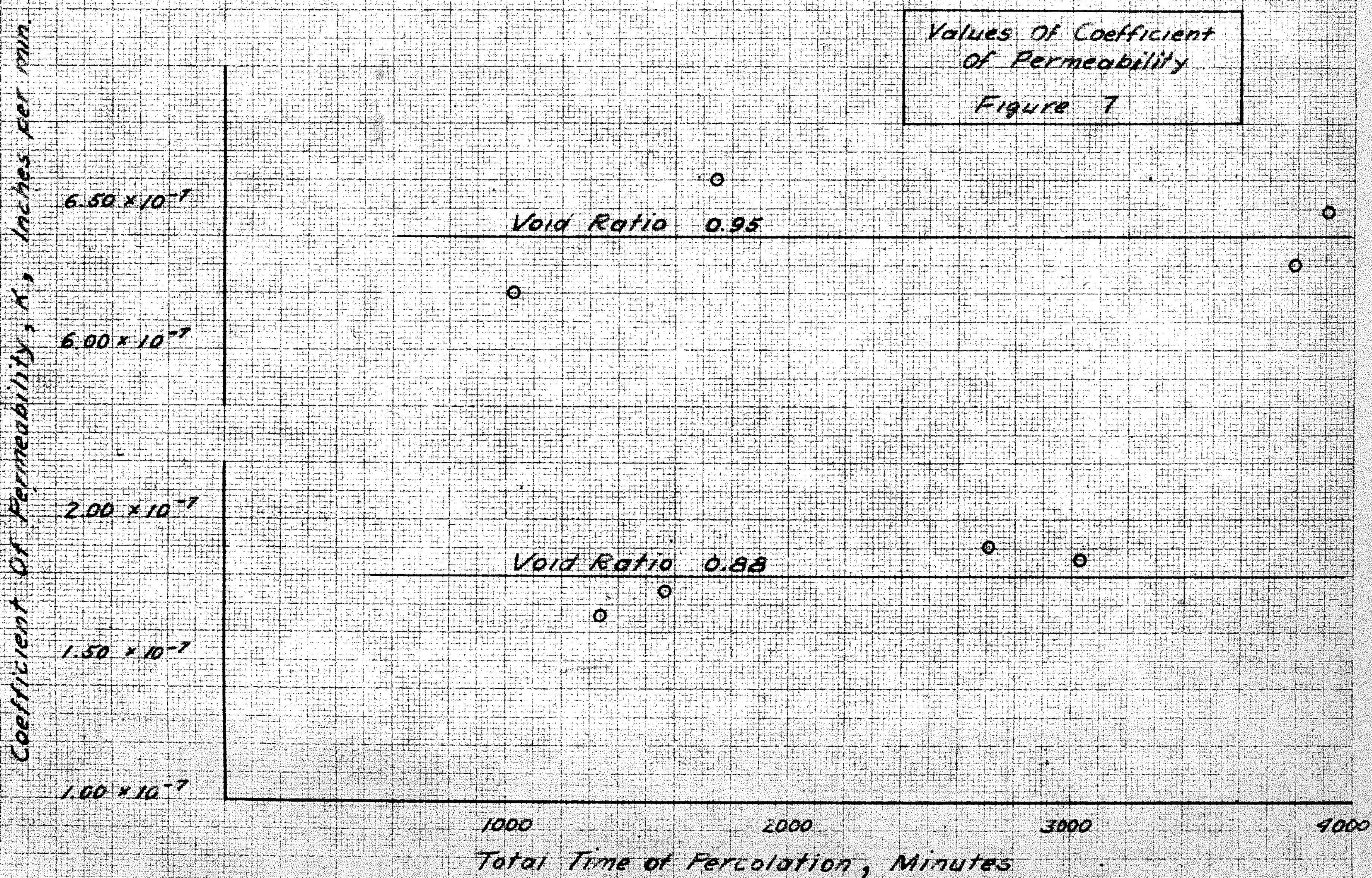
Using a value of K equal to 1.8×10^{-7} inches per minute, the value of M was determined as 1.87×10^{-3} inches squared per minute. This agreed favorably with values of 1.71×10^{-3} inches squared per minute obtained in the manner described in paragraph 102, Krynine[9] for 30 and 40 per cent consolidation and time values gained from the experimental time consolidation curve illustrated by Figure 8. A mean value for M or 1.6×10^{-3} inches squared per minute was used for computation.

Figures 9, 10 and 11 illustrate the comparison between theoretical and experimental time settlement curves. Since drainage took place at the edges of each ring as well as through the sand drain, it was necessary to use values of 2a equal to one-half the prototype diameter of the corresponding test ring.

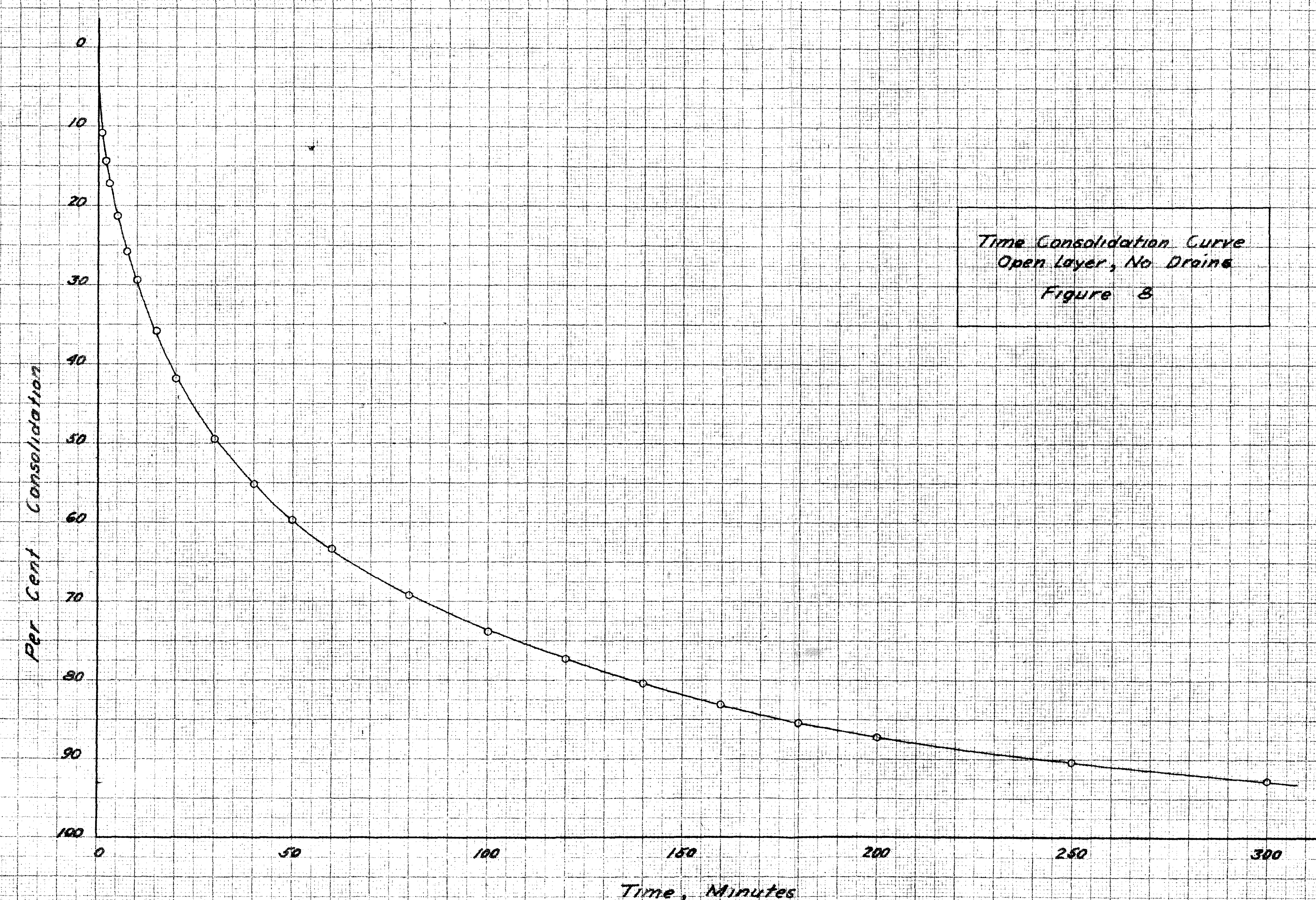


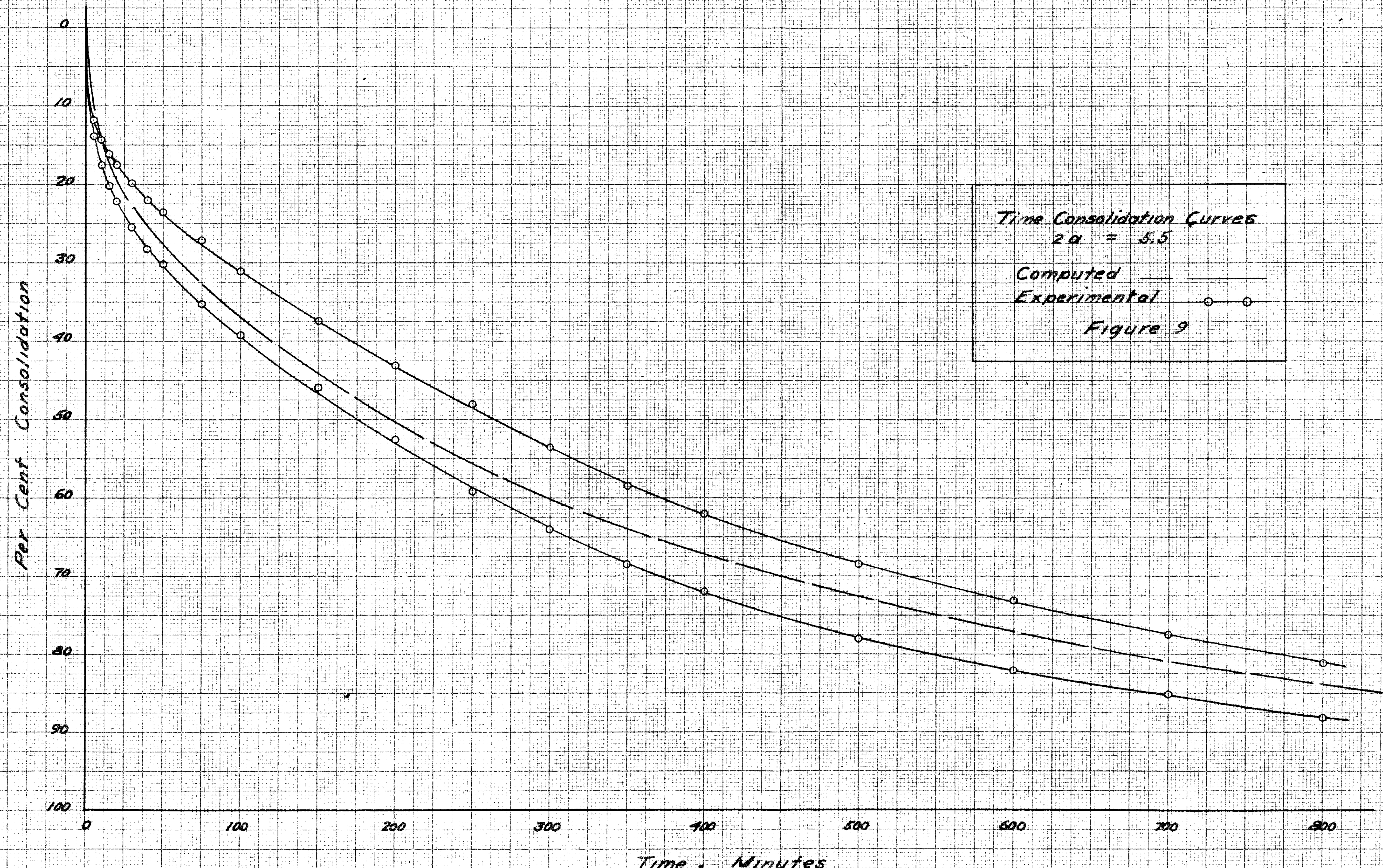
Void Ratio Pressure Curve

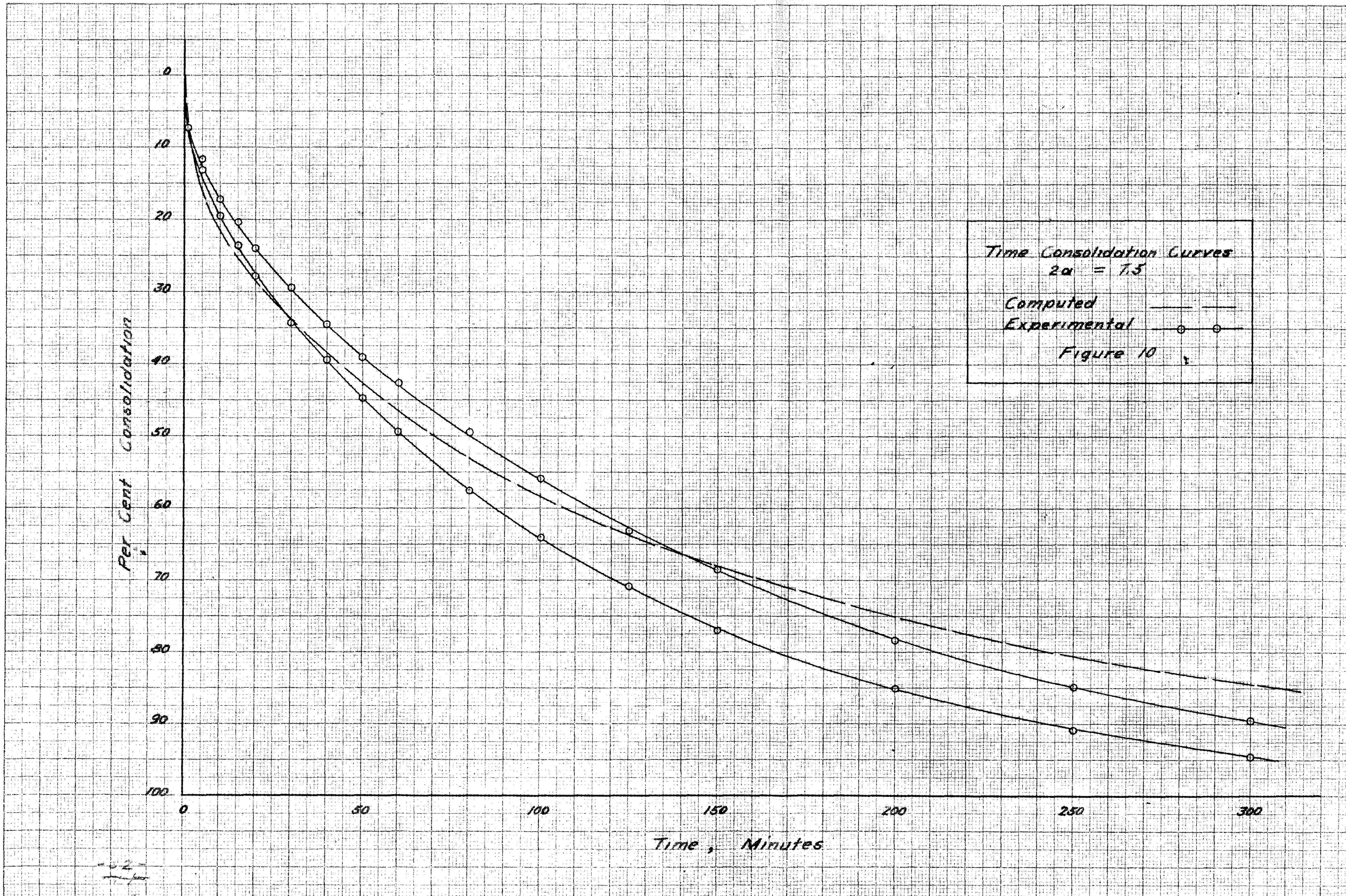
Figure 6



Values Of Coefficient Of Permeability
Figure 7
Void Ratio 0.95
Void Ratio 0.88
6.50 × 10⁻⁷
6.00 × 10⁻⁷
2.00 × 10⁻⁷
1.50 × 10⁻⁷
1.00 × 10⁻⁷
Coefficient Of Permeability, K, Inches Per min.
1000
2000
3000
4000
Total Time of Percolation, Minutes


Time Consolidation Curve
Open Layer, No Drains
Figure 8
Per Cent Consolidation
0
10
20
30
40
50
60
70
80
90
100
0
50
100
150
200
250
300
Time, Minutes


Time Consolidation Curves
2a = 5.5
Computed
Experimental
Figure 9
Per Cent Consolidation
Time , Minutes
0
10
20
30
40
50
60
70
80
90
100
0
100
200
300
400
500
600
700
800


Time Consolidation Curves
2a = 7.5
Computed
Experimental
Figure 10
Per Cent Consolidation
0
10
20
30
40
50
60
70
80
90
100
0
50
100
150
200
250
300
Time, Minutes

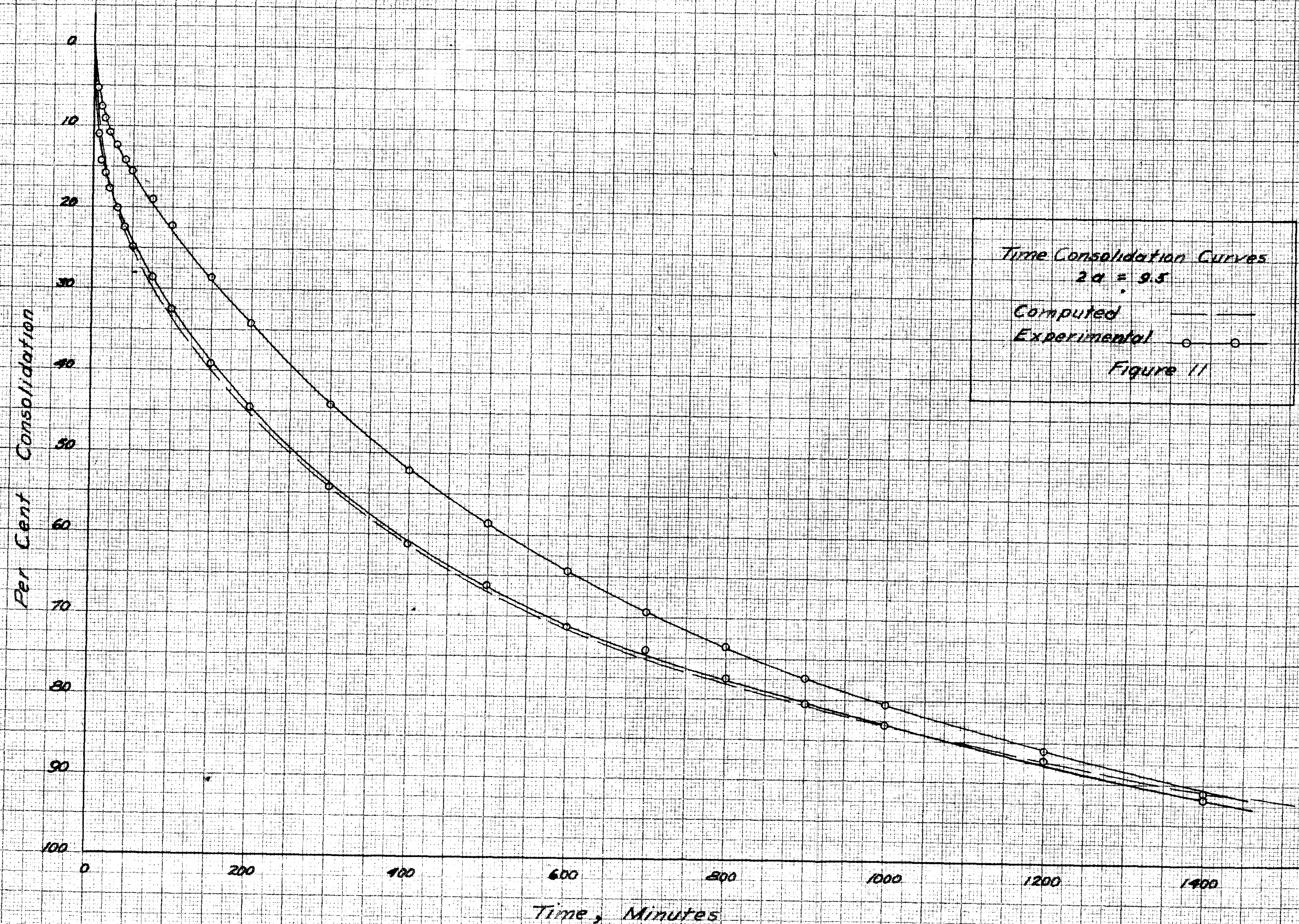

Time Consolidation Curves
2a = 9.5
Computed
Experimental
Figure 11
Per Cent Consolidation
0
10
20
30
40
50
60
70
80
90
100
0
200
400
600
800
1000
1200
1400
Time, Minutes

Figure 12

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