AN EXPERIMENTAL INVESTIGATION OF TURBULENCE MIXING

as a factor in the

TRANSPORTATION OF SEDIMENT IN OPEN CHANNEL FLOW

Thesis by

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SYNOPSIS

This investigation was conducted for the purpose of making a thorough study of the fundamental properties of the turbulence mixing process involved in the flow of water in an open channel.

A channel $10\frac{1}{2}$ in. wide, 10 in. deep, and 40 ft. long was constructed in such a manner that illuminated globules formed by the injection of an immiscible fluid into the water could be photographed. A statistical analysis of the displacements of the globules from the origin yielded the intensity of turbulence and the correlation of successive velocities of a particle of fluid.

Through these measurements of the intensity of turbulence and the correlation of velocities, the nature of the turbulence in open channel flow was determined. The results of the investigation led to the conclusion that similarity of diffusion exists throughout the vertical center line of open channel flow. Due to this similarity, the mixing length may be proportional to the root-mean-square velocity fluctuation and the mixing coefficient proportional to the mean-square velocity fluctuation or energy of turbulence. The concept of a particle of fluid losing its identity in the surrounding fluid was found to be untrue for the flows investigated herein.

This study was conducted under the direction of Robert T. Knapp, Associate Professor of Hydraulic Engineering, California Institute of Technology.

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The turbulence mixing of flowing water has for a long time been of interest to the fluid technician and the hydraulic engineer. Inherent in the turbulence of the water, this mixing is responsible for the existence of many internal phenomena such as apparent shearing stress, convective heat transfer, and sediment suspension.

Mixing of water may be characterized by the mixing coefficient, or more technically, the turbulence diffusion coefficient. A detailed investigation of the diffusion coefficient of the fluid itself is necessary before we can determine whether this coefficient applies directly to the various mixing phenomena. Also essential is a knowledge of the variation of the mixing coefficient from point to point, for without that, integration of the fundamental equation for sediment suspension is impossible.

Some observations[1] on diffusion in water have been made, but most investigations have been conducted in air, either in the wind tunnel[2] or the atmosphere[3].

THEORETICAL CONSIDERATIONS

The diffusion theory for the turbulent motion of a fluid is closely analogous to the theory for the diffusion of gases and of particles in Brownian movement. For this reason the latter theory will be reviewed first.

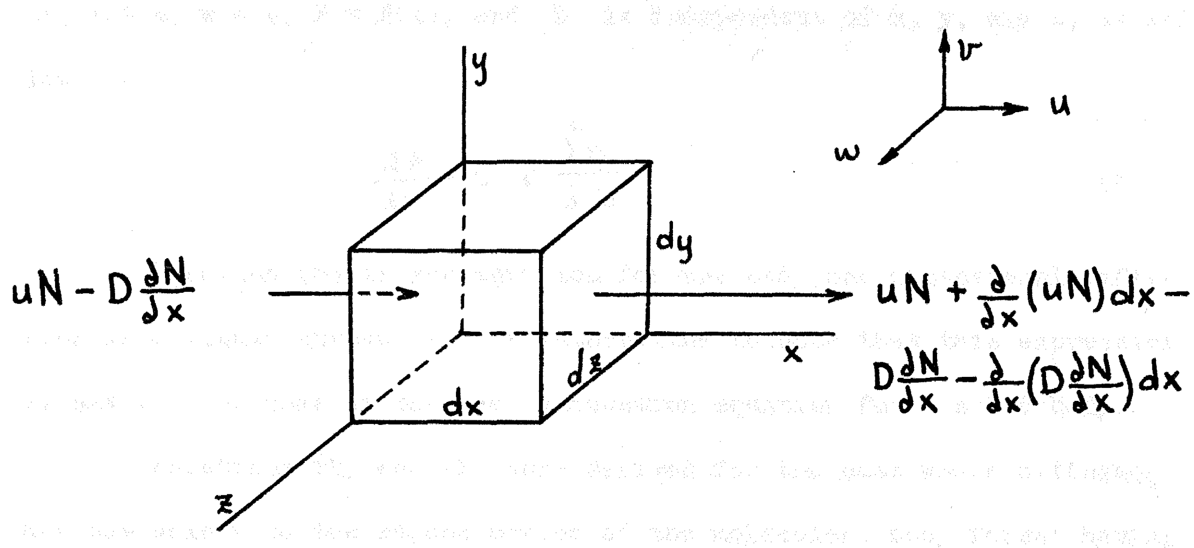
Molecular Diffusion--Kinetic Theory.

From experiments[4] on diffusion it has been assumed that, for the steady state, the number of molecules n diffusing through a given area $dx\,dy$ in a time t may be expressed by the relation

$$n = -D\frac{\partial N}{\partial z}\,t\,dx\,dy \tag{1}$$

where N is the concentration of molecules at the point in question. The constant D depends upon the gas in question, the temperature, and to a slight degree upon the concentration.

Assuming that equation (1) holds for particles suspended in a fluid and N is a function of time and space, consider the diffusion into and out of an elementary cube $dx\,dy\,dz$ in a moving fluid.



In the x--direction the number of particles flowing into the cube per unit of time is $\left(uN - D\frac{\partial N}{\partial x}\right)dy\,dz$; out of the cube, $\left[uN + \frac{\partial}{\partial x}(uN)dx - D\frac{\partial N}{\partial x} - \frac{\partial}{\partial x}\left(D\frac{\partial N}{\partial x}\right)dx\right]dy\,dz$. The excess flowing in is $\left[\frac{\partial}{\partial x}\left(D\frac{\partial N}{\partial x}\right) - \frac{\partial}{\partial x}(uN)\right]dx\,dy\,dz$. Similar expressions may be obtained for the y and z directions. The sum of the excesses in the three directions should then be equated to $\frac{\partial N}{\partial t}\,dx\,dy\,dz$; hence,

$$\frac{\partial N}{\partial t}dxdydz = \Big[-\frac{\partial}{\partial x}(uN) - \frac{\partial}{\partial y}(vN) - \frac{\partial}{\partial z}(wN) + \frac{\partial}{\partial x}\Big(D\frac{\partial N}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(D\frac{\partial N}{\partial y}\Big) + \frac{\partial}{\partial z}\Big(D\frac{\partial N}{\partial z}\Big)\Big]\,dx\,dy\,dz.$$

or

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} + w\frac{\partial N}{\partial z} + N\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\Big) = \frac{\partial}{\partial x}\Big(D\frac{\partial N}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(D\frac{\partial N}{\partial y}\Big) + \frac{\partial}{\partial z}\Big(D\frac{\partial N}{\partial z}\Big)$$

Assuming the fluid to be incompressible, $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$, and we deduce then, the general equation for diffusion in a liquid stream.

$$\frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} + w\frac{\partial N}{\partial z} = \frac{\partial}{\partial x}\Big(D\frac{\partial N}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(D\frac{\partial N}{\partial y}\Big) + \frac{\partial}{\partial z}\Big(D\frac{\partial N}{\partial z}\Big) \qquad (2)$$

If $v = o$, $w = o$, $N = N(z)$, and D is independent of x, y, and z, it follows that

$$\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial z^2} \qquad (3)$$

Equation (3) is the equation for unsteady one dimensional diffusion in a liquid stream. It is interesting to note that this expression is exactly the same as the heat conduction equation for a solid body.

Equations (2) and (3) were derived for the case where diffusion was due solely to the random motion of the molecules, body forces having been neglected.

Applying the fundamental diffusion law (1) to the condition where the particles have a definite velocity η in the direction of the z - axis, equation (3) will become

$$\frac{\partial N}{\partial t} = \eta\frac{\partial N}{\partial z} + \frac{\partial}{\partial z}\Big(D\frac{\partial N}{\partial z}\Big) \qquad (4)$$

and for the steady state,

$$\eta \frac{\partial N}{\partial z} = -\frac{\partial}{\partial z}\left(D \frac{\partial N}{\partial z}\right) \tag{5}$$

Upon integration with respect to z ,

$$\eta N = -D \frac{\partial N}{\partial z} + C$$

To justify (1) for equilibrium, c = o, and

$$\eta N = -D \frac{\partial N}{\partial z} \tag{6}$$

Equation (6) is the law governing the suspension of minute particles in a liquid at rest, suspension being due purely to molecular collisions. It should be obvious that the diffusion coefficient varies with the size of the suspended particles. The form of equation (6) was checked experimentally by Perrin[5].

Evaluation of D.

Evaluation of the coefficient D for molecular diffusion may be made by following the method given by Loeb[6] in which the diffusion of similar gases in two connected vessels is considered. Diffusion is assumed to have continued for a sufficiently long time to attain an equilibrium state such that $\frac{\partial N}{\partial t} = 0$. As the molecules of the gas interdiffuse, let N be the number of molecules per unit volume at a certain section. For a distance L on either side of this section the concentration will be $N + L\frac{dN}{dz}$ and $N - L\frac{dN}{dz}$ respectively. If L is the mean free path, one-sixth of the molecules moving with a velocity $\bar{c}$ will have velocities directed so as to pass through dx dy as their next free path. In time dt the total number of molecules having crossed dx dy from one side will be 1/6 dx dy $\bar{c}$ dt $(N + L \frac{dN}{dz})$ and from the other 1/6 dx dy $\bar{c}$ dt $(N - L \frac{dN}{dz})$. The total net transfer will then be $- 1/3$ dx dy $\bar{c}$ dt $L \frac{dN}{dz}$. By comparison with equation (1) it is seen that

$$D = 1/3 \ \bar{c} \ L \qquad (7)$$

Thus, the mean velocity and the mean free path play an important part in the diffusion of molecules.

When particles are suspended in a liquid, the coefficient of diffusion may be determined by measurement of displacements of the particles in a certain time interval. Einstein[7] developed the following method of approach.

Considering diffusive action in one direction only, suppose that the number of particles at any place and time is N (x, t). Of the particles found between x and x + dx at time t, let the fraction $\phi(\Delta)\, d\Delta$ be found between Δ and $\Delta + d\Delta$ at the time $t + \tau$. It is assumed that τ is small but of such a magnitude that succession movements of a particle are independent of each other. The number of particles between x and x + dx is then given by

$$N(x, t+\tau)dx = dx \int_{-\infty}^{+\infty} N(x+\Delta, t)\, \phi(\Delta)\, d\Delta$$

If τ is very small, we can put

$$N(x, t+\tau) = N(x, t) + \tau \frac{dN}{dt}$$

Expanding $N(x + \Delta, t)$ in powers of Δ,

$$N(x+\Delta, t) = N(x,t) + \Delta \frac{\partial}{\partial x} N(x,t) + \frac{\Delta^2}{2!} \frac{\partial^2}{\partial x^2} N(x,t) + \cdots\cdots$$

and placing under the integral

$$N(x,t) + \tau \frac{\partial N}{\partial t} = N \int_{-\infty}^{+\infty} \phi(\Delta) d\Delta + \frac{\partial N}{\partial x} \int_{-\infty}^{+\infty} \Delta \phi(\Delta) d\Delta + \frac{\partial^2 N}{\partial x^2} \int_{-\infty}^{+\infty} \frac{\Delta^2}{2!} \phi(\Delta) d\Delta + \cdots\cdots$$

If on the right-hand side of this expression the higher order terms may be neglected, and the second, fourth, etc., terms are dropped, since $(x) = \phi(-x)$ and remembering that

$$\int_{-\infty}^{+\infty} \phi(\Delta) d\Delta = 1$$

we get

$$\tau \frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} \int_{-\infty}^{+\infty} \frac{\Delta^2}{2} \phi(\Delta) d\Delta .$$

Since $\int_{-\infty}^{+\infty} \Delta^2 \phi(\Delta)\, d\Delta = \overline{\Delta^2}$, the mean-square displacement, therefore,

$$\tau \frac{\partial N}{\partial t} = \frac{1}{2} \overline{\Delta^2} \frac{\partial^2 N}{\partial x^2} \qquad (8)$$

Comparing equation (8) with equation (3), we recognize that

$$D = \frac{1}{2} \frac{\overline{\Delta^2}}{\tau} \qquad (9)$$

It should be noted that $\overline{\Delta^2}$ is merely the mean-square displacement of particles from a plane source in the short interval of time τ .

If the problem is treated as that of diffusion from a source we can then say that the total displacement X in time $n\tau = t$ is $x_1 + x_2 + x_3 + \ldots + x_n$, and the mean-square displacement $\overline{X^2} = \sum_1^n \overline{x_i^2}$, the product terms vanishing because of the independence of successive displacements. Again, because of independence of displacements $\overline{x_i^2}$ is constant and independent of the particular interval considered, so that

$$\sum_{1}^{n} \overline{x_i^2} = n\,\overline{x_1^2}$$

therefore,

$$\frac{\overline{X^2}}{t} = \frac{n\,\overline{x_1^2}}{t} = \frac{\overline{x_1^2}}{\tau} = \frac{\overline{\Delta^2}}{\tau} = \text{Constant} \qquad (10)$$

Since Δ Will vary with the size of the material suspended, D will depend upon Δ as well as the properties of the liquid.

Transfer Phenomena

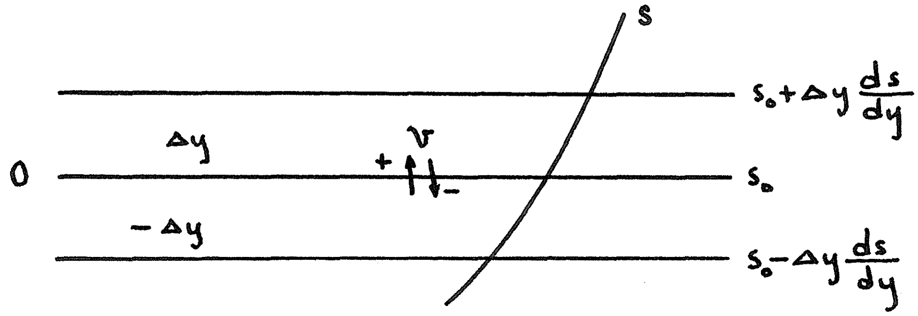
Fluids in turbulent motion exhibit a turbulence diffusion in additon to a diffusion due to molecular activity. The turbulence diffusion will depend upon the Reynolds number of the flow and the relative boundary roughness. The relative effect of the turbulence and molecular diffusion will depend upon the above condition and also the entity diffused. For example, salt and heat lend themselves rapidly to molecular diffusion, whereas sand grains are affected only by turbulence; molecular momentum exchange in the form of viscosity has an appreciable influence only at low flows.

Since turbulent motion is random as is molecular motion, one would suspenct that an equation similar to equation (1) could be set up expressing the turbulence diffusion S of any local entity s, whence

$$S = - \epsilon \frac{ds}{dy} \qquad (11)$$

In this expression, ϵ and y are more universally used than D and z.

The derivation of equation (11) and the significance of ϵ are easily shown as follows:



Let $s = s\,(y)$ be the average concentration of the entity s at a particular point O in the fluid. Then the amount of s passing through an area A in unit time at any instant is $\int_0^A v(s_o+\Delta y\frac{ds}{dy})dA$. The transfer per unit area per unit time is then

$$\frac{1}{A}\int_0^A v(s_o+\Delta y\frac{ds}{dy})\,dA = \frac{1}{A}\int_0^A v\,s_o\,dA + \frac{1}{A}\int_0^A v\,\Delta y\frac{ds}{dy}\,dA$$
$$= \overline{v\Delta y}\,\frac{ds}{dy}.$$

since $\frac{1}{A}\int_0^A v\,s_o\,dA = 0$ over a sufficiently large area A. Using numerical values for v and Δy, and substituting ℓ for Δy, and noticing that $+\Delta y$ is always associated with $-v$ and vice versa

$$S = -\,\overline{v\ell}\,\frac{ds}{dy} \tag{12}$$

Comparing equation (11) with equation (10)

$$\varepsilon = \overline{v\ell} \tag{13}$$

It is seen that ε, the mixing coefficient, is the mean product of the fluctuating velocity and corresponding distance of travel.

Applying equation (10) to the momentum of a flowing stream, and balancing forces, we deduce the apparent shearing stress at a point.

$$\tau = -\varepsilon \frac{d(\rho U)}{dy} \tag{14}$$

On application to sediment transfer we get upon equilibrium

$$\eta N = -\varepsilon \frac{dN}{dy} \tag{15}$$

which is identical with equation (6).

Equations (11), (14), and (15) are very useful qualitatively, but as yet they have not produced anything quantitatively. This is due to the fact that the variation of ε as a function of y is not known. There is also some question as to whether ε may be used universally once it is defined, or whether a constant of proportionality must be applied in each case. At present, experiments[8] are being conducted to compare the ε's in equations (14) and (15). In any case, a more fundamental study must be made of ε, and its variation as a function of the vertical coordinate y.

Turbulence Diffusion--Continuous Movements

Two important differences exist between molecular and turbulent motion: (1) in molecular motion the movement of particles is interrupted by collisions, whereas, in turbulent motion the movement is continuous, and (2) the length of the molecular mean free path is infinitesimally small, whereby the size of the turbulence eddies is comparatively large. Since the size of an average eddy is of the same order of magnitude as the measurable displacement distances, successive movements are no longer mutually independent phenomena, and therefore, the history of a particle must be taken into account. On this basis Taylor[9-10] developed a theory of diffusion for the continuous movement of a turbulent fluid by using the correlation between the velocity of a particle of fluid at any instant and velocity of the same particle after a time interval ξ .

Consider the random migration of particles in the y direction and let v be the instantaneous velocity parallel to that direction. The correlation R_ξ between the velocity of a particle of fluid at time t and the velocity of the same particle after an interval of time ξ is given by

$$R_\xi = \frac{\overline{v_t v_{t+\xi}}}{\sqrt{\overline{v_t^2}}\sqrt{\overline{v_{t+\xi}^2}}}$$

If the turbulence is considered uniform throughout the field and R_ξ is an even function of ξ, then $\sqrt{\overline{v_t^2}} = \sqrt{\overline{v_{t+\xi}^2}} = \sqrt{\overline{v_{t-\xi}^2}}$ and

$$R_\xi = \frac{\overline{v_t v_{t-\xi}}}{\overline{v_t^2}}$$

Consider the value of the definite integral

$$\int_0^t \overline{v_t v_{t-\xi}}\, d\xi .$$

By the definition of R_ξ , we have

$$\int_0^t \overline{v_t v_{t-\xi}}\, d\xi = \int_0^t \overline{v_t^2}\, R_\xi\, d\xi .$$

Hence, since $\overline{v_t^2}$ does not vary with t ,

$$\int_0^t \overline{v_t v_{t-\xi}}\, d\xi = \overline{v_t^2} \int_0^t R_\xi\, d\xi \qquad (16)$$

but

$$\int_0^t \overline{v_t v_{t-\xi}}\, d\xi = \overline{v_t \int_0^t v_{t-\xi}\, d\xi} = \overline{v_t Y} = \overline{v Y}$$

where Y is the displacement of a particle in time t.

Hence,

$$\overline{v^2}\int_0^t R_\xi \, d\xi = \overline{vY} = \frac{1}{2}\frac{d\overline{Y^2}}{dt} \tag{17}$$

whence,

$$\overline{Y^2} = 2\,\overline{v^2}\int_0^T\!\!\int_0^t R_\xi \, d\xi \, dt \tag{18}$$

The problem of diffusion in a uniform turbulent field is now resolved to the consideration of a single quantity, namely, the correlation coefficient between the velocity of a particle at one instant and that after an interval of time ξ.

Equation (16) also contains the turbulence intensity $\sqrt{\overline{v^2}}$. This could be measured by observing the displacement of particles so close to a point that $R_\xi \rightarrow 1$. It follows that

$$\overline{Y^2} = \overline{v^2}\,T^2$$

or

$$\sqrt{\overline{v^2}} = \frac{\sqrt{\overline{Y^2}}}{T}$$

If the fluid is moving with the velocity U in a direction transverse to the migration $X = U\,T$, and

$$\sqrt{\overline{v^2}} = \frac{\sqrt{\overline{Y^2}}}{X}\,U \tag{19}$$

More interesting is the case when $R_\xi \rightarrow 0$. If we can assume that a time interval T_1 exists such that the velocity of a particle at the end of the interval T_1 has no correlation with the velocity at the beginning, then $\int_0^{T_1} R_\xi \, d\xi$ is finite and

$$\overline{v^2}\int_0^{T_1} R_\xi \, d\xi = \overline{vY} = \text{Constant} \tag{20}$$

Equation (20) permits us to define a length ℓ_1 by the relation

$$\ell_1 = \sqrt{\overline{v^2}}\int_0^{T_1} R_\xi \, d\xi \tag{21}$$

thus,

$$\ell_1\sqrt{\overline{v^2}} = \overline{vY} = \frac{1}{2}\frac{d\overline{Y^2}}{dt} \tag{22}$$

and

$$\overline{Y^2} = 2\ell_1\sqrt{\overline{v^2}}\,t + C \tag{23}$$

Equation (23) is the equation of the straight line to which the $\overline{Y^2}$ diffusion curve becomes asymtotic.

When t is very large, $\sqrt{\overline{Y^2}}$ is proportional to the square root of the time so that on comparison with equation (10) we approach the condition of diffusion of particles in Brownian movement. It is, therefore, seen that $\ell_1 \sqrt{\overline{v^2}}$ or $\overline{vY}$ plays the same part as D for molecular movement, where ℓ_1 is analogous to the mean free path and $\sqrt{\overline{v^2}}$ analogous to the mean velocity. From equation (22) it is seen that ℓ_1 is not a true length but must be called an effective diffusion length because it contains implicity the correlation between v and Y.

It should be observed that $\overline{vY}$ appears in such a form that it can be measured without recourse to equations (14), (15), etc. Besides, we have broken $\overline{vY}$ into two additional measurable quantities so that one can make a deeper insight into the mixing properties of the turbulence. ε is now equal to $\sqrt{\overline{v^2}}\ \ell_1$.

PHYSICAL INTERPRETATIONS

The coefficient of diffusion can be seen to have the dimensions of a velocity times a length. Indeed, in both molecular and turbulent motion the coefficients were found to be composed of characteristic velocities and lengths. Little thought will soon convince one that the movement of any entity from one point to another depends upon the speed with which the quantity is carried and the number of interruptions experienced. The quantity rate of transfer, however, will depend upon the gradient of the entity at the point in question, because, due to the gradient more of the entity is carried past the point from one side than the other.

The coefficient of correlation between successive velocities of a particle of fluid has shown its significance in the theory of diffusion. By following successive paths of particles of the fluid it is possible to obtain the value of ℓ_1 $\sqrt{\overline{v^2}}$ or $\overline{vY}$ by use of equation (22).

Correlation is a measure of the association of variables with one another. The measure is proportional to the algebraic mean product of the variables. In the case of the successive velocities of a particle at the beginning and end of an interval of time ξ, the correlation is indicated by $\overline{v_t \, v_{t+\xi}}$. Of course, a sufficient number of measurements must be taken to insure a true mean.

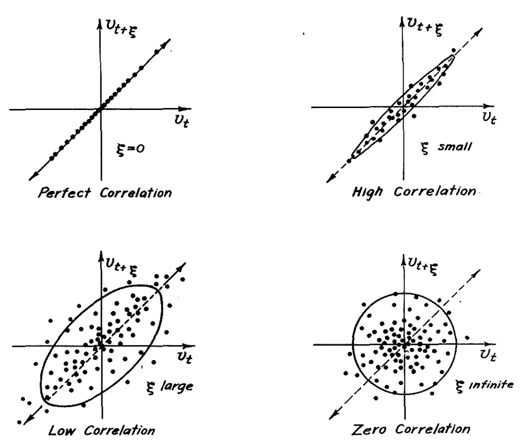


Fig. 1. Schematic representation of correlation of successive velocities of a fluid particle.

Figure 1 is a schematic picture showing various correlations between velocities, the high correlation corresponding to a short time interval, the low, corresponding to a long interval. The dots are imaginary measurements, the value of the correlation being obtained indirectly by other means in this paper. In order to make the correlation coefficient dimensionless, $\overline{v_t \, v_{t+\xi}}$ is divided by the product of the standard deviations $\sqrt{\overline{v_t^2}} \cdot \sqrt{\overline{v_{t+\xi}^2}}$.

In a similar sense, the correlation coefficient may be interpreted as a measure of predictability; i.e., from the velocity at one instant, how well one can predict the velocity at a later instant. If the correlation is unity one can predict with sureness, but if the correlation is zero there is no chance of prediction.

There is one important limitation to the correlation coefficient as defined above: it can be applied only to data which scatter about a straight line. If the data have a curved trend, a correction must be applied. However, in random fluctuations which are encountered in turbulent motion there is no reason to believe that the trend will be anything but linear. In the case of correlating velocities of the same particle the regression is obviously linear.

Many other types of correlations exist between velocities in a turbulent field. For details see reference 1 and 9.

Equation (21) defines a length, $\lambda_1 = \sqrt{\overline{v^2}} \int_0^{T_1} R_\xi \, d\xi$. This may be considered as being the length which the fluid technicians and engineers call the "mixing length". The "mixing length" is interpreted physically as either a measure of the average size of the eddies or the effective transverse distance the fluid masses are carried before losing their identity in the surrounding fluid. If the former interpretation is given, λ_1 would be approximately half the average size of the eddies, since $\int_0^T R_\xi \, d\xi$ may be thought of as an average measure of the time a particle takes to travel to the top or bottom of the eddy and $\sqrt{\overline{v^2}}$ is the mean velocity of travel. Measurements[11] in the wind tunnel tend to substantiate this concept. On the other hand, if the latter interpretation is given, the integral is the effective time that a particle requires to penetrate the surrounding fluid far enough to lose its initial identity. It is necessary to note that the time of penetration is given by ξ for $R_\xi = 0$, whereas, the integral takes into account a correlation between the velocity of penetration and the time for penetration. As mentioned before, the "mixing length" has meaning only when a correlation is implied.

SCOPE

This paper is concerned with the discussion of an investigation into the nature of diffusion due to turbulence in open channel flow. Theoretical considerations are reviewed for the purpose of acquainting the reader with the necessary fundamental concepts and technical terms.

Although the variation of turbulence intensity was measured, particular attention was focused upon the measurement and variation of the Taylor length λ_1, with the hope that it would throw some light on the significance of the "mixing length".

Two channel conditions were used in the investigation, smooth walls and bottom, and rough walls and bottom. The maximum depth was held constant at 0.65 feet. In the smooth channel, three discharges were studied, 0.58 c.f.s., 1.08 c.f.s., and 1.76 c.f.s.; in the rough channel one discharge, 1.51 cubic feet per second. The flow in the rough channel had the same slope as the high flow in the smooth channel. Measurements were made at five points in the center of the channel for the lowest flow, while only two points were investigated in the remaining flows. The Reynolds numbers ranged from 97,000 to 294,000.

LABORATORY EXPERIMENTS

Apparatus

The experiments were conducted in a closed circuit flume, (see figures 2, 3, 4, and 5) 10½ in. wide, 10 in. high, and 40 ft. long. The slope was adjustable by means of a jack and pivot combination. On each side of the flume was a glass window 5 ft. long and 7 ft. from the downstream end. Also at the downstream end was a motor and propeller pump assembly the discharge of which could be varied from ½ to 3 cubic feet per second.

Smooth walls and bottom were obtained by application of bitumastic paint. Rough conditions were produced by covering soft bitumastic paint with sand grains having a mean size of 0.890 mm. and a geometric standard deviation of 1.18. Figure 6 shows a representative sample of roughness.

Quantities of flow were set by a venturi meter which had been

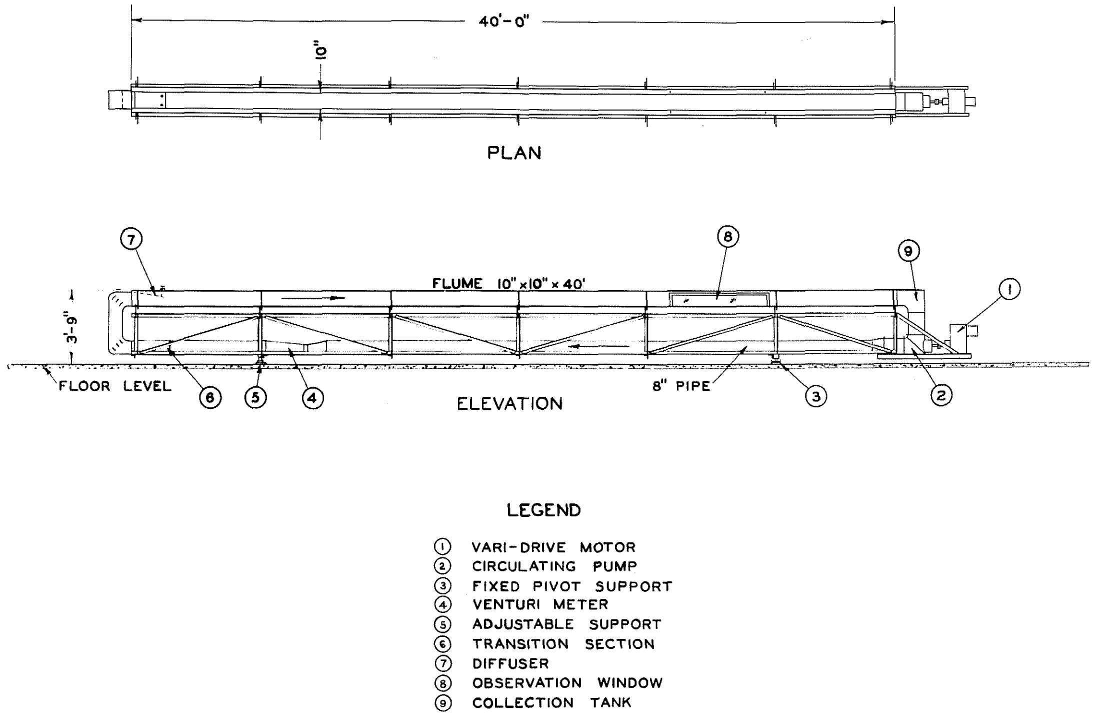


Fig. 2. Diagramatic sketch of 10" closed-circuit flume.

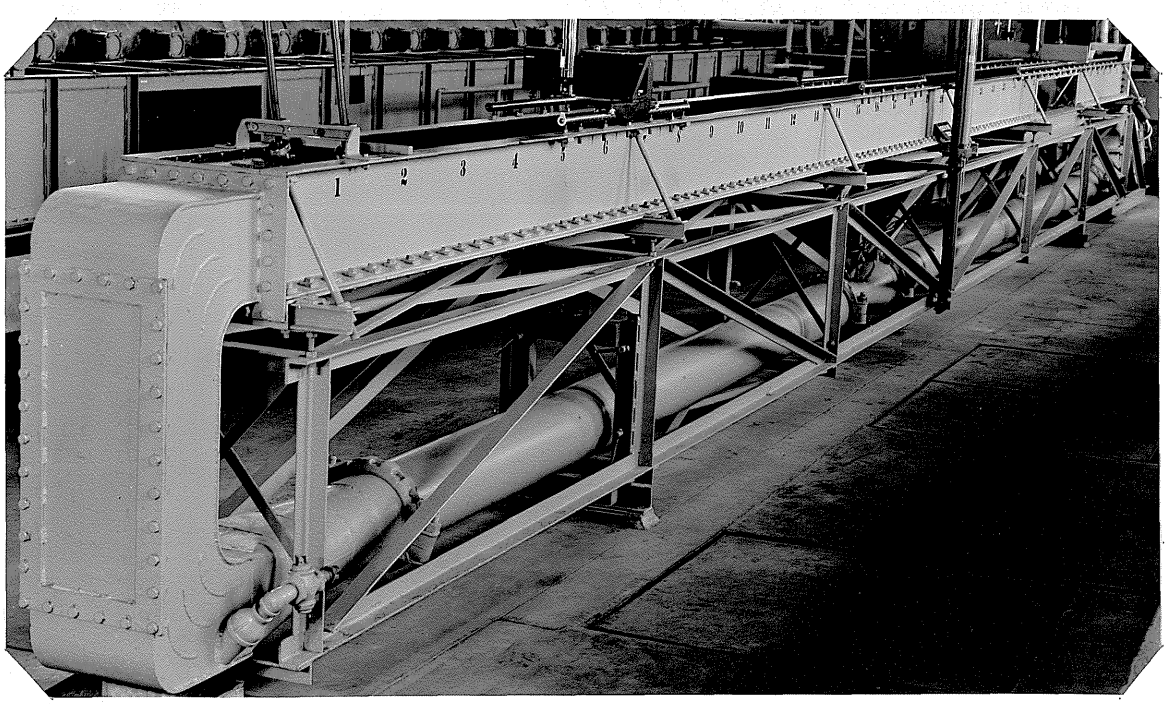


Fig. 3. Side-view of flume showing adjustable and pivot supports, observation window and venturi meter.

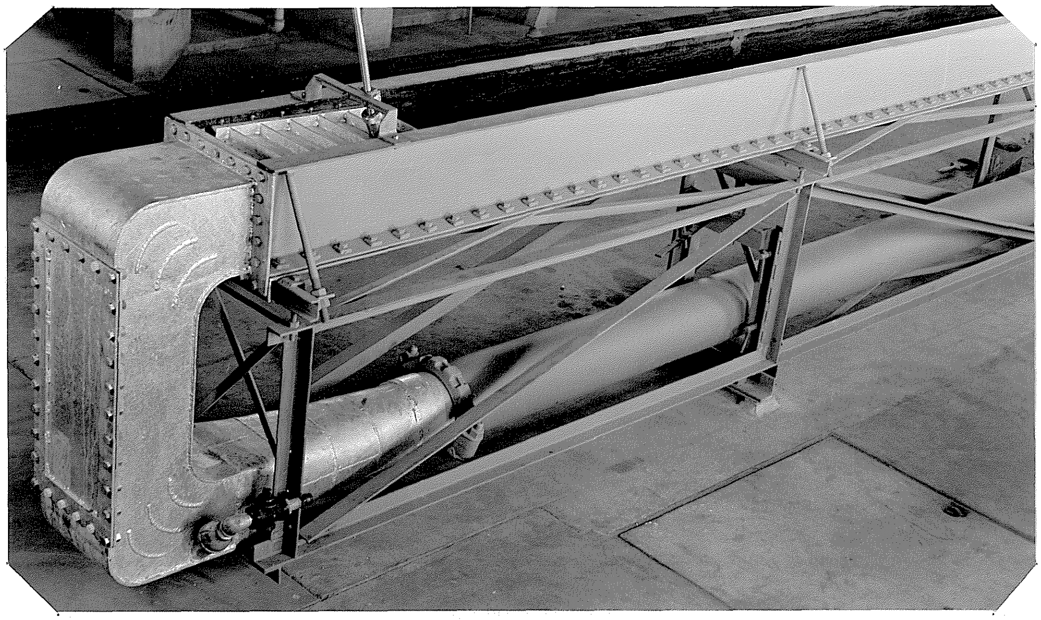


Fig. 4. Upstream end of flume showing diffuser, transition section and varied corners.

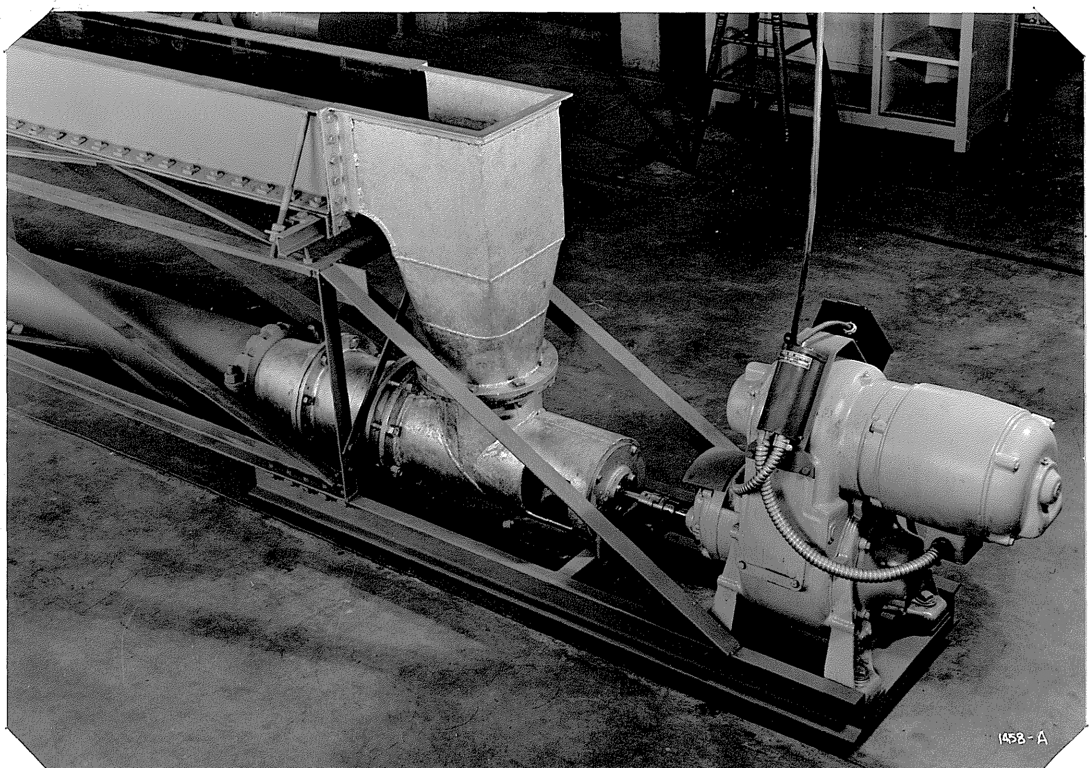


Fig. 5. Downstream-end of flume showing collection tank, vari-drive motor and circulating pump.

Fig. 6. Representative sand roughness sample. Magnified 10 times.

previously calibrated by either pitot tube or miniature current-meter measurements. The depth was regulated by adding small quantities of water to the flow.

Figure 7 shows injection apparatus in position. A mixture of carbontetrachloride and benzene adjusted to the same specific gravity as the water was injected into the flow through a small 24-gauge stainless steel tube having an internal diameter of 0.0123 inches. Upon leaving the tube, the mixture broke up into small immiscible globules which could be traced photographically. A burette tube was used to provide the head necessary for the proper injection velocity. Alignment of the injection tube was determined by means of an adjustable plate supported in the flow downstream from the tube. The small immiscible globules of carbontetrachloride and benzene were photographed by a Leica camera using Kodak plus X film.

Figure 8 gives the calibration of the flume for smooth conditions.

Procedure

A particular discharge and depth of water at uniform flow was established by adjusting the slope of the flume and speed of the pump. The injection tube, connected to the burette tube by a flexible conduit,

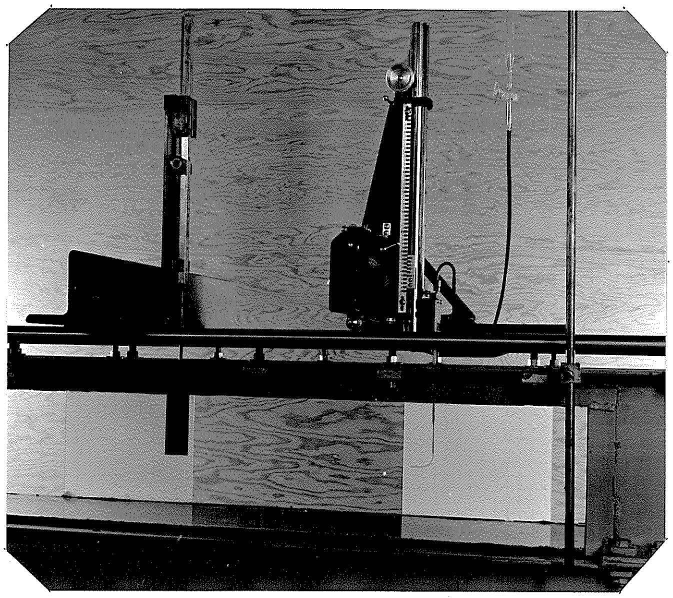


Fig. 7. Injection apparatus.

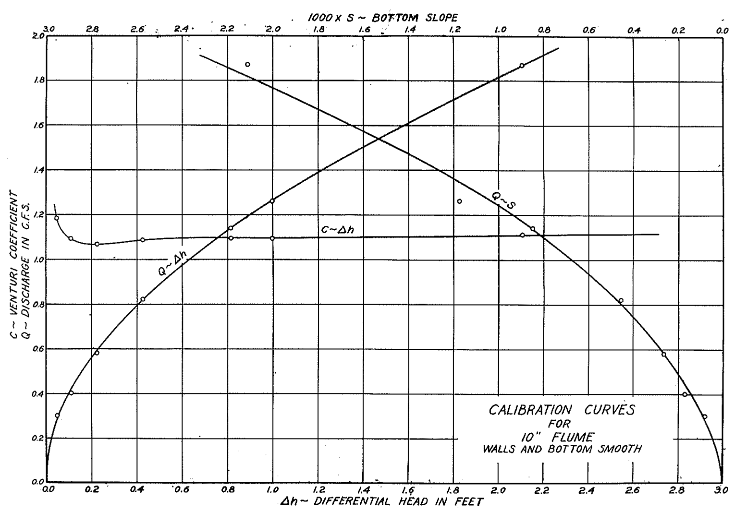


Fig. 8. Calibration curves for the 10" closed-circuit flume.

was fixed at the desired depth. The alignment plate was then inserted 15 in. downstream from the injector and at the same depth. A sample of water was then taken from the flume. This sample was used to make a final test on the relative density of a previously prepared mixture of carbontetrachloride and benzene. Knowing the velocity of the water at the desired depth from previous calibration and the diameter of the injector from measurement, the height of the injection fluid in the burette tube was adjusted to produce an effluent velocity equal to that of the water.

With the diaphram set at f. 18 and 1/1000 second, the camera was placed on one side of the flume. A bank of three flood lamps in a horizontal row was placed on the other side of the flume as close as possible to the plane of alignment of the edge of the area to be photographed and the camera. Then a wooden strip was placed in front of the glass on the camera side to shield the lens from the glare of the lamps.

After the flow became steady, the immiscible fluid was injected into the water. A roll of pictures was then taken by hand in rapid succession. According to preliminary studies on the amount of data required for a good statistical mean-square-displacement curve, from four to five rolls of film were used at each point for a particular flow. Between rolls, it was necessary to clean out the returning globules. This was accomplished by running the water through two layers of cheese cloth.

Figure 9 shows points of three typical instantaneous photographs of diffusion at the same injection depth and discharge.

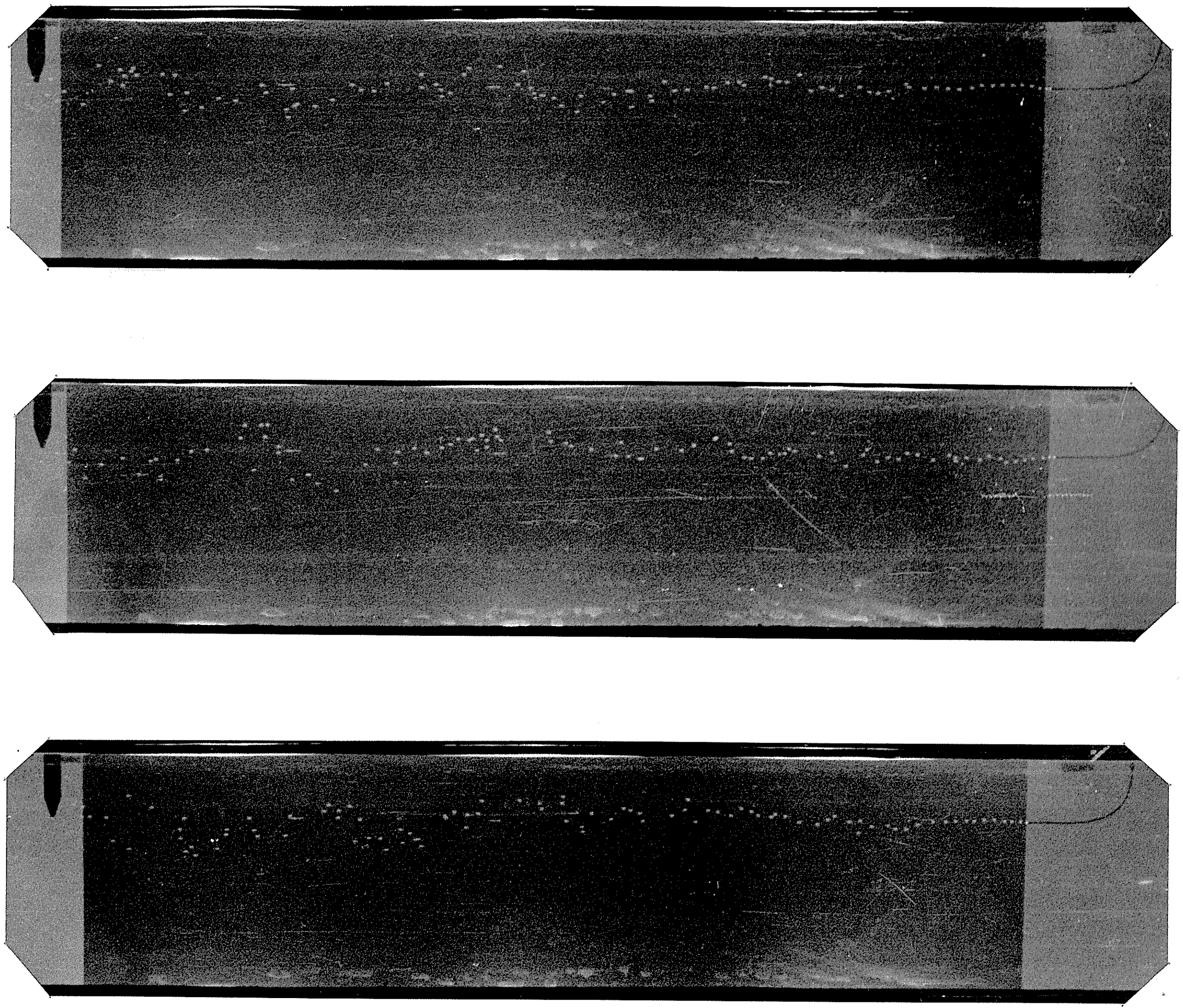


Fig. 9. Typical diffusion photographs. Note the periodic wandering which gives some indication of a definite turbulence pattern.

After development, the films were projected on a screen to exactly full size. At various distances downstream from the injector tube, the vertical positive and negative displacements within a narrow band were read off the screen to the nearest 1/100-inch by means of co-ordinate paper. For the study of the two lower discharges, displacements were obtained at the following downstream points: $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3,--13 inches, and as far as 30 inches at one depth. For the higher discharges these points were used: 1/8, $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3,--13 inches, or 1/8, $\frac{1}{4}$, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, 3, 4, 6, 8--14 and one case to 32 inches downstream.

ANALYSIS OF DATA

Method

The analysis of the data was purely statistical. At each depth for a certain discharge the data was taken off the film as described above and averaged algebraically. The process of averaging eliminated the possibility of extraneous mean-vertical currents affecting the mean displacement of the globules. All the data for the various distances downstream were then squared to the nearest 0.001 $(in.)^2$ for distances of 1 in. or more from the tube, and to the nearest 0.0001 $(in.)^2$ for points $\frac{1}{2}$-in. away or nearer. The mean of these squares was determined by dividing the sum of the total number of squares. The final procedure in the analysis was to subtract the square displacement of the particles of fluid over a period of time T or distance downstream $X = U T$.

For all depths and discharges, the mean-square displacement was plotted on logarithmic paper against the distance downstream. In every case, this plot yielded a definite curve which was most useful in studying the turbulence and its diffusion characteristics. A plot of the square root of the curve near the injection point was used to obtain the intensity of turbulence $\frac{\sqrt{\overline{v^2}}}{U}$ through equation (17) by extrapolation to the origin. The curve itself gave the curvature of the absolute mean-square displacement curve by the relation

$$\frac{d^2\overline{Y^2}}{dX^2} = \frac{1}{U^2}\frac{d^2\overline{Y^2}}{dt^2} = \frac{\overline{Y^2}}{X^2}n(n-1) \tag{24}$$

where n is the slope of the logarithmic curve. The curvature in turn, led to the value of the correlation coefficient R_ξ through the derivative of equation (17), namely,

$$\frac{1}{2}\frac{d^2\overline{Y^2}}{dt^2} = \overline{v^2}\,R_\xi \tag{25}$$

Solving for R_ξ and using equation (24),

$$R_\xi = \frac{1}{2}\left(\frac{U}{\sqrt{\overline{v^2}}}\right)^2 \frac{d^2\overline{Y^2}}{dX^2} \tag{26}$$

$$= \frac{1}{2}\left(\frac{U}{\sqrt{\overline{v^2}}}\right)^2 \frac{\overline{Y^2}}{X^2}n(n-1) \tag{27}$$

or with the help of equation (19),

$$R_\xi = \frac{1}{2}\left(\frac{X}{\sqrt{\overline{Y^2}}}\right)^2_{X=0} \frac{\overline{Y^2}}{X^2}n(n-1) \tag{28}$$

Since it is desirable to plot R_ξ versus ξ, we use

$$\xi = \frac{X}{U} \tag{29}$$

Equations (19), (28), and (29) are the final ones used in this investigation. In using these expressions, the assumption of uniform turbulence was made. The assumption was good near the surface of the water.

Presentation of Results

Figures 10 to 13 show the mean-square displacement curves. Each point represents an average of about four hundred measurements.

Table I gives the results of measurements of $\sqrt{\overline{v^2}}$ near the injection point. In column 4 are the percentages of turbulence at the origin as given by measurements at various distances from the origin.

Curves of variation of $\sqrt{\overline{v^2}}$, ℓ_1 and $\sqrt{\overline{v^2}} \cdot \ell_1$ at the low flow are shown in figure 14.

Figure 15 shows the correlation curves for each flow studied.

DISCUSSION OF RESULTS

Mean Displacement Curves

It can be seen from figures 10 to 13 that the data plot on definite straight lines except near the injection point. Since the plots are on logarithmic paper, the straight lines signify that the mean-square displacement curves are exponential of the form $\overline{Y^2} = c\,X^n$.

Very near to the origin where $R_\xi \rightarrow 1$, the mean-square displacement should be proportional to the square of the distance or in other words parabolic. The $\overline{Y^2}$ curve should then plot with a slope of 2 on log paper. In the other extreme, far from the origin where $R_\xi \rightarrow 0$, $\overline{Y^2}$

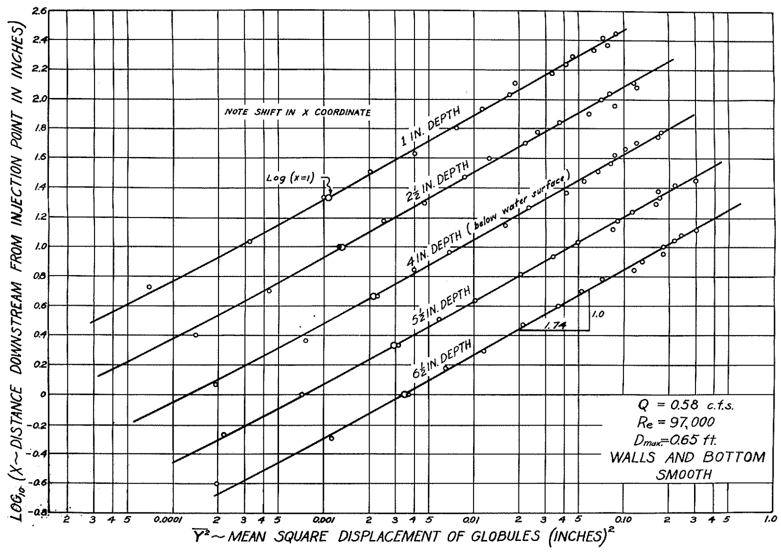


Fig. 10. Mean-square vertical displacement curves at various injection depths for Q = 0.58 c.f.s., walls and bottom smooth.

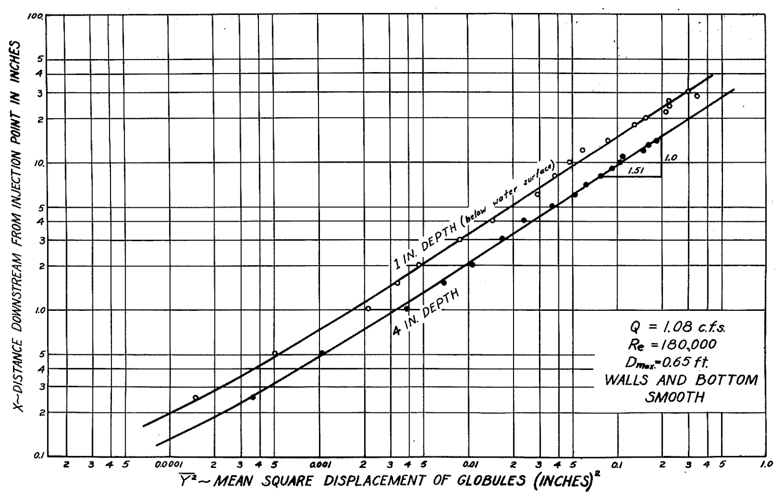


Fig. 11. Mean-square vertical displacement curves at two injection depths for Q = 1.08 c.f.s.

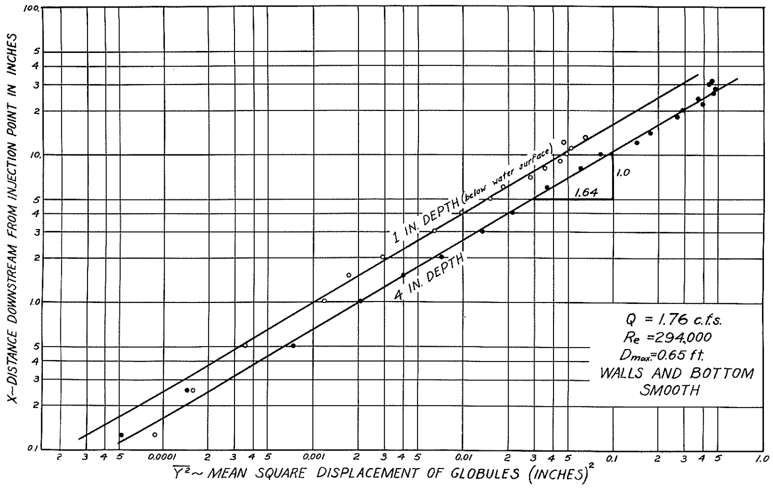


Fig. 12. Mean-square vertical displacement curves at two injection depths for Q = 1.76 c.f.s.

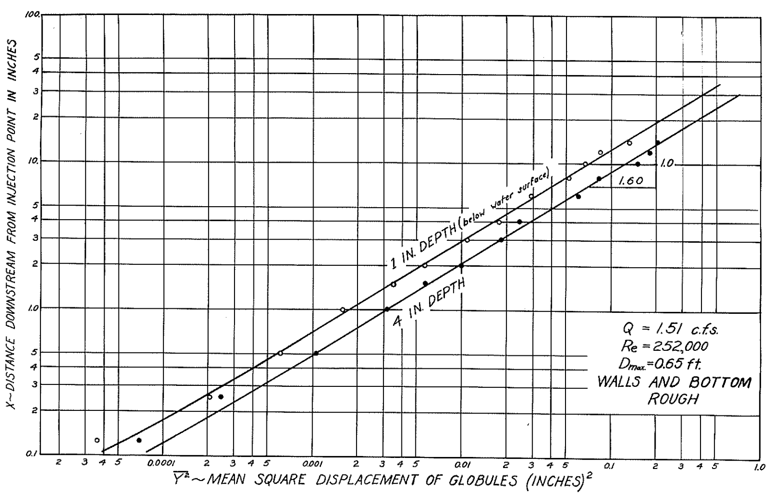


Fig. 13. Mean-square vertical displacement curves at two injection depths for Q = 1.51 c.f.s., walls and bottom rough.

should be a linear function of distance and a log plot would yield a curved line approaching a slope of $n = 1$. With these limitations in mind a curve was drawn on the plotted points. An approach to the proper curvature was noted for small values of X, but no systematic curvature was evident for large values of X. Even for such extreme distances downstream as 30 inches there was no pronounced tendency.

There may be some question as to the effect of a vertical variation of turbulence on the mean displacement. True, a vertical variation did exist and this also implied a variation in diffusion strength. In the lowest flow there was a marked tendency for the mean globule path to fall at the 1 in. and $6\frac{1}{2}$ in. depths. In all the remaining flows the 1 in. depth showed an upward displacement within 9 in. of the injector and a downward displacement for the remaining distance, evidently showing an effect of the injector. At the 4-in. depth in the high flows the mean movement was always upward. In any case the maximum mean-displacement was 0.26 in. in 32 inches. It is doubtful whether the movement is due to a turbulence variation or a consistent vertical mean-motion in the channel. At any rate the mean-displacement correction had to be made for definitely observed movements due to the presence of the tube. Besides, the corrected data do plot with striking characteristics.

That the straight line portions of the curves are parallel for each flow is very interesting. This means that at any vertical section of channel the mean-square displacements at corresponding depths are proportional.

Further inspection of the curves for all smooth channel discharges shows that for corresponding depths of 1 in. and 4 inches the spacing of the curves is the same. Therefore, it can probably be concluded that for any flow at a given maximum depth and channel roughness, the mean-square displacements are proportional for corresponding depths. In fact it so happens that for the 1 in. and 4 in. depths the ratio is 2. The rough channel curves give evidence that the latter similarity conclusion cannot be extended to cover changes in roughness.

The slopes of the curves do not bear any relation to one another.

Sutton[12] from data collected by Richardson and Proctor[3] on the diffusion of balloons in the atmosphere, found a linear variation very similar to the variations found in the investigation in water. Assuming that $R_{\xi} \propto (U\xi)^{-p}$, he obtained by use of equation (18)

$$\overline{Y^2} \propto (U\xi)^{2-p} \qquad (30)$$

or by equation (29)

$$\overline{Y^2} = K\,(X)^m \qquad (31)$$

where K is a proportionality factor and m is substituted for $2-p$. Using equation (31) as a guide, Sutton plotted $\overline{Y^2}$ versus X logarithmically and got $m = 1.75$. This is in perfect agreement with the author's results on the low flow. However, it does not agree with the results on the remaining flows. Obviously the factor K depends upon the intensity of turbulence at the point in question.

It cannot be over-emphasized that this investigation confirms Sutton's theory of diffusion as a function of distance rather than time.

This means that turbulence diffusion in an open channel is analogous to the diffusion in the atmosphere.

Intensity of Turbulence

The intensity of turbulence at a point is determined by extrapolation of the square root of the mean-square curve to the origin. The slope of the root curve at the origin is then the intensity of the turbulence $\frac{\sqrt{\overline{v_o^2}}}{U}$ referred to the mean velocity of flow. (In the present case the transverse velocity fluctuations are referred to the mean longitudinal velocity). If the actual displacements are used to find the mean fluctuating velocity $\sqrt{\overline{v_o^2}}$, table I shows that accuracy attained as the origin is approached. Columns 8 and 9 indicate that in the two lower flows the $\frac{1}{4}$-in. distance is reliable whereas, in the lighter flows the 1/8-in. distance is necessary.

1	2	3	4	5	6	7	8	9	10	11
Q in c.f.s.	S Slope	Depth in Inches	$\frac{\sqrt{\overline{v_o^2}}}{U}$ %	U	$\sqrt{\overline{v_o^2}}=v_o'$ at origin	$\frac{v_o'}{v_{1/2}'}$ %	$\frac{v_o'}{v_{1/4}'}$ %	$\frac{v_o'}{v_{1/8}'}$	$\left(\frac{v_o'}{U}\right)_1 / \left(\frac{v_o'}{U}\right)_4$	$\frac{(v_o')_1}{(v_o')_4}$
Smooth 0.58	0.00027	1	3.75	1.14	0.0427	87	97		0.696 (1 to 4)	0.703
		2½	4.25	1.14	0.0485	86	98			
		4	5.38	1.13	0.0608	87	97			
		5½	6.15	1.06	0.0652	89	98			
		6½	6.50	0.96	0.0624	90	98			
Smooth 1.08	0.00080	1	5.40	2.21	0.1195	86	94		0.696	0.713
		4	7.75	2.16	0.1675	86	95			
Smooth 1.76	0.0022	1	4.52	3.45	0.1560	79	87	98	0.696	0.713
		4	6.50	3.36	0.2185	78	87	95		
Rough 1.51	0.0022	1	5.96	3.01	0.1793	81	90	97	0.727	0.737
		4	8.20	2.97	0.2435	79	88	94		

TABLE I.–COMPLETE RESULTS OF TURBULENCE INTENSITY MEASUREMENTS

The complete results of intensity of turbulence measurements appears in table I. The percentage intensity is given in column 4. Column 6 gives the absolute mean vertical velocity fluctuations. The effect of roughness is evident.

Figure 14 shows a plot of column 6. The shape of this curve, which has been checked by other experiments, references (13), and (14), signifies that most of the energy of turbulence occurs and is generated near the boundary. The turbulence energy near and at the free surface is the result of diffusive migration of eddies from the bottom or side walls. However, as we have shown, the turbulence energy plays only one part of the diffusion phenomenon; the other part is discussed later.

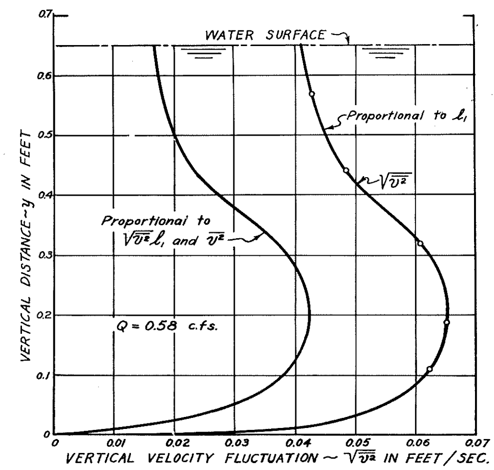


Fig. 14. Mixing properties of turbulence in open channel flow.

It is interesting to note from column 10 that the ratios of intensities at the 1-in. and 4-in. depths are the same for all smooth channel flows. This results, of course, from the spacing of the displacement curves. The ratios of the absolute values of the turbulence agree very well according to column 11, and it might appear that the absolute values vary inversely with the fourth root of the depths. However, other ratios for the same flow do not substantiate this latter relationship.

An inconsistency appears in the variation of the intensity with discharge; the intermediate smooth flow has a greater intensity than the high smooth flow. However, the absolute values are in the expected order of magnitude. The inconsistency is probably due to vibration of the injection tube, because at this particular pump speed some vibration was noticeable although apparently remedied at the time. But since the vibration depends somewhat upon the depth of the tube in the flow, there may be some question as to whether the vibration was actually present or whether the increase in turbulence was part of the nature of the flume as a whole.

Correlation Curves

Figure 15 shows curves of correlation coefficients for vertical velocities at all points measured. As stated above, these curves were obtained by means of equations (28) and (29).

These curves yield three important conclusions: 1) that of correlation R_{ξ} is reasonably independent of the depth of flow, 2) that the curve falls off very quickly for small values of ξ, and 3) that the curve never gets very near to $R_{\xi} = 0$. The first conclusion is a direct result of the similarity of the mean-square displacement curves. In fact, for each flow the R_{ξ} curves could have been brought together by intentional selection of the intensity of turbulence. The second conclusion means that very small eddies are present which destroy the early high correlation and diffuse the particles of fluid rapidly. These small eddies are either generated by the stem of the injection tube, or are inherent in the flow. The third conclusion drawn from the curves signifies that no matter how far the particles of fluid travel these velocity correlations never get to zero, at least within the distances included in this

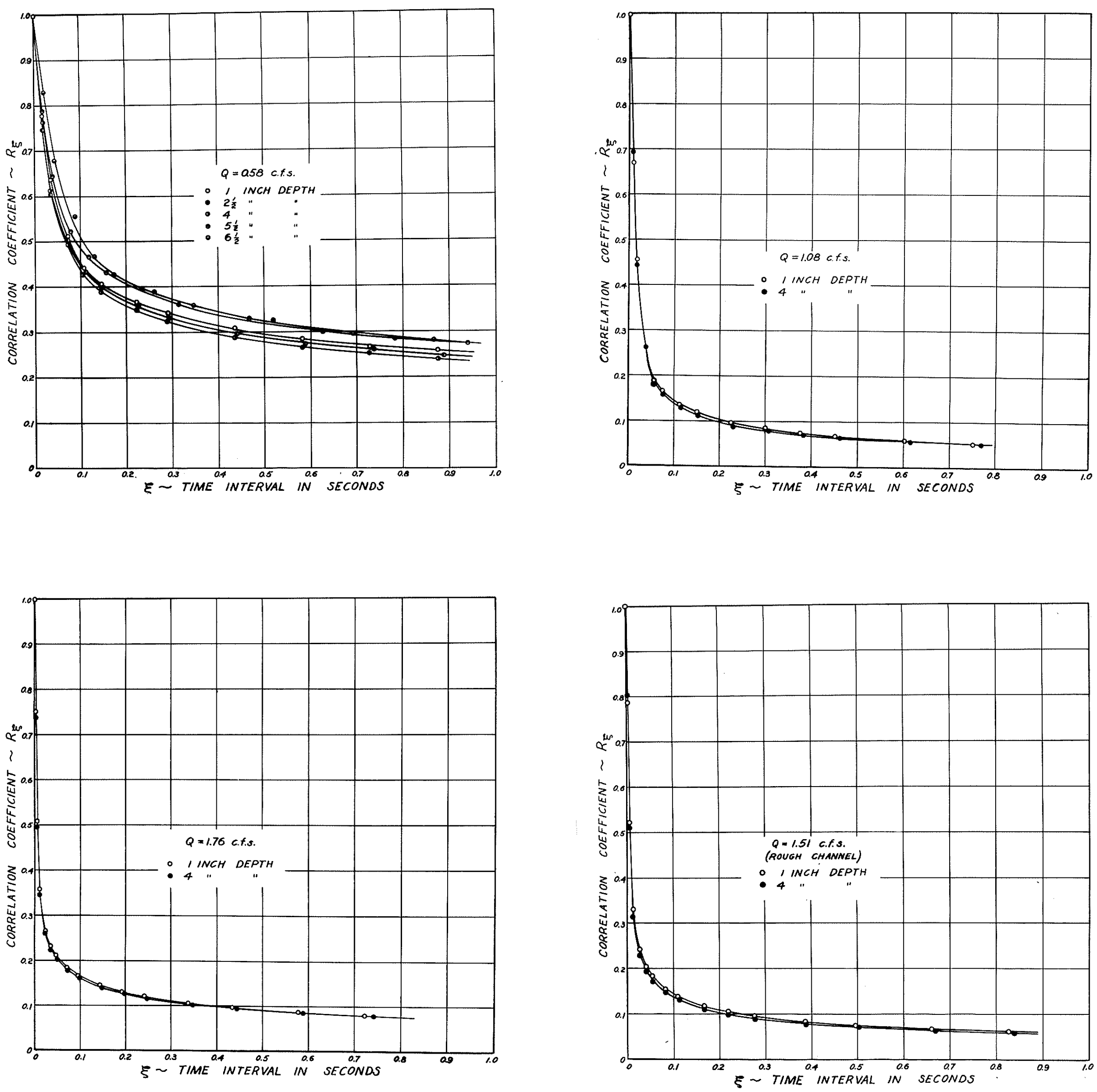


Fig. 15. Velocity correlation curves for various depths, discharges and channel roughnesses.

investigation. In other words a particle never loses its initial identity as far as velocities are concerned, or better, as the particles spread apart, larger and larger eddies show their effect. It is evident, therefore, that the flow must be made up of variable-sized eddies, since very small eddies would quickly diffuse particles when close together, and large eddies would tend to keep the particles moving when far apart.

Mixing Length

It was thought that the measurement of Taylor's ℓ_1, would yield considerable information on the significance of the "mixing length". Inspection of the correlation curves show that a quantitative value for ℓ_1, cannot be attained, at least within the measurements made in this investigation.

As stated in the previous section, the correlation curves demonstrate that the size of the eddies in our channel has apparently no upper limit, comparable to the situation in the atmosphere as found by Sutton. This is probably to be expected as there was really no "a priori" reason why the eddies should have a definite scale or size, i.e., there was no honeycomb or grid placed in the channel. The eddies were created only by the channel boundaries unless some eddies generated by the pump and elbows prevailed in the flow at the measuring section. At least we are certain of one thing and that is: the velocity distribution curves (see figures 16 and 17) at the test section appeared to be fully developed although quite different from those 13 feet upstream. No velocities were measured below the section to test for further development.

Another important conclusion may be drawn with respect to the

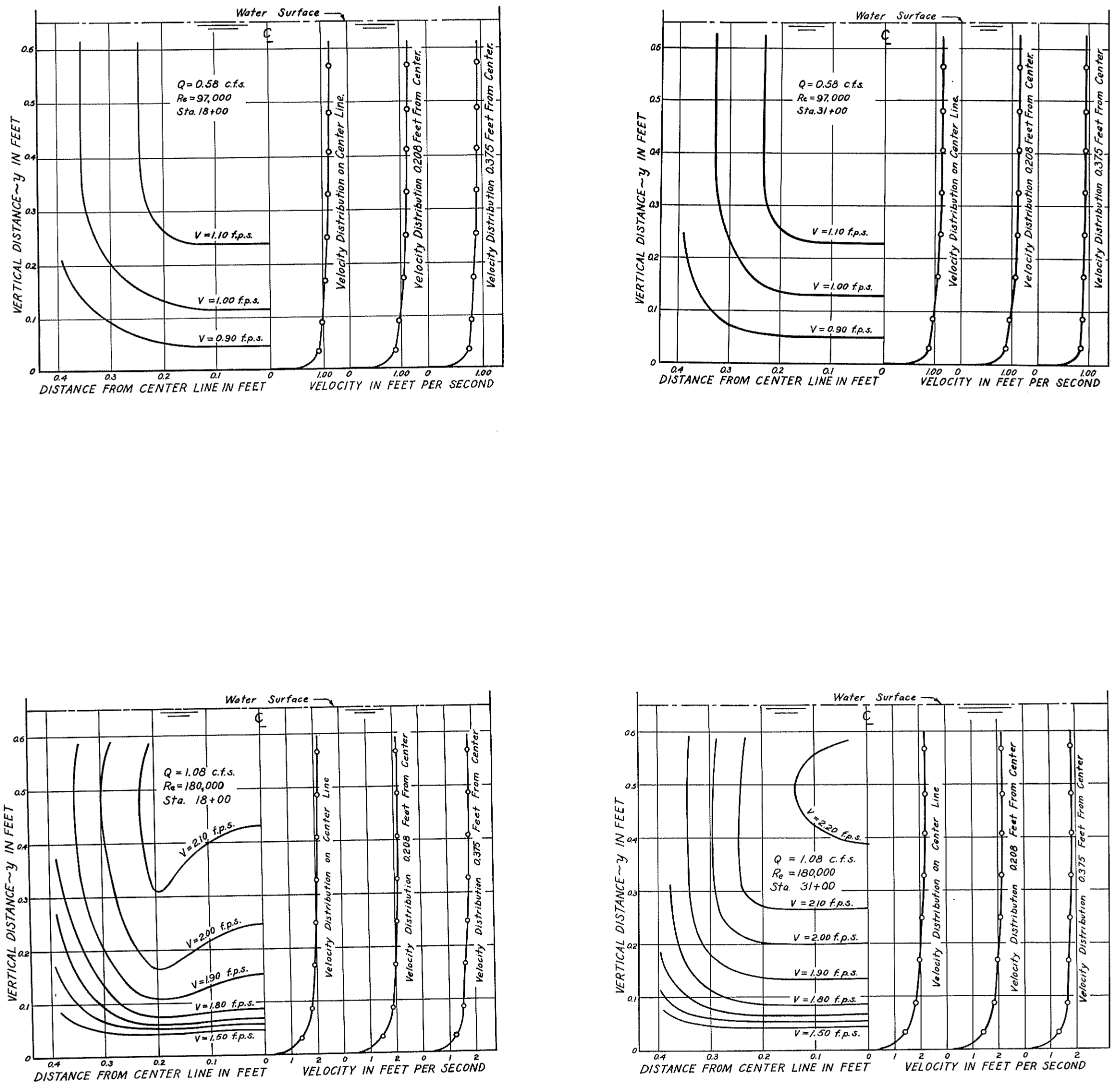


Fig. 16. Velocity traverses.

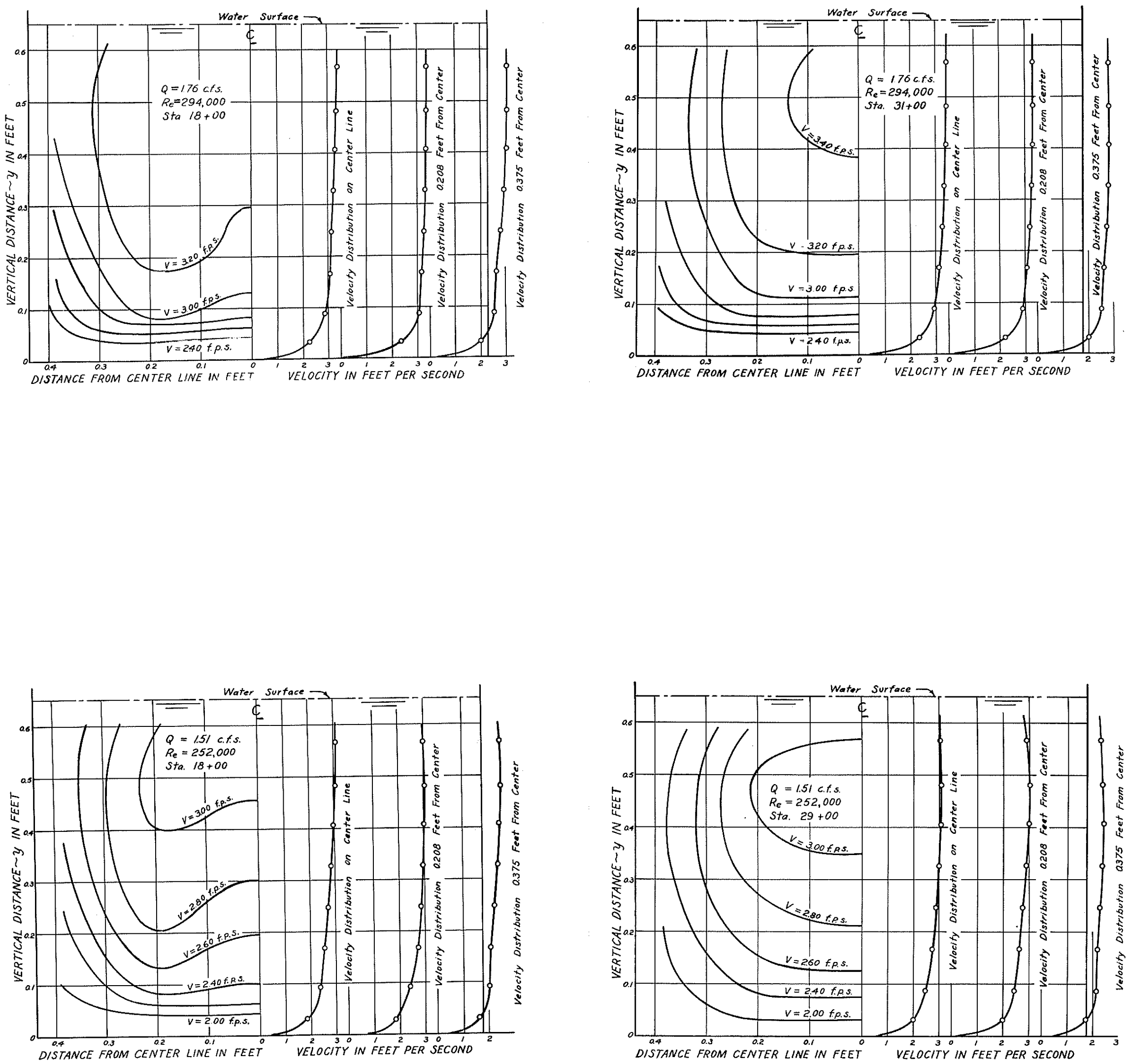


Fig. 17. Velocity traverses.

correlation curve. If it is true that the mixing length is defined by equation (21), and that the correlation does go to zero after a finite time, then, since the curves are reasonably independent of the injection depth we can conclude that the mixing length is proportional to the root-mean-square value of the turbulence fluctuations, as shown in figure 14. This variation does not agree very well with results obtained by other methods[15].

After all, the time interval ξ is less than one second in all investigated cases. However, if the correlation values did go to zero for intervals as large as ξ = 10 or 20 seconds, and this is not an unreasonable duration of time, the areas under the curves would be reasonable and the resulting ℓ_1's would be of an expected order of magnitude.

As mentioned before, the size of the eddies is large with respect to the measured displacements, and therefore, successive displacements cannot be expected to be independent of one another. If diffusion measurements were made in a large tank many feet in diameter and with the same turbulence properties as prevail in the 10-inch flume, then there is no doubt that the mean-square displacement would become proportional to time as is the case in molecular measurements.

Diffusion Coefficient

The "mixing coefficient" or more technically, the diffusion coefficient is given by the mean product of the velocity fluctuation and the depth of penetration. This was shown to be equal to $\sqrt{\overline{v^2}}\,\ell_1$.

If it is assumed that $R_\xi \rightarrow 0$ for ξ very large, $\int_0^t R_\xi \, d\xi$

becomes finite, and ε becomes proportional to $\overline{v^2}$ or varies with the energy of turbulence as shown in figure 14. The results are consistent with those obtained by other means.[16]

However, if the mean-square displacement curve does follow the logarithmic linear law, it is obvious that no definite scale can be assigned to the eddies and, therefore, the diffusion coefficient defined above is likewise indefinite. But we know that this cannot be so, since it is obvious that a finite coefficient must exist in order to transfer momentum and sediment.

RESUME AND CONCLUSIONS

A laboratory experiment was conducted for the purpose of measuring certain diffusion properties of water flowing in an open channel with the aim of attaining a more fundamental understanding of phenomena such as apparent shearing stress, convective heat transfer, and sediment suspension.

The experimental work consisted of the measurement of displacements of small immiscible globules injected at various depths in four different discharges with two channel roughnesses.

Theory, reviewed to acquaint the reader with the necessary basic concepts and terminology, was applied to determine the variation of intensity of turbulence, Taylor's ℓ_1, and the diffusion coefficient.

The results of the investigation lead to the following conclusions:

1. Within the range of conditions tested in the laboratory,

the ratio of the mean-square vertical displacement at corresponding depths in any vertical section of any flow with fixed boundary conditions is independent of the distance from the origin.

2. In any case, the mean-square displacement is an exponential function of the distance from the source, except very near to the source.

3. The diffusion exhibited by water flowing in an open channel is similar to atmospheric diffusion.

4. The intensity and energy of turbulence are greatest near the boundary.

5. The effect of roughness is to increase the intensity of turbulence.

6. The correlation of successive velocities of a particle of fluid falls off rapidly for short-time intervals but never attains the value of zero. This means that there exists a large variation in the size of eddies, similar to the condition in the atmosphere.

7. From result (6), a particle of fluid never loses its velocity identity in the surrounding fluid in the open channel flow investigated herein.

8. The correlation coefficient is independent of the depth for any particular flow.

9. Taylor's length ℓ_1, which may be interpreted as the "mixing length", is proportional to the root-mean-square velocity fluctuation, provided that the correlation coefficient R_ξ becomes zero after a finite interval of time.

10. The diffusion coefficient $\sqrt{\overline{v^2}}\,\ell_1$, according to result

(9) is proportional to the energy of the vertical component of the fluctuating velocity.

The shortcomings of the laboratory technique used in this investigation should not be overlooked. They are: 1) the insertion of an injection tube which disturbs the flow, 2) vibration of the tube, 3) inertial effect of the globules at the origin, 4) the size of the globules which may be as large as some of the smallest eddies, and 5) the tediousness of the analysis of the data.

It should be remembered that the results of this study were limited by the number of conditions investigated. Future work of this type should be done in other channels as well as the channel used herein, to completely understand the nature of the turbulence of open channel flow.

Acknowledgments.

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