

TRANSFER OF STRESS FROM MAIN BEAMS TO INTERMEDIATE  
STIFFENERS IN METAL SHEET COVERED  
BOX BEAMS

Thesis by  
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California Institute of Technology  
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I. TABLE OF CONTENTS:

	<u>SECTION</u>	<u>PAGE</u>
I.	Acknowledgments	2
II.	Summary	3
III.	Introduction	4
IV.	Experimental Investigation	5
V.	Theoretical Calculations	11
VI.	Discussion of Results	18
VII.	References	20

## I. ACKNOWLEDGMENTS:

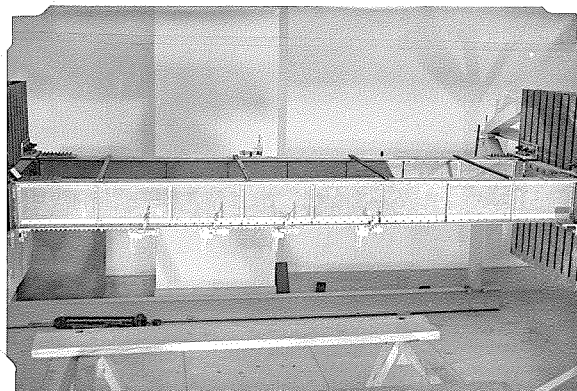
The authors wish to thank Dr. von Karman, Director of the Guggenheim Aeronautical Laboratory at the California Institute of Technology, for his advice and aid in working out the theoretical treatment of the investigation. They wish to thank Dr. E.E.Sechler, under whose direction the research was carried out, for his valuable assistance during the research, and his careful reviews of the completed manuscript. Acknowledgment is also due Dr. A.L.Klein of the Institute and the Douglas Aircraft Company, Inc., for the careful preparation of the equipment tested, and to Messers R.J.White and H.M.Antz for their work in setting up the apparatus.

## II. SUMMARY:

The authors have obtained by experimental methods an effective shear modulus for the sheet in a stiffened plane sheet beam combination under bending loads. For the combinations tested it was found that the modulus decreases rapidly under light loadings from the elastic value to some asymptotic value depending upon the sheet thickness. The thick sheet combination gave higher values of the effective shear modulus than the thin sheet.

### III. INTRODUCTION:

As a part of the investigation of the stress distribution in metal covered wings leading to the most efficient distribution of materials, experiments are being made in the Guggenheim Aeronautics Laboratory with flat plates reenforced with stiffeners. White and Antz (Ref.I) investigated reenforced flat plates under concentrated end loads. The work covered by this paper has extended the investigation to include the case of two built-up beam sections, connected by a stiffened plane sheet, and subjected to a uniform bending moment. The transfer of compressive stress from the main spars to the intermediate stiffeners has been studied with the idea of finding a rational method of calculating the load in the stiffeners. Two cover sheet sizes were tested, one of which was heavy enough to stay out of the wave state under moderate loading, while the other was light enough to go into the wave state early in the loading. The stiffener size and arrangement on the two panels were the same.



#### IV. EXPERIMENTAL INVESTIGATION:

The test specimen consisted of two main spars 96" long and 8" deep spaced 26" apart, and a bottom cover sheet 26" X 72" secured to the flange angles by 8-32 bolts spaced  $\frac{3}{4}$ " apart. The four stiffeners consisted of extruded bulb angles spaced 5" apart and riveted with  $\frac{3}{32}$ " rivets spaced at 1" intervals. Both ends of the spars were rigidly attached to the testing machine as was also one end of the cover sheet. The other end of the cover panel (and its reinforcing stiffeners) was free corresponding to an infinitely weak rib system near the tip of a wing. At intervals along the span chordwise spacers were used to prevent rotation of main spars. (See Fig. I)

FIG. 1

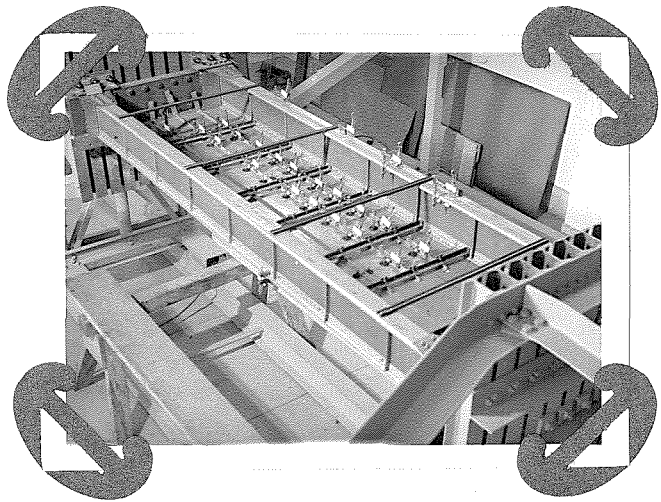
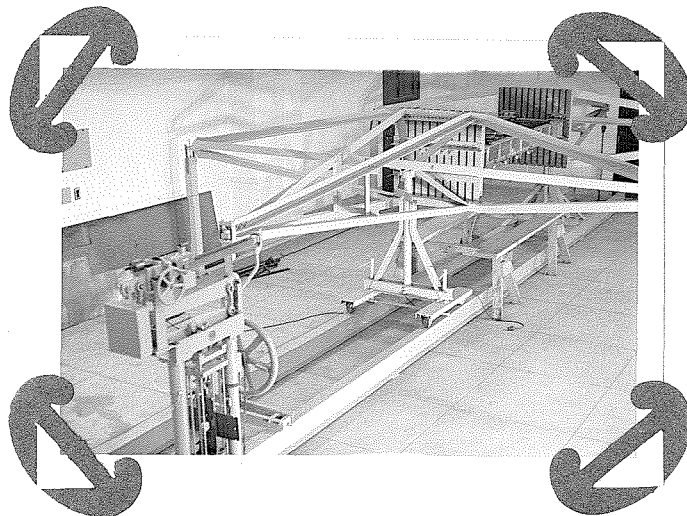


FIG. 2



The beam was mounted in the 500,000 in. lbs. Torsion-Bending Testing Machine shown in figure 2\*. The machine "floated" laterally, ie, with no lateral restraint, thus eliminating torsional loads.

The bending moments were obtained by applying forces on the end of each of the testing machine lever arms. The amount of force applied was indicated by an Ames Dial Gauge measuring the lateral deflection of initially bent steel springs. (See Fig. 3). Calibration curves of pounds force versus gauge reading were obtained by calibrating the gauges in an ordinary tensile testing machine.

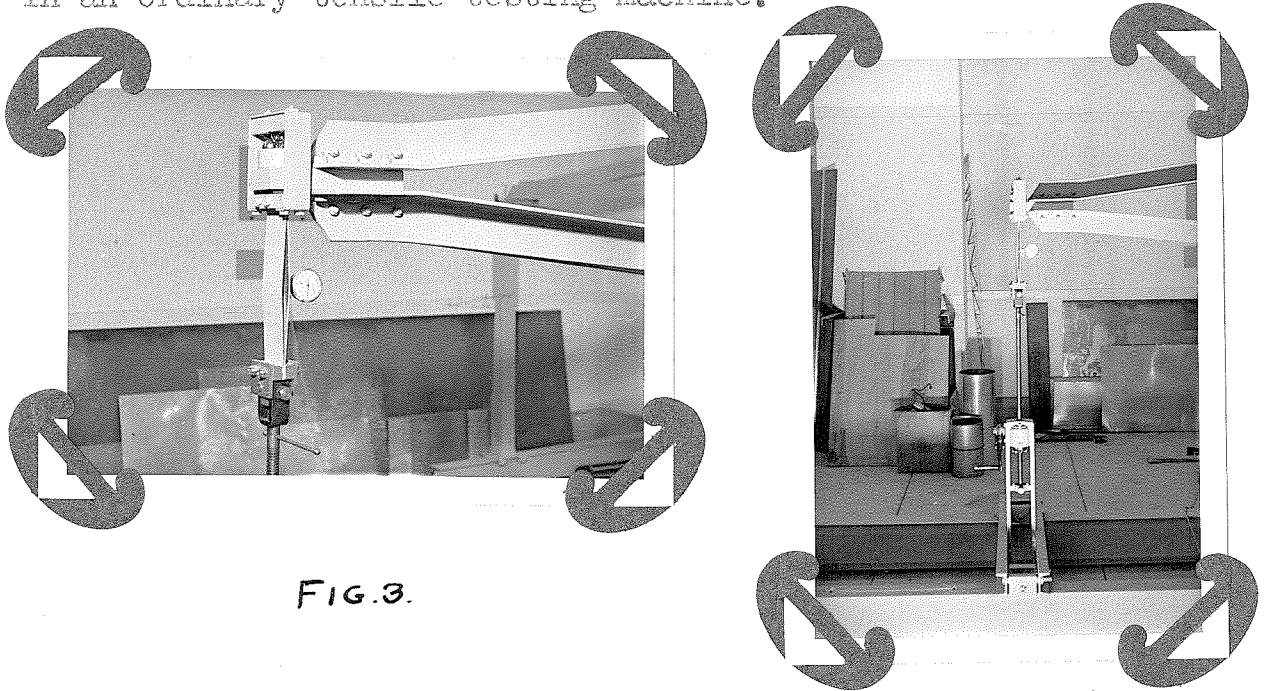
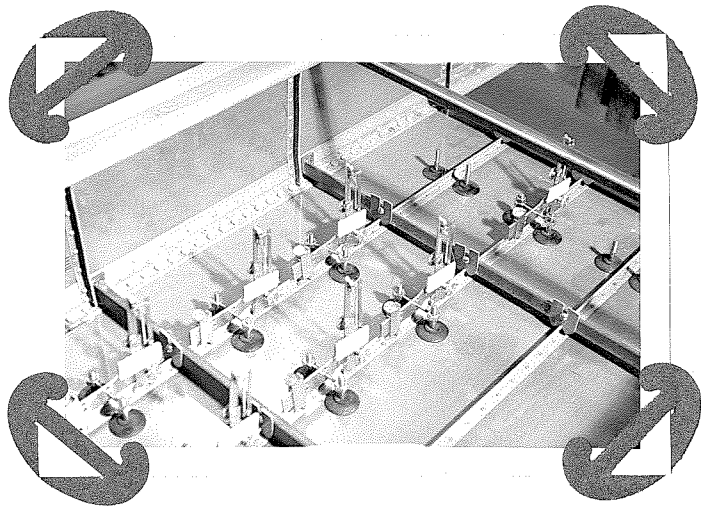


FIG. 3.

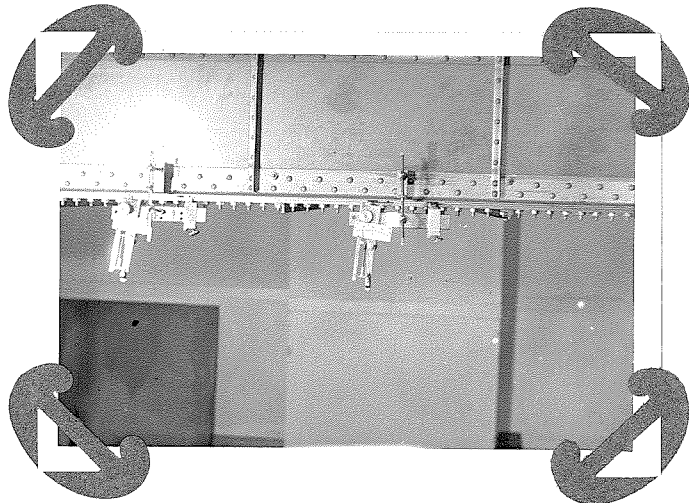
\*For a description of this apparatus used in torsion see Reference 2.

The stress distribution was obtained by measuring deformations with Huggenberger Tensometers on the beam caps and stiffener legs. The gauge length was 8 cm. These Tensometers, whose scale readings are approximately 1200 times the measured deflections, were accurately calibrated with an interferometer. The tensometers placed on the stiffeners were held in place by means of rubber suction cups moistened with glycerine, as seen in figure 4, while those on the main spar were held by a special clamp shown in figure 5.

*FIG. 4*



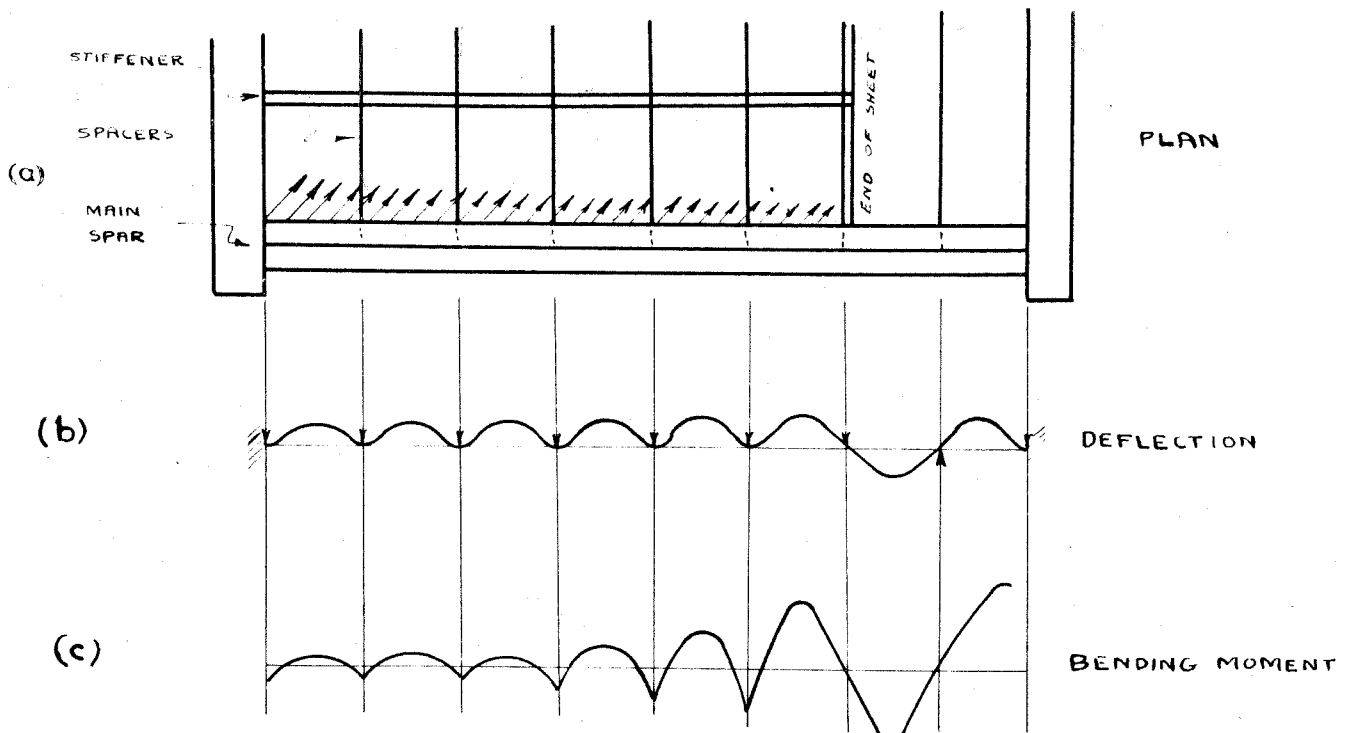
*FIG. 5*



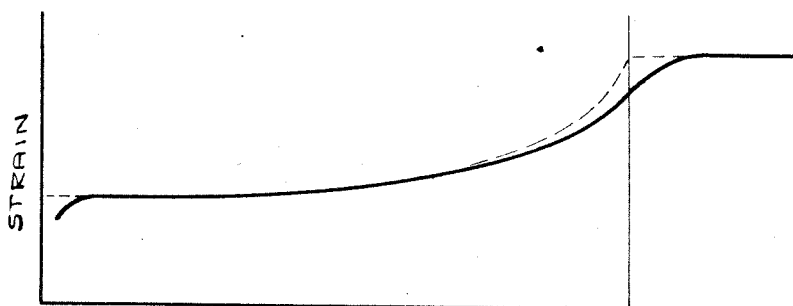


From the general nature of the set up and from the previous experiments one would expect that strain readings taken along the bottom flange would produce a graph of the general shape of the solid curve of figure 6. But with the tensometers placed at position "A" along the flange the resulting curve was the dotted line of figure 6. However, by taking two sets of readings at each station, one at "B" and another at "C" and extrapolating to the vertical neutral axis at the center of the web, "D", the resulting points produced the smooth curve shown by the solid line. This indicated the presence of and eliminated the effects of chordwise bendings. A similar effect was noted on the intermediate stiffeners.

The following diagram shows how the chordwise bending stresses arise.



The tension field in the sheet loads the main spar laterally, or chordwise, while the chordwise spacers act as supports of a continuous beam. The deflection curve thus produced is of the form shown at (b) and the bending moment of the general shape shown at (c). The stresses caused by this bending are superimposed upon the stresses from the applied moments, the combination producing the dotted curve of Fig. 6.



At the point on the main spar where the sheet ends a sudden change in the moment of inertia and neutral axis takes place. In an elastic body at such discontinuities as this the stress instead of making a sharp change takes up the load more evenly and reaches its final value about a spar depth further on. A similar discontinuity exists at the fixed end of the sheet where a securing angle changes the section properties of the beam.

In converting strain into stress the value of Young's Modulus for dural was taken as  $10.4 \times 10^6$  lbs/sq. in.

Figures (12) to (27) show the stress distribution for various bending loads and thicknesses of sheet. It is of interest to note that the stresses at any station vary directly with the bending moment for both the thin and thick sheet, and while the thick sheet extrapolates to zero, the thin sheet does not. This is thought to be due to the fact that some initial stresses existed in the thin sheet set up in the unloaded condition. (See Fig. 12 to 14 and 20 to 22).

The authors have no explanation for the peculiar shape of the stress curves of stiffener "C" of the thick sheet near the free end.

## V. THEORETICAL CALCULATIONS:

In order that the results obtained here may be of some use to the designer who is looking for a method of predicting the stresses in the stiffeners, an attempt is made to find an effective shear modulus,  $G'$ , or better, the ratio  $\frac{G'}{E}$ .

Figure (7) is a plan view of the beam, divided into a number of transverse strips arbitrarily chosen in 5" widths. As the test beam is symmetrical about its longitudinal axis only one half is shown. Stress distribution curves are also indicated.

$A_A$ ,  $A_B$ , and  $A_C$  are areas of stiffeners.

A, B, and C respectively.

Let us consider a strip across the beam of width ( $\delta$ ).

The basic relationship between shear stress ( $\tau$ ) and strain ( $\gamma$ ) is

$$\tau = G\gamma$$

Where  $G$  = shear modulus.

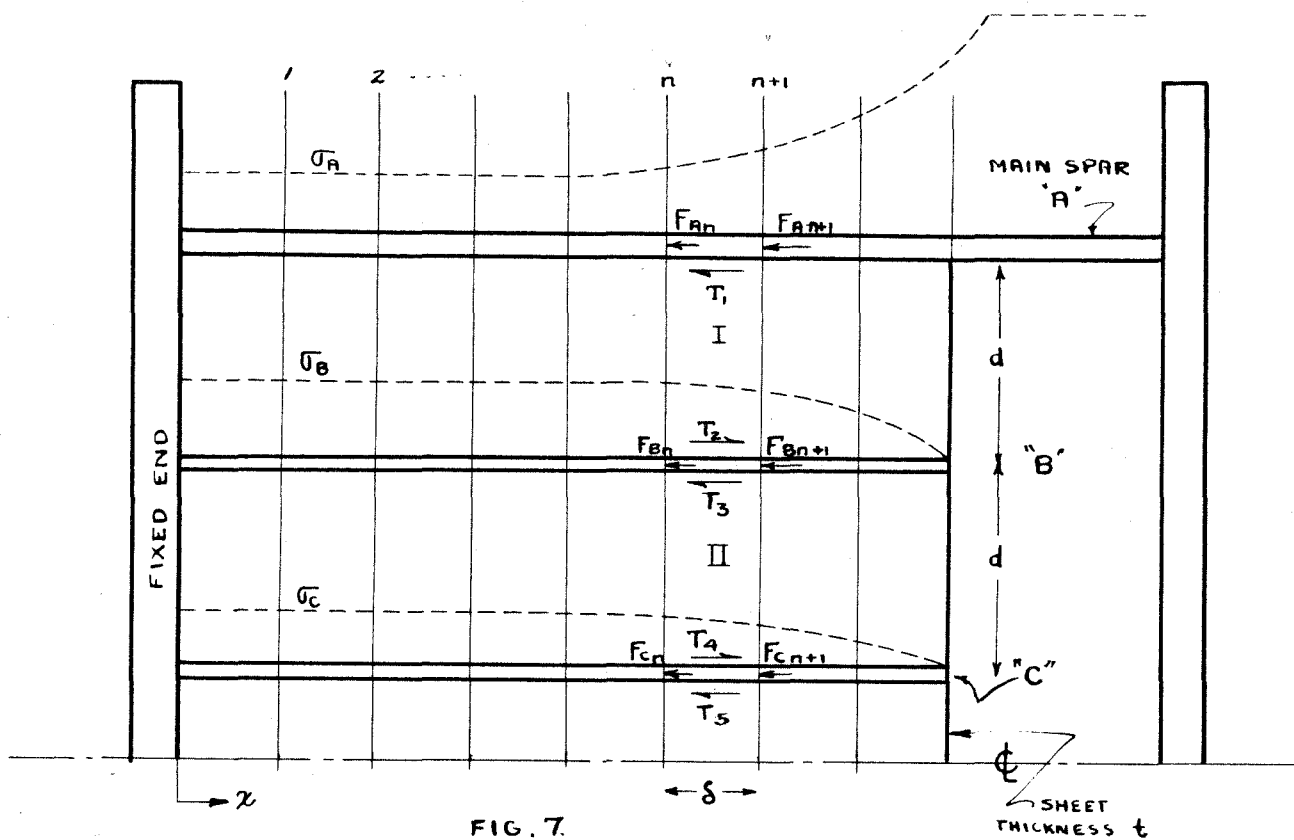


FIG. 7.

AT STATION "n"

$$\text{FORCE IN "A"} = F_{An} = \sigma_{An} A_A$$

$$\text{SHEARING FORCE IN "A", } T_1 = F_{An+1} - F_{An} = A_A (\sigma_{An+1} - \sigma_{An})$$

$$\text{SIMILARLY, IN STIFFENER "B", } F_{Bn} = \sigma_{Bn} A_B, \quad F_{Bn+1} = \sigma_{Bn+1} A_B$$

$$T_2 - T_3 = F_{Bn} - F_{Bn+1} = A_B (\sigma_{Bn} - \sigma_{Bn+1})$$

$$\text{AND IN "C", } F_{Cn} = \sigma_{Cn} A_C$$

$$F_{Cn+1} = \sigma_{Cn+1} A_C$$

$$T_4 - T_5 = F_{Cn} - F_{Cn+1} = A_C (\sigma_{Cn} - \sigma_{Cn+1})$$

BUT BECAUSE OF SYMMETRY ABOUT LONGITUDINAL  $\zeta$ ,  $T_5 = 0$

$$\text{WE CAN WRITE } \textcircled{1} \frac{T_1 + T_2}{2} \cdot \frac{1}{\delta t} = \tau_1 = G_1 \gamma_1 \quad \text{FOR PANEL I}$$

$$\textcircled{2} \frac{T_3 + T_4}{2} \cdot \frac{1}{\delta t} = \tau_2 = G_2 \gamma_2 \quad \text{" " II}$$

SUBTRACTING,  $\textcircled{1} - \textcircled{2}$ , AND ASSUMING THAT PANELS I + II ARE IN

THE SAME STATE, IN WHICH  $G_1 = G_2 = G'$  WE GET

$$\textcircled{3} T_1 + (T_2 - T_3) - T_4 = 2 \delta t G' (\gamma_1 - \gamma_2)$$

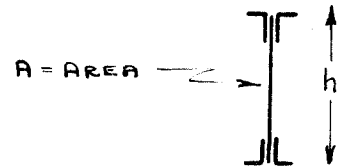
$$\text{OR } A_A (\sigma_{An+1} - \sigma_{An}) + A_B (\sigma_{Bn} - \sigma_{Bn+1}) - A_C (\sigma_{Cn} - \sigma_{Cn+1}) = 2 \delta t G' (\gamma_1 - \gamma_2)$$

$(A_R)$  is the "effective" cross-section area of the main spar and it was obtained as follows: Consider a section at  $x$  inches from the fixed end of the sheet. Assuming that the stress across the depth of the beam has a linear variation the stress at the bottom fibers is composed of three parts:

- 1- Compression due to the uniform bending moment  $M_0$ .
- 2- Tension due to the bending moment produced by the shearing force  $T_1$
- 3- Tension due to the shearing force  $T_1$ , acting as an end load

Then the compressive stress ( $\sigma_R$ ) is

$$\sigma_R = \frac{M_0 c}{I} - \frac{h}{2} T_1 \frac{c}{I} - \frac{T_1}{A}$$



The change in  $\sigma_R$  in a small length  $dx$  is

$$\frac{d\sigma_R}{dx} = - \left[ \frac{ch}{2I} + \frac{1}{A} \right] \frac{dT_1}{dx} = \frac{d}{dx} \left[ \frac{T_1}{\text{EFFECTIVE AREA}} \right] \quad \text{OR}$$

$$\frac{d}{dx} [\sigma_R A_R] = \frac{dT_1}{dx}$$

$$A_R = \frac{2AI}{2I + Ach} = \frac{A}{1 + \frac{ChA}{2I}} = \frac{A}{1 + \frac{Ch}{2\rho^2}}$$

$$\rho = \sqrt{\frac{I}{A}}$$

The effective area computed in this way is 0.568 sq. in. and the area of the stiffener cross-section is 0.044 sq. in. All items on the left side of equation (3) are known. We must now find  $(\delta_1 - \delta_2)$ . We assume that the angle  $(\delta)$  and its tangent are equal for small deformations.

$$\delta_1 = \frac{l_1}{d} \quad \text{and} \quad \delta_2 = \frac{l_2}{d}$$

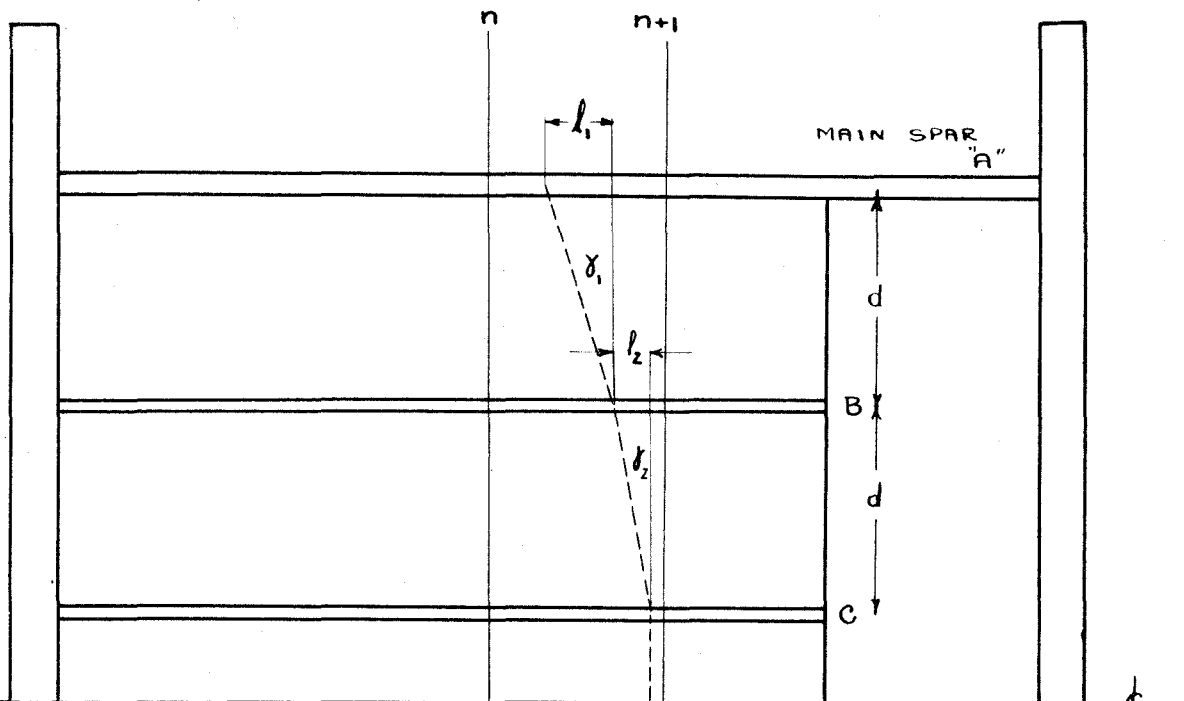
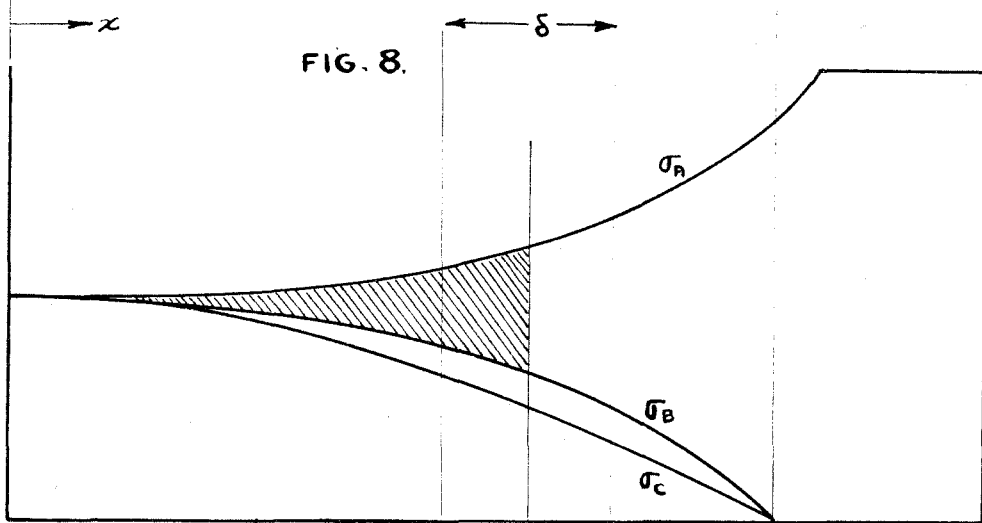


FIG. 8.



$$l_1 = \left[ \frac{\sigma_{A_1} + \sigma_{A_2}}{2} \cdot \frac{\delta}{E} + \frac{\sigma_{A_2} + \sigma_{A_3}}{2} \cdot \frac{\delta}{E} + \dots + \frac{\sigma_{A_{n-1}} + \sigma_{A_n}}{2} \cdot \frac{\delta}{E} + \frac{\sigma_{A_n} + \sigma_{A_{n+1}}}{2} \cdot \frac{\delta}{E} \right] - \left[ \frac{\sigma_{B_1} + \sigma_{B_2}}{2} \cdot \frac{\delta}{E} + \dots + \frac{\sigma_{B_n} + \sigma_{B_{n+1}}}{2} \cdot \frac{\delta}{E} \right]$$

$$= \frac{\delta}{E} \left[ \frac{\sigma_{A_1}}{2} + \sigma_{A_2} + \dots + \sigma_{A_n} + \frac{\sigma_{A_{n+1}}}{2} \right] - \left[ \frac{\sigma_{B_1}}{2} + \sigma_{B_2} + \dots + \sigma_{B_n} + \frac{\sigma_{B_{n+1}}}{2} \right]$$

$$= \frac{1}{E} \left[ \text{AREA UNDER } \sigma_A \text{ CURVE TO } (n+1) \right] - \left[ \text{AREA UNDER } \sigma_B \text{ CURVE TO } (n+1) \right]$$

But since an average value of  $\delta_1$  should be taken over the interval between (n) and (n+1) the area shown in Fig. 8 should be taken.

$$(4) \quad \delta_1 = \frac{l_1}{d} = \frac{1}{dE} \left[ \begin{array}{l} \text{area under } \sigma_A \text{ curve to } (n+\frac{1}{2}) \\ - \text{area under } \sigma_B \text{ curve to } (n+\frac{1}{2}) \end{array} \right]$$

Similarly, for  $\delta_2$

$$(5) \quad \delta_2 = \frac{l_2}{d} = \frac{1}{dE} \left[ \begin{array}{l} \text{area under } \sigma_B \text{ curve to } (n+\frac{1}{2}) \\ - \text{area under } \sigma_C \text{ curve to } (n+\frac{1}{2}) \end{array} \right]$$

In our case we have assumed  $\delta = 5$  inches  
and our stiffener spacing is  $d = 5$  inches

We now have ( $\delta_1 - \delta_2$ ) in terms of E. Substituting in equation (3) we get the equation for  $\frac{G'}{E}$

$$(6) \quad \frac{G'}{E} = \frac{A_A(\sigma_{A_{n+1}} - \sigma_{A_n}) + A_B(\sigma_{B_n} - \sigma_{B_{n+1}}) - A_C(\sigma_{C_n} - \sigma_{C_{n+1}})}{\frac{2\delta t}{d} [X - Y]}$$

where

X = area between  $\sigma_A$  and  $\sigma_B$  curves to  $(n+\frac{1}{2})$

Y = area between  $\sigma_B$  and  $\sigma_C$  curves to  $(n+\frac{1}{2})$

The areas were obtained from the stress distribution curves with a planimeter. Results of the computations by this formula are plotted in figures 9 to 11.



## SAMPLE CALCULATIONS

For a section at 85% of the span from the fixed end of the 0.025 sheet subjected to a bending moment of 120,000 in.lbs.  $n = 60$  inches and  $n+1 = 65$  inches and the numerator of equation (6) will be:

$$\begin{aligned}
 A (\sigma_{A_{n+1}} - \sigma_{A_n}) &= 0.568 (12000 - 12750) = 426 \text{ lbs.} \\
 A (\sigma_{B_n} - \sigma_{B_{n+1}}) &= 0.044 ( 5100 - 3450 ) = 72 \text{ lbs.} \\
 A (\sigma_{C_n} - \sigma_{C_{n+1}}) &= 0.044 ( 3500 - 2000 ) = - 66 \text{ lbs.} \\
 &\hspace{15em} \text{-----} \\
 &\hspace{15em} 432 \text{ lbs.}
 \end{aligned}$$

For the denominator:

$$\begin{aligned}
 X &= 5.80 \text{ sq.in.} \\
 Y &= 1.12 \text{ sq.in.} \\
 X-Y &= 4.68 \text{ sq.in. and since one square inch of chart area} \\
 &\quad \text{represents 2,000 lbs/sq.in. times 10 inches, or} \\
 &\quad 20,000 \text{ lbs/in.}
 \end{aligned}$$

$$X-Y = 4.68 \times 20,000 = 93,600 \text{ lbs/in.}$$

$$\frac{2 \delta t}{d} ( X-Y ) = \frac{2 \times 5 \times 0.025}{5} \times 93,600 = 4680 \text{ lbs.}$$

Then,

$$\frac{G'}{E} = \frac{432}{4680} = 0.092$$

## VI. DISCUSSION:

It is seen that  $\frac{G'}{E}$  decreases with increasing bending moment and also decreases with the distances from the fixed end of the sheet. It is of interest to note that the curve when extrapolated to zero moment (Fig. 9) reaches the elastic value of  $\frac{G'}{E}$  where, taking  $\mu = 0.30$

$$\frac{G'}{E} = \frac{1}{2(1+\mu)} = 0.385$$

The thick sheet gives higher values of  $\frac{G'}{E}$  than the thin sheet, (Fig. 11) but they were not nearly as constant as those for the thin sheet. This is due to the fact that the stresses in the thick sheet and stiffeners were not as high, and small errors in stress difference and areas have much greater effect. Much better accuracy would have undoubtedly been attained had the thick sheet been subjected to a higher bending load, but, as can be seen from figure 6, the stresses in the main spar caps near the end were 20,000 lbs/sq. in., beyond which there was danger of some permanent set, which we wished to avoid.

More experimenting will be necessary with other stiffener number, size and spacings, lengths of panel and unequal end moments to verify these results. The application of torsional loads and the testing of cambered sheet in a similar manner offer further extensions of this problem.

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VII. REFERENCES:

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2. Donnell, L.H. : Stability of Thin-walled Tubes under Torsion. N.A.C.A. Technical Report No. 479.

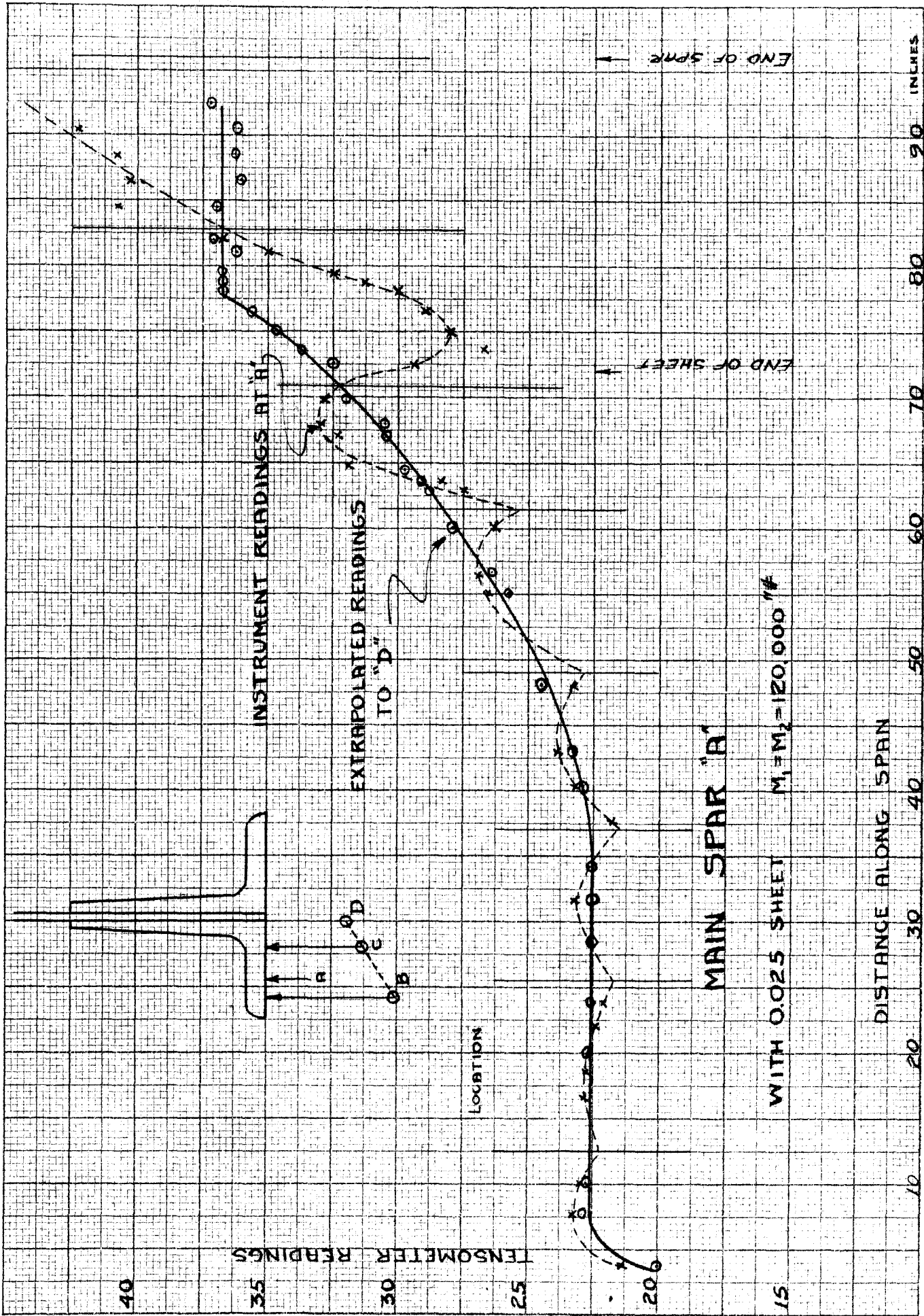


FIG. 6.

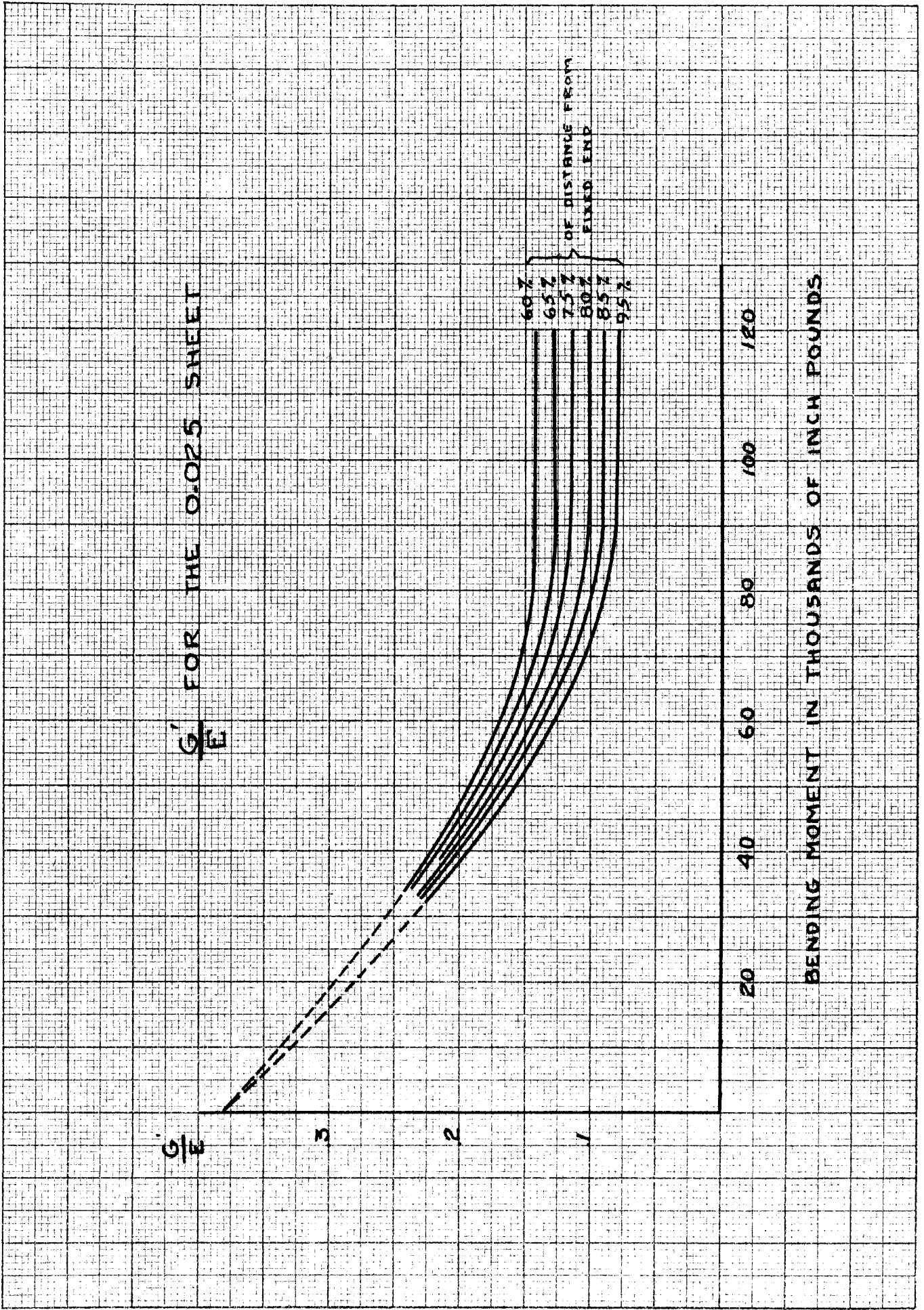


FIG. 9.

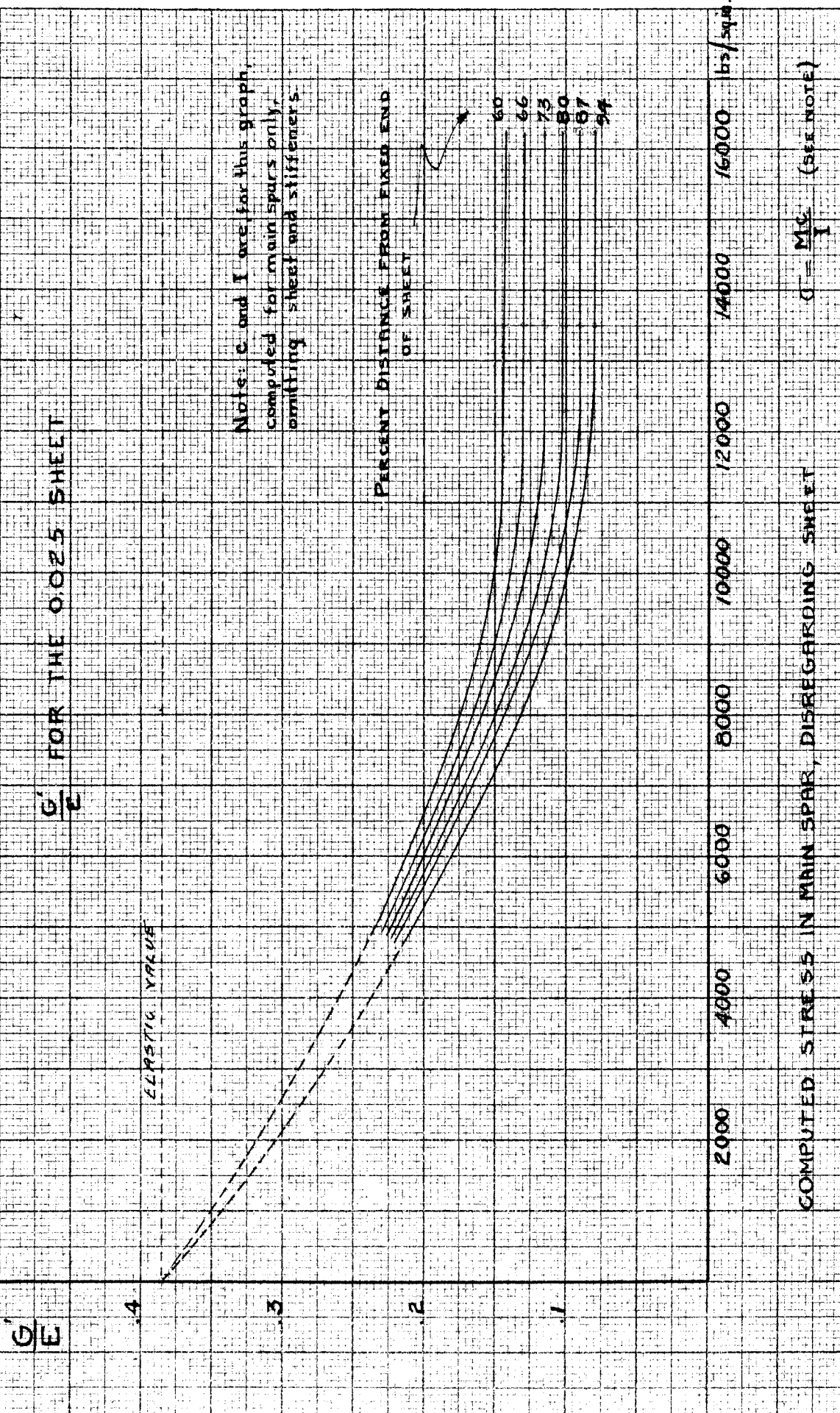


FIG. 9 A.

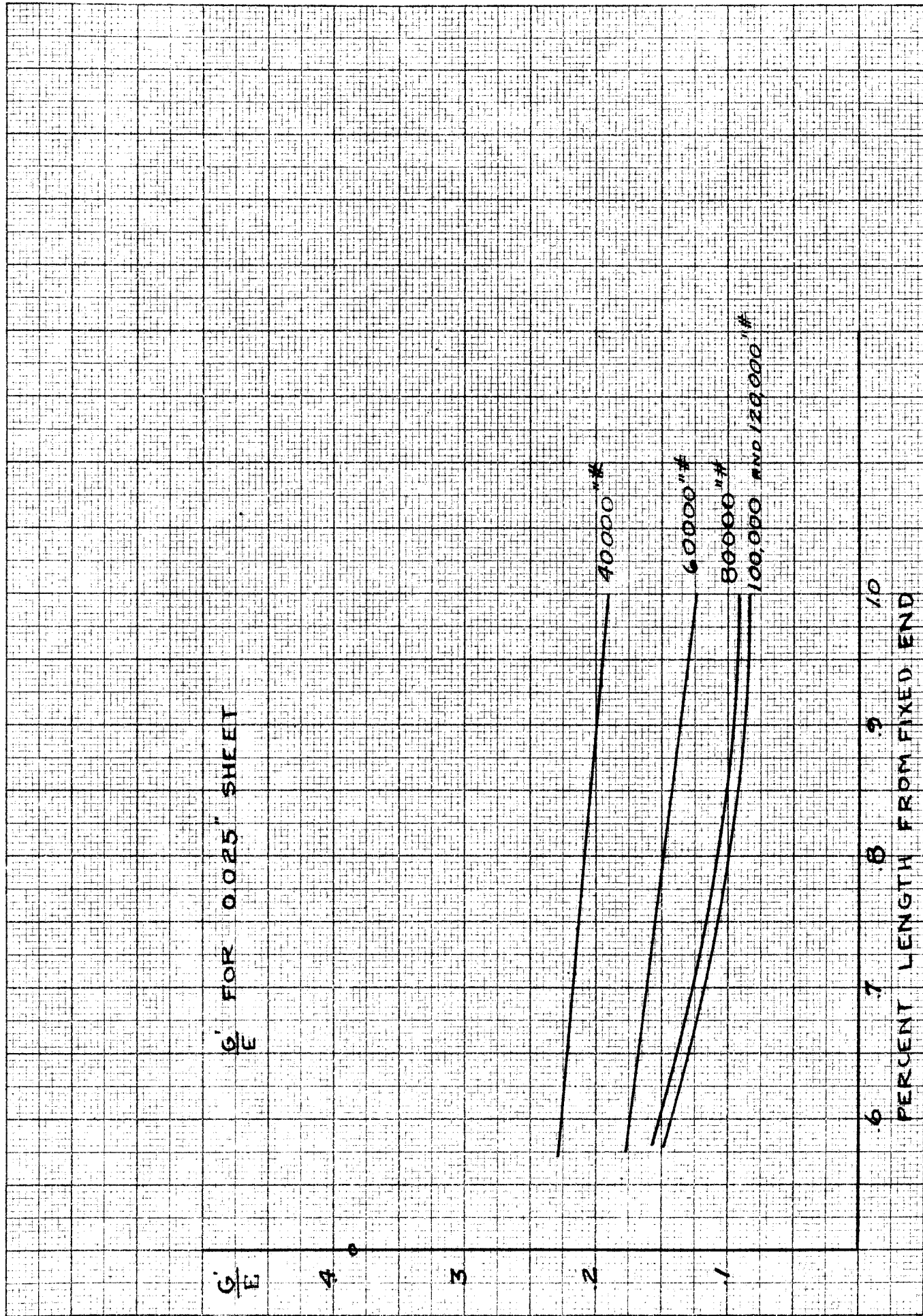


FIG. 10.



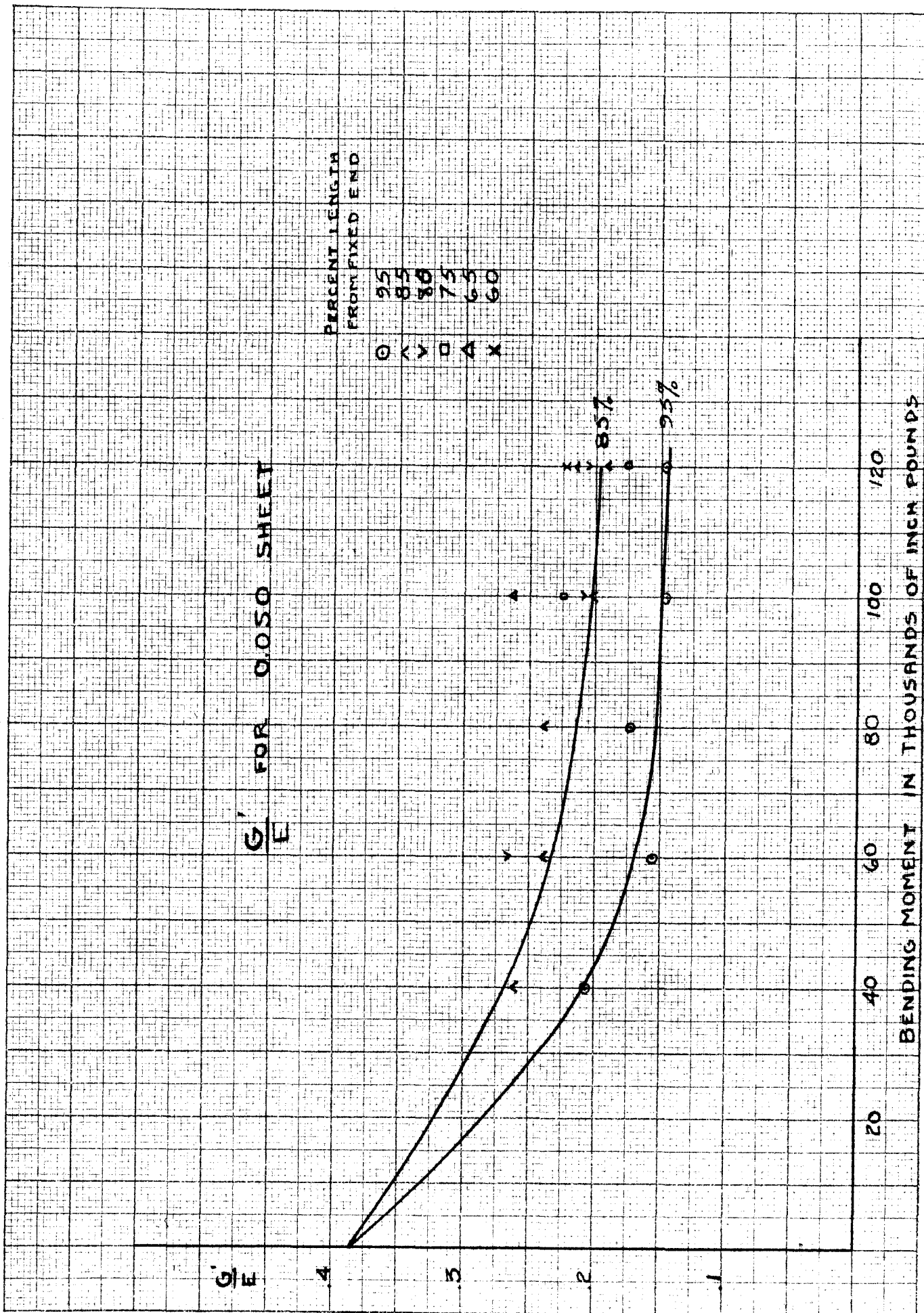


FIG. II.

FIG. 12

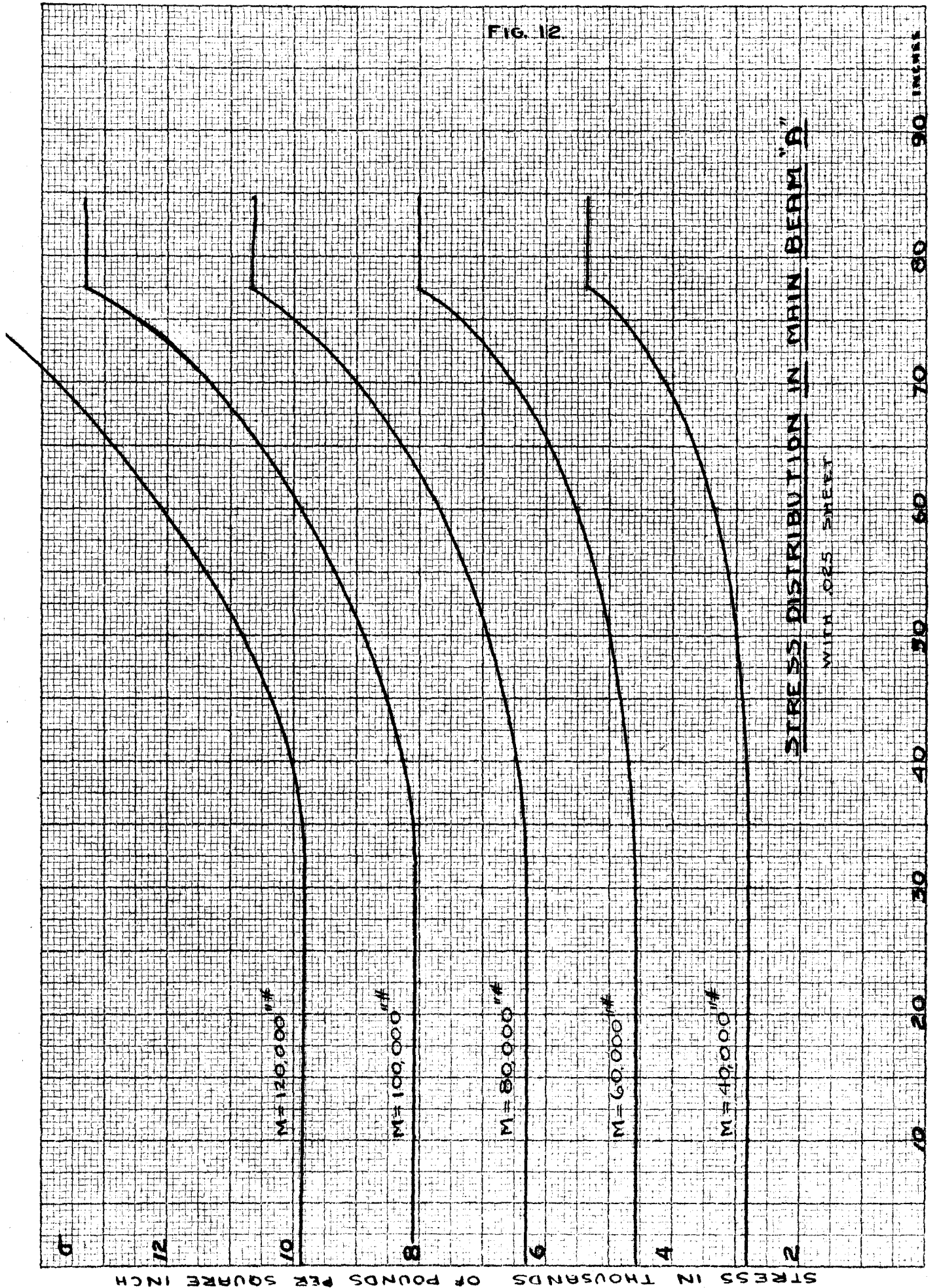
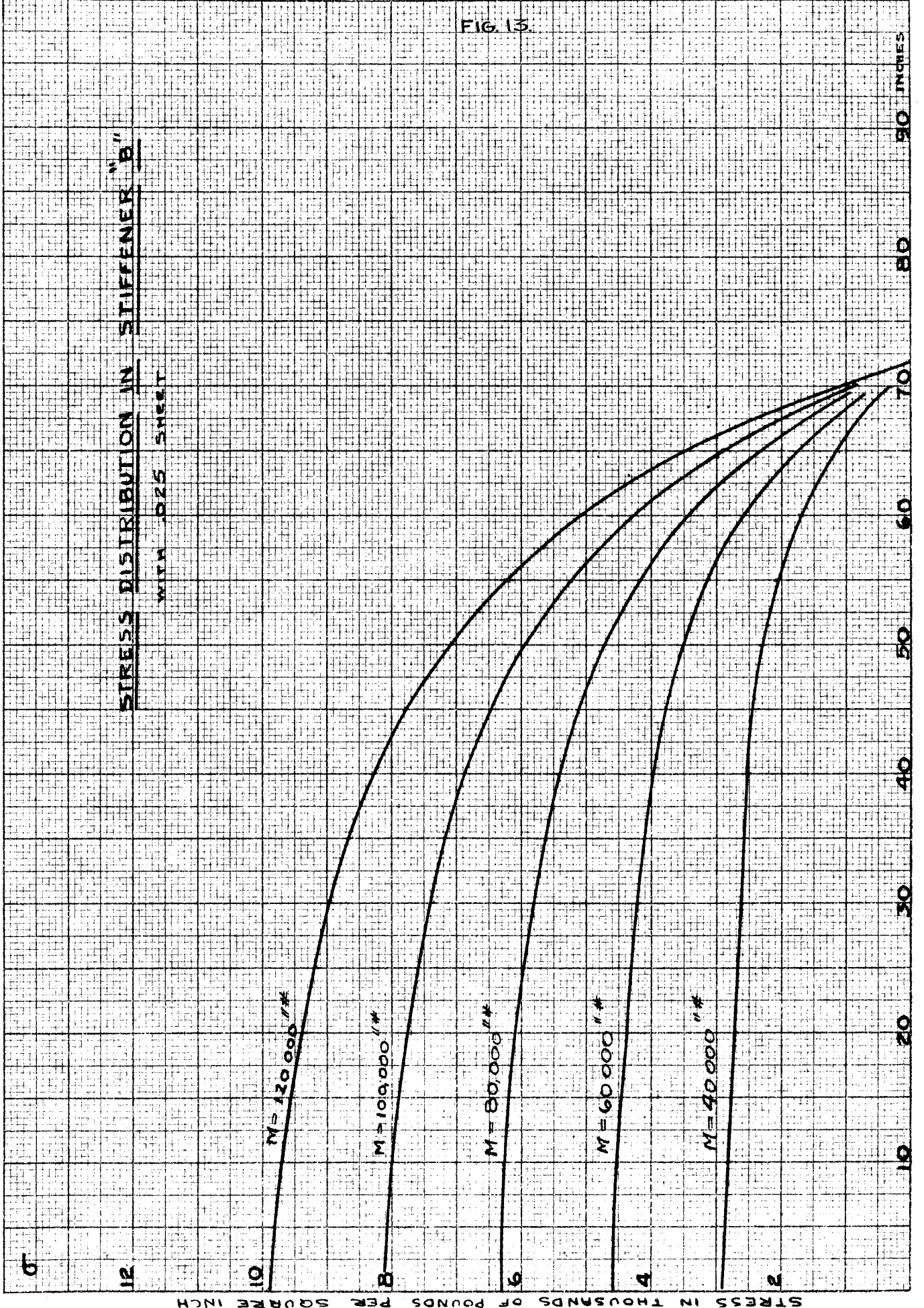


FIG. 13.

STRESS DISTRIBUTION IN STIFFENER "B"

WITH .025 SHEET



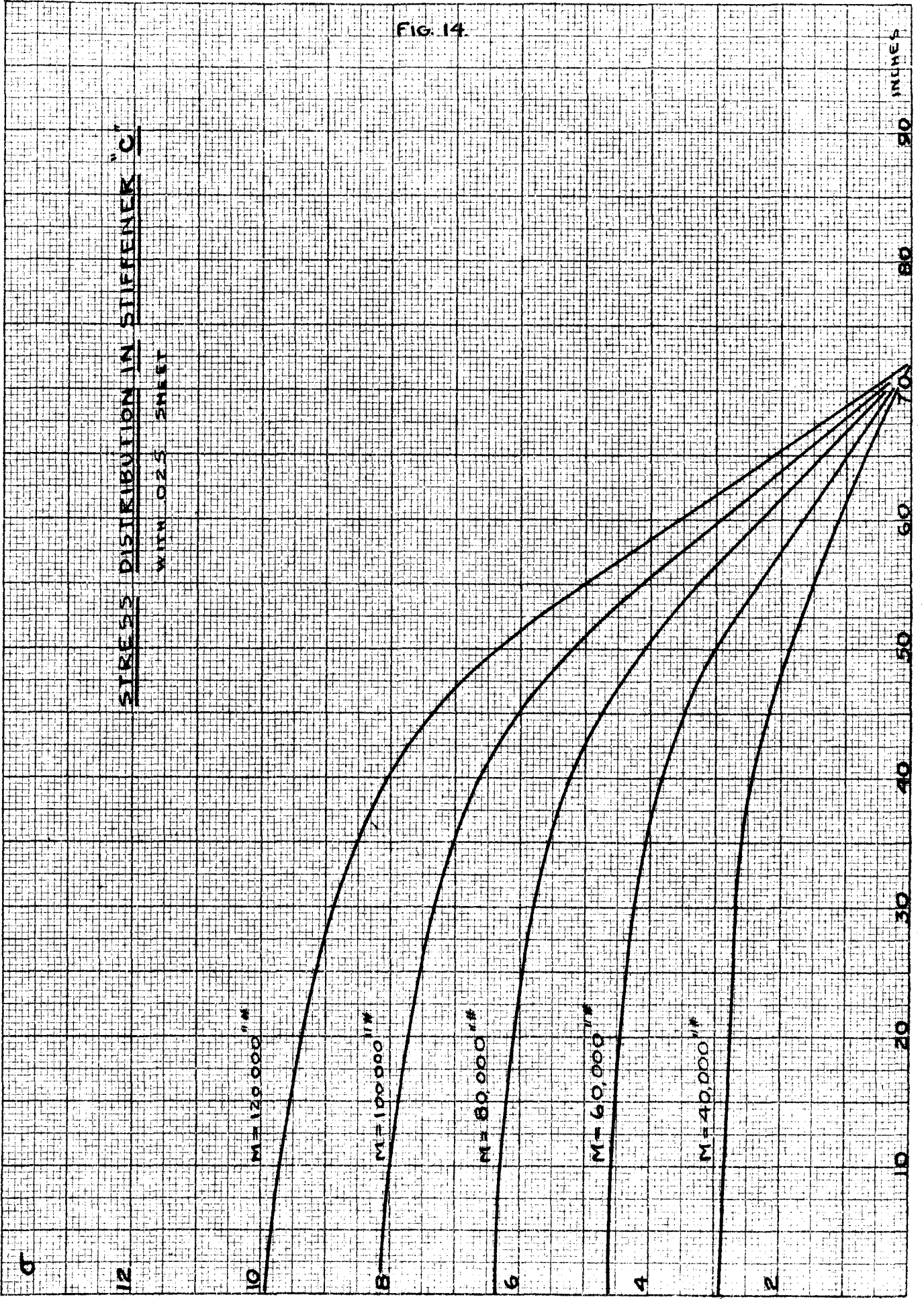
DISTANCE ALONG SPAN

STRESS IN THOUSANDS OF POUNDS PER SQUARE INCH

Fig. 14.

STRESS DISTRIBUTION IN STIFFENER "C"

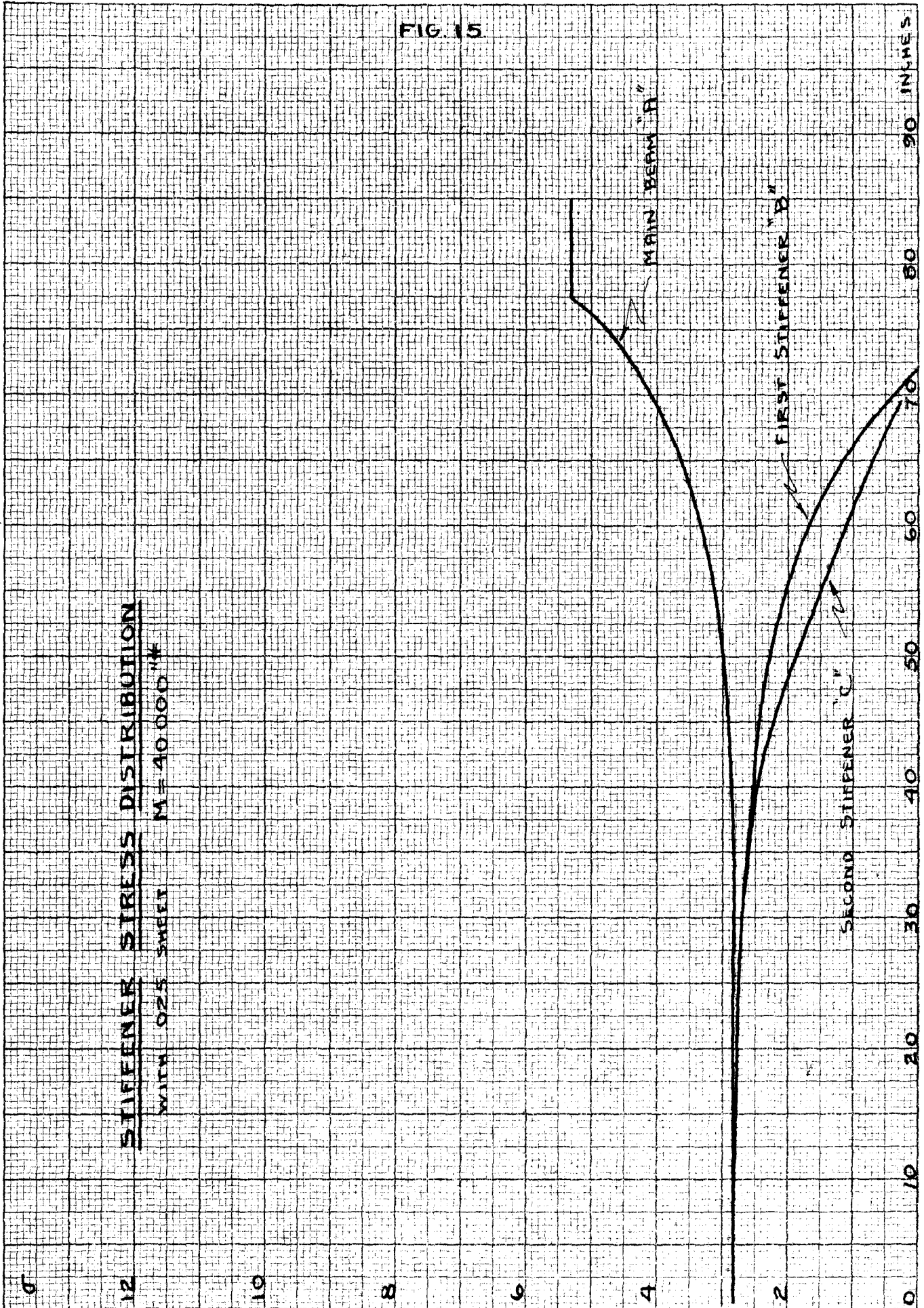
WITH 0.25 SHEET



STRESS IN THOUSANDS OF POUNDS PER SQUARE INCH

DISTANCE ALONG SPAN

STRESS IN THOUSANDS OF POUNDS PER SQUARE INCH

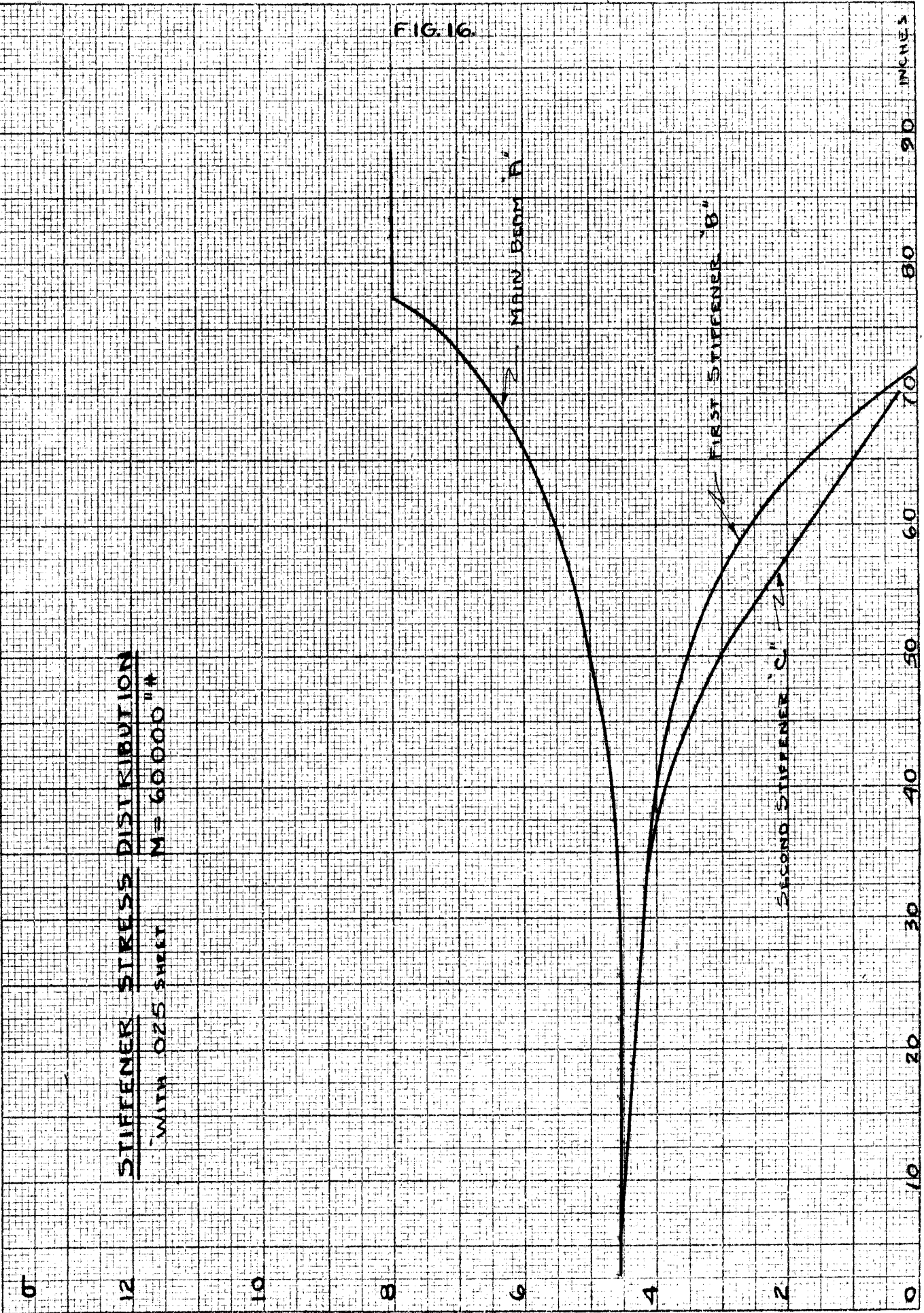


DISTANCE ALONG SPAN

STRESS IN THOUSANDS OF POUNDS PER SQUARE INCH

STIFFENER STRESS DISTRIBUTION  
WITH 0.25 SHEET  $M = 60,000 \text{ " #}$

FIG. 16

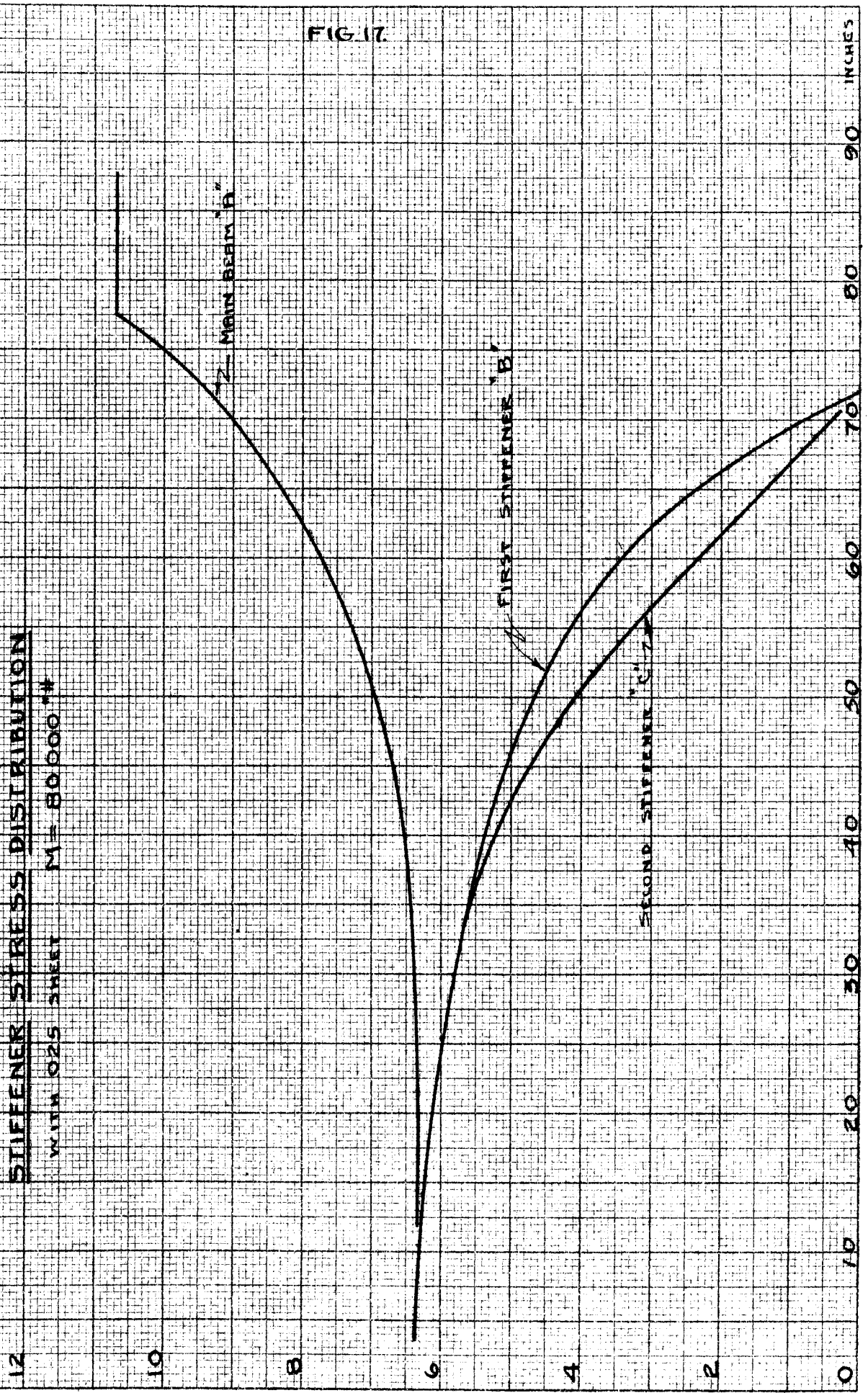


DISTANCE ALONG SPAN

FIG. 17

$\sigma$  IN THOUSANDS  
OF POUNDS PER  
SQ. IN.

**STIFFENER STRESS DISTRIBUTION**  
WITH 0.25 SHEET  $M = 80,000 \text{ IN}^2$



DISTANCE ALONG SPAN

INCHES

FIG. 18.

STIFFENER STRESS DISTRIBUTION

WITH 0.25 SHEET M-100,000 #/IN<sup>2</sup>

STRESS IN THOUSANDS OF POUNDS PER SQUARE INCH

DISTANCE ALONG SPAN

INCHES

MAIN BEAM "A"

FIRST STIFFENER "B"

SECOND STIFFENER "C"

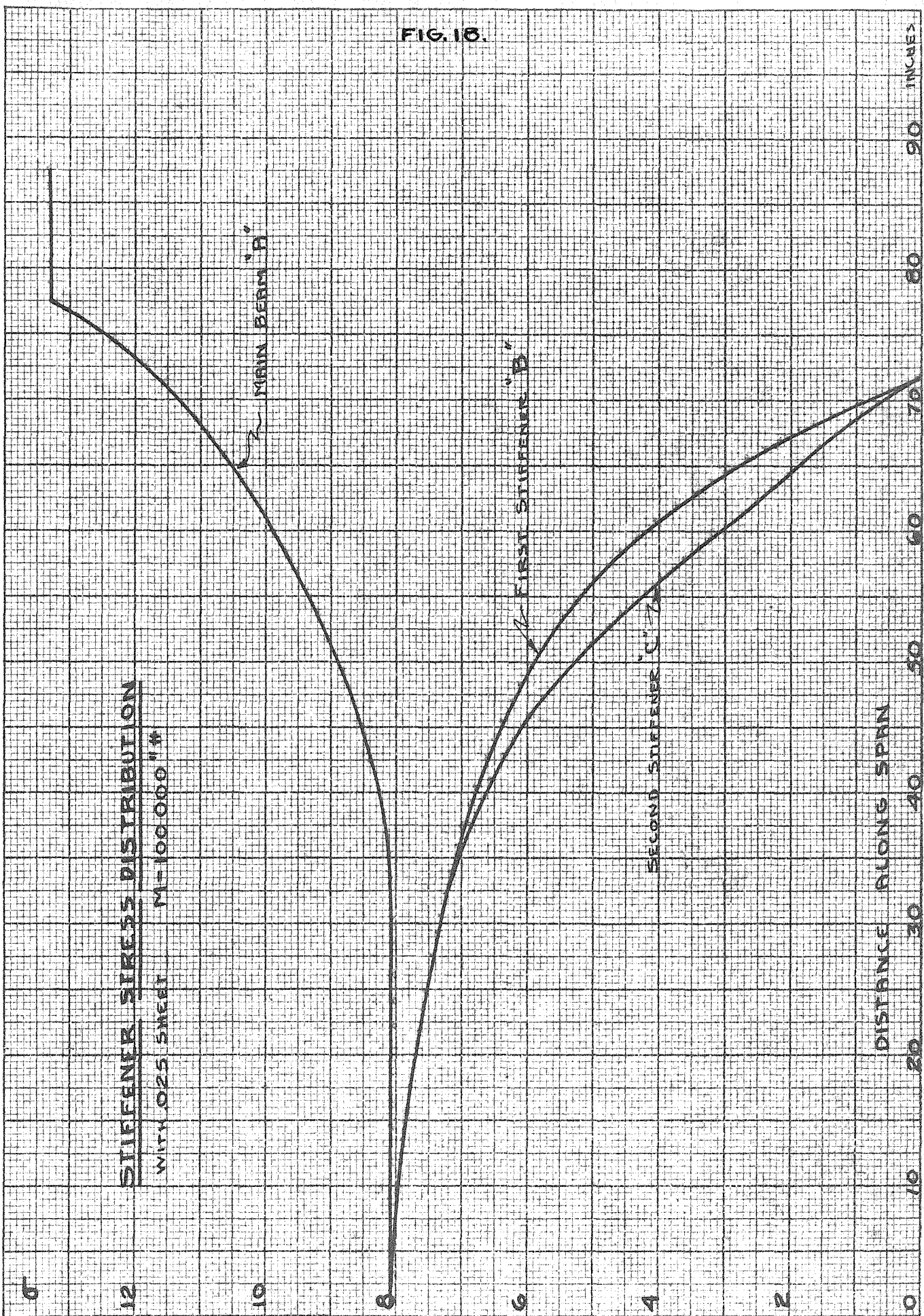




FIG. 19.

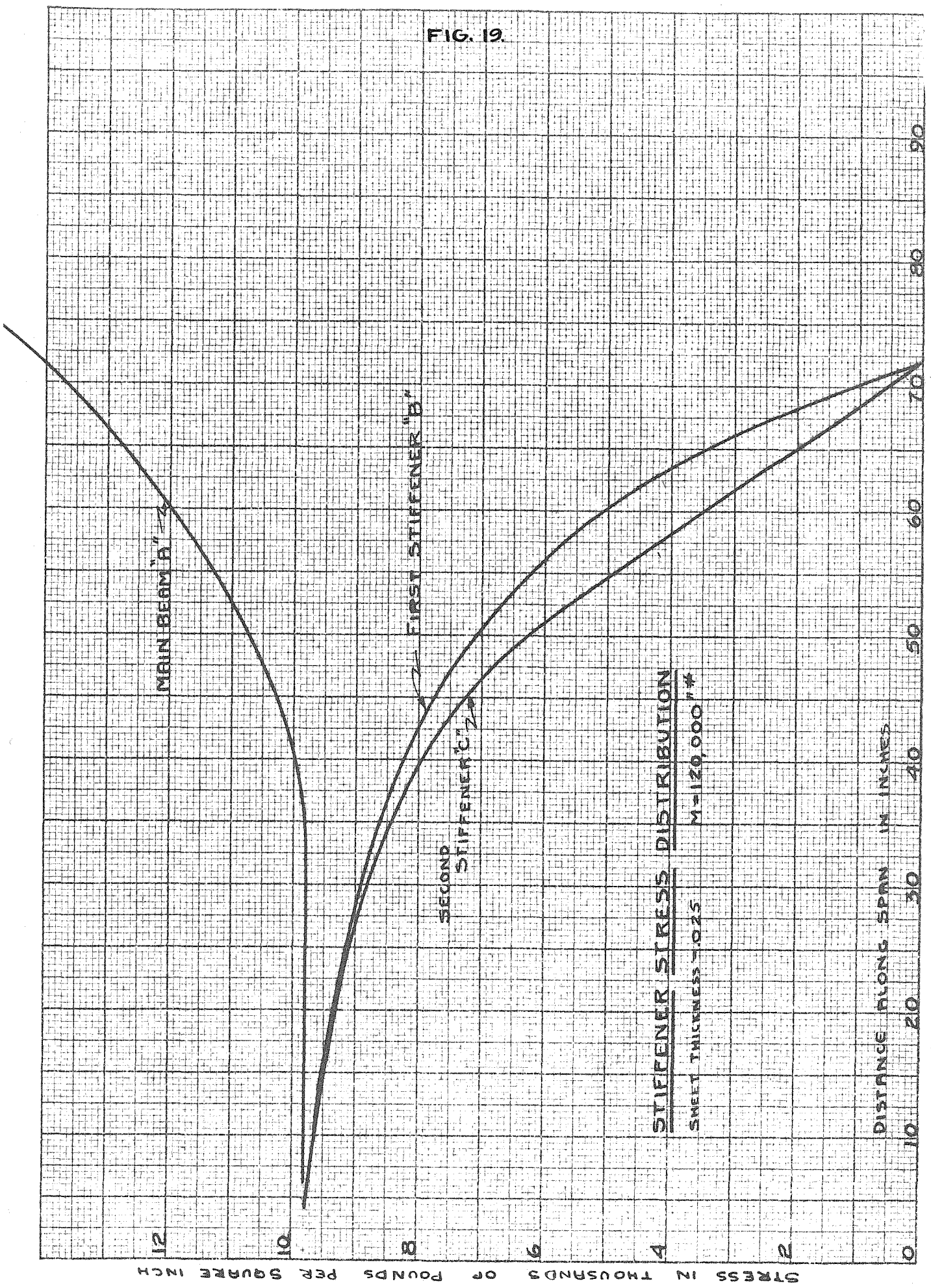
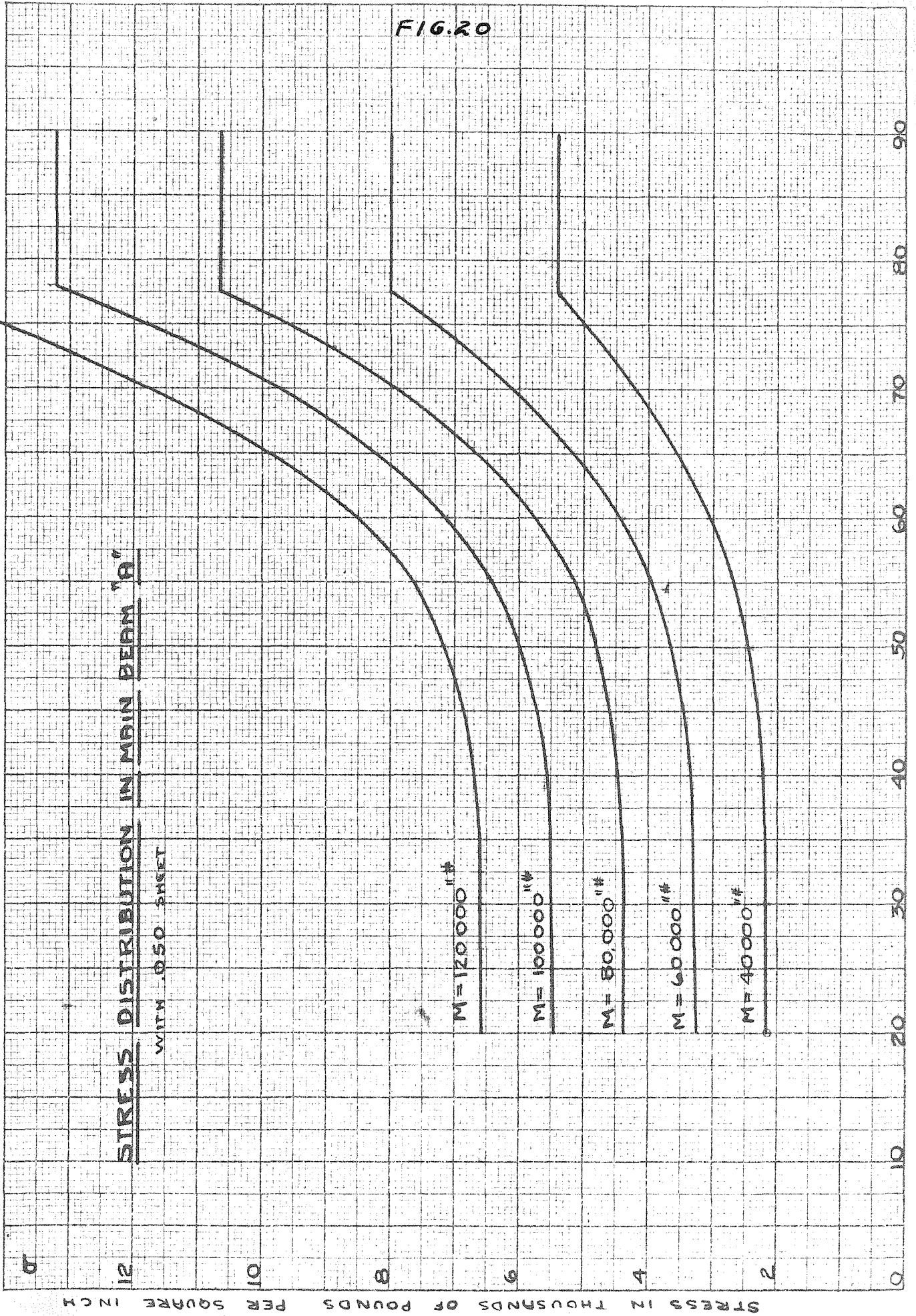


FIG. 20

STRESS DISTRIBUTION IN MAIN BEAM "A"

WITH .050 SHEET



DISTANCE ALONG SPAN IN INCHES

FIG. 21

STRESS DISTRIBUTION IN STIFFENER "B"

WITH .050 SHEET

POUNDS PER  
SQUARE INCH

9

12,000

10,000

8,000

6,000

4,000

2,000

M = 120,000 "#

M = 100,000 "#

M = 80,000 "#

M = 60,000 "#

M = 40,000 "#

DISTANCE ALONG SPAN

10

20

30

40

50

60

70

80

90

INCHES

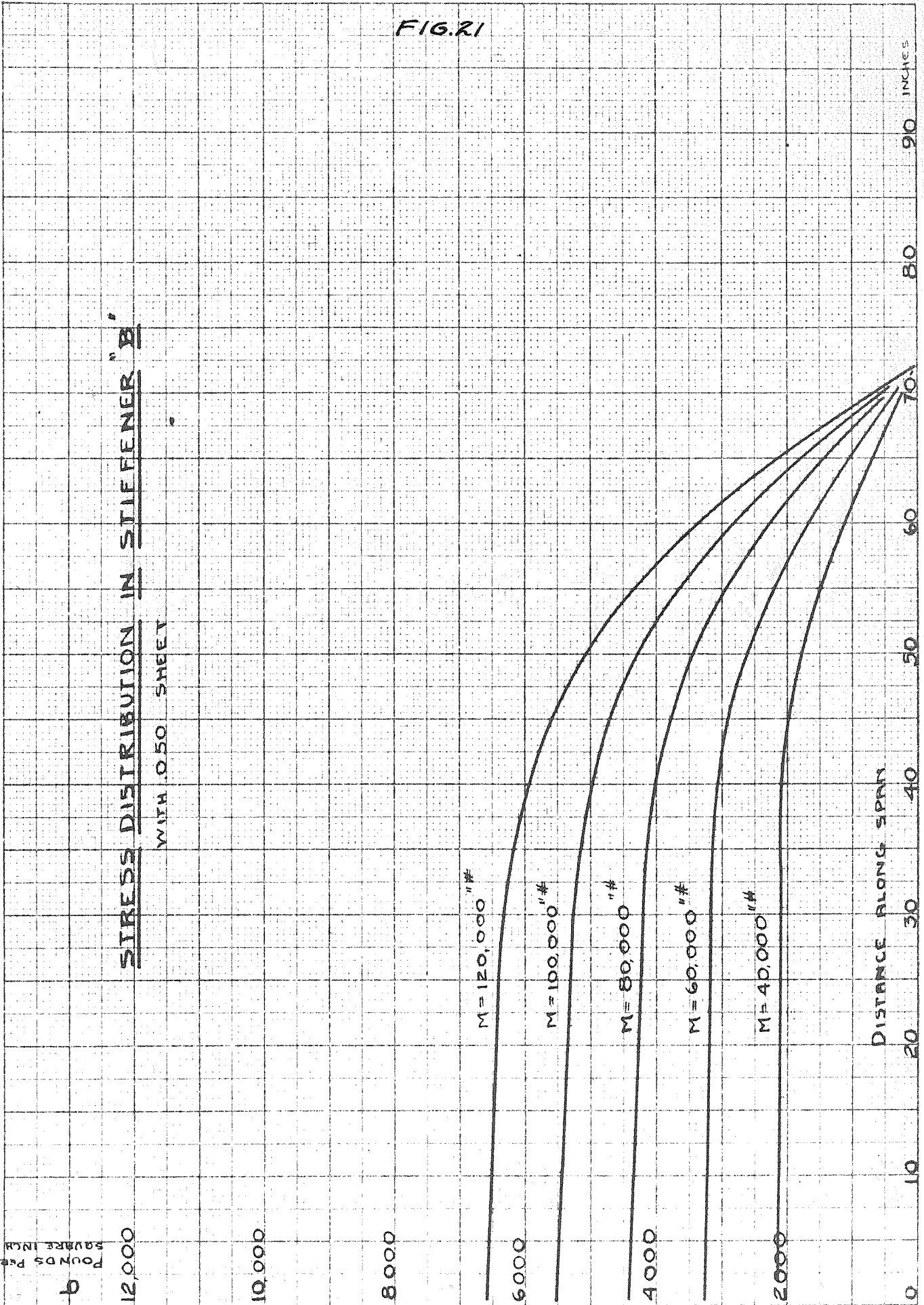


FIG. 22

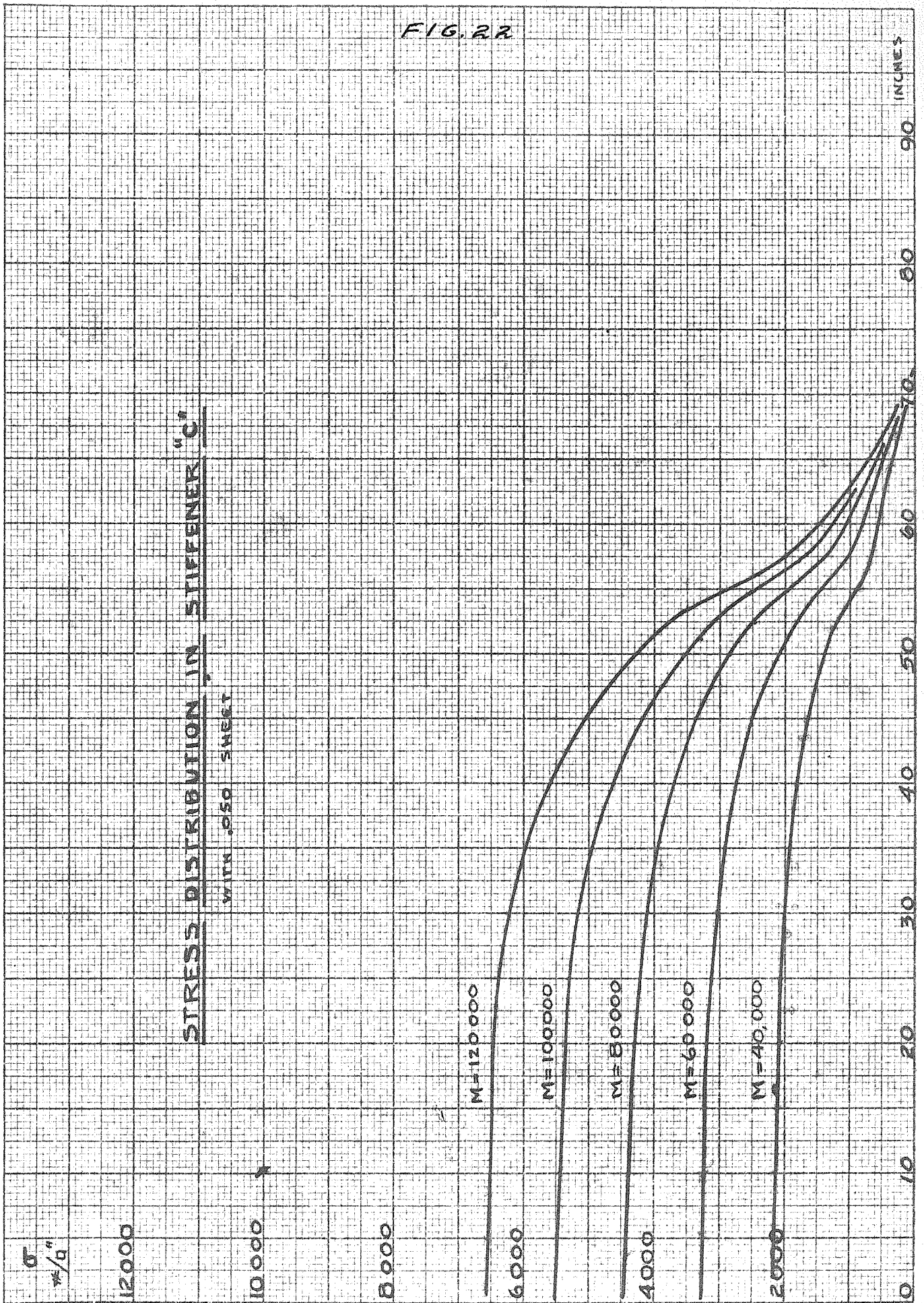
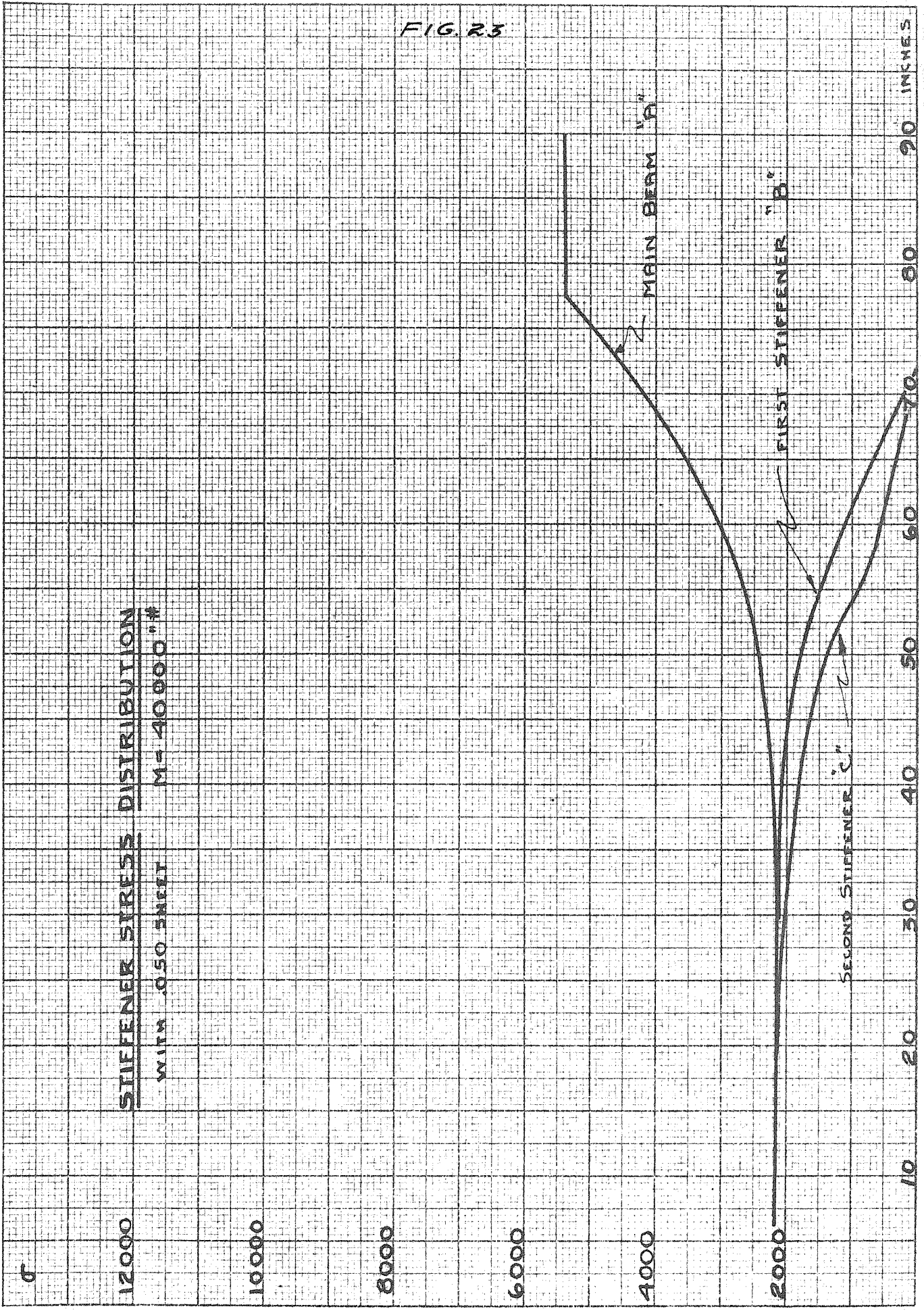


FIG. 23



STIFFENER STRESS DISTRIBUTION  
WITH .050 SHEET M-40000 "#

$\sigma$

STRESS IN #/sq in

DISTANCE ALONG SPAN

INCHES

FIG. 24

STIFFENER STRESS DISTRIBUTION  
WITH .050 SHEET      M = 60,000 "K"

σ

12000  
10000  
8000  
6000  
4000  
2000

STRESS  
# / IN<sup>2</sup>

MAIN BEAM "A"

FIRST STIFFENER "B"

SECOND STIFFENER "C"

INCHES

10    20    30    40    50    60    70    80    90

DISTANCE ALONG SPAN

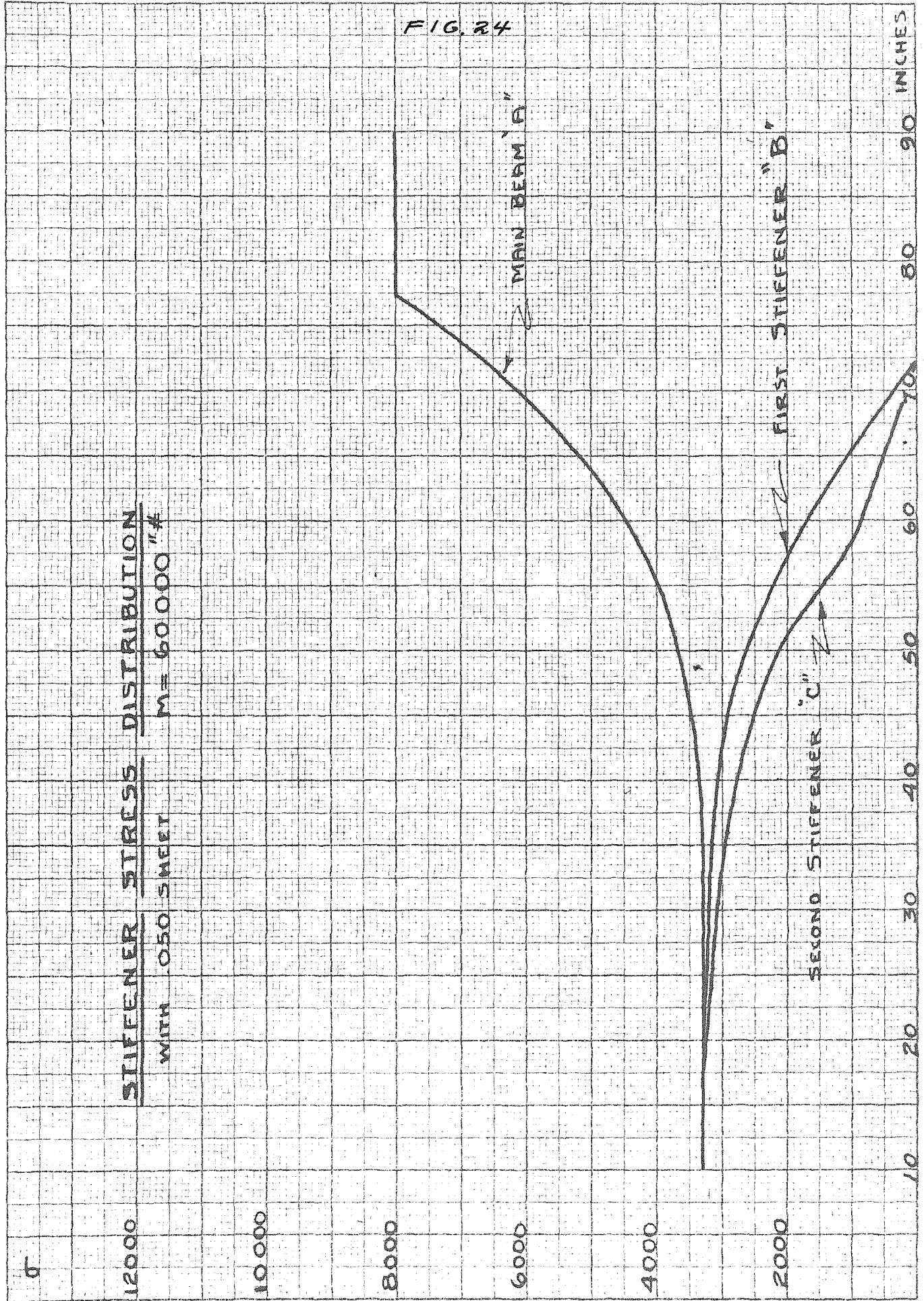
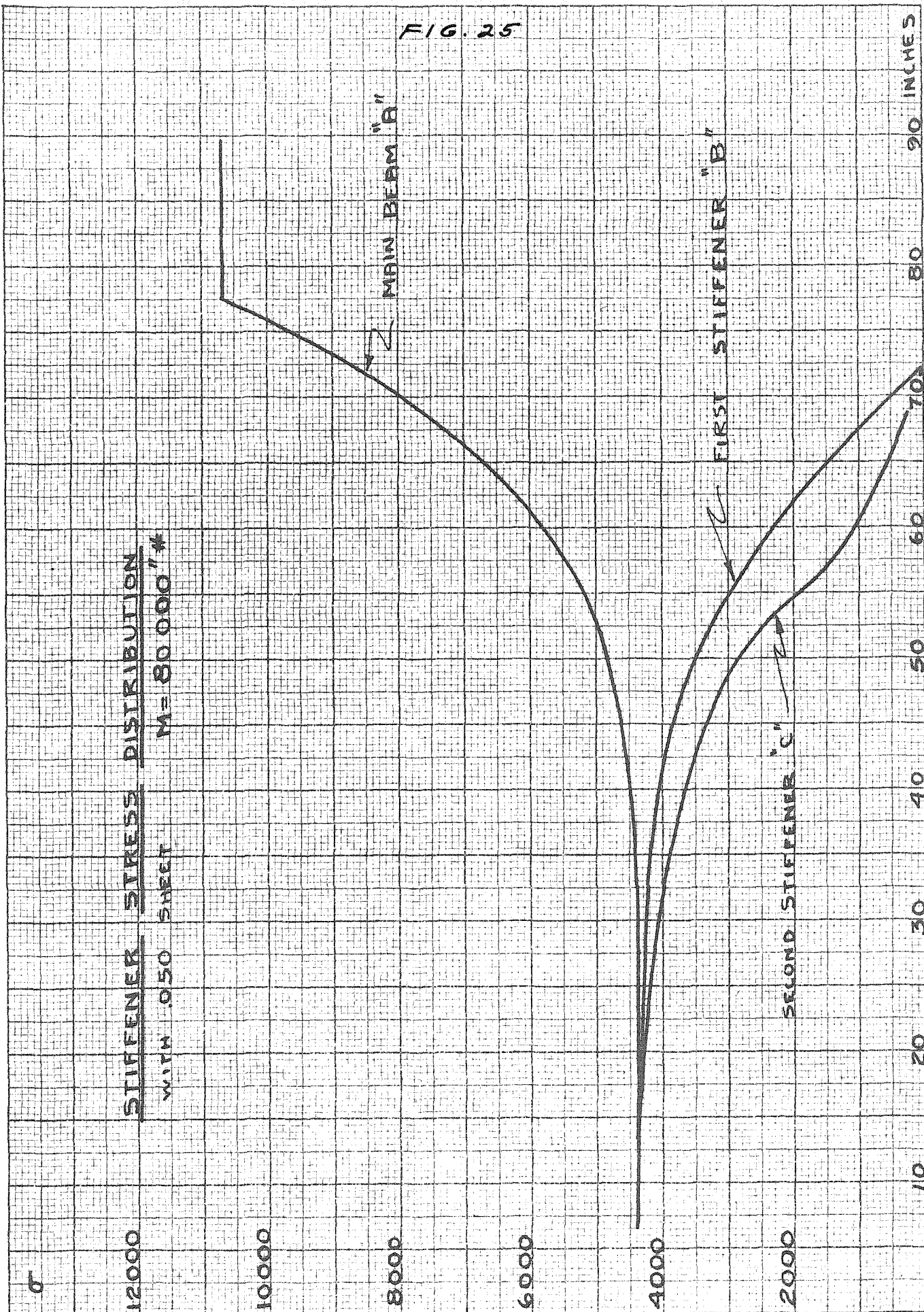


FIG. 25



DISTANCE ALONG SPAN

FIG. 26

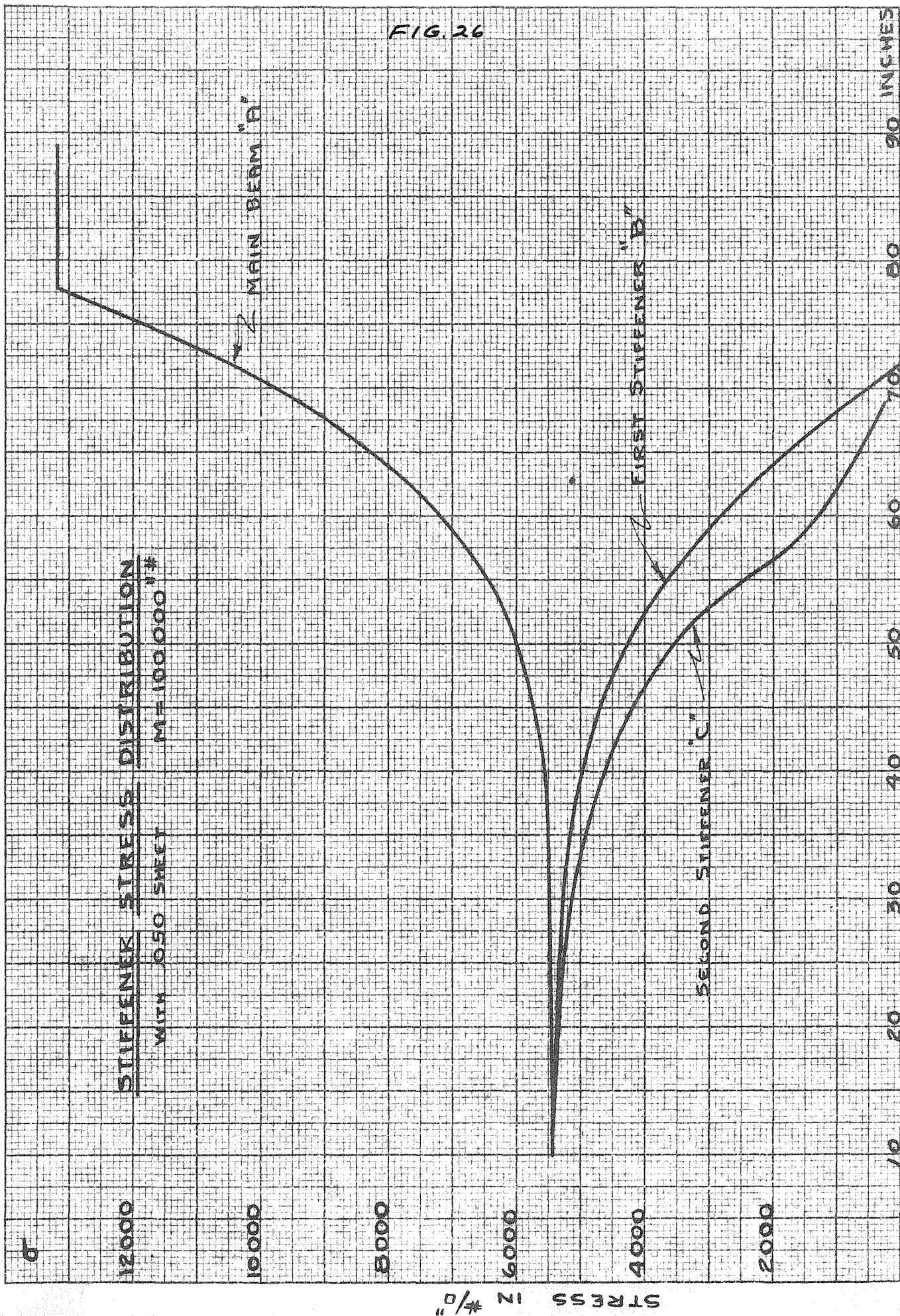




FIG. 27

