CALCULATION OF THE N*(1238) ELECTROMAGNETIC MASS DIFFERENCES BY S-MATRIX METHODS

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Frank Stephen Henyey

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ABSTRACT

We calculate the electromagnetic mass differences within the N*(1238) isomultiplet, using the S-matrix theory of perturbations developed by Dashen and Frautschi. An ambiguity in the choice of methods for calculating the effect of one photon exchange, and lack of sufficient data on the rho meson lead to rather large uncertainties in the predicted mass differences. Our results are consistent with experimental determinations of the differences, which also have rather large uncertainties.

TABLE OF CONTENTS

I. INTRODUCTION

In this paper we attempt to calculate the electromagnetic mass differences within the $N^*(1238)$ isomultiplet, using the S-matrix method of Dashen and Frautschi.[1,2] This method has recently been applied to other problems involving mass and coupling differences.[3-7] In this calculation the N^* is considered to be a resonance in the pion-nucleon scattering amplitude, with nucleon exchange providing the major force. We include all low-energy forces such as nucleon exchange and ρ exchange, as well as "driving term" forces involving a photon.

The first experimental values of the N^* mass differences have recently been measured.[8,9] When the uncertainties of such measurements become smaller, a comparison with our calculation will provide a test of the Dashen-Frautschi method. In particular, this comparison will test the "driving terms", which we include, but which have not been needed in most previous applications of this method.

There already exist predictions for the N^* mass differences given by octet dominance theories.[10-12] However, these predictions (both the equal spacing rule and the magnitude of the spacings) are not reliable since the N and N^* violate SU(3) badly. In particular, the N^* is largely decoupled from strange particle channels which would be equal in strength to the non-strange channels if SU(3) symmetry were exact.[5] In the present calculation, strange particle channels are neglected, and thus the strong SU(3) breaking is roughly taken into account.[13]

This calculation gives no reliable quantitative results because of two major difficulties. Of these, the more serious is the infrared divergence of one photon exchange, which provides one of the larger contributions to the N^* mass differences. In an exact calculation, the infrared divergence of the photon exchange is presumably cancelled by infrared divergent parts of more distant singularities of the S-matrix, so that the mass differences are finite. This property, however, is not automatically ensured in approximations to the S-matrix theory. To make practical estimates of electromagnetic effects in the S-matrix approach, one must attempt, therefore, to subtract infrared divergences in such a way that the remainder in the more distant singularities has a reasonable size. Dashen and Frautschi suggest two methods to do this.[1] Unfortunately, at least one of these methods does not leave small remainders in the distant singularities, since the remainder of photon exchange has different signs for the two methods.[14]

The second major difficulty is the lack of sufficient data on the ρ meson couplings and mass differences, leading to a large uncertainty in that combination of N^* mass differences which transforms like isospin 2.

The organization of this paper can be summarized as follows. The basic equations for the calculation are described in Section II. In Section III we discuss the effect of photon exchange and the handling of the infrared divergence. Sections IV and V contain calculations of the effect of nucleon and pion mass shifts respectively. In

Section VI, we calculate the effects of ρ exchange in terms of ρ couplings and ρ mass shifts and coupling shifts. In Section VII we study the effect of πNN coupling shifts. In Section VIII we calculate the effect of the γN intermediate states in the s and u channels. Section IX contains a discussion of other possible low-energy effects and a summary of our numerical results.

II. BASIC EQUATIONS FOR THE CALCULATION

We assume that if there were no electromagnetic interactions, charge independence would hold exactly. Both nucleons would have a mass M, all pions a mass μ, and all N^*'s a mass M^*. In this unperturbed model we define the amplitude for scattering of pions by nucleons in the channel ($J = 3/2^+$, $T = 3/2$) containing the N^*:

$$A(W) = \rho(W)\, e^{i\eta} \sin\eta \quad . \tag{1}$$

We choose

$$\rho(W) = \frac{W^2}{M^2} \frac{1}{(W-M)^2-\mu^2} \frac{1}{q(W)} , \tag{2}$$

where W is the total energy in C.M., and $q(W) = q$ is the C.M. momentum given by

$$q^2 = \frac{[(W-M)^2 - \mu^2]\,[(W+M)^2 - \mu^2]}{4W^2} \quad . \tag{3}$$

This choice of $\rho(W)$ is made for ease of comparison with Dashen's results for the neutron-proton mass difference.[3] Note that the choice for $\rho(W)$ for which the amplitude has no kinematical singularities should be[15]

$$\rho_1(W) = \frac{4M^2}{q^3\,[(W+M)^2-\mu^2]} \quad . \tag{4}$$

The ratio of this amplitude to ours is

$$\frac{\rho_1(W)}{\rho(W)} = \frac{16\,M^4}{[(W+M)^2-\mu^2]^2} \quad . \tag{5}$$

Equation (2) leads to incorrect D-wave threshold properties (at $W = -M \pm \mu$). However, we are not interested in D waves. The behavior of A(W) at W = 0 is[16] $W^{-2\alpha(0)+1}$, where $\alpha(s)$ is the leading Regge trajectory with the quantum numbers of the N^*. We neglect this singularity because it is more distant than many other singularities which we are neglecting.

The choice of the unperturbed D function has an important effect on the calculated mass differences.[17] We cannot simply use the Omnès formula

$$D = \exp\left[-\frac{1}{\pi} \int_{M+\mu}^{\infty} \frac{\eta(W')\,dW'}{W' - W}\right] \tag{6}$$

for three reasons. First, we do not have good data for the phase shift η at energies above ~1500 MeV. Second, above inelastic threshold the phase shift has various possible definitions. Third, even if we could calculate a D function with the Omnès formula, it might not be the best to use. Any function with $D(M^*) = 0$ and $D'(M^*) \neq 0$ would give the same answer for the mass shift in an exact calculation. In our approximate calculation we want to have the total of uncalculated contributions to be as small as possible, so that the calculated contributions give the best possible estimate of the mass shifts. There are two types of uncalculated contributions. First are the uncalculated left-hand singularities and the singularities on the right due to inelasticity. Second is the cut on the right due to the deviation of the phase of D(W) from the phase of $A^{-1}(W)$. Making $\frac{D^2}{W-M^*}$ smaller in the regions on the left where singularities are neglected makes the effect of these singularities smaller but tends to make the phases of $D(W)$ and $A^{-1}(W)$ differ more on the right. Conversely, making the difference in the phases of $D(W)$ and $A^{-1}(W)$ smaller makes the effect of the corresponding cut smaller, but tends to make $\frac{D^2}{W-M^*}$ larger on the left. Therefore, it is necessary to compromise on the extent to which these two conditions are met. In the compromise, there is no point in making the phase of D on the right good any farther from M^* than the distance to the neglected singularities on the left.[18]

In practice, D must also be easy to use. We choose the Balázs form for D,

$$D = \frac{W_o - M^*}{W_o - W} (W - M^*) \quad . \tag{7}$$

This form will only be used on the left where, with the proper choice of W_o, it is supposed to approximate a D function satisfying the compromise. The smaller $W_o - M^*$ is, the smaller $\frac{D^2}{W-M^*}$ is on the left. However, if W_o is too small, the low-energy contributions to δM^* will be sensitive to changes in the value of W_o. Therefore, if W_o is too small, there must be important uncalculated higher energy effects, since an exact calculation would give δM^* independent of W_o. The value of W_o is fixed by choosing W_o as small as possible, subject to the condition that δM^* be relatively insensitive to changes of W_o. This leads to the choice[7] $W_o = 7/3\ M^*$ which is used for numerical results reported.

The Dashen-Frautschi method is based on the use of a dispersion relation for the quantity $D^2\ \delta A$ where δA is the change in amplitude due to electromagnetism in our case. Analyzing the singularity of δA at $W = M^*$, they find that

$$\delta M^* = \frac{1}{2\pi i R^*} \oint_L \frac{\delta A(W')D^2(W')}{W' - M^*} dW' + \frac{1}{\pi R^*} \int_R \frac{\mathrm{Im}\,[D^2(W')\,\delta A(W')]}{W' - M^*} dW' , \tag{8}$$

where R^* is the residue of the N^* pole. In the ensuing discussion we will often use the following simplified evaluation of Eq. (8) based on the Balázs form for D. For any term δA_i in $\delta A = \sum_i \delta A_i$ not having a pole at $W = M^*$ we can deform the contour in Eq. (8) to circle the point $W = W_o$. Then we find

$$\delta M_i^* = \frac{1}{R^*} (W_o - M^*)^2 \frac{d}{dW_o} (W_o - M^*)\, \delta A_i\,(W_o) \ . \tag{9}$$

This inversion can be used only when we want to keep the integration over the entire cut of δA_i. For some approximations to δA_i, the integrals in Eq. (8) will diverge unless truncated and Eq. (9) cannot be used.

The singularities we include result from photon exchange; shifts in the masses of the scattering pion and nucleon; N exchange mass and coupling shifts; ρ exchange mass and coupling shifts; and $N\gamma$ intermediate states in the s and u channels. Low-energy effects not included are strange particles whose couplings to the πN channel are small because of the breaking of SU(3);[5] N^* exchange, which has a crossing factor of 1/9; $N^*\gamma$ intermediate states, which we later show are expected to have smaller contributions than the numerically small contributions of $N\gamma$ states; and shifts of any non-resonant strong scattering amplitude.

It is convenient to separate the mass splittings into parts which transform differently under isospin. The driving terms of each type only contribute to mass splittings of the same type, and the bootstrap effects only connect mass and coupling splittings of the same type. We resolve the four N^* masses into an average mass M^* and three N^* splittings δM_1, δM_2, δM_3 which transform as isospin 1, 2, and 3 respectively. δM_1, δM_2, and δM_3 are normalized to give the following relations:

$$\begin{aligned} M^{++} &= M^* + 3/2\ \delta M_1 - 1/2\ \delta M_2 - 1/2\ \delta M_3 , \\ M^{+} &= M^* + 1/2\ \delta M_1 + 1/2\ \delta M_2 + 3/2\ \delta M_3 , \\ M^{0} &= M^* - 1/2\ \delta M_1 + 1/2\ \delta M_2 - 3/2\ \delta M_3 , \\ M^{-} &= M^* - 3/2\ \delta M_1 - 1/2\ \delta M_2 + 1/2\ \delta M_3 , \end{aligned} \tag{10}$$

where M^{++}, M^+, M^0, M^- are the masses of the various members of the multiplet. Inverting this set of equations, we get

$$\begin{aligned} M^* &= \frac{M^{++} + M^{+} + M^{0} + M^{-}}{4} , \\ \delta M_1 &= \frac{3(M^{++} - M^{-}) + (M^{+} - M^{0})}{10} , \\ \delta M_2 &= \left(\frac{M^{+} + M^{0}}{2}\right) - \left(\frac{M^{++} + M^{-}}{2}\right) , \\ \delta M_3 &= \frac{3(M^{+} - M^{0}) - (M^{++} - M^{-})}{10} . \end{aligned} \tag{11}$$

The electromagnetic current acts twice in the lowest order of the fine structure constant, and is comprised of parts which transform as isospin 0 and 1 only. Therefore, isospin 3 shifts cannot occur

($\delta M_3 = 0$), and we have the additional relation

$$\delta M_1 = (M^{+} - M^{0}) = \frac{M^{++} - M^{-}}{3} \quad . \tag{12}$$

III. PHOTON EXCHANGE

The amplitude for photon exchange is

$$\delta T(s,t) = 4\pi\alpha[F_\pi(t)\ (q_{1\mu}+q_{2\mu})]\ \frac{1}{t-\lambda^2}[iF_1(t)\gamma_\mu - \frac{iF_2(t)}{2M}\sigma_{\mu\nu}k_\nu]\ ; \tag{13}$$

where we give the photon a (fictitious) mass λ; and where F_π, F_1, F_2 are the form factors of the pion and nucleon;[19] k_μ is the momentum transfer; $t = -k_\mu k_\mu$; and $q_{1\mu}$, $q_{2\mu}$ are the pion initial and final momenta respectively. Since Eq. (13) for δT includes the form factors, it really includes contributions from t-channel processes in addition to photon exchange, as for example the part of ρ exchange which comes from $\pi + \pi \rightarrow \rho_o \rightarrow \gamma \rightarrow N + \overline{N}$ and $\pi + \pi \rightarrow \gamma \rightarrow \rho_o \rightarrow N + \overline{N}$.

The pion must couple to an isovector photon. Therefore, only isoscalar nucleon form factors appear in the $\Delta I = 1$ amplitude and only isovector nucleon form factors appear in the $\Delta I = 2$ amplitude. In addition, there are factors of 1/3 and -2/3 in projecting out the $\Delta I = 1$ and $\Delta I = 2$ isospin amplitudes respectively (using the normalization given by Eqs. (10) and (11)).

Projecting out the $J = 3/2^{+}$ amplitude (Eq. (1)) and the ΔI amplitudes, we find[20]

$$\delta A = \frac{\alpha}{12M^2} \begin{pmatrix} 1 \text{ for } \Delta I = 1 \\ -2 \text{ for } \Delta I = 2 \end{pmatrix} \int_{-1}^{1} dx \, \frac{F_\pi(t)}{t-\lambda^2} ,$$

$$\Bigg\{ F_1(t) \left[\frac{(W+M)^2-\mu^2}{(W-M)^2-\mu^2} (W-M)\, x + \frac{3x^2-1}{2} (W+M) \right] + \tag{14}$$

$$+ \frac{F_2(t)}{2M} \left[\frac{3x^2-2x-1}{2} \right] \left[(W+M)^2-\mu^2 \right] \Bigg\} ,$$

where $x = 1 + \frac{t}{2q^2}$ is the cosine of the C.M. scattering angle. The appropriate nucleon form factors are to be used as explained above. We use the approximations[21]

$$F_\pi(t) = \frac{m_\rho^2}{m_\rho^2 - t} \tag{15}$$

and

$$F_i(t) = F_i(0) \frac{m_s^2}{m_s^2 - t} \quad ; \qquad i = 1, 2 \tag{16}$$

with

$$m_\rho^2 = 30\,\mu^2 ,$$

$$\tag{17}$$

$$m_s^2 = 20\,\mu^2 .$$

Then

$$\delta A = \frac{-\alpha}{24 M^2 q^2} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \Bigg\{ F_1^{s,v}(0) \left[\ell n \frac{1+4q^2/\lambda^2}{1+4q^2/m_s^2} - \frac{m_s^2}{m_\rho^2 - m_s^2} \ell n \frac{1+4q^2/m_s^2}{1+4q^2/m_\rho^2} \right]$$

$$\times \left[\frac{2W(W^2-M^2-\mu^2)}{(W-M)^2 - \mu^2} \right] + F_1^{s,v}(0) \left[\frac{(W+M)^2-\mu^2}{(W-M)^2-\mu^2} \; (W-M) \right.$$

$$\left(- \frac{m_s^2 \, m_\rho^2}{2q^2(m_\rho^2 - m_s^2)} \; \ell n \; \frac{1 + 4q^2/m_s^2}{1 + 4q^2/m_\rho^2} \right) + \frac{3}{2} \, (W+M) \left(- \frac{m_s^2 \, m_\rho^2}{2q^2(m_\rho^2 - m_s^2)} \right)$$

$$\left. \left(\left(2 + \frac{m_s^2}{2q^2} \right) \ell n \left(1 + \frac{4q^2}{m_s^2} \right) - \left(2 + \frac{m_\rho^2}{2q^2} \right) \ell n \left(1 + \frac{4q^2}{m_\rho^2} \right) \right) \right]$$

$$+ \frac{F_2^{s,v}(0)}{2M} \frac{m_\rho^2 \, m_s^2}{2q^2(m_\rho^2 - m_s^2)} \left[-[(W+M)^2 - \mu^2] \left[\left(2 + \frac{3}{2} \frac{m_s^2}{2q^2} \right) \ell n \left(1 + \frac{4q^2}{m_s^2} \right) \right. \right.$$

$$\left. - \left(2 + \frac{3}{2} \; \frac{m_\rho^2}{2q^2} \right) \ell n \left(1 + \frac{4q^2}{m_\rho^2} \right) \right]$$

$$\left. \left. + \frac{[(W+M)^2 - \mu^2]^2}{4W^2} \left[\left(1 + \frac{m_s^2}{2q^2} \right) \ell n \left(1 + \frac{4q^2}{m_s^2} \right) - \left(1 + \frac{m_\rho^2}{2q^2} \right) \ell n \left(1 + \frac{4q^2}{m_\rho^2} \right) \right] \right] \Bigg\} \quad . \tag{18}$$

In this equation, we put the photon mass λ equal to zero everywhere except in the infrared divergent logarithm. The factor 1 and the s (isoscalar) superscript on the nucleon form factors refer to the case $\Delta I = 1$. Similarly, the factor -2 and the v (isovector) superscript refer to the case $\Delta I = 2$.

The term proportional to F_2 is not infrared divergent.[22] In addition, F_2^s is very small, and the term involving it is therefore neglected. The term proportional to F_2^v can be evaluated using Eq. (9) and leads to a value $\delta M_2 = -0.9$ MeV. We neglect the second term involving F_1 which is also not infrared divergent, and is numerically very small. The remaining term is

$$\delta A_{\mathrm{infra}} = -\frac{\alpha W\,(W^2-M^2-\mu^2)}{12\,M^2 q^2[(W-M)^2-\mu^2]}\left[\ln\frac{1+4q^2/\lambda^2}{1+4q^2/m_s^2} - \frac{m_s^2}{m_\rho^2-m_s^2}\ln\frac{1+4q^2/m_s^2}{1+4q^2/m_\rho^2}\right] . \tag{19}$$

This expression has branch points at $q^2 = \lambda^2/4$, $q^2 = m_s^2/4$, and $q^2 = m_\rho^2/4$. Because we put $\lambda = 0$ whenever possible, it also has poles at $W = M \pm \mu$. The singularities of $\delta A_{\mathrm{infra}}$ in the W plane are shown in Fig. 1.

Now we work out and compare the two methods which Dashen and Frautschi gave for removing the infrared divergence. The first method for evaluating the mass shift is to give the photon mass the value

$$\lambda = \frac{2q(M^*)}{e} \tag{20}$$

inside the logarithm.[1,3] We then use Eq. (9) to evaluate δM^*. We find $\delta M_1 = -0.46$ MeV ($\Delta I = 1$) and $\delta M_2 = 0.92$ MeV ($\Delta I = 2$).

The second method for evaluating the mass shift is to define the amplitude[1]

$$\delta\hat{A} = \delta A - \rho(W)\, e^{2i\eta}\, \delta\eta_B \qquad , \tag{21}$$

where $\delta\eta_B$ is the change in phase shift from the Born approximation terms alone. For the Born terms, $\delta A_B = \rho(W)\, \delta\eta_B$. Therefore, defining $\delta\hat{A}_B \equiv \delta A_B - \rho(W)\, e^{2i\eta}\, \delta\eta_B$ (so that for non-Born terms, $\delta\hat{A} = \delta A$), we find

$$\delta\hat{A}_B = -2i\, e^{i\eta} \sin\eta\, \delta A_B = -2i \frac{A(W)}{\rho(W)}\, \delta A_B \quad . \tag{22}$$

In particular,

$$\delta\hat{A}_{infra} = -2i \frac{A(W)}{\rho(W)}\, \delta A_{infra} \quad . \tag{23}$$

We use a two pole approximation for A(W):

$$A(W) = -\frac{\frac{4}{9}R}{W-M} + \frac{R^*}{W-M^*} \tag{24}$$

where $R = -3f^2/\mu^2 \approx -0.24/\mu^2$ is the residue of the nucleon pole in the $J = 1/2^+$, $T = 1/2$ amplitude. The factor $-(4/9)R$ comes from crossing. We have left out the crossed N^* pole, which has a crossing factor of 1/9. The reciprocal bootstrap gives $(4/9)R \approx R^*$ in agreement with experiment. The singularities of $\frac{D^2}{W-M^*}\, \delta\hat{A}_{infra}$ are shown in Fig. 2. The poles at $W = M \pm \mu$ in Fig. 1 and the $\ell n\lambda$ contribution of the cut in δA_{infra} are removed by the factor $1/\rho(W)$. There are no poles and no right cut introduced by $A(W)/\rho(W)$. The nucleon exchange pole is removed by the zero in δA_{infra} at $\sqrt{M^2-\mu^2} \approx M$. The pole at M^* and the right cut are removed by $D^2/W-M^*$. There is, however, a new cut for $W < M - \mu$. We find the contribution of δA_{infra} to δM^* to be[23]

$$\delta M^{*} = \frac{\alpha}{3}\int_{\mathcal{D}L} \frac{dW}{\sqrt{\mu^{2}-(W-M)^{2}}}\left(\frac{D}{W-M^{*}}\right)^{2}\left[\left(1-\frac{\frac{4}{9}R}{R^{*}}\right)(W-M)+\frac{\frac{4}{9}R}{R^{*}}(M^{*}-M)\right]$$

$$\frac{1}{2i}\left[\ell n\ \frac{1+4q^{2}/\lambda^{2}}{1+4q^{2}/m_{s}^{2}} - \frac{m_{s}^{2}}{m_{\rho}^{2}-m_{s}^{2}}\ \ell n\ \frac{1+4q^{2}/m_{s}^{2}}{1+4q^{2}/m_{\rho}^{2}}\right] \quad . \qquad (25)$$

The integral around the cut in $\delta\hat{A}_{infra}$ corresponding to the cut in δA_{infra} can be explicitly evaluated.[24] It involves the imaginary part of the logarithms, which are finite and constant in the limit $\lambda \to 0$. Numerically, its contribution to δM_1 is 0.29 MeV and to δM_2 is -0.58 MeV. The remaining cut, due to the cut in $\sqrt{\mu^2-(W-M)^2}$ for $W < M - \mu$, involves the real part of the logarithms, and is therefore infrared divergent. The magnitude of the contribution from this cut can be estimated using the first method by giving the photon a mass $\frac{2q(M^*)}{e}$. Of course, this does not necessarily give the correct sign, since the two methods disagree in sign. In this way we find this contribution to be about ± 0.03 MeV (in δM_1) and therefore negligible. The values of the one photon exchange contribution to δM_1 and δM_2 are shown in Table 1.

The proper choice between the two methods is not clear. Of the two, the $\delta\hat{A}$ method should be more convergent (i.e., the sum of neglected distant singularities should be small).[1] This method also agrees with the non-relativistic argument that a system of two like charges (the N^{*++}) should have a higher energy, and a system of unlike charges (part of the N^{*0}) should have a lower energy, than a system involving a neutral particle (the N^{*+} and N^{*-}). The other, the photon mass method, gives a contribution whose sign stays the same (opposite the non-relativistic sign) for any photon mass less than $\sim 3\ \mu$. This method was used by Dashen to get the result agreeing with experiment for the n-p mass difference.[3]

Since the proper choice of methods for the N^* is unclear, it is useful to review the situation for the one photon exchange contribution to the neutron-proton mass difference. This situation is essentially the same as for the N^*. The F_2 term is not infrared divergent, and is numerically small. The sign of the mass shift is different for the two different methods. The $\delta\hat{A}$ method agrees in sign with the non-relativistic argument, but requires larger contributions of opposite sign from other singularities in order to agree with the experimental value. The sign of the photon mass method agrees with the experimental sign for a large range of values of the photon mass.

IV. EFFECTS OF NUCLEON MASS SHIFTS

The n-p mass difference contributes to the $\Delta I = 1$ mass shifts of the N^* in two independent ways. One is that the position of the N exchange pole gets shifted, while the other is that the kinematics is changed by the external nucleon mass shift. These contributions have already been calculated by Dashen and Frautschi.[4,25] However, we wish to introduce a minor refinement. The calculation is conveniently carried out by considering $\Delta I = 0$ mass shifts first, and then using group theory to find that $\Delta I = 1$ shifts. For $\Delta I = 0$, the exchanged nucleon mass shift gives a contribution $A_{\Delta I=0}^{N^*N\,\mathrm{exch}}\,\delta M$ to δM^* where

$$A_{\Delta I=0}^{N^*N\,\mathrm{exch}} = -\frac{4}{9}\,\frac{R}{R^*}\,\frac{M^*}{M}\,\frac{d}{dW}\,\frac{D^2(W)}{W-M^*}\Bigg|_{W=M} \quad . \tag{26}$$

Using the Balázs form Eq. (7) for D, we find $A_{\Delta I=0}^{N^*N\,\mathrm{exch}} = -0.65$. The effect of the external N mass shift is found by mass scale invariance. M^* must be a homogeneous function of degree 1 of all masses. Therefore,

$$M^* = (A_{\Delta I=0}^{N^*N\,\mathrm{exch}} + A_{\Delta I=0}^{N^*N\,\mathrm{ext}})\,M + A_{\Delta I=0}^{N^*N^*}\,M^* + A_{\Delta I=0}^{N^*\pi}\mu + A_{\Delta I=0}^{N^*\rho}\,m_\rho + \ldots \tag{27}$$

We neglect N^* exchange because of the small crossing factor. We anticipate the results to be given in Sections V and VI that $A^{N^*\pi}$ and $A^{N^*\rho}$ are small enough to be neglected. Therefore, $A_{\Delta I=0}^{N^*N\,\mathrm{ext}} = \frac{M^*}{M} - A_{\Delta I=0}^{N^*N\,\mathrm{exch}} = 1.98$. The group theoretic factors to convert these results to $\Delta I = 1$ are given in Ref. 4. The results are

$$\delta M_1 = \left(\frac{1}{3} A_{\Delta I=0}^{N^*N_{ext}} - \frac{1}{3} A_{\Delta I=0}^{N^*N_{exch}} \right) (m_p - m_n) = 0.88\ (m_p - m_n) \quad . \tag{28}$$

Using the experimental value $m_p - m_n = -1.3$ MeV, we find that $\delta M_1 = -1.07$ MeV.

V. EXTERNAL PION MASS SHIFT

The external pion mass shift affects the N^* mass shift in two ways. First, it modifies the kinematics of crossing in that both the positions and values of the left-hand singularities are shifted. Second, a shift in μ introduces a cut on the right since the kinematic factor $\rho(W)$ depends explicitly on μ.

As with the nucleon, we first consider the $\Delta I = 0$ mass shift. We make the transition to $\Delta I = 2$ after we add the contributions from the left and right. Since the pion mass difference transforms as $\Delta I = 2$, there is no effect on the $\Delta I = 1$ mass shift of the N^*.

We treat the left-hand singularities first. If $|W - M| \gg \mu$, the value of μ has little effect on the kinematics of crossing. Therefore, only singularities near $W = M$ have an effect. Again, because of the small crossing factor, we neglect N^* exchange, so that we are left with only N exchange. N exchange leads to a short cut which we replace by a pole. However, in so doing we must keep the dependence of the position and residue of this pole on the pion mass (in lowest order). The cut in the vicinity of $W = M$ for the nucleon exchange amplitude is given by[15]

$$\mathrm{IM}\ A_N = \frac{\pi R}{6q^2}\left[-\frac{(W+M)^2-\mu^2}{(W-M)^2-\mu^2}\,(W-M)\,a + (W+M)\,\frac{3a^2-1}{2}\right], \quad (29)$$

for

$$M - \frac{\mu^2}{M} < W < \sqrt{M^2 + 2\mu^2} \quad . \quad (30)$$

In Eq. (29),

$$a = \frac{W^2 - M^2 - 2\mu^2}{2q^2} - 1 \qquad . \tag{31}$$

We replace this cut by a pole $R'/W-M'$. Im A_N is sufficiently well behaved so that placing the equivalent pole at the center of the cut leads to an error of order μ^4/M^3 in the position of the pole. Therefore, correct to order μ^2/M, the pole is at $M' = M$. The residue R' of the equivalent pole is given by

$$R' = -\frac{1}{\pi} \int_{M-\frac{\mu^2}{M}}^{\sqrt{M^2+2\mu^2}} \mathrm{Im}\, A_N \, dW \qquad . \tag{32}$$

We expand Im A_N in a power series in μ and $\epsilon = W-M$. We find

$$\mathrm{Im}\, A_N = \frac{\pi R M}{6\mu^2} \left[1 + \frac{M^2\epsilon^2}{\mu^4} + \frac{7}{2}\frac{\epsilon^2}{\mu^2} - \frac{9}{2}\frac{M\epsilon^3}{\mu^4} + \frac{\mu^2}{4M^2} + \frac{6M^2\epsilon^4}{\mu^6} + \ldots\right] \tag{33}$$

so that

$$R' = -\frac{4}{9} R \left[1 + \frac{127}{80}\frac{\mu^2}{M^2} + \ldots .\right] \qquad . \tag{34}$$

We now differentiate $R'/W-M'$ with respect to μ and find

$$\delta A = -\frac{\frac{4}{9}R}{W-M} \frac{127}{40} \frac{\mu\, \delta\mu}{M^2} \qquad . \tag{35}$$

From Eq. (9), we find

$$\delta M^* = -\frac{\frac{4}{9}R}{R^*} \left(\frac{W_o - M^*}{W_o - M}\right)^2 \frac{(M^* - M)\mu}{M^2} \frac{127}{40} \delta\mu = -0.11\ \delta\mu \tag{36}$$

where we have put $(4/9)\, R = R^*$, $W_o = (7/3)\, M^*$.

Next we consider the right cut introduced by the pion mass shift. Since we have replaced the entire right cut of D by a single pole, we cannot expect Eq. (8) to give a reliable result. For a kinematic shift, the integral over the right can be transformed into[26]

$$\delta M^* = \frac{1}{R^*} \frac{1}{\pi} \int_{M+\mu}^{\infty} \frac{N^2(W)\ \delta\left(\frac{1}{\rho(W)}\right)}{W - M^*}\ dW \tag{37}$$

where

$$N(W) = A(W)\, D(W) \quad . \tag{38}$$

Although N(W) does not have a right cut, by using Eq. (7) for D and Eq. (24) for A in Eq. (38), we obtain an approximation for it having a singularity on the right. Therefore, we use the integral equation for N,

$$N(W) = \frac{1}{\pi} \int_L \frac{D(W')\ \mathrm{Im}\, A(W')}{W' - W}\ dW' \tag{39}$$

with these approximations for A and D, finding

$$N(W) = - \frac{\frac{4}{9} R\ D(M)}{W - M} \quad . \tag{40}$$

By differentiating Eq. (2), we find

$$\delta\left(\frac{1}{\rho(W)}\right) \approx - 3\mu\ \delta\mu\ q\, \frac{M^2}{W^2} \quad . \tag{41}$$

Substituting Eqs. (40) and (41) into Eq. (37), and numerically integrating, we find $\delta M^* = 0.12\ \delta\mu$.

There is an almost complete cancellation between the right cut and left cut. The sum is $\delta M^* = 0.01\ \delta\mu$. The $\Delta I = 2$ ratio $\delta M_2/\mu^+ - \mu^0$ is just a group theoretic factor of order one times $\delta M^*/\delta\mu \doteq 0.01$. Therefore, the effect of pion mass shifts on N^*

mass shifts is negligible. The cancellation that occurs is probably not fortuitous. Another calculation of $\delta M^{*}/\delta\mu$ with a different approximation for D and A, gives the same cancellation. These approximations include the nucleon exchange pole and artificial short range poles in A, and a subtraction in D. Elastic unitarity was satisfied at low energies by solution of the N/D equations. The residues of the poles and the subtraction constant were chosen to give the N^{*} its physical energy and width, and to give the correct very low-energy behavior for the 3-3 phase shift. This approximation could not be used throughout the calculation since the resulting D was very large at high energies.

VI. RHO EXCHANGE

We use a narrow resonance approximation for the rho in the t-channel. Then the amplitude for rho exchange in the N^* channel of πN scattering is given by Frautschi and Walecka[15] in terms of two coupling constants, γ_1 and γ_2, which are related to electric-like and magnetic-like couplings. In order to obtain $\Delta I = 0$ mass shifts, we differentiate this amplitude and use Eq. (9) to evaluate the mass shift. We find

$$\delta M^* = (0.068\ \gamma_1 - 0.007\ \gamma_2)\ \delta m_\rho - 0.65\ \mu\ \delta\gamma_1 + 0.10\ \mu\ \delta\gamma_2\ . \tag{42}$$

In order to obtain the contribution from δm_ρ to the $\Delta I = 2$ N^* mass difference (there is no $\Delta I = 1$ contribution since the ρ mass difference is pure $\Delta I = 2$, and also the group theoretic factor for $\Delta I = 1$ is zero) we use the bubble diagram shown in Fig. 3.[27] The vertices A, A' have Clebsch-Gordan coefficients $C_{\frac{1}{2},1}(\frac{3}{2}\ i,\ j\,k)$ and $C_{\frac{1}{2},1}(\frac{3}{2}\ i,\ n\,m)$ respectively. The vertices B, B' have Clebsch-Gordan coefficients $C_{\frac{1}{2},\frac{1}{2}}(1\ \ell,\ j\ {-n})$ and $C_{1,1}(1\ \ell,\ {-k}\ m)$ respectively. In crossing to the t-channel, we cross the nucleon N_n and the pion π_k, which introduces a factor $(-)^{n+\frac{1}{2}}\ (-)^k$. Therefore,

$$\delta M_i^* = \sum_{j,k,\ell,m,n} \text{const}\ (-)^{k+n+\frac{1}{2}}\ C_{\frac{1}{2},1}(\tfrac{3}{2}\ i,\ j\ k)\ C_{\frac{1}{2},1}(\tfrac{3}{2}\ i,\ n\ m)$$

$$C_{\frac{1}{2},\frac{1}{2}}(1\ \ell,\ j\ {-n})\ \ C_{1,1}(1\ \ell,\ {-k}\ m)\ \ \delta m_{\rho^\ell}\ . \tag{43}$$

The evaluation of Eq. (43) gives

$$\delta M^{++} = \delta M^{-} = \left(\frac{\delta M^{*}}{\delta m_{\rho}}\right)_{\Delta I=0} \delta m_{\rho^0}$$

$$\delta M^{+} = \delta M^{0} = \left(\frac{\delta M^{*}}{\delta m_{\rho}}\right)_{\Delta I=0} \left[-\frac{1}{3}\, \delta m_{\rho^0} + \frac{2}{3}\, (\delta m_{\rho^+} + \delta m_{\rho^-}) \right] , \tag{44}$$

so that

$$\delta M_2 = (\delta M^{+} - \delta M^{++}) = -\frac{4}{3}\, (\delta m_{\rho^0} - \delta m_{\rho^\pm}) \left(\frac{\delta M^{*}}{\delta m_{\rho}}\right)_{\Delta I=0} . \tag{45}$$

The physical values must not be used for δm_{ρ}, $\delta\gamma_1$, and $\delta\gamma_2$ in Eqs. (42) and (45), since part of their effect is included through the use of form factors in photon exchange. This part consists of the contributions to the ρ^0 mass and coupling shifts coming from the one photon intermediate state. In particular, the one photon state may cause the dominant part of the coupling shifts.

In order to calculate the effect of ρ exchange, we need estimates of γ_1, γ_2, δm_ρ, $\delta\gamma_1$, and $\delta\gamma_2$. We turn first to estimating γ_1 and γ_2. These are the residues at the ρ pole of the $\pi + \pi \rightarrow N + \overline{N}$ amplitudes Γ_1, $M\Gamma_2$ defined by Frazer and Fulco.[28] These are conveniently found in terms of the residues of the ρ poles in the nucleon isovector form factors and the pion form factor. The imaginary part of the nucleon form factors for small positive t are given (in our normalization) by[29]

$$\mathrm{Im}\, F_1^{V} = -K(t)\; F_\pi^{*}\, \Gamma_1 ,$$

$$\mathrm{Im}\, F_2^{V} = -K(t)\; F_\pi^{*}\, M\Gamma_2 , \tag{46}$$

where

$$K(t) = \frac{(t - 4\mu^2)^{3/2}}{4t^{\frac{1}{2}}} \quad . \tag{47}$$

Near the ρ pole,

$$\begin{aligned}
F_1{}^v &= \frac{f_1 m_\rho^2}{m_\rho^2 - t - i m_\rho \Gamma} \quad , \\
F_2{}^v &= \frac{f_2 m_\rho^2}{m_\rho^2 - t - i m_\rho \Gamma} \quad , \\
F_\pi &= \frac{f_\pi m_\rho^2}{m_\rho^2 - t - i m_\rho \Gamma} \quad , \\
\Gamma_1 &= \frac{\gamma_1/\pi}{m_\rho^2 - t - i m_\rho \Gamma} \quad , \\
M\Gamma_2 &= \frac{\gamma_2/\pi}{m_\rho^2 - t - i m_\rho \Gamma} \quad .
\end{aligned} \tag{48}$$

Picking out the coefficient of the ρ pole in Eq. (46), we find

$$\begin{aligned}
\gamma_1 &= -\pi \frac{f_1}{f_\pi} \frac{m_\rho \Gamma}{K(m_\rho^2)} = -2.5 \frac{f_1}{f_\pi} \quad , \\
\gamma_2 &= -\pi \frac{f_2}{f_\pi} \frac{m_\rho \Gamma}{K(m_\rho^2)} = -2.5 \frac{f_2}{f_\pi} \quad .
\end{aligned} \tag{49}$$

If we assume that $F_1{}^v$, $F_2{}^v$, F_π are all proportional between the ρ pole and $t = 0$, we find that $f_1 : f_2 : f_\pi = F_1(0) : F_2(0) : F_\pi(0) = 1 : 3.70 : 1$, and that therefore $\gamma_1 = -2.5$, $\gamma_2 = -9.2$. From Eqs. (42) and (45), we find the ρ mass shift contribution to δM_2 to be

$$\delta M_2 = 0.14\ (\delta m_{\rho^0} - \delta m_{\rho^\pm}) \quad . \tag{50}$$

This result, however, is rather sensitive to the assumed extrapolation, which is over a distance of $30\,\mu^2$ in t.

There exist both theoretical and experimental estimates of $\delta m_{\rho^0} - \delta m_{\rho^\pm}$. Beder[29] has estimated $\delta m_{\rho^0} - \delta m_{\rho^\pm} \approx 13$ MeV from an S-matrix calculation. Moreover, he found the $\rho^0 \rightarrow \gamma \rightarrow \rho^0$ contribution, which we must not include, to be small. Equation (50) then gives $\delta M_2 = 1.8$ MeV.

Several experiments[30] have reported a $\rho^0 - \rho^\pm$ mass difference. Although the measured values of the average ρ mass differ among them, there is general agreement on the value of the mass difference. The averaged value for $\delta m_{\rho^0} - \delta m_{\rho^\pm}$ is -2 MeV[31] (compared with Beder's +13). The existence of an isoscalar $\pi\pi$ resonance with a mass slightly lower than the ρ ($m_{scalar} \approx 720$ MeV) could easily lower the observed ρ^0 mass significantly. If the discrepancy between the theoretical and experimental values persists, the status of the $\rho^0 \rightarrow \gamma \rightarrow \rho^0$ contribution becomes unclear. Conceivably, a small mass difference could be due to a large cancellation between $\rho^0 \rightarrow \gamma \rightarrow \rho^0$ and all other terms. Therefore, the experimental value does not provide a reliable estimate of the contribution to the N^* mass shifts.

Although the ρ mass shifts might have an important effect on N^* mass shifts, the numerical value of $\delta M^*/\delta m_\rho$ for $\Delta I = 0$ (0.10 for proportional form factors) gives a relatively small contribution in the calculation done in Section IV of the effect of the external nucleon mass shift.

The status of the ρ coupling shifts is somewhat better.

Coupling shifts generally turn out to be less than about 1% of the coupling. Except for the part of the coupling shift we include through use of form factors in photon exchange, we expect this to be true for the ρ. Then the ρ coupling shifts give a contribution to δM^* of less than about 1 MeV.

VII. πNN COUPLING SHIFTS

The nucleon exchange pole of A is $-(4/9)\,\frac{R}{W-M}$. Therefore, a shift in the coupling introduces a change in the amplitude:

$$\delta A = -\frac{4}{9}\,\frac{\delta R}{W-M} \qquad . \tag{51}$$

Equation (8) then gives the N^* mass shift for $\Delta I = 0$:

$$\delta M^* = -\frac{4}{9}\,(M^*-M)\,\frac{R}{R^*}\left(\frac{D(M)}{M^*-M}\right)^2 \frac{\delta R}{R} \qquad . \tag{52}$$

With the same approximations as before, Eq. (52) becomes $\delta M^* = 1.57\,\mu\,\frac{\delta R}{R}$. Since R is proportional to g^2/M^2, where g is the pseudoscalar πNN coupling constant, and since the shift in $M(\sim 0.15\%)$ can be neglected, it follows that $\delta M^* = 430\ \mathrm{MeV}\,\frac{\delta g}{g}$.

The group theoretic factors are found from the bubble diagram in Fig. 4 wherein all vertices have the usual Clebsch-Gordan factors.[31] Crossing the pions introduces a factor $(-)^{m+k}$. In addition, vertex B has a factor $\delta g/g$. (Also, adding a perturbation of the coupling at vertex B' does not change the ratios of the induced $\Delta I = 0$, $\Delta I = 1$, and $\Delta I = 2$ mass shifts.) The result is

$$\begin{aligned}
\delta M^{++} &= \left(\frac{\delta g_{\pi^-\overline{p}n}}{g}\right)\frac{\delta M^*}{\delta g/g} \qquad ,\\
\delta M^{+} &= \left(\frac{1}{3}\,\frac{\delta g_{\pi^0\overline{p}p}}{g} + \frac{1}{3}\,\frac{\delta g_{\pi^0\overline{n}n}}{g} + \frac{1}{3}\,\frac{\delta g_{\pi^-\overline{p}n}}{g}\right)\frac{\delta M^*}{\delta g/g} \ ,\\
\delta M^{0} &= \left(\frac{1}{3}\,\frac{\delta g_{\pi^0\overline{p}p}}{g} + \frac{1}{3}\,\frac{\delta g_{\pi^0\overline{n}n}}{g} + \frac{1}{3}\,\frac{\delta g_{\pi^+\overline{n}p}}{g}\right)\frac{\delta M^*}{\delta g/g} \ ,\\
\delta M^{-} &= \left(\frac{\delta g_{\pi^+\overline{n}p}}{g}\right)\frac{\delta M^*}{\delta g/g} \qquad .
\end{aligned} \tag{53}$$

By CP invariance, $g_{\pi^+ n\bar{p}}$ and $g_{\pi^- p\bar{n}}$ are equal.[7] Therefore, there are no $\Delta I = 1$ mass shifts introduced by the coupling shifts. The $\Delta I = 2$ shift is

$$\delta M_2 = \left(\frac{1}{3} \frac{\delta g_{\pi^0 p\bar{p}}}{g} + \frac{1}{3} \frac{\delta g_{\pi^0 n\bar{n}}}{g} - \frac{2}{3} \frac{\delta g_{\pi^- p\bar{n}}}{g} \right) \frac{\delta M^*}{\delta g/g}$$

$$= \frac{1}{3} \frac{\delta g_2^2}{g^2} \frac{\delta M^*}{\delta g/g} = \frac{1}{3} \frac{\delta g_2^2}{g^2} \; 430 \text{ MeV} \qquad . \qquad (54)$$

This defines δg_2^2. The relevant coupling shift has been estimated in two ways. In the first place, from the NN scattering lengths Riazuddin[32] finds $\delta g_2^2/g^2 \approx 1.5\%$, after correcting the scattering lengths for all other known effects. In the second place, Dashen, Dothan, Frautschi, and Sharp[7] estimated the octet ($\Delta I = 1$) coupling shifts in the S-matrix theory. They predicted that the $\Delta I = 1$ shifts are substantially larger than the $\Delta I = 2$ shifts because the former undergo double enhancement. By this they mean that enhanced mass shifts occur in a driving term for enhanced coupling shifts. In the case of the $\Delta I = 2$ shifts, neither the mass shifts which occur in the driving terms for the coupling shifts, nor the coupling shifts themselves, are enhanced. Except for the cases in which the unperturbed coupling is very small (e.g., $\bar{N}N\eta$), no doubly enhanced coupling shift is larger than 2%.

The estimate of the coupling shift from the scattering lengths gives a mass shift $\delta M_2 \approx 2$ MeV. The estimate of Dashen, Dothan, Frautschi, and Sharp gives a substantially smaller shift.

VIII. Nγ INTERMEDIATE STATES

In this section we work out the contribution of the Nγ intermediate states in the s and u channels in terms of photoproduction amplitudes. We then evaluate these contributions using the photoproduction amplitudes given by Chew, Goldberger, Low, and Nambu[33] (CGLN).

We first find the contribution from the s-channel Nγ state. This state only occurs in the N^{*+} and N^{*0} channels, and therefore leads to $\Delta I = 0$ and $\Delta I = 2$ shifts. The cut in δA is given by

$$\text{Im } \delta A = \frac{4}{3} q(W)\ \rho(W)\ k(W)\ \left(|M_{1+}|^2 + 3\ |E_{1+}|^2 \right) \quad . \tag{55}$$

In this equation, q(W) and $\rho(W)$ are defined in Eqs. (3) and (2), k(W) is the C.M. momentum in the Nγ state given by

$$k(W) = \frac{W^2 - M^2}{2W} \quad , \tag{56}$$

and M_{1+} and E_{1+} are photoproduction multipoles (for isospin 3/2) defined, for example, in CGLN. The cut given by Eq. (55) goes from $W = M$ to $W = \infty$. We use the multipoles as given by CGLN. These are

$$M_{1+} = e^{i\eta} qkef \left[\frac{(g_p - g_n) \sin \eta}{4f^2 M^2 q^3} + \frac{1}{3} \cos \eta\ F_M \right] \quad , \tag{57}$$

$$E_{1+} = e^{i\eta} qkef \left[-\frac{1}{3} \cos \eta\ F_Q \right] \quad , \tag{58}$$

where

$$F_Q = \frac{1}{w^2}\left[1 - \frac{3}{4V^2}\left(1 + \frac{1-V^2}{2V}\,\ell n\,\frac{1-V}{1+V}\right)\right] \quad , \tag{59}$$

$$F_M = \frac{3}{4q^2}\left[1 + \frac{1-V^2}{2V}\,\ell n\,\frac{1-V}{1+V}\right] \quad , \tag{60}$$

$$w = \sqrt{q^2 + \mu^2} \quad , \tag{61}$$

$$V = q/w \quad . \tag{62}$$

For the 3-3 phase shift, we use the approximation

$$\tan \eta = \frac{0.3\, q^3/\mu^2}{(M^* - W)\,(1 + 0.8\, q^2/\mu^2)} \quad . \tag{63}$$

These approximations lead to an integrand in Eq. (8) which has a very small, but non-zero, limit as $W \to \infty$. Therefore, we truncate the integral and do not use Eq. (9). The value of the integral is insensitive to the truncation energy. The major contribution comes from low energies, so the choice of D function is of no importance for this term. We do the integration numerically and find $\delta M \approx 0.3$ MeV.

We now find the contribution from the u-channel $N\gamma$ state. Since isotopic spin is not conserved, we cannot use the ordinary crossing matrix. Instead, we use the bubble diagram in Fig. 5 to find the effect of crossing. Vertices A and A' have Clebsch-Gordan coefficients and a factor $(-)^{m+k}$ comes from crossing the pions. Vertices B and B' have photoproduction amplitudes for $\gamma + N_\ell \to \pi_{-m} + N_j$ and $\gamma + N_\ell \to \pi_{-k} + N_n$ respectively. The complex conjugate of either one of these amplitudes is used (consistently). Finally the ordinary angular momentum crossing matrix and kinematical factors appear. The results are as follows: For $\Delta I = 1$,

$$\mathrm{Im}\ \delta A = \frac{16}{9}\ \frac{W^2}{M^2}\ \frac{(3M-W)(M-W)}{(W-M)^2 - \mu^2}\ \mathrm{Re}\Big[(M_{1+}^{0})^* M_{1+}^{-} + 3(E_{1+}^{0})^* E_{1+}^{-} + 2(M_{1-}^{0})^* M_{1-}^{-}\Big] \; ; \qquad (64)$$

and for $\Delta I = 2$,

$$\mathrm{Im}\ \delta A = \frac{4}{9}\frac{W^2}{M^2}\ \frac{(3M-W)(M-W)}{(W-M)^2 - \mu^2}\ \mathrm{Re}\Big[|M_{1+}^{+}|^2 + 3\,|E_{1+}^{+}|^2 + 2\ |M_{1-}^{+}|^2 - |M_{1+}^{-}|^2 - 9\ |E_{1+}^{-}|^2 - 6\ |M_{1-}^{-}|^2 + 2(M_{1+}^{+})^* M_{1+}^{-} + 6\ (E_{1+}^{+})^* E_{1+}^{-} + 4\ (M_{1-}^{+})^* M_{1-}^{-}\Big] \quad . \qquad (65)$$

We use static crossing, in which the multipoles are evaluated at an energy of 2M - W. As before, we do the integration of Eq. (8) numerically and find δM_1 very small and $\Delta M_2 \approx -0.2$ MeV. Therefore, the total effect of $N\gamma$ states is small.

We now consider $\gamma\pi N$ intermediate states in the s and u channels. We expect these to be dominated by the $N^*\gamma$ state. The amplitude is large only when the energy is close to M^* where we expect it to be comparable to the $N\gamma$ amplitude at an energy near M. In the s-channel, the zero of $D^2/W-M^*$ will make the integrand of Eq. (8) small. The u-channel $N^*\gamma$ contribution will be smaller than the u-channel $N\gamma$ contribution (neglecting isospin factors) because of the W^2/M^2 factor in $\rho(W)$ (Eq. (2)). At the position of the crossed

N^* pole, this factor is about 1/2. We expect, therefore, that each term contributed by $\gamma\pi N$ states to be smaller than the corresponding γN term, which is itself rather small.

IX. OTHER LOW-ENERGY STATES AND SUMMARY

There remain other low-energy states whose effects we do not calculate. The first of these is the $\pi\gamma$ state in the t-channel. If $\pi + \pi \rightarrow \pi + \gamma$ is dominated by $\pi + \pi \rightarrow$ vector meson $\rightarrow \pi + \gamma$ and $N + \overline{N} \rightarrow \pi + \gamma$ is dominated by $N + \overline{N} \rightarrow$ vector meson $\rightarrow \pi + \gamma$, then $\pi\gamma$ exchange is similar to γ exchange, and the ratio of $\delta A_{\pi\gamma}$ to δA_{γ} is expected to be of order[3] $\frac{1}{2} \frac{\mu^4}{24} \frac{1}{m_\rho^4}$ so that the $\pi\gamma$ cut will have a negligible effect.

Closely related is the vector meson-γ state. We do not calculate its effect, since all hadron-γ states previously calculated have had small effects, and this singularity is more distant than the $\pi\gamma$ singularity. Similarly, we neglect all multiparticle-γ states.

The last remaining effect is that caused by coupling shifts in ω and φ exchange. We expect these to be similar to ρ exchange. Except for the part included in photon exchange through use of form factors, their effect should be small.

In this calculation, we do not arrive at a single set of predictions. Therefore, in Table 2 we summarize our results by giving the minimum and maximum estimates for each term. Two of the entries in the table require explanation. For photon exchange, the <u>minimum</u> δM_1 corresponds to the <u>maximum</u> δM_2 (photon mass method), while the <u>maximum</u> δM_1 corresponds to the <u>minimum</u> δM_2 ($\delta\hat{A}$ method). The δM_2 entries for the ρ mass shift were found by taking the estimate 1.8 MeV given by one particular set of assumptions and arbitrarily assigning a 100% uncertainty to this estimate.

Our two estimates for δM_1 are not extremely different. From their theory of octet dominance, Dashen and Frautschi[4] give an upper bound of 8.4 MeV for the mass difference $M^- - M^{++} = -3\ \delta M_1$. This is larger than, and has the same sign as our estimates of 4.8 MeV and 2.4 MeV for this difference. Either of our estimates is consistent with the experimental results of Gidal, Kernan, and Kim[8] who found $M^- - M^{++} = 7.9 \pm 6.8$ MeV.

Our result for δM_2 is extremely uncertain. The only conclusion we can make is that there is no evidence that δM_2 is small, as is predicted by octet dominance theories. As δM_2 is so uncertain, our range of values includes the experimental value given by Olsson,[9] who found $M^{++} - M^0 = 2\ \delta M_1 - \delta M_2 = -0.45 \pm 0.85$ MeV.

TABLE 1

One photon exchange contributions to δM.

		F_1 part	F_2 part	Total
photon mass method	δM_1	-0.46 MeV	~ 0	-0.5 MeV
	δM_2	0.92 MeV	-0.9 MeV	0 MeV
$\delta\hat{A}$ method	δM_1	0.29 MeV	~ 0	0.3 MeV
	δM_2	-0.58 MeV	-0.9 MeV	-1.5 MeV

TABLE 2

Summary of numerical estimates of N^* mass shifts (in MeV). See text for explanation of photon exchange and ρ mass shift entries.

Effect	δM_1 minimum	δM_1 maximum	δM_2 minimum	δM_2 maximum
photon exchange	-0.5	+0.3	-1.5	0
nucleon mass shift	-1.1	-1.1	0	0
πNN coupling shift	0	0	small	2.0
π mass shift	0	0	small	small
ρ mass shift	0	0	0	3.6
ρ coupling shift	small	small	small	small
γN : s-channel	0	0	0.3	0.3
γN : u-channel	small	small	-0.2	-0.2
TOTAL	-1.6	-0.8	-1.4	5.7

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14. It has already been noticed that this holds for the n-p mass difference. See, e.g., G. Barton, Phys. Rev. 146, 1149 (1966).

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16. The possibility of poles at W = 0 in the amplitude $e^{i\eta}\sin\eta/q$, discussed in Ref. 15, is ruled out by D. Freedman and J. Wang, Berkeley preprint (1966).

17. G. Shaw and D. Wong, Phys. Rev. 147, 1028 (1966).

18. For a more complete discussion of the choice of D, see Appendix B of Ref. 7.

19. The normalization of the nucleon form factors are:

$$F_1^s(0) = F_1^p(0) + F_1^n(0) = 1 \quad , \quad F_2^s(0) = F_2^p(0) + F_2^n(0) = 0.12 \ ,$$

$$F_1^v(0) = F_1^p(0) - F_1^n(0) = 1 \quad , \quad F_2^v(0) = F_2^p(0) - F_2^n(0) = 3.70 \ .$$

20. The necessary equations, and the definition of the amplitude T of Eq. (13) are given in Ref. 15.

21. This choice is the same as that made by Dashen (Ref. 3) for F_π and F_1.

22. F. Gilman, private communication.

23. We have replaced $M^2 + \mu^2$ by M^2 and $(W+M)^2 - \mu^2$ by $(W+M)^2$.

24. Equation (9) cannot be used since the cut in Eq. (25) has been divided into two parts.

25. Their normalization of mass shifts is different from ours. The correspondence is as follows, their quantity given first:

Nucleon: $\delta M_1 = \sqrt{2}\ \delta M$

$$\delta M_3 = \frac{1}{\sqrt{2}}\,(m_p - m_n)$$

N^{*}: $\delta M_1^{*} = 2\ \delta M^{*}$

$$\delta M_3^{*} = \sqrt{5}\ \delta M_1$$

$$\delta M_5^{*} = -\ \delta M_2$$

$$\delta M_7^{*} = -\ \sqrt{5}\ \ \delta M_3$$

26. R. Dashen and S. Frautschi, Phys. Rev. 137, B1318 (1965).

27. The use of bubble diagrams to find group theoretic factors is explained in Ref. 26.

28. W. Frazer and J. Fulco, Phys. Rev. 117, 1609 (1960).

29. D. Beder, UCRL report 16465 (1965).

30. Summarized and averaged by A. Rosenfeld et al., UCRL report 8030 (rev.) (August 1966).

31. Even the perturbed vertex has a Clebsch-Gordan factor, since we normalize the unperturbed coupling constants so that they are all equal.

32. Riazuddin, Nucl. Phys. 7, 217 (1958).

33. G. Chew, M. Goldberger, F. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

FIGURE CAPTIONS

Figure 1: The singularities of the infrared divergent part of δA from photon exchange. x's are poles, dots are branch points, and the broken lines connecting them are cuts. q^2 is real on the circle.

Figure 2: The singularities of $\frac{D^2}{W-M^*}\ \delta\hat{A}_{infra}$.

Figure 3: Bubble diagram for ρ exchange mass shifts.

Figure 4: Bubble diagram for πNN coupling shifts.

Figure 5: Bubble diagram for $N\gamma$ state in u-channel.

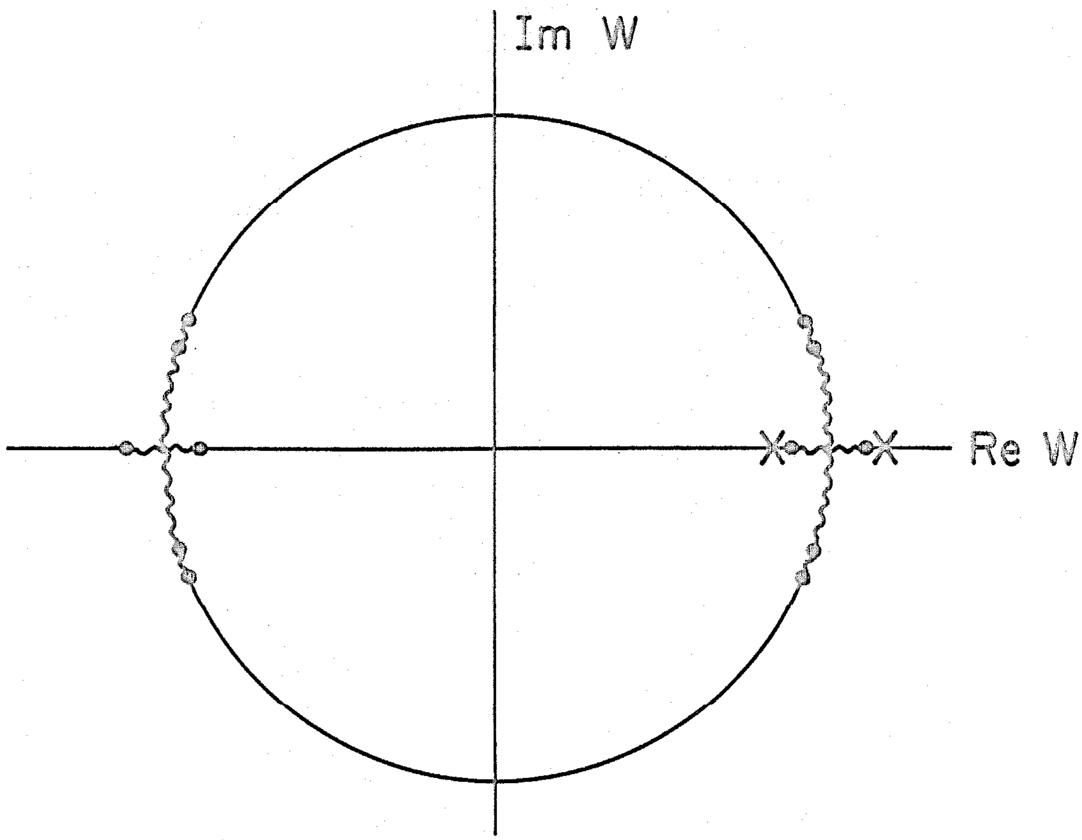

Fig. 1

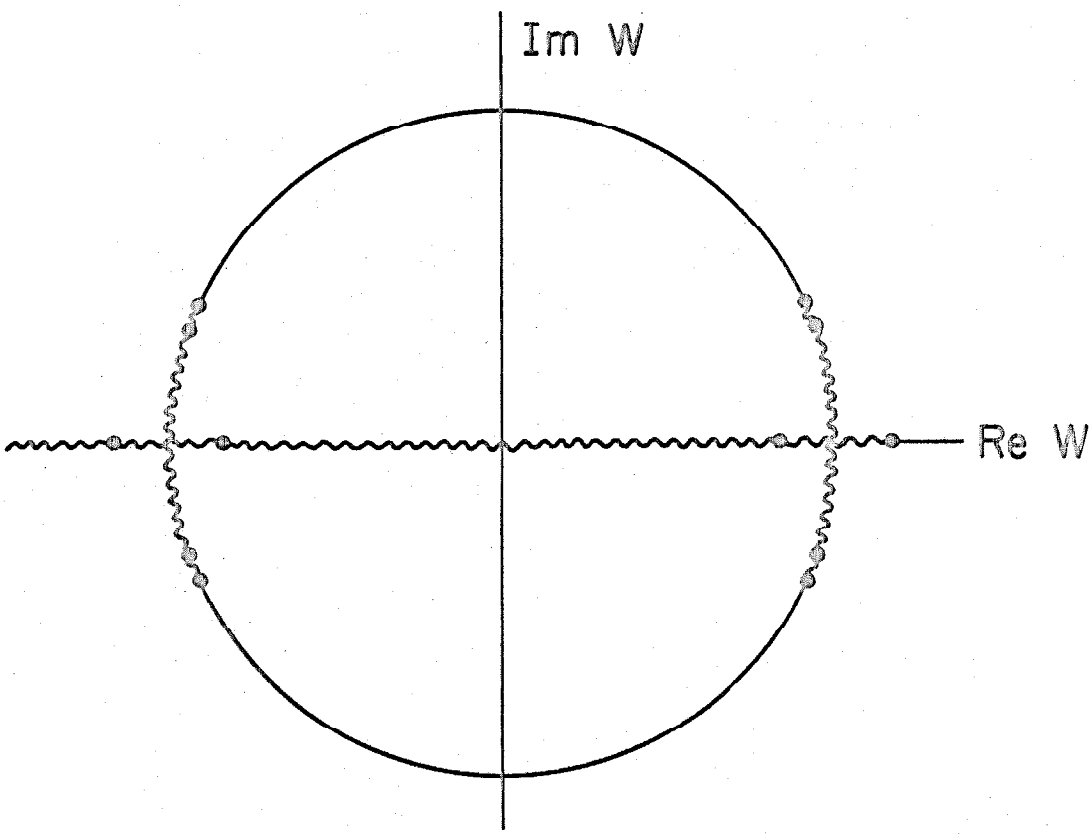

Fig. 2

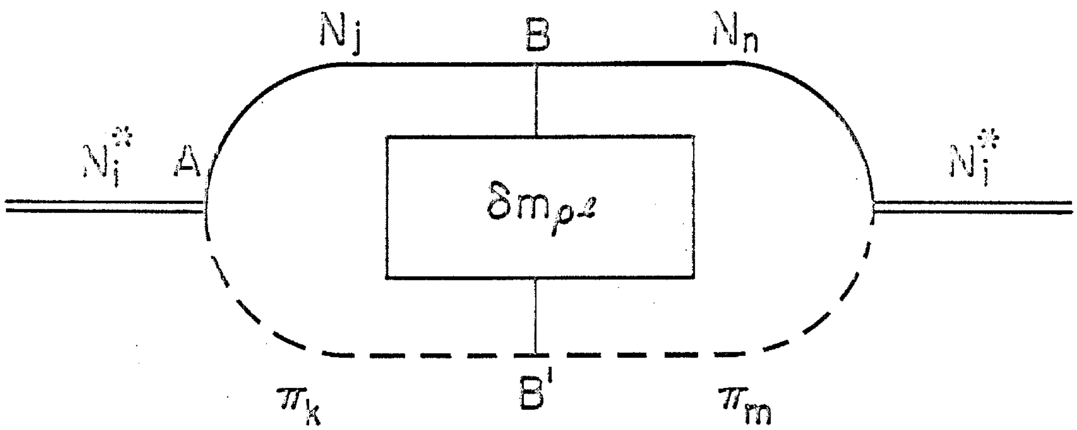
N_j
B
N_n
N_i^*
A
$\delta m_{\rho\ell}$
N_i^*
π_k
B'
π_m

Fig. 3

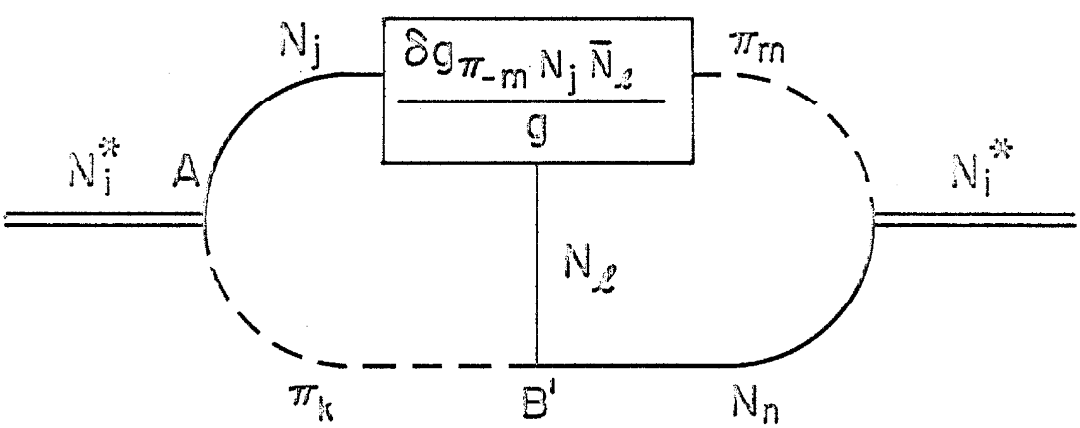
N_j
$\dfrac{\delta g_{\pi_{-m} N_j \bar{N}_\ell}}{g}$
π_m
N_i^*
A
N_i^*
N_ℓ
π_k
B'
N_n

Fig. 4

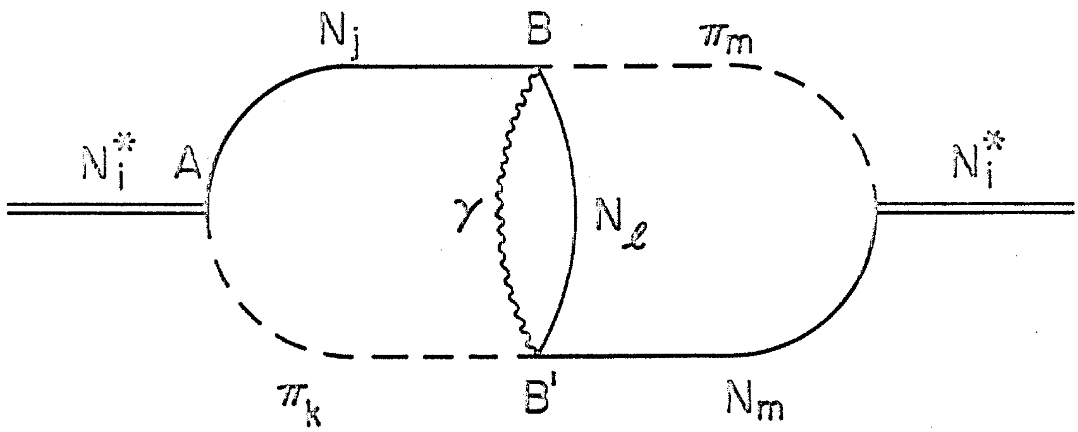

Fig. 5