THE RESPONSE OF NONLINEAR
MULTI-STORY STRUCTURES SUBJECTED
TO EARTHQUAKE EXCITATION

Thesis by

Melbourne Fernald Giberson

In Partial Fulfillment of the Requirements

For the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1967

(Submitted May 15, 1967)

ii

ACKNOWLEDGMENTS

ABSTRACT

The dynamic responses of a 20-story nonlinear structural frame representative of a modern high rise building are analyzed with the aid of a digital computer. Related analytical studies of continuous systems are carried out. Quantitative information is provided on the importance of a wide range of modes to the various responses of a multi-story structure during an earthquake. The effect of yielding on the response is observed. The magnitude of the structural responses are compared with common measurements of earthquake strength.

At the ends of each girder and column of the structural frame are yield hinges which have bilinear bending moment-rotation hysteretic characteristics. Two beam models having such characteristics are studied; one of these models can treat curvilinear hysteretic behavior. Three definitions of ductility factor are discussed, one of which is applicable to both bilinear and curvilinear hysteresis loops. In the computer program, the frame is subjected to the time history of an earthquake accelerogram, the equations of motion are stepwise integrated, and the various structural responses - displacement, bending moments, incurred yielding, etc. - are determined.

The agreement between the response parameters resulting from excitation by seven different earthquake ground motions indicates that these response characteristics are determined more by the properties of the structure than by the earthquake. These results throw some light on extreme value statistics of the response of yielding structures subjected to earthquakes. The characteristic patterns observed in the computed responses of the nonlinear structure can be related to analytical studies of linear elastic, shear-type, uniform and tapered continuous cantilever beams.

iv
TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

Until recently, comparatively little has been known about the response of multi-story structures subjected to earthquakes. Consequently, in regions where earthquakes have occurred relatively frequently, building codes imposed stringent height limits on structures. In the past few years, however, there has been a significant improvement in the understanding of how structures respond to earthquakes, and, as a result, in some cities height restrictions have been relaxed for structures which are designed on the basis of the dynamic forces, deformation, and yielding which would be induced by a strong earthquake.

A common type of high-rise building being constructed today (1967) uses glass and light-weight walls extensively, relying on large columns and girders for support. The investigation described in this report was undertaken in order to provide additional information on how this kind of multi-story structure responds to earthquake excitation.

The initial work on problems encountered in designing earthquake resistant multi-story structures was done during the 1920's and early 1930's by K. Muto[3] and K. Suyehiro[4] of Japan and M. A. Biot[5,6] and J. R. Freeman[7] in the United States. Although the equations of motion for earthquake excited multi-story structures were well known, the time history solutions of the responses of multi-story structures could not be obtained because neither analog

nor digital computers were available to make the extensive calcu-
lations necessary.

Through the years the one degree of freedom system has
received extensive study by several investigators using analytical
techniques and, as they become available, analog and digital com-
puters. Among them are D. E. Hudson[8], G. W. Housner[9],
T. K. Caughey[10], G. V. Berg[11], S. S. Thomaides[11],
J. Penzien[12], W. D. Iwan[13], Paul C. Jennings[14], C. V.
Chelapati[15], and L. D. Lutes[16]. In addition to direct application
to the one degree of freedom system, the results of their investiga-
tions also provide significant insight into the understanding of how
multi-story structures respond to earthquake excitation.

One early study in multi-story structures appeared in 1938.
In that study, L. S. Jacobsen and R. S. Ayre[17] experimentally
determined the dynamic shears for at least the first four modes of a
sixteen-story model of an office building using a decaying sinusoidal
base excitation of finite duration. The results indicated that the
higher modes are important for shear forces in the upper floors.

With the advent of analog and digital computers, the equations
of motion of multi-story structures subjected to earthquake excitation
could be solved. The earlier research work on multi-story structures
using these tools was performed by T. P. Tung[18], Richard L.
Jennings[19,20], N. M. Newmark[18,20,23], R. W. Clough[21],
J. A. Blume[22,23], J. E. Goldberg[24], J. L. Bogdanoff[24],
Z. L. Moh[24], L. Corning[23], J. Penzien[25] and many others.
More recent work of this nature has been carried out by J. I.

Bustamente[26], H. Umemura[27], Y. Osawa[27], A. Shibata[27], and G. V. Berg[28], to mention a few.

A number of researchers have visualized this problem as a wave propagation problem in a continuous cantilever beam. Among them are K. Muto[3,29], H. Suyehiro[4], H. M. Westergaard[30], L. S. Jacobsen[31], K. Kanai[32], G. N. Bycroft[33,34,35], M. J. Murphy[35] and L. W. Harrison[35].

During the five years preceding 1967 a large amount of research on multi-story structures using high capacity-high speed digital computers has been done by R. W. Clough[36,37,38,39,1] and his co-workers E. L. Wilson[37,38,39], I. P. King[37,38], K. L. Benuska[39], and T. Y. Lin & Associates[2].

Many of these authors have noted that several modes can be excited in a multi-story structure and that the "higher" modes must be considered in the interfloor shear force in the upper portion of the structure. In this report, quantitative information is provided on the importance of all modes including the "higher" ones to the various responses of a multi-story structure during an earthquake.

In Chapter II two different models of a nonlinear beam which can be used to represent each girder and column in a structural frame are presented and compared. One beam model can have different curvilinear or bilinear bending moment-rotation hysteresis loops located at the yield joints at each end of the beam. The other model is restricted to having bilinear hysteresis loops at the ends. Three definitions of ductility factor applicable to these beams are given and discussed.

4

The properties of the class of nonlinear multi-story structures considered in this report are listed in Chapter III. In addition, the damping mechanisms are described and the equations of motion for this system are derived. Using the Wilson-Clough integration technique[40], the equations of motion are put into a form suitable for solution by the 7090/7094 digital computer at the Computer Center of the California Institute of Technology.

As an overall check on the computer program for analyzing the response of nonlinear multi-story structures, several tests on it were carried out which are briefly described in Chapter IV. One of these was a comparison with results for a corresponding system presented in the FHA Study[1].

In Chapter V uniform and tapered, shear-type, linear elastic, continuous cantilever beams are analytically studied in order to determine the importance of "higher modes" to the responses, i.e., displacement, strain, shear force, and total acceleration, at various positions in an earthquake excited structure. "Whiplash" is discussed and a definition suggested. A method of determining "equivalent" modal participation factors for nonlinear structures is presented and examples of the "equivalent" modal contributions to displacements and interfloor displacements (strain) are given.

Chapter VI treats the response results for a nonlinear twenty story structure subjected to earthquake excitation. The effect of yielding is observed by comparing the responses for the nonlinear structure with those for the corresponding linear structure. Time history response plots, displacement envelopes, and ductility factors

for the nonlinear structure subjected to the entire duration of several earthquakes are presented and analyzed. The effects of various modes are observed by comparing time history response plots of the multi-story structure with those for one degree of freedom linear oscillators representing the individual modes. Three common measurements of the strength of an earthquake are given and are compared with the magnitudes of the corresponding structural responses for the earthquakes used.

CHAPTER II

MODELS OF BEAMS WITH NONLINEAR CHARACTERISTICS
AND ASSOCIATED DEFINITIONS OF DUCTILITY FACTOR

2.1 Introduction

The primary purpose of this chapter is to describe and com-
pare two different models of a nonlinear beam which can be used to
represent each girder and column in a discretized model of a non-
linear tall structure. One beam model has identical bilinear bending
moment-end rotation hysteresis loops at the yield joints located at
each end of the beam. The other model is more versatile since it
can have different curvilinear or bilinear bending moment-end
rotation hysteresis loops at each end.

In order to develop concepts and terminology useful in dis-
cussing yielding beams, simple yielding systems are studied first.
In this study the term "simple yielding system" always refers to a
system with only one yield joint. A yield joint is similar to a
Coulomb slider, but is more general since it can have a curvilinear
force-deflection (or bending moment-rotation) relationship. In
particular, certain nomenclature pertaining to hysteresis loops is
presented, two different simple yielding systems are studied, and
three definitions of ductility factor for these simple systems are
given. Two independent models of beams with yield joints at each
end are presented as extensions from the simple yielding systems.
Three definitions of ductility factor which apply to yielding beams are
also discussed.
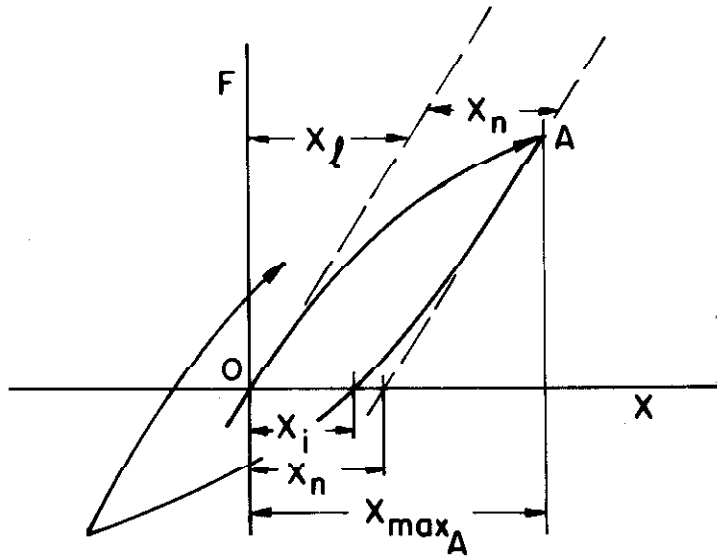
Fig. 2.1 Curvilinear
Hysteresis Loop
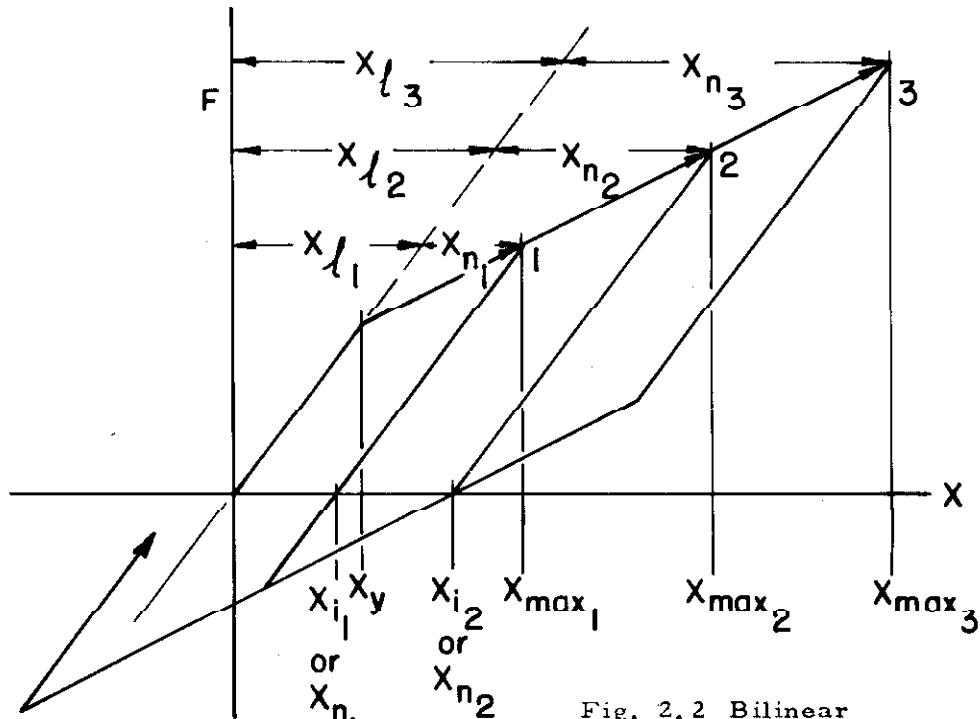
Fig. 2.2 Bilinear
Hysteresis Loop

$X_{i_1} \leqslant X_{i_2} = X_{i_3}$

2.2 Nomenclature Pertaining to Hysteresis Loops

The nomenclature pertaining to hysteresis loops as used in this report is defined in this section. A typical curvilinear hysteresis loop is shown in Fig. 2.1 and a typical bilinear hysteresis loop is depicted in Fig. 2.2. For both of these loops, the following nomenclature applies:

$$|x|_{max} \qquad = \text{maximum absolute displacement,}$$

$$x_y \qquad = \text{displacement at yield (yield level),}$$

$$x_n \qquad = \text{nonlinear displacement: departure from the initial tangent (absolute value),}$$

$$x_\ell = |x|_{max} - x_n \qquad = \text{linear displacement: displacement along the initial tangent at the force level of } |x|_{max} \text{ (absolute value),}$$

$$x_i \qquad = \text{irrecoverable displacement or permanent set (absolute value), and}$$

$$x_r = |x|_{max} - x_i \qquad = \text{recoverable displacement (absolute value).}$$

Hence,

$$x_\ell + x_n = x_r + x_i ,$$

or

$$x_r = x_\ell + x_n - x_i . \tag{2.1}$$

From Fig. 2.1 and Fig. 2.2 it is seen that

$$x_i \leq x_n \qquad \text{and} \qquad x_\ell \leq x_r . \tag{2.2}$$

Note that for the bilinear hysteresis loop of Fig. 2.2 with

$$|x|_{max} \leq x_{max_2} , \tag{2.3}$$

the irrecoverable displacement is identical to the nonlinear displacement, and the linear displacement is identical to the recoverable displacement:

$$x_i = x_n \quad \text{and} \quad x_\ell = x_r \ . \tag{2.4}$$

The bilinear hysteresis loop with the maximum displacement restricted by Eq. 2.3 is the one of primary interest in this report.

Looking at Fig. 2.1 and Fig. 2.2, it appears that in many instances the irrecoverable displacement can be much smaller than the corresponding nonlinear displacement. Furthermore, failure is probably more closely related to the nonlinear displacement than to the irrecoverable displacement. Consequently, one definition of ductility based on the nonlinear displacement rather than the irrecoverable displacement is given in the next section.

2.3 Three Definitions of "Ductility Factor" for Simple Yielding Systems

In this report, the term "ductility factor" is used as a measure of the amount of yielding incurred in a system. However, a "ductility factor" has no precise meaning until the method of measuring it has been defined. The purpose of this section is to make three definitions of the term "ductility factor" for the nonlinear spring of the simple yielding system of Fig. 2.3. All three definitions apply to the bilinear hysteresis loop shown in Fig. 2.4 and one of them also applies to the more general curvilinear hysteresis loop of Fig. 2.5.

X
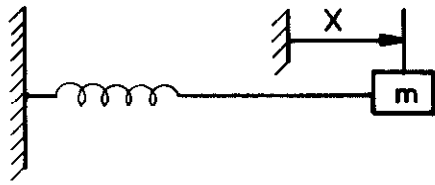
Fig. 2.3 Simple Yielding System
With Nonlinear Spring

m

F

F_y

a

b

k_2

k

O

X_i
or
X_n

c

X_y

δ

X_{max}

X

$X_{max\,2}$

d

f

e
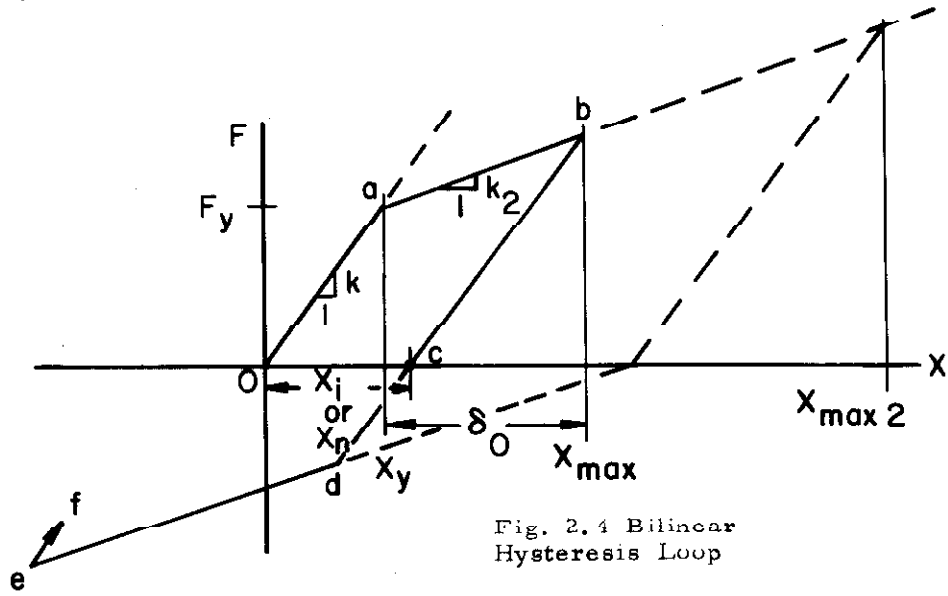
Fig. 2.4 Bilinear
Hysteresis Loop

X_ℓ

X_O

b

f

O

c

$X_{max} - X_O$

X_O

X
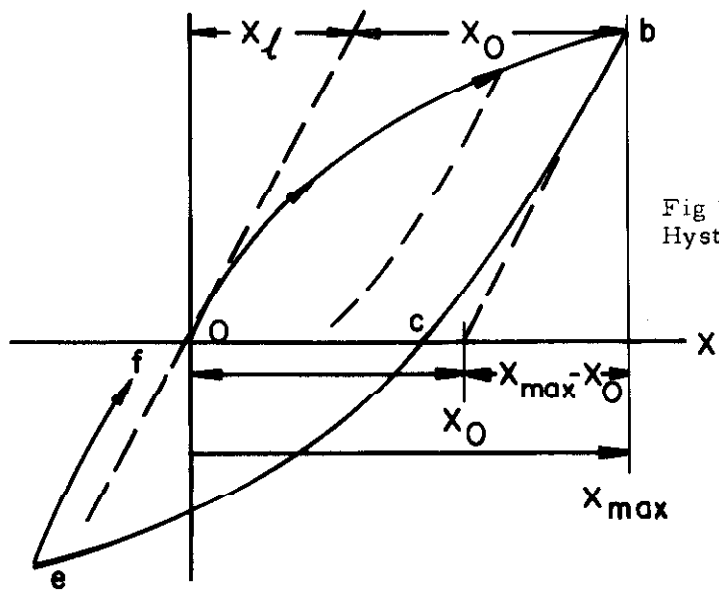
Fig 2.5 Curvilinear
Hysteresis Loop

e

X_{max}

The next section, Section 2.4, treats two systems composed of linear springs and Coulomb sliders which can be used to model a simple yielding system with bilinear hysteretic characteristics. However, for the purposes of this section, it is only necessary to consider the simple yielding system of Figs. 2.3, 2.4, and 2.5.

For the nonlinear spring of the system of Fig. 2.3, one possible hysteresis loop is the path 0, a, b, c, d, e, f in Fig. 2.4. On this path only linear displacement is incurred until the displacement x exceeds x_y at point (a), whereupon it yields. At point (b) the displacement $|x|_{max}$ is composed of the linear displacement, x_y, and additional displacement incurred during yielding, δ_o, where

$$\delta_o = |x|_{max} - x_y \; .$$

Furthermore, using the terminology of Section 2.2, δ_o has both "linear" and "nonlinear" components. Now, if the spring is released so that the force becomes zero, the equilibrium displacement is the irrecoverable displacement x_i, at point (c). Note that for this example, $|x|_{max}$ is less than x_{max_2}; consequently, the nonlinear displacement x_n and irrecoverable displacement x_i are equal:

$$x_n = x_i \quad (if \; |x|_{max} \leq x_{max_2}) \; . \tag{2.5}$$

From geometrical considerations, x_n is found from Fig. 2.4 to be

$$x_n = \left(1 - \frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.6}$$

Hence, the additional linear displacement incurred in going from point (a) to point (b) is

$$\delta_o - x_n = \left(\frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.7}$$

Therefore, the total linear displacement incurred in traversing from (0) to (a) to (b) is

$$x_\ell = x_y + \left(\frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.8}$$

The first definition of ductility factor, μ_1, is simply a measure of the maximum absolute displacement at point (b), $|x|_{max}$, (see Fig. 2.4) with respect to the yield displacement, x_y, without regard to the second slope, k_2:

$$\mu_1 = \frac{|x|_{max}}{x_y} \tag{2.9}$$

or

$$\mu_1 = 1 + \frac{\delta_o}{x_y} \quad . \tag{2.10}$$

Although Eq. 2.10 can be used to measure the yielding for any bilinear hysteresis loop, its most appropriate application is to the ideally elasto-plastic hysteresis loop, i.e., a bilinear hysteresis loop with the second slope identically equal to zero, $k_2 \equiv 0$. In this

case, all of the displacement from the yield displacement at point (a)
to point (b) is nonlinear:

$$x_n = \delta_o = |x|_{max} - x_y , \quad (k_2 \equiv 0) \qquad (2.11)$$

and the total linear displacement at point (b) is the yield displacement,
x_y:

$$x_\ell = x_y \qquad (k_2 \equiv 0) . \qquad (2.12)$$

Furthermore, the total energy dissipated, E_D in a complete, sym-
metric hysteresis loop with $k_2 \equiv 0$ is proportional to $(\mu_1 - 1)$ and is
given by

$$E_D = 4F_y \cdot \delta_o \qquad (k_2 \equiv 0) \qquad (2.13)$$

where

$$F_y = k \cdot x_y . \qquad (2.14)$$

The amount of energy dissipated is of interest since, in some cases,
dissipating energy by yielding can limit the amplitudes of the
response.

Now, for systems with $k_2 \neq 0$, a second definition of ductility
factor which measures the nonlinear displacement (instead of the
maximum absolute displacement) at point (b) with respect to the yield
displacement is

$$\mu_2 = 1 + \frac{x_n}{x_y} \qquad (2.15)$$

which, by substituting Eq. 2.6 for x_n, becomes

$$\mu_2 = 1 + \left(1 - \frac{k_2}{k}\right)\frac{\delta_o}{x_y} \; . \tag{2.16}$$

In this case, the total energy dissipated in a complete symmetric hysteresis loop is proportional to $(\mu_2 - 1)$:

$$E_D = 4F_y \cdot \left(1 - \frac{k_2}{k}\right) \cdot \delta_o \; . \tag{2.17}$$

For the first two definitions of ductility factor, it is necessary to have a well-defined yield level. However, most curvilinear hysteresis loops do not have an inherently well-defined yield level. Nevertheless, for any hysteresis loop, except one with a vertical initial tangent, the nonlinear and linear displacements are well defined. For these loops a third definition of ductility factor relating the maximum absolute displacement, $|x|_{max}$, at point (b) (see Fig. 2.4) to the linear displacement, x_ℓ, can be made:

$$\mu_3 = \frac{|x|_{max}}{x_\ell}$$

or

$$\mu_3 = 1 + \frac{x_n}{x_\ell} \; . \tag{2.18}$$

However, the energy dissipated either in a curvilinear or in a bilinear hysteresis loop cannot be directly related to $(\mu_3 - 1)$ as was done above for $(\mu_2 - 1)$ in Eq. (2.17).

When used for the bilinear hysteresis loop of Fig. 2.4, the

third definition can be written in another form. For the maximum

absolute displacement in this loop (at point (b)), x_ℓ and x_n are

given by Eq. 2.6 and Eq. 2.8, respectively. Substituting these into

Eq. 2.18, μ_3 becomes

$$\mu_3 = 1 + \frac{\left(1 - \frac{k_2}{k}\right) \cdot \delta_0}{x_y + \left(\frac{k_2}{k} \cdot \delta_0\right)} \quad . \tag{2.19}$$

In order to show how these three definitions differ for bilinear

hysteresis loops, the following numerical examples are considered:

Let $x_y = 1$ and $|x|_{max} = 5$ for systems with $k_2/k = 0.05$ and

$k_2/k = 0.95$. The various ductility factors are:

Definition	$\frac{k_2}{k} = 0.05$	$\frac{k_2}{k} = 0.95$
$\mu_1 \equiv 1 + \frac{\delta_0}{x_y}$	5.0	5.0
$\mu_2 \equiv 1 + \left(1 - \frac{k_2}{k}\right) \cdot \frac{\delta_0}{x_y}$	4.8	1.2
$\mu_3 \equiv 1 + \frac{\left(1 - \frac{k_2}{k}\right) \cdot \delta_0}{x_y + \left(\frac{k_2}{k} \cdot \delta_0\right)}$	4.17	1.17

(Note that when $k_2 \equiv k = 0$, $\mu_1 = \mu_2 = \mu_3$.) From these examples, one

can see that the choice of the definition of ductility factor can make a

sizeable difference in the resulting numerical value.

If only bilinear hysteresis loops are of interest, the second

definition, μ_2, is a better indication of the amount of yielding

incurred as well as of the energy dissipated than the first, μ_1. On

the other hand, if one wishes to compare the ductility factors of

curvilinear systems to those of bilinear systems, the third definition,

μ_3, is best.

2.4 Two Systems for Modeling a Simple Yielding System With Bilinear Hysteretic Characteristics

This section treats two systems composed of linear springs

and Coulomb sliders (see Fig. 2.6 and Fig. 2.7) which are used to

model the simple yielding system of Fig. 2.3 with the bilinear

hysteresis loop of Fig. 2.4. The purpose of studying these systems

is to form a base from which to extend to the more complex problem

of nonlinear beams.

As seen in Figs. 2.6, 2.7, 2.8 and 2.9, the primary difference

between these two simple yielding systems is the arrangement of the

springs and Coulomb sliders. In one, the springs are in series, and

in the other, the springs are in parallel. Consequently, the general

equations relating forces to displacements as well as those relating

the ductility factors to the displacement incurred during yielding have

different functional dependencies for the two systems. Concerning

the functional dependence of the ductility factor for the springs-in-

series system of Fig. 2.6 and Fig. 2.8, the incurred yielding (the

amount of slip in the Coulomb slider), δ', is purely the nonlinear[*]

displacement and the corresponding linear[*] displacement is

---

[*]See Section 2.2 for definitions of linear and nonlinear displacements.
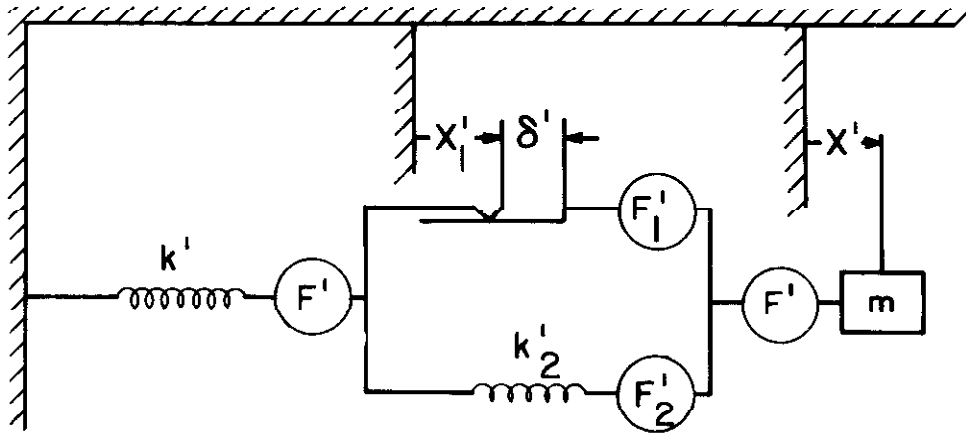
Fig. 2.6   Springs-in-Series
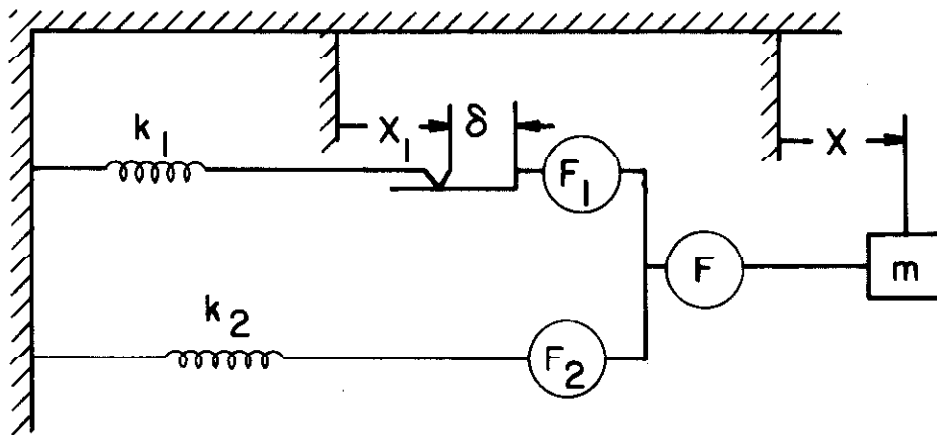
Fig. 2.7 Springs-in-Parallel

SPRINGS-IN-SERIES

SPRINGS-IN-PARALLEL

Fig. 2.8a Spring k'

Fig. 2.9a Spring k_1 and Coulomb Slider in Series
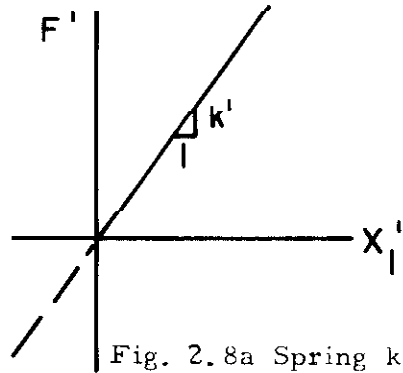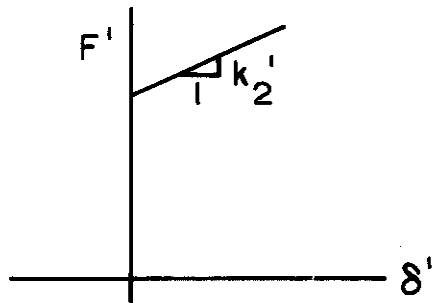
Fig. 2.8b Spring k_2' and Coulomb Slider in Parallel

Fig. 2.9b Spring k_2

$X' = X_1' + \delta'$

Fig. 2.8c Bilinear Hysteresis Loop.
$(\frac{1}{k_2''} = \frac{1}{k'} + \frac{1}{k_2'})$

Fig. 2.9c Bilinear Hysteresis Loop.
$(k = k_1 + k_2)$

$$|x|_{max} - \delta' \quad .$$
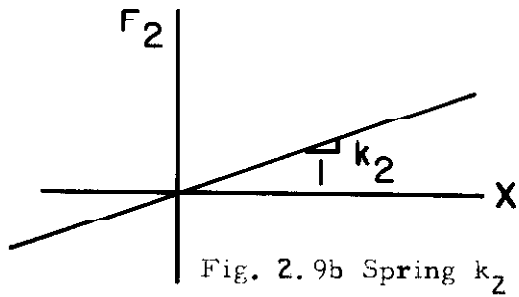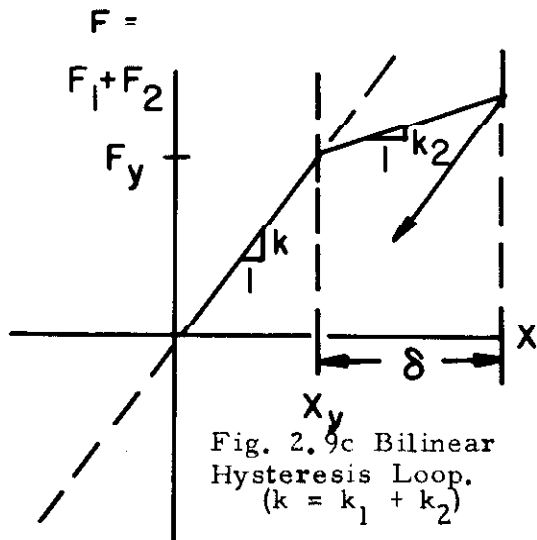
Since δ', as shown in Fig. 2.8c, corresponds to x_n of Fig. 2.4,
the ductility factor for this system by the third definition, Eq. 2.19,
is

$$\mu_{3_{series}} = 1 + \frac{\delta'}{|x|_{max} - \delta'}$$

or

$$\mu_{3_{series}} = 1 + \frac{\delta'}{x_1'} \quad . \tag{2.20}$$

In the springs-in-parallel system of Figs. 2.7 and 2.9, the displace-
ment incurred during yielding, δ, is the sum of both the nonlinear
and the linear displacements incurred by the entire system beyond
the yield point. Since δ of Fig. 2.9c corresponds to δ_o of Fig. 2.4,
the ductility factor for this system, by the third definition, Eq. 2.19,
is

$$\mu_{3_{parallel}} = 1 + \frac{\left(1 - \frac{k_2}{k}\right)\delta}{x_y + \frac{k_2}{k}\delta} \tag{2.21}$$

where $k = k_1 + k_2$.

In the following vis-à-vis presentation, the different functional
dependencies of the forces upon displacements in the two systems are
studied.

Springs-in-Series (Fig. 2.6 and Fig. 2.8)	Springs-in-Parallel (Fig. 2.7 and Fig. 2.9)

Springs-in-Series

(Fig. 2.6 and Fig. 2.8)

General equations:

$$F' = F_1' + F_2';$$

$$x' = x_1' + \delta' ;$$

in incremental form:

$$\Delta F' = \Delta F_1' + \Delta F_2'; \qquad (2.22.1)$$

$$\Delta x' = \Delta x_1' + \Delta \delta' . \qquad (2.23.1)$$

For small displacements:

$$F' = k'x' .$$

Spring k':

$$F' = k'x_1' ;$$

$$\Delta F' = k'\Delta x_1' . \qquad (2.24.1)$$

Spring k_2':

$$F_2' = k_2'\delta' ;$$

$$\Delta F_2' = k_2'\Delta \delta' . \qquad (2.25.1)$$

Coulomb slider:

If F_y' is the yield limit for the force F', then

$$F_{1y}' = F_y'$$

Springs-in-Parallel

(Fig. 2.7 and Fig. 2.9)

General equations:

$$F = F_1 + F_2 :$$

$$x = x_1 + \delta ;$$

in incremental form:

$$\Delta F = \Delta F_1 + \Delta F_2; \qquad (2.22.2)$$

$$\Delta x = \Delta x_1 + \Delta \delta . \qquad (2.23.2)$$

For small displacements:

$$F = kx$$

where

$$k = k_1 + k_2 ;$$

or

$$k_1 \equiv qk, \quad k_2 \equiv pk ,$$

and

$$p + q = 1 .$$

Spring k_1:

$$F_1 = k_1 x_1 ;$$

$$\Delta F_1 = k_1 \Delta x_1 . \qquad (2.24.2)$$

Spring k_2:

$$F_2 = k_2 x ;$$

$$\Delta F_2 = k_2 \Delta x . \qquad (2.25.2)$$

Coulomb slider:

If F_y is the yield limit for the force F, then

$$F_{1y} = q \cdot F_y$$

Springs-in-Series, cont.

is the yield limit for the Coulomb slider:

$$|F_1'| \leq F_{1y}'.$$

Yield Criteria:

When

$$|F_1'| < F_{1y}' \quad \text{(linear state)},$$
$$\Delta\delta' = 0.$$

Hence,

$$\Delta F' = k' \cdot \Delta x'. \qquad (2.26.1)$$

When

$$|F_1'| = F_{1y}' \quad \text{(nonlinear state)}$$
$$\Delta F_1' = 0.$$

Hence, from Eq. 2.22.1,

$$\Delta F' = \Delta F_2'$$

and with Eq. 2.25.1,

$$\Delta F' = k_2' \cdot \Delta\delta'. \qquad (2.27.1)$$

Also, in this state

$$\Delta F' = k_2'' \cdot \Delta x'$$

where k_2'', as shown in Fig. 2.8c, is given by

$$\frac{1}{k_2''} = \frac{1}{k'} + \frac{1}{k_2'}.$$

Springs-in-Parallel, cont.

is the yield level for the Coulomb slider:

$$|F_1| \leq F_{1y}.$$

Yield Criteria:

When

$$|F_1| < F_{1y} \quad \text{(linear state)},$$
$$\Delta\delta = 0.$$

Hence,

$$\Delta x_1 = \Delta x, \text{ and}$$
$$\Delta F = k \cdot \Delta x. \qquad (2.26.2)$$

When

$$|F_1| = F_{1y} \quad \text{(nonlinear state)}$$
$$\Delta F_1 = 0.$$

Hence,

$$\Delta x_1 = 0$$

and

$$\Delta\delta = \Delta x.$$

From Eq. 2.22.2, with $\Delta F_1 = 0$,

$$\Delta F = \Delta F_2 \qquad (2.27.2)$$

and

$$\Delta F = k_2 \Delta x,$$

i.e., any increase of the force above the yield level is done via spring k_2 only.

In order to make the bilinear spring systems appear the same to mass (m) in Fig. 2.6 and Fig. 2.7, let

$$x' = x , \quad (\Delta x' = \Delta x) ;$$

and

$$F' = F ; \quad (\Delta F' = \Delta F) .$$

Then, from Eq. 2.26.1 and Eq. 2.26.2 for the linear state, it is found that

$$k' = k$$

and, correspondingly, for the nonlinear state, it is found that

$$k_2'' = k_2 .$$

Hence, by substitution, an equation for k_2' in terms of k and k_2 arises:

$$\frac{1}{k_2'} = \frac{1}{k_2} - \frac{1}{k} .$$

This shows that it is possible to adjust the spring constants of these two simple yielding systems so that they appear the same from the outside in both states of yield. As will be seen in the next section, it is not possible to adjust the spring constants of the two models of nonlinear beams so that they appear the same in all states of yield.

2.5 Two Models of Nonlinear Beams

The two beam models presented in this section have nonlinear hysteretic bending moment-end rotation characteristics; consequently, they are more complicated than the simple yielding systems of the previous sections. It should be noted that the "one-component" beam model, Fig. 2.10, is an extension of the springs-in-series system,

$$\omega_i' = \omega_i - \alpha_i$$

$$\omega_j' = \omega_j - \alpha_j$$

$$k = \frac{4EI}{l}$$
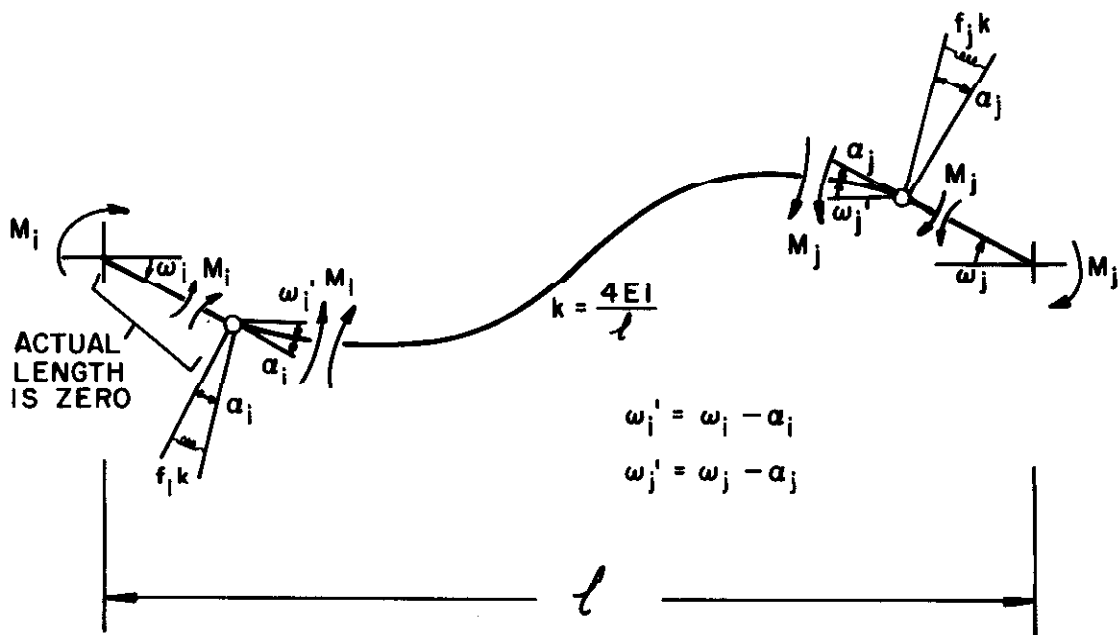
Fig. 2.10 One-component Model
of a Nonlinear Beam

$$\omega_i = \omega_i' + a_i$$
$$\omega_j = \omega_j' + a_j$$
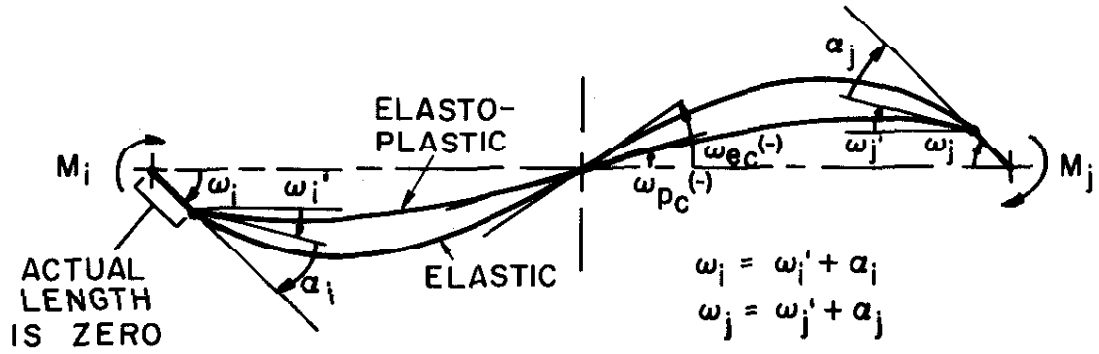
Fig. 2.11a Two-component Model
of a Nonlinear Beam
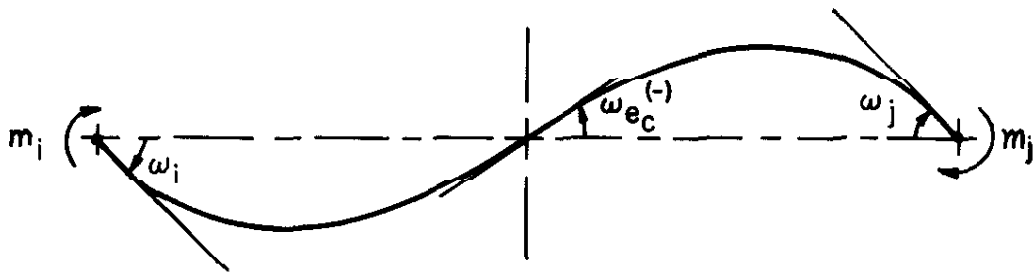
Fig. 2.11b The Linear Component
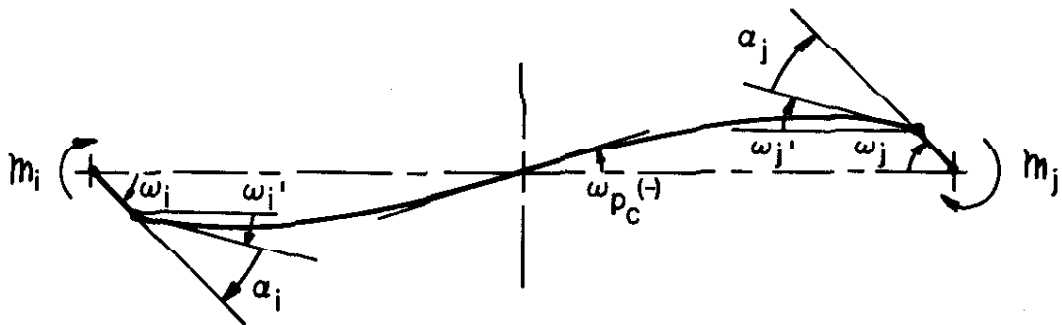
Fig. 2.11c The Ideally Elasto-plastic Component
(Note that the plastic angles a_i and a_j occur only in this component.)

Fig. 2.6 and the "two-component" beam model, Fig. 2.11, is an

extension of the springs-in-parallel system, Fig. 2.7.

As seen in Fig. 2.10 and Fig. 2.11, the one-component beam

is the more appealing model of a yielding column or girder from

the physical point of view. Furthermore, nonlinear (bilinear and

curvilinear) hysteresis loops can be used with the one-component

beam, the only restriction being that the initial slopes of the hysteresis

loops at both ends of the same beam must be equal. On the other hand,

the two-component model can have only bilinear hysteresis loops at

each end. Although the yield levels can differ, these loops must have

identical initial and identical second slopes. For this model, the

second slope of the loop is determined by the stiffness, pk, of the

linear component of the beam.

Nevertheless, the two-component beam has been used by other

investigators[*] in the study of nonlinear tall structures with results that

are very similar to those obtained using the corresponding one-com-

ponent beam. Consequently, both of these beam models are studied

in the following vis-à-vis presentation.

In the linear state both beam models have stiffness

$$k = \frac{4EI}{\ell}$$

where

E is Youngs modulus (lbs/in^2) ,

---

[*]R. W. Clough and K. L. Benuska, FHA Study, reference 1.

I is the area moment of inertia (in^4), and

ℓ is the length of the beam (in) .

The shear deflection in the beam is neglected.

Although the same symbols are used in the moment-rotation

equations for both models, some of the definitions differ slightly,

particularly for the plastic angles α. As seen in Figs. 2.10 and 2.11,

the symbols have the following meanings:

M_i, M_j	bending moments at the ends (i) and (j) (same in both models),
m_i, m_j	bending moments at the ends of the linear component of the two-component model,
$\bar{m}_i, \bar{m}_j$	bending moments at the ends of the elasto-plastic component of the two-component model,
ω_i, ω_j	end rotations (same in both models),
ω_i', ω_j'	end rotations of central beam in one-component model, or end rotations of central beam of elasto-plastic component in two-component model,
α_i, α_j	incurred plastic angle at the ends in the one-component model (note correspondence to δ' in Fig. 2.8c); or incurred plastic angle at the ends of the elasto-plastic component in the two-component model (note correspondence to δ in Fig. 2.9c).

One-component Model (Fig. 2.10)	Two-component Model (Fig. 2.11)
Stiffness distribution:	Stiffness distribution:
The central beam has stiffness k. The end beams have zero length and are dropped from further consideration.	The stiffness k of the total beam is apportioned to the two components as follows: linear component: $k_{linear} = p \cdot k$; elasto-plastic component: $k_{elasto-plastic} = q \cdot k$; where $p + q = 1$; (usually $p \approx 0.05$).
Fundamental Bending Moment-End Rotation Equations:	Fundamental Bending Moment-End Rotation Equations: linear component: $m_i = pk(\omega_i + \frac{1}{2}\omega_j)$ $m_j = pk(\frac{1}{2}\omega_i + \omega_j)$ or, in incremental form, $\Delta m_i = pk(\Delta\omega_i + \frac{1}{2}\Delta\omega_j)$ $\Delta m_j = pk(\frac{1}{2}\Delta\omega_i + \Delta\omega_j)$ elasto-plastic component: $m_i = qk(\omega_i' + \frac{1}{2}\omega_j')$ $m_j = qk(\frac{1}{2}\omega_i' + \omega_j')$ where $q = 1 - p$. From Fig. 2.11a, $\omega_i' = \omega_i - \alpha_i$

One-component Model, cont.

Central beam

$$M_i = k(\omega'_i + \tfrac{1}{2}\omega'_j)$$

$$M_j = k(\tfrac{1}{2}\omega'_i + \omega'_j) .$$

From Fig. 2.10,

$$\omega'_i = \omega_i - \alpha_i$$

and

$$\omega'_j = \omega_j - \alpha_j .$$

By substitution, the fundamental bending moment-end rotation equations for this beam model are obtained:

$$M_i = k[(\omega_i - \alpha_i) + \tfrac{1}{2}(\omega_j - \alpha_j)]$$
$$M_j = k[\tfrac{1}{2}(\omega_i - \alpha_i) + (\omega_j - \alpha_j)] \qquad (2.28.1)$$

or, in incremental form:

$$\Delta M_i = k[(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j)] \qquad (2.29.1)$$
$$\Delta M_j = k[\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j)] .$$

Two-component Model, cont.

and

$$\omega'_j = \omega_j - \alpha_j .$$

By substitution, then,

$$m_i = qk[(\omega_i - \alpha_i) + \tfrac{1}{2}(\omega_j - \alpha_j)]$$

$$m_j = qk[\tfrac{1}{2}(\omega_i - \alpha_i) + (\omega_j - \alpha_j)]$$

or, in incremental form:

$$\Delta m_i = qk[(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j)]$$

$$\Delta m_j = qk[\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j)]$$

For the total beam:

$$M_i = m_i + m_i$$

$$M_j = m_j + m_j ;$$

similarly in incremental form.

By addition, the fundamental bending moment-end rotation equations for this beam model are obtained:

$$M_i = k[(\omega_i - q\alpha_i) + \tfrac{1}{2}(\omega_j - q\alpha_j)]$$
$$M_j = k[\tfrac{1}{2}(\omega_i - q\alpha_i) + (\omega_j - q\alpha_j)] \qquad (2.28.2)$$

or, in incremental form:

$$\Delta M_i = k[(\Delta\omega_i - q\Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - q\Delta\alpha_j)] \qquad (2.29.2)$$
$$\Delta M_j = k[\tfrac{1}{2}(\Delta\omega_i - q\Delta\alpha_i) + (\Delta\omega_j - q\Delta\alpha_j)] .$$

The purpose of writing these equations for both beam models in incremental form is that in this form it is possible to solve the equations of motion using finite integration techniques assuming that the state of yield remains constant throughout each time increment. As seen above, the incremental bending moments, ΔM, are related to both the incremental rotations, $\Delta \omega$, and incremental plastic angles, $\Delta \alpha$. Now, if the state of yield is known at the beginning of the time increment, it is possible to establish beforehand an equation of the form

$$\Delta \alpha = \Delta \alpha (\Delta \omega_i, \Delta \omega_j)$$

relating the incremental plastic angles to the end rotations. Using these equations it is possible to eliminate the incremental plastic angles from the incremental moment-rotation equations resulting in equations of the form:

$$\Delta M = \Delta M(\Delta \omega_i, \Delta \omega_j)$$

which are valid for each time increment.

The criteria for establishing the state of yield at the time t (at the beginning of a time increment) are based upon the bending moment at time t and the last incremental bending moment prior to time t. These criteria, which are the same for both beam models, are (see Fig. 2.12) the following:
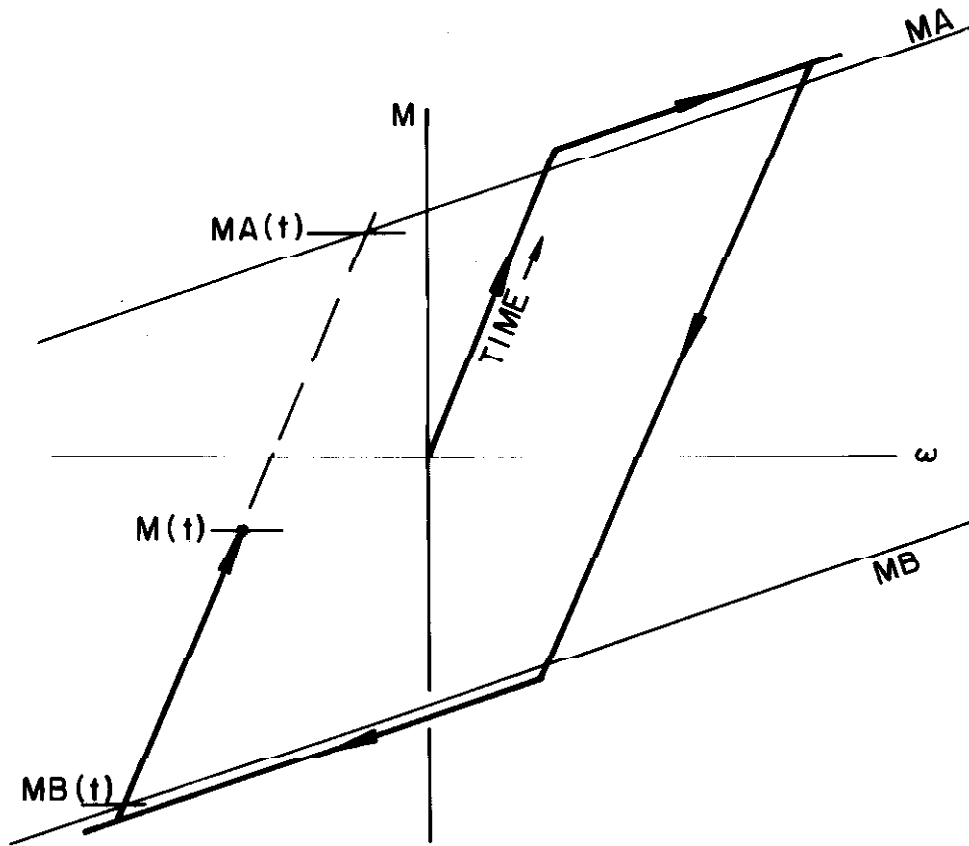
Fig. 2.12 Bilinear Hysteresis Loop Showing "Overshooting"
Upon Entering the Nonlinear State of Yield and "Backtracking"
Upon Returning to the Linear State

If $MB(t) < M(t) < MA(t)$, then the state of yield is linear;

$$\text{if} \left\{ \begin{array}{l} \phantom{\text{or }} M(t) \geq MA(t) \text{ and } \Delta M(t) > 0, \\ \text{or} \\ \phantom{\text{or }} M(t) \leq MB(t) \text{ and } \Delta M(t) < 0 \end{array} \right\} \left\{ \begin{array}{l} \text{then the state of} \\ \\ \text{yield is nonlinear;} \end{array} \right.$$

where

$M(t)$:   total bending moment at time t ,

$MA(t)$: upper yield bending moment at time t ,

$MB(t)$: lower yield bending moment at time t, and

$\Delta M(t)$: the last incremental bending moment prior to time t.

Inherent in this procedure is the phenomena of "overshooting" the

yield limit upon entering the nonlinear state and "backtracking" upon

returning to the linear state, as shown in Fig. 2.12. This results

from the assumption that the state of yield remains constant through-

out the time increment.

Although the yield criteria and the general form of the equa-

tions relating moments and plastic angles to rotations are the same

for both beams, the actual equations for each beam are different as

shown below.

One-component Model	Two-component Model
When the state of yield is <u>linear</u> at end (i) and/or end (j), the corresponding incremental plastic angle must be zero:	When the state of yield is <u>linear</u> at end (i) and/or end (j), the corresponding incremental plastic angle must be zero:
at end (i), $\Delta\alpha_i = 0$: and/or	at end (i), $\Delta\alpha_i = 0$: and/or

One-component Model, cont.

at end (j), $\Delta\alpha_j = 0$.

When the state of yield is non-linear at end (i) and/or end (j), the corresponding incremental bending moment is proportional to the incremental plastic angle:

at end (i), $\Delta M_i = f_i k \Delta\alpha_i$;

and/or

at end (j), $\Delta M_j = f_j k \Delta\alpha_j$;

where f_i and f_j are independent. These equations are analogous to Eq. 2.27.1 of the springs-in-series system of the previous section.

Two-component Model, cont.

at end (j), $\Delta\alpha_j = 0$.

When the state of yield is non-linear at end (i) and/or end (j), the corresponding incremental total bending moment equals the incremental bending moment of the linear component, and the incremental bending moment in the elasto-plastic component is zero:

at end (i), $\Delta M_i = \Delta m_i$,

$$\Delta \bar{m}_i = 0 ;$$

and/or

at end (j), $\Delta M_j = \Delta m_j$,

$$\Delta \bar{m}_j = 0 .$$

These equations are analogous to Eq. 2.27.2 of the springs-in-parallel system in the previous section.

Since f_i and f_j are independent, it is possible to use a curvilinear hysteresis loop with the one-component model. The method of determining f_i and f_j is discussed in the next section. On the other hand, for the two-component model, Δm_i and Δm_j are dependent upon the same stiffness parameter (p). Consequently,

this model can have hysteresis loops at the ends with <u>only</u> the two

slopes (k) and (pk) although the yield levels may be different.

Since there are two possible states of yield for each yield

joint, there are four possible states of yield for a beam. These

states, denoted by (a), (b), (c), and (d), are described below.

One-component Model

State (a) - linear at (i) and (j):

$$\Delta\alpha_i = 0 \; ;$$

$$\Delta\alpha_j = 0 \; .$$

State (b) - nonlinear at (i);
linear at (j):

$$\Delta M_i = f_i k \Delta\alpha_i \; ;$$

$$\Delta\alpha_j = 0 \; .$$

State (c) - linear at (i);
nonlinear at (j):

$$\Delta\alpha_i = 0 \; ;$$

$$\Delta M_j = f_j k \Delta\alpha_j \; .$$

State (d) - nonlinear at (i) and (j):

$$\Delta M_i = f_i k \Delta\alpha_i \; ;$$

$$\Delta M_j = f_j k \Delta\alpha_j \; .$$

The functional dependencies of

the incremental plastic angles

upon rotations are as follows:

Two-component Model

State (a) - linear at (i) and (j):

$$\Delta\alpha_i = 0 \; ;$$

$$\Delta\alpha_j = 0 \; .$$

State (b) - nonlinear at (i);
linear at (j):

$$\Delta M_i = \Delta m_i; \quad (\Delta\alpha_i \neq 0) \; ;$$

$$\Delta\alpha_j = 0 \; .$$

State (c) - linear at (i);
nonlinear at (j):

$$\Delta\alpha_i = 0 \; ;$$

$$\Delta M_j = \Delta m_j \; ; \quad (\Delta\alpha_j \neq 0) \; .$$

State (d) - nonlinear at (i) and (j):

$$\Delta M_i = \Delta m_i \; ; \quad (\Delta\alpha_i \neq 0) \; ;$$

$$\Delta M_j = \Delta m_j \; ; \quad (\Delta\alpha_j \neq 0) \; .$$

The functional dependencies of

the incremental plastic angles

upon rotations are as follows:

One-component Model, cont.	Two-component Model, cont.

One-component Model, cont.

<u>State (a) - linear at (i) and (j):</u>

$$\Delta\alpha_i = 0 \ ;$$

$$\Delta\alpha_j = 0 \ .$$

<u>State (b) - nonlinear at (i);</u>
<u>linear at (j):</u>

from state (b) above and

Eq. 2.29.1,

$$k[(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}\Delta\omega_j] = f_i k \Delta\alpha_i \ .$$

Hence,

$$\Delta\alpha_i = \left(\frac{1}{1+f_i}\right)(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ ;$$

$$\Delta\alpha_j = 0 \ .$$

<u>State (c) - linear at (i) ;</u>
<u>nonlinear at (j) :</u>

(similar to state (b); replace (i)

by (j)):

$$\Delta\alpha_i = 0 \ ;$$

$$\Delta\alpha_j = \left(\frac{1}{1+f_j}\right)(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j)$$

<u>State (d) - nonlinear at (i) and (j):</u>

from state (d) above and

Eq. 2.29.1,

$$k[(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j)]$$
$$= f_i k \Delta\alpha_i$$
$$k[\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j)]$$
$$= f_j k \Delta\alpha_j$$

Two-component Model, cont.

<u>State (a) - linear at (i) and (j):</u>

$$\Delta\alpha_i = 0 \ ;$$

$$\Delta\alpha_j = 0 \ .$$

<u>State (b) - nonlinear at (i);</u>
<u>linear at (j):</u>

from state (b) above and

Eq. 2.29.2

$$k[(\Delta\omega_i - q\Delta\alpha_i) + \tfrac{1}{2}\Delta\omega_j] = pk(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ .$$

Hence,

$$\Delta\alpha_i = (\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ ;$$

$$\Delta\alpha_j = 0 \ .$$

<u>State (c) - linear at (i) ;</u>
<u>nonlinear at (j):</u>

(similar to state (b); replace (i)

by (j)):

$$\Delta\alpha_i = 0 \ ;$$

$$\Delta\alpha_j = (\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j) \ .$$

<u>State (d) - nonlinear at (i) and (j):</u>

from state (d) above and

Eq. 2.29.2,

$$k[(\Delta\omega_i - q\Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - q\Delta\alpha_j)]$$
$$= pk(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j)$$
$$k[\tfrac{1}{2}(\Delta\omega_i - q\Delta\alpha_i) + (\Delta\omega_j - q\Delta\alpha_j)]$$
$$= pk(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j)$$

One-component Model, cont.

rearranging:

$$(1+f_i)\Delta\alpha_i + \tfrac{1}{2}\Delta\alpha_j = \Delta\omega_i + \tfrac{1}{2}\Delta\omega_j \ ;$$

$$\tfrac{1}{2}\Delta\alpha_i + (1+f_j)\Delta\alpha_j = \tfrac{1}{2}\Delta\omega_i + \Delta\omega_j \ .$$

Hence,

$$\Delta\alpha_i = \frac{\left(1+\tfrac{4}{3}f_j\right)\Delta\omega_i + \tfrac{2}{3}f_j\Delta\omega_j}{1+\tfrac{4}{3}(f_i+f_j+f_i\cdot f_j)} \ ;$$

$$\Delta\alpha_j = \frac{\tfrac{2}{3}f_i\Delta\omega_i + \left(1+\tfrac{4}{3}f_i\right)\Delta\omega_j}{1+\tfrac{4}{3}(f_i+f_j+f_i\cdot f_j)} \ .$$

By substitution, the $\Delta\alpha$'s can be eliminated from the incremental moment-rotation equations:

State (a) - linear at (i) and (j):

$$\Delta M_i = k(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ ;$$

$$\Delta M_j = k(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j) \ .$$

State (b) - nonlinear at (i); linear at (j):

$$\Delta M_i = \left(\frac{1}{1+f_i}\right)k(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ ;$$

$$\Delta M_j = k\left[\tfrac{1}{2}\left(\frac{1}{1+f_i}\right)\Delta\omega_i + \left(\frac{3+4f_i}{4(1+f_i)}\right)\Delta\omega_j\right].$$

Two-component Model, cont.

rearranging:

$$(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j) = 0 \ ;$$

$$\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j) = 0 \ .$$

Hence.

$$\Delta\alpha_i = \Delta\omega_i \ ;$$

$$\Delta\alpha_j = \Delta\omega_j \ .$$

By substitution, the $\Delta\alpha$'s can be eliminated from the incremental moment-rotation equations:

State (a) - linear at (i) and (j):

$$\Delta M_i = k(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ ;$$

$$\Delta M_j = k(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j) \ .$$

State (b) - nonlinear at (i); linear at (j):

$$\Delta M_i = pk(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) \ ;$$

$$\Delta M_j = k\left[\tfrac{1}{2}p\Delta\omega_i + \left(1-\tfrac{q}{4}\right)\Delta\omega_j\right].$$

One-component Model, cont.	Two-component Model, cont.

State (c) - linear at (i); nonlinear at (j): | State (c) - linear at (i); nonlinear at (j):

$$\Delta M_i = k\left[\left(\frac{3+4f_j}{4(1+f_j)}\right)\Delta\omega_i + \tfrac{1}{2}\left(\frac{f_j}{1+f_j}\right)\Delta\omega_j\right];$$

$$\Delta M_j = \left(\frac{f_j}{1+f_j}\right)k(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j).$$

$$\Delta M_i = k\left[\left(1 - \tfrac{q}{4}\right)\Delta\omega_i + \tfrac{1}{2}p\Delta\omega_j\right]\;;$$

$$\Delta M_j = pk(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j).$$

State (d) - nonlinear at (i) and (j): | State (d) - nonlinear at (i) and (j):

$$\Delta M_i = \frac{f_i k\left[\left(1+\tfrac{4}{3}f_j\right)\Delta\omega_i + \tfrac{2}{3}f_j\Delta\omega_j\right]}{1 + \tfrac{4}{3}(f_i + f_j + f_i\cdot f_j)}\;;$$

$$\Delta M_j = \frac{f_j k\left[\tfrac{2}{3}f_i\Delta\omega_i + \left(1+\tfrac{4}{3}f_i\right)\Delta\omega_j\right]}{1 + \tfrac{4}{3}(f_i + f_j + f_i\cdot f_j)}.$$

$$\Delta M_i = pk(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j)\;;$$

$$\Delta M_j = pk(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j).$$

$$(2.30.1) \qquad\qquad (2.30.2)$$

Since the incremental bending moment-end rotation equations have a regular pattern for all four states of yield, the following matrix equations using the "effective stiffness" parameters S_A, S_B and S_C can be established:

One-component Model	Two-component Model

$$\begin{Bmatrix}\Delta M_i \\ \Delta M_j\end{Bmatrix} = \begin{bmatrix} S_A & S_B \\ S_B & S_C \end{bmatrix} \begin{Bmatrix}\Delta\omega_i \\ \Delta\omega_j\end{Bmatrix}$$

$$(2.31.1)$$

$$\begin{Bmatrix}\Delta M_i \\ \Delta M_j\end{Bmatrix} = \begin{bmatrix} S_A & S_B \\ S_B & S_C \end{bmatrix} \begin{Bmatrix}\Delta\omega_i \\ \Delta\omega_j\end{Bmatrix}$$

$$(2.31.2)$$

One-component Model, cont.			Two-component Model, cont.		

where

	s_A	s_B	s_C		s_A	s_B	s_C
state of yield:				state of yield:			
a	k	$\frac{1}{2}k$	k	a	k	$\frac{1}{2}k$	k
b	$\dfrac{f_i k}{(1+f_i)}$	$\dfrac{1}{2}\dfrac{f_i k}{(1+f_i)}$	$\dfrac{(3+4f_i)k}{4(1+f_i)}$	b	pk	$\frac{1}{2}pk$	$(1-\frac{q}{4})k$
c	$\dfrac{(3+4f_j)k}{4(1+f_j)}$	$\dfrac{1}{2}\dfrac{f_j k}{(1+f_j)}$	$\dfrac{f_j k}{(1+f_j)}$	c	$(1-\frac{q}{4})k$	$\frac{1}{2}pk$	pk
d	$\dfrac{f_i(1+\frac{4}{3}f_j)k}{D}$	$\dfrac{\frac{2}{3}f_i f_j k}{D}$	$\dfrac{f_j(1+\frac{4}{3}f_i)k}{D}$	d	pk	$\frac{1}{2}pk$	pk

where

$$D = 1 + \frac{4}{3}(f_i + f_j + f_i \cdot f_j)$$

Note that for bilinear hysteresis loops, which are of primary concern here, $f_i = f_j = f$. As a result, some of the effective stiffnesses can be simplified.

In order to identically match the bilinear hysteretic response of the two beam models, an (f:p) relationship must be found which equates the corresponding incremental moment-rotation equations for each of the four states of yield. Although one (f:p) relationship forms a match between the two beam models for yield states (a), (b),

and (c), and another (f:p) relationship forms a match for yield states (a) and (d), it is <u>not</u> possible to find an (f:p) relationship which forms a match for all four states of yield.

However, in order to compare the response of a nonlinear tall structure using the one-component model to the response of the same nonlinear tall structure using the two-component model, it is necessary to choose the most representative (f:p) relationship. Since it is common in a tall structure for both ends of a girder to rotate symmetrically and to yield at the same time, the (f:p) relationship chosen is the one which matches the incremental moment-rotation equations for yield states (a) and (d). With

$$\Delta M_i = \Delta M_j = \Delta M$$

and

$$\Delta \omega_i = \Delta \omega_j = \Delta \omega \ ,$$

the following moment-rotation equations are obtained for yield state (d). For the one-component model, Eq. 2.30.1 yields

$$\Delta M = \frac{f}{(1 + \frac{2}{3} f)} k \Delta \omega \ ; \qquad (2.32.1)$$

and for the two-component model, Eq. 2.30.2 yields

$$\Delta M = 1.5 \ pk \Delta \omega \ . \qquad (2.32.2)$$

By equating the incremental bending moments and end rotations for the two models, the desired (f:p) relationship is found

$$f = \frac{1.5 \, p}{1 - p} \qquad \text{or} \qquad p = \frac{f}{1.5 + f} \qquad .$$

For example, if $p = 0.050$, then $f = 0.079$.

2.6 Hysteresis Loops and Simple Yielding Systems for Nonlinear Beams

In order to draw an exact two-dimensional hysteresis loop for either end of a beam, it is necessary to have the same functional relationship between the bending moment and the curvature for all states of yield. Such a hysteresis loop, Fig. 2.13d, can be established for end (i) of the one-component model of Fig. 2.13a by defining the "simple yielding beam" shown in Fig. 2.13b having the following properties:

M_i	the bending moment at end (i) of the one-component model,
α_i	the plastic angle at end (i) of the one-component model,
ω_{ii}	the rotation of the simple yielding beam having bending moment M_i and plastic angle α_i,
$\omega'_{ii} = \omega_{ii} - \alpha_i$	an angle representing the curvature at end (i) of the central beam of the one-component model, and
$\omega_{ji} = \omega_i - \omega_{ii}$	the angle representing the cross-effect of end (j) on end (i) where ω_i is the end rotation of the one-component beam.

A similar "simple yielding beam" can be defined for end (j) of the one-component model having the above properties but with (j) and (i) interchanged. By using the simple yielding beam for end (i),
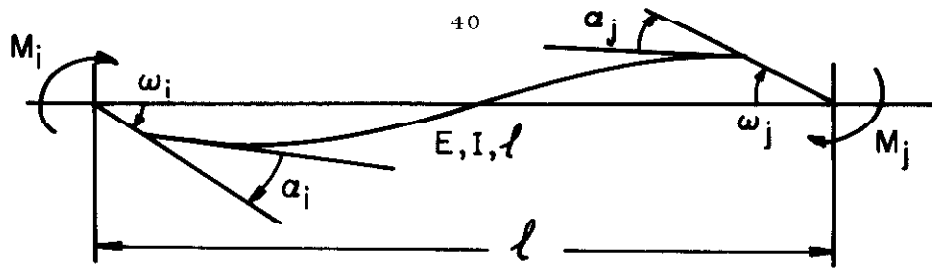
Fig. 2.13a One-component Model
of a Nonlinear Beam
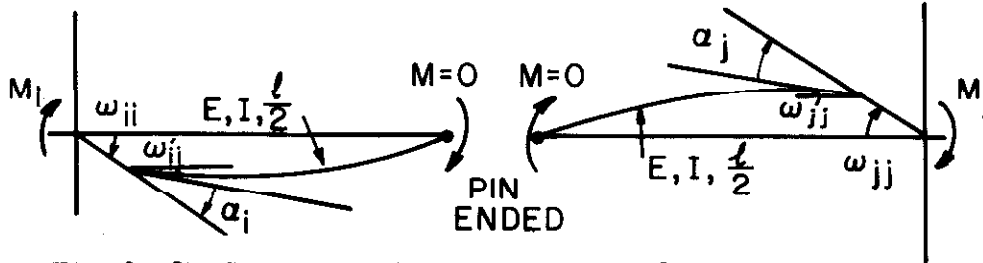
Fig. 2.13b Simple Yielding
Beam for End (i)

Fig. 2.13c Simple Yielding
Beam for End (j)

$$p = \frac{f}{1.5+f}$$

WHERE
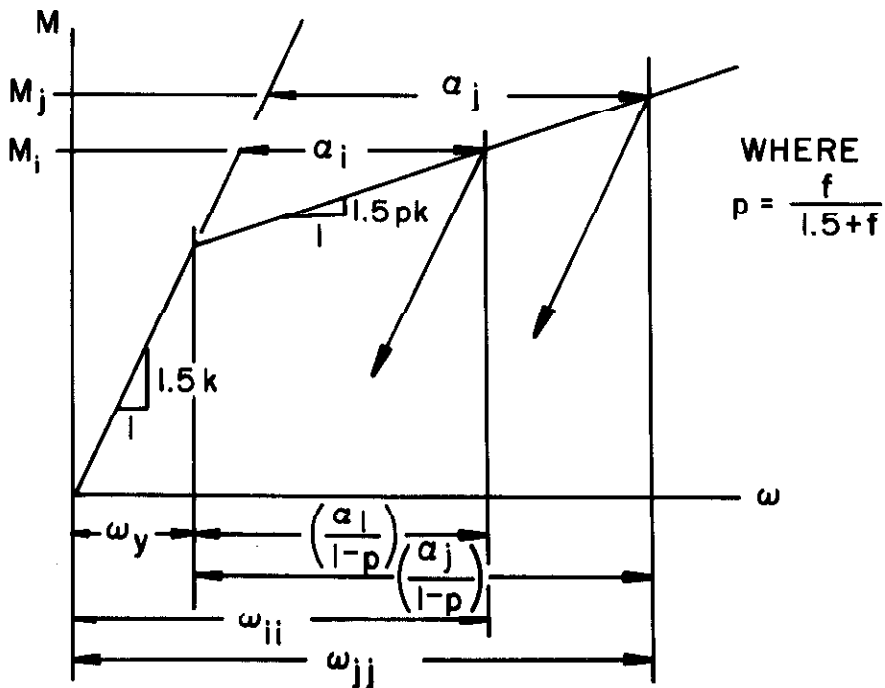
Fig. 2.13d Bilinear Hysteresis Loop for the Simple
Yielding Beams for Ends (i) and (j) of the One-
component Model of a Nonlinear Beam

the bending moment M_i can be directly related to ω'_{ii} (see

Fig. 2.13b) which represents the curvature of the central beam at

end (i) as shown below (similarly for end (j)).

The bending moment-end rotation equations for the simple

yielding beams are:

for end (i), $M_i = 1.5 \, k(\omega_{ii} - \alpha_i)$;

$$(2.33)$$

for end (j), $M_j = 1.5 \, k(\omega_{jj} - \alpha_j)$.

Furthermore, in the nonlinear state of yield, the equations relating

incremental bending moments to incremental plastic angles for the

simple yielding beams are the same ones used at the ends of the one-

component model:

at end (i), $\Delta M_i = f_i k \Delta \alpha_i$;

$$(2.34)$$

at end (j), $\Delta M_j = f_j k \Delta \alpha_j$.

By equating Eq. 2.33 to Eq. 2.28.1, the general moment rota-

tion equations for M_i and M_j, the following equations for the simple

yielding beams are obtained:

$$1.5k(\omega_{ii} - \alpha_i) = k[\, (\omega_i - \alpha_i) + \tfrac{1}{2}(\omega_j - \alpha_j)] \; ;$$

$$(2.35)$$

$$1.5k(\omega_{jj} - \alpha_j) = k[\, \tfrac{1}{2}(\omega_i - \alpha_i) + (\omega_j - \alpha_j)] \; .$$

Hence, equations for ω_{ii} and ω_{jj} are found:

$$\omega_{ii} = \frac{2}{3} \left[\omega_i + \frac{1}{2}\alpha_i + \frac{1}{2}(\omega_j - \alpha_j) \right] \; ;$$

$$\tag{2.36}$$

$$\omega_{jj} = \frac{2}{3} \left[\frac{1}{2}(\omega_i - \alpha_i) + \omega_j + \frac{1}{2}\alpha_j \right] \; .$$

Since

$$\omega_i = \omega_{ii} + \omega_{ji}$$

and

$$\omega_j = \omega_{jj} + \omega_{ij} \; ,$$

an equation relating the cross-effect angles is obtained:

$$0 = \omega_{ij} + \omega_{ji} \; ,$$

or

$$\omega_{ij} = - \omega_{ji} \; . \tag{2.37}$$

This means that the effect of the rotation at end (i) on end (j) is just the negative of the effect of the rotation at end (j) on end (i). Since a definite relationship exists between the bending moment M_i and the angle ω_{ii} and between M_j and ω_{jj} for all states of yield, it is possible to draw a meaningful bilinear hysteresis loop with the abscissa labeled ω_{ii} and ω_{jj} as is done in Fig. 2.13d for both ends of the one-component model of a nonlinear beam. In this case (see Section 2.4),

$$f_i = f_j = f \qquad \text{and} \qquad p = \frac{f}{1.5 + f} \; .$$

The above derivation of the equations for ω_{ii}, ω_{jj}, and ω_{ij} (ω_{ji}) are also valid for a curvilinear hysteresis loop such as the one
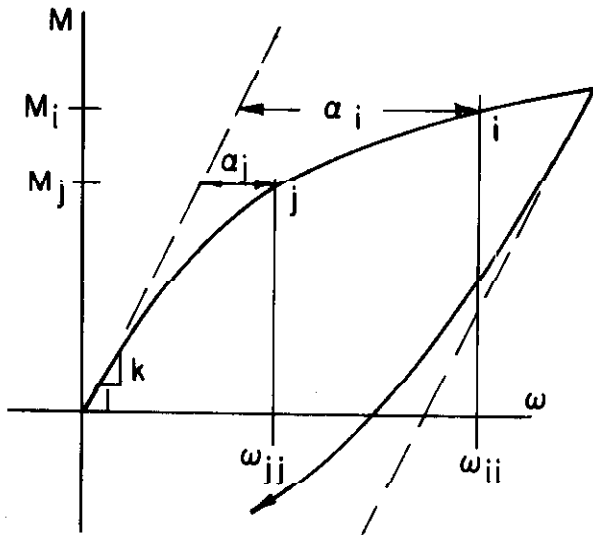
Fig. 2.14a Curvilinear Moment-
Rotation Hysteresis Loop of the
Simple Yielding Systems for
Ends (i) and (j)
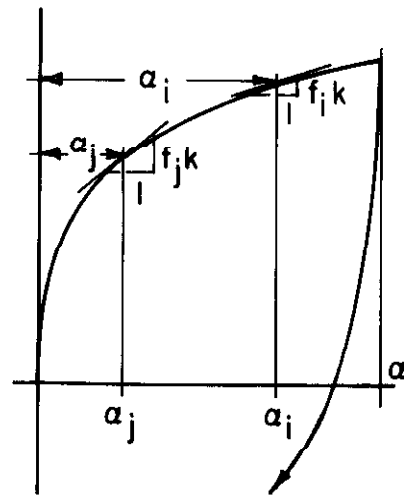
Fig. 2, 14b Associated Moment-
Plastic Angle Hysteresis Loop
of the Simple Yielding Systems
for Ends (i) and (j)
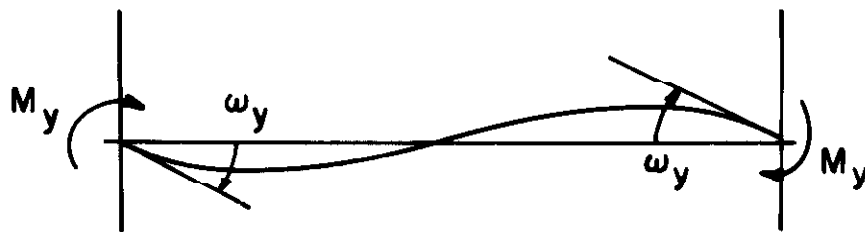
Fig. 2.15 Configuration of Beam
at Point of Incipient Yielding

shown in Fig. 2.14a. Such freedom of choice of the hysteresis loop makes the one-component model quite versatile. The points (i) and (j) on the hysteresis loop in Fig. 2.14a represent the moments, rotations, and plastic angles of the ends of a one-component beam at a particular point in time. The values of f_i and f_j to be used in the next time increment are found from the slopes at the corresponding points (i) and (j) on the associated $M - \alpha$ hysteresis loop of Fig. 2.14b.

In the two-component model, the total bending moment at an end is dependent upon the curvatures of both components of the beam at that end. Since these two curvatures have different functional dependencies on the end rotations and plastic angles, when a beam encounters multiple states of yield (at different times), it becomes impossible to draw a two-dimensional hysteresis loop for the two-component model such as was done above for the one-component model.

2.7 Ductility Factors for Nonlinear Beams

In the previous section (Section 2.6) it was shown that by using the concept of simple yielding beams, bilinear and curvilinear hysteresis loops with precise meanings could be found for each end of a one-component model of a nonlinear beam. Consequently, the three ductility factors for simple yielding systems given in Section 2.3 can be directly applied to either end of a one-component beam. Even though an exact hysteresis loop cannot be found for the two-component

beam, two ductility factors can be defined for the two-component
model which approximate the corresponding definitions for the
one-component model.

It should be recalled that the definition of the plastic angle (α)
for the two-component model is different from the definition of (α)
for the one-component model. Since no definite hysteresis loop can
be drawn for the two-component model which is valid for all states of
yield, one can only obtain an approximate relationship between the
incurred plastic angles, α, of the two systems. Since the beam
usually yields approximately symmetrically, one can approximately
relate the α's of the two systems by

$$\alpha_{\substack{\text{one-}\\\text{component}}} \doteq (1-p)\alpha_{\substack{\text{two-}\\\text{component}}} \qquad . \qquad (2.38)$$

The incipient yield angle ω_y appears in the definitions which
are used for bilinear hysteretic systems. Knowing the yield moment
M_y, ω_y is found from either a simple yielding beam or from the
entire beam rotating symmetrically as shown in Fig. 2.15. In either
case, the equation relating M_y to ω_y is

$$M_y = \frac{6EI}{l}\,\omega_y$$

One-component Model	Two-component Model

One-component Model

The <u>first definition</u> of ductility factor measures the maximum absolute value of the end rotation of the simple yielding beam with respect to the incipient yield angle (for end (i)):

$$\mu_{1i} = \frac{|\omega_{ii}|_{max}}{\omega_y} \quad , \quad (2.39.1)$$

but since

$$\omega_{ii} = \omega_y + \left(\frac{\alpha_i}{1 - p}\right) , \quad (2.40.1)$$

where

$$p = \frac{f}{1.5 + f} ,$$

Eq. 2.39.1 becomes

$$\mu_{1i} = 1 + \frac{\left|\frac{\alpha_i}{(1 - p)}\right|_{max}}{\omega_y} \quad .(2.41.1)$$

The <u>second definition</u> measures the maximum absolute value of the nonlinear angle, which is the plastic angle (α) for this model, with respect to the incipient yield

Two-component Model

The <u>first definition</u> measures the maximum absolute value of the end rotation of the elasto-plastic component (or of the total beam) with respect to the incipient yield angle of the total beam (for end (i)):

$$\mu_{1i} = \frac{|\omega_i|_{max}}{\omega_y} \quad , \quad (2.39.2)$$

but since

$$\omega_i = \omega_y + \alpha_i , \quad (2.40.2)$$

Eq. 2.39.2 becomes[*]

$$\mu_{1i} = 1 + \frac{|\alpha_i|_{max}}{\omega_y} \quad . \quad (2.41.2)$$

Using Eq. 2.38, it is seen that Eq. 2.41.1 and Eq. 2.41.2 are approximately equal.

The <u>second definition</u> measures the maximum absolute value of the (approximate) nonlinear angle, which is

$$(1 - p) \cdot \alpha ,$$

---

[*]This is the definition of ductility factor (or ductility requirement) used in the FHA Study, reference 1.

<div style="float:left; width:50%">

One-component Model, cont.

angle (for end (i)):

$$\mu_{2i} = 1 + \frac{|\alpha_i|_{max}}{\omega_y} \qquad . \quad (2.42.1)$$

The <u>third definition</u> measures the

maximum absolute value of the

total end rotation of the simple

yielding beam with respect to the

linear end rotation (for end (i)):

$$\mu_{3i} = \left| \frac{\omega_{ii}}{\omega_{ii} - \alpha_i} \right|_{max} \qquad . \quad (2.43.1)$$

The form of Eq. 2.43.1 is useful

for curvilinear hysteresis loops;

however, for bilinear hysteresis

loops, Eq. 2.40.1 can be used to

eliminate ω_{ii}. The resulting

equation is (for end (i)):

$$\mu_{3i} = 1 + \left| \frac{\alpha_i}{\omega_y + \frac{p\alpha_i}{(1-p)}} \right|_{max} \qquad .$$

$$(2.44.1)$$

Eq. 2.44.1 is the one used in the

</div>

<div style="float:right; width:50%">

Two-component Model, cont.

where (α) is the plastic angle for

this model, with respect to the

incipient yield angle (for end (i)):

$$\mu_{2i} = 1 + \frac{(1-p)|\alpha_i|_{max}}{\omega_y} . (2.42.2)$$

Using Eq. 2.38, it is seen that

Eq. 2.42.1 and Eq. 2.42.2 are

approximately equal.

The <u>third definition</u> of ductility

factor is not written for the two-

component model since it is im-

possible to use this model for

curvilinear hysteresis loops.

</div>

48

One-component Model, cont. | Two-component Model, cont.

computer studies for finding the ductility factors of one-component beams. The primary reason for using this definition is that it can be used for curvilinear hysteresis loops as well.

In this chapter two models of a nonlinear beam were discussed and compared. It was seen that the one-component model was more realistic from the physical point of view. Furthermore, the one-component model was shown to be more versatile since it could treat curvilinear hysteresis loops whereas the two-component model was restricted to bilinear hysteresis loops. Consequently, the one-component model is considered to be the better model of a nonlinear beam and is therefore used in the computer studies of nonlinear tall structures for this report.

In the next chapter the equations of motion of a nonlinear tall structure are derived wherein either of these beam models can be used to represent the girders and columns of the structure.

CHAPTER III

EQUATIONS OF MOTION FOR NONLINEAR
TALL STRUCTURES

3.1   Introduction

The purposes of this chapter are to describe the properties of a class of nonlinear tall structures, to present the damping mechanisms appropriate to such structures, to derive the equations of motion, to discuss the integration techniques, and to show how the equations of motion are solved with the aid of a digital computer.

3.2   A Class of Nonlinear Tall Structures

The class of nonlinear structures considered in this report is intended to be representative of modern high-rise buildings with glass and other lightweight walls such as are being built today (1967). This class of structures is characterized by the following assumed properties.

1. The foundation is infinitely rigid.

2. The structure is symmetric in plan view; hence, torsional deformation is neglected.

3. The girders provide all of the stiffness to the floors and they are flexible (not infinitely rigid).

4. The columns provide all of the stiffness to the walls.

5. There no shear walls; hence, the structures are of the moment-resisting type. This means that the structures resist deformation only by the moments developed at the
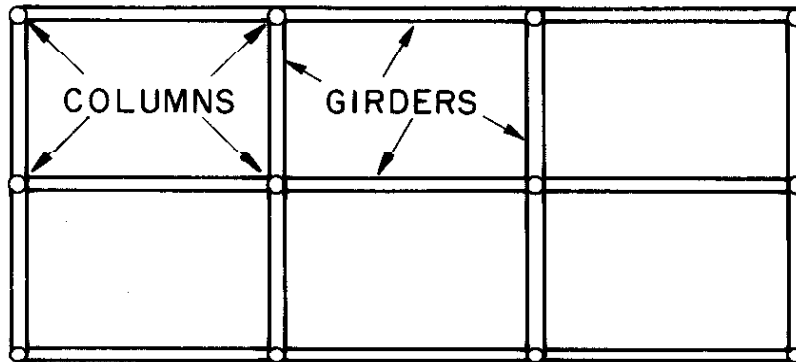
Fig. 3.1a Plan View of a
Moment-resisting Frame
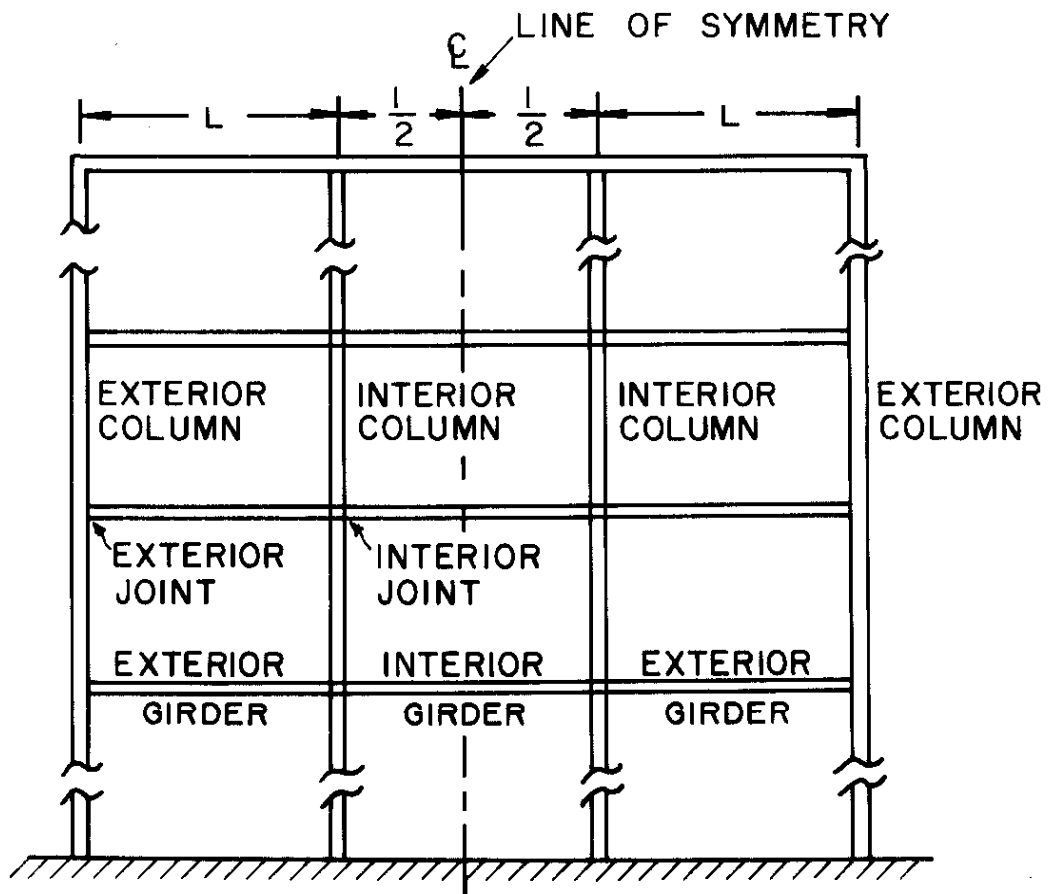
Fig. 3.1b Side View of a
Moment-resisting Frame

ends of the girders and columns.

6. Shear deflection in the girders and columns is neglected.

7. As modeled in Chapter II, the girders and columns can yield at each end according to a bilinear bending moment-end rotation hysteresis loop.

8. There is no extension (or contraction) between any joints within the same floor.

9. There is no extension (or contraction) between floors since it is assumed that the structures are infinitely rigid in the vertical direction.

10. All mass is concentrated at the floor levels.

11. The mass of each floor moves only horizontally. Vertical motion is not considered because of property (9). Although the joints rotate, the rotational inertia associated with each joint is neglected.

12. Gravitational effects are negligible for the base over-turning moment.

The particular computer program developed in this study can analyze only one (two-dimensional) structural frame at a time. In addition to the properties listed above, the structural frame must also have the following:

A. It must have three bays (four columns) and be symmetric with respect to the centerline as shown in Fig. 3.1b.

B. The structural properties of the frame are representative of the three-dimensional structure being considered.

Such a three-dimensional structure is shown in Fig. 3.1a.

C.   The columns are uniformly spaced L inches apart.

D.   There are twenty floors above ground level. With minor

alteration, however, the computer program can handle

an arbitrary number of floors.

Several properties of the frame are optional:

E.   The mass of each floor;

F.   The distance between any two adjacent floors:

G.   The stiffness properties of the girders and columns in

the linear state of yield;

H.   The ratio of the second slope to the first slope of the

bilinear hysteresis loops (see Chapter II); and

I.   The yield moments in the hysteresis loops.

Since the frame is subjected to an earthquake accelerogram

in the computer program, it is of interest to note the properties of

the accelerograms which can be used:

J.   The time history must be digitized.

K.   The time increments can be either uniform or nonuniform.

L.   A multiplicative amplitude scale factor can be used.

One frame having the above structural properties is exten-

sively studied in this report. This is the A/20/2.2/2/6 frame which

was designed by R. W. Clough and K. L. Benuska and was used in their

studies for the Federal Housing Administration.* An advantage of

using this particular frame is that a basis for comparison exists for

_____

*See Section 2.2 of the FHA Study, reference 1.

verifying that the computer program for analyzing nonlinear frames written for this report operates properly (see Section 4.5). Independent studies using this program with various modifications can then be made with confidence in the results.

This frame was designed by means of static analysis using a digital computer. The design moments and design forces in the frame were found in a conventional manner by taking into consideration static gravity loads plus lateral loads as specified by the Uniform Building Code (of 1965). The design of the girders and columns is such that after the static gravity loads (but not the lateral loads) are applied, the positive and the negative yield bending moment for each hysteresis loop have the same absolute value, M_y, as shown in Fig. 2.12.

The identifying numbers of this frame have the following significance:

A: structural properties are tapered;

20: number of floors above ground level;

2.2: (approximate) period of the fundamental mode;

2: ratio of the yield moments to the corresponding design moments in the girders; and

6: ratio of the yield moments to the corresponding design moments in the columns.

For completeness, the structural properties of this frame are given in Fig. 3.2. The stiffness and strength properties of structures are often unrelated and this is reflected in this frame.

A section of a frame having the properties listed above is

54

Mass	Stiffness Properties	Floor	Strength Properties
$\dfrac{(\text{lbs-sec}^2)}{\text{in}}$	Young's Modulus $E = 1.925 \times 10^6$ lbs/in^2	Number from Ground	Yield Moments at the Ends of the Girders and Columns as Shown Below, (in-lbs) $\times 10^{-3}$
half floor	Area Moments of Inertia of the Girders and Columns as Shown Below, (in^4) $\times 10^{-3}$		

Left frame (spans 240" and 120"):

Mass	Left col I	Girder (240")	Mid col I	Girder (120")
230	10	40	20	40
260	10	40	20	40
260	10	40	20	40
260	15	40	30	40
290	15	60	30	60
290	15	60	30	60
290	30	60	60	60
300	30	60	60	60
300	30	60	60	60
300	45	60	90	60
370	45	80	90	80
370	45	80	90	80
370	60	80	120	80
380	60	80	120	80
380	60	80	120	80
380	100	80	200	80
410	100	100	200	100
410	100	100	200	100
410	120	100	240	100
420	120	100	240	100

Right frame (yield moments):

Floor	Col	Girder	Col	Girder
20	1590	1080	1770	987
	2330	1820	3210	1670
18	3010	2300	4620	2160
	3780	2700	5900	2570
16	4480	3350	7440	3210
	5140	3790	8810	3640
14	6280	4220	10200	4080
	6700	4580	11400	4470
12	7200	4960	12400	4850
	8170	5180	13900	5081
10	8500	5980	14600	5860
	9040	6210	15700	6090
8	9730	6520	16700	6410
	10020	6780	17500	6680
6	10210	7016	17900	6910
	11410	7020	19800	6940
4	11000	7700	19100	7620
	11090	7660	19400	7570
2	12060	7620	20600	7540
	17600	7230	34200	7131

Ground

Fig. 3.2. Structural Properties of the A/20/2.2/2/6 Frame. Only Half of the Frame is Shown Since it is Symmetric With Respect to the Centerline (see Fig. 3.1b). The Height Between Floors is 144 Inches Except Between Ground and the First Floor Which is 180 Inches.
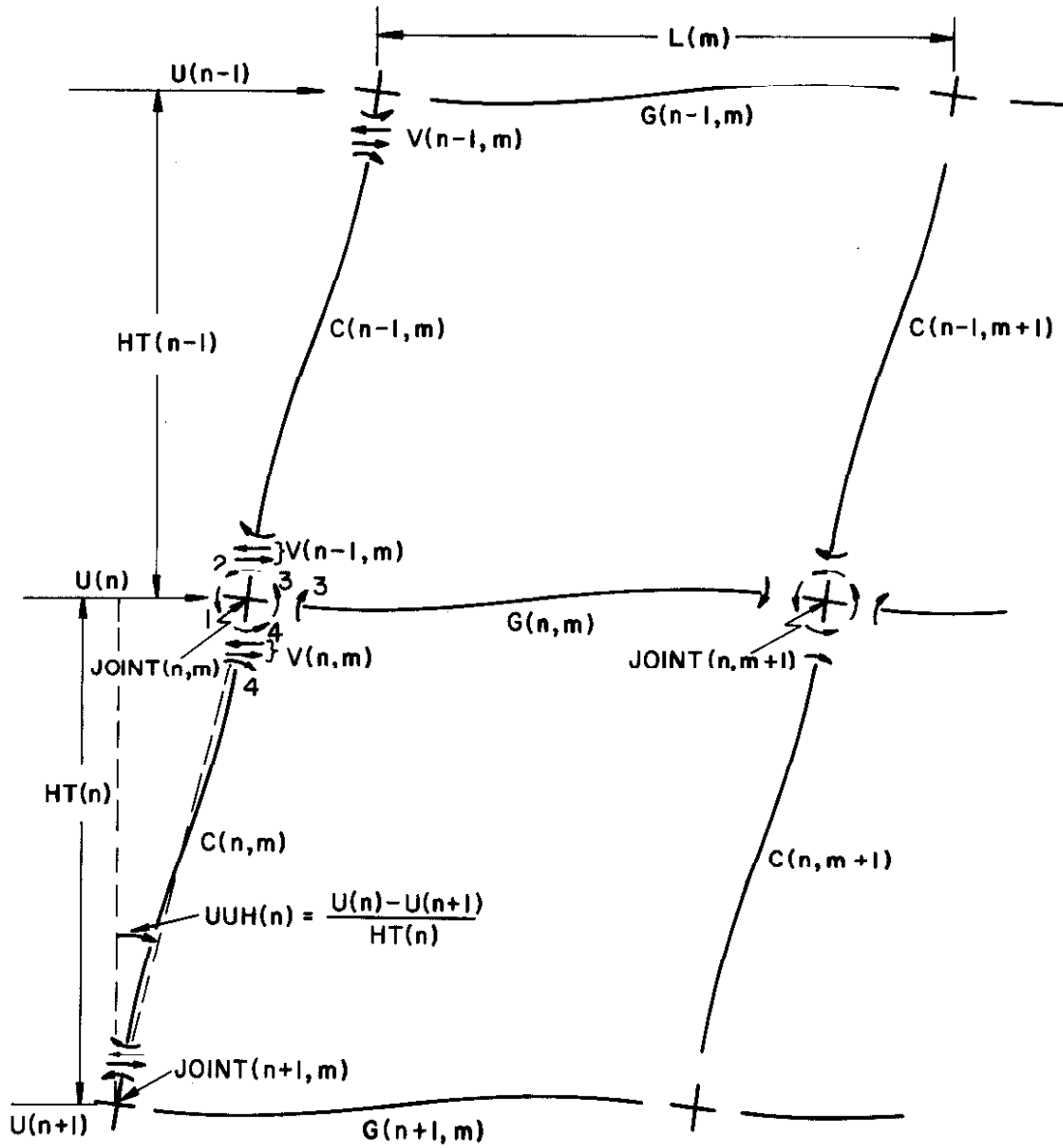
Fig. 3.3 Section of a Typical Moment-resisting Frame
in a Deformed Position
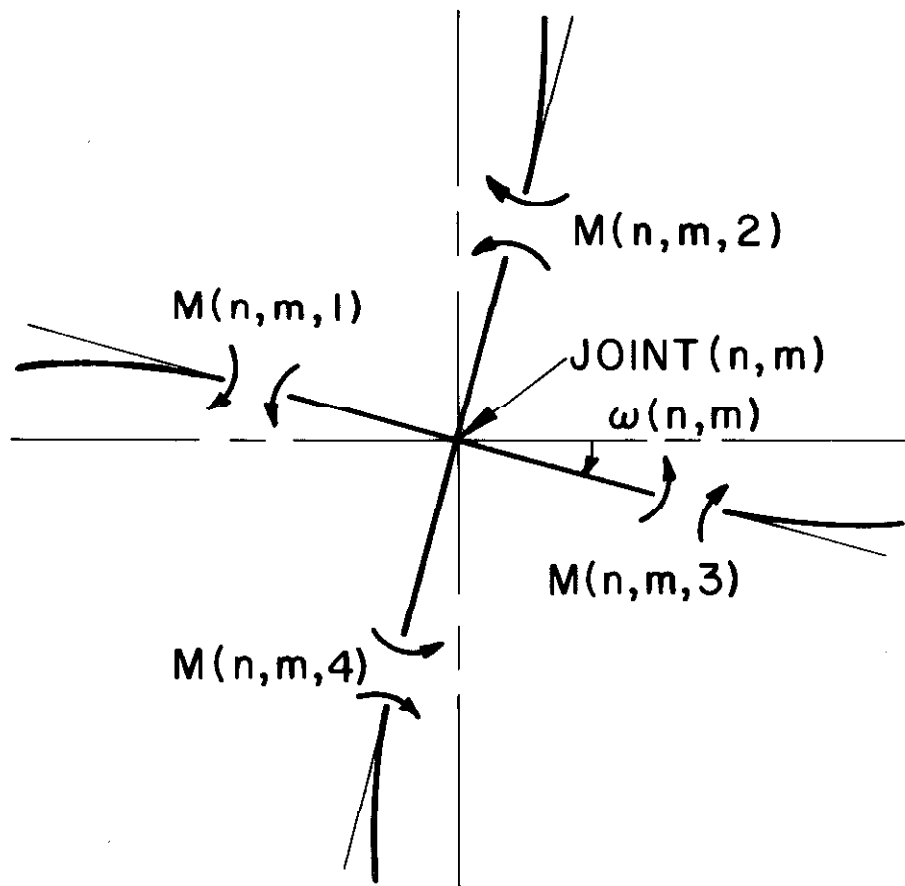
Fig. 3.4 Bending Moments M(n,m,j)
at Joint (n,m)

shown in a deformed position in Fig. 3.3 and a typical joint is shown

in a rotated position in Fig. 3.4. The coordinates, moments and

forces used in this chapter are depicted in Figs. 3.3 and 3.4 in the

positive direction unless labeled to the contrary. These coordinates,

etc., are defined below.[*]

n	: index, the floor number starting from the roof;
m	: index, the column line starting from the left;
j (j = 1,2,3,4)	: index, the number of the intersecting beam (column or girder);
joint (n,m)	: the joint formed by the intersection of girder (n) and column (m);
$\omega(n,m)$	: the angular rotation of joint (n,m);
$\alpha(n,m,j)$	: the plastic angle of the jth beam at joint (n,m) (see Section 2.5);
M(n,m,j)	: the (total) bending moment of the jth beam at joint (n,m);
V(n,m)	: the shear force in column C(n,m);
HT(n,m)	: the height between floor number (n) and (n+1) (numbered from roof);
U(n)	: the horizontal displacement of floor (n); and
$UUH(n) = \dfrac{U(n) - U(n+1)}{HT(n)}$	: the interfloor shear angle between floors (n) and (n+1).

Usually floors are numbered from the ground up as shown in

Fig. 3.2. However, this method of numbering is not convenient for

the computer program since the natural sequence of most operations

is from the roof down. Consequently, in the derivation of the

---

[*]Some of these coordinates, etc., are labeled in "Fortran notation,"
e.g., UUH(n): the interfloor shear angle, since this type of notation
is used in the resulting computer program (see Appendix E).

equations of motion, the floors are numbered from the roof down, as shown in Fig. 3.3.

3.3 Damping

Because of the difficulty of determining the damping mechanisms in structures, conventional approximations are made. Two approximate damping mechanisms which can be adapted to the equations of motion are described below.

1. As a structure vibrates during an earthquake, energy is dissipated aerodynamically. This dissipation is approximately proportional to the product of the area of the face of the structure perpendicular to the motion and the absolute (or total) velocity of the face. However, for the studies performed here, a pseudo-aerodynamic energy dissipation is assumed which uses the velocity relative to ground. Assuming that the area of the face at a given floor level is proportional to the mass of the associated floor, "mass proportional" viscous damping is obtained. The expression for this type of damping, for an N-floor structure is:

$$\alpha M \dot{\underline{U}} \qquad (3.1)$$

where

α: scalar constant,

M: mass matrix $(N \times N)$, and

$\dot{\underline{U}}$: vector $(N \times 1)$ of the velocities of the floors relative to the ground.

2. When a structure vibrates in an earthquake, the interfloor displacements are large enough to cause the walls and partitions to crack and to rub together, thus dissipating energy. This type of damping in a structure is proportional to the relative interfloor velocity, and a convenient approximation is the expression

$$\beta K \dot{\underline{U}} \qquad (3.2)$$

where

β: a scalar constant,

K: a stiffness matrix $(N \times N)$, and

$\dot{\underline{U}}$: vector $(N \times 1)$ of the velocities of the floors relative to ground.

The form of the stiffness matrix which can be used in Expression 3.2 depends to some degree upon the method of solving the equations of motion. Since the equations of motion are nonlinear, they are solved by means of an incremental integration approach (see Sections 3.4 and 3.5). In this case, if K is a tridiagonal stiffness matrix such as the one for a structure with rigid girders[*], Expression 3.2 can be readily adapted to the equations of motion. This type of damping is often referred to as "stiffness proportional" viscous damping.

These two damping mechanisms together comprise "Rayleigh damping" for the structure with rigid girders.

---

[*]The $(N \times N)$ stiffness matrix for an N-floor structure with flexible girders is, in general, full.

Assume for the moment that the structure with flexible girders remains linear (does not yield) and that the damping matrix, C, is given by

$$C = \alpha M + \beta K'$$ (3.3)

where

α, β : scalar constants;

M : diagonal mass matrix (N × N); and

K' : stiffness matrix (N × N) for the structure with <u>flexible</u> girders.

The resulting linear homogeneous matrix equation of motion is

$$M\ddot{\underline{U}} + (\alpha M + \beta K')\dot{\underline{U}} + K'\underline{U} = 0$$ (3.4)

where

$\underline{U}$ is the displacement vector (N × 1) .

In this case, classical normal modes can be found.[44] Hence, for mass proportional damping ($\beta = 0$) only, it can be shown that the fraction of critical damping in the nth mode is

$$\xi_n^m = \frac{\alpha}{2\omega_n} \ , \quad n = 1, 2, \ldots, N \ ,$$

where ω_n is the natural frequency (circular); therefore,

$$\xi_1^m > \xi_2^m > \xi_3^m > \ldots > \xi_N^m \ . \quad \text{(See footnote, page 61.)}$$

Furthermore, for stiffness proportional damping ($\alpha = 0$) only, it can be shown that

$$\xi_n^s = \frac{\beta \omega_n}{2} \quad , \quad n = 1, 2, \ldots, N \; ;$$

therefore,

$$\xi_1^s < \xi_2^s < \xi_3^s < \ldots < \xi_N^s \; . \quad *$$

When written in the form of the incremental equations of motion for structures with flexible girders, the Rayleigh damping described above in Expressions 3.1 and 3.2 appears as

$$\Delta \underline{D} = [\, \alpha M + \beta K\,] \, \Delta \underline{\dot{U}} \qquad (3.5)$$

where

$\Delta \underline{D}$: incremental damping force vector $(N \times 1)$;

α, β : scalar constants;

M : diagonal mass matrix $(N \times N)$:

K : tridiagonal stiffness matrix $(N \times N)$ for the structure with rigid girders; and

$\Delta \underline{\dot{U}}$: vector $(N \times 1)$ of the velocities of the floors relative to ground.

---

*For the A/20/2.2/2/6 structure described in the previous section (Section 3.2),

$$\omega_n = 2.84, \; 6.91, \; 11.5, \; 16.0, \; 20.9, \; \ldots \; .$$

Hence, for mass proportional damping, with 0.10 fraction of critical damping in the fundamental mode,

$$\xi_n^m = 0.10, \; 0.041, \; 0.025, \; 0.018, \; 0.014, \; \ldots \; ,$$

and for stiffness proportional damping with 0.10 fraction of critical damping in the fundamental mode,

$$\xi_n^s = 0.10, \; 0.24, \; 0.40, \; 0.56, \; 0.74, \; \ldots \; .$$

In the next section it will be seen how Eq. 3.5 is incorporated into the equations of motion for this class of structures.

3.4 Equations of Motion

In this section the equations of motion are derived for the structural frame described in Section 3.2 having the damping mechanisms of Section 3.3. Two general equations are derived. One of these is for the angular rotation of the joints and the other governs the horizontal motion of the floors. The equation for the rotation of the joints is derived from a balance of bending moments at the ends of the girders and columns at a typical joint and the equation of motion for the floors is derived from a balance of forces on a typical floor. Since the frame is symmetric, as shown in Fig. 3.1b or in Fig. 3.5, these equations of motion are written for only one-half of the frame (the left half) taking symmetry into consideration.

Both of these equations use incremental bending moment-end rotation equations for the girders and columns in the frame. In Chapter II, the moment-rotation equations, Eq. 2.29.1 and Eq. 2.29.2, are developed for beams without making a distinction between columns and girders. However, for the present purpose, a distinction must be made since the interfloor shear angle appears in the moment-rotation equations for the columns but does not appear in the equations for the girders. Using the one-component beam model, the incremental moment-rotation equations for the girders and columns which intersect to form joint (n,m) (see Fig. 3.4) are given in Eq. 3.6. Similar equations can be written for the two-

component beam model.

$$\Delta M(n,m,1) = k_G(n,m-1)\left\{[\Delta\omega(n,m) - \Delta\alpha(n,m,1)]\right.$$

$$\left. + \tfrac{1}{2}[\Delta\omega(n,m-1) - \Delta\alpha(n,m-1,3)]\right\} \;;$$

$$\Delta M(n,m,2) = k_C(n-1,m)\left\{[\Delta\omega(n,m) - \Delta UUH(n-1) - \Delta\alpha(n,m,2)]\right.$$

$$\left. + \tfrac{1}{2}[\Delta\omega(n-1,m) - \Delta UUH(n-1) - \Delta\alpha(n-1,m,4)]\right\} \;;$$

$$(3.6)$$

$$\Delta M(n,m,3) = k_G(n,m)\left\{[\Delta\omega(n,m) - \Delta\alpha(n,m,3)]\right.$$

$$\left. + \tfrac{1}{2}[\Delta\omega(n,m+1) - \Delta\alpha(n,m+1,1)]\right\} \;;$$

$$\Delta M(n,m,4) = k_C(n,m)\left\{[\Delta\omega(n,m) - \Delta UUH(n) - \Delta\alpha(n,m,4)]\right.$$

$$\left. + \tfrac{1}{2}[\Delta\omega(n+1,m) - \Delta UUH(n) - \Delta\alpha(n+1,m,2)]\right\} \;;$$

where

$k_G(n,m)$: stiffness of girder $G(n,m)$[*] in the linear state of yield[**] ;

$k_C(n,m)$: stiffness of column $C(n,m)$ in the linear state of yield.

Just as the $\Delta\alpha$'s were eliminated from the incremental bending moment-end rotation equations in Chapter II, the $\Delta\alpha$'s can be eliminated from Eq. 3.6. Consequently, equations analogous to

———————————

[*]See Section 3.2 for nomenclature of $G(n,m)$; $C(n,m)$.

[**]See Section 2.5 for nomenclature of "linear state of yield."

Eq. 2.31.1 relating the incremental bending moments to end rotations

via effective stiffness parameters can be written

for the girders:

$$\begin{Bmatrix} \Delta M(n,m,3) \\ \Delta M(n,m+1,1) \end{Bmatrix} = \begin{bmatrix} SGA(n,m) & SGB(n,m) \\ SGB(n,m) & SGC(n,m) \end{bmatrix} \begin{Bmatrix} \Delta\omega(n,m) \\ \Delta\omega(n,m+1) \end{Bmatrix} \quad ; \quad (3.7)$$

for the columns:

$$\begin{Bmatrix} \Delta M(n,m,4) \\ \Delta M(n+1,m,2) \end{Bmatrix} = \begin{bmatrix} SCA(n,m) & SCB(n,m) \\ SCB(n,m) & SCC(n,m) \end{bmatrix} \begin{Bmatrix} \Delta\omega(n,m) - \Delta UUH(n) \\ \Delta\omega(n+1,m) - \Delta UUH(n) \end{Bmatrix} .$$

$$(3.8)$$

In Eq. 3.7, $SGA(n,m)$ is the Fortran name for the effective stiffness

parameter which corresponds to S_A in Eq. 2.31.1 (or Eq. 2.31.2)

for the girder $G(n,m)$ having stiffness $k_G(n,m)$ in the linear state

of yield. A similar correspondence applies for the other effective

stiffnesses of the girders and columns.

One of the properties of the class of nonlinear structures

considered is that the rotational inertia associated with each joint

is neglected (see Section 3.2). Hence, the sum of the bending

moments at the ends of the girders and columns at each joint is zero

(see Fig. 3.4):

$$\sum_{j=1}^{4} \Delta M(n,m,j) = 0 \quad . \tag{3.9}$$

By substituting Eq. 3.7 and Eq. 3.8 with appropriate indices, n, m,

and j, into Eq. 3.9, the general equation for the incremental rotation

of joint (n,m) is found:

$$\Delta\omega(n,m) = - \frac{\begin{bmatrix} + SGB(n,m-1)\cdot\Delta\omega(n,m-1) + SGB(n,m)\cdot\Delta\omega(n,m+1) \\ + SCB(n-1,m)\cdot\Delta\omega(n-1,m) + SCB(n,m)\cdot\Delta\omega(n+1,m) \\ - [SCB(n-1,m) + SCC(n-1,m)]\cdot\Delta UUH(n-1) \\ - [SCA(n,m) + SCB(n,m)]\cdot\Delta UUH(n) \end{bmatrix}}{[SGC(n,m-1) + SGA(n,m) + SCC(n-1,m) + SCA(n,m)]}$$

$$(3.10)$$

For the left half of the symmetrical three bay frame of Fig. 3.5, the joints $(n,2)$ and $(n,3)$ are seen to be typical of the joints of column lines $m = 2$ and $m = 3$, respectively. By considering the characteristics of each of these two joints, different equations for the incremental joint rotations of each column line are derived.

Because the girders $G(n,1)$, $n = 1, \ldots, 22$, do not exist in the structure (see Fig. 3.5), an equation for the incremental joint rotation $\Delta\omega(n,2)$ is found by removing the constants $SGB(n,1)$ and $SGC(n,1)$ from Eq. 3.10. Hence, for each joint of column line $m = 2$:

$$\Delta\omega(n,2) = - \frac{\begin{bmatrix} + SGB(n,2)\cdot\Delta\omega(n,3) \\ + SCB(n-1,2)\cdot\Delta\omega(n-1,2) + SCB(n,2)\cdot\Delta\omega(n+1,2) \\ -[SCB(n-1,2) + SCC(n-1,2)]\cdot\Delta UUH(n-1) \\ -[SCA(n,2) + SCB(n,2)]\cdot\Delta UUH(n) \end{bmatrix}}{[SGA(n,2) + SCC(n-1,2) + SCA(n,2)]} \quad .(3.11)$$

For column line $m = 3$, the symmetry of the frame is considered in either of two ways. The first and simpler method is to look at the entire frame and to observe that
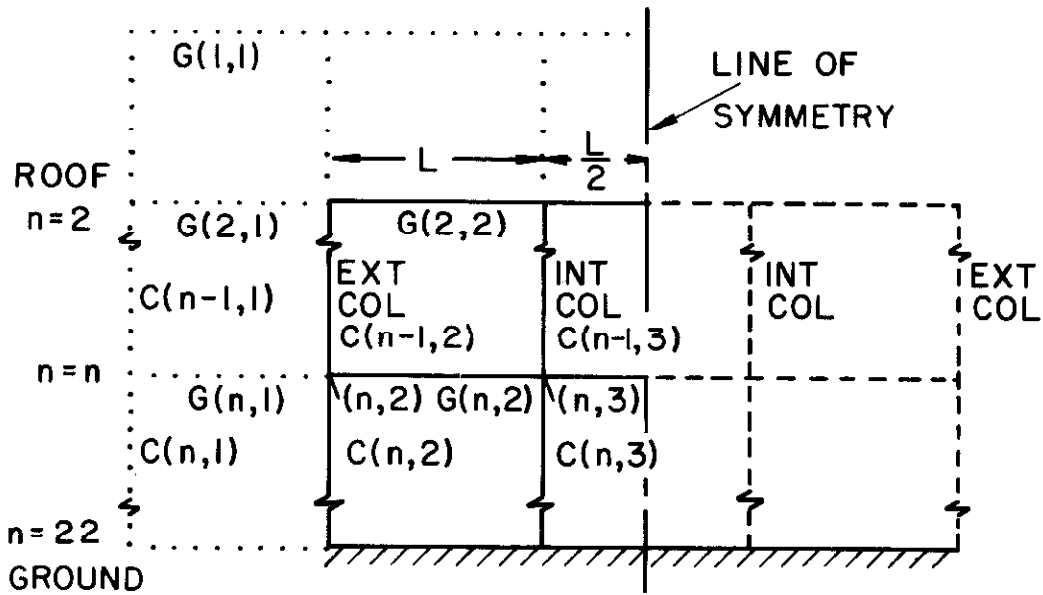
Fig. 3.5 (·····) Dummy Columns and Girders With Stiffnesses Identically Equally to Zero. (Computer Program Artifice)
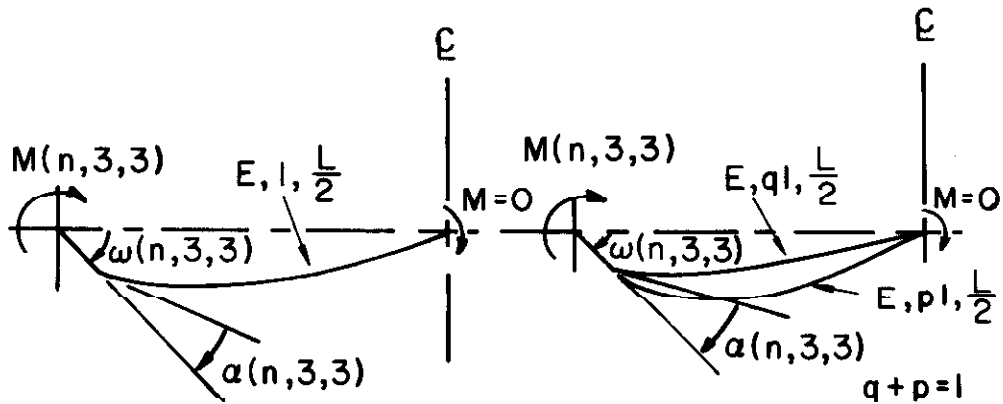
Fig. 3.6a Treatment of Symmetry for One-component Model of Girder G(n, 3). Moment at Centerline Equals Zero.

Fig. 3.6b Treatment of Symmetry for Two-component Model of Girder G(n, 3). Moment at Centerline of Each Component Equals Zero.

$$\Delta\omega(n,4) = \Delta\omega(n,3) . \qquad\qquad (3.12)$$

Consequently, for the interior girders, $G(n,3)$, $n = 2, \ldots , 21$,
a simplified bending moment equation is obtained for $\Delta M(n,3,3)$
by substituting Eq. 3.12 into Eq. 3.7) :

$$\Delta M(n,3,3) = [SGA(n,3) + SGB(n,3)] \cdot \Delta\omega(n,3) . \qquad (3.13)$$

Because the joints at both ends of each interior girder rotate identi-
cally the same amount, only the two yield states (a) and (d) need to be
considered for either beam model:

State (a) - linear at both ends:

$$\Delta M(n,3,3) = 1.5 \cdot k \cdot \Delta\omega(n,3) ; \qquad\qquad (3.14)$$

State (d) - nonlinear at both ends:

$$\Delta M(n,3,3) = 1.5 \cdot p \cdot k \cdot \Delta\omega(n,3) ; \qquad\qquad (3.15)$$

(see Eq. 2.32.1 and Eq. 2.32.2 at the end of Section 2.5). In the
second method the girders are cut at the centerline and pinned so
that the bending moment at the centerline remains identically zero.
For the one-component model this is shown in Fig. 3.6a. For the
two-component model, in order to properly treat symmetry, each
component must be separately pinned at the centerline as shown in
Fig. 3.6b. If, instead, the two components are first connected
together and then pinned, certain problems arise, which are dis-
cussed in Appendix C. Assuming symmetry is considered correctly,
the second method yields the same incremental moment-rotation
equations as the first.

68

Since there are only two states of yield for the interior girders, Eq. 3.14 and Eq. 3.15 are of the form

$$\Delta M(n,3,3) = SGA(n,3) \cdot \Delta\omega(n,3) \qquad (3.16)$$

which uses only one effective stiffness parameter: SGA(n,3). Hence, for the interior girders, SGB(n,3) is unnecessary and is merged into SGA(n,3). The new values of SGA(n,3) are given (for both beam models) by:

state of yield	SGA(n,3)
linear :	$1.5 \cdot k$
nonlinear :	$1.5 \cdot p \cdot k$

Using Eq. 3.16 for $\Delta M(n,3,3)$ instead of Eq. 3.13, the following equation is obtained for each joint of column line $m = 3$:

$$\Delta\omega(n,3) = - \frac{\begin{bmatrix} + SGB(n,2)\cdot\Delta\omega(n,2) \\ + SCB(n-1,3)\cdot\Delta\omega(n-1,3) + SCB(n,3)\cdot\Delta\omega(n+1,3) \\ -[SCB(n-1,3) + SCC(n-1,3)]\cdot\Delta UUH(n-1) \\ -[SCA(n,3) + SCB(n,3)]\cdot\Delta UUH(n) \end{bmatrix}}{[SGC(n,2) + SGA(n,3) + SCC(n-1,3) + SCA(n,3)]} \qquad (3.17)$$

The differential equation of motion for a typical floor of the structural frame is derived from a balance of forces wherein the sum of the shear forces in the columns, V, damping forces, D, and initial force, I, acting on each floor must be zero. These forces, along with the displacement, U, are depicted in incremental form in Fig. 3.7 and are positive as shown.

$\Delta D(n-1)$;
$\Delta I(n-1)$; $\Delta U(n-1)$

$\Delta V(n-1,m)$

$C(n-1,m)$

$\Delta D(n)$;
$\Delta I(n)$; $\Delta U(n)$

$\Delta V(n-1,m)$ $\Delta V(n-1,m+1)$

$\Delta V(n,m)$ $\Delta V(n,m+1)$

$C(n,m)$

$\Delta D(n+1)$;
$\Delta I(n+1)$; $\Delta U(n+1)$

$\Delta V(n,m)$
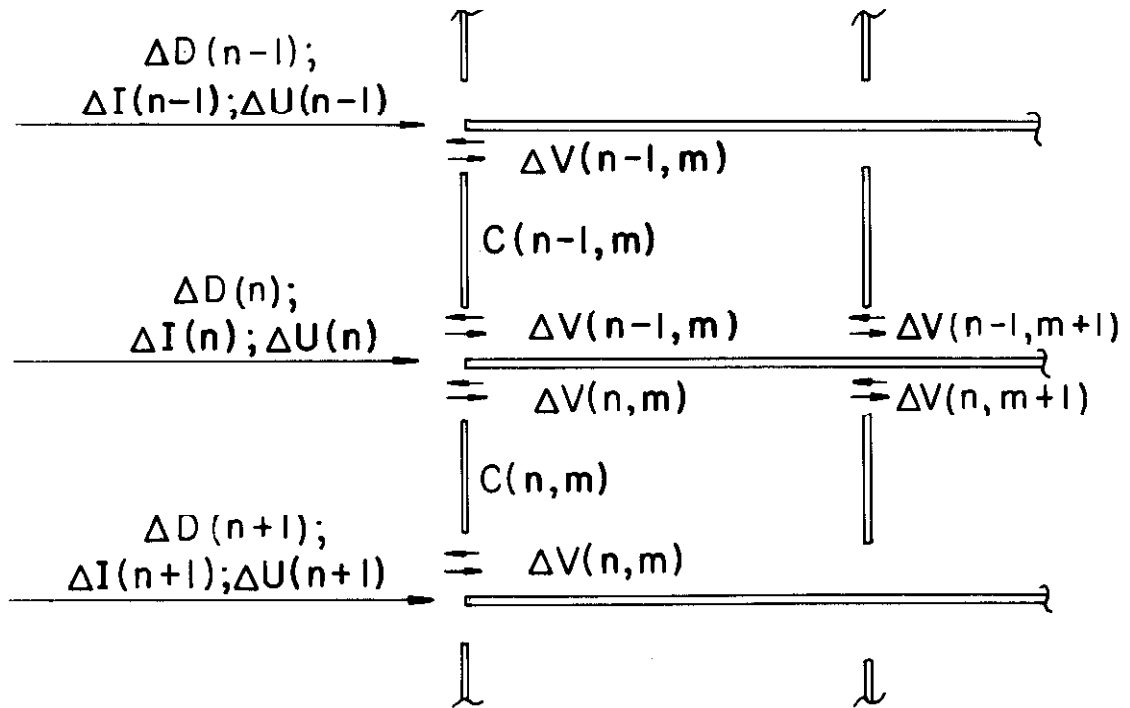
Fig. 3.7

Several constants having Fortran names are used in the equations of motion. Besides those introduced previously, the following ones are used:

$$SCAB(n,m) \quad = \quad SCA(n,m) + SCB(n,m) \quad ;$$

$$SCBC(n,m) \quad = \quad SCB(n,m) + SCC(n,m) \quad ;$$

$$SCABH(n,m) \quad = \quad \frac{SCAB(n,m)}{HT(n)} \quad ;$$

$$SCBCH(n,m) \quad = \quad \frac{SCBC(n,m)}{HT(n)} \quad ;$$

$$(3.18)$$

$$SCD(n) \quad = \quad \sum_{m=2}^{3} [SCAB(n,m) + SCBC(n,m)] \quad ; \quad \text{and}$$

$$SCDHH(n) \quad = \quad \frac{SCD(n)}{[HT(n)]^2} \quad .$$

The incremental shear force in column $C(n,m)$ is

$$\Delta V(n,m) = - \left[\frac{\Delta M(n,m,4) + \Delta M(n+1,m,2)}{HT(n)} \right] \quad . \quad (3.19)$$

Substituting from Eq. 3.8 and from the list of Fortran constants, Eq. 3.18, Eq. 3.19 becomes

$$\Delta V(n,m) = - \frac{1}{HT(n)} \left[\begin{array}{l} SCAB(n,m) \cdot [\Delta\omega(n,m) - \Delta UUH(n)] \\ + SCBC(n,m) \cdot [\Delta\omega(n+1,m) - \Delta UUH(n)] \end{array} \right] \quad ;$$

$$(3.20)$$

where

$$\Delta UUH(n) = \frac{\Delta U(n) - \Delta U(n+1)}{HT(n)} \quad . \quad (3.21)$$

The resultant incremental shear force acting on (the left half of) the

n^{th} floor of the structural frame in Fig. 3.7 is given by the expression

$$\sum_{m=2}^{3} [\Delta V(n-1,m) - \Delta V(n,m)] \quad .\tag{3.22}$$

Using Eq. 3.20 for ΔV and Eq. 3.21 for ΔUUH, and substituting from the list of Fortran constants, Eq. 3.18, an equation for the resultant incremental shear force acting on the nth floor is found:

$$\sum_{m=2}^{3} [\Delta V(n-1,m) - \Delta V(n,m)]$$

$$= - \sum_{m=2}^{3} SCABH(n-1,m) \cdot \Delta\omega(n-1,m) - \sum_{m=2}^{3} SCBCH(n-1,m) \cdot \Delta\omega(n,m)$$

$$+ \sum_{m=2}^{3} SCABH(n,m) \cdot \Delta\omega(n,m) + \sum_{m=2}^{3} SCBCH(n,m) \cdot \Delta\omega(n+1,m)$$

$$+ SCDHH(n) \cdot \Delta U(n+1) + SCDHH(n-1) \cdot \Delta U(n-1)$$

$$- [SCDHH(n) + SCDHH(n-1)] \cdot \Delta U(n) \quad .\tag{3.23}$$

The incremental damping force acting on the nth floor is found from Eq. 3.5:

$$\Delta D(n) = \left\{ \begin{array}{l} - 2\xi_1^m \omega_1 M(n) \cdot \Delta\dot{U}(n) \\[2ex] + \left(\dfrac{2\xi_1^s}{\omega_1}\right) \cdot \left\{ \begin{array}{l} + SCDHH(n-1) \cdot \Delta\dot{U}(n-1) \\[1ex] -[SCDHH(n-1) + SCDHH(n)] \cdot \Delta\dot{U}(n) \\[1ex] + SCDHH(n) \cdot \Delta\dot{U}(n+1) \end{array} \right. \end{array} \right\} ;\tag{3.24}$$

where

ξ_1^m :   the fraction of critical damping for the fundamental mode of the linear structure with only mass proportional damping;

ξ_1^s :   the fraction of critical damping for the fundamental mode of the linear structure with only stiffness proportional damping;

ω_1 :   circular frequency of the fundamental mode,

$M(n)$:   mass of one half of the nth floor, and

$\Delta \dot{U}(n)$:   incremental velocity of the nth floor relative to the foundation.

Although both mass proportional and stiffness proportional damping are included in the equations of motion, only mass proportional damping is used in the computer program for this report (see Subroutine CONV in Appendix E).

The incremental inertial force, ΔI, acting on the nth floor is given by

$$\Delta I(n) = -M(n) \cdot (\Delta \ddot{U}(n) + \Delta \ddot{y}) \qquad (3.25)$$

where

$M(n)$:   mass of one half of the nth floor,

$\Delta \ddot{U}(n)$:   incremental acceleration of the nth floor relative to the foundation; and

$\Delta \ddot{y}$:   incremental acceleration of the foundation.

Since the incremental force balance for the nth floor is

$$\Delta I(n) + \Delta D(n) + \sum_{m=2}^{3} [\Delta V(n-1,m) - \Delta V(n,m)] = 0 , \qquad (3.26)$$

by substituting from Eq. 3.23, Eq. 3.24, and Eq. 3.25, the differential equation of motion in incremental form is obtained:

$$M(n) \cdot \Delta \ddot{U}(n)$$

$$+ \left[2\xi_1^m \cdot \omega_1 \cdot M(n) + 2\left(\frac{\xi_1^s}{\omega_1}\right) \cdot (SCDHH(n) + SCDHH(n-1)) \right] \cdot \Delta \dot{U}(n)$$

$$+ [SCDHH(n) + SCDHH(n-1)] \cdot \Delta U(n)$$

$$+ \sum_{m=2}^{3} [SCBCH(n-1,m) - SCABH(n,m)] \cdot \Delta \omega(n,m)$$

$$- \left(\frac{2\xi_1^s}{\omega_1}\right) \cdot SCDHH(n-1) \cdot \Delta \dot{U}(n-1) - SCDHH(n-1) \cdot \Delta U(n-1)$$

$$+ \sum_{m=2}^{3} SCABH(n-1,m) \cdot \Delta \omega(n-1,m)$$

$$- \left(\frac{2\xi_1^s}{\omega_1}\right) \cdot SCDHH(n) \cdot \Delta \dot{U}(n+1) - SCDHH(n) \cdot \Delta U(n+1)$$

$$- \sum_{m=2}^{3} SCBCH(n,m) \cdot \Delta \omega(n+1,m)$$

$$= - M(n) \cdot \Delta \ddot{y} . \qquad (3.27)$$

In this section, three equations of motion have been derived for the left half of the nth floor of the structural frame taking into consideration the symmetrical properties of the frame. In the next section, the methods used to solve the set of $3N$ equations of motion

for an N-floor nonlinear frame are discussed.

3.5   Solution of the Equations of Motion

As previously mentioned, because of yielding, the stiffness properties of the girders and columns change as a function of time during a strong earthquake. Hence, the equations of motion are nonlinear, which eliminates, for all practical purposes, solutions using modal analysis methods. Nevertheless, the nonlinear equations of motion can be solved by means of an integration technique which uses finite increments of time. In the integration technique used here, it is assumed that the stiffness properties of each girder and column in the structure remain constant throughout each time increment, but that they can change from one time increment to the next. Consequently, within each time increment, the equations of motion are linear.

One criteria for determining the magnitude "ΔT" of the time increment is

$$\Delta T = \frac{1}{10} \, \tau_{20}$$

where τ_{20} is the period of the 20th mode of the linear structure. It should be mentioned that in the studies for this report the A/20/2.2/2/6 structure is analyzed with

$$\Delta T - 0.005 \text{ sec} \approx \frac{1}{6} \, \tau_{20} \, .$$

The integration technique used to solve the equations of

motion is one presented by E. L. Wilson and R. W. Clough[40]

and is similar to the Newmark β-method[42,43] with $\beta = 1/6$. For

this method, it is assumed that:

    1. the acceleration is linear within each time interval; and

    2. the acceleration, velocity and displacement at the
       beginning of the time interval are known.

With these assumptions, for each floor, a set of equations relating

the incremental acceleration, velocity, and displacement that occur

during the tth time interval can be found. One form of this set of

equations is the following:

$$\Delta U(n)_t = \Delta U(n)_t \; :$$

$$\Delta \dot{U}(n)_t = \frac{3}{\Delta t} \Delta U(n)_t + B(n) \; : \qquad (3.28)$$

$$\Delta \ddot{U}(n)_t = \frac{6}{(\Delta t)^2} \Delta U(n)_t + A(n) \; :$$

where

$$A(n) \quad = - \frac{6}{\Delta t} \ddot{U}(n)_{(t-\Delta t)} - 3\dot{U}(n)_{(t-\Delta t)} \; ;$$

$$B(n) \quad = - 3\dot{U}(n)_{(t-\Delta t)} - \frac{\Delta t}{2} \ddot{U}(n)_{(t-\Delta t)} \; ;$$

    $t-\Delta t$:   time at the beginning of the increment; and

    t :   time at the end of the increment.

A(n) and B(n) appear as constants in Eq. 3.28 since they are deter-

mined at the beginning of the time increment. Using Eq. 3.28, the

incremental acceleration and incremental velocity are eliminated

from the equations of motion leaving only the incremental displace-

ment; hence, it is said to be the "ΔU" form. Furthermore,

Eq. 3.28 can be manipulated into any one of six forms, ΔU, $\Delta \dot{U}$, $\Delta \ddot{U}$, U, $\dot{U}$, or $\ddot{U}$, where the name of the form corresponds to the variable that remains in the equations of motion.

Using Eq. 3.28, the equation of motion, Eq. 3.27, becomes

$$\left\{ \begin{array}{l} \left(\dfrac{6}{(\Delta t)^2} + \dfrac{6\xi_1^m \omega_1}{\Delta t} \right) \cdot M(n) \\ \\ \quad + \left(1 + \dfrac{6\xi_1^s}{\Delta t \cdot \omega_1} \right) \cdot (SCDHH(n) + SCDHH(n-1)) \end{array} \right\} \cdot \Delta U(n)$$

$$+ \sum_{m=2}^{3} [\, SCBCH(n-1,m) - SCABH(n,m)\,] \cdot \Delta \omega(n,m)$$

$$- \left(1 + \dfrac{6\xi_1^s}{\Delta t \cdot \omega_1} \right) \cdot SCDHH(n-1) \cdot \Delta U(n-1)$$

$$+ \sum_{m=2}^{3} SCABH(n-1,m) \cdot \Delta \omega(n-1,m)$$

$$- \left(1 + \dfrac{6\xi_1^s}{\Delta t \cdot \omega_1} \right) \cdot SCDHH(n) \cdot \Delta U(n+1)$$

$$- \sum_{m=2}^{3} SCBCH(n+1,m) \cdot \Delta \omega(n+1,m)$$

$$= - [\, \Delta \ddot{y} + A(n) + 2 \cdot \xi_1^m \cdot \omega_1 \cdot B(n)\,] \cdot M(n)$$

$$+ \left(\dfrac{2\xi_1^s}{\omega_1} \right) \cdot \left[\begin{array}{l} -[SCDHH(n) + SCDHH(n-1)] \cdot B(n) \\ \\ + SCDIII(n-1) \cdot B(n-1) + SCDHH(n) \cdot B(n+1) \end{array} \right]$$

$$(3.29)$$

where the incremental displacements and joint rotations are the only unknowns.

Two methods for solving the equations of motion, Eq. 3.11, Eq. 3.17, and Eq. 3.29, for a nonlinear tall structure are considered here. One is entitled relaxation-iteration and the other, matrix substitution. In the relaxation-iteration method, for each time increment, the three equations of motion are solved for each floor beginning with the top floor (roof) keeping all of the other floors fixed. Then, the equations for the adjacent floor are solved keeping all of the other floors, including the roof, fixed. This procedure is continued sequentially until the equations for all the floors have been "relaxed." By iteration, or repetition of this relaxation process until a convergence criteria is satisfied, the incremental displacements and rotations are found for this time increment. Then the succeeding time interval is considered and the relaxation-iteration process is applied again.

In the matrix substitution method,[37] which is described in more detail later in this section, a matrix equation is written involving a large tridiagonal matrix with 3×3 matrices as elements. For each time increment the equations of motion are solved according to a recursion relationship. No iteration is necessary.

In summary, any one of three forms, ΔU, $\Delta \dot{U}$, or $\Delta \ddot{U}$, of the integration technique can conceivably be used with either one of the two methods, iteration or matrix, for solving the incremental equations of motion.

Four of the combinations were studied: $\Delta \ddot{U}$-iteration, $\Delta \ddot{U}$-matrix, ΔU-iteration, and ΔU-matrix. Of these, only the

$\Delta \ddot{U}$-matrix does not work well. In this case, some of the

3×3 matrices are ill-conditioned since the relative magnitudes of

the elements may be on the order of 10^6. The other three combi-

nations yield approximately the same answers (resulting displace-

ments are the same to about 4 decimal places) and take about the

same amount of computer time, using single precision arithmetic

(8 decimal digits). The combination used in all of the computer

studies presented in this report is the ΔU-matrix method using

double precision arithmetic* (16 decimal digits) for solving the

incremental equations of motion and finding the displacements, joint

rotations and interfloor displacements. However, the bending

moments, shear forces, etc., are calculated using single precision.

Throughout this chapter the roof has been numbered $n = 2$

and the first floor above ground level $n = 21$ for reasons mentioned

earlier. However, there is one exception: for the matrix operations

described below, $n = 1$ corresponds to the roof and $n = 20$ cor-

responds to the first floor above the ground level. It should be

mentioned that this notation, including the exception for the matrix

operations, is consistent with the computer program.

The three equations of motion for the nth floor of the

structural frame, Eq. 3.11, Eq. 3.17 and Eq. 3.29, can be written

together as a matrix equation, Eq. 3.30, which is of the form of

Eq. 3.31. The equations of motion for a tall structure are obtained

by writing Eq. 3.31 for each floor. Together, the equations for a

---

*By using double precision instead of single precision arithmetic,
the computing time for the usual run, i.e., without Subroutine MODE,
was increased approximately 75%.

$$
\left[
\begin{array}{ccc}
-\left(1+\dfrac{6\xi_1^2}{\Delta t \cdot \omega_1}\right) \cdot \text{SCDHH}(n-1) & +\text{SCABH}(n-1,2) & +\text{SCABH}(n-1,3) \\
-\text{SCBCH}(n-1,2) & +\text{SCB}(n-1,2) & 0 \\
-\text{SCBCH}(n-1,3) & 0 & +\text{SCB}(n-1,3)
\end{array}
\right.
$$

$$
\left.
\begin{array}{cc}
\left(1+\dfrac{6\xi_1^s}{\Delta t\omega_1}\right)\left(\text{SCDHH}(n-1)+\text{SCDHH}(n)\right) & +\text{SCBCH}(n-1,2) \\
\ +M(n)\left(\dfrac{6}{\Delta t}+\dfrac{6\xi_1^m \omega_1}{\Delta t}\right) & \\
+\text{SCBCH}(n-1,2) & +\text{SCC}(n-1,2) \\
-\text{SCABH}(n,2) & +\text{SGA}(n,2) \\
& +\text{SCA}(n,2) \\
+\text{SCBCH}(n-1,3) & +\text{SGB}(n,2) \\
-\text{SCABH}(n,3) &
\end{array}
\right.
$$

$$
\left.
\begin{array}{cc}
+\text{SCBCH}(n-1,3) & -\left(1+\dfrac{6\xi_1^s}{\Delta t\cdot\omega_1}\right)\cdot \\
-\text{SCABH}(n,3) & \text{SCDHH}(n) \\
+\text{SGC}(n,2) & +\text{SCABH}(n,2) \\
+\text{SGC}(n,3) & \\
+\text{SGA}(n,3) & \\
+\text{SCA}(n,3) & +\text{SCABH}(n,3)
\end{array}
\right.
$$

$$
\left.
\begin{array}{cc}
-\text{SCBCH}(n,2) & -\text{SCBCH}(n,3) \\
0 & 0 \\
+\text{SCB}(n,2) & +\text{SCB}(n,3)
\end{array}
\right]
\cdot
\left\{
\begin{array}{c}
\Delta U(n-1) \\
\Delta\omega(n-1,2) \\
\Delta\omega(n-1,3) \\
\Delta U(n) \\
\Delta\omega(n,2) \\
\Delta\omega(n,3) \\
\Delta U(n+1) \\
\Delta\omega(n+1,2) \\
\Delta\omega(n+1,3)
\end{array}
\right\}
$$

$$
=
\left\{
\begin{array}{c}
-M(n)\cdot\left[\Delta\ddot{y}\cdot A(n)+2\xi_1^m\cdot\omega_1\cdot B(n)\right] \\
+\left(\dfrac{2\xi_1^s}{\omega_1}\right)\left[\begin{array}{c}+\text{SCDHH}(n-1)\cdot B(n-1)\\ -[\text{SCDHH}(n)+\text{SCDHH}(n-1)]\cdot B(n)\\ +\text{SCDHH}(n)\cdot B(n+1)\end{array}\right] \\
0 \\
0
\end{array}
\right\}
$$

(3.30)

which is of the form

$$
\left[C_{n-1}^T \quad S \quad C_n\right]
\left\{
\begin{array}{c}
DR(n-1) \\
DR(n) \\
DR(n+1)
\end{array}
\right\}
= DP(n)
$$

(3.31)

where

$$
DR(n) = \left\{
\begin{array}{c}
\Delta U(n) \\
\Delta\omega(n,2) \\
\Delta\omega(n,3)
\end{array}
\right\}
$$

and

$$
DP(n) = \left\{
\begin{array}{c}
-M(n)\cdot\left[\Delta\ddot{y}+A(n)+2\xi_1^m\cdot\omega_1\cdot B(n)\right]+\left(\dfrac{2\xi_1^s}{\omega_1}\right)\left[\begin{array}{c}+\text{SCDHH}(n-1)\cdot B(n-1)\\ -[\text{SCDHH}(n)+\text{SCDHH}(n-1)]\cdot B(n)\\ +\text{SCDHH}(n)\cdot B(n+1)\end{array}\right] \\
0 \\
0
\end{array}
\right\}
$$

twenty floor structure form the tridiagonal matrix equation,

Eq. 3.32:[*]

$$
\begin{bmatrix}
S_1 & C_1 & & & & & & \\
C_1^T & S_2 & C_2 & & & & 0 & \\
& C_2^T & S_3 & C_3 & & & & \\
& & & \bullet & & & & \\
& & & & \bullet & & & \\
& & & & & \bullet & & \\
& & & & C_{17}^T & S_{18} & C_{18} & \\
& 0 & & & & C_{18}^T & S_{19} & C_{19} \\
& & & & & & C_{19}^T & S_{20}
\end{bmatrix}
\cdot
\begin{Bmatrix}
DR_1 \\
DR_2 \\
DR_3 \\
\bullet \\
\bullet \\
\bullet \\
DR_{18} \\
DR_{19} \\
DR_{20}
\end{Bmatrix}
=
\begin{Bmatrix}
DP_1 \\
DP_2 \\
DP_3 \\
\bullet \\
\bullet \\
\bullet \\
DP_{18} \\
DP_{19} \\
DP_{20}
\end{Bmatrix}
$$

(3.32)

This set of equations is solved by successive substitution followed by successive back-substitution as shown below. The equation for $n = 1$ is

$$ S_1 \cdot DR_1 + C_1 \cdot DR_2 = DP_1 \, . $$

Multiplying by S_1^{-1}, an equation for DR_1 is obtained:

$$ DR_1 = -S_1^{-1} \cdot C_1 \cdot DR_2 + S_1^{-1} \cdot DP_1 \, . \qquad (3.33) $$

DR_1, which is composed of the unknown displacement and joint rotations for floor $n = 1$, is eliminated from the equation for floor $n = 2$:

---

[*]This closely follows the matrix-substitution method presented by R. W. Clough, I. P. King, and E. L. Wilson in reference 37.

$$C_1(-S_1^{-1} \cdot C_1 \cdot DR_2 + S_1^{-1} \cdot DP_1) + S_2 \cdot DR_2 + C_2 \cdot DR_3 = DP_2$$

or

$$\overline{S}_2 \cdot DR_2 + C_2 \cdot DR_3 = \overline{DP}_2 \tag{3.34}$$

where

$$\overline{S}_2 = S_2 - C_1^T \cdot S_1^{-1} \cdot C_1$$

and

$$\overline{DP}_2 = DP_2 - C_1^T \cdot S_1^{-1} \cdot DP_1 \ .$$

From Eq. 3.34 a recursive relationship is seen for the nth floor:

$$\overline{S}_n \cdot DR_n + C_n \cdot DR_{n+1} = \overline{DP}_n \tag{3.35}$$

where

$$\overline{S}_n = S_n - C_{n-1}^T \cdot S_{n-1}^{-1} \cdot C_{n-1}$$

and

$$\overline{DP}_n = DP_n - C_{n-1}^T \cdot S_{n-1}^{-1} \cdot DP_{n-1} \ .$$

Noting that both C_{20} and DR_{21} are nonexistent for a 20 floor structure, the equation for floor $n = 20$ is

$$\overline{S}_{20} \cdot DR_{20} = \overline{DP}_{20} \tag{3.36}$$

where $\overline{S}_{20}$ and $\overline{DP}_{20}$ are found by successive substitution. Since the only unknown in Eq. 3.36 is DR_{20}, it can be found from the equation

$$DR_{20} = \overline{S}_{20}^{-1} \cdot \overline{DP}_{20} \ . \tag{3.37}$$

Having DR_{20}, by means of back-substitution using the recursive relationship

$$DR_n = \bar{S}_n^{-1}(\overline{DP}_n - C_n \cdot DR_{n+1}) \tag{3.38}$$

which is an inverse form of Eq. 3.35, solutions are found for DR_{19}, $DR_{18} \ldots DR_1$. In this manner, the incremental equations of motion are solved for each time increment.

In this chapter, the class of nonlinear tall structures treated in this report was described in detail, and the equations of motion for this class of structures were derived using the bending moment-end rotation equations for beams developed in Chapter II. Also, the method used to solve the equations of motion with the aid of a digital computer was outlined.

In order to verify that the resulting computer program operates properly, a number of tests were made. These are discussed in the next chapter.

CHAPTER IV

VERIFICATION OF THE COMPUTER PROGRAM

4.1 Introduction

The computer program developed in connection with this
report is based upon the equations of motion derived in the previous
chapter. As an overall check on the program, several tests cover-
ing various aspects of it were made. The purpose of this chapter
is to briefly describe these tests.

4.2 Error Checks in the Program

In the computer solution of the equations of motion, one can
anticipate certain round-off error. To evaluate these errors, the
"error parameters" $\epsilon_1(n,m)$ and $\epsilon_2(n)$ were introduced into the
moment balance and force balance equations, respectively.

For each time increment, the equations of motion were solved
for the joint rotations and the displacements of the floors using
double precision arithmetic (16 decimal digits). The individual bend-
ing moments and forces were found for the girders and columns of
the frame using single precision arithmetic (8 decimal digits) since
the nature of the round-off error in these computations is not so
critical. Having determined the moments and forces , the error
$\epsilon_1(n,m)$ in the bending moment balance at each joint in the frame
is found according to Eq. 4.1, and the error $\epsilon_2(n)$ in the balance
of forces acting on each floor is found according to Eq. 4.2:[*]

[*]See Chapter III for the terminology used in these equations.

$$\sum_{j=1}^{4} M(n,m,j) = \epsilon_1(n,m) ; \qquad (4.1)$$

$$I(n) + D(n) + \sum_{m=2}^{3} [V(n-1,m) - V(n,m)] = \epsilon_2(n,m) \qquad (4.2)$$

In the printed output at the end of the program, the maximum absolute values of $\epsilon_1(n,m)$ and $\epsilon_2(n)$ are printed along with the times at which they occur. For a typical computer run using an earthquake accelerogram, it is found that

$$\left| \epsilon_1(n,m) \right|_{max} \sim 10^{-6} \left| M(n,m,j) \right|_{max} ; \qquad (4.3)$$

and

$$\left| \epsilon_2(n) \right|_{max} \sim 10^{-6} \left| \sum_{m=2}^{3} V(n,m) \right|_{max} . \qquad (4.4)$$

It should be noted that for single precision arithmetic, the limit on accuracy is 10^{-8}.

If the values of the error parameters in the printed output of a computer run are consistent with the typical values given in Eq. 4.3 and Eq. 4.4, then there is reason to accept the results of the run. On the other hand, unusual behavior of the error parameters would lead one to suspect some type of program or computer malfunction. Such difficulties were not encountered in the computer runs for the present investigation.

4.3   The Wilson-Clough Integration Technique[40]

To investigate the properties of the Wilson-Clough integration technique, several different programs based on it were examined. At the same time, other programs were developed which used different integration techniques to solve the equations of motion for either the same or an equivalent system. These programs are described below, and the results are compared.

The equations of motion of the damped, three-mass, bilinear hysteretic system of Fig. 4.1, excited by a constant frequency, sinusoidal base acceleration $\ddot{y}(t)$, were solved using three different computer programs. The first program, explicitly written for this system, solved the equations of motion by means of a third order Runge-Kutta integration technique.* The second program, also explicitly developed for this system, used the Wilson-Clough integration technique. The third program was an adaptation of the regular program for analyzing nonlinear frames developed in connection with this report. In this adaptation the number of floors was reduced from twenty to three, and the girders were made rigid. The resulting model is shown in Fig. 4.2. The same approximately steady state initial conditions and the same sinusoidal excitation were used for all three programs. In each case the integration was carried out for fifteen cycles of the input acceleration. The resulting time history plots of the corresponding displacements were within

*This program, written by Dr. W. D. Iwan of California Institute of Technology, uses the standard integration subroutine of the California Institute of Technology Computing Center.
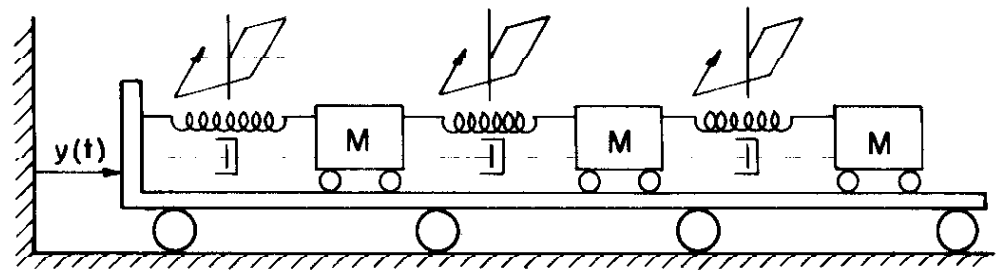
Fig 4.1 Three-mass, Damped, Base Excited System
With Bilinear Hysteretic Springs
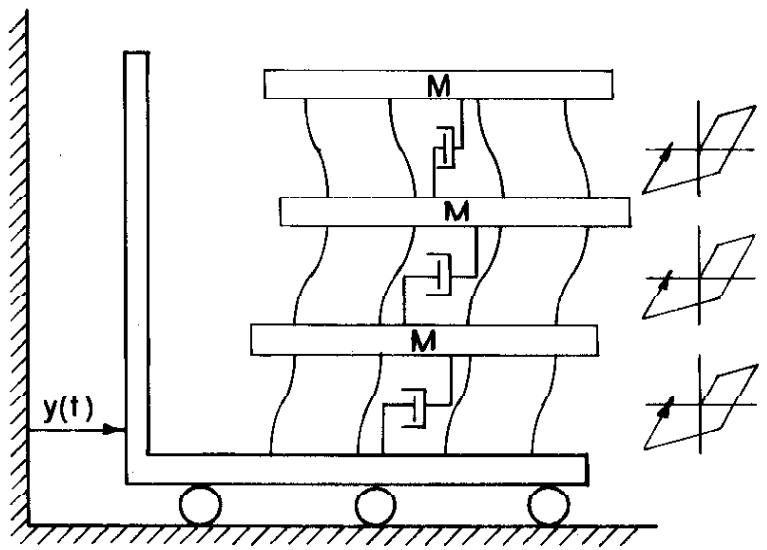
Fig. 4.2 Three-mass, Damped, Base Excited System
With Bilinear Hysteretic Springs
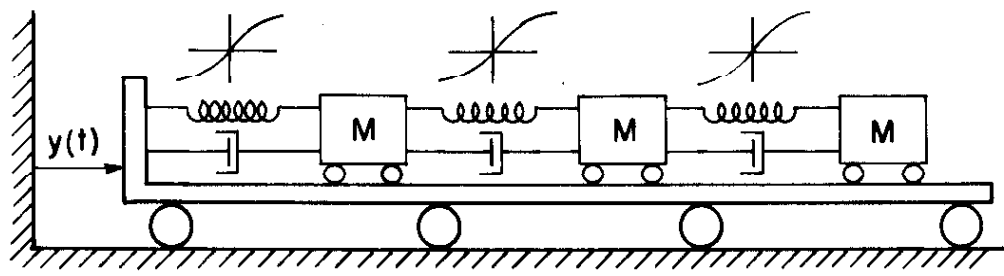
Fig. 4.3 Three-mass, Damped, Base Excited System
With Cubic Nonhysteretic Springs

2% of each other throughout the entire length of the computer runs.

A second system used for comparing the first two programs described above consisted of the damped, three-mass system with cubic-nonhysteretic springs shown in Fig. 4.3. Again, the resulting time history plots of the corresponding displacements were within 2% of each other for the fifteen cycles of the computer run. Even with zero initial conditions, this system achieves steady state motion after nine cycles.

Because of the consistency of the above results, it would appear that the Wilson-Clough integration technique is suitable for the present investigation.

4.4   Excitation of the Linear A/20/2.2/2/6 Frame Near Its Third Natural Frequency

For an additional test of the more complicated system later used in the nonlinear analysis, the computer program was modified so that the base excitation was a constant frequency, sinusoidal accelerogram. Included in this program was a means of checking that the response was approximately steady state.

The A/20/2.2/2/6 frame was used with the yield condition suppressed so that the system remained linear. The specified damping in the fundamental mode was 0.10 fraction of critical. Since mass proportional damping only was used, the effective damping in the higher modes is reduced, being 0.025 fraction of critical in the third mode. The frequencies of the half power points of the steady state resonance curve for a single degree of freedom system

having as a natural frequency the third modal frequency of the frame were analytically calculated (see Table 4-1).

Three computer runs were made in this test: one at each of the three frequencies given in Table 4.1. In each run the initial conditions were zero and the amplitude of the sinusoidal forcing function was the same. After the steady state criteria[*] were satisfied, the amplitude of the displacement of the 20th floor and the phase angle between that floor and the forcing function were found. The three resulting amplitudes, normalized with respect to the amplitude for the third natural frequency, along with the corresponding phase angles are given in Table 4-1.

For a one degree of freedom system at steady state, the normalized amplitudes at the half-power points are 0.707 instead of 0.69 and 0.77, and the phase angles are 45° and 135° instead of 47° and 140°, respectively. The unnormalized displacement of the 20th floor at the third natural frequency as calculated by the computer was 5% smaller than that found by analytical means using eigenvalue theory wherein it was assumed that the contributions to the excitation by all modes except the third were negligible.

---

[*]Within any one complete cycle of the input acceleration, if
$$|U\ MAX(n) + U\ MIN(n)| < 0.05 \cdot U\ MAX(n)$$
for each floor, where

U MAX(n) : the most positive displacement for the nth floor during that cycle, and

U MIN(n) : the most negative displacement for the nth floor during that cycle;

then an approximation to steady state is obtained.

TABLE 4-1

	Lower half-power point	Third natural frequency	Upper half-power point
Circular frequency	11.1905	11.4748	11.7591
Theoretical amplitude, normalized	0.707	1.00	0.707
Amplitude of the displacement of the 20th floor, normalized (computer output)	0.69	1.00	0.77
Theoretical phase angle	45°	90°	135°
Phase angle between displacement of the 20th floor and the forcing function* (computer output)	47°	90°	140°

These discrepancies in amplitudes and phase angles can be explained by the following factors:

1. Because the steady state criteria are not exact, the results of the computer program are only approximately steady state.

2. In the analytical calculations it is assumed that only one mode--the third--contributes to the excitation. However, in this test, base excitation is used which does not excite a pure mode in a multi-degree of freedom structure.

---

*The forcing function used here is

$$f_n(t) = - M(n)\ddot{y}(t)$$

where

$M(n)$: mass of one half of the nth floor, and
$\ddot{y}(t)$: base acceleration.

Consequently, even though the third mode is the dominant

one in this frequency range, the others do contribute.

Since the discrepancies in the results are small and can be

explained by these factors, this test provides further assurance that

the computer program, including the mass proportional damping

mechanism, is working properly.

Another point which reflects upon the accuracy of the computer

program concerns the joint rotations of the linear $A/20/2.2/2/6$

frame. The stiffness properties of this frame (see Fig. 3.2) are

designed[*] so that at each joint within the same floor the ratio of the

sum of the column stiffnesses to the sum of the girder stiffnesses is

the same. For example, for the first floor above the ground, this

ratio for each joint is 2.4. Hence, as long as this frame remains

linear, the rotations of the joints within the same floor should be

equal. Even though the rotation of each joint is calculated independ-

ently, it is seen in the computer output that the rotations of joints

within the same floor are the same to eight significant decimal digits,

thus giving an additional check on the program. However, such a

check cannot be made for the particular nonlinear frame studied in

this report. For this frame, the strength properties are not directly

proportional to the corresponding stiffness properties, and, there-

fore, the rotations within the same floor would not be expected to be

equal. This difference in joint rotations within the same floor of the

nonlinear frame can be seen in the time history plots of Chapter VI.

---

[*]See Section 2.2 of the FHA Study, reference 1.

It is especially evident in the plots for the joints of the 20th floor.

4.5 A Comparison With Results Presented in the FHA Study

In another comparison test, the A/20/2.2/2/6 frame was subjected to earthquake excitation and the resulting responses were compared with corresponding ones presented in the FHA Study. Since the yield criteria and the treatment of symmetry used in the regular nonlinear analysis program for this report differ from those used in the program for the FHA Study, the computer program used for this test includes modifications to remove these differences.

It is the author's understanding that the computer program used for the FHA Study is basically the same one presented in a report to the Office of Civil Defense by T. Y. Lin and Associates.[2] That program can treat an arbitrary number of girders and columns, but since the two-component beam model is used (see Section 2.5), the symmetrical properties of the three bay (four column) frame cannot be treated exactly by cutting the interior girders at the centerline as is done there. Nevertheless, this is an expedient approximation for that program. The problems that are encountered by using such an approximation are discussed in Appendix C. In order to account for the treatment of symmetry used in the FHA Study, the two-component beam model is used in the program for this test with appropriate modifications to the incremental moment-rotation-plastic angle equations.

As mentioned above, the yield criteria used in the regular

nonlinear analysis program for this report differ from those used in the program for the FHA Study. In order that the criteria be the same in both computations, the criteria used in the FHA Study are adopted for purposes of the comparison test. It is noted that the yield criteria used in the FHA program can, in special cases, lead to incorrect responses. The plastic angles and resulting ductility factors are particularly sensitive to this difficulty. The special situations under which these problems may become troublesome are discussed in Appendix B.

The same input data consisting of the structural properties and the accelerogram of the El Centro (N-S) earthquake of 18 May 1940 were used in both computations. This accelerogram is the same as the one used at the Earthquake Engineering Laboratory of the California Institute of Technology, but rounded off to four decimal places.

The responses obtained from the modified program using this input data are the same as those (within the tolerance of the plots) presented in the FHA Study.

In this Chapter several different tests of the computer program were outlined and the results of each test indicate that the program performs as expected. Consequently, it is believed that the computer program for determining the response of nonlinear tall structures subjected to earthquake excitation operates properly and provides the desired accuracy.

CHAPTER V

MODAL ANALYSIS OF CONTINUOUS CANTILEVER BEAMS AND OF TALL STRUCTURES

5.1   Introduction

In the past a number of authors, M. A. Biot,[6] G. N. Bycroft,[34] R. W. Clough,[36] R. L. Jennings,[19] K. Kanai,[32] K. Muto,[29] H. M. Westergaard[30] and others have mentioned that "higher modes" are important and that "whiplash" is observed in tall structures. This chapter is intended to determine the conditions under which higher modes are important as well as to suggest a definition of whiplash for tall structures.

By studying continuous beams and discretized models of tall structures, it is possible to gain a fuller understanding of why the displacements, strains (interfloor displacements), shear forces, and total accelerations have differing dependencies upon the various modes as well as upon the position in the structure. Two linear elastic continuous cantilever beams which represent limiting cases of tall structures are studied. In one, the stiffness and mass distributions are uniform, while in the other, these distributions taper to a point at the top. Furthermore. a discretized model of a typical structural frame $(A/20/2.2/2/6)^*$ is also studied by modal analysis. In Section 5.2 the continuous beams, which are subjected to an ensemble of earthquakes, are analyzed. In particular, the average

---

*See Section 3.2 for a description of the structural properties.
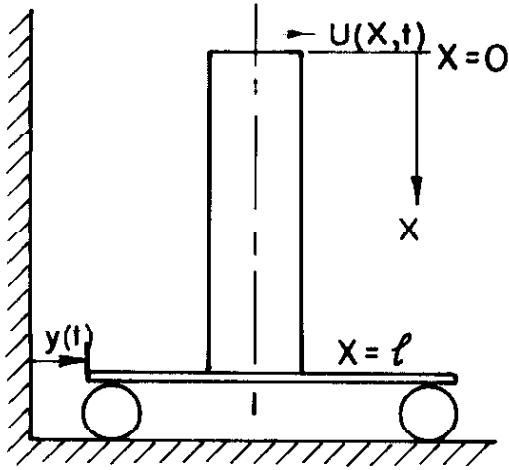
Fig. 5.1 Uniform
Cantilever Beam
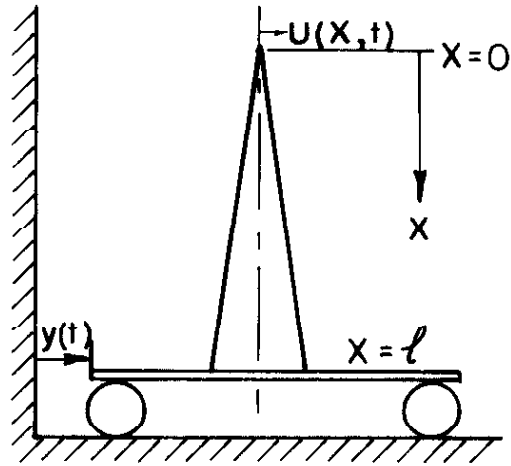
Fig. 5.2 Tapered
Cantilever Beam

Average
Pseudo-
velocity
Spectra,

S_v , $(\frac{in}{sec})$

Undamped Natural Period, Sec
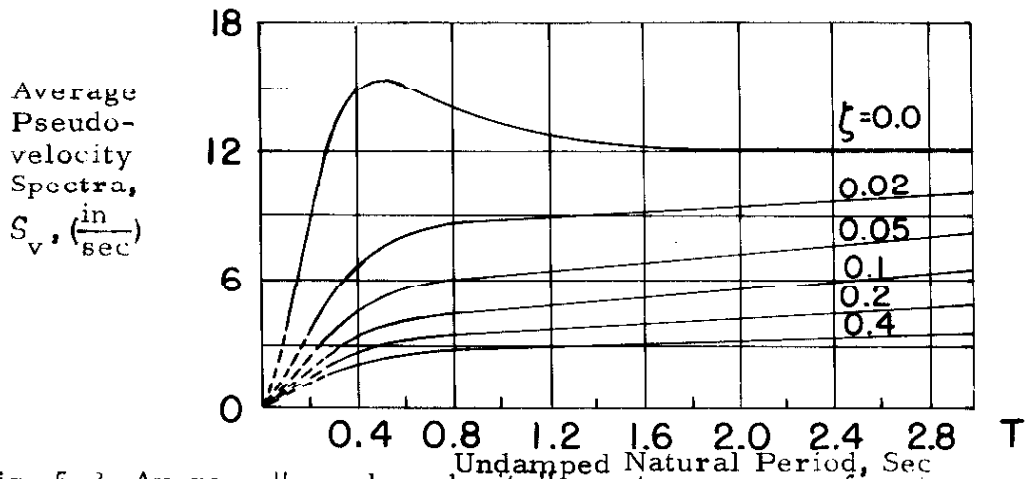
Fig. 5.3 Average "pseudo-velocity" spectrum curves for strongest ground motion. These curves are the average of the four strongest ground motions so far recorded: El Centro, California, May 18, 1940; El Centro, California, Dec. 30, 1934; Olympia, Washington, Apr. 13, 1949; Taft, California, July 21, 1952. Before averaging, each spectrum was normalized so that there was a unit area under the zero-damped curve. To correspond to the respective ground motions, the ordinates must be multiplied by factors as follows: El Centro, 1940 (2.7); El Centro, 1934 (1.9); Olympia, 1949 (1.9); Taft, 1952 (1.6). (Reference 9).

maximum absolute modal contributions to the displacements, strains, shear forces, and total accelerations are related to the average pseudo-velocity spectra S_v. In this chapter, the phrase "average maximum absolute" means average over an ensemble of earthquakes of the maximum absolute value over time of each member of the ensemble. In order to show how these average maximum absolute modal contributions depend upon the taper of the beam and upon the position $(\frac{x}{\ell})$ in the beam (see Fig. 5.1) some of them are later presented in tabular form.

In Section 5.3, the method used to obtain "equivalent" modal participation factors and the "equivalent" modal contributions to displacements and to interfloor displacements for yielding structures is treated. The results obtained by using this method for a typical yielding structural frame are then discussed.

5.2 Modal Analysis of Continuous Beams

One possible model for approximating a tall structure of uniform properties subjected to horizontal earthquake excitation is the shear-type continuous cantilever beam shown in Fig. 5.1. The general equation for this type of cantilever beam excited by the base acceleration $\frac{d^2 y}{dt^2}(t)$ is

$$\frac{\partial}{\partial x}\left(K(x)\,\frac{\partial u}{\partial x}(x,t)\right) = \gamma(x)\left(\frac{\partial^2 u}{\partial t^2}(x,t) + \ddot{y}\right) \tag{5.1}$$

with the boundary conditions:

at x = 0 , $\frac{\partial u}{\partial x} = 0$; and

at x = ℓ , u = 0 ;

where

K(x) is the shear stiffness in units of lbs ;

$\gamma(x)$ is the mass density in units of $\frac{mass}{unit\ length}$

u(x,t) is the relative displacement; and

$$\ddot{y} = \frac{d^2 y}{dt^2} (t).$$

For the uniform beam, the properties of stiffness and mass density are constants:

$$K(x) = K_o , \quad and$$

$$\gamma(x) = \gamma_o .$$

The velocity of wave propagation, c, is defined by

$$\frac{1}{c^2} = \frac{\gamma_o}{K_o} .$$

By substituting these values into Eq. 5.1, the general equation for a uniform beam is obtained:

$$\frac{\partial^2 u}{\partial x^2} (x,t) - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} (x,t) = \frac{1}{c^2} \ddot{y}(t) . \qquad (5.2)$$

The boundary conditions remain the same:

x = 0 $\frac{\partial u}{\partial x} = 0$, and

x = ℓ u = 0 .

The solution of this eigenvalue problem with zero initial conditions is

$$u(x,t) = \frac{2c}{\ell} \sum_{n=1}^{\infty} \frac{(-1)^n}{\omega_n^2} \cos\left(\frac{\omega_n x}{c}\right) \int_0^t \ddot{y}(\tau) \cdot \sin\left(\omega_n(t-\tau)\right) d\tau \quad (5.3)$$

and the eigenfrequencies are given by

$$\frac{\omega_n \ell}{c} = \frac{(2n-1)\pi}{2} \quad , \quad n = 1,2,3,\ldots \quad . \quad (5.4)$$

In this section the primary interest in using modal analysis is in making comparisons between the average maximum absolute values of the individual modal contributions of the responses, and not in finding the sums of the modal contributions. Furthermore, there is some question as to the absolute convergence of the total acceleration series. This is discussed briefly later. Consequently, only the nth terms of the various responses are considered.

From Eq. 5.4, the contribution of the nth mode to the displacement is

$$u_n(x,t) = \frac{2c}{\ell} \frac{(-1)^n}{\omega_n^2} \cos\left(\frac{\omega_n x}{c}\right) \int_0^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau. \quad (5.5)$$

Differentiation of Eq. 5.5 with respect to x gives the contribution of the nth mode to the strain:

$$\frac{\partial u_n}{\partial x}(x,t) = \frac{2}{\ell} \frac{(-1)^{n+1}}{\omega_n} \sin\left(\frac{\omega_n x}{c}\right) \int_0^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \quad . \quad (5.6)$$

It is noted that strain for a continuous model is analogous to the interfloor displacement of a model of a structure.

The contribution of the nth mode to the shear force is obtained by multiplying Eq. 5.6 by K_o:

$$K_o \frac{\partial u_n}{\partial x}(x,t) = \frac{2K_o}{\ell} \frac{(-1)^{n+1}}{\omega_n} \sin\left(\frac{\omega_n x}{c}\right) \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \quad (5.7)$$

From Eq. 5.2 it is seen that the total acceleration is given by

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \ddot{y}(t) = c^2 \frac{\partial^2 u}{\partial x^2}(x,t) \quad (5.8)$$

and the nth modal contribution to the total acceleration is[*]

$$c^2 \frac{\partial^2 u_n}{\partial x^2}(x,t) = \frac{2c}{\ell}(-1)^{n+1} \cos\left(\frac{\omega_n x}{c}\right) \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \quad (5.9)$$

By taking the maximum value over time of the absolute value of Eqs. 5.5, 5.6, 5.7 and 5.9, one finds the maximum absolute value of the nth modal contribution to each response for a given position $(\frac{x}{\ell})$ in the beam. Note that a term common to all of these equations is the maximum over time of the absolute value of the integral

$$S_v(\omega_n) = \max_{0 < t} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \quad (5.10)$$

which is sometimes called the "spectral velocity." Physically,

---

[*]Absolute convergence of this series is discussed below.

$S_v(\omega_n)$ is the absolute value of the maximum pseudo-velocity for an undamped single degree of freedom oscillator with a natural frequency ω_n (or period T_n)[*] subjected to an earthquake accelerogram $\ddot{y}(t)$. Curves of $S_v(T)$ vs. period T have been made for an ensemble of earthquakes with several damping factors.[9] By averaging $S_v(T)$ over the ensemble, average spectral pseudo-velocity curves are obtained which are shown in Fig. 4.3. From these curves one can find the average over an ensemble of earthquakes of the maximum absolute value of the pseudo-velocity of a single degree of freedom oscillator with a given period and fraction of critical damping.

Hence, the average maximum absolute values of the nth modal contributions to the various responses are as follows:

for displacement:

$$\overline{\left|u_n(x)\right|}_{\substack{max \\ 0 < t}} = \frac{2\ell}{c} \frac{4}{\pi^2(2n-1)^2} \cos\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \tag{5.11}$$

for strain:

$$\overline{\left|\frac{\partial u_n}{\partial x}(x)\right|}_{\substack{max \\ 0 < t}} = \frac{2}{c} \frac{2}{\pi(2n-1)} \sin\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \tag{5.12}$$

---

[*]For linear systems "pseudo-velocity" equals the displacement times the natural frequency. Studies comparing pseudo-velocity spectra to velocity spectra for a one degree of freedom linear system excited by an ensemble of earthquakes have been performed by C. V. Chelapati, reference 15.

for shear force:

$$\left| K_o \frac{\partial u_n}{\partial x} (x) \right|_{\substack{max \\ 0 < t}} = \frac{2K_o}{c} \frac{2}{\pi(2n-1)} \sin\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \qquad (5.13)$$

and for total acceleration:

$$\left| c^2 \frac{\partial^2 u_n}{\partial x^2} (x) \right|_{\substack{max \\ 0 < t}} = \frac{2c}{l} \cos\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \quad . \qquad (5.14)$$

In Fig. 5.3 it is seen that for periods greater than 0.3 seconds (frequency less than $\frac{2\pi}{0.3}$ radians per second) and for very lightly damped systems the average spectral velocity $S_v(\omega)$ is approximately a constant, to be denoted by $\overline{S}_v$. Since the periods of only the first five modes of the $\Lambda/20/2.2/2/6$ structure are larger than 0.3 sec, only the first five modes of the continuous beams are considered below.

Looking again at Fig. 5.3, one sees that for periods less than 0.3 sec, as the periods approach zero ($\omega \rightarrow \infty$), the average spectral velocity $S_v(\omega)$ decreases approximately in proportion to T. From Eq. 5.4 then,

$$S_v(\omega_n) \sim \frac{1}{n} \quad .$$

This rate of decrease of $S_v(\omega_n)$ is sufficient to ensure absolute convergence of summations of Eqs. 5.5, 5.6, and 5.7 for displacement, strain, and shear force. However, there may be some

question of the absolute convergence of a summation of Eq. 5.9 in

order to obtain the total acceleration, even though the average

maximum absolute contribution of the nth mode does decrease as

(n) increases beyond some value. By assuming that the base

excitation $\ddot{y}(t)$ has a bounded frequency spectrum, absolute con-

vergence can be assured.

In order to show more explicitly the dependence of the

average maximum absolute values of these responses upon position

in the uniform beam, Eqs. 5.11, 5.12, 5.13, and 5.14 are evaluated

at seven different positions in the beam with $S_v(\omega_n)$ replaced by

$\overline{S}_v$. The results are tabulated in Tables 5-2, 5-4, 5-6, and 5-8.

From these tables several general trends are seen regarding the

dependencies of the various responses upon the modes and upon the

position $(\frac{x}{\ell})$ in the beam.

Table 5-2 shows that the displacements can be expected to be

dominated by the first two or three modes, particularly in the upper

portion of the beam. In Tables 5-4 and 5-6 it is seen that the strains

and shear forces in the lower portion of a uniform beam are highly

dependent on the first few modes, while in the upper portion many

modes can be expected to contribute significantly.

From Table 5-8 for the total acceleration it is seen that

throughout the structure all modes can be expected to contribute.

Note, however, that near the base of the beam the expected contri-

bution of the first mode is the smallest of the first five modes.

In order to show the effect of taper on the response of a con-

tinuous cantilever beam, subjected to earthquake excitation, the

beam of Fig. 5.2 is examined.

In this case, the stiffness $K(x)$ and the density per unit length $\gamma(x)$ are functions of x:

$$K(x) = K_o \frac{x}{l}$$

$$\gamma(x) = \gamma_o \frac{x}{l} \ .$$

However, the velocity of wave propagation, c, is constant since

$$\frac{1}{c^2} = \frac{\gamma(x)}{K(x)} = \frac{\gamma_o}{K_o} \ .$$

By substituting these values into the general shear beam equation, Eq. 5.1, the equation for the tapered beam is obtained:

$$\frac{1}{x} \frac{\partial}{\partial x} \left(x \frac{\partial u}{\partial x} (x,t) \right) - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} (x,t) = \frac{1}{c^2} \ddot{y} \tag{5.15}$$

with the same boundary conditions:

at $x = 0$, $\qquad \frac{\partial u}{\partial x} = 0$; and

at $x = l$, $\qquad u = 0$.

The solution of this eigenvalue problem with zero initial conditions is

$$u(x,t) = -\frac{2c}{l} \sum_{n=1}^{\infty} \frac{1}{\omega_n^2} \frac{J_o\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n l}{c}\right)} \int_0^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \tag{5.16}$$

and the frequency equation is

$$J_o\left(\frac{\omega_n \ell}{c}\right) = 0 \quad . \tag{5.17}$$

The roots of the Bessel function J_o are given in Table 5-1.

TABLE 5-1

Natural Frequencies of Uniform and Tapered Beams

	Uniform Beam		Tapered Beam	
	$\cos\left(\frac{\omega_n \ell}{c}\right) = 0$	Frequency Ratios	$J_o\left(\frac{\omega_n \ell}{c}\right) = 0$	Frequency Ratios
n	$\left(\frac{\omega_n \ell}{c}\right)$		$\left(\frac{\omega_n \ell}{c}\right)$	
1	1.57	1	2.4	1.00
2	4.71	3	5.52	2.30
3	7.85	5	8.65	3.61
4	10.99	7	11.8	4.92
5	14.13	9	14.9	6.21
6	17.27	11	18.1	7.54

Again, because the primary interest is in making comparisons between the average maximum absolute values of the individual modal contributions, the nth terms of the various responses are discussed.

From Eq. 5.16, the nth term of the displacement for the tapered beam is

$$u_n(x,t) = -\frac{2c}{l} \frac{1}{\omega_n^2} \frac{J_o\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n l}{c}\right)} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \qquad .(5.18)$$

By differentiating Eq. 5.18 with respect to x, the nth modal contribution to the strain is obtained:

$$\frac{\partial u_n}{\partial x}(x,t) = \frac{2}{l} \frac{1}{\omega_n} \frac{J_1\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n l}{c}\right)} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \qquad (5.19)$$

The contribution of the nth mode to the shear force in a tapered beam is found by multiplying Eq. 5.19 by $K_o \cdot \frac{x}{l}$:

$$K_o \frac{x}{l} \frac{\partial u_n}{\partial x}(x,t) = \frac{x}{l} \frac{2K_o}{l} \frac{1}{\omega_n} \frac{J_1\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n l}{c}\right)} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau. \qquad (5.20)$$

From Eq. 5.15 it is seen that the total acceleration for the tapered beam is given by

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \ddot{y}(t) = \frac{c^2}{x} \frac{\partial}{\partial x}\left(x \frac{\partial u(x,t)}{\partial x}\right) . \qquad (5.21)$$

Hence, the nth modal contribution to the total acceleration is[*]

$$\frac{c^2}{x} \frac{\partial}{\partial x}\left(x \frac{\partial u_n}{\partial x}(x,t)\right) = \frac{2c}{l} \frac{J_o\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n l}{c}\right)} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \qquad (5.22)$$

---

[*]See p.100 for discussion of absolute convergence of this series.

By taking the maximum over time of the absolute value of Eqs. 5.18, 5.19, 5.20 and 5.22; using Eq. 5.10; and letting $z_n = \dfrac{\omega_n \ell}{c}$; the average maximum absolute values of the contributions of the nth mode to the various responses are obtained:

for displacement:

$$\overline{\left. |u_n(x)| \right|_{\substack{max \\ 0<t}}} = \frac{2\ell}{c} \frac{1}{z_n^2} \frac{|J_1\left(z_n\frac{x}{\ell}\right)|}{|J_1(z_n)|} S_v(\omega_n) \quad , \tag{5.23}$$

for strain:

$$\overline{\left. \left| \frac{\partial u_n(x)}{\partial x} \right| \right|_{\substack{max \\ 0<t}}} = \frac{2}{c} \frac{1}{z_n} \frac{|J_1\left(z_n\frac{x}{\ell}\right)|}{|J_1(z_n)|} S_v(\omega_n) \quad , \tag{5.24}$$

for the shear force:

$$\overline{\left. \left|K_o \frac{x}{\ell} \frac{\partial u_n}{\partial x}(x)\right| \right|_{\substack{max \\ 0<t}}} = \frac{x}{\ell} \frac{2K_o}{c} \frac{1}{z_n} \frac{|J_1\left(z_n\frac{x}{\ell}\right)|}{|J_1(z_n)|} S_v(\omega_n) \quad , \tag{5.25}$$

and for total acceleration:

$$\overline{\left. \left| \frac{c^2}{x} \frac{\partial}{\partial x} \left(x \frac{\partial u_n}{\partial x}(x) \right) \right| \right|_{\substack{max \\ 0<t}}} = \frac{2c}{\ell} \frac{|J_o\left(z_n\frac{x}{\ell}\right)|}{|J_1(z_n)|} S_v(\omega_n) \quad . \tag{5.26}$$

In order to show the dependencies of displacements, strains, shear forces, and total accelerations upon the first five modes and upon the position $(\frac{x}{l})$ in the tapered beam, Eq. 5.23, 5.24, 5.25 and 5.26 with $S_v(\omega_n)$ replaced by $\overline{S_v}$ have been tabulated in Tables 5-3, 5-5, 5-7 and 5-9. The general trends regarding the dependencies of the responses upon the modes of and upon the position $(\frac{x}{l})$ in the tapered beam are approximately the same as for the uniform beam with two exceptions. These exceptions are the increased emphasis on the higher modes in all of the responses in the tapered beam and the fact that the shear force in the tapered beam is proportional to

$$\frac{x}{l}\left(\frac{\partial u}{\partial x}\right)$$

whereas the shear force in the uniform beam is directly proportional to the strain.

The total acceleration coefficients of the uniform and tapered beams in Tables 5-8 and 5-9 are interesting since they indicate that each of the first five modes of the uniform beam can be expected to contribute about the same amount, except near the base where the contribution of the first mode becomes small. At the top of the tapered beam, however, the higher modes can be expected to dominate the total acceleration.

It appears, then, that a mass particle at the top of a severely tapered structure can be expected to feel very high accelerations. Furthermore, since it is more closely related to strain than to

TABLE 5-2

Displacement Coefficients of $\dfrac{2\ell}{c}\bar{S}_v$

for Uniform Beam from Eq. 5.11

$\dfrac{x}{\ell}$	Mode				
	1	2	3	4	5
0.0	0.405	0.045	0.016	0.008	0.005
0.167	0.391	0.032	0.004	0.002	0.004
0.333	0.351	0.000	0.014	0.007	0.000
0.500	0.287	0.032	0.011	0.006	0.004
0.667	0.203	0.045	0.008	0.004	0.005
0.833	0.105	0.032	0.016	0.008	0.004
1.000	0.000	0.000	0.000	0.000	0.000

Displacement Coefficients of $\dfrac{2\ell}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\dfrac{x}{\ell}$	Mode				
	1	2	3	4	5
0.0	1.000	0.111	0.040	0.020	0.012
0.167	1.000	0.081	0.011	0.005	0.009
0.333	1.000	0.000	0.040	0.020	0.000
0.500	1.000	0.111	0.040	0.020	0.012
0.667	1.000	0.222	0.040	0.020	0.025
0.833	1.000	0.304	0.149	0.076	0.034
1.000	0.000	0.000	0.000	0.000	0.000

TABLE 5-3

Displacement Coefficients of $\frac{2\ell}{c}\bar{S}_v$

for Tapered Beam from Eq. 5.23

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.0	0.333	0.096	0.049	0.031	0.022
0.167	0.320	0.077	0.027	0.008	0.001
0.333	0.282	0.031	0.011	0.012	0.004
0.500	0.223	0.016	0.018	0.004	0.006
0.667	0.151	0.038	0.004	0.006	0.005
0.833	0.074	0.029	0.014	0.007	0.003
1.000	0.000	0.000	0.000	0.000	0.000

Displacement Coefficients of $\frac{2\ell}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.0	1.000	0.290	0.148	0.093	0.065
0.167	1.000	0.241	0.084	0.024	0.003
0.333	1.000	0.108	0.038	0.044	0.014
0.500	1.000	0.073	0.079	0.017	0.026
0.667	1.000	0.254	0.027	0.042	0.035
0.833	1.000	0.387	0.196	0.098	0.040
1.000	0.000	0.000	0.000	0.000	0.000

TABLE 5-4

Strain Coefficients of $\frac{2}{c}\bar{S}_v$

for Uniform Beam from Eq. 5.12

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.0	0.000	0.000	0.000	0.000	0.000
0.167	0.165	0.150	0.123	0.088	0.050
0.333	0.318	0.212	0.064	0.045	0.071
0.500	0.450	0.150	0.090	0.064	0.050
0.667	0.551	0.000	0.110	0.079	0.000
0.833	0.615	0.150	0.033	0.024	0.050
1.000	0.637	0.212	0.127	0.091	0.071

Strain Coefficients of $\frac{2}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.0	0.000	0.000	0.000	0.000	0.000
0.167	1.000	0.911	0.746	0.533	0.304
0.333	1.000	0.667	0.200	0.143	0.222
0.500	1.000	0.333	0.200	0.143	0.111
0.667	1.000	0.000	0.200	0.143	0.000
0.833	1.000	0.244	0.054	0.038	0.081
1.000	1.000	0.333	0.200	0.143	0.111

TABLE 5-5

Strain Coefficients of $\frac{\ell}{c}\bar{S}_v$

for Tapered Beams from Eq. 5.24

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	0.157	0.220	0.234	0.211	0.162
0.333	0.296	0.310	0.162	0.014	0.105
0.500	0.400	0.225	0.077	0.108	0.041
0.667	0.457	0.033	0.134	0.078	0.018
0.833	0.462	0.137	0.025	0.032	0.057
1.000	0.416	0.181	0.116	0.085	0.067

Strain Coefficients of $\frac{\ell}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	1.000	1.398	1.486	1.342	1.030
0.333	1.000	1.047	0.548	0.048	0.356
0.500	1.000	0.563	0.192	0.270	0.103
0.667	1.000	0.072	0.294	0.170	0.039
0.833	1.000	0.296	0.053	0.068	0.123
1.000	1.000	0.436	0.278	0.204	0.161

TABLE 5-6

Shear Force Coefficients of $\dfrac{2K_o}{c}\bar{S}_v$

for Uniform Beam from Eq. 5.13

$\dfrac{x}{l}$	Mode				
	1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	0.165	0.150	0.123	0.088	0.050
0.333	0.318	0.212	0.064	0.045	0.071
0.500	0.450	0.150	0.090	0.064	0.050
0.667	0.551	0.000	0.110	0.079	0.000
0.833	0.615	0.150	0.033	0.024	0.050
1.000	0.637	0.212	0.127	0.091	0.071

Shear Force Coefficients of $\dfrac{2K_o}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\dfrac{x}{l}$	Mode				
	1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	1.000	0.911	0.746	0.533	0.304
0.333	1.000	0.667	0.200	0.143	0.222
0.500	1.000	0.333	0.200	0.143	0.111
0.667	1.000	0.000	0.200	0.143	0.000
0.833	1.000	0.244	0.054	0.038	0.081
1.000	1.000	0.333	0.200	0.143	0.111

TABLE 5-7

Shear Force Coefficients of $\dfrac{2K_o}{c}\bar{S}_v$

for Tapered Beam from Eq. 5.25

$\dfrac{x}{\ell}$	Mode				
	1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	0.026	0.037	0.039	0.035	0.027
0.333	0.099	0.103	0.054	0.005	0.035
0.500	0.200	0.113	0.038	0.054	0.021
0.667	0.304	0.022	0.090	0.052	0.012
0.833	0.385	0.114	0.020	0.026	0.047
1.000	0.416	0.181	0.116	0.085	0.067

Shear Force Coefficients of $\dfrac{2K_o}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\dfrac{x}{\ell}$	Mode				
	1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	1.000	1.398	1.486	1.342	1.030
0.333	1.000	1.047	0.548	0.048	0.356
0.500	1.000	0.563	0.192	0.270	0.103
0.667	1.000	0.072	0.294	0.170	0.039
0.833	1.000	0.296	0.053	0.068	0.123
1.000	1.000	0.436	0.278	0.204	0.161

TABLE 5-8

Total Acceleration Coefficients of $\frac{2c}{l}\bar{S}_v$

for Uniform Beam from Eq. 5.14

$\frac{x}{l}$	Mode				
	1	2	3	4	5
0.000	1.000	1.000	1.000	1.000	1.000
0.167	0.966	0.707	0.259	0.259	0.707
0.333	0.866	0.000	0.866	0.866	0.000
0.500	0.707	0.707	0.707	0.707	0.707
0.667	0.500	1.000	0.500	0.500	1.000
0.833	0.259	0.707	0.966	0.966	0.707
1.000	---------------- base acceleration -------------				

Total Acceleration Coefficients of $\frac{2c}{l}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{l}$	Mode				
	1	2	3	4	5
0.000	1.000	1.000	1.000	1.000	1.000
0.167	1.000	0.732	0.268	0.268	0.732
0.333	1.000	0.000	1.000	1.000	0.000
0.500	1.000	1.000	1.000	1.000	1.000
0.667	1.000	2.000	1.000	1.000	2.000
0.833	1.000	2.732	3.732	3.732	2.732
1.000	--------------- base acceleration --------------				

TABLE 5-9

Total Acceleration Coefficients of $\frac{2c}{\ell}\bar{S}_v$

for Tapered Beam from Eq. 5.26

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.000	1.926	2.939	3.684	4.302	4.842
0.167	1.850	2.349	2.003	1.049	0.206
0.333	1.629	0.931	0.805	1.724	0.896
0.500	1.290	0.495	1.312	0.520	1.311
0.667	0.874	1.170	0.302	0.872	1.180
0.833	0.427	0.870	1.085	1.009	0.664
1.000	---------------base acceleration --------------				

Total Acceleration Coefficients of $\frac{2c}{\ell}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode				
	1	2	3	4	5
0.000	1.000	1.526	1.912	2.233	2.513
0.167	1.000	1.270	1.083	0.567	0.112
0.333	1.000	0.571	0.494	1.058	0.550
0.500	1.000	0.384	1.017	0.403	1.016
0.667	1.000	1.339	0.346	0.998	1.350
0.833	1.000	2.039	2.541	2.365	1.555
1.000	---------------base acceleration --------------				

displacement, structural damage appears to be more dependent upon the higher modes than upon the lower ones, particularly in the upper portion of a severely tapered structure.

From these tables of response coefficients it can be concluded that in order to minimize the strains and total accelerations in a structure, particularly in the upper portion, it is best to have the stiffness and mass distributions as uniform as possible.

It is also clear from these tables that the extent to which "higher modes" are important depends upon the response of interest, the taper of the structure, and the position within the structure.

Often an analogy is made between either excessive displacements (relative to ground), structural damage, or total accelerations near the top of a building and an energy pulse traveling toward and being reflected from the tip of a whip. These phenomena are said to result from "whiplash" of the structure. Since structural damage and total acceleration are directly related to excessive interfloor displacement and do not necessarily result from large displacements, a suggested definition of "whiplash" in structures is the magnitude of the interfloor displacements.

5.3 Modal Analysis of Discretized Models of Tall Structures

The application of eigenvalue theory to the discretized model of a structure is covered in this section. Using the equations developed here, a computer program subroutine was written which determines the "equivalent" modal participation factors, "equivalent"

modal contributions to the displacements and "equivalent" modal contributions to the interfloor displacements for arbitrarily selected floors in the structure. Using such results a comparison can be made with the results for the continuous beams.

Usually, modal analysis is restricted to structures that remain linear. However, modal analysis can also be useful for structures which yield. Since the eigenvectors of the linear structure form a complete set of base vectors, it is possible to expand any displacement vector in terms of these eigenvectors in order to find what will be called the "equivalent" modal participation factors. These factors can then be used to find "equivalent" modal contributions to the displacements and to the interfloor displacements. In this sense, the concept of the modal participation factor has the same significance for a yielding structure as for a linear one.

It should be noted that, for the excitation used, all of the columns in the $A/20/2.2/2/6$ structure remain linear except for those between the upper two or three floors. Furthermore, all of the girders yield--but not necessarily all at the same time. Hence, from an overall point of view, the structure is reasonably linear when compared to one in which all of the girders and columns yield at the same time.

The eigenvectors of a linear structure with N floors are found as follows: The homogeneous equation of motion is

$$M \ddot{\underline{u}} + K \underline{u} = 0 \qquad (5.27)$$

where

$\underline{u}$:   displacement vector $(N \times 1)$,

M :   diagonal mass matrix $(N \times N)$, and

K :   symmetric and positive definite stiffness matrix $(N \times N)$.

Letting

$$\underline{u} = M^{-\frac{1}{2}}\underline{\xi} \tag{5.28}$$

and premultiplying by $M^{-\frac{1}{2}}$, Eq. 5.27 becomes

$$I\ddot{\underline{\xi}} + \overline{K}\underline{\xi} = 0 \tag{5.29}$$

where

$$\overline{K} = M^{-\frac{1}{2}}KM^{-\frac{1}{2}} .$$

Since $\overline{K}$ is symmetric and positive definite, a theorem of matrix algebra guarantees the existence of a unitary transformation matrix ϕ such that

$$\phi^T\phi = I \quad \text{and} \quad \phi^T\overline{K}\phi = \begin{bmatrix} \ddots & & \\ & \omega_n^2 & \\ & & \ddots \end{bmatrix} . \tag{5.30}$$

Therefore, letting

$$\underline{\xi} = \phi\underline{\eta} \tag{5.31}$$

and premultiplying by ϕ^T, Eq. 5.29 becomes

$$\ddot{\underline{\eta}} + \begin{bmatrix} \ddots & & \\ & \omega_n^2 & \\ & & \ddots \end{bmatrix}\underline{\eta} = 0 \tag{5.32}$$

which is a diagonalized, or separated equation. That is, the solution for the nth modal participation factor η_n is independent of every other mode.

By substituting Eq. 5.31 into Eq. 5.28, an equation relating the displacement vector, $\underline{u}$, to the modal participation factors, $\underline{\eta}$ is obtained:

$$\underline{u} = M^{-\frac{1}{2}} \phi \underline{\eta} \; . \tag{5.33}$$

The inverse of Eq. 5.33 is

$$\underline{\eta} = \phi^{T} M^{+\frac{1}{2}} \underline{u} \; . \tag{5.34}$$

Equations 5.33 and 5.34 are the ones used in the modal analysis of a tall structure.

As discussed in Section 3.5, a direct integration method is used in the computer program to solve the equations of motion for the yielding structure. Then, knowing the displacement vector $\underline{u}$, Eq. 5.34 is used to find the vector, $\underline{\eta}$, of "equivalent" modal participation factors. For η_n, Eq. 5.34 becomes

$$\eta_n = \sum_{k=1}^{20} \phi_{kn} M_k^{+\frac{1}{2}} u_k \tag{5.35}$$

for a structure with 20 floors. Similarly, for the displacement, u_j, of the jth floor, Eq. 5.33 becomes

$$u_j = \sum_{n=1}^{20} M_j^{-\frac{1}{2}} \phi_{jn} \eta_n \; . \tag{5.36}$$

Consequently, after obtaining η_n, the "equivalent" contribution of the nth mode to the displacement of the jth floor, u_{jn}, is found

from

$$u_{jn} = M_j^{-\frac{1}{2}} \phi_{jn} \eta_n \; . \tag{5.37}$$

Using Eq. 5.37, the "equivalent" contribution of the nth mode to the interfloor displacement, UU_{jn},* between floors (j) and (j+1) is found by subtraction:

$$UU_{jn} = u_{jn} - u_{(j+1)n} \tag{5.38}$$

As mentioned earlier, the interfloor displacement of the discretized structure corresponds to the strain in a continuous beam.

Only one modal analysis study of a yielding structure was made on the computer because of the extensive time involved.** In this study the A/20/2.2/2/6 structure with 0.0035 fraction of critical damping*** in the first mode was subjected to the four seconds of the El Centro Earthquake (N-S) of 18 May 1940. Furthermore, the two-component beam model described in Appendix C and the yield criteria with the "test bending moment" described in Appendix B were used. Although the beam model and yield criteria were later

---

*UU is the Fortran name for interfloor displacement. Also, j = 2 at the top floor (roof) and j = 22 at the ground level. See Section 3.2 for more details of notation.

**The modal analysis is performed by subroutine MODE, which is optional. A listing of this subroutine is provided in Appendix E-- Listing of the Computer Program.

***This fraction of critical damping is believed to be the same as that used by Clough and Benuska for their studies with essentially no damping in the FHA Study (reference 1).

TABLE 5-10

Modal Contributions to the Largest Displacements for the A/20/2.2/2/6 Structure

Floor	$\frac{x}{l}$	Mode							Displacement (inches)	Time of Occurrence
		1	2	3	4	5	6	7		
19	.05	-12.54	+.841	-.640	-.120	+.065	+.013	-.035	-12.3	3.155 sec
14	.30	- 9.52	-.121	+.316	+.076	+.143	-.042	-.254	- 9.12	3.230 sec
9	.55	- 5.93	-.597	-.133	-.136	-.154	+.014	+.044	- 6.86	3.155 sec
4	.80	- 2.058	-.331	-.713	+.063	+.135	+.035	+.007	- 2.92	3.010 sec

Modal Contributions Normalized to the Contribution of the First Mode

Floor	$\frac{x}{l}$	Mode						
		1	2	3	4	5	6	7
19	.05	1.00	.067	.051	.010	.005	.001	.003
14	.30	1.00	.013	.033	.008	.015	.004	.027
9	.55	1.00	.101	.022	.023	.026	.002	.007
4	.80	1.00	.161	.346	.031	.066	.017	.003

TABLE 5-11

Modal Contributions to the Largest Inter-floor Displacements for the A/20/2.2/2/6 Structure

Between Floors	$\frac{x}{\ell}$	Mode							Interfloor Displacement (inches)	Time of Occurrence
		1	2	3	4	5	6	7		
18-19	.08	-.038	-.132	-.373	-.360	-.199	+.120	+.020	-1.11	2.525 sec
13-14	.33	-.662	+.122	+.035	-.106	-.041	-.053	+.032	-.519	3.210 sec
8- 9	.58	-.685	-.001	-.018	-.103	+.012	+.019	+.062	-.756	3.335 sec
3- 4	.83	-.617	-.085	-.080	-.011	-.020	-.039	-.026	-.957	3.160 sec

Modal Contributions Normalized to the Contribution of the First Mode

Between Floors	$\frac{x}{\ell}$	Mode						
		1	2	3	4	5	6	7
18-19	.08	1.00	3.478	9.847	9.488	5.256	3.161	.536
13-14	.33	1.00	.184	.053	.160	.062	.030	.048
8- 9	.58	1.00	.001	.026	.151	.017	.027	.090
3- 4	.83	1.00	.137	.130	.018	.032	.064	.042

refined (see Chapter II and Appendix C, respectively), it is felt that
the general trends of the results presented below are, nevertheless,
correct.

The entries in Table 5-10 are the "equivalent" modal contri-
butions to the displacements of the arbitrarily selected floors at
the time when the entire displacement (not each modal contribution)
reaches its maximum absolute value. Similarly, the entries in
Table 5-11 are the "equivalent" modal contributions to the selected
interfloor displacements at the time when the entire interfloor dis-
placement (not each modal contribution) reaches its maximum
absolute value. Hence, Tables 5-10 and 5-11 are not directly com-
parable to the corresponding tables for the continuous beams.

At various times during this study, modes as high as the
eighth were found to contribute as much as 2% to the displacements.
And what is more important, modes as high as the 13th were found
to contribute similar amounts to the interfloor displacements. If
one either increased the level of the mass proportional damping, or
preferably, introduced stiffness proportional damping, the number
of modes which contribute significantly would probably decrease.

In Table 5-10 it is seen that, for the instances presented, the
fundamental mode dominates the displacements throughout the
structure. It is also seen that the higher modes contribute very little
to the displacements in the upper portion but somewhat more in the
lower portion.

On the other hand, for the cases presented in Table 5-11, the

third and fourth modes contribute the largest amounts to the inter-

floor displacement between the 18th-19th floor (which is the largest

one occurring anywhere in the structure) with the fundamental mode

contributing a relatively small amount. In the lower portion, how-

ever, the fundamental is the most important contributor.

Even though the results of this computer study are not directly

comparable to the studies of continuous beams, the general trends

of the modal contributions are the same. That is, (1) the fundamental

mode dominates the displacements, particularly in the upper floors;

and (2), for the interfloor displacements, the higher modes usually

contribute more than the fundamental in the upper floors, but in the

lower floors, the fundamental mode is more important.

In order to learn more about these trends, the time history

plots of numerous responses are analyzed in the next chapter.

CHAPTER VI

RESPONSE RESULTS

6.1 Introduction

The purpose of this chapter is to present response results
from computer studies of the A/20/2.2/2/6 structural frame
(Sec. 3.2) subjected to earthquake excitation. These studies are
carried out by the computer program described earlier in the report
(Sec. 3.5) using the one-component beam model (Sec. 2.5) for the
girders and columns of the frame. The plots given here are time
histories of bending moments, interfloor shear forces, joint rotations,
total accelerations, displacements and interfloor displacements for a
number of stations in the structure.

The first section of this chapter shows the effect of yielding
on the response of the structural frame. The excitation used in this
investigation is the first four seconds of the accelerogram of the
El Centro (N-S) earthquake of 18 May 1940. This is the strongest
portion of the earthquake. In a later study in which the nonlinear
frame was subjected to the entire length of the earthquake, it was
observed that the maximum values of most of the responses
occurred during the first four seconds, although several occurred
afterwards. It is believed, however, that results based on the first
four seconds are indicative of the effect of yielding.

The second section treats the response results for the non-
linear frame subjected to the entire length (30 sec) of the Jennings
pseudo-earthquake number 6. In addition to time history plots, the

TABLE 6-1

Nomenclature Used in the Time History
Plots in Chapter VI and in Appendix A

TOTAL ACCEL 17F : The total horizontal acceleration of the 17th floor

HORIZ DISP 4F : The horizontal displacement of the 4th floor relative to ground

OVRTURNG MOMT-BASE : The overturning moment at the base of the three bay structure

SHEAR FORCE 18-19F : One-half of the interfloor shear force between the 18th and 19th floors (from only the two columns used in the analysis)

INT-FL DISP GD-1F : The interfloor displacement between the ground and the first floor

JNT ROT'N 17F-XC : The rotation of the joint formed by the intersection of the exterior girder of the 17th floor and the exterior column line

BND MOMT 17F-XG-XC : The bending moment at station (e)

PL ANGLE 4F-XG-XC : The plastic angle at station (f)

PL INDEX 4F-XG-XC : The plastic index at station (f). This index indicates when yielding occurs and the direction of incremental yielding

BND MOMT 20F-IC-BT : The bending moment at station (d)

MOD JNT ROT 20F-IC : The rotation of the joint formed by the intersection of the 20th floor and the interior column line modified to include the effect of the interfloor shear angle (see Eq. 3.8)

PL ANGLE 2F-XG-IC : The plastic angle at station (g)

BND MOMT 2F-IG-IC : The bending moment at station (h)
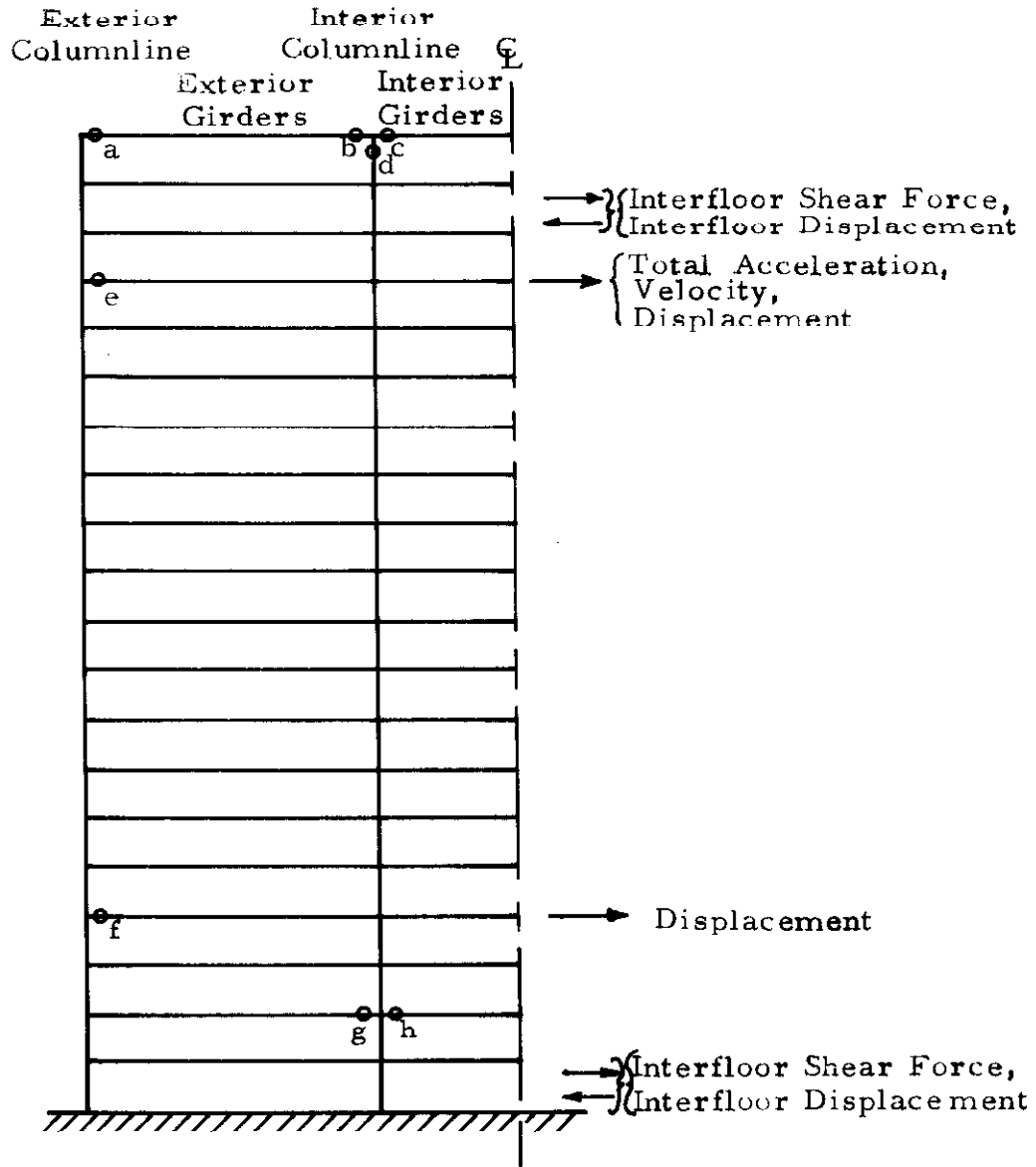
Fig. 6.1 o : Stations for Which the Bending Moment, Joint Rotation (or Modified Joint Rotation), Plastic Angle, and Plastic Index are Plotted .

displacement envelope and the ductility factors are shown. Response plots for similar studies of this frame using the entire length (29.38 sec) of the El Centro (N-S) earthquake of 18 May 1940 and several other pseudo-earthquakes are given in Appendix A.

The nomenclature used in these plots is described in Table 6-1. The unit system used is "in-lbs-sec" with the joint rotations given in radians. The stations for which the time histories of the responses are plotted are shown in Fig. 6.1.

6.2 Comparison of the Linear and Nonlinear Responses of a Structural Frame

In this section a comparison is made between the responses of the linear A/20/2.2/2/6 structural frame and the corresponding nonlinear frame. Each system had 0.10 fraction of critical damping in the fundamental mode and was subjected to approximately the first four seconds of the El Centro (N-S) earthquake of 18 May 1940.

The time histories of the total acceleration of the 17th floor, the horizontal displacements of the 17th floor and the 4th floor as well as the overturning moment at the base of each frame are shown in Fig. 6.2a and Fig. 6.2b. For both the linear and the nonlinear cases, the displacement of the 17th floor and the overturning moment at the base are primarily characterized by the frequency of the fundamental mode. In the time history of the displacement of the 4th floor, higher frequencies are observed in addition to the fundamental for both the linear and nonlinear cases. Many frequencies are noted in the time history of the total acceleration of the 17th floor. These

observations are in agreement with those for the continuous cantilever beam of Chapter V.

Comparing the total acceleration of the 17th floor of the linear structure with that of the nonlinear structure, two features are noted which call for some discussion. The first is that an unusual peak occurs at 2.7 sec in the nonlinear plot in Fig. 6.2b which does not occur in the corresponding linear response, Fig. 6.2a. The second concerns the relatively larger high frequency components of the nonlinear plot.

For the above calculations mass proportional damping was used but no stiffness proportional damping was introduced. As a result, the higher modes are not so effectively damped as the lower. Consequently, there is a limitation on the model for the representation of certain responses which exhibit significant high frequencies.

The first feature to be discussed concerns the unusual peak in the total acceleration record of the 17th floor in the nonlinear frame. This peak has an absolute value of 0.81g and is the maximum absolute value of the total acceleration for any floor during this run for the nonlinear case. However, for the linear frame, the maximum absolute value of the total acceleration at the 17th floor is 0.58g while at the 20th floor it is 1.0g. Since the maximum absolute value of the acceleration of the ground is 0.32g, it appears possible to have amplifications of the total acceleration on the order of 2 or more in this particular nonlinear structure. As a result of yielding, the amplitude and phase of the shear force waves in the nonlinear case

129

LINEAR RESPONSE, FRAME A/20/2.2/2/6. BI-LIN = 1.0. FR CR DAMP =.10. EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

0 SEC   1 SEC   2 SEC   3 SEC   4 SEC

386.06
TOTAL ACCEL 17F

0

-386.06
10.0
HORIZ DISPL 17F

0

-10.0
4.0
HORIZ DISPL 4F

0

-4.0
1000000192.
OVRTURNG MOMT-BASE

0

-1000000192.

0 SEC   1 SEC   2 SEC   3 SEC   4 SEC

Fig. 6.2a

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

0 SEC   1 SEC   2 SEC   3 SEC   4 SEC

386.06
TOTAL ACCEL 17F

0

-386.06
10.0
HORIZ DISPL 17F

0

-10.0
4
HORIZ DISPL 4F

0

-4
1000000192.
OVRTURNG MOMT-BASE

0

-1000000192.

0 SEC   1 SEC   2 SEC   3 SEC   4 SEC

Fig. 6.2b

LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN = 1.0, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.3a

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.3b

131

LINEAR RESPONSE. FRAME A/20/2.2/2/6. BI-LIN = 1.0. FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.4a

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.4b   Station (e)

LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN = 1.0, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.5a

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.5b Station (f)

LINEAR RESPONSE.   FRAME A/20/2.2/2/6,   BI-LIN = 1.0,   FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.6a

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. 6.6b   Station (d)

TABLE 6-2

Floor	Sum of Shear* Forces Above and Below Floor (at Yield Level) lbs×10^{-4}	Weight* of Floor lbs×10^{-4}	Estimate of Maximum Absolute Value of Total Acceleration fraction g	Maximum Absolute Value of Total Acceleration from a Non-linear Study Using El Centro (N-S), 18 May 1940 fraction g
20	4.31**	8.88	0.49	0.54
	12.00	10.04	1.20	0.58
18	18.29	10.04	1.82	0.45
	24.04	10.04	2.39	0.81
16	29.99	11.20	2.67	0.65
	35.93	11.20	3.21	0.61
14	42.27	11.20	3.77	0.66
	48.03	11.58	4.14	0.45
12	52.36	11.58	4.52	0.53
	57.87	11.58	4.99	0.50
10	62.73	14.28	4.39	0.48
	67.44	14.28	4.72	0.43
8	71.06	14.28	4.97	0.39
	74.92	14.67	5.10	0.45
6	77.26	14.67	5.26	0.39
	82.39	14.67	5.61	0.45
4	85.15	15.83	5.37	0.50
	84.15	15.83	5.31	0.37
2	87.71	15.83	5.54	0.41
	102.92	16.21	6.34	0.37
Ground	-	-	-	0.32

*Calculations based on half of the A/20/2.2/2/6 frame (see Fig. 3.2).
**This one uses the yield moment in the 20th floor girder at the exterior column since it is lower than the one in the exterior column.

are changed from those in the linear case. From a review of the plots for this study, it appears that in the nonlinear case at 2.7 sec, shear force waves fortuitously meet at the 17th floor in a manner sufficient to cause a high total acceleration.

To explore the way in which interfloor shear forces and accelerations may add up at various points in the structure, Table 6-2 has been compiled. The estimate of the maximum absolute value of the total acceleration of a floor is found by dividing the sum of the yield levels of the shear forces in the columns directly above and below that floor by the mass of the floor. These estimates of the maximum total acceleration that the structure could sustain are very high: from 0.49g at the roof to 6.3g near the base. This only means that with respect to total acceleration the columns near the base of the structure are much stronger than would be required for actual earthquake excitation. As shown in Table 6-2 for the El Centro (N-S) earthquake of 18 May 1940, the total acceleration of the 20th floor was calculated to be 0.54g, and near the base it was calculated to be 0.37g. The 0.54g value is in excess of the estimated 0.49g because interfloor shear forces occurred between the 19th and 20th floors in excess of those sufficient to cause yielding in the columns.

The second point to be discussed concerns the larger high frequency components observed in the total acceleration for the non-linear case in comparison with that for the linear case. This is likely to be a consequence of the sharp corner at the yield level in the bi-linear hysteresis loop. A similar phenomenon was observed in the

previous investigation of the three-mass system with bilinear

hysteretic springs shown in Fig. 4.1. In that case it was noted that

higher frequency components corresponding to the higher natural

modes appeared whenever a spring passed into the yield range. This

type of response corresponds to that which would result from a step

in the forcing function, i.e., the response would contain all of the

natural frequencies of the system. The total acceleration of a real

structure may or may not exhibit this high frequency characteristic

depending upon the properties of the structure at the yield level.

An estimate of the effect of such a sharp corner on the response of a

tall structure might be obtained in a future study using curvilinear

moment-rotation hysteresis loops such as shown in Fig. 2.14.

In order to observe the effect of yielding on other responses of

the structure, Figs. 6.2a and 6.2b are again considered. Comparing

the horizontal displacement of the 17th floor for the linear case with

that for the nonlinear case, one notes that there is little difference

in the maximum displacements or in the time at which they occur.

However, it is seen in Fig. A.1.3 of Appendix A that, as a result of

yielding, a permanent displacement of about four inches is incurred

in the displacement of the 17th floor. Similarly, the maximum

absolute displacement of the 4th floor in the linear and the nonlinear

cases are approximately equal and occur at about the same time. In

the nonlinear case, a permanent displacement of about one inch is

incurred.

The maximum absolute value of the overturning moment at

the base, Fig. 6.2, is reduced about 30% by yielding. Since the

girders yield, the moments at the ends of the girders and the axial forces in the columns are limited. Consequently, the base overturning moment is also limited. Note that gravitational effects are not considered in the calculation of the base overturning moment of this model.

In the plots of the interfloor shear forces and interfloor displacements, Fig. 6.3, changes in the shape of the shear waves as a result of yielding can be seen. In particular, a shear wave pulse with a period of approximately 0.5 sec, about the period of the third mode, can be traced and changes in its shape observed. This shear force pulse appears between the ground and the 1st floor from 1.6 sec to 2.1 sec in both the linear and nonlinear cases. In the linear case, this pulse appears approximately 0.6 sec later between the 18th and 19th floors. However, in the nonlinear case, the shear force pulse is observed between the 18th and 19th floors with different characteristics than those observed in the linear case. In comparison with the linear case, the amplitude is reduced 50%, the shape is different, and the peaks are delayed by approximately 0.1 sec. This delay is such that the peak in the shear force between the 18th-19th floors at 2.7 sec coincides with the 0.81g peak in the total acceleration plot in Fig. 6.26. The effect of yielding on the wave shape is even more noticeable after 2.8 sec in these time history plots.

Similar effects are observed for the bending moments at different stations in the structure as shown in Figs. 6.4, 6.5 and 6.6. Furthermore, it is seen that the time histories of the bending moments,

joint rotations, and plastic angles differ from one station to another

in both the linear and in the nonlinear cases.

To summarize the effects of yielding on the response of the

A/20/2.2/2/6 frame subjected to the strongest portion of the El Centro

earthquake, it was observed that yielding limits the maximum abso-

lute values of the shear forces to about 60% of the corresponding

values in the linear case. However, yielding was not so effective in

limiting displacements, interfloor displacements, and joint rotations.

For these, the maximum absolute values were reduced to only about

80% of the linear values. Although the maximum absolute values of

the total acceleration may be reduced by yielding, it was shown that

for the structural properties used, this frame could withstand total

horizontal accelerations in excess of 6g near the base and in excess

of 1g near the top without the columns yielding (the girders do yield).

It was also seen that during the time of severe yielding, the phase of

the response was significantly altered.

In the next section, the response of this nonlinear structure to

a pseudo-earthquake is presented. By a visual analysis of the time

history plots of various responses, it is possible to obtain an under-

standing of the frequency components expected in each response.

6.3 Response of the Nonlinear A/20/2.2/2/6 Frame Subjected to Earthquake Excitation

Several response studies of the bilinear hysteretic

A/20/2.2/2/6 frame subjected to different earthquakes are performed

using the computer program developed in connection with this report.

In addition to the El Centro (N-S) earthquake of 18 May 1940, six of
the pseudo-earthquakes generated from a random process by P. C.
Jennings are used. Since the corresponding responses of the
structure are similar in amplitude and frequency content for the
pseudo-earthquakes used, the plots for one earthquake only are
analyzed in this section. The plots for the other earthquakes are given
in Appendix A.

The pseudo-earthquake accelerograms have spectral proper-
ties similar to those for the recorded strong motion accelerograms
of real earthquakes. However, the pseudo-earthquake accelero-
grams are more nearly uniform in amplitude over time than are the
real ones. In an attempt to obtain approximately the same amount of
yielding in the structure for the pseudo-earthquakes as occurred in
the structure for the El Centro earthquake, they were scaled so that
the rms value of each accelerogram is approximately 1.2 times that
of El Centro. The actual rms values used are given in Table 6-3. In
all cases the bilinearity (the ratio of the second slope to the first in
the hysteresis loop) was 0.05. The fraction of critical damping in the
fundamental mode was 0.10 in all runs except one which used 0.05
fraction of critical damping in order to observe the effect of damping
on the response. A summary of the combinations of parameters used
in the studies is shown in Table 6-3.

The response plots for the nonlinear A/20/2.2/2/6 frame using
pseudo-earthquake number 6 with 0.10 fraction of critical damping in
the fundamental mode are presented in this section. In addition, the
displacement responses (in inches) of seven one-degree of freedom

TABLE 6-3

Earthquake	Fraction Critical Damping in Frame	rms Scale Factor (with respect to El Centro)	Max. Abs. Acceleration of Earthquake (frac. grav.)	Spectrum Intensity $SI_{0.0}$	Max. Abs. Displacement (inches)	Max. Abs. Interfloor Displacement (inches)	Maximum Ductility Factors		
							All Girders	Interior Column	Exterior Column
El Centro (N-S) 18 May 1940	0.10	1.0	0.316	8.94	11.52	1.29	4.44	2.99	0.83
Pseudo-Earthquake No. 1	0.10	1.15	0.246	11.45	8.98	0.98	3.58	1.74	0.71
Pseudo-Earthquake No. 2	0.10	1.18	0.267	13.23	14.43	1.31	5.00	3.11	0.81
Pseudo-Earthquake No. 4	0.10	1.26	0.267	12.21	10.96	0.93	3.96	2.02	0.80
Pseudo-Earthquake No. 5	0.10	1.19	0.271	12.25	9.69	0.91	3.63	2.23	0.76
Pseudo-Earthquake No. 6	0.10	1.22	0.313	10.96	8.91	0.99	4.01	2.58	0.78
Pseudo-Earthquake No. 6	0.05	1.22	0.313	10.96	10.28	1.08	4.32	2.91	0.80
Pseudo-Earthquake No. 7	0.10	1.20	0.328	12.06	8.52	0.97	3.75	2.53	0.79

Notes:

These pseudo-earthquakes were chosen by flipping coins.

In all cases the bilinearity is 0.05.

Max. Abs. = Maximum Absolute.

140

141

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6. BI-LIN =.05. FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.7

Fig. 6.8

Fig. 6.9

Fig. 6.10

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.11   Station (e)

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.12   Station (f)

144

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.13   Station (h)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.14 Station (g)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.15   Station (a)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.16   Station (b)

146

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.17   Station (c)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2

Fig. 6.18   Station (d)

147

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6
BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10
(JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6 , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

Fig. 6.19

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. 6.20a   Fig. 6.20b

linear oscillators subjected to the same excitation are presented in

Fig. 6.8. There is a one-to-one correspondence between these

oscillators and the first seven modes of the linear A/20/2.2/2/6

frame with respect to period and damping. For example, the first

oscillator has the period and the fraction of critical damping of

mode 1. In Fig. 6.8 the response of this oscillator is labeled

$$1 \quad T = 2.21 \quad DAMP = .100 \quad .$$

The time history of the displacement of the 17th floor is shown

in Fig. 6.9. By comparison with the response of the linear oscillator

corresponding to the first mode of the linear frame, henceforth called

the first modal oscillator, it is seen that the two are closely corre-

lated. This is in accordance with the analytical results for continuous

cantilever beams in Chapter V which indicate that the horizontal

displacements in the upper porition of the structure are expected to be

dominated by the first few modes, primarily the fundamental.

By design, the distribution of yield bending moments in the

A/20/2.2/2/6 frame is such that throughout most of the structure the

girders yield at a much lower level than the neighboring columns. The

purpose for such a design is "to maintain vertical load integrity of the

structure."[*] For all of the earthquake excitations used, the interior

columns of the nonlinear structure yield in only the upper 2 or 3 floors

and the girders spend at most 10% of the time in yield. Consequently,

the time histories of the gross responses of this particular structure,

---

[*]Clough and Benuska, FHA Study, p.1-7.

such as the displacements, should be highly correlated with those of the linear structure. If the yield moment distribution were changed so that more of the columns yielded, a lower correlation with the response of the linear system would probably occur.

The overturning moment at the base, Fig. 6.7, is observed to be closely correlated with the displacement at the 17th floor and with the response of the first modal oscillator (with opposite sign). This correlation should exist for the following reason: When a floor is displaced, shear forces develop in the girders which appear as forces in the axial direction in the columns. Only when many of the floors are displaced in the same direction, such as in the fundamental mode, do the axial column forces add together to cause a significant resultant moment at the base.

A comparison of the 4th floor displacement with the 17th floor displacement, Fig. 6.9, indicates that higher frequency response (corresponding to the higher modes in a linear structure) of the structure influences the displacement (not interfloor displacement) of the 4th floor more than it influences the displacement of the 17th floor. This is in agreement with the studies of continuous beams.

In addition to a fundamental modal response component, higher frequency response is observed in the plot of the velocity (relative to ground) of the 17th floor. This is expected since the velocity is a first derivative of the displacement and differentiations characteristically tend to amplify higher frequencies.

As seen in Fig. 6.9, the total acceleration, which is the sum

of the acceleration relative to ground and the acceleration of the
ground, has many frequency components. Again, this is in agree-
ment with the results of the continuous beam studies.

In Fig. 6.10, the time histories of the shear force and of
the interfloor displacements between the 18th and 19th floors as well
as between the ground and 1st floors are shown. By comparison, it
is seen that the time history of the shear force between the 18th and
19th floors and the time history of the corresponding interfloor dis-
placement are very similar in appearance. The small differences
between them occur because the shear force is a function of the joint
rotations in addition to the interfloor displacement and because
permanent displacement occurs as a result of yielding.

In the propagation of the first shear force pulse from the base
to the 19th floor in the linear A/20/2.2/2/6 frame a time lag of about
0.6 seconds is observed. This time lag is seen throughout most of
the earthquake, but it is usually not so clear. For the shear-type
continuous cantilever beams described in Chapter V, the time neces-
sary for a shear force pulse to travel from the base to the top is:

for the uniform beam:

$$\frac{T_1}{4} = 0.55 \text{ sec} \text{ and}$$

for the tapered (to a point) beam:

$$\frac{T_1}{2.6} = 0.85 \text{ sec}$$

where T_1 is the period of the fundamental mode in each case. Since

this frame has mass and stiffness properties which are partially

tapered (Fig. 3.2), the observed time lag of 0.6 sec is in good agree-

ment with those predicted for the continuous beams. In these plots

it is seen that the shear force and interfloor displacement between

the ground and first floor have a strong dependence upon the funda-

mental mode. However, many modes contribute to the shear force

and interfloor displacement near the top of the frame.

Time history plots of the bending moment, joint rotation,

plastic angle, and plastic index are shown for several stations in the

frame.* Each of the figures 6.11 through 6.18 corresponds to one of

these stations. If the station is in a girder, the joint rotation is given;

if the station is in a column, the modified joint rotation (Eq. 3.8) taking

into account the interfloor displacement is plotted. During yielding

the plastic index is (+1) or (-1) depending upon the direction in which

the associated plastic angle moves. For example, at station (e) in the

exterior girder in the 17th floor, Fig. 6.11, the initial yielding occurs

with negative bending moment; hence, the incurred plastic angle is

negative and the plastic index is (-1). By making visual comparisons

of the time history plot of the bending moment at station (e), Fig. 6.11,

with the response of each of the linear modal oscillators, Fig. 6.8,

correlations can be observed which indicate the importance of the

modes to the response at that station. Between 3 sec and 10 sec a

---

*All of these time history plots were made on the California Computer
Products "Model 763 Plotter" programmed with the IBM 7094-7040
Computer at the California Institute of Technology Computing Center.
For the 30 seconds of the earthquake there were 6000 time incre-
ments. After every 4th time increment, a point was plotted; hence,
50 points per earthquake second were plotted.

strong correlation with the 2nd modal response is observed; hence,
it is concluded that the 2nd mode is responsible for most of the
yielding during this time at this station. Between 13 sec and 19 sec,
correlation with the first modal response occurs. Similarly, it is
concluded that during this interval the first mode is responsible for
most of the yielding at this station. At times combinations of modes
are responsible for the yielding since the contribution of each mode
by itself does not appear to be sufficient to cause yielding.

For an accurate determination of the relative importance of
each mode at a particular instant, it is usually necessary to make a
modal decomposition of the motion of the structure at that instant.
Only at the times when one or two modes dominate a response does
it appear possible to visually determine the importance of these
modes. Such a modal analysis is made for this nonlinear structure
in each time increment throughout the first four seconds of the
El Centro earthquake according to a technique described in Section
5.3. The modal contributions to the largest displacements and inter-
floor displacements that occurred in that analysis are given in
Table 5-10 and Table 5-11. The results presented in those tables
indicate that often more modes than can be observed by the eye are
important for the interfloor displacements. However, because exten-
sive computer time is used in making modal decompositions, an
analysis of this type was not performed for the present study. Conse-
quently, in this chapter the discussion of modal contributions (or
frequency components) is based upon visual analysis.

At station (f) in the 4th floor, the fundamental mode appears to be the largest contributor to the bending moment and, hence, is the most important for yielding at this station. At times, however, the second and higher modes are observed to add to the fundamental mode, thereby contributing to the yielding.

As the frame is deformed, the ends of the girders within the same floor may or may not yield simultaneously depending upon the distribution of strength properties, i.e., yield moments, in that portion of the frame. In the lower portion, the yield moments in the interior and exterior girders within the same floor are approximately equal and the yield moments in the girders are lower than those in the neighboring columns. As a result, the columns remain linear and the girders yield. In the time history plots, Fig. 6.13 and Fig. 6.14, for stations (h) and (g), it is observed that the girders in the second floor yield almost simultaneously.

Near the top of the frame the yield moments within the same floor differ by as much as 10%. In the top two floors the yield moments in the girders approximate the yield moments in the neighboring columns. Consequently, in this portion of the frame, the girders within the same floor usually do not yield at the same time or by the same amount. This can be seen by comparing the time history plots, Fig. 6.15 and Fig. 6.16, of the plastic angles incurred at the opposite ends of the exterior girder in the 20th floor. (Note that different scale factors are used in the various plots for the 20th floor.)

For stations (h) and (g) in the second floor, there are some

TABLE 6-4

Times at which the most positive and most negative displacements and interfloor displacements occur for each floor of the A/20/2.2/2/6 frame with 0.10 fraction of critical damping (mass proportional) in the fundamental mode. Pseudo-earthquake number 1 is used. See Fig. A.2.3 in Appendix A for plots of corresponding displacements at the 4th and 17th floors and Fig. A.2.4 for the interfloor displacements between ground and the 1st floor and between the 18th and 19th floors.

Time (sec) of Occurrence of the Most Negative:		· Floor	Time (sec) of Occurrence of the Most Positive:	
Displacement	Interfloor Displacement		Displacement	Interfloor Displacement
14.6 sec		20	18.0 sec	
	29.5 sec			24.5 sec
14.6		19	18.0	
	10.1			24.5
14.6		18	18.0	
	29.6			24.5
4.0		17	18.0	
	29.0			3.2
4.0		16	25.3	
	4.1			3.2
3.9		15	25.3	
	4.1			18.0
3.8		14	25.3	
	12.0			18.0
3.8		13	25.3	
	12.0			25.3
3.8		12	15.6	
	12.0			25.4
3.8		11	13.1	
	16.5			25.4
3.8		10	13.1	
	16.5			25.4
3.8		9	13.1	
	3.8			15.3
21.5		8	13.1	
	3.8			15.6
21.5		7	13.1	
	9.7			13.2
21.5		6	13.1	
	9.7			13.1
21.5		5	13.1	
	16.3			13.1
21.5		4	13.0	
	21.5			13.1
21.4		3	13.0	
	21.5			13.0
21.4		2	13.0	
	21.4			13.0
21.4		1	13.0	
	21.4	Ground		13.0

high frequency components in the bending moments, but it appears that most of the yielding occurs in connection with the response of the fundamental mode. On the other hand, in the time history plots of the bending moments in the 20th floor, many frequencies are observed. It is not obvious that any one frequency dominates the bending moments or the yielding in the 20th floor. Furthermore, the times at which yielding occurs in the 20th floor are different from those at which yielding occurs in either the 2nd, 4th, or the 17th floor.

In Fig. 6.19a the displacement envelope is given. Although the points are connected together, this does not necessarily imply that either all of the most positive displacements or all of the most negative displacements occur at the same time. Typically three or four times are represented for either side of the envelope as shown in Table 6-4 for the study using pseudo-earthquake number 1. However, in the study being analyzed in this section, which uses pseudo-earthquake number 6, all floors do reach their most positive displacements at the same time and the most negative displacements at the same time.

The 20th floor (roof) usually, but not always, is the floor which incurs the maximum absolute displacement of any floor in the frame during an earthquake. As indicated in Table 6-3, for the various studies using 0.10 fraction of critical damping in the frame, the maximum absolute displacements range between 8.5 and 14.4 inches. The ductility factors for the individual girders and columns are shown in

Fig. 6.20a and Fig. 6.20b, respectively. The ductility factors

greater than 1.0 indicate that yielding occurred and are calculated

according to Eq. 2.44.1. However, when yielding does not occur,

the ductility factors are less than 1.0 and are given by the maximum

absolute values of the ratios of the incurred bending moments to

the corresponding yield bending moments. For the girders, the

ductility factor for each end is shown, but for the columns, only the

maximum value that occurs at either end is plotted. The maximum

values of the ductility factors for each study are presented in

Table 6-3. For the pseudo-earthquakes, with 0.10 fraction of critical

damping in the frame, the maximum values of the ductility factors

for the girders range from 3.6 to 5.0. For the exterior columns,

which do not yield, the maximum values range from 0.71 to 0.81.

For the interior columns, which yield in the upper two or three floors

only, the maximum values are between 1.74 and 3.11.

This range in the maximum values of the structural responses

appears to result from statistical variation. Such a range might be

expected because the exact time histories of the earthquake accelero-

grams differ and because there is some variation in the statistics

of the properties of the accelerograms.

In order to observe the effect of viscous damping on the

response of this frame, a computer run using pseudo-earthquake

number 6 with 0.05 fraction of critical damping was made. The

plots for this run are shown in Fig. A.6.1 through Fig. A.6.8 in

Appendix A. The effect of viscous damping is seen by comparing

the various responses in those plots with the corresponding ones

presented in this section. The summary of these response results tabulated in Table 6-3 indicates that by decreasing the fraction of critical damping from 0.10 to 0.05, the maximum absolute values of the various responses increase: the displacement by 15%; the interfloor displacement by 9%; the ductility factors in the girders by 8%, in the interior columns by 12%, and in the exterior columns (moment ratio) by 3%.

6.4 Relationships Between Structural Responses and the Strength of an Earthquake

Three common measurements of the strength of an earthquake are the following:

1. The maximum absolute value of the ground acceleration during the earthquake;

2. The rms value of the acceleration of the earthquake:

$$\left[\frac{1}{30} \int_0^{30} [\ddot{y}(t)]^2 \, dt \right]^{\frac{1}{2}} ;$$

3. The spectrum intensity of the earthquake: The spectrum intensity SI_ξ is defined as the area under the velocity spectrum curve* from period $T = 0.1$ to $T = 2.5$ for a

---

*If $\dot{z}$ is the relative velocity of a linear, base excited oscillator having period T and fraction of critical damping ξ, then the velocity spectrum value for this system is

$$S_{vel}(\xi, \frac{2\pi}{T}, t') = |\dot{z}|_{max}$$

where t' is the length of the earthquake excitation. Other points of the same velocity spectra curve are obtained by using different natural periods of the system. The average spectra curves in Fig. 5.3 are based upon the "pseudo-velocity" spectrum values

$$S_{ps-vel}(\xi, \omega, t') = \omega |z|_{max} .$$

158

given damping factor, ξ, and earthquake excitation[9,14]:

$$SI_\xi = \int_{T=0.1}^{T=2.5} S_{vel}(\xi, \frac{2\pi}{T}, 30) \, dT$$

The numerical values of these three strength measurements of the earthquakes used to excite the nonlinear structure are listed in Table 6-3. Note that the spectrum intensities are given for $\xi = 0.0$.

In an attempt to find a means for predicting the magnitude of structural response, it is of interest to compare the earthquake strengths as defined above with the magnitudes of the corresponding structural responses. Unfortunately, it is seen that none of these three measurements correctly predicts the trend of the maximum absolute values of either the displacements or ductility factors of the structure.

The first two measurements, maximum absolute value and rms value of an earthquake, do not take any properties of the structure into consideration and, therefore, they should not be expected to accurately estimate the maximum absolute value of the various responses of a tall structure. However, if the same earthquake is used with two different amplitude scale factors, the response corresponding to the larger scale factor can be expected to be larger.

The period range chosen for the spectrum intensity includes many of the natural periods of a typical tall structure. However, because the modes of a particular structure are not specifically considered, the spectrum intensity should not be expected to accurately predict the amplitude of the response of that structure. Nevertheless,

the spectrum intensity may be indicative of the amplitude of the average response of numerous structures taken together.

Another method for estimating the maximum values of the horizontal displacement, $y_{j_{max}}$, in the structure is[9]

$$y_{j_{max}} = \left\{ \sum_{i=1}^{20} \left[\frac{\sum_{k=1}^{20} \phi_{ki} M_k}{\sum_{k=1}^{20} \phi_{ki}^2 M_k} \cdot \frac{\phi_i}{\omega_i} S_{ps-vel_i} \right]^2 \right\}^{\frac{1}{2}}$$

where

$\quad$ j $\quad$: floor number,

$\quad$ i $\quad$: mode number,

$\quad$ S_{ps-vel_i} $\quad$: the pseudo-velocity spectrum value at T_i,

$\quad$ ϕ_{ki} $\quad$: the kth component of the ith mode shape, and

$\quad$ ω_i $\quad$: the ith modal frequency.

Since this method takes the modes of the structure into account, it might be more successful in predicting the trends of the maximum absolute displacements of the structure as a function of the earthquakes. It is noted that this method does not consider the time dependence of the modes. From the results of Section 6.3, it appears necessary to take the time dependence into consideration in order to accurately predict the maximum values of the various responses. One such method of approximating the magnitude of a given response in a structure might use the maximum absolute value of a time dependent sum of the linear modal responses, such as shown in Fig. 6.8, with

each mode weighted according to its expected contribution to the response of interest. The abilities of these methods to predict the trends could be evaluated in a future study by applying them to the structure and earthquakes discussed in this chapter.

CHAPTER VII

CONCLUSIONS

The following is a summary of the conclusions reached for the investigation described in this report:

1) Since the one-component nonlinear beam model can have a different curvilinear or bilinear hysteresis loop at each end, it is more versatile than the two-component model which is restricted to bilinear hysteresis loops at the ends.

2) Of the three definitions of ductility factor presented, only one is adaptable to both bilinear and curvilinear hysteresis loops. In the computer studies, this one was used with the one-component model having bilinear hysteretic behavior.

3) As an overall check on the computer program for analyzing the response of a nonlinear multi-story structure subjected to earthquake excitation, several tests covering various aspects of the computations were made. The results of each test indicate that the program operates properly and provides the desired accuracy.

4) For a nonlinear structure, it is possible to find "equivalent" modal participation factors which describe the contributions of the various modes to the structural response. These can be obtained by expanding the displacement vector for the nonlinear structure in terms of the eigenvectors of the linear system.

Such an analysis indicates that more modes may contribute to the interfloor displacements than can be visually observed in the time history response plots, particularly in the upper portion of the structure.

5) The effects of yielding on the response of the A/20/2.2/2/6 frame subjected to the first four seconds of the El Centro (N-S) earthquake of 18 May 1940 are, generally,

   (a) to reduce the forces and moments to approximately 60% of the values for the linear system, and

   (b) to reduce the displacements, interfloor displacements, and total accelerations to about 80% of the corresponding values for the linear system.

   During severe yielding, the shapes of the response time history plots are significantly changed from the shapes of the corresponding responses for the linear structure.

6) In different computer runs, the nonlinear A/20/2.2/2/6 frame with 0.10 fraction of critical damping in the fundamental mode was subjected to the entire El Centro (N-S) earthquake of 18 May 1940 and to six pseudo-earthquakes. The time history plots for a given response at a given position in the structure were compared for the earthquakes used. Characteristic patterns of response behavior were noted which were in qualitative agreement with similar responses of linear elastic, shear-type, continuous cantilever beams. A brief summary of

these characteristic response patterns follows:

(a) For the overturning moment at the base, the fundamental mode dominates.

(b) For the horizontal displacements in the upper portion, the fundamental mode dominates; in the lower portion, the fundamental mode is important, but the 2nd and 3rd modes are also significant.

(c) For the interfloor displacements in the upper portion, as many as eight modes may be important. The fundamental mode decreases in importance approaching the roof. In the lower portion, the fundamental mode increases in importance approaching the base, but the 2nd, 3rd and 4th modes also contribute significantly.

(d) For the total accelerations, frequencies of almost all modes are observed. This is partly a consequence of the relatively low damping in the higher modes.

(e) Results of the studies of the continuous systems indicate that, with increasing taper of the structure, the higher modes become more important for all of the responses throughout the structure.

The distribution of ductility factors in the girders are similar for the different earthquakes used. Column ductility factor distributions were also similar for the different earthquakes.

Because these results are consistent for all of the earthquakes used, it is believed that the characteristic patterns of

modal contributions to the responses and the shapes of the

ductility factor distributions are a function more of the

structure than of the earthquake.

The maximum absolute value of a particular structural

response differs between earthquakes because this value is highly

dependent upon:

(a) the extent to which individual modes are excited at a

particular time, and

(b) the phase relationships between the contributing modes at

that time.

These modal responses differ from one earthquake to another

because the exact time histories of the earthquakes are different.

7) Comparing maximum responses of the yielding structure for the

series of pseudo-earthquakes having similar statistics, the

following percentage deviations were found:

    (a) maximum absolute displacements   50%

    (b) maximum absolute interfloor displacements   40%

    (c) girder ductility factors   35%

    (d) interior column ductility factors   55%

    (e) exterior column ductility factors   15%

8) The effect of decreasing the fraction of critical damping from

0.10 to 0.05 in the fundamental mode of this particular structure

excited by pseudo-earthquake number 6 is to increase the maxi-

mum values of the displacement response by 15% and of the

ductility factor by 8%.

APPENDIX A

RESPONSE PLOTS FOR A NONLINEAR MULTI-STORY STRUCTURE SUBJECTED TO SEVERAL EARTHQUAKE EXCITATIONS

In this appendix response plots for the A/20/2.2/2/6 structure subjected to several earthquakes are presented. Sets of response plots consisting of the displacement envelope, distributions of ductility factors in the girders and in the columns, and time history response plots for various stations in the structure are given for seven structure-earthquake combinations. The positions in the structure of the stations indicated in the plots are shown in Fig. 6.1. The structural and earthquake parameters used are listed in the heading at the top of each plot. They are also tabulated in Table 6-3.

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6

BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10

EL CENTRO EARTHQUAKE (N·S), 18 MAY 1940, SCALE = 1.0

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

Fig. A.1.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
○ : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
○ : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. A.1.2.a

Fig. A.1.2.b

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. A.1.3

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. A.1.4

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. A.1.5 Station (d)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, EL CENTRO EARTHQUAKE (N-S), 18 MAY 40, SCALE =1.0

Fig. A.1.6 Station (e)

169

Fig. A.1.7   Station (f)

Fig. A.1.8   Station (h)

170

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6
BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10
(JENNINGS) PSEUDO-EARTHQUAKE NUMBER I , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

Fig. A.2.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. A.2.2.a

Fig. A.2.2.b

171

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 1, SCALE =1.2

Fig. A.2.3

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 1, SCALE =1.2

Fig. A.2.4

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 1, SCALE =1.2

Fig. A.2.5   Station (d)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 1, SCALE =1.2

Fig. A.2.6   Station (e)

Fig. A.2.7   Station (f)
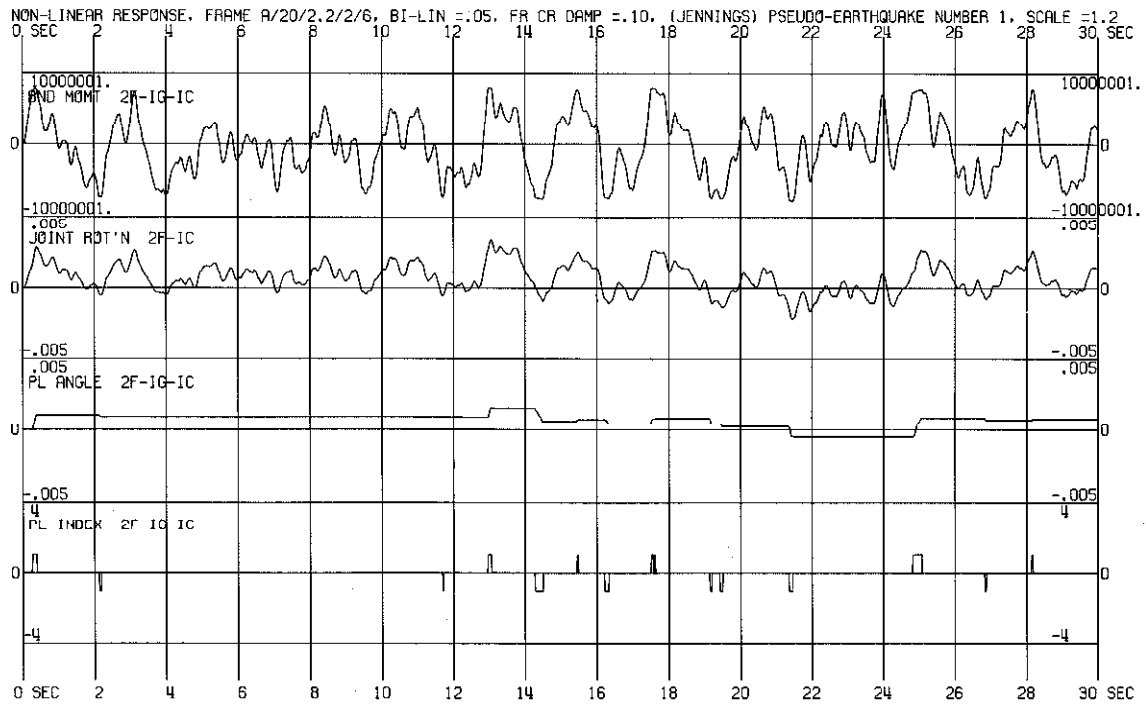
Station A.2.8   Station (h)

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6

BILINEARITY - 0.05, FRACTION OF CRITICAL DAMPING = 0.10

(JENNINGS) PSEUDO-EARTHQUAKE NUMBER 2 , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES
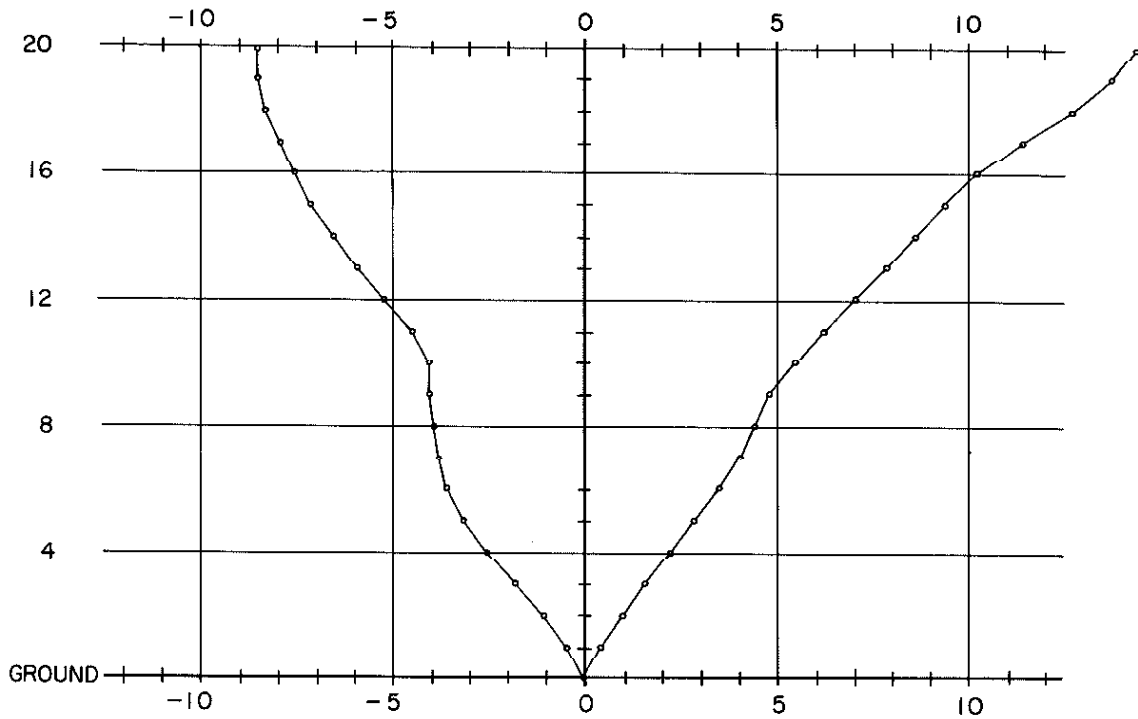
Fig. A.3.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. A.3.2.a

Fig. A.3.2.b

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 2, SCALE =1.2

Fig. A.3.3

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 2, SCALE =1.2

Fig. A.3.4

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 2, SCALE =1.2

Fig. A.3.5   Station (d)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 2, SCALE =1.2

Fig. A.3.6   Station (e)

177
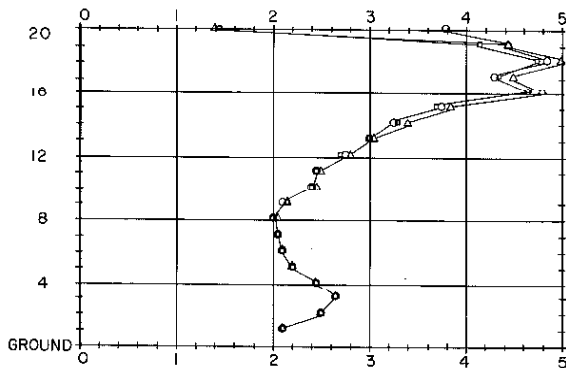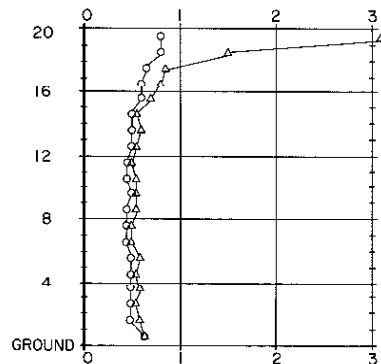
Fig. A.3.7   Station (f)

Fig. A.3.8   Station (h)

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6

BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10

(JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4 , SCALE = I.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

Fig. A.4.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. A.4.2.a

Fig. A.4.2.b

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4, SCALE =1.2

Fig. A.4.3

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4, SCALE =1.2

Fig. A.4.4

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4, SCALE =1.2

Fig. A.4.5   Station (d)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4, SCALE =1.2

Fig. A.4.6   Station (e)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4, SCALE =1.2
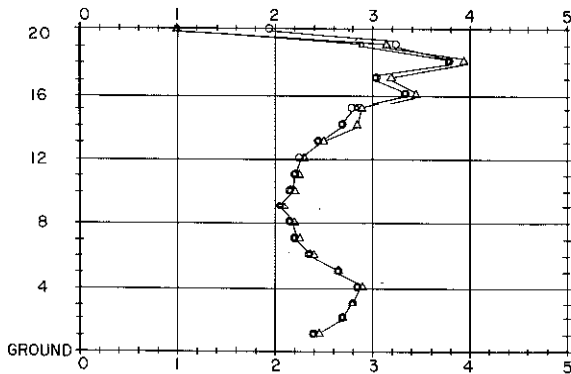
Fig. A.4.7   Station (f)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 4, SCALE =1.2
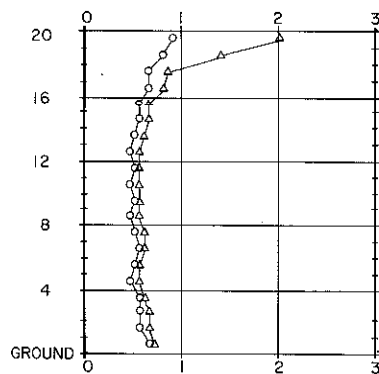
Fig. A.4.8   Station (h)

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6

BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10

(JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5 , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

Fig. A.5.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. A.5.2.a

Fig. A.5.2.b

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5, SCALE =1.2

Fig. A.5.3

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5, SCALE =1.2

Fig. A.5.4

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5, SCALE =1.2

Fig. A.5.5   Station (d)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5, SCALE =1.2

Fig. A.5.6   Station (c)

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6. BI-LIN =.05. FR CR DAMP =.10. (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5. SCALE =1.2
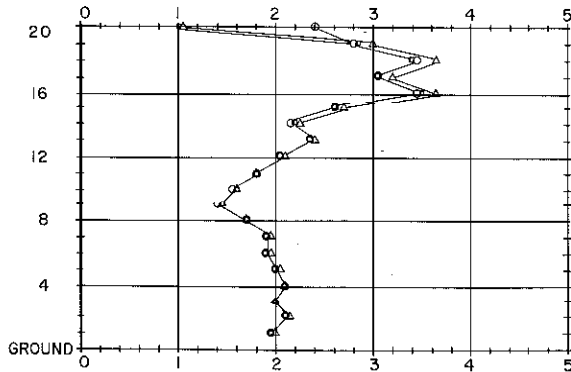
Fig. A.5.7   Station (f)

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6. BI-LIN =.05. FR CR DAMP =.10. (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 5. SCALE =1.2
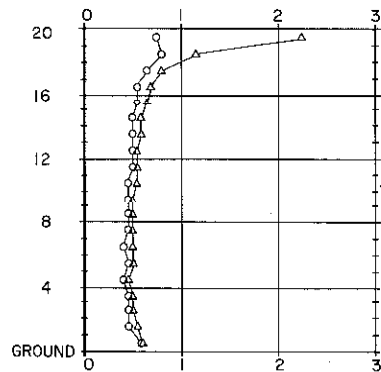
Fig. A.5.8   Station (h)

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6

BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.05

(JENNINGS) PSEUDO−EARTHQUAKE NUMBER 6 , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

Fig. A.6.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN
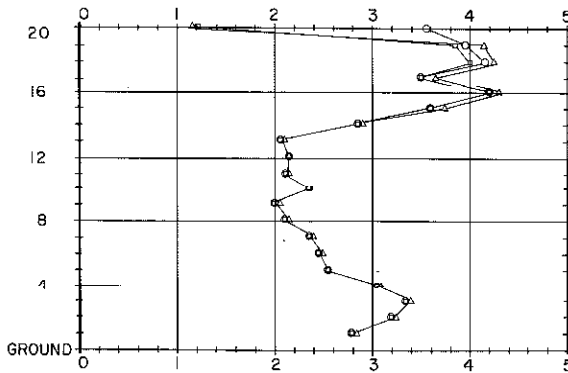
COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN
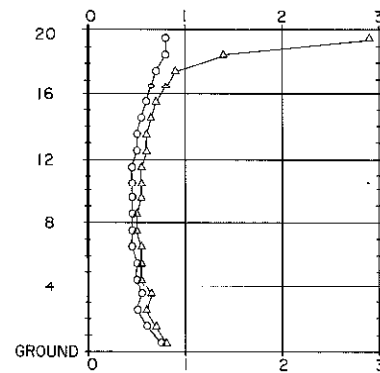
Fig. A.6.2.a

Fig. A.6.2.b

Fig. A.6.3

Fig. A.6.1

188
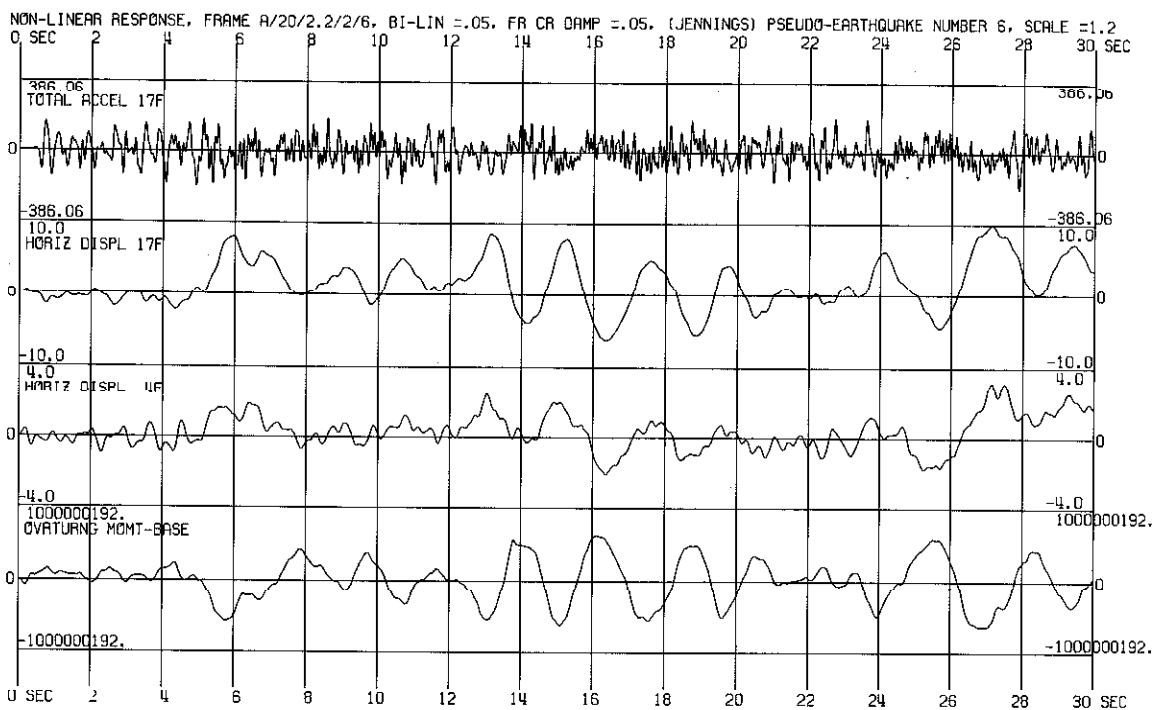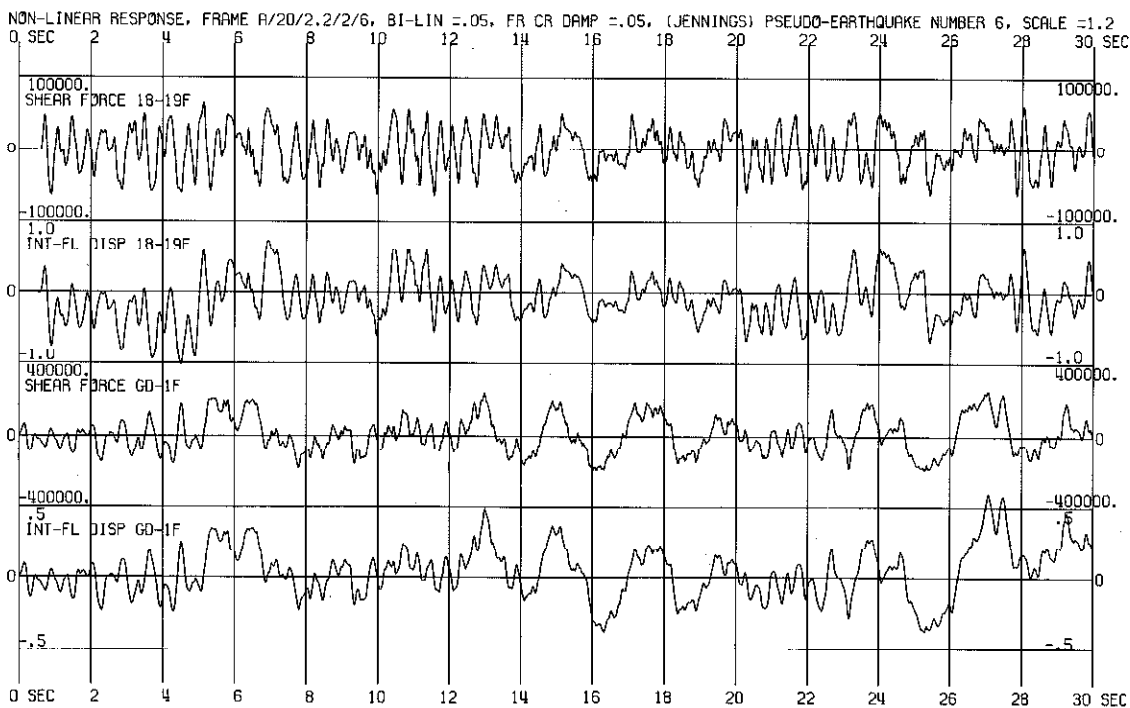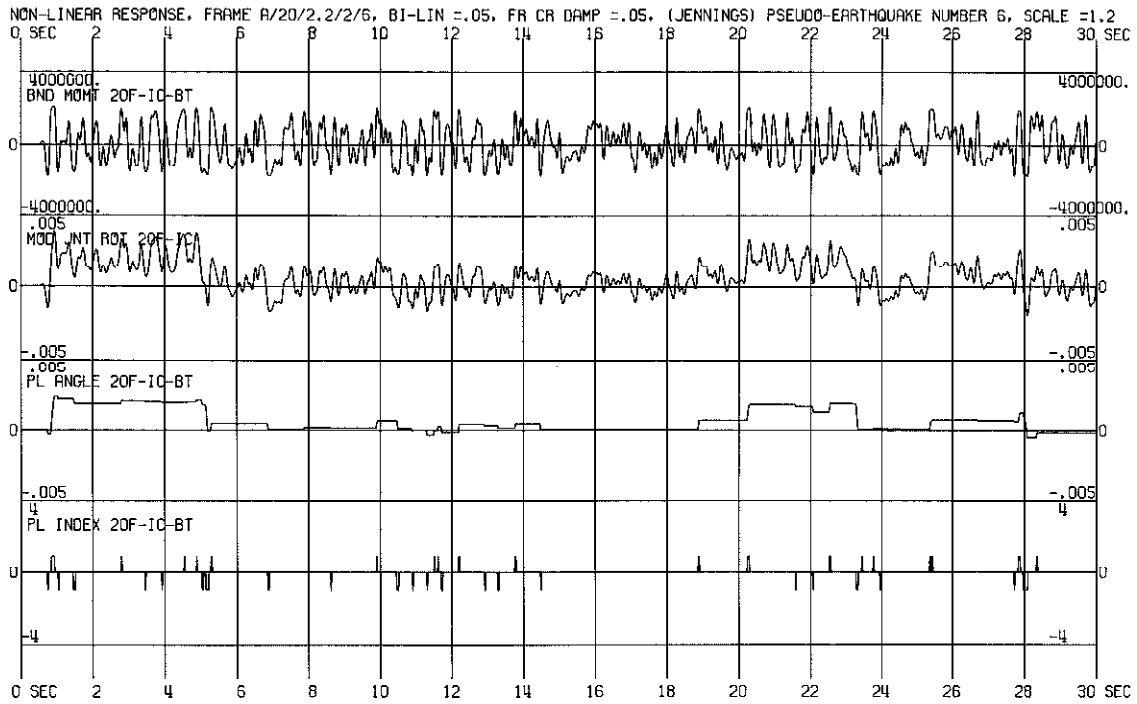
Fig. A.6.5   Station (d)

Fig. A.6.6   Station (e)

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6. BI-LIN =.05, FR CR DAMP =.05. (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6. SCALE =1.2

Fig. A.6.7   Station (f)

NON-LINEAR RESPONSE. FRAME A/20/2.2/6. BI-LIN =.05, FR CR DAMP =.05. (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE =1.2
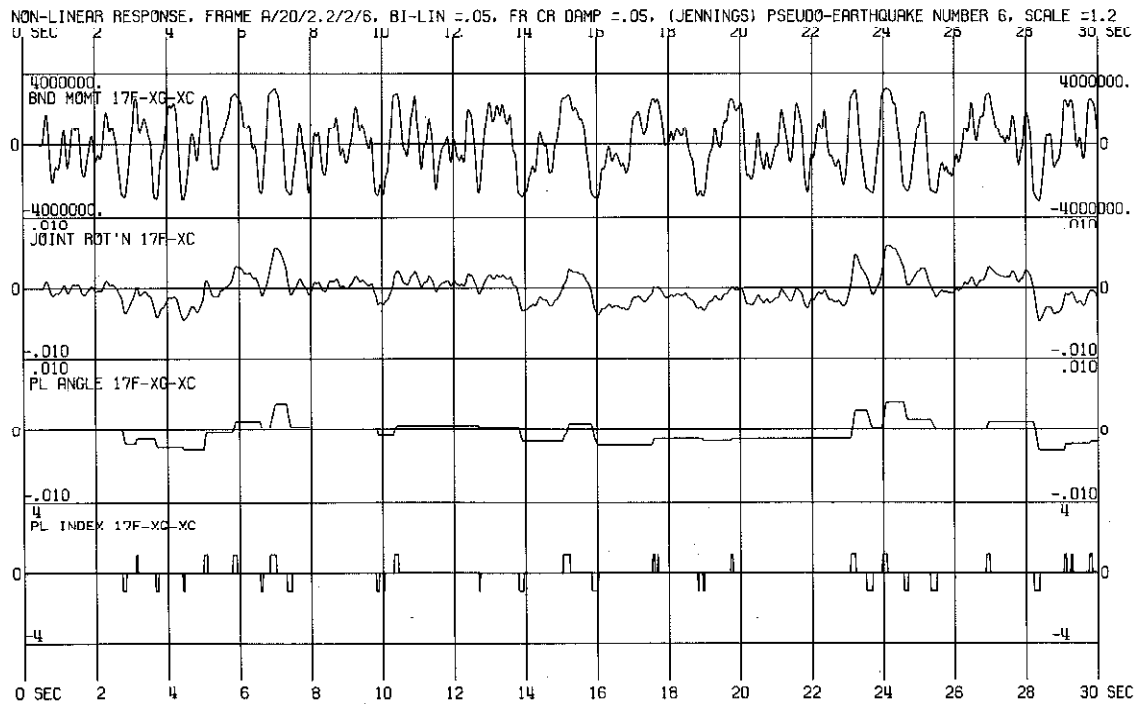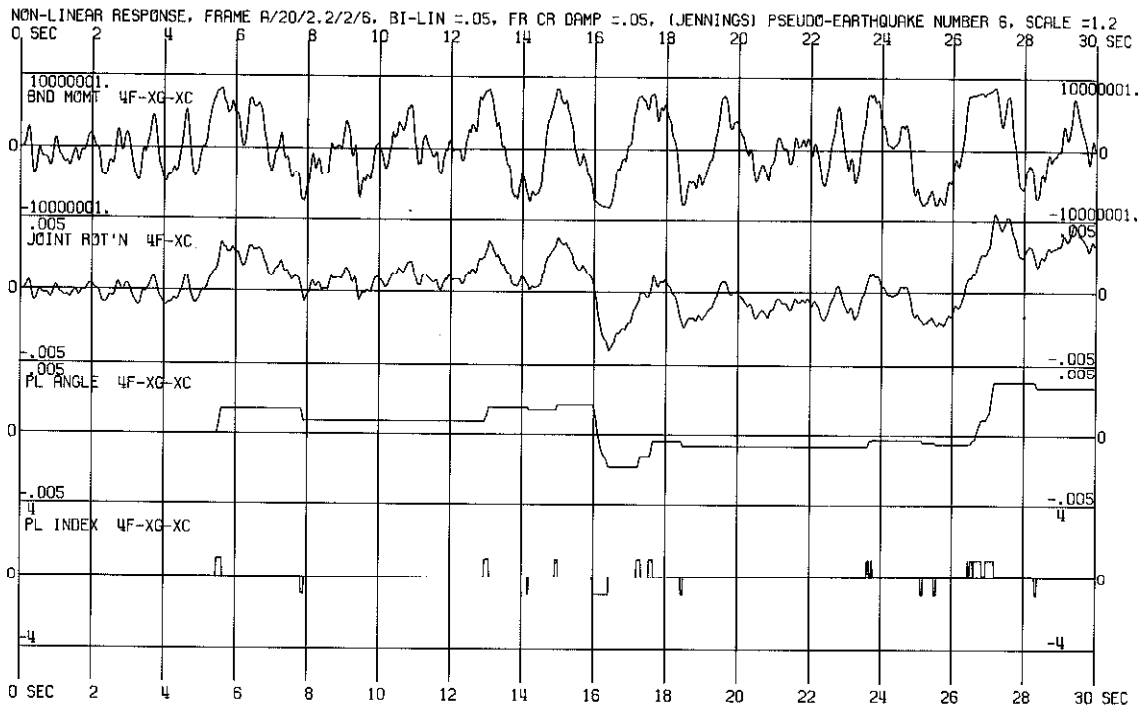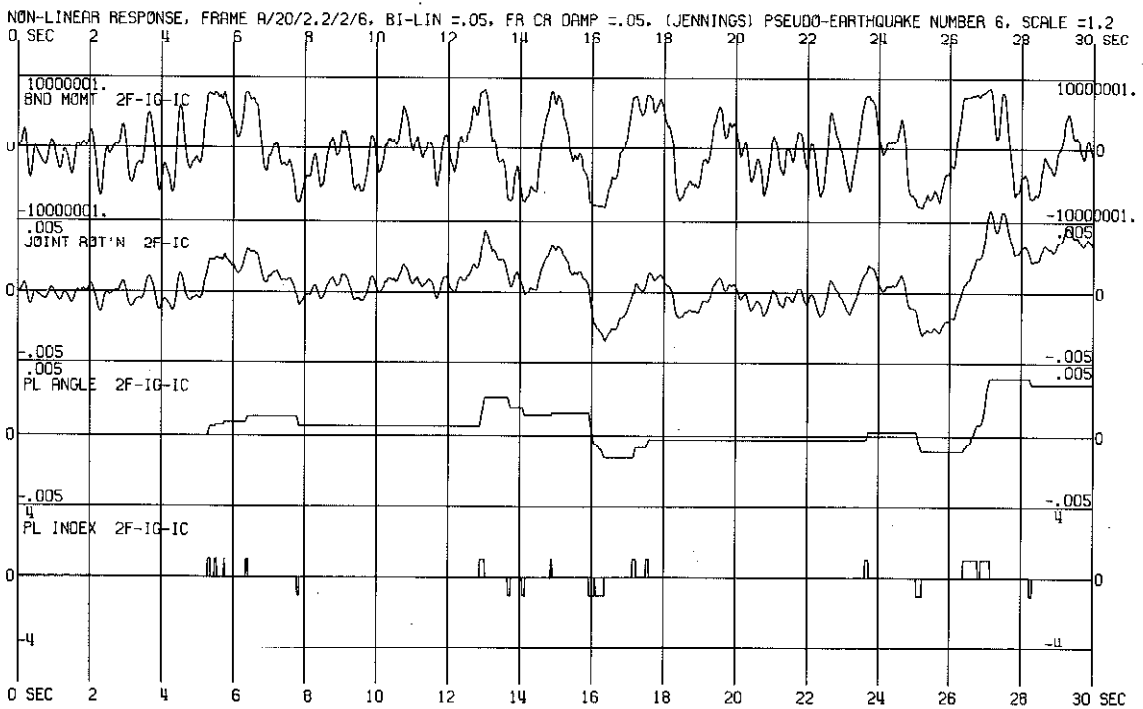
Station A.6.8   Station (h)

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6

BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10

(JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7 , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES
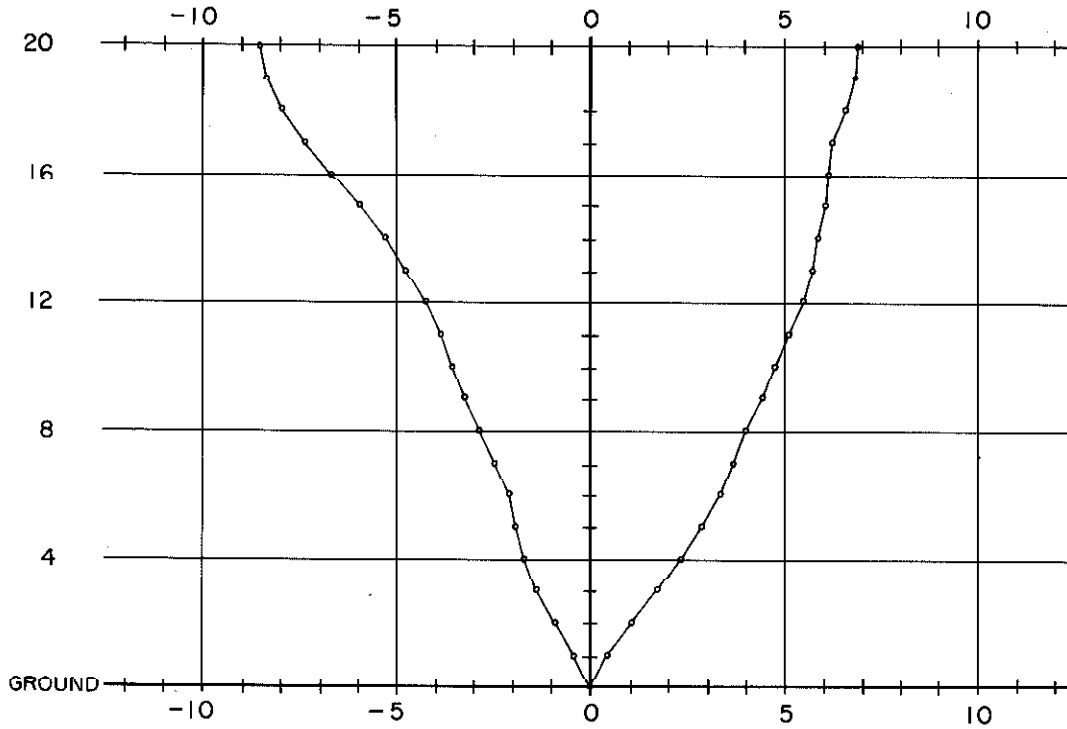
Fig. A.7.1

EXTERIOR AND INTERIOR GIRDER
DUCTILITY FACTORS
O : IN EXTERIOR GIRDER AT EXTERIOR COLUMN
□ : IN EXTERIOR GIRDER AT INTERIOR COLUMN
△ : IN INTERIOR GIRDER AT INTERIOR COLUMN

COLUMN DUCTILITY FACTORS
O : EXTERIOR COLUMN
△ : INTERIOR COLUMN

Fig. A.7.2.a

Fig. A.7.2.b

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7, SCALE =1.2

Fig. A.7.3

NON-LINEAR RESPONSE. FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7, SCALE =1.2

Fig. A.7.4

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7, SCALE =1.2

Fig. A.7.5 Station (d)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7, SCALE =1.2

Fig. A.7.6 Station (e)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7, SCALE =1.2

Fig. A.7.7   Station (f)

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05, FR CR DAMP =.10, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 7, SCALE =1.2
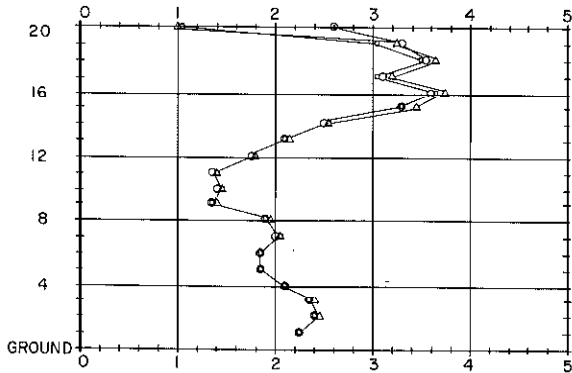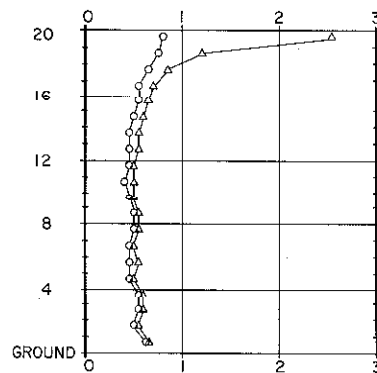
Fig. A.7.8   Station (h)

APPENDIX B

DISCUSSION OF THE YIELD CRITERIA USED IN THE FHA STUDY

The purpose of this appendix is to describe the special situation which causes the yield criteria used in the computer program for the FHA Study to lead to computational difficulties. Instead of the actual bending moment, this criteria compares a corresponding "test bending moment" with the yield bending moment in order to determine whether the end of a beam is in the linear range or in the yield range. By following the actual and corresponding test bending moments for a number of time increments along the paths shown in Fig. B.1a and Fig. B.1b, respectively, the special situation can be seen.

Usually, the bending moments are written as a function of the rotations and plastic angles of both ends of the beam; however, a simpler form in which the moment is a function of the rotation and plastic angle at one end only is sufficient for the present purpose. The equations written below are those for a simplified two-component beam. The usual moment-rotation equations for a two-component beam are described in Section 2.5.
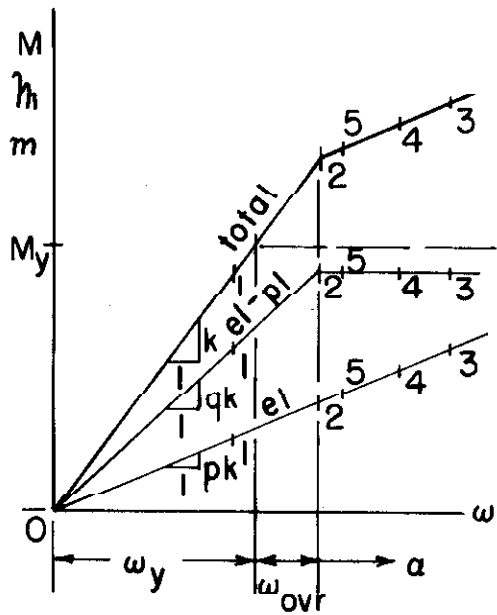
Fig. B.1a Actual
Bending Moment
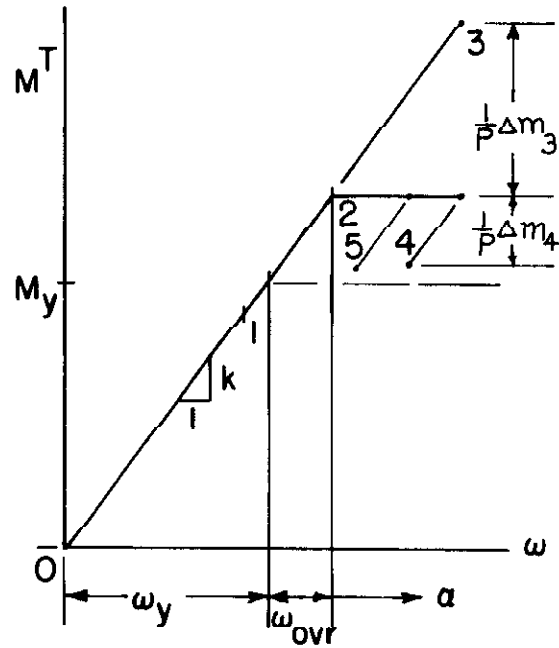
Fig. B.1b Test
Bending Moment

Actual Bending Moment	Test Bending Moment
(See Fig. B.1a)	(See Fig. B.1b)

General equations:	General equations:
elasto-plastic component:	test bending moment:
$m_t = qk(\omega_t - \alpha_t)$;	$M_t^T = k(\omega_t - \alpha_{t-\Delta t})$;
elastic component:	however, with
$m_t = pk(\omega_t)$;	$\omega_t = \omega_{t-\Delta t} + \Delta\omega_t$,
total actual bending moment:	$M_t^T = k(\omega_{t-\Delta t} - \alpha_{t-\Delta t}) + k\Delta\omega_t$,
$M_t = k(\omega_t - q\alpha_t)$;	or
end rotation:	$M_t^T = \frac{1}{q} m_{t-\Delta t} + \frac{1}{p} \Delta m_t$
$\omega_t = \omega_{t-\Delta t} + \Delta\omega_t$;	where
plastic angle:	m_t, m_t, p, q, and t
$\alpha_t = \alpha_{t-\Delta t} + \Delta\alpha_t$;	are the same as those for the
t = 1,2,3,... the integration	"actual bending moment."
step number, i.e., the time	
at the end of the increment; and	
p + q = 1 (usually p $\approx$ 0.05).	

Follow the process for

a number of time increments.

t = 0, initial point.	t = 0
$\omega_o = 0$,	Initially in the linear range;
$\alpha_o = 0$,	hence,
$m_o = 0$,	PLASTIC INDEX (1) = 0.

Actual Moment, cont.	Test Moment, cont.

$$m_o = 0, \text{ and}$$

$$M_o = 0.$$

Integration step number 2,

calculate $\Delta\omega_1$.

t = 1

	t = 1
Since PLASTIC INDEX (1) = 0,	$M_1^T = \dfrac{1}{q} m_o + \dfrac{1}{p} \Delta m_1$,

$$\Delta\alpha_1 = 0.$$

with

$$\therefore \ \alpha_1 = 0,$$

$$m_o = 0,$$

$$m_1 = qk(\omega_1 - 0),$$

$$\Delta m_1 = pk\Delta\omega_1 ,$$

$$m_1 = pk\omega_1, \text{ and}$$

and

$$M_1 = k\omega_1 \ .$$

$$\Delta\omega_1 = \omega_1 ,$$

$$(M_1 = M_1^T)$$

$$M_1^T = k\omega_1 \ .$$

Compare M_1^T with M_y. From

point (t = 1) in Fig. B.1b, find

that $M_1^T < M_y$.

$\therefore$ PLASTIC INDEX (2) = 0, i.e.,

in the linear range for integration

step number 2.

Integration step number 2,

calculate $\Delta\omega_2$.

Actual Moment, cont.	Test Moment, cont.

Actual Moment, cont.

$t = 2$

Since PLASTIC INDEX (2) = 0,

$\Delta\alpha_2 = 0.$

$\therefore \alpha_2 = 0,$

$m_2 = qk(\omega_2 - 0),$

$m_2 = pk\omega_2,$ and

$M_2 = k\omega_2$.

$(M_2 = M_2^T)$

Test Moment, cont.

$t = 2$

$M_2^T = \frac{1}{q} m_1 + \frac{1}{p} \Delta m_2$

With

$m_1 = qk\omega_1$,

$\Delta m_2 = pk\Delta\omega_2$,

and

$\Delta\omega_2 = \omega_2 - \omega_1$,

it follows that

$M_2^T = k\omega_2.$

Compare M_2^T with M_y. From

point (t = 2) in Fig. B.1b, find that

$M_2^T > M$.

$\therefore$ PLASTIC INDEX (3) = 1, i.e.,

in the yield range for integration

step number 3. From Fig. B.1b,

note that

$M_2^T = M_y + k\omega_{ovr}$

where the overshoot ω_{ovr} is

$\omega_{ovr} = \omega_2 - \omega_y$.

Integration step number 3,

calculate $\Delta\omega_3$.

Actual Moment, cont.

$t = 3$

Since PLASTIC INDEX (3) = 1,

$\Delta\alpha_3 = \Delta\omega_3 = \omega_3 - \omega_2$.

Since $\alpha_3 = \alpha_2 + \Delta\alpha_3$,

and $\alpha_2 = 0$,

it follows that

$\alpha_3 = \omega_3 - \omega_2$.

Furthermore,

$m_3 = qk\omega_2$,

or

$m_3 = m_2$;

$m_3 = pk\omega_3$;

and

$M_3 = k(\omega_3 - q\alpha_3)$.

Also,

$M_3 = M_2 + \Delta M_3$

or

$M_3 = M_2 + \Delta m_3$

where

$\Delta m_3 = pk\Delta\omega_3$.

Test Moment, cont.

$t = 3$

$M_3^T = \frac{1}{q} m_2 + \frac{1}{p} \Delta m_3$

With

$m_2 = qk\omega_2$,

$\Delta m_3 = pk\Delta\omega_3$,

it follows that

$M_3^T = k\omega_3$.

Compare M_3^T with M_y. From

Fig. B.1b, find that $M_3^T > M_y$.

$\therefore$ PLASTIC INDEX (4) = 1, i.e.,

in the yield range for integration

step number 4.

Note that

$M_3^T = M_2^T + k\Delta\omega_3$.

Since

$M_2^T = M_y + k\omega_{ovr}$,

$M_3^T = M_y + k(\omega_{ovr} + \Delta\omega_3)$.

If

$(\omega_{ovr} + \omega_3) > 0$,

then

$M_3^T > M_y$.

As seen in Fig. B.1b,

$\Delta\omega_3 = \omega_3 - \omega_2 > 0$.

In this instance, the criteria indi-

cate that the system is in the

Actual Moment, cont.	Test Moment, cont.

Actual Moment, cont.

Test Moment, cont.

yield range during the next time interval, as it should be. Hence, PLASTIC INDEX (4) = 1.

Integration step number 4,

calculate $\Delta\omega_4$.

Actual column:

$t = 4$

Since PLASTIC INDEX (4) = 1,

$\Delta\alpha_4 = \Delta\omega_4 = \omega_4 - \omega_3$,

$\mathbb{m}_4 = \mathbb{m}_2$.

Also,

$M_4 = M_3 + \Delta m_4$

where

$\Delta m_4 = pk\Delta\omega_4$.

In this time increment "back-tracking" occurred, i.e.,

$\Delta\omega_4 < 0$,

as shown in Fig. B.1b. Hence,

$\alpha_4 < \alpha_3$

and $M_4 < M_3$.

The first increment of back-tracking is unavoidable but additional ones are undesirable.

Test column:

$t = 4$

$M_4^T = \frac{1}{q}\mathbb{m}_3 + \frac{1}{p}\Delta m_4$.

With

$\mathbb{m}_3 = \mathbb{m}_2 = qk\omega_2$

and

$\Delta m_4 = pk\Delta\omega_4$,

it follows that

$M_4^T = k\omega_2 + k\Delta\omega_4$.

Since

$k\omega_2 = M_y + k\omega_{ovr}$,

$M_4^T = M_y + k(\omega_{ovr} + \Delta\omega_4)$.

As shown in Fig. B.1b,

$\Delta\omega_4 < 0$,

but

$\omega_{ovr} + \Delta\omega_4 > 0$.

Consequently, the criteria for yielding:

$M_4^T > M_y$

Actual Moment, cont.	Test Moment, cont.
	is still satisfied indicating that the system continues in the yield range:
	PLASTIC INDEX (5) = 1.

As long as

$$\omega_{ovr} + \Delta\omega_t > 0$$

for

$$t = 4, 5, 6, \ldots \quad ,$$

the criteria for being in the yield range will be satisfied even though

$$\Delta\omega_t < 0 \quad \text{for} \quad t = 4, 5, 6, \ldots \quad .$$

In this special situation, the bending moment as well as the yield angle will continue to "backtrack," that is, decrease according to the equations for the yield range when they should have returned to the linear range. The system will __not__ return to the linear range until some $\Delta\omega_t$ is sufficiently negative so that

$$\omega_{ovr} + \Delta\omega_t < 0 \ .$$

Fortunately, $\Delta\omega_t$ is usually sufficiently negative, or what is the same, ω_{ovr} is usually sufficiently small so that this problem with the yield criteria does not often cause serious difficulties.

APPENDIX C

DISCUSSION OF THE TREATMENT OF SYMMETRY
FOR THE TWO-COMPONENT BEAM MODEL

The purpose of this appendix is to discuss an approximate treatment of symmetry for the two-component model of the interior girder of a three-bay (four-column) structural frame. This approximation is convenient for the computer program used for the FHA Study[1]. In this treatment, the interior girder is cut at the center-line and the resulting "centerline" end, end (c), is treated the same as the end of any other beam. That is, the ends of both components of the girder are connected together and then pinned so that they rotate identically, as shown in Fig. C.1b. In this case, the sum of the bending moments of <u>both</u> components of the girder at end (c) is zero.

The correct treatment of symmetry is achieved by cutting the girder at the centerline and <u>separately</u> pinning the resulting end of each component so that they can rotate independently. In this case, the bending moment at end (c) of each component is zero, as shown in Fig. C.1a.

The nomenclature used in this appendix closely follows that for the two-component beam in Chapter II, with the usage for each system being indicated in Fig. C.1a and Fig. C.1b. The appropriate boundary conditions are also given in these figures.

The bending moment-end rotation-plastic angle equations for both the correct and approximate treatments of symmetry are given
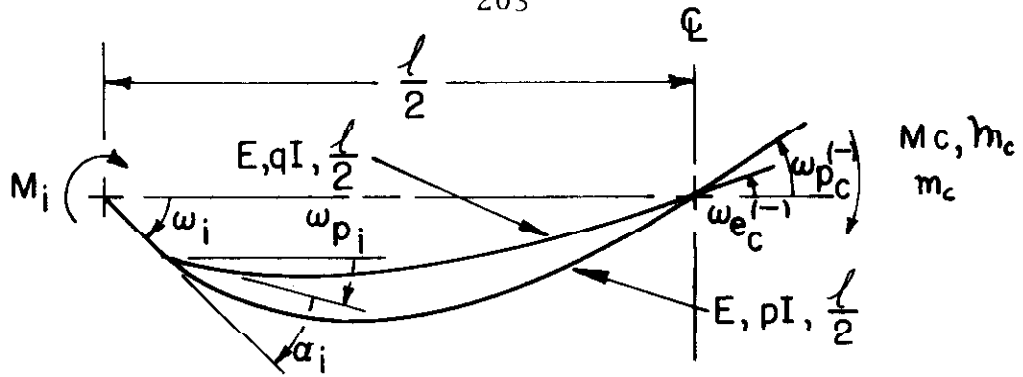
left end:

$$\Delta M_i = \Delta \bar{m}_i + \Delta m_i \; ;$$

$$\Delta \omega_i = \Delta \omega_{p_i} + \Delta \alpha_i \; ;$$

$$\Delta \alpha_i \neq 0 \; .$$

"centerline" end:

$$\Delta \bar{m}_c = 0 \; ; \; \Delta m_c \neq 0 \; ;$$

$$\omega_{e_c} \neq \omega_{p_c} \; ; \; \Delta \alpha_c = 0 \; .$$

Fig. C. 1a Treatment of Symmetry
of an Interior Girder

left end:

$$\Delta M_i = \Delta \bar{m}_i + \Delta m_i \; ;$$

$$\Delta \omega_i = \Delta \omega_{p_i} + \Delta \alpha_i \; ;$$

$$\Delta \alpha_i \neq 0 \; .$$
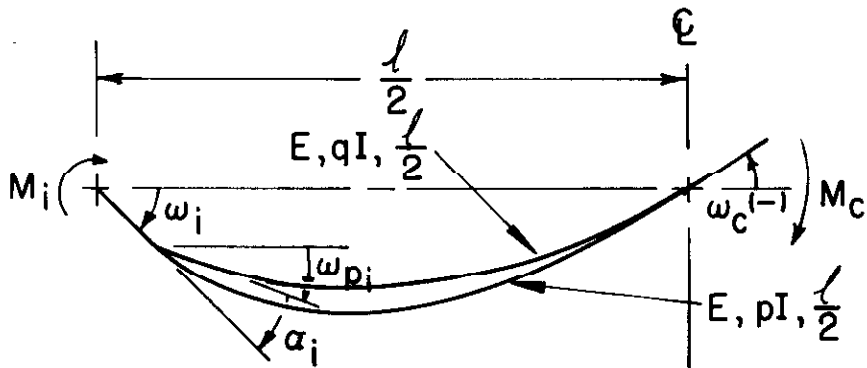
"centerline" end:

$$\Delta M_c = \Delta \bar{m}_c + \Delta m_c = 0 \; ;$$

$$\Delta \bar{m}_c = - \Delta m_c \; ; \; \Delta \alpha_c = 0 \; ;$$

$$\Delta \omega_c = \Delta \omega_{p_c} = \Delta \omega_{e_c} \; .$$

Fig. C. 1b Approximate Treatment of Symmetry
of an Interior Girder

below. In both systems, the stiffness

of the elastic component is:

$$pk' = p\left(\frac{EI}{\ell/2}\right) \; ;$$

and of the elasto-plastic component is:

$$qk' = q\left(\frac{EI}{\ell/2}\right) \; ;$$

where $p + q = 1$. In this case $k' = 2k$ where

$$k = \frac{EI}{\ell}$$

and ℓ is the length of the entire interior girder. Since no yielding

occurs at end (c) for either system, only two yield cases are con-

sidered: end (i) is either linear or yielding.

Correct Treatment	Approximate Treatment
(See Fig. C.1a)	(See Fig. C.1b)
Boundary Conditions:	Boundary Conditions:
end (i):	end (i):
$\Delta M_i = \Delta \bar{m}_i + \Delta m_i$,	$\Delta M_i = \Delta \bar{m}_i + \Delta m_i$,
$\Delta \omega_{p_i} = \Delta \omega_i - \Delta \alpha_i$;	$\Delta \omega_{p_i} = \Delta \omega_i - \Delta \alpha_i$;
end (c):	end (c):
$\Delta \bar{m}_c = 0$,	$\Delta M_c = \Delta \bar{m}_c + \Delta m_c = 0$
$\Delta m_c = 0$,	or
$\Delta \omega_{p_c} \neq \Delta \omega_{e_c}$.	$\Delta \bar{m}_c = -\Delta m_c$,
With these boundary conditions,	and
the general moment-rotation	$\Delta \omega_{p_c} = \Delta \omega_{e_c} = \Delta \omega_c$.

Correct Treatment, cont.

equations in incremental form

are:

for the elasto-plastic component:

$$\Delta \bar{m}_i = qk'[\Delta \omega_i - \Delta \alpha_i) + \tfrac{1}{2} \Delta \omega_{p_c}]$$

$$\Delta \bar{m}_c = qk'[\tfrac{1}{2}(\Delta \omega_i - \Delta \alpha_i) + \Delta \omega_{p_c}] \ ;$$

for the elastic component:

$$\Delta m_i = pk'[\Delta \omega_i + \tfrac{1}{2} \Delta \omega_{e_c}]$$

$$\Delta m_c = pk'[\tfrac{1}{2} \Delta \omega_i + \Delta \omega_{e_c}] \ .$$

Apply boundary conditions at

end (c):

$$\Delta \bar{m}_c = 0$$

and

$$\Delta m_c = 0 \ .$$

Hence,

$$\Delta \omega_{p_c} = -\tfrac{1}{2}(\Delta \omega_i - \Delta \alpha_i) \ .$$

$$\Delta \omega_{e_c} = -\tfrac{1}{2} \Delta \omega_i \ .$$

For the total beam at end (i):

$$\Delta M_i = \Delta \bar{m}_i + m_i \ .$$

Eliminating $\Delta \omega_{p_c}$ and $\Delta \omega_{e_c}$, this becomes

$$\Delta M_i = \tfrac{3}{4} k'(\Delta \omega_i - q\Delta \alpha_i) \ .$$

Approximate Treatment, cont.

With these boundary conditions,

the general moment-rotation

equations in incremental form

are:

for the elasto-plastic component:

$$\Delta \bar{m}_i = qk'[(\Delta \omega_i - \Delta \alpha_i) + \tfrac{1}{2} \Delta \omega_c]$$

$$\Delta \bar{m}_c = qk'[\tfrac{1}{2}(\Delta \omega_i - \Delta \alpha_i) + \Delta \omega_c] \ ;$$

for the elastic component:

$$\Delta m_i = pk'[\Delta \omega_i + \tfrac{1}{2} \Delta \omega_c]$$

$$\Delta m_c = pk'[\tfrac{1}{2} \Delta \omega_i + \Delta \omega_c] \ .$$

Apply boundary conditions at

end (c):

$$\Delta M_c = \Delta \bar{m}_c + \Delta m_c = 0 \ .$$

Hence

$$\Delta \omega_c = -\tfrac{1}{2}(\Delta \omega_i - q\Delta \alpha_i).$$

For the total beam at end (i):

$$\Delta M_i = \Delta \bar{m}_i + \Delta m_i \ .$$

Eliminating $\Delta \omega_c$, this becomes

$$\Delta M_i = \tfrac{3}{4} k'(\Delta \omega_i - q\Delta \alpha_i) \ .$$

Correct Treatment, cont.	Approximate Treatment, cont.
Since	Since
$k' = 2k$,	$k' = 2k$,
where $k = \frac{EI}{\ell}$ is the stiffness of the entire interior girder,	where $k = \frac{EI}{\ell}$ is the stiffness of the entire interior girder.
$\Delta M_i = 1.5(\Delta \omega_i - q\Delta \alpha_i)$.	$\Delta M = 1.5(\Delta \omega_i - q\Delta \alpha_i)$.
The moment-rotation equation for the total beam at end (i) for the linear case with	The moment-rotation equation for the total beam at end (i) for the linear case with
$\Delta \alpha_i = 0$,	$\Delta \alpha_i = 0$,
is	is
$\Delta M_i = 1.5\, k\Delta \omega_i$.	$\Delta M_i = 1.5\, k\Delta \omega_i$.

These equations for both systems are

the same in the linear case.

Incremental plastic angle at end (i). During yielding, for the two-component beam (see Sec. 2.5),	Incremental plastic angle at end (i). During yielding, for the the two-component beam (see Sec. 2.5),
$\Delta M_i = \Delta m_i$	$\Delta M_i = \Delta m_i$
or	or
$1.5k[\Delta \omega_i - q\Delta \alpha_i] = 1.5pk\Delta \omega_i$.	$1.5k[\Delta \omega_i - q\Delta \alpha_i] =$ $1.5pk[\Delta \omega_i + \frac{q}{3}\Delta \alpha_i]$.

Correct Treatment, cont.

Hence

$$\Delta \alpha_i = \Delta \omega_i \ .$$

Moment-rotation equation for the total beam at end (i) during yielding. By eliminating $\Delta \alpha_i$, this equation becomes:

$$\Delta M_i - 1.5 \, pk \Delta \omega_i \ .$$

Approximate Treatment, cont.

Hence

$$\Delta \alpha_i = \left(\frac{3}{3+p} \right) \Delta \omega_i \ .$$

Moment rotation equation for the total beam at end (i) during yielding. By eliminating $\Delta \alpha_i$, this equation becomes:

$$\Delta M_i = \left(\frac{6}{3+p} \right) pk \Delta \omega_i \ .$$

For $p = 0.05$, as used in this report,

$$\Delta M_i = 1.97 \, pk \Delta \omega_i .$$

Comparing these two moment-rotation equations, it is observed that for the value of (p) used, the effective stiffness of the approximate technique during yielding is about 4/3 times that for the correct method. Because each joint of the frame is in the linear case about 90% of the time, and because the equations are the same for both systems in the linear case, only small differences in the structural responses occur. By comparing the computer results for the two methods with each other, these differences are seen to be on order of 1%, being small enough to justify the use of the approximate technique for treating symmetry.

APPENDIX D

FUTURE STUDIES

As a result of the investigation described in this report, several analyses are suggested which would provide additional understanding of how multi-story structures respond to earthquake excitation. These are briefly mentioned below.

1) Stiffness proportional damping in the form of Eq. 3.24 could be added to the computer program, thereby making it possible to observe the effect of this type of damping on the response. Combinations of the two types of damping would make it possible to match more closely the modal damping distribution in actual buildings.

2) An energy balance subroutine could be developed and added to the program. Such a balance might consist of the energy in (and out) at the base, dissipation by yielding, dissipation by damping mechanisms, kinetic energy and (recoverable) potential energy. Such an analysis would provide insight into the distribution of energy dissipation in a multi-story structure.

3) Instead of bilinear hysteresis loops which inherently have a sharp corner at the yield level, curvilinear hysteresis loops could be used with the one-component beam model. This would permit the effect of the sharp corner upon the response of the structure to be observed.

4) Analyses of nonlinear elastic, shear type, base excited, continuous cantilever beams might be considered. The excitation could be either pulse, sinusoidal, or earthquake.

5) Static moments could be applied to the ends of the girders to account for static loads in the structure. This would require the use of the entire frame because the yielding characteristics would no longer be symmetric with respect to the centerline.

APPENDIX E

LISTING OF THE COMPUTER PROGRAM

The computer program used for determining the response of nonlinear multi-story structures subjected to earthquake excitation is listed in this appendix. A flow chart is also provided. Numerous comment cards are used so that the program will be understandable to the reader with some background in Fortran II or Fortran IV.

This program was written in Fortran IV for use on the IBM 7090/7094 Computer at the Computer Center of the California Institute of Technology. The computer has a 32,000-cell memory core which is filled by this program and the necessary library subroutines. Alterations may be necessary to adapt this program to the computers at other installations.

A typical run in which the nonlinear A/20/2.2/2/6 frame was subjected to 30 seconds of pseudo-earthquake took 40 minutes of computer time. Since the time increment is 0.005 sec (earthquake time), there were 6000 integration steps. After every fourth time interval, most of the responses of the structure were written onto binary tape. After every second of earthquake time, the most positive and most negative (cumulative) values of all of the responses, along with the times at which they occurred, were written out on paper.

The resulting 2400 ft, 560 bit-per-inch binary tape was scanned to obtain the time histories of the responses to be plotted. The time history for each response was punched into a deck of cards.

The plots were made from these decks. The time necessary to scan a tape for 30 responses was about 10 minutes. Each page of four response plots took the computer 30 sec to set up; however, it took 7 minutes to actually plot.

When a modal analysis was made, the "equivalent" modal participation factors, the "equivalent" modal contributions to four displacements and to four interfloor displacements were found. This information was calculated and written out for every time increment. Such an analysis doubles the computer time required. One run of this nature was made for the first four seconds of the El Centro (N-S) earthquake of 18 May 1940.
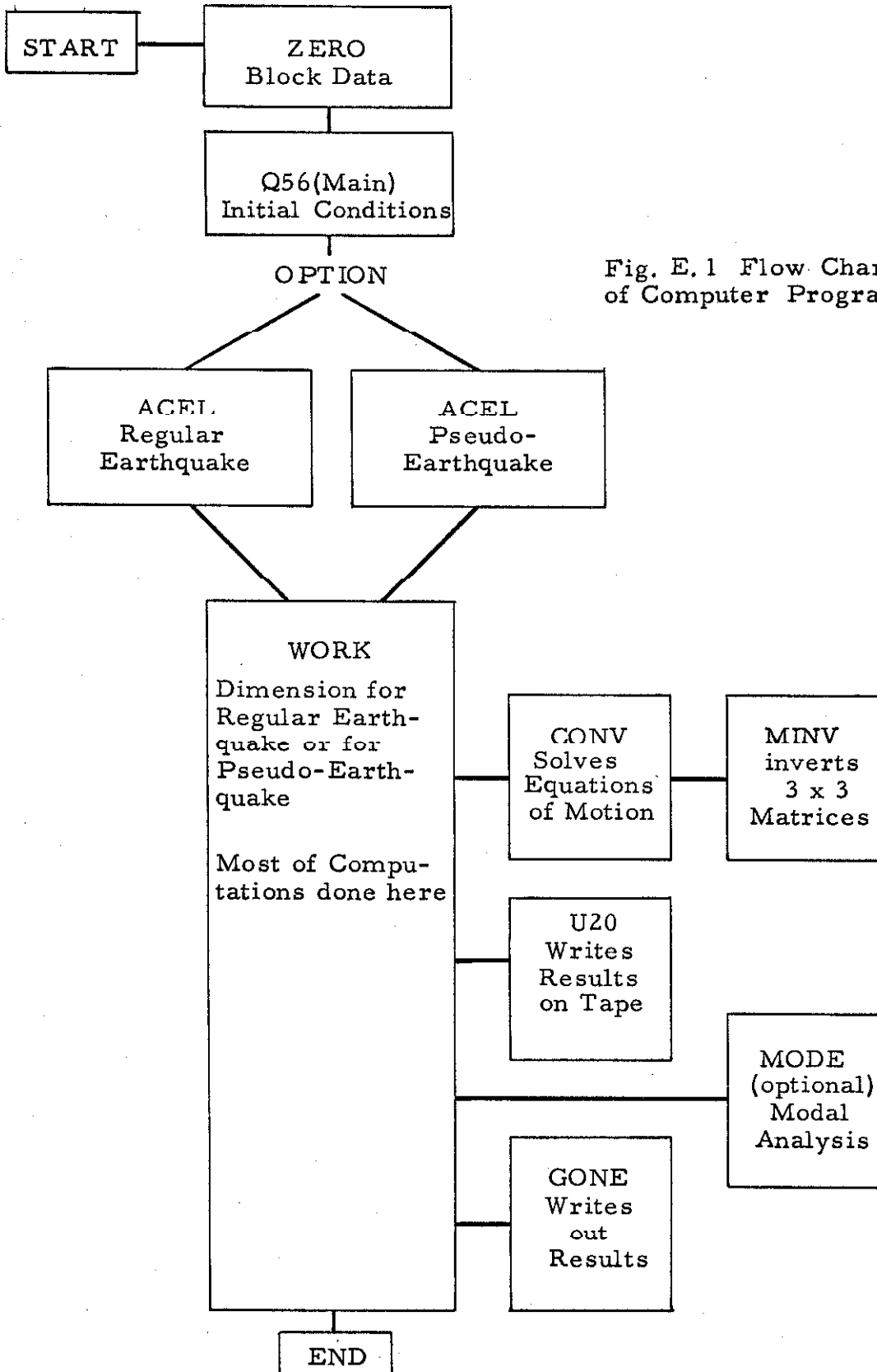
Fig. E.1 Flow Chart
of Computer Program

```
$IBFTC Q56    DECK *INITIAL VALUES AND STRUCTURAL CONSTANTS*    Q56  MAIN   1
C***************************************************************       MAIN   2
C    THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES     MAIN   3
C    OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.  MAIN   4
C                                                                      MAIN   5
C    THE BILINEAR HYSTERETIC BENDING MOMENT - JOINT ROTATION - YIELD   MAIN   6
C    ANGLE RELATIONSHIP AT EACH END OF EACH BEAM (GIRDER AND COLUMN),  MAIN   7
C    THE DUCTILITY FACTOR, AND THE CRITERIA FOR ESTABLISHING THE       MAIN   8
C    STATE OF YIELD ARE BASED UPON THE 'SINGLE COMPONENT BEAM' MODEL   MAIN   9
C    BY GIBERSON.                                                      MAIN  10
C                                                                      MAIN  11
C    THE PURPOSE OF THE MAIN PROGRAM IS TO READ IN STRUCTURAL AND      MAIN  12
C    OTHER DATA, TO EVALUATE THE STRUCTURAL CONSTANTS, STIFFNESS       MAIN  13
C    PARAMETERS, AND INITIAL YIELD PARAMETERS.                         MAIN  14
C                                                                      MAIN  15
C    THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE   MAIN  16
C    F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INST.-   MAIN  17
C    TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                         MAIN  18
C                        LAST REVISION - 11 JANUARY 1967               MAIN  19
C                                                                      MAIN  20
C***************************************************************       MAIN  21
C                                                                      MAIN  22
C    MUST USE 'ZCORE' OPTION ON IBJOB CARD.  THIS PLACES A 'O' (ZERO)  MAIN  23
C    IN EACH MEMORY CELL OF CORE PRIOR TO LOADING.                     MAIN  24
C                                                                      MAIN  25
C***************************************************************       MAIN  26
C                                                                      MAIN  27
C              CONTROL PARAMETERS                                      MAIN  28
C                                                                      MAIN  29
C  NAMEQK    DESCRIPTION OF THE EARTHQUAKE EXCITATION RECORD           MAIN  30
C  TIMEND    TIME AT END OF INTEGRATION  (TIMEND .LE. ENDREC)          MAIN  31
C  P         RATIO OF SECOND SLOPE TO FIRST                            MAIN  32
C            =1.5*P/(1.0-P) , USED IN EQN *QB* WHEN YIELDING           MAIN  33
C  IELPL     =0 FOR ELASTIC RESPONSE, =1 FOR NON-LINEAR RESPONSE       MAIN  34
C  LRTEST    WRITE RESULTS ON TAPE EVERY 'LRTEST' TIME INTERVALS       MAIN  35
C  NAMEST    DESCRIPTION OF THE STRUCTURE ANALYZED                     MAIN  36
C  ZO        NUMBER OF FLOORS IN STRUCTURE                             MAIN  37
C  N=2       TOP FLOOR                                                 MAIN  38
C  N=21      FIRST FLOOR                                               MAIN  39
C  N=22      GROUND LEVEL                                              MAIN  40
C  NCARD     DIMENSION OF THE ARRAYS IN THE VERTICAL DIRECTION IS  22  MAIN  41
C  DBDT      NUMBER OF POINTS OF ACCELERATION RECORD READ IN           MAIN  42
C  ENDREC    TIME INTERVAL (DOUBLE PRECISION) USED IN INTEGRATION      MAIN  43
C            END OF TIME OF ACCELERATION RECORD AS SET-UP BY SUBROU-   MAIN  44
C            TINE 'ACCIL'  (TIMEND .LE. ENDREC)                        MAIN  45
C  IOPTON    =0 FOR NO PRINT OUT OF ACCELEROGRAM, =1 FOR PRINT OUT     MAIN  46
C  SCALE     RATIO OF RMS OF ACCELEROGRAM USED TO ACCELEROGRAM OF      MAIN  47
C            EL CENTRO (N-S), 18 MAY 1940, (CALTECH RECORD)            MAIN  48
C                                                                      MAIN  49
C***************************************************************       MAIN  50
C                                                                      MAIN  51
C              UNITS OF VALUES READ IN                                 MAIN  52
C                                                                      MAIN  53
C  ZETA      FRACT OF CRIT DAMPING MODE 1                              MAIN  54
C  FUNDFQ    NONE     RAD/SEC   FUND FREQ OF LINEAR STRUCTURE          MAIN  55
C  CE,CI,GE,GI         IN**4    MOM OF INERTIA OF COLS AND GIRDS       MAIN  56
C  BMA(N,M,JI)         KIP*IN   MAG OF ORIGINAL YIELD MOM              MAIN  57
C  ZMIN)               KIP*(SEC**2)/IN   1/2 OF THE MASS OF THE FLOOR  MAIN  58
C  HT(N)               INCHES   HEIGHT BETWEEN FLOORS                  MAIN  59
C  E                   KIPS/(IN**2)   YOUNGS MODULUS                   MAIN  60
C  CL                  INCHES   DIST BETWEEN COLS. IN FRAME            MAIN  61
C                                                                      MAIN  62
C    THE UNIT SYSTEM USED IN ALL COMPUTATIONS AND PRINT-OUTS IS        MAIN  63
C    'INCH,LBS,SEC'.                                                   MAIN  64
C                                                                      MAIN  65
C***************************************************************       MAIN  66
C                                                                      MAIN  67
C    MUST HAVE INPUT CARDS IN THIS ORDER                               MAIN  68
C                                                                      MAIN  69
C                                                                      MAIN  70
```

```
C    NAMEQK                                                            MAIN  71
C    TIMEND,P,IELPL,LRTEST                                             MAIN  72
C    ZETA,FUNDFQ                                                       MAIN  73
C    NAMEST                                                            MAIN  74
C    E,ZL                                                              MAIN  75
C    STRUCTURAL CONSTANTS DECK                                         MAIN  76
C    NCARD,DBDT,ENDREC,IOPTON,SCALE                                    MAIN  77
C    TIME - ACCELERATION DECK                                          MAIN  78
C    MODAL ANALYSIS DECK - OPTIONAL                                    MAIN  79
C    EIGENVECTORS AND EIGENVALUES COME FROM PROGRAM 'Q33'              MAIN  80
C    STATE OF YIELD ARE BASED UPON, SEE SUBROUTINE 'MODE'              MAIN  81
C    FOR EIGENVECTOR NORMALIZATION, SEE SUBROUTINE 'MODE'              MAIN  82
C                                                                      MAIN  83
C***************************************************************       MAIN  84
C                                                                      MAIN  85
C    DIMENSION CE(22),CI(22),GE(22),GI(22)                             MAIN  86
C                                                                      MAIN  87
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),      MAIN  88
     2 GA(22,3),CA(22,3),HT(22),HI(22),ZN(22),A(5),I8),P,F,ZETA,FUNDFQ, MAIN  89
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                    MAIN  90
C                                                                      MAIN  91
      COMMON/MNGONE/ YIELD(22,3,4)                                     MAIN  92
C                                                                      MAIN  93
C***** READ IN DATA, AND EVALUATE OTHER CONSTANTS *****               MAIN  94
C                                                                      MAIN  95
      READ(5,1010) NAMEQK                                              MAIN  96
 1010 FORMAT(13A6)                                                     MAIN  97
C                                                                      MAIN  98
      READ(5,1020) TIMEND,P,IELPL,LRTEST                               MAIN  99
 1020 FORMAT(10X,2F10.5,2I10)                                          MAIN 100
C                                                                      MAIN 101
      READ(5,1030) ZETA,FUNDFQ                                         MAIN 102
 1030 FORMAT(10X,2F10.0)                                               MAIN 103
      READ(5,1011) NAMEST                                              MAIN 104
      FORMAT(10A6)                                                     MAIN 105
      READ(5,1030) E,ZL                                               MAIN 106
 1011 READ(5,1050) (CE(N),BMA(N,2,4),CI(N),BMA(N,3,4),GE(N),BMA(N,2,3), MAIN 107
     2  GI(N),BMA(N,3,3),HT(N),ZM(N), N=2,21)                          MAIN 108
 1050 FORMAT(8F8.0,2F6.0,4X)                                           MAIN 109
      ZK(P=1000.0                                                      MAIN 110
      AL=1.0/ZL                                                        MAIN 111
      E=2*ZKIP                                                         MAIN 112
      EL=E*AL                                                          MAIN 113
      ALAMDA = ZETA*FUNDFQ                                            MAIN 114
      BLAMDA = 2.0*ZETA*FUNDFQ                                        MAIN 115
C                                                                      MAIN 116
C***** EVALUATE YIELD MOMENTS, GIRDER AND COLUMN CONSTANTS, AND *****  MAIN 117
C***** INCIPIENT YIELD ANGLES                                         MAIN 118
C                                                                      MAIN 119
      DO 50 N=2,21                                                    MAIN 120
      N1=N+1                                                          MAIN 121
      ZM(N)=ZM(N)*KIP                                                 MAIN 122
      H(N) = 1.0/HT(N)                                                MAIN 123
      BMA(N,2,4)=ZKIP*BMA(N,2,4)                                      MAIN 124
      BMA(N,3,4)=ZKIP*BMA(N,3,4)                                      MAIN 125
      BMA(N,2,3)=ZKIP*BMA(N,2,3)                                      MAIN 126
      BMA(N,3,3)=ZKIP*BMA(N,3,3)                                      MAIN 127
      BMA(N,3,1)=BMA(N,2,3)                                           MAIN 128
      GA(N,2)=4.0*IL*GE(N)                                           MAIN 129
      GA(N,3)=4.0*IL*GI(N)                                           MAIN 130
      EHE/HT(N)                                                       MAIN 131
      CA(N,2)=4.0*EH*CE(N)                                           MAIN 132
      CA(N,3)=4.0*EH*CI(N)                                           MAIN 133
      BMA(N1,2,2)=BMA(N,2,4)                                          MAIN 134
      BMA(N1,3,2)=BMA(N,3,4)                                          MAIN 135
      DO 40 M=2,3                                                     MAIN 136
      YIELD(N,M,3)= BMA(N,M,3)/(1.5*GA(N,M))                         MAIN 137
      YIELD(N,M,4)= BMA(N,M,4)/(1.5*CA(4,M))                         MAIN 138
      YIELD(N1,M,2 =YIELD(N,M,4)                                     MAIN 139
   40 CONTINUE                                                        MAIN 140
```

```
50      YIELD(N,3,1)=YIELD(N,2,3)                                      MAIN 141
        CONTINUE                                                       MAIN 142
C                                                                      MAIN 143
        DO 60 M=1,3                                                    MAIN 144
        DO 60 J=1,4                                                    MAIN 145
        DO 60 N=2,22                                                   MAIN 146
        BMB(N,M,J) = -BMA(N,M,J)                                       MAIN 147
60      CONTINUE                                                       MAIN 148
C                                                                      MAIN 149
C...... EVALUATE BILINEAR STIFFNESS PARAMETERS 'F, A, S' ........      MAIN 150
C                                                                      MAIN 151
        F = 1.5*P/(1.0-P)                                              MAIN 152
C                                                                      MAIN 153
        A(1) = 1.0/(1.0 + F)                                           MAIN 154
        A(2) = 0.5*A(1)                                                MAIN 155
        D = 1.0 + 2.6666667*F + 1.3333333*F*F                         MAIN 156
        A(3) = (1.0 + 1.3333333*F)/D                                   MAIN 157
        A(4) = 0.6666667*F/D                                           MAIN 158
        A(5) = (1.0 + 2.0*F)/D                                         MAIN 159
C                                                                      MAIN 160
        S(3) = F*A(1)                                                  MAIN 161
        S(4) = F*A(2)                                                  MAIN 162
        S(5) = (0.75 + F)/(1.0 + F)                                   MAIN 163
        S(6) = F*A(3)                                                  MAIN 164
        S(7) = F*A(4)                                                  MAIN 165
        S(8) = F*A(5)                                                  MAIN 166
C                                                                      MAIN 167
C...... WRITE OUT INPUT DATA AND INITIAL CONDITIONS ..........        MAIN 168
C                                                                      MAIN 169
        WRITE(6,1069) A,S,F,P,D                                        MAIN 170
1069    FORMAT(10X,1PE17.7))                                          MAIN 171
C                                                                      MAIN 172
        WRITE(6,1003) NAMEST,P,ZETA,FUNDFQ,TIMEND,NAMEQK               MAIN 173
1003    FORMAT(1H1 ,10A6,15H,BILINEARITY = ,F5.3,16H,FRAC CR DAMP = , MAIN 174
       2 F7.5,13H,FUND FREQ = ,F7.5, 8H RAD/SEC ,//,                   MAIN 175
       3 5X, 15HUSING THE FIRST ,F10.5, 15H SECONDS OF THE , 13A6,2X,//) MAIN 176
        WRITE(6,1060)                                                  MAIN 177
1060    FORMAT( 132H STORY    CE      BMA     YIELD     CI      BMA    MAIN 178
       2 YIELD     GE      BMA      YIELD     GI      BMA     YIELD    MAIN 179
       3 HT      ZM )                                                  MAIN 180
        NN=21                                                          MAIN 181
        DO 70 N=2,22                                                   MAIN 182
        NN = NN-1                                                      MAIN 183
        WRITE(6,1070) NN,CE(N),BMA(N,2,4),YIELD N,2,4),CI(N),BMA(N,3,4), MAIN 184
       2 YIELD(N,3,4),GE(N),BMA(N,2,3),YIELD(N,2,3),GI(N),BMA(N,3,3),  MAIN 185
       3 YIELD(N,3,3),HT(N),ZM(N)                                      MAIN 186
1070    FORMAT(IX,I3,4(0PF3.0,0PFF1.0,1PE10.3), 0P2F6.0)              MAIN 187
70      CONTINUE                                                       MAIN 188
C                                                                      MAIN 189
        WRITE(6,1002)                                                  MAIN 190
1002    FORMAT(1H0)                                                   MAIN 191
        WRITE(6,1080)                                                  MAIN 192
1080    FORMAT(12X,7HGA(N,2), 10X,7HGA(N,3), 10X,7HGA(N,2), 10X,7HGA(N,3)) MAIN 193
        NN=21                                                          MAIN 194
        DO 80 N=2,22                                                   MAIN 195
        NN = NN-1                                                      MAIN 196
        WRITE(6,1090)NN,CA(N,2),CA(N,3),GA(N,2),GA(N,3)               MAIN 197
1090    FORMAT(2X,I3,1P4E17.7)                                        MAIN 198
80      CONTINUE                                                       MAIN 199
C                                                                      MAIN 200
        CALL ACEL                                                      MAIN 201
        CALL WORK                                                      MAIN 202
        STOP                                                           MAIN 203
        END                                                            MAIN 204
```

```
$IBFTC ACEL     DECK *ACCELEROGRAM FROM REGULAR EARTHQUAKE*        Q56 ACRG    1
      SUBROUTINE ACEL                                                 ACRG    2
C                                                                     ACRG    3
C*********************************************************************ACRG    4
C                                                                     ACRG    5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES   ACRG    6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.ACRG    7
C                                                                     ACRG    8
C     EACH OF THE  TWO  'ACEL' SUBROUTINES SETS-UP AN ACCELEROGRAM    ACRG    9
C     WITH UNITS OF 'FRACTIONS OF G' AT EQUAL TIME INTERVALS 'DBDT'   ACRG   10
C     FOR USE BY SUBROUTINE 'WORK'.  THE DIFFERENCE BETWEEN THEM IS   ACRG   11
C     THE FORMAT OF THE INPUT ACCELEROGRAM.                           ACRG   12
C        THE '.JENNINGS) PSEUDO-EARTHQUAKE' IS GIVEN WITH VALUES OF   ACRG   13
C     ACCELERATION IN UNITS OF '=T/(SEC**2)' AT EQUAL TIME INTER-     ACRG   14
C     VALS OF 0.025 SEC.  THE CORRESPONDING TIMES MUST BE GENERATED.  ACRG   15
C        THE REGULAR EARTHQUAKE ACCELEROGRAMS ARE GIVEN IN            ACRG   16
C     'TIME-ACCELERATION' PAIRS AT UNEQUAL TIME INTERVALS AND THE     ACRG   17
C     UNITS OF ACCELERATION ARE 'FRACTIONS OF G'.                     ACRG   18
C                                                                     ACRG   19
C                                                                     ACRG   20
C     THIS 'ACEL' SUBROUTINE HANDLES THE 'REGULAR EARTHQUAKES'.       ACRG   21
C                                                                     ACRG   22
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE ACRG   23
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INS I- ACRG   24
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                       ACRG   25
C                   LAST REVISION - 4 DECEMBER 1966                   ACRG   26
C*********************************************************************ACRG   27
C                                                                     ACRG   28
      DOUBLE PRECISION DBTIME,DBDT,ED,DBTIM3,DBZK                     ACRG   29
C                                                                     ACRG   30
      COMMON/ACEWORK/  DBTIME,DBDT,ED,DBTIM3,IOPTON,                  ACRG   31
     2 TIM3(601),DDY3(601),DBY1(601), KEND                           ACRG   32
C                                                                     ACRG   33
      READ(5,1110)  NCARD,DBDT,ENDREC,IOPTON,SCALE                    ACRG   34
 1110 FORMAT(I10,D10.0,F10.0,I10,F10.0)                               ACRG   35
      READ(5,1120) (TIM3(I),DDY3(I),I=1,NCARD                         ACRG   36
 1120 FORMAT(2X,2F10.0)                                               ACRG   37
      ED = DBLE(ENDREC)/DBDT                                          ACRG   38
      DT = SNGL(DBDT)                                                 ACRG   39
      KEND = SNGL(ED) + .5                                            ACRG   40
      INC = KEND - 1                                                  ACRG   41
      WRITE(6,1130) NCARD,DT,ENDREC,INC                               ACRG   42
 1130 FORMAT(1H1, 25H NO. TIME-ACCEL. PAIRS = ,I4,//,                 ACRG   43
     2 26H     EACH TIME INCREMENT = ,F12.8, 5H SEC. ,//,             ACRG   44
     3 26H     END OF RECORD = ,F12.8, 5H SEC. , //,                  ACRG   45
     4 26H     NO. TIME INCREMENTS = ,I4  )                           ACRG   46
      TIME = TIM3(1)                                                  ACRG   47
      DBTIM3 = DBLE(TIM3(1))                                          ACRG   48
      DBTIME = DBTIM3                                                 ACRG   49
      L = 1                                                           ACRG   50
      K = 1                                                           ACRG   51
  110 L = L+1                                                         ACRG   52
      DDT = TIM3(L) - TIM3(L-1)                                       ACRG   53
      DDP = DDY3(L) - DDY3(L-1)                                       ACRG   54
      IF(DDT.LE.0.0)  GO TO 110                                       ACRG   55
      SLOPE = DDP/DDT                                                 ACRG   56
  120 IF(TIME.GT.TIM3(L))  GO TO 110                                  ACRG   57
      DDY1(K) =(DDY3(L-2) + (TIME - TIM3(L-1))*SLOPE)*SCALE           ACRG   58
      IF(IOPTON.EQ.0)  GO TO 125                                      ACRG   59
      WRITE(6,1140)    K,TIME,DDY1(K)                                 ACRG   60
 1140 FORMAT(  8X,I4,2X,2E15.8)                                       ACRG   61
  125 CONTINUE                                                        ACRG   62
      ZK = K                                                          ACRG   63
      DBZK = DBLE(ZK)                                                 ACRG   64
      DBTIME = DBTIM3 + DBZK*DBDT                                     ACRG   65
      TIME = SNGL(DBTIME)                                             ACRG   66
      K = K+1                                                         ACRG   67
      IF(KEND.GE.K)  GO TO 120                                        ACRG   68
  130 CONTINUE                                                        ACRG   69
      RETURN                                                          ACRG   70
      END                                                             ACRG   71
```

```
$IBFTC ACEL    DECK *ACCELEROGRAM FROM PSEUDO-EARTHQUAKE*          Q56 ACPS   1
      SUBROUTINE ACEL                                                   ACPS   2
C***********************************************************************ACPS   3
C                                                                       ACPS   4
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES;    ACPS   5
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.  ACPS   6
C                                                                       ACPS   7
C                                                                       ACPS   8
C     EACH OF THE TWO 'ACEL' SUBROUTINES SETS-UP AN ACCELEROGRAM        ACPS   9
C     WITH UNITS OF 'FRACTIONS OF (* AT EQUAL TIME INTERVALS 'DBDT'     ACPS  10
C     FOR USE BY SUBROUTINE 'WORK', THE DIFFERENCE BETWEEN THEM IS      ACPS  11
C     THE FORMAT OF THE INPUT ACCELEROGRAM.                            ACPS  12
C     THE '(JENNINGS) PSEUDO-EARTHQUAKE' IS GIVEN WITH VALUES OF        ACPS  13
C     ACCELERATION IN UNITS OF 'FT/(SEC**2)' AT EQUAL TIME INTER-       ACPS  14
C     THE THE  CORRESPONDING TIMES MUST BE GENERATED.                  ACPS  15
C     THE REGULAR EARTHQUAKE ACCELEROGRAMS ARE GIVEN IN                 ACPS  16
C     'TIME-ACCELERATION' PAIRS AT UNEQUAL TIME INTERVALS AND THE       ACPS  17
C     UNITS OF ACCELERATION ARE 'FRACTIONS OF G'.                       ACPS  18
C                                                                       ACPS  19
C     THIS 'ACEL' SUBROUTINE HANDLES THE '(JENNINGS) PSEUDO-EARTHQUAKES'ACPS  20
C                                                                       ACPS  21
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE   ACPS  22
C     F. GIBELSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-   ACPS  23
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                         ACPS  24
C              LAST REVISION - 4 DECEMBER 1966                          ACPS  25
C                                                                       ACPS  26
C***********************************************************************ACPS  27
C                                                                       ACPS  28
      EQUIVALENCE (DDY1(4811),DDY3(1))                                  ACPS  29
C                                                                       ACPS  30
      DOUBLE PRECISION DBTIME,DBDT,ED,DBTIM3,DBZK                       ACPS  31
      DOUBLE PRECISION CBDT3,DBZII                                      ACPS  32
C                                                                       ACPS  33
      DIMENSION NAME(13),TIM3(1201),DDY3(1201)                          ACPS  34
C                                                                       ACPS  35
      COMMON/ACEWOR/ DETIME,DBDT,ED,DBTIM3,IOPTON,                      ACPS  36
     2 DDY1(6)11), KEND                                                 ACPS  37
C                                                                       ACPS  38
      READ(5,1110) NCARD,DBDT,ENDREC,IOPTON,SCALE                       ACPS  39
 1110 FORMAT(I10,D10.0,F10.0,I10,F10.0)                                 ACPS  40
      READ(5,1120) (TIM3(I),DDY3(I),I=1,NCARD)                          ACPS  41
C1120 FORMAT(10X,2F10.0)                                                ACPS  42
C                                                                       ACPS  43
C .... THE FOLLOWING STATEMENTS READ IN THE PSUEDO-EARTHQUAKES .....    ACPS  44
C                                                                       ACPS  45
      READ(5,1123) NAME                                                 ACPS  46
 1113 FORMAT(13A6)                                                      ACPS  47
      READ(5,1125) DDY3                                                 ACPS  48
 1125 FORMAT(5F11.7,14X)                                                ACPS  49
      DBDT3 = 0.025D+00                                                 ACPS  50
      SSCALE = 3.16*SCALE                                               ACPS  51
      DO 100 I = 1,NCARD                                                ACPS  52
      ZII = I - 1                                                       ACPS  53
      DBZII = DBLE(ZII)                                                 ACPS  54
      TIM3(I) = SNGL(DBZII*DBDT3)                                       ACPS  55
  100 CONTINUE                                                          ACPS  56
      WRITE(6,1123) NAME                                                ACPS  57
      WRITE(6,1127) (I,TIM3(I),DDY3(I),I=1,NCARD)                       ACPS  58
 1117 FORMAT(10X,I5,F12.6,F12.7)                                        ACPS  59
      DO 105 I=1,NCARD                                                  ACPS  60
      DDY3(I) = DDY3(I)*SSCALE/32.171667                                ACPS  61
  105 CONTINUE                                                          ACPS  62
      DDT = SNGL(DBDT3)                                                 ACPS  63
C                                                                       ACPS  64
      ED = DBLE(ENDREC)/DBDT                                            ACPS  65
      DT = SNGL(DBDT)                                                   ACPS  66
      KEND = SNGL(ED) + 1.5                                             ACPS  67
      INC = KEND - 1                                                    ACPS  68
      WRITE(6,1130) NCARD,DT,ENDREC,INC                                 ACPS  69
 1130 FORMAT(1H1, 25H NO. TIME-ACCEL. PAIRS = ,I4,//,                   ACPS  70
     2 26H   EACH TIME INCREMENT = ,F12.8, 5H SEC. ,//,                 ACPS  71
     3 26H      END OF RECORD = ,F12.8, 5H SEC. , //                    ACPS  72
     4 26H   NO. TIME INCREMENTS = ,I4 )                                ACPS  73
      TIME = TIM3(L)                                                    ACPS  74
      DBTIM3 = DBLE(TIM3(1))                                            ACPS  75
      DBTIME = DBTIM3                                                   ACPS  76
      L = 1                                                             ACPS  77
      K = 1                                                             ACPS  78
  110 L = L+1                                                           ACPS  79
C     DDT = TIM3(L) - TIM3(L-1)                                         ACPS  80
      DCP = DDY3(L) - DDY3(L-1)                                         ACPS  81
      IF(DDT.LE.0.0)  GO TO 110                                         ACPS  82
      SLOPE = DCP/DDT                                                   ACPS  83
      IF(TIME.GT.TIM3(L))   GO TO 110                                   ACPS  84
  120 DDY1(K) = DDY3(L-1) + (TIME - TIM3(L-1))*SLOPE                    ACPS  85
      IF(IOPTON.EQ.0)   GO TO 125                                       ACPS  86
      WRITE(6,1140)   K,TIME,DDY1(K), TIM3(L),DDY3(L)                   ACPS  87
 1140 FORMAT(  8X,I4,2X,1P4E15.8)                                       ACPS  88
  125 CONTINUE                                                          ACPS  89
      ZK = K                                                            ACPS  90
      DBZK = DBLE(ZK)                                                   ACPS  91
      DBTIME = DBTIM3 + DBZK*DBDT                                       ACPS  92
      TIME = SNGL(DBTIME)                                               ACPS  93
      K = K+1                                                           ACPS  94
      IF(KEND.GE.K)   GO TO 120                                         ACPS  95
  130 CONTINUE                                                          ACPS  96
      RETURN                                                            ACPS  97
      END                                                               ACPS  98
```

```
$IBFTC WORK    DECK      ** MOST OF THE WORK IS DONE HERE**    Q56    WORK   1
      SUBROUTINE WORK                                                 WORK   2
C                                                                     WORK   3
C*********************************************************************WORK   4
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES   WORK   5
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.WORK   6
C                                                                     WORK   7
C     THE PURPOSE OF SUBROUTINE 'WORK' IS TO EVALUATE THE PARAMETERS  WORK   8
C     NEEDED BY SUBROUTINE 'CONV' IN ORDER TO SOLVE THE EQUATIONS OF  WORK   9
C     MOTION FOR EACH TIME INTERVAL.  THEN THE VARIOUS RESPONSES ARE  WORK  10
C     MONITORED CUMULATIVELY FROM TIME ZERO FOR THE MOST POSITIVE AND WORK  11
C     MOST NEGATIVE VALUES AND THE TIMES THEY OCCUR, AFTER EVERY      WORK  12
C     SECOND OF REAL TIME AND AT THE END OF THE RUN THESE MAXIMA AND  WORK  13
C     TIMES ARE WRITTEN OUT BY SUBROUTINE 'GOKE'.                     WORK  14
C                                                                     WORK  15
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE WORK  16
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- WORK  17
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                       WORK  18
C           LAST REVISION - 11 JANUARY 1967                           WORK  19
C                                                                     WORK  20
C*********************************************************************WORK  21
      DOUBLE PRECISION DBU,DBUU                                       WORK  22
      DOUBLE PRECISION DBTIME,DBOT,ED,DBTIM3,DBZJJ                    WORK  23
C                                                                     WORK  24
      DIMENSION  TACC(22),AXCLD(22,3),GSR(22,3),SHEAR(22),            WORK  25
     2 L(22,3,4),ALP(22,3,4),BM(22,3,4),BMLAS(22,3,4),OMT(22,3),      WORK  26
     3 OTMOM(22),SRT(22)                                              WORK  27
C                                                                     WORK  28
      COMMON/HRKGON/UMAX(22),UMIN(22),UMIN(22),TMXV(22),TMNV(22),     WORK  29
     2 VMAX(22),VMIN(22),TMXUU(22),TMNUU(22),                         WORK  30
     3 UUMAX(22),UUMIN(22),TMXSR(22),TMNSR(22),                       WORK  31
     4 WMAX(22),WMIN(22),TMXW(22),TMNW(22),                           WORK  32
     5 ALPMAX(22,3,4),ALPMIN(22,3,4),TMXALP(22,3,4),TMNALP(22,3,4),   WORK  33
     6 BMMAX(22,3,4),BMMIN(22,3,4),TMXBM(22,3,4),TMNBM(22,3,4),       WORK  34
     7 TACCMX(22),TACCMN(22),TMXTA(22),TMNTA(22),                     WORK  35
     8 GSRMX(22,3),GSRMN(22,3),TMXSR(22,3),TMNGSR(22,3),              WORK  36
     9 CLDMAX(22,3),CLDMIN(22,3),TMXCLD(22,3),TMNCLD(22,3),           WORK  37
     1 OTMMX(22),OTMMN(22),TMXOTM(22),TMNOTM(22),                     WORK  38
     2 ABSBMT(22,3),TABSBM(22,3),ABSSRT(22),TABSSR(22)                WORK  39
C                                                                     WORK  40
      COMMON/NNWCGN/NAMEST(10),NAMIQK(13),BMA(22,3,4),BMB(22,3,4),    WORK  41
     2 GA(22,3),CA(22,3),HT(22),HI(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ,WORK 42
     3 ZL,AL,IMEND,ALAMDA,BLAMDA,ELPL,TIM1,RTEST                      WORK  43
C                                                                     WORK  44
C****** THERE ARE TWO OPTIONS FOR THE SECOND CARD OF 'COMMON/ACEWOR/  WORK  45
C****** ONE FOR '(JENNINGS) PSEUDO-EARTHQUAKES, THE OTHER FOR REG-    WORK  46
C****** ULAR EARTHQUAKES.  SEE EITHER SUBROUTINE 'ACEL' FOR FURTHER   WORK  47
C****** EXPLANATION.***********************************                WORK  48
C                                                                     WORK  49
      COMMON/ACEWOR/  DBTIME,DBOT,ED,DBTIM3,IOPTON,                   WORK  50
     2 DDY16(1),JEND                      (USED FOR REG EQKS)         WORK  51
     2 TIM3(601),DDY3(601),DDY1(601),JJEND                            WORK  52
C                                                                     WORK  53
      COMMON/VKCONV/SCA(22,3),SCB(22,3),SCC(22,3),SCABH(22,3),        WORK  54
     2 SCBCH(22,3),SCDHH(22),SGA(22,3),SGB(22,3),SGC(22,3),           WORK  55
     3 DU(22),UI(22),DUUH(22),JUH(22),DW(22),V(22),                   WORK  56
     4 DACC(22),ACC(22),DW(22,3),W(22,3),    DDDY,DT,KC,N22           WORK  57
C                                                                     WORK  58
      COMMON/CONMOD/ DBL(22),DBUU(22)                                 WORK  59
C                                                                     WORK  60
      COMMON/VKGMIN/LLX,LLI,LLB,LLG,LLD,LLP, LGD(2),LINE(129)         WORK  61
C                                                                     WORK  62
      ZG=386.J6                                                       WORK  63
C                                                                     WORK  64
      JJ = 1                                                          WORK  65
      ITIME = 1                                                       WORK  66
      ITIME = ITIME                                                   WORK  67
                                                                      WORK  68
                                                                      WORK  69
                                                                      WORK  70

      DT = SNGL(DBOT)                                                 WORK  71
      DBTIME = DBTIM3                                                 WORK  72
      TIM1 = SNGL(DBTIME)                                             WORK  73
C                                                                     WORK  74
      DCY1ZG = DDY1(1)*ZG                                             WORK  75
      DO 5 N=2,21                                                     WORK  76
      ACC(N) = -DDY1ZG                                                WORK  77
C                                                                     WORK  78
    5 WRITE(20) NAMEST,NAMEQK                                         WORK  79
C                                                                     WORK  80
      WRITE(20)  JJ,LL,L,SHEAR,BM,OTMOM,AXCLD,ACC,TACC,V,             WORK  81
     2 CBU,DBUU,W,JUH,ALP,TIM1,DT,DDY1(JJ)                            WORK  82
C                                                                     WORK  83
 1001 WRITE(6,1001)                                                   WORK  84
      FORMAT(1H1)                                                     WORK  85
C                                                                     WORK  86
      WRITE(6,1075) DT                                                WORK  87
 1075 FORMAT(5H DT = ,1PE15.7)                                        WORK  88
C                                                                     WORK  89
      WRITE(6,1076)    JJ   TIM1         DD/1            U(2)       LI )WORK 90
 1076 FORMAT( 48H                                                 )   WORK  91
C                                                                     WORK  92
      WRITE(6,1077)  JJ,TIM1,DDY1(JJ),U(2),L1                         WORK  93
 1077 FORMAT( 1X,I4,1F10.6,1F2E15.7,I3)                               WORK  94
C                                                                     WORK  95
C...... BEGIN NEW TIME INTERVAL ...................................   WORK  96
   10 CONTINUE                                                        WORK  97
      ZJJ = JJ                                                        WORK  98
      DBZJJ = DBLE(ZJJ)                                               WORK  99
      DBTIME = DBTIM3 + DBZJ.*DBOT                                    WORK 100
      TIM1 = SNGL(DBTIME)                                             WORK 101
      J.M1 = JJ                                                       WORK 102
      J. = JJ+1                                                       WORK 103
      DDDY = (DDY1(JJ) - DDY1(JJM1))*ZG                               WORK 104
      IF(ABS(DDT).GT.0.000001)  GO TO 80                              WORK 105
      DO 70 N=2,21                                                    WORK 106
      ACC(N) = ACC(N) - DDDY                                          WORK 107
   70 CONTINUE                                                        WORK 108
      WRITE(6,1111)  LLX,TIM,DT,DDDY                                  WORK 109
 1111 FORMAT(2X,A1,2X,2F12.5.18X,1PE18.7)                             WORK 110
      GO TO 10                                                        WORK 111
   80 CONTINUE                                                        WORK 112
C...... EVALUATE 'EFFECTIVE STIFFNESSES    SCA, SCB, SCC' FOR COLUMNS ...WORK113
C                                                                     WORK 114
      DO 150 M=2,3                                                    WORK 115
      DO 145 N=2,21                                                   WORK 116
      CANM = CA(N,M)                                                  WORK 117
      I.(LIN,M,4).NE.0)  GO TO 120                                    WORK 118
      I.(LIN+1,M,2).NE.0)  GO TO 110                                  WORK 119
C------------ NC HINGES IN COLUMN                                     WORK 120
      SCB(N,M) = S(4)*CANM                                            WORK 121
      SCC(N,M) = S(3)*CANM                                            WORK 122
      GO TO 140                                                       WORK 123
  120 I.(LIN+1,M,2).NE.0) GO TO 130                                   WORK 124
C------------ HINGE IN COLUMN AT (N,M,4)                              WORK 125
      CANM = CA(N,M)                                                  WORK 126
      SCA(N,M) = S(3)*CANM                                            WORK 127
      SCB(N,M) = S(4)*CANM                                            WORK 128
      SCC(N,M) = S(5)*CANM                                            WORK 129
      GO TO 140                                                       WORK 130
C------------ HINGES IN COLUMN AT BOTH ENDS ------------------------- WORK 131
  130 SCA(N,M) = S(6)*CANM                                            WORK 132
      SCB(N,M) = S(7)*CANM                                            WORK 133
      SCC(N,M) = SCA(N,M)                                             WORK 134
  140 CONTINUE                                                        WORK 135
      SCABH(N,M)=(SCA(N,M)+SCB(N,M))*HIN)                             WORK 136
      SCBCH(N,M)=(SCB(N,M)+SCC(N,M))*HIN)                             WORK 137
C...... SCDHH(N) IS EVALUATED BELOW IN STMT 195 .................     WORK 138
  145 CONTINUE                                                        WORK 139
                                                                      WORK 140
```

```
150   CONTINUE
C
C..... EVALUATE 'EFFECTIVE STIFFNESSES   SGA, SGB, SGC' FOR GIRDERS ...
C
      DO 200 N=2,21
      GAN2 = GA(N,2)
      IF(L(N,2,3).NE.0) GO TO 170
      IF(L(N,3,1).NE.0) GO TO 160
C----------NO HINGES IN GIRDER
      SGA(N,2) = GAN2
      SGB(N,2) = 0.5*GAN2
      SGC(N,2) = GAN2
      GO TO 185
C----------HINGE IN GIRDER AT (N,3,1)
160   SGA(N,2) = S(5)*GAN2
      SGB(N,2) = S(4)*GAN2
      SGC(N,2) = S(3)*GAN2
      GO TO 185
170   IF(L(N,3,1).NE.0)   GO TO 180
C----------HINGE IN GIRDER AT (N,2,3)
      SGA(N,2) = S(3)*GAN2
      SGB(N,2) = S(4)*GAN2
      SGC(N,2) = S(5)*GAN2
      GO TO 185
C----------HINGE AT BOTH ENDS OF GIRDER
180   SGA(N,2) = S(6)*GAN2
      SGB(N,2) = S(7)*GAN2
      SGC(N,2) = S(6)*GAN2
185   IF(L(N,3,3).NE.0)   GO TO 190
C..... SGA(N,3) CONTAINS SGB(N,3) DUE TO SYMMETRY OF THE MIDDLE BAY...
      SGA(N,3) = 1.5*GA(N,3)
      GO TO 195
190   SGA(N,3) = S(8)*GA(N,3)
195   SCDHH(N) =(SCABH(N,2) + SCBCH(N,2) + SCAFH(N,3) +H(N)H(N)
200   CONTINUE
C
      CALL CONV
C
C....... SUBROUTINE 'CONV' SOLVES THE EQUATIONS OF MOTION FOR THE ......
C....... VARIABLES 'DACC,DV,DU,DUUH,DW' AND OTHERS .......
C
C....... MONITOR FOR MAXIMUM AND MINIMUM 'JOINT ROTATIONS - W ' .......
C
      DO 220 M=2,3
      DO 215 N=2,21
      IF(WMAX(N,M).GE.W(N,M)) GO TO 210
      WMAX(N,M)=W(N,M)
      TMX(N,M)=TIM1
      GO TO 211
210   CONTINUE
      IF(WMIN(N,M).LE.W(N,M)) GO TO 211
      WMIN(N,M)=W(N,M)
      TMN(N,M)=TIM1
211   CONTINUE
215   CONTINUE
220   CONTINUE
C
      IF(LL.EQ.0)  GO TO 105
C
C....... EVALUATE 'YIELD ANGLES'  IN COLUMNS FOR THIS TIME INTERVAL ....
C
      DO 250 M=2,3
      DO 245 N=2,21
      N1 = N+1
      IF(L(N,M,4).NE.0) GO TO 235
      IF(L(N1,M,2).NE.0) GO TO 230
C----------NO HINGES IN COLUMN
      GO TO 245
C----------HINGE IN COLUMN AT (N+1,M,2)
230   ALP(N,M,2) = ALP(N,M,2) + A(2)*(DW(N,M)-DUUH(N)) +
                     A(1)*(DW(N1,M)-DUUH(N))
C
235   IF(L(N1,M,2).NE.0) GO TO 240
C----------HINGE IN COLUMN AT (N,M,4)
      ALP(N,M,4) = ALP(N,M,4) + A(1)*(DW(N,M)-DUUH(N)) +
                     A(2)*(DW(N1,M)-DUUH(N))
      GO TO 245
C----------HINGES IN COLUMN AT BOTH ENDS
240   ALP(N,M,4) = ALP(N,M,4) + A(3)*(DW(N,M)-DUUH(N)) +
                     A(4)*(DW(N1,M)-DUUH(N))
      ALP(N1,M,2) = ALP(N1,M,2) + A(4)*(DW(N,M)-DUUH(N)) +
                     A(3)*(DW(N1,M)-DUUH(N))
245   CONTINUE
250   CONTINUE
C
C....... EVALUATE 'YIELD ANGLES'  IN GIRDERS FOR THIS TIME INTERVAL ....
C
      DO 300 N=2,21
      IF(L(N,2,3).NE.0)  GO TO 270
      IF(L(N,3,1).NE.0)  GO TO 260
C----------NO HINGES IN GIRDER
      GO TO 285
C----------HINGE IN GIRDER AT (N,3,1)
260   ALP(N,3,1) = ALP(N,3,1) + A(2)*DW(N,2) + A(1)*DW(N,3)
      GO TO 285
270   IF (L(N,3,1).NE.0)  GO TO 280
C----------HINGE IN GIRDER AT (N,2,3)
      ALP(N,2,3) = ALP(N,2,3) + A(1)*DW(N,2) + A(2)*DW(N,3)
      GO TO 285
C----------HINGE AT BOTH ENDS OF GIRDER
280   ALP(N,2,3) = ALP(N,2,3) + A(3)*DW(N,2) + A(4)*DW(N,3)
      ALP(N,3,1) = ALP(N,3,1) + A(4)*DW(N,2) + A(3)*DW(N,3)
C....... EVALUATE 'ALP' FOR GIRDERS OF MIDDLE BAY .......
285   IF( L(N,3,3).EQ.0)  GO TO 300
C----------HINGES AT BOTH ENDS OF GIRDER  'IF(L(N,3,3)=1)'
      ALP(N,3,3) = ALP(N,3,3) + A(5)*DW(N,3)
300   CONTINUE
305   CONTINUE
C
C....... SAVE VALUES OF BENDING MOMENTS AT BEGINNING OF THIS 'DT'....
C
      DO 310 M=2,3
      DO 310 J=1,4
      DO 310 N=2,22
      BMLAST(N,M,J) = BM(N,M,J)
310   CONTINUE
C....... EVALUATE 'BENDING MOMENTS' IN COLUMNS AT END OF THIS 'DT'....
C
      DO 345 N=2,3
      DO 340 N=2,21
      N1=N+1
      BM(N,M,4) = CA(N,M)*((W(N,M) -UUH(N) - ALP(N,M,4)) +
      2 (0.5*(W(N1,M) -ALP(N,M,2))))
      BM(N1,M,2) = CA(N,M)*(W(N1,M)-UUH(N)- ALP(N1,M,2)) +
      2 (0.5*(W(N,M) - ALP(N,M,4)))
340   CONTINUE
345   CONTINUE
C
C....... EVALUATE  BENDING MOMENTS' IN GIRDERS AT END OF THIS 'DT'....
C
      DO 350 N=2,2.
      BM(N,2,3) = GA(N,2)*((W(N,2) - ALP(N,2,3)) +
      2 (0.5*(W(N,3)) - ALP(N,3,1)))
      BM(N,3,1) = GA(N,2)*((W(N,3) - ALP(N,3,1)) +
      2 (0.5*(W(N,2) - ALP(N,2,3)))
      BM(N,3,3) = .5*GA(N,3)*(W(N,3) - ALP(N,3,3))
350   CONTINUE
C
C....... MASTER CONTROL SECTION FOR EVALUATING THE ' STATE OF YIELD '
```

```
      IF(IELPL.EQ.0) GO TO 336                                          WORK 281
      LL1 = L1                                                          WORK 282
      L1 = 0                                                            WORK 283
      DO 335 M=2,3                                                      WORK 284
      DO 334 J=1,4                                                      WORK 285
      IF((M+J).EQ.3) GO TO 334                                          WORK 286
      DBM = BM(N,M,J) - BALASTN(N,M,J)                                  WORK 287
      IF(L(N,M,J).EQ.0) GO TO 315                                       WORK 288
      BMAIN(N,M,J) = BMAIN(N,M,J) + DBM                                 WORK 289
      BMBIN(N,M,J) = BMBIN(N,M,J) + DBM                                 WORK 290
  315 CONTINUE                                                          WORK 291
C                                                                       WORK 292
C....... UPDATE THE 'PLASTIC INDEX - L(N,M,J)' FOR NEXT TIME INTERVAL   WORK 293
C                                                                       WORK 294
      IF(BM(N,M,J).GE.BMAIN(N,M,J) .AND. DBM.GT.0.0)  GO TO 320         WORK 295
      IF(BM(N,M,J).LE.BMBIN(N,M,J) .AND. DBM.LT.0.0)  GO TO 325         WORK 296
      L(N,M,J) = 0                                                      WORK 297
      GO TO 330                                                         WORK 298
  320 L(N,M,J) = +1                                                     WORK 299
      L1 = 1                                                            WORK 300
      GO TO 330                                                         WORK 301
  325 L(N,M,J) = -1                                                     WORK 302
      L1 = 1                                                            WORK 303
  330 CONTINUE                                                          WORK 304
  334 CONTINUE                                                          WORK 305
  335 CONTINUE                                                          WORK 306
  336 CONTINUE                                                          WORK 307
C                                                                       WORK 308
C......... END OF MASTER CONTROL SECTION ............................   WORK 309
C                                                                       WORK 310
      DDY1ZG = DDY1(JJ)*ZG                                              WORK 311
      DO 450 N=2,22                                                     WORK 312
C                                                                       WORK 313
C...... MONITOR FOR   MAXIMUM AND MINIMUM  'TOTAL ACCELERATION - TACC'  WORK 314
C                                                                       WORK 315
      TACC(N) = ACC(N) + DDY1ZG                                         WORK 316
      IF(TACCMX(N).GE.TACC(N)) GO TO 355                                WORK 317
      TACCMX(N)=TACC(N)                                                 WORK 318
      TMXTAC(N)=TIM1                                                    WORK 319
      GO TO 360                                                         WORK 320
  355 CONTINUE                                                          WORK 321
      IF(TACCMN(N).LE.TACC(N)) GO TO 360                                WORK 322
      TACCMN(N)=TACC(N)                                                 WORK 323
      TMNTAC(N)=TIM1                                                    WORK 324
  360 CONTINUE                                                          WORK 325
C                                                                       WORK 326
C...... MONITOR FOR   MAXIMUM AND MINIMUM  'VELOCITY - V '              WORK 327
C                                                                       WORK 328
      IF(VMAX(N).GE.V(N)) GO TO 365                                     WORK 329
      VMAX(N)=V(N)                                                      WORK 330
      TMXV(N)=TIM1                                                      WORK 331
      GO TO 370                                                         WORK 332
  365 CONTINUE                                                          WORK 333
      IF(VMIN(N).LE.V(N)) GO TO 370                                     WORK 334
      VMIN(N)=V(N)                                                      WORK 335
      TMNV(N)=TIM1                                                      WORK 336
  370 CONTINUE                                                          WORK 337
C                                                                       WORK 338
C...... MONITOR FOR   MAXIMUM AND MINIMUM  'DISPLACEMENT - U '          WORK 339
C                                                                       WORK 340
      IF(UMAX(N).GE.U(N)) GO TO 375                                     WORK 341
      UMAX(N)=U(N)                                                      WORK 342
      TMXU(N)=TIM1                                                      WORK 343
      GO TO 380                                                         WORK 344
  375 CONTINUE                                                          WORK 345
      IF(UMIN(N).LE.U(N)) GO TO 380                                     WORK 346
      UMIN(N)=U(N)                                                      WORK 347
      TMNU(N)=TIM1                                                      WORK 348
  380 CONTINUE                                                          WORK 349
C                                                                       WORK 350
C........ EVALUATE 'SHEAR' AND                                          WORK 351
C........ MONITOR FOR MAX AND MIN   INTER-FLOOR SHEAR 'SHEAR' (: COL) .....WORK 352
C                                                                       WORK 353
      IF(N.EQ.22) GO TO 390                                             WORK 354
      N1=N+1                                                            WORK 355
      SHEAR(N) = -(BM(N,2,4) + BM(N,3,4 + BM(N1,2,2)+BM(N1,3,2))*H(N)   WORK 356
      IF(SRMAX(N).GE.SHEAR(N)) GO TO 385                               .WORK 357
      SRMAX(N)=SHEAR(N)                                                 WORK 358
      TMXSR(N)=TIM1                                                     WORK 359
      GO TO 390                                                         WORK 360
  385 CONTINUE                                                          WORK 361
      IF(SRMIN(N).LE.SHEAR(N)) GO TO 390                                WORK 362
      SRMIN(N)=SHEAR(N)                                                 WORK 363
      TMNSR(N)=TIM1                                                     WORK 364
  390 CONTINUE                                                          WORK 365
C                                                                       WORK 366
C...... MONITOR FOR   MAXIMUM AND MINIMUM  'INTER-FLOOR DISP. - UU' ....WORK 367
C                                                                       WORK 368
      IF(UUMAX(N).GE.UU(N)) GO TO 395                                   WORK 369
      UUMAX(N)=UU(N)                                                    WORK 370
      TMXUU(N)=TIM1                                                     WORK 371
      GO TO 400                                                         WORK 372
  395 CONTINUE                                                          WORK 373
      IF(UUMIN(N).LE.UU(N)) GO TO 400                                   WORK 374
      UUMIN(N)=UU(N)                                                    WORK 375
      TMNUU(N)=TIM1                                                     WORK 376
  400 CONTINUE                                                          WORK 377
C                                                                       WORK 378
C........ EVALUATE '  CHECK-SUM' OF SHEAR FORCES ON EACH FLOOR ....... .WORK 379
C                                                                       WORK 380
      SR(N) = ZM(N)*(TACC(N) + BLAMDA*V(N)) + SHEAR(N) - SHEAR(N-1)     WORK 381
      IF(ABSSRT(N).GE.ABS(SRT(N))) GO TO 445                            WORK 382
      ABSSRT(N) = ABS(SRT(N))                                           WORK 383
      TABSSR(N) = TIM1                                                  WORK 384
  445 CONTINUE                                                          WORK 385
  450 CONTINUE                                                          WORK 386
C                                                                       WORK 387
      DO 500 M=2,3                                                      WORK 388
      DO 495 J=1,4                                                      WORK 389
      DO 490 N=2,22                                                     WORK 390
      IF(L1.EQ.0) GO TO 480                                             WORK 391
C                                                                       WORK 392
C......MONITOR FOR   MAXIMUM AND MINIMUM  'YIELD ANGLE - ALP '  .....   WORK 393
C                                                                       WORK 394
      IF(ALPMAX(N,M,J).GE.ALP(N,M,J)) GO TO 470                         WORK 395
      ALPMAX(N,M,J)=ALP(N,M,J)                                          WORK 396
      TMXALP(N,M,J)=TIM1                                                WORK 397
      GO TO 475                                                         WORK 398
  470 CONTINUE                                                          WORK 399
      IF(ALPMIN(N,M,J).LE.ALP(N,M,J)                                    WORK 400
      ALPMIN(N,M,J)=ALP(N,M,J)                                          WORK 401
      TMNALP(N,M,J)=TIM1                                                WORK 402
  475 CONTINUE                                                          WORK 403
  480 CONTINUE                                                          WORK 404
C                                                                       WORK 405
C...... MONITOR FOR   MAXIMUM AND MINIMUM  'BENDING MOMENT - BM '       WORK 406
C                                                                       WORK 407
      IF(BMMAX(N,M,J).GE.BM(N,M,J)) GO TO 485                           WORK 408
      BMMAX(N,M,J)=BM(N,M,J)                                            WORK 409
      TMXBM(N,M,J)=TIM1                                                 WORK 410
      GO TO 490                                                         WORK 411
  485 CONTINUE                                                          WORK 412
      IF(BMMIN(N,M,J).LE.BM(N,M,J)) GO TO 490                           WORK 413
      BMMIN(N,M,J)=BM(N,M,J)                                            WORK 414
      TMNBM(N,M,J)=TIM1                                                 WORK 415
  490 CONTINUE                                                          WORK 416
  495 CONTINUE                                                          WORK 417
  500 CONTINUE                                                          WORK 418
C                                                                       WORK 419
                                                                        WORK 420
```

```
C...........  EVALUATE  'GIRDER SHEAR FORCES, COLUMN AXIAL FORCES, AND        WORK 421
C............................  OVERTURNING MOMENTS'  .......................   WORK 422
C                                                                             WORK 423
      DO 510 N=2,22                                                           WORK 424
      GSR(N,2) = -((BMIN,2,3) + BM(N,3,1)) *AL)                               WORK 425
      GSR(N,3) = -(2.0*BMIN,3,3)*AL)                                          WORK 426
      AXCLD(N,2) = AXCLD(N-1,2) +  GSR(N,2)                                   WORK 427
      AXCLD(N,3) = AXCLD(N-1,3) + (GSR(N,3) -  GSR(N,2))                      WORK 428
      OTMOM(N) = ZL*(AXCLD(N-1,3) + 3.0*AXCLD(N-1,2))                         WORK 429
     2          + 2.0*(BMIN,2,2) + BM(N,3,2))                                 WORK 430
C                                                                             WORK 431
C........  MONITOR FOR MAXIMUM AND MINIMUM 'OVERTURNING MOMENTS - OTMOM'       WORK 432
C                                                                             WORK 433
      IF(OTMMX(N).GE.OTMOM(N)) GO TO 505                                      WORK 434
      OTMMX(N) = OTMOM(N)                                                     WORK 435
      TMXOTM(N) = TIM1                                                        WORK 436
      GO TO 510                                                               WORK 437
  505 CONTINUE                                                                WORK 438
      IF(OTMMN(N).LE.OTMOM(N))  GO TO 510                                     WORK 439
      OTMMN(N) = OTMOM(N)                                                     WORK 440
      TMNOTM(N) = TIM1                                                        WORK 441
  510 CONTINUE                                                                WORK 442
C                                                                             WORK 443
C........  EVALUATE 'CHECK-SUM' OF BENDING MOMENTS AT EACH JOIN"               WORK 444
C                                                                             WORK 445
      DO 550 M=2,3                                                            WORK 446
      DO 545 N=2,22                                                           WORK 447
      BMT(N,M) = 0.0                                                          WORK 448
      DO 520 J=1,4                                                            WORK 449
      BMT(N,M) = BMT(N,M) + BM(N,M,J)                                         WORK 450
  520 CONTINUE                                                                WORK 451
      IF(ABSBMT(N,M).GE.ABS(BMT(N,M)) )  GO TO 525                            WORK 452
      ABSBMT(N,M) = ABS(BMT(N,M))                                             WORK 453
      TABSBM(N,M) = TIM1                                                      WORK 454
  525 CONTINUE                                                                WORK 455
C                                                                             WORK 456
C........  MONITOR FOR MAXIMUM AND MINIMUM 'GIRDER SHEAR - GSR '.......        WORK 457
C                                                                             WORK 458
      IF(GSRMX(N,M).GE.GSR(N,M)) GO TO 530                                    WORK 459
      GSRMX(N,M)=GSR(N,M)                                                     WORK 460
      TMXGSR(N,M)=TIM1                                                        WORK 461
      GO TO 535                                                               WORK 462
  530 CONTINUE                                                                WORK 463
      IF(GSRMN(N,M).LE.GSR(N,M))  GO TO 535                                   WORK 464
      GSRMN(N,M)=GSR(N,M)                                                     WORK 465
      TMNGSR(N,M)=TIM1                                                        WORK 466
  535 CONTINUE                                                                WORK 467
C                                                                             WORK 468
C........  MONITOR FOR MAXIMUM AND MINIMUM 'AXIAL COLUMN LOAD - AXCLD'         WORK 469
C                                                                             WORK 470
      IF(CLDMAX(N,M).GE.AXCLD(N,M)) GO TO 540                                 WORK 471
      CLDMAX(N,M =AXCLD(N,M)                                                  WORK 472
      TMXCLD(N,M =TIM1                                                        WORK 473
      GO TO 545                                                               WORK 474
  540 CONTINUE                                                                WORK 475
      IF(CLDMIN(N,M).LE.AXCLD(N,M))  GO TO 545                                WORK 476
      CLDMIN(N,M =AXCLD(N,M)                                                  WORK 477
      TMNCLD(N,M =TIM1                                                        WORK 478
  545 CONTINUE                                                                WORK 479
  550 CONTINUE                                                                WORK 480
C                                                                             WORK 481
      CALL MODE                                                               WORK 482
C                                                                             WORK 483
      LWRITE = LWRITE + 1                                                     WORK 484
      IF(LWRITE.LT.LRTEST)  GO TO 560                                         WORK 485
      LWRITE = 0                                                              WORK 486
      WRITE(6,107)  JJ,TIM1,DDYI(JJ),U(2),LL1                                 WORK 487
C                                                                             WORK 488
      WRITE(20)  JJ,LL1,L,SHEAR,BM,OTMOM,AXCLG,ACC,TACC,V,                    WORK 489
     2 DBU,DBUU,V,UUH,ALP,TIM1,DT,DDYI(JJ)                                    WORK 490

  560 CONTINUE                                                                WORK 491
C                                                                             WORK 492
C.........  END OF TIME INCREMENT ......................................      WORK 493
C                                                                             WORK 494
      IF(ABS(TIM1-ZTIME).GT.2.5000002E-01.AND.TIM1.LT.TIMEND) GO TO 10        WORK 495
      ITIME = ITIME + 1                                                       WORK 496
      ZTIME = ITIME                                                           WORK 497
      CALL GONE                                                               WORK 498
      WRITE(6,1001)                                                           WORK 499
C                                                                             WORK 500
      IF(TIM1.LT.TIMEND)  GO TO 10                                            WORK 501
C                                                                             WORK 502
      CALL GONE                                                               WORK 503
      WRITE(6,1090)  JJ                                                       WORK 504
 1090 FORMAT(10H0 J,END =    ,I4)                                             WORK 505
C                                                                             WORK 506
C.........  TERMINATION OF RUN ......                                         WORK 507
C                                                                             WORK 508
      RETURN                                                                  WORK 509
      END                                                                     WORK 510
```

```fortran
$IBFTC CONV    DECK    ***MATRIX** **DU** **DBLE PRECISION***   Q56  CONV   1
      SUBROUTINE CONV                                                CONV   2
C                                                                    CONV   3
C*********************************************************************CONV  4
C   THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES    CONV   5
C   OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION. CONV   6
C                                                                    CONV   7
C   THE PURPOSE OF SUBROUTINE 'CON/' IS TO SOLVE THE EQUATIONS OF    CONV   8
C   MOTION FOR EACH TIME INTERVAL AND TO RETURN UPDATED RESPONSES TO CONV   9
C   SUBROUTINE 'WORK'.                                               CONV  10
C              D O U B L E   P R E C I S I O N                       CONV  11
C                                                                    CONV  12
C                                                                    CONV  13
C   THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE  CONV  14
C   F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-  CONV  15
C   TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                        CONV  16
C          LAST REVISION - 4 DECEMBER 1966.                          CONV  17
C                                                                    CONV  18
C*********************************************************************CONV  19
C                                                                    CONV  20
C   NOTE ON PREFIX LETTERS..                                         CONV  21
C     'D'    INCREMENTAL VARIABLE                                    CONV  22
C     'DB'   DOUBLE PRECISION VARIABLE                               CONV  23
C     'DBD'  DOUBLE PRECISION INCREMENTAL VARIABLE                   CONV  24
C                                                                    CONV  25
C   NOTE.. 'COMMON/ACEWOR/' IS USED TO COMMUNICATE ONLY             CONV  26
C     THE VALUE OF 'DBDT' TO 'SUBR CONV' FROM 'SUBR ACEL'.          CONV  27
C     THIS MEANS (1) WHAT FOLLOWS 'DBDT' IN 'COMMON/ACEWOR/'        CONV  28
C     IS IMMATERIAL TO 'SUBR CONV' AND HENCE IS NOT USED-           CONV  29
C     (2) CONSEQUENTLY 'SUBR CONV' MUST BE LOADED AFTER ANY         CONV  30
C     OTHER SUBROUTINE THAT HAS THE COMPLETELY CORRECT              CONV  31
C     'COMMON/ACEWOR/' AS IT IS USED IN COMMUNICATING               CONV  32
C     BETWEEN 'SUBR ACEL' AND 'SUBR WORK'.                          CONV  33
C                                                                    CONV  34
C*********************************************************************CONV  35
C                                                                    CONV  36
C                                                                    CONV  37
C   VARIABLES USED HEREIN                                            CONV  38
C                                                                    CONV  39
C   U     HORIZONTAL DISPLACEMENT RELATIVE TO GROUND                 CONV  40
C   V     VELOCITY RELATIVE TO GROUND                                CONV  41
C   ACC   ACCELERATION RELATIVE TO GROUND                           CONV  42
C   H     JOINT ROTATION                                            CONV  43
C   UU    INTERFLOOR DISPLACEMENT                                    CONV  44
C                                                                    CONV  45
C*********************************************************************CONV  46
C                                                                    CONV  47
      DOUBLE PRECISION DBTIME                                        CONV  48
C                                                                    CONV  49
      DOUBLE PRECISION    C(20,3),SINVI(20,3),CTS1(20,3),DR(21,3),DP(20,3),  CONV 50
     2 USEDI(3,3),USEDP1(1),CTS1(20,3,3),S(20,3,3),          DBDUUH(22),CONV 51
     3 AACC(22), BVEL(22),DBZM(22),DBH(22),DBJU(22),                CONV  52
     4 DBUUH(22),DBV(22),DBACC(22),DBW(22,3),         SS,SSINV,  DBU,DBUU CONV 53
     5 DBLAMD,DBDT,C1,C2,C3,C4,C5,DBDDDY,                        CONV  54
C                                                                    CONV  55
      COMMON/ACEWOR/DBTIMI,DBDT                                      CONV  56
C                                                                    CONV  57
C***. USE DUMMY NAME '((8)' INSTEAD OF 'S(8)' IN 'COMMON/MNWCGN/'...*CONV  58
C                                                                    CONV  59
      COMMON/MNWCGN/NAMES1(10),NAMECK(13),BMA(22,3,4),BMI(22,3,4),  CONV  60
     2 GA(22,3),CA(22,3),HT(22),H(22),ZM(22),A(5),Z(8),P,F,ZETA,FUNDFQ,CONV 61
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                 CONV  62
C                                                                    CONV  63
      COMMON/WKCONV/SCA(22,3),SCB(22,3),SCC(22,3),SCABH(22,3),      CONV  64
     2 SCBCH(22,3),SCDHH(22),SGA(22,3),SGB(22,3),SGC(22,3),         CONV  65
     3 DU(22),UU(22),DUUH(22),UUH(22),DV(22),V(22),                 CONV  66
     4 DACC(22),ACC(22),DV(22,3),W(22,3),         DDDY,DT,K,N22     CONV  67
C                                                                    CONV  68
      COMMON/CONMOD/ DBU(22),DBUU(22)                               CONV  69
C                                                                    CONV  70
C                                                                    CONV  71
      COMMON/CONMIN/SS(3,3),SSINV(3,3),N                            CONV  72
C                                                                    CONV  73
      IF(LL1.EQ.1) GO TO 1601                                       CONV  74
      LL1=1                                                         CONV  75
      WRITE(6,1600)                                                 CONV  76
 1600 FORMAT(50H 'DU'   VIA MATRICES USING DOUBLE PRECISION      )  CONV  77
      DBLAMD = DBLE(BLAMDA)                                         CONV  78
      C1 = 3.0D+00                                                  CONV  79
      DO 690 N=2,2L                                                 CONV  80
      DBZM(N) = DBLE(ZM(N))                                         CONV  81
      DB4(N) = DBLE(H(N))                                           CONV  82
      DBACC(N) = DBLE(ACC(N))                                       CONV  83
  690 CONTINUE                                                      CONV  84
 1601 CONTINUE                                                      CONV  85
      C2 = DBDT/2.0D+00                                             CONV  86
      C3 = 6.0D+00/DBDT                                             CONV  87
      C4 = C1/DBDT                                                  CONV  88
      C5 = C3/DBDT                                                  CONV  89
      DBDDDY = DBLE(DDDY)                                           CONV  90
      DO 700 N=1,20                                                 CONV  91
      N1=N+1                                                        CONV  92
      AACC(N1) =-C3*DBV(N1) - C1*DBACC(N1)                          CONV  93
      BVEL(N1) =-C1*DBV(N1) - C2*DBACC(N1)                          CONV  94
C                                                                    CONV  95
C..... EVALUATE 'S'  - THE STIFFNESS MATRICES ON THE MAIN DIAGONAL . CONV  96
C                                                                    CONV  97
      S(N,1,1) = DBLE(SCDHH(N) + SCDHH(N1)) + ( C5 + C4*DBLAMD)*DBZM(N1) CONV 98
      S(N,2,2) = DBLE( SCC(N,2) + SGA(N,2) + SCA(N,2) )             CONV  99
      S(N,3,3) = DBLE( SGC(N1,2) + SCC(N,3) + SGA(N1,3) + SCA(N1,3) ) CONV 100
      S(N,1,2) = DBLE( SCBCHN,2) - SCABH(N1,2) )                    CONV 101
      S(N,1,3) = DBLE( SCBCHN,3) - SCABH(N1,3) )                    CONV 102
      S(N,2,3) = DBLE ( SGB(N1,2) )                                 CONV 103
      S(N,2,1) = S(N,1,2)                                           CONV 104
      S(N,3,1) = S(N,1,3)                                           CONV 105
      S(N,3,2) = S(N,2,3)                                           CONV 106
C                                                                    CONV 107
C..... EVALUATE 'C'  - THE OFF DIAGONAL STIFFNESS MATRICES ........ CONV 108
C                                                                    CONV 109
      C(N,1,1) = DBLE( -SCDHH(N1) )                                 CONV 110
      C(N,2,2) = DBLE( SCB(N,2) )                                   CONV 111
      C(N,3,3) = DBLE( SCU(N,3) )                                   CONV 112
      C(N,1,2) = DBLE( -SCBCHN,2) )                                 CONV 113
      C(N,1,3) = DBLE( -SCBCH(N1,3) )                               CONV 114
      C(N,2,1) = DBLE( SCABH(N1,2) )                                CONV 115
      C(N,3,1) = DBLE( SCABH(N1,3) )                                CONV 116
C                                                                    CONV 117
C..... EVALUATE 'DP' VECTOR OF EFFECTIVE INCREMENTAL FORCES ....... CONV 118
C                                                                    CONV 119
      DPIN,1) = -(DBZM(N1)*( DBDDDY  + AACC(N1) + DBLAMD*BVEL(N1) ) ) CONV 120
      DPIN,2) = 0.0D+00                                             CONV 121
      DPIN,3) = 0.0D+00                                             CONV 122
  700 CONTINUE                                                      CONV 123
C                                                                    CONV 124
      DO 770 N=2,2L                                                 CONV 125
      NM=N-1                                                        CONV 126
      DO 720 I=1,3                                                  CONV 127
      DO 720 J=1,3                                                  CONV 128
      SSI(I,J) = S(NM1,I,J)*1.0D-06                                 CONV 129
  720 CONTINUE                                                      CONV 130
C                                                                    CONV 131
      CALL MINV                                                     CONV 132
C                                                                    CONV 133
      DO 725 I=1,3                                                  CONV 134
      DO 725 J=1,3                                                  CONV 135
      SINV(NM1,I,J) = SSINV(I,J)*1.0D-06                            CONV 136
      SINV(NM1,2,1) = SINV(NM1,1,2)                                 CONV 137
      SINV(NM1,3,1) = SINV(NM1,1,3)                                 CONV 138
      SINV(NM1,3,2) = SINV(NM1,2,3)                                 CONV 139
  725 CONTINUE                                                      CONV 140
      DO 730 I=1,3                                                  CONV 141
      DO 730 J=1,3                                                  CONV 142
      CTS1(NM1,I,J) = 0.0                                           CONV 143
      DO 730 K=1,3                                                  CONV 144
```

```
$IBFTC MINV   DECK *INVERTS '3X3 SYMMETRIC' MATRIX, DOUBLE PREC*   Q56 MINV  1
      SUBROUTINE MINV                                                      MINV  2
C**********************************************************************   *MINV  3
C                                                                         MINV  4
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE /ARIOUS RESPONSES       MINV  5
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.    MINV  6
C                                                                         MINV  7
C     THE PURPOSE OF SUBROUTINE 'MINV' IS TO INVERT A '3X3 SYMMETRIC'     MINV  8
C     MATRIX.  THE INPUT MATRIX IS 'SS' AND THE INVERSE MATRIX RE-        MINV  9
C     TURNED TO SUBROUTINE 'CINV' IS 'SSINV'.                            MINV 10
C     CAUTION.. 'SSINV' RETURNS WITH ONLY THE UPPER TRIANGLE FILLED      MINV 11
C     SINCE IT IS SYMMETRI..                                            MINV 12
C                                                                        MINV 13
C                                                                        MINV 14
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE    MINV 15
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-    MINV 16
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                          MINV 17
C                    LAST REVISION - 4 DECEMBER 1966                     MINV 18
C                                                                        MINV 19
C**********************************************************************   *MINV 20
C                                                                        MINV 21
      DOUBLE PRECISION SS,SSINV,ADG,DETT                                 MINV 22
      COMMON/CONMIN/SS(3,3),SSINV(3,3),N                                 MINV 23
      DIMENSION ADG(3,3)                                                 MINV 24
      ADG(1,1) = (SS(2,2)*SS(3,3)) - (SS(2,3)*SS(2,3))                   MINV 25
      ADG(2,2) = (SS(1,1)*SS(3,3)) - (SS(1,3)*SS(1,3))                   MINV 26
      ADG(3,3) = (SS(1,1)*SS(2,2)) - (SS(1,2)*SS(1,2))                   MINV 27
      ADG(1,2) = (SS(1,3)*SS(2,3)) - (SS(1,2)*SS(3,3))                   MINV 28
      ADG(1,3) = (SS(1,2)*SS(2,3)) - (SS(1,3)*SS(2,2))                   MINV 29
      ADG(2,3) = (SS(1,2)*SS(1,3)) - (SS(1,1)*SS(2,3))                   MINV 30
      DETT = SS(1,1)/( (ADG(3,3)*ADG 2,2) ) - (ADG(2,3)*ADG(2,3) ) )     MINV 31
      IF(DABS(DETT).LT.1.0D-06)  GO TO 44                                MINV 32
      WRITE(6,1004) DETT,N                                              MINV 33
1004  FORMAT(1H1,10X,8H DETT =   ,1PE15.7,10X,3H N=,I14)                 MINV 34
      STOP                                                              MINV 35
44    CONTINUE                                                          MINV 36
      SSINV(1,1) = ADG(1,1)*CETTT                                       MINV 37
      SSINV(2,2) = ADG(2,2)*CETTT                                       MINV 38
      SSINV(3,3) = ADG(3,3)*CETTT                                       MINV 39
      SSINV(1,2) = ADG(1,2)*LETTT                                       MINV 40
      SSINV(1,3) = ADG(1,3)*LETTT                                       MINV 41
      SSINV(2,3) = ADG(2,3)*LETTT                                       MINV 42
      RETURN                                                            MINV 43
      END                                                               MINV 44
```

```
730   CTSI(NM1,I,J) = CTSI(NM1,I,J) + (C(NM1,K,I)*SINV(NM1,K,J))        CONV 141
      CONTINUE                                                          CONV 142
      IF(N.EQ.21)  GO TO 771                                            CONV 143
      DO 735 I=1,3                                                      CONV 144
      DO 735 J=1,3                                                      CONV 145
      USEDI(I,J) = 0.0                                                  CONV 146
      DO 735 K=1,3                                                      CONV 147
735   USEDI(I,J) = USEDI(I,J) + (CTSI(NM1,I,K)*C(NM1,K,J))              CONV 148
      DO 740 I=1,3                                                      CONV 149
      DO 740 J=1,3                                                      CONV 150
740   SIN(N,I,J) = SIN(N,I,J) - USEDI(I,J)                              CONV 151
      DO 750 I=1,3                                                      CONV 152
      USEDPI(I) = 0.0                                                   CONV 153
      DO 750 J=1,3                                                      CONV 154
750   USEDPI(I) = USEDPI(I) + (CTSI(NM1,I,J)*DP(NM1,J))                 CONV 155
      DO 755 I=1,3                                                      CONV 156
755   DP(N,I) = DP(N,I) - USEDPI(I)                                     CONV 157
770   CONTINUE                                                          CONV 158
771   CONTINUE                                                          CONV 159
      CONTINUE                                                          CINV 160
C                                                                       CCNV 161
C..... BACK-SUBSTITUTE TO OBTAIN 'DR' VECTOR OF INCR DISP AND ROTNS...CCNV 162
      DO 805 I=1,3                                                      CONV 163
      DR(20,I) = 0.0                                                    CONV 164
      DO 805 J=1,3                                                      CONV 165
805   DR(20,I) = DR(20,I) + (SINV(20,I,J)*DP(20,J))                     CONV 166
      DO 840 NN=1,19                                                    CONV 167
      N = 20-NN                                                         CONV 168
      DO 820 I=1,3                                                      CONV 169
      USEDPI(I) = 0.0                                                   CONV 170
      DO 810 J=1,3                                                      CONV 171
810   USEDPI(I) = USEDPI(I) + (C(N,I,J)*DR(N+1,J))                      CONV 172
820   DP(N,I) = DP(N,I) - USEDPI(I)                                     CUNV 173
      DO 835 I=1,3                                                      CONV 174
      DR(N,I) = 0.0                                                     CONV 175
      DO 835 J=1,3                                                      CCNV 176
835   DR(N,I) = DR(N,I) + (SINV(N,I,J)*DP(N,J))                         CCNV 177
840   CONTINUE                                                          CCNV 178
C                                                                       CCNV 179
      DO 850 N=2,21                                                     CCNV 180
      NM1=N-1                                                           CONV 181
      DBU(N) = DR(NM1,1)                                                CONV 182
      DBU(N) = DBU(N) + DBDU(N)                                         CONV 183
      DBW(N,2) = DBW(N,2) + DR(NM1,2)                                   CONV 184
      DBW(N,3) = DBW(N,3) + DR(NM1,3)                                   CONV 185
      DBACC(N) = DBACC(N) + AACC(N) + C5*DBDU(N)                        CONV 186
      DBV(N) = DBV(N) +3VEL(N) + (4*DBDU(N)                             CONV 187
      U(N) = SNGL(DBU(N))                                               CONV 188
      W(N,2) = SNGL(DBW(N,2))                                           CONV 189
      W(N,3) = SNGL(DBW(N,3))                                           CONV 190
      DW(N,2) = SNGL( DR(NM1,2) )                                       CONV 191
      DW(N,3) = SNGL( DR(NM1,3) )                                       CONV 192
      V(N) = SNGL(DBV(N))                                               CONV 193
      ACC(N) = SNGL(DBACC(N))                                           CONV 194
850   CONTINUE                                                          CONV 195
      N1 = N+1                                                          CONV 196
      DO 860 N=2,21                                                     CONV 197
      DBUU(N) = DBU(N) - DBU(N1)                                        CONV 198
      DBDUUH(N) = (DBDU(N) - DBDU(N1) )*DBHN(N)                         CONV 199
      DBUUH(N) = DBUUH(N) + DBDUUH(N)                                   CONV 200
      DBUH(N) = SNGL(DBDUUH(N))                                         CONV 201
      UUH(N) = SNGL(DBUUH(N))                                           CONV 202
      UU(N) = SNGL(DBUU(N))                                             CONV 203
860   CONTINUE                                                          CONV 204
C                                                                       CONV 205
C..... END OF 'MATRIX SOLUTION' OF EQUATIONS OF MOTION FOR THIS 'DT' .CONV 206
C                                                                       CONV 207
      RETURN                                                            CONV 208
      END                                                               CONV 209
```

```
$IBFTC GONE    DECK    **WRITES OUT RESULTS OF PROGRAM**       Q56 GONE    1
      SUBROUTINE GONE                                              GONE    2
                                                                   GONE    3
C*****************************************************************GCNE    4
C                                                               **GCNE    5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES GONE   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION. GONE 7
C                                                                   GONE    8
C     THE PURPOSE OF THIS SUBROUTINE IS TO EVALUATE THE DUCTILITY  GONE    9
C     FACTORS, TO WRITE OUT THE MOST POSITIVE AND MOST NEGATIVE VALUES GONE 10
C     OF THE VARIOUS RESPONSES ALONG WITH THE TIMES AT WHICH THEY  GONE   11
C     OCCUR, AND TO PLOT OUT THE DISPLACEMENT ENVELOPE AND THE COLUMN GONE 12
C     AND GIRDER DUCTILITY FACTORS.                                GONE   13
C                                                                   GONE   14
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE GONE 15
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- GONE 16
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                    GONE   17
C           LAST REVISION - 11 JANUARY 1967                        GONE   18
C                                                                   GCNE   19
C*****************************************************************GONE    20
C                                                                   GONE   21
      DIMENSION DUCTIL(22,3,4)                                     GONE   22
C                                                                   GONE   23
      COMMON/ANWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4), GCNE   24
     2 GA(22,1),CA(22,3),HT(22),HI(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ, GCNE 25
     3 ZL,AL,IMEND,ALAMDA,BLAMDA,ELPL,TIM1,LRTEST                  GCNE   26
C                                                                   GONE   27
      COMMON/IRKGGN/UMAXI(22),UMIN(22),TMXUI(22),TMNUI(22),        GONE   28
     2 VMAX(22),VMIN(22),TMXVI(22),TMNVI(22),                      GONE   29
     3 UUMAX(22),UUMIN(22),TMXUU(22),TMNUU(22),                    GONE   30
     4 SRMAX(22),SRMIN(22),TMXSR(22),TMNSR(22),                    GONE   31
     5 WMAX(22),WMIN(22,3),TMXW(22,3),TMNW(22,3),                  GONE   32
     6 ALPMAX(22,3,4),ALPMIN(22,3,4),TMXALP(22,3,4),TMNALP(22,3,4),GONE   33
     7 BMMAX(22,3,4),BMMIN(22,3,4),TMXBM(22,3,4),TMNBM(22,3,4),    GCNE   34
     8 TACCMX(22),TACCMN(22),TMXTAC(22),TMNTAC(22),                GONE   35
     9 GSRMX(22,3),GSRMN(22,3),TMXGSR(22,3),TMNGSR(22,3),          GONE   36
     1 CLDMAX(22,3),CLDMIN(22,3),TMXCLD(22,3),TMNCLD(22,3),        GONE   37
     2 OTMMX(22),OTMMN(22),TMXOTM(22),TMNOTM(22),                  GONE   38
     3 ABSBMT(22,3),TABSBM(22,3),ABSSRT(22),TABSSR(22)             GONE   39
C                                                                   GONE   40
      COMMON/ANGONE/ YIELDI(22,3,4)                                GONE   41
C                                                                   GONE   42
      COMMON/JKGNIN/LLX,LLI,LLB,LLG,LLD,LLP, LGD(2),LINE(129)      GCNE   43
C                                                                   GONE   44
C.........  DUCTILITY BY GIBERSON ........................        GCNE   45
C                                                                  *GONE   46
      DO 600 K=2,3                                                 GONE   47
      DO 595 I=1,4                                                 GONE   48
      DO 590 M=2,22                                                GONE   49
      IF(-ALPMIN(N,M,J) + ALPMAX(N,M,J).GT.0.0)  GO TO 570        GONE   50
      IF(-BMMIN(N,M,J).GT.BMMAX(N,M,J))  GO TO 550                GONE   51
      DUCTIL(N,M,J)=BMMAX(N,M,J)/BMA(N,M,J)                       GCNE   52
      GO TO 590                                                   GONE   53
  550 DUCTIL(N,M,J)=BMMIN(N,M,J)/BMB(N,M,J)                       GONE   54
      GO TO 590                                                   GONE   55
C                                                                   GONE   56
  570 CONTINUE                                                     GONE   57
      IF(-ALPMIN(N,M,J).GT.ALPMAX(N,M,J))  GO TO 580             GCNE   58
      DUCALP = ALPMAX(N,M,J)                                      GONE   59
      GO TO 585                                                   GONE   60
  580 DUCALP = -ALPMIN(N,M,J)                                     GONE   61
  585 DUCTIL(N,M,J) = 1.0 + DUCALP*(YIELDI(N,M,J) + (P*DUCALP)/(1.0-P)) GONE 62
C                                                                   GONE   63
  590 CONTINUE                                                     GONE   64
  595 CONTINUE                                                     GONE   65
  600 CONTINUE                                                     GONE   66
C                                                                   GONE   67
C.......... WRITE OUT MAXIMUM AND MINIMUM VALUES AND CORRESPONDING TIMES ....... .GONE 68
C.......... 'MAXIMUM' CORRESPONDS TO 'MOST POSITIVE VALUE' ......GONE 69
C.......... 'MINIMUM' CORRESPONDS TO 'MOST NEGATIVE VALUE' ......GONE 70
```

```
C                                                                   GONE   71
 1001 WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK              GONE   72
 1001 FORMAT(1H1 ,10A6,15H,BILINEARITY = ,F5.3,1bH,FRAC CR DAMP =  , GONE 73
     2 -7.5,13H,FUND FREQ = ,F7.5, 8H RAD/SEC ,//,                 GONE   74
     3 3X, 15HUSING THE FIRST ,F10.5, 15H SECONDS OF THE , 13A6,2X,//) GONE 75
 1300 WRITE(6,1300)                                                GONE   76
 1300 FORMAT(20H LMAX,TMXU,UMIN,TMNU,55X,27HTACCMX,TMXTAC,TACCMN,TMNTAC,)GONE 77
      NN = 21                                                      GONE   78
      DO 710 N=2,22                                                GONE   79
      NN = NN -1                                                   GONE   80
      WRITE(6,1311) NN,UMAX(N),TMXU(N),UMIN(N),TMNU(N),TACCMX(N),  GONE   81
     2 TMXTAC(N),TACCMN(N),TMNTAC(N)                               GONE   82
 1311 FORMAT((1X,I3,4(7X,1PE15.7,2X,0PF8.5) ))                     GONE   83
  710 CONTINUE                                                     GONE   84
C                                                                   GONE   85
 1002 WRITE(6,1002)                                                GONE   86
 1002 FORMAT(1H0)                                                  GONE   87
 1320 WRITE(6,1320)                                                GONE   88
 1320 FORMAT(24H UUMAX,TMXUU,UUMIN,TMNUU,51X,23HSRMAX,TMXSR,SRMIN,TMNSR)GONE 89
      NN = 21                                                      GONE   90
      DO 720 N=2,22                                                GONE   91
      NN = NN - 1                                                  GONE   92
      WRITE(6,1311) NN,LUMAX N),TMXUU(N),UUMIN(N),TMNUU(N), SRMAX(N), GONE 93
     2 MXSR(N),SRMIN(N),TMNSR(N)                                   GCNE   94
  720 CONTINUE                                                     GONE   95
C                                                                   GONE   96
 1330 WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK             GONE   97
      WRITE(6,1330)                                                GCNE   98
 1330 FORMAT(40H (WMAX,  TMXW,  WMIN,  TMNW,  M=2,3)              GGNE   99
      NN = 21                                                      GONE  100
      DO 730 N=2,22                                                GONE  101
      NN = NN - 1                                                  GONE  102
      WRITE(6,1311) NN,(WMAX N,M),TMXW(N,M),WMIN(N,M),TMNW(N,M),M=2,3)GCNE 103
  730 CONTINUE                                                     GONE  104
C                                                                   GONE  105
 1340 WRITE(6,1002)                                                GONE  106
      WRITE(6,1340)                                                GONE  107
 1340 FORMAT(30H VMAX,    TMX'V,    VMIN,    TMNV              )  GONE  108
      NN = 21                                                      GONE  109
      DO 740 N=2,22                                                GONE  110
      NN = NN - 1                                                  GONE  111
      WRITE(6,1345) NN,VMAX(U),TMXV(N),VMIN(N),TMNV(N)            GCNE  112
 1345 FORMAT((1X,I3,2(7X,1PE15.7,2X,0PF8.5) ))                    GONE  113
  740 CONTINUE                                                     GONE  114
C                                                                   GONE  115
 1350 WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK             GCNE  116
      WRITE(6,1350)                                                GCNE  117
 1350 FORMAT(36H  ((ALPMAX(N,M,J),J=1,4),M=2,3)            )      GONE  118
      NN = 21                                                      GCNE  119
      DO 750 N=2,22                                                GONE  120
      NN = NN - 1                                                  GONE  121
      WRITE(6,1313) NN,((ALPMAX(N,M,J),J=1,4),M=2,3)              GONE  122
 1312 FORMAT((1X,I3,2(4X,1P4(15.7) ))                             GONE  123
  750 CONTINUE                                                     GONE  124
C                                                                   GCNE  125
 1360 WRITE(6,1002)                                                GONE  126
      WRITE(6,1360)                                                GCNE  127
 1360 FORMAT(36H  ((TMXALP(N M,J),J=1,4),M=2,3)            )      GCNE  128
      NN = 21                                                      GONE  129
      DO 760 N=2,22                                                GONE  130
      WRITE(6,1313)  NN,((TMXALP(N M,J),J=1,4),M=2,3)            GONE  131
 1313 FORMAT((1X,I3,2(4F15.7,4X) ))                               GCNE  132
  760 CONTINUE                                                     GONE  133
C                                                                   GCNE  134
 1370 WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK             GONE  135
      WRITE(6,1370)                                                GONE  136
 1370 FORMAT(36H  ALPMIN(N,M,J),J=1,4),M=2,3)                      GONE  137
      NN = 21                                                      GONE  138
      DO 770 N=2,22                                                GONE  139
                                                                   GONE  140
```

```
        NN = NN - 1
770     WRITE(6,1312)   NN,((ALPM(N,M,J),J=1,4),M=2,3)
        CONTINUE
C
        WRITE(6,1002)
        WRITE(6,1380)
1380    FORMAT(35H  TMNALP(N,M,J),J=1,4),M=2,3)
        NN = 21
        DU 780 N=2,22
        NN = NN - 1
780     WRITE(6,1313)   NN,((TMNALP(N,M,J),J=1,4),M=2,3)
        CONTINUE
C
        WRITE(6,1001)   NAMEST,P,ZETA,FUNDFQ,TIM,NAMEQK
        WRITE(6,1390)
1390    FORMAT(50H  (GSRMX,  TMXGSR,  GSRMN,  'MNGSR ),  M=2,3)
        NN = 21
        DO 790 N=2,22
        NN = NN - 1
790     WRITE(6,1311)   NN,(GSRMX(N,M),TMXGSR(N,M),GSRMN(N,M),TMNGSR(N,M),
     2  M=2,3)
        CONTINUE
C
        WRITE(6,1002)
        WRITE(6,1400)
1400    FORMAT(50H  (CLDMAX,  TMXCLD,  CLDMIN,  TMNCLD),  M=2,3)
        NN = 21
        DO 800 N=2,22
        NN = NN - 1
800     WRITE(6,1311)   NN,(CLDMAX(N,M),TMXCLD(N,M),CLDMIN(N,M),TMNCLD(N,M),
     2  M=2,3)
        CONTINUE
C
        WRITE(6,1001)   NAMEST,P,ZETA,FUNDFQ,TIM,NAMEQK
        WRITE(6,1410)
1410    FORMAT(26H  (BMMAX(N,M,J),J=1,4),M=2,3)
        NN = 21
        DO 810 N=2,22
        NN = NN - 1
810     WRITE(6,1312)   NN,((BMMAX(N,M,J),J=1,4),M=2,3)
        CONTINUE
C
        WRITE(6,1002)
        WRITE(6,1420)
1420    FORMAT(26H  (TMXBM(N,M,J),J=1,4),M=2,3)
        NN = 21
        DO 820 N=2,22
        NN = NN - 1
820     WRITE(6,1312)   NN,((TMXBM(N,M,J),J=1,4),M=2,3)
        CONTINUE
C
        WRITE(6,1001)   NAMEST,P,ZETA,FUNDFQ,TIM,NAMEQK
        WRITE(6,1430)
1430    FORMAT(26H  (BMMIN(N,M,J),J=1,4),M=2,3)
        NN = 21
        DO 830 N=2,22
        NN = NN - 1
830     WRITE(6,1312)   NN,((BMMIN(N,M,J),J=1,4),M=2,3)
        CONTINUE
C
        WRITE(6,1002)
        WRITE(6,1440)
1440    FORMAT(26H  (TMNBM(N,M,J),J=1,4),M=2,3)
        NN = 21
        DO 840 N=2,22
        NN = NN - 1
840     WRITE(6,1313)   NN,((TMNBM(N,M,J),J=1,4),M=2,3)
        CONTINUE
C
        WRITE(6,1001)   NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK

        WRITE(6,1445)
1445    FORMAT(20X,80H  SUM-CHECK OF ALL OF THE FORCES ACTING ON EACH FLOOR
     2  AND THE TIME OF OCCURRENCE  )
        NN=21
        DO 845 N=2,22
        NN=NN-1
845     WRITE(6,1345)   NN,ABSSR(N),TABSSR(N)
C       CONTINUE
        WRITE(6,1002)
        WRITE(6,1450)
1450    FORMAT(15X,90H  SUM-CHECK OF THE BENDING MOMENTS AT EACH  'JOINT(1,
     2  M1'  AND THE TIME OF OCCURRENCE  )
        NN = 21
        DO 850 N=2,22
        NN = NN - 1
850     WRITE(6,1345)   NN,(ABSBMT(N,M),TABSBM(N,M),M=2,3)
C       CONTINUE
        WRITE(6,1001)   NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK
        WRITE(6,1460)
1460    FORMAT(10X,100H  OVERTURNING MOMENTS IN 'INCH LBS' AND THE TIME OF
     2  OCCURRENCE .. OTMMX,TMXOTM,  OTMMN,TMNOTM  )
        NN=21
        DO 855 N=2,22
        NN=NN-1
855     WRITE(6,1345)   NN,OTMMX(N),TMXOTM(N),OTMMN(N),TMNOTM(N)
C       CONTINUE
        WRITE(6,1002)
        WRITE(6,1465)
1465    FORMAT(42H  (DUCTIL(N,M,J),J=1,4),M=2,3)  ,GIBERSON  )
        NN = 21
        DO 860 N=2,22
        NN = NN - 1
860     WRITE(6,1313)   NN,((DUCTIL(N,M,J),J=1,4),M=2,3)
C       CONTINUE
        WRITE(6,1001)   NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK
        WRITE(6,1470)
1470    FORMAT(40X,65H  MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN IN
     2  CHES  )
        WRITE(6,1480)
1480    FORMAT(132H  -16 -15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4
     2  -3 -2 -1  0  1  2  3  4  5  6  7  8  9 10 11 12 13
     3  14  15  16  )
1485    FORMAT(3313X,1H+)
        NN=21
        DO 610 N=2,21
        NN=NN-1
        LINE(65)=LLF
        I=(UMIN(N).LE.-26.0)   GO TO 605
        I=(FIX(4.0+UMIN(N)  -0.5) + 65
        LINE(I)=LLD
605     I=(UMAX(N).CT.16.0)   GO TO 608
        J=IFIX(4.04UMAX(N) + 0.5) + 65
        LINE(J)=LLD
608     DO 610 N=2,21
        NN=NN-1
        WRITE(6,1490)   NN,LINE
1490    FORMAT(1X,I2,129A1)
        LINE(I)=LLB
610     LINE(J)=LLB
        CONTINUE
        LINE(65)=LLF
        WRITE(6,1495)   LGD,LINE
1495    FORMAT(1X,2I,129A1)
        LINE(65)=LLF
        WRITE(6,1485)
C
        WRITE(6,1002)
```

```
1497  WRITE(6,1497)                                                      GONE 281
      FORMAT(45X,18HSTORY UMIN-INCHES, 10X, 12H UMAX-INCHES        )      GONE 282
      NN=21                                                              GONE 283
      DO 611 N=2,21                                                      GONE 284
      NN=NN-1                                                            GONE 285
      WRITE(6,1498)  NN,UMIN(N),UMAX(N)                                  GONE 286
1498  FORMAT(47X,I2, 2X,F12.6,10X,F12.6  )                               GONE 287
611   CONTINUE                                                           GONE 288
C                                                                        GONE 289
                                                                         GONE 290
1500  WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                    GONE 291
      WRITE(6,1500)                                                      GONE 292
      FORMAT(20X,78H  COLUMN DUCTILITY FACTORS,     X = EXT COL,  I =     GONE 293
     2NT COL, GIBERSON              ,//)                                 GONE 294
1520  WRITE(6,1520)                                                      GONE 295
      FORMAT(            132H  0                 1                        GONE 296
     2                                             4                     GONE 297
     3                        6                                    5     GONE 298
     4                                                              )    GONE 299
1525  WRITE(6,1525)                                                      GONE 300
      FORMAT(3X,1H+,6(19X,1H+)  )                                        GONE 301
      NN=21                                                              GONE 302
      DO 620 N=2,22                                                      GONE 303
      NN=NN-1                                                            GONE 304
      IF(N.EQ.2)  GO TO 616                                              GONE 305
      LINE(21)=LLP                                                       GONE 306
      IF(DUCTIL(N,2,2).GT.6.4)  GO TO 612                                GONE 307
      I= DUCTIL(N,2,2)*20.0 +1.5                                         GONE 308
      LINE(I)=LLX                                                        GONE 309
612   IF(DUCTIL(N,3,2).GT.6.4)  GO TO 615                                GONE 310
      J= DUCTIL(N,3,2)*20.0 +1.5                                         GONE 311
      LINE(J)=LLI                                                        GONE 312
      IF(N.EQ.22)   GO TO 522                                            GONE 313
615   WRITE(6,1490) NN,LINE                                             GONE 314
      LINE(I)=LLB                                                        GONE 315
      LINE(J)=LLB                                                        GONE 316
616   LINE(21)=LLP                                                       GONE 317
      IF(DUCTIL(N,2,4).GT.6.4)  GO TO 618                                GONE 318
      I= DUCTIL(N,2,4)*20.0 +1.5                                         GONE 319
      LINE(I)=LLX                                                        GONE 320
618   IF(DUCTIL(N,3,4).GT.6.4)  GO TO 619                                GONE 321
      J= DUCTIL(N,3,4)*20.0 +1.5                                         GONE 322
      LINE(J)=LLI                                                        GONE 323
619   WRITE(6,1490) NN,LINE                                             GONE 324
      LINE(I)=LLB                                                        GONE 325
      LINE(J)=LLB                                                        GONE 326
620   CONTINUE                                                           GONE 327
622   WRITE(6,1495) LGD,LINE                                             GONE 328
      LINE(I) = LLB                                                      GONE 329
      LINE(J) = LLB                                                      GONE 330
      LINE(21)=LLB                                                       GONE 331
      WRITE(6,1525)                                                      GONE 332
      WRITE(6,1520)                                                      GONE 333
C                                                                        GONE 334
1528  WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                    GONE 335
      WRITE(6,1528)                                                      GONE 336
      FORMAT(14H MAXIMUM VALUE OF  COLUMN DUCTILITY FACTORS BETWEEN ADJAC GONE 337
     2CENT FLOORS,    X = EXT COL,  I = INT COL,  GIBERSON               GONE 338
     )                                                                   GONE 339
      WRITE(6,526)                                                       GONE 340
1526  FORMAT(3X,1H+,6(19X,1H+),//)                                       GONE 341
      NN=21                                                              GONE 342
      DO 660 N=2,21                                                      GONE 343
      NN=NN-1                                                            GONE 344
      N1=N+1                                                             GONE 345
      LINE(21)=LLP                                                       GONE 346
      IF(DUCTIL(N,2,4).GT.DUCTIL(N1,2,2))  GO TO 635                     GONE 347
      I=(DUCTIL(N,2,4)*20.0 ) + 1.5                                      GONE 348
      GO TO 640                                                          GONE 349
635   I=(DUCTIL(N1,2,2)*20.0) + 1.5                                      GONE 350
640   IF(DUCTIL(N,3,4).GT.DUCTIL(N1,3,2))  GO TO 645                     GONE 350
      J=(DUCTIL(N1,3,2)*20.0) + 1.5

      GO TO 650                                                          GCNE 351
645   J=(DUCTIL(N,,3,4)*20.0) + 1.5                                      GCNE 352
650   LINE(I)=LLX                                                        GONE 353
      LINE(J)=LLI                                                        GONE 354
      WRITE(6,1540)  NN,LINE                                             GONE 355
      LINE(I)=LLB                                                        GONE 356
      LINE(J)=LLB                                                        GONE 357
660   CONTINUE                                                           GONE 358
      WRITE(6,1525)                                                      GONE 359
      WRITE(6,1520)                                                      GONE 360
C                                                                        GONE 361
      WRITE(6,1001)       NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK              GONE 362
      WRITE(6,1530)                                                      GONE 363
1530  FORMAT(132HEITERIOR AND INTERIOR GIRDER DUCTILITY FACTORS,  X = EX GONE 364
     2T GIR AT EXT COL,  . = EXT GIR AT INT COL,  I = INT  GIR AT INT COL.GONE 365
     3 GIBERSON    )                                                     GONE 366
      WRITE(6,1520)                                                      GONE 367
      WRITE(6,1526)                                                      GONE 368
      NN=21                                                              GONE 369
      DO 700 N=2,22                                                      GONE 370
      NN=NN-1                                                            GONE 371
      LINE(21)=LLP                                                       GONE 372
      IF(DUCTIL(N,,3).GT.6.4)  GO TO 630                                 GONE 373
      I= DUCTIL(N,2,3)*20.0 + 1.5                                        GONE 374
      LINE(I)=LLX                                                        GONE 375
680   IF(DUCTIL(N,,1).GT.6.4)  GO TO 630                                 GONE 376
      J= DUCTIL(N,,1)*20.0 + 1.5                                         GONE 377
      LINE(J)=LLD                                                        GONE 378
690   IF(DUCTIL(N,3,3).GT.6.4)  GO TO 695                                GONE 379
      K= DUCTIL(N,3,3)*20.0 + 1.5                                        GONE 380
      LINE(K)=LLI                                                        GONE 381
695   WRITE(6,1540) NN,LINE                                             GONE 382
1540  FORMAT(1X,I2,129A1,//)                                            GONE 383
      LINE(I)=LLB                                                        GONE 384
      LINE(J)=LLB                                                        GONE 385
      LINE(K)=LLB                                                        GONE 386
700   CONTINUE                                                           GONE 387
      LINE(21)=LLP                                                       GONE 388
      WRITE(6,1550)    LGD,LINE                                          GONE 389
1550  FORMAT(1X,2A1,129A1,//)                                           GONE 390
      WRITE(6,1525)                                                      GONE 391
      WRITE(6,1520)                                                      GONE 392
      LINE(21)=LLB                                                       GONE 393
C                                                                        GONE 394
      RETURN                                                             GONE 395
      END                                                                GONE 396
```

```
$IBFTC MODE    DECK **MODAL ANALYSIS - DBLE PREC**              Q56 MODE    1
      SUBROUTINE MODE                                              MODE    2
C*******************************************************************MODE    3
C***************************************************************MODE    4
C                                                                  MODE    5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES MODE  6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION. MODE 7
C                                                                  MODE    8
C     THE PURPOSE OF SUBROUTINE 'MODE' IS TO MAKE A MODAL ANALYSIS MODE   9
C     AFTER EACH TIME INCREMENT OF THE HORIZONTAL DISPLACEMENT VECTOR MODE 10
C     (20X1) WITH RESPECT TO THE MODES OF THE LINEAR STRUCTURE WITH  MODE 11
C     FLEXIBLE GIRDERS. THE EIGENVECTOR CORRESPONDING TO THE LARGEST MODE 12
C     EIGENVALUE IS READ IN FIRST, THEY DIAGONALIZE THE TRANSFORMED MODE 13
C     STIFFNESS MATRIX M**(-1/2) * K * M**(-1/2) AND ARE NORMALIZED MODE 14
C     SUCH THAT (EIGENVECTOR TRANSPOSED)*(EIGENVECTOR) = I .        MODE 15
C                                                                  MODE   16
C                    D O U B L E   P R E C I S I O N               MODE   17
C     DOUBLE PRECISION PREFIXES ARE NOT GENERALLY USED HEREIN.     MODE   18
C                                                                  MODE   19
C     AN OPTIONAL DUMMY SUBROUTINE 'MODE' IS AVAILABLE WHEN A MODAL MODE  20
C     ANALYSIS IS NOT DESIRED.                                     MODE   21
C                                                                  MODE   22
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE MODE 23
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- MODE 24
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                    MODE   25
C                    LAST REVISION - 4 DECEMBER 1966              MODE    26
C                                                                  MODE   27
C*******************************************************************MODE   28
C******************************************************************MODE    29
      DOUBLE PRECISION         AIDA(20), ZMHAF(21),EVALU(20),      MODE   30
     2 EVECT(20,20),CKVECT(20),UM(4,20),   UUM(4,20), TUUM(4),     MODE   31
     3 AMEV(20,20),DMEV(20,20),EVTM(20,20),       ZZM,ZMH,ABSAMV   MODE   32
     4 ,DBU,DBU                                                    MODE   33
C                                                                  MODE   34
      DIMENSION SEVECT(20)                                         MODE   35
C                                                                  MODE   36
      COMMON/MNVCGN/NAMEST(10),NAMEGK(13),BMA(22,3,4),BMB.22,3,4), MODE   37
     2 GA(22,3),CA(22,3),FT(22),ZM(22),H(22),A(5),S(8),P,F,ZETA,FUNOFQ, MODE 38
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                MODE   39
C                                                                  MODE   40
      COMMON/COMMOD/ DBU(22),DBUU(22)                              MODE   41
C                                                                  MODE   42
C     INITIALIZE THE MODAL ANALYSIS MATRICES (TO STATEMENT 669) ...MODE   43
C                                                                  MODE   44
      IF(LL1.EQ.1)  GO TO 669                                      MODE   45
      LL1 = 1                                                      MODE   46
      DO 605  N=1,20                                               MODE   47
      ZZM= DBLE(1.0/ZM(N+1))                                       MODE   48
      ZMHAF(N) = FDSQ(ZZM)                                         MODE   49
      NN = 21-N                                                    MODE   50
C                                                                  MODE   51
C     READ IN EIGENVALUES AND EIGENVECTORS OF LINEAR STRUCTURE ....MODE   52
C                                                                  MODE   53
      READ(5,1210) EVAL                                            MODE   54
      EVALU(NN) = DBLE(EVAL)                                       MODE   55
      READ(5,1210) (SEVECT(J),J=1,20)                              MODE   56
 1210 FORMAT(6D12.8X)                                              MODE   57
      DO 603 J=1,20                                                MODE   58
      EVECT(J,NN) = DBLE(SEVECT(J))                                MODE   59
  603 CONTINUE                                                     MODE   60
  605 CONTINUE                                                     MODE   61
      DO 640 I=1,20                                                MODE   62
      ZMH = ZMHAF(I)                                               MODE   63
      DO 630 J=1,20                                                MODE   64
      AMEV(I,J) = ZMH*EVECT(I,J)                                   MODE   65
  630 CONTINUE                                                     MODE   66
  640 CONTINUE                                                     MODE   67
C                                                                  MODE   68
C     WRITE OUT SELECTED (ARBITRARY) MODES AS A CHECK ............MODE    69
C                                                                  MODE   70
```

```
C                                                                  MODE   71
      DO 645 J=1,17,8                                              MODE   72
      ABSAMV= 0.0D+00                                              MODE   73
      DO 643 K=1,20                                                MODE   74
      IF(ABSAMV.GE.DABS(AMEV(K,J)))  GO TO 642                     MODE   75
      ABSAMV =DABS(AMEV(K,J))                                      MODE   76
  642 CONTINUE                                                     MODE   77
  643 CONTINUE                                                     MODE   78
      DO 644 K=1,20                                                MODE   79
      CKVECT(K) = AMEV(K,J)/ABSAMV                                 MODE   80
  644 CONTINUE                                                     MODE   81
      WRITE(6,1215) J,(K,CKVECT(K),K=1,20)                         MODE   82
 1215 FORMAT( 9H1    MODE    ,I4,/,(5X,I5,5X,1PE15.7) ;            MODE   83
  645 CONTINUE                                                     MODE   84
C                                                                  MODE   85
C.... EVALUATE (TRANSPOSED EIGENVECTOR)*(MASS**(1/2)) ............MODE    86
C                                                                  MODE   87
      DO 660  J=1,20                                               MODE   88
      ZMH =  ZMHAF(J)                                              MODE   89
      DO 650  I=1,20                                               MODE   90
      EVTM(I,J) = EVECT(J,I)/ZMH                                   MODE   91
  650 CONTINUE                                                     MODE   92
  660 CONTINUE                                                     MODE   93
C                                                                  MODE   94
C.... EVALUATE THE MODAL PARTICIPATION FACTOR 'AIDA' .............MODE    95
C                                                                  MODE   96
  669 CONTINUE                                                     MODE   97
      DO 680  I=1,20                                               MODE   98
      AIDA(I) = 0.0D+00                                            MODE   99
      DO 670  J=1,20                                               MODE  100
      AIDA(I) = AIDA(I) + EVTM(I,J)*DBU(J+1)                       MODE  101
  670 CONTINUE                                                     MODE  102
  680 CONTINUE                                                     MODE  103
      WRITE(6,1220) AIDA                                           MODE  104
 1220 FORMAT(7H0 AIDA   ,/,(4X,1PE17.7))                          MODE  105
C                                                                  MODE  106
C.... EVALUATE THE MODAL CONTRIBUTIONS TO THE DISPLACEMENTS AND ..MODE   107
C     INTER-FLOOR DISPLACEMENTS OF SELECTED (ARBITRARY) FLOORS    MODE  108
C                                                                  MODE  109
      JN = 0                                                       MODE  110
      DO 690 I = 2,17,5                                            MODE  111
      JN=JN+1                                                      MODE  112
      TUM(JN) = 0.0D+00                                            MODE  113
      TUUM(JN) = 0.0D+00                                           MODE  114
      DO 685  J=1,20                                               MODE  115
      UM(JN,J)= AMEV(I,J)*AIDA(J)                                  MODE  116
      UUM(JN,J) = (AMEV(I+1,J) - AMEV(I,J))*AIDA(J)                MODE  117
      TUM(JN) = TUM(JN) + UM(JN,J)                                 MODE  118
      TUUM(JN) = TUUM(JN) + UUM(JN,J)                              MODE  119
  685 CONTINUE                                                     MODE  120
  690 CONTINUE                                                     MODE  121
      WRITE(6,1230) (N,(UM(JN,N),  UUM(JN,N),JN=1,4),N=1,20)       MODE  122
 1230 FORMAT(( 2X,I2, 4(2X,1P2E15.7) ))                           MODE  123
      N = 21                                                       MODE  124
      WRITE(6,1230) N,(TUM(JN), TUUM(JN),JN=1,4)                   MODE  125
      WRITE(6,1230) N, (DBU(J),DBUU(J),J=3,18,5)                   MODE  126
      WRITE(6,1001)                                                MODE  127
 1001 FORMAT(1H1)                                                  MODE  128
      RETURN                                                       MODE  129
      END                                                          MODE  130
```

```
$IBFTC MODE    DECK *THIS IS A DUMMY SUBROUTINE*              Q56 DUMY   1
       SUBROUTINE MODE                                           DUMY   2
C*********************************************************************DUMY  3
C*********************************************************************DUMY 4
C                                                                 DUMY   5
C  THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES; DUMY   6
C  OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.DUMY  7
C                                                                 DUMY   8
C  DUMMY SUBROUTINE - USED WHEN A MODAL ANALYSIS IS -NOT- WANTED).DUMY   9
C                                                                 DUMY  10
C  THIS PROGRAM INCLUDING ALL SUBROUTINES (AS WRITTEN BY MELBOURNE DUMY 11
C  F. GIBERSCN, DEPARTMENT OF APPLIED MECHNICS, CALIFORNIA INSTI- DUMY  12
C  TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                      DUMY  13
C                                                                 DUMY  14
C*********************************************************************DUMY 15
C                                                                 DUMY  16
       RETURN                                                     DUMY  17
       END                                                        DUMY  18


$IBFTC ZERO    DECK *INITIALIZING BY 'BLOCK DATA'**           Q56 ZERO   1
C*********************************************************************ZERO  2
C*********************************************************************ZERO  3
C                                                                 ZERO   4
C  THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES  ZERO   5
C  OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.ZERO  6
C                                                                 ZERO   7
C  THE PURPOSE OF 'BLOCK DATA' IS TO PLACE PRESCRIBED NON-ZERO    ZERO   8
C  VALUES IN CERTAIN MEMORY CELLS AT THE TIME OF LOADING.         ZERO   9
C                                                                 ZERO  10
C  THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE ZERO 11
C  F. GIBERSCN, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- ZERO 12
C  TUTE OF TECHNOLOGY, PASADENA. CALIFORNIA.                      ZERO  13
C             LAST REVISION - 4 DECEMBER 1966                     ZERO  14
C                                                                 ZERO  15
C*********************************************************************ZERO 16
C                                                                 ZERO  17
C  BLOCK DATA                                                     ZERO  18
C                                                                 ZERO  19
       COMMON/WKGNIN/LLX,LLI,LLB,LLS,LLD,LLP, LGD(2),LINE(129)    ZERO  20
C                                                                 ZERO  21
       COMMON/ANWCGN/NAMEST(10),NAM=QK(13),BMA(22,3,4),BMB(22,3,4), ZERO 22
      2 GA(22,3),CA(22,3),HT(22),ZM(22),AI5),S(8),P,F,ZETA,FLNOFQ, ZERO 23
      3 ZL,AL,TIMEND,ALANDA,BLAMDA,(ELPL,TIMI,ARTEST             ZERO  24
C                                                                 ZERO  25
       DATA LL,LLI,LLB,LLS,LLD,LLP/1HX,1HI,1H,,1H*,1H-,1H+/       ZERO  26
      2 LGD(1), LGD(2)/1HG,1HD/,  (LINE(I),I=1,129) /129*1H /     ZERO  27
      3 (((BMAIN,M,J),N=1,22 ),M=1,3)/264*1.0/                   ZERO  28
       END                                                        ZERO  29
```

THE FOLLOWING IBMAP 'TAPE CONTROL' PROGRAM IS USED BY PROGRAM '456'.

```
$IBMAP  U20     DECK                                          U20  1
.UN20.  ENTRY   .UN20.                                        U20  2
X       PZE     X                                             U20  3
.UN20.  FILE    RESULT TAPE,B.1),HOLD,BLK=256,BIN,MOUNT,OUTPUT U20 4
        END                                                  U20  5
```

REFERENCES

1. Clough, R. W., K. L. Benuska and T. Y. Lin & Associates. FHA Study of Seismic Design Criteria for High-Rise Buildings, (HUD TS-3), Federal Housing Administration, Washington, D.C., (August 1966).

2. T. Y. Lin & Associates. A Computer Program to Analyze the Dynamic Response of High Rise Buildings to Nuclear Blast Loading, Vol. I: Elastic Analysis, Vol. II: Dynamic Nonlinear Response Analysis. (OCD-OS-63-44), Office of the Secretary of the Army, Department of the Army, Washington, D.C., (October 1963).

3. Sano, R., and K. Muto. Earthquake and Typhoon Resistant Construction, (Japanese), Tokyo, Japan, 1935.

4. Suyehiro, K. "Engineering Seismology," ASCE, Proceedings, v.58, no.5, pt.2, (May 1932), 110p.

5. Biot, M. A. "Theory of Elastic Systems Vibrating Under Transient Impulse With an Application to Earthquake Proof Buildings," National Academy of Science, Proceedings, v.19 (1933) 262-268.

6. Biot, M. A. "Analytical and Experimental Methods in Engineering Seismology," ASCE, Transactions, v.108 (1943) 262-268.

7. Freeman, J. R. Earthquake Damage and Earthquake Insurance, N.Y., McGraw-Hill, 1932.

8. Hudson, D. E. "Response Spectrum Techniques in Engineering Seismology." In World Conference on Earthquake Engineering, Berkeley, California, 1956, Proceedings. San Francisco, Earthquake Engineering Research Institute, 1956, Paper No. 4.

9. Housner, G. W. "Vibration of Structures Induced by Seismic Waves, Part I: Earthquakes," (Chapter 50). In Shock and Vibration Handbook, v.3. Edited by Cyril M. Harris and Charles E. Crede. N.Y., McGraw-Hill, 1961.

10. Caughey, T. K. "Random Excitation of a System with Bilinear Hysteresis," Journal of Applied Mechanics, v.27, no.4 (December 1960) 649-652.

11. Berg, G. V. and S. S. Thomaides. "Energy Consumption by Structures in Strong Motion Earthquakes." In World Conference on Earthquake Engineering, 2d, Tokyo and Kyoto, Japan 1960, Proceedings. Tokyo, Science Council of Japan, 1960.

12. Penzien, J. "Dynamic Response of Elasto-Plastic Frames," ASCE, Structural Division, Journal, v.86, no.ST7 (July 1960) 81-94.

13. Iwan, W. D. The Dynamic Response of Bilinear Hysteretic Systems, Earthquake Engineering Research Laboratory (Ph.D. Thesis), California Institute of Technology, 1961.

14. Jennings, Paul C. Response of Simple Yielding Structures to Earthquake Excitation, Earthquake Engineering Research Laboratory (Ph. D. Thesis),California Institute of Technology, 1963.

15. Chelapati, C. V. Response of Simple Inelastic Systems to Ground Motion, Ph.D. Thesis, University of Illinois, Urbana, Illinois, 1962.

16. Lutes, L. D. Stationary Random Response of Bilinear Hysteretic Systems, Earthquake Engineering Research Laboratory (Ph.D. Thesis), California Institute of Technology, 1967.

17. Jacobsen, L. S. and R. S. Ayre. "Experimentally Determined Dynamic Shears in a Fifteen Story Model," Seismological Society of America, Bulletin, v.28, no.4 (October 1938) 269-311.

18. Tung, T. P. and N. M. Newmark. "Numerical Analysis of Earthquake Response of a Tall Structure," Seismological Society of America, Bulletin, v.45, no.4 (October 1955) 269-278.

19. Jennings, Richard L. The Response of Multistoried Structures to Strong Ground Motion, M.S. Thesis, University of Illinois, Urbana, Illinois, 1958.

20. Jennings, Richard L. and N. M. Newmark. "Elastic Response of Multi-story Shear Beam Type Structures Subjected to Strong Ground Motion." In World Conference on Earthquake Engineering, 2d, Tokyo and Kyoto, Japan, 1960, Proceedings, Tokyo, Science Council of Japan, 1960, v.2, 699-718.

21. Clough, R. W. "On the Importance of Higher Modes of Vibration in the Earthquake Response of a Tall Building," Seismological Society of America, Bulletin, v.45, no.4 (October 1955) 289-301.

22. Blume, J. A. "Structural Dynamics in Earthquake-Resistant Design," ASCE, Structural Division, Journal, v.84, no.ST4 (July 1958) Proc. Paper 1695.

23. Blume, J. A., L. Corning and N. M. Newmark. Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motions. Chicago, Illinois, Portland Cement Association, 1961.

24. Goldberg, J. E., J. L. Bogdanoff and Z. L. Moh. "Forced Vibration and Natural Periods of Tall Building Frames," Seismological Society of America, Bulletin, v.49, no.1 (1959) 33-48.

25. Penzien, J. "Elasto-Plastic Response of Idealized Multi-Story Structures." In World Conference on Earthquake Engineering, 2d, Tokyo and Kyoto, Japan, 1960, Proceedings, Tokyo, Science Council of Japan, 1960, v.2, Session II, 739-760.

26. Bustamante, J. I. "Seismic Shears and Overturning Moments in Buildings." In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.3, Session IV, 144-160.

27. Umemura, H., Y. Osawa and A. Shibato. "Study on Shearing Forces in Structures Caused by Medium Earthquakes Recorded in Japan." In World Conference on Earthquake Engineering, 3d, 1965, Auckland and Wellington, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.3, Session IV, 539-571.

28. Berg, G. V. "Response of Multi-Story Structures to Earthquakes," ASCE, Engineering Mechanics Division, Journal, v.87, no.EM2 (April 1961) 1-16.

29. Muto, K. "Recent Trends in High Rise Building Design in Japan." (Special Address) In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.1, 119-146.

30. Westergaard, H. M. "Earthquake Shock Transmission in Tall Buildings," Engineering News-Record, v.111 (November 30, 1933) 654-656.

31. Jacobsen, L. S. "Vibrational Transfer from Shear Buildings to Ground," ASCE, Engineering Mechanics Division, Journal, v.90, no.EM3 (June 1964) 21-38.

32. Kanai, K. "Some New Problems of Seismic Vibrations of a Structure." In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.2, Session II, 260-275.

33. Bycroft, G. N. "Yield Displacements in Multi-Story Aseismic Design," Seismological Society of America, Bulletin, v.50, no.3 (July 1960) 441-453.

34. Bycroft, G. N. "Effect of Stiffness Taper in Aseismic Design," Seismological Society of America, Bulletin, v.50, no.4 (October 1960) 537-552.

35. Murphy, M. J., G. N. Bycroft and L. W. Harrison. "Electrical Analog for Earthquake Shear Stresses in a Multi-Story Building." In World Conference on Earthquake Engineering, Berkeley, California, USA, 1956, Proceedings, San Francisco, Earthquake Engineering Research Institute, 1956, Paper No.9.

36. Clough, R. W. "Earthquake Analysis by Response Spectrum Superposition," Seismological Society of America, Bulletin, v.52, no.3 (July 1962) 647-660.

37. Clough, R. W., I. P. King and E. L. Wilson. "Large Capacity Multistory Frame Analysis Programs," ASCE, Structural Division, Journal, v.89, no.ST4 (August 1963) 179-204.

38. Clough, R. W., I. P. King and E. L. Wilson. "Structural Analysis of Multistory Buildings," ASCE, Structural Division, Journal, v.90, no.ST3 (June 1964) 19-34.

39. Clough, R. W., K. L. Benuska and E. L. Wilson. "Inelastic Earthquake Response of Tall Buildings." In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.2, Session II, 68-89.

40. Clough, R. W. and E. L. Wilson. "Dynamic Response by Step-by-Step Matrix Analysis," Symposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, 1962.

41. Richard, R. M. and J. E. Goldberg. "Analysis of Nonlinear Structures, Force Method," ASCE, Structural Division, Journal, v.91, no.ST6 (December 1965) 33-48.

42. Melin, J. "Numerical Integration by the Beta Method." American Society of Civil Engineers, Conference on Electronic Computation, 1st, 1958, Kansas City, Missouri, 227-236.

43. Newmark, N. M. "A Method of Computation for Structural Dynamics," ASCE, Engineering Mechanics Division, Journal, v.85, no. EM3 (July 1959) 67-94.

44. O'Kelley, M. E. J. Vibration of Viscously Damped Linear Dynamic Systems, Dynamics Laboratory (Ph.D. Thesis), California Institute of Technology, 1964.

Several textbooks on the design of indeterminant structures such as those modeled in this report are available. Among them are the following:

45. Ferguson, P. M. Reinforced Concrete Fundamentals, (1st ed., 1958), (2d ed., rev. 1965), N.Y., J. Wiley.

46. Hurty, W. C. and M. F. Rubenstein. Dynamics of Structures, Englewood Cliffs, N.J., Prentice-Hall, 1964.

47. Timoshenko, S. and D. H. Young. Theory of Structures, (2nd ed.), N.Y., McGraw-Hill, 1965.

Two bibliographies of research work covering the entire earthquake engineering field are noted:

48. Hollis, E. P. Bibliography of Engineering Seismology (2nd ed.), San Francisco, Earthquake Engineering Research Institute, 1966.

49. Rosenblueth, E. "Earthquake Resistant Design," Applied Mechanics Review, v.14, no.12 (December 1961) 923-926.