ALLOCATION AND COMPUTATION IN RAIL NETWORKS: A BINARY CONFLICTS ASCENDING PRICE MECHANISM (BICAP) FOR THE DECENTRALIZED ALLOCATION OF THE RIGHT TO USE RAILROAD TRACKS

Thesis by

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Abstract:

The thesis addresses problems that surfaced as part of the proposal to deregulate access to railroads in Sweden. Skepticism exists about the feasibility and efficiency of competitive processes for access to the publicly owned track network. The skepticism is related to the capacity of any competitive process to solve certain technical problems that stem from performance criteria (efficiency, safety), informational requirements (values of track access are initially known only to the operators) and computational requirements. In the thesis, auction-like processes are developed for allocating the rights to operate trains on the track and for procuring the necessary computational effort to solve a related optimization problem inherent in the track auction process. The processes are tested in a series of human subject laboratory experiments. The data are examined to determine the degree to which the evaluative criteria are met and the degree to which the performance of the processes are consistent with the behavioral principles on which they are based.

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CHAPTER 1. Introduction and Overview

1.1 Introduction

The research in this thesis is concerned with identifying and testing a process for the allocation of access to a railroad track network. Specific economic and technical issues are identified that are of interest in current debates concerning railroad allocation in Sweden. These issues provide a very specific application framework that challenges any institutional design effort and guides the research of this thesis. The thesis rests on the assumption that progress in understanding institutional processes to solve general problems can best be attained by the study of specific examples, together with the related study of the institutions and processes that are naturally suggested by the issues that the examples force. The hope is that issues emerging from the examination of the specific application framework will facilitate a deeper understanding of theory and institutions than will be achieved by a study of general issues alone.

This chapter serves as an introduction to a specific allocation problem considered in the thesis: the problem of decentralized allocation of access rights on a railroad. The chapter is divided into four sections. The next section is an introduction to the applied problem. That section is followed by a brief discussion of other studies that contain mechanisms that might be applied to the same problem. Those studies contain many elements that are used in the institutions that are proposed in this study. Hopefully, the institutional process and theory discussed in this thesis will ultimately contribute towards a more comprehensive understanding of resource allocation processes and general theories regarding how these processes function. The third section is a discussion of a general background problem of the allocation of uses of shared resources that sets the stage for all of these studies and a great many more. The final section is a detailed outline of the chapters of the thesis.

1.2 Decentralization of Swedish Railways

In 1992, Swedish Parliament voted to privatize and deregulate railroad operations[1]. The agency responsible for rail operations, Banverket, will retain ownership of the track. The Parliament ordered Banverket to identify a mechanism that would tend to allocate access of track to those who valued it the most. At this point researchers are investigating different processes for allocating access to interested agents. The existence of such a mechanism and the likelihood that such a mechanism would be superior to systems of administratively based rules is unknown and a point of political controversy[2]. The previous mechanism for allocation of the Swedish railways

1 This was part of the budgetary bill for 1992, prop.1991/92:100, sup. 7.

2 In a transportation committee report (SOU1993:13. ³Ökad konkurrens på järnvägen" [translated: Increased competition in the railway industry]), some criticisms of market based systems include claims that "Allocation models based on price mechanisms have not yet been developed in the sector," and that "The absence of any international experience is seen as the strongest argument against an allocation model based on pricing systems." (Translation provided by J.E. Nilsson, personal correspondence). A consulting report prepared for the Swedish transportation committee by Coopers

involved a system of priorities for trains. There is a desire by opponents of decentralization to retain the priority scheme, whereas proponents of decentralization would like to discard it.

Major issues in this debate are outlined in Chapter 3. These issues set forth the challenge for those that would offer any decentralized process as a replacement for the current system of priorities. A presumption is implicit in the arguments of decentralization critics that no set of decentralized institutions can perform the tasks they pose. It is at this joining of arguments that laboratory methods become important. Laboratory evidence for the existence of such an allocation mechanism would be useful in the following sense: If it can be shown that decentralized mechanisms exist for laboratory testbeds that solve some of the types of technical and economic problems inherent in the railway allocation problem, then such decentralized options for solving the Swedish problem can not be dismissed immediately as being impossible or impractical, without further study. Arguments that decentralized mechanisms for rail allocation simply could not possibly exist because of such technical or economic problems could be refuted using the evidence from the laboratory demonstration. This would move the argument in the following sense: by showing that decentralized allocation is possible in the laboratory, it can no

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and Lybrand is also mildly critical of pricing. Coopers and Lybrand begin by explaining that auction based market pricing tends to result in allocation to the user who values the item the most, but "... auction can not be used for railway capacity since there are no independent units of capacity to bid for." (p.293) The claim is that the difficulty is commodity space related: "These train paths can not be treated as independent units, since they are not interchangeable, and depend on the specification of all other paths in the integrated timetable. There is therefore no common unit of capacity on a mixed-use railway which can be allocated to owners, priced and traded among a number of buyers and sellers." (p.291) longer be claimed that either (i) no example of decentralized allocation process for railroad scheduling problems exists, or that (ii) decentralized allocation is impossible on general principles (in which case it would also be impossible in the laboratory). This would force opponents of decentralization to more closely consider either the possibility of decentralized allocation or the issues preventing it. The goal of the research is not to solve the Swedish rail problem in the laboratory, but instead to simply move the argument - to show that decentralized allocation processes are not necessarily impossible and are worth additional study.

This thesis provides such a demonstration. The demonstration has two major objectives. The first objective is to show *existence*: an allocation mechanism can be found that performs as desired in the laboratory testbeds. The second objective is to show *design consistency*: that the mechanism works according to behavioral principles that are consistent with the design.

The substance of Chapter 5 is to describe such a process. However, the question of existence should address something more than one example of a single mechanism. The more robust the example, the more the burden of proof is pushed to critics of decentralization to explain or expand upon their claims. In addition, a more robust example can serve as a starting point for researchers who wish to develop a process for actual use in the field. If a decentralized process exists, several robustness issues become relevant: Is the example capable of being generalized to more complex environments or do inherent technical limitations exist? Does the mechanism always, usually, or seldom produce efficient outcomes in the testbed environments? If efficient allocations do not always occur, can conjectures be formed regarding the causes of inefficient outcomes?

The question of robustness also prompts an investigation of design consistency. If the agent behavior observed in an institutional process is consistent with theoretical models upon which the design of the institution is based, then robustness might be expected to extend to parameter values not actually tested in the experiments. For instance, if efficient allocations occur when the mechanism reaches an appropriate type of equilibrium (e.g., competitive equilibrium, Nash equilibrium, dominant strategy equilibrium), then a test for design consistency would include a test to see if the mechanism reaches these equilibria. Equilibrium concepts are introduced in Chapter 2 and Chapter 5. Tests are conducted in Chapters 6 and 8.

Another important robustness consideration involves the problem of size. The idea is that although the processes are not tested in environments having as many tracks or trains as the Swedish rail system, it is in principle possible to design processes that are somewhat robust to increases in scale. Chapter 4 partially addresses the size issue by showing how the structure of the feasible set might allow a parallel computing approach to scheduling. Chapter 7 extends the discussion to a process that involves procured computation.

The conclusion of the thesis is that an institution exists that, at least in the laboratory, performs as required. Efficiency is extremely high (97% average) in the Chapter 6 experiments, and fairly high (above 90% average) in the more complex experiments of Chapter 8. Techniques to deal with computational difficulties, described in Chapter 7, can be seen working, as expected, in the experiments of Chapter 8. Design consistency is very strong in the experiments of Chapter 6. Design consistency in Chapter 8 is strong in some areas but not in others. Problems seem to be associated with agents who have the high value for multiple trains, perhaps identifying a type of "network monopoly" problem. Chapter 9 provides the bulk of concluding remarks, by returning to each of the issues introduced in Chapter 3 and explaining how they were handled in the research.

1.3 Related Experimental Studies

The railroad allocation problem combines an information revelation problem with a special mathematical framework: that of a knapsack problem. Three other experimental studies[3] take a very similar technical approach to allocation problems.

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[3] In particular: allocation of airport slots in Rassenti, Smith, and Bulfin (1982); allocation of "uncertain and unresponsive" resources in Banks, Ledyard, and Porter (1989); and the assignment problem of Olson (1992) and Olson and Porter (1994). There are a large number of other studies that address similar problems of shared resources and also discuss other application frameworks where computer assisted market or auction processes are developed. The AUSM institution developed in Banks, Ledyard, and Porter (1989) was later used by Plott and Porter (1990) to study the allocation of resources on the planned NASA space station. A number of studies involve transportation applications and complementary goods. Grether, Isaac, and Plott (1981) were the first to experimentally examine the possibility of allocation of airport slots through independent markets. Access to resource pipelines

The information revelation problem is the classic private values environment. A brief description of the three studies will be provided, and then important facts relevant for creating a decentralized allocation mechanism for railroads will be discussed.

Rassenti, Smith, and Bulfin (1982) consider a combinatorial sealed bid auction for allocating airport takeoff and landing slots when flights require packages of such slots. In the RSB study, the knapsack problem enters because the mechanism maximizes a set of bids for flights relative to the constraints that flights do not over demand airport slot resources. Banks, Ledyard, and Porter (1989) compare various mechanisms (markets, the Iterated Vickrey Groves (IVG) mechanism, the Adaptive User Selection Mechanism (AUSM)) for allocating resources when agents have preferences over packages of the resources. The knapsack problem enters in the IVG and AUSM mechanisms because agents bid on packages of resources and the mechanisms involve finding a set of bids to accept that maximize bid revenue while satisfying the supply constraints. Olson (1992) and Olson and Porter (1994) consider the *assignment problem*, an allocation problem where each agent receives a single resource from a set of resources. Bidding mechanisms for the assignment problem include the knapsack constraints specifying that each user may be allocated only one of the resources.

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has been considerd by Plott (1988); McCabe, Rassenti, and Smith (1992); and Rassenti, Reynolds and Smith (1994). Plott considers a type of tantonnement, whearas the later papers are computer assisted markets. Ledyard, Porter, and Rangel (1994, forthcoming) are analyzing a set of computer-assisted barter markets actually used to allocate space and other resources on the Cassini space probe mission. Nilsson (1994) is experimenting with iterated Vickrey-Groves allocation mechanisms for access to railways, using methodology and techniques similar to those introduced here.

Unlike the other studies, Olson (1992) considers the theoretical possibilities for non-monetary allocation mechanisms. Several such schemes exist involving "chits" and rough equivalents of ranking or voting. Often they are used within a larger institution where money-based internal markets are not thought to be feasible (such as the case with project groups at NASA). Obviously, one reason agents might prefer these schemes over a more efficient, money based scheme is that the non-monetary processes allow agents to escape monetary payment for the resources being used.

An important insight from Olson is that an incentive compatible, non-monetary scheme for allocating access to railroads, where each agent's access is limited to running one train[4], would, at best, perform like the old priority scheme. Olson shows that the Gibbard-Satterthwait theorem for social choice implies that any non-monetary, incentive compatible allocation mechanism for the assignment problem is equivalent to a *serial dictatorship.*

In serial dictatorship, agents take turns choosing their most preferred allocation until all agents have an assignment of resources. This mechanism is essentially a priority scheme where agents choose in order of their priority ranking. The serial dictator priority scheme differs somewhat from the priority scheme used to allocate the Swedish rail systems: in serial dictatorship agents have priority to choose trains or

4 The one-train constraint could be envisioned as a type of anti-monopoly or fairness constraint. In any event, the constraint is necessary to impose it to make the rail allocation problem into the type of general assignment problem studied by Olson.

other resources; in the second scheme it is the trains which have priority. Like the prioritized rail system, the efficiency of serial dictatorship depends substantially on the particular assignment of priorities. If priorities are assigned correctly, the efficiency can be quite high, even optimal. If priorities are assigned randomly, then the efficiency is generally quite low.

Two general classes of institutions utilizing monetary transfers are *market-based* and *auction-based* mechanisms. In laboratory experiments for shared resources involving knapsack problems, more success has been obtained through auction-based mechanisms than with market-based mechanisms.

In a market-based mechanism, agents would be given some initial endowment of rail resources[5] and would be allowed to trade resources and cash in a set of independent markets. The prices in each market are set by the agents through their bid and ask offers. Banks et al. suggest that independent markets could be inefficient when agents have preferences over packages of resources. In their independent market experiments, goods are strong complements with indivisibilities. Banks et al. purposefully choose preferences such that prices that support the optimal allocation as a competitive equilibrium do not exist. If resources correspond to the right to have a train at a particular section of track and a particular time, a package of several resources might be necessary in order to be able to run a train. Preferences of train

[5] In particular, the endowment of resources that existed under the status quo is typically suggested as a means for obtaining cooperation from all agents.

operators over resources would likely involve a strong complements case such as Banks et al. constructed. In such cases independent markets can not be expected to provide efficient allocations. In a series of experiments, Banks et al. show that market inefficiencies do in fact occur.

In an auction-based mechanism, agents would submit bids on either trains or on packages of resources necessary to run trains, depending upon the rules of the mechanism. The auction outcome is to allocate resources to a set of high bidders, determined through solution of the knapsack problem. The specific auction rules vary in the different experimental studies and will now be considered in a bit more detail.

Banks et al. show that in the same strong complement environments that caused inefficiency in the independent markets case, two auction-based mechanisms produce more efficient allocations. These mechanisms are the Iterative Vickrey-Groves (IVG) and the Adaptive User Selection Mechanism (AUSM). Both of these auction mechanisms are iterative mechanisms in which bidding on packages of resources continues over several stages. Rassenti, Smith, and Bulfin, in a somewhat earlier paper, propose a type of sealed bid auction over uses of airport slots and consider fairly similar types of strong complement environments.

The IVG mechanism is implemented in the Banks et al. experiments as the following tantonnement (i)-(iii): (i) All agents submit bids over all packages of resources. (ii)

The allocation of resources that maximized the sum of the bids is computed along with a set of Vickrey-Groves second-prices, and this information was relayed back to agents. (iii) The auction continues in batches until no one changes their bid. Bids on packages of resources can be raised or lowered. The IVG mechanism has a dominant strategy, incentive compatible equilibria based on a special second-price payment rule. It is in each agent's best interests to bid his or her value for the resource packages because the bid determines only an upper bound on price. The actual price is determined by excluded agents' bids.

The AUSM mechanism is an increasing, first price iterative auction in packages of resources. Agents specify their bids for packages of resources. As bids arrive, the allocation of resources that maximizes the sum of the bids is computed. Once bids exhaust supply, a new bid is either accepted and several old bids rejected, or the new bid is rejected. The auction ends when no agent makes an acceptable bid for a given period of time. The efficiency of the AUSM mechanism is shown to be substantially higher than the efficiency of markets. Banks et al. consider a variation of the AUSM mechanism in their experiments that further increases efficiencies in their experimental findings: the AUSM/Q mechanism. In AUSM/Q, rejected bids go to a queue. The queue is a bulletin board where agents may recombine rejected bids with their own bids and create new, group bids. The queue enables higher efficiencies by permitting cooperation that in many cases is necessary to displace large packages.

While AUSM/Q does not guarantee that such cooperation will occur, such cooperation is impossible under AUSM.

A sealed bid auction mechanism for airport slots is proposed by Rassenti, Smith, and Bulfin where the bids are on flight paths and the constraints take into consideration the takeoff and landing slot requirements. This mechanism can be converted to one involving trains quite easily: the terms "train path" can be substituted for flight path and the "train track resource requirements" can be substituted for the takeoff and landing slot requirements.

All of the auction-like processes mentioned above have the need to solve a knapsack problem to determine the set of bids that are accepted. The knapsack problem is an NP-complete computational problem that should be considered a serious challenge for the designer. NP-complete computational problems have the property that as a problem size variable increases, the computer time required for solution quickly outgrows even the best facility.

Because the computational issues involve costs and tradeoffs[6], an economic approach is necessary. What is being suggested here is not solely the need to consider the

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[6] For example, an agent might already have private information about the solution to a particular knapsack problem. There would be a tradeoff between getting the cost of getting agents to reveal such information and the cost of using computing machines. Often the issue of computation is considered technical in nature (e.g., the computer program is not fast enough) and solutions are also technical (e.g., change the algorithm, use approximations, increase CPU speed or memory). These technical solutions, however, also imply certain economic costs as well as benefits.

feasibility of an institution's computational requirements[7], but also the need to consider how those requirements are generated and satisfied. There is a demand and supply of computation in institutions, a sort of secondary economic allocation problem that may sometimes need to be considered. In most cases inefficiency in this secondary problem is unimportant in comparison with the need for efficiency in allocating the primary resources. With NP-complete problems like knapsack, the importance of the secondary problem can grow quite rapidly. To give an example of what is meant, consider the following two approaches taken in the literature:

Rassenti et al. claim computational feasibility for their mechanism given estimates of the number of variables required in the airport slots application. They do not address the problem of what may happen should scale increase in the future. Consider how computational demand and supply incentives may be inappropriate in this mechanism. Agents in the combinatorial sealed bid auction may submit as many bids as they wish. As they do so, the computational cost of the knapsack problem is increasing, perhaps exponentially. The agents, however, do not pay any of the computational cost. Computation is supplied by the seller or airport authority until it is infeasible for them to do so. Therefore, agents may over demand computation relative to what is optimal for a particular case.

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7 The need to consider the computational requirements of an economic institution dates back at least as far as Hayek (1935), who uses computational requirements as an argument against the possibility of efficient socialism.

Banks et al. with the AUSM queue, allow agents to voluntarily perform the task of solving much of the knapsack problem. The queue consists of a list of rejected bids that individually can not beat any accepted bid. However, combinations of rejected bids might beat an accepted bid, and agents are allowed to search the queue and resubmit such combinations. Any agent who has rejected bids in the queue is essentially demanding computational resources, and any agent searching the queue for recombination opportunities is supplying computational resources. The only incentive to the agents for performing the search is the possibility of receiving an improved allocation. Therefore, computation is being supplied only when moderately cost-effective, and primarily by those agents who are also demanders of computation.

One would expect the efficiency of the computational process to be better in AUSM/Q than in the combinatorial sealed bid auction, because the supply of computation in AUSM/Q is responsive to cost. Demand for computation may not be responsive to costs, but in AUSM/Q not all demands are necessarily satisfied by the suppliers. Overall it would appear that AUSM/Q is an improvement, since it does address the computational problem in a way that AUSM alone does not.

The successes and failures in the various experiments suggest some direction for constructing an efficient, decentralized mechanism for allocating access to railroad track. Hints from the previous studies can suggest how an efficient allocation mechanism for access to railroad tracks might be constructed.

The evidence would seem to indicate that sealed bid auctions are less desirable for two reasons: The Rassenti et al. experiments produce somewhat lower efficiencies than the Banks et al. AUSM/Q experiments, suggesting that an iterative auction approach allows bidders to better coordinate their actions. The Rassenti et al. bid message space is also too large to be practicable: each agent must communicate contingent bids for all possible routes on which they are interested. In the iterative auction mechanisms, agents quickly learn the routes on which they do not have high values. In this way the number of routes on which an agent must bid is greatly reduced.

The particular elements that are influenced from the literature are the choice of an auction-like mechanism rather than a market-like mechanism (from Banks et al. demonstration of poor market performance in the presence of complements), the need to structure the problem so that the effects of complements are minimized, and the need to consider both the computational requirements and the computational economics of the mechanism.

Varying the approach taken here from that in previous studies creates the opportunity for both successfully demonstrating existence of a decentralized allocation mechanism for accessing railroad networks and for contributing towards general knowledge in the allocation of shared resources. The next section will briefly consider these general issues.

1.4 The General Problem of Shared Resource Allocation

A railroad can be thought of as an example of a shared resource. Several agents can share access to the tracks, relative to some intercompatibility constraints, e.g., collision avoidance and safety criteria. A shared resource allocation environment might be characterized using three crude properties that the railroad allocation problem shares in common with some of the allocation problems considered in other studies:

- There is a shared facility with a finite number of alternative uses and agents who value these uses. There is a set of compatibility constraints indicating the limits of the facility in supporting the different uses.

- Information concerning the value of possible uses of the shared facility is dispersed among the agents in the environment. Different institutional processes (e.g., markets, auctions, voting, surveys) will result in differing degrees of revelation of values from agents.

- Given a value for each use of the facility, the optimal set of uses is defined as the set that maximizes the sum of the agents' individual values for the different uses of the facility, while satisfying the compatibility constraints. This maximization problem, given the values, is an integer programming problem known as the *knapsack* problem. The maximization problem itself can become very difficult, regardless of how[8] the individual agents' use values are obtained or approximated.

The fact that a pattern of common properties can be identified in the experimental literature for vastly different applications suggests the possibility of starting with these common properties and discussing general techniques for the allocation of 'shared resources'. The primary goal of this research has to do with demonstrating the existence of decentralized mechanisms for allocating access to a railroad, a specific type of shared resource. In accomplishing this goal, some headway will be made towards addressing the general issues.

This research, which is primarily the study of a specific decentralized mechanism for a special environment, has distinguishing properties that separate it from the other studies. This allows for the study to be interpreted as a contribution towards the more general problem involving the allocation of shared resources. As suggested in the conclusion to the previous section, two important distinguishing properties involve the commodity space and the handling of computational complexity.

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[8] In particular, the knapsack problem exists independent of the allocation mechanism and independent of the informational issues. It can be ignored by some institutions, e.g., random selection or lottery, but it can not be avoided if efficiency is a goal of the institution. For example, if a central authority attempts to calculate individual's values for various uses through some combination of engineering and econometric analysis, the knapsack problem must still be solved to obtain the best set of uses.

In other studies, a set of exclusive resources available within the shared resource were already defined. An agent's use of the shared resource required packages of the exclusive resources. Agents had to be allowed to make contingent package bids to avoid problems associated with complements. The knapsack problem is introduced through the need to maximize the sum of bids in an auction, where bids are contingent on receiving entire packages.

A similar perspective can be taken for the allocation of access to railroad tracks. In such a case the exclusive resources would be the use of individual, short, train track segments for brief segments of time. A train would require a package of these resources in order to be allowed to run out its schedule on the tracks.

A somewhat different approach can be taken by assuming that all infeasibilities on the track are the result of a conflict between two trains. The exclusive resources produced by the shared set of tracks are defined to be the rights to run particular trains. The problem of complementary resources vanishes so long as agents have additive preference values over sets of trains. The knapsack problem remains, because given a set of bids for the trains, not all combinations of trains are possible and maximizing bid revenue for an auction will involve a set of integer constraints.

The knapsack problem would appear to be inevitable so long as the resource is allocated through an auction which maximizes the sum of bids. Since the knapsack problem is known to be an NP-complete computational problem, approximations might eventually be necessary, and it is important that these be carried out in an unbiased manner. In addition, it is desirable to end computations when marginal computational cost outweighs any allocational benefits.

This research addresses computational issues by the same approach used to decide resource allocation issues: an auction. The task of computing approximate solutions to the knapsack problem is delegated to the agents through a type of procurement auction, called the Computation Procuring Dutch Auction (CPDA). In this procurement auction the solution to the knapsack is computed solely by the agents. Agents receive a reward for supplying information concerning changes to the allocation that improve the sum of bids. The reward is bounded by the amount of the improvement, so that supply of computation reacts to computational costs. Only one agent can get the reward for a given piece of information, so there is a competitive pressure to be the first to submit. Information supplied by agents is made public. In this way, the procurement auction is structured to encourage both competition and cooperation in solving the knapsack problem.

Further details of the technique are left for Chapter 7, along with a crude theory of its operation. CPDA can probably be generalized to other allocation environments, although it is defined in terms of the rail allocation problem. Chapter 8 consists of a test of BICAP+CPDA in the experimental laboratory.

The remainder of the research is largely concerned with the application framework: demonstrating existence of a decentralized allocation mechanism for access to railroads. One of the advantages of such an approach is the ability to occasionally bypass large general problems by returning, when necessary, to specific properties of the application framework of railroads.

1.5 Outline of Chapters

Chapter 2 introduces terminology and notation upon which the thesis is based. Terms such as "environment" and "mechanism" are discussed and defined here.

Chapter 3 defines a set of performance function requirements relevant to the debate over decentralized allocation of railroads. Basic concepts such as trains, tracks, allocations and feasibility of allocations are discussed and formalized. A series of experimental testbed environments is defined that reflect these issues. Each testbed consists of a set of train routes and agent train values. These testbeds will be the ones used in later experiments. The testbeds are crucial, and must be simple enough for an initial investigation while still containing essential difficulties thought to be problematic in railroad allocation. They are the economic environments that provide the properties that will "stress test" the mechanisms developed in the later chapters.

Chapter 4 shows that essential characteristics of a feasible set of allocations for railroad environments can be described by a set of "binary conflicts." The binary conflicts can be represented by a binary conflict graph. This graph has certain useful properties, which may aid in necessary computations in the mechanism.

Chapter 5 describes a mechanism, the Binary Conflict Ascending Price (BICAP) mechanism, which is designed to yield high efficiency allocations in binary conflict environments. BICAP is essentially an auction where the constraints implied by the binary conflicts are taken into account. The BICAP mechanism is first defined, and then a crude theory of its operational properties is presented. BICAP does not have good computational properties, because computation is supplied centrally without regard to cost. This computational difficulty is addressed in Chapters 7 and 8.

Chapter 6 describes the results of initial laboratory experiments that apply the BICAP mechanism of Chapter 5 to a simple testbed from Chapter 3. The experiments show that BICAP produces allocations of extremely high efficiency and that it appears to exhibit design consistency: it works for the reasons discussed in BICAP's theory of operation as defined in Chapter 5.

In Chapter 7 the difficulties that might arise due to computational complexities involving size and schedule interdependencies are considered. Chapter 7 suggests a Computation Procuring Dutch Auction (CPDA) for dealing with problematic optimization computations required by BICAP. CPDA not only procures the computation needed to compute potential allocations in BICAP, but also determines when the mechanism should terminate. A theory of operations is developed for BICAP+CPDA showing that such computation should take place and in a cost effective manner.

Chapter 8 describes the results of experiments that apply the BICAP+CPDA mechanism of Chapter 7 to the three-track testbeds from Chapter 3. The goal is to show that BICAP+CPDA performs as well as BICAP even though computation is done by the agents rather than by a central authority. Three categories of results are addressed: performance of BICAP+CPDA allocations, performance of CPDA in inducing computation by agents, and comparison of final bid prices and bidding behavior in BICAP with and without CPDA. BICAP+CPDA produces favorable results in the performance categories, but allocation efficiency is not as high as in the BICAP experiments. The BICAP+CPDA closing prices are very close to the closing prices BICAP produced in the cases where the testbeds are comparable. Some puzzling results concern the observation that BICAP+CPDA fails to reach one stage Nash equilibria outcomes (defined in Chapters 2, 5) that BICAP without CPDA does reach in comparable environments. The failure to reach one stage Nash equilibria is partially explained in terms of a type of local monopoly power present in the agent incentive parameters in the testbeds.

CHAPTER 2. General Notation and Concepts

As noted in Chapter 1, the problem of allocating access to a railroad involves an information and incentive problem and a computational problem. The purpose of this chapter is to introduce notation that allows a formal discussion of the railroad allocation problem and the institutional processes that are developed in the thesis. The notation chosen is fairly standard in the mechanism and social choice literature. The concepts that are found in the literature[9] are very general and facilitate a rigorous representation of institutional and behavioral processes.

The idea that information is dispersed among agents in an economy is modeled by an *economic environment* $e=(e_1,...,e_i,...)$ [Hurwicz(1960)]. The different sections of this chapter will provide interpretations of such abstract concepts. A set of agents $\boldsymbol{I}$ are participants in the economic environment. Each agent $i \in \boldsymbol{I}$ is described by private characteristics e_i . As will be made clear in the next section, each agent's

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9 The literature is quite broad and covers a number of interesting applications. Hurwicz (1960) introduces the model used here for economic environments and also introduces a tantonnement model of institutional processes he calls an adjustment process. Reiter (1977) defined a mechanism originally as a summary of the adjustment process. Smith (1982) extends the mechanism model to cover institutions that do not equilibrate like a tantonnement. The environment model is used in other related disciplines such as social choice. In the social choice literature, an environment consists of agents and their (private) preferences. One can then ask whether social preferences are well defined and whether social choice processes are incentive compatible. A good summary to the mechanism literature can be found in Groves and Ledyard (1984). There are important later developments regarding processes that are based on Nash equilibria models of behavior: Moore and Repullo (1988) provide an existence proof that almost any social choice rule can be implemented as a mechanism in subgame perfect equilibria. Palfrey and Srivastava consider implementation in undominated strategies. The Nash equilibrium implementation results are in general much more permissive than the dominant strategy implementation summarized in Groves and Ledyard (1984). Therefore, models of agent behavior in this research will concentrate on Nash-type models.

characteristics are private information for that agent. Agent's characteristics usually include preferences, technology, and initial resource endowments, but can include any other characteristics that are important in modeling agent behavior. The economic environment completely characterizes all the important issues of an allocation problem, and is the basis of all subsequent analysis.

Section 2.1 defines the term 'railroad network' for the purposes of the formal definitions, and gives a brief example. The class of economic environments for the railroad network is defined in section 2.2. After defining this class of environments, additional definitions are built upon the environment in the following order: Section 2.3 defines the feasible allocations of resources for the railroad environment. Section 2.4 defines standard performance criteria, e.g., Pareto-optimality, for allocations based on the agents' preferences in the environment. Section 2.5 defines an iterative mechanism model for describing institutions for allocating the railroad. Section 2.6 defines concepts related to individual and aggregate agent behavior in a mechanism.

2.1 Physical Feasibility Requirements for Railroads

The most common use of a railroad network is to support a train traveling between two locations. It is important that a model of a railroad's physical constraints take into consideration all uses (e.g. track maintenance) and not just uses for trains. For that reason, a *railroad network* is defined as a type of shared resource that has private '*uses*'. This concept of a use is distinct from the resources that might facilitate or enable that use.

As an example of a use f, consider a train following a specific path over the tracks. At every moment in time the train has a position on the tracks, a speed, and a length. Another example of a use would be a maintenance crew clearing debris, or perhaps ice, from the track. For the most part, uses will be exclusive. This means that at most one agent may use the railroad network in that very specific way. For example, if f is a train at a specific position on the track, it is physically impossible for another train to be at the same position for the trivial reason that two objects can not be at the same exact place and at the same time (a collision between trains will be something different and will be discussed later). The set of all uses of the railroad network (or, perhaps, all the uses of interest, or all the permitted uses, etc.) will be called $\boldsymbol{F}$. A formal definition follows:

Definition. A *use* $f \in \boldsymbol{F}$ is an application that is feasible in the absence of other uses, and exclusive in the sense that an application consisting of two instances of f is not feasible.

An agent $i \in \boldsymbol{I}$ might not use the railroad network at all, or might use the network for just one use (or train), or might have a whole fleet of trains on the network. The set of uses of the network by agent $i \in \boldsymbol{I}$ will be called agent i's individual schedule $\boldsymbol{F}_i$.

The system schedule $\boldsymbol{F}_S$ will be the set of all uses of the network by any agent. For example, if agent 1 runs train A, and agent 2 runs trains B and C, then the individual schedule run by agent 1 is $\boldsymbol{F}_1 = \{ A \}$, the individual schedule run by agent 2 is $\boldsymbol{F}_2=\{B,C\}$ and the system schedule is $\boldsymbol{F}_S = \{ A, B, C \}$. A formal definition follows.

<u>*Definition.*</u> An *individual schedule* $\boldsymbol{F}_i$ is a set of uses by the agent $i \in \boldsymbol{I}$. A *system schedule* $\boldsymbol{F}_S \subseteq \boldsymbol{F}$ is a set of uses by all agents $i \in \boldsymbol{I}$, i.e. $\boldsymbol{F}_S = \cup_{i \in \boldsymbol{I}} \boldsymbol{F}_i$.

A railroad can be described as having a set of uses $\boldsymbol{F}$ and a family of feasible system schedules $\mathcal{S}$. The family of feasible system schedules indicates what sets of trains or other uses are permissible together on the railroad network, and is partially due to physical constraints and partially a policy issue. Safety criteria and similar issues will play a crucial role in determining the exact form of the set of feasible system schedules, and these safety criteria may be somewhat dependent on the terrain and other specifics of the tracks involved. Here these terms are merely defined, with further investigations delayed to chapters 3 and 4.

<u>*Definition.*</u> A *railroad network* can be described by an ordered pair $(\boldsymbol{F}, \mathcal{S})$: a set of *uses* $\boldsymbol{F}$ and a family of *feasible system schedules* $\mathcal{S} \subseteq 2^{\boldsymbol{F}}$. A *feasible system schedule* is a system schedule that passes all safety and other requirements for implementation.

As an example, once again consider three trains, A,B, and C, but now place some restrictions on what is considered feasible. Uses A and B are northbound trains with well defined timetables. Use C is a southbound train with some well defined timetable. According to the definitions above, $\boldsymbol{F}$ consists of the set {A,B,C}. Suppose the system schedule {A,C} involves a fatal collision between the trains A and C, and therefore is not feasible for implementation. Then {A,C} $\notin \mathcal{S}$. Alternatively, the system schedule {B,C} may involve trains that leaves at times far enough apart that there is no possibility of collision. In that case, {B,C} $\in \mathcal{S}$. The important properties of the railroad network, in terms of what is or is not feasible, are completely characterized by $(\boldsymbol{F}, \mathcal{S})$.

2.2 The Railroad Economic Environment

To consider a 'railroad allocation problem' as separate from the allocation of the economy as a whole, a particular economic problem is isolated and exogenous influences will be summarized as parameters of the problem.

Begin by considering only those agents directly involved in the allocation process. For railroad allocation, this consists of a track authority, who will be called agent S(seller), and a set $\boldsymbol{I} = \{ 1,2,3,4,..., |I| \}$ of train operator agents (buyers). The effect of all other classes of agents (e.g., agents purchasing cargo or passenger services from the train operators) will be assumed to be summarized in the valuations of the trains for the agents in ***I***. How the model takes into account agents values for trains will be discussed shortly.

The agents in ***I*** acquire track access from agent S, who owns or manages the rail network. The agents in ***I*** use track access as a factor good, along with other factors such as engines and cars, in order to operate trains on the track. The agents then sell tickets, transport cargo, or provide some valuable product or service resulting from their train operations. These sales occur at some other 'train output' market, the exact nature of which will be omitted from the scope of the model.

Under certain simplifying assumptions[10], the implications of these 'train output' markets can be summarized as a matrix $\underline{\mathbf{V}}$ giving the revenue of each agent for operating each train. The element $V_{jf} \in \Re$ would give agent j's net revenue for operating train $f \in \boldsymbol{F}$. Agent j's revenues for the various trains is summarized as a vector of train values $\mathbf{V}_j$. Each agent's vector of train revenue values is assumed to be private information for that agent and known with certainty[11], at least initially, only by that agent. As mentioned above, the matrix $\underline{\mathbf{V}}$ is assumed to summarize a number of

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10 The assumptions are that the net revenue to an agent for operating trains is separable across agents (i.e., no externalities), and the net revenue is additive over trains operated by an agent.

11 Assuming train revenues are known with certainty is primarily a simplifying assumption, but may have an additional interpretation relevant for deregulation in Sweden. Coopers and Lybrand point out that large number of commuter rail services have a single buyer, the local or regional government who subsidizes the services (p. 281). If the subsidy is fixed and costs of operation are known fairly well, then the revenue an operator would expect from operating these services may be known quite well, perhaps even with certainty.

parameters not included in the model: all economic incentives for running a train except for the payment to acquire track access.

As the seller is a public agency, it will be assumed that the seller's characteristics are public knowledge. In addition, certain physical constraints about operation of trains on the track will also be considered public knowledge, e.g., how collisions and safety violations between trains can be determined. The seller's policies concerning the use of the track and the rules of any process by which resources are allocated to the agents are all assumed to be public knowledge.

There are many policies by which the seller could allocate access to the track. A fairly broad class of policies of interest involves the seller dividing up the railroad network into resources or rights of some type, which then might be sold or otherwise allocated to the agents. This set of resource rights will be called Λ. The rights allocated to agent i will be called Λ_i. It is important to realize that the choice of Λ is part of the policy debate. Different choices may lead to different conclusions later regarding the possibility of decentralized trade, safety, or other considerations (in particular, the definitions of Λ and the other policy variables that follow will leave open the possibility of infeasible system schedules or other problems that might result from poor policy choices). The Coopers and Lybrand report can be interpreted as taking the position that there may be no 'suitable' choice of Λ. Disregarding 'suitability' for now, it must be clear that many choices are possible: Λ could consist of the rights to operate trains for one hour intervals on specific miles of tracks, or Λ could consist of several types of permits that must be obtained from separate agencies, or Λ could consist of a priority ranking, etc.

The types of resource rights that are of interest here will have a separability property. The set of rights that an agent must obtain to run a train will be independent of the rights or trains run by any other agents. Associated with an agent buyer in possession of a set of rights Λ_i will be a set of uses as dictated by the resource policy. An individual buyer, with resource rights Λ_i, will be permitted to choose any desired schedule from the uses in $\Gamma_B(\Lambda_i)$. The notation $\Gamma_B(\Lambda_i)$ represents a set of uses that is available to buyer i by virtue of access to resource rights Λ_i. The mapping Γ_B from resource rights to uses, like Λ, is a policy variable. It serves to define the meaning of the resource rights to the agents. For example, if Λ consists of permits that must be obtained from a number of agencies, then $\Gamma_B(\Lambda_i)$ consists of the set of trains for which all the permits have been obtained. If Λ consists of permissions to use specific tracks for specific time intervals, then $\Gamma_B(\Lambda_i)$ consists of all trains that can be run using only the track time intervals in Λ_i. The agent may then pick any subset of these trains to run on the tracks.

In addition to the constraints on the operators, the seller may be disallowed from selling certain combinations of resources, e.g. those likely to result in a collision or hazard. It may be desirable, as part of a policy, to allow the seller to distribute the rights to run trains {A,B,C} or {D,E,F} but not {A,B,C,D,E,F} because in the later case some collisions will occur. The family of allowed resource distributions can be thought of as a family $\Gamma_S \subseteq 2^{\Lambda}$ consisting of subsets of Λ indicating what sets of resources may be sold.

The ideas discussed roughly in the three preceding paragraphs can be formalized in the following definition of a *resource rights policy.*

<u>*Definition.*</u> A *resource rights policy* for the railroad network ($\boldsymbol{F}$, $\mathcal{S}$) consists of a triplet (Λ, Γ_B , Γ_S), where

Λ is a set of *resource rights.*

Each resource right may be either:

retained by the seller, or

provided to at most one agent $i \in \boldsymbol{I}$.

$\Gamma_B : 2^{\Lambda} \to 2^{\boldsymbol{F}}$ is the *buyers' technological constraint.*

An agent $i \in \boldsymbol{I}$ is allowed to operate the individual schedule $\boldsymbol{F}_i$ if and only if agent i obtains resource rights Λ_i such that $\boldsymbol{F}_i \subseteq \Gamma_B(\Lambda_i)$.

$\Gamma_S \subseteq 2^{\Lambda}$ is the *seller's technological constraint.*

A sale of the resources in Λ_i to each agent $i \in \boldsymbol{I}$ is permissible under the technology Γ_S if and only if $\Lambda_S \in \Gamma_S$, where $\Lambda_S = \cup_{i \in \boldsymbol{I}} \Lambda_i$.

Once a resource rights policy has been defined, a commodity space also exists, and consists of individual schedules, individual resource rights allowing those schedules, and a numeraire (cash). This commodity space is defined below:

Definition. An agent *commodity bundle* consists of a triplet $(\boldsymbol{F}_i, \Lambda_i, x_i) \in 2^{\boldsymbol{F}} \otimes 2^{\Lambda} \otimes \Re$, where:

$\boldsymbol{F}_i \subseteq \boldsymbol{F}$ is an individual schedule,

$\Lambda_i \subseteq \Lambda$ is a set of resources, and

x is an amount of a numeraire good (cash).

Once a commodity bundle for an agent has been defined, it is possible to define more standard notation in a manner common in the mechanism literature. In the literature an agent is typically defined as having preferences over commodity bundles, an initial endowment commodity bundle, and a means (if any) of changing certain bundles into others. Each of these three agent characteristics will be briefly discussed. An agent is defined as having preferences over commodity bundles.

An agent, for instance, may prefer \$300 and running train route A to \$500 and running train route C, because train route C will yield more than \$200 more operating profit than train route A. Recall that the matrix V_{if} gives the dollar value to agent i of running train f. In this case, an agent's preferences are over the individual schedules and numeraire goods. Agents prefer different bundles of resource rights only to the extent that it affects what individual schedules are permitted.

It will be assumed that agents start with no initial endowment of resource rights and an empty individual schedule. The reader is reminded that one of the goals of the thesis research is to find a suitable resource rights policy and an institution for allocating the rights to the agents.

The ability of agents to transform commodity bundles will be limited to what is obviously implied by the resource rights policy. That is, if an agent has accumulated resource rights that allow choices of schedules in {A,B,C}, then the agent may change his individual schedule to {A}, or {A,B}, or {A,B,C}, or ∅, or any other subset of {A,B,C}.

We are now prepared to discuss the general notation that was described by Hurwicz.

Definition. An *agent characteristic* $e_k = (\geq_k, \omega_k, \gamma_k)$ consists of a transitive preference relation $\geq_k$ over commodity bundles, an initial endowment commodity bundle ω_k, and a rule γ_k indicating an agent's feasible transformations of the commodity bundle.

A set of agents, a railroad network, a resource rights policy, and a matrix of agent train values form a complete specification of the economic environment, in the sense of Hurwicz (1960). The economic environment summarizes the "initial conditions" present before the agents and seller interact through some allocation institution or mechanism. The information in the economic environment will be necessary to be able to discuss the features and likely outcomes of mechanisms to be defined later in the text.

Definition. An Additive Separable Railroad Economic Environment e for a set of agents $\boldsymbol{I}$, in a railroad network $(\boldsymbol{F}, \mathcal{S})$, with a resource rights policy $(\Lambda, \Gamma_B, \Gamma_S)$, and agent train values $\underline{\mathbf{V}}$, is defined as a list $e=(e_S ; e_1, ..., e_{|I|})$ of agent characteristics $e_k = (\succeq_k, \omega_k, \gamma_k)$, where :

for agent S,

$(\boldsymbol{F}_i', \Lambda_i', x') \succeq_S (\boldsymbol{F}_i, \Lambda_i, x)$ if and only if $x' \geq x$

$\omega_S = (\varnothing, \varnothing, x_{S0})$

Under γ_S a seller may transform the initial commodity bundle ω_S to $\omega_S' = (\varnothing, \Lambda_S, x_{S0})$ if and only if $\Lambda_S \in \Gamma_S$

for agents $i \in \boldsymbol{I}$,

$(\boldsymbol{F}_i', \Lambda_i', x_i') \succeq_i (\boldsymbol{F}_i, \Lambda_i, x_i)$ if and only if $\left(x_i' + \sum_{f \in F_i'} V_{if} \right) \geq \left(x_i + \sum_{f \in F_i} V_{if} \right)$

$\omega_i = (\varnothing, \varnothing, x_{i0})$

Under γ_i agent $i \in \boldsymbol{I}$ may transform a commodity bundle $(\boldsymbol{F}_i, \Lambda_i, x_i)$ into $(\boldsymbol{F}_i', \Lambda_i, x_i)$ if and only if $\boldsymbol{F}_i' \in \Gamma_B(\Lambda_i)$.

2.3 Allocations and Feasibility

Following Hurwicz, an allocation consists of a commodity bundle for each agent in $\boldsymbol{I}+\{S\}$.

<u>Definition:</u> An *allocation* a for a railroad economic environment is a list of commodity bundles. $a=(a_S ; a_1 , \ldots , a_i)$, where:

(i) for each agent $j \in I$

$a_j = (\boldsymbol{F}_j, \Lambda_j , t_j)$ where

$\boldsymbol{F}_j \subseteq \boldsymbol{F}$ indicates the individual schedule of uses for agent j.

$\Lambda_j \subseteq \Lambda$ indicates the resource rights allocated to agent j, and

$t_j \in \Re$ indicates agent j's cash transfer[12]

(ii) for the seller S,

$a_S = (\boldsymbol{F}_S, \Lambda_S , t_S)$ where

$\boldsymbol{F}_S$ is the system schedule of uses of the railroad network.

$\Lambda_S \subseteq \Lambda$ indicates the track resources supplied by the seller.

$t_S \in \Re$ indicates the seller's net cash transfer.

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12 It is necessary to select a standard for determining the sign of the transfers 't'. The sign of the change in the agent's cash holdings will determine the sign of the transfer. That is, '+' indicates that the agent receives money, and '-' indicates that the agent pays money.

Note that an allocation, as defined, is just a list of numbers and sets. Very few constraints are imposed. For example, there is no restriction in the definition above requiring the cash transfers balance out, or for the system schedule to equal the union of the individual schedules. A feasible allocation is an allocation that actually is consistent with the interpretations of each element given above. A feasible allocation will have to satisfy two groups of constraints: budget constraints and physical compatibility constraints.

The budget constraints are concerned with establishing that an allocation conforms to basic definitions of system vs. individual schedule, seller resource provision vs. agent resource rights , and balance of cash transfers (total (negative) transfers of buyer agents to obtain resources plus the sellers transfer equals zero) and with the constraints regarding the resource rights policy (Λ, Γ_B , Γ_S) (that buyers individual schedules are permitted given their resource rights, and that the seller only distributes an allowed set of resources). The budget constraints do not guarantee feasible system allocations. Information about $\boldsymbol{F}$ or $\mathcal{S}$ does not appear in the budget constraints.

The physical compatibility constraints are concerned with establishing that an allocation results in a feasible system schedule for the railroad network ($\boldsymbol{F}$,$\mathcal{S}$). Identical uses may not appear in multiple agents' individual schedules and the system schedule must be one of those in the family $\mathcal{S}$.

Note that the physical compatibility constraints do not guarantee that the budget constraints are satisfied, just as the budget constraints do not guarantee that the physical compatibility constraints are satisfied. As mentioned before, the set of feasible allocations will satisfy both sets of constraints. The formal definitions are provided below:

Definition: The *budget constraints* (B1)-(B6) imposed by a resource rights policy $(\Lambda, \Gamma_B, \Gamma_S)$ upon an allocation $a=(a_S; a_1, \ldots, a_i)$ are:

(B1) $\Lambda_S \in \Gamma_S$ (seller's production constraint)

(B2) $\forall j,k \in I, j \neq k \Rightarrow \Lambda_j \cap \Lambda_k = \varnothing$ (exclusivity of track resources)

(B3) $\cup_{j \in I} \Lambda_j = \Lambda_S$ (budget balance in track resources)

(B4) $-\sum_{j \in I} t_j = t_S$ (budget balance in cash)

(B5) $\boldsymbol{F}_i \subseteq \Gamma_B(\Lambda_i)$. (buyer's resource use constraint[13])

(B6) $\boldsymbol{F}_S = \cup_{j \in \boldsymbol{I}} \boldsymbol{F}_i$ (system schedule definition)

Definition. The *physical compatibility constraints* (P1)-(P2) imposed by a railroad network $(\boldsymbol{F}, \mathcal{S})$ upon an allocation $a=(a_S; a_1, \ldots, a_i)$ are:

(P1) $\forall j,k \in \boldsymbol{I}, j \neq k \Rightarrow \boldsymbol{F}_j \cap \boldsymbol{F}_k = \varnothing$

(P2) $(\cup_{i \in \boldsymbol{I}} \boldsymbol{F}_i) \in \mathcal{S}$

13 Note that a budget constraint for the buyers (of the form $t_j \leq \omega_{j\Re}$) is not imposed. Effects of low initial endowments were not considered to be of interest in this study: demands of sufficiently endowed buyers or buyers with good access to capital markets will not be affected by this type of constraint.

Definition. The set of feasible allocations $\mathcal{F}(e)$ is the set of allocations that satisfy both the budget constraints and the physical compatibility constraints.

To hint at things to come, it might be possible to be able to choose the resource rights policy so that every allocation that satisfies the budget constraints also satisfies the physical compatibility constraints. How this might be accomplished is delayed for discussion in Chapter 4, where the notions of resource policies and feasibility will be revisited.

2.4 Measures of Allocation Performance: Value, Optimality, and Efficiency

Recall that the process desired by the Swedes is one that allocates the use of track to those who value it the most. Below, it is shown that this is consistent with maximizing the sum of seller's and buyers' profits and with Pareto-optimality. One claim in the debates has been that there may be conflicts between 'commercial' and 'socio-economic' goals[14]. This section shows that the 'commercial' goal of achieving Pareto-optimal allocations is identical to the 'socio-economic' goal of maximizing the total value of the usage of the trains. Of course, there may still be conflicts between 'commercial' and 'socio-economic' goals if important externalities exist (e.g., pollution) and also depending on precisely what it means to maximize

[14] Coopers and Lybrand state that "... One of the potential difficulties with introducing competition into the rail industry is the conflict between the commercial way in which operators are expected to make decisions and the socio-economic role of Banverket (the rail agency) as provider of the infrastructure. Operators will wish to make decisions which, at the margin, are in their commercial interests, while Banverket, and many of the buyers of passenger services, will wish to ensure that social benefits are maximized." (p.283)

social benefits in the Swedish sense. For this initial study externalities are assumed to be nonexistent, and the goal of the next few paragraphs is to characterize the Pareto-optimal set of allocations and the related efficiency measure to be used in evaluating the experimental results.

Definition. The *agent's profit* $\Pi_j(a;\underline{V})$ of agent $j \in \boldsymbol{I}$ at a feasible allocation $a \in \mathcal{F}(e)$ is

$$\Pi_j(a;\underline{V}) = \left(\sum_{f \in F_j} V_{jf} \right) + t_j .$$

Definition. The *total value* $\upsilon(a;\underline{V})$ of a feasible allocation $a \in \mathcal{F}(e)$ in an environment e is $\upsilon(a;\underline{V}) = \sum_{j \in I} \sum_{f \in F_j} V_{jf}$.

Proposition 2.1. The *total value* $\upsilon(a;\underline{V})$ of a feasible allocation $a \in \mathcal{F}(e)$ in an environment e is equal to the sum of buyers' profit and the transfer to the seller.

$$\upsilon(a;\underline{V}) = t_S + \sum_{j \in I} \Pi_j(a;\underline{V}) .$$

Proof. By definition $\upsilon(a;\underline{V}) = \sum_{j \in I} \sum_{f \in F_j} V_{jf}$.

Because a is feasible the budget balance condition (B4) $t_S + \sum_{j \in I} t_j = 0$ is satisfied. Therefore, add this equation to both sides of $\upsilon(a;\underline{V})$ yielding

$$\upsilon(a;\underline{V}) = t_S + \sum_{j \in I}(t_j + \sum_{f \in F_j} V_{jf}).$$

From the definition of profit Π_j ,

$$\upsilon(a;\underline{V}) = t_S + \sum_{j \in I}\Pi_j(a;\underline{V}) \bullet$$

Definition. The *Pareto-optimal set* of feasible allocations $\mathcal{P}(e)$ in an environment e is the set of feasible allocations having the property that there exist no feasible allocations that make all/one[15] agents better off while making none worse off.

Proposition 2.2. $\mathcal{P}(e) = \{ a \in \mathcal{F}(e) : a \text{ maximizes } \upsilon(a;\underline{V}) \}$.

Proof. It must be established that (i) $a \in \mathcal{F}(e)$, a maximizes $\upsilon(a;\underline{V}) \Rightarrow a \in P(e)$,

and (ii) $a \in \mathcal{P}(e) \Rightarrow a \in \mathcal{F}(e)$, a maximizes $\upsilon(a;\underline{V})$

(i) If a maximizes $\upsilon(a;\underline{V})$, then there are no feasible allocations which provide larger total profit. Any other feasible allocation must have equal or lower total profit. Therefore, any other feasible allocation must either produce revenues equivalent to a or must make at least one agent worse off. Therefore, $a \in \mathcal{P}(e)$.

(ii) From the definitions it is known that $a \in \mathcal{P}(e) \Rightarrow a \in \mathcal{F}(e)$. Suppose $a \in \mathcal{P}(e)$ and a does not maximize $\upsilon(a;\underline{V})$. Then let $a' \in \mathcal{F}(e)$ maximize $\upsilon(a';\underline{V})$. Since $\upsilon(a';\underline{V})$ is invariant to balanced transfers, set the transfer so that each agent receives

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[15] When the environment has transfers of divisible cash, the Strong and Weak versions of Pareto-optimality are equivalent. If an agent can be made better off while making no one worse off, then by adjusting transfers all agents can be made better off. If all agents can be made better off, then by adjusting transfers the group gain can be distributed to one agent. If payoffs are made in discrete units (as is done later), then some slight modifications to this proof are necessary and only Weak Pareto-Optimality can be implied.

what would have been received under allocation a, plus an equal share of the difference between $\upsilon(a';\underline{\mathbf{V}})$ and $\upsilon(a;\underline{\mathbf{V}})$. Then a′ is a Pareto-improvement to a. But this is a contradiction to the assumption that $a \in \mathcal{P}(e)$. Therefore, the assumption that there exists an allocation $a \in \mathcal{P}(e)$ that does not maximize $\upsilon(a;\underline{\mathbf{V}})$ is false.

Therefore, $a \in \mathcal{P}(e) \Rightarrow$ a maximizes $\upsilon(a;\underline{\mathbf{V}})$.

Notice that the Pareto-optimal set of allocations $\mathcal{P}(e)$ does not in general contain "corners" such as giving all the rights to run trains to one operator. Corners can only be an artifact of a particular choice of $\underline{\mathbf{V}}$. Corners do not occur in general because cash transfers are not bounded in the feasible set of allocations. If the feasible set of allocations included a cash budget constraint for the buyers, then corner solutions could occur more easily. The lack of bounds on cash transfers implicitly assumes that operators can always access capital to buy out inefficient operations.

The lack of corner solutions means that the Pareto-optimal set of allocations usually contains a single unique set of permitted trains and an agent to operate each train permitted on the track[16]. Different Pareto-optimal allocations differ only in the cash transfers that agents must pay to receive allocations. Because of this uniqueness, the standard concept of efficiency can be defined.

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[16] In the experiments, ties in total redemption value between different schedules were not permitted when the train values $\underline{\mathbf{V}}$ were generated. This will be discussed more fully in Section 3.4: Testbed Preferences for Trains. Even with ties, notice that the efficiency formula will give the same result since only the total value is relevant.

Definition. The *efficiency* of an allocation a in the environment e, denoted Eff(a,e), is defined as the ratio Eff(a,e) = $\upsilon(a;\underline{\mathbf{V}}) / \upsilon(a';\underline{\mathbf{V}})$, where a′ is any Pareto-optimal allocation $a' \in \mathcal{P}(e)$.

2.5 Mechanisms for Rail Allocation

The term mechanism indicates a formal model of an institution. The mechanism formalism defined here is quite general and proceeds similar to Smith (1982). Mechanisms are defined as a type of state-machine[17]. The class of mechanisms of interest to the railroad allocation problem will be narrowed shortly so that issues of behavior and equilibria can be more easily addressed.

Definition: An *iterative mechanism* $(\boldsymbol{M},\boldsymbol{S},s_0,\boldsymbol{S}^*,\zeta,O^*,T)$ for a class E of economic environments with agents $\boldsymbol{I}$ and feasible allocations $\mathcal{F}(e)$ consists of:

(i) A set $\boldsymbol{M}$ consisting of messages

$\boldsymbol{M}_i \subseteq \boldsymbol{M}$ denotes the messages that agent i may send.

$\boldsymbol{M}$ includes messages by nature (e.g., that time has elapsed).

(ii) A set of states $\boldsymbol{S}$

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[17] A state-machine is a machine whose description is summarized by a set of states and a rule explaining how the state evolves, perhaps under the presence of outside stimuli. The state-machine approach is useful for describing a number of physical, mathematical, biological, and cognitive processes. The "object oriented programming" revolution in computer languages (such as C++) aessentially involves forcing programmers to model the modules of their programs as families of state-machines. Marschak (1977) suggests that state-machines could be used to describe economic processes and mechanisms in his discussion of Reiter (1977).

(iii) An initial state $s_0 \in \boldsymbol{S}$

The mechanism begins operation in the initial state.

(iv) Set of Terminal States $\boldsymbol{S}^*$

Indicates states where the mechanism terminates and produces an outcome.

(v) Agent Feedback Information $\zeta_j(s)$

Each agent j receives information $\zeta_j(s)$ about the state s.

If $\zeta_j(s)=s$ for all agents j, then the state is public information.

(vi) Outcome Allocation Rule O*: $\boldsymbol{S}^* \rightarrow \mathcal{J}(e)$

Associates each terminal state with a feasible allocation.

(vii) Transition Rule: T: $\boldsymbol{S} \otimes \boldsymbol{M} \rightarrow \boldsymbol{S}$

indicates how the state evolves when messages are received.

(viii) The following algorithm

Begin Mechanism

Step 1: Set the state of the mechanism $s=s_0$.

Step 2: While $s \notin \boldsymbol{S}^*$, repeat the following block of steps

Step (2.a): Wait for a message m to be received.

Step (2.b): Update the state according to the transition rule: $s^{new}=T(s^{old},m)$.

step (2.c): Send the agents information $\zeta_j(s^{new})$.

Step 3: [When this step is reached, $s \in S^*$] Calculate an outcome allocation o*=O*(s) according to the final state the mechanism reached.

Step 4: Send the outcome allocation information o* back to the agents.

End Mechanism

A specific class of mechanisms, called Soft Terminating Iterative Mechanisms (STIMs), terminate only when activity ceases for a given period of time. Because of the termination rule, one-stage Nash equilibria and Core states of STIM mechanisms are fairly straightforward to identify.

<u>Definition:</u> An iterative mechanism is a *soft-terminating iterative mechanism* (STIM) if the following conditions are satisfied:

(i) The mechanism state can be written $s=(\tau,\text{--})$ where -- represents other state variables. The initial state $s_0=(\tau_0,\text{--})$ has $\tau_0 > 0$. The variable τ is called the *mechanism timer*.

(ii) The set of terminal mechanism states $S^* = \{s \in S\text{: } \tau=0\}$, i.e., the mechanism terminates if and only if the mechanism timer τ reaches zero.

(iii) The transition rule T resets τ to τ_0 whenever any other state variables are changed.

(iv) The message space ***M*** and transition rule T allow for a time message from nature that decrements the value of τ.

(v) The transition rule T affects τ only through (iii) and (iv) above.

2.6 Individual Agent Behavior

This section briefly touches on some notation and concepts that will be more fully developed in Chapter 5 and 7 regarding the types of behavioral models used in the thesis. These models begin by making some definitions regarding potential outcomes and allocations at each state of the mechanism. Soft terminating mechanisms can be modeled by considering beliefs and behavior when the mechanism timer τ is very small and about to expire. The definitions below describe a myopic form of expectations: they are based on what would occur if no agent takes any additional action. An agent is modeled as choosing between whether to allow the mechanism to terminate or whether to take some action that will prevent termination.

Definition. For a STIM, the *potential allocation* $O^P(s)$ at a state s is the value of the outcome rule $O(s/\tau=0)$ where $s/\tau=0$ indicates the terminal state obtained by setting the τ component of s equal to 0.

Definition: An agent's *potential profit* $\pi_j^P(s;\underline{\mathbf{V}})$ at a state s in a is the profit the agent would earn under the potential allocation, $\pi_j^P(s;\underline{\mathbf{V}}) = \Pi_j(O^P(s) ; \underline{\mathbf{V}})$

It is possible to classify messages available to agents by their affect on the mechanism state and the potential allocation. This approach is motivated by an observation of actual behavior rather than by any formal theory. The definitions, however, are useful in describing possible stopping points of the mechanism.

Definition. In a STIM $\mathcal{M}$, at each state s the message space $\boldsymbol{M}$ can be partitioned into three subsets of messages:

(i) the *set of pivotal messages* $\boldsymbol{M}_{\Pi}(s)$; These messages change the mechanism state and potential allocation.

$$\boldsymbol{M}_{\Pi}(s) = \{m \in \boldsymbol{M} : O^{p}(T(s,m)) \neq O^{p}(s)\}$$

(ii) the *set of neutral messages* $\boldsymbol{M}_{N}(s)$; These messages change the mechanism state but do not change the potential allocation.

$$\boldsymbol{M}_{N}(s) = \{ m \in \boldsymbol{M} : O^{P}(T(s,m)) = O^{P}(s) , T(s,m) \neq s \}$$

(iii) the set of null messages $\boldsymbol{M}_{\varnothing}(s)$; These messages change neither the state nor the potential allocation.

$$\boldsymbol{M}_{\varnothing}(s) = \{ m \in \boldsymbol{M} : T(s,m)=s \}$$

Statements regarding equilibria in mechanisms can be phrased in terms of the properties of available pivotal or neutral bids and the measure of potential profit:

Definition. In an environment e, the set of *Nash-1 Stationary Equilibria*, NE1($\mathcal{M}$,$\underline{\mathbf{V}}$,π^P), of a soft-terminating iterative mechanism $\mathcal{M}$ is the set of states s such that there do not exist pivotal messages that can increase some agent's potential profit:

$$NE1(\mathcal{M}, \underline{\mathbf{V}}, \pi^P) = \{ s \in S : \forall j \in I, \forall m \in (\boldsymbol{M}_\Pi(s) \cap \boldsymbol{M}_j), \pi^P_j(T(s,m);\underline{\mathbf{V}}) < \pi^P_j(s;\underline{\mathbf{V}}) \}.$$

The NE1 concept could be easily generalized to include sets of bids from a single agent. For the purposes here, there is no advantage to the additional notation at this point. NE1 outcomes represent possible stopping points of the mechanism because no agent has a pivotal message to send that increases its immediate potential profit. If an agent did have such a pivotal message, it is reasonable to expect the agent to submit it and the mechanism to continue. Therefore the only "reasonable" stopping points must be NE1.

Definition. In an environment e, the *set of Core outcomes* of a soft-terminating iterative mechanism $\mathcal{M}$, Co($\mathcal{M}$, e, π^P), is a set of states s such that, for any coalition of agents and any set of messages from the coalition of agents, an increase in the potential profits of all agents in the coalition is not possible.

Consider briefly the nature of dynamic processes that might lead to NE1 or Core outcome states. Together with the previous definitions of pivotal, neutral, and null messages, it is possible to define processes that might explain how equilibria such as NE1 or Core are reached.

A *pivotal process* is a particular description of dynamic agent behavior where agents always submit a bid from an available potential profit increasing pivotal bid sequence rather than allowing the mechanism to terminate. In Chapter 5 it will be shown that such a process yields NE1 outcomes in a particular auction-like process. Unfortunately, there will probably be many more NE1 than Core outcomes, and it is desirable that a mechanism has a possible dynamic that will lead agents away from inefficient NE1 to the Core. The *strong neutral process* , defined in Chapter 5, fills this need and is described briefly in the next paragraph.

Recall that in a STIM, any non-null message will reset the timer. In the absence of potentially profitable pivotal messages for individuals, as is the case at an NE1, agents might negotiate joint behavior by submitting neutral bids. These messages would both act as signaling behavior from one agent to another and reset the mechanism timer so that other agents could have time to consider their situation. Eventually the series of (individually) neutral messages might form a pivotal joint message and move the mechanism state towards the Core. This dynamic, which relies on the submission of neutral bids, will be called the *strong neutral process* and will be shown, in Chapter 5, to possibly lead to more efficient outcomes than the pivotal process.

CHAPTER 3. Performance Requirements and the Elements of Experimental Testbeds

Figure 3.1 shows a map of the rail network in Sweden. Much of the track in outlying areas is single track, which can only support travel in a single direction at a time. Double track can be thought of as equivalent to two single tracks, and is typically used to provide bi-directional traffic flow. Gothenburg and Malmo are major seaports, and are connected to Stockholm by double track. Many of the rural areas have only single track. The network of rails is complex even though Sweden is a relatively small country, and it follows immediately that size and complexity due to size are serious problems.

This chapter is organized as follows: Section 1 is devoted to introducing basic concepts regarding trains, schedules, allocations, and feasibility. Physical constraints are introduced that the allocation process must satisfy. Section 2 defines a list of important economic and technical issues in the debates over rail decentralization and defines a rail scheduling problem that contains all of these issues. This list of issues and the associated testbed form the central problem to be considered in the rest of the research. Section 3 is devoted to issues of size and computational difficulties due to size, and creates several experimental testbeds of varying size and complexity using the rail scheduling problem of Section 2 as a building block. The mechanism design and experimentation efforts of the remainder of the thesis are organized around these testbeds.

3.1 Basic Physical Requirements: The Feasibility of Train Schedules

As multiple tracks can be considered to be a collection of single tracks, it would appear sufficient to consider, at least initially, the scheduling problem for single tracks in isolation. Also, since over 80% of the total track mileage in Sweden is single track, a study of scheduling on single tracks is directly relevant to much of the network. Figure 3.2 shows a very simplified rail scheduling diagram for a small period of time on a hypothetical single track rail line. The horizontal and vertical axes of the scheduling diagram correspond to time and location respectively. The vertical line has a location, Stockholm, at the bottom and a location, Borlange, at the top. At the middle point of the line, Uppsala, a single sidetrack is assumed to exist[18]. The locations on this simple system of railroad tracks can be indexed as the set X.

A single train can be interpreted as a function from time to a location on a system of tracks. In the notation to be used, a train is a function, f(t), where t is

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[18] In reality, the Borlange to Stockholm route is an important traffic corridor and has multiple track, not single track as used in the above scheduling diagram. The use of names of major Swedish cities here and elsewhere is solely for motivating the examples. The locations could have been called City X and City Y and Sidetrack Z, but with the trains also using letter names one quickly runs out of letters. The reader is reminded that the purpose of the research is not to simulate decentralized allocation of the actual Swedish rail network. Instead, the purpose of the research is but an initial exploration of the possibilty of decentralized allocation of a rail network. The research is being motivated through a series of example "testbed" rail network environments rather than appeal to the actual rail network in use in Sweden.

understood to be an element of a well defined set[19], $\boldsymbol{T}$, such as time of day and f(t) is understood to be a point on a graph representing locations, $\boldsymbol{X}$, on a system of railroad tracks. For example, in Figure 3.2 the line G represents a train that starts early and moves from south to north along the track. The curve A is a train that would start later than G and travels faster reaching Borlange before G would arrive. Curve B is also a train that moves from south to north, but the horizontal portion indicates that it pulls to the sidetrack and stops at Uppsala. Curve C is a train that would start in the north and move southward along the tracks. Seven different trains are shown in Figure 3.2.

From the figure, a notion of feasibility of an allocation can be obtained. If both trains A and train C operated, there would be a head-on collision at the location and time of intersection of the two lines. Similarly, C and G would involve a collision as would A and G. Because A is faster than G, it would run into the rear of G at the time and location of the intersection. Some collisions can be avoided if a sidetrack exists. Thus trains B and C do not collide because B pulls to the sidetrack at Uppsala and lets train C pass. Notice the line representing train B is horizontal, indicating that the train is not moving. The train waits for train C to pass before continuing on to Borlange.

[19] In this notation I ignore the fact that only certain functions can represent trains. Technical restrictions (e.g., piecewise continuity) on the mathematical representations are not imposed because they play no real role in the analysis that follows.

Trains and collisions are not the only consideration for feasibility. Track congestion can be a problem especially if equipment failures occur. Stopping distances require a safety margin between trains, especially in the winter when friction is low. Thus feasibility can involve constraints that require that either train A or train B can operate, but not both. These two trains would leave Stockholm so close together that safety regulations would be violated.

A feasible schedule is a set of trains that do not collide and do not violate other side conditions like safety regulations. Constructing the set of feasible schedules[20], i.e., those with no collisions, is essentially the construction of a "production possibility set" for use of the track. As might be obvious from Figure 3.2, the set of feasible schedules and thus any production possibilities set, is neither smooth nor convex. In Chapter 4, analysis of the feasible set will reveal a "binary conflict property" that will be useful in characterizing the feasible set and in the construction of the BICAP allocation mechanism in Chapter 5. For now it is sufficient to note the requirements that the application framework of railroads imposes on the feasible set: that collisions must not occur, and that other constraints can be included as necessary.

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[20] Which will be a set of sets.

3.2 Technical and Economic Issues: Performance Requirements

Historically, scheduling has been seen primarily as a technical problem and not as an economic/political problem. Without the aid of computer technology[21], scheduling is an incredibly complicated task, and has concentrated on identifying a feasible schedule and then modifying it incrementally when changes are necessary. In Sweden, scheduling has typically consisted of ranking trains in priority and then resolving conflicts as they occur (requiring one train to wait or excluding trains) based on the priority ranking. Schedules are fine tuned by rules developed in administrative committees, and new services are added incrementally into previous schedules as possible.

The priority system has been criticized for many reasons, many of which can be summarized as saying that access to track is not allocated to the users who value access the highest[22]. The system is defended by those who hint that because of

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[21] For a discussion of computerized rail scheduling techniques, see the review article of Petersen, Taylor, and Martin (1986). Many models of rail operations seem to fix trains at their maximum possible speed while in motion. A notable exception is Kraay, Harker, and Chen (1989), who study how train speed should be varied to meet an objective function based on travel time and fuel consumption given the constraints that trains must stop for meets and passes to occur. In general, this literature assumes the viewpoint of a central dispatcher who wishes to maximize some function, and has some sort of administrative powers. The literature does not address issues of decentralized agents with conflicting objectives that are determining the allocation through some competitive process.

[22] The current system of priorities involves three classes. The priority ranking order is express trains first, general passenger trains second, and freight trains third. Coopers and Lybrand admit that "Unfortunately, such a hierarchy of priorities, if rigidly applied, can not allways reflect the relative value of services, since there are some marginally profitable inter-city (passenger) services and some highly profitable freight services. " In general they claim that the priority ranking does approximate the general trend in the cost of adjusting schedules: the freight train usually has less strict scheduling and can be delayed at lower cost than the passenger train.

certain technical features, nothing else is likely to work (see note [2] above). No one can be quoted as taking such an extreme position, but the references to "lack of international experience" with any alternate institutions indicate that a type of skepticism exists that any new process needs to overcome before it can be seriously studied. This section contains ten issues that have surfaced in the controversy. The first six are issues raised by those who defend the current system of priorities against those who advocate decentralized and competitive access to the use of tracks. The final four are issues raised by those who criticize the system of priorities as being insensitive to efficiency improving possibilities.

In discussing the issues, the railroad system in Figure 3.2 will be used to illustrate the points. In addition the values in Table 3.1 will be used. Table 3.1 outlines values placed on trains by ten potential users numbered as agent 0 through agent 9. Each agent has an additive preference for the seven trains labeled A through G.

The issues listed here and the example from Figure 3.2 and Table 3.1 are more than just an illustration. The set of train schedules from Figure 3.2 will be used in the next section to define the 1T7[23] experimental testbed. The 1T7 testbed will be used in the experiments of Chapter 5, and as a building block in creating the three-track

[23] Train schedules in the experimental testbeds are identified by a number indicating the number of tracks, then "T," then the number of trains under consideration. Since Figure 4 has one track with seven proposed trains, it will be called the 1T7 testbed. While many other scheduling environments are also possible with one track and seven trains, the only such scheduling environment used in the discussion in the thesis is the scheduling environment of Figure 4. Therefore the reader should interpret the term "1T7 testbed" as referring to the particular train schedules shown in Figure 4.

testbeds that are used in the experiments of Chapter 8. The preferences in Table 3.1 are taken from one of the patterns of preferences that will exist in the experimental testbed. The point of this section and the next is not only to explain the controversy but to also show that the controversial elements are actually present in the testbed.

Non-Track Constraints. How can safety considerations and other non-track constraints be guaranteed if decentralized competitive allocation takes place? Consider again Figure 3.2. It is stated on Figure 3.2 that trains such as the pair A and B, the pair C and D and the pair E and F are too close and thus can not be operated together. If one runs then the other can not run without violating a safety standard. This a constraint on the Feasible Set of Allocations and any mechanism for allocation must be capable of recognizing such constraints. Efficient allocation would require that no such pairs operate and that any process of insuring that non-track constraints be satisfied should not prohibit more than is necessary.

Schedule Interdependency. The network in Figure 3.2 suggests the many complications that arise from schedule interdependencies[24]. Suppose agent 1 operates early from Stockholm and from a choice of A or B wants to take A. Agent 2 operates from Borlange early and prefers C from a choice of C or D. However, 2 is persuaded by 1 to choose D, which does not conflict with A. However, a choice of

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[24] Coopers and Lybrand state: "There is a high degree of interdependence between trains sharing the same network. Trains cannot easily pass each other, particularly on the single track lines which make up a high proportion of Sweden's network. Neither is there space for trains to queue. Therefore, a change in the plan for one service often means that the plans of many other services have to be altered to fit and provide a workable timetable." (p.291)

D has an impact on agent 3 who operates from Stockholm at a time later than 1, and who wants to choose E from the two options available, E and F. If agent 2 runs train D, then agent 3 can not run train E because they are in conflict.

Revelation of Values. How can the private values of independent train operators become exposed and used in a competitive process?[25] A type of "free rider problem" or a game of "chicken" seems to exist. Consider an agent that would like to implement train G. This train is in conflict with all of the trains in the set { A,B,C,D,E,F }. If G is to operate, it must somehow preclude all of these trains, or if one or more of these trains operate, then train G must be precluded. If the set of rights to operate trains {A,B,C,D,E,F} are held by different operators, then they must either be paid, thereby creating a "holdout problem" for the operator of G who must strike a price with each individually, or if the operator of G has the right to operate, then these different operators must collectively pay the G operator, thereby creating a type of public goods problem among themselves. In both cases the independent operators could have an incentive to misrepresent their true values of operations.

Resource and Market Fragmentation. If a classical market process is to be used, then the number of potential markets would be large. How would the resources be

[25] Coopers and Lybrand are concerned with insuring that the market process is not abused. "(bidding) process ... will need to address a number of practical issues, including: (a) limiting the potential for gaming by operators by providing misleading information, such as understating or overstating requirements, aimed at influencing other operator's bids during the various stages of the bidding process." The interesting question is why they do not believe that such problems would plague an administrative system. In the auction process to be studied, agents bid on pre-specified routes, so misspecification of routes is not an important concern at this point. Agents could underbid or overbid, but there are consequences for bidding too little or too much.

defined? It is possible to divide the tracks into mile by time squares and have a market for each. Given the system represented by Figure 3.2, a natural division would have the track divided into three segments (Stockholm - Uppsala, sidetrack at Uppsala, Uppsala - Borlange) and time divided into four segments (morning, mid-day, evening and night). This would create 12 resources. For the example this number of markets might not be so onerous, but for more complex tracks this is going to require a large number of markets, possibly raising transactions costs[26] to both operators and the seller.

Strong Complements. If the "multiple independent market" approach mentioned in the paragraph above is used, then some resources will be strong complements. As a result, competitive equilibria may not exist that support the Pareto-optimal outcome. In addition, complements can cause difficulty in dynamic models of market price convergence.[27] To demonstrate some potential difficulties, only a static notion of equilibrium is necessary. Consider the following example of the problems involved when strong complements are combined with indivisibility in a railroad allocation problem.

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[26] Coopers and Lybrand assert that "The transactions costs of developing and managing the process for an effective price-based system are likely to be high and may offset any efficiency gains over an administered system based on rules and negotiation ..." (p.294)

[27] Arrow and Hurwicz(1958) discuss the dynamic stability of market prices in the two-goods case and conclude that complements and stability are mutually exclusive.

Suppose only routes G, A, and E from Figure 3.2 are of value and that the commodity space is as defined above. Under the circumstances assume that the whole track is sold and as defined by two times of day, morning and mid-day. This gives only two commodities. Assume that agent 1 is willing to pay $10 for both the morning and mid-day use in order to operate G, but otherwise places no value on the tracks. Agent 2 would pay $7 for either the morning or the mid-day in order to run train A or train E but does not want to run both. The optimal allocation is for agent 1 to own both and for agent 2 to have neither. Yet, competitive prices for both morning and mid-day must be above $7 to exclude agent 2, but at such prices agent 1 does not wish to buy. The optimum can not be supported as a competitive equilibrium.

Competitive Equilibrium Existence. The presence of strong complements and indivisibilities can affect the existence of any static competitive equilibria. In the previous example involving strong complements, no competitive equilibria exist[28]. At any set of prices there will always be excess demand or excess supply of track time slots. It was shown in the previous example that the optimal allocation (where both slots are allocated to agent 1) can not be supported as a competitive equilibria because at prices where agent 1 will demand both slots, agent 2 will demand one of the slots as well. For prices where agent 1 does not demand but agent 2 does, agent 2

28 Coopers and Lybrand do not consider a market model where tracks access is broken up into small time-distance segments. But they do claim that "The process for defining the products to be traded is complex, it is difficult for market-clearing prices to emerge, and attempts to simplify the task may lead to inefficient distortions." (p.293) The non-existence of market clearing prices may be a real problem if strong complements are involved.

demands only one slot. This leaves an excess supply of one slot. Therefore, there are no competitive equilibria prices, as all prices lead to excess demand or supply.

Now consider some issues from proponents of decentralization. Decentralized allocation might improve upon some known failings of the (previously used) priority system of allocation. If efficiency is 100%, then the mechanism would circumvent all such failings, but substantial improvements might still be gained from an inefficient mechanism that avoids certain major difficulties about to be discussed.

Priority and Substitution Between Users or User Types. Suppose an agent is given priority over other agents. If G is the most valuable route to any user with priority then it would be implemented. For example, if agent 0 is given the right of priority for a single train such as G then, as can be ascertained from Table 3.1, train G would operate at a value of 1604. But there are many options that have greater value than G. In particular, B, C, and E held by agents 1, 0 and 2 respectively have a combined value of 3022. Under the priority system, there is no incentive for the three trains run by different users to be substituted.

Priority and Combining Trains. Suppose that fast trains have priority over slow trains and that agent 0 is operating fast trains but had no priority for a slower train such as G. As can be seen from Table 3.1, the value for G to agent 0 is 1604. The best feasible fast trains to this agent is the set of three trains {B,C,E}, that total in value to 1134. The agent has no incentive to combine trains if the result is a slower train because priority, and thus the trains, would be lost.

Priority Gives no Incentive to Wait. If agent 7 has priority with north to south fast trains, then the agent has no incentive to delay and wait. Given the preferences of Table 3.1, agent 7 would operate train A even though another agent such as agent 0 must delay and run train D rather than train C. Agent 0 values train C by a difference of 337 over D while agent 7 values A , which forces agent 0 to delay, to train B, in which agent 7 waits, by only a margin of 102. Thus an allocation in which Agent 7 waits as opposed to Agent 0 would increase total value by 335. With priorities there is no incentive for this to take place.

Priority Systems Do Not Respond to Changing Circumstances. If the track authority always assigns priorities correctly, then an efficient allocation is often possible and depends on the ability of the priority rule system to span all feasible schedules. However, to assign priorities correctly, the track authority must gather the necessary information from independent operators or operating divisions in order to make these decisions. It may not always be in the interests of the operators to truthfully reveal this information. Furthermore, as circumstances change, the information must be gathered again and again. Apparently, the criticism that "access to track is not allocated to the users who value it the most" directly attacks the ability of the track authority to gather this information using the current administrative processes.

Changing circumstances in the testbeds are introduced by using the additional sets of train values in Table 3.2 that, in experiments, would be used in different periods.

3.3. Size and the Experimental Testbeds

To address the problem of size, several experimental testbeds are required in which size and computational difficulty related to size are varied. The effort of section 3 identified a set of non-size issues that proponents and opponents of decentralization feel are important in rail allocation. The set of train schedules and redemption values defined in the previous section will be known as the 1T7 testbed.

As the 1T7 testbed contains economic problems thought to be important in arguments concerning decentralization, the question is how to build upon this environment by both maintaining all of its current technical and economic difficulties and increasing the size and complexities inherent due to size. Two larger testbeds that will be constructed from the 1T7 testbed will be known as the three-track testbeds, and, in particular, the 3ST7 and 3NST7 testbeds (3 tracks, Separable or Non-Separable, 7 trains per track). The 3ST7 testbed will consist of three copies of the 1T7 testbed on three separate, independent tracks. This results in increased size over the 1T7 testbed as there are now 21 trains instead of 7 trains. The 3NST7 testbed consists of the same three copies of the 1T7 testbed with interdependencies between the schedules of trains on the different tracks. This results in increased size over the

1T7 testbed, but also increased schedule interdependencies over the 3ST7 testbed. Not only is size increased but also the computational difficulties due to the additional schedule interdependencies.

The 3ST7 testbed is constructed as follows. Consider the railroad scheduling diagram of Figure 3.3. The system of tracks consists of three geographically isolated single track lines. One track line connects Stockholm to Borlange (in the central part of Sweden), the second track line connects Umea to Lulea (in northern Sweden), and the third track line connects Malmo to Gothenburg (in southern Sweden). The three single track lines are assumed to be geographically isolated and not interconnected. Because of the separation, there are no constraints which affect scheduling on one track given scheduling on a different track. Suppose that for each line operators are interested in running trains exactly like the trains A-G of the 1T7 testbed. On the Stockholm-Borlange line call these trains A-G, on the Umea-Lulea line call these trains H-N and on the Malmo-Gothenburg line call these trains O-V.

Because there are now three times as many trains, three times as many train redemption values are needed. The redemption values for the 1T7 testbed are reused for the three-track testbeds in an obvious way: each independent of the three-track in the three-track redemption values takes its train redemption values from a different set of redemption values for the 1T7 testbed. The correspondence between the redemption values in the two testbeds is given in Tables 4-6.

Clearly, this environment still addresses all the issues of the previous section, but size has been increased through duplication. This increase in size should not introduce new economic problems, if formulations of the feasible set of allocations and formulations of allocation mechanisms in the next chapters can exploit the nature of this duplicated environment.

As well as suggesting behavior to be expected of characterizations of mechanisms and feasible sets, the nature of the 3ST7 environment suggests that further modifications are necessary to the testbed to claim that the problems of increased size have actually been adequately addressed. One of the primary computational problems related to size is the issue of schedule interdependency. It is this issue that complicates the form of the feasible set of allocations in section 2. To increase schedule interdependency, it is necessary to add scheduling conflicts to the diagrams above and cause scheduling on the three independent tracks to become interdependent.

The 3NST7 testbed starts with the 3ST7 testbed and adds conflicts between trains on different track lines. This can be motivated as follows: the cities in the diagrams finally become part of a larger rail network. The 21 trains in the scheduling diagram are expanded to include new service areas. In these new areas additional conflicts occur between trains. It should be clear that with enough additional tracks and enough extension of the time period, any set of additional conflicts could have been chosen for study. The conflicts between the pairs of trains in {(AH),(AJ),(AO),(BP),(CI),(CK),(CQ),(DR),(ES),(FT) } were chosen because of their effect of greatly increasing the interdependence of schedules among the trains rather than for their ability to represent any particular configurations of trains and track.

3.4 Testbed Preferences for Trains

The train values $\underline{V}$ for the different periods of the 1T7 testbed are given in Table 3.2. The 3ST7 and 3NST7 testbeds also use these values. As was previously discussed, each period of the three track experiments took train values from three of the periods from the one track experiments; the details of this can be found in Tables 3.3-3.5. The details of why and how these particular values were generated will now be addressed.

Issues in section 3.2 play a major role in defining certain patterns of train values that are of particular interest in the Swedish debate. The question of whether a decentralized allocation institution can perform well in terms of numerical efficiency does not constrain the form that the train values take. What constrains the form of the train values is the need to question the performance of the institution, in the laboratory, with regard to the specific issues and situations of the previous section.

In the testbed the train values are determined by five of these constraints. Each of these constraints and situations will now be briefly discussed.

Priority and Waiting. One criticism of priority rules is that no one with priority ever has the incentive to wait. To advance the argument, it is necessary to test whether the decentralized institution can avoid this problem. The criticism of priority contains an implicit assumption about the value of trains, that waiting decreases train value but the priority rule does not always make the right trains wait. A test of the decentralized institution is only interesting when this assumption is satisfied. If the trains B,D,F are interpreted as delayed versions of the trains A,C,E respectively, then a natural condition is that for each agent $i \in I$, $V_{iA} > V_{iB}$, $V_{iC} > V_{iD}$, and $V_{iE} > V_{iF}$.

Revelation of Values and Coordination. Recall that an interesting coordination problem exists when V_{iG} is very large for some agents, but train G is not optimal. Investigation of these cases requires that the value of G be high enough so that the coordination problem exists, but not so high that G is in the optimal allocation. A value of G around 60% of the optimal allocation value was generally chosen.

Revelation of Values and Conflicts of Interest. Each agent may have the maximum value (among all agents) for up to one pair of {A,B}, {C,D}, and {E,F}. Allowing agents to have the high value for both trains in a conflicting pair enables the study of agent bidding behavior when he might be in a situation to bid against himself. Note that in the 3 track testbed experiments, an agent might have up to three pairs of high values, one pair on each track.

Efficiency and Salience. For efficiency to be a salient measure, there must be some separation between the values of the best and second best outcomes. To put this another way, if the efficiency of the 2nd best allocation is 99.9%, it would not be significant from an efficiency viewpoint which allocation was chosen. The values were constructed so that there was such a minimum separation, about 500 francs (experimental currency) or $2.50-$5.

Elimination of Experimenter Effects. It is desirable to try to eliminate any unintended consistent bias on the form of train values due to the experimenter. For this reason the train values were chosen at random by computer, until the four constraints above were satisfied.

Figure 3.1: Map of Swedish Rail Network.

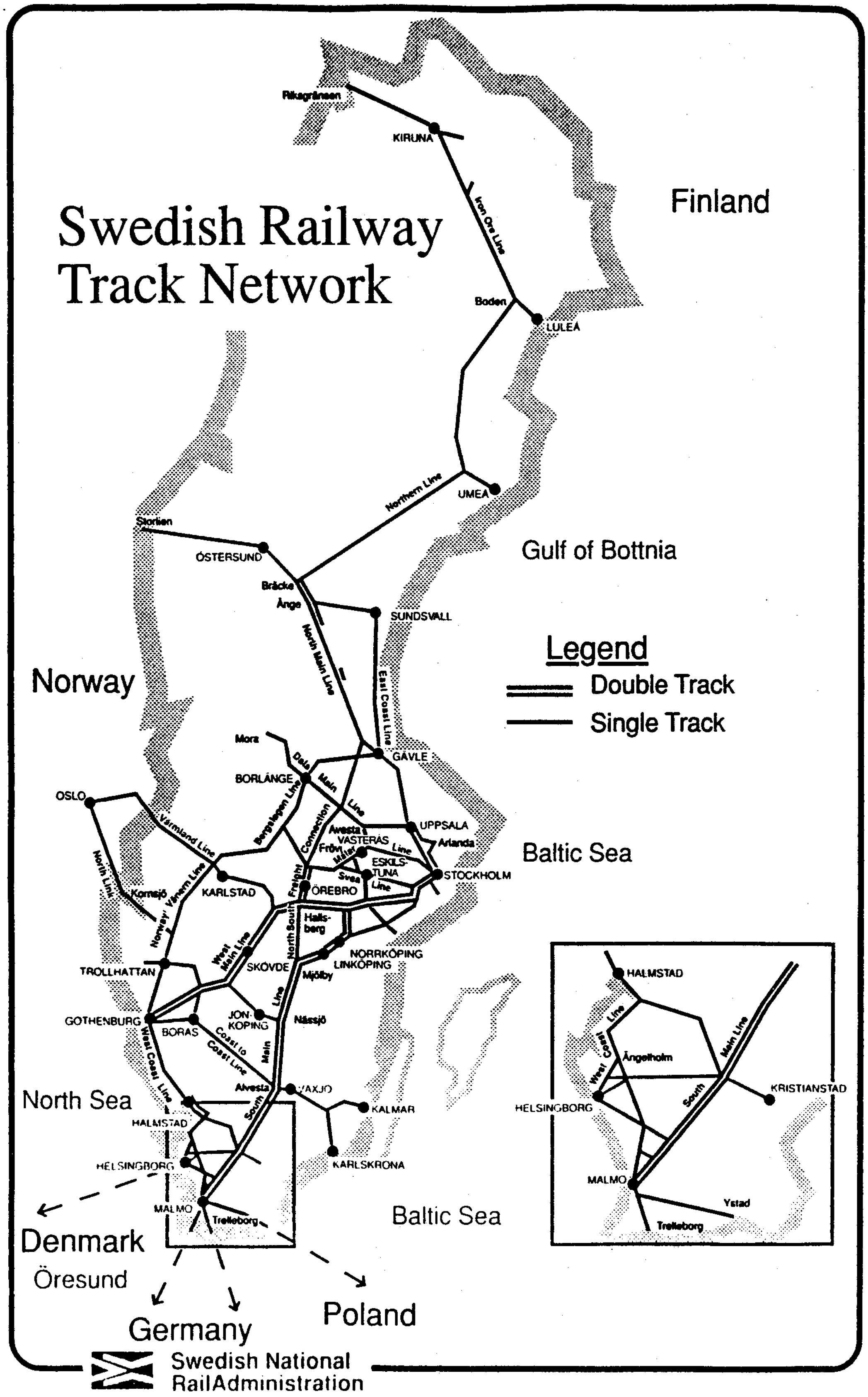
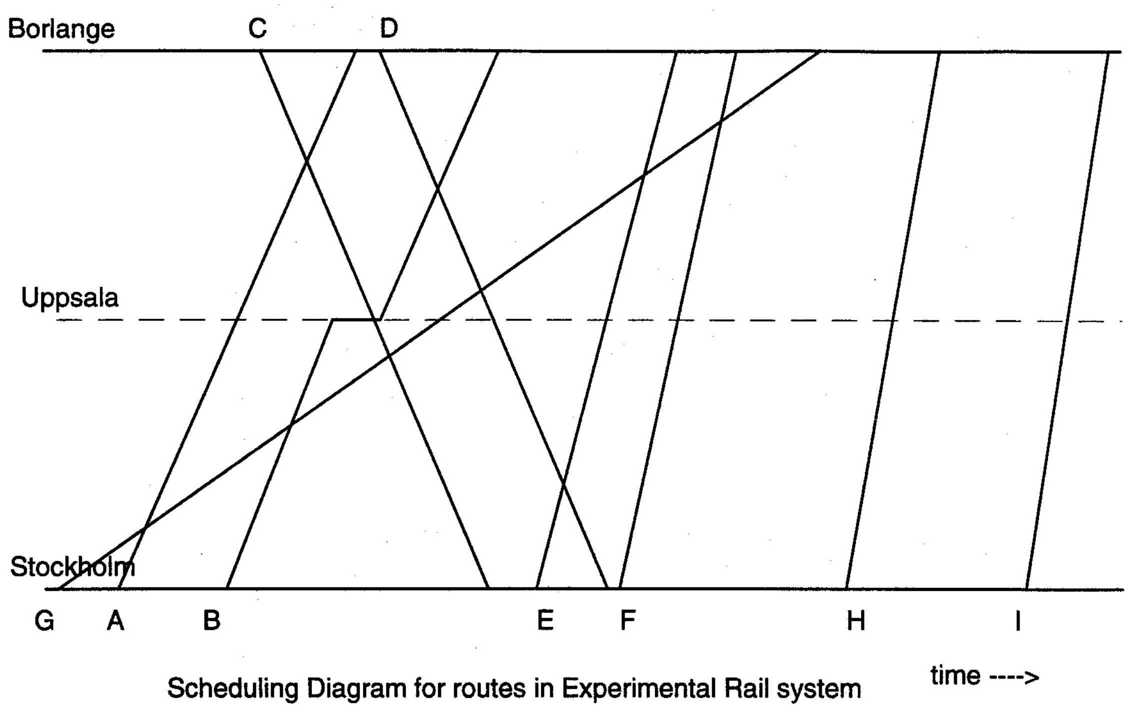


Figure 3.2: The 1T7 Testbed Rail Environment.



Scheduling Diagram for routes in Experimental Rail system

Non-Track Constraints:
Trains A and B may not both be scheduled to run.
Trains C and D may not both be scheduled to run.
Trains E and F may not both be scheduled to run.

Figure 3.3: Train Schedules for the Three-Track Testbeds.

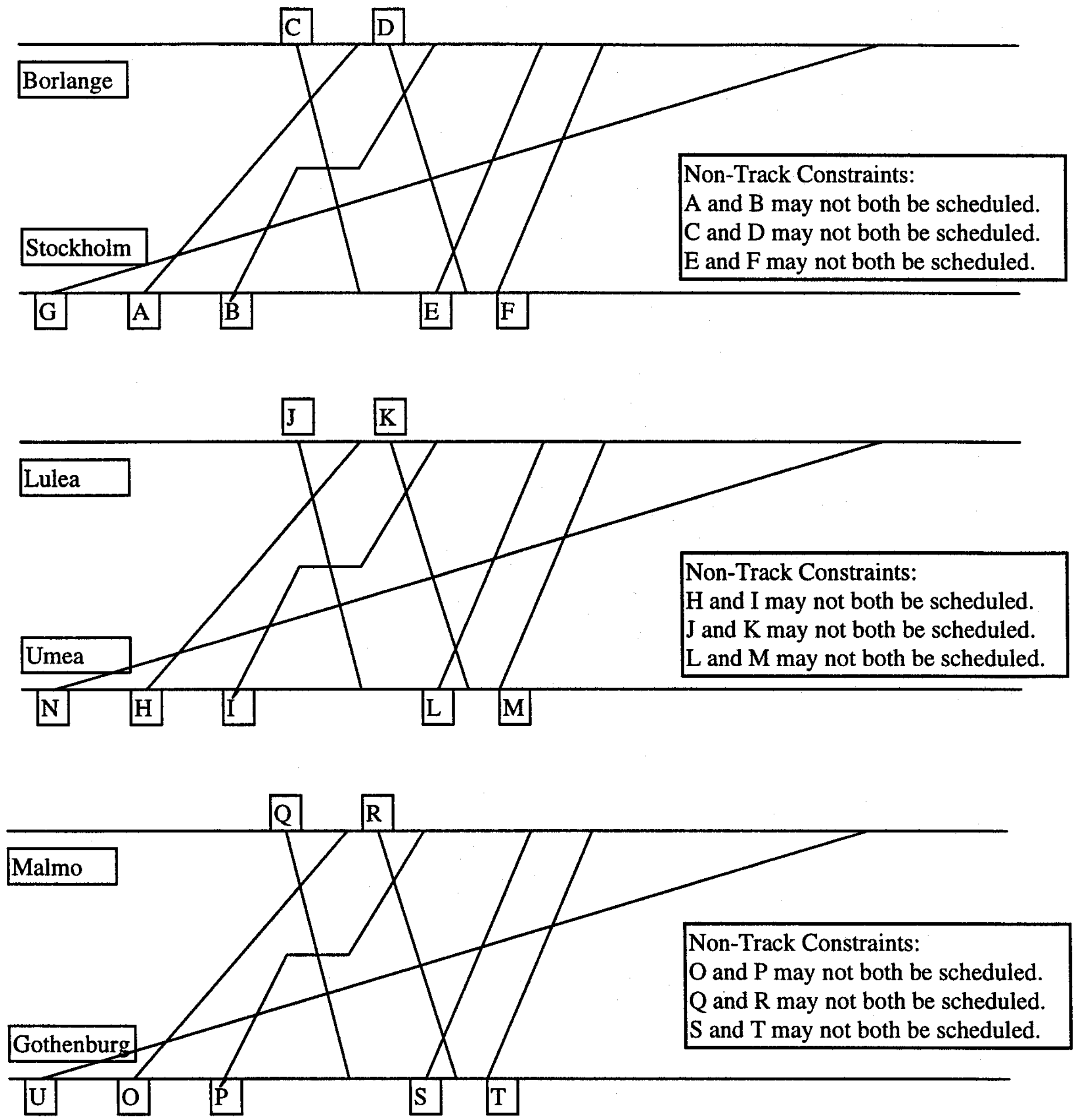


Table 3.1: Example Values for Trains

Agent Id#	A	B	C	D	E	F	G
0	332	232	878	708	746	426	2619
1	946	521	321	241	739	265	2491
2	302	198	307	270	1013	645	1329
3	1699	645	307	206	306	217	509
4	1282	454	1634	1447	341	134	2543
5	801	354	933	465	936	561	2339
6	389	242	387	117	583	348	423
7	320	132	1405	974	528	360	594
8	708	332	309	188	1635	1421	2005
9	372	277	341	138	395	284	1549

Table 3.2: Train Redemption Values V_{jf} For Each Period in the 1T7 Testbed

Period 1

Agent id#	A	B	C	D	E	F	G
0	332	232	878	708	746	426	2619
1	946	521	321	241	739	265	2491
2	302	198	307	270	1013	645	1329
3	1699	645	307	206	306	217	509
4	1282	454	1634	1447	341	134	2543
5	801	354	933	465	936	561	2339
6	389	242	387	117	583	348	423
7	320	132	1405	974	528	360	594
8	708	332	309	188	1635	1421	2005
9	372	277	341	138	395	284	1549

Period 2

Agent id#	A	B	C	D	E	F	G
0	368	133	683	346	320	108	1604
1	1124	980	319	269	340	291	93
2	303	219	335	168	1359	641	373
3	305	171	371	149	524	177	466
4	403	325	463	237	475	382	124
5	692	487	320	267	1027	515	1625
6	405	315	370	194	375	284	570
7	413	311	417	343	430	377	531
8	558	340	354	270	577	224	304
9	362	154	320	96	312	206	1710

Period 3

Agent id#	A	B	C	D	E	F	G
0	425	365	360	116	500	310	598
1	319	241	337	263	463	194	1843
2	528	382	350	117	306	206	1570
3	1858	615	840	662	384	264	412
4	456	376	1227	964	315	105	206
5	660	405	342	217	328	169	1336
6	413	227	314	248	368	257	382
7	448	290	371	274	943	774	1387
8	312	267	1025	657	482	341	247
9	300	109	451	244	309	257	1731

Period 4

Agent id#	A	B	C	D	E	F	G
0	1020	410	788	594	356	187	48
1	883	553	1193	381	537	310	392
2	516	334	768	385	309	106	1533
3	362	147	446	151	455	249	1401
4	496	348	303	128	1300	430	918
5	334	258	312	228	300	174	1386
6	516	222	386	139	1067	812	2057
7	366	157	309	245	652	290	607
8	319	158	597	499	306	247	1135
9	1371	1105	615	439	410	277	130

Period 5

Agent id#	A	B	C	D	E	F	G
0	680	501	347	121	318	283	1589
1	645	302	302	121	340	299	606
2	341	189	699	518	363	153	1636
3	365	151	599	193	873	557	1039
4	650	246	505	255	576	300	1395
5	2108	700	384	263	321	175	1616
6	436	349	726	235	580	356	1999
7	568	438	1162	873	369	246	34
8	301	103	465	194	570	281	1295
9	648	527	760	634	315	267	1470

Period 6

Agent id#	A	B	C	D	E	F	G
0	438	342	353	176	1005	603	514
1	565	398	419	151	405	141	114
2	788	459	675	334	514	360	67
3	300	219	462	179	389	305	214
4	305	111	671	327	342	218	143
5	374	294	669	272	785	471	864
6	527	360	385	218	500	245	1340
7	309	174	347	124	690	243	956
8	408	340	325	231	342	227	645
9	353	210	1341	749	645	397	724

Period 7

Agent id#	A	B	C	D	E	F	G
0	1444	581	308	174	452	270	401
1	480	288	337	224	838	554	54
2	1685	550	648	292	509	418	41
3	635	558	301	127	473	283	710
4	305	220	1071	931	486	266	1260
5	971	394	538	256	335	218	698
6	740	614	415	319	519	301	25
7	835	447	315	127	361	229	331
8	540	341	307	144	517	211	174
9	325	198	316	107	557	169	1133

Table 3.3: Redemption Values for Period 1, Three-track Testbed Environments

Project	Highest Redemption Value	Id #	Redemption values equal to those in 1T7 environment	
			Train	Period
A	1699	3	A	1
B	645	3	B	1
C	1634	4	C	1
D	1447	4	D	1
E	1635	8	E	1
F	1421	8	F	1
G	2619	0	G	1
H	1124	1	A	2
I	980	1	B	2
J	683	0	C	2
K	346	0	D	2
L	1359	2	E	2
M	641	2	F	2
N	1710	9	G	2
O	1858	3	A	3
P	615	3	B	3
Q	1227	4	C	3
R	964	4	D	3
S	943	7	E	3
T	774	7	F	3
U	1843	1	G	3

Table 3.4: Redemption Values for Period 2, Three-track Testbed Environments

Project	Highest Redemption Value	Id #	Redemption values equal to those in 1T7 environment	
			Train	Period
A	1371	9	A	4
B	1105	9	B	4
C	1193	1	C	4
D	594	0	D	4
E	1300	4	E	4
F	812	6	F	4
G	2057	6	G	4
H	2108	5	A	5
I	700	5	B	5
J	1162	7	C	5
K	873	7	D	5
L	873	3	E	5
M	557	3	F	5
N	1999	6	G	5
O	788	2	A	6
P	459	2	B	6
Q	1341	9	C	6
R	749	9	D	6
S	1005	0	E	6
T	603	0	F	6
U	1340	6	G	6

Table 3.5: Redemption Values for Period 3, Three-Track Testbed Environments

Project	Highest Redemption Value	Id #	Redemption values equal to those in 1T7 environment	
			Train	Period
A	1685	2	A	7
B	614	6	B	7
C	1071	4	C	7
D	931	4	D	7
E	838	1	E	7
F	554	1	F	7
G	1260	4	G	7
H	1858	3	A	3
I	615	3	B	3
J	1227	4	C	3
K	964	4	D	3
L	943	7	E	3
M	774	7	F	3
N	1843	1	G	3
O	1371	9	A	4
P	1105	9	B	4
Q	1193	1	C	4
R	594	0	D	4
S	1300	4	E	4
T	812	6	F	4
U	2057	6	G	4

CHAPTER 4. Characterizing Feasible Allocations of Trains: The Binary Conflict Property

Before mechanisms for allocation can be discussed, a representation of the set of feasible schedules $\mathcal{S}$ for a railroad network ($\boldsymbol{F}$, $\mathcal{S}$) is necessary. In particular $\mathcal{S}$ or a simple function that will provide it is needed for all the experimental testbeds. A specification for the resource rights policy (Λ, Γ_B , Γ_S) may also prove helpful in simplifying the construction of an allocation mechanism.

The chapter is organized as follows. Section 4.1 examines two simple choices for a resource rights policy, and makes a decision regarding what resource rights policy will be chosen for the research. Section 4.2 examines a special property of the set of feasible schedules of the Chapter 3 testbeds. This property will be used to characterize the feasible set in the chapters that follow. Section 4.3 further examines the properties of binary conflicts, via an examination of a related graph.

4.1 Choosing a Resource Rights Policy

Consider two simple choices of a resource rights policy. Resources could be specified as either the right to run entire trains or the right to use specific slots of track-time. Setting up the set of resources in the latter manner will result in a large number of resources. Strong complements and other undesirable properties will be present, as shown in examples of Chapter 3. In addition, Banks et al. (1989) consider exactly such a specification for allocating "uncertain and unresponsive" resources on NASA missions. They resolve many of the difficulties involving strong complements by allowing all-or-nothing bids contingent on receiving an entire bundle of resources. Such an approach has an immediate interpretation in terms of trains: one defines a resource for the use of each location $\boldsymbol{X}$ on the track at each time $\boldsymbol{T}$, so that $\Lambda=\boldsymbol{X}\otimes\boldsymbol{T}$. Resources would also have to be defined so that non-track constraints could be satisfied. Bids would involve agents specifying a willingness to pay for a specific all-or-none package of resources. As this study is not an attempt to duplicate their work, a somewhat different approach will be taken: a resource will be the right to run a given train on the tracks, so that $\Lambda=\boldsymbol{F}$. The major effects of this choice of resources upon the definitions of section 2.2 are considered below.

Definition. A *direct* resource rights policy $(\Lambda, \Gamma_B, \Gamma_S)$ for a railroad network $(\boldsymbol{F}, \mathcal{S})$ satisfies:

$$\Lambda = \boldsymbol{F},$$

$$\Gamma_B(\Lambda') = \Lambda', \quad \forall \Lambda' \subseteq \Lambda \ (= \boldsymbol{F})$$

$$\Gamma_S = \mathcal{S}$$

Proposition 4.1. If any subset of a feasible schedule is also a feasible schedule, then for a direct resource rights policy on a railroad network, the physical compatibility constraints are implied by the budget constraints

Proof. It will be shown that budget constraint (B5) and (B2) implies physical compatibility constraint (P1), and budget constraints (B5), (B6) , (B3) and (B1) imply physical compatibility constraint (P2).

Starting with budget constraint (B5)

$\boldsymbol{F}_i \subseteq \Gamma_B(\Lambda_i)$, and since in a direct resource rights policy, $\Gamma_B(\Lambda') = \Lambda'$, then $\boldsymbol{F}_i \subseteq \Lambda_i$. This implies $(\boldsymbol{F}_j \cap \boldsymbol{F}_k) \subseteq (\Lambda_j \cap \Lambda_k)$, but budget constraint (B2) states

(B2) $\forall j,k \in I, j \neq k \Rightarrow \Lambda_j \cap \Lambda_k = \varnothing$

which implies $\forall j,k \in I, j \neq k \Rightarrow \boldsymbol{F}_j \cap \boldsymbol{F}_k = \varnothing$.

This is physical compatibility constraint (P1).

Starting from budget constraint (B5) and $\boldsymbol{F}_i \subseteq \Lambda_i$, and applying it to

(B6) $\boldsymbol{F}_S = \cup_{j \in I} \boldsymbol{F}_i$, yields $\boldsymbol{F}_S \subseteq \cup_{j \in I} \Lambda_i$.

But (B3) and (B1) say that $\cup_{j \in I} \Lambda_j = \Lambda_S \in \Gamma_S$, so

$\boldsymbol{F}_S \subseteq \Lambda_S \in \Gamma_S$.

But in a direct resource rights policy $\Gamma_S = \mathcal{S}$, so if $\mathcal{S}$ has the additional property that

$\boldsymbol{F}'_S \subseteq \boldsymbol{F}_S, \boldsymbol{F}_S \in \mathcal{S} \Rightarrow \boldsymbol{F}'_S \in \mathcal{S}$

then physical compatibility constraint

(P2) $(\cup_{i \in I} \boldsymbol{F}_i) \in \mathcal{S}$ is satisfied •

Proposition 4.1 implies, under suitable assumptions, a direct resource rights policy will allow the search for a resource allocation mechanism to be confined to those that satisfy budget constraints. It is unnecessary to consider mechanisms that take further steps to insure physical compatibility.

The assumption that all subsets of feasible schedules are themselves feasible schedules is equivalent to the idea that infeasibility is the result of a conflict between uses or trains. In the case of trains, it would appear that collisions and other types of infeasibilities can be described by a set of two-train, or binary conflicts. This idea is more fully investigated in the next section.

4.2 Conflicts and The Binary Conflict Environment

If the basic allocational unit is the right to run a train, the form of the constraints on the feasible set of allocations are closest to the constraints considered in the airport slots problem of Rassenti, Smith, and Bulfin (1982). The constraints are *integer programming* constraints indicating that certain combinations of (train schedules, airplane schedules) are infeasible. Both the rail allocation and the airport slots problem have a *conflict property* : an allocation is infeasible if and only if it contains a conflict. The feasible sets for the specific testbed environments developed in Chapter 2 are shown to have a *binary conflict property* which will be exploited in the design of the mechanism and the experiments.

The *binary conflict property* is that infeasibility of a schedule necessarily involves a conflict between two trains. If many trains collide, then two trains also collide and two is unacceptable. If many trains travel too close, then two trains travel too close and two is unacceptable. Thus any conflict implies that a pair is in conflict. This is an assumption which is known to be true only in the testbed environments. Whether it is true in the field is a question for railroad engineers, but one suspects that suitable specifications of the sets of times and locations could cause all conflicts to be of the binary type. Much depends on the exact form that the non-track constraints take.

A *Binary Conflicts Technical Environment* as applied to a rail allocation problem is defined as a quintuplet BCTE($\boldsymbol{T}, \boldsymbol{X}, \boldsymbol{I}, \boldsymbol{F}, \boldsymbol{C}$). $\boldsymbol{T}$ is a set of times necessary to describe the paths of trains. $\boldsymbol{X}$ is the system of tracks, the set of locations for trains. $\boldsymbol{I}$ is the set of individual agents that would like to have access to the tracks. $\boldsymbol{F}$ is a set of all trains that might operate on $\boldsymbol{X}$, i.e., $\boldsymbol{F} \subseteq \{ f(t): \boldsymbol{T} \rightarrow \boldsymbol{X} \}$. $\boldsymbol{C}$ is a set of binary conflicts, a subset of $\boldsymbol{F} \otimes \boldsymbol{F}$, that specifies pairs of trains that are incompatible. Incompatibility means that the two trains would collide if run or that if the two trains run then some other safety standard would be violated. For example, if $y=f^{A}(t) = f^{B}(t)$ then the two trains A and B would collide at location y at time t. If all trains are compatible, then $\boldsymbol{C}$ is the empty set. Symmetry would be a natural property of $\boldsymbol{C}$.

As shown in the following definition, only the sets ***I, F*** and ***C*** from the binary conflict technical environment BCTE(***T, X , I , F , C***) are necessary to determine the set of feasible schedules.

<u>Definition:</u> In the case of binary conflicts , the *binary conflicts feasible set of schedules* $\mathcal{S}_{BC}$(**F,C**) is the set of train schedules with no conflicts. That is, $\mathcal{S}_{BC}(\mathbf{F},\mathbf{C}) = \{ \mathbf{F'} \subseteq \mathbf{F} : (\mathbf{F'} \otimes \mathbf{F'}) \cap \mathbf{C} = \varnothing \}$. The *binary conflicts feasible allocations* $\mathfrak{I}_{BC}$***(I,F,C)*** are the allocations under the direct resource rights policy that satisfy the budget constraints defined in Chapter 2, using (***F***, $\mathcal{S}_{BC}$(**F,C**)) as the definition of the railroad network.

4.3 Properties of Binary Conflict Technical Environments: The Binary Conflict Graph

Because of the nonconvexities, the concept of a binary conflict environment and the use of the graph theoretic formulations will facilitate the computation of solutions to the problems that will exist. A *binary conflict graph* contains, in graphical form, the essential features of a binary conflict environment necessary for determining feasibility. Each train in ***F*** is represented as a point in the graph and each conflict in ***C*** is represented as a line that connects the pair of points representing the conflicting pair of trains. The only significant parts of the graph are the points and whether given points are connected by lines.

The physical arrangement of points on the graph or the fact that lines cross each other is coincidental and does not convey any additional information. It is important to realize that the graph simply conveys information about a binary conflict environment. It is not an extension to the environment. Any general statement about properties of the graphs is actually a statement about binary conflict environments in general, although it may be easier to see the logic of the statements by looking at the graphs.

Figures 6 , 7 and 8 show the binary conflict graph for the 1T7, 3ST7 and 3NST7 testbeds respectively. Some useful properties of these graphs for the different testbeds become immediately apparent.

One of the most useful properties of the binary conflict graphs is that the exclusion or inclusion of a train from a schedule immediately results in another binary conflict graph. For instance, if train A is included in a schedule in the 1T7 testbed, then we know that all the trains that conflict with A are infeasible: namely trains B and C. Trains B and C and the lines representing conflicts due to B and C can then be removed from the binary conflict graph resulting in a reduced allocation problem.

Notice in all the testbeds, schedules of trains A and E are interdependent in the following sense. Suppose that the current schedule has trains {B,C,E} running, but because of problems train A must be substituted for train B. If train A is run instead of train B, then train C can not be run and train D could be run instead. Running train D, however, would preclude train E. Thus a decision to run train A can affect the decision of whether or not to run train E even though trains A and E do not conflict directly. Schedule interdependency is only possible if the trains share some mutual conflicts, that is indicated in the conflict graph in the figures by a path of lines from train A to train E. This suggests a general method for using the binary conflict graph to determine whether schedules are interdependent:

Test for Schedule Interdependence: In order for the scheduling of train x to affect the feasibility of scheduling another train y, it is necessary and sufficient that, in the conflict graph, either (i) train x and y are directly connected by a line; or (ii) there is a path of lines in the conflict graph going from x, through other points, and ending at y.

If schedules are not interdependent, then they are independent. In the binary conflict graph this is indicated by a lack of line paths. Because the tracks in the 3ST7 testbed are defined to be geographically isolated, scheduling of trains on one track is independent from scheduling on another track. In particular, scheduling of trains {A,...,G}, { H,...,N} and {O,...,U} are mutually independent. This is indicated in the binary conflict graph by the fact that there these groups of trains are path disconnected. There is no line path from a point in {A,...,G} through any other points in the conflict graph to a point in {H,...,N} or {O,...,U}. This observation suggests a test for schedule independence based on conflict graphs:

Test for Schedule Independence: If X_1 and X_2 are subsets of the conflict graph and there is no path of lines in the conflict graph connecting any point in X_1 with any point in X_2, then the feasibility of scheduling trains in X_1 is completely independent of the trains scheduled in X_2 and vice-versa.

Information concerning whether two groups of trains can be scheduled independently without consideration of each other is an important piece of information for an allocation mechanism, as is being able to easily construct a reduced allocation problem when decisions regarding a few trains have been made. The binary conflict graphs doubtlessly have other properties that can aid in the design of optimization processes for markets or other institutions.

Figure 4.1: Binary Conflict Graph for the 1T7 Testbed Environment

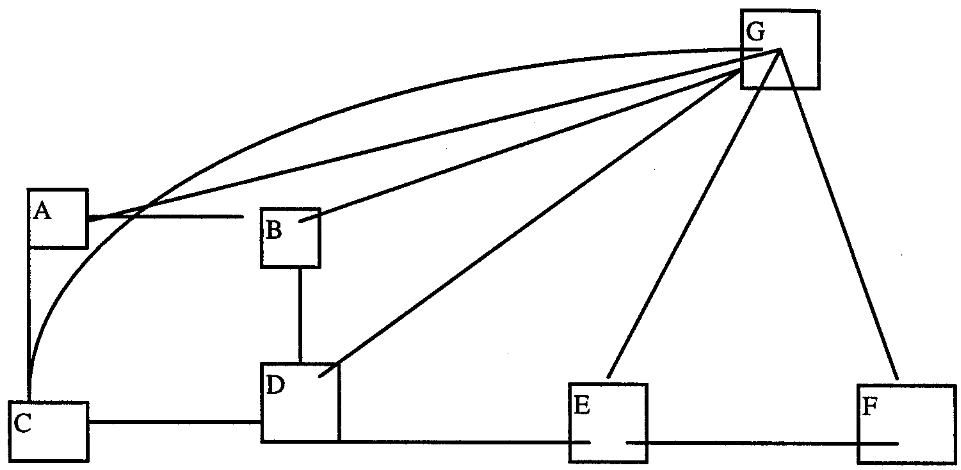


Figure 4.2: Binary Conflict Graph for the 3ST7 Testbed Environment

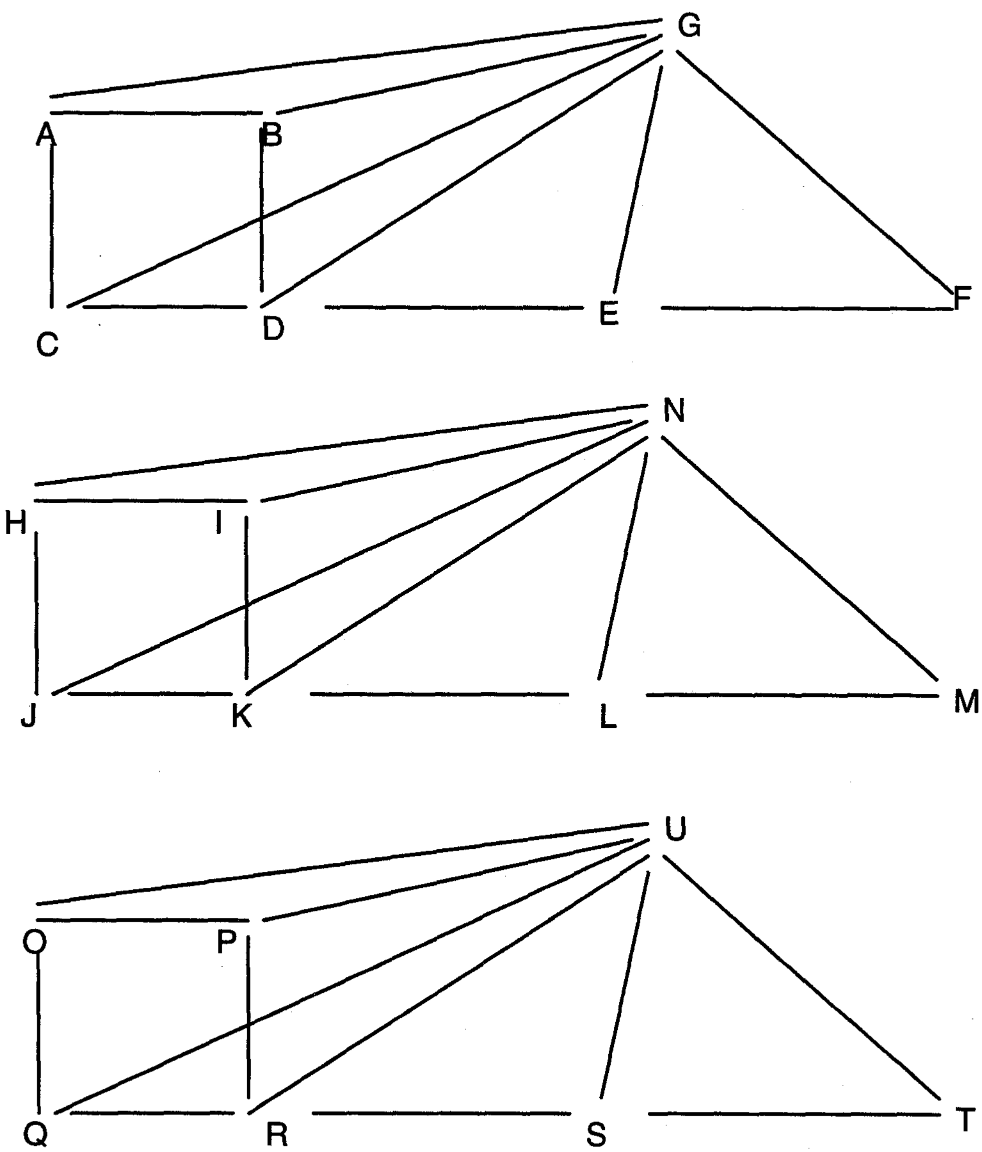
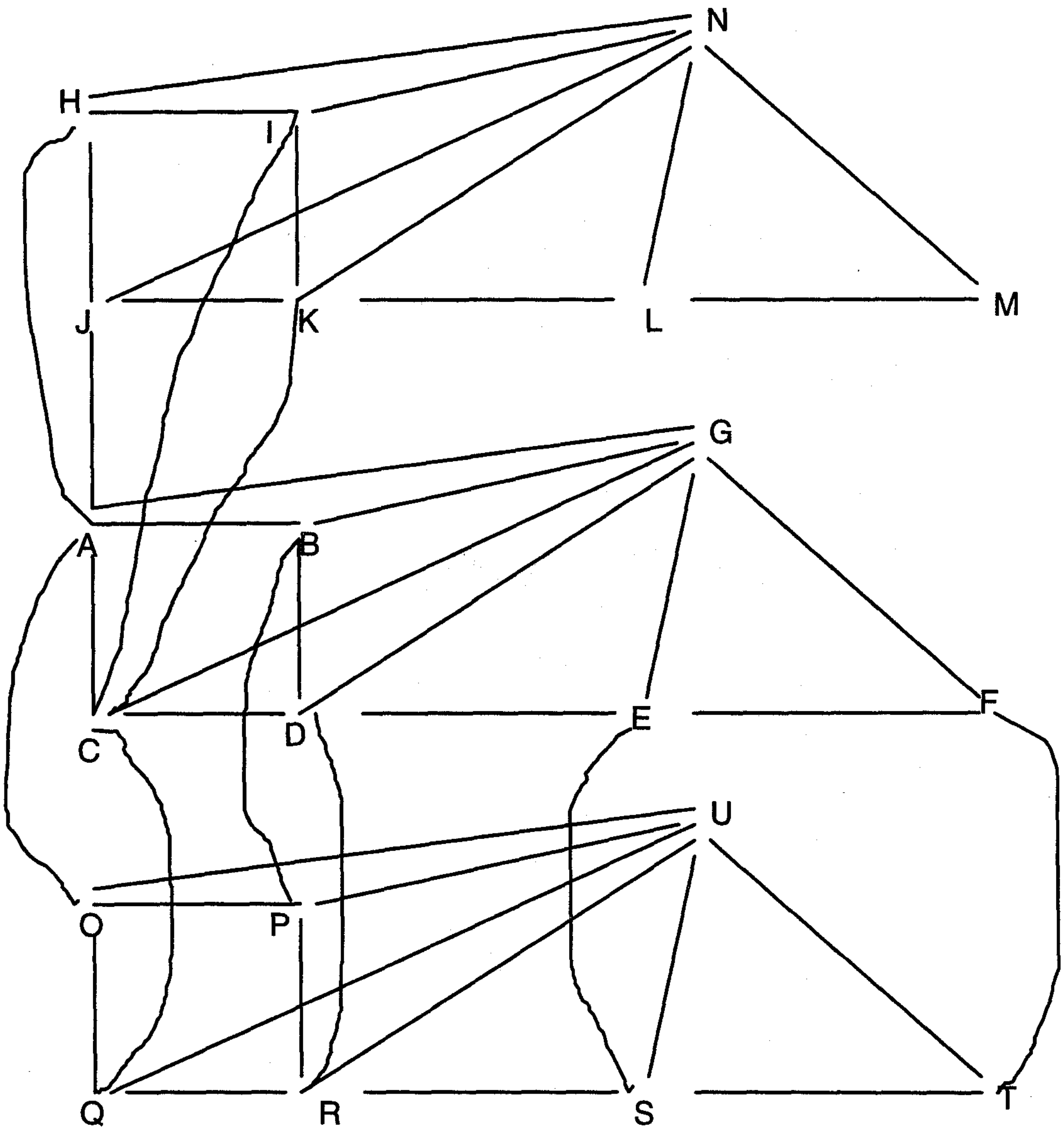


Figure 4.3: Binary Conflict Diagram for the 3NST7 Testbed Environment



CHAPTER 5. The Binary Conflicts Ascending Price (BICAP) Mechanism

While the BICAP auction was not developed through the application of mechanism theory, the language of that theory is useful for describing the auction. Consequently, much of that language is used in this chapter. A mechanism involves three essential rough elements: a set of feasible outcome allocations, a message space through which agents interact with each other and with the allocation authority, and an algorithm specifying when messages are sent and received by the mechanism and how they eventually determine an outcome from the feasible set of allocations. In Chapter 4 the feasible set of allocations for rail allocation was shown to be derived from binary conflicts. The rail allocation mechanism now presented in this chapter, the Binary Conflicts Ascending Price Mechanism [BICAP], essentially implements an auction-like process in an environment with binary conflicts.

In the BICAP mechanism each agent submits bids for trains in a continuous time auction. The highest bid on a train prevails as the potential winner and cancels all lower bids for the train. At every point in time the potential allocation is a set of bids that has no conflict and has the maximum sum of bids of any feasible allocation. The process of bidding continues until some pre-specified time has elapsed with no bids taking place. Formally the mechanism is defined as an iterative mechanism (see Chapter 2).

5.1 Formal Specification of the Binary Conflict Ascending Price Mechanism

Using the notation developed in Chapter 2, the essential elements of the Binary Conflict Ascending Price (BICAP) mechanism are as follows:

1. The message space $\boldsymbol{M}$ consists of:

    the bid messages $(b,f)_i$

        $b \in \Re$ is an amount offered by agent $i \in \boldsymbol{I}$ on the train route $f \in \boldsymbol{F}$.

    a message from nature indicating the elapse of time

2. A mechanism state $s=(\tau,\boldsymbol{B},\boldsymbol{H},\mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}])$ consists of

    τ: the time remaining in the auction

    $\boldsymbol{B}$: a vector of highest bid prices; $B_f \in \Re^+$ is the high bid for train $f \in \boldsymbol{F}$.

    $\boldsymbol{H}$: a vector of high bidders; $H_f \in I$ is the high bidder for train $f \in \boldsymbol{F}$.

    $\mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}]$: the potential allocation as a function of $\boldsymbol{B}$ and $\boldsymbol{H}$

    The set $\boldsymbol{S}$ of mechanism states is the set of all s.

3. The initial mechanism state $s_0=(\tau=\tau_0,\boldsymbol{B}=0,\boldsymbol{H}=0, \mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}]=\text{null allocation})$.

At the start of the mechanism:

The timer τ is set to τ_0 seconds.

The high bid prices $\boldsymbol{B}$ and bidders $\boldsymbol{H}$ are set to 0.

The potential allocation is the null allocation that all agents receive nothing

and pay nothing.

4. The set of terminal mechanism states $\boldsymbol{S}^*$ is the subset of $\boldsymbol{S}$ obtained by making the restriction $\tau=0$, i.e.,

$$\boldsymbol{S}^* = \{s^* \in \boldsymbol{S}: \tau=0\}.$$

The mechanism terminates as soon as the mechanism timer reaches 0 seconds. BICAP is a soft terminating mechanism as defined in Chapter 2.

5. Throughout the auction the binary conflict technical environment $(\boldsymbol{F},\boldsymbol{C})$ and the mechanism state variables τ, $\boldsymbol{B}$, $\boldsymbol{H}$, and $\wp^*[\boldsymbol{B},\boldsymbol{H}]$ are all common knowledge. The agent feedback information functions are defined so that the mechanism state is publicly broadcast whenever it changes.

6. At a terminal state $s^* \in \boldsymbol{S}^*$, the mechanism outcome allocation rule is

$$O^*(s^*) = \wp^*[\boldsymbol{B},\boldsymbol{H}].$$

This is equivalent to saying that when the mechanism terminates, the potential allocation becomes the actual allocation.

7. The mechanism transition rule $T:\boldsymbol{S}\otimes\boldsymbol{M}\rightarrow\boldsymbol{S}$ that describes how the mechanism state evolves in response to messages is specified as follows:

If m= bid message $(b,f)_j$ then:

If $b < B_f$: T(s,m)=s [bids lower than the current bids have no effect]

If $b > B_f$: s^{new}=T(s,m), where

components of s^{new} are denoted by the "new" superscript:

$\tau^{new} = \tau_0$ [the timer is reset]

$B^{new}{}_f = b$ [the bid becomes the new train high bid]

$H^{new}{}_f = j$ [the bidder becomes the new train high bidder]

$\mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}]$ must be recomputed from $\boldsymbol{B}^{new}$ and $\boldsymbol{H}^{new}$.

If m = time message from nature, then $\tau^{new} = \tau - 1$.

Determination of $\mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}]$ is by Centralized[29] Optimization. The mechanism involves centralized optimization if $\mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}]$ corresponds to a feasible allocation that maximizes the sum of stated willingness to pay B_f.

That is, $\mathcal{P}^*[\boldsymbol{B},\boldsymbol{H}] = (a_S ; a_1, ..., a_i) \in \mathcal{F}\!\mathcal{A}(\boldsymbol{I}, \boldsymbol{F}, \boldsymbol{C})$; $a_j = (\Lambda_j, t_j) \in 2^F \otimes \Re$:

(i) Λ_S maximizes $\sum_{f \in \Lambda_S} B_f$

(ii) $t_S = \sum_{f \in \Lambda_S} B_f$

(iii) $\forall j \in I$, $\Lambda_j = \Lambda_S \cap \{ f \in \boldsymbol{F} : H_f = j \}$

(iv) $\forall j \in I$, $t_j = -\sum_{f \in \Lambda_i} B_f$

[29] The term centralized here refers to the way in which optimization is performed. In Centralized Optimization some central authority in charge of the market must bear the cost of calculating the potential allocations given the current bids and communicating this information to the buyers. Because the scheduling problem is NP-complete, this cost could increase exponentially with the number of trains. Some type of decentralized optimization or approximation scheme for the potential allocations might be an appropriate topic for future research should computational cost issues be considered a serious problem.

In summary, the mechanism works as a set of simultaneous ascending auctions. Each auction is for a different train $f \in \boldsymbol{F}$, and so there could be as many auctions as there are possible trains in $\boldsymbol{F}$. A bid is submitted in real time, and with each bid the mechanism determines if the new bid is higher than the old bid for the train on which the new bid was submitted. Only the highest bids are kept as information by the mechanism. After the high bid changes on any train, the mechanism then determines the set of trains that maximize the total value of the track sale given the existing bids. This set of bids is announced by the mechanism as the potential allocation. The potential allocation becomes the outcome if no more bids arrive during some pre-specified period of time.

5.2 Models of Performance and Agent Behavior

In this section, some behavioral models of the BICAP mechanism will be constructed and examined. Performance of the system will be evaluated primarily in terms of its capacity to produce an efficient allocation. The modeling will be used as a check for design consistency. The question is whether the mechanism is operating according to the principles that were the underpinning of the design. Since there is no fully worked out theory about the behavior of such complex mechanisms in complex environments, an approach less ambitious than a general and rigorously tested theory must be used. The questions that the models will help answer are does it work and does it work for the right reasons. If it works but for the wrong reasons, then one should be very cautious about whether or not it might work in other environments in which it had not been tested, especially environments more complex than the simple laboratory testbeds.

The primary evaluative tool will be the *efficiency* of the final allocation as defined in section 2.4. The system will operate at 100% efficiency if the sum of the private values of operators in the final allocation is the maximum possible over the feasible possibilities. Notice that under the conditions of the environment this is not a simple task. The values of agents are known only to themselves and they are never asked to communicate these values to anyone. Thus the process must behave *as if* it knew the values and *as if* it could solve the related constrained maximization problem even though the values are never communicated as such.

The modeling effort developed in the following paragraphs rests upon the NE1 concept developed in section 2.6 together with conditions for which the mechanism will converge to such an equilibrium. There are many such equilibria, not all of which are efficient. A testable conjecture is developed about how the mechanism, might, nonetheless, still reach the efficient equilibria. Finally, implications for revelation of values and closing prices are discussed. As will be discussed in the experimental results of the next chapter, much of the behavior of the mechanism is captured by these simple models.

The modeling effort begins by applying the Nash-1 Stationary equilibrium concept to the BICAP mechanism. The definitions in section 2.6 are for abstract classes of allocation mechanisms, and need to be applied to the BICAP mechanism in particular. Recall that in these models, agent behavior is a function only of the environment (through agent's private values $\mathbf{V}_j$) and the current state (τ,$\boldsymbol{B}$,$\boldsymbol{H}$, $\mathcal{P}$*[$\boldsymbol{B}$,$\boldsymbol{H}$]) of the mechanism[30]. The bids in the BICAP mechanism are classified by similar (but not identical) criteria to those developed in Section 2.6 for messages. The strong pivotal, strong neutral, etc., bid message categories for the BICAP mechanism are given in the table below. These bid message categories play a central role in defining the Nash-1 stationary equilibria (NE1) of BICAP along with various associated behavioral assumptions and processes.

BICAP Bid Classifications. Bids $(b^*,f^*)_i$ are classified according to their effect on the potential allocation to agent i:

(i) A *null* bid satisfies $b^* \leq B_{f^*}$. A null bid is a bid for a train that is lower than the current bid for that train. Null bids are ignored by the BICAP mechanism.

(ii) A *strong pivotal* bid satisfies $B_{f^*} < b^* \leq V_{if}$ and $\pi^P_i(T((b^*,f^*)_i,s)) > \pi^P_i(s)$. It results in a change in the potential allocation and an increase in agent i's potential profit.

(iii) A *strong neutral* bid satisfies $B_{f^*} < b^* \leq V_{if}$ and

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[30] In particular it is important to remind the reader that the models of behavior do not allow strategies to be conditioned on full histories of the mechanism or beliefs about others strategies or train value vectors. This does preclude many types of strategic behavior. Nevertheless the analysis will be confined to this limited set for purposes of definitions and modeling.

$\pi^P_i(T((b^*,f^*)_i,s)) = \pi^P_i(s)$. It leaves the potential allocation and agent i's profit unchanged, but does replace the current bid for the train f* and could, possibly, increase the agent's potential profit if the bid were to become included in the potential allocation at a later time.

(iv) A dominated bid satisfies either $b^* > V_{if}$ or $\pi^P_i(T((b^*,f^*)_i,s)) < \pi^P_i(s)$. It is a bid that either lowers agent i's potential profit or is capable of doing so should the potential allocation change so that the bid is accepted.

The definition of NE1 equilibria in Section 2.6 involves the absence of pivotal mechanism messages that are profitable for any agent. Using the BICAP bidding classification above, the abstract NE1 definition can be restated for BICAP as follows:

Nash-1 Stationary Equilibria. NE1 states of the BICAP mechanism are those for which no pivotal bids exist for any agent.

Proposition 5.1. In the testbed environment NE1 exist.

Proof: The allocation and bids resulting from every agent bidding its value V_{if} for each train f is trivially NE1. Because of the first price rule and the fact that bids must be increasing, no opportunity for profit exists, and the result is therefore NE1. Therefore, NE1 outcomes always exist. •

Corollary 5.1.1 The efficient outcome, the one that places the trains in the hands of agents with the highest values, can be supported as a NE1.

Proof: The proof of 5.1 describes an NE1 that would yield the optimal allocation, because the bid maximizing potential allocation is trivially the optimal allocation when the bids are equal to the values $\underline{\mathbf{V}}$. •

Two types of "gradient processes" will be introduced, the pivotal process and the strong neutral process, along with underlying behavioral assumptions that support them. The idea is that if agents follow a type of "gradient method" that involves making pivotal bids when they exist, then the stopping point of BICAP will be an NE1 defined above. The idea is not necessarily that agents optimize over all pivotal bids, but simply that agents select some pivotal bid to avoid ending the mechanism, and that agents avoid submitting dominated bids as either a strategy or a mistake. The following paragraphs will make the ideas clear.

The Strong Pivotal Offer Hypothesis (SPOH). An agent is said to satisfy the strong pivotal offer hypothesis if the agent never submits dominated bids and the agent always submits some pivotal bid available to him, rather than let the BICAP auction end.

The Exhaustive Offer Hypothesis (EOH). An agent is said to satisfy the exhaustive offer hypothesis if the agent never submits dominated bids and the agent always submits some strong pivotal or strong neutral bid available to him, rather than let the BICAP auction end.

Note that an agent satisfying the EOH also satisfies the SPOH, because the behavioral hypothesis in both cases requires that the agent not allow the auction to end when a pivotal bid is available. Dynamic processes are assembled below by assuming that all agents in the mechanism bid in according to the SPOH or EOH.

Definition. Group agent behavior under BICAP is said to correspond to a *pivotal process* if all agents satisfy the Pivotal Offer Hypothesis.

Definition. Group agent behavior under BICAP is said to correspond to a strong neutral process if all agents satisfy the Exhaustive Offer Hypothesis.

The hypotheses suggests one way in which NE1 outcomes might occur in practice. An absence of dominated bids will guarantee that the process terminates. If individuals do not pass up the opportunity to make pivotal bids then the process will terminate at a NE1. Thus, if the number of pivotal and strong neutral bids vastly outweighs the number of dominated bids, then it is reasonable to expect that the result will be at or close to an NE1 outcome.

However, to get convergence to a NE1, BICAP must stop. Neither the SPOH or EOH guarantee this. For example, if two agents had a value of 100 for an item, an infinite sequence of bids such as 99.9, 99.99, 99.999, ... by the two agents would satisfy SPOH but not result in a stopping point for the mechanism. One way to obtain a stopping point is to require finiteness and discreteness in the train values and the BICAP bids. The discreteness requirement below places a constraint upon both the bids in the BICAP process and on the types of private value environments that must be satisfied for the results that follow to be true.

Discreteness Requirement. There is a minimum unit with which payments and values can be measured. The train value matrix $\underline{\mathbf{V}}$ is finite and integer valued with respect to this unit of measurement. Bids in the BICAP mechanism are required to be integer valued and finite with respect to this unit of value.

Proposition 5.2. If the discreteness requirement is satisfied and BICAP follows a pivotal process, then the outcome will be NE1.

Proof. First it must be shown that (i) an outcome exists, then it must be shown that (ii) the outcome is NE1.

(i) For an outcome to exist the mechanism must stop at some set of bids. Only non-null bids allow the auction to continue. Under the SPOH agents do not make dominated bids. Because of the discreteness requirement, there are only a finite number of non-dominated, non-null bids possible. In particular, if redemption values for k trains are bounded by v, then a sequence of no more than kv non-dominated, non-null bids can occur. Therefore, the auction will stop at some set of bids and so an outcome exists.

(ii) Whenever a pivotal bid exists for some agent, the SPOH implies that the agent will submit the pivotal bid and cause the auction to continue. Therefore when the auction ends, the SPOH implies the absence of pivotal bids for all agents. Recall that when no pivotal bids exist for any agent, that is the definition of an NE1 outcome.

Therefore, given discreteness and agents following a pivotal process, the outcome exists and is NE1. •

Although Proposition 5.2 shows how NE1 outcomes might be obtained, convergence to an allocation supported by an NE1 is not necessarily satisfactory performance. For example, in the experimental testbeds, a high bid on train "G" could represent an NE1 outcome even if "G" were not in the optimal allocation. Since "G" conflicts with many other trains, it could be the case that a single agent acting on his own could not profitably bid high enough to change the potential allocation. In such a case, there exists a coalition of agents who could bid on the different trains that conflict with "G" and change the potential allocation, even though no agent individually can affect the outcome in their favor. Perhaps some of the agents might bid cooperatively in these circumstances, thereby avoiding suboptimal NE1 outcomes.

Rather than allowing the market to close when an agent can not make a pivotal bid, the agent might submit a strong neutral bid, precommitting himself to a higher level of potential payment on some unallocated train. This might occur as a type of negotiation or signaling with the other agents or it might be a reflex reaction to the current bids.

The strong neutral process captures some of this intuition about how inefficient equilibria might be avoided.. The idea is that if all agents submit all available strong neutral bids, then this will tend to move the mechanism state away from some of the suboptimal NE1 outcomes. This idea is made formal by Proposition 5.3.

Proposition 5.3. If the discreteness requirement is satisfied and BICAP follows a strong neutral process then the outcome will be a 100% efficient, optimal NE1 if and only if strong neutral bids can disrupt every inefficient NE1 that can be reached with undominated bids.

Proof. A strong neutral process is also a pivotal process by definition. Therefore, from Proposition 5.2 the BICAP outcome must be a NE1. First sufficiency will be shown and then necessity. The strong neutral process may only stop when all pivotal and strong neutral bids have been submitted. If whenever an inefficient NE1 is reached there exists a set of strong neutral that will change the BICAP outcome, then the process can not stop at an inefficient NE. Since the process must stop at some NE1 (by Proposition 5.2), it must stop at the 100% efficient NE1. This shows that the condition above is sufficient to obtain a 100% efficient NE1. The condition is necessary because the strong neutral process could otherwise reach an inefficient NE1and then stop, if some strong neutral bids available at the NE1 were not capable of disrupting it. •

The conditions of Proposition 5.3 may or may not be met. If the conditions are not met, the likelihood that the BICAP mechanism will stop at an efficient or an inefficient NE1 is not known, but can be considered as an empirical question to be answered by the data. For now it is possible to briefly explore some conditions under which the conditions of Proposition 5.3 are met. Examples exist where the conditions are not met[31].

To meet the conditions of Proposition 5.3 it is necessary to identify a new sub-class of bids, *path-undominated* bids, that includes all the pivotal bids but only some of the strong-neutral bids. Since the strong neutral process can not end with pivotal or strong-neutral bids unsubmitted it can not end with path-undominated bids not submitted. The path-undominated bids will be used in Proposition 5.4 to identify

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31 Consider the following situation: the trains in the 1T7 testbed have current bids of A-400, B-400, C-350, D-450, E-100, and F-450. Suppose agent 1 has the high redemption values on trains A and E of 1000 on each and AE is the optimal allocation. To secure the allocation A, agent 1 need only bid 401 on A. To secure the allocation E, agent 1 need only bid 451 on E. But to obtain AE agent 1 must bid at least 751 for A and 901 for E. A sequence of pivotal bids that secures allocation AE is for agent 1 to bid 751 for A and then to bid 901 for E. However, consider instead what happens if agent 1 bids 401 for A. Now if the agent bids either 452 for E or 901 for E, he makes a dominated bid. If the agent tries to increase the bid on A to 901, that is also a dominated bid. In this situations BICAP could be stuck at an inefficient NE1 outcome and strong neutral or pivotal bids will not necessarily remedy the situation.

when a set of bids that are simultaneously strong-neutral could be submitted as a sequence[32] of strong-neutral and pivotal bids.

Definition. A *path* from s to s′ is a sequence of states and bids

$\{ (s(t) ; (b(t),f(t))_{j(t)}) : t=1,\ldots,T \}$ such that:

(i) $s(1) = T(s,(b(1),f(1))_{j(1)})$

(ii) $s(t) = T(s(t-1),(b(t),f(t))_{j(t)}),\ 1 < t \leq T$

(iii) $s' = s(T)$

Definition. A bid $(b,f)_i$ is *path-dominated* at a BICAP state s if there exists a path from the state s to any state s′ where:

(i) the potential allocations at s and s′ are the same, i.e. $\mathcal{P}=\mathcal{P}'$

(ii) $(b,f)_i$ is a dominated bid at state s′.

Definition. A bid is *path-undominated* at state s if it is not path-dominated at s.

Proposition 5.4. At some BICAP state s, let $\mathcal{B}$ be a set of path-undominated bids. Then any ordered sequence of bids $\mathcal{B}^{seq} = \{ (b_1,f_1)_{i1} , \ldots , (b_K,f_K)_{iK} \} \subseteq \mathcal{B}$, where the

[32] This issue can be motivated with a simple example. Suppose at some instant it is a strong neutral bid for agent 1 to bid 1000 on A and agent 2 to bid 1000 on E, because either of these bids would not affect the potential allocation alone. Suppose, however, the joint effect of these two bids is to lower potential profits for both agents by changing the potential allocation away from an allocation that is very profitable for them by changing the potential allocation from something else to AE. So, as soon as one bid is submitted, the other bid becomes a dominated bid, not a strong neutral bid, for the remaining agent. From this example, one can see that simultaneous sets of strong neutral bids are sometimes not implementable as sequences of strong neutral bids.

last bid and <u>only</u> the last bid $(b_K,f_K)_{iK} \in \mathcal{B}^{seq}$ changes the potential allocation has the property that each bid 1,...,K-1 is either null or strong neutral in the order it is submitted and bid K is either pivotal or strong neutral when it is submitted.

Proof. Since it is assumed that the bids 1,...,K-1 in the sequence $\mathcal{B}^{seq}$ do not change the potential allocation, and it is known that, that the bids are not dominated bids, then the bids must be strong neutral bids or null bids. Before bid K is submitted the potential allocation is the same as when bid 1 is submitted, so since bid K is path-undominated it can not be a dominated bid. Since bid K changes the allocation, it is not a null bid. Therefore bid K is either a pivotal bid or a strong neutral bid. •

Lemma 5.5. Let BICAP be at a state s with bids B_f, potential allocation $\mathcal{P}$, and potential profits π_i for each agent. V_{if} is the private value agent i receives for running train f. Then

$$\sum_{f \in \Lambda_S} B_f + \sum_{i \in I} \pi_i = \upsilon(\mathcal{P}; \underline{V})$$

Proof.

From the definition of allocation value in Section 2.4,

$$\upsilon(\mathcal{P}; \underline{V}) = \sum_{i \in I} \sum_{f \in \Lambda_i} V_{if}$$

From the definition of profit (also in Section 2.4),

$$\pi_i = \Pi_i(\mathcal{P}; \underline{V}) = \sum_{f \in \Lambda_i} (V_{if} - B_f)$$

So $\sum_{i \in I} \pi_i = \sum_{i \in I} \sum_{f \in \Lambda_i} (V_{if} - B_f) = \upsilon(\mathcal{P}; \underline{V}) - \sum_{i \in I} \sum_{f \in \Lambda_i} B_f$

The last double sum is the sum of the bids on trains allocated to a certain agent i, summed over all agents i. But since a train may be allocated to at most one agent, this is just the sum of bids on trains that are allocated to someone, the sum $\sum_{f\in\Lambda_S} B_f$. Therefore, $\sum_{i\in I}\pi_i = \upsilon(\mathcal{P};\underline{V}) - \sum_{f\in\Lambda_S} B_f$, or , rearranging terms,

$$\sum_{f\in\Lambda_S} B_f + \sum_{i\in I}\pi_i = \upsilon(\mathcal{P};\underline{V}) \bullet$$

<u>*Proposition 5.6.*</u> For any pivotal process let a* represent an inefficient allocation that can be supported as a NE1 of the process. Suppose at allocation a*, the bids satisfy $B_f^* > V^{(2)}_f$, where $V^{(2)}_f$ is the 2nd highest social value V_{if} for the train f among the agents $i\in \boldsymbol{I}$. If the optimal allocation provides each agent with the rights to at most one train route[33] in $\boldsymbol{F}$, then a set of path-undominated bids $\mathcal{B}$ exist for the agents that change the potential allocation away from a*.

<u>*Proof.*</u>

Let a*=(Λ_j^* , t_j^*) be the allocation at state s*.

Suppose at this NE1 state, $\mathcal{B}$ consists of all the non-null bids in the collection $\{\ (b = V_{if} - \pi_i^{P*}, f)_i : i\in I\ , f\in \boldsymbol{F}- \Lambda_i^*\ , \pi_i^{P*} = \sum_{f\in\Lambda_i^*} V_{if} - B_f^*\ \}$. Note that π_i^{P*} is the potential profit for agent i at the potential allocation a*. The set of such constant-profit bids has the following properties:

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33 It is not very difficult to apply the scheduling independence concepts from Chapter 4 to expand the applicability of the proposition to include cases where each agent has the rights to at most one train route from each disconnected subgraph of the binary conflict graph. This is not done here because it would complicate the notation of the proof and is not essential at this point.

(i) The bids are path-undominated bid at B_f*. As long as the allocation is a*, the bids, if accepted, always maintain the profits that existed under allocation a* at bids B_f*.

(ii) To show that the set of bids $\mathcal{B}$ causes the allocation to change from a*, it is necessary and sufficient to show that after the bids in $\mathcal{B}$ are submitted, a different set of trains, Λ_S', has a higher sum of bids than the set of train Λ_S* that are allocated at a*. Create any path starting from s*, using all the bids in $\mathcal{B}$. Let the endpoint of this path be the state s**, and the bids at s** be B_f^{**}.

In a pivotal process no one submits a dominated bid, so by definition each agent can only bid as high as their redemption value. Therefore, for each train $f \in \boldsymbol{F}$ the high bidders H_f^* at bids B_f* and the high bidders H_f** at bids B_f** must be the same, the agent with the highest social value for the train f.

It can now be shown that the set of trains Λ_S^O allocated at the optimal allocation a^O gives a higher sum of bids at B_f** than the set of trains Λ_S*, and therefore the allocation must change away from a*. To show this, it will be posed as a logical question (5.5.1) and the proof will proceed by identifying an equivalent inequality that is true.

Is $\left(\sum_{f \in \Lambda_S{}^O} B_f ** \right) > \left(\sum_{f \in \Lambda_S *} B_f ** \right)$? (5.5.1)

To begin, it is useful to establish that $\mathcal{B}$ does not contain bids on the trains in Λ_S^*. Because $B_f^* > V^{(2)}_f$, there do not exist agents $i \in \boldsymbol{I}$ and trains $f \in \boldsymbol{F}$ for which $f \in \Lambda_S^*$, $f \notin \Lambda_i^*$, and $V_{if} > B_f^*$. Note that these conditions are necessary (but not sufficient) conditions for a bid to be in $\mathcal{B}$, so there are no bids on any of the trains in Λ_S^* in $\mathcal{B}$. Therefore, $B_f^{**} = B_f^* \ \forall f \in \Lambda_S^*$ (5.5.2).

Applying (5.5.2) to (5.5.1) yields the condition:

(5.5.1) true if and only if (5.5.3) : $\left(\sum_{f \in \Lambda_S{}^O} B_f{}^{**}\right) > \left(\sum_{f \in \Lambda_S{}^*} B_f{}^{*}\right)$.(5.5.3)

Apply Lemma 5.5 to both sums in the inequality in of 5.5.3. Let π_i^* be the profits at the allocation a^* with bids B_f^*, and let π_i^{**O} be the profits at the allocation a^O with bids B_f^{**}. Then, (5.5.1) true if and only if

(5.5.4): $\upsilon(a^O;\underline{V}) - \sum_{i \in I} \pi_i{}^{**O} > \upsilon(a^*;\underline{V}) - \sum_{i \in I} \pi_i{}^*$ (5.5.4)

Since $\upsilon(a^O;\underline{V}) > \upsilon(a^*;\underline{V})$ by assumption, a sufficient condition for (5.5.4) to be true can be found: (5.5.4) true if $\forall i \in \boldsymbol{I}$ condition (5.5.5) below is true:

$\pi_i^{**O} \leq \pi_i^*$ (5.5.5).

The proof proceeds by partitioning $\boldsymbol{I}$ into four subsets $\boldsymbol{I}_{0,0}, \boldsymbol{I}_{0,1}, \boldsymbol{I}_{1,0}, \boldsymbol{I}_{1,1}$ where (5.5.5) will then be shown to be true in each subset. Define the four subsets as follows:

$\boldsymbol{I}_{0,0} = \{ i \in \boldsymbol{I}: \Lambda_i^* = \varnothing = \Lambda_i^O \}$

(5.5.5) is trivially true $\forall$ i$\in$ $\boldsymbol{I}_{0,0}$, since $\pi_i^{**O} = \pi_i^* = 0$.

$\boldsymbol{I}_{0,1} = \{ i \in \boldsymbol{I}: \Lambda_i^* = \varnothing \neq \Lambda_i^O \}$

In these cases $\pi_i^* = 0$, and this implies that $\mathcal{B}$ contains bids where these agents in $\boldsymbol{I}_{0,1}$ bid their entire social values V_{if}. Therefore, $\pi_i^{**O} = 0$ $\forall$i$\in$ $\boldsymbol{I}_{0,1}$, and (5.5.5) holds here as well.

$\boldsymbol{I}_{1,0} = \{ i \in \boldsymbol{I}: \Lambda_i^* \neq \varnothing = \Lambda_i^O \}$

(5.5.5) is trivially true $\forall$ i$\in$ $\boldsymbol{I}_{1,0}$, since $\pi_i^{**O} = 0 \leq \pi_i^*$.

$\boldsymbol{I}_{1,1} = \{ i \in \boldsymbol{I}: \Lambda_i^* \neq \varnothing, \Lambda_i^O \neq \varnothing \}$

Showing (5.5.5) is true $\forall$ i$\in$ $\boldsymbol{I}_{1,1}$ is somewhat more complicated than the previous three cases. The easiest way to prove it is to assume that it is not true and look for a contradiction. Assuming (5.5.5) is false for some i, $\exists$ i $\in$ $\boldsymbol{I}_{1,1}$ such that $\pi_i^{**O} > \pi_i^*$. Now at the optimal allocation it is assumed that each agent is allocated at most one train. Suppose that for the agent i above, this is the train f. Then for $\pi_i^{**O} > \pi_i^*$ it must be true that the highest bid agent i ever submits on train f satisfies $b < V_{if} - \pi_i^*$. But this can not be true, since the definition of $\mathcal{B}$ implies that $B_f^{**} \geq V_{if} - \pi_i^*$. Therefore there is a contradiction and (5.5.5) can not be false for any i$\in$ $\boldsymbol{I}_{1,1}$, and so (5.5.5) must be true $\forall$ i$\in$ $\boldsymbol{I}_{1,1}$.

Because the union $\boldsymbol{I}_{0,0} \cup \boldsymbol{I}_{0,1} \cup \boldsymbol{I}_{1,0} \cup \boldsymbol{I}_{1,1} = \boldsymbol{I}$, this shows that (5.5.5) is true $\forall$i$\in$$\boldsymbol{I}$. Therefore (5.5.4) is also true, and so (5.5.1) is true. (5.5.1) says that at the bids B_f^{**}, the allocation a* no longer gives a maximum sum of bids. Therefore the allocation will change. •

These theoretical results give empirical predictions for the chapters that follow. They would predict BICAP to reach efficient NE1 outcomes in the 1T7 and 3ST7 testbeds but not necessarily in the 3NST7 testbeds. In the 1T7 and 3ST7 testbeds each agent, at the optimal allocation, receives at most one train in the 1T7 testbed and one train per independent track in the 3ST7 experiments. In the 3NST7 testbeds agent receive multiple trains on interdependent tracks at the optimal allocations

Whether the strong neutral process or the pivotal process is more descriptive of the actual behavior is an empirical question. These questions, along with the question of overall performance of the mechanism, will be addressed by the experiments of the next chapter.

CHAPTER 6. An Experimental Investigation of the BICAP Mechanism

This chapter details the results of laboratory experiments where the mechanism of the previous section was applied to the environment of 1T7 testbed environment from Chapter 3. Only the 1T7 testbed seemed appropriate for initial testing. If the mechanism performed well by generating efficient allocations and doing so for reasons consistent with the theory of operations of BICAP, then and only then would tests with the larger testbeds be appropriate.

The results of the experiments confirm that, in fact, a decentralized mechanism can solve some of the technical aspects of the rail scheduling problem and yield efficient allocations. In the laboratory experiments, BICAP allocations are 97% efficient on average. Design consistency appears strong: Outcomes correspond to one-stage Nash equilibria. Evidence exists that the process of convergence is essentially as captured by the pivotal process introduced in the previous section. In addition, inefficient NE1 seem to be avoided because of a high degree of revelation in the bid prices.

6.1 Methodological Details

The experiment was conducted using the one track testbed scheduling problem and train values substantially as discussed in Chapter 3. Two additional trains, H and I, were added to the testbed that had no conflicts or interaction with the other trains. These trains were included both as a control and as a means to provide some of the agents who could not, because of their redemption values, profitably obtain a train in A...G an opportunity to obtain train H or I instead. The trains H and I should not have affected the experimental outcomes for A-G and do not affect the validity of the testbed in addressing the issues listed in Chapter 3. If anything, they add a control to the experiment: Because the trains have no conflicts and are always allocated, BICAP acts like an iterative 1st price auction for these two trains. If BICAP had failed to produce efficient allocations for trains A through G, then perhaps bidding behavior on trains H and I could provide clues as to errors that may have occurred.

Although the BICAP mechanism was implemented as defined in Chapter 5, the formal features of the mechanism in Chapter 5 leave unspecified choices which must be made to implement the mechanism on a computer network. These choices involve details in the development of computer software for use by human agents: layout of keys and screens, the detail of error messages when invalid actions are taken, etc. Each agent was stationed at a personal computer that was attached to other agents through a token ring network. Figure 6.1 is a representation of the screen as seen by a subject. In the experiment the options of value were called projects as opposed to trains. On the actual screen, different project lines were in different colors to aid in reading the table. The status column indicates the potential allocation. If a bidder had a high bid for one or more projects, those bids were tagged on the screen, as is project

"C" in Figure 6.1. Bids are entered by pressing the key corresponding to the project, and then entering a value. The bid value and project may be edited, or the bid may be deleted. A special key (F1) must be pressed to actually send the bid into the mechanism, at which point it is checked against the high bid. If the new bid beats the high bid, it is sent to the other screens and becomes binding (if accepted in the final allocation) until replaced by a higher bid.

When entering bids into the experimental software, subjects had difficulty with typographical errors. It was easy for subjects to create typographical errors by forgetting to hit a key that deleted previous input. This tended to cause false large bids to be entered into the system, such that a subject would lose $10 to $50 if forced to honor the bid. When such an error occurred, the experimenter reset the experimental software. The subjects were instructed that the period would start over and that they should continue using the same incentive value sheet. The possibility exists that subjects created false typographical errors to delay the mechanism, but there are no obvious profit opportunities from using such a delay strategy since incentives are the same when the period is restarted. Only the error free run of each period was considered valid data. It is possible that there are effects of comfort and ergonomics, and that different arrangements of command keys and display screens could result in better performance by allowing the agent to focus better on the task at hand.

A total of three experimental sessions were conducted. Subjects were Caltech students recruited through an announcement on the campus computer network. Procedures in each of the three experiments are essentially identical. Each of the three sessions lasted approximately 2 to 2 1/2 hours and required ten subjects. Each session consisted of seven periods.

Each subject received common instructions included in Appendix A, as well as an individual incentive table and common supplies (e.g., scratch paper, pocket calculator). Because of space, the individual incentive information tables were not included in Appendix A, but are essentially individual sheets showing each agent their train redemption values from Table 3.2 of Chapter 3. Each table consisted of 20 pages. Each page consisted of an individual firm's incentives for the routes (projects) for one period. Only the first 7 pages were actually used.

No mention is made of trains or scheduling in the experimental instructions. The language of the experiment is "project" for train route, and "combination of projects" for train schedule. Language was chosen to make the experiment independent from the specific industrial application, in an attempt to eliminate any effects of pre-conceived notions subjects may have about railroad operations.

6.2 Experimental Results: Performance

Table 6.1 summarizes the experimental parameters and results simultaneously. Only the highest and second highest redemption values are thought to be important for determining prices, allocation and strategic behavior. There are seven different periods (different redemption value parameters) that were repeated in identical sequence in three different experiments.

The Table is read as follows. The first row contains for the first period (of all three experiments) the high redemption value for each of the nine individual routes, the identification number of the agent that held the high value, and the optimum system schedule that is the maximum valued feasible schedule. Reading across the row, the optimal schedule is {A, D, F, H, I}; and the maximum redemption value for route A is 1699 held by participant number 3, etc. Row 2 contains the second highest redemption values. Row 3 shows the actual schedule that resulted in period 1 of experiment 1 ({A,D,F,H,I} - which was optimal), and it shows, for example, that the maximum bid for route A was 1300 tendered by participant 3. Rows 4 and 5 show the period 1 data for experiments 2 and 3 respectively. Row 6 starts the enumeration of the same data for period 2. The table continues through period 7.

The first result suggests that the mechanism is successful in producing efficient allocations for the rail allocation problem. Efficiencies are calculated from the table, using the standard definition of efficiency from section 2.4: the efficiency is the ratio of the total of redemption values for agents at the outcome allocation divided by the maximum possible total value of redemption values. The maximum possible total is attained at the optimal allocation.

Observation 6.1. The outcome allocations produced by the mechanism are often 100% efficient. Inefficient outcome allocations occasionally occur. Efficiency for the 1T7 testbed routes A-G averaged 97%.

Support. The table directly supports three statements concerning efficiency. (i) Inefficient outcomes are rare. In 18 out of 21 experimental trials, the mechanism resulted in the optimal allocation. Only 3 of the 21 trials, only period 2-experiment 3, period 3-experiment 2, and period 5-experiment 3 resulted in allocations that were not optimal. Thus, for trains A-G the efficiency is 100% in 86% of the experimental trials. (ii) For the 3 inefficient trials in periods 2, 3, and 5, efficiency for trains A through G is at 0.82,0.65, and 0.93 respectively. This yields an average efficiency of 97% for trains A-G for the experiment. (iii) Trains H and I are always allocated in the optimal manner. •

6.3 Experimental Results: Design Consistency

Given that the mechanism is efficient, the question of design consistency is now addressed. It is not sufficient that the outcomes are efficient, but rather they should be efficient for theoretically understandable reasons. This increases the guarantee that the mechanism will behave similarly in other environments that are untested but theoretically similar.

To begin this examination, Table 6.2 represents a measure of 'distance' of outcomes from NE1 outcomes in terms of the potential profitability of pivotal responses. The entries in the Table are the maximum potentially profitable pivotal response available to any agent. In a sense the entries are the maximum opportunity cost of stopping (assuming that the process would go only one step more). For each agent, a search was made for the most potentially profitable pivotal bid at the final bid prices for each period. A maximum was then taken over all the agents for that period, and the amount of this potentially foregone profit along with the agent id number with the corresponding pivotal bid opportunity were tabulated and entered in Table 6.2. Entries of zero correspond to NE1 outcomes, since a zero entry is only possible if there are no remaining pivotal bidding opportunities at the close of each trial. Positive entries represent possibilities for profit, and are stated in Francs. (Franc conversion rates varied; in the experiments francs were worth \$0.005-\$0.02 or so.)

Typically in experiments there is an unknown variable subjective cost for getting agents to take any action. That is, if only $0.10 is to be made by pressing the keys, it is possible that the agent will not take any action. Taking these costs into consideration suggests a classification of outcomes from Table 6.2 into strict NE1 and "thick indifference" NE1 type outcomes. Consideration of both as degrees of NE1 behavior yields Observation 6.2.

Observation 6.2. Outcomes tend to be NE1 Equilibria of the one stage game.

Support. Consider the entries in Table 6.3 that are a classification of the outcomes taken from Table 6.2. A little over one-third of the periods results in strict NE1. Approximately 71% have deviation less than 50Fr ($0.25-$0.50). This leaves only 29% of the periods resulting in outcomes that were not NE1 or "near" NE1 in the sense that the maximum opportunity cost of a move was low. •

The property is strengthened by the fact that the outcomes that are near NE1 tend to be core outcomes. Given the bids expressed in the mechanism at the final outcome, no coalition could construct a joint bid unavailable to members acting alone, that would produce benefits for some members of the coalition and hurt none of the other members of the coalition.

Observation 6.3. Outcomes tend to be in the core of BICAP.

<u>*Support.*</u> Core outcomes are NE1 outcomes that are coalition-proof in the sense that a coalition of members has no profitable opportunities available to it at the final prices and allocations that the individual members could not carry out unilaterally. This result was obtained by brute force, i.e., a computer algorithm was used to do an exhaustive search over all coalitions for profit opportunities, given the final prices and allocations in each trial. No additional opportunities for profitable bids, outside those available to individual agents, were found. •

An efficient outcome does not require NE1 behavior, just as NE1 behavior does not guarantee efficiency due to the existence of multiple equilibria. However, if the conjectures behind the design of the mechanism presented in the preceding section are correct, one would expect there to be a correlation between NE1 and efficiency. An examination of the inefficient outcomes yields the following observation:

<u>*Observation 6.4.*</u> Inefficient NE1 do not occur.

<u>*Support.*</u> In the environments studied, inefficient outcomes coincide with failure to converge to an NE1. Period 2-experiment 3, period 3-experiment 2, and period 5-experiment 3 resulted in inefficient outcomes. In Table 6.2, these three trials account for the three largest deviations from an NE1 outcome. •

Corollary Observation. If outcomes are inefficient then they are not NE1. •

The Corollary above together with Observation 6.2 present natural questions. Why does the process result in NE1 outcomes and why among those does it seek only the efficient outcome? Is this a lucky accident or is it related to the game theoretic structure of the problem? The answer to the first question is suggested by the NE1 convergence property of the discrete pivotal process shown in Proposition 5.2. The answer to the second question is more elusive, but the other propositions in Chapter 5 support the idea that some inefficient NE1 might be avoided by the strong neutral process.

To facilitate an investigation of the bidding dynamics, Table 6.4 was compiled. Every non-null individual bid was recorded by a computer during the experiments. Classifications of these bids into the strong-pivotal, strong-neutral, and dominated categories are done and recorded in the Table. This analysis yields the following observation.

Observation 6.5. Convergence to NE1 is governed by the *discrete pivotal process.*

Support. If the process is operative, then convergence will be to an NE 1 by virtue of Proposition 5.2. Two assumptions of the process involve behavior. One of the assumptions, is that individuals will not let the process stop if strong-pivotal bids exist. The tendency for this property to be satisfied is established by Observation 6.2. The second property is that individuals not tender dominated bids. The data in Table 6.4 reveal that dominated bids account for 0-6% of the bidding activity. Thus the tendency of bidding behavior is to tender non-dominated bids as required by the process. With the tendency for both of the properties of the process to be satisfied, one can conclude that it is operative and thus characterizes equilibration. •

Because the assumptions for the class of discrete pivotal processes seem to be satisfied, a search of the bidding data is now conducted to see if the type of dynamics in operation can be narrowed further. Observation 6.6 shows that the strong neutral bid processes are good candidates. The case is made by eliminating from consideration those dynamics that suggest no bids will be made unless it changes the state in a favorable way for an agent. Strong neutral bids are those (non-dominated bids) that place the bidder as the high bidder for a train that is not part of the potential allocation even after the bid is tendered. Thus, the bid does not change the potential allocation. Such bidding is not explained by the pivotal process dynamics, so the concept adds behavior that is otherwise absent from consideration. The data from Table 6.4 will be used to show that strong neutral bids are frequently made. The data from Table 6.5 will be used to show that the dynamics exhibit no tendency to stop at a NE1 the first time such a state is reached.

Observation 6.6. From among many possible discrete pivotal dynamic processes, the class of strong neutral processes receives support as an explanation of the convergence path.

Support. First, strong neutral bids exist as a substantial feature of bidding behavior. On average, strong neutral bids consist of approximately one-third of all bidding behavior. Note from the Table 6.4 that in some periods there are more strong neutral bids than pivotal bids, and in others, vice versa. Thus, even though with the same period parameters, different experiments can have substantially different ratios of pivotal bids to neutral bids. A tendency exists for a substantial portion of bids to be strong neutral. Second, a tendency exists for the dynamics to not stop at NE1 when they are attained. Thus, the dynamics tend to not be the discrete pivotal processes that limit behavior to NE1 reaction functions. Table 6.5 examines the frequency with which bids place the mechanism at an NE1 intermediate outcome. No attempt is made to distinguish between NE1 outcomes that support the same allocation, but differ slightly in bid price, and those that produce different allocations. According to Table 6.5, either the mechanism never reaches an NE1 outcome, or it passes through multiple NE1 outcomes. Only in 2 of the 21 cases, namely period 3-experiment 3, and period 6-experiment 2, does the mechanism stop at the first NE1 outcome reached. Since the class of discrete processes where strong neutral bids are not used must stop at the first NE1 outcome encountered, those dynamic processes can be discarded from consideration. Thus, the presence of multiple NE1 intermediate outcomes and the use of strong neutral bids at intermediate NE1 outcomes suggest a refinement like the strong neutral process. •

The fact of efficiency, when taken with Observations 6.5 and 6.6, creates an interesting picture of the dynamics. The mechanism wanders over the allocations until the optimal set of trains is discovered. The bidding then proceeds to advance until an allocation of the set of trains and a set of bids is attained that supports the optimal as an NE1. The process of the 'discovery' of the optimum must be associated with a process of preference revelation and coordination. We have no rigorous theory about how this might take place, but the intuition rests on the submission of strong neutral bids that reveal the social opportunity cost of the allocation. The basic intuition is that strong neutral bids will be made as a type of negotiation process driven by the possibility that the auction will terminate unless a bid is made. By making bids on unallocated trains, an agent is contributing to the 'public good' of defeating the current allocation. The strong neutral bid process holds that a person will reveal rather than let the market close. When the possibility for strong neutral bids is exhausted, then all excluded agents have revealed the maximum any would ever be willing to pay for the excluded trains. Since the potential allocation is of higher value than any allocation possible from the excluded trains, the final allocation must necessarily be efficient if the excluded agents are fully revealing their willingness to pay.

If the NE1 property of the dynamics of the discrete pivotal process guarantees that the high value bidder has the final standing bid for allocated trains; and, if the high value bidder has the final standing bid for excluded trains, then whatever trains are allocated will be allocated to those who value them the most. The only question is how the proper set of trains might be chosen. If, in addition, the excluded agents bid as high as their redemption value, then the operation of the BICAP mechanism assures that the proper allocation will be chosen and that an efficient allocation will be the final result. Thus, several measurements of value revelation are suggested by this logic. For excluded trains, does the high value bidder have the final standing bid? How high is this bid, either as a percentage of the agent's redemption value or as a distance from it?

A complication in the parameters makes analysis difficult. Often an agent will have high redemption values on a pair of trains that are in conflict. In this case, it may not be to that agent's advantage to have the high bid for both trains, since he would, in effect, be bidding against himself. An opportunity cost exists for the agent that lowers the agent's willingness to bid on the excluded train by the amount of potential profit on the allocated train for which he has the standing high bid. This potential profit would be foregone if the allocation switched to the other train. Thus the data can be divided into the 'clean cases' for posing the questions and the 'unclean cases'. The clean cases are selected pairs in selected periods where the conflicts happened to not exist. The revelation conjecture that follows summarizes the weight of the assessment of clean cases.

Revelation Conjecture. Social opportunity costs of allocation are revealed through the operation of the BICAP mechanism.

Support. Different agents have the high value for trains A and B in period 7, for trains C and D in period 4, trains E and F (also) in period 4, and for train G in periods { 1,2,3,5,6 }. In all other periods the same agent will have high values for a pair of conflicting trains. In period 7, train B is excluded from the allocation, and in period 4 trains D and F are excluded while G is excluded in all periods. Aggregation over the periods and trains above provides 24 excluded train "clean cases" for analysis of the conjecture. In these cases the high value individual tends to hold the high bid and also reveals the value to the mechanism. The results are: (i) the high value agent has the high closing bid in 18/24 or 75% of the 24 excluded train cases; (ii) on average the excluded agent bid 93.8% of the high redemption value for the 24 excluded train cases[34]. Thus, the social opportunity cost is revealed in these cases.

The "unclean cases" are more difficult. Revelation behavior is different when the holder of the high redemption value for one train that is included in the potential allocation also holds the high redemption value on another train that is excluded from

[34] The overbid by agent 10 in period 4, experiment 2, train D was counted as a bid for the full amount of the redemption value, 594, and not the amount of 920 which was bid. Using the amount of 920 would have raised this figure still further.

the potential allocation. If all unclean cases are aggregated (all cases in which the high value agent has the high value for a pair of conflicting items), then in a total of 33 out of 60 cases the high value agent has the final bid on the excluded item in the pair. Thus in 27 of 60 unclean cases, the high bidders on the excluded items are not the agent with the highest redemption value. In this sense the social opportunity cost information is not revealed to the mechanism. However, the revelation is as one might expect in an auction-like process: if the high redemption value agent does not bid, the agent holding the second highest redemption value can be expected to do so. The second high value agent has the final bid on the excluded item in an additional 18 out of 60 cases. Bid revelation by one of the top two value agents then yields a total of 51 out of 60 cases. This suggests that while agents may not bid as high as the highest redemption value on excluded routes, the second highest redemption value holder is being revealed.

The natural question now is whether the redemption values are being revealed. For this a measure is developed.

For each train, set

$$d_2(f) = \max(0, \text{2nd highest redemption value for } f - B(f))$$

If the final bid is above the second highest redemption value, then $d_2(f)=0$. d_2 is thus a measure of the amount by which the bid is less than the second highest redemption value.

Table 6.6 compares bid prices with redemption values, in such a way as to pool data across all periods. On average, when optimal allocations occur, revelation of values is near the second highest redemption value. This occurs to within an average of 10Fr on allocated routes and 28Fr on unallocated routes. Depending upon the individual this amounts to something on the order of a nickel to a quarter on an item that is worth several dollars. •

The pattern of results provides much evidence of design consistency. The reasons for the efficient allocations are for theoretically understandable reasons. Agents do not limit their behavior to reaction functions that only make themselves better off. They take actions that make no changes in their own well-being but, depending upon the actions taken by others, might make themselves better off. This dynamic leads away from inefficient allocations that otherwise might exist as equilibria. The nature of BICAP is such that it pits competitors against each other such that values become revealed to the mechanism and then it uses that information to move the system in a dynamic in the direction of optimal allocations. The analysis also suggests that the nature of potential inefficiencies might be related to agents with the high value on multiple trains that conflict. Having a degree of 'market power' they might not bid against themselves, and as a result they prevent some degree of information revelation. This lack of efficient operations is clearly a parameter issue and not an issue related to the principles upon which the mechanism design rests. Nevertheless, it is important to note that while from the point of view of design consistency this issue surfaces, it was not generally a problem in the operations of the mechanism since the mechanism operated at near 100% efficiency.

Figure 6.1: BICAP Computer Screen

id number: 7

Project	Current High Bid	Bidder ID#	Status	
A	225	3	ACCEPTED	
B	438	3		
C	80	7		<- yours
D	500	8	ACCEPTED	
E	300	9		
F	290	2	ACCEPTED	
G	600	5		
H	50	1	ACCEPTED	
I	75	4	ACCEPTED	

To enter a bid for a project, press its corresponding key A-I.

Table 6.1: Closing Prices, Allocations, and Redemption Values for 1T7 Testbed Experiments

Period	High Feasible Package	A	B	C	D	E	F	G	H	I
1 - high redemption values	ADFHI	1699-3	645-3	1634-4	1447-4	1635-8	1421-8	2619-10	1432-5	1318-5
1 - 2nd highest redemption values	BCEHI	1282-4	521-1	1405-7	974-7	1013-2	645-2	2543-4	888-9	1231-10
1 - data from experiment 1	ADFHI	1300-3	601-1	1401-4	1295-4	731-1	800-8	2601-10	890-5	1250-5
1 - data from experiment 2	ADFHI	1300-3	520-1	1300-7	974-4	1112-8	900-8	2600-10	888-5	1250-5
1 - data from experiment 3	ADFHI	1500-3	520-1	1000-4	1000-4	1100-8	1000-8	2615-10	900-5	1250-5
2 - high redemption values	BCEHI	1124-1	980-1	683-10	346-10	1359-2	641-2	1710-9	430-5	259-6
2- 2nd highest redemption values	BCEHI	692-5	487-5	463-4	343--7	1027-5	515-5	1625-5	319-9	173-4
2 - data from experiment 1	BCEHI	1090-1	486-1	610-10	340-7	1030-2	514-5	1610-9	320-5	240-6
2 - data from experiment 2	BCEHI	1000-1	500-1	660-10	330-10	1000-2	505-5	1660-9	320-5	190-6
2 - data from experiment 3	AEHI	1100-1	400-5	682-10	345-10	1059-2	300-4	1610-9	333-5	200-6
3 - high redemption values	ADFHI	1858-3	615-3	1227-4	964-4	943-7	774-7	1843-1	886-2	849-8
3 - 2nd highest redemption values	BCEHI	660-5	405-5	1025-8	662-3	500-10	341-8	1731-9	757-6	759-5
3 - data from experiment 1	ADFHI	1050-3	600-3	1100-4	811-4	500-7	341-7	1840-1	700-2	790-8
3 - data from experiment 2	BCEHI	675-3	381-5	1025-4	656-8	600-7	400-7	1830-1	786-2	790-8
3 - data from experiment 3	ADFHI	1000-3	364-10	1050-4	675-4	550-7	350-7	1560-2	786-2	770-8

period	high feasible package	A	B	C	D	E	F	G	H	I
4 - high redemption values	BCEHI	1371-9	1105-9	1193-1	594-10	1300-4	812-6	2057-6	1123-3	1258-2
4 - 2nd highest redemption values	BCEHI	1020-10	553-1	788-10	499-8	1067-6	430-4	1533-2	254-8	368-5
4 - data from experiment 1	BCEHI	1000-10	726-9	790-1	594-10	1050-4	500-6	1450-6	400-3	400-2
4 - data from experiment 2	BCEHI	901-10	600-9	730-1	920-10	1299-4	805-6	2040-6	300-3	458-2
4 - data from experiment 3	BCEHI	1150-9	550-9	800-1	500-10	1100-4	255-9	1900-6	260-3	458-2
5 - high redemption values	ADFHI	2108-5	700-5	1162-7	873-7	873-3	557-3	1999-6	755-4	1102-4
5 - 2nd highest redemption values	BCEHI	680-10	527-9	760-9	634-9	580-6	356-6	1636-2	554-6	758-9
5 - data from experiment 1	ADFHI	1320-5	525-9	932-7	654-7	850-3	360-3	1589-10	550-4	800-4
5 - data from experiment 2	ADFHI	1101-5	527-9	800-7	634-7	750-3	360-3	1999-6	555-4	800-4
5 - data from experiment 3	ADFHI	1500-5	520-9	1135-7	520-9	700-3	360-3	1995-6	575-4	760-4
6 - high redemption values	BCEHI	788-2	459-2	1341-9	749-9	1005-10	603-10	1340-6	358-1	676-6
6 - 2nd highest redemption values	BCEHI	565-1	398-1	675-2	334-2	785-5	471-5	956-7	353-6	394-1
6 - data from experiment 1	BCEHI	701-5	425-2	676-9	310-4	791-10	400-5	1340-6	353-1	450-6
6 - data from experiment 2	BCEHI	700-2	409-2	700-9	334-2	800-10	360-2	1300-6	355-1	400-6
6 - data from experiment 3	BCEHI	700-2	400-2	750-9	400-9	780-10	400-5	1000-6	353-1	400-6
7 - high redemption values	ADFHI	1685-2	614-6	1071-4	931-4	838-1	554-1	1260-4	1483-10	1465-10

7 - 2nd highest redemption values	ADFHI	1444-10	581-10	648-2	319-6	557-9	418-2	1133-9	164-8	392-3
7 - data from experiment 1	ADFHI	1450-2	560-1	610-1	350-4	730-1	410-1	710-1	165-10	396-10
7 - data from experiment 2	ADFHI	1485-2	551-10	600-4	400-4	570-1	370-1	700-3	221-10	391-10
7 - data from experiment 3	ADFHI	1485-2	510-10	648-2	450-4	625-1	420-1	1133-9	170-10	400-10

Table 6.2: Unrealized Profitable Opportunities at Closing in 1T7 Testbed Experiments

Period	Experiment 1		Experiment 2		Experiment 3	
	Possible Profit	Id #	Possible Profit	Id#	Possible Profit	Id#
1	0		0		0	
2	0		26	5	538	1
3	56	6	913	3	0	
4	16	6	57	0	10	6
5	30	6	105	7	352	7
6	0		0		4	5
7	7	5	47	5	0	

Table 6.3: Classification of final allocations as NE1 equilibria

'd' corresponds to the unrealized individual profit opportunity in Table 6.2.

Equilibrium Classification	#cases
Strict NE1 equilibrium (d=0)	8
NE1 if subjects have thick indifference (1<d<50)	7
Borderline cases (d=56,57)	2
Not NE1 (d>100)	4

Table 6.4: Classification of individual bids

Period	Experiment	Bid Event Counts		
		Dominated	Neutral	Pivotal
1	1	6	121	83
	2	2	20	60
	3	1	28	45
2	1	12	101	91
	2	1	44	63
	3	0	42	90
3	1	4	104	111
	2	3	81	142
	3	0	19	73
4	1	2	12	25
	2	4	54	74
	3	1	27	62
5	1	3	46	93
	2	3	66	93
	3	2	36	70
6	1	4	15	74
	2	6	34	129
	3	0	21	66
7	1	1	44	94
	2	6	37	75
	3	1	17	24
Totals		62	969	1637

Table 6.5: NE1 Outcomes During Each Period

Period	Experiment	Reached NE1 Outcome States	
		Strict (d=0)	Near (d<50)
1	1	17	17
	2	2	9
	3	8	9
2	1	6	19
	2	0	4
	3	0	0
3	1	0	0
	2	0	0
	3	1	3
4	1	0	10
	2	0	0
	3	0	2
5	1	0	13
	2	0	0
	3	0	0
6	1	4	10
	2	1	7
	3	0	4
7	1	0	4
	2	0	1
	3	4	8

Table 6.6: Comparison of Bid Prices and the Second Highest Redemption Values

Optimal Schedule	Allocated Schedule	#Trials		$d_2(A)$	$d_2(B)$	$d_2(C)$	$d_2(D)$	$d_2(E)$	$d_2(F)$	$d_2(G)$	$d_2(H)$	$d_2(I)$
ADFHI	ADFHI	11	Avg	0	15.8	44.6	10.4*	25.6*	0.7*	97.6	5.6	0.1
			Max	0	71	405	114*	282*	8*	433	57	1
BCEHI	BCEHI	8	Avg	17.4	0.5	7.3*	5.0	6.1	54.9	12.3	0	0
			Max	119	3	58*	24	27	175	83	0	0
BCEHI	AEHI	1 (period 2-exp. 3)	---	0	87	0	0	0	215	15	0	0
ADFHI	BCEHI	1 (period 3-exp. 2)	---	0	24	0	6	0	0	0	0	0

Note: * --- This deviation value is entirely due to one experimental trial.

CHAPTER 7. Decentralized Computation in BICAP: The Computation Procuring Dutch Auction (CPDA)

When BICAP is used in a computationally difficult environment, the calculation of potential allocations that maximize bid revenue may exceed the limits of computational facilities and cause the mechanism to fail. One type of failure is for the algorithm to stop functioning due to need for additional computing resources, e.g., memory. A more typical mode of failure might be that the optimization algorithm exceeds a reasonable time limit for computation. Another way to say this is that the use of centralized optimization in BICAP results in a mechanism that can overdemand computational resources relative to what is economical. Not only may the level of computation be too high, centralized optimization may not necessarily produce the calculations at lowest cost if the mechanism authority (the seller) does not keep abreast of all the latest technical innovations in computers and algorithms to do the optimizations.

One means for decentralized computation is some type of procurement auction. Such an auction might tend to identify the low cost agents, who would be given an economic incentive to perform the computations. This chapter formalizes this technique as the Computation Procuring Dutch Auction [CPDA]. The BICAP mechanism of Chapter 5 is then modified in a minimal way to include CPDA. The new mechanism obtained in this way will be referred to as BICAP+CPDA.

This chapter is outlined as follows. Section 7.1 gives an informal introduction to the CPDA technique, section 7.2 formally applies it to BICAP, and section 7.3 gives a theory of CPDA's operation. A short sketch of the intuition behind BICAP+CPDA follows, which will be expanded upon in section 7.3.

If one assumes that CPDA works as well as centralized optimization, then, theoretically, it can be shown that the minimal modifications in BICAP necessary to include CPDA do not substantially alter bidding incentives. Can one expect CPDA to function well, i.e., do potential allocations under CPDA correspond to those that would have been calculated by computer? It is difficult to answer this question, but one can show the following: Under CPDA, if agents believe that their peers have similar information, then it is often[35] a dominant (in the sense of increasing profits) strategy for the agents to submit known improvements to the potential allocation. In cases where it might seem that an agent has a perceived conflict-of-interest, the proposal bonus for submitting the improvement eventually exceeds the loss accruing to the agent. The BICAP+CPDA mechanism will then be tested in Chapter 8 to see to what extent the theoretical assertions can be verified in the laboratory.

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[35] The Improvement Characterization Theorem, in this chapter, provides this result.

7.1 The Computation Procuring Dutch Auction [CPDA]

This section informally describes the Computation Procuring Dutch Auction [CPDA]. CPDA is a Dutch clock auction for procuring computational activities in a mechanism. CPDA transfers the computationally difficult task, that of searching all feasible allocations for those that maximize the sum of the current bids, from the mechanism authority (the seller) to the agents. Agents are then provided with incentives that may encourage them to perform the search in a manner that forces them to consider costs and benefits of such activities.

Under CPDA, the mechanism supplies to agents the information they need to search for improvements to the potential allocation. The set of high bids and the conflicts between trains are public knowledge as well as the current potential allocation. The potential allocation is initially set to the null allocation where no one receives the rights to run any train. Agents propose improvements to this allocation. An improvement consists of a feasible set of trains that would produce higher bid revenue, given the current bids, than the current potential allocation. Submission of an improvement changes the potential allocation. The agent submitting the improvement receives a *proposal bonus*, which is a percentage of the instantaneous increase[36] in bid

[36] As improvements are submitted, some agents may also wish to raise their bids. This further raises the bid revenue to the seller, and one could argue that this increase in revenue is in part due to the submission of the improvements. In calculating the proposal bonus, only the increase in bid revenue given the current bids is considered.

revenue resulting from the improvement. This percentage is called the *bonus percentage.*

The bonus percentage starts at 0% and increases slowly up to 100% as time passes without bids or improvements being submitted. When a bid or improvement is submitted, the bonus percentage is reset to 0%. In this way the bonus percentage causes the incentive scheme above to act as a Dutch clock auction.[37]

The goals of CPDA are to encourage both cooperation and competition between agents in submitting improvements and to also cause the amount of resources devoted to searching for improvements to respond to benefits and costs of such activities. Briefly consider the intuition behind how these goals might be accomplished under the above specified incentives. The intuition below will be developed into a theory of operation in a later section, after CPDA is merged into the BICAP mechanism.

CPDA encourages cooperation by making public knowledge any improvements to the potential allocation that are submitted. CPDA creates a parallel processing environment where agents willingly compute increasingly better approximations. CPDA encourages competition through the Dutch clock auction structure of the incentives: since only one agent will be paid for an improvement, it is to an agent's advantage to hold back information regarding improvements only until he or she

[37] If the bonus percentage did not reset when bids are entered, then an agent could submit a bid which would change the bid maximizing potential allocation, immediately submit an improvement. This would effectively give the bidder a discount on his bid, perhaps a very lage discount.

believes other agents might also have the knowledge and be about to submit it. Such competition helps to lower computational cost by encouraging computation by the fastest least cost agents. Because bonuses paid to agents for finding improvements are always less than the change in bid revenue, agents must consider the benefits and costs of searching for improvements in their decision to undertake computational tasks.

One potential difficulty with CPDA is that duplication of computational effort among agents is possible. Agents competing for proposal bonuses might choose to simultaneously work on finding similar improvements. Practical, incentive compatible coordination mechanisms to avoid this problem might be difficult to construct. This research simply defines and tests CPDA and does not attempt to solve this coordination problem.

7.2 Modifying BICAP to Include CPDA

This section details the changes made in the BICAP mechanism to include CPDA. Recall that a mechanism involves three essential elements: a set of feasible outcome allocations, a message space through which agents interact, and an outcome rule that specifies how these messages determine a unique outcome from the feasible set of allocations. In the BICAP mechanism each agent submits bids for projects (trains) in a continuous time auction. The highest bid on a project prevails as the potential winner and cancels all lower bids for the project. The process of bidding continues until some pre-specified time has elapsed with no bids taking place. These features of the BICAP mechanism are retained in the BICAP+CPDA mechanism. The only changes made are those necessary to allow agents to take over the task of determining, through the CPDA incentive process, the potential allocation. Formally, the mechanism is outlined by the following statements, much of which is reproduced from the BICAP discussion of Chapter 5.

The essential elements of the Binary Conflict Ascending Price with Computation Procuring Dutch Auction (BICAP+CPDA) mechanism are as follows:

1. The message space $\boldsymbol{M}$ consists of the following messages[38] for each individual:

    Bids $(b,f)_i$; b is a bid (in cash) by agent i for the train route f.

    System Proposals $[\boldsymbol{Q}]_i$; $[\boldsymbol{Q}]_i$ is a request by agent i that the allocation be changed so that the trains in $\boldsymbol{Q} \subseteq \boldsymbol{F}$ are allocated to their respective high bidders.

    A time message from nature indicating the elapse of time

2. A mechanism state $s=(\tau,\boldsymbol{B},\boldsymbol{H}, \wp, \boldsymbol{v})$ consists of

    (i) the original BICAP state variables τ, $\boldsymbol{B}$, $\boldsymbol{H}$, $\wp$

    (ii) a vector of earnings $\boldsymbol{v}$ of agent proposal bonuses

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38 The different symbols '()' and '[]' will help to distinguish the messages in discussions that follow.

3. The initial mechanism state $s_0 = (\tau=\tau_0, \boldsymbol{B=0}, \boldsymbol{H=0}, \rho^*=$null allocation$, \boldsymbol{v=0})$.

At the start of the mechanism:

(i) The variables τ, $\boldsymbol{B}$, $\boldsymbol{H}$, ρ^* are set as they were in BICAP.

(ii) The agent proposal bonuses are set to zero.

4. The set of terminal mechanism states $\boldsymbol{S^*}$ is the subset of $\boldsymbol{S}$ obtained by making the restriction $\tau=0$, i.e.,

$$\boldsymbol{S^*} = \{s^* \in \boldsymbol{S}: \tau=0\}.$$

Like BICAP, BICAP+CPDA is also a soft terminating mechanism.

5. Throughout the auction, $(\boldsymbol{X,I,F,C})$, and the mechanism state variables τ, $\boldsymbol{B}$, $\boldsymbol{H}$, and ρ^* are all common knowledge. The agent feedback information functions are defined so that these elements of the mechanism state are publicly broadcast whenever it changes. This is also exactly as is in BICAP.

6. At a terminal state $s^* \in \boldsymbol{S^*}$, the mechanism outcome allocation rule is

$$O^*(s^*) = \rho^* \text{ “+” } \boldsymbol{v}.$$

The outcome allocation consists of the allocation ρ^* of rail resources and money transfers added[39] to the monetary transfers necessary for the agents to receive their CPDA proposal bonus earnings $\boldsymbol{v}$ from the seller.

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[39] There are two ways to handle the payment of the CPDA proposal bonuses. Either they can be included in the specification of ρ^* or they can be made separate. If they are made separate, then technically the cash component of the outcome allocation from the mechanism is the sum of the train and transfer payment allocations in BICAP and the proposal bonus allocationsin CPDA.

7. The <u>mechanism transition rule</u> $T{:}\boldsymbol{S}\otimes\boldsymbol{M}\rightarrow\boldsymbol{S}$ that describes how the mechanism state evolves in response to messages is specified as follows:

If m = bid message $(b,f)_j$ from agent $j\in I$, then:

If $b > B_f$: $s^{new}=T(s,m)$, where components of s^{new} are denoted by the "new" superscript:

$\tau^{new} = \tau_0$ [the timer is reset]

$B^{new}{}_f = b$ [the bid becomes the new train high bid]

$H^{new}{}_f = j$ [the bidder becomes the new train high bidder]

If $f\in \Lambda_S$, $\wp^{\bullet new}$ changes from $\wp^{\bullet}$ so that:

$$\Lambda_{H_f}^{new} = \Lambda_{H_f} - \{f\}$$

$$\Lambda_j^{new} = \{f\} \cup \Lambda_j$$

$$t_j^{new} = \sum_{\{f':H_{f'}=j\}} B_{f'}^{new}$$

$$t_S^{new} = t_S + (b - B_f)$$

Otherwise, $\wp^{\bullet new} = \wp^{\bullet}$.

If $b \leq B_f$:

$T(s,m)=s$ [bids lower than the current bids have no effect]

If m = system schedule proposal message $[\boldsymbol{Q}]_i$, then:

If

$(\boldsymbol{Q} \otimes \boldsymbol{Q}) \cap \boldsymbol{C} = \varnothing$ [Proposal is Feasible] and

$$\sum_{f \in Q} B_f > t_S$$, where t_S is the seller's transfer from $\mathcal{P}^*$. [Improving]

then

$s^{new} = T(s,m)$

where

$\mathcal{P}^{*new} = (a_S^{new} ; a_1^{new}, ..., a_{|I|}^{new}) \in \mathfrak{I}_{BC}(\boldsymbol{I,F,C})$;

$\forall k \in I + \{S\},\ a_k = (\Lambda_j^{new}, t_j^{new}) \in \boldsymbol{2^F} \otimes \Re$:

(i) $\Lambda_S^{new} = \boldsymbol{Q}$ [system schedule proposal is accepted]

(ii) $t_S^{new} = \sum_{f \in \Lambda_S^{new}} B_f$

(iii) $\forall j \in I,\ \Lambda_j^{new} = \Lambda_S^{new} \cap \{ f \in \boldsymbol{F} : H_f = j \}$

(iv) $\forall j \in I,\ t_j^{new} = - \sum_{f \in \Lambda_j^{new}} B_f$

$v_i^{new} = v_i + ((\tau_0 - \tau)/\tau_0) (t_S^{new} - t_S)$

If m = time message from nature, then $\tau^{new} = \tau - 1$.

<u>*Remark.*</u> Combining the Chapter 2 definitions with (1)-(7) above, the potential profits vector $\pi_j^P(s;\underline{\mathbf{V}})$ in the BICAP+CPDA mechanism is, by definition,

$\pi^P(s;\underline{\mathbf{V}}) = \Pi(O^P(s) ; \underline{\mathbf{V}}) = \Pi(\mathcal{P}; \underline{\mathbf{V}}) + \boldsymbol{v}$.

Definition. $\pi^{TP}(s;\underline{V}) = \Pi(\, P;\, \underline{V}\,)$ and $\pi^{CP}(s) = v$. The term *train potential profits* or *BICAP potential profit* refers to $\pi^{TP}(s;\underline{V}) = \Pi(\, P;\, \underline{V}\,)$ and *CPDA bonuses* or *proposal bonuses* refer to $\pi^{CP}(s) = v$. Notice that $\pi^{P}(s;\underline{V}) = \pi^{TP}(s;\underline{V}) + \pi^{CP}(s)$.

7.3 Models of Performance and Agent Behavior

The original analysis of the BICAP mechanism involved identification of NE1 outcomes and conjectures regarding dynamic processes that might cause the mechanism to select the NE1 that yield the optimal allocation. All of the analysis involving bidding carries over, given a key assumption: that CPDA does, in fact, induce agents to calculate and submit improvements to the potential allocation so that the final potential allocation is virtually equivalent to that expected under BICAP's system of centralized optimization. This section will be concerned with trying to justify such an assumption. To begin, a few definitions are needed.

Definition. The *set of feasible improvements* at state s, denoted FI(s), is the set of non-null proposal messages for the BICAP+CPDA mechanism (i.e. those that increase the sum of the bids of the allocated trains):

$FI(s) = \{\, \boldsymbol{F}_S \subseteq \boldsymbol{F} : T(s,[\, \boldsymbol{F}_S\,]_i) \neq s\,\}$.

Definition. The *set of known improvements* for agent j at state s, $KI_j(s)$, is the subset of FI(s) that agent j may propose (or is "aware of").

The following definition and remark show that, along a bidding path where the agents are fully aware of the implications of their bid on the bid maximizing potential allocation, a model of "search" for improvements is unnecessary.

Definition. Suppose, in BICAP+CPDA, agent i submits a BICAP-pivotal bid $(b,f)_i$ that changes the bid maximizing system schedule from Λ_S to Λ_S'. Agent i is referred to as the *inside bidder* until either: (i) other bids are submitted, by any agent, that change the bid maximizing system schedule, or (ii) the improvement message $[\Lambda_S']_j$ is submitted by some agent j. The improvement message $[\Lambda_S']_i$ for agent i is called the *inside improvement.*

Proposition 7.1. If inside improvements are always known improvements for inside bidders, then a potential-profit maximizing inside bidder will always submit the inside improvement rather than let the mechanism terminate.

Proof. Submitting the inside improvement always improves the inside bidder's potential profit. Failing to do so leaves uncaptured the potential profit that the bid was meant to capture. Therefore a potential-profit maximizing agent will always submit any inside improvement when he is an inside bidder, rather than let the mechanism terminate. •

In difficult computational environments it is possible that a bidder will not realize all the implications of his or her bid, and will not submit an associated improvement. This creates the possibility that some improvements will not be submitted unless some agent searches for them. It is then necessary to investigate and characterize search and reporting processes. To begin, consider the following definitions.

Some feasible improvements will raise potential profits for an agent, and some will not. If agent j believed with certainty that another agent was about to submit a known improvement that would lower agent j's potential profit, then agent j would have a dominant strategy to submit the improvement before the other agent. In that way, agent j would at least gain the proposal bonus from the improvement. This is similar to avoiding a sunk cost fallacy[40].

The above argument suggests that one possible theory of operation of CPDA could be built on two hypotheses:

<u>*Complete Search Hypothesis.*</u> There exists a $\tau_\varepsilon > 0$ such that when the mechanism timer is at $\tau < \tau_\varepsilon$, $\cup_{j \in I} KI_j(s) = FI(s)$.

40 Agent j behaves like he is avoiding a sunk cost fallacy, where the "sunk cost" is the potential profit j would obtain from a potential allocation that is transient. The fallacy in this case is that j's claiming the proposal bonus causes the loss of the potential profit from the previous potential allocation. While de facto true by the rules of the mechanism, we are assuming that had j not submitted the improvement, someone else would, and the potential profit from the previous potential allocation would still be lost bu t agent j would not have gained the proposal bonus.

Full Reporting Beliefs Hypothesis. There exists a $\tau_\varepsilon > 0$ such that when the mechanism timer is at $\tau < \tau_\varepsilon$, each agent $j \in I$ believes that every known improvement $KI_j(s)$ will be submitted by some other agent unless precluded by another submission.

Proposition 7.2. Under the hypotheses of Complete Search and Full Reporting Beliefs, the mechanism can only terminate when the potential allocation is the bid maximizing potential allocation.

Proof. Under the condition that the potential allocation is not the bid maximizing potential allocation, it will be shown that the mechanism can not terminate. This implies that the mechanism can only terminate at a bid maximizing potential allocation. Suppose that the potential allocation is not bid maximizing. Then feasible improvements exist. For the mechanism to terminate, the mechanism timer τ must reach zero. When the mechanism timer falls below τ_ε , under the Complete Search hypothesis some agent will know any feasible improvement and under the Full Reporting Beliefs hypothesis each agent will have a dominant strategy to submit some known improvement. The first submission will reset the mechanism timer, and the process will repeat. Therefore, the mechanism can not terminate until the set of feasible improvements is empty. This occurs at a potential allocation that maximizes the sum of bids. •

The two hypotheses used to obtain the result are quite strong and probably are not satisfied by the human agents in the experiments. Note, however, that full reporting beliefs is a self-sustaining hypothesis in the following sense. Consider a Bayesian agent who has beliefs over what known improvements will be submitted, and who updates beliefs via Bayes rule. A Bayesian agent whose beliefs are characterized by full reporting beliefs never tests those beliefs. To test his beliefs the agent would have to find an improvement, refrain from submitting it, and then observe the mechanism terminating with no one submitting it. Since the agent has a dominant strategy to always submit improvements, the agent who has full reporting beliefs never tests or changes his beliefs.

Proposition 7.2 is unsatisfying in its assumptions about beliefs. Another model can be created to predict success of CPDA. The model now developed will begin by strengthening the search hypothesis to the Universal Search Hypothesis below but the model will not assume the Full Reporting Beliefs Hypothesis.

Universal Search Hypothesis. There exists a $\tau_\varepsilon > 0$ such that when the mechanism timer is at $\tau < \tau_\varepsilon$, $KI_j(s) = FI(s)\ \forall j \in I$.

Now assume that agents do not submit improvements that will lower their potential profit and do submit improvements that will increase their potential profit. If the Universal Search Hypothesis is assumed, will all known improvements be submitted? Will the mechanism potential allocations still reach the bid maximizing potential allocations? To answer these questions, it is necessary to examine whether improvements exist that do not improve potential profit for any agent. Such improvements will be called *universally opposed* because no agent would be willing to submit it.

<u>*Definition.*</u> An non-null improvement message $[\underline{Q}]_i$ is *universally opposed* at a state $s' \in S$ in BICAP+CPDA if the total of the train potential profit change and the maximum possible CPDA proposal bonus is non-positive for every agent, i.e.

$\forall i,\ \Delta\pi_i^P = (\ \pi_i^{TP}(T(s',[\underline{Q}]_i);\underline{V}) - \pi_i^{TP}(s';\underline{V})\ ) + (\ t_S'' - t_S'\ ) < 0$, where:

t_S'' is the total transfer to the seller at the potential allocation at state $s''=T(s', [\underline{Q}]_i)$, and t_S' is the total transfer to the seller at the potential allocation at state s'.

The success of CPDA hinges on the set of universally opposed improvements being small or having some other interesting property. It will be shown that universally opposed improvements lower the sum of the values of the trains that are allocated (potential allocational efficiency) even though the sum of the bids is increased.[41] This is an interesting result because none of the agents actually know which improvements raise or lower the sum of the values of the trains. Different trains are generally allocated to different agents and the train value information is private. Nevertheless, all universally opposed improvements have this property. The mechanism causes agents' incentives to take this information into account even though no agent need

[41] This is possible when there are high bids for train routes that are inefficient. An extreme example is the mistake of large overbidding on a worthless route.

realize consciously that this is happening.[42] By avoiding these improvements, CPDA could possibly have the effect of raising the efficiency of the mechanism outcome allocation.

Theorem (Improvement Characterization Theorem). At a BICAP+CPDA mechanism state s′, a non-null improvement message [*Q*] is universally opposed only if two conditions (i)-(ii) are met:

(i) $\pi_i^{TP}(T(s',[Q]_i);\underline{V}) < \pi_i^{TP}(s';\underline{V}) \ \forall\, i \in I$

[each agent's train potential profits is lowered].

(ii) $\upsilon(O^P(T(s',[Q]_i));\underline{V}) < \upsilon(O^P(s');\underline{V})$

[total value of the potential allocation is lowered]

Proof. Let ′ denote relevant variables before the improvement is submitted and ″ denote relevant variables after the improvement is submitted. For instance, $s''=T(s',[Q]_i)$. In particular let $\upsilon'= \upsilon(O^P(s');\underline{V})$ and $\upsilon''= \upsilon(O^P(s'');\underline{V})$ be the total redemption values of the potential allocations before and after the improvement is submitted. By definition, a non-null proposal message $[Q]_i$ is universally opposed if $\forall i$,

$$\Delta\pi_i^P = (\, \pi_i^{TP}(T(s',[Q]_i);\underline{V}) - \pi_i^{TP}(s';\underline{V})\,) + (\, t_S'' - t_S'\,) < 0.$$

Recall that the first term, in parenthesis in the sum, is the change in train potential profit for agent i under the improvement. The second term in the sum is the maximum possible bonus agent i could receive under CPDA for the improvement.

42 Therefore, an infomration transfer process similar to the one Hayek(1945) described for markets would seem to be at work.

For a non-null improvement, the BICAP+CPDA transition rule $T(s',[Q]_i)$ implies that $t_S'' > t_S'$. Therefore the second term above, $(t_S'' - t_S')$, is necessarily positive. The first term, the train potential profits, must be negative to cancel out the incentive from the second term; otherwise some agent will have positive potential profit from the improvement and have a dominant strategy to submit the improvement. Condition (i) says that this first term is negative for all agents, and therefore is a necessary condition for an improvement to be universally opposed.

To show Condition (ii) is necessary, we use the same technique as in Proposition 5.6: that total bid must equal the total redemption value υ of the allocation minus the sum of all agents' train profits. This is an accounting identity and is true no matter what the allocation is. Symbolically:

$$t_S'' = \upsilon'' - \sum_{j \in I} \pi_j^{TP}(s'')$$

$$t_S' = \upsilon' - \sum_{j \in I} \pi_j^{TP}(s')$$

Then, applying this accounting identity to $\Delta\pi_i^P$, we initially have

$$\Delta\pi_i^P = (\pi_i^{TP}(s'';\underline{V}) - \pi_i^{TP}(s';\underline{V})) + (t_S'' - t_S')$$

and after the substitution for t_S above, we have

$$\Delta\pi_i^P = (\pi_i^{TP}(s'';\underline{V}) - \pi_i^{TP}(s';\underline{V})) + \upsilon'' - \sum_{j \in I} \pi_j^{TP}(s'';\underline{V}) - (\upsilon' - \sum_{j \in I} \pi_j^{TP}(s';\underline{V}))$$

regrouping terms

$$\Delta\pi_i^P = (\upsilon'' - \upsilon') - \sum_{\substack{j \in I \\ j \neq i}} (\pi_j^{TP}(s''; \underline{V}) - \pi_j^{TP}(s'; \underline{V})) \,.$$

Note that the first term is just the change in total redemption value under the allocation. The second term is minus the sum of all potential profit changes for all other agents except i under the improvement. However, we know that all potential profit changes for all other agents are negative, so the term in brackets is negative and makes a positive contribution to $\Delta\pi$. Therefore, $\upsilon'' < \upsilon'$ is a necessary condition for an improvement to be universally opposed, and this condition is the same as condition (ii) above. •

It is important to point out that condition (i) of the theorem predicts the absence of universally opposed improvements in the experimental testbeds that were constructed in Chapter 3. In the experimental testbeds, all allocations leave some agents with no trains. Because there are always agents who receive no train rights, condition (i) is never satisfied because there are some agents whose train potential profits do not strictly decrease. This means that in the experimental testbeds, agents only need to be capable of Universal Search for CPDA to function successfully. Reporting must automatically follow since no improvement is universally opposed.

The next chapter will experimentally test the performance of BICAP+CPDA in the three-track testbeds, and this will allow an opportunity to investigate whether theoretical expectations adequately explain behavior of human agents.

CHAPTER 8. An Experimental Test of the BICAP+CPDA Mechanism

Three questions of primary interest to be addressed by the next series of experiments are whether the BICAP+CPDA mechanism performs as well as the BICAP mechanism in efficiency of allocation, whether computation under BICAP+CPDA is performed in a timely manner and at low cost, and whether there are any noticeable differences in distribution of surplus or bidding behavior under the BICAP and BICAP+CPDA mechanisms. The first two questions are primarily performance issues, and the last question, the comparison test, addresses some elements of design consistency.

The comparison empirically examines the assumption that BICAP+CPDA is a minimal modification of the BICAP bidding incentives. Recall that the idea behind BICAP+CPDA is to improve computational performance but retain as much of the original bidding incentives that made BICAP efficient. If the incentives are the same, then one might reasonably expect the economic results to be the same as well[43].

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[43] In the interest of completeness, note that such a statement is actually being tested along with the performance of BICAP/BICAP+CPDA. The comparison relies on at least two auxilliary assumptions: (i) that incentives for running trains on each train track line in the testbed environment of the 3ST7 experiments faithfully duplicate the incentive structures that existed for operators of trains A-G in the 1T7 testbed environment and that the different train track lines are indeed independent of each other, and that (ii) nearly identical incentive structures produce nearly identical results. In paricular (ii) assumes that a number of details (time, place, personal identity of agents, wording of instructions) that are different between this series of experiments and the BICAP experiments do not matter. If the results are compared between BICAP+CPDA experiments using the 3ST7 testbed above and the 1T7 experiments, and fundamental aspects of the results are nearly identical, then the Assumptions (i) and (ii) must have been satisfied to a reasonable degree.

The experimental results show that BICAP+CPDA performs almost as well as BICAP. Efficiencies will be high, but not as high as BICAP in the 1T7 testbed experiments. Decentralized computation of potential allocation calculations will be shown to be working well. The BICAP+CPDA mechanism will produce very similar total revenues in the comparable portions of the 3ST7 and the 1T7 testbed environments, and thus BICAP and BICAP+CPDA might be said to have equal political compatibility at least in terms of revenue production for the government track owner[44]. There will be some puzzling mixed results in the design consistency section. Overall, however, BICAP+CPDA could be given a positive evaluation. The experiments also provide a proof of principal that CPDA can be added to a mechanism to decentralize mechanism computation without hurting allocation performance.

8.1 Experimental Methodology

The experiments involve testing the BICAP+CPDA mechanism of Chapter 7 in the three-track testbed environments from Chapter 3. Recall that there are two different three-track testbeds, labeled 3ST7 and 3NST7. The 3ST7 environment is nothing

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[44] Ledyard points out that it could be the case that it is common knowledge that mechanism A is more efficient than mechanism B, but transition from mechanism B to mechanism A is politically infeasible because such a transition would change the distribution of surplus away from agents or groups of agents that are decisive about what mechanism is used for allocation. To recognize this problem, he defines a change in mechanism to be a 'politically compatible' transition if it does not make any politically decisive coalitions worse off. Unless a mechanism leaves everyone indifferent or everyone better off, political compatibility will depend on the political environment. If the BICAP+CPDA experiments produce very similar results to the BICAP experiments, (e.g., efficiencies, revenue generated for the seller, closing prices are nearly identical), then the opportunities of any agent are the same either under either BICAP or BICAP+CPDA, and the political compatibility of the two mechanism, when compared against some status quo must be similar. This criterion may be important for those who wish to use BICAP, BICAP+CPDA, or something similar to replace the priority system for allocation of track time in Sweden.

more than three independent copies of the 1T7 testbed environment used in the Chapter 6 experiments, but operating on separate tracks. Direct comparisons of economic variables, such as total revenue, will be made between the Chapter 6 experiments and the 3ST7 experiments reported here. Similarity in results between BICAP and BICAP+CPDA is expected. The 3NST7 testbed is the 3ST7 testbed with additional conflicts between trains on the previously separate, independent train tracks. Because of the additional conflicts, the 3NST7 experiments are not economically comparable. Experiments with the 3NST7 testbed do demonstrate the response of the BICAP+CPDA mechanism to a more difficult computational problem.

The BICAP+CPDA mechanism was implemented for the most part as specified in Chapter 7. One important variation is there is a set I_P of agents who are allowed to propose improvements, and this is varied in the two 3ST7 experiments. This variable tests effects of CPDA and does not affect underlying properties of the rail testbeds. The 3ST7-1 testbed uses $I_p=I_B$, which means that any agent could propose an improvement. In the 3ST7-2 testbed I_p consisted of three special, independent agents {#21,#22,#23} who do not otherwise take part in the experiments (i.e., they do not have redemption values for trains and are not buyers), and so only these three agents could propose improvements. In the 3NST7 environments there is only one specification: $I_P=I_B$.

Varying I_P enables one to see how limiting the set of proposers affects the mechanism outcome. In particular, does the mechanism provide sufficient incentives for otherwise indifferent agents to submit proposals for improvements? Can these agents, being small in number, extract a large portion of the surplus through some type of Cournot effect? This variation only occurred for the 3ST7 testbeds.

The rules of the BICAP+CPDA mechanism are implemented in the experimental instructions essentially as described in Chapter 7, albeit in a simplified form. The agents saw these rules, simplified and reworded, in their instructions, which are included in the Appendices B and C. Essentially the procedures are virtually identical to those of the experiments in Chapter 6.

Computer screens displaying the mechanism were substantively equivalent to the BICAP screens in Chapter 6 except that 21 trains were displayed instead of 9, and a special command for submitting improvements to the potential allocation was added. The agent's computer system would check and report on validity of an improvement when it was submitted, and invalid improvements were rejected with an appropriate error message. However in no way did the computer equipment suggest improvements to agents. Agents had pencil, paper, calculator, and their own personal abilities upon which to rely to try to find improvements to the potential allocation.

Experiments were carried out using Caltech undergraduate and graduate students as agents. The students were recruited through an announcement on a campus computer network. A total of four experimental sessions were conducted, one session of 3ST7-1, one session of 3ST7-2, and two sessions of 3NST7 that are labeled 3NST7-1 and 3NST7-2 for consistency. Each session lasted approximately two to three hours and procedures in each of the four experiments were essentially identical. Table 8.1 summarizes the testbed parameters for each experiment.

Results will be discussed in the following order: first, performance measures of the final allocations will be addressed, then performance measures involving the computation of potential allocations. Then, a comparison with BICAP is conducted. The total revenue and bidding behaviors are compared between the results from the 3ST7 testbeds and the results from the 1T7 testbed. As BICAP+CPDA is supposed to be a minimal modification of BICAP, it is expected that these behaviors are similar. The resulting similarity in size and distribution of surplus would also suggest that BICAP and BICAP+CPDA are equal in Ledyard's criterion of political compatibility. Finally, an examination of design consistency issues examines classifications of bidding behavior and NE1 stationary equilibria.

8.2 Experimental Results: Allocational Performance

Two measures of the efficiency of allocations in the experiments using the BICAP+CPDA mechanism are presented in Table 8.2. The first measure is the standard measure of efficiency defined in section 2.4, which was used in describing the BICAP experimental results in Chapter 5. The second measure is a rank measure of the total redemption value against redemption values in a special class *of maximal agent allocations*, defined formally below. This rank measure is a new concept developed here, and incorporates some information missing from efficiency ratios[45]. First, the maximal rank measure will be formally defined and then compared with the use of efficiency ratios. Then the data in Table 8.2 will be evaluated.

<u>*Definition*</u>. A *maximal agent allocation* a* has the property that agents hold the high social value V_{if} for every train that they are allocated under a*. That is,

$\forall i \in \boldsymbol{I}, f \in \Lambda_i, V_{if} = \max_{j \in \boldsymbol{I}} V_{jf}$.

45 One criticism with the use of efficiency ratios is that it does not give an idea of the difficulty of finding or negotiating superior allocations. While efficiency is one way of ranking allocations, it provides no clue of whether a specific outcome, say an efficiency of 0.80, is very good or very poor. To determine whether 0.80 efficiency is good or poor an economist often needs to understand economic aspects (e.g., are there public goods, externalities, or other economic phenomena that make negotiating difficult?) as well as computational aspects (e.g., is it difficult to search for a better allocation? are there many better allocations or only a few?) of the environment. The rank in maximal agent allocations is a measure of performance that incorporates some of the information above, in particular a measure of the number of allocations that are superior, into the measurement.

<u>*Definition.*</u> The *maximal rank* Mrk(a*) of an allocation a* is

(i): for maximal agent allocations a*, the order a* appears in the set of maximal agent feasible allocations sorted by efficiency, with the optimal allocation being rank 1, the second most efficient allocation being rank 2, etc.

(ii): for allocations a* that are not maximal agent allocations, the maximal rank is defined via interpolation. Let a^*_H be the maximal allocation whose efficiency is closest to, but above or equal to Eff(a*). Let a^*_L be a maximal allocation whose efficiency is closest to, but below Eff(a*).

$$Mrk(a^*) = \frac{(Eff(a^*) - Eff(a^*_L))Mrk(a^*_H) + (Eff(a^*_H) - Eff(a^*))Mrk(a^*_L)}{Eff(a^*_H) - Eff(a^*_L)}$$

Here are some simple examples of calculating Mrk(a*). The optimal allocation is assigned the rank '1'. The next best maximal agent allocation is given the rank '2' and so forth. Allocations that are not maximal agent allocations have a fractional rank indicating that its total agent redemption value is between the values for two maximal agent allocations. For example, if the experimental allocation has a total redemption value of 5679, the 20th maximal agent allocation has a total redemption value of 5680 and the 21st maximal agent allocation has a total redemption value of 5670, then the ranking would be 20.1, indicating that the experimental allocation is 10% of the way between the 20th maximal agent allocation and the 21st maximal agent allocation.

Figure 8.1 compares Eff(a*) with maximal rank, Mrk(a*) using the 3NST7-1 period 1 testbed as a motivating example. Notice that all of the maximal rank allocations are above 0.5 in efficiency. The slope change in Fig 8.1 near the optimal allocation seem to indicate more difficulty of getting an allocation, with say Eff(a*)>0.95, where there are few maximal agent allocations compared to getting an efficiency of Eff(a*)>0.85 where there are many maximal agent allocations.

In the experiments, BICAP+CPDA performed fairly well in all testbeds in terms of both Eff(a*) and Mrk(a*), although some initial periods do yield low efficiency results. The low efficiency results are due to agents submitting dominated bids. Some agents bid a higher amount than their redemption value on the train. An open question is whether this might be a learning effect[46] that would go away with more periods or better training. Table 8.2 shows the optimal allocation, the experimental result, and its ratings under the two performance measures.

In cases where the efficiency is low, overbidding was a problem in the 3ST7-2 experiment, especially in the first period, where agent 0 overbid and acquired trains A, P, and Q that should have been purchased by agents 3, 3, and 4 respectively. Trains where overbidding caused lower efficiencies are indicated with a '*' in Table 8.2. A summary of overbidding in the various experiments is given in Table 8.3. If

46 Sometimes agents tend to try unusual or unprofitable strategies more often in the first period(s) of an experiment, and then adopt more rational strategies as they learn about incentives. In principle, a series of new experiments with the order of the periods reversed could distinguish between a learning effect and phenomonae that are dependent on the testbed environment

overbidding is considered to be a type of agent error in the experiment[47], then overall the efficiencies are fairly high when agents do not commit these errors. This observation is summarized as Observation 8.1.

Observation 8.1. The outcome allocations produced by the BICAP+CPDA mechanism tend to be high in efficiency.

Support. From Table 8.2, in 8 of the 11 cases efficiency is above 0.93 and allocation rank is above the 4th maximal agent allocation out of (3ST7 125 / 3NST7 139) feasible maximal agent allocations. In the remaining 3 cases, efficiency is 0.82 (3ST7-1 period 01), 0.67 (3ST7-2 period 02) and 0.77 (3NST7-1 period 03) and allocations ranks are poor. The first two inefficient cases seem to be largely due to overbidding by certain agents, but the third inefficient case can not be explained by overbidding. By experiment, the average efficiencies are 0.933 (3ST7-1), 0.869 (3ST7-2), 0.892 (3NST7-1), and 0.973 (3NST7-2). Over all experiments and periods, the average efficiency was 0.912. •

In comparison to the earlier BICAP experiments, BICAP+CPDA is measurably lower in average efficiency but still does quite well. Recall that only the 3ST7-1 and 3ST7-2 testbeds have the structure of conflicts and redemption values of the three

[47] In individual BICAP or BICAP+CPDA periods overbidding should never be profitable. Overbidding decreases profitability for several agents, especially the one overbidding, in the period that it occurs. Because redemption values are different across periods and because this is common knowledge among the agents, it is expected that "repeated-play" effects are minimal and are not a factor in explaining overbidding. The only remaining source for overbidding is agent error.

simultaneous independent periods of the 1T7 testbeds used in the BICAP experiments, so comparisons can only take place between the BICAP/1T7 and BICAP+CPDA/3ST7-1,2 treatments. The comparison is summarized as Observation 8.2.

Observation 8.2. (2.i) The outcome allocations produced by the BICAP+CPDA mechanism in the 3ST7 environments are not as efficient, on average, as the allocations produced in the BICAP mechanism experiments in the 1T7 testbeds. (2.ii) However, examples exist for certain individual trains and periods, where BICAP+CPDA does produce higher efficiencies than BICAP did.

Support. (2.i) Recall that each of the three groups of trains, {A..G}, {H..N}, and {O..U}, in the 3ST7 testbeds corresponds to the trains {A...G} from different periods of the 1T7 testbed. From Chapter 6, the average efficiency of the BICAP allocations for trains A-G was 0.97 whereas the average efficiency in the 3ST7 testbeds for BICAP+CPDA is 0.90. (2.ii) Here is one example where BICAP+CPDA outperformed BICAP. In BICAP, experiment 3-period 2, trains {A,E} are allocated but the optimal allocation is for trains {B,C,E} to be allocated. In the BICAP+CPDA experiments the redemption values and conflicts for these trains are duplicated in period 1 of both 3ST7-1 and 3ST7-2 experiments. The trains A and E in the Chapter 6 experiments correspond to H and L, respectively, here, while B,C,E in the Chapter 6 experiments corresponds to I,J,L here. In both 3ST7-1 and 3ST7-2, {I,J,L} is part of the BICAP+CPDA mechanism allocation. Therefore, BICAP+CPDA produced the efficient allocation of these trains whereas BICAP did not. •

Averaged data indicates that the 3NST7 testbeds produced higher efficiency results than the 3ST7 testbeds, but because of the overbidding it is interesting to look at data conditional on high efficiency of allocation (say above 0.90). The idea is to look at mechanism performance in a regime where it would appear that agents understand the incentives and the mechanism is performing adequately. Conditional on high efficiency, the opposite is true. Allocations in the 3ST7 testbeds are of higher efficiency and allocation rank than allocations in the 3NST7 testbeds. The difference is small in terms of absolute efficiency, but larger when allocation rank is examined. If computational difficulties affect allocations, then this effect is in the direction expected. In the two 3ST7 testbeds, the proposers were all agents in 3ST7-1 but restricted to three non-buyers in 3ST7-2. Conditional on high efficiency, 3ST7-1 allocation performance is higher than 3ST7-2. Again the effect is small but suggestive. Observation 8.3 summarizes these findings.

Observation 8.3. Conditional on high efficiency (>0.90), BICAP+CPDA mechanism allocation performance is higher in the 3ST7 environments than in the 3NST7 environments. Performance also decreases in the 3ST7 environments when the set of agents who may propose mechanism improvements is restricted.

Support. The high efficiency requirement eliminates 3ST7-1 period 01, 3ST7-2 period 01, and 3NST7-1 period 03. In the remaining 8 periods, allocation rank in maximal allocations averages 1.295/125 for the 3ST7-1 environment (anyone may propose improvements), 1.785/125 for the 3ST7-2 environment (only the 3 non-buyers may propose improvements), and 2.52/139 for the 3NST7 environments. In terms of efficiencies, the relevant conditional averages are 0.988 for 3ST7-1, 0.967 for 3ST7-2, and 0.963 for 3NST7-1 and 2 combined. •

Given that efficiency is high, there appear to be small effects relating efficiency to who may propose improvements and effects relating efficiency to the computational difficulty of the environment. The fact that efficiency goes down when the number of proposers is reduced indicates the importance of having many agents competing to submit potential allocation improvements. The fact that the efficiency goes down with computational difficulty is probably a predictable tradeoff in computationally difficult environments.

The next area of performance to be investigated is computational performance.

8.3 Experimental Results: Computational Performance

Measures of computational performance specific to computer technology are not applicable to describing the computational performance of human agents in

BICAP+CPDA. Clearly any computer could have found the bid maximizing potential allocation in this environment much faster than the agents, who were working with calculators and pencil and paper. Consideration of the goal of the computations yields some more appropriate measures of performance.

The goal of the computations in BICAP+CPDA is to find the potential allocation yielding the highest possible total bid revenue, given a set of current bids. In finding potential allocations, a reasonable tradeoff between computational cost and the total bid revenue for the potential allocation is expected. In these experiments, computational cost is internal to the agents, who perform computations by hand. There is no way to know an agent's internalized computational cost in the experiment, but the cost of the computational incentives paid to the agents can be measured. In simple environments, the bid maximizing potential allocation should be found and the computational incentives paid to agents should be low. Three measures of computational performance will be examined in this chapter: proposal computational efficiency, improvement lag, and cost of proposal improvements.

<u>*Definition.*</u> The *proposal computation efficiency* Pce(s) at a state s in BICAP+CPDA,

is the ratio $Pce(s) = \dfrac{\sum_{f \in \Lambda_S} B_f}{\max_{\{\Lambda_S' \subseteq F : (\Lambda_S' \otimes \Lambda_S') \cap C = \emptyset\}} \sum_{f \in \Lambda_S'} B_f}$ of the sum of bids at the current

potential allocation to the sum of bids at the bid maximizing potential allocation.

Proposal efficiency is the ratio of the sum of agent's current bids for the current potential allocation to the maximum possible sum of agent's current bids over all feasible allocations. When the proposal efficiency is 1.0, the potential allocation is the bid maximizing potential allocation[48]. Figures 8.2 through 8.5 show the proposal efficiency as a function of time in the experiment. Notice that in each graph the proposal efficiency reaches 1.0, then experiences short down turns and then returns to 1.0. The short down turns are caused by bids that change the bid maximizing potential allocation away from the current potential allocation, and the returns to 1.0 can be caused either by agents' submission of improvements or by agents' submission of bids that return the bid maximizing potential allocation to its previous value. To determine which of these is the case requires a more detailed examination of the data, and further definitions.

Table 8.4 lists all bids that changed the bid maximizing potential allocation and all improvements to the potential allocation for Period 1 of the 3ST7-1 testbed experiment. The table is read as follows: Experiment Time measure time in seconds for the experiment. Only differences in the time variable are significant ; the starting value is not necessarily zero at the beginning of the experiment. The potential allocation is the current potential allocation submitted by agents through CPDA. The

[48] Warning: Do not confuse the concepts of bid maximizing and social value maximizing allocations. The bid maximizing potential allocation is not the same concept as the optimal allocation or allocation of highest social efficiency. The bid maximizing potential allocation depends on the current bids and there may be high bids for trains that are only present in inefficient allocations. The optimal (social) allocation does not depend on the current bids, but on the agents private values $\underline{\mathbf{V}}$.

ID# column indicates the agent that submitted a bid and changed the bid maximizing potential allocation, or that submitted an improvement and changed the potential allocation. The bonus percentage column gives the incentive rate on the CPDA clock at the time when the improvement was submitted. In the interests of space, only one table is provided.

From Table 8.4 one can see that, at the beginning of the period, the potential allocation computed by agents does not exactly track the changes in the bid maximizing potential allocation. However, beginning at T=1962 the bid maximizing potential allocation and the potential allocation matched, and whenever the bid maximizing potential allocation changed, an improvement was submitted to update the potential allocation to the new bid maximizing potential allocation.

To measure the degree to which the potential allocation tracks the bid maximizing potential allocation, the *improvement lag* is defined below as the time between the bid maximizing potential allocation changing and an improvement being submitted so that the potential allocation changes to match it. If the bid maximizing potential allocation changes two or more times without the potential allocation tracking the change, then that bid maximizing potential allocation is said to be *skipped.* These definitions will now be made more precise.

Definition. The *improvement lag* is the total elapsed time between two ends of a path of bid and improvement messages that have the following properties:

(i) the path starts at a state s where Pce(s)=1.

(ii) the first message in the path is a bid that changes the bid maximizing potential allocation.

(iii) no other bid message in the path changes the bid maximizing potential allocation.

(iv) the last message in the path is an improvement proposal message that returns proposal efficiency to 1.

Definition. A path satisfying (i), (ii) and (iv) above but for which (iii) is false is said to involve *skipped improvements.*

Recall from Chapter 7 that an agent who by submitting a bid changes the bid maximizing potential allocation is called the *inside bidder,* reflecting the possibility that this bidder knows that his bid changed the bid maximizing potential allocation. Inside bidders have an interest in submitting improvements and changing the potential allocation so that their bid is included in the allocation. Tables 8.5 through 8.8 detail the time series of proposal efficiency lags, the id numbers of the inside bidder and the agent proposing the improvement in the various experiments. *Skipped N* in the time column indicates the number N of bid maximizing potential allocations that were skipped before the potential allocation once again matched the bid maximizing potential allocation.

The tables and figures suggest the following result regarding computational performance:

Observation 8.4. Under CPDA, agents' submission of improvements causes potential allocations to reach and track the bid maximizing potential allocations.
Support. From Figs. 8.2 through 8.4 it is seen that the proposal efficiency rises to 1.0 and then stays close to 1.0 for all experiments. Tables 17 through 20 reveal that once Pce reaches 1.0, the potential allocation tends to track the bid maximizing potential allocation. Few bid maximizing potential allocation changes are skipped after the initial convergence, and the improvement lags are often quite low (under ten seconds in over 50% of cases in the tables). •

The previous result states that the CPDA incentives are producing high quality computational outcomes in a timely fashion in both the 3ST7 and 3NST7 environments. For the next result the cost of eliciting these outcomes is examined.

Table 8.9 presents the earnings for each agent due to proposal improvement bonuses. Due to a bug in the experimental software, agents sometimes earned more than these amounts for improvements submitted simultaneously with other agents. Only the first agent was supposed to receive a bonus, but the software error created a situation where a second agent could also receive a bonus if they pressed the keys almost simultaneously with the first agent while the screen was updating. The erroneous bonuses were removed from the data and are not reflected in the table above, but they may have affected behavior. When the error occurred bonuses were somewhat larger than specified in the rules. The error became noticeable to the agents during the "3NST7" testbeds, but would appear to cause an incentive to wait and not submit improvements, instead waiting for another agent to do so and then trying to time entry to collect the larger bonus. The bug would tend to raise proposal efficiency lags and with it proposal improvement earnings. Nevertheless, earnings from CPDA proposal bonuses are low, which is stated as Observation 8.5.

Observation 8.5. Under CPDA, cost of paying incentives for computing potential allocations is low.

Support. The total cost of incentives paid to agents under CPDA, as a percentage of total bid revenue, was below 3% in each period for all 3ST7 testbed experiments and below 4.9% in each period for all 3NST7 testbed experiments. •

One reason for having the set of proposers be three independent proposers in 3ST7-2 instead of all agents as in 3ST7-1 and 3NST7-1,2 was to see if a small set of agents, responsible for the potential allocation, could reach some type of collusive outcome.

If agents in I_P withhold submitting improvements to the potential allocation until the CPDA clock gave a near 100% bonus rate, they could extract large amounts of surplus as CPDA computational incentives. Table 8.9 shows that this is not the case, and this is stated as Observation 8.6.

Observation 8.6. Under CPDA, small sets of $|I_P| >= 3$ proposers are sufficient for competition in submitting improvements. No evidence exists that when $|I_P| = 3$, proposers were able to collude and gain higher CPDA incentives than would be available in a competitive environment.

Support. From Table 8.9, in the 3ST7-2 testbed, which had only three agents capable of proposing improvements, the total cost of CPDA incentives was the lowest of all testbeds and ranged from 0.2% to 0.6% of total bid revenue. •

The discussion will now turn to results regarding revenue generation and closing bid prices. The preceding results show that BICAP+CPDA is about as efficient as BICAP, and that CPDA works well in providing economic incentives for computational activities. To demonstrate that BICAP+CPDA is a successful modification of BICAP, it remains to show that revenue to the seller and opportunities for purchasing train rights in the two mechanisms are roughly equivalent. The intent is to take a first step in showing that modification of BICAP by adding CPDA did not alter incentives for bidding in any noticeable way and that bidding behavior under either mechanism is virtually identical. If this is true then one would expect BICAP+CPDA to produce efficient allocations in any situation in which BICAP produces them.

8.4 Experimental Results: Does CPDA Alter BICAP Revenue Generation and Closing Prices?

The results in this section are based on a comparative study of the BICAP+CPDA experiments and the BICAP experiments. It will be shown that total revenues and train prices generated by the BICAP+CPDA mechanism in the 3ST7 environments are generally fairly close to those that occurred in the BICAP experiments[49]. If the changes in the BICAP mechanism necessary to include CPDA have only minimal consequences for bidding behavior, then one would expect revenues and prices to be similar in similar testbeds across the two sets of experiments.

The experiments show that incorporation of CPDA into BICAP does not radically alter the ability of the mechanism to raise revenue or the likely distributions of surplus. Since these important economic properties are unaffected, this supports an argument for the study of the incorporation of decentralized computation into other electronic market institutions that have been proposed.

[49] It is not meaningful in an economic sense to directly compare the 3NST7 environment revenues and train prices with those from the 3ST7 or 1T7 environments. Although the redemption values are the same, the conflicts are different, and the sets of feasible allocations based upon the conflicts are different. This changes the optimal allocation and the set of prices that support it.

Table 8.10 compares bid revenues predicted from the BICAP experiments with the BICAP+CPDA bid revenues in the 3ST7-1 and 3ST7-2 testbed experiments. Recall that each period in the 3ST7 testbeds corresponds to three periods in the 1T7 testbeds. The average predicted revenue for a period in the 3ST7 testbed consists of the averages, across all BICAP-1T7 experiments, of the sums of bid revenues attained in the three corresponding[50] periods of the BICAP experiments that make up one period of the BICAP+CPDA experiments. Standard deviations are calculated for these average revenue predictions using the BICAP data. Because sample sizes are small (only three BICAP experiments), the sample standard deviations are provided as a rough measure of variability of the results and not as a tool for making rigorous statistical inferences. This variation might be economic or procedural in nature, and might be a possible indicator for variability in revenue in the new experiments as well. From the table one can see that the 3ST7 testbed revenues are within a few standard deviations of the predictions made using the BICAP-1T7 data. That suggests the following result:

Observation 8.7. In simple environments having identical (but relabeled) conflicts and redemption values, total bid revenue produced by the BICAP+CPDA mechanism is close to the total bid revenue produced by the BICAP mechanism.

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[50] This correspondence was defined and discussed in the testbed constructions of Chapter 3, most notably Section 3.3 and Tables 3.3 to 3.6.

Support. Table 8.10 compares the BICAP revenues to the BICAP+CPDA revenues. Differences appear to be on the order of a few standard deviations in a small sample size. Half (3/6) of the 3ST7 experimental testbed periods produce revenues within 1.0 standard deviations of the revenue predictions, and (5/6) of the 3ST7 experimental testbed periods produce revenues within 3.0 standard deviations of the revenue predictions. •

Table 8.11 shows a comparison of mechanism closing bid prices for trains in the BICAP experiments and the BICAP+CPDA 3ST7 testbed experiments. The BICAP price data is averaged, relabeled to correspond to its role in the new testbed, and presented alongside counterpart data produced in the 3ST7 testbed environment[51]. To give an idea of the variance in the BICAP data, which might be due to either economic or procedural effects, the sample standard deviation is given alongside the average. Once again, small sample sizes and unknown error distributions make it impossible to extract statistically rigorous inferences from the data, so such inferences will not be attempted but are a topic for future work. However, one can make a rough examination to see to what extent prices in the two BICAP+CPDA 3ST7 experiments are in the range produced in the BICAP-1T7 data. This rough examination yields the following observation.

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[51] For example, period 2 train C in the 1ST7 testbed corresponds to period 1 item J in the three-track testbed environments. Recall that the relationship between trains in the two environments is discussed in Chapter 3.

Observation 8.8. In environments having identical (but relabeled) conflicts and redemption values, the majority of final bid prices for trains produced by the BICAP+CPDA mechanism tend to be close to the final bid prices for trains produced by the BICAP mechanism.

Support. In Table 8.11, there are six trials, two experiments 3ST7-1 and 3ST7-2 and three periods for each experiment. In each trial there are 21 trains. In four of the six trials, a majority of the trains have closing bid prices that are within one standard deviation of the BICAP train closing bid price results. Overall, two-thirds of the trains have closing bid prices that are within two standard deviations of the BICAP results. •

Additionally, many of the outlyers in Table 8.11 can be explained by overbidding in the new experiments or unusually small variance in the BICAP experiments. In 6 of the 33 cases where train closing prices were further than two standard deviations of the BICAP results, an agent overbid his/her redemption value. While there may still be a significant deviation for some of the cases, most of the cases produce closing bid prices under BICAP+CPDA are reasonably close to those produced in BICAP.

By showing that the revenues generated by BICAP+CPDA are close to the revenues generated by BICAP in similar environments, one can argue that a seller trying to decide from this data which mechanism to use on the basis of revenue generation alone would be hard pressed to show conclusive reasons for preference of BICAP over BICAP+CPDA or vice versa. Buyers who might have a political input into what mechanism will be used would also have a difficult time showing, from this data, that the mechanisms produce any sort of bias in closing prices. Opportunities for revenue for the seller and for purchasing trains for the buyers remain much the same under both the BICAP and the BICAP+CPDA mechanisms in the simple environments tested here. The importance of these results is that it shows that a smart market mechanism can be modified to go from centralized computation (BICAP) to decentralized computation (BICAP+CPDA) without somehow perverting the allocational performance of the mechanism. This strengthens the need for similar study of other smart market mechanisms, especially those actually under consideration for use in high computational cost environments.

It is not sufficient, however, to show that the mechanism performs well in both allocation and computational respects and that closing bid prices are unaltered by the CPDA modification. It is also necessary to show that design consistency is maintained, that the modifications have not changed basic aspects of bidding behavior and Nash-1 Stationary Equilibria (NE1) on which the initial theoretical predictions of BICAP's high efficiency rest. That is the purpose of the next section.

8.5 Experimental Results: Design Consistency and the NE1 Puzzle

Examining the BICAP+CPDA experiments for design consistency produces mixed results. Bidding behavior in the BICAP+CPDA experiments seems to roughly correspond to the strong neutral process, but NE1 outcomes are not being attained. In the only case where a NE1 outcome was attained, the outcome was not 100% efficient. This contrasts against the Chapter 6 BICAP experiments where most outcomes were at or near[52] NE1 and every outcome that was not a 100% efficient outcome was also not an NE1 outcome. Thus while the efficiencies of BICAP+CPDA are almost as high as the efficiencies in BICAP, and the closing bid prices for BICAP+CPDA in the 3ST7 testbeds are very similar to those obtained in the BICAP experiments, theoretical design consistency is not as strong with the BICAP+CPDA mechanism as with the BICAP mechanism. The difficulties above, that inefficient NE1 are occurring and that NE1 are not obtained as often, will be referred to as the NE1 puzzle. The purpose of this section is to provide the data leading to the NE1 puzzle, evaluate its relevance, and suggest a partial explanation.

Table 8.12 shows that the majority of bids in the BICAP+CPDA experiments are pivotal and neutral bids.

However, agents are leaving some pivotal bids unsubmitted, and NE1 profiles of bids are not being reached when the mechanism terminates. Table 8.13 shows the

52 'Near NE1' means that pivotal bids were worth no more than 50Fr.

unsubmitted pivotal bids that existed for each agent at the close. A "---" means that an agent did not have any pivotal bidding opportunities at the close. Notice that there is often at least one agent who, instead of allowing the mechanism to close, could profitably bid on some trains, increasing their potential profit by several dollars over what they received in the experiments. The only exact NE1 equilibria occurred in period 1 of the 3ST7-1 testbed. However, this NE1 equilibria resulted in an inefficient allocation.

One difference between the 1T7 and the three track environment is that, in the three track environments, agents may have the high value V_{if} on several trains. In the 1T7 environment the agents may have a high value of V_{if} on at most two trains, and these are trains that conflict and are not jointly feasible. It turns out that the agents who have high values on multiple trains are associated with these initially puzzling design consistency results. These agents will be referred to as multitrain agents. To examine this effect, first a few definitions are needed.

Definition. The *high value count* HVC(i) of an agent $i \in \boldsymbol{I}$ is the number of trains $f \in F$ for which i has the high value, i.e. $m_i = |\{ f \in \boldsymbol{F} : \forall j \in \boldsymbol{I} \;\; V_{if} \geq V_{jf} \}|$

Definition. An agent $i \in \boldsymbol{I}$ is a *multitrain agent* if $\text{HVC}(i) \geq 2$.

Examples of multitrain agents not submitting pivotal bids exist in the original BICAP experiments. The non-NE1 outcomes in Period 2-Experiment 3, Period 3-Experiment 2, and Period 5-Experiment 3 in BICAP in the 1T7 testbed are all due to a multitrain bidder who had the high redemption value on two conflicting trains. These agents did not place certain pivotal bids, instead allowing the mechanism to terminate. These cases also correspond to the only three trials where there were outcomes that were not 100% efficient.

Given these facts about the BICAP experiments, it would not be suprising to learn that multitrain agents cause difficulties in the BICAP+CPDA experiments.

By classifying the cases where non-NE1 behavior according to HVC, a large step forward is made in explaining the NE1 puzzle. Table 8.14 shows the proportion of cases (trains times periods) where agents failed to submit pivotal bids, broken down to give the number of cases for each possible value of HVC(i). Figure 8.6 displays the foregone increase in potential profit in these cases, again using HVC(i) as the explanatory variable.

Total environmental complexity increases whenever multitrain-agent complexities increase, e.g., when agents have the highest redemption values on more trains, and when overall computational difficulty for the economic environment increases.

Observation 8.9. Under BICAP+CPDA, increasing either the interdependence of train scheduling or increasing HVC(i) will tend to increase the propensity of an agent to leave pivotal bids unsubmitted at the mechanism close.

Support. From Table 8.14 it is seen that the proportions tend to increase in going from either 3ST7 to 3NST7 testbeds or in increasing the number of trains for which an agent holds the highest redemption value. •

Observation 8.10. Under BICAP+CPDA, HVC(i) is positively correlated with the potential profit value of pivotal bids that agents leave unsubmitted at the mechanism close.

Support. From Figure 8.6 it is seen that the foregone potential profit values tend to lie along an ascending path in the number of trains for which an agent holds the highest redemption value. About two-thirds of cases (13/17) where forgone potential profit is greater than 100Fr come from the 3NST7 testbed environments. •

These observations suggest that the NE1-puzzle is at least partially explainable in terms of multitrain-agent phenomena. Whether this occurs because of increased computational complexity to the agent or because of strategic considerations is unknown because there is no way to observe whether it is the case that agents did not know pivotal bids existed (computational complexity) or if they knew the bids existed but did not wish to submit them (strategic complexity). If the multitrain-agent phenomena is computational in nature, then means may exist to obtain better performance in the mechanism. If the multitrain-agent phenomena is strategic in nature, then there is no reason to believe that better performance can be obtained given the pattern of values V_{if}.

The increase in the computational difficulty of the overall environment from the 3ST-7 experiments to the 3NST7 experiments does enhance the effects. If these are computational effects, then it might be diminished in environments where agents had access to computer assistance in determining their bids. In a field application one expects that this would be the case: agents would provide themselves with whatever equipment was most cost effective in helping them bid effectively. In the experiments agents were restricted to use of pencil and paper and calculator to determine their bids. The multitrain agent may find it cumbersome to search for pivotal bids in this manner. This suggests that the NE1 puzzle may be largely a computational effect, but additional experiments would be necessary to confirm that this is the case and for now it is an open question.

8.6 Conclusions

This chapter began with the question of whether BICAP+CPDA could pass a proof of principle test. That is, did removing the centralized computation of potential allocations from BICAP and replacing it with a system of decentralized incentives, CPDA, result in a mechanism that worked well? Would the BICAP+CPDA mechanism allocate resources efficiently, and elicit computational work from the agents effectively? Would these changes result in a mechanism where the bidding behaviors observed in BICAP are preserved or would there be large differences?

The answers to most of these questions are positive. BICAP+CPDA does allocate resources fairly efficiently. It is effective in eliciting the computational work from the agents: agents found the bid maximizing potential allocations and provided them at low cost. For the most part BICAP+CPDA produces similar outcomes to BICAP in terms of closing bid prices and allocations when the mechanisms are operated in comparable economic environments. All of these positive results occurred in an environment that contained all the economic issues that existed in the BICAP train experiments, which included elements that critics of decentralized allocation thought were problematic in the rail environment. Therefore, the effectiveness of BICAP+CPDA was not merely tested in an environment where it would necessarily be successful, but in an environment with known, problematic allocational and computational issues that must be overcome.

Mixed results enter when the design consistency tests for BICAP are applied to BICAP+CPDA: BICAP+CPDA does not seem to produce NE1 outcomes. Some conjectures are offered that might suggest certain elements in the environment are problematic, i.e., multitrain-agents. More experiments should be conducted to try to ascertain whether the failure is absent in the absence of multitrain-agents and whether the failure is due to computational or strategic issues when multitrain-agents are present.

On the whole, BICAP+CPDA would appear to pass a proof-of-principle test. BICAP+CPDA was designed to automatically adjust to problems of computational scale, and tests towards larger and larger scales would now appear to be appropriate. Problems will be encountered and could be addressed by different modifications to the mechanism.

Figure 8.1 Efficiency of the Maximal Allocations vs. Allocation Rank.

3NST7-1 Experimental Testbed, Period 1.

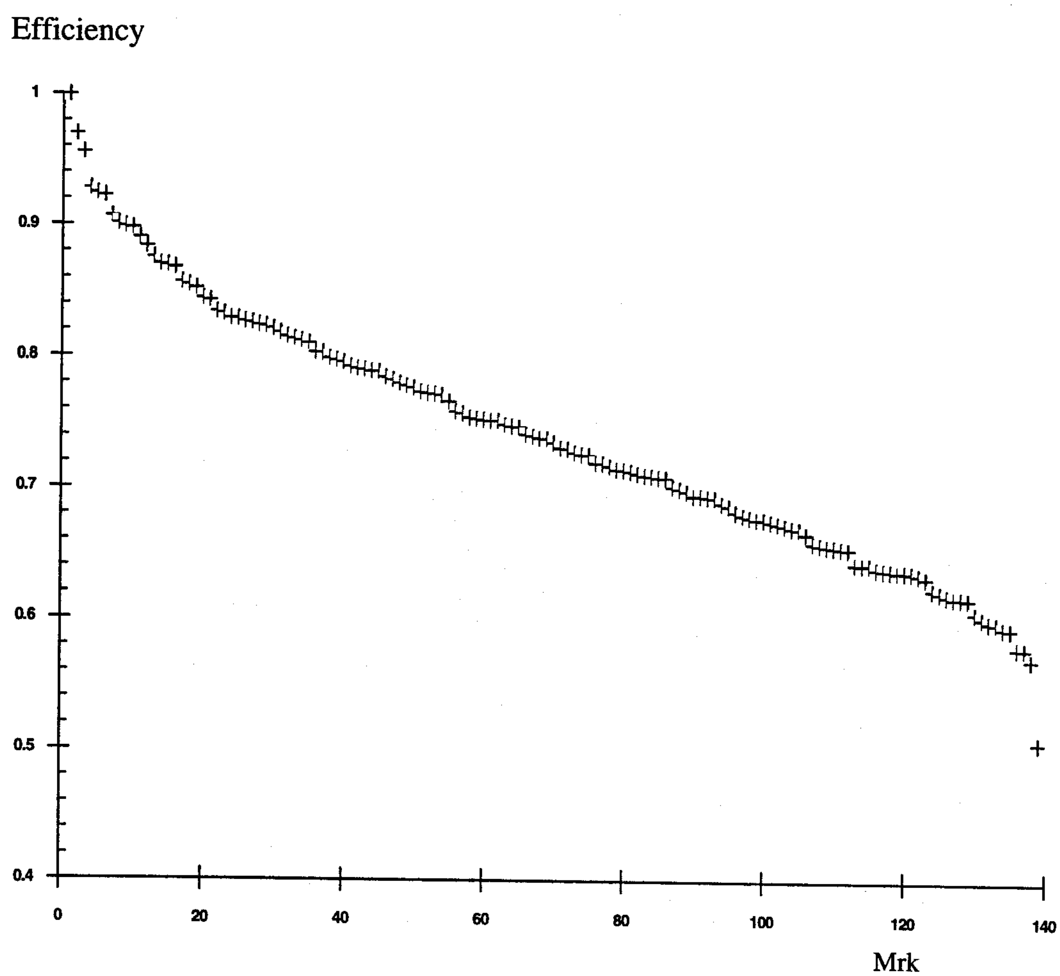


Figure 8.2: Proposal Efficiency in 3ST7-1 Experiment

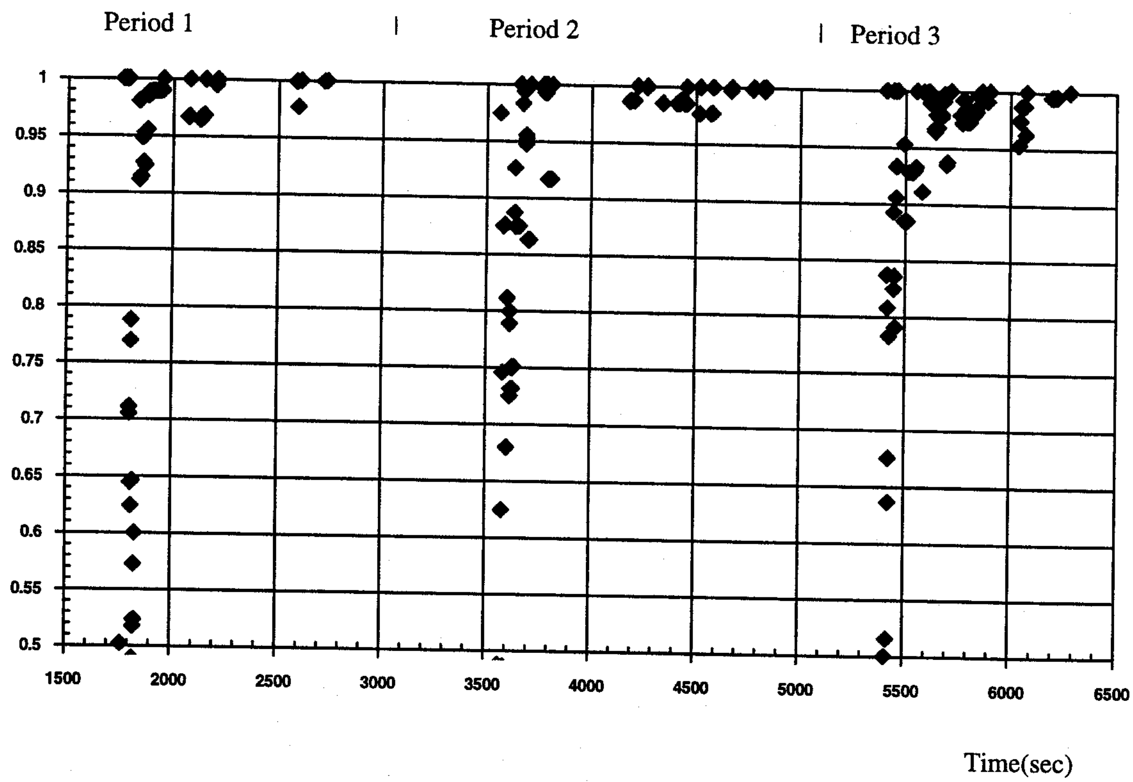
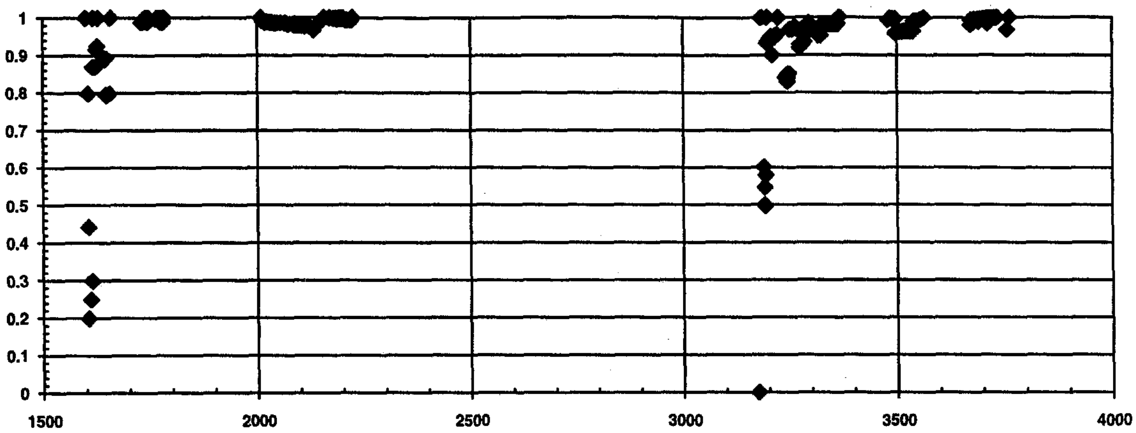
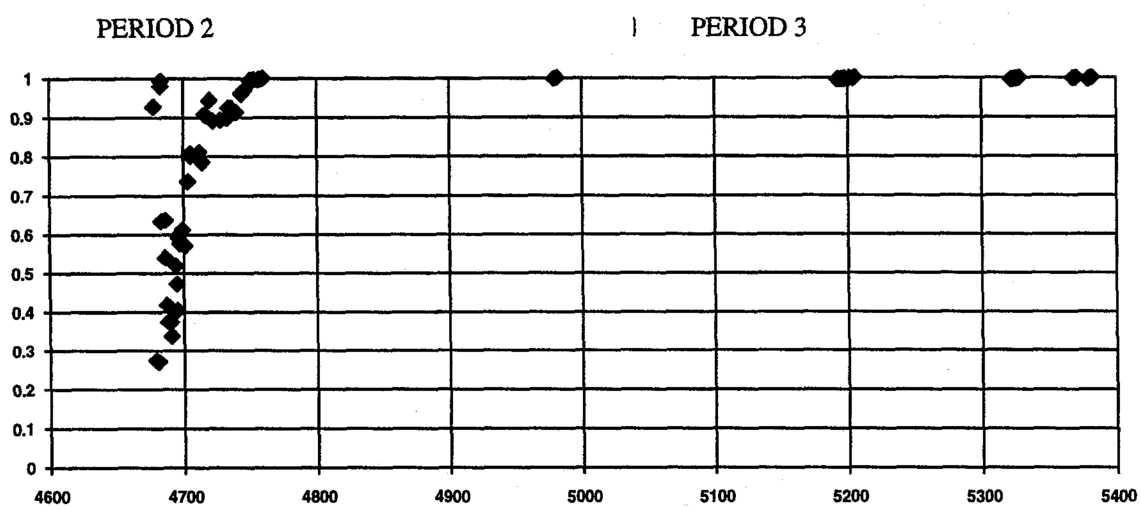


Figure 8.3 : Proposal Efficiency in the 3ST7-2 Experiment



Proposal Efficiency in Period 1, 3ST7-2 Experiment.



Proposal Efficiency in Periods 2, 3 ; 3ST7-2 Experiment.

Figure 8.4: Proposal Efficiency for 3NST7-1 Experiment.

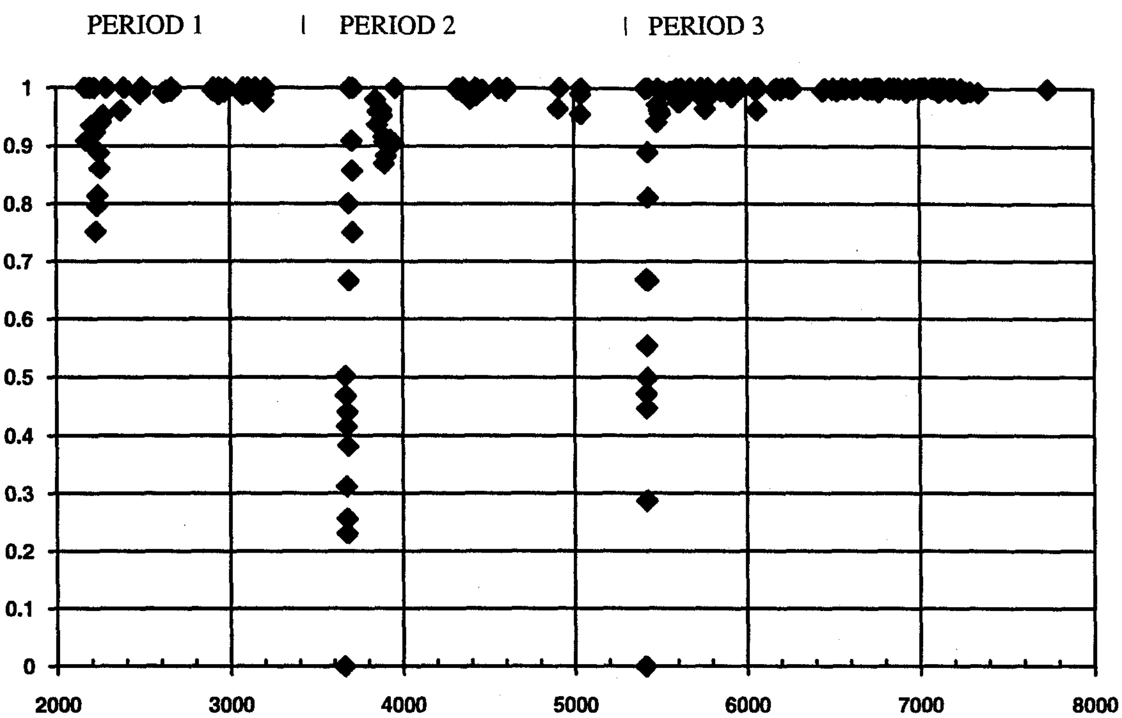


Figure 8.5: Proposal Efficiency for 3NST7-2 Experiment

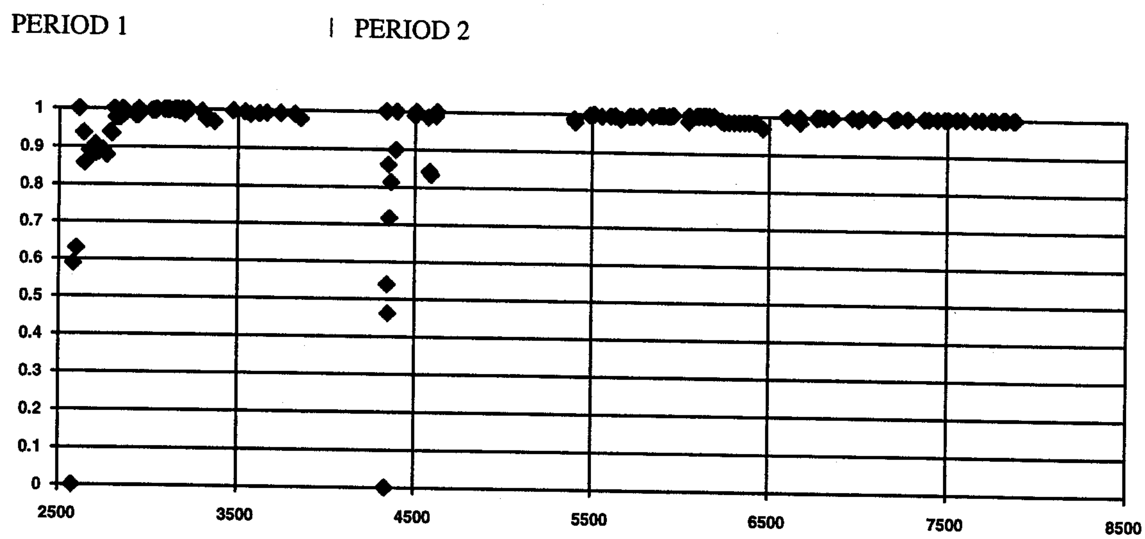


Figure 8.6: Missed Pivotal Bidding Opportunities at Closing: Possible Increase in Potential Profit vs. HVC for Each Agent

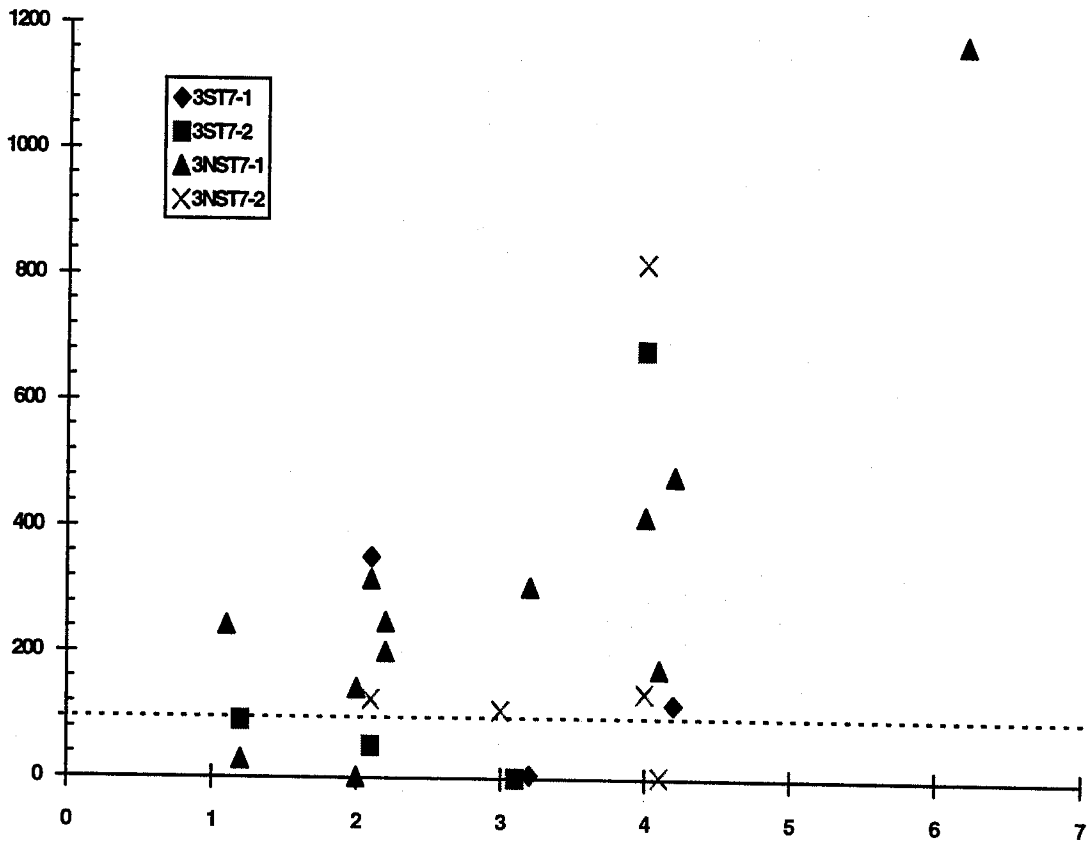


Table 8.1: Summary of Experiments Performed with Three-Track Testbeds

Experiment	Environment	Proposers P	Date	Periods	Subjects
3ST7-1	3ST7 (separable)	all buyers	7/20/94	3	10 Caltech students
3ST7-2	3ST7 (separable)	3 completely independent agents	7/23/94	3	13 Caltech Student
3NST7-1	3NST7 (interdependent)	all buyers	8/9/94	3	10 Caltech Students
3NST7-2	3NST7 (interdependent)	all buyers	8/10/94	2*	10 Caltech Students

*Time did not allow running period 3.

Table 8.2: Efficiencies of Final Allocations in the Three-Track Testbed Experiments

Experiment	Period	Optimal System Allocation	System Allocation realized in Experiment	Allocation Efficiency	Allocation Rank in Maximal Allocations
3ST7-1	01	ADFIJLORT	BCEIJLP*QS	0.824	37.22/125
	02	BCEHKMPQS	BCEHKMPQS	0.976	1.59/125
	03	ADFHKMPQS	ADFHKMPQS	1.0	1/125
3ST7-2	01	ADFIJLORT	A*DFIJ*LP*Q*S	0.674	108.97/125
	02	BCEHKMPQS	BCEHKM*PQS	0.962	1.95/125
	03	ADFHKMPQS	ADFHK*MPQS	0.971	1.62/125
3NST7-1	01	BCEHLORT	ADFILPQS	0.970	2/139
	02	BCEHLORT	GHKMPQS	0.936	3.75/139
	03	DFHKMPQS	ADFNPQS	0.769	74.7/139
3NST7-2	01	BCEHLORT	BCEHLORT	0.946	3.34/139
	02	BCEHLORT	BCEHLORT	1.0	1/139

* --- An agent overbid (bid higher than his/her redemption value) on this project and purchased it, causing an allocation with lower efficiency than the optimal allocation.

Table 8.3: Overbidding at the Final Allocation

Experiment	Period	Project	Project Awarded in Outcome Allocation	Agent Overbidding	Agent's Project Bid	Agent's Project Redemption Value
3ST7-1						
	01					
		P	Y	9	1000	109
	02					
		D	N	8	512	499
3ST7-2						
	01					
		A	Y	0	2000	332
		J	Y	2	500	335
		P	Y	0	650	365
		Q	Y	0	1226	360
	02					
		M	Y	5	600	175
		U	N	6	1600	1340
	03					
		K	Y	3	1000	662
3NST7-1						
	02					
		D	N	2	766	385
		N	N	3	2411	1039
		U	N	3	1800	214
3NST7-2						
	02					
		E	Y	4	1331	1300
		N	N	6	2000	1999

Table 8.4: Time Series of Bid Maximizing Allocations vs. BICAP+CPDA Potential Allocations, 3ST7-1 Experiment

	Experiment Time(sec)	Bid Maximizing Potential Allocation [BMPA]	Potential Allocation [PA]	ID# of Agent Who Changed BMPA or PA	Bonus Percentage when Improvement Submitted
PERIOD01					
	1756	BENU		5	
	1759	AHU		3	
	1763		A	0	6
	1765	ANO		3	
	1772		AO	0	11
	1773		AO	3	0
	1775	AHU		2	
	1789	ANO		2	
	1792	BCFNOT		7	
	1800		ADFIJLPQS	4	0
	1805		CHL	5	6
	1806	BCFNPQT		2	
	1807	BCENPQT		7	
	1811	BCFNPQT		7	
	1818	BCFNU		2	
	1819	BCFHMU		1	
	1823		ADFIJLPQS	4	6
	1826	BCFHLU		1	
	1839		BCEHLU	0	15
	1842	BCFIJLU		7	
	1859	BCFHKMU		7	
	1860	BCFIJLU		4	
	1867	BCFHKMU		7	
	1870	BCFIJLU		3	
	1880		BCEHKMU	0	6
	1890	BCFHKMU		2	
	1962	BCEHKMU		5	
	2071	GHKMU		0	
	2078		GHKMU	0	1
	2128	GNU		0	
	2151		GNU	0	6
	2205	BCENU		8	
	2212		BCENU	8	11
	2588	BCENPQS		7	
	2607		BCENPQS	8	20
	2721	BCEIJLPQS		8	
	2730		BCEIJLPQS	8	8

Table 8.5: Proposal Efficiency Lags, 3ST7-1 Experiment

	Time (sec)	Proposal Efficiency Lag (sec)	ID# of Inside Bidder	ID# of Agent Proposing Improvement
PERIOD01				
	skipped 19			
	2078	7	0	0
	2151	23	0	0
	2212	7	8	8
	2607	19	7	8
	2730	9	8	8
PERIOD02				
	skipped 10			
	3659	19	7	0
	skipped 1			
	3706	20	5	0
	3771	3	9	9
	3782	4	5	5
	3811	18	0	0
	4219	37	6	8
	4265	14	9	9
	4455	115	6	8
	4520	5	4	4
	4581	12	1	4
	4669	2	9	6
	4776	3	2	3
	4821	1	8	8
	4833	3	9	3
PERIOD03				
	skipped 9			
	5553	58	7	5
	skipped 13			
	5861	18	9	5
	5898	7	3	3
	6074	36	4	3
	6228	36	6	5
	6279	6	2	5

Table 8.6: Proposal Efficiency Lags, 3ST7-2 Experiment

	Time (sec)	Proposal Efficiency Lag	ID# of Inside Bidder	ID# of Agent Proposing Improvement
PERIOD01				
	skipped 8			
	1656	13	2	21
	skipped 5			
	1864	83	1	23
	skipped 4			
	2154	23	4	22
	skipped 1			
	2222	59	4	21
PERIOD02				
	skipped 6			
	3195	3	7	21
	skipped 5			
	3364	67	6	22
	3561	81	6	21
	skipped 5			
	3760	3	5	21
PERIOD03				
	skipped 11			
	4981	2	3	23
	5204	12	4	23
	skipped 2			
	5369	1	1	23
	5381	2	2	23

Table 8.7: Proposal Efficiency Lags, 3NST7-1 Experiment

	Time (sec)	Proposal Efficiency Lag	ID# of Inside Bidder	ID# of Agent Proposing Improvement
PERIOD01				
	skipped 5			
	2287	59	1	3
	2389	17	2	3
	2494	10	5	5
	2664	46	3	4
	2907	3	3	3
	2941	2	0	0
	2981	11	3	3
	3082	3	0	0
	3108	5	3	4
	3154	7	0	0
	3209	13	8	6
PERIOD02				
	skipped 7			
	3715	2	6	2
	skipped 1			
	3960	104	5	5
	skipped 4			
	4429	32	9	0
	4563	166	9	1
	4606	2	3	4
	4915	7	2	1
	5042	3	5	1
PERIOD03				
	skipped 4			
	5434	4	2	0
	skipped 5			
	5621	13	9	5
	5673	48	8	5
	skipped 2			
	5739	3	6	4
	skipped 1			
	5777	12	1	1
	5865	12	2	2
	5927	9	6	6
	5957	16	1	7
	6051	3	6	6
	6070	7	9	1
	6177	12	6	0
	6205	2	4	2
	6251	2	6	1
	6266	1	4	5
	6455	9	7	3
	6502	1	4	9
	6554	27	7	6

	6577	1	4	9
	6624	4	6	6
	6640	1	4	9
	6693	2	6	9
	6708	1	4	9
	6730	1	3	9
	6748	1	4	2
	6768	1	7	5
	skipped 1			
	6855	2	7	9
	6879	1	4	8
	6880	2	4	2
	6925	1	7	8
	skipped 2			
	7014	1	2	2
	7015	2	2	1
	7016	3	2	5
	7034	1	7	8
	skipped 2			
	7091	4	1	8
	7114	1	7	8
	7115	2	7	5
	7121	6	9	8
	skipped 2			
	7200	1	7	5
	7242	1	9	5
	skipped 1			
	7572	2	1	8
	7623	1	7	3
	7624	2	7	5
	7625	3	7	8
	7637	3	1	5

Table 8.8: Proposal Efficiency Lags, 3NST7-2 Experiment

	Time (sec)	Proposal Efficiency Lag	ID# of Inside Bidder	ID# of Agent Proposing Improvement
PERIOD01				
	skipped 2			
	2604	21	0	5
	skipped 8			
	2849	32	5	5
	2942	17	8	8
	skipped 1			
	3041	22	2	5
	3089	4	5	5
	skipped 5			
	3371	74	3	4
	3475	3	4	4
	skipped 1			
	3896	324	3	8
PERIOD02				
	skipped 5			
	4392	36	2	4
	skipped 4			
	4620	30	5	5
	5167	577	5	4
	skipped 2			
	5874	320	6	5
	5885	4	5	7
	5906	1	4	4
	5934	4	2	7
	5944	1	1	3
	6049	3	0	8
	6095	2	4	4
	6103	3	6	7
	6129	1	4	4
	6137	3	3	7
	6158	1	4	4
	6167	3	0	7
	6526	296	6	8
	skipped 2			
	6771	4	3	8
	6809	1	0	7
	6857	1	5	0
	6897	1	7	4
	6940	4	3	0
	7003	4	6	6
	7087	1	5	8
	skipped 1			
	7282	3	3	8
	7330	1	6	7

	7331	2	6	8
	7372	1	3	0
	7397	1	6	8
	7398	2	6	4
	7399	3	6	7
	7442	2	3	8
	7486	1	6	6
	7504	1	3	8
	7555	1	6	6
	7556	2	6	7
	7602	12	3	3
	7658	1	6	6
	7659	2	6	8
	7660	3	6	7
	skipped 1			
	7747	1	6	6
	7748	2	6	8
	7749	3	6	7
	7765	1	3	8
	7811	1	6	7
	7812	2	6	6
	7828	2	3	8
	7871	1	6	6
	7893	13	3	3

Table 8.9: Agent Earnings ν for Submission of Improvements by Period and Experiment.

Experiment	Period	id 0	id 1	id 2	id 3	id 4	id 5	id 6	id 7	id 8	id 9	Total
	01											
3ST7-1		184	0	0	0	11	6	0	0	31	0	232
3ST7-2			21	17	1							39
3NST7-1		95	69	0	123	5	28	30	0	0	0	350
3NST7-2		0	0	0	0	12	67	12	0	40	0	131
	02											
3ST7-1		75	1	0	0	75	0	0	0	18	0	169
3ST7-2			5	0	7							12
3NST7-1		0	44	34	100	0	14	8	0	0	114	314
3NST7-2		0	0	0	4	10	12	23	0	32	3	84
	03											
3ST7-1		43	0	8	18	27	23	0	0	0	0	119
3ST7-2			0	14	29							43
3NST7-1		35	13	4	9	1	22	2	1	4	15	106

Table 8.10: Comparison of Bid Revenues Generated in the 3ST7-1 and 3ST7-2 Experiments with Predictions Using Bid Revenues from the BICAP/1T7 Experimental Data

PERIOD	Avg. Predicted Revenue, Based on BICAP/1T7 Experiments	Standard Deviation Across BICAP/1T7 Experiments	3ST7-1 Revenue	3ST7-2 Revenue
01	7581	199	7734	8413
02	6727	179	7247	6753
03	6898	159	6803	7174

Table 8.11: Final Bid Prices in the 3ST7 Testbed Environments vs. BICAP/1T7 Results

3ST7 PERIOD 01		Avg.Price BICAP 1T7 [Centralized Opt.]	Std. Dev. BICAP 1T7	Prices 3ST7-1 [Buyers Propose]	Prices 3ST7-2 [Special Proposers]
	A	1367	115	900	2000
	B	547	47	526	520
	C	1234	208	1410	1405
	D	1090	178	1000	1020
	E	981	216	1012	950
	F	900	100	359	645
	G	2605.	8	2610	2527
	H	1063	55	630	850
	I	462	54	525	720
	J	651	36	600	500
	K	338	8	300	250
	L	1030	30	1030	1032
	M	440	121	500	475
	N	1627	29	1625	1616
	O	908	203	440	1000
	P	448	131	1000	650
	Q	1058	38	1030	1226
	R	714	85	235	920
	S	550	50	601	620
	T	364	32	350	400
	U	1743	159	1678	1840

3ST7 PERIOD 02		Avg.Price BICAP 1T7 [Centralized Opt.]	Std. Dev.	Prices 3ST7-1 [Buyers Propose]	Prices 3ST7-2 [Special Proposers]
	A	1017	125	1320	1100
	B	625	91	600	600
	C	773	38	1000	800
	D	671	220	512	400
	E	1150	131	1250	1104
	F	520	276	800	812
	G	1797	308	2057	2057
	H	1307	200	1500	1200
	I	524	4	236	438
	J	956	169	572	800
	K	603	72	520	640
	L	767	76	500	800
	M	360	0	357	600
	N	1861	236	1900	1999
	O	700	1	500	600
	P	411	13	400	400

	Q	709	38	820	676
	R	348	47	231	321
	S	790	10	800	733
	T	387	23	232	320
	U	1213	186	1000	1600

Final Bid Prices in the 3ST7 Testbed Experiments vs. BICAP Results.

3ST7 PERIOD 03		Avg.Price BICAP 1T7 [Centralized Opt.]	Std. Dev.	Prices 3ST7-1 [Buyers Propose]	Prices 3ST7-2 [Special Proposers]
	A	1473	20	1450	1450
	B	540	27	614	614
	C	619	25	1000	650
	D	400	50	330	400
	E	642	81	518	700
	F	400	26	419	420
	G	848	247	700	900
	H	908	203	1100	1000
	I	448	131	350	400
	J	1058	38	1100	1070
	K	714	85	670	1000
	L	550	50	490	500
	M	364	32	370	390
	N	1743	159	1732	1730
	O	1017	125	1000	1200
	P	625	91	605	600
	Q	773	38	799	824
	R	671	220	550	499
	S	1150	131	1060	1090
	T	520	276	800	812
	U	1797	308	1700	2057

Table 8.12: Classification of bids (Counts, with Percentages in Brackets) in Three-Track Testbed Experiments

Period	Experiment	BICAP -PIVOTAL	BICAP-NEUTRAL	BICAP-DOMINATED
01				
	3ST7-1	254 [0.799]	53 [0.167]	11 [0.035]
	3ST7-2	270 [0.465]	239 [0.411]	72 [0.124]
	3NST7-1	84 [0.506]	78 [0.470]	4 [0.024]
	3NST7-2	80 [0.362]	130 [0.588]	11 [0.050]
02				
	3ST7-1	212 [0.721]	68 [0.231]	14 [0.048]
	3ST7-2	269 [0.536]	216 [0.430]	17 [0.034]
	3NST7-1	138 [0.414]	191 [0.574]	4 [0.012]
	3NST7-2	229 [0.512]	189 [0.423]	29 [0.065]
03				
	3ST7	196 [0.613]	121 [0.378]	3 [0.009]
	3ST7-2	256 [0.540]	195 [0.411]	23 [0.049]
	3NST7-1	129 [0.373]	210 [0.607]	7 [0.020]

Table 8.13: Unsubmitted Pivotal Bids at Close in Three-Track Testbed Experiments.

3ST7-1	Period	Agent	Train	Pivotal Bid	Increase in Potential Profit
	01				
		0	-	-	-
		1	-	-	-
		2	-	-	-
		3	-	-	-
		4	-	-	-
		5	-	-	-
		6	-	-	-
		7	-	-	-
		8	-	-	-
		9	-	-	-
	02				
		0	-	-	-
		1	-	-	-
		2	-	-	-
		3	-	-	-
		4	-	-	-
		5	-	-	-
		6	-	-	-
		7	K	521	352
		8	-	-	-
		9	-	-	-
	03				
		0	-	-	-
		1	E	585	118
		2	-	-	-
		3	-	-	-
		4	-	-	-
		5	-	-	-
		6	S	1061	6
		7	-	-	-
		8	-	-	-
		9	-	-	-

3ST7-2					
	01				
		0 [overbid PQ]	T	577	884
		1	-	-	-
		2	-	-	-
		3	O	1177	681
		4	-	-	-
		5	-	-	-
		6	-	-	-
		7	-	-	-
		8	-	-	-
		9	-	-	-
	02				

		0	D	593	1
		1	-	-	-
		2	-	-	-
		3	-	-	-
		4	-	-	-
		5	S	734	51
		6	-	-	-
		7	-	-	-
		8	-	-	-
		9	-	-	-
	03				
		0	R	503	91
		1	-	-	-
		2	-	-	-
		3	-	-	-
		4	-	-	-
		5	-	-	-
		6	-	-	-
		7	-	-	-
		8	-	-	-
		9	-	-	-

3NST7-1					
	01				
		0	-	-	-
		1	-	-	-
		2	-	-	-
		3	O	1642	418
		4	-	-	-
		5	-	-	-
		6	-	-	-
		7	D	831	143
		8	Q	1023	2
		9	-	-	-
	02				
		0	-	-	-
		1	-	-	-
		2	-	-	-
		3	-	-	-
		4	E	1057	243
		5	-	-	-
		6	F	614	175
		7	K	556	317
		8	-	-	-
		9	-	-	-
	03				
		0	-	-	-
		1	N	1754	482
		2	A	1608	29
		3	-	-	-
		4	D	308	1173

		5	-	-	-
		6	S	753	306
		7	M	573	201
		8	-	-	-
		9	P	856	249

Experiment	Period	Agent	Best Response Bid on Project	Amount to Bid	Change in Potential Profit
3NST7-2					
	01				
		0	G	2694	108
		1	-	-	-
		2	-	-	-
		3	R	526	136
		4	A [53]	1346	819
		5	-	-	-
		6	-	-	-
		7	-	-	-
		8	-	-	-
		9	-	-	-
	02				
		0	-	-	-
		1	-	-	-
		2	-	-	-
		3	-	-	-
		4	-	-	-
		5	-	-	-
		6	G	2053	4
		7	K	748	125
		8	-	-	-
		9	-	-	-
	03				

[53] By bidding on A , agent 4 can change his allocation to A,D,Q.

Table 8.14: Number of Cases Where Agents Fail to Supply All Pivotal Bids, vs. Agent High Value Count

High Value Count →	0	1	2	3	4	6
Experiment ↓						
3ST7-1	0/5 (0%)	1/5 (20%)	1/10 (10%)	1/4 (25%)	1/5 (20%)	0/1 (0%)
3ST7-2	0/5 (0%)	1/5 (20%)	1/10 (10%)	1/4 (25%)	1/5 (20%)	0/1 (0%)
3NST7-1	0/5 (0%)	2/5 (40%)	5/10 (50%)	1/4 (25%)	3/5 (60%)	1/1 (100%)
3NST7-2	0/3 (0%)	0/3 (0%)	1/7 (14%)	1/3 (33%)	3/4 (75%)	0/0 (------)

CHAPTER 9. Concluding Remarks

This thesis began with the goal of providing a demonstration that a decentralized mechanism exists for allocating the right to access tracks on a railroad. Specific issues concerning rail allocation were enumerated in Chapter 3 and experimental laboratory testbeds were created that included each issue. The remainder of the chapters develops and tests the BICAP and BICAP+CPDA mechanisms in laboratory testbed environments from Chapter 3.

The performance of the mechanism in the laboratory testbeds provides the desired demonstration: a practical, efficient, decentralized mechanism does exist for allocating the right to access tracks on a railroad, at least in the experimental laboratory railroad environments. If it were impossible to create such an allocation mechanism because of one or more issues in Chapter 3, then either the search for such a mechanism or the laboratory experiments involving the BICAP mechanism would have failed. Because the experiments show that high efficiency allocations were achieved, the burden of proof is shifted to opponents of decentralization to provide additional issues not present in the Chapter 3 testbeds in order to argue that decentralized mechanisms are, nevertheless, impossible in the field.

While the efficiency of the allocations and the design consistency tests show that a mechanism exists that performs well in spite of all the issues of Chapter 3, it is useful to briefly review each issue and note where in the analysis it was addressed and solved. In this way some insight into the process of mechanism design and testing is provided. Some issues were essentially design constraints on the feasible set or on the rules of the mechanism and were solved through specification of the feasible set or the mechanism. Other issues required experimental evidence of performance. Some issues are of a mixed variety and involve both specification and performance.

Size. Size and computational issues related to size were resolved by modifying the BICAP mechanism to the BICAP+CPDA mechanism defined in Chapter 7. The modifications change responsibility for difficult computational tasks: while BICAP involves difficult optimization tasks being performed by the mechanism authority, BICAP+CPDA involves a series of approximations to the optimum being computed voluntarily by the agents through the CPDA incentive system. Computation stops when agents no longer find a sufficient incentive for continuing. Therefore, computation can not hold up the mechanism, as in BICAP. The BICAP+CPDA mechanism was tested in Chapter 8. It produced high efficiency allocations and bidding behavior mostly similar to BICAP, although there were some puzzling design consistency problems. The BICAP+CPDA mechanism can not fail to produce allocations because of size and related computational difficulties, and since it performs comparably to BICAP, size alone can not be an issue.

Non-Track Constraints. The issue of non-track constraints involved the fact that not all infeasibilites occur because of collisions between trains on the track. This issue was revolved by including all such constraints when the set of feasible allocations for the tracks was defined. Essentially this problem can be solved with the use of the binary conflicts methodology for representing feasibility. Because the non-track constraints are included in the binary conflicts representing infeasibilities that BICAP is designed to avoid, the non-track constraints are always considered.

Schedule Interdependency. All testbeds contained considerable schedule interdependency. Interdependency is an essential feature of the railroad environment and can not be assumed away in the specification of feasible sets of allocations or in the design of mechanisms. Therefore, it is a performance issue: did it cause problems obtaining efficient allocations? Schedule interdependency was not seen to be a problem in producing efficient allocations when agents have the high value for only a few trains. In cases described in Chapter 6 and Chapter 8, certain agents who had the high bid on several trains that involved interdependencies sometimes did not bid high enough on certain trains to obtain an efficient allocation. However, this would appear to be more of a type of network monopoly problem than a problem with schedule interdependency per se.

Revelation of Values. The principles of operation of the BICAP mechanism detailed in Chapter 5 suggest that revelation of values should be high. The experimental analysis of BICAP in Chapter 6 show a trend in the data towards full revelation of train value in the bids of excluded agents. The agents included in the allocation tend to be the high value agents. Therefore, a theory of revelation of values is designed into the BICAP mechanism and seems to be occurring in the experiments.

Resource and Market Fragmentation. If a classical market process of multiple, independent markets for small slots of track time is to be used, then the number of potential markets would be large. Such a process was avoided. Instead, in BICAP simultaneous, interdependent auctions existed for the rights to run each train. This kept the number of markets small and ensured that agents always obtained usable track time slots.

Strong Complements. If the "multiple independent market" approach mentioned in the paragraph above is used, then strong complements will be present and, as a result, prices may not exist that support a Pareto-optimal outcomes as a competitive equilibrium. It is the act of splitting up the resources into the necessary small chunks to create multiple independent markets that introduces strong complements. The existence of the strong complements problem is, therefore, a function of the design of the allocation mechanism, and in the BICAP mechanism the problem was avoided by only allowing bidding on resources associated with running a complete train.

Competitive Equilibrium Existence. It is known that competitive equilibria may fail to exist in environments containing indivisibilities and non-convexities. However, issues regarding competitive equilibria in the sense of Walrasian equilibria are only of direct relevance in a market or market-like mechanism. They not important for the BICAP allocation mechanism, that is not a market but an auction. Walrasian equilibria and Edgeworth box type constructions do not apply and are not appropriate for its analysis. In a market-like mechanism the existence of competitive equilibria would be crucial to showing that the mechanism would eventually terminate. By choosing a mechanism that was auction-like instead of market-like, the problem that competitive equilibria might not exist was avoided.

Non-existence of the CE signals a type of difficulty present in the underlying environment. In some sense, it is useful to show that the mechanism can perform allocations in circumstances in which a market could not perform. Although competitive equilibria are not an issue for BICAP, the existence of Nash or Nash-like (NE1) equilibria is an issue. The existence and properties of NE1 equilibria for the BICAP mechanism are established in Chapter 5[54] and whether the mechanism reaches such equilibria in the experiments is investigated in Chapters 6 and 8.

The next three issues turn on the operation of priority systems. Decentralization advocates claim that the current system of priority has consequences that can be

[54] In particular it is shown that BICAP always has many NE1 equilibria. The optimal allocation is supportable as a NE1 equilibria. Conjectures are offered concerning bidding processes that might lead to selection of NE1 supporting the optimal allocation.

avoided by a decentralized system. A demonstration that such consequences can be avoided is necessary.

Priority and Substitution Between Users or User Types. Suppose any agent is given priority. If G was the most valuable route to any user with priority, then it would be implemented. For example, if agent 0 was given the right of priority for a single train such as G then, as can be ascertained from Table 2.1, train G would operate at a value of 1604. But there are many options that have greater value than G. In particular, trains B, C, and E, held by agents 1, 0 and 2 respectively, have a combined value of 3022. Given such a priority system, there is no incentive for the three trains run by different users to be substituted. In the BICAP mechanism agents 1 and 2 have an incentive to bid high enough so that agent 0 would prefer to run train C rather than train G. In the experiments this can be observed to be the case.

Priority and Combining Trains. Suppose that fast trains have priority over slow trains and that agent 0 is operating fast trains but had no priority for a slower train such as G. As can be seen from the Table 2.1, the value for G to agent 0 is 1604 while the value of the best feasible fast trains to this agent is the set of three trains B, C, and E that total to 1134. The agent has no incentive to combine trains if the result is a slower train because priority and thus the trains would be lost. In the BICAP mechanism such peculiar incentives for an agent, due to retaining priority, are eliminated and the agent is free to attempt to maximize his profit by bidding on whatever trains are desired.

Priority Gives no Incentive to Wait. If agent 7 has priority with north to south fast trains, then the agent has no incentive to delay and wait. Given the preferences of Table 3.1, agent 7 would operate train A even though another agent such as agent 0 must delay and run train D rather than train C. Agent 0 values train C by a difference of 337 over D while agent 7 values A (that forces agent 0 to delay) to train B (where agent 7 waits) by only a margin of 102. Thus an allocation where Agent 7 waits as opposed to Agent 0 would increase total value by 335. With priorities there is no incentive for this to take place.

Priority Systems Do Not Respond to Changing Circumstances. The criticism that "access to track is not allocated to the users who value it the most" seems to directly attack the ability of the track authority to gather information needed to assign priority using the current administrative processes. Changing circumstances in the experimental testbeds are introduced by varying the train redemption values from period to period. BICAP is designed so that it is in agent's interest to reveal enough information concerning train values to identify the optimal allocation, and the experiments show that bidding is in fact responsive to changes in the train redemption values from period to period.

Certainly, issues do exist in the field that were not covered in Chapter 3. For example, the demonstration involved train values that were additive and separable for each agent. Interesting types of rail network externalities exist indicating that this is not always the case, and future experiments should be performed now to address such issues.

In closing, this thesis opens opportunity for dialogue between proponents and opponents of decentralization and both theoretical and experimental economists to further define other important features of rail allocation that should be included in testbeds. Theoretical economists might try their hand at designing mechanisms that would perform at higher efficiencies or would be more robust to new problems and issues. Experimentalists could conduct additional tests incorporating the new testbeds or new mechanism features. Eventually, through a series of such dialogues, a mechanism could be obtained that would be appropriate for the Swedish problem and rail allocation in general.

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APPENDIX A. EXPERIMENTAL INSTRUCTIONS. BICAP. 7/7/93. 1ST7 TESTBED

Introduction

This is an experiment in the economics of market decision making. The instructions are simple, and if you follow them carefully and make good decisions you might earn money which will be paid to you in cash.

In this experiment, we are going to conduct a computerized market over a sequence of trading periods. The items to be sold are called projects, and are designated by letters of the alphabet (project A, project B, project C, etc...). You may try to purchase any number of projects as you wish. The value to you of any particular project is detailed on an attached set of redemption value sheets. Notice that these sheets are labeled period.1, period.2, etc... Notice that the redemption values vary from period to period. During the experiment, pay careful attention to make sure you are using the correct sheet in evaluating which project(s) you wish to purchase. [note: the information on the redemption sheets is your own private information. do not reveal it to anyone.] At the end of each period, project(s) you have purchased are redeemed by the experimenter for the amounts indicated on these sheets.

Your profits in a period, then are determined by the difference in the redemption amount you receive for the projects you purchased and the amount you paid for them.

i.e. **your profit = (total project redemption value) - (total purchase price)**

Each project can be sold to one and only one buyer during each period. The projects are sold via an auction, carried out using the computer terminals. You will have an opprotunity to bid on each project as many times as you wish. To bid, follow the instructions at the bottom of the screen. Bids are not binding until the SEND key is hit. Bids which are lower than the current bid on the screen are ignored. Once your bid for a project is sent into the system, and becomes the current bid, you are obligated to honor it until someone else bids higher on the same project, at which point it is deleted from the system.

There is an additional complication. Not all combinations of projects are possible. For example, it could be that if X is sold, that Y or Z can not be sold. Incompatible groups of projects are detailed on an attached sheet. The computer will use the bidding information to determine which group of projects to sell to maximize the amount of money collected from buyers. The set of high bids which would actually be accepted by the computer at any particular time is displayed on the computer screen and updated along with any new bids.

At the end of each period, the computer notifies each buyer of any successful bids. Unsuccessful bids are not displayed. At this time, buyers should fill out their BUYER RECORD SHEET and calculate any profits (or losses) from the period.

Currency:

The currency used in these markets is "francs." At the end of the experiment francs will be

converted to dollars at the rate of: ________________ francs equals one dollar.
[the exchange rate is also private information. do not reveal it to other participants.]

None of the following pairs are feasible, nor is any combination containing one or more
of these pairs:

A,B
A,C
A,G

B,A
B,D
B,G

C,A
C,D
C,G

D,B
D,C
D,E
D,G

E,D
E,F
E,G

G,A
G,B
G,C
G,D
G,E
G,F

H: nothing conflicts with H. H is allways feasible.

I: nothing conflicts with I. I is allways feasible.

Examples:

{A,D,F} is feasible since neither A,D , A,F or D,F are listed above.

{B,D,F} is not feasible since B,D is listed above as being impossible

ID NUMBER: ________ PERIOD#	LIST PROJECTS PURCHASED	TOTAL PURCHASE PRICE	TOTAL REDEMPTION VALUE	PROFIT OR LOSS(-)

APPENDIX B. EXPERIMENTAL INSTRUCTIONS. BICAP+CPDA. 7/20/94,7/23/94. 3ST7 TESTBEDS

Introduction

This is an experiment in the economics of market decision making. The instructions are simple, and if you follow them carefully and make good decisions you might earn money which will be paid to you in cash.

In this experiment, we are going to conduct a computerized market over a sequence of trading periods. The items to be sold are called projects, and are designated by letters of the alphabet (project A, project B, project C, etc...). You may try to purchase any number of projects as you wish. The value to you of any particular project is detailed on an attached set of redemption value sheets. Notice that these sheets are labeled period.1, period.2, etc... Notice that the redemption values vary from period to period. During the experiment, pay careful attention to make sure you are using the correct sheet in evaluating which project(s) you wish to purchase. [note: the information on the redemption sheets is your own private information. do not reveal it to anyone.] At the end of each period, project(s) you have purchased are redeemed by the experimenter for the amounts indicated on these sheets.

Your trading profits in a period, then are determined by the difference in the redemption amount you receive for the projects you purchased and the amount you paid for them.

i.e. **trading profit = (total project redemption value) - (total purchase price)**

Each project can be sold to one and only one buyer during each period. The projects are sold via an auction, carried out using the computer terminals. You will have an opprotunity to bid on each project as many times as you wish. To bid, follow the instructions at the bottom of the screen. Bids are not binding until the SEND key is hit. Bids which are lower than the current bid on the screen are ignored. Once your bid for a project is sent into the system, and becomes the current bid, you are obligated to honor it until someone else bids higher on the same project, at which point the lower bid is deleted from the system.

There is an additional complication. Not all combinations of projects are possible. For example, it could be that if X is sold, that Y or Z can not be sold. Incompatible groups of projects are detailed on the blackboard.

The computer will accept proposals for which objects should be sold. To make a proposal, use the [F5] key, type in the proposal by listing the object letters which should be accepted, then hit the [F1] key. You may propose any set of projects that you wish, but there must not be any incompatibilities, and the value of the proposal, given the current bids, must exceed the value of the current proposal. The period begins with the current proposal being to sell nothing.

You are paid a bonus for making proposals. This bonus is equal to a percentage of the amount by which you improved the value of the current proposal. The percentage varies and depends on the time left on the period timer. This percentage is displayed on your screen.

At the beginning of each period, a period timer is set to ________ seconds and is reset to this value whenever an acceptable bid or proposal is made. When the timer reaches 0, the period closes.

At the end of each period, the computer notifies each buyer of any successful bids. Unsuccessful bids are not displayed. At this time, buyers should fill out their BUYER RECORD SHEET and calculate any profits (or losses) from the period.

Currency:

The currency used in these markets is "francs." At the end of each period of the experiment francs will be converted to dollars. This will occur according to the following formula:

Losses: $0.02 * francs

1-10 francs: $0.20 * francs, or 5 francs = $1

10-infinity francs: $1.80 + 0.02 * francs .

Therefore , if you gain 10 francs either formula gives $2.00

Gain 100 francs, $3.80.

Gain 500 francs, $11.80.

If you lose 50 francs, then thats $1.

Remember, conversion to dollars occurs at the end of each PERIOD of the experiment.

APPENDIX C. EXPERIMENTAL INSTRUCTIONS. BICAP+CPDA 8/9/94, 8/10/94. 3NST7 TESTBED

INSTRUCTIONS

This is an experiment in the economics of market decision making. The instructions are simple, and if you follow them carefully and make good decisions you might earn money which will be paid to you in cash.

In this experiment, we are going to conduct a computerized market over a sequence of trading periods. The items to be sold are called projects, and are designated by letters of the alphabet (project A, project B, project C, etc...). You may try to purchase any number of projects as you wish. The value to you of any particular project is detailed on your attached set of redemption value sheets. The redemption values vary from period to period and from person to person. You must pay careful attention to make sure you are using the correct period number sheet in evaluating which project(s) you wish to purchase. [note: the information on the redemption sheets is your own private information. do not reveal it to anyone.] At the end of each period, project(s) you have purchased are redeemed by the experimenter for the amounts indicated on these sheets.

Your trading profits in a period are determined by the difference in the redemption amount you receive for the projects you purchased and the amount you paid for them.

i.e. **trading profit = (total project redemption value) - (total purchase price)**

For example, if BUYER 43 purchases project C in the market for 500 and project N for 200 and her redemption value from her sheet is 750 for C and 300 for N , then BUYER 43's trading profit is
750 (value of C) - 500 (payment for C) +300 (value of N) -200(payment for N) = 350 (profit) .

Each project can be sold to one and only one buyer during each period. The projects are sold via an auction, carried out using the computer terminals. Buyers will have an opprotunity to bid on each project as many times as they wish. To bid, follow the instructions at the bottom of the screen. Bids are not binding until the SEND key is hit. Bids which are lower than the current bid on the screen are ignored. Once a bid for a project is sent into the system, and becomes the current bid, the bidder is obligated to honor it until someone else bids higher on the same project, at which point the lower bid is deleted from the system.

There is an additional complication. Not all combinations of projects are possible. For example, it could be that if X is sold, that Y or Z can not be sold. Incompatible groups of projects are detailed on an attached sheet.

The computer will accept proposals for which objects should be sold. The PROPOSERs earn profit for making proposals which are ACCEPTED for consideration. At the end of the period, the computer will use the best proposal submitted to determine which projects it will sell.

A PROPOSAL consists of a list of proposed objects to be sold by the computer. The computer allways keeps the current proposal on display. Projects included in the proposal are green on the display and items which were not included are red. At the beginning of the period, the current proposal is the proposal that none of the projects are sold.

A new proposal is ACCEPTED if there are no incompatible groups of projects in the proposal, and if the value of the new proposal given the current bids is higher than the value of the current proposal.

The proposer earns profit for ACCEPTED proposals. For each ACCEPTED proposal, a PROPOSAL BONUS is paid. Bonuses accumulate over the period.

PROPOSAL BONUS = amount of improvement X bonus percentage.

The bonus percentage starts at 0% and rises as time left on the PERIOD TIMER decreases. When the timer indicates half the time left, the bonus will be 50%, and when the timer indicates 1 second left, the bonus will be close to 100%.

If you wish to make a proposal, type in the proposal by listing the object letters which should be accepted, then hit the [F1] key. You may propose any set of projects that you wish, but it will not be ACCEPTED unless it meets the criteria above (no conflicts, improves sum of bids). The period begins with the current proposal being to sell nothing.

It is important to emphasize the difference between Proposals and Bids. Remember that a PROPOSAL is a recommendation to the computer concerning which projects it should sell, given the BIDS already entered into the system. These bids might be your own, or they might be another BUYERs bids. Since projects are sold to the highest bidder, it is allways necessary to BID on projects which you are attempting to purchase. The projects you wish to purchase must also be in the best proposal received by the end of the period in order for you to actually purchase the projects. However, this proposal does not need to be made by the same person who is bidding on the projects.

At the beginning of each period, a PERIOD TIMER is set to __60___ seconds and is reset to this value whenever an acceptable bid or proposal is made. When the timer reaches 0, the period closes.

At the end of each period, the computer notifies each buyer of any successful bids. Successful bid(s) must be paid, and the bidder receives the indicated projects.

Unsuccessful bids are not displayed. Unsuccessful bidders pay nothing, and receive nothing.

At the end of the period, buyers should fill out their BUYER RECORD SHEET and calculate any profits (or losses) from the period. The total bonus from proposals is also displayed on the screen when the period closes, and this should be included in the profit calculation.

Currency:

The currency used in these markets is "francs." At the end of each period of the experiment francs will be converted to dollars. This will occur according to the following formula:

Losses: $0.02 * francs

1-10 francs: $0.20 * francs, or 5 francs = $1

10-infinity francs: $1.80 + 0.02 * francs .

Therefore , if you gain 10 francs either formula gives $2.00

Gain 100 francs, $3.80.

Gain 500 francs, $11.80.

If you lose 50 francs, then thats $1.

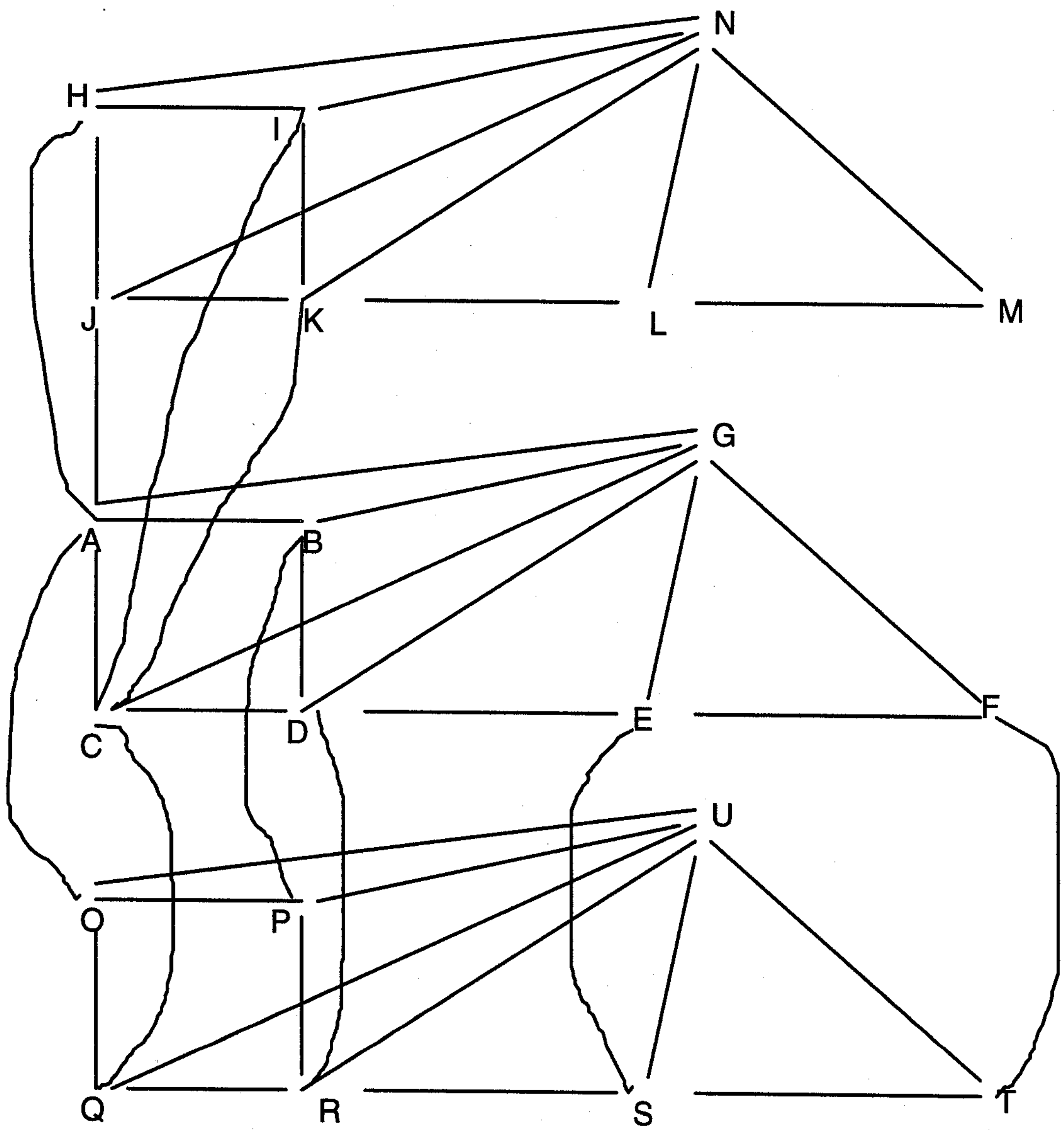
Remember, conversion to dollars occurs at the end of each PERIOD of the experiment.

INCOMPATIBLE PROJECTS.

Any proposal that contains an incompatible pair is not feasible.

The incompatible pairs of projects are shown via the following graph(s).

An incompatible pair of projects are directly joined by a line. For instance A,C is an incompatible pair because a line directly connects A and C, but A and E are compatible because there is not a line between A and E.



Examples:

{A,D,F} is feasible since neither A,D , A,F or D,F are connected by lines in the figures above.

{B,D,F} is not feasible since B,D is connected by a line.