ON THE RADIATION PATTERNS OF INTERFACIAL ANTENNAS

Thesis by

Nader Engheta

In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1982

(Submitted May 18, 1982)

ACKNOWLEDGMENTS

I would like to dedicate this work and to express my sincere gratitude and deep appreciation to my advisor, Professor Charles Herach Papas, for his continued guidance, support, and encouragement during the course of this work. I would also like to thank Dr. Charles Elachi of the Jet Propulsion Laboratory, California Institute of Technology for his interest, encouragement, and support during this work.

I would like to acknowledge with thanks Ms. Janice L. Tucker for her careful typing of the manuscript.

ABSTRACT

The radiation pattern of an interfacial radiating source is obtained for the case where the source is an infinitely long line source lying along the plane interface of two dielectric half-spaces; for the case where the source is an infinitesimal electric dipole vertically located on the interface; and for the case where the dipole is lying horizontally along the interface. For all the three cases, it is found that the radiation pattern at the interface has a null (interface extinction). For the infinitely long line source, it is obtained that the pattern in the upper half-space, whose index of refraction is taken to be less than that of the lower half-space, has a single lobe with a maximum normal to the interface, and that the pattern in the lower half-space (subsurface region) has two maxima straddling symmetrically a minimum. Interpretation of these results in terms of ray optics, Oseen's extinction theorem, and the Cerenkov effect are given. For the vertical dipole, it is found that the radiation pattern along the dipole axis has a null. For the horizontal dipole, it is obtained that the pattern in the upper half-space has a single lobe whose maximum is normal to the interface; that in the lower half-space, in the plane normal to the interface and containing the dipole, the pattern has three lobes; whereas in the plane normal to the interface and normally bisecting the dipole, the pattern has two maxima located symmetrically about a minimum. Interpretation of these results in terms of the Cerenkov effect is also given.

TABLE OF CONTENTS

I. INTRODUCTION

Many of the problems of remote sensing are problems of electromagnetic wave propagation in inhomogeneous media [1]. Among these problems, the interface problems are the ones which mostly attract the attention of the engineers and physicists who work in electrodynamics [2]. By the interface problems, we mean the problems of reflection and refraction of the electromagnetic wave illuminating the interface of two homogeneous media. This interface can be geometrically smooth or rough. In the case of the smooth interface, the problem amounts to the Fresnel problem [3] of reflection and refraction at a plane interface and was solved by Fresnel and presented by him in a celebrated memoir to the French Academy in 1823 [4]. The Fresnel problem is the most basic interface problem. In the case of the rough interface, there are some studies on the modelling of the rough surfaces [5-6]. Therefore, there are different theoretical models which can simplify the geometry of the problem and thus can lead to finding the reflection and refraction of the wave. One of the techniques that can be useful for solving the problems of the scattering from the rough surfaces is the equivalent current source technique which was used by Marcuse [7] and also by Elachi and Yeh [8]. In using this technique, the slightly rough surface is replaced by a smooth surface and the equivalent sources located on interface of the smooth surface. We call these sources the interfacial sources.

Therefore, the original problem reduces to the problem of finding the radiation of the sources which lie on the interface of two homogeneous media. This is exactly the purpose of the present work. The problem of finding the radiation of a source close to the interface of two media is not a new problem [9-12]. But in the previous studies on this problem a relatively high conductivity in at least one of the two media is taken into consideration. Consequently, this consideration leads to the surface wave along the interface and the no-loss case cannot be obtained from it by letting the conductivity approach zero. In our present work, we consider two lossless dielectric media and we have zero conductivity in the two media. Therefore the results are different from those obtained by considering a high conductivity. We consider three cases of the interfacial radiation source (see Fig. 1).

In Chapter II, we consider the infinitely long line source lying along the plane interface of two dielectric half-spaces. It is clear that this problem is two dimensional and amounts to one of solving a scalar Helmholtz equation. The radiation pattern and emitted power of such an interfacial line source are obtained in the two media. A posteriori, we shall construct a suitable ray optical description of the phenomenon.

In Chapter III, we have an infinitesimal electric dipole vertically located on the plane interface of the two media. This problem is three-dimensional. We shall use the electric Hertz vector

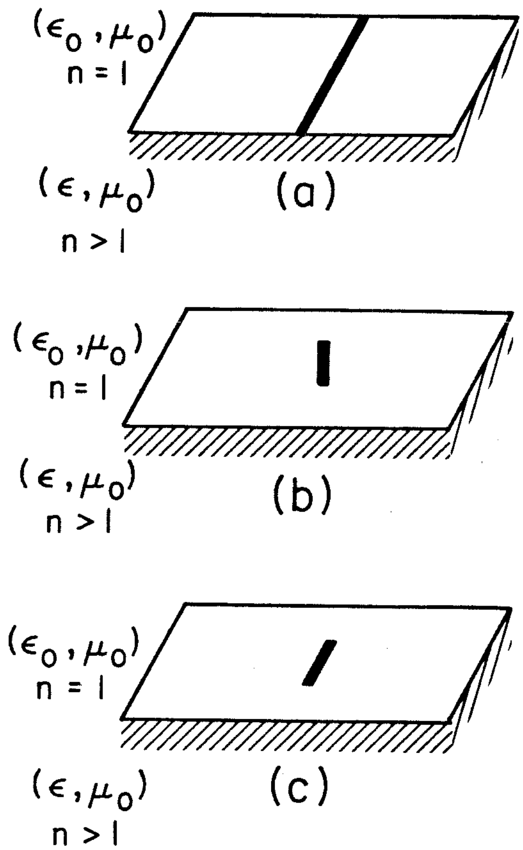

Fig. 1. Three interfacial radiating sources: (a) infinitely long line source, (b) infinitesimal vertical dipole, and (c) infinitesimal horizontal dipole.

and its integral representation.

In Chapter IV, we consider the interfacial horizontal dipole directed parallel to the x-axis (see Fig. 13). The radiation pattern and emitted power for both cases, interfacial vertical and horizontal dipoles, are obtained in the two half-spaces. Finally, in Chapter V, general conclusions for these problems are given.

II. INTERFACIAL LINE SOURCE

IIA. Formulation of the Problem

In this chapter, we intend to calculate the radiation pattern and emitted power of a line source lying along the plane interface of two homogeneous half spaces. To formulate the problem mathematically, we introduce a Cartesian coordinate system x,y,z wherein the z axis lies along the axis of the line source and the plane of the interface is given by the coordinate surface y = 0. Since the problem is a two-dimensional one, the far-zone field is in the form of a cylindrical wave. Therefore, to handle the far-zone field we find it convenient to also introduce a cylindrical coordinate system ρ,ϕ,z where $\rho \cos \phi = x$, $\rho \sin \phi = y$, and $-\pi \leq \phi \leq \pi$. (See Fig. 2).

We use MKS system of units and assume that the current density of the line source is given by

$$\underline{J}\,(x,y;t) = \text{Re}\left[\underline{e}_z\; I\; \delta(x)\delta(y)e^{-i\omega t}\right], \tag{2A.1}$$

where "Re" is the shorthand for the "real part of", $\underline{e}_z$ is the unit vector along the z-axis, $\delta(x)$ and $\delta(y)$ are Dirac delta functions, ω is the angular frequency of the oscillation, and I is the total current. We take the index of refraction, n, to be 1 in the upper half-space ($y > 0$) and greater than 1 in the lower half-space ($y < 0$). Although this means that the upper half-space is a vacuum and the lower half-space or subsurface region is a dielectric, our analysis will hold true for any two dielectric half-spaces whose indices of refraction are in the ratio of n to 1.

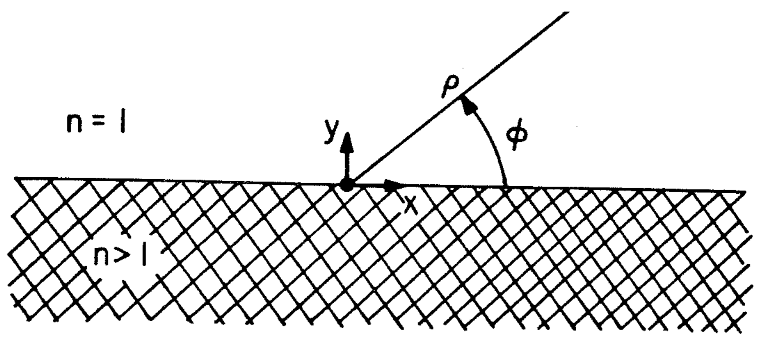

Fig. 2. Line source lies along z axis. In the upper half-space ($y > 0$) the index of refraction n is equal to 1, and in the lower half-space ($y < 0$) n is greater than 1.

Since the problem is linear and the line source is a monochromatic source with the angular frequency ω, all the field quantities have the same angular frequency and they can be written as follows:

$$\underline{E}\,(x,y;t) = \mathrm{Re}\left[\underset{\sim}{E}\,(x,y)e^{-i\omega t}\right], \tag{2A.2}$$

$$\underline{H}\,(x,y;t) = \mathrm{Re}\left[\underset{\sim}{H}\,(x,y)e^{-i\omega t}\right], \tag{2A.3}$$

where $\underset{\sim}{E}\,(x,y)$ and $\underset{\sim}{H}\,(x,y)$ are the phasors of electric and magnetic fields respectively. The field quantities obey Maxwell's equations [13]:

$$\nabla \times \underset{\sim}{E} = i\omega\mu\underset{\sim}{H}\,, \tag{2A.4}$$

$$\nabla \times \underset{\sim}{H} = \underset{\sim}{J} - i\omega\varepsilon\underset{\sim}{E}\,, \tag{2A.5}$$

$$\nabla \cdot (\varepsilon\underset{\sim}{E}) = \rho\,, \tag{2A.6}$$

$$\nabla \cdot (\mu\underset{\sim}{H}) = 0\,, \tag{2A.7}$$

where $\underset{\sim}{J}$ is the total macroscopic current density as a source, ε and μ are the dielectric constant and permeability of the medium respectively, and ρ is the total macroscopic charge density.

From the symmetry of the configuration, it is clear that the electromagnetic field of the line source is independent of z. Therefore, from the continuity equation

$$\nabla \cdot \underset{\sim}{J} + i\omega\rho = 0 \tag{2A.8}$$

we can write

$$\rho = 0\,. \tag{2A.9}$$

Since $\underset{\sim}{J}$ has only z-component and all the quantities are independent of z, we can conclude that the vector potential $\underline{A}$ has only one component which is A_z.

From

$$\underline{H} = \frac{1}{\mu} \nabla \times \underline{A} , \qquad (2A.10)$$

we can conclude that the magnetic field has only the components $H_\rho(\rho,\phi)$ and $H_\phi(\rho,\phi)$. Consequently, from equation (2A.5) we can see that the electric field has only the component $E_z(\rho,\phi)$; therefore, in this configuration the electromagnetic field of the line source has only the components $E_z(\rho,\phi)$, $H_\rho(\rho,\phi)$, and $H_\phi(\rho,\phi)$. For such an electromagnetic field the equation (2A.4) yields the relations

$$H_\rho = \frac{1}{i\omega\mu} \frac{1}{\rho} \frac{\partial}{\partial\phi} E_z , \qquad (2A.11)$$

$$H_\phi = \frac{i}{\omega\mu} \frac{\partial}{\partial\rho} E_z \qquad (2A.12)$$

which express the magnetic field in terms of E_z. Hence, since E_z is the only component of the electric field and since the magnetic field components can be derived from E_z using (2A.11) and (2A.12), we see that our problem can be formulated in terms of E_z alone.

From the Maxwell equations $\nabla \times \underset{\sim}{E} = i\omega\mu \underset{\sim}{H}$ and $\nabla \times \underset{\sim}{H} = J - i\omega\varepsilon \underset{\sim}{E}$, it follows that

$$\nabla \times \nabla \times \underset{\sim}{E} - k^2 \underset{\sim}{E} = i\omega\mu \underset{\sim}{J} , \qquad (2A.13)$$

where $k^2 = \omega^2\mu\varepsilon$. Moreover, from the Maxwell equation $\nabla\cdot(\varepsilon \underset{\sim}{E}) = 0$ it follows that $\nabla\varepsilon\cdot\underset{\sim}{E} + \varepsilon\nabla\cdot\underset{\sim}{E} = 0$. Because here $\nabla\varepsilon$ is perpendicular to $\underset{\sim}{E}$, the term $\nabla\varepsilon\cdot\underset{\sim}{E}$ disappears and we have

$$\nabla\cdot\underset{\sim}{E} = 0. \qquad (2A.14)$$

It is clear from (2A.1), (2A.13), and (2A.14) that E_z must satisfy

$$\nabla^2 E_z + k^2 E_z = -i\omega\mu \, I \, \delta(x) \, \delta(y), \qquad (2A.15)$$

where $\mu = \mu_o$ everywhere, $k^2 = k_o^{\;2} = \omega^2 \mu_o \varepsilon_o$ for $y > o$, and $k^2 = n^2 k_o^{\;2}$ for $y < o$. Here μ_o, ε_o denote the permeability and the dielectric constant of free-space. We shall denote E_z in the upper half-space by E_{z1} and E_z in the subsurface region by E_{z2}.

Thus our problem amounts to one of finding the solution of (2A.15), but in solving this equation, we have to consider the Sommerfeld radiation condition [14] which is

$$\lim_{\rho \to \infty} \sqrt{\rho} \left(\frac{\partial E_z}{\partial \rho} - ik \, E_z \right) \to 0, \qquad (2A.16)$$

for the cylindrical wave. The following boundary conditions should also be satisfied [15]

$$E_{z1} = E_{z2} \, , \qquad (2A.17)$$

$$\frac{\partial}{\partial \phi} E_{z1} = \frac{\partial}{\partial \phi} E_{z2} \, , \qquad (2A.18)$$

along the interface y = 0.

IIB. Method of Integral Transform

To solve the Helmholtz equation (2A.15), we express E_z as a Fourier integral [16], that is, we write

$$E_z = \int_{-\infty}^{\infty} V(y,h) e^{ihx} \, dh \qquad (2B.1)$$

By substituting this expression into (2A.15) and by recalling that

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ihx} \, dh \qquad (2B.2)$$

we find that

$$\int_{-\infty}^{\infty} \left(\frac{d^2}{dy^2} V(y,h) - h^2 V(y,h) + k^2 V(y,h)\right)e^{ihx}\, dh =$$

$$- \frac{i\omega\mu_o}{2\pi} I \int_{-\infty}^{\infty} \delta(y) e^{ihx}\, dh \;. \qquad (2B.3)$$

From the orthogonality and completeness of the functions e^{ihx} [17], we can write

$$\frac{d^2}{dy^2} V(y,h) - h^2 V(y,h) + k^2 V(y,h) = \frac{- i\omega\mu_o I}{2\pi} \delta(y) \;, \qquad (2B.4)$$

where for the upper half-space ($y > 0$) we have $k^2 = k_o^2$ and denote V by V_1, and where for the lower half-space ($y < 0$) we have $k^2 = n^2 k_o^2$ and denote V by V_2. Since E_z must satisfy the radiation condition the solutions of (2B.4) must likewise satisfy the radiation condition of $y = \infty$ and $y = -\infty$. Accordingly, the appropriate solutions of (2B.4) must have the form

$$V_1 = A\, e^{-\sqrt{h^2 - k_o^2}\, y} \qquad (y > 0) \;, \qquad (2B.5a)$$

$$V_2 = B\, e^{\sqrt{h^2 - n^2 k_o^2}\, y} \qquad (y < 0) \;. \qquad (2B.5b)$$

In these two formulas (2B.5a), (2B.5b), when we have the negative sign under the $\sqrt{\quad}$, we bring $-i$ out to satisfy the radiation condition. These two solutions should satisfy the boundary conditions. Across the interface E_z, and hence V, must be continuous, i.e. at $y = 0$ we must have $V_1 = V_2$. Therefore,

$$A = B \;. \qquad (2B.6)$$

To find the value of A or B we integrate (2B.4) with respect to y, from $-\Delta y$ to $+\Delta y$. Since Δy is a vanishingly small quantity and since V is a continuous function of y throughout the range of integration, the second term on the left hand side of (2B.4) integrates to zero and we are left with

$$\frac{d}{dy} V \Big|_{y = +\Delta y} - \frac{d}{dy} V \Big|_{y = -\Delta y} = - \frac{i\omega\mu_o I}{2\pi} . \tag{2B.7}$$

By substituting (2B.5a) and (2B.5b) into this relation and by taking the limit $\Delta y \to 0$, we find that

$$A = B = \frac{i\omega\mu_o I}{2\pi} \quad \frac{1}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}} . \tag{2B.8}$$

Thus

$$V_1 = \frac{i\omega\mu_o I}{2\pi} \quad \frac{\exp[-\sqrt{h^2-k_o{}^2}\; y]}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}} \quad (y > 0) , \tag{2B.9a}$$

$$V_2 = \frac{i\omega\mu_o I}{2\pi} \quad \frac{\exp[\sqrt{h^2-n^2k_o{}^2}\; y]}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}} \quad (y < 0) . \tag{2B.9b}$$

Finally, by substituting V_1 and V_2 back into (2B.1) we obtain

$$E_{z1} = \frac{i\omega\mu_o I}{2\pi} \int_{-\infty}^{\infty} \frac{\exp[-\sqrt{h^2-k_o{}^2}\; y + ihx]}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}} dh \quad (y > 0), \tag{2B.10a}$$

$$E_{z2} = \frac{i\omega\mu_o I}{2\pi} \int_{-\infty}^{\infty} \frac{\exp[\sqrt{h^2-n^2k_o{}^2}\; y + ihx]}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}} dh \quad (y < 0). \tag{2B.10b}$$

These are the required solutions of (2A.15).

The integrals are difficult to evaluate exactly for all values of x and y. However, as shown in the Appendix A, an exact evaluation is possible for points along the interface (y = 0) and yields

$$E_z = E_{z1} = E_{z2} = \frac{\omega\mu_o I}{2(n^2-1)k_o}\left[\frac{1}{|x|} H_1^{(1)}(k_o|x|) - \frac{n}{|x|} H_1^{(1)}(nk_o|x|)\right] . \tag{2B.11}$$

From equation (2A.4), we obtain

$$H_\phi = H_{\phi 1} = H_{\phi 2} = \frac{iI}{2(n^2-1)}\left[\frac{n^2}{|x|} H_2^{(1)}(nk_o|x|) - \frac{1}{|x|} H_2^{(1)}(k_o|x|)\right] \tag{2B.12}$$

for y = 0 and all x.

To gain a description of E_z off the interface we resort to an asymptotic evaluation of the integrals.

IIC. Radiation Pattern

To determine the nature of the far-zone radiation field of the line source it is most convenient to use the cylindrical coordinates (ρ,ϕ). In view of this, in the integrands of (2B.10a) and (2B.10b) we replace x by $\rho \cos \phi$, and y by $\rho \sin \phi$, and thus obtain the desired integral representations of $E_{z1}(\rho,\phi)$ and $E_{z2}(\rho,\phi)$ in cylindrical coordinates:

$$E_{z1} = \frac{i\omega\mu_0 I}{2\pi} \int_{-\infty}^{\infty} \frac{\exp[-\sqrt{h^2-k_0{}^2}\,\rho \sin \phi + ih\rho \cos \phi]}{\sqrt{h^2-k_0{}^2} + \sqrt{h^2-n^2k_0{}^2}} dh \quad (0 \leq \phi \leq \pi) , \tag{2C.1}$$

$$E_{z2} = \frac{i\omega\mu_0 I}{2\pi} \int_{-\infty}^{\infty} \frac{\exp[\sqrt{h^2-n^2k_0{}^2}\,\rho \sin \phi + ih\rho \cos \phi]}{\sqrt{h^2-k_0{}^2} + \sqrt{h^2-n^2k_0{}^2}} dh \quad (-\pi \leq \phi \leq 0). \tag{2C.2}$$

In the integral representation of $E_{z1}(\rho,\phi)$ we divide the range of integration into three subranges so that

$$E_{z1}(\rho,\phi) = \int_{-\infty}^{-k_0} f(h)\, dh + \int_{-k_0}^{k_0} f(h)\, dh + \int_{k_0}^{\infty} f(h)\, dh, \tag{2C.3}$$

where f(h) is a shorthand for the integrand. For $k_0\rho \to \infty$, all three integrals on the right hand side can be evaluated, as shown in the Appendix B. However, it can be seen beforehand that the integrals for the subranges $(-\infty \leq h \leq -k_0)$ and $(k_0 \leq h \leq \infty)$ are negligibly small compared to the integral for the middle subrange $(-k_0 \leq h \leq k_0)$.

Thus, by retaining only the middle subrange integral, which we denote by

$$F_1^{mid} = \frac{-\omega\mu_o I}{2\pi} \int_{-k_o}^{k_o} \frac{\exp[i\sqrt{k_o^2-h^2}\,\rho \sin\phi + ih\rho\cos\phi]}{\sqrt{k_o^2-h^2} + \sqrt{n^2 k_o^2-h^2}}\, dh,$$

$$(0 \le \phi \le \pi) \qquad (2C.4)$$

and introducing the variable α, which is defined by $\sin\alpha = h/k_o$, we get

$$E_{z1} \sim \frac{\omega\mu_o I}{2\pi(n^2-1)} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\alpha - \sqrt{n^2-\sin^2\alpha})\cos\alpha \cdot \exp[ik_o\rho\sin(\alpha+\phi)]d\alpha \qquad (2C.5)$$

for $k_o\rho \to \infty$ and $0 \le \phi \le \pi$. Applying the method of stationary phase, we find that (2C.5) yields the following expression for E_{z1} in the far-zone of the upper half-space:

$$E_{z1} \sim \frac{\omega\mu_o I}{\sqrt{2\pi}} \frac{1}{(n^2-1)} (\sin^2\phi - \sin\phi \cdot \sqrt{n^2-\cos^2\phi}) \frac{e^{ik_o\rho - i\pi/4}}{\sqrt{k_o\rho}} \qquad (2C.6)$$

for $k_o\rho \to \infty$ and $0 \le \phi \le \pi$. (See the Appendix B).

In the integral representation of $E_{z2}(\rho,\phi)$ we similarly divide the range of integration into subranges. That is, we write

$$E_{z2}(\rho,\phi) = \int_{-\infty}^{-nk_o} g(h)\, dh + \int_{-nk_o}^{-k_o} g(h)\, dh + \int_{-k_o}^{k_o} g(h)\, dh + \int_{k_o}^{nk_o} g(h)\, dh + \int_{nk_o}^{\infty} g(h)\, dh, \qquad (2C.7)$$

where g(h) is a shorthand for the integrand. For $nk_o\rho \to \infty$ all five integrals can be handled, as shown in the Appendix C. It turns out that the first and fifth integrals are negligibly small for all values of ϕ in the lower half-space, i.e. for $-\pi \le \phi \le 0$. The second integral is the only one that contributes to the far-zone field in the sector $-\pi \le \phi \le -\pi + \phi_c$, and the fourth integral is the only one that contributes to the far-zone field in the sector $-\phi_c \le \phi \le 0$. Here ϕ_c is the critical angle given by $\cos \phi_c = 1/n$. The third integral is the only one that contributes to the far-zone field in the dihedral sector $-\pi + \phi_c \le \phi \le -\phi_c$. (See the Appendix C for details of the calculations).

Thus, in the third integral, which we denote by

$$F_2^{mid} = \frac{-\omega\mu_o I}{2\pi} \int_{-k_o}^{k_o} \frac{\exp[-i\sqrt{n^2k_o{}^2-h^2}\,\rho \sin \phi + ih\rho \cos \phi]}{\sqrt{k_o{}^2-h^2} + \sqrt{n^2k_o{}^2-h^2}}\, dh \qquad (2C.8)$$

for the dihedral region ($nk_o\rho \to \infty$ and $-\pi + \phi_c \le \phi \le -\phi_c$), we introduce the variable α by $\sin\alpha = \frac{h}{nk_o}$. Therefore, we get

$$F_2^{mid} = \frac{+\omega\mu_o nI}{2\pi(n^2-1)} \int_{-\sin^{-1}\frac{1}{n}}^{\sin^{-1}\frac{1}{n}} \left(\sqrt{1-n^2\sin^2\alpha} - n\cos\alpha\right) \cos\alpha \cdot$$

$$\exp[ink_o\rho \sin(\alpha-\phi)]\, d\alpha \, . \tag{2C.9}$$

Applying the method of stationary phase, we find that (2C.9) yields the following expression for E_{z2} in the far-zone of the dihedral region of lower half-space:

$$E_{z2} \sim \frac{-\omega\mu_o I}{\sqrt{2\pi}} \frac{n}{(n^2-1)} \left(n \sin^2\phi + \sin\phi \sqrt{1-n^2 \cos^2 \phi}\right) \frac{e^{ink_o\rho - i\pi/4}}{\sqrt{nk_o\rho}} \, . \tag{2C.10}$$

Using the similar procedure, we get the following expression for E_{z2} for the other two sectors of lower half-space ($nk_o\rho \to \infty$, $-\phi_c \leq \phi \leq 0$, $-\pi \leq \phi \leq -\pi + \phi_c$)

$$E_{z2} = \frac{-\omega\mu_o I}{\sqrt{2\pi}} \frac{n}{(n^2-1)} \left(n \sin^2\phi + i \sin\phi \cdot \sqrt{n^2 \cos^2\phi - 1}\right) \frac{e^{ink_o\rho - i\pi/4}}{\sqrt{nk_o\rho}} \, . \tag{2C.11}$$

By use of equation (2A.4) we can find H_ϕ from a knowledge of E_z. In the far-zone, we get

$$H_{\phi 1} = \frac{-\omega\sqrt{\mu_o \varepsilon_o} I}{\sqrt{2\pi}} \frac{1}{(n^2-1)} (\sin^2\phi - \sin\phi \cdot \sqrt{n^2 - \cos^2\phi}) \frac{e^{ik_o\rho - i\pi/4}}{\sqrt{k_o\rho}} \qquad (2C.12)$$

in the upper half-space, and

$$H_{\phi 2} = \frac{\omega\sqrt{\mu_o \varepsilon_o} I}{\sqrt{2\pi}} \frac{n^2}{(n^2-1)} (n \sin^2\phi + \sin\phi \cdot \sqrt{1 - n^2 \cos^2\phi}) \frac{e^{ink_o\rho - i\pi/4}}{\sqrt{nk_o\rho}} \qquad (2C.13)$$

for the dihedral region of lower half-space ($nk_o\rho \to \infty$ and $-\pi + \phi_c \leq \phi \leq -\phi_c$), and

$$H_{\phi 2} = \frac{\omega\sqrt{\mu_o \varepsilon_o} I}{\sqrt{2\pi}} \frac{n^2}{(n^2-1)} (n \sin^2\phi + i \sin\phi \sqrt{n^2 \cos^2\phi - 1}) \frac{e^{ink_o\rho - i\pi/4}}{\sqrt{nk_o\rho}} \qquad (2C.14)$$

for the other two sectors of lower half-space ($nk_o\rho \to \infty$, $-\phi_c \leq \phi \leq 0$, $-\pi \leq \phi \leq -\pi + \phi_c$). We notice that in the far-zone, we have

$$H_{\phi 1} = -\sqrt{\frac{\varepsilon_o}{\mu_o}} E_{z1} \qquad (2C.15)$$

in the upper half-space, and

$$H_{\phi 2} = -n\sqrt{\frac{\varepsilon_o}{\mu_o}} E_{z2} \qquad (2C.16)$$

in the lower half-space. From these relations we see that in the far-zone the Poynting vector is real and has only a ρ component, S_ρ.

Since S_ρ is given by [18]

$$S_{\rho 1} = \frac{1}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\;|E_{z1}|^2 \tag{2C.17}$$

in the upper half-space, and by

$$S_{\rho 2} = \frac{1}{2}\; n\sqrt{\frac{\varepsilon_o}{\mu_o}}\;|E_{z2}|^2 \tag{2C.18}$$

in the lower half-space, we find by substituting (2C.6) into (2C.17) that

$$S_{\rho 1} = \frac{\omega\mu_o I^2}{4\pi\rho}\;\frac{1}{(n^2-1)^2}\;[\sin^2\phi - \sin\phi \cdot \sqrt{n^2-\cos^2\phi}]^2 \tag{2C.19}$$

for the upper half-space ($k_o\rho \to \infty$, $0 \le \phi \le \pi$). By substituting (2C.10) into (2C.18) we find that

$$S_{\rho 2} = \frac{\omega\mu_o I^2}{4\pi\rho}\;\frac{n^2}{(n^2-1)^2}\;[n\;\;\sin^2\phi + \sin\phi \cdot \sqrt{1-n^2\;\cos^2\phi}]^2 \tag{2C.20}$$

for the dihedral region ($nk_o\rho \to \infty$, $-\pi + \phi_c < \phi < -\phi_c$); and by substituting (2C.11) into (2C.18) we find that

$$S_{\rho 2} = \frac{\omega\mu_o I^2}{4\pi\rho}\;\frac{n^2}{(n^2-1)^2}\;[n^2\;\sin^4\phi + \sin^2\phi \cdot (n^2\;\cos^2\phi - 1)] \tag{2C. 21}$$

for the both sectors ($nk_o\rho \to \infty$, $-\phi_c \le \phi \le 0$ and $-\pi \le \phi \le -\pi + \phi_c$).

From (2C.19), (2C.20), and (2C.21) we can sketch the radiation pattern (S_ρ versus ϕ) of the line source (see Fig. 3). We see that at the interface ($\phi = 0$, $\phi = \pi$) the radiation pattern disappears, i.e. $S_\rho = 0$, and that in the upper half-space the radiation pattern consists of a single lobe, the maximum (A) of which lies along the line $\phi = \pi/2$ and has the value

$$(S_{\rho 1})_{max} = \frac{\omega\mu_o I^2}{4\pi\rho} \frac{1}{(n+1)^2} \quad . \tag{2C.22}$$

In the lower half-space, at the angles $\phi = -\phi_c$ and $\phi = -\pi + \phi_c$, $S_{\rho 2}$ has peaks (C and D) whose values are given by

$$(S_{\rho 2})_{peak} = \frac{\omega\mu_o I^2}{4\pi\rho} \, . \tag{2C.23}$$

Between the critical angles along the line $\phi = -\pi/2$, $S_{\rho 2}$ has a mimimum (B) whose value is given by

$$(S_{\rho 2})_{min} = \frac{\omega\mu_o I^2}{4\pi\rho} \frac{n^2}{(n+1)^2} \quad . \tag{2C.24}$$

As n increases, the radiation pattern in the upper half-space shrinks, i.e., the lobe gets smaller and point A moves downward. In the lower half-space the behaviour of the radiation pattern is not so simple: as n increases, the dihedral angle between $\phi = -\pi + \phi_c$ and $\phi = -\phi_c$ decreases, point B moves downward, and points C and D move closer together.

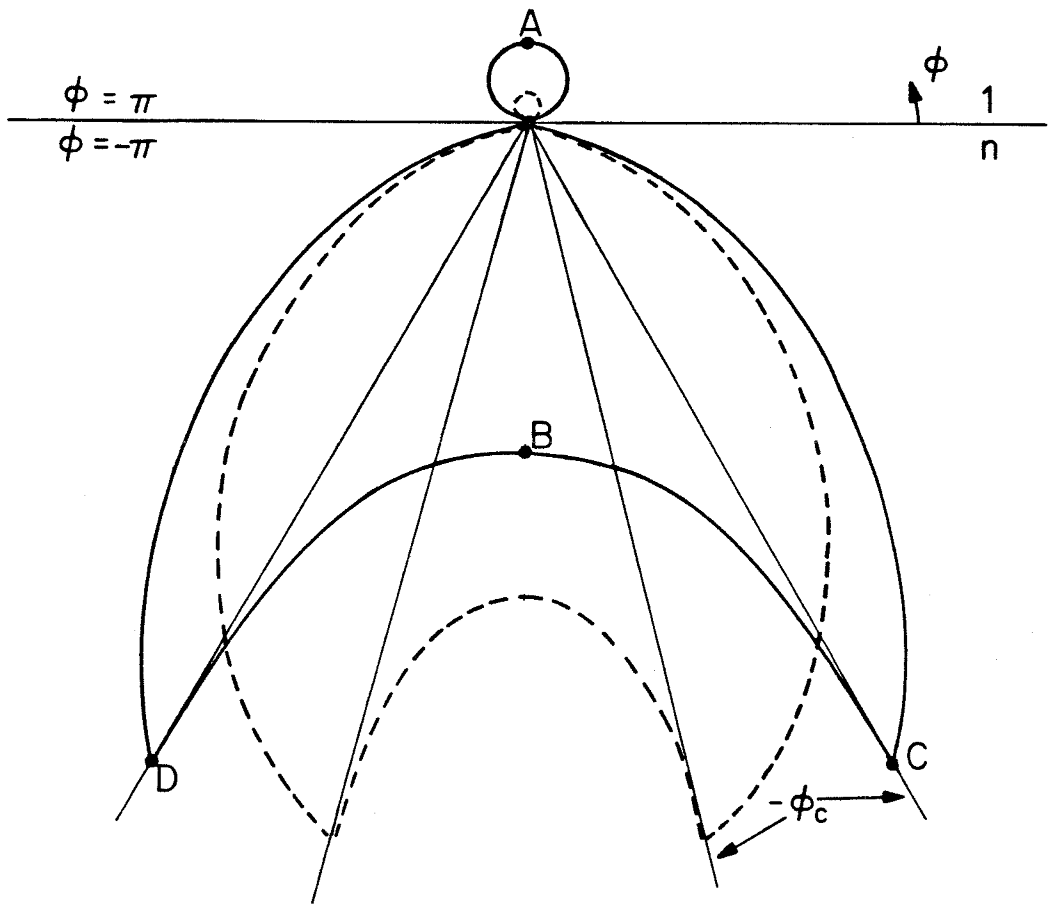

Fig. 3. Radiation patterns for n = 2 (continuous line) and n = 4 (broken line). As the index of refraction n of subsurface region increases, the lobe in the upper half-space shrinks, the dihedral angle between the two (Cerenkov) peaks D and C decreases, and the point B moves downward. For all values of $n > 1$, there is a broad null along the interface. Here $\cos \phi_c = 1/2$ (continuous line) and $\cos \phi_c = 1/4$ (broken line).

II D. Limiting Cases

We have found the expressions for the electromagnetic field and the Poynting vector of the line source lying along the interface of two homogeneous half-spaces. If the index refraction of the lower medium approaches one, we must get the expressions of the electromagnetic field and the Poynting vector of the line source in free-space.

To show that, we intend to find the limit of E_{z1} when $n \to 1$. Using L'Hospital rule [19], we get

$$\lim_{n \to 1} E_{z1} = \frac{\omega\mu_o I}{\sqrt{2\pi}} \frac{e^{ik_o\rho - i\pi/4}}{\sqrt{k_o\rho}} \left. \frac{\frac{\partial(\text{numerator})}{\partial n}}{\frac{\partial(\text{denominator})}{\partial n}} \right|_{n=1} . \tag{2D.1}$$

Therefore, we obtain

$$\lim_{n \to 1} E_{z1} = \frac{-\omega\mu_o I}{2\sqrt{2\pi}} \frac{e^{ik_o\rho - i\frac{\pi}{4}}}{\sqrt{k_o\rho}} \tag{2D.2}$$

The above expression is the far-zone electric field that would be radiated by the line source if it were in a homogeneous dielectric [see the Appendix D].

By using the similar approach, we can see that

$$\lim_{n \to 1} E_{z2} = -\frac{\omega\mu_o I}{2\sqrt{2\pi}} \frac{e^{ik_o\rho - i\frac{\pi}{4}}}{\sqrt{k_o\rho}} , \tag{2D.3}$$

which matches with the far-zone electric field that would be radiated by the line source if it were in a homogeneous dielectric.

We find the limit of the Poynting vectors when n approaches one. Thus, we obtain

$$\lim_{n \to 1} S_{\rho 1} = \frac{\omega\mu_o I^2}{16\pi\rho} \tag{2D.4}$$

$$\lim_{n \to 1} S_{\rho 2} = \frac{\omega\mu_o I^2}{16\pi\rho} \tag{2D.5}$$

we know that the Poynting vector ($S_{\rho o}$) associated with a line source in free-space is (see the Appendix D)

$$S_{\rho o} = \frac{\omega\mu_o I^2}{16\pi\rho} \qquad (k_o\rho \to \infty) \tag{2D.6}$$

By comparing (2D.4), (2D.5), and (2D.6), we can see that

$$\lim_{n \to 1} S_{\rho 1} = \lim_{n \to 1} S_{\rho 2} = S_{\rho o} \tag{2D.7}$$

II E. Radiated Power

The time-average power radiated into the upper half-space is given by [20]

$$P_1 = \int_0^{\pi} S_{\rho 1} \, \rho d\phi \qquad (k_o \rho \to \infty) \tag{2E.1}$$

and that radiated into the lower half-space is given by

$$P_2 = \int_{-\pi}^{0} S_{\rho 2} \, \rho d\phi \qquad (nk_o \rho \to \infty) \quad . \tag{2E.2}$$

Since the lower half-space is divided into three parts it is of interest to see how much each part contributes to P_2. Accordingly, we write P_2 as the sum of three integrals:

$$P_2 = \int_{-\pi}^{-\pi + \phi_c} S_{\rho 2} \, \rho d\phi + \int_{-\pi + \phi_c}^{-\phi_c} S_{\rho 2} \, \rho d\phi + \int_{-\phi_c}^{0} S_{\rho 2} \, \rho d\phi \quad . \tag{2E.3}$$

The second integral, which we denote by P_{2d}, gives the power radiated into the dihedral region ($-\pi + \phi_c \leq \phi \leq -\phi_c$); and the first and third integrals, which we denote by P_{2a} and P_{2b} respectively, give the power radiated into the regions between the dihedral region and the interface. That is,

$$P_2 = P_{2a} + P_{2d} + P_{2b} \quad . \tag{2E.4}$$

Substituting (2C.19) into (2E.1) we find that

$$P_1 = \frac{\omega\mu_o I^2}{4\pi(n^2 - 1)^2} \left[\frac{\pi}{4} - \frac{\pi}{2} n^2 + \frac{\pi}{4} n^4 + 2n^2\phi_c - \frac{1}{2} n^4\phi_c - n^2 \sin(2\phi_c) + \frac{n^4}{8} \sin(4\phi_c)\right], \tag{2E.5}$$

where $\cos \phi_c = 1/n$. From this relation we see that

$$P_1 \rightarrow o \text{ for } n \rightarrow \infty \tag{2E.6}$$

and

$$P_1 \rightarrow \frac{\omega\mu_o I^2}{16} = \frac{1}{2} P_o \text{ for } n \rightarrow 1 \tag{2E.7}$$

where P_o (= $\omega\mu I^2/8$) denotes the time-average power that would be radiated by the line source if it were in an homogeneous dielectric. [See the Appendix D].

By substituting (2C.20) into the second integral on the right-hand side of (2E.3) we find that

$$P_{2d} = \frac{\omega\mu_o I^2}{4\pi} \frac{n^2}{(n^2-1)^2} \left[-\frac{\pi}{2} + \frac{\pi}{4} n^2 + \frac{\pi}{4n^2} + \frac{1}{2} (n^2+1) \sin(2\phi_c) - \frac{n^2}{2} \phi_c - \phi_c - \frac{n^2}{8} \sin(4\phi_c)\right], \tag{2E.8}$$

and by substituting (2C.21) into the first and third integrals on the right-hand side of (2E.3) we find that

$$P_{2a} = P_{2b} = \frac{\omega\mu_o I^2}{4\pi} \frac{n^2}{(n^2 - 1)^2} \left[\frac{n^2}{2} \phi_c - \frac{1}{2} \phi_c + \frac{1}{4} (1 - n^2) \sin(2\phi_c) \right]. \tag{2E.9}$$

From (2E.4), (2E.8), and (2E.9) it follows that

$$P_2 = \frac{\omega\mu_o I^2}{4\pi} \frac{n^2}{(n^2-1)^2} \left[-\frac{\pi}{2} + \frac{\pi}{4} n^2 + \frac{\pi}{4n^2} + \frac{n^2}{2} \phi_c - 2\phi_c + \sin(2\phi_c) - \frac{n^2}{8} \sin(4\phi_c) \right]. \tag{2E.10}$$

Consequently,

$$P_2 \to P_o \text{ for } n \to \infty \tag{2E.11}$$

$$P_2 \to \frac{1}{2} P_o \quad \text{for } n \to 1. \tag{2E.12}$$

We notice that

$$P_1 + P_2 = P_o = \frac{\omega\mu_o I^2}{8} \tag{2E.13}$$

As shown in Fig. 4, P_1 smoothly decreases from $(1/2)P_o$ to zero and P_2 smoothly increases from $(1/2)P_o$ to P_o, as n increases. For any value of n, $P_1 + P_2 = P_o$; and $P_2 > P_1$ for $n > 1$. This means that more power is radiated into the lower half-space where $n > 1$ than into the upper half-space where $n = 1$, and that for the large n most of the power is radiated into the subsurface region [21].

As shown in Figs. 5 and 6, as n increases from 1 to ∞, P_{2d} decreases smoothly from $(1/2)P_o$ to zero and $P_{2a}(=P_{2b})$ increases from zero to $(1/2)P_o$. We note that for $n > 1$, $P_{2d} > P_1$, and P_1 vanishes

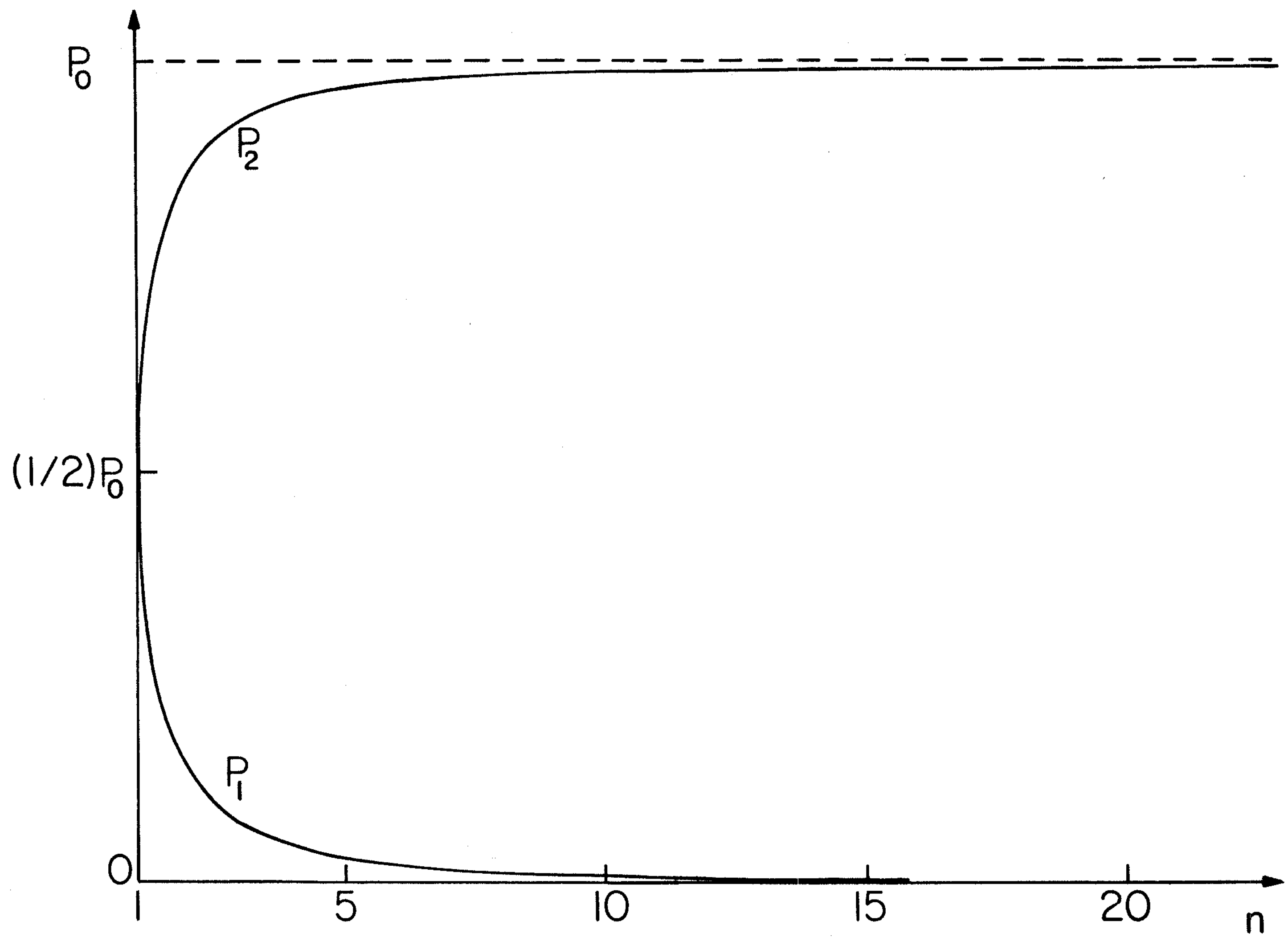

Fig. 4. P_1 (lower curve) and P_2 (upper curve) versus n. P_o ($= \omega\mu_o I^2/8$) denotes power radiated by the line source in a homogeneous dielectric.

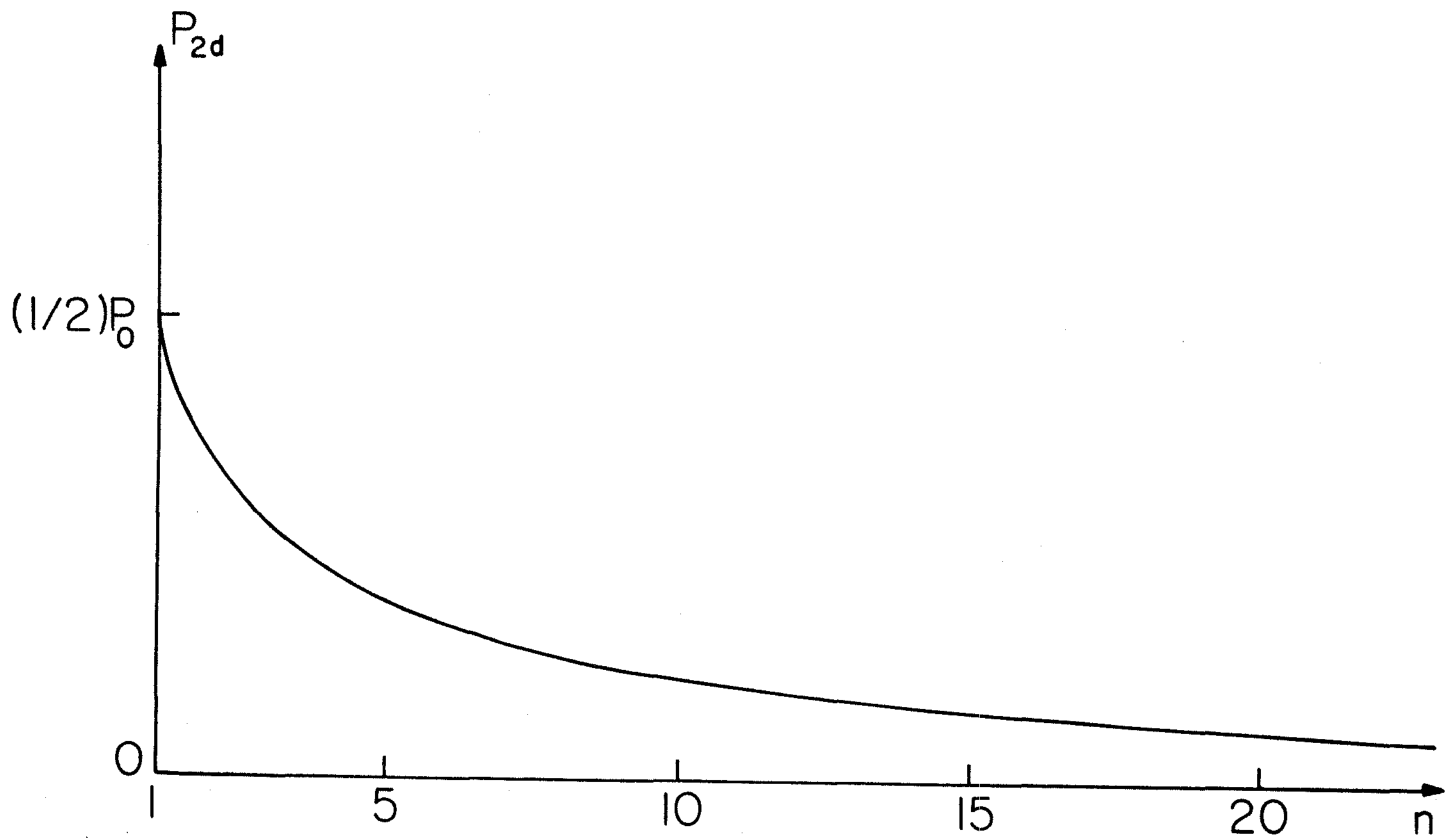

Fig. 5. P_{2d}, power radiated into dihedral angle, versus n. As n increases, P_{2d} decreases from $(1/2)P_o$ to zero. For any value of $n > 1$, P_{2d} is greater than P_1 of Fig. 4.

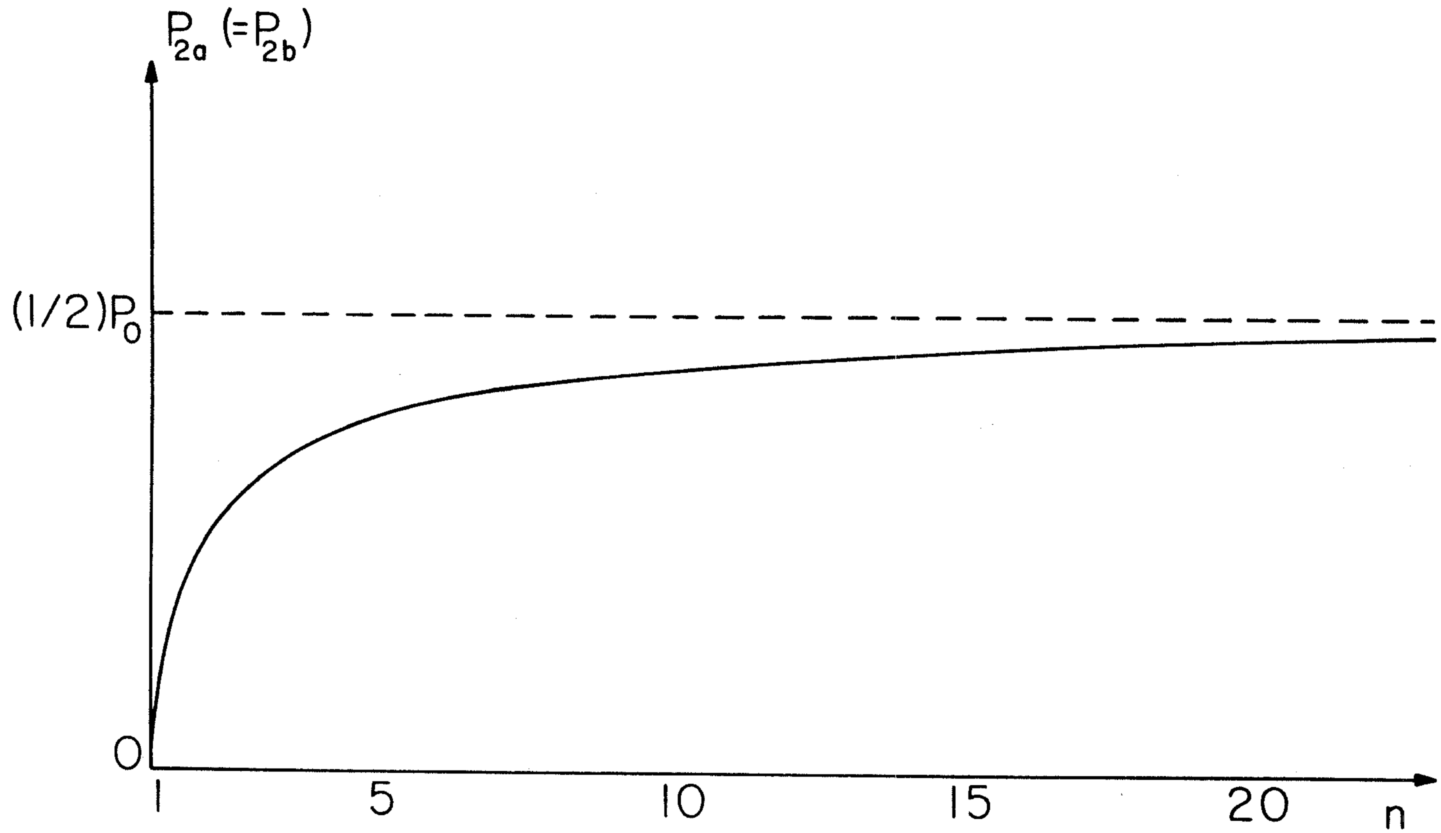

Fig. 6. P_{2a} (= P_{2b}), power radiated in region between interface and direction of (Cerenkov) peak, versus n. As n increases, P_{2a} increases from zero and approaches (1/2)P_o asymptotically.

faster than P_{2d}, as n increases.

IIF. Ray Optical Description

We can describe the foregoing results in terms of optical rays and the Fresnel coefficients of reflection and transmission.

To do so, we must take the source to be a little below the interface, where $n > 1$. Thus, the far-zone field in the subsurface region is the sum of two rays: the direct ray from the source and the ray reflected by the interface; and the far-zone field in the upper half-space is the ray transmitted from the lower half-space to the upper half-space. (See Fig. 7).

Accordingly, the far-zone field in the subsurface region is given by

$$E_{z2} = \sqrt{\frac{P_0}{n\pi\rho}} \sqrt[4]{\frac{\mu_0}{\varepsilon_0}} \left[1 + R(\theta)\right] e^{ink_0\rho} , \tag{2F.1}$$

where P_0 denotes power radiated by the line source in an homogeneous dielectric, θ is the angle of incidence, and $R(\theta)$ is the Fresnel reflection coefficient. Moreover, the far-zone field in the upper half-space is given by

$$E_{z1} = \sqrt{\frac{P_0}{n\pi\rho}} \sqrt[4]{\frac{\mu_0}{\varepsilon_0}} \, T(\gamma) \sqrt{\frac{\Delta\theta}{\Delta\gamma} \frac{\Delta L'}{\Delta L}} \, e^{ik_0\rho} , \tag{2F.2}$$

where γ is the angle of refraction, and $T(\gamma)$ is the Fresnel transmission coefficient. The meaning of the small quantities $\Delta\theta$,

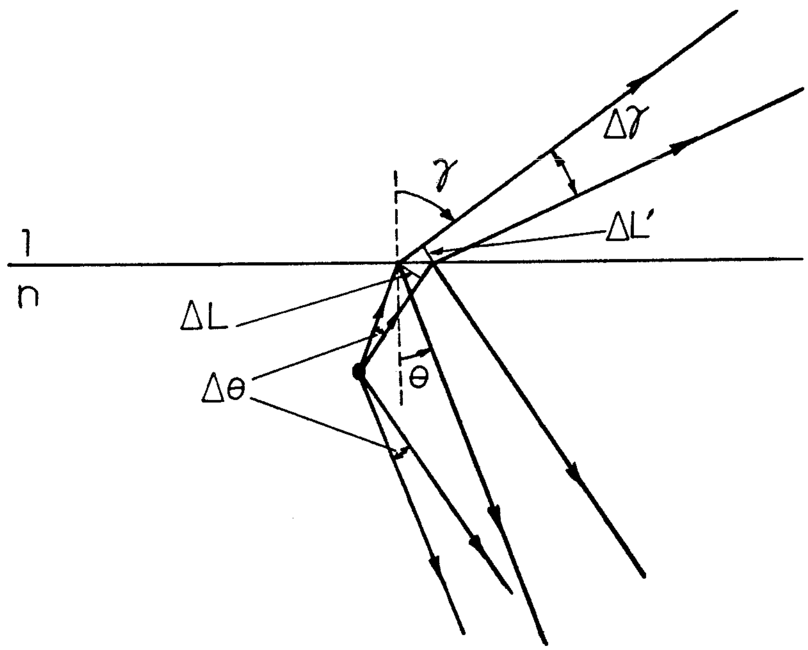

Fig. 7. Ray diagram for case of source a little below interface.

$\Delta\gamma$, $\Delta L'$ and ΔL is illustrated in Fig. 7 . [Appendix E for the details].

From Fig. 7 and from Snell's law, which expresses the relation between the angles θ and γ, we see that

$$\frac{\Delta L'}{\Delta L} = \frac{\cos\gamma}{\cos\theta} = \frac{n\cos\gamma}{\sqrt{n^2 - \sin^2\gamma}} , \tag{2F.3}$$

$$\frac{\Delta\theta}{\Delta\gamma} = \frac{\cos\gamma}{n\cos\theta} = \frac{\cos\gamma}{\sqrt{n^2 - \sin^2\gamma}} . \tag{2F.4}$$

We invoke Snell's law to write $R(\theta)$ as a function of θ only and $T(\gamma)$ as a function of γ only, [22], viz.

$$T(\gamma) = \frac{2\sqrt{n^2 - \sin^2\gamma}}{\sqrt{n^2 - \sin^2\gamma} + \cos\gamma} \tag{2F.5}$$

for $-\frac{\pi}{2} \leq \gamma \leq \frac{\pi}{2}$,

$$R(\theta) = \frac{n\cos\theta - \sqrt{1-n^2\sin^2\theta}}{n\cos\theta + \sqrt{1-n^2\sin^2\theta}} \tag{2F.6}$$

for the dihedral region ($-\theta_c \leq \theta \leq \theta_c$ where $\sin\theta_c = \frac{1}{n}$), and

$$R(\theta) = \frac{n\cos\theta - i\sqrt{n^2\sin^2\theta-1}}{n\cos\theta + i\sqrt{n^2\sin^2\theta-1}} \tag{2F.7}$$

for the other two sectors of lower half-space ($\theta_c \leq \theta \leq \frac{\pi}{2}$, $-\frac{\pi}{2} \leq \theta \leq -\theta_c$).

We find by substituting (2F.3), (2F.4), and (2F.5) into (2F.2) that

$$E_{z1} = -\sqrt{\frac{P_o}{\pi\rho}} \sqrt[4]{\frac{\mu_o}{\varepsilon_o}} \frac{2}{(n^2 - 1)} \left[\cos^2\gamma - \cos\gamma\sqrt{n^2 - \sin^2\gamma}\right] e^{ik_o\rho} \tag{2F.8}$$

for the upper half-space. By substituting (2F.6) into (2F.1) we get

$$E_{z2} = \sqrt{\frac{P_o}{n\pi\rho}} \sqrt[4]{\frac{\mu_o}{\varepsilon_o}} \frac{2n}{(n^2 - 1)} \left[n\cos^2\theta - \cos\theta\sqrt{1 - n^2\sin^2\theta}\right] e^{ink_o\rho} \tag{2F.9}$$

for the dihedral region; and by substituting (2F.7) into (2F.1) we find that

$$E_{z2} = \sqrt{\frac{P_o}{n\pi\rho}} \sqrt[4]{\frac{\mu_o}{\varepsilon_o}} \frac{2n}{(n^2 - 1)} \left[n\cos^2\theta - i\cos\theta\sqrt{n^2\sin^2\theta - 1}\right] e^{ink_o\rho} \tag{2F.10}$$

for the other two sectors of the lower half-space.

Since $P_o = \frac{\omega\mu_o I^2}{8}$, $\gamma = \frac{\pi}{2} - \phi$, and $\theta = \frac{\pi}{2} + \phi$, where ϕ is the angle shown in Fig. 2 , (2F.8), (2F.9), and (2F.10) become

$$E_{z1} = -I\sqrt{\frac{\omega\mu_o}{2\pi\rho}} \sqrt[4]{\frac{\mu_o}{\varepsilon_o}} \frac{1}{(n^2 - 1)} \left[\sin^2\phi - \sin\phi\sqrt{n^2 - \cos^2\phi}\right] e^{ik_o\rho} \tag{2F.11}$$

for the upper half-space ($o \leq \phi \leq \pi$);

$$E_{z2} = I\sqrt{\frac{\omega\mu_o}{2n\pi\rho}}\ \sqrt[4]{\frac{\mu_o}{\varepsilon_o}}\ \frac{n}{(n^2-1)}\left[n\sin^2\phi + \sin\phi\sqrt{1-n^2\cos^2\phi}\right]e^{ink_o\rho}$$

(2F.12)

for the dihedral region ($-\pi + \phi_c \leq \phi \leq -\phi_c$ where $\cos\phi_c = \frac{1}{n}$); and

$$E_{z2} = I\sqrt{\frac{\omega\mu_o}{2n\pi\rho}}\ \sqrt[4]{\frac{\mu_o}{\varepsilon_o}}\ \frac{n}{(n^2-1)}\left[n\sin^2\phi + i\sin\phi\sqrt{n^2\cos^2\phi - 1}\right]e^{ink_o\rho}$$

(2F.13)

for the sectors ($-\phi_c \leq \phi \leq o$ and $-\pi \leq \phi \leq -\pi + \phi_c$).

Except for the ignorable constant phase difference, we see that (2F.11), (2F.12), and (2F.13) are identical to (2C.6), (2C.10), and (2C.11) respectively. This means that ray optics can be made to yield the radiation patterns we get from a field theoretic approach.

If the source is taken to be a little above the interface, the correct result is obtained for the upper half-space and for the dihedral region of the lower half-space. However, there is no field radiated in the regions between the critical angles and the interface. This is a result of the fact that if the source is in the upper half-space no wave can be excited in the regions above the critical angle. Therefore, this configuration will only partially duplicate the results derived in the previous sections.

IIG. Physical Remarks

By looking at the radiation pattern, we see that the radiation pattern disappears at the interface ($\phi = 0$, $\phi = \pi$), i.e. $S_\rho = 0$. This is reasonable, because in the far-zone we have a TEM wave since in the far-zone H_ρ is negligibly small compared to E_z and H_ϕ. We know that a TEM wave propagates with the velocity of light in the medium. Accordingly, the TEM wave in the upper half-space propagates with the velocity of light in the upper half-space, i.e. $v_{ph1} = c$, whereas the TEM wave in the lower half-space propagates with the velocity of light in that medium, i.e. $v_{ph2} = c_2 = c/n$. To satisfy the boundary conditions along the interface E_z and H_ϕ must be continuous along the interface. But the TEM wave in the upper half-space propagates faster than that in the lower half-space. Therefore the two TEM waves will not be continuous along the interface unless their intensities are zero (see Fig. 8). Consequently we have zero intensity for the TEM waves along the interface and the radiation pattern disappears at the interface.

The radiation pattern in the upper half-space resembles the broadside radiation pattern of a tapered distribution of sources lying along the plane of the interface. However, the radiation pattern in the lower half-space is not so simple, for it appears to be a combination of two patterns, one being the pattern of a tapered distribution of sources lying in the plane of the interface, and the

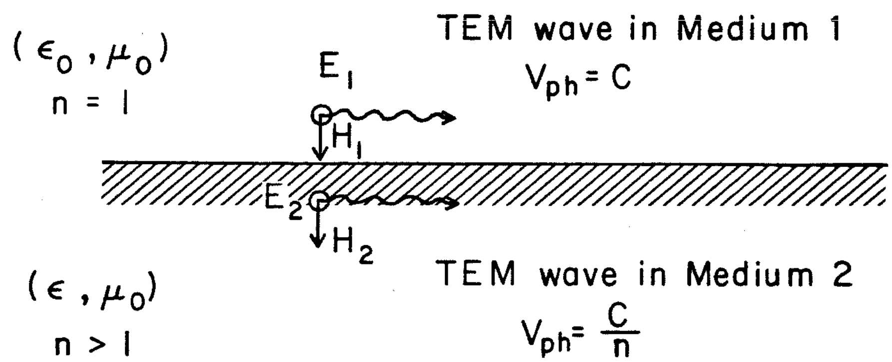

Fig. 8. Impossibility of matching two TEM waves with nonzero amplitudes and different phase velocities.

other being the pattern of sources moving radially outward along the interface. The peaks of the subsurface radiation pattern seem to be generated by these moving sources. To physically justify the general features of these radiation patterns, we invoke the extinction theorem [23] and the Cerenkov effect [24].

We assume the incident field to be the field that the line source would emit in vacuum. That is, the incident wave is assumed to be a cylindrical wave emanating from the line source in vacuum. This wave's phase velocity in the near-zone of the line source is a complicated function of ρ, but in the far-zone of the line source its phase velocity is constant and equal to c. As this incident wave sweeps through the dielectric of the lower half-space, two waves, α and β, are generated by the induced dipoles. For points in the dielectric, but not near the interface, the α wave, which has the phase velocity c, completely cancels the incident wave, whereas the β wave, which has the phase velocity c/n, gives rise to the resultant subsurface radiation pattern. For points in upper half-space, but not near the interface, the β wave is vanishingly small, and the α wave combines destructively with the incident wave to yield the resultant radiation pattern in the upper half-space.

For points close to the interface, both α and β waves exist. They combine with the incident wave to yield a composite field which on the interface is given by (2B.11). Along the interface, the incident and α waves combine to give $(1/\rho)H_1^{(1)}(k_0\rho)$ and the β wave contributes

$(n/\rho)H_1^{(1)}(nk_0\rho)$. Thus we see that the far-zone field along the interface is zero because the combination of the incident and α waves disappears and the β wave disappears by itself.

Mathematically, the patterns in the upper and lower half-spaces can be considered as patterns produced by stationary sources distributed along the interface and by sources moving along the interface. According to (2B.11) these moving sources have two different velocities, v_1 and v_2, given by the far-zone phase velocities of $H_1^{(1)}(k_0\rho)$ and $H_1^{(1)}(nk_0\rho)$. That is, $v_1 = \omega/k_0$ and $v_2 = \omega/nk_0$. Clearly, neither v_1 nor v_2 is greater than the velocity c of light in the upper half-space. However, v_1 is greater than the velocity c/n of light in the lower half-space. Consequently, these moving sources produce Cerenkov-like radiation in the lower half-space (in the directions $\phi = -\phi_c$ and $\phi = -\pi + \phi_c$) but not in the upper half-space. This is the reason we have peaks in the subsurface radiation pattern and no peaks in the radiation pattern in the upper half-space.

IIH. Conclusions to Chapter II

Starting from the Maxwell equations we have calculated the radiation pattern and emitted power of a line source lying along the plane interface of two dielectric half-spaces. Also, we have shown that is is possible to describe these results in terms of ray optics, provided we take the position of the source to be a little below the interface but not exactly on it. From our calculations one can draw the following conclusions:

In the upper half-space where $n = 1$ the radiation pattern is a single lobe which resembles the radiation pattern of a tapered broadside array. Accordingly, from above one would "see" not a line source but a tapered broadside array. In the lower half-space where $n > 1$ the radiation pattern is not so simple; it consists of two equal maxima (peaks) symmetrically located about a minimum. At the interface itself the radiation pattern is zero.

Clearly, when $n = 1$ the power P_1 radiated into the upper half-space is equal to the power P_2 radiated into the lower half-space. However, as n increases, P_1 decreases, P_2 increases, and $P_1 + P_2$ remains constant. For $n > 1$ the line source radiates more power into the lower half-space than into the upper half-space.

III. INTERFACIAL VERTICAL DIPOLE

IIIA. Formulation of the Problem

In this chapter, we have an infinitesimal electric dipole located on the plane interface of two dielectric half-spaces and directed normal to the interface. We calculate the radiation pattern of this dipole antenna. To formulate the problem mathematically, as we did in chapter two, we introduce a Cartesian coordinate system, x,y,z wherein the xy plane is the plane of the interface of two homogeneous half-spaces and the vertical infinitesimal electric dipole is located at the origin and directed parallel to the z axis. Moreover, to handle the far-zone field, which shall have the form of a spherical wave, we find it convenient to introduce also a spherical coordinate system r,θ,ϕ where $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, and $0 \leq \theta \leq \pi$, $0 \leq \phi \leq 2\pi$. In addition to these two coordinate systems, since we shall use Sommerfeld's method [25], we introduce a cylindrical coordinate system ρ,ϕ,z where $x = \rho\cos\phi$, $y = \rho\sin\phi$. (See Fig. 9).

In using the MKS system of units, the current density of the electric dipole is given by

$$\underline{J}\,(x,y,z;t) = \mathrm{Re}\left[\underline{e}_z I_0 \delta(x)\delta(y)\delta(z)e^{-i\omega t}\right] \quad , \qquad (3A.1)$$

where "Re" is the shorthand for "the real part of", $\underline{e}_z$ is the unit vector along the z axis, $\delta(x)$, $\delta(y)$, $\delta(z)$ are Dirac delta functions, ω is the angular frequency of the oscillation, and I_0 denotes $i\Delta\ell$ where

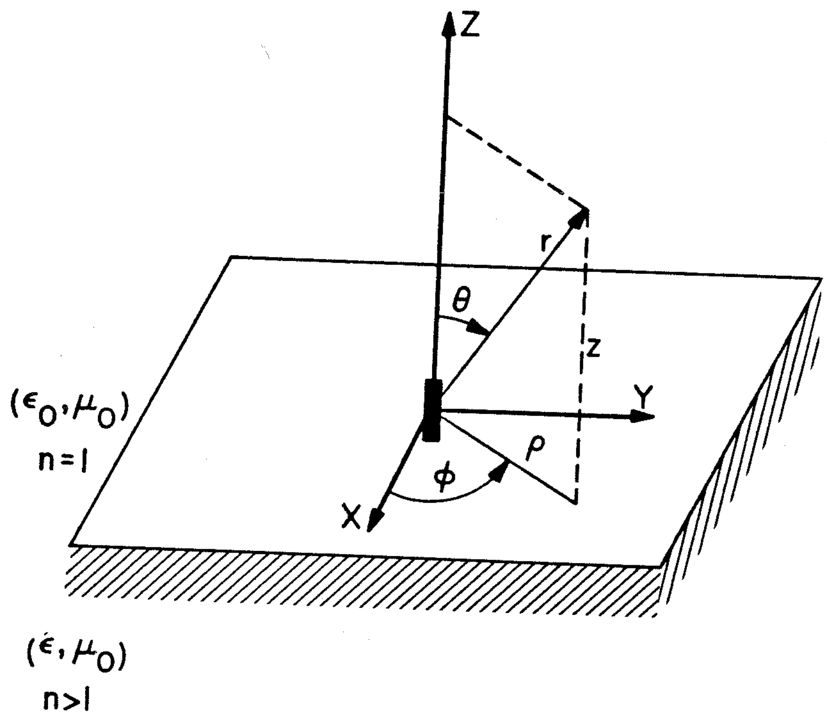

Fig. 9. The infinitesimal electric dipole is located on the interface and is normal to it. The dielectric constant of the upper half-space ($z > 0$) is ε_0 and that of the subsurface region ($z < 0$) is ε which is greater than ε_0.

i is the total current and $\Delta\ell$ is the length of the dipole. As we did in chapter two, we take the index of refraction, n, to be 1 in the upper half-space (z > 0) and greater than 1 in the lower half-space (z < 0). Accordingly, the dielectric constant of the upper half-space is $\varepsilon_1 = \varepsilon_0$ and that of the lower half-space is $\varepsilon_2 = n^2\varepsilon_0$ where ε_0 is the permittivity of free-space. Although this means that the upper half-space is a vacuum and the lower half-space is a dielectric, our analysis will hold true for any two dielectric half-spaces whose indices of refraction are in the ratio of n to 1.

From the Maxwell equations

$$\nabla \times \underset{\sim}{H} = \underset{\sim}{J} - i\omega\varepsilon\underset{\sim}{E} , \tag{3A.2}$$

$$\nabla \times \underset{\sim}{E} = i\omega\mu\underset{\sim}{H} , \tag{3A.3}$$

it follows that

$$\nabla \times \nabla \times \underset{\sim}{E} - k^2\underset{\sim}{E} = i\omega\mu\underset{\sim}{J} , \tag{3A.4}$$

$$\nabla \times \nabla \times \underset{\sim}{H} - k^2\underset{\sim}{H} = \nabla \times \underset{\sim}{J} , \tag{3A.5}$$

where $k^2 = \omega^2\mu\varepsilon$ and μ is the permeability of the medium. It is clear from (3A.1) and (3A.4) that $\underset{\sim}{E}$ should satisfy

$$\nabla \times \nabla \times \underset{\sim}{E} - k^2\underset{\sim}{E} = i\omega\mu I_0\delta(x)\delta(y)\delta(z)\underline{e}_z , \tag{3A.6}$$

where $\mu = \mu_0$ everywhere, $k^2 = k_0{}^2 = \omega^2\mu_0\varepsilon_0$ for the upper half-space (z > 0) and $k^2 = n^2k_0{}^2$ for the subsurface region (z < 0). We shall denote $\underset{\sim}{E}$ in the upper half-space by $\underset{\sim}{E}_1$ and $\underset{\sim}{E}$ in the lower half-space by $\underset{\sim}{E}_2$. To solve equation (3A.6) we must consider the Sommerfeld radiation condition [25] for the spherical wave in the spherical coordinate system. That is

$$\lim_{r \to \infty} r\left(\frac{\partial E_i}{\partial r} - ikE_i\right) \to 0 \quad , \qquad (3A.7)$$

where E_i is any component of the electric field. In addition to the radiation condition, the solution to the equation (3A.6) must satisfy the boundary conditions. Accordingly, we consider the continuity of the tangential electric and magnetic fields at the plane interface of the two media.

To solve the problem, we cannot use the method of separation of variables, because of the unusual boundary requirements on the fields along the surface. Instead we intend to use the electric Hertz vector and its integral representation [27]. Accordingly, we introduce the electric Hertz vector $\underset{\sim}{\Pi}$. This vector is connected to the electric and magnetic fields by the following relations [28]:

$$\underline{E} = \nabla(\nabla\cdot\underline{\Pi}) - \mu\varepsilon \frac{\partial^2 \underline{\Pi}}{\partial t^2} \quad , \qquad (3A.8)$$

$$\underline{H} = \varepsilon\nabla \times \frac{\partial \underline{\Pi}}{\partial t} \quad . \qquad (3A.9)$$

In the time harmonic case with the angular frequency ω, we get

$$\underset{\sim}{E} = \nabla(\nabla\cdot\underset{\sim}{\Pi}) + k^2\underset{\sim}{\Pi} \qquad (3A.10)$$

$$\underset{\sim}{H} = \frac{k^2}{i\omega\mu_o} \nabla \times \underset{\sim}{\Pi} \tag{3A.11}$$

Using Sommerfeld's method, we can see that the z component of the vector $\underset{\sim}{\Pi}$ is the only component that is necessary. (See Appendix F). Therefore, we take

$$\underset{\sim}{\Pi} = \underline{e}_z \Pi_z \ . \tag{3A.12}$$

It can be easily shown that the boundary conditions for the Π_z along $z = 0$ are

$$\Pi_{1z} = n^2 \Pi_{2z} \ , \tag{3A.13a}$$

$$\frac{\partial \Pi_{1z}}{\partial_z} = \frac{\partial \Pi_{2z}}{\partial_z} \ , \tag{3A.13b}$$

(Appendix F) where Π_{1z} and Π_{2z} are the z component of the Hertz vectors in the upper and lower media respectively. From (3A.6), (3A.10), and (3A.12), it follows that

$$\nabla^2 \Pi_z + k^2 \Pi_z = \frac{-i\omega\mu}{k^2} I_o \ \delta(x) \ \delta(y) \ \delta(z) \tag{3A.14}$$

where Π_z is the z component of $\underset{\sim}{\Pi}$ in the cylindrical coordinate system. Thus, our problem amounts to one of finding the solution of (3A.14) that gives outgoing spherical waves in the far-zone and satisfies the boundary conditions (3A.13a) and (3A.13b) along $z = 0$.

We can find by the Sommerfeld method that the solutions for Π_{1z} and Π_{2z} are

$$\Pi_{1z} = \frac{i\, I_o}{8\pi\omega\varepsilon_o} \int_W \frac{2n^2 h \exp[-\sqrt{h^2 - k_o^2}\, z]}{n^2\sqrt{h^2 - k_o^2} + \sqrt{h^2 - n^2 k_o^2}}\, H_o^{(1)}(h\rho)dh\,, \quad (3A.15)$$

$$\Pi_{2z} = \frac{i\, I_o}{8\pi\omega\varepsilon_o} \int_W \frac{2h \exp[\sqrt{h^2 - n^2 k_o^2}\, z]}{n^2\sqrt{h^2 - k_o^2} + \sqrt{h^2 - n^2 k_o^2}}\, H_o^{(1)}(h\rho)dh\,, \quad (3A.16)$$

where $H_o^{(1)}$ is the zeroth order Hankel function of the first kind, h is a complex variable and W is the path of integration in the complex h-plane. For $Re(h) < 0$, W goes parallel and slightly above the real axis and for $Re(h) > 0$ it goes parallel and slightly below the real axis. The two pieces of the path are connected smoothly at $h = 0$.

The integrals are difficult to evaluate exactly for all values of ρ and z. To have a description of Π_z, we resort to an asymptotic evaluation of the integrals.

IIIB. Π_z in the Upper Half-Space

To find an asymptotic evaluation of integral (3A.15), we use the spherical coordinate system and we replace ρ by $r\sin\theta$, and z by $r\cos\theta$. Accordingly, we obtain

$$\Pi_{1z} = \frac{i\, I_o}{8\pi\omega\varepsilon_o} \int_W \frac{2n^2 h \exp[-\sqrt{h^2 - k_o^2}\, r\cos\theta]}{n^2\sqrt{h^2 - k_o^2} + \sqrt{h^2 - n^2 k_o^2}}\, H_o^{(1)}(hr\sin\theta)dh \tag{3B.1}$$

for $0 \leq \theta \leq \frac{\pi}{2}$.

As can be shown, this integral along the path W will reduce to the following integral over the interval $0 \leq h \leq \infty$:

$$\Pi_{1z} = \frac{i\, I_o}{2\pi\omega\varepsilon_o} \int_0^\infty \frac{n^2 h \exp[-\sqrt{h^2 - k_o^2}\, r\cos\theta]}{n^2\sqrt{h^2 - k_o^2} + \sqrt{h^2 - n^2 k_o^2}}\, J_o(hr\sin\theta)dh \tag{3B.2}$$

where J_o is the zeroth order Bessel function. Here the path of integration is the real axis of h from 0 to ∞ and is indented from below the branch points at $h = k_o$ and $h = nk_o$. We express the Bessel function by its integral representation [29], i.e.

$$J_o(hr\sin\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[ihr\sin\theta\sin\beta]d\beta \ . \tag{3B.3}$$

By substituting (3B.3) into (3B.2) we find

$$\Pi_{1z} = \frac{i\,I_o}{4\pi^2\omega\varepsilon_o}\int_0^{\infty}\int_{-\pi}^{\pi} \frac{n^2 h\,\exp[-\sqrt{h^2 - k_o^2}\,r\cos\theta + ihr\sin\theta\sin\beta]}{n^2\sqrt{h^2 - k_o^2} + \sqrt{h^2 - n^2 k_o^2}}\, d\beta dh. \tag{3B.4}$$

This integral can be divided into two integrals, viz.

$$\Pi_{1z} = \int_0^{k_o}\int_{-\pi}^{\pi} f(h,\beta)dhd\beta + \int_{k_o}^{\infty}\int_{-\pi}^{\pi} f(h,\beta)dhd\beta\,. \tag{3B.5}$$

In the first integral, which we denote by F_1

$$F_1 = \frac{-\,I_o}{4\pi^2\omega\varepsilon_o}\int_0^{k_o}\int_{-\pi}^{\pi} \frac{n^2 h\exp[i\sqrt{k_o^2-h^2}\,r\cos\theta + ihr\sin\theta\sin\beta]}{n^2\sqrt{k_o^2-h^2} + \sqrt{n^2k_o^2-h^2}}\, dhd\beta\,. \tag{3B.6}$$

We introduce the variable α by $\sin\alpha = \frac{h}{k_o}$, and thus from (3B.6) obtain

$$F_1 = \frac{-\,I_o}{4\pi^2\omega\varepsilon_o}\int_0^{\frac{\pi}{2}}\int_{-\pi}^{\pi} \frac{n^2 k_o\sin\alpha\cos\alpha\exp[ik_o r(\sin\alpha\sin\theta\sin\beta + \cos\alpha\cos\theta)}{n^2\cos\alpha + \sqrt{n^2-\sin^2\alpha}}\, d\alpha d\beta\,. \tag{3B.7}$$

As we notice, in (3B.6) the factor $\sqrt{h^2-k_o^2}$ has been replaced by $(-i)\sqrt{k_o^2-h^2}$ for the subrange $0 \le h \le k_o$, because the solutions (3A.15) and (3A.16) must satisfy the radiation condition. We invoke the theory of asymptotic expansions of double integrals by N. Chako [30]. In this theory, we have an integral with the form

$$I = \iint g(u,v)e^{ikrf(u,v)}dudv \qquad (3B.8)$$

and we like to have an asymptotic evaluation of this integral for $kr \to \infty$. To do so, we must find the so-called critical points. Chako showed that if we find a critical point of the first kind (u_o,v_o), which satisfies

$$\left.\frac{\partial f}{\partial u}\right|_{\substack{u = u_o \\ v = v_o}} = o \, , \qquad (3B.9)$$

$$\left.\frac{\partial f}{\partial v}\right|_{\substack{u = u_o \\ v = v_o}} = o \, , \qquad (3B.10)$$

in the interval of integration, then we will have the following leading term for the asymptotic evaluation of the integral when $kr \to \infty$. That is

$$I \sim \frac{2\pi i \sigma}{\sqrt{|\alpha\beta - \gamma^2|}} \, g(u_o,v_o) \, \frac{e^{ikrf(u_o,v_o)}}{kr} \, , \qquad (3B.11)$$

where $$\alpha = \left.\frac{\partial^2 f}{\partial u^2}\right|_{\substack{u = u_o \\ v = v_o}} \, , \qquad (3B.12)$$

$$\beta = \left.\frac{\partial^2 f}{\partial v^2}\right|_{\substack{u = u_o \\ v = v_o}} \, , \qquad (3B.13)$$

$$\gamma = \left.\frac{\partial^2 f}{\partial u \partial v}\right|_{\substack{u = u_o \\ v = v_o}} , \tag{3B.14}$$

and

$$\sigma = \begin{cases} +1 \text{ for } \alpha\beta > \gamma^2, \ \alpha > o \\ -1 \text{ for } \alpha\beta > \gamma^2, \ \alpha < o \\ -i \text{ for } \alpha\beta < \gamma^2 \end{cases} . \tag{3B.15}$$

Applying this theory to integral (3B.7), we find that the critical point of the first kind

$$\begin{cases} \beta = \frac{\pi}{2} \\ \alpha = \theta \end{cases} \tag{3B.16}$$

lies in the interval of integration. Therefore, we obtain the following expression for integral F_1 in the far-zone of the upper half-space,

$$F_1 \sim \frac{iI_o}{2\pi\omega\varepsilon_o} \frac{n^2\cos\theta}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \frac{e^{ik_o r}}{r} . \tag{3B.17}$$

Using the similar procedure and applying the theory of asymptotic expansion to the second integral of (3B.5), which we denote by

$$F_2 = \frac{iI_o}{4\pi^2\omega\varepsilon_o} \int_{k_o}^{\infty} \int_{-\pi}^{\pi} \frac{n^2 h \exp[-\sqrt{h^2-k_o^2}\, r\cos\theta + ihr\sin\theta\sin\beta]}{n^2\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}} \, d\beta dh , \tag{3B.18}$$

we find that the critical point of the first kind does not lie in the region of integration. Therefore the leading term of the integral F_2 does not have the order $\frac{1}{k_o r}$, and it decays more rapidly than $\frac{1}{k_o r}$. Consequently, the following expression is the asymptotic expression for Π_{1z} in the far-zone of the upper half-space,

$$\Pi_{1z} \sim \frac{iI_o}{2\pi\omega\varepsilon_o} \frac{n^2\cos\theta}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \frac{e^{ik_o r}}{r}. \tag{3B.19}$$

By substituting (3B.19) into (3A.10) and (3A.11) we obtain the electric and magnetic fields in the far-zone of the upper half-space, i.e.

$$E_{1\theta} \sim \frac{-iI_o}{2\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{n^2 k_o \sin\theta\cos\theta}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \frac{e^{ik_o r}}{r}, \tag{3B.20}$$

$$H_{1\phi} \sim \frac{-iI_o}{2\pi} \frac{n^2 k_o \sin\theta\cos\theta}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \frac{e^{ik_o r}}{r}. \tag{3B.21}$$

As we see from (3B.20) and (3B.21), the far-zone electric and magnetic fields in the upper half-space are related by the following relation

$$E_{1\theta} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_{1\phi} \tag{3B.22}$$

IIIC. Π_z in the Lower Half-Space

By using a similar procedure, Π_{2z} can be written as follows:

$$\Pi_{2z} = \frac{i\, I_o}{4\pi^2\omega\varepsilon_o} \int_0^\infty \int_{-\pi}^{\pi} \frac{h \exp[\sqrt{h^2 - n^2k_o^2}\, r\cos\theta + ihr\sin\theta\sin\beta]}{n^2\sqrt{h^2 - k_o^2} + \sqrt{h^2 - n^2k_o^2}}\, d\beta dh \tag{3C.1}$$

for $\frac{\pi}{2} \leq \theta \leq \pi$. We divide the range of integration over h into three subranges, that is, we write

$$\Pi_{2z} = \int_0^{k_o}\int_{-\pi}^{\pi} g(h,\beta)dhd\beta + \int_{k_o}^{nk_o}\int_{-\pi}^{\pi} g(h,\beta)dhd\beta + \int_{nk_o}^{\infty}\int_{-\pi}^{\pi} g(h,\beta)dhd\beta \tag{3C.2}$$

where $g(h,\beta)$ is a shorthand for the integrand. To have an asymptotic evaluation for Π_{2z}, as we did in IIIB, we invoke the Chako theory of asymptotic expansion. Applying this theory to the third integral of (3C.2), we find that the critical point of the first kind does not lie in the interval of integration. Therefore the integral for the subrange $nk_o \leq h < \infty$ does decay more rapidly than $\frac{1}{nk_o r}$ for all values of θ in the lower half-space ($\frac{\pi}{2} \leq \theta \leq \pi$). By defining the variable α by $\sin\alpha = \frac{h}{nk_o}$, the first and second integrals of (3C.2) can be written as follows:

$$I = \frac{-I_o}{4\pi^2\omega\varepsilon_o}\int_0^{\theta_c}\int_{-\pi}^{\pi} \frac{nk_o \sin\alpha\cos\alpha \exp[ink_o r(\sin\alpha\sin\theta\sin\beta - \cos\alpha\cos\theta)]}{n\sqrt{1-n^2\sin^2\alpha} + \cos\alpha}\, d\alpha d\beta$$

$$+ \frac{iI_o}{4\pi^2\omega\varepsilon_o}\int_{\theta_c}^{\frac{\pi}{2}}\int_{-\pi}^{\pi} \frac{nk_o \sin\alpha\cos\alpha \exp[ink_o r(\sin\alpha\sin\theta\sin\beta - \cos\alpha\cos\theta)]}{n\sqrt{n^2\sin^2\alpha - 1} - i\cos\alpha}\, d\alpha d\beta\,, \tag{3C.3}$$

where $\sin\theta_c = \frac{1}{n}$ and $\frac{\pi}{2} \leq \theta \leq \pi$. To have a leading term of order $\frac{1}{nk_o r}$ for $nk_o r \to \infty$, the critical points of the first kind must be in the interval of integration [26]. The critical point of the first kind of integrals (3C.3) has the form

$$\begin{cases} \beta = \frac{\pi}{2} \\ \alpha = \pi - \theta \end{cases} \tag{3C.4}$$

Thus if θ lies in the interval $\pi - \theta_c \leq \theta \leq \pi$, the leading term will result from the first integral of (3C.3). However, if θ lies in the interval $\frac{\pi}{2} \leq \theta \leq \pi - \theta_c$, the leading term will come from the second integral of (3C.3). Therefore, we have

$$\Pi_{2z} \sim \frac{-iI_o}{2\pi\omega\varepsilon_o}\; \frac{\cos\theta}{n\sqrt{1-n^2\sin^2\theta} - \cos\theta}\; \frac{e^{ink_o r}}{r} \tag{3C.5}$$

for $nk_o r \to \infty$ and $\pi - \theta_c \leq \theta \leq \pi$; and

$$\Pi_{2z} \sim \frac{-I_o}{2\pi\omega\varepsilon_o} \frac{\cos\theta}{n\sqrt{n^2\sin^2\theta - 1} + i\cos\theta} \frac{e^{ink_o r}}{r} \tag{3C.6}$$

for $nk_o r \to \infty$ and $\frac{\pi}{2} \leq \theta \leq \pi - \theta_c$.

We get by substituting (3C.5) and (3C.6) into (3A.10) and (3A.11) the electric and magnetic fields in the far-zone of the subsurface region ($z < 0$). That is

$$E_{2\theta} \sim i \frac{I_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{n^2 k_o \cos\theta\sin\theta}{n\sqrt{1 - n^2\sin^2\theta} - \cos\theta} \frac{e^{ink_o r}}{r} , \tag{3C.7}$$

$$H_{2\phi} \sim i \frac{I_o}{2\pi} \frac{n^3 k_o \cos\theta\sin\theta}{n\sqrt{1 - n^2\sin^2\theta} - \cos\theta} \frac{e^{ink_o r}}{r} \tag{3C.8}$$

for $nk_o r \to \infty$ and $\pi - \theta_c \leq \theta < \pi$; and

$$E_{2\theta} \sim \frac{I_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{n^2 k_o \cos\theta\sin\theta}{n\sqrt{n^2\sin^2\theta - 1} + i\cos\theta} \frac{e^{ink_o r}}{r}, \tag{3C.9}$$

$$H_{2\phi} \sim \frac{I_o}{2\pi} \frac{n^3 k_o \cos\theta\sin\theta}{n\sqrt{n^2\sin^2\theta - 1} + i\cos\theta} \frac{e^{ink_o r}}{r} \tag{3C.10}$$

for $nk_o r \to \infty$ and $\frac{\pi}{2} \leq \theta \leq \pi - \theta_c$.

As we see from (3C.7), (3C.8), (3C.9), and (3C.10), the far-zone electric and magnetic fields in the lower half-space obey the following relation

$$E_{2\theta} = \frac{1}{n}\sqrt{\frac{\mu_o}{\varepsilon_o}}\,H_{2\phi} \tag{3C.11}$$

IIID. Radiation Pattern

Having the electric and magnetic fields in the far-zone of both half-spaces we can see that the Poynting vector has only an r component, S_r.

Since S_r is given by

$$S_r = \frac{1}{2}\,\mathrm{Re}(E_\theta H_\phi^*), \tag{3D.1}$$

we find by substituting (3B.20) and (3B.21) into (3D.1) that

$$S_{r1} = \frac{I_o^{\,2}}{8\pi^2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\;\frac{n^4 k_o^{\,2}\sin^2\theta\cos^2\theta}{(n^2\cos\theta + \sqrt{n^2 - \sin^2\theta})^2}\;\frac{1}{r^2} \tag{3D.2}$$

for the upper half-space ($k_o r \to \infty$, $0 \leq \theta \leq \frac{\pi}{2}$). By substituting (3C.7) and (3C.8) into (3D.1), we obtain

$$S_{r2} = \frac{I_o^{\,2}}{8\pi^2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\;\frac{n^5 k_o^{\,2}\sin^2\theta\cos^2\theta}{(n\sqrt{1 - n^2\sin^2\theta} - \cos\theta)^2}\;\frac{1}{r^2} \tag{3D.3}$$

for $nk_o r \to \infty$ and $\pi - \theta_c \leq \theta \leq \pi$; and by substituting (3C.9) and (3C.10) into (3D.1), we get

$$S_{r2} = \frac{I_o^2}{8\pi^2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \frac{n^5 k_o^2 \sin^2\theta\cos^2\theta}{n^2(n^2\sin^2\theta - 1) + \cos^2\theta}\ \frac{1}{r^2} \tag{3D.4}$$

for $nk_o r \to \infty$ and $\frac{\pi}{2} \leq \theta \leq \pi - \theta_c$.

From (3D.2), (3D.3) and (3D.4) we sketch the radiation pattern of the vertical infinitesimal electric dipole (S_r versus θ only, because from the symmetry of configuration all the quantities are independent of ϕ) (see Fig. 10a and 10b). At the interface ($\theta = \frac{\pi}{2}$) and along the dipole axis ($\theta = 0$, $\theta = \pi$), the radiation pattern is zero. The dotted curve (L) is the locus of the maxima of the radiation pattern in the upper half-space, as n increases from one to infinity. In the lower half-space, the maximum of S_{r2} lies along the conical angle $\theta = \pi - \theta_c$ and as n increases the angle θ_c decreases.

IIIE. Limiting Cases

We have obtained the expressions for the electromagnetic field and the Poynting vector of the infinitesimal electric dipole which is vertically located on the plane interface of two dielectric half-spaces. If the index of refraction of the lower medium approaches one, we will get the electromagnetic field and the Poynting vector of the infinitesimal electric dipole in free-space.

To show that, we find the limit of $E_{1\theta}$ when $n \to 1$. Therefore, from (3B.20) we get

$$\lim_{n \to 1} E_{\theta 1} = \frac{-iI_o k_o}{4\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\sin\theta\,\frac{e^{ik_o r}}{r}\ . \tag{3E.1}$$

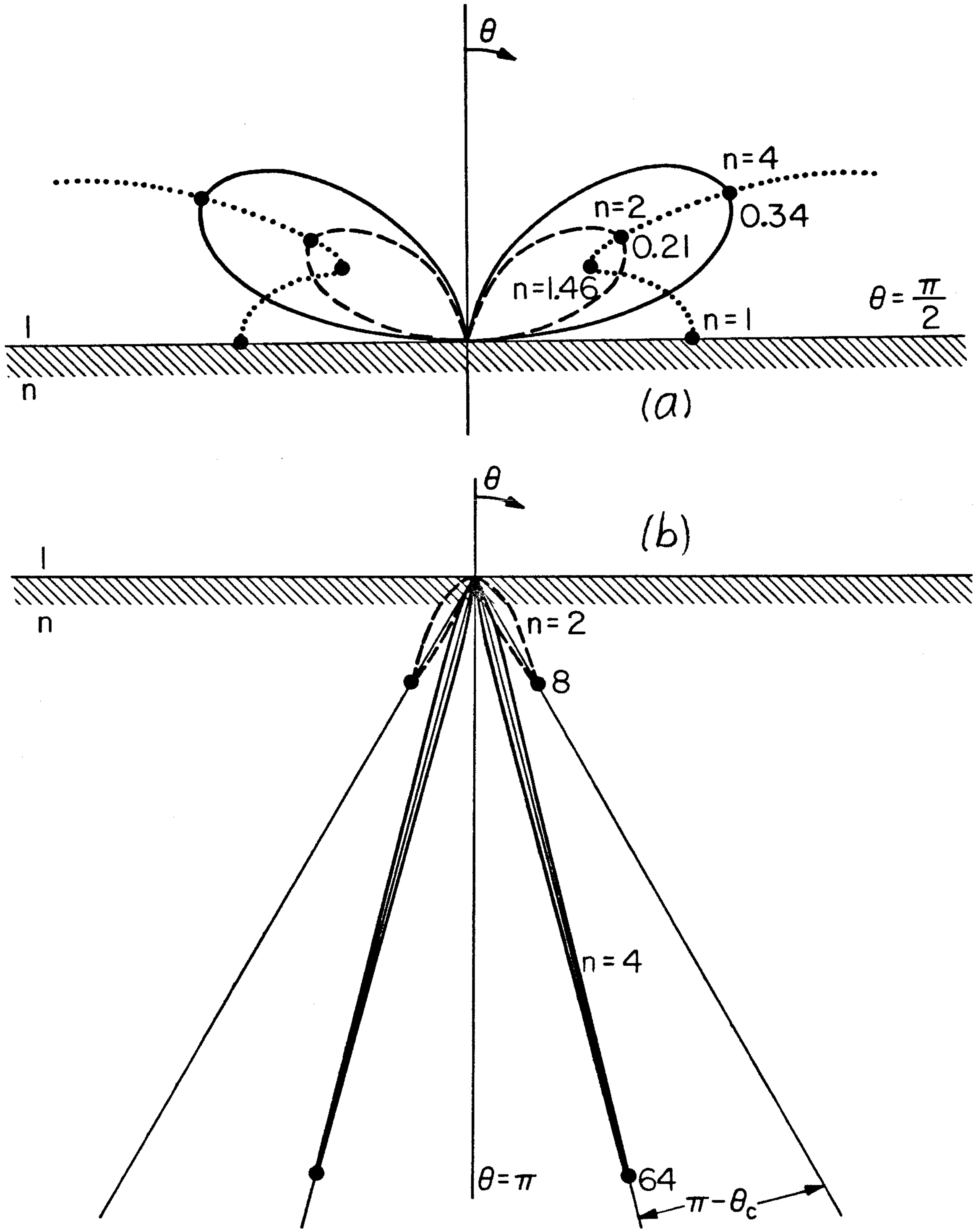

Fig. 10. Radiation pattern of the vertical dipole of Fig. 9 (a) in the upper half-space, (b) in the lower half-space for n=4 (continuous line) and n=2 (broken line). The dotted curve (L) is the locus of the maxima of the pattern in the upper half-space, as n increases from one to infinity. Note that the scale in Fig. 10(a) is different from that of Fig. 10(b).

The above expression is the far-zone electric field that would be radiated by the infinitesimal electric dipole if it were in free-space. [See Appendix D].

By using the similar approach, from (3C.7) we can see that

$$\lim_{n \to 1} E_{\theta 2} = \frac{-iI_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin\theta \frac{e^{ik_o r}}{r} , \tag{3E.2}$$

which agrees with the far-zone electric field that would be radiated by the infinitesimal electric dipole if it were in free-space.

We obtain the limit of the Poynting vectors when n approaches 1. Thus we get

$$\lim_{n \to 1} S_{r1} = \frac{I_o^2 k_o^2}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin^2\theta ; \tag{3E.3}$$

$$\lim_{n \to 1} S_{r2} = \frac{I_o^2 k_o^2}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin^2\theta . \tag{3E.4}$$

We know that the Poynting vector (S_{ro}) associated with an infinitesimal electric dipole in free-space is [Appendix D]

$$S_{ro} = \frac{I_o^2 k_o^2}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin^2\theta . \tag{3E.5}$$

By comparing (3E.3), (3E.4), and (3E.5), we can see that

$$\lim_{n \to 1} S_{r1} = \lim_{n \to 1} S_{r2} = S_{ro} \tag{3E.6}$$

IIIF. Radiated Power

The time-average power radiated into the upper half-space is given by

$$P_1 = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} S_{r1}\, r^2 \sin\theta d\theta d\phi \qquad (k_o r \to \infty), \tag{3F.1}$$

and that radiated into the lower half-space is given by

$$P_2 = \int_{\frac{\pi}{2}}^{\pi} \int_0^{2\pi} S_{r2}\, r^2 \sin\theta d\theta d\phi \qquad (nk_o r \to \infty). \tag{3F.2}$$

We find by substituting (3D.2) into (3F.1) that

$$P_1 = \frac{I_o^{\,2}}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \int_0^{\frac{\pi}{2}} \frac{n^4 k_o^{\,2} \sin^3\theta \cos^2\theta d\theta}{(n^2\cos\theta + \sqrt{n^2 - \sin^2\theta})^2} . \tag{3F.3}$$

For P_2, we divide the range of integration over θ into two subranges, i.e.

$$P_2 = \int_{\frac{\pi}{2}}^{\pi-\theta_c} \int_0^{2\pi} S_{r2}\, r^2 \sin\theta d\theta d\phi + \int_{\pi-\theta_c}^{\pi} \int_0^{2\pi} S_{r2}\, r^2 \sin\theta d\theta d\phi . \tag{3F.4}$$

From (3D.3), (3D.4) and (3F.4), we obtain

$$P_2 = \frac{I_o^{\,2}}{4\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}} \int_{\frac{\pi}{2}}^{\pi-\theta_c} \frac{n^5 k_o^{\,2} \sin^3\theta \cos^2\theta d\theta}{(n\sqrt{1-n^2\sin^2\theta} - \cos\theta)^2} +$$

$$+ \frac{I_o^{\,2}}{4\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}} \int_{\pi-\theta_c}^{\pi} \frac{n^5 k_o^{\,2} \sin^3\theta \cos^2\theta d\theta}{n^2(n^2\sin^2\theta - 1) + \cos^2\theta} \; . \qquad \text{(3F.5)}$$

It is useful to have the plots of P_1 and P_2 versus n. Figs.11 and 12 show the sketch of P_1 and P_2 versus n. As n approaches 1, both P_1 and P_2 approach $\frac{1}{2} P_o$ where $P_o (= \frac{I_o^2 k_o^2}{12\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}})$ denotes the time-average power that would be radiated by an electric dipole if it were in free-space. [See Appendix D]. As n increases from 1 to ∞, P_1 first decreases rapidly and then increases, and for $n \to \infty$, P_1 approaches $2P_o$. As n increases, P_2 smoothly increases.

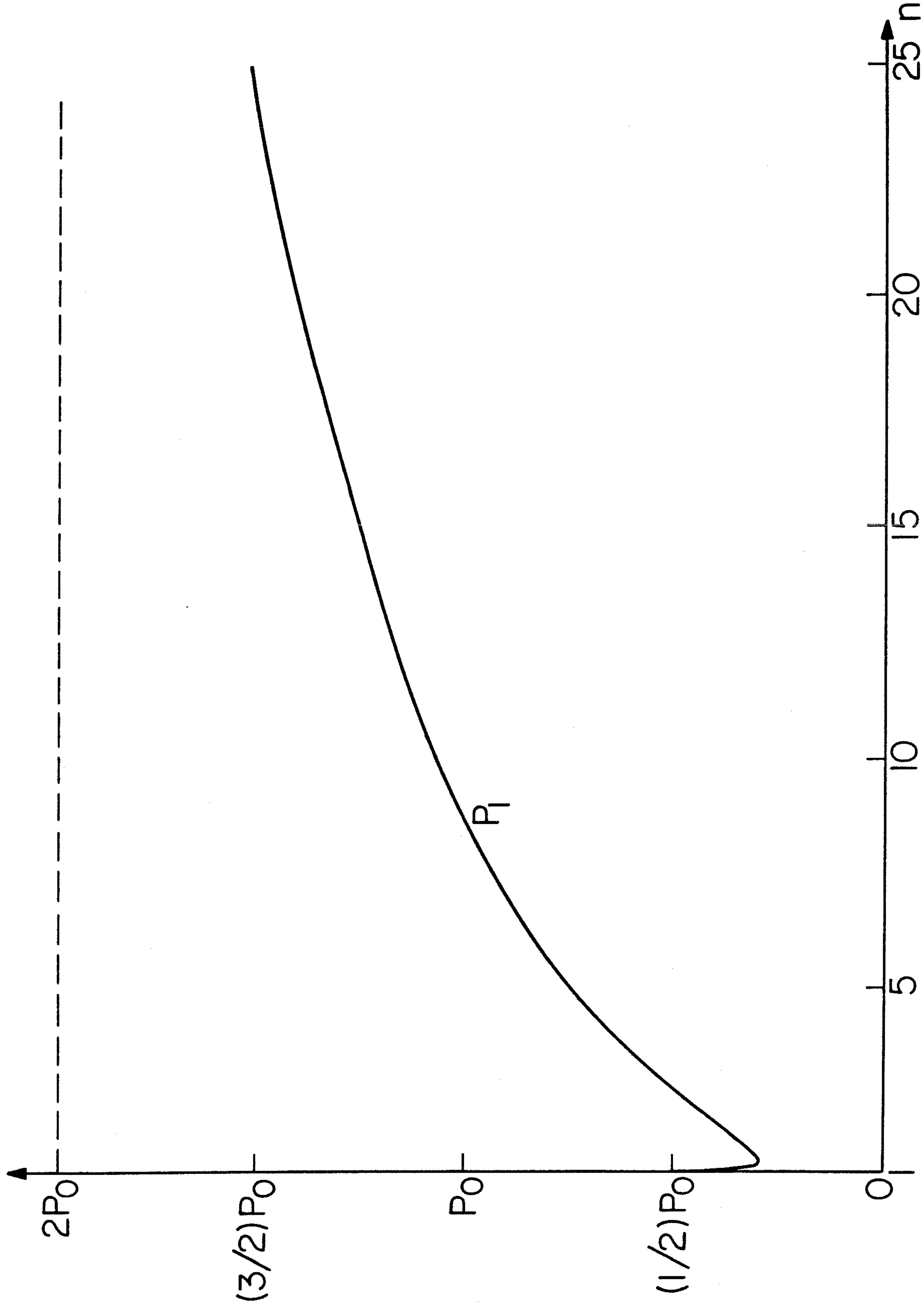

Fig. 11. P_1 the power of the vertical dipole radiated into the upper half-space. P_o denotes power radiated by the electric dipole in free-space.

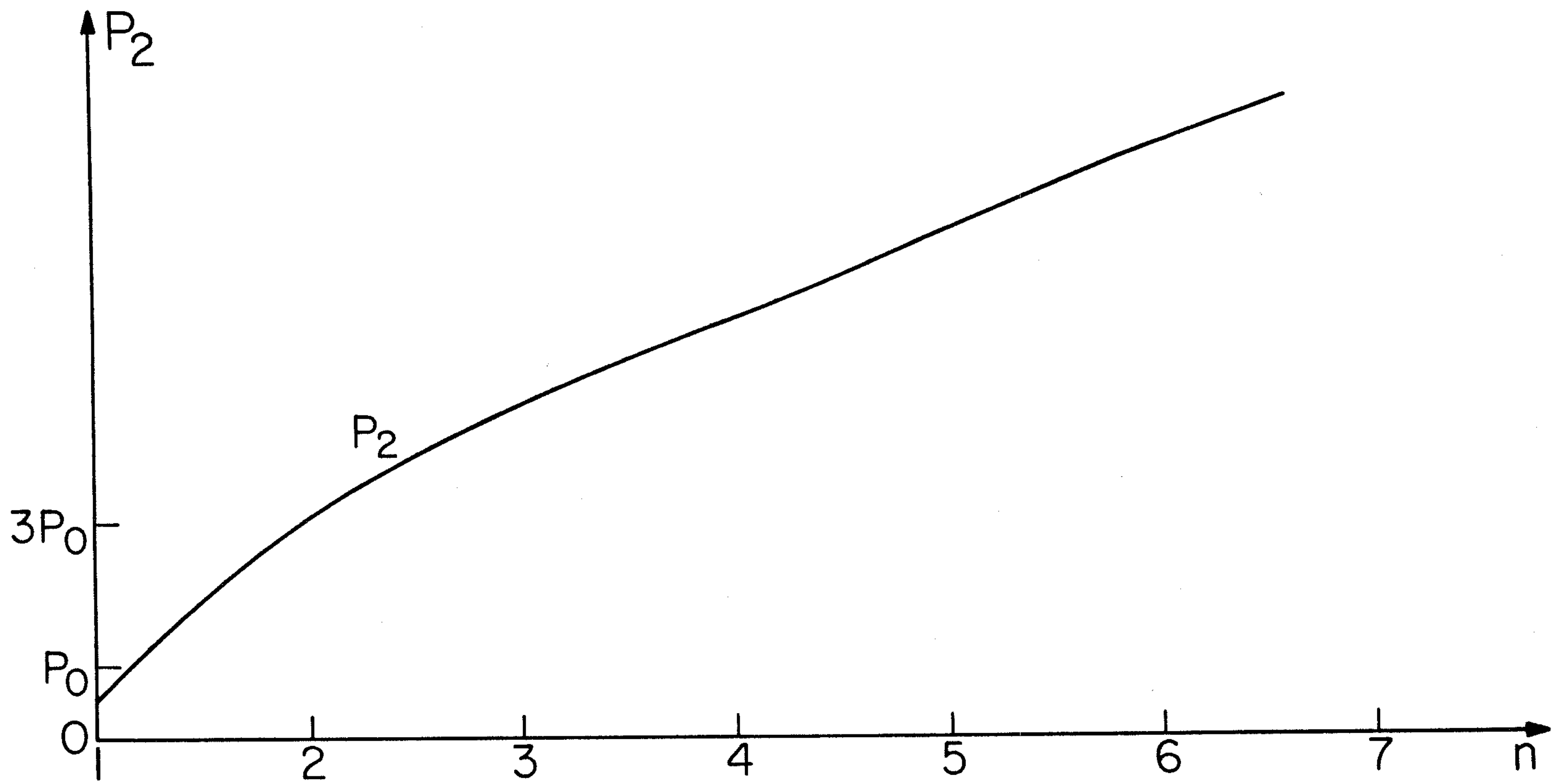

Fig. 12. P_2 the power of the vertical dipole radiated into the lower half-space. When n=1 $P_2=(1/2)P_o$. As n increases, P_2 increases. For any value of $n > 1$, P_2 is greater than P_1 of Fig. 11.

IIG. Conclusions to Chapter III.

We have found the radiation pattern and emitted power of an infinitesimal electric dipole for the case where the dipole is vertically located on the plane interface of two dielectric half-spaces.

For this case, the radiation pattern has nulls along the interface and along the dipole axis; the pattern in the upper half-space has a maximum which in amplitude and direction depends on n; and the pattern in the lower half-space has a maximum which also depends on n.

For this vertical dipole, as n increases from 1 to ∞, P_2, the power radiated into the lower half-space, increases monotonically whereas P_1, the power radiated into the upper half-space, first decreases and then increases and approaches $2P_0$ where P_0 is the time-average power that would be radiated by the dipole if it were in free space.

IV. INTERFACIAL HORIZONTAL DIPOLE

IVA. Formulation of the Problem

In this chapter, we have an infinitesimal electric dipole lying along the plane interface of the two dielectric half-spaces. We intend to calculate the radiation pattern of this dipole antenna and sketch its emitted power into two half-spaces in terms of the index of refraction of the lower medium. As before, we use Cartesian, spherical, and cylindrical coordinate systems. The dipole is located at the origin but now is directed parallel to the x axis (see Fig.13). Accordingly, in using MKS system of units, the current density of the electric dipole is given by

$$\underline{J}\ (x,y,z;t) = \mathrm{Re}\left[\underline{e}_x\ I_0\ \delta(x)\ \delta(y)\ \delta(z)\ e^{-i\omega t}\right] \qquad (4A.1)$$

where $\underline{e}_x$ is the unit vector along the x axis, and other symbols have already been defined in Chapter III. In this case, we also take the index of refraction, n, to be 1 in the upper half-space ($z > 0$) and greater than 1 in the lower half-space ($z < 0$).

By following the procedure we used in dealing with the vertical dipole in Chapter III we see that the problem amounts to one of finding the solution to the following equation

$$\nabla \times \nabla \times \underset{\sim}{\Pi} - k^2 \underset{\sim}{\Pi} - \nabla(\nabla\cdot\underset{\sim}{\Pi}) = \frac{i\omega\mu}{k^2}\ I_0\ \delta(x)\ \delta(y)\ \delta(z)\ \underline{e}_x\ , \qquad (4A.2)$$

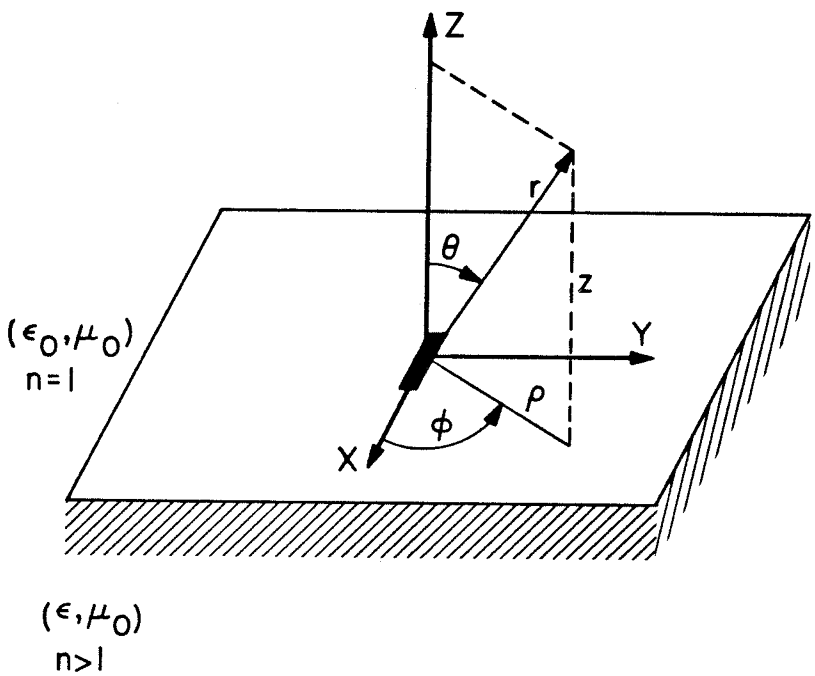

Fig. 13. The infinitesimal electric dipole is located on the interface and is directed parallel to x axis. In the upper half-space ($z > 0$) the dielectric constant is ε_0 and in the lower half-space ($z < 0$) the dielectric constant is ε which is greater than ε_0.

where $k^2 = k_o{}^2 = \omega^2\mu_o\varepsilon_o$ for the upper half-space ($z > 0$) and $k^2 = n^2k_o{}^2$ for the subsurface region ($z < 0$). We again denote $\underset{\sim}{\Pi}$ in the upper half-space by $\underset{\sim}{\Pi}_1$ and $\underset{\sim}{\Pi}$ in the lower half-space by $\underset{\sim}{\Pi}_2$.

To satisfy the boundary conditions along the interface, both the x and z components of $\underset{\sim}{\Pi}$ are necessary (see Appendix F). Accordingly, we take

$$\underset{\sim}{\Pi} = \Pi_x \underline{e}_x + \Pi_z \underline{e}_z \ . \tag{4A.3}$$

It can be shown that the boundary conditions for the Π_x along the $z = 0$ are

$$\Pi_{1x} = n^2 \Pi_{2x} \tag{4A.4a}$$

$$\frac{\partial \Pi_{1x}}{\partial z} = n^2 \frac{\partial \Pi_{2x}}{\partial z} \ , \tag{4A.4b}$$

and the boundary conditions for the Π_z along $z = 0$ are

$$\Pi_{1z} = n^2 \Pi_{2z} \tag{4A.5a}$$

$$\frac{\partial \Pi_{1z}}{\partial z} - \frac{\partial \Pi_{2z}}{\partial z} = \frac{\partial \Pi_{2x}}{\partial x} - \frac{\partial \Pi_{1x}}{\partial x} \ . \tag{4A.5b}$$

(See Appendix F). We get from (4A.2) the equations for Π_x and Π_z:

$$\nabla^2\Pi_x + k^2\Pi_x = \frac{-i\omega\mu I_o}{k^2}\ \delta(x)\ \delta(y)\ \delta(z)\ , \tag{4A.6}$$

$$\nabla^2\Pi_z + k^2\Pi_z = 0\ , \tag{4A.7}$$

where $\mu = \mu_o$ everywhere, $k^2 = k_o{}^2 = \omega^2\mu_o\varepsilon_o$ for the upper half-space ($z > 0$) and $k^2 = n^2k_o{}^2$ for the subsurface region ($z < 0$). Following the procedure we used in dealing with the vertical dipole in Chapter III and considering the boundary conditions (4A.4a) through (4A.5b) we obtain that the solutions of Π_{1x}, Π_{1z}, Π_{2x}, and Π_{2z} are [31],

$$\Pi_{1x} = \frac{i\ I_o}{4\pi\omega\varepsilon_o} \int_W \frac{h \exp[-\sqrt{h^2-k_o{}^2}\ z]}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}}\ H_o^{(1)}(h\rho)\ dh\ , \tag{4A.8}$$

$$\Pi_{1z} = \frac{-i\ I_o}{4\pi\omega\,\varepsilon_o k_o{}^2} \cos\phi \int_W \frac{h^2(\sqrt{h^2-k_o{}^2} - \sqrt{h^2-n^2k_o{}^2})\exp[-\sqrt{h^2-k_o{}^2}\,z]}{n^2\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}} \cdot H_1^{(1)}(h\rho)\ dh, \tag{4A.9}$$

$$\Pi_{2x} = \frac{i\ I_o}{4\pi\omega n^2\varepsilon_o} \int_W \frac{h \exp[\sqrt{h^2-n^2k_o{}^2}\ z]}{\sqrt{h^2-k_o{}^2} + \sqrt{h^2-n^2k_o{}^2}}\ H_o^{(1)}(h\rho)\ dh\ , \tag{4A.10}$$

$$\Pi_{2z} = \frac{-\, i\, I_o}{4\pi\omega\varepsilon_o n^2 k_o^2} \cos\phi \int_W \frac{h^2\left(\sqrt{h^2-k_o^2} - \sqrt{h^2-n^2k_o^2}\right) \exp\left[\sqrt{h^2-n^2k_o^2}\, z\right]}{n^2 \sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}} \cdot H_1^{(1)}(h\rho)\, dh \qquad (4A.11)$$

where $H_1^{(1)}$ is the first order Hankel function of the first kind, h is a complex variable and W is the path of integration which we used in dealing with the vertical dipole.

An exact evaluation of Π_x is possible for points along the interface (z = 0) and yields

$$\Pi_x = \Pi_{x1} = \Pi_{x2} = \frac{I_o}{2\pi(n^2-1)\omega\varepsilon_o\rho} \left[h_1^{(1)}(k_o\rho) - n^2 h_1^{(1)}(nk_o\rho)\right], \qquad (4A.12)$$

where $h_1^{(1)}$ is the first order spherical Hankel function of the first kind. (See Appendix G). To have a description of Π_x and Π_z, we must evaluate the integrals asymptotically.

IVB. Π_x and Π_z in the Upper Half-Space

To evaluate integrals (4A.8), (4A.9) asymptotically, we use the spherical coordinate system where $\rho = r\sin\theta$ and $z = r\cos\theta$. Accordingly, we get

$$\Pi_{1x} = \frac{i\, I_o}{4\pi\omega\varepsilon_o} \int_W \frac{h \exp\left[-\sqrt{h^2-k_o^2}\, r\cos\theta\right]}{\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}}\, H_o^{(1)}(hr\sin\theta)\, dh\,, \qquad (4B.1)$$

$$\Pi_{1z} = \frac{-\,i\,I_o}{4\pi\omega\varepsilon_o k_o^2}\cos\phi \int_W \frac{h^2\left(\sqrt{h^2-k_o^2} - \sqrt{h^2-n^2k_o^2}\right)\exp\left[-\sqrt{h^2-k_o^2}\,r\cos\theta\right]}{n^2\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}} \cdot H_1^{(1)}(hr\sin\theta)\,dh \qquad (4B.2)$$

for $0 \leq \theta \leq \frac{\pi}{2}$. The integrals along the path W can be reduced to the following integrals over the interval $0 \leq h \leq \infty$; that is

$$\Pi_{1x} = \frac{i\,I_o}{2\pi\omega\varepsilon_o}\int_0^\infty \frac{h\exp\left[-\sqrt{h^2-k_o^2}\,r\cos\theta\right]}{\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}}\,J_o\,(hr\sin\theta)\,dh\ , \qquad (4B.3)$$

$$\Pi_{1z} = \frac{-\,i\,I_o}{2\pi\omega\varepsilon_o k_o^2}\cos\phi \int_0^\infty \frac{h^2\left(\sqrt{h^2-k_o^2} - \sqrt{h^2-n^2k_o^2}\right)\exp\left[-\sqrt{h^2-k_o^2}\,r\cos\theta\right]}{n^2\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}} \cdot J_1(hr\sin\theta)\,dh \qquad (4B.4)$$

for $0 \leq \theta \leq \frac{\pi}{2}$, where J_o and J_1 are the zeroth and first order Bessel functions respectively. Here the path of integration is the real axis of h from 0 to ∞ and is indented from below the branch points at $h = k_o$ and $h = nk_o$. We invoke the integral representation of J_1 [32], i.e.

$$J_1(hr\sin\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[ihr\sin\theta\sin\beta - i\beta]\,d\beta\ . \qquad (4B.5)$$

By substituting (3B.3) and (4B.5) into (4B.3) and (4B.4) we find

$$\Pi_{1x} = \frac{+\, i\, I_o}{4\pi^2 \omega \varepsilon_o} \int_0^{\infty} \int_{-\pi}^{\pi} \frac{h \exp[-\sqrt{h^2-k_o^2}\, r\cos\theta + ihr\sin\theta\sin\beta]}{\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}}\, dh d\beta \,, \tag{4B.6}$$

$$\Pi_{1z} = \frac{-\, i\, I_o}{4\pi^2 \omega \varepsilon_o k_o^2} \cos\phi \int_0^{\infty} \int_{-\pi}^{\pi} \frac{h^2(\sqrt{h^2-k_o^2} - \sqrt{h^2-n^2k_o^2})}{n^2\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}} \times \tag{4B.7}$$

$$\cdot \exp[-\sqrt{h^2-k_o^2}\, r\cos\theta + ihr\sin\theta\sin\beta - i\beta]\, dh d\beta$$

for $0 \leq \theta \leq \frac{\pi}{2}$. Each of the above integrals can be divided into two integrals, viz.

$$\Pi_{1x} = \int_0^{k_o} \int_{-\pi}^{\pi} u(h,\beta)\, dh d\beta + \int_{k_o}^{\infty} \int_{-\pi}^{\pi} u(h,\beta)\, dh d\beta \,, \tag{4B.8}$$

$$\Pi_{1z} = \int_0^{k_o} \int_{-\pi}^{\pi} w(h,\beta)\, dh d\beta + \int_{k_o}^{\infty} \int_{-\pi}^{\pi} w(h,\beta)\, dh d\beta \,. \tag{4B.9}$$

In the first integrals, which we denote by F_{1x} and F_{1z}

$$F_{1x} = \frac{-\, I_o}{4\pi^2 \omega \varepsilon_o} \int_0^{k_o} \int_{-\pi}^{\pi} \frac{h \exp[i\sqrt{k_o^2-h^2}\, r\cos\theta + ihr\sin\theta\sin\beta]}{\sqrt{k_o^2-h^2} + \sqrt{n^2k_o^2-h^2}}\, dh d\beta \,, \tag{4B.10}$$

$$F_{1z} = \frac{-iI_o}{4\pi^2\omega\varepsilon_o k_o^2} \cos\phi \int_0^{k_o}\int_{-\pi}^{\pi} \frac{h^2(\sqrt{k_o^2-h^2} - \sqrt{n^2k_o^2-h^2})}{n^2\sqrt{k_o^2-h^2} + \sqrt{n^2k_o^2-h^2}} \quad .$$

$$\exp[i\sqrt{k_o^2-h^2}\, r\cos\theta + ihr\sin\theta\sin\beta - i\beta]\, dh d\beta \qquad (4B.11)$$

we introduce the variable α by $\sin\alpha = \frac{h}{k_o}$, and thus from (4B.10) and (4B.11) obtain

$$F_{1x} = \frac{-I_o k_o}{4\pi^2\omega\varepsilon_o} \int_0^{\frac{\pi}{2}}\int_{-\pi}^{\pi} \frac{\sin\alpha\cos\alpha\exp[ik_o r(\sin\alpha\sin\theta\sin\beta + \cos\alpha\cos\theta)]}{\cos\alpha + \sqrt{n^2-\sin^2\alpha}}\, d\alpha d\beta \,,$$

(4B.12)

$$F_{1z} = \frac{-iI_o k_o}{4\pi^2\omega\varepsilon_o} \cos\phi \int_0^{\frac{\pi}{2}}\int_{-\pi}^{\pi} \frac{\sin^2\alpha\cos\alpha(\cos\alpha - \sqrt{n^2-\sin^2\alpha})}{n^2\cos\alpha + \sqrt{n^2-\sin^2\alpha}} \quad .$$

$$\exp[ik_o r(\sin\alpha\sin\theta\sin\beta + \cos\alpha\cos\theta) - i\beta] d\alpha d\beta \quad .$$

(4B.13)

We notice that in (4B.10) and (4B.11) the factor $\sqrt{h^2-k_o^2}$ has been replaced by $(-i)\sqrt{k_o^2-h^2}$ for the subrange $0 \leq h \leq k_o$, because the solutions (4A.8) and (4A.9) must satisfy the radiation condition. Applying the theory of asymptotic expansions of the double integrals, we find that the critical point of the first kind

$$\begin{cases} \beta = \frac{\pi}{2} \\ \alpha = \theta \end{cases} \tag{4B.14}$$

lies in the interval of integration. Hence, the integrals F_{1x} and F_{1z} in the far-zone of the upper half-space are

$$F_{1x} \sim \frac{iI_o}{2\pi\omega\varepsilon_o} \frac{\cos\theta}{\cos\theta + \sqrt{n^2-\sin^2\theta}} \frac{e^{ik_o r}}{r}, \tag{4B.15}$$

$$F_{1z} \sim \frac{iI_o}{2\pi\omega\varepsilon_o} \cos\phi\sin\theta\cos\theta \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \frac{e^{ik_o r}}{r}. \tag{4B.16}$$

Following the similar procedure and applying the same theory of asymptotic expansions to the second integrals of (4B.8) and (4B.9) which we deonte by

$$F_{2x} = \frac{iI_o}{4\pi^2\omega\varepsilon_o} \int_{k_o}^{\infty}\int_{-\pi}^{\pi} \frac{h\exp[-\sqrt{h^2-k_o^2}\, r\cos\theta + ihr\sin\theta\sin\beta]}{\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}}\, dh d\beta \tag{4B.17}$$

$$F_{2z} = \frac{-iI_o}{4\pi^2\omega\varepsilon_o k_o^{\,2}} \cos\phi \int_{k_o}^{\infty}\int_{-\pi}^{\pi} \frac{h^2(\sqrt{h^2-k_o^{\,2}} - \sqrt{h^2-n^2k_o^{\,2}})}{n^2\sqrt{h^2-k_o^{\,2}} + \sqrt{h^2-n^2k_o^{\,2}}} \quad .$$

$$\exp[-\sqrt{h^2-k_o^{\,2}}\, r\cos\theta + ihr\sin\theta\sin\beta - i\beta]\, dh d\beta \, . \tag{4B.18}$$

We see that the critical point of the first kind does not lie in the region of integration. Therefore the leading terms of the integrals F_{2x} and F_{2z} do not have the order $\frac{1}{k_o r}$ and they decay more rapidly than $\frac{1}{k_o r}$. As a result, the following expressions are the asymptotic expressions for Π_{1x} and Π_{1z} in the far-zone of the upper half-space,

$$\Pi_{1x} \sim \frac{i\ I_o}{2\pi\omega\varepsilon_o} \ \frac{\cos\theta}{\cos\theta + \sqrt{n^2-\sin^2\theta}} \ \frac{e^{ik_o r}}{r}, \tag{4B.19}$$

$$\Pi_{1z} \sim \frac{i\ I_o}{2\pi\omega\varepsilon_o} \cos\phi\sin\theta\cos\theta \ \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \ \frac{e^{ik_o r}}{r} \, . \tag{4B.20}$$

By substituting (4B.19) and (4B.20) into (3A.10) and (3A.11) we obtain the electric and magnetic fields in the far-zone of the upper half-space;

$$E_{1\theta} \sim \frac{i\, I_o k_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\left\{\frac{\cos^2\theta}{\cos\theta+\sqrt{n^2-\sin^2\theta}} - \sin^2\theta\cos\theta\,\frac{\cos\theta-\sqrt{n^2-\sin^2\theta}}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}\right\}\cdot\cos\phi\,\frac{e^{ik_o r}}{r}, \tag{4B.21}$$

$$E_{1\phi} \sim \frac{-\, i\, I_o k_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\,\frac{\cos\theta\sin\phi}{\cos\theta+\sqrt{n^2-\sin^2\theta}}\,\frac{e^{ik_o r}}{r}, \tag{4B.22}$$

$$H_{\theta 1} \sim \frac{i\, I_o k_o}{2\pi}\,\frac{\cos\theta\sin\phi}{\cos\theta+\sqrt{n^2-\sin^2\theta}}\,\frac{e^{ik_o r}}{r}, \tag{4B.23}$$

$$H_{\phi 1} \sim \frac{i\, I_o k_o}{2\pi}\left\{\frac{\cos^2\theta}{\cos\theta+\sqrt{n^2-\sin^2\theta}} - \sin^2\theta\cos\theta\,\frac{\cos\theta-\sqrt{n^2-\sin^2\theta}}{n^2\cos\theta+\sqrt{n^2-\sin^2\theta}}\right\}\cdot\cos\phi\,\frac{e^{ik_o r}}{r}. \tag{4B.24}$$

It can be seen that the far-zone electric and magnetic fields in the upper half-space satisfy the following relations

$$E_{1\theta} = \sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_{1\phi} \tag{4B.25}$$

$$E_{2\phi} = -\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_{1\theta} \tag{4B.26}$$

IVC. Π_x and Π_z in the Subsurface Region

In using a similar procedure and using asymptotic evaluation of double integrals, Π_{2x} and Π_{2z} in the far zone of the subsurface region can be written as follows:

$$\Pi_{2x} \sim \frac{-\,i\,I_o}{2\pi\omega n\varepsilon_o}\;\frac{\cos\theta}{\sqrt{1-n^2\sin^2\theta}-n\cos\theta}\;\frac{e^{ink_or}}{r}, \tag{4C.1}$$

$$\Pi_{2z} \sim \frac{-\,i\,I_o}{2\pi\omega n\varepsilon_o}\sin\theta\cos\theta\cos\phi\;\frac{\sqrt{1-n^2\sin^2\theta}+n\cos\theta}{n\sqrt{1-n^2\sin^2\theta}-\cos\theta}\;\frac{e^{ink_or}}{r} \tag{4C.2}$$

for $nk_or \to \infty$, and $\pi-\theta_c \le \theta \le \pi$ where $\sin\theta_c = \frac{1}{n}$; and

$$\Pi_{2x} \sim \frac{-\,I_o}{2\pi\omega n\varepsilon_o}\;\frac{\cos\theta}{\sqrt{n^2\sin^2\theta-1}+in\cos\theta}\;\frac{e^{ink_or}}{r}, \tag{4C.3}$$

$$\Pi_{2z} \sim \frac{-\,i\,I_o}{2\pi\omega n\varepsilon_o}\sin\theta\cos\theta\cos\phi\;\frac{\sqrt{n^2\sin^2\theta-1}-in\cos\theta}{n\sqrt{n^2\sin^2\theta-1}+i\cos\theta}\;\frac{e^{ink_or}}{r} \tag{4C.4}$$

for $nk_or \to \infty$, and $\frac{\pi}{2} \le \theta \le \pi - \theta_c$.

We obtain by substituting (4C.1), (4C.2), (4C.3), (4C.4) into (3A.10) and (3A.11), the electric and magnetic fields in the far-zone of the lower half-space,

$$E_{2\theta} \sim \frac{inI_ok_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\left\{\sin^2\theta\cos\theta\;\frac{\sqrt{1-n^2\sin^2\theta}+n\cos\theta}{n\sqrt{1-n^2\sin^2\theta}-\cos\theta} - \frac{\cos^2\theta}{\sqrt{1-n^2\sin^2\theta}-n\cos\theta}\right\}\cos\phi\;\frac{e^{ink_or}}{r}, \tag{4C.5}$$

$$E_{2\phi} \sim \frac{inI_o k_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\;\frac{\cos\theta\sin\phi}{\sqrt{1-n^2\sin^2\theta}-n\cos\theta}\;\frac{e^{ink_o r}}{r}, \tag{4C.6}$$

$$H_{2\theta} \sim \frac{-iI_o k_o n^2}{2\pi}\;\frac{\cos\theta\sin\phi}{\sqrt{1-n^2\sin^2\theta}-n\cos\theta}\;\frac{e^{ink_o r}}{r}, \tag{4C.7}$$

$$H_{2\phi} \sim \frac{iI_o k_o n^2}{2\pi}\left\{\sin^2\theta\cos\theta\,\frac{\sqrt{1-n^2\sin^2\theta}+n\cos\theta}{n\sqrt{1-n^2\sin^2\theta}-\cos\theta} - \frac{\cos^2\theta}{\sqrt{1-n^2\sin^2\theta}-n\cos\theta}\right\}\cos\phi\;\frac{e^{ink_o r}}{r} \tag{4C.8}$$

for $nk_o r \to \infty$, and $\pi - \theta_c \leq \theta \leq \pi$; and

$$E_{2\theta} \sim \frac{inI_o k_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\left\{\sin^2\theta\cos\theta\,\frac{\sqrt{n^2\sin^2\theta-1}-in\cos\theta}{n\sqrt{n^2\sin^2\theta-1}+i\cos\theta} + i\,\frac{\cos^2\theta}{\sqrt{n^2\sin^2\theta-1}+in\cos\theta}\right\}\cos\phi\;\frac{e^{ink_o r}}{r}, \tag{4C.9}$$

$$E_{2\phi} \sim \frac{nI_o k_o}{2\pi}\sqrt{\frac{\mu_o}{\varepsilon_o}}\;\frac{\cos\theta\sin\phi}{\sqrt{n^2\sin^2\theta-1}+in\cos\theta}\;\frac{e^{ink_o r}}{r}, \tag{4C.10}$$

$$H_{2\theta} \sim \frac{-I_o k_o}{2\pi}\;\frac{n^2\cos\theta\sin\phi}{\sqrt{n^2\sin^2\theta-1}+in\cos\theta}\;\frac{e^{ink_o r}}{r}, \tag{4C.11}$$

$$H_{2\phi} \sim \frac{i\, I_o k_o n^2}{2\pi} \left\{ \sin^2\theta\cos\theta \, \frac{\sqrt{n^2\sin^2\theta - 1} - in\cos\theta}{n\sqrt{n^2\sin^2\theta - 1} + i\cos\theta} + i \frac{\cos^2\theta}{\sqrt{n^2\sin^2\theta - 1} + in\cos\theta} \right\} \cos\phi \, \frac{e^{ink_o r}}{r}, \tag{4C.12}$$

for $nk_o r \to \infty$, and $\frac{\pi}{2} \le \theta \le \pi - \theta_c$.

We can see that the far-zone electric and magnetic fields in the lower half-space are related by the simple relations, i.e.

$$E_{2\theta} = \frac{1}{n}\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_{2\phi} \tag{4C.13}$$

$$E_{2\phi} = -\frac{1}{n}\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_{2\theta} \tag{4C.14}$$

IVD. Radiation Pattern

Since in the far-zone, we have two components for the electric field and two components for the magnetic field, we can write the Poynting vector as follows:

$$S_r = \frac{1}{2}\,\mathrm{Re}\,[E_\theta H_\phi^{*} - E_\phi H_\theta^{*}]\,. \tag{4D.1}$$

We find by substituting (4B.21), (4B.22), (4B.23) and (4B.24) into (4D.1) the Poynting vector in the upper half-space, i.e.

$$S_{r1} = \frac{I_o^2 k_o^2}{8\pi^2 r^2}\sqrt{\frac{\mu_o}{\varepsilon_o}} \left\{ \left[\frac{\cos^2\theta}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} - \sin^2\theta\cos\theta \, \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \right]^2 \cos^2\phi + \frac{\cos^2\theta\sin^2\phi}{(\cos\theta + \sqrt{n^2 - \sin^2\theta})^2} \right\} \tag{4D.2}$$

for $0 \leq \theta \leq \frac{\pi}{2}$, $0 \leq \phi \leq 2\pi$ and $k_o r \rightarrow \infty$. By substituting (4C.5), (4C.6), (4C.7) and (4C.8) into (4D.1) we get the Poynting vector S_{r2}

$$S_{r2} = \frac{I_o^2 k_o^2 n^3}{8\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \Bigg\{ \Bigg[\sin^2\theta\cos\theta \frac{\sqrt{1-n^2\sin^2\theta} + n\cos\theta}{n\sqrt{1-n^2\sin^2\theta} - \cos\theta} - \frac{\cos^2\theta}{\sqrt{1-n^2\sin^2\theta} - n\cos\theta} \Bigg]^2 \cos^2\phi + \frac{\cos^2\theta\sin^2\phi}{(\sqrt{1-n^2\sin^2\theta} - n\cos\theta)^2} \Bigg\} \tag{4D.3}$$

for $nk_o r \rightarrow \infty$, $0 \leq \phi \leq 2\pi$ and $\pi - \theta_c \leq \theta \leq \pi$; and by substituting (4C.9), (4C.10), (4C.11) and (4C.12) into (4D.1) we obtain

$$S_{r2} = \frac{I_o^2 k_o^2 n^3}{8\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \Bigg\{ \frac{(n^2-1)\sin^4\theta\cos^2\theta\cos^2\phi - 2\cos^2\phi\sin^2\theta\cos^4\theta}{n^2(n^2\sin^2\theta-1) + \cos^2\theta} + \frac{\cos^4\theta\cos^2\phi + \sin^2\phi\cos^2}{(n^2-1)} \Bigg\} \tag{4D.4}$$

for $nk_o r \rightarrow \infty$, $0 \leq \phi \leq 2\pi$ and $\frac{\pi}{2} \leq \theta \leq \pi - \theta_c$.

Since the Poynting vector is a function of θ and ϕ, we sketch, from (4D.2), (4D.3) and (4D.4), the radiation pattern of the horizontal infinitesimal electric dipole for three different values of ϕ (S_r versus θ, for the $\phi = 0, \frac{\pi}{4}$, and $\frac{\pi}{2}$) (Figs. 14a,b, 15a,b, 16a,b). At the interface ($\theta = \frac{\pi}{2}$), the radiation pattern is zero for all values of ϕ. In the upper half-space, the radiation pattern consists of a single lobe, the maximum of which lies along the line $\theta = 0$ and has the value

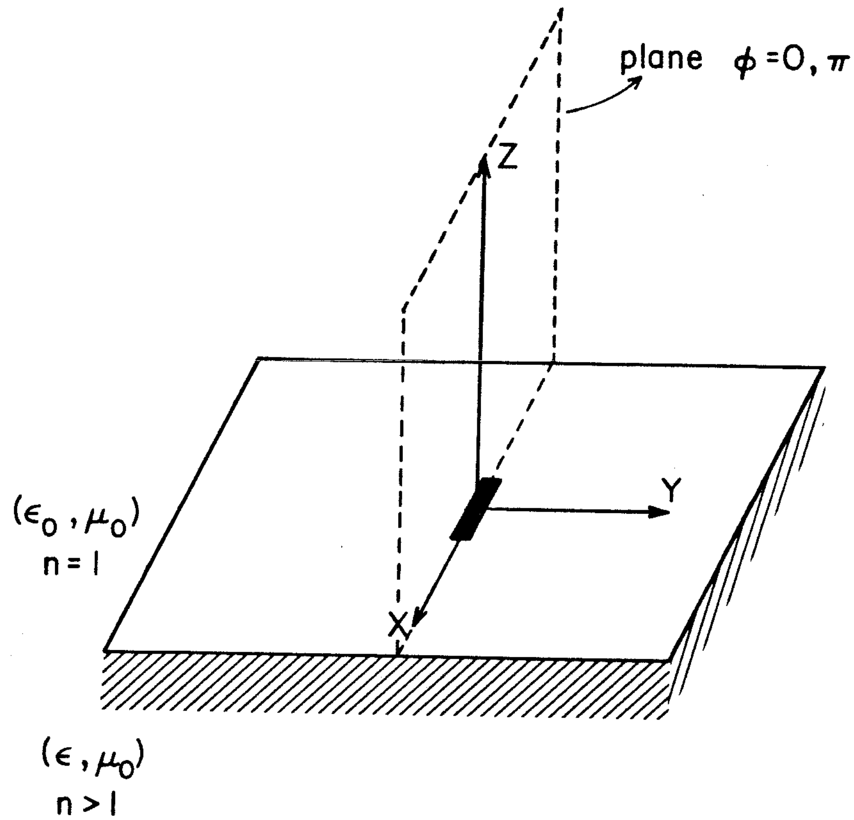

Fig. 14a. The mathematical plane normal to the interface and containing the dipole (plane $\phi = 0,\pi$).

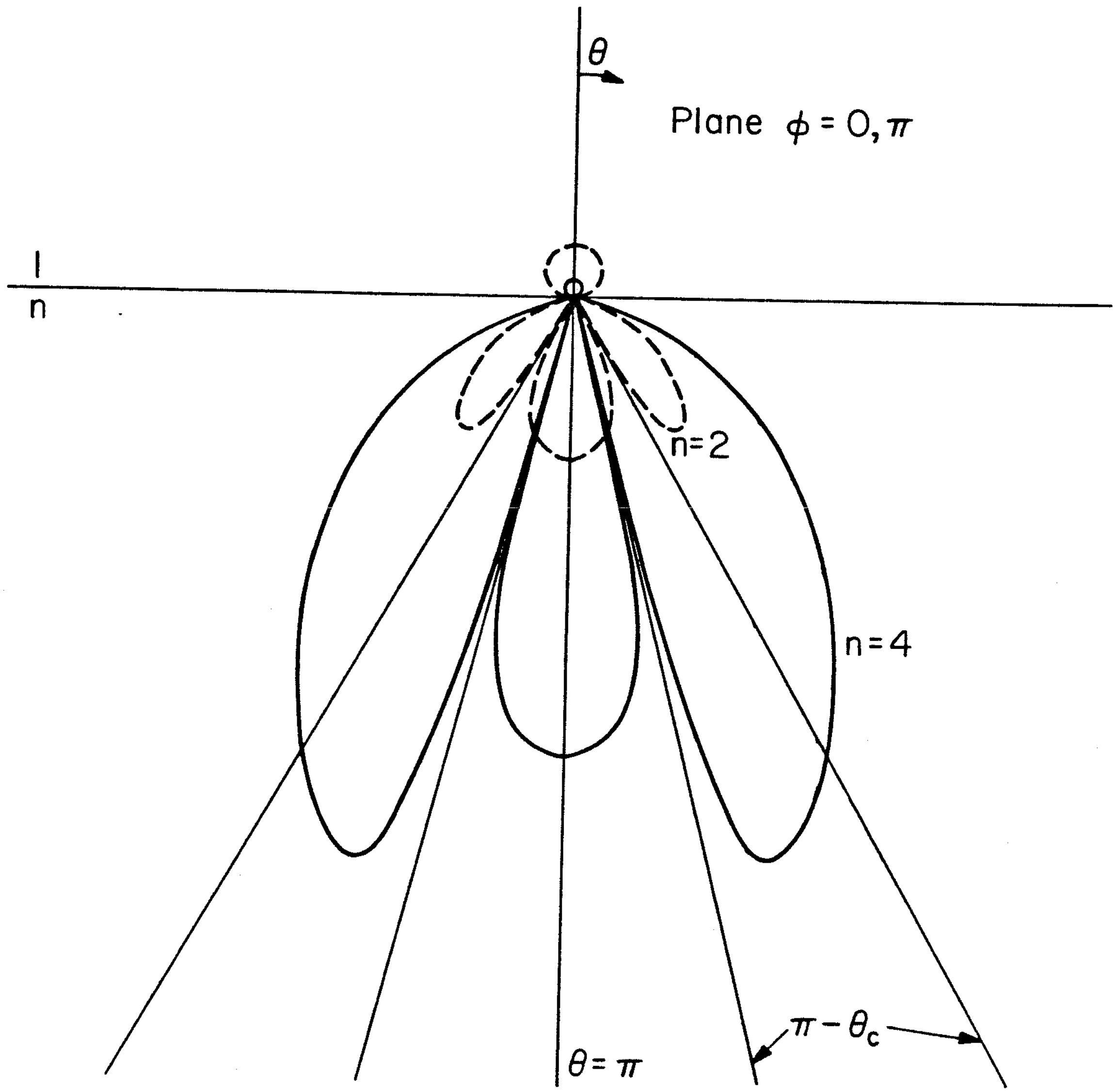

Fig. 14b. Radiation pattern of the horizontal dipole in the mathematical plane shown in Fig. 14a. For any value of n, there are nulls along the interface and along the angle $\theta = \pi - \theta_c$ in the plane $\phi = 0, \pi$. Here $\sin\theta_c = \frac{1}{4}$ (continuous line) and $\sin\theta_c = \frac{1}{2}$ (broken line).

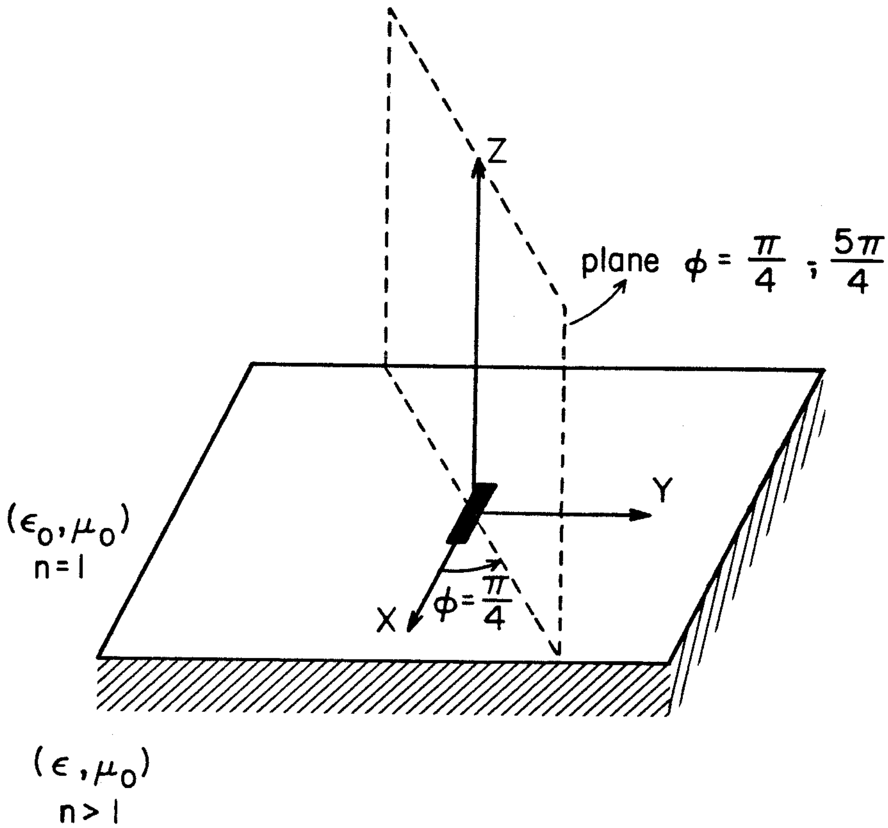

Fig. 15a. The mathematical plane normal to the interface and bisecting the dipole with angle $\phi = \frac{\pi}{4}$.

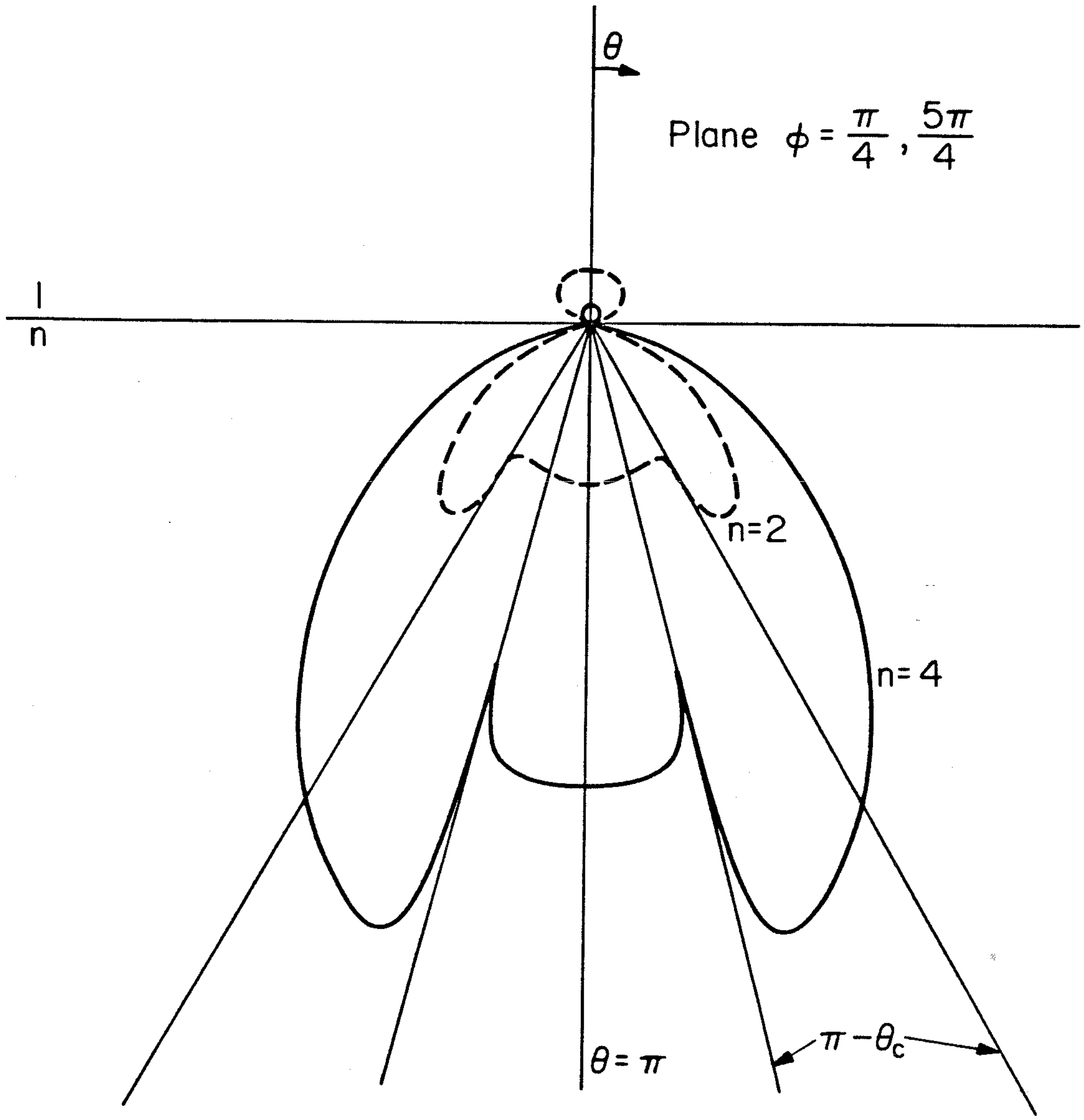

Fig. 15b. Radiation pattern of the horizontal dipole in the mathematical plane shown in Fig. 15a for n=4 (continuous line) and n=2 (broken line).

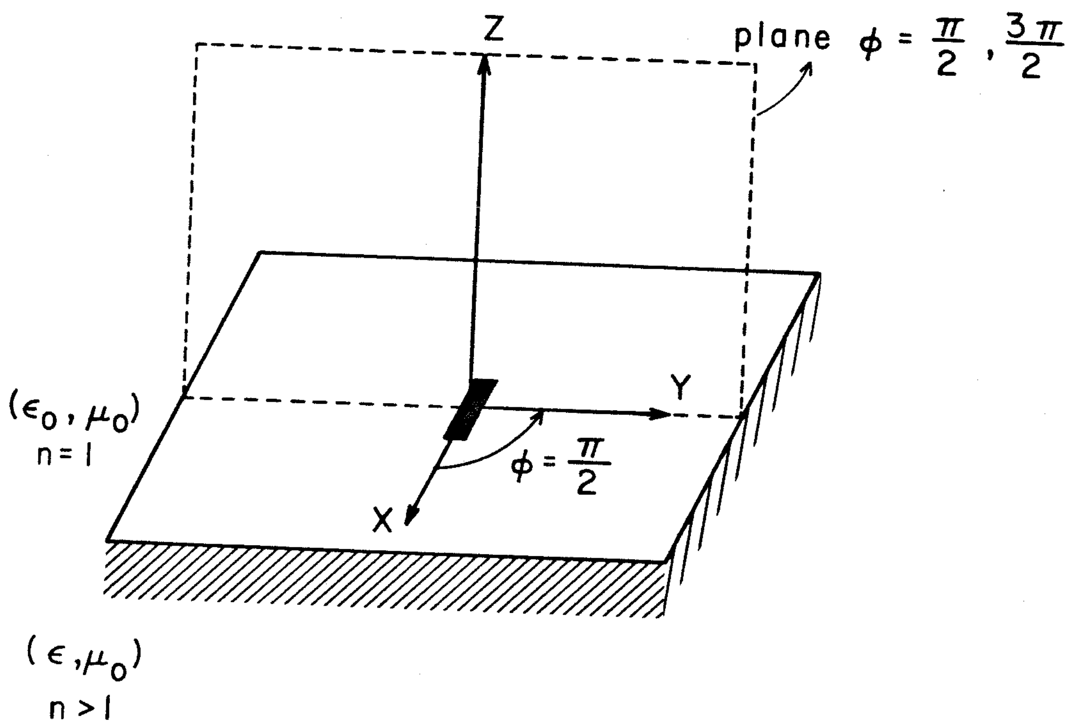

Fig. 16a. The mathematical plane normal to the interface and normally bisecting the dipole.

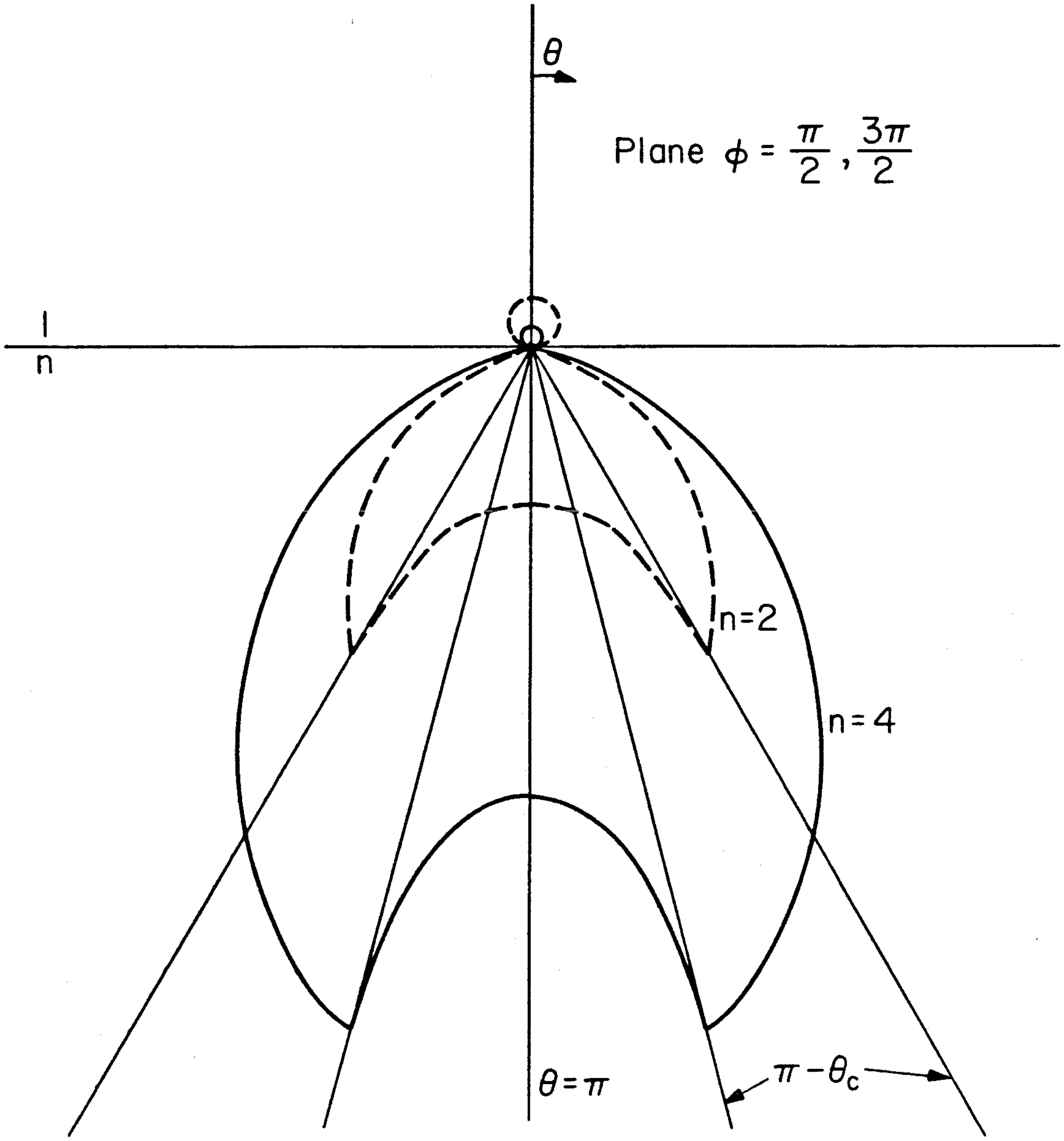

Fig. 16b. Radiation pattern of the horizontal dipole in the mathematical plane shown in Fig. 16a for n=4 (continuous line) and n=2 (broken line).

$$(S_{r1})_{peak} = \frac{I_o^2 k_o^2}{8\pi^2 r^2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \frac{1}{(1+n)^2} \ . \qquad (4D.5)$$

In the lower half-space, along the line $\theta = \pi$, S_{r2} has the value

$$(S_{r2})_{\theta=\pi} = \frac{I_o^2 k_o^2}{8\pi^2 r^2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\ \frac{n^3}{(1+n)^2} \ . \qquad (4D.6)$$

At the angle $\theta = \pi - \theta_c$, the radiation pattern has a strange characteristic. At $\theta = \pi - \theta_c$ and $\phi = 0$ and π the radiation pattern has a null. As ϕ increases from 0 to $\frac{\pi}{2}$, S_{r2} increases and at $\phi = \frac{\pi}{2}$, it has a maximum whose value is given by

$$(S_{r2})_{\theta=\pi-\theta_c\,,\ \phi=\frac{\pi}{2}} = \frac{I_o^2 k_o^2}{8\pi^2 r^2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\ n \qquad (4D.7)$$

As n increases, the radiation pattern in the upper half-space shrinks. In the lower half-space, as n increases, the cone whose vertex angle is $2\theta_c$ becomes more narrow.

IVE. Limiting Cases

Knowing the expressions for the electromagnetic field and the Poynting vector of the infinitesimal electric dipole which is lying horizontally along the interface of the two dielectric half-space, we like to show that if the index of refraction of the lower medium approaches 1, we will obtain the electromagnetic field and the Poynting vector of the infinitesimal electric dipole in free-space.

To do so, we find the limit of $E_{1\theta}$ and $E_{1\phi}$, when $n \to 1$. Therefore, from (4B.21) and (4B.22) we obtain

$$\lim_{n \to 1} E_{1\theta} = \frac{iI_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \cos\theta\cos\phi \, \frac{e^{ik_o r}}{r}, \tag{4E.1}$$

$$\lim_{n \to 1} E_{1\phi} = \frac{-iI_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin\phi \, \frac{e^{ik_o r}}{r}, \tag{4E.2}$$

and from (4C.5), (4C.6) we get

$$\lim_{n \to 1} E_{2\theta} = \frac{iI_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \cos\theta\cos\phi \, \frac{e^{ik_o r}}{r} \tag{4E.3}$$

$$\lim_{n \to 1} E_{2\phi} = \frac{-iI_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin\phi \, \frac{e^{ik_o r}}{r} \tag{4E.4}$$

which agree with the electromagnetic field that would be radiated by the infinitesimal electric dipole if it were in free-space.

We calculate the limit of the Poynting vector when n approaches 1. Therefore, from (4D.2) and (4D.3) we get

$$\lim_{n \to 1} S_{r1} = \frac{I_o^{\,2} k_o^{\,2}}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \, [\cos^2\theta\cos^2\phi + \sin^2\phi] , \tag{4E.5}$$

$$\lim_{n \to 1} S_{r2} = \frac{I_o^{\,2} k_o^{\,2}}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \, [\cos^2\theta \mathrm{cso}^2\phi + \sin^2\phi] . \tag{4E.6}$$

We know that the Poynting vector (S_{ro}) associated with an infinitesimal electric dipole directed along the x axis of a Cartesian coordinate system in free space is (see Appendix D)

$$S_{ro} = \frac{I_o^{\,2} k_o^{\,2}}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \, [\cos^2\theta\cos^2\phi + \sin^2\phi] . \tag{4E.7}$$

By comparing (4E.5), (4E.6), and (4E.7), we can see that

$$\lim_{n \to 1} S_{r1} = \lim_{n \to 1} S_{r2} = S_{ro} \tag{4E.8}$$

IVF. Radiated Power

Substituting (4D.2) into (3F.1), and (4D.3) and (4D.4) into (3F.4) we can find the time-average power radiated into the upper and lower half-spaces.

$$P_1 = \frac{I_o{}^2 k_o{}^2}{8\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \int_0^{\frac{\pi}{2}} \left\{ \left[\frac{\cos^2\theta}{\cos\theta + \sqrt{n^2-\sin^2\theta}} - \sin^2\theta\cos\theta \, \frac{\cos\theta - \sqrt{n^2-\sin^2\theta}}{n^2\cos\theta + \sqrt{n^2-\sin^2\theta}} \right]^2 \right.$$

$$\left. + \frac{\cos^2\theta}{(\cos\theta + \sqrt{n^2-\sin^2\theta})^2} \right\} \sin\theta d\theta \; , \tag{4F.1}$$

$$P_2 = \frac{I_o{}^2 k_o{}^2 n^3}{8\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \int_{\frac{\pi}{2}}^{\pi-\theta_c} \left\{ \frac{(n^2-1)\sin^4\theta\cos^2\theta - 2\sin^2\theta\cos^4\theta}{n^2(n^2\sin^2\theta-1) + \cos^2\theta} \right.$$

$$\left. + \frac{\cos^4\theta + \cos^2\theta}{(n^2-1)} \right\} \sin\theta d\theta$$

$$+ \frac{I_o{}^2 k_o{}^2 n^3}{8\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \int_{\pi-\theta_c}^{\pi} \left\{ \left[\sin^2\theta\cos\theta \, \frac{\sqrt{1-n^2\sin^2\theta} + n\cos\theta}{n\sqrt{1-n^2\sin^2\theta} - \cos\theta} \right. \right.$$

$$\left. \left. - \frac{\cos^2\theta}{\sqrt{1-n^2\sin^2\theta} - n\cos\theta} \right]^2 + \frac{\cos^2\theta}{(\sqrt{1-n^2\sin^2\theta} - n\cos\theta)^2} \right\} \sin\theta d\theta \, . \tag{4F.2}$$

Here we have the plots of P_1 and P_2 versus n. Fig. 17a and b illustrate the sketch of P_1 and P_2 versus n. As n increases from 1 to ∞, P_1

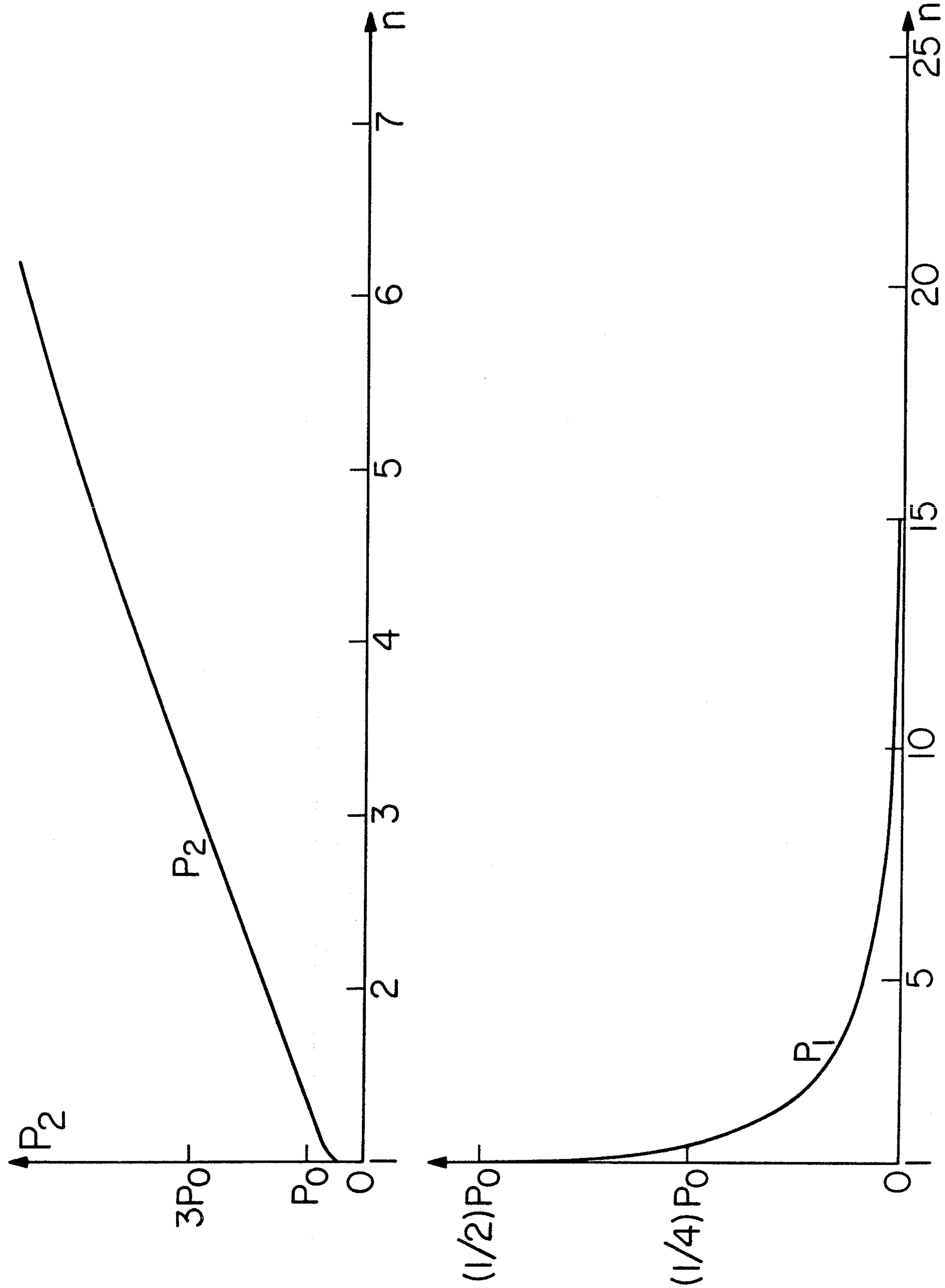

Fig. 17. Power of the horizontal dipole (a) P_1 radiated into the upper half-space, (b) P_2 radiated into the lower half-space.

smoothly decreases and P_2 increases. For any value of n, P_2 is always greater than P_1. This means that more power is radiated into the lower half-space (n > 1) than into the upper half-space (n = 1). As n approaches 1, both P_1 and P_2 approach $\frac{1}{2} P_0$ where $P_0 = \frac{I_0{}^2 k_0{}^2}{12\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}}$.

IVG. Physical remarks

The radiation pattern for the horizontal case in the upper half-space resembles the broadside radiation pattern of a tapered distribution of horizontal dipoles along the plane of the interface. However, the radiation pattern in the lower half-space appears to be a combination of two patterns, one being the pattern of a tapered distribution of dipoles lying in the plane of the interface and the other being the pattern of sources moving radially outward along the interface. It appears that the peaks and nulls of the subsurface region are generated by these moving sources. To explain the features of the radiation patterns we invoke the Cerenkov effect [33]. According to (4A.12) the moving sources have two different velocities given by the far-zone phase velocities of $h_1^{(1)}(k_0\rho)$ and $h_1^{(1)}(nk_0\rho)$, that is, by $V_1 = \frac{\omega}{k_0}$ and $V_2 = \frac{\omega}{nk_0}$. Neither V_1 nor V_2 is greater than the velocity of light in the upper half-space. However, V_1 is greater than the velocity c/n of light in the lower half-space. Therefore, these moving sources produce Cerenkov-like radiation in the lower half-space (in the conical direction $\theta = \pi - \theta_c$). For $\phi = \frac{\pi}{2}$, $\frac{3\pi}{2}$ and $\phi = 0, \pi$ this radiation respectively adds to and subtracts from the radiation due to the apparent tapered distribution of dipoles. Thus, there are peaks in the plane $\phi = \frac{\pi}{2}, \frac{3\pi}{2}$ and nulls in the plane $\phi = 0, \pi$ in the conical direction $\theta = \pi - \theta_c$.

IVH. Conclusions to Chapter IV

We have gotten the radiation pattern and emitted power of an infinitesimal electric dipole for the case where the dipole is lying horizontally along the interface.

For this horizontal dipole, in the upper half-space, the radiation pattern is a single lobe which resembles the radiation pattern of a tapered broadside array of horizontal dipoles; and the pattern in the subsurface region ($n > 1$) has three lobes in the plane normal to the interface and containing the dipole, whereas in the plane normal to the interface and normally bisecting the dipole the pattern has two maxima symmetrically located about a minimum.

As n increases, P_1, the power radiated into the upper half-space, decreases and P_2, the power radiated into the lower half-space, increases. For $n > 1$ P_2 is greater than P_1.

V. GENERAL CONCLUSIONS

Starting from the Maxwell equations we have obtained the radiation pattern and emitted power of an interfacial radiating source for the case where the source is an infinitely long line source lying along the plane interface of two dielectric half-spaces; for the case where the source is an infinitesimal electric dipole vertically located on the interface; and for the case where the dipole is lying horizontally along the interface.

For the infinitely long line source, from our calculation one can draw the following conclusions:

In the upper half-space where $n = 1$ the radiation pattern is a single lobe which resembles the radiation pattern of a tapered broadside array. Accordingly, from above one would "see" not a line source but a tapered broadside array. In the lower half-space where $n > 1$ the radiation pattern consists of two equal maxima symmetrically located about a minimum. At the interface itself the radiation pattern is zero. We have also shown that it is possible to describe these results in terms of ray optics, provided we take the position of the source to be a little below the interface but not exactly on it. Clearly, when $n = 1$ the power P_1 radiated into the upper half-space is equal to the power P_2 radiated into the lower half-space. However, as n increases, P_1 decreases, P_2 increases, and $P_1 + P_2$ remains constant. For $n > 1$ the line source radiates more power into the lower half-space than into the upper half-space.

For the case of the vertical dipole, the radiation pattern disappears along the interface and along the dipole axis; the pattern in the upper half-space has a maximum which in amplitude and direction depends on n; and the pattern in the lower half-space has a maximum which also depends on n. As n increases from 1 to ∞, P_2 increases monotonically, whereas P_1 first decreases and then increases and approaches $2P_0$ where P_0 is the time-average power that would be radiated by the dipole if it were in free-space. For $n > 1$, P_2 is greater than P_1.

For the case of the horizontal dipole, in the upper half-space, the radiation pattern is a single lobe which resembles the radiation pattern of a tapered broadside array of horizontal dipoles; and the pattern in the subsurface region ($n > 1$) has three lobes in the plane normal to the interface and containing the dipole, whereas in the plane normal to the interface and normally bisecting the dipole the pattern has two maxima symmetrically located about a minimum. As n increases, P_1 decreases and P_2 increases. For $n > 1$, P_2 is greater than P_1.

APPENDIX A

The Exact Evaluation of the Electric Field Along the Interface Due to the Interfacial Line Source

In this appendix, we exactly evaluate the electric field along the interface due to the line source lying along the interface of two dielectric half-spaces.

By substituting $y = 0$ into (2B.10a) or (2B.10b) we obtain

$$E_z = E_{z1} = E_{z2} = \frac{i\omega\mu_o I}{2\pi} \int_{-\infty}^{\infty} \frac{\exp(ihx)}{\sqrt{h^2-k_o^2} + \sqrt{h^2-n^2k_o^2}} \, dh \cdot \qquad (1)$$

Formula (1) can be written as follows:

$$E_z = E_{z1} = E_{z2} = \frac{i\omega\mu_o I}{2\pi(n^2-1)k_o^2} \int_{-\infty}^{\infty} \left(\sqrt{h^2-k_o^2} - \sqrt{h^2-n^2k_o^2}\right) \exp(ihx) \, dh. \qquad (2)$$

This integral can be divided into two integrals, viz.

$$E_z = \frac{i\omega\mu_o I}{2\pi(n^2-1)k_o^2} \int_{-\infty}^{\infty} \left(\sqrt{h^2-k_o^2}\right) \exp(ihx) \, dh$$

$$- \frac{i\omega\mu_o I}{2\pi(n^2-1)k_o^2} \int_{-\infty}^{\infty} \left(\sqrt{h^2-n^2k_o^2}\right) \exp(ihx) \, dh. \qquad (3)$$

In (3) we denote the first and second integrals by I_1 and I_2, respectively. In I_1 we divide the range of integration into three subranges that is, we write

$$I_1 = \int_{-\infty}^{-k_o} u(h)dh + \int_{-k_o}^{k_o} u(h)dh + \int_{k_o}^{\infty} u(h)dh, \qquad (4)$$

where u(h) is a shorthand for the integrand. We denote the integral for the middle subrange ($-k_o \leq h \leq k_o$) by I_1^{mid} and the first and third integrals by I_1^{α} and I_2^{β}, respectively.

In the integral

$$I_1^{mid} = \frac{\omega\mu_o I}{2\pi(n^2-1)k_o^2} \int_{-k_o}^{k_o} (\sqrt{k_o^2-h^2}) \exp(ihx)\, dh\, , \qquad (5)$$

by introducing the variable ℓ, which is defined by $\ell = h/k_o$, we get

$$I_1^{mid} = \frac{\omega\mu_o I}{2\pi(n^2-1)} \int_{-1}^{+1} \sqrt{1-\ell^2} \exp(ik_o x\ell)d\ell\ . \qquad (6)$$

As we notice, the factor $\sqrt{h^2-k_o^2}$ has been replaced by $(-i)\sqrt{k_o^2-h^2}$ for the subrange $-k_o \leq h \leq k_o$, because the solutions (2B.5a) and (2B.5b) must satisfy the radiation condition. Using the table of integrals [34], we find that

$$I_1^{mid} = \frac{\omega\mu_o I}{2(n^2-1)} \frac{1}{k_o x} J_1(k_o x). \qquad (7)$$

By introducing the variable $\ell = \frac{h}{k_o}$ in the integral

$$I_1^{\beta} = \frac{i\omega\mu_o I}{2\pi(n^2-1)k_o^{2}} \int_{k_o}^{\infty} \left(\sqrt{h^2-k_o^{2}}\right) \exp(ihx)\, dh\,, \qquad (8)$$

and $\ell = -\frac{h}{k_o}$ in the integral

$$I_1^{\alpha} = \frac{i\omega\mu_o I}{2\pi(n^2-1)k_o^{2}} \int_{-\infty}^{-k_o} \left(\sqrt{h^2-k_o^{2}}\right) \exp(ihx)\, dh\,, \qquad (9)$$

we get

$$I_1^{\alpha} = \frac{i\omega\mu_o I}{2\pi(n^2-1)} \int_{1}^{\infty} \sqrt{\ell^2-1}\, \exp(-ik_o x\ell) d\ell\,, \qquad (10)$$

$$I_1^{\beta} = \frac{i\omega\mu_o I}{2\pi(n^2-1)} \int_{1}^{\infty} \sqrt{\ell^2-1}\, \exp(ik_o x\ell) d\ell\,. \qquad (11)$$

Using the tables of integrals and functions [34] and [35], we obtain

$$I_1^{\alpha} = -\frac{\omega\mu_o I}{4(n^2-1)k_o x}\, H_1^{(2)}(k_o x)\,, \qquad (12)$$

$$I_1^{\beta} = \frac{\omega\mu_o I}{4(n^2-1)k_o x}\, H_1^{(1)}(k_o x)\,, \qquad (13)$$

where $H_1^{(1)}(k_o x)$ and $H_1^{(2)}(k_o x)$ are the Hankel functions of the first and second kind, respectively.

We find by substituting (7), (12), and (13) into (4) that

$$I_1 = \frac{\omega\mu_0 I}{2(n^2-1)} \frac{1}{k_0 x} H_1^{(1)} (k_0 x) \text{ (for } x > 0) . \tag{14}$$

From symmetry we can conclude that

$$I_1 = \frac{\omega\mu_0 I}{2(n^2-1)} \frac{1}{k_0|x|} H_1^{(1)} (k_0|x|) \text{ (for } y = 0, \text{ all } x). \tag{15}$$

By following a similar procedure, we get

$$I_2 = - \frac{\omega\mu_0 I}{2(n^2-1)} \frac{n}{k_0|x|} H_1^{(1)} (nk_0|x|) \text{ (for } y = 0, \text{ all } x). \tag{16}$$

Hence, the electric field along the interface ($y = 0$) becomes

$$E_z = \frac{\omega\mu_0 I}{2(n^2-1)k_0} \left[\frac{1}{|x|} H_1^{(1)} (k_0|x|) - \frac{n}{|x|} H_1^{(1)} (nk_0|x|) \right] \tag{17}$$

(for $y = 0$, all x).

By use of equation (2A.4) we can find $H\phi$ from a knowledge of E_z. Accordingly, we obtain

$$H\phi = \frac{i\, I}{2(n^2-1)} \left[\frac{n^2}{|x|} H_2^{(1)} (nk_0|x|) - \frac{1}{|x|} H_2^{(1)} (k_0|x|) \right] \tag{18}$$

(for $y = 0$, all x).

Appendix B

Asymptotic Evaluation of the Electric Field in the Upper Half-Space in the Problem of Interfacial Line Source

To find E_{z1} in the far-zone, we must use an asymptotic evaluation of integral (2B.10a). To do so, we divide the integral in (2B.10a) into three subranges. That is, we write

$$E_{z1} = \int_{-\infty}^{-k_o} f(h)dh + \int_{-k_o}^{k_o} f(h)dh + \int_{k_o}^{\infty} f(h)dh \tag{1}$$

where f(h) is a shorthand for the integrand. We denote the integral for the middle subrange ($-k_o \leq h \leq k_o$) by F_1^{mid} and the first and third integrals by F_1^{α} and F_1^{β}, respectively.

In the integral

$$F_1^{mid} = \frac{-\omega\mu_o I}{2\pi} \int_{-k_o}^{k_o} \frac{\exp[i\sqrt{k_o^2-h^2}y + ihx]}{\sqrt{k_o^2-h^2} + \sqrt{n^2k_o^2-h^2}} dh, \quad (y > 0) \tag{2}$$

we introduce the variable α, which is defined by $\sin\alpha = \frac{h}{k_o}$, and use $x = \rho\cos\phi$ and $y = \rho\sin\phi$. Therefore we obtain

$$F_1^{mid} = \frac{\omega\mu_o I}{2\pi(n^2-1)} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\cos\alpha - \sqrt{n^2-\sin^2\alpha})\cos\alpha \exp[ik_o\rho\sin(\alpha+\phi)]d\alpha \tag{3}$$

for $(0 \leq \phi \leq \pi)$.

Applying the method of stationary phase [36], we find that the stationary point is given by $\alpha = \frac{\pi}{2} - \phi$ and lies in the interval $\frac{-\pi}{2} \leq \alpha \leq \frac{\pi}{2}$. Hence, for $k_o\rho \to \infty$, the leading term of F_1^{mid} is

$$F_1^{mid} \sim \frac{\omega\mu_o I}{\sqrt{2\pi}} \frac{1}{(n^2 - 1)} (\sin^2\phi - \sin\phi\sqrt{n^2 - \cos^2\phi}) \frac{e^{ik_o\rho - i\frac{\pi}{4}}}{\sqrt{k_o\rho}} \qquad (4)$$

$$(\text{for } k_o\rho \to \infty) .$$

In the integral

$$F_1^{\beta} = \frac{i\omega\mu_o I}{2\pi} \int_{k_o}^{\infty} \frac{\exp[-\sqrt{h^2 - k_o^{\ 2}}\, y + ihx]}{\sqrt{h^2 - k_o^{\ 2}} + \sqrt{h^2 - n^2 k_o^{\ 2}}} dh, \ (y > 0) \qquad (5)$$

by introducing the variable s, which is defined by $\cosh s = \frac{h}{k_o}$, we obtain

$$F_1^{\beta} = \frac{i\omega\mu_o I}{2\pi(n^2 - 1)} \int_0^{\infty} (\sinh s - \sqrt{\cosh^2 s - n^2}) \sinh s \exp[ik_o\rho\cosh(s + i\phi)] ds \qquad (6)$$

The saddle points of the integrand are $s = -i\phi \pm p\pi$ (p is an integer). In using the steepest descent method [36], we deform the integration path, which runs from 0 to ∞ along the real axis of s, to one which does not cross any branch cut and consists of two parts: one is along the imaginary axis of s; and the other one passes through one of the saddle points and approximates the constant-phase contour near that saddle point. From the study of the steepest descent method, for

$k_o\rho \to \infty$, we find that the leading term does not have the order of $\frac{1}{\sqrt{k_o\rho}}$. It will decay more rapidly than $\frac{1}{\sqrt{k_o\rho}}$. Consequently, for $k_o\rho \to \infty$ the integral F_1^{β} is negligibly small compared to the integral F_1^{mid}. Using a similar method for F_1^{α}, we find that for $k_o\rho \to \infty$ the integral F_1^{α} is also negligible compared to F_1^{mid}. Therefore, we obtain

$$E_{z1} \sim \frac{\omega\mu_o I}{\sqrt{2\pi}} \, \frac{1}{(n^2-1)} \left(\sin^2\phi - \sin\phi\sqrt{n^2-\cos^2\phi}\right) \frac{e^{ik_o\rho - i\frac{\pi}{4}}}{\sqrt{k_o\rho}} \qquad (7)$$

for $k_o\rho \to \infty$ and $0 \leq \phi \leq \pi$.

Appendix C

Asymptotic Evaluation of the Electric Field in the Lower Half-Space in the Problem of Interfacial Line Source

To find the electric field in the lower half-space ($-\pi \leq \phi \leq 0$), we use an approach similar to one we used in the Appendix B.

In the integral representation of E_{z2} (2B·10b) we divide the range of integration into five subranges:

$$E_{z_2} = \int_{-\infty}^{-nk_o} g(h)dh + \int_{-nk_o}^{-k_o} g(h)dh + \int_{-k_o}^{k_o} g(h)dh + \int_{k_o}^{nk_o} g(h)dh + \int_{nk_o}^{\infty} g(h)dh. \tag{1}$$

Here g(h) is a shorthand for the integrand. Using a similar procedure, we see that for $nk_o\rho \to \infty$ the first and fifth integrals are negligibly small compared to the other three integrals for all values of ϕ in the lower half-space ($-\pi \leq \phi \leq 0$). By introducing the variable α, defined by $\sin\alpha = h/nk_o$ and by using $x = \rho\cos\phi$ and $y = \rho\sin\phi$, we can invoke the method of stationary phase to evaluate the second, third, and fourth integrals for $nk_o\rho \to \infty$. To have a leading term of order $\frac{1}{\sqrt{nk_o\rho}}$ for $nk_o\rho \to \infty$, the stationary point must lie in the interval of integration [38]. Therefore, if ϕ lies in the interval $-\pi \leq \phi \leq -\pi + \phi_c$ or $-\phi_c \leq \phi \leq 0$ where $\cos\phi_c = \frac{1}{n}$, the leading term will come from the second or fourth integral, respectively. However, if ϕ lies in the interval $-\pi + \phi_c \leq \phi \leq -\phi_c$ the leading term will come from the third integral. Therefore, we get

$$E_{z_2} \sim -\frac{\omega\mu_o I}{\sqrt{2\pi}} \frac{n}{(n^2 - 1)} (n \sin^2 \phi + \sin \phi \sqrt{1-n^2\cos^2\phi}) \frac{e^{ink_o\rho - i\pi/4}}{\sqrt{nk_o\rho}}, \quad (2)$$

for the dihedral region ($nk_o\rho \to \infty$ and $-\pi + \phi_c \leq \phi \leq -\phi_c$); and

$$E_{z_2} \sim -\frac{\omega\mu_o I}{\sqrt{2\pi}} \frac{n}{(n^2 - 1)} (n \sin^2\phi + i \sin\phi\sqrt{n^2\cos^2\phi-1}) \frac{e^{ink_o\rho - i\pi/4}}{\sqrt{nk_o\rho}} \quad (3)$$

for the other two sectors of the lower half-space ($nk_o\rho \to \infty$, $-\phi_c \leq \phi \leq 0$, $-\pi \leq \phi \leq -\pi + \phi_c$).

Appendix D

Far-Zone Electric Fields and Poynting Vectors of a Radiating Line Source and Infinitesimal Electric Dipole in Free-Space

1. Radiating Line Source in Free-Space

We consider an infinitely long straight wire in free-space which lies along the z axis of a Cartesian coordinate system. We also introduce a cylindrical coordinate system ρ, ϕ, z where $x = \rho\cos\phi$, and $y = \rho\sin\phi$. (Fig. 18a).

We intend to solve the Helmholtz wave equation in free-space with the source $\underline{J}$ which can be expressed by

$$\underline{J} = \underline{e}_z I\delta(x)\delta(y)e^{-i\omega t} . \tag{1}$$

Accordingly, we can write from (2A.13) that

$$\nabla \times \nabla \times \underset{\sim}{E} - k_0{}^2\underset{\sim}{E} = i\omega\mu_0 I\delta(x)\delta(y) . \tag{2}$$

Since $\rho = 0$, we can write

$$\nabla \cdot (\varepsilon_0 \underset{\sim}{E}) = 0 \quad . \tag{3}$$

ε_0 is a constant. Thus, it follows that

$$\nabla \cdot \underset{\sim}{E} = 0 . \tag{4}$$

From (2), (4) it follows that

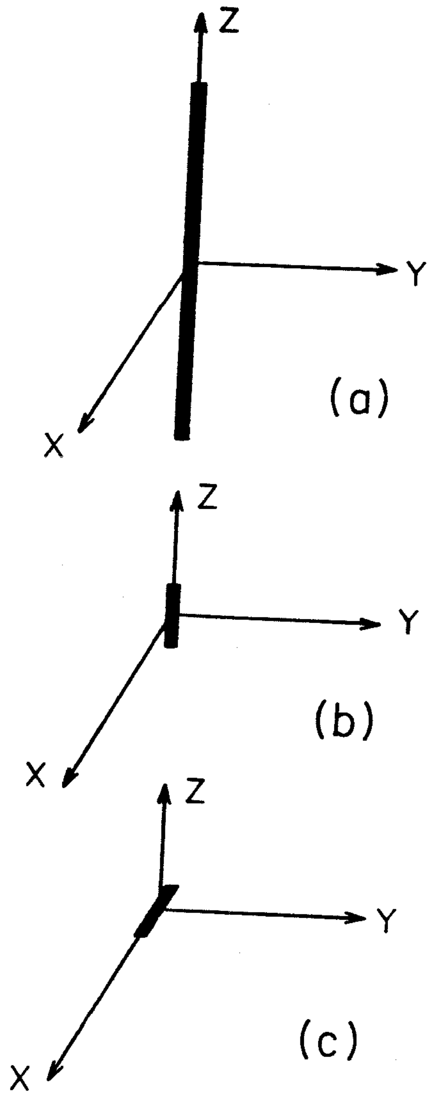

Fig. 18. (a) infinitely long line source in free-space, (b) and (c) infinitesimal electric dipole in free-space directed parallel to the z and x axes respectively.

$$\nabla^2 \underset{\sim}{E} + k_o{}^2 \underset{\sim}{E} = -i\omega\mu_o I\delta(x)\delta(y)\underline{e}_z \cdot \tag{5}$$

From the symmetry of the configuration, we can see that $E_z(\rho)$ and $H\phi(\rho)$ are the only components of the electromagnetic field of a line source in free-space. Thus

$$\nabla^2 E_z + k_o{}^2 E_z = -i\omega\mu_o I\delta(x)\delta(y) \tag{6}$$

we know that the Green's function for the two-dimensional wave equation, which is

$$(\nabla^2 + k_o{}^2)\, G(\underline{\rho}-\underline{\rho}') = -\delta(\underline{\rho}-\underline{\rho}')\ , \tag{7}$$

can be expressed by

$$G(\rho-\rho') = \frac{i}{4} H_o{}^{(1)}(k_o|\underline{\rho}-\underline{\rho}'|) \tag{8}$$

By comparing (6) and (8), we obtain

$$E_z(\rho) = \frac{-\omega\mu_o I}{4} H_o{}^{(1)}(k_o\rho)\ . \tag{9}$$

For the far-zone, we use the far-zone asymptotic expression of the zeroth order Hankel function of the first kind, which can be written as

$$H_o{}^{(1)}(k_o\rho) \sim \sqrt{\frac{2}{\pi k_o\rho}}\ e^{i(k_o\rho-\pi/4)}$$

$$k_o\rho >> 1 \tag{10}$$

Therefore, from (9) and (10) it follows that

$$E_z(\rho) = \frac{-\omega\mu_0 I}{2\sqrt{2\pi k_0 \rho}} \; e^{i(k_0\rho - \pi/4)} \tag{11}$$

which is the far-zone electric field that would be radiated by a line source if it were in free-space.

To find the Poynting vector, we substitute (11) into (2C.17). Thus we obtain

$$S\rho_0 = \frac{\omega\mu_0 I^2}{16\pi\rho}. \tag{12}$$

The time-average power radiated into free-space by a line source can be found as follows

$$P = \int_0^{2\pi} S\rho_0 \rho d\phi = \frac{\omega\mu_0 I^2}{8} \tag{13}$$

2. Radiating Infinitesimal Electric Dipole in Free-Space

2a. Dipole is Parallel to the z Axis

We consider an infinitesimal electric dipole located at the origin of a Cartesian coordinate system and directed parallel to the z axis. This dipole is in free-space (see Fig. 18b). The exact expression of the electric field of this dipole with the electric dipole moment P_z is calculated by Papas [38]. By introducing a spherical coordinate system r, θ, ϕ, we can write the θ-component of the electric field of this dipole, i.e.

$$E_\theta = -\frac{1}{\varepsilon} P_z \sin\theta \left(\frac{ik_o}{r} - \frac{1}{r^2} + k_o^{\,2}\right) \frac{e^{ik_o r}}{4\pi r} \quad . \tag{14}$$

For the far-zone ($k_o r >> 1$), the above expression approximates to

$$E_\theta \sim -\frac{k_o^{\,2} P_z}{4\pi\varepsilon} \frac{e^{ik_o r}}{r} \sin\theta \quad . \tag{15}$$

We know the relation among the electric dipole moment P_z, the total current i, the length of the dipole $\Delta\ell$. i.e.

$$\underline{P} = \underline{e}_z \frac{i}{\omega} i\Delta\ell \ . \tag{16}$$

$i\Delta\ell$ is already denoted by I_o. Therefore, by substituting $i\Delta\ell$ by I_o in (16), we obtain

$$P_z = \frac{i}{\omega} I_o \ . \tag{17}$$

From (15) and (17), we find

$$E_\theta \sim -\frac{ik_o I_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{e^{ik_o r}}{r} \sin\theta \tag{18}$$

$k_o r >> 1$

The above expression is the far-zone electric field of an infinitesimal electric dipole parallel to the z axis in free-space.

To find the Poynting vector, we substitute (18) into (2C.17), thus we obtain

$$S_{ro} = \frac{I_o^2 k_o^2}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin^2\theta \ . \qquad (19)$$

The time-average power radiated into free-space by an infinitesimal electric dipole can be obtained as follows

$$P_o = \int_0^{2\pi}\int_0^{\pi} S_{ro}\, r^2 \sin\theta d\theta d\phi = \frac{I_o^2 k_o^2}{12\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \qquad (20)$$

2b. Dipole is Parallel to the x Axis

In this case, we have the infinitesimal electric dipole lying parallel to the x axis of the Cartesian coordinate system (see Fig. 18c). Following the similar procedure, we can write the electric field expression of this dipole with electric dipole moment P_x[38], i.e.

$$E_\theta = \frac{1}{\varepsilon} P_x \cos\theta\cos\phi \left(\frac{ik_o}{r} - \frac{1}{r^2} + k_o^2\right) \frac{e^{ik_o r}}{4\pi r}, \qquad (21)$$

$$E_\phi = -\frac{1}{\varepsilon} P_x \sin\phi \left(\frac{ik_o}{r} - \frac{1}{r^2} + k_o^2\right) \frac{e^{ik_o r}}{4\pi r} \ . \qquad (22)$$

From (21) and (22), we can find the far-zone electric field ($k_o r >> 1$), i.e.

$$E_\theta \sim \frac{1}{\varepsilon} P_x k_o^2 \cos\theta\cos\phi \frac{e^{ik_o r}}{4\pi r}, \qquad (23)$$

$$E_\phi \sim -\frac{1}{\varepsilon} P_x k_o^{\,2} \sin\phi \frac{e^{ik_o r}}{4\pi r} . \tag{24}$$

The relation similar to (16) can be expressed by

$$\underline{P} = \underline{e}_x \frac{i}{\omega} i \Delta \ell = \underline{e}_x \frac{i}{\omega} I_o \tag{25}$$

Therefore,

$$P_x = \frac{i}{\omega} I_o \tag{26}$$

By substituting (26) into (23) and (24), we obtain

$$E_\theta \sim \frac{i I_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \cos\theta\cos\phi \frac{e^{ik_o r}}{r} , \tag{27}$$

$$E_\phi \sim \frac{-i I_o k_o}{4\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sin\phi \frac{e^{ik_o r}}{r} . \tag{28}$$

$k_o r \gg 1$

The above expressions are the far-zone electric field of an infinitesimal electric dipole parallel to the x axis in free-space.

The Poynting vector can be found by substituting (27) and (28) into (4D.1), i.e.

$$S_{ro} = \frac{I_o^{\,2} k_o^{\,2}}{32\pi^2 r^2} \sqrt{\frac{\mu_o}{\varepsilon_o}} (\cos^2\theta\cos^2\phi + \sin^2\phi) . \tag{29}$$

The total time-average power radiated into free-space by this dipole is also

$$P_o = \frac{I_o{}^2 k_o{}^2}{12\pi} \sqrt{\frac{\mu_o}{\varepsilon_o}} \quad . \tag{30}$$

Appendix E

Finding the Intensity of the Electric Field of a Line Source in Free-Space from its Power

We consider the line source in Fig. 18a. We know that the time-average power radiated into free-space by this line source can be expressed by

$$P = \int_0^{2\pi} S_{\rho o}\ \rho d\phi \ . \tag{1}$$

But $S_{\rho o}$ is independent of ϕ. Therefore, we get

$$P = 2\pi\rho\ S_{\rho o}\ . \tag{2}$$

For this line source, $S_{\rho o}$ can be simply found from (2C.17), i.e.

$$S_{\rho o} = \frac{1}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\ |E_{zo}|^2 \tag{3}$$

where E_{zo} is the electric field of the line source in free-space. By substituting (3) into (2), we obtain

$$P = \pi\rho\sqrt{\frac{\varepsilon_o}{\mu_o}}\ |E_{zo}|^2 . \tag{4}$$

Therefore

$$|E_{zo}| = \sqrt{\frac{P}{\pi\rho}}\;\sqrt[4]{\frac{\mu_o}{\varepsilon_o}}\;. \tag{5}$$

As can be seen, the intensity of the electric field of the line source in free-space can be expressed in terms of the time-average power. We can not obtain the phase of the electric field only from a knowledge of the power.

Appendix F

Boundary Conditions for the Electric Hertz Vector

In this appendix, we study the boundary conditions for the electric Hertz vector. In Chapters III and IV we used the electric Hertz vector. Consequently, we must know the boundary conditions which the Hertz vector ought to satisfy.

According to Maxwell, the tangential components of the electric and magnetic fields must be continuous along the interface of two media in Chapter III. From the symmetry of the configuration in the problem of Chapter III, it is clear that the tangential components of the electric and magnetic fields along the interface are only $E_\rho(\rho,z)$ and $H_\phi(\rho,z)$ in the cylindrical coordinate system. Therefore, on the interface (z = 0) we must have the following relations

$$E_{1\rho} = E_{2\rho} \;, \tag{1a}$$

$$H_{1\phi} = H_{2\phi} \;, \tag{1b}$$

at z = 0. We know that the electric and magnetic fields can be derived from the electric Hertz vector. We can write

$$\underset{\sim}{E} = k^2 \underset{\sim}{\Pi} + \nabla(\nabla \cdot \underset{\sim}{\Pi}) \;, \tag{2}$$

$$\underset{\sim}{H} = \frac{k^2}{\mu_o i\omega} \nabla \times \underset{\sim}{\Pi} \quad . \tag{3}$$

Now, we assume

$$\underset{\sim}{\Pi} = \Pi_z \underline{e}_z \tag{4}$$

From (2), (3), and (4), we can express the electric and magnetic fields in terms of Π_z. That is

$$E_\rho = \frac{\partial}{\partial \rho} \frac{\partial \Pi_z}{\partial z} , \tag{5}$$

$$H_\phi = - \frac{k^2}{\mu_o i\omega} \frac{\partial \Pi_z}{\partial \rho}, \tag{6}$$

$$E_z = k^2 \Pi_z + \frac{\partial^2 \Pi_z}{\partial z^2} . \tag{7}$$

By using (5), (6) and the boundary conditions (1a) and (1b) we obtain

$$\frac{\partial}{\partial \rho} \frac{\partial \Pi_{1z}}{\partial z} = \frac{\partial}{\partial \rho} \frac{\partial \Pi_{2z}}{\partial z} \tag{8a}$$

$$\frac{-k_o^{\ 2}}{\mu_o i\omega} \frac{\partial \Pi_{1z}}{\partial \rho} = \frac{-n^2 k_o^{\ 2}}{\mu_o i\omega} \frac{\partial \Pi_{2z}}{\partial \rho} \tag{8b}$$

for the points on the interface (z = 0). After simplification, we get

$$\frac{\partial}{\partial\rho}\frac{\partial\Pi_{1z}}{\partial z} = \frac{\partial}{\partial\rho}\frac{\partial\Pi_{2z}}{\partial z} \tag{9a}$$

$$\frac{\partial\Pi_{1z}}{\partial\rho} = n^2\frac{\partial\Pi_{2z}}{\partial\rho} \tag{9b}$$

We integrate both (9a) and (9b) with respect to ρ. The constants of the integration are zero, because all the expressions approach zero for $\rho \to \infty$. Hence, we obtain

$$\Pi_{1z} = n^2\Pi_{2z} \tag{10a}$$

$$\frac{\partial\Pi_{1z}}{\partial z} = \frac{\partial\Pi_{2z}}{\partial z} \quad . \tag{10b}$$

The above expressions are the boundary conditions for the electric Hertz vector. As can be noticed, only z component of vector $\underset{\sim}{\Pi}$ in the problem of Chapter III is necessary and sufficient to produce all the nonvanishing components of the electromagnetic field in that problem and to satisfy all the boundary conditions.

In Chapter IV, interfacial horizontal dipole, the problem is slightly different. Following a similar procedure, we assume that the Hertz vector has only x component which is parallel to the axis of the dipole. Accordingly, we can write the components of the electric field in terms of Π_x in the Cartesian coordinate system, i.e.

$$E_x = k^2\Pi_x + \frac{\partial^2\Pi_x}{\partial x^2} , \qquad (11)$$

$$E_y = \frac{\partial^2\Pi_x}{\partial y\partial x} , \qquad (12)$$

$$E_z = \frac{\partial^2\Pi_x}{\partial z\partial x} . \qquad (13)$$

To satisfy boundary conditions for $z = 0$, the x and y components of $\underset{\sim}{E}$ must be continuous on the interface of the two media in the problem of Chapter IV. Thus, we obtain

$$k_o{}^2\Pi_{1x} + \frac{\partial^2\Pi_{1x}}{\partial x^2} = n^2k_o{}^2\Pi_{2x} + \frac{\partial^2\Pi_{2x}}{\partial x^2} , \qquad (14a)$$

$$\frac{\partial^2\Pi_{1x}}{\partial y\partial x} = \frac{\partial^2\Pi_{2x}}{\partial y\partial x} . \qquad (14b)$$

From (14b), we deduce the continuity of Π_x on the interface. But using this continuity in (14a) implies that $n = 1$ which is contradictory with the original assumption in the problem. (We have $n > 1$ for the lower medium). Therefore, taking only the x component of $\underset{\sim}{\Pi}$ is not enough and we must also take its z component. Thus, we assume

$$\underset{\sim}{\Pi} = \Pi_x\underline{e}_x + \Pi_z\underline{e}_z . \qquad (15)$$

By substituting (15) into (2) we obtain

$$E_x = k^2\Pi_x + \frac{\partial}{\partial x}(\nabla\cdot\underset{\sim}{\Pi}) \tag{16}$$

$$E_y = \frac{\partial}{\partial y}(\nabla\cdot\underset{\sim}{\Pi}) \;. \tag{17}$$

On the interface (z = 0) we have the continuity of E_y and E_x. Hence, we get

$$k_o^{\,2}\Pi_{1x} + \frac{\partial}{\partial x}(\nabla\cdot\underset{\sim}{\Pi}_1) = n^2k_o^{\,2}\Pi_{2x} + \frac{\partial}{\partial x}(\nabla\cdot\underset{\sim}{\Pi}_2) \tag{18}$$

$$\frac{\partial}{\partial y}(\nabla\cdot\underset{\sim}{\Pi}_1) = \frac{\partial}{\partial y}(\nabla\cdot\underset{\sim}{\Pi}_2) \tag{19}$$

From (19), we get

$$\nabla\cdot\underset{\sim}{\Pi}_1 = \nabla\cdot\underset{\sim}{\Pi}_2 \tag{20}$$

Using (20) and (18), we deduce that

$$\Pi_{1x} = n^2\Pi_{2x} \tag{21}$$

We find, by substituting (15) into (3), that

$$H_x = \frac{k^2}{i\mu_o\omega}\,\frac{\partial\Pi_z}{\partial y} \tag{22}$$

$$H_y = \frac{k^2}{i\mu_o\omega}\left(\frac{\partial\Pi_x}{\partial z} - \frac{\partial\Pi_z}{\partial x}\right) \tag{23}$$

$$H_z = \frac{k^2}{i\mu_o\omega}\left(\frac{\partial\Pi_y}{\partial x} - \frac{\partial\Pi_x}{\partial y}\right) \tag{24}$$

Along the interface, we have $H_{x1} = H_{x2}$. Thus, from (22), we get

$$\Pi_{1z} = n^2\Pi_{2z} \ . \tag{25}$$

The y component of the magnetic field must be continuous on the boundary ($z = 0$). Therefore, from (23), we find

$$\frac{\partial\Pi_{1x}}{\partial z} = n^2 \frac{\partial\Pi_{2x}}{\partial z} \ . \tag{26}$$

Finally, from (20), (21), (25) and (26), we can write the boundary conditions for the vector $\underset{\sim}{\Pi}$ in Chapter IV, i.e.

$$\Pi_{1x} = n^2\Pi_{2x} \ , \tag{27}$$

$$\frac{\partial\Pi_{1x}}{\partial z} = n^2 \frac{\partial\Pi_{2x}}{\partial z} \ , \tag{28}$$

$$\Pi_{1z} = n^2\Pi_{2z} \ , \tag{29}$$

$$\frac{\partial\Pi_{1z}}{\partial z} + \frac{\partial\Pi_{1x}}{\partial x} = \frac{\partial\Pi_{2z}}{\partial z} + \frac{\partial\Pi_{2x}}{\partial x} \ . \tag{30}$$

Appendix G

The Exact Evaluation of Π_x Along the Interface Due to the Interfacial Horizontal Dipole

We intend to exactly evaluate the x component of the electric Hertz vector along the interface of the two media in Chapter IV.

To do so, we substitute $\theta = \frac{\pi}{2}$ into (4B.3). Thus, we get

$$\Pi_x = \Pi_{x1} = n^2\Pi_{x2} = \frac{iI_o}{2\pi\omega\varepsilon_o}\int_0^\infty \frac{h\, J_o(hr)}{\sqrt{h^2-k_o^2}+\sqrt{h^2-n^2k_o^2}}\, dh \,. \qquad (1)$$

The above formulas can be written as follows:

$$\Pi_x = \frac{iI_o}{2\pi\omega k_o^2\varepsilon_o(n^2-1)}\int_0^\infty h\left(\sqrt{h^2-k_o^2} - \sqrt{h^2-n^2k_o^2}\right) J_o(hr)dh \,. \qquad (2)$$

This integral can be divided into two integrals, i.e.

$$\Pi_x = \frac{iI_o}{2\pi\omega\varepsilon_o k_o^2(n^2-1)}\int_0^\infty h\sqrt{h^2-k_o^2}\, J_o(hr)dh$$

$$- \frac{iI_o}{2\pi\omega\varepsilon_o k_o^2(n^2-1)}\int_0^\infty h\sqrt{h^2-n^2k_o^2}\, J_o(hr)dh \,. \qquad (3)$$

In (3) we denote the first integral by I_1 and the second integral by I_2. In I_1, we divide the range of integration into two subranges. That is, we write

$$I_1 = \int_0^{k_o} u(h)dh + \int_{k_o}^{\infty} u(h)dh \,, \tag{4}$$

where u(h) is a shorthand for the integrand. We denote the integral for the subrange ($0 \leq h \leq k_o$) by I_1^a and the other one by I_1^b.

In the integral

$$I_1^a = \frac{I_o}{2\pi\omega\varepsilon_o k_o^2 (n^2-1)} \int_0^{k_o} h \sqrt{k_o^2 - h^2}\, J_o(hr)dh, \tag{5}$$

we introduce the variable ℓ by $\ell = \frac{h}{k_o}$. Therefore, we obtain

$$I_1^a = \frac{I_o k_o}{2\pi\omega\varepsilon_o (n^2-1)} \int_0^1 \ell \sqrt{1-\ell^2}\, J_o(k_o r\ell)d\ell \,. \tag{6}$$

Using the table of integrals, we find that

$$I_1^a = \frac{I_o k_o \sqrt{\pi}}{2\pi\omega\varepsilon_o (n^2-1) \sqrt{2}} \; \frac{1}{(k_o r)^{3/2}} J_{3/2}(k_o r) \tag{7}$$

where $J_{3/2}(k_o r)$ is the Bessel function of the order $\frac{3}{2}$.

By introducing the variable $\ell = \frac{h}{k_o}$ in the integral

$$I_1^b = \frac{iI_o}{2\pi\omega k_o^2\varepsilon_o(n^2-1)} \int_{k_o}^{\infty} h\sqrt{h^2-k_o^2}\, J_o(hr)dh, \qquad (8)$$

we get

$$I_1^b = \frac{iI_o k_o}{2\pi\omega\varepsilon_o(n^2-1)} \int_1^{\infty} \ell\sqrt{\ell^2-1}\, J_o(k_o r\ell)d\ell\ . \qquad (9)$$

Using the tables of integrals and functions, we obtain

$$I_1^b = \frac{iI_o k_o\sqrt{\pi}}{2\pi\omega\varepsilon_o(n^2-1)\sqrt{2}} \frac{1}{(k_o r)^{3/2}} N_{3/2}(k_o r) \qquad (10)$$

where $N_{3/2}(k_o r)$ is the Neumann function of the order 3/2.

We find, by substituting (7) and (10) into (4), that

$$I_1 = \frac{I_o k_o\sqrt{\pi}}{2\pi\omega\varepsilon_o(n^2-1)\sqrt{2}} \frac{1}{(k_o r)^{3/2}} H_{3/2}^{(1)}(k_o r) \qquad (11)$$

where $H_{3/2}^{(1)}(k_o r)$ is the Hankel function of the first kind. We know that

$$h_1^{(1)}(k_o r) = \sqrt{\frac{\pi}{2k_o r}}\, H_{3/2}^{(1)}(k_o r) \qquad (12)$$

where $h_1^{(1)}(k_o r)$ is the first order spherical Hankel function of the first kind. From (11) and (12), we get

$$I_1 = \frac{I_o}{2\pi\omega\varepsilon_o(n^2-1)r} h_1^{(1)}(k_o r) \qquad (13)$$

By following a similar procedure, we obtain

$$I_2 = -\frac{I_o n^2}{2\pi\omega\varepsilon_o(n^2-1)r} h_1^{(1)}(nk_o r) . \qquad (14)$$

By substituting (13) and (14) into (3), we can find Π_x along the interface, i.e.

$$\Pi_x = \frac{I_o}{2\pi\omega\varepsilon_o(n^2-1)r} [h_1^{(1)}(k_o r) - n^2 h_1^{(1)}(nk_o r)] \qquad (15)$$

REFERENCES

1. Dag T. Gjessing, Remote Surveillance by Electromagnetic Waves, Ann Arbor Science Publisher Inc. Ann Arbor (1978).

2. V. A. Fock, "The current distribution induced by a plane wave on the surface of a conductor," Zhurn. Eksp. Teor. Fiz 15, 693-702 (1945).

3. E. C. Jordan and K. G. Balmain, Electromagnetic Waves and Radiating Systems, Prentice-Hall, Inc. Englewood Cliffs, New Jersey (1968).

4. E. T. Whittaker, From Euclid to Eddington, Cambridge University Press, London (1949).

5. P. Beckmann, The Depolarization of Electromagnetic Waves, The Golem Press, Boulder, Colorado (1968).

6. P. Beckmann and A. Spizzichino, The Scattering of Electromagnetic Waves from Rough Surfaces, The Macmillan Company, New York (1963).

7. D. Marcuse, Light Transmission Optics, New York (1972). See also D. Marcuse, Theory of Dielectric Optical Waveguides, New York (1974).

8. C. Elachi and C. Yeh, "Mode conversion in periodically disturbed thin-film waveguides," Journal of Applied Physics, 45 (8), 3494-3499, August 1974.

9. A. Sommerfeld, Partial Differential Equations in Physics, Academic Press, New York (1949).

10. A. Baños, Dipole Radiation in the Presence of a Conducting Half-Space, Pergamon Press, New York (1966).

11. J. Wait, Electromagnetic Waves in Stratified Media, Macmillan Co., New York (1962).

12. L. M. Brekhovskikh, Waves in Layered Media, Academic Press, New York (1960).

13. C. H. Papas, Theory of Electromagnetic Wave Propagation, McGraw-Hill, New York (1965).

14. A. Sommerfeld, op. cit., p. 193.

15. J. A. Stratton, Electromagnetic Theory, McGraw-Hill Book Company, New York (1941).

16. R. M. Whitmer, "Fields in nonmetallic waveguides," Proc. IRE. 36, 1105-1109 (1948).

17. F. B. Hildebrand, Advanced Calculus for Applications, Prentice-Hall, Inc. Englewood Cliffs, New Jersey (second edition 1976).

18. J. D. Jackson, Classical Electrodynamics, John Wiley & Sons, Inc. New York (second edition 1975).

19. F. B. Hildebrand, op. cit. p. 604.

20. J. D. Jackson, op. cit. p. 237.

21. R. W. P. King and G. S. Smith, Antennas in Matter, MIT Press, Cambridge (1981).

22. M. Born and E. Wolf, Principles of Optics, Pergamon Press, New York (sixth edition 1980).

23. M. Born and E. Wolf, op. cit. p. 100.

24. C. H. Papas, op. cit. p. 47.

25. A. Sommerfeld, op. cit. p. 246.

26. A. Sommerfeld, op. cit. p. 193.

27. A. Sommerfeld, op. cit. p. 247.

28. J. A. Stratton, op. cit. p. 29.

29. G. N. Watson, Theory of Bessel Functions, Cambridge University Press (second edition 1966).

30. N. Chako, C. R. Acad. Sci., Paris, 247 (1958), 436, 580, 637. See also M. Born and E. Wolf, op. cit. p. 753.

31. A. Sommerfeld, op. cit. p. 257.

32. G. N. Watson, op. cit.

33. C. H. Papas, op. cit. p. 47.

34. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, Academic Press, New York (1965).

35. M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions, Dover Publications, Inc., New York (1972).

36. C. M. Bender and S. A. Orszag, Advanced Mathematical Methods for Scientists and Engineers, McGraw-Hill Book Company, New York (1978).

37. C. M. Bender and S. A. Orszag, op. cit. p. 276.

38. C. H. Papas, op. cit. p. 92.