ASPECTS OF HEAVY QUARK PHYSICS

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To my Mother and Father, for getting me to this point in spacetime

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ABSTRACT

The $\frac{1}{m}$ Wisgur corrections to semileptonic decay form factors for the $\Omega_b \to \Omega_c e\bar{\nu}$ system are enumerated, and a general theorem on the vanishing of all $\frac{1}{m}$ corrections at threshold is derived. The contribution of charged higgs scalars to the neutron electric dipole moment in multi-higgs models is also examined, and found to be near present experimental limits.

CONTENTS

I. ASPECTS OF HEAVY QUARK PHYSICS

In the Standard Model and its generalizations, charge conjugation and parity violating (CP violating) processes arise because of complex couplings involving the higgs, fermion, and other fields. In the Minimal Standard Model, the complex couplings in the quark mass matrices are shifted into the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix by field redefinitions. Extensions of the Standard Model typically have new CP violating complex couplings (e.g., higgs mass matrices in multi-higgs models, or squark mass and kinetic energy terms in supersymmetric models), which cannot be incorporated into the CKM matrix.

Processes involving CP violation therefore offer the possibility of distinguishing between Standard and non-Standard Model physics once the CKM mixing angles are sufficiently well determined. Theoretical methods allowing more precise experimental extraction of particularly important CKM angles (such as V_{cb} and V_{ub}) set the stage for sensitive tests of physics beyond the Standard Model; the heavy quark symmetry of Isgur and Wise[1-3], to be explored in Chapters II-IV, is such a method. Hypothetical heavy quark processes leading to CP violation too large to be explained by the CKM matrix also provide mechanisms for probing physics beyond the Standard Model; the contribution to the neutron electric dipole moment of Weinberg's[4] three-gluon operator, examined in Chapters V and VI, is one such process.

Although both topics above feature heavy quarks, they cast the quarks in very different roles. The heavy quark symmetry arises as a simplification of Quantum Chromodynamics (QCD) in the large-mass limit, while the importance of the top and bottom quarks to electric dipole moments is due to the large coupling of massive quarks to higgs particles.

Because the top quark is expected to decay via W bosons before it has time to hadronize, the heavy quark symmetry will not be useful in predicting quantities pertaining to top physics. The bottom quark, on the other hand, appears ideally suited for the application of the large mass limit to QCD, as done in Chapters II through IV.

II. INTRODUCTION TO HEAVY QUARK SYMMETRY

1. Ideas behind the Heavy Quark Expansion

The basic idea behind the heavy quark formalism is that QCD displays new symmetries in the limit of infinitely heavy quark masses. We can make this more precise by considering a meson, say a $\bar{B}$ meson, and letting the mass of the bottom quark m_b go to infinity with the meson four-velocity v^μ fixed. In this limit, the heavy quark carries nearly all of the meson's momentum, and we can write the heavy quark momentum as $p^\mu = m_B v^\mu + k^\mu$. The residual momentum k^μ measures how far the heavy quark is off-shell, and should be of order the QCD scale Λ_{QCD}. One pictures the meson as consisting of a bowling ball (the heavy quark) and many ping pong balls (the light quarks and gluons).

Eventually we will make an effective Lagrangian to describe the dynamics of this system, but already we can see some of the salient features such a description will have. Since the light degrees of freedom can transfer only order Λ_{QCD} of momentum to the heavy quark, the four-velocity of the heavy quark will be conserved. The effective Lagrangian will therefore have a velocity superselection rule specifying that heavy quarks of different velocities do not interact with each other via nonperturbative strong physics. It is important to note that $\frac{k^\mu}{m_b} << 1$ always holds, since the transfer of large momentum by an infinite number of soft gluons results in the disintegration of the meson and is therefore irrelevant to our problem.

Another feature of the $m_b \to \infty$ limit appears when we go to the meson rest frame, where the heavy quark acts as a static color source. Here, the heavy quark flavor is irrelevant to dynamics. Further, in the presence of the color magnetic field that is due to the light degrees of freedom, a gluon with momentum Λ_{QCD} cannot flip the spin of the infinitely heavy quark. In the infinite mass limit, then, the spin of the heavy quark is conserved. The resulting $SU(2N)$ spin-flavor symmetry (dubbed the Wisgur symmetry after M. Wise and N. Isgur) will be manifested in the effective Lagrangian we derive in the next section, but first we look at some immediate experimental implications.

To the extent that the b and c quarks are very heavy, nonperturbative QCD treats them the same. The light degrees of freedom in a B or corresponding D meson will thus be in the same state. This implies, for flavor-changing current J^μ acting on states normalized to twice their mass, that

$$\frac{\langle O|J^\mu|B(v)\rangle}{\sqrt{2m_B}} = \frac{\langle O|J^\mu|D(v)\rangle}{\sqrt{2m_D}}. \tag{2.1.1}$$

Defining the decay constants f_X by

$$\langle O|J^\mu|X\rangle = f_X p_X^\mu, \tag{2.1.2}$$

where p_X is the momentum of meson X, we see that the decay constants scale as

$$f_B = \sqrt{\frac{m_D}{m_B}} f_D. \tag{2.1.3}$$

Preliminary lattice calculations make (2.1.3) suspect[5]. Whether the B and D mesons are heavy enough for the above relation to apply with any accuracy must wait for more lattice calculations or experiments, although we will see how to estimate errors in the next section.

Another implication comes from the spin symmetry, which relates matrix elements involving vector and pseudoscalar mesons. The spin operator for charm quarks

$S_z = \frac{1}{2}\int d^3x c^\dagger \sigma_z c$, whose commutation relations with flavor-changing currents are known, is a symmetry operator of the effective Hamiltonian. Writing the spin part of the D meson wave function as $|D\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ and of the vector meson as $|D^*\rangle = |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$, we see that the spin operator changes the pseudoscaler into a vector meson: $S_z|D\rangle = \frac{1}{2}|D^*\rangle$. Then, for example,

$$\langle D|V_3|B\rangle = 2\langle D^*|[S_3, V_3]|B\rangle = -\langle D^*|A_0|B\rangle, \tag{2.1.4}$$

where V_μ, A_μ are the μ components of the vector and the axial flavor-changing currents, respectively. Similarly to the pseudoscalar case above, this relation holds because the state of the light degrees of freedom for the D and D^* are identical for infinite c quark mass.

A variety of relations can be derived by using commutation relations such as those above, but a simpler method exists. By examining the QCD Lagrangian in the large-mass limit, an effective field theory that duplicates the QCD results, order by order in $\frac{\Lambda_{QCD}}{m}$, can be constructed[2].

2. Derivation of the Effective Lagrangian

The spin-flavor symmetry arises because nonperturbative gluons transfer momentum that is small compared to the mass of the heavy quark. In principle, one arrives at the effective Lagrangian $\mathcal{L}_{\text{eff}}$ for nonperturbative, strong interactions by integrating hard gluons out of the QCD Lagrangian. In practice, one matches matrix elements in the two theories to determine the coupling constants in $\mathcal{L}_{\text{eff}}$. For example, the QCD propagator for a heavy quark with momentum $p^\mu = m_B v^\mu + k^\mu$ is

$$\frac{i(\not{p}+m)}{p^2-m^2} = \frac{i(m\not{v}+\not{k}+m)}{2mv\cdot k+k^2} = (\frac{1+\not{v}}{2})\frac{i}{v\cdot k} + \mathcal{O}(\frac{\Lambda_{QCD}}{m}). \tag{2.2.1}$$

Because pair creation of heavy quarks is suppressed, we need use only the positive-frequency component of the heavy quark field. Then $\not{v}Q = Q + \mathcal{O}(\frac{\Lambda_{QCD}}{m})$. Since heavy quarks are always nearly on-shell, the projection operator $\frac{1+\not{v}}{2}$ in the propagator may be set to one. Thus, the leading-order, heavy quark propagator is $\frac{i}{v\cdot k}$. Similarly, since

$$\bar{Q}(p')\gamma_\mu Q(p) = \bar{Q}(\frac{p'+p}{2m})Q + \mathcal{O}(\frac{\Lambda_{QCD}}{m}), \tag{2.2.2}$$

the gluon vertex to order one is

$$igT^a v_\mu, \tag{2.2.3}$$

where g is the strong coupling constant and T^a is a SU(3) generator.

These Feynman rules are reproduced to lowest order by $\mathcal{L}_v = i\bar{h}_v v\cdot Dh_v$ if we take h_v to be a rescaled heavy quark field with residual momentum k. The Lagrangian can be made Lorentz-invariant by integrating over all velocities

$$\mathcal{L}_{\text{eff}} = \int \frac{dv}{2v^0}\mathcal{L}_v. \tag{2.2.4}$$

This evasion of the No-Go Theorem, which forbids mixing of internal and spacetime symmetries, is accomplished by using an infinite number of fields, one for each four-velocity. For additional flavors, one simply sums over the flavor index, making explicit the spin and flavor symmetry.

A simple method for deriving from QCD the $\mathcal{O}(\frac{\Lambda_{QCD}}{m})$ symmetry-breaking corrections to $\mathcal{L}_{\text{eff}}$ is to define the rescaled quark field in terms of the on-shell QCD heavy quark field c(x) by shifting away its large momentum dependence and projecting out the component that behaves like a particle of momentum mv :

$$h_v^{(c)}(x) = (\frac{1+\not{v}}{2})c(x)e^{im_c v\cdot x}. \tag{2.2.5}$$

Since we are not interested in antiquarks, we will drop the negative frequency component of c(x) by hand. Inverting the definition gives a $\frac{1}{m}$ expansion for the QCD field

$$c(x) = e^{-im_c v\cdot x}(1-\frac{i\not{D}}{2m})^{-1}h_v = e^{-im_c v\cdot x}\left[1+\frac{i\not{D}}{2m}+(\frac{i\not{D}}{2m})^2+\cdots\right]h_v. \qquad (2.2.6)$$

Making this substitution in the QCD Lagrangian gives

$$\mathcal{L}_{\text{eff}} = \bar{h}^{(c)}_{v'} iv'\cdot D h^{(c)}_{v'} - \frac{1}{2m_c}\bar{h}^{(c)}_{v'}\left[D^2+\tfrac{1}{2}g_s\sigma_{\mu\nu}G^{\mu\nu}\right]h^{(c)}_{v'} + (b,v\leftrightarrow c,v'), \qquad (2.2.7)$$

in which $G^{\mu\nu}$ denotes the gluon field-strength tensor, and we have treated the c and b quarks as heavy. The form of the b quark correction is the same as that of the c quarks, but the coefficients will be different at the charm-mass scale because of renormalization from the b to c scale. Since $\mathcal{O}(\frac{\Lambda_{QCD}}{m})$ corrections are dominated by the charm quark, we will not need the $\mathcal{O}(\frac{\Lambda_{QCD}}{m_b})$ corrections. To examine weak meson decays at first order will, however, require the $\mathcal{O}(\frac{\Lambda_{QCD}}{m_c})$ corrections to flavor-changing currents. Substituting Equation (2.2.6) into vector and axial currents gives[6]

$$\begin{aligned} V^\mu_{\text{weak}}(x) &= \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{\frac{-6}{25}} \bar{h}^{(c)}_{v'}\left[\gamma^\mu - \frac{i}{2m_c}\overleftarrow{\not{D}}\,\gamma^\mu\right]h^{(b)}_v(x)e^{-i(m_b v - m_c v')\cdot x} \\ A^\mu_{\text{weak}}(x) &= \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{\frac{-6}{25}} \bar{h}^{(c)}_{v'}\left[\gamma^\mu\gamma_5 - \frac{i}{2m_c}\overleftarrow{\not{D}}\,\gamma^\mu\gamma_5\right]h^{(b)}_v(x)e^{-i(m_b v - m_c v')\cdot x} \end{aligned} \quad . \qquad (2.2.8)$$

The above results are valid to $\mathcal{O}(\frac{\Lambda_{QCD}}{m_c})$, at the charm mass scale. Since relations between physical quantities are renormalization point-independent, our computations at this scale will be relevant for predictions at any scale. The factor involving strong coupling constants comes from strong renormalization of the Lagrangian between the b and c mass scales, and is numerically equal to about 1.1 .

An immediate check on the validity of the $\frac{1}{m}$ expansion for b and c quarks can now be made by examining the B^* to B versus D^* to D mass differences. At leading-order we expect degenerate masses for the pseudoscalar and vector mesons, with corrections of order Λ_{QCD}. Since the B mass splitting is about 45 MeV and the D mass splitting is 145 MeV, this agrees with leading order predictions and a value of $\Lambda_{QCD} = 200 - 300 MeV$. At first order, the mass splitting scales as $\frac{1}{m}$ times a strong renormalization factor of 1.15, so we expect the mass splitting ratio to be

$$\frac{m_{B^*} - m_B}{m_{D^*} - m_D} = \frac{m_D}{1.15 m_B}. \tag{2.2.9}$$

In this case, the leading $\frac{\Lambda_{QCD}}{m}$ correction is less than 10%.

This prediction is a simple one which is duplicated by the nonrelativistic quark model. Less obvious, more useful predictions arise when we apply the heavy quark formalism to semileptonic decays.

3. B → D Semileptonic Decay in the Wisgur Limit

Possibly the most important semileptonic decay to consider is $B \to D$. The rate for this decay is proportional to the Kobayashi-Maskawa-Cabbibo mixing angle V_{cb}, but because the hadronic part of the matrix element is unknown, it is difficult to extract the mixing angle from experimental data. Current models of the hadronic matrix element leave a theoretical uncertainty in V_{cb} of around 30%, although even the error is difficult to estimate. The systematic, QCD-based, $\frac{\Lambda_{QCD}}{m}$ expansion can do better than this, and is expected eventually to allow the extraction of V_{cb} with errors of $\mathcal{O}(\frac{\Lambda^2_{QCD}}{m^2})$, a few percent. Although the calculation has been done to $\mathcal{O}(\frac{\Lambda_{QCD}}{m})$[7], we will reproduce here only the leading-order analysis[8].

The $B \to D$ decay factorizes (up to electromagnetic corrections) into hadronic and leptonic matrix elements. A typical hadronic matrix element such as

$$H^{\mu} = \langle D(v')|V^{\mu}_{weak}(0)|B(v)\rangle \tag{2.3.1}$$

is constrained by the Wisgur symmetry to transform under spin-flavor rotations in a manner dictated by the Wigner-Eckart Theorem and the transformation properties of the heavy quarks in B, D and V^{μ}. Further, this matrix element will be related to transition elements involving B^* or D^*, as well as A^{μ}_{weak}, because the light degrees of freedom for the B, B^*, D, and D^* mesons are identical.

Lorentz invariance allows the experimentally relevant matrix elements to be parameterized by six form factors, which may be defined by

$$\langle D(v')|V^{\mu}|B(v)\rangle = f_+(v+v')^{\mu} + f_-(v-v')^{\mu}, \tag{2.3.2}$$

$$\langle D^*(v'),\epsilon|A^{\mu}|B(v)\rangle = a_0\epsilon^{*\mu} + a_+\epsilon^*\cdot v(v+v')^{\mu} + a_-\epsilon\cdot v(v-v')^{\mu}, \tag{2.3.3}$$

$$\langle D^*(v'),\epsilon|V^{\mu}|B(v)\rangle = g\epsilon^{\mu\alpha\beta\delta}\epsilon^*_{\alpha}v_{\beta}v'_{\delta}, \tag{2.3.4}$$

where ϵ^{μ} is the polarization vector for the D^* meson, and V^{μ} and A^{μ} are weak vector and axial currents.

In contrast, Wisgur symmetry will permit all six of these form factors to be written in terms of one unknown function $\xi(v\cdot v')$. A useful mnemonic for doing the group theory that leads to this result is to construct interpolating fields

$$B = \bar{h}^{(b)}_{\alpha}\bar{l}_{\alpha}, \tag{2.3.5}$$

and

$$D^* = \bar{h}^{(c)}_{\alpha}\not{\epsilon}_{\alpha\beta}\gamma^5_{\beta\delta}\bar{l}_{\delta}, \tag{2.3.6}$$

where the Dirac indices α, β, δ are implicitly summed, and l, t represent the light degrees of freedom.

Using these fields and extracting the Dirac structures that are relevant to heavy quark spin transformations, we may write schematically

$$\begin{aligned}\langle D^*(v')|\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}|B(v)\rangle &= \langle 0|t\gamma^5 \not\epsilon^* h_{v'}^{(c)}\bar{h}_{v'}^{(c)}\Gamma h_v^{(b)}\bar{h}_v^{(b)}\bar{l}|0\rangle \\ &= \langle 0|t_\alpha \bar{l}_\beta|0\rangle[\gamma^5 \not\epsilon^* \tfrac{1+\not v'}{2}\Gamma\tfrac{1+\not v}{2}]_{\alpha\beta}.\end{aligned} \tag{2.3.7}$$

Here we have contracted the heavy quark fields and have retained all momentum integrations and unknown strong interaction physics in a nonperturbative matrix element, which can be parameterized by

$$P_{\alpha\beta} = \langle 0|t_\alpha \bar{l}_\beta|0\rangle = A\delta_{\alpha\beta} + B\not v_{\alpha\beta} + C\not v'_{\alpha\beta}, \tag{2.3.8}$$

in which A,B, and C are arbitrary functions of $v \cdot v'$. However, since the three functions simply add together after the contraction in Equation (2.3.7), $P_{\alpha\beta}$ reduces to

$$P_{\alpha\beta} = \xi(v \cdot v')\delta_{\alpha\beta}. \tag{2.3.9}$$

Since the light degrees of freedom are the same for all of the matrix elements under consideration, similar treatments will give other matrix elements in terms of the same unknown function ξ, dubbed the Isgur-Wise function. The results are[8]

$$\begin{aligned}&\langle D(v')|V^\mu|B(v)\rangle = \sqrt{m_c m_b}\,\xi(v \cdot v')(v + v')^\mu \\ &\langle D^*(v'), \epsilon|A^\mu|B(v)\rangle = \sqrt{m_c m_b}\,\xi(v \cdot v')[\epsilon^{*\mu}(1 + v \cdot v') - v'^\mu v \cdot \epsilon^*]\,\cdot \\ &\langle D^*(v'), \epsilon|V^\mu|B(v)\rangle = -i\sqrt{m_c m_b}\,\xi(v \cdot v')\epsilon^{\mu\alpha\beta\delta}\epsilon^*_\alpha v_\beta v'_\delta\end{aligned} \tag{2.3.10}$$

The reduction of six form factors to one unknown function is an impressive feat of heavy quark symmetry, but it is not the best. Since $\bar{c}\gamma^0 c$ is the generator of charm-quark number, it acts trivially on states. However, Wisgur symmetry also tells us its matrix element in terms of the Isgur-Wise function. Since

$$\begin{aligned}\langle D(v)|\bar{c}\gamma^0 c|D(v)\rangle &= 2m_c v^0 \\ &= m_c \xi(v\cdot v) Tr\left[\tfrac{1+\not{v}}{2}\gamma^0\tfrac{1+\not{v}}{2}\right] \qquad (2.3.11)\\ &= 2m_c v^0 \xi(1),\end{aligned}$$

the Isgur-Wise function is normalized to one at threshold ($v = v'$). The significance is that $B \to D$ decay rates at threshold are now completely predicted in terms of one unknown quantity, the mixing angle V_{cb}. This allows extraction of V_{cb} from experimental data.

Previously, theoretical errors were comparable to or larger than current experimental data. Currently, using the heavy quark formalism, theoretical errors are expected to be a few percent, much better than existing experimental errors. Much higher production of B mesons is expected in the next few years (e.g., at Cleo, B factories, etc.), so it seems likely that we will know V_{cb} quite well, quite soon.

The reason we expect only a few percent theoretical error is that somewhat surprisingly, the $\mathcal{O}(\frac{\Lambda_{QCD}}{m})$ corrections to the normalization of ξ at threshold also vanish[7]! Indeed, it turns out[9,10] that similar statements can be made about heavy Lambda and Omega systems. We demonstrate the techniques of $\mathcal{O}(\frac{\Lambda_{QCD}}{m})$ calculations and the vanishing of threshold corrections for the Omega system in the next Chapter.

III. Semileptonic Decay of Ω_b*

1. Introduction

Recent advances in heavy quark physics[1–3] increase the predictive power of QCD and allow the determination of previously unknown, strong matrix elements. This is accomplished by treating the heavy quark as infinitely massive compared to the QCD scale. In this limit, QCD contains an additional SU(2N) spin-flavor symmetry[1], where N is the number of heavy quarks. This symmetry has been exploited to derive relations among form factors† in numerous systems[12–14,7], including $B \to D$, $\Lambda_b \to \Lambda_c$, and $\Omega_b \to \Omega_c$. The calculations can be understood in terms of an effective field theory[2], which incorporates SU(2N) breaking terms as perturbations in a $1/m$ expansion.

Because we expect the expansion parameter $\Lambda_{QCD}/m_c \approx 1/5$, it is desirable to include the $1/m_c$ corrections. This is particularly important for extraction of the Kobayashi-Maskawa mixing angle V_{cb} from baryon systems, where the relevant corrections may be significantly larger (typically a factor of two compared to mesons with similar light-quark content). It is a testament to the power of the above method that predictive power remains even at $\mathcal{O}(\frac{1}{m_c})$. Indeed, for the $B \to D$ and $\Lambda_b \to \Lambda_c$ systems, all $\mathcal{O}(\frac{1}{m_c})$ corrections vanish at threshold[7,9], allowing the possibility of determining V_{cb} to this order.

*from G. Boyd and D. Brahm, *Phys. Lett.* **B254**:468 (1991).

†The heavy quark symmetry was present in earlier phenomenological models, such as that of Ref. [11].

In this letter, we calculate the $\mathcal{O}(\frac{1}{m_c})$ corrections to the $\Omega_b \to \Omega_c$ weak form factors. As in the two cases mentioned above, relations between form factors can be derived at this order, and all $\mathcal{O}(\frac{1}{m_c})$ corrections vanish at threshold. The normalization and much of the notation we use will follow that of Reference [14]. After recapitulating the leading order results and displaying the $\mathcal{O}(\frac{1}{m_c})$ effective Lagrangian and flavor changing current, we consider the various unknown matrix elements both at and away from threshold. The results are tabulated in Tables 1 and 2.

We will denote the Ω states by Ω^M, with $M = 1$ corresponding to Ω and $M = 2$ to Ω^*. The tensors B^M_μ that describe the Ω^M states are

$$B^1_\mu(v,s) = \tfrac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 u(v,s), \qquad B^2_\mu(v,s) = u_\mu(v,s). \tag{3.1.1}$$

Here, u_μ is the Rarita-Schwinger spinor for the Ω^*, and flavor indices have been suppressed. The tensors obey

$$\not{v} B^M_\mu(v,s) = B^M_\mu(v,s), \quad v^\mu B^M_\mu(v,s) = 0, \quad \gamma^\mu B^2_\mu = 0 \tag{3.1.2}$$

The vector V^μ and axial A^μ current form factors are defined by

$$\begin{aligned}
\langle\Omega_c(v',s')|V^\mu|\Omega_b(v,s)\rangle &= \bar{u}(v',s')\left[F_1\gamma^\mu + F_2 v^\mu + F_3 v'^\mu\right]u(v,s) \\
\langle\Omega_c(v',s')|A^\mu|\Omega_b(v,s)\rangle &= \bar{u}(v',s')\left[G_1\gamma^\mu\gamma_5 + G_2 v^\mu\gamma_5 + G_3 v'^\mu\gamma_5\right]u(v,s) \\
\langle\Omega^*_c(v',s')|V^\mu|\Omega_b(v,s)\rangle &= \bar{u}_\lambda(v',s')\left[N_1 v^\lambda\gamma^\mu\gamma_5 + N_2 v^\lambda v^\mu\gamma_5 + N_3 v^\lambda v'^\mu\gamma_5 + N_4 g^{\lambda\mu}\gamma_5\right]u(v,s) \\
\langle\Omega^*_c(v',s')|A^\mu|\Omega_b(v,s)\rangle &= \bar{u}_\lambda(v',s')\left[K_1 v^\lambda\gamma^\mu + K_2 v^\lambda v^\mu + K_3 v^\lambda v'^\mu + K_4 g^{\lambda\mu}\right]u(v,s)
\end{aligned} \tag{3.1.3}$$

The leading-order results can be parameterized by two unknown functions[14] (see the first two columns of Table 1):

$$\langle\Omega^M_c(v',s')|\bar{h}^{(c)}_{v'}\Gamma h^{(b)}_v|\Omega^N_b(v,s)\rangle = C\,\bar{B}^M_\mu(v',s')\Gamma B^N_\nu\left[-g^{\mu\nu}\xi_1(w) + v^\mu v'^\nu\xi_2(w)\right] \tag{3.1.4}$$

$$C = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25} = 1.1. \tag{3.1.5}$$

The coefficient C is the strong renormalization correction evaluated at the scale $\mu = m_c$. We are free to evaluate all matrix elements at $\mu = m_c$ because relations between physical quantities are independent of the subtraction scale. To leading order, the form factors are uniquely determined at threshold by $\xi_1(1) = 1$.

To find the leading $1/m_c$ corrections, we start with the effective Lagrangian[9,15]

$$\mathcal{L} = \bar{h}_v^{(b)} iv \cdot D h_v^{(b)} + \bar{h}_v^{\prime(c)} iv' \cdot D h_{v'}^{(c)} + \frac{1}{2m_c} \bar{h}_{v'}^{(c)} \left[(v' \cdot D)^2 - D^2 - \tfrac{1}{2} g_s \sigma_{\mu\nu} G^{\mu\nu} \right] h_{v'}^{(c)} \tag{3.1.6}$$

in which $G^{\mu\nu}$ denotes the gluon field-strength tensor, and the flavor-changing currents

$$\begin{aligned} V^\mu &= C \bar{h}_{v'}^{(c)} \left[\gamma^\mu - \frac{i}{2m_c} \overleftarrow{\not{D}} \gamma^\mu \right] h_v^{(b)} \\ A^\mu &= C \bar{h}_{v'}^{(c)} \left[\gamma^\mu \gamma_5 - \frac{i}{2m_c} \overleftarrow{\not{D}} \gamma^\mu \gamma_5 \right] h_v^{(b)} \end{aligned} \tag{3.1.7}$$

at the scale $\mu = m_c$. The $(v' \cdot D)^2$ term appearing in the interaction Lagrangian $\mathcal{L}'$ (the third term of Equation (3.1.6)) can be ignored because the equations of motion imply that an insertion of $(v' \cdot D) h_{v'}^{(c)}$ in a physical amplitude suppresses the amplitude by $\mathcal{O}(\frac{1}{m_c})$.

Insertions of the interaction Lagrangian with the dimension-three part of the currents give rise to the unknown, time-ordered matrix elements

$$\begin{aligned} I_1^{MN}(w;\Gamma) &= \frac{-ig_s}{4m_c} \langle \Omega_c^M(v',s')| \mathrm{T} \left\{ \int \mathrm{d}^4x \left(\bar{h}_{v'}^{(c)} \sigma_{\mu\nu} G^{\mu\nu} h_{v'}^{(c)} \right)(x) \left(\bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} \right)(0) \right\} |\Omega_b^N(v,s)\rangle \\ I_3^{MN}(w;\Gamma) &= \frac{-i}{2m_c} \langle \Omega_c^M(v',s')| \mathrm{T} \left\{ \int \mathrm{d}^4x \left(\bar{h}_{v'}^{(c)} D^2 h_{v'}^{(c)} \right)(x) \left(\bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} \right)(0) \right\} |\Omega_b^N(v,s)\rangle \end{aligned} \tag{3.1.8}$$

Since D^2 transforms trivially under the spin symmetries, I_3^{MN} contributes as does the right hand side of Equation (3.1.4), but with new functions $\tilde{\xi}_1(w)$ and $\tilde{\xi}_2(w)$.

The dimension-four part of the currents introduces

$$I_2^{MN}(w;\gamma^\lambda\Gamma)_\lambda = \frac{-1}{2m_c}\,\langle\Omega_c^M(v',s')|\bar{h}_{v'}^{(c)}\,i\overleftarrow{D}_\lambda\gamma^\lambda\Gamma\,h_v^{(b)}|\Omega_b^N(v,s)\rangle. \tag{3.1.9}$$

2. The $G^{\mu\nu}$ Contribution

The Isgur-Wise symmetry constrains the form of I_1^{MN} to be

$$I_1^{MN}(w;\Gamma) = \bar{B}_\mu^M(v',s')\,\sigma^{\lambda\rho}\,\frac{1+\not{v}'}{2}\,\Gamma B_\nu^N(v,s)\,M_{\lambda\rho}^{\mu\nu}. \tag{3.2.1}$$

where $M_{\lambda\rho}^{\mu\nu}$ is the most general tensor (antisymmetric in λ,ρ), which can be constructed from v_α, v'_α and $g_{\alpha\beta}$. $M_{\lambda\rho}^{\mu\nu}$ may not contain v'_μ or v_ν because of Eq. (3.1.2). Further, since $[\gamma_\mu,\,\not{v}](1+\not{v}) = 2(1-\not{v})(\gamma_\mu+v_\mu)$, v'_λ and v'_ρ are also disallowed. We may therefore write

$$M_{\lambda\rho}^{\mu\nu} = \tfrac{-3i}{2}\left[\eta_1(w)g_\lambda^\mu g_\rho^\nu + \eta_2(w)g_\rho^\mu v'^\nu v_\lambda + \eta_3(w)g_\lambda^\nu v^\mu v_\rho\right] \tag{3.2.2}$$

without loss of generality. Of these three new functions, only η_1 contributes at $w=1$.

Some algebra gives

$$\begin{aligned} I_1^{11}(w;\Gamma) = \bar{u}(v',s')\gamma_5\{&-\eta_1\left[\gamma_\mu\Gamma\gamma^\mu + \Gamma\not{v}' + \not{v}\Gamma + w\Gamma\right] + \eta_2\left[(\not{v}+w)\Gamma(\not{v}'+w)\right] \\ &- \tfrac{1}{2}\eta_3\left[(w^2-1)\gamma_\mu\Gamma\gamma^\mu - (\not{v}+w)\Gamma - (1+w\not{v})\Gamma\not{v}'\right]\}\gamma_5 u(v,s) \\ I_1^{21}(w;\Gamma) = \bar{u}_\mu(v',s')\{&-\tfrac{\sqrt{3}}{2}\eta_1\Gamma(\gamma^\mu+v^\mu) + \tfrac{\sqrt{3}}{2}\eta_2 v^\mu\Gamma(\not{v}'+w) \\ &+ \tfrac{\sqrt{3}}{2}\eta_3 v^\mu\left[(w-\not{v})\gamma^\nu\Gamma\gamma_\nu - \Gamma - \not{v}\Gamma\not{v}'\right]\}\gamma_5 u(v,s) \end{aligned} \tag{3.2.3}$$

The functions η_1 and η_2 contribute as do ξ_1 and ξ_2, respectively, but with a proportionality constant dependent on the final spin state. They can therefore be eliminated from the form factors for either decay by redefinitions of ξ_1 and ξ_2, although we will not do so at this time.

3. The $\not{D}$ Contribution

To examine $I_2^{MN}(w;\gamma^\lambda\Gamma)_\lambda$ (see Eq. (3.1.9)), we first look at

$$\begin{aligned} I_2^{MN}(w;\Gamma)_\lambda &= \frac{-1}{2m_c}\langle\Omega_c^M(v',s')|\bar{h}_{v'}^{(c)} i\overleftarrow{D}_\lambda \Gamma h_v^{(b)}|\Omega_b^N(v,s)\rangle \\ &= \bar{B}_\mu^M(v',s')\,\Gamma\, B_\nu^N(v,s)\; P_\lambda^{\mu\nu} \end{aligned} \tag{3.3.1}$$

where the most general form for $P_\lambda^{\mu\nu}$ is

$$P_\lambda^{\mu\nu} = \tfrac{-3}{2}\left[\kappa_1 v'^\nu v^\mu v_\lambda + \kappa_2 v'^\nu v^\mu v'_\lambda + \kappa_3 g^{\mu\nu} v_\lambda + \kappa_4 g^{\mu\nu} v'_\lambda + \kappa_5 g_\lambda^\mu v'^\nu + \kappa_6 g_\lambda^\nu v^\mu\right]. \tag{3.3.2}$$

In the $w\to 1$ limit, the part of $P_\lambda^{\mu\nu}$ that gives a nonvanishing contribution to $I_2^{MN}(1;\Gamma)$ depends only on $(\kappa_3+\kappa_4)$. However, since $v'^\lambda I_2^{MN}(w;\Gamma)_\lambda = 0$,

$$w\kappa_1 + \kappa_2 + \kappa_6 = 0, \qquad w\kappa_3 + \kappa_4 = 0, \tag{3.3.3}$$

so I_2^{MN} gives no contribution to the current matrix elements at threshold. Our six unknown functions reduce to two after application of the usual[7,9] trick

$$\begin{aligned} \frac{-1}{2m_c}\langle\Omega_c(v',s')|i\partial_\mu(\bar{h}_{v'}^{(c)}\Gamma\bar{h}_v^{(b)})|\Omega_b(v,s)\rangle &= \frac{\bar{\Omega}}{2m_c}(v'-v)_\mu\,\langle\Omega_c(v',s')|\bar{h}_{v'}^{(c)}\Gamma\bar{h}_v^{(b)}|\Omega_b(v,s)\rangle \\ &= I_2^{11}(w;\Gamma)_\mu + \tilde{I}_2^{11}(w;\Gamma)_\mu \end{aligned}, \tag{3.3.4}$$

where $\tilde{I}_2^{MN}(w;\Gamma)_\mu$ is defined identically to $I_2^{MN}(w;\Gamma)_\mu$, except that the D_μ acts on the bottom quark field, and to the order we are working, $\bar{\Omega} = m_{\Omega_b} - m_b = m_{\Omega_c} - m_c \approx 1$ GeV. Dotting Equation (3.3.4) with v^μ, using Equation (3.1.4), and noting $v^\mu \tilde{I}_2^{MN}(w;\Gamma)_\mu = 0$, we get

$$\begin{aligned} \kappa_3 &= -\frac{\bar{\Omega}}{3m_c(1+w)}\xi_1 & \kappa_4 &= \frac{\bar{\Omega}w}{3m_c(1+w)}\xi_1 \\ \kappa_5 &= \frac{\bar{\Omega}(1-w)}{3m_c}\xi_2 - (\kappa_1 + w\kappa_2) & \kappa_6 &= -(w\kappa_1+\kappa_2). \end{aligned} \tag{3.3.5}$$

A similar argument shows that $\tilde{I}_2^{MN}(w;\Gamma)$, like its companion, gives no contribution at w=1.

Thus, the dimension-four part of the current gives vanishing contribution to $\Omega^N \to \Omega^M$ matrix elements at threshold.

4. Normalization at Threshold

By comparing the vector current to the QCD charm-quark symmetry generator, we get two normalization conditions at threshold (there is no implied sum over M):

$$\begin{aligned}\langle\Omega_c^M(v,s)|\bar{c}\gamma^0 c|\Omega_c^M(v,s)\rangle = 1 = \langle\Omega_c^M(v,s)|\mathrm{T}\{(1+i\mathcal{L}')\bar{h}_v^{(c)}\gamma^0 h_v^{(c)}\}|\Omega_c^M(v,s)\rangle \\ - \tfrac{1}{2m_c}\langle\Omega_c^M(v,s)|\bar{h}_v^{(c)}(i\overleftarrow{\not{D}}\gamma^0 - i\gamma^0\overrightarrow{\not{D}})h_v^{(c)}|\Omega_c^M(v,s)\rangle \cdot \\ = \xi_1(1) + 2I_1^{MM}(1;\gamma^0) + 2I_3^{MM}(1;\gamma^0) + I_2^{MM}(1;\gamma^\lambda\gamma^0)_\lambda - \tilde{I}_2^{MM}(1;\gamma^0\gamma^\lambda)_\lambda\end{aligned} \tag{3.4.1}$$

Substituting $I_1^{11}(1;\gamma^0) = -3\eta_1(1)$, $I_1^{22}(1;\gamma^0) = \frac{3}{2}\eta_1(1)$, $I_2^{MM}(1;\gamma^0) = \tilde{I}_2^{MM}(1;\gamma^0) = 0$, and $I_3^{MM}(1;\gamma^0) = \tilde{\xi}_1(1)$, these conditions reduce to

$$\xi_1(1) = 1, \qquad \tilde{\xi}_1(1) - 3\eta_1(1) = 0, \qquad \tilde{\xi}_1(1) + \tfrac{3}{2}\eta_1(1) = 0. \tag{3.4.2}$$

Thus $\eta_1(1) = \tilde{\xi}_1(1) = 0$, and all $\mathcal{O}(\frac{1}{m_c})$ corrections vanish at threshold. In another paper[16] we explore the generality of this result. The distinct measurable quantities at threshold to $\mathcal{O}(\frac{1}{m_c^2})$ are [C is defined in (3.1.5)]

$$F_1(1) + F_2(1) + F_3(1) = C, \quad G_1(1) = {}^{-}\tfrac{1}{3}C, \quad K_4(1) = \tfrac{2}{\sqrt{3}}C. \tag{3.4.3}$$

5. Conclusions

Since $\tilde{\xi}_1(1) = 0$, we may absorb the contribution of $I_3^{MN}(w;\Gamma)$ into the definitions of $\xi_1(w)$ and $\xi_2(w)$ without affecting their normalization at $v = v'$. Then away from $w = 1$, we have seven unknown functions and one dimensional constant describing fourteen weak decay form factors. The results for the vector and axial currents are listed in Table 1. Recall that η_i and κ_i are $\mathcal{O}(\Lambda_{QCD}/m_c)$, while ξ_i are $\mathcal{O}(1)$. The relevant form factor is $C = 1.1$ times the sum of the nine entries; *e.g.*,

$$F_1 = 1.1 \times \Big[\tfrac{-w}{3}(1+\tfrac{\bar{\Omega}}{2m_c})\xi_1(w) + \tfrac{w^2-1}{3}(1+\tfrac{3\bar{\Omega}}{2m_c})\xi_2(w) + w\eta_1(w) + (1-w^2)\eta_2(w)$$
$$+ (w+1)\kappa_1(w) + w(w+1)\kappa_2(w)\Big] \qquad (3.5.1)$$

We can simplify these results by defining $\xi_i' = \xi_i - 3\eta_i$ and $\xi_i'' = \xi_i + 3\eta_i$ for $i = 1, 2$. The new relations, correct to $\mathcal{O}(\frac{1}{m_c})$, are listed in Table 2.

Predictions for form-factor relations are easier to construct in the Ω^* system since fewer nonperturbative functions contribute. For example, measurements of N_4 over a range of recoil momenta and a measurement of K_4 at a single kinematic point determine $\xi_1''(w)$ and $\bar{\Omega}$. Values of K_4 at subsequent kinematic points are then predicted by the results in Table 2.

At threshold, the $\mathcal{O}(\frac{1}{m_c})$ form-factor predictions coincide with the $\mathcal{O}(1)$ predictions. Perturbative corrections have been calculated[12] and are easily incorporated into the results of this paper. Thus, if the Ω_b is seen in the near future, threshold measurements of the decays discussed here should yield a value for the Kobayashi-Maskawa angle V_{cb} with theoretical uncertainties of order $\frac{1}{m_c^2}$. While these uncertainties are likely to be larger in the present system than in, say, the B-D system, alternate systems capable of extracting V_{cb} have additional importance in understanding higher-order corrections. The existence of three separate systems in which rates are predicted to $\mathcal{O}(\frac{1}{m_c})$ provides a useful laboratory for both theoretical and experimental physics.

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7. Tables of Form Factors

	ξ_1	ξ_2	$\frac{\Omega}{2m_c}\xi_1$	$\frac{\Omega}{2m_c}\xi_2$	η_1	η_2	η_3	κ_1	κ_2
F_1	$-\frac{w}{3}$	$\frac{w^2-1}{3}$	$-\frac{w}{3}$	w^2-1	w	$1-w^2$	0	$w+1$	$w(w+1)$
G_1	$-\frac{w}{3}$	$\frac{w^2-1}{3}$	$\frac{w}{3}\frac{1-w}{1+w}$	$(w-1)^2$	w	$1-w^2$	0	$w-1$	$w(w-1)$
F_2	$\frac{2}{3}$	$\frac{2(1-w)}{3}$	$\frac{-2}{3(w+1)}$	0	-2	$2w-2$	$1-w$	w	1
G_2	$\frac{2}{3}$	$-\frac{2(w+1)}{3}$	$\frac{2}{3(w+1)}$	0	-2	$2w+2$	$w+1$	$-w$	-1
F_3	$\frac{2}{3}$	$\frac{2(1-w)}{3}$	$\frac{2}{3}$	$2-2w$	-2	$2w-2$	$1-w$	$-2-w$	$-1-2w$
G_3	$-\frac{2}{3}$	$\frac{2(w+1)}{3}$	$\frac{2}{3}\frac{1-w}{1+w}$	$2w-2$	2	$-2w-2$	$-w-1$	$2-w$	$2w-1$
N_1	$\frac{-1}{\sqrt{3}}$	$\frac{w-1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}(w-1)}{2}$	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}w}{2}$
K_1	$\frac{-1}{\sqrt{3}}$	$\frac{w+1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}\frac{1-w}{1+w}$	0	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}(w+1)}{2}$	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}w}{2}$
N_2	0	0	$\frac{2}{\sqrt{3}(w+1)}$	0	0	0	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$
K_2	0	0	$\frac{2}{\sqrt{3}(w+1)}$	0	0	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
N_3	0	$\frac{2}{\sqrt{3}}$	0	0	0	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$
K_3	0	$\frac{-2}{\sqrt{3}}$	0	0	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
N_4	$\frac{-2}{\sqrt{3}}$	0	$\frac{-2}{\sqrt{3}}$	0	$-\sqrt{3}$	0	0	0	0
K_4	$\frac{2}{\sqrt{3}}$	0	$\frac{-2}{\sqrt{3}}\frac{1-w}{1+w}$	0	$\sqrt{3}$	0	0	0	0

Table 1: Form Factors from Original Functions

The form factors are given by $C = 1.1$ times the sum of the entries; see Eq. (3.5.1).

	ξ_1'	ξ_2'	$\frac{\bar{\Omega}}{2m_c}\xi_1'$	$\frac{\bar{\Omega}}{2m_c}\xi_2'$	η_3	κ_1	κ_2
F_1	$-\frac{w}{3}$	$\frac{w^2-1}{3}$	$-\frac{w}{3}$	w^2-1	0	$w+1$	$w(w+1)$
G_1	$-\frac{w}{3}$	$\frac{w^2-1}{3}$	$\frac{w}{3}\frac{1-w}{1+w}$	$(w-1)^2$	0	$w-1$	$w(w-1)$
F_2	$\frac{2}{3}$	$\frac{2(1-w)}{3}$	$\frac{-2}{3(w+1)}$	0	$1-w$	w	1
G_2	$\frac{2}{3}$	$-\frac{2(w+1)}{3}$	$\frac{2}{3(w+1)}$	0	$w+1$	$-w$	-1
F_3	$\frac{2}{3}$	$\frac{2(1-w)}{3}$	$\frac{2}{3}$	$2-2w$	$1-w$	$-2-w$	$-1-2w$
G_3	$-\frac{2}{3}$	$\frac{2(w+1)}{3}$	$\frac{2}{3}\frac{1-w}{1+w}$	$2w-2$	$-w-1$	$2-w$	$2w-1$

	ξ_1''	ξ_2''	$\frac{\bar{\Omega}}{2m_c}\xi_1''$	$\frac{\bar{\Omega}}{2m_c}\xi_2''$	η_3	κ_1	κ_2
N_1	$\frac{-1}{\sqrt{3}}$	$\frac{w-1}{\sqrt{3}}$	$\frac{-1}{\sqrt{3}}$	0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}w}{2}$
K_1	$\frac{-1}{\sqrt{3}}$	$\frac{w+1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}\frac{1-w}{1+w}$	0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}w}{2}$
N_2	0	0	$\frac{2}{\sqrt{3}(w+1)}$	0	$\sqrt{3}$	$-\sqrt{3}$	$\sqrt{3}$
K_2	0	0	$\frac{2}{\sqrt{3}(w+1)}$	0	$-\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
N_3	0	$\frac{2}{\sqrt{3}}$	0	0	$\sqrt{3}$	$\sqrt{3}$	$-\sqrt{3}$
K_3	0	$\frac{-2}{\sqrt{3}}$	0	0	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}$
N_4	$\frac{-2}{\sqrt{3}}$	0	$\frac{-2}{\sqrt{3}}$	0	0	0	0
K_4	$\frac{2}{\sqrt{3}}$	0	$\frac{-2}{\sqrt{3}}\frac{1-w}{1+w}$	0	0	0	0

Table 2: Form Factors from Shifted Functions

For each decay separately, the functions η_1 and η_2 can be absorbed into ξ_1 and ξ_2, respectively, without changing the normalization condition $\xi_1(1) = 1$. We define $\xi_i' \equiv \xi_i - 3\eta_i$ and $\xi_i'' \equiv \xi_i + \frac{3}{2}\eta_i$.

IV. VANISHING OF $\frac{1}{m}$ CORRECTIONS AT THRESHOLD*

1. Introduction

Recent advances in heavy quark physics[1–3,17] increase the predictive power of QCD and allow the determination of previously unknown hadronic transition amplitudes. To lowest order, this is accomplished by exploiting the $SU(2N)$ spin-flavor symmetry[1] present in a system with N heavy quarks to derive relations among form factors in numerous systems[12–14,18], including $B \to D$, $\Lambda_b \to \Lambda_c$, and $\Omega_b \to \Omega_c$. Higher-order corrections can be calculated and understood in terms of an effective field theory[2,15], which incorporates $SU(2N)$ breaking terms as perturbations in a $\lambda \equiv \Lambda_{\rm QCD}/m$ expansion. In all three systems, it is found[7,9,10] that all $\mathcal{O}(\lambda)$ corrections vanish at threshold.

We show that this is a general result that applies to any hadronic transition amplitude in which the initial and final particles are composed of a heavy quark or scalar plus light degrees-of-freedom in an arbitrary spin state. This result holds because the leading order corrections to the effective current make no contribution-at threshold. This allows use of the Ademollo-Gatto Theorem[19], which says that the leading corrections to the effective Lagrangian also make no contribution.

The outline of this chapter is as follows: We first write down the effective Lagrangian and current, and make explicit the $SU(4)$ symmetry present to leading order. We examine the consequences of normalization conditions at threshold and compare

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them to similar conditions in chiral $SU(3)_{\text{flavor}}$. In Section 3 we remove obstacles to forming an SU(4) current algebra valid to $\mathcal{O}(\lambda)$, namely, the current corrections. We then apply the methods of Ademollo and Gatto in Sections 4 and 5, concluding that all $\mathcal{O}(\lambda)$ corrections vanish at threshold. We call this result Luke's Theorem. The theorem is generalized to heavy scalars in Section 6. Finally, we summarize our findings in Section 7.

2. Luke's Theorem: A First View

The effective theory is written in the fields $h_v^{(c)}$, satisfying $\not{v} h_v^{(c)} = h_v^{(c)}$, given by

$$h_v^{(c)} = \left(\tfrac{1+\not{v}}{2}\right) c\, e^{im_c v\cdot x} = (1 - \tfrac{i\not{D}}{2m_c}) c e^{im_c v\cdot x} \qquad (4.2.1)$$

so that

$$c = e^{-im_c v\cdot x}(1 - \tfrac{i\not{D}}{2m_c})^{-1} h_v^{(c)} = e^{-im_c v\cdot x} \sum_{n=0}^{\infty} (\tfrac{i\not{D}}{2m_c})^n h_v^{(c)} \qquad (4.2.2)$$

and similarly for $h_v^{(b)}$. By inserting the $\frac{1}{m}$ expansion into the QCD Lagrangian, we derive the effective Lagrangian to $\mathcal{O}(\lambda)$*,

$$\mathcal{L}' = \bar{h}_{v'}^{(c)} iv' \cdot D h_{v'}^{(c)} - \frac{1}{2m_c} \bar{h}_{v'}^{(c)} \left[D^2 + \tfrac{1}{2} g_s \sigma_{\mu\nu} G^{\mu\nu}\right] h_{v'}^{(c)} + (b, v \leftrightarrow c, v') \qquad (4.2.3)$$

in which $G^{\mu\nu}$ denotes the gluon field-strength tensor, and the flavor-changing currents become, to $\mathcal{O}(\lambda)$,

$$\begin{aligned} V^\mu_{\text{weak}}(x) &= \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{\frac{-6}{25}} \bar{h}_{v'}^{(c)} \left[\gamma^\mu - \frac{i}{2m_c} \overleftarrow{\not{D}} \gamma^\mu\right] h_v^{(b)}(x) e^{-i(m_b v - m_c v')\cdot x} \\ A^\mu_{\text{weak}}(x) &= \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{\frac{-6}{25}} \bar{h}_{v'}^{(c)} \left[\gamma^\mu \gamma_5 - \frac{i}{2m_c} \overleftarrow{\not{D}} \gamma^\mu \gamma_5\right] h_v^{(b)}(x) e^{-i(m_b v - m_c v')\cdot x} \end{aligned} \qquad (4.2.4)$$

*Our definition (4.2.1) differs slightly from that of References [2,15,9], but this alters the Lagrangian only by terms which vanish by the equations of motion.

at the scale $\mu = m_c$.

Conservation of charm states that the matrix element of $\bar{c}\gamma^0 c$ is unity order by order in $\frac{1}{m}$. Schematically,

$$\langle \bar{c}\gamma^0 c\rangle = 1 = \langle \bar{h}\gamma^0 h\rangle + \tfrac{1}{2m}\langle i\mathcal{L}'\bar{h}\gamma^0 h + \bar{h}i\not{D}\gamma^0 h\rangle \tag{4.2.5}$$

between identical states. This was first used by Luke[7] to show the vanishing of $\mathcal{O}(\lambda)$ corrections to the B-D threshold amplitudes. In general, a vanishing $\not{D}$ contribution implies a vanishing $\mathcal{L}'$ contribution to the vector transition element between states of identical spin, but does not obviously rule out nonzero contributions to transitions involving spin flip. Rather than try to rule out such contributions with the usual tensor methods, we will approach the problem using current algebra.

We may rewrite the effective Lagrangian as

$$\mathcal{L} = \bar{\Psi} iv\cdot D\Psi - \bar{\Psi}M\not{D}\not{D}\Psi, \tag{4.2.6}$$

which has the SU(4) Noether currents

$$j^\mu_a = \bar{\Psi}\gamma^\mu t_a\Psi - i\bar{\Psi}M\not{D}\gamma^\mu t_a\Psi, \tag{4.2.7}$$

where $M = \frac{1}{2}\,diag[\frac{1}{m_c}, \frac{1}{m_c}, \frac{1}{m_b}, \frac{1}{m_b}]$, $\bar{\Psi} = (\bar{h}^{(c)}_+, \bar{h}^{(c)}_-, \bar{h}^{(b)}_+, \bar{h}^{(b)}_-)$, $h^{(q)}_{+(-)}$ is the rescaled q field with spin up (down), and the t_a are the SU(4) generators.

To leading order, the Noether currents form a SU(4) algebra

$$[j^0_a(\vec{x}), j^0_b(\vec{y})]_{e.t.} = if_{abc}j^{0c}(\vec{x})\delta^3(\vec{x}-\vec{y}). \tag{4.2.8}$$

It is convenient to work in the meson rest frame, where the relation between effective currents and QCD currents J (assembled from QCD fields c and b) which change heavy flavor from q_i to q_j is $J(x) = e^{-i(m_{q_i}-m_{q_j})t}j(x)$. There is a strong renormalization factor [see Eq. (4.2.4)], which will remain implicit.

The heavy quark symmetry is strongly reminiscent of the approximate chiral (u, d, s) light quark symmetry. In chiral SU(3), the current algebra was used by Ademollo and Gatto[19] to show that the vector current transition amplitudes are uncorrected to $\mathcal{O}(\frac{m_s}{\Lambda_{QCD}})$ at $q^2 = 0$. No analogous statement can be made for axial chiral currents because the $SU(3)_{\text{axial}}$ is spontaneously broken, a hindrance that is absent in the heavy quark case. On the other hand, the heavy quark symmetry-breaking term involves derivatives of the fields, which alters the Noether current at $\mathcal{O}(\lambda)$. We address this issue first.

3. The $\not{D}$ Lemma

In order to apply the method of Ademollo and Gatto to $\mathcal{O}(\lambda)$ corrections in the Lagrangian, we must first show that the $\not{D}$ corrections to the current give no contribution to threshold transition elements. We will do this for baryons and mesons separately.

A useful mnemonic for doing the SU(4) group theory is to construct interpolating fields for the external states, contract the heavy fields with the current, and shift all information about the light degrees of freedom into a nonperturbative matrix element. The interpolating field for a spin $n - \frac{1}{2}$ baryon may be written

$$B(v) = \bar{l}^{\mu_1\mu_2\cdots\mu_n} \bar{h}^v_\alpha u_b^{\alpha(r)}(v) \epsilon^{(r)}_{\mu_1\mu_2\cdots\mu_n}, \tag{4.3.1}$$

where l represents the light degrees of freedom, $u_b^{\alpha(r)}(v)\epsilon^{(r)}_{\mu_1\mu_2\cdots\mu_n}$ describes the polarization of the baryon, r is summed over heavy quark spins, the μ_i are Lorentz indices, and α, β are Dirac indices. The light polarization tensor satisfies $v^{\mu_i}\epsilon_{\mu_1\cdots\mu_i\cdots\mu_n} = 0$. The spin $n + \frac{1}{2}$ baryon with heavy quark spin parallel to the same light degrees of freedom has an interpolating field

$$B^*(v) = l^{\mu_1\mu_2\cdots\mu_n} h^v \gamma^{\mu_{n+1}} \gamma_5 u_b^{(r)}(v) \epsilon^{(r)}_{\mu_1\mu_2\cdots\mu_n\mu_{n+1}}, \tag{4.3.2}$$

where Dirac indices are now suppressed.

A flavor-changing transition amplitude between spin $n - \frac{1}{2}$ baryons B_b and B_c thus contains

$$I_\lambda = \langle B_c(v')|\bar{h}^{v'}_{(c)} D_\lambda \Gamma h^v_{(b)}|B_b(v)\rangle = P_\lambda^{\mu_1\cdots\mu_{2n}} \epsilon^*_{\mu_1\cdots\mu_n} \epsilon_{\mu_{n+1}\cdots\mu_{2n}} \bar{u}_c(v') \tfrac{1+\not{v}'}{2} \Gamma \tfrac{1+\not{v}}{2} u_b(v), \tag{4.3.3}$$

where Γ is a collection of gamma matrices, and the nonperturbative matrix element may be schematically denoted by

$$P_\lambda^{\mu_1\cdots\mu_{2n}} = \langle 0|l^{\mu_1\cdots\mu_n} D_\lambda l^{\mu_{n+1}\cdots\mu_{2n}}|0\rangle. \tag{4.3.4}$$

This is the $\not{D}$ contribution. At threshold ($v = v'$), the index λ can be carried only by v_λ or $g_\lambda^{\mu_i}$, but in the latter case at least one of the remaining upper indices must be carried by v^{μ_j}, and from $v^{\mu_j}\epsilon_{\mu_j\cdots} = 0$ this vanishes. Thus $P_\lambda^{\mu_1\cdots\mu_{2n}} = v_\lambda X^{\mu_1\cdots\mu_{2n}}$.

The equations of motion imply that an on-shell transition amplitude with an insertion of $v \cdot Dh$ is suppressed by $\mathcal{O}(\lambda)$. Then $v^\lambda I_\lambda = 0$, forcing $X^{\mu_1\cdots\mu_{2n}} = 0$. Since the nonperturbative matrix elements for transitions involving B^* are the same, the $\not{D}$ contribution to any transition element in the $B - B^*$ system vanishes at threshold.

If one sums over initial and final spins, the above argument extends trivially to meson systems. Amplitudes between states of definite polarization require a bit more work.

The light degrees of freedom, with half-integral spin, can be written using only Dirac indices. For example, the interpolating field for a spin-n meson in which the light degrees of freedom carry spin $n+\frac{1}{2}$ is written

$$M = l^{\alpha_1\cdots\alpha_{2n}\beta}\epsilon_{\mu_1\cdots\mu_n}\gamma^{\mu_1}_{\alpha_1\alpha_2}\cdots\gamma^{\mu_n}_{\alpha_{2n-1}\alpha_{2n}}h^v_\beta. \tag{4.3.5}$$

The corresponding spin $n+1$ meson is

$$M^* = l^{\alpha_1\cdots\alpha_{2n}\beta}\epsilon_{\mu_1\cdots\mu_n\mu_{n+1}}\gamma^{\mu_1}_{\alpha_1\alpha_2}\cdots\gamma^{\mu_n}_{\alpha_{2n-1}\alpha_{2n}}(\gamma^{\mu_{n+1}}\gamma^5 h^v)_\beta. \tag{4.3.6}$$

At $v=v'$, an amplitude between spin n mesons looks like

$$\begin{aligned} I'_\lambda &= \langle M(v')|\bar{h}^{v'}D_\lambda\Gamma h^v|M(v)\rangle \\ &= P_\lambda^{\alpha_1\cdots\alpha_{4n}\beta_1\beta_2}\epsilon^*_{\mu_1\cdots\mu_n}\epsilon_{\mu_{n+1}\cdots\mu_{2n}}\gamma^{\mu_1}_{\alpha_1\alpha_2}\cdots\gamma^{\mu_{2n}}_{\alpha_{4n-1}\alpha_{4n}}\left(\tfrac{1+\not v}{2}\Gamma\tfrac{1+\not v}{2}\right)_{\beta_1\beta_2} \end{aligned} \tag{4.3.7}$$

If one of the particles is spin $n+1$, the Γ is replaced by $\Gamma\gamma^\kappa\gamma^5$, and one of the polarization tensors acquires an extra index κ. This will make no difference to the argument presented below.

The nonperturbative matrix element $P_\lambda^{\alpha_1\cdots\alpha_{4n}\beta_1\beta_2}$ now carries Dirac indices, which must be paired in Kronecker deltas $\delta_{\alpha_i\alpha_j}$ or contracted velocities $\not v_{\alpha_i\alpha_j}$. It will turn out that a $\not v$ is always equivalent to some combination of Kronecker deltas. The free index λ can be carried only by v_λ or γ_λ, so the amplitude will be a sum of terms, each of which contains either v_λ or γ_λ. As in the baryon case, $v^\lambda I'_\lambda = 0$ implies $I'_\lambda = 0$ if I'_λ is proportional to v_λ. We proceed to show that this is the case.

A given term in the amplitude containing γ_λ will be a product of traces of gamma matrices γ^{μ_i}, γ_λ, contracted velocities $\not v$, and $P^+\Gamma P^+$, where $P^\pm = \frac{1\pm\not v}{2}$ are velocity projection operators. The gamma matrices may be transposed, but this does not obstruct our proof. This can be seen by choosing the Majorana basis in which $\gamma^k = (\gamma^k)^T$, for spatial index k, and then going to the rest frame of the decaying meson. In this frame the naught components of the polarization tensors are zero, so the γ^{μ_i}

have only spatial indices. The argument below then gives $I'_\lambda = 0$ in the rest frame, and therefore in all frames.

A term containing γ_λ will be either of the form

$$\epsilon_{\mu_i \dots} \epsilon^*_{\mu_j \dots} \operatorname{Tr}[\gamma \cdots \gamma] \cdots \operatorname{Tr}[\gamma \cdots \gamma_\lambda] \operatorname{Tr}[\gamma \cdots \gamma \, P^+ \Gamma P^+\} \tag{4.3.8}$$

or of the form

$$\epsilon_{\mu_i \dots} \epsilon^*_{\mu_j \dots} \operatorname{Tr}[\gamma \cdots \gamma] \cdots \operatorname{Tr}[\gamma \cdots \gamma] \operatorname{Tr}[\gamma \cdots \gamma_\lambda \cdots \gamma \, P^+ \Gamma P^+\}. \tag{4.3.9}$$

All gamma matrices written without Lorentz indices are contracted with polarization tensors. There are no insertions of $\not v$ because any trace with a contracted velocity $\not v$ vanishes unless the trace contains either γ_λ, in which case the $\not v$ converts the γ_λ into v_λ, or $P^+ \Gamma P^+$, in which case the $\not v$ anticommutes through until it is absorbed by a projection operator.

Since the trace in line (4.3.8) containing γ_λ contains an odd number of contracted gamma matrices, the trace with projection operators also contains an odd number of contracted gamma matrices. However, $\gamma^\mu P^+ = P^- \gamma^\mu + v^\mu$, so this trace vanishes.

The last trace in line (4.3.9) contains an even number of contracted gamma matrices. By anticommuting $\frac{1+\not v}{2}$ around the trace, we convert γ_λ into v_λ, showing that I_λ is proportional to v_λ. Then $v^\lambda I'_\lambda = 0$ forces $I'_\lambda = 0$, completing our proof.

The above argument also implies the vanishing of some types of $\mathcal{O}(\lambda^2)$ threshold corrections, such as insertions of $\mathcal{L}'$ with the $\mathcal{O}(\lambda)$ part of the current $\langle \mathcal{L}' \not D \Gamma \rangle$, although this appears academic at present.

4. The Ademollo-Gatto Theorem

The heavy quark symmetry induces an SU(4)[18] among hadron quadruplets, such as $\{B, D, B^*, D^*\}$. We study, as an example, the vector-current matrix element for $B \to D$,

$$\langle D(v')|V^{\mu}_{\text{weak}}|B(v)\rangle = F_{+}(v\cdot v')(v+v')^{\mu} + F_{-}(v\cdot v')(v-v')^{\mu}. \tag{4.4.1}$$

We want to find $F_{+}(1)$ to $\mathcal{O}(\lambda)$. We use the $\mathrm{SU}(2)_{\text{vector}}$ subalgebra consisting of

$$Q^{+}_{v} = \int \mathrm{d}^3x \tfrac{1}{\sqrt{2}} \bar{h}^{(b)}_{v}(x)\gamma^0 h^{(c)}_{v}(x), \qquad Q^{-}_{v} = \int \mathrm{d}^3x \tfrac{1}{\sqrt{2}} \bar{h}^{(c)}_{v}(x)\gamma^0 h^{(b)}_{v}(x),$$

$$K_v = \int \mathrm{d}^3x \tfrac{1}{2}[\bar{h}^{(b)}_{v}(x)\gamma^0 h^{(b)}_{v}(x) - \bar{h}^{(c)}_{v}(x)\gamma^0 h^{(c)}_{v}(x)]. \tag{4.4.2}$$

Comparing to (4.2.4) and using the $\not{v}$ lemma, we see that between states of the same multiplet, Q^{-}_{v} is the charge corresponding to V^{μ}_{weak} to $\mathcal{O}(\lambda)$, up to a phase. To the same order, $K_v = \frac{1}{2}(b^{\dagger}b - c^{\dagger}c)$.

We sandwich the vector commutation relation

$$[Q^{+}_{v'}, Q^{-}_{v}] = K_v \delta_{vv'} \tag{4.4.3}$$

between $\langle B(v')|$ and $|B(v)\rangle$, and on the left side insert a complete set of states. It is convenient to work in the meson rest-frame (where $v^0 = 1$), and in a volume V; then states are normalized $\langle B|B\rangle = 2\mathrm{V}$, and we get

$$\frac{1}{2\mathrm{V}} \sum_{\alpha, v''} \langle B(v')|Q^{+}_{v'}|\alpha(v'')\rangle \, \langle \alpha(v'')|Q^{-}_{v}|B(v)\rangle - (Q^{+} \leftrightarrow Q^{-}) = \langle B(v)|K_v|B(v)\rangle. \tag{4.4.4}$$

To $\mathcal{O}(\lambda)$, the right side equals V.

The matrix elements on the left side can be written

$$\langle \alpha|Q^{\pm}|B\rangle = \frac{\langle \alpha|[H_{\text{eff}}, Q^{\pm}]|B\rangle}{E_{\text{eff}}(\alpha) - E_{\text{eff}}(B)} = \frac{\lambda \, \langle \alpha|[H'_{\text{eff}}, Q^{\pm}]|B\rangle}{E_{\text{eff}}(\alpha) - E_{\text{eff}}(B)}, \tag{4.4.5}$$

where H'_{eff} is the symmetry-breaking part of the effective Hamiltonian H_{eff}, and E_{eff} is the eigenvalue of H_{eff}; *e.g.*, $E_{\text{eff}}(B) = m_B - m_b$. When $|\alpha\rangle$ is not in the quadruplet, the matrix element is $\mathcal{O}(\lambda)$, and since these matrix elements appear in pairs – one insertion of $\mathcal{L}'$ to leave the quadruplet and another to return – the contribution is $\mathcal{O}(\lambda^2)$. Therefore, we keep only intermediate states within the quadruplet, of which only $|D\rangle$ contributes. The left side of Eq. (4.4.4) thus becomes

$$\frac{1}{2\mathrm{V}}\left|\langle D(v)|Q_v^-|B(v)\rangle\right|^2. \tag{4.4.6}$$

Eq. (4.4.4) then becomes

$$\frac{1}{2\mathrm{V}}[\sqrt{2}\mathrm{V}F_+(1)]^2 = \mathrm{V}, \tag{4.4.7}$$

from which we conclude that

$$F_+(1) = 1 + \mathcal{O}(\lambda^2). \tag{4.4.8}$$

Alternately, we could study the $B \to D^*$ transition, using

$$Q_v^{5+} = \int \mathrm{d}^3x \tfrac{1}{\sqrt{2}}\bar{h}_v^{(b)}(x)\gamma^3\gamma^5 h_v^{(c)}(x), \qquad Q_v^{5-} = \int \mathrm{d}^3x \tfrac{1}{\sqrt{2}}\bar{h}_v^{(c)}(x)\gamma^3\gamma^5 h_v^{(b)}(x), \tag{4.4.9}$$

which satisfy the axial commutation relation

$$[Q_{v'}^{5+}, Q_v^{5-}] = K_v\delta_{vv'}. \tag{4.4.10}$$

Now the form factor of the weak axial current A^3 is shown to be normalized to unity at threshold and uncorrected to $\mathcal{O}(\lambda)$.

5. The Wigner-Eckart Theorem

We generalize to an arbitrary mesonic or baryonic quadruplet $\{B, C, B^*, C^*\}$, with highest spin $s+1$. For given current and external states, the transition element is parameterized by velocity-dependent form factors which reduce at threshold to a single, reduced matrix element

$$\begin{array}{lll} \langle C|V^0_{\rm weak}|B\rangle = F_{BC}, & \langle C^*|V^0_{\rm weak}|B\rangle = 0, & \langle C^*|V^0_{\rm weak}|B^*\rangle = F_{B^*C^*}, \\ \langle C|A_{\rm weak}|B\rangle = G_{BC}, & \langle C^*|A_{\rm weak}|B\rangle = G_{BC^*}, & \langle C^*|A_{\rm weak}|B^*\rangle = G_{B^*C^*} \end{array} . \qquad (4.5.1)$$

A transition amplitude is the product of a reduced matrix element times a Clebsch-Gordan coefficient, *e.g.*,

$$\langle C^*(s+1)|A^3|B^*(s+1)\rangle = G_{B^*C^*}\,\langle s+1, s+1|1,0;\; s+1, s+1\rangle \qquad (4.5.2)$$

in the standard notation[20]. The Clebsch-Gordan coefficients are trivial in the vector case, since $V^0_{\rm weak}$ is spin-0.

We can sandwich the vector commutation relation between $|B\rangle$ states to show that $F_{BC} = 1$, and between $|B^*\rangle$ states to show that $F_{B^*C^*} = 1$. The sandwiched axial commutation relation between $|B^*(s{+}1)\rangle$ states gives, from Eq. (4.5.2), $G_{B^*C^*} = \langle s{+}1, s{+}1|1,0;\; s{+}1, s{+}1\rangle^{-1}$, while between $|B^*(s)\rangle$ states and between $|B(s)\rangle$ states we get, respectively,

$$\begin{array}{c} |\langle s{+}1, s|1,0;\; s{+}1, s\rangle G_{B^*C^*}|^2 + |\langle s{+}1, s|1,0;\; s,s\rangle G_{BC^*}|^2 = 1 \\ |\langle s{+}1, s|1,0;\; s,s\rangle G_{BC^*}|^2 \quad + \quad |\langle s,s|1,0;\; s,s\rangle G_{BC}|^2 \quad = 1 \end{array} , \qquad (4.5.3)$$

where we have used $\langle C(m')|A^3|B^*(m)\rangle^* = \langle C^*(m)|A^3|B(m')\rangle$ in the second term of the first equation. Thus, all the reduced matrix elements are determined to $\mathcal{O}(\lambda)$.

6. Luke's Theorem for Scalars

The above analysis is not limited to heavy fermions. For a heavy particle of arbitrary spin, one expects the leading term in the effective Lagrangian to contain

$v \cdot D$, and higher-order terms to be quadratic in derivatives. These are precisely the conditions employed is the $\not{l}\not{p}$ lemma to demonstrate the nonrenormalization of the effective current at threshold. The Ademollo-Gatto theorem can then be applied to the relevant spin-flavor symmetry.

The case of a heavy color triplet scalar has been done to leading order by Georgi and Wise[21]. They use effective scalar fields $\chi_v = e^{im_\chi v \cdot x}\chi$ in terms of which the effective Lagrangian is

$$\mathcal{L}_v = \bar{\Psi}_v M iv \cdot D\Psi_v - \bar{\Psi}_v N\Psi_v. \tag{4.6.1}$$

The heavy quark fields h_v have been assembled with the scalars in $\bar{\Psi} = (\bar{h}_v, \chi^*)$, the kinetic-energy term involves $M = diag[1, 2m_\chi]$, and the breaking term is $N = diag[\frac{\not{l}\not{p}\not{l}\not{p}}{2m_c}, iD^2]$.

At leading order, the above Lagrangian is invariant under SU(3) $\otimes$ U(1) transformations

$$\delta\Psi_v = i\begin{pmatrix} \epsilon \cdot S & E\sqrt{2m_\chi} \\ \frac{E}{\sqrt{2m_\chi}} & \epsilon_s \end{pmatrix}\Psi_v, \tag{4.6.2}$$

where S^μ are the $SU(2)_{\text{spin}}$ generators on the quark and E is an infinitesimal spinor obeying $\not{v}E = E$. The Noether current is

$$j^\mu = \bar{\Psi}_v M v^\mu t_a \Psi_v + i\bar{\Psi}_v \tilde{N} t_a \Psi_v, \tag{4.6.3}$$

where the t_a are the SU(3) generators and $\tilde{N} = diag[\frac{\not{l}\not{p}\gamma^\mu}{2m_c}, iD^\mu]$. Since the equations of motion imply that $v \cdot D\chi = 0$ when inserted in a physical matrix element, the current is uncorrected at threshold by the $\not{l}\not{p}$ lemma. The Ademollo-Gatto theorem then asserts vanishing $\mathcal{L}'$ corrections at threshold.

7. Conclusions

To lowest order, threshold transitions between heavy quark systems can be described by reduced matrix elements with known normalization. In the B-D system, for example, the only such element is the Isgur-Wise function[1], and this normalization allows extraction of the Kobayashi-Maskawa mixing angle V_{cb} with corrections of $\mathcal{O}(\lambda)$. Luke[7] has shown that this normalization is unchanged even at $\mathcal{O}(\lambda)$, allowing correspondingly reliable extraction of V_{cb}. We have shown that these matrix elements are uncorrected at $\mathcal{O}(\lambda)$ in general, for heavy quark systems of arbitrary spin.

This result includes both the correction to the currents, by our $\not{D}$ lemma, and corrections arising from insertions of the Lagrangian ($\mathcal{L}'$), by the Ademollo-Gatto Theorem. We have generalized it to heavy scalar particles, and we expect it to apply for heavy particles with general spin. The statement that $\mathcal{O}(\lambda)$ corrections vanish for arbitrary spin systems is Luke's Theorem.

Luke's Theorem unifies existing $\mathcal{O}(\lambda)$ calculations. However, while it applies in principle to any heavy quark system, in practice the experimentally accessible systems are those already examined. Its present value is thus largely pedagogical, unless new particles (squarks, technibosons, heavy four-quark bags, etc.) are discovered, a prospect with perhaps a glimmer of hope.

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postulating the result of the present paper.

V. THE NEUTRON ELECTRIC DIPOLE MOMENT

1. Standard Model Predictions

The Wisgur symmetry is unnecessary for analysis of some heavy quark processes. Contributions of top quark loops to the neutron electric dipole moment (nedm) can be studied perturbatively. Such studies are sensitive to new physics at the weak scale.

In the Standard Model with $\theta_{QCD} = 0$, the only CP-violating parameter is a complex phase in the CKM matrix. Since electric dipole moments are CP-violating quantities, Standard Model generation of a nedm involves W bosons. Unitarity of the CKM matrix requires at least two W loops, so that a nedm is order α^2_{weak}. It turns out[22] that the nedm is suppressed by an additional factor of $\alpha_{strong}(M_W)$. The end result is that nedm generation via edm's of fundamental quarks in the Standard Model is predicted to be less than about $10^{-34} e \cdot cm$. A larger contribution comes from baryon-meson loops[23], but these are also small: $|d| \lesssim 10^{-31} e \cdot cm$.

The current experimental limit on the nedm d is[24] $|d| \lesssim 10^{-25} e \cdot cm$. A measurement in the near future of a nonzero nedm would therefore signal the existence of physics beyond that of the Standard Model.

2. Multi-Higgs Physics

Until recently, most extensions of the Standard Model were believed to predict nedm's much smaller than the experimental limit (although one exception was the Minimal Supersymmetric Model, which was actually constrained by experiment[25]).

The reason is that most CP-violating operators one can write down for an effective theory below the W and Higgs masses are suppressed by either small CKM angles or powers of light quark to Higgs mass ratios.

The exception is the three-gluon operator $GG\tilde{G}$, where G is the gluon field-strength tensor and $\tilde{G}$ is its dual. This operator, as noted by Weinberg[4], is unsuppressed by powers of light quark masses and will typically dominate contributions to the nedm. It can appear in the low-energy effective Lagrangian of theories with several Higgs particles. Multi-Higgs theories generally contain additional complex couplings in the Higgs mass matrix if there are three or more Higgs doublets in the theory, or if there are two Higgs doublets and additional Higgs singlets.

When the heavy particles (top quarks, W and Z bosons, and Higgs) are integrated out of the theory, the operators with complex couplings induce nonrenormalizable CP-violating operators such as $GG\tilde{G}$. In two-Higgs models with additional singlets, only neutral Higgs contribute to $GG\tilde{G}$, which in turn contributes to the wavefunction of the neutron nonperturbatively. Using naive dimensional analysis[26] to estimate the effect of the three-gluon operator on the neutron, one arrives at a predicted nedm very near current experimental thresholds.

In theories with more than two Higgs doublets, charged Higgs can also induce the three-gluon operator. Indeed, over much of parameter space, charged Higgs processes dominate[27]. This is the topic of the next chapter.

VI. Effective Hamiltonian for the Electric Dipole Moment of the Neutron*

The standard model with three generations of quarks and leptons and a single Higgs doublet has only two sources of CP violation: the vacuum angle for the strong interactions $\theta_{\rm QCD}$[28] and the phase δ in the Kobayashi-Maskawa matrix[29]. The stringent experimental limit on the electric dipole moment of the neutron, $d_n \lesssim 10^{-25}\ e-cm$[24], gives rise to the bound $\theta_{\rm QCD} \lesssim 10^{-9}$ on the strong interaction vacuum angle[30]. The phase δ, however, is not restricted by the present limit on the electric dipole moment of the neutron, and in the minimal standard model it must be the source of the CP violation observed in kaon decays[31].

Understanding the conservation of CP by the strong interactions is an important problem in particle physics. Speculative explanations for the smallness of $\theta_{\rm QCD}$ do exist. For example, there could be a U(1) Peccei-Quinn symmetry[32] that is spontaneously broken at a large energy scale. This converts $\theta_{\rm QCD}$ to a dynamical variable, which is determined to be near zero by minimizing the vacuum energy. Alternatively, if quantum fluctuations in the topology of spacetime occur, the wave function of the universe may be infinitely strongly peaked on the subspace of universes where $\theta_{\rm QCD}$ is near zero[33].

In the standard model only one Higgs doublet is required spontaneously to break SU(3)×SU(2)×U(1) to the low-energy gauge group SU(3)×U(1) and to give mass to the quarks and leptons. However, there is no compelling physical reason for such a minimal Higgs sector. Extensions of the standard model motivated by the hierarchy

*C.G. Boyd, A. Gupta, S. Trivedi and M. Wise, *Phys. Lett.* **B241**:584 (1990).

puzzle typically have a much more complicated Higgs sector. One of the simplest extensions of the Higgs sector is the addition of more doublets. With one doublet the Higgs sector contributes a single real scalar to the spectrum of the theory. It automatically has no flavor-changing couplings to quarks; its coupling to quarks is proportional to their mass matrices so the same transformation that diagonalizes the quark mass matrices diagonalizes its couplings. With n doublets the physical degrees of freedom arising from the Higgs sector are $(2n-1)$ neutral scalars and $(n-1)$ charged scalars. It is no longer automatic that the tree level couplings of the neutral scalars are flavor diagonal. However, flavor-changing tree level couplings of the neutral scalars are absent if the up-type quark Yukawa couplings involve only one of the doublets and the down-type quark Yukawa couplings also involve only one of the doublets[34]. We consider a model of this type with n doublets $H_j, j = 1, \dots n$ where H_1 gives mass to the up-type quarks, H_2 gives mass to the down-type quarks, and the remaining $n-2$ doublets do not Yukawa couple to the quarks. For three or more doublets, there are new phases in the Higgs sector that contribute to CP violation[35]. In this Chapter we consider the influence of these new phases on the electric dipole moment of the neutron.

In this model the coupling of the charged Higgs $H_j^{(\pm)}$ to the quarks is given by the Lagrangian density

$$\mathcal{L} = -\frac{1}{2v_1^*} H_1^{(+)}(\bar{u},\bar{c},\bar{t})\ M_U\ V(1-\gamma_5)\begin{pmatrix} d \\ s \\ b \end{pmatrix} + \frac{1}{2v_2^*} H_2^{(+)}(\bar{u},\bar{c},\bar{t})\ V\ M_D(1+\gamma_5)\begin{pmatrix} d \\ s \\ b \end{pmatrix} + \text{h.c.} \quad , \tag{6.0.1}$$

where

$$H_j = \begin{pmatrix} H_j^{(0)} \\ H_j^{(-)} \end{pmatrix} \quad , \quad H_j^{(-)\dagger} = H_j^{(+)} \tag{6.0.2}$$

and $\langle H_j^{(0)} \rangle = v_j$. In Eq. (6.0.1), V is the Kobayashi-Maskawa matrix and

$$M_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \quad , \quad M_D = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \quad . \tag{6.0.3}$$

The quark fields in Eq. (6.0.1) are mass eigenstate fields, but that is not true of the Higgs fields. The mass eigenstate charged scalars $\phi_j^{(+)}$ are related to the $H_j^{(+)}$ by a $n \times n$ unitary transformation

$$\phi_j^{(+)} = \sum_{k=1}^{n} Y_{jk}^{-1}\ H_k^{(+)} = \sum_{k=1}^{n} Y_{kj}^*\ H_k^{(+)} \quad . \tag{6.0.4}$$

One of the fields $\phi_j^{(+)}$ is the Goldstone boson associated with the spontaneous breakdown of SU(2)×U(1) to $\mathrm{U}(1)_{\mathrm{e.m.}}$. Without loss of generality, we take it to be $\phi_1^{(+)}$, so that

$$Y_{k1}^* = g_2\ v_k/\sqrt{2}\ M_W \quad . \tag{6.0.5}$$

Combining Eqs. (6.0.1), (6.0.4) and (6.0.5) gives[36]

$$\mathcal{L} = \sum_{k=2}^{n} \frac{g_2\, \phi_k^{(+)}}{2\sqrt{2}\, M_W} \left\{ -\left(\frac{Y_{1k}}{Y_{11}}\right) (\bar{u},\bar{c},\bar{t})\, M_U\, V(1-\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix} \right. \tag{6.0.6}$$

$$\left. + \left(\frac{Y_{2k}}{Y_{21}}\right) (\bar{u},\bar{c},\bar{t})\, V\, M_D(1+\gamma_5) \begin{pmatrix} d \\ s \\ b \end{pmatrix} \right\} + \text{h.c.}$$

In unitary gauge the field $\phi_1^{(+)}$ becomes the longitudinal component of the W-boson field and so it is omitted from Eq. (6.0.6). (Its couplings are independent of Y.) For $n \geq 3$ the matrix Y contains phases that cannot be removed by redefining the phases of the fields $\phi_j^{(+)}$ and hence are a source of CP violation.

Integrating out the scalar degrees of freedom (we assume that none of the scalars is significantly lighter than the W-boson), the top-quark and the W- and Z-gauge bosons, generates an effective Hamiltonian for CP violating quantities. In the case of the electric dipole moment of the neutron, one is interested in flavor-singlet operators that violate parity P and time-reversal T. In addition to renormalizing the value of the vacuum angle θ_{QCD} and the Kobayashi-Maskawa phase δ, this procedure will generate nonrenormalizable operators whose matrix elements contribute to the electric dipole moment of the neutron. The nonrenormalizable operators of dimension six that can contribute to the electric dipole moment of the neutron (and are not suppressed by light up, down and strange quark masses) are

$$O_1 = ig^3\, \text{Tr}[G_{\mu\rho}\, G_\nu{}^\rho\, G_{\lambda\sigma}]\, \epsilon^{\mu\nu\lambda\sigma} \tag{6.0.7a}$$

$$O_2 = g\, m_b\, \bar{b}\, T^a\, \sigma_{\mu\nu}\, b\, G^a_{\lambda\sigma}\, \epsilon^{\mu\nu\lambda\sigma} \tag{6.0.7b}$$

$$O_3 = e\, m_b\, \bar{b}\, \sigma_{\mu\nu}\, b\, F_{\lambda\sigma}\, \epsilon^{\mu\nu\lambda\sigma} \tag{6.0.7c}$$

, where F is the electromagnetic field-strength tensor. In Eq. (6.0.7) we have included a factor of m_b with O_2 and O_3, since a factor of m_b must accompany a (single) right-handed bottom quark. This factor increases the dimension of O_2 and O_3 by one unit. We have neglected other dimension-six operators (e.g., the color-electric dipole moment of the charm quark and four-quark operators) whose coefficients are suppressed (in the multi-Higgs models considered here) by factors of $(m_b/M_W)^2$ or $(m_c/M_W)^2$. The effective Hamiltonian density is

$$H_{\text{eff}} = \sum_{i=1}^{3} C_i(\mu)\, O_i(\mu) \quad . \tag{6.0.8}$$

The coefficients $C_i(M_W)$ have a perturbative expansion in $\alpha_s(M_W)$.

In Ref. [4] it was noted that the operator O_1 gives a contribution to the electric dipole moment of the neutron that is not suppressed by light (compared with the QCD scale) quark masses. Its coefficient $C_1(M_W)$ has been computed and arises from two-loop graphs involving neutral scalar exchange[4,37]. Charged scalar exchange does not contribute (again neglecting terms suppressed by $(m_b/M_W)^2$ and $(m_c/M_W)^2$) to $C_1(M_W)$.

The coefficient $C_2(M_W)$ can be computed from the one-loop graph in Fig. 1. In this case, the contribution from charged scalar exchange dominates over neutral scalar exchange. To compute $C_2(M_W)$ we work off-shell, matching the amputated one-particle-irreducible $b \to b+$ gluon Green function in the complete theory with that in the effective theory. Since we are working off-shell, the graph in Fig. 1 can contribute not only to the coefficient of O_2 but also to the coefficient of the operator $O_4 = \bar{b}\not{D}\not{D}\gamma_5 b$, which becomes a mass term when the equations of motion are applied. As noted in Ref. [38], the contribution of O_2 can be isolated by focussing on the Lorentz structure $\gamma_\mu \not{k}\, \gamma_5$, where μ is the gluon vector index and k is its four-momentum.

Using this procedure (and the approximation $|V_{tb}|^2 \sim 1$), we find that

$$C_2(M_W) = \frac{G_F}{\sqrt{2}} \frac{1}{16\pi^2} \sum_{k=2}^{n} Im\left\{\left(\frac{Y_{2k}}{Y_{21}}\right)\left(\frac{Y_{1k}}{Y_{11}}\right)^*\right\} \cdot x_k \left\{\frac{1}{(x_k-1)^3}\, \ell n\, x_k + \frac{1}{2(x_k-1)^2}(x_k-3)\right\}, \tag{6.0.9}$$

where

$$x_k = m_t^2/m_{\phi_k}^2 \quad . \tag{6.0.10}$$

With a subtraction point equal to M_W (the W-boson mass), there are large logarithms of M_W divided by the QCD scale in the perturbative expansion of the matrix elements of the operators O_1, O_2 and O_3. These can be transferred from the matrix elements of the operators to their coefficients C_j by using the renormalization group equations to move μ down to the scale of the strong interactions. The anomalous dimension for O_1 was calculated in Ref. [39] and the anomalous dimension for O_2 was calculated in ref. [40]. Using these results, it follows that for $M_W > \mu > m_b$,

$$C_1(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{54/23} C_1(M_W) \quad , \tag{6.0.11a}$$

$$C_2(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{14/23} C_2(M_W) \quad . \tag{6.0.11b}$$

Since $C_2(M_W)$ arises at two loops while $C_1(M_W)$ arises at one loop, we have neglected in Eq. (6.0.11a) possible mixing under renormalization of O_2 with O_1 (operator O_1 cannot mix with O_2 because with the factor of m_b removed, it is only dimension five).

At the scale $\mu = m_b$, it is appropriate to go over to an effective four-quark theory. In the effective four-quark theory, only the operator O_1 survives. However, because the coefficient $C_1(M_W)$ arises at two-loops, it is important to include the contribution to the coefficient of O_1 (in the effective four-quark theory) that comes from matching the one-loop Feynman diagrams in Figs. 2 and 3 with the tree level amputated three-gluon Greens function of O_1. In Figs. 2 and 3 the shaded square denotes an insertion of O_2. Explicit calculation of Fig. 2 gives

$$
\begin{aligned}
-\frac{g^3}{6\pi^2}\,\mathrm{Tr}\Big(T^d[T^f,T^e]\Big)\Big[&2\epsilon_\beta{}^{\nu\lambda}{}_\alpha\, p_\lambda\, r_\nu p_\gamma \;+\; \epsilon_\beta{}^{\nu\lambda}{}_\alpha\, p_\lambda\, r_\nu\, r_\gamma \\
&+\epsilon_\gamma{}^{\nu\lambda}{}_\alpha\, p_\lambda\, r_\nu\, p_\beta \;-\; \epsilon_\gamma{}^{\nu\lambda}{}_\alpha\, p_\lambda\, r_\nu\, r_\beta \;+\; \epsilon_\gamma{}^{\nu\lambda}{}_\beta\, r_\lambda\, p_\nu\, p_\alpha \\
&+2\epsilon_\gamma{}^{\nu\lambda}{}_\beta\, r_\lambda\, p_\nu\, r_\alpha \;-\; 2\epsilon_{\beta\gamma}{}^{\lambda}{}_\alpha\Big(p_\lambda(r^2+2r\cdot p)+r_\lambda\big(p^2+2(r\cdot p)\big)\Big)\Big]
\end{aligned}
\quad , \qquad (6.0.12)
$$

while explicit calculation of Fig. 3 gives

$$
-\frac{g^2}{6\pi^2}\,\mathrm{Tr}\Big(T^d[T^f,T^e]\Big)\epsilon_{\beta\gamma}{}^{\lambda}{}_\alpha\Big(r_\lambda(p^2+2p\cdot r)+p_\lambda(r^2+2p\cdot r)\Big) \quad . \qquad (6.0.13)
$$

Comparing the sum of (12) and (13) with the amputated three gluon tree level Greens function of O_1 gives that in the effective four-quark theory,

$$
C_1(m_b)=\left[\frac{\alpha_s(M_W)}{\alpha_s(M_b)}\right]^{54/23}\left\{C_1(M_W)+\frac{1}{12\pi^2}\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{-40/23}C_2(M_W)\right\} \quad . \qquad (6.0.14)
$$

Note that the operator O_3 does not match onto O_1 at one-loop, so its effects can be neglected (they will be suppressed by powers of the QCD scale divided by the bottom quark mass). Finally, moving the subtraction point μ below the charm-quark mass, we go over to an effective three-quark theory where

$$
\begin{aligned}
C_1(\mu)=&\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{54/23}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{54/25}\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right]^{54/27} \\
&\cdot\left\{C_1(M_W)+\frac{1}{12\pi^2}\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{-40/23}C_2(M_W)\right\} \quad .
\end{aligned}
\qquad (6.0.15)
$$

Eqs. (6.0.15) and (6.0.9) are the main results of this letter.

The strong interaction corrections suppress the effects of $C_1(M_W)$ relative to those of $C_2(M_W)$ by the factor $[\alpha_s(M_W)/\alpha_s(m_b)]^{40/23} \sim 0.3$. Also, there are special cases, for example, when $|Y_{21}| << 1$, where a further enhancement of $C_2(M_W)$ occurs.

Towards the completion of this work we received a copy of Ref. [41], where $C_1(M_W)$ is computed in minimal low-energy supergravity models. There it is also noted that at the weak scale, the color-electric dipole moment of heavy quarks (compared with the QCD scale) should, in principle, be included in the effective Hamiltonian. Similar work to that presented here has been done independently by M. Dine and W. Fishler (CCNY-HEP-89/21 and UTTG-03-90) and E. Braaten, C.S. Li and T.C. Yuan[39]. We thank them for discussing their work with us prior to its publication.

Fig. 1. Feynman diagram that determines the value of $C_2(M_W)$.

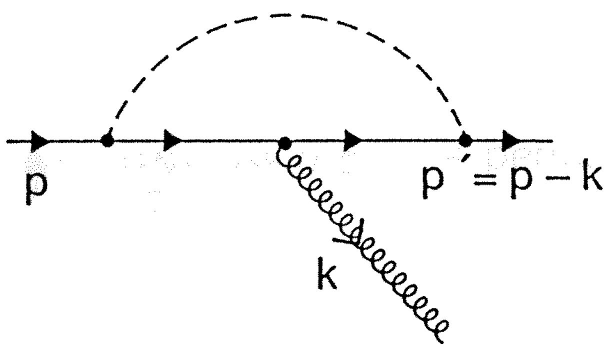

FIGURE 1

Fig. 2. Some of the Feynman diagrams that determine the matching between $C_2(m_b)$ in the effective five-quark theory and $C_1(m_b)$ in the effective four-quark theory.

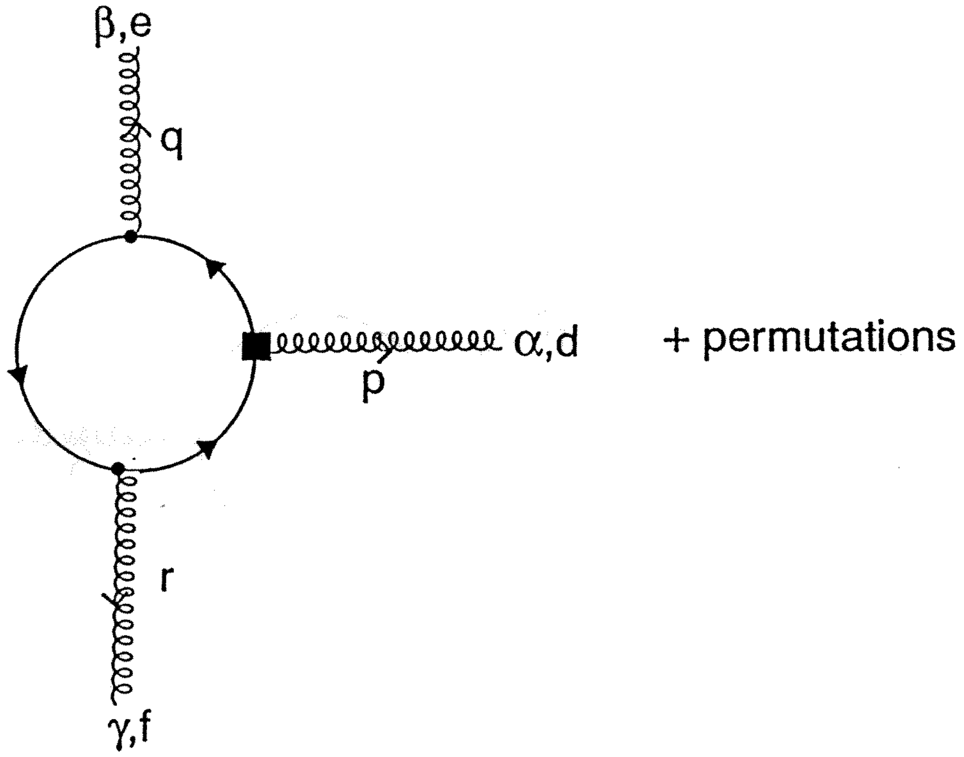

FIGURE 2

Fig. 3. Remainder of the Feynman diagrams that determine the matching between $C_2(m_b)$ in the effective five-quark theory and $C_1(m_b)$ in the effective four-quark theory.

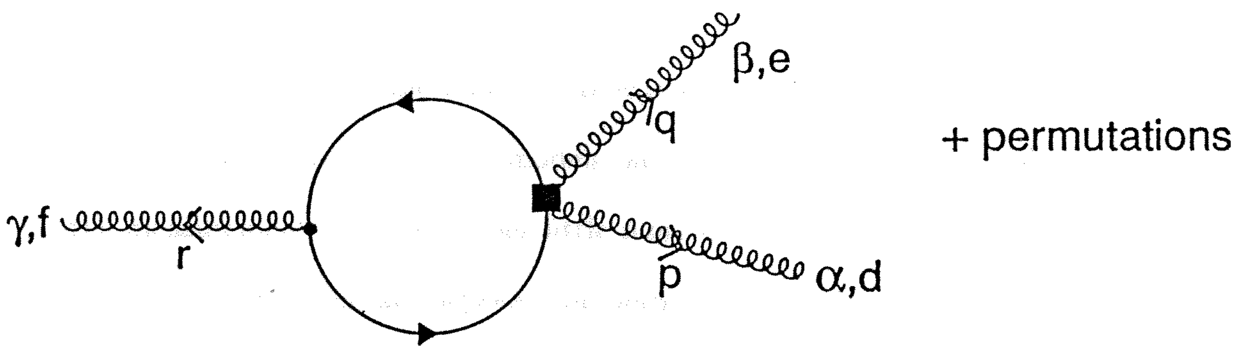

FIGURE 3

VII. Outlook

What have we learned from the preceding chapters? For the neutron electric dipole moment, we know that a value of $10^{-26}e \cdot cm$ is incompatible with the Minimal Standard Model, but is compatible with multi-Higgs models with or without charged Higgs exchange. The effect of charged Higgs on the neutron electric dipole moment is determined by integrating the heavy Higgs out of the theory, thus generating a color magnetic moment for the b quark. After renormalizing down to the b mass scale and integrating out the b quark, Weinberg's three gluon operator is induced. This operator then feeds into the neutron wavefunction to produce an electric dipole moment.

In addition to simple multi-Higgs extensions of the Standard Model, left-right symmetric and supersymmetric models can induce similarly large dipole moments, so the measurement of a nedm does little more than rule out the Standard Model. It is difficult to differentiate between competing models using only the neutron electric dipole moment.

One reason for this difficulty is the need for some estimation scheme, such as naive dimensional analysis, to determine the effect of the Weinberg operator on the neutron wavefunction. This problem is not present for the electron electric dipole moment, to which operators similar to Weinberg's can induce a moment near current experimental limits[42]. This allows quantitative information about new physics to be extracted from the electron electric dipole moment, although it will still be difficult to rule out competing models because of the presence of free parameters in most theories.

More precise statements about physics beyond the Standard Model can be made from CP-violating processes once the CKM angles are determined accurately. A useful tool for theoretical extraction of the angle V_{cb} from semileptonic flavor-changing decays is the Isgur-Wise heavy quark expansion (which has many other useful applications, of course).

This method uses an expansion in $\frac{\Lambda_{QCD}}{m}$ to make statements about nonperturbative, hadronic matrix elements. As seen previously, $\frac{1}{m}$ corrections can be enumerated, and form factors parameterized in terms of unknown Isgur-Wise functions. In the case of $\Omega_b \rightarrow \Omega_c$, the fourteen form factors can be described by seven unknown functions and one unknown constant with errors of order $\frac{1}{m^2}$.

For the semileptonic decays of Omegas, Lambdas, and B mesons, all $\frac{1}{m}$ corrections vanish at maximum momentum transfer. This allows extraction of V_{cb} to a few percent, because the remaining Isgur-Wise functions are normalized to unity. This is, in fact, a general property, as embodied in Luke's Theorem.

The existence of techniques such as those above improves the likelihood that data from the next generation of experiments will be instrumental in confirming, or more hopefully, initiating the downfall of the Standard Model.

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