A Theoretical Study of Political Institutions and Economic Policies

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Dedication

To my family,

Shi Xiaolin, Chen Fengliao and Chen Sen

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ABSTRACT

This dissertation consists of two relatively independent chapters that study the effects of political institutions on economic policies.

Chapter 1 studies the privatization policies of maximizing politicians in a tightly managed transition economy under different political institutions. The majority of literature pertaining to privatization policies ignores the political constraints and the motivation of the politicians. In this dissertation, we consider two types of politicians, a Niskanen-style Bureaucrat who maximizes a surplus budget subject to the constraint of staying in office, and a Populist who maximizes consumer welfare subject to the constraint of a balanced budget. Other things being equal, the Bureaucrat will privatize the sector (firms) with the least market power and the largest subsidy first. The Populist will adopt the same policy, if the marginal costs of products in the private sectors are not too high with respect to the marginal utilities. We also show that controlled privatization is easier and faster in less democratic societies.

Chapter 2 examines the effects that political processes, i.e., electoral systems and legislative processes, have on income taxation and public good allocation. We characterize the equilibrium income tax schedules under two types of political institutions. It is shown that, when there is a single district, for the two party plurality system the equilibrium income tax schedule is equivalent to an optimal tax schedule that puts equal weight over the whole population; when there are multiple districts, however, the simplest subgame perfect stationary equilibrium tax schedule of the stochastic leg-

islative game is equivalent to an optimal tax schedule that puts more welfare weight on the subsets of the population whose legislators are in the winning coalition of the legislature. Thus, the social welfare functions in the optimal taxation literature can be endogenously determined by explicitly modelling the political processes that determines them.

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Chapter 1

The Optimal Choice of Privatization

1.1 Introduction

Because of the universally recognized deficiencies of state-owned enterprises compared to private enterprises, there is almost no controversy over the necessity to transform central-planned economies into market economies. The controversy lies in how to perform the transition and the extent of the transition. This chapter builds on a model of transition economy in which a Public Servant, i.e., a politician, with different objectives and under different political institutions, must decide which enterprises to privatize first.

The sequence suggested by some economists and practiced in some of the Eastern

European countries roughly follows the size of different sectors (Li 1989; Blommesteine and Marrese [ed.] 1991): rapid privatization of small businesses first; establishment of a social safety net; demonopolization; privatization of medium state-owned enterprises; and last, privatization of large state-owned enterprises.

In discussing these policies, the political constraint is often ignored. In particular, there is no formal model in the privatization literature that incorporates the influence of politics on privatization policies. We need to remember that politicians choose the privatization policies. Therefore, it is important to see what kind of policies a maximizing politician would choose under different political institutions.

In this chapter, we set up a model to test the rationale of the sequences suggested above by economists of central planned economies and to study what kind of choice a Public Servant with different objectives would choose. Some features of the model, including the compensation scheme, are abstracted from the Chinese experience. We want to see what kind of choice is optimal for a Public Servant in the context of a controlled privatization process. Section 2 introduces a two period model of a highly simplified transition economy. Section 3 presents the analysis of the problem of the consumer, the firm, and the Public Servant respectively. Two types of Public Servants are considered: one who maximizes the surplus budget subject to the constraint of staying in office – the Bureaucrat, and one who maximizes consumer welfare subject to the constraint of a balanced budget – the Populist. Section 4 contains the main results of the model: other things being equal, the Bureaucrat will privatize the sector (firms) with the least market power[1] and the largest subsidy first. The Populist will adopt the same policy, if the marginal costs of products in the private sectors are not too high. Therefore, the result is quite robust to the specification of the politician's objectives. Also, we show that it can be relatively easier and faster to privatize in a less democratic society. Finally, in Section 5, we discuss the limitations and possible extensions of the model and conclude the chapter.

1.2 Setup and Basic Assumptions

This section presents a simplified two period model of a transition economy that consists of I consumers, $N+1$ sectors of firms, and a Public Servant.

Consumers have different utility functions and incomes, which are exogenously given. At time t, consumer i is rationed to a fixed amount of products from the public sectors at fixed prices. Because of the low prices and minimum amounts supplied, we assume that he buys all the quantities that are rationed to him. This assumption closely approximates the actual situations in many central planned economies. He uses the rest of his income to choose consumption bundles from the products of the private sectors to maximize his utility.

Assumption 1 *Each consumer has a quasilinear utility function,* $u_i(q_0, q_1, ..., q_N) = \mu_i(q_1, ..., q_N) + q_{i0}$*, where* q_1*, ...* q_N *are the amounts of products 1, ..., N he consumes,*

[1] This concept is introduced later. It basically captures the competitiveness of the sector and the elasticity of the product. The higher the market power of a sector, the more it can raise the price above marginal cost.

and q_0 *is a numeraire good* ($p_0 = 1$) *s.t.*

(i) $u : R_+^{N+1} \rightarrow R_+^1$ *is monotonically increasing;*

(ii) u *is twice continuously differentiable, and strictly concave;*

(iii) u *satisfies the following Inada conditions:*

$$lim_{q_j \downarrow 0} \frac{\partial}{\partial q_j} u_i(q_0, ..., q_N) = +\infty, \ lim_{q_j \uparrow \infty} \frac{\partial}{\partial q_j} u_i(q_0, ..., q_N) = 0.$$

The numeraire commodity is assumed to be already produced by the private sector by time $t-1$. It can be thought of as some nonperishable foodstuff.

Firms produce N distinct products and are hence divided into N sectors. Each sector j consists of L_j identical firms, where $L_j \in [1, +\infty)$. When $L_j = 1$, sector j is a monopoly; when L_j is large enough (approaching infinity), sector j is competitive; when L_j is between 1 and infinity, it can have different degrees of competitiveness (Tirole 1988). This also implies that all firms within a sector have the same cost function. At time t - 1, k sectors are public (denoted by sector 1, ... k), and N-k+1 sectors are private (denoted by sectors $k + 1$, $k + 2$, ..., N, 0). In each period the Public Servant can choose to privatize one or more sectors. At time t, the Public Servant decides whether to continue privatizing, and which sector(s) to privatize. Without loss of generality, we assume sector k is picked. We can then study the characteristics of k and the influence of its privatization on the changes in consumer welfare.

Assume that a firm lj in public sector j fulfills quota $\bar{Q}_{lj}$ imposed by the Public

Servant, and sells its output at a fixed price $\bar{P}_j$, which is below the market-clearing price. This assumption reflects a basic feature of centrally planned economies, where prices are fixed for historical reasons and reflect neither cost nor market demand[2]. Therefore, the total output of sector j is $\bar{Q}_j = \bar{Q}_{lj} L_j$. For simplicity, assume the population in the economy is fixed. Therefore, we can assume that $\bar{Q}_j$ and $\bar{Q}_{lj}$ are fixed as long as sector j remains public, since the quota is decided by rationing over the total population.

Assumption 2 *All firms in the same sector have the same cost function:*
C_j*:* $R^1_+ \to R^1_+$ *is differentiable, monotonic, and convex, for* $j = 0, 1, \cdots, N$.

This assumption simplifies the definition of the market power index defined and used in Section 1.3. After sector j is privatized, firm lj's objective becomes profit maximization. Let π_{lj} be the firm's profit function. It chooses the optimal output Q_{lj} and sells it at the market-clearing price P_j.

Let $P_j(Q_1, ..., Q_N)$ be sector j's inverse demand function. To ensure the existence of a Cournot equilibrium (Novshek 1985), we need the following assumption.

Assumption 3 $P_j(\vec{Q})$ *is twice continuously differentiable, monotonic, and sastisfies* $\frac{\partial P_j(\vec{Q})}{\partial Q_j} + Q_j \frac{\partial^2 P_j(\vec{Q})}{\partial Q_j^2} \leq 0$, *which requires the inverse demand function to be concave.*

[2] There are cases when there is no demand for the output of certain goods at the state-set prices, mostly in the production sector. In these cases, privatization may actually decrease the prices or change the products to something demanded by the market. Since there is little ambiguity in the privatization of these sectors and consumers do not need to be compensated for their privatization, we assume that they are privatized already and hence do not focus our attention on these cases.

Note that the cost function of firm lj does not change before or after privatization. Here we implicitly assume that technology does not change. What is changed is the production quantity and price, which is adjusted for the purpose of profit maximization. This implies that the objective functions of the firms change after privatization, but any efficiency gain occurs after the transition period[3].

In order to simplify the structure of the model, we assume that none of the products of the N sectors are substitutes for each other. They can be either independent or complements. Another way to think about this assumption is to group all the substitutes in the economy in the same sector and treat them as one product.

Assumption 4 $\frac{\partial Q_i}{\partial P_j} \leq 0, \forall i \neq j.$

This is equivalent to saying that the cross elasticity of any two products $\varepsilon_{ij} = -\frac{\partial q_i}{\partial P_j}\frac{P_j}{q_i} \geq 0, \forall i \neq j,\ i,j = 1,2,...,n$, which implies that the consumers' utility functions need to satisfy the following condition: $\frac{\partial^2 u_i(\vec{q})}{\partial q_i \partial q_l} \leq 0, \forall l = 1,...,N.$

We consider two types of Public Servant. Either type knows the distribution of consumers' utility functions[4] and of share ownership, and the matrices of supply and demand elasticities of every product. The reason for this assumption is to see what would be his best policy if he has enough information. At any given time t, he makes three decisions – whether to continue privatizing, which sector to privatize, and how to compensate the consumers. In order to concentrate on the characteristics of the

[3]Here we do not want to make *ad hoc* assumptions about the effects of privatization on cost, efficiency, or quality.

[4]But he does not know the exact utility function of each consumer.

transition period, we neglect some other important functions of the government, such as public good provision, and assume that the Public Servant's only functions are privatization and compensation. We use a parameter d to characterize the political institutions, where d is the percentage of consumers he needs to satisfy in order to stay in office.

1.3 Analysis of the Model

1.3.1 The Firm's Problem

Public firm lj in sector j is given the quota $\bar{Q}_{lj}$. Assume each firm is given the same quota, i.e., $\bar{Q}_{lj} = \bar{Q}_{kj}$, for all k, l. Suppose it can fulfill the quota and sell its output at the fixed price $\bar{P}_j$. Then it will provide revenue (or require subsidy) in the amount B_{lj}, where

$$B_{lj} = \bar{P}_j \bar{Q}_{lj} - C_j(\bar{Q}_{lj}).$$

After the firm is privatized, it becomes a profit maximizer. It chooses its optimal output Q_{lj} to maximize its profit. The price of product j is determined by the total output of the sector, which depends on the decisions of the other identical firms in the same sector and the total output of other sectors. Note that by Assumption 1, consumers all have quasilinear utility functions, so the inverse demand functions exist. We use Cournot equilibrium analysis for the private firms' decisions.

Firm lj chooses the optimal output Q_{lj} in order to

$$max_{Q_{lj}} P_j(Q_1, ..., Q_N)Q_{lj} - C_j(Q_{lj}).$$

From Assumption 2 and Assumption 3, the second order condition for the above maximization problem is satisfied, so we only need to look at the first order condition, which is,

$$P_j + Q_{lj}\left[\frac{\partial P_j}{\partial Q_j} + \sum_{h\neq j}\frac{\partial P_j(\vec{Q})}{\partial Q_h}\frac{\partial Q_h}{\partial Q_j}\right] - MC_j = 0,$$

where MC_j is the marginal cost of firms in sector j. Rearranging terms we get

$$\frac{P_j(\vec{Q}) - MC_j}{P_j(\vec{Q})} = \frac{1}{L_j}\left[\frac{1}{\varepsilon_{jj}} + \sum_{h\neq j}\frac{1}{\varepsilon_{jh}}\right] = \frac{1}{L_j}\sum_{h=1}^{N}\frac{1}{\varepsilon_{jh}} \equiv \alpha_j,$$

where ε_{jj} is the own elasticity of demand at Q_j, and ε_{jh} is the cross elasticity of demand between product j and product h. Since all firms of the same sector are identical, namely, they all have the same cost functions, the market share of firm lj equals the inverse of the number of firms in sector j, i.e., $\frac{Q_{lj}}{Q_j} = \frac{1}{L_j}$. Call α_j firm lj's *market power index*, which also characterizes sector j's market power. Alternatively, the above equation can be expressed as $P_j = \frac{MC_j}{1-\alpha_j}$, which will be used later.

Note that the market power index is quite general with regard to the degree of competitiveness in a sector. When $L_j = 1$, the above formula becomes the monopoly pricing formula. On the other hand, if $L_j \to \infty$, the equilibrium converges to the Cournot competitive equilibrium (Tirole 1988). Therefore, the market power index shows how much in equilibrium a sector can raise the price of its product above its marginal cost. This is inversely related to the number of firms in the sector and the elasticities of demand of the product.

1.3.2 The Consumer's Problem

In order to study the effects of the privatization of a certain sector, say k, on the change of a consumer's utility, we study his maximization problem in two arbitrarily chosen contiguous time periods, *t-1* and *t*.

At time *t-1*, sectors *1*, ..., k are in the public sector, fulfilling quotas; sectors *0*, *k+1*, ..., N are in the private sector, maximizing profits. Consumer i's rationed quantities of products *1* through k are $\bar{q}_1$, ..., $\bar{q}_k$, which are allocated equally to everybody in the economy. In reality, the allocations vary from person to person according to age, sex and other personal characteristics. Here, for simplicity of analysis, and also because we can not distinguish among individual consumers, we assume an equal allocation.

At time t, if another sector, say, sector k, is privatized, consumer i is given compensation T for the price increase in product k and the price changes in the other private sectors. At the same time, he can buy shares in the newly privatized sector. So he chooses q_{ik}^t, ..., q_{iN}^t, q_{i0}^t in order to

$$max_{\{q_{ij}^t\}_{j=0,k}^N} \quad u_i(\bar{q}_1, ..., \bar{q}_{k-1}, q_{ik}^t, ..., q_{iN}^t) + q_{i0}^t \tag{1.1}$$

$$s.t. \quad \sum_{j=0,k}^N P_j^t q_{ij}^t = y_i + \sum_{j=k}^N \theta_{ij}(1-\tau_j)\pi_j^t - \theta_{ik}S_k - \sum_{j=1}^{k-1} \bar{P}_j \bar{q}_j + T \equiv y_i^t \tag{1.2}$$

where S_k is the total revenue from the sale of sector k, τ_j is the tax rate of sector j, and θ_{ij} is consumer i's proportion of shares in sector j. From the budget constraint, consumer i's income comes from two sources: his exogenously given income y_i, which can be interpreted as wage and other personal endowments, and his share of the after tax profit from the private sector, $\sum_{j=k}^{N} \theta_{ij}(1-\tau_j)\pi_j^t$. His *effective income*, y_i^t, with which he can choose his consumption bundle among products produced in the private sector, is total income less the expenditure on rationed products.

Therefore, his indirect utility function is

$$v_i(\vec{P}^t, y_i^t) = \phi_i(\vec{P}^t) + y_i^t,$$

where $\phi_i(\vec{P}^t) = \mu_i(\bar{q}_1, ..., \bar{q}_{k-1}, q_{ik}^t(\vec{P}^t), ..., q_{iN}^t(\vec{P}^t)) - \sum_{j=k}^{N} P_j^t q_{ij}^t(\vec{P}^t)$, and where $\vec{P}^t$ is the shorthand for the vector of all prices at time t. Since consumer i has a quasi-linear utility function, his indirect utility function can be written in two parts, with effective income separate from $\phi_i(\vec{P}^t)$, and with the demand function independent of income.

Consumer i's indirect utility function at time *t-1* is obtained similarly. Since consumer i has one less degree of freedom and less purchasing power at time *t-1* compared to time t, his effective income at time *t-1* is

$$y_i^{t-1} \equiv y_i + \sum_{j=k+1}^{N} \theta_{ij}(1-\tau_j)\pi_j^{t-1} - \sum_{j=1}^{k} \bar{P}_j \bar{q}_j. \tag{1.3}$$

His indirect utility function at time *t-1* is

$$v_i(\vec{P}^{t-1}, y_i^{t-1}) = \phi_i(\vec{P}^{t-1}) + y_i^{t-1},$$

where $\phi_i(\vec{P}^{t-1}) = \mu_i(\bar{q}_1, \cdots, \bar{q}_k, q_{ik+1}^{t-1}(\vec{P}^{t-1}), \cdots, q_{iN}^{t-1}(\vec{P}^{t-1})) - \sum_{j=0,k+1}^{N} P_j^{t-1} q_{ij}^{t-1}(\vec{P}^{t-1})$.

For each consumer i, we can calculate the minimal amount of compensation necessary to keep him on the same indifference curve as he was before sector k was privatized by equating his indirect utility function at time t to that at *t-1*,

$$v_i(\vec{P}^t, y_i^t) = v_i(\vec{P}^{t-1}, y_i^{t-1}), \quad \text{i.e.,}$$

$$\phi_i(\vec{P}^t) + y_i^t = \phi_i(\vec{P}^{t-1}) + y_i^{t-1},$$

Plugging in the definition of y_i^t and y_i^{t-1} from Equation 1.2 and Equation 1.3, and rearranging terms, we get the *individual consumer's minimal compensation*,

$$T_i^* = \phi_i(\vec{P}^{t-1}) - \phi_i(\vec{P}^t) + \sum_{j=k+1}^{N} \theta_{ij}(1-\tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}[S_k - (1-\tau_k)\pi_k^t] - \bar{P}_k\bar{q}_k.$$

The first two terms, the *price effect*, are the change in his indirect utility due to price changes; the next two terms, the *profit effect*, show the consumer's income changes due to the changes in his after tax profit shares, where $\sum_{j=k+1}^{N} \theta_{ij}(1-\tau_j)(\pi_j^{t-1} - \pi_j^t)$ is the total change in i's shares of after tax profits in the old private sectors as a result of privatizing sector k, and $\theta_{ik}[S_k - (1-\tau_k)\pi_k^t]$ is his total payment for his shares in the newly privatized sector less his share of the after-tax profit in this sector. Note that the profit effect can be either positive or negative. So can be the individual consumers' minimal compensations. T_i^* is an important expression in the later analysis of the Public Servant's problem.

Figure 1.1 shows the consumer's consumption before and after privatization in a simple two-good economy. At time *t-1*, sector 2 is private, while sector 1 is public. Since $\bar{q}_1$ is the rationed amount, the consumer's consumption bundle $(\bar{q}_1, q_2^{t-1})$ usually is not the tangency point. At time t, sector 1 is privatized. The price of product 1 goes up to the market clearing price P_1, and the price of product 2 also changes. With the new price ratio and effective income, the consumer maximizes his utility subject to his budget constraint. For some consumers, the new consumption bundle can lie on a higher indifference curve; for others, it can lie on a lower indifference curve. The minimal compensation, T^*, shows the amount of transfer needed to get the consumer to the tangent point consumption bundle, (q_1^t, q_2^t), on the previous indifference curve. Note that it could be positive, zero or negative.

1.3.3 The Public Servant's Problem

The Public Servant is a highly simplified representation of the government. At any given time t, he decides whether to continue privatizing, and, if yes, what sector(s) to privatize and how to compensate the consumers. Assume at time t his budget comes from three sources:

(1) Revenue and subsidies from the public sectors, $\sum_{j=1}^{k-1} B_j$;

(2) Revenue from the sale of the public sector k, S_k; and

(3) Taxes from the private sectors, $\sum_{j=k}^{N} \tau_j \pi_j^t$.

It would be interesting to understand the details of the sale process. But since those depend on the bargaining power of the seller and the buyers, the future profitability of the firms and a number of other political considerations, we do not study these in this chapter. Instead we assume that sales revenue has the following relationship with after tax profit, $S_k = (1+\epsilon_k)(1-\tau_k)\pi_k^t$, where $\epsilon_k \in R^1$ represents the difference between the sale amount and the actual after tax profit due to the bargaining power of the buyers and seller, political considerations or other factors.

One type of Public Servant, **the Bureaucrat**, has the objective of maximizing the surplus budget, i.e., total budget less total consumer compensation, subject to the constraint of staying in office. The surplus can be used to build up the Bureaucracy, or on personal gratification, if he is a corrupt bureaucrat[5]. This objective function can be justified under a range of circumstances (Niskanen 1971). In a society with elections, suppose that consumers/voters use a retrospective voting rule, i.e., they will vote for the Bureaucrat if they occupy the same or a higher utility curve in this period as in the last period, and vote against him if they are on a lower utility curve (Fiorina 1981). Denote the proportion of votes the Bureaucrat has to get to stay in office as d. Note that different democratic systems can have different d's. Even in

[5]Note that the use of the surplus budget does not affect the quantities of public sector goods provided, or the price paid.

a society without elections, the Bureaucrat needs to satisfy a certain percentage of consumers to be able to stay in office, though this d could be much lower than the one in a democratic society. For example, suppose that consumers in a society without elections judge the Bureaucrat's policy in a similar retrospective way as those in a democratic society, and they can throw the Bureaucrat out of the office, by revolt or other means, whenever the percentage of dissatisfied consumers exceeds $1 - d$. Then, the Bureaucrat's constraint is to satisfy at least d percent of the consumers to stay in office.

Assume that the Public Servant knows the distribution of the consumers' utility functions, but does not know the utility functions of individual consumers. In each period, therefore, he compensates everybody the same amount[6]. Depending on their utility functions, some consumers will be better off and some will be worse off after the privatization and the compensation than they were before.

To formalize the problem, let the Bureaucrat choose the sector and the level of consumer compensation to

$$max \quad \sum_{j=1}^{k} B_j + \sum_{j=k+1}^{N} \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - I \cdot T \quad (B1)$$

$$s.t. \qquad \frac{1}{I} |i : T \geq T_i^*| \geq d, \qquad (B2)$$

where $I \cdot T$ is the total transfers, and the constraint means that at least d of the voters

[6]This is a feasible and practical compensation scheme. It is used in China after each successive "price liberalization" reform.

are content with the level of compensation offered by the government.

For comparison with the behavior of the Bureaucrat, we model another kind of Public Servant, **the Populist**, whose objective function is to maximize popularity or consumer welfare, subject to a balanced budget,

$$max \qquad W^t = \sum_i v_i^t \qquad (P1)$$

$$s.t. \quad \sum_{j=1}^{k} B_j + \sum_{j=k+1}^{N} \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] = I \cdot T \quad (P2)$$

In the next section, we will analyse the decisions of both types of Public Servant and compare their optimal behavior.

1.4 Main Results and Discussion

To get the final results about the Public Servant's optimal privatization policy, the first step is to analyse how an individual consumer's minimal compensation changes with the different characteristics of a sector. Since the Public Servant does not know each individual's utility function, but knows the distribution of their utility functions, the second step is to get the minimal aggregate compensation from the distribution of utility functions and the political constraint, d. The third step is to derive the main results about the Public Servant's optimal privatization policy. Then we show some results on the effects of political institutions on the privatization process.

1.4.1 Individual Consumer's Minimal Compensation

In order to study the Public Servant's decision, we need to know how an individual consumer's minimal compensation changes with the characteristics of sector k, α_k.

Proposition 1 *When a sector k is privatized, and consumer i's share in sector k is sufficiently small, then other things being constant, the minimal individual compensation increases with an increase in P_k^t, and with an increase in the market power of sector k, and vice versa, i.e.,*

$$\frac{\partial T_i^*}{\partial P_k^t}, \frac{\partial T_i^*}{\partial \alpha_k} \begin{cases} \geq 0 & \text{if } \epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0 \text{ and } \theta_{ik} \leq A_i, \text{ or } \epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0 \ ; \\ < 0 & \text{if } \epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0 \text{ and } \theta_{ik} > A_i; \text{ where} \end{cases}$$

$$A_i = \frac{\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} + \sum_{j=k+1}^{N} \theta_{ij}(1-\tau_j)\frac{\partial \pi_j^t}{\partial P_k^t}}{\epsilon_k(1-\tau_k)\frac{\partial \pi_k^t}{\partial P_k^t}}.$$

Proof: Substituting $S_k = (1+\epsilon_k)(1-\tau_k)\pi_k$ into T_i^*, we get

$$T_i^* = \phi_i(\vec{P}^{t-1}) - \phi_i(\vec{P}^t) + \sum_{j=k+1}^{N} \theta_{ij}(1-\tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}\epsilon_k(1-\tau_k)\pi_k^t - \bar{P}_k\bar{q}_k.$$

Differentiate T_i^* with respect to P_k^t,

$$\frac{\partial T_i^*}{\partial P_k^t} = -\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} - \sum_{j=k+1}^{N} \theta_{ij}(1-\tau_j)\frac{\partial \pi_j^t}{\partial P_k^t} + \theta_{ik}\epsilon_k(1-\tau_k)\frac{\partial \pi_k^t}{\partial P_k^t}.$$

Since $\frac{\partial \phi_i(\vec{P}^t)}{\partial P_k^t} \leq 0$, the first term is positive.

Differentiating $\pi_{lj} = P_j Q_{lj}(\vec{P}) - C_j(Q_{lj}(\vec{P}))$ with respect to P_k, we have

$$\frac{\partial \pi_{lj}}{\partial P_k} = \frac{\partial Q_{lj}(\vec{P})}{\partial P_k^t}[P_j - MC_j(Q_{lj})] \leq 0,$$

by Assumption 4, so the second term is also positive.

Since the profit in sector k can increase or decrease with an increase in the price of product k, the sign of $\frac{\partial \pi_k^t}{\partial P_k^t}$ is ambiguous.

(1) If $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0$, then we have $\frac{\partial T_i^*}{\partial P_k^t} \geq 0$.

(2) If $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0$, however, the exact change in the amount of compensation caused by the change in the price of product k depends on the proportion of shares he holds in sector k.

When $0 \leq \theta_{ik} \leq A_i$, we have $\frac{\partial T_i^*}{\partial P_k^t} \geq 0$. On the other hand, if $\theta_{ik} > A_i$, we have $\frac{\partial T_i^*}{\partial P_k^t} < 0$.

Since $P_j = \frac{MC_j}{1-\alpha_j}$, it follows that $\frac{\partial P_k^t}{\partial \alpha_k} > 0$. We know that $\frac{\partial T_i^*}{\partial \alpha_k} = \frac{\partial T_i^*}{\partial P_k^t}\frac{\partial P_k^t}{\partial \alpha_k}$. Therefore, the sign of $\frac{\partial T_i^*}{\partial \alpha_k}$ is the same as the sign of $\frac{\partial T_i^*}{\partial P_k^t}$. ∎

Intuitively, for a small shareholder or for somebody who does not hold any shares in the newly privatized sector, the price effect dominates the profit effect – he mainly suffers from the price increase as a consumer; for a large shareholder, however, the profit effect dominates the price effect.

Figure 1.2 illustrates Proposition 1 with a simple computer simulation. The economy consists of 100 consumers and two goods. Consumer i's utility function takes the form of $u_i = a_i\sqrt{q_1} + (1 - a_i)\sqrt{q_2}$, where the indices $a_i \in [0,1]$ are generated randomly by the computer. At time *t-1*, both sector *1* and *2* are public. We normalize $\bar{P}_1 = \bar{P}_2 = \bar{q}_2 = 1$. At time t, sector 2, is privatized, but sector 1 is still public. Let the tax rate be 0.3, and ϵ_2 be 0. Let sector 2 have a cubic cost function, $C_2 = .04q_2^3 - .9q_2^2 + 10q + 5$. For a randomly picked consumer i, we give him different proportions of shares in sector k, and plot out how his minimal compensation, T_i^*, changes with the change in P_k, when his proportion of shares, $\theta = 0, 0.1, 0.3, 0.5$. We can see that when $\theta = 0, 0.1$, his minimal compensation increases with an increase in P_k. When $\theta = 0.3$, the cutpoint, the graph goes to the other direction from $P_k = 2$. When $\theta = 0.5$, this large shareholder's minimal compensation decreases with an increase in P_k. Note that this is only a 100-consumer economy. In a large economy, the threshold should be much smaller.

1.4.2 Minimal Aggregate Compensation

Since the Public Servant does not know each individual's utility functions and shares, he can only make his decision from the aggregate behavior of the individual's minimal compensation. In what follows, we derive the minimal aggregate compensation to all individuals.

Recall the form of the minimal individual compensation, T_i^*,

$$T_i^* = \phi_i(\vec{P}^{t-1}) - \phi_i(\vec{P}^t) + \sum_{j=k+1}^{N} \theta_{ij}(1 - \tau_j)(\pi_j^{t-1} - \pi_j^t) + \theta_{ik}\epsilon_k(1 - \tau_k)\pi_k^t - \bar{P}_k\bar{q}_k.$$

Note that there are two kinds of distributions in the above expression, the indirect utility function $\phi(\cdot)$ and the consumer's proportion of shares in a private sector, $\theta_{\cdot j}$. So in order to know the distribution of T_i^*, we need to know the distributions of $\phi_i(\vec{P}^{t-1}) - \phi_i(\vec{P}^t)$ and $\theta_{\cdot j}$.

Different individuals usually have different utility functions. Let $\mathcal{F}$ be all possible functional forms of $\phi(\cdot)$, and let Φ be the admissible set of $\mathcal{F}$, i.e., $\Phi \subset \mathcal{F} : \mathcal{R}^{\mathcal{N}} \to \mathcal{R}$. We can label the indirect utility functions in Φ by ω. Let the index set Ω be a subset of the real line, i.e., $\omega \in \Omega \subset R$. We assume that $\phi(\cdot, \omega)$ depends continuously on index ω, and that $\frac{\partial \phi(\cdot,\omega)}{\partial \omega} > 0$. ω has cumulative distribution function $M(\omega)$.

Let $\theta_{\cdot j} \sim F_j, j = k, k+1, ..., N$. Let the joint distribution of $\theta_k, ..., \theta_N$ be $F(\theta)^{n-k+1}$, where $\theta \in \Theta$, and Θ is the admissible set of $\theta_{\cdot j}$. We employ the following notation: $\vec{\beta} = (b_k, b_{k+1}, ..., b_N)$, where the b_j's are the coefficients of the $\theta_{\cdot j}$'s; $c \equiv \bar{P}_k \bar{q}_k$, which is a constant because it is the expenditure on the rationed allotment of product k. For simplicity of calculation, assume that ω and θ_j are independent of each other. Suppose $T_i^* \sim G(\cdot, \vec{\beta})$, then for any given level of compensation T, the cumulative distribution function of T_i^*, i.e., the percentage of consumers for which $T_i^* \leq T$, is

$$G(T, \vec{\beta}) = \int \cdots \int_{\Theta} \left[\int_{\{\omega \in \Omega : \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) + \vec{\beta}\vec{\theta} - c \leq T\}} dM(\omega) \right] dF(\theta)^{n-k+1}.$$

It follows that the cumulative distribution of T_i^* can be expressed in terms of the distribution of ω and $\vec{\theta}$. This facilitates our method of solving the Public Servant's problem.

In the Bureaucrat's problem, the constraint, (B2), is equivalent to $G(T, \vec{\beta}) \geq d$. His constrained maximization problem, (B1) and (B2), can be converted into one of unconstrained maximization by finding the minimal T, T_{min}, to keep him in office. Therefore, he chooses the public sector k to

$$max \sum_{j=1}^{k} B_j + \sum_{j=k+1}^{N} \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - I \cdot T_{min}, \qquad \text{(B3)}$$

where T_{min} is the solution to $G(T_{min}, \vec{\beta}) = d$. We call T_{min} the *minimal aggregate compensation* in a transition economy.

Figure 1.3 illustrates the concept of the minimal aggregate compensation by using the same economy as in Figure 1.2, by calculating the minimal individual compensation for all 100 consumers and plotting out the cumulative distribution function. Then for any given level of d, the proportion of consumers to be left not worse off by the privatization of sector k, there is a corresponding T_{min}, so that at least d percent of the consumers are better off.

The following proposition characterizes the properties of T_{min} — how it changes with the changes in the underlying parameters.

Proposition 2 *In a large population, the minimal aggregate compensation, T_{min}, increases with an increase in the market power of sector k, α_k, i.e.,*

$$\frac{\partial T_{min}}{\partial \alpha_k} \geq 0,$$

(1) if $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0$, *or*

(2) if $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0$ *and* $Prob\{\theta_k \leq \underline{A}\} = 1$, *which holds in a large population; where*

$$\underline{A} = \frac{\frac{\partial \phi(\vec{P}^t, \bar{\omega})}{\partial P_k^t}}{\epsilon_k (1-\tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}}.$$

Proof: Since

$$G(T_{min}, \vec{\beta}) = \int \cdots \int_{\Theta} \left[\int_{\{\omega \in \Omega : \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) + \vec{\beta}\vec{\theta} - c \leq T_{min}\}} dM(\omega) \right] dF(\theta)^{n-k+1} = d,$$

Let $T \equiv \phi(\vec{P}^{t-1}, \omega) - \phi(\vec{P}^t, \omega) + \vec{\beta}\vec{\theta} - c$. First, we want to show that $\frac{\partial T}{\partial P_k^t} \geq 0$, $\forall \omega$, $\vec{\theta}$, if $Prob\{\theta_k \leq \underline{A}\} = 1$.

Differentiating T with respect to P_k^t, we get

$$\frac{\partial T}{\partial P_k^t} = -\frac{\partial \phi(\vec{P}^t, \omega)}{\partial P_k^t} - \sum_{j=k+1}^{N} \theta_j (1-\tau_j) \frac{\partial \pi_j^t}{\partial P_k^t} + \theta_k \epsilon_k (1-\tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}.$$

From the proof of Proposition 1, we know that the first two terms are both positive, while the sign of the third term is ambiguous.

(1) If $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} \geq 0$, then we have $\frac{\partial T}{\partial P_k^t} \geq 0$.

(2) If $\epsilon_k \frac{\partial \pi_k^t}{\partial P_k^t} < 0$, we need more conditions to decide the sign of $\frac{\partial T}{\partial P_k^t}$. Let $A = \frac{\frac{\partial \phi(\vec{P}^t, \omega)}{\partial P_k^t} + \sum_{j=k+1}^{N} \theta_j (1-\tau_j) \frac{\partial \pi_j^t}{\partial P_k^t}}{\epsilon_k (1-\tau_k) \frac{\partial \pi_k^t}{\partial P_k^t}}$. Note that there are two kinds of distributions in A: the index of utility functions, ω, and the proportion of shares in the existing private sectors, θ_j, $j = k+1, ..., N$. Since $\phi(\vec{P}^t, \omega)$ is decreasing in P, and strictly increasing

in ω, we use $\underline{\omega}$ to denote the highest absolute value of $\frac{\partial \phi(\vec{P}^t,\omega)}{\partial P_k^t}$, and $\overline{\omega}$ to denote its lowest absolute value. Then it follows that the lower bound of A is

$$\underline{A} = \frac{\frac{\partial \phi(\vec{P}^t,\bar{\omega})}{\partial P_k^t}}{\epsilon_k(1-\tau_k)\frac{\partial \pi_k^t}{\partial P_k^t}}.$$

Therefore, if $Prob\{\theta_k \leq \underline{A}\} = 1$, we have $\frac{\partial T}{\partial P_k^t} \geq 0$, $\forall \omega$, $\vec{\theta}$.

Next, we want to show that T_{min} has a similar property. It follows from the first part of the proof that for any $P_k^{'t} \geq P_k^t$, if $Prob\{\theta_k \leq \underline{A}\} = 1$, we have $T^{'} \geq T$, $\forall \omega$, $\vec{\theta}$. We want to show that $T^{'}_{min} \geq T_{min}$, where $T^{'}_{min}$ satisfies

$$G'(T^{'}_{min}, \vec{\beta}^{'}) = \int \cdots \int_{\Theta} \left[\int_{\{\omega \in \Omega : \phi(\vec{P}^{t-1},\omega) - \phi(\vec{P^{'}}^{t},\omega) + \vec{\beta}^{'}\vec{\theta} - c \leq T^{'}_{min}\}} dM(\omega) \right] dF(\theta)^{n-k+1} = d.$$

Suppose not, then $T^{'}_{min} < T_{min}$. Let $A = \{\omega \in \Omega : T^{'} \leq T_{min}\}$, and $B = \{\omega \in \Omega : T \leq T_{min}\}$. Since $T^{'} \geq T$, it follows that $A \subseteq B$. We know that

$$\int \cdots \int_{\Theta} \left[\int_{B} dM(\omega) \right] dF(\theta)^{n-k+1} = d.$$

It follows that

$$\int \cdots \int_{\Theta} \left[\int_{A} dM(\omega) \right] dF(\theta)^{n-k+1} \leq d.$$

Let $C = \{\omega \in \Omega : T^{'} \leq T^{'}_{min}\}$. Since $T_{min} > T^{'}_{min}$, we have $C \subset A$. Therefore,

$$\int \cdots \int_{\Theta} \left[\int_{C} dM(\omega) \right] dF(\theta)^{n-k+1} < d,$$

but this contradicts the definition of T'_{min}. So $T'_{min} \geq T_{min}$. Then we have $\frac{\partial T_{min}}{\partial P^t_k} \geq 0$, and equivalently, $\frac{\partial T_{min}}{\partial \alpha_k} \geq 0$.

Finally, we want to show that $Prob\{\theta_k \leq \underline{A}\} = 1$ holds in a large population.

$$Prob\{\theta_k \leq \underline{A}\} = 1 - Prob\{\theta_k > \underline{A}\} = 1 - \frac{|i : \theta_{ik} > \underline{A}|}{I}.$$

Since $\sum_{i=1}^{I} \theta_{ik} = 1$, $|i : \theta_{ik} > \underline{A}| < \min \text{int}[\frac{1}{\underline{A}}]$, which is bounded and independent of I. Therefore, as $I \to +\infty$, $\frac{|i:\theta_{ik}>\underline{A}|}{I} \to 0$, we have

$$Prob\{\theta_k \leq \underline{A}\} = 1 - \frac{|i : \theta_{ik} > \underline{A}|}{I} = 1.$$

∎

The intuition behind this result is quite clear. In a large population most people will own a very small percentage of the total shares, a percentage that approximates zero. It follows from Proposition 1 that for small shareholders, whose price effect dominates their profit effect, the minimal individual compensation increases as the price increases. Therefore, on the aggregate level, the minimal aggregate compensation, T_{min}, increases with an increase in the market power of sector k, α_k.

1.4.3 The Public Servant's Optimal Behavior

This section contains two main propositions of the chapter – the optimal choice of the Bureaucrat and the Populist.

Define the Bureaucrat's *incremental budget* between period t and *t-1* as

$$IB \equiv SB(t) - SB(t-1) = \sum_{j=k+1}^{N} \tau_j(\pi_j^t - \pi_j^{t-1}) + [-B_k + S_k + \tau_k \pi_k^t] - I \cdot T_{min}.$$

Therefore, he will privatize another sector k if and only if there exists a sector such that $IB \geq 0$.

When $IB \geq 0$ is satisfied, the Bureaucrat will choose the public sector that gives him the highest surplus budget. Define the maximal budget at time t as

$$SB^* = \sum_{j=1}^{k} B_j + \sum_{j=k+1}^{N} \tau_j \pi_j^t + [-B_k + S_k + \tau_k \pi_k^t] - I \cdot T_{min}.$$

It is obvious that maximizing SB^* is equivalent to maximizing IB. We would like to know the characteristics of sector k that give the Bureaucrat his maximal surplus budget. That is, what kind of sector would he like to choose?

Proposition 3 (The Bureaucrat) *For a Bureaucrat, in a large population, his maximal budget increases with a decrease in the market power of sector k; the incremental budget increases with an increase in the amount of subsidy sector k receives from the government, i.e.,*

$$\frac{\partial SB^*}{\partial \alpha_k} = \frac{\partial IB}{\partial \alpha_k} \leq 0, \ \textit{if} \ Prob\{\theta_k \leq \underline{A}\} = 1; \ \textit{and} \ \frac{\partial IB}{\partial B_k} = -1.$$

Proof: Differentiating SB^* and IB with respect to P_k^t, we get

$$\frac{\partial SB^*}{\partial P_k^t} = \frac{\partial IB}{\partial P_k^t} = \sum_{j=k+1}^{N} \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t} - I \frac{\partial T_{min}}{\partial P_k^t}$$
$$= I \left[\frac{1}{I} \left(\sum_{j=k+1}^{N} \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t} \right) - \frac{\partial T_{min}}{\partial P_k^t} \right].$$

When $I \to +\infty$, we have $\frac{1}{I}\left(\sum_{j=k+1}^{N} \tau_j \frac{\partial \pi_j^t}{\partial P_k^t} + \tau_k \frac{\partial \pi_k^t}{\partial P_k^t}\right) \to 0$; also, from Proposition 2, it follows that when $I \to +\infty$, $Prob\{\theta_k \leq \underline{A}\} = 1$, and thus $\frac{\partial T_{min}}{\partial P_k^t} \geq 0$. Therefore, $\frac{\partial SB^*}{\partial P_k^t} = \frac{\partial IB}{\partial P_k^t} \leq 0$, and $\frac{\partial SB^*}{\partial \alpha_k} = \frac{\partial IB}{\partial \alpha_k} \leq 0$.

Differentiating IB with respect to B_k, we get $\frac{\partial IB}{\partial B_k} = -1$. ∎

This proposition tells us that in a large population, the Bureaucrat will gain most by first privatizing the public sector with the least market power and the largest subsidy, if all other characteristics of the public sectors are the same. By privatizing the more competitive sector, the price increase as a result of the privatization will be relatively lower. Therefore, the Bureaucrat does not need to compensate the consumers as much, so he can skim off the cream via minimizing the transfers he pays to maintain a certain percentage, *d*, of consumers not worse off by his policy.

In comparison with the Bureaucrat, we can study the optimal policy of the Populist. His constrained maximization problem, (P1) and (P2), can be converted into one of unconstrained maximization by noting that

$$W^t = \sum_i v_i(\vec{P}^t, y_i^t)$$

$$= \sum_i \mu_i(\bar{q}_1, ..., \bar{q}_{k-1}, q_{ik}^t(\vec{P^t}), ..., q_{iN}^t(\vec{P^t})) - \sum_{j=k}^{N} P_j^t Q_{ij}^t(\vec{P^t}) + \sum_i y_i + \sum_{j=k}^{N}(1-\tau_j)\pi_j^t$$
$$-S_k - I \cdot \sum_{j=1}^{k-1} \bar{p}_j \bar{q}_j + I \cdot T$$
$$= \sum_i \mu_i(\bar{q}_1, ..., \bar{q}_{k-1}, q_{ik}^t(\vec{P^t}), ..., q_{iN}^t(\vec{P^t})) - \sum_{j=k}^{N} C_j(Q_j) + \bar{Z},$$

where $\bar{Z} = \sum_i y_i + \sum_{j=1}^{k-1} B_j - \sum_{j=1}^{k-1} \bar{p}_j \bar{Q}_j$ is a constant. Therefore, his unconstrained maximization problem is

$$max \sum_i \mu_i(\bar{q}_1, ..., \bar{q}_{k-1}, q_{ik}^t(\vec{P^t}), ..., q_{iN}^t(\vec{P^t})) - \sum_{j=k}^{N} C_j(Q_j) \quad \text{— (P3).}$$

Define the *incremental welfare* between period *t* and *t-1* as

$$IW = \sum_i [v_i(\vec{P^t}, y_i^t) - v_i(\vec{P}^{t-1}, y_i^{t-1})].$$

Therefore, he will privatize another sector k if and only if there exists a sector such that $IW \geq 0$.

Proposition 4 (The Populist) *For a Populist, the maximal social welfare increases with a decrease in the market power of sector* k, *if the sum of weighted marginal utilities for products in the private sectors is greater than or equal to their marginal costs, and vice versa; while the incremental social welfare increases with an increase* *in subsidy sector* k *receives when it belongs to the public sector.*

$$\partial W^t/\partial \alpha_k \leq 0, \quad \text{if } \sum_i \sum_{j=k}^N (-\tfrac{\partial q_{ij}}{\partial P_k^t})(\mu_{ij} - MC_j) \geq 0$$

$$\partial W^t/\partial \alpha_k > 0, \qquad \text{otherwise;}$$

$$\frac{\partial IW}{\partial B_k} = -1,$$

where μ_{ij} *is consumer* i*'s marginal utility with respect to product* j.

Proof:

$$\frac{\partial W^t}{\partial P_k^t} = \sum_{j=k}^N \sum_i \frac{\partial \mu_i}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial P_k^t} - \sum_{j=k}^N MC_j \frac{\partial Q_j}{\partial P_k^t}$$
$$= \sum_{j=k}^N \sum_i \frac{\partial q_{ij}}{\partial P_k^t} [\mu_{ij} - MC_j].$$

∎

The above proposition shows that holding constant the other characteristics of the firms in the public sector, if the marginal costs of products in the private sectors are not too high relative to marginal utilities, a Populist will privatize the public sector with the least market power and the largest subsidy first. Intuitively, when the marginal costs of products in the private sectors are not too high, the dominating effect in the social welfare function is the sum of individuals' indirect utility functions (see (P3)), whose magnitude increases with a decrease of the market power of the newly privatized sector due to the price effect. In this case, privatizing the more competitive sector is the socially most efficient way of privatization. When the marginal costs of products in the private sectors are too high with respect to marginal utilities, the dominating effect in the social welfare function is the sum of costs in the private sectors. In this case the Populist will privatize the most monopolistic sector first, since the increase in its prices causes a decrease in the demand of other products[7], which causes a decrease in the total cost of production, and hence an increase in the total social welfare. This seemingly perverse result makes sense because the cost of production affects the profit of private firms, all of which goes to the consumers either as profit shares or as compensations from the profit tax to the state. Therefore, total social welfare improves.

From the above two propositions, the optimal privatization policy is quite robust to the specification of the politician's objectives. Under ordinary situations, exploiting the more nearly competitive pricing is a faster route to efficiency gains for either type: the one who wants to allocate the efficiency gains to enough citizens at lowest possible rent transfer, and the one who wants to maximize social welfare when the marginal costs in the existing private sectors are not too high. An interesting but probably unusual case arises when the marginal costs in the private sectors are too high relative to the marginal utilities, when the Populist gains more by privatizing the monopolistic sectors first.

[7] They are not substitutes, by Assumption 4.

1.4.4 Political Institutions and the Optimal Policy

The above analysis of the Bureaucrat holds the political institution, the percentage of support needed to retain power, constant. It is interesting to know how the privatization processes differ under different political institutions. The following proposition will help us understand how the characteristics of political institutions might affect the Bureaucrat's behavior.

Proposition 5 *The maximal surplus budget increases with a decrease in the threshold of the satisfaction level, d, i.e.,* $\frac{\partial SB^*}{\partial d} < 0$.

Proof: Differentiate SB^* with respect to d, and apply $\partial T_{min}/\partial d \geq 0$. ∎

This proposition shows that in an economy with a smaller d, i.e., a less democratic society, the Bureaucrat actually benefits more from the privatization process. If the surplus budget becomes his personal property, he becomes richer consequently. If it is used to ease the operation of the Bureaucracy or privatization process, it could be relatively easier and faster to privatize in a less democratic society.

1.5 Conclusions

From the analysis of the strategies of Public Servants with different objective functions, we can see that the comparative statics results are very similar under ordinary situations. Among the public sectors with all other characteristics the same, each will choose to privatize the sector with the least market power and the largest subsidy from the state. Intuitively speaking, this is the "cheapest" way to privatize from either Public Servant's point of view.

This is a two-period static model. We assumed that the number of firms remained the same. Our results suggest that from the Public Servant's point of view, he should encourage measures that can drive down the market power of a sector. An important issue about transition is whether or not large enterprises should be broken up before privatization. Since demonopolization will drive down the market power of that sector, the answer is positive. Another important issue is entry. If we allow entry into the model, it also drives down the market power of any sector, $\alpha_j = \frac{1}{L_j} \sum_h \frac{1}{\varepsilon_{jh}}$.

Going back to the sequencing policy discussed in the introduction, we can see that the size of a sector is not the only factor that should be taken into consideration in the Public Servant's optimal policy. Other important factors, such as the subsidy a sector gets, the elasticity of demand of the product, and the competitiveness of a sector (the latter two are included in the concept of the market power index) should all be taken into consideration.

Another assumption is that all goods are non-substitutes to each other. Substitutes are grouped in the same sector. A more realistic approach would do away with this assumption. Such a model would be more complicated, and we are not sure how the result would change. In our model, the wage income of the consumers and the cost functions of firms are taken as exogenously given. Future work should be done to make these factors endogenous within the economy. Some preliminary thinking suggests that the privatization of a sector in the economy would lead to a total change in the supply and demand of labor, and hence to a change in wage income. Therefore, for consumers in a transition economy, both the compensation from the government and the change in their wage income will be the decisive factor in coping with price increases.

Though not exactly a model of the Chinese reform, it sheds some light on the sequence of reform policies in price liberalization and partial privatization in China over the past decade (Wang and Chern 1992). The first sectors that were partially privatized or removed of state-controlled prices were those producing more elastic goods and more competitive, such as the TV industry. The basic consumption goods, such as rice or meat, were the last ones that were removed of state controls. Earlier attempts to remove the state subsidies and price controls of basic consumption goods in some cities resulted in such massive dissatisfaction and complaints that the government had to resume state control. From the analysis of this model, we can see that careful choice of the sector(s) to be privatized can influence political stability and consumer welfare in general. Though the political and economic situations in Eastern Europe and ex-Soviet republics are different from China[8], some of the schemes and sequencing considerations from the Chinese experience can still provide some practical lessons on the likely success and failure of transition toward a market economy, such as the influence of a more gradual and controlled set of economic transition poli-

[8]Most notably, the democratization process that accompanied economic reforms in Eastern Europe and ex-Soviet Union did not occur in China, where strong government control have been maintained in enforcing reform policies.

cies on political stability and the influence of a strong and relatively less democratic government control on the resulting economics policies. The latter point can be seen from comparing the different outcomes of the Shatalin's 500 Day Plan with those of the Chinese reform policies in the 80s.

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Figure 1.1: Consumption Before and After Privatization of Sector 1

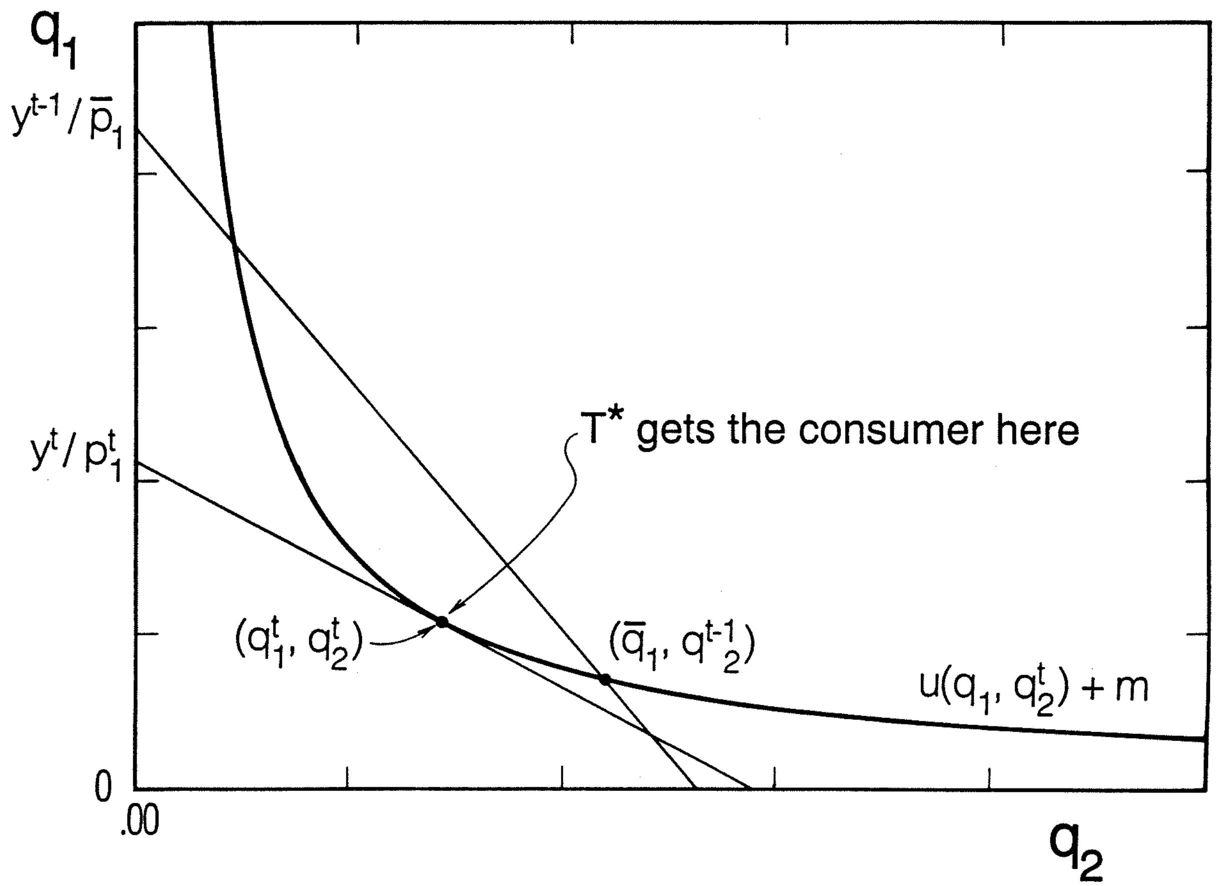


Figure 1.2: Minimal Individual Compensation

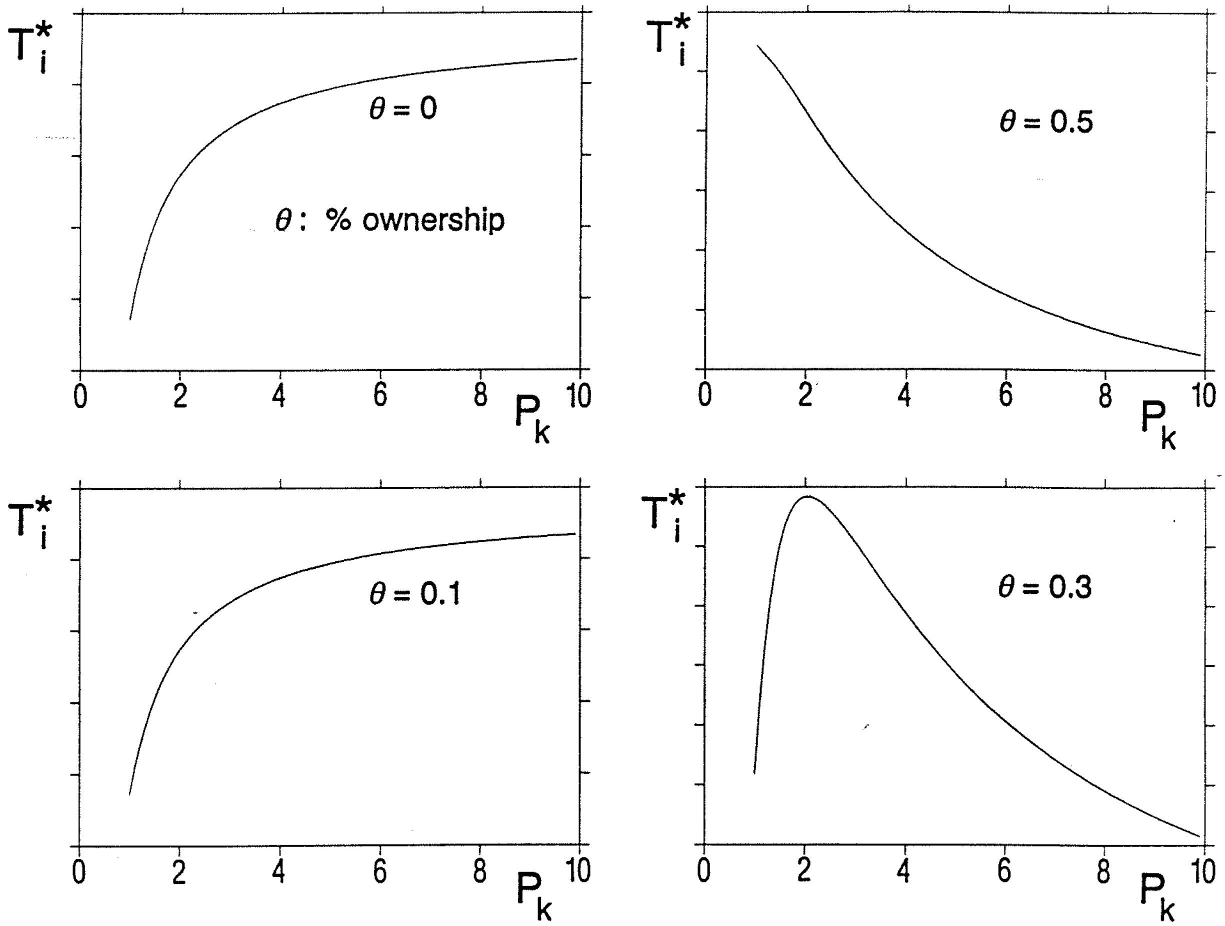
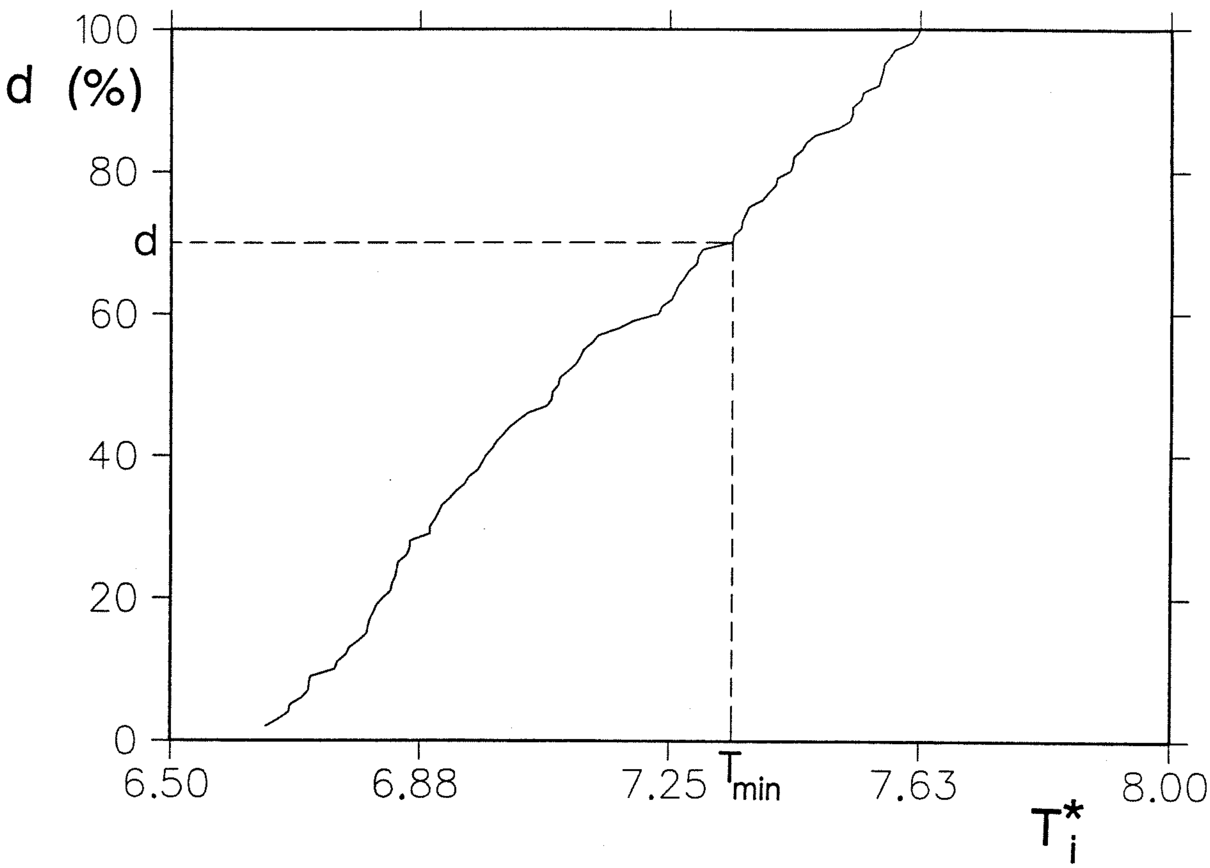


Figure 1.3: Minimal Aggregate Compensation



Chapter 2

Political Institutions and Income Taxation

2.1 Introduction

The goal of this chapter is to understand the effect of political institutions on income tax structures and the level of public goods provided and, in doing so, to merge the economics of the optimal income taxation approach with political science models of voting and legislative choice.

The optimal income taxation literature, starting from Mirrlees (1971), studies the features of income tax schedules, which arise when a social planner maximizes an exogenously given social welfare function, subject to incentive compatibility constraints and an exogenously given revenue requirement. These models have some good features: (1) they recognize that individuals have different productivity, or wage rates; (2) individual labor supply depends on the tax schedule, so incentive effects are taken into consideration; (3) most of them start with unrestricted tax schedules, without *a priori* limitations. The main shortcomings to these models are the neglect of institutional constraints and the exogeneity of the social welfare function. In practice, most public policies concerning income taxation and public goods provision are determined through political institutions, such as direct democracy or legislative processes. We will see that by incorporating these institutional features, social welfare functions can be endogenously determined.

There exists a relatively small literature (Roberts 1977, Kramer and Snyder 1988, Cukierman and Meltzer 1991, Berliant and Gouveia 1991, etc.) that models income tax schedules as the outcome of political processes. But all of these researchers only model simple majority rule. And nearly all results focus on the median voter. Due to the nonexistence of majority rule equilibrium when the dimension of the issue space exceeds one[1], these models either start with a restricted set of tax schedules, such as a linear tax, or put restrictions on the environment. And most of them abstract from the economics and incentive problems inherent in the income tax problem.

This chapter tries to combine the more realistic features of both literatures. Individuals in the economy have different productivity/wage rates; their labor supply depends on the tax schedule, and therefore incentive effects are incorporated. We do

[1] See McKelvey 1979.

not restrict the class of tax schedules so that the tax schedule is the result of the forces caused by political institutions. We compare two types of political institutions: a two-party plurality system under single district, i.e., simple majority rule, and a two-party plurality system under multiple districts with a legislature deciding the final policy outcome.

By explicitly modelling the political institutions, we can characterize the equilibrium tax schedules and conditions under which they are optimal, and thereby endogenously determine the social welfare function. Under plurality rule, the equilibrium tax schedule of two candidate competition (the single district scenario) is compared with the equilibrium outcome from a legislative process when there are multiple districts. We establish that each equilibrium is equivalent to an optimal tax schedule for some welfare weights. Furthermore, we show the equilibrium which arises in a two-candidate, single-district competition puts equal welfare weight over the whole population, while the equilibrium tax schedule of the legislative process puts more weight on those subsets of the population whose legislators are in the majority coalition.

In Section 2.2 we construct a general equilibrium model where the amount of public good level is endogenously determined. Section 2.3 includes a survey of voting models, with special emphasis on the probabilistic voting model and an extension of the equilibrium result to a general functional space. Section 2.4 presents a characterization of the equilibrium income tax schedules under two party plurality system for a single district, and that of a stochastic legislative game when there are multiple districts. Optimality conditions for these equilibria are also determined, thus establishing the relationship between these positive models and traditional optimal income taxation models. In Section 2.5 we present a numerical example of the equilibrium income tax under the two political systems. Section 2.6 concludes the chapter.

2.2 The Model

A general equilibrium model is constructed in which the amount of public good level is endogenously determined. The general problem analysed in this section uses a framework similar to that of Mirrlees (1971), but includes a public good, financed by the tax revenue instead of having an exogenous revenue requirement[2]. This model serves as a building block for the latter part when we introduce the political institutions. It turns out that the two political institutions we consider will be special cases of the optimal tax model, in the sense that the equilibrium tax schedules from political processes are as if some social welfare functions are maximized.

Suppose individuals are identified by a single parameter, $\omega \in \Omega_0 = [\omega_0, \bar{\omega}] \subset R_{++}$, which can be interpreted as the wage rate or ability level of an individual. Assume that $\omega \sim F(\cdot)$, and that ω has a density function $f(\omega)$, and $f(\omega) > 0$ a.s. on Ω_0. Call an individual whose ability-parameter is ω a ω-person. The individual parameter, ω,

[2]Brito and Oakland (1977) also model a public good in their optimal income taxation model, but it is not explicitly financed by the tax revenue.

is private information, but its distribution is common knowledge. There are three commodities: a consumption good, $x \in R_+$, labor, $l \in [0,1)$, and a public good, $y \in R_+$. Let $I(\omega) = \omega l$ be the income of the ω-person. The utility function, $u(x,l,y)$ satisfies the following assumptions.

Assumption 5 $u(x,l,y) = x + \nu(l,y)$, *where* $\nu(\cdot,\cdot)$ *is concave,* C^3, $u_2 = \nu_1 < 0$, $u_3 = \nu_2 > 0$, *and satisfies the Inada conditions:*

$$\lim_{l \to 1} u_2(x,l,y) = -\infty; \quad \lim_{y \to 0} u_3(x,l,y) = \infty.$$

Assumption 6 *The marginal utility for private good consumption decreases with an increase of labor; the marginal utility of leisure is convex.*

$$u_{211} \leq 0; \quad u_{222} \leq 0.$$

Assumption 6 is introduced to avoid bunching of individuals when using the first order approach to solve the optimal taxation problem.

Let $I(\omega) = \omega l$ be the income of the ω-person. Then $I : \Omega \to \mathcal{I}$ is defined as the *income function*, where $\mathcal{I} \subset \mathcal{R}$ is the set of all possible incomes. Let $\mathcal{T} \subset \mathcal{R}$ be the set of all possible taxes. Define $T : \mathcal{I} \to \mathcal{T}$ as the *income tax function.*

Assumption 7 *The income tax function, $T(I)$, is lower semicontinuous.*

We use the revelation principle to analyse the general equilibrium optimal taxation problem[3].

Define a *revenue requirement function,* $\tau : \Omega \to \mathcal{T}$. The problem of taxation of income (the indirect mechanism) is transformed to the direct mechanism: an agent reports his type, ω, based on which he is required to have income, $I(\omega)$, and pay taxes, $\tau(\omega)$. We want to find a tax function T that implements τ in the sense that $T(I(\omega; T)) = \tau(\omega)$. The revenue requirement function satisfies the following assumption:

Assumption 8 *The revenue requirement function, $\tau : \Omega \to \mathcal{T}$, is lower semicontinuous, and bounded below, i.e.,*

$$\tau(\omega) > -\int_{\Omega} \omega dF(\omega).$$

In order to implement the revenue requirement function, $\tau(\omega)$, by means of an income tax, $T(I)$, we need a monotonicity condition.

Lemma 1 (Monotonicity) *Assumption 5 is sufficient to ensure $I(\omega)$ an increasing function, and therefore to implement $\tau(\omega)$ by means of an income tax.*

[3]I thank Miguel Gouveia for pointing out this approach to me. See Berliant and Gouveia (1992).

Proof: From Assumption 5, $u(x, y, l)$ is C^3, and

$$u(x, y, l) = \nu(l, y) + \omega l - \tau \equiv V(\omega, l, y) - \tau.$$

Therefore the Spence-Mirrlees Condition is satisfied, i.e.,

$$\frac{\partial^2 V}{\partial \omega \partial l}(\omega, l) = \frac{\partial}{\partial \omega}(\omega + \nu_l) = 1 > 0.$$

From Proposition 1 of Rochet '87, $l(\cdot)$ is rationalizable, i.e., $(l(\cdot), \tau(\cdot))$ is truthfully implementable in dominant strategies, if and only if $l(\cdot)$ is nondecreasing.

Since $I(\omega) = \omega l(\omega)$, and $l(\omega) \in [0, 1)$, $I(\omega)$ is increasing except possibly in the interval $[\omega_0, \underline{\omega}]$, where $I(\omega) = 0$. In this model, we treat the flat interval as one point, i.e., $\tau(\omega)$ is the same for all $\omega \in [\omega_0, \underline{\omega}]$. Therefore we can concentrate on the interval $[\underline{\omega}, \overline{\omega}] \equiv \Omega$, where $I(\omega)$ is increasing. Then we can invert the income function, $I(\omega)$, and get $\omega = \eta(I)$, and therefore, $T(I) = \tau(\eta(I))$, so we can implement a revenue requirement function by an income tax function. ∎

Lemma 1 shows that, in equilibrium, after all behavioral adjustments, income must be an increasing function of ability.

Given a revenue requirement function, $\tau(\omega)$, and an income function, $I(\omega)$, a ω-person chooses to report his type, ω', to maximize his utility,

$$\max_{\omega'} u(I(\omega') - \tau(\omega'), \frac{I(\omega')}{\omega}, y).$$

Solving this problem gives us an individual's optimal reported type, ω, and thus, his optimal amount of income, $I(\omega)$, his optimal supply of labor, $l(\omega)$, and the individual's private good consumption, $x(\omega) = I(\omega) - \tau(\omega)$. The total supply of labor adjusted for quality is $L^s = \int \omega l(\omega) dF(\omega)$, the aggregate demand for the private good is $X^d = \int_\Omega x(\omega) dF(\omega)$, and the total tax revenue is $\int_\Omega \tau(\omega) dF(\omega)$.

On the production side, assume that firms are price-takers. The input for the production of the private good is labor which, adjusted for quality, equals $L = \int_\Omega \omega l(\omega) dF(\omega)$. The public good is produced from the private good.

Assume that all firms are identical and that they maximize profit by choosing the optimal amount of labor input in the production of the private good and the public good. The production functions of the private good and the public good are assumed to be linear. The total amount of private good produced is $X^T = aL$, and the total amount of public good produced from the private good is $y = b(X^T - X)$. Normalize the price of the private good to 1, and let the price of the public good be p. The firm's problem can be expressed as the following,

$$\begin{aligned} \max \quad & X^s + py - L^d \\ s.t. \quad & X^T = aL^d \\ & y = b(X^T - X^s). \end{aligned}$$

In equilibrium, the firm's profit is zero, and demand equals supply in all markets.

So we have

$$a(X^s + py) = X^s + \frac{y}{b},$$

and hence

$$a = 1,\ p = \frac{1}{b}.$$

The government uses the tax revenue to purchase the public good. Therefore, we have a balanced budget constraint, $py = \int_\Omega \tau(\omega))dF(\omega)$.

2.3 Properties of Voting Equilibria

We want to study the equilibria of two types of political institutions. This section lay a foundation for studying these political equilibria. We start with a survey of voting models for those who may be unfamiliar with that literature. Then we extend a result from the probabilistic voting models to cover the case in which the policy belongs to a functional space, which is used later in characterizing the equilibrium tax policies.

2.3.1 Voting Models

In our problem of voting over the income tax schedules, we do not want to restrict the tax schedule *a priori* to one dimension. Existing voting models have different results when the issue space exceeds one dimension.

There are mainly three types of voting models, based on different behavioral assumptions. The first kind, used in most of the voting literature, is the **deterministic**

voting model, which assumes no uncertainty. A voter votes for an alternative, T_j, if $u(T_j) \geq u(T_i)$, for any $T_i \neq T_j$. When the policy space is more than one dimension, a majority cycle usually prevails[4]. Equilibrium does not usually exist.

When we introduce uncertainty into voters' decision processes, which maybe a descriptively more accurate representation of the real decision processes, we can establish the existence of a voting equilibrium.

One approach in Ledyard (1984) uses the **Bayesian voting model**, where Bayesian equilibrium analysis is used, and voters can abstain. In the resulting equilibrium, both candidates adopt the same platform that maximizes a social welfare function. The analysis is based on an individual being pivotal in an election, which is not applicable when we have a continuum of voters/consumers.

An alternative way of modeling voting is the **probabalistic voting model**[5]. We will briefly go over the underlying rationale for this approach. This approach can be understood as reflecting candidates' uncertainty about whom the individual voters will vote for. Assume that an individual's choice probabilities are "proportional to his strength of preferences" (Coughlin and Nitzan 1981).

Consider an electorate where everyone votes. In the two candidate case, this means that the probability with which an individual ω chooses candidate i, $P^i(T_1, T_2, \omega)$, satisfies

$$P^1(T_1, T_2, \omega) + P^2(T_1, T_2, \omega) = 1.$$

[4]See, e.g., McKelvey 1979.

[5]For a comprehensive treatment of this subject, see Coughlin 1992.

The individual-ω's utility from candidate i's platform is

$$\mu(T_i, \omega) = u(T_i, \omega) \exp(\epsilon_i), i = 1, 2.$$

Assuming that the error term, ϵ, is distributed logistically, we get the individual choice probabilities on any pair of platforms as[6]

$$P^i(T_1, T_2, \omega) = \frac{u(T_i, \omega)}{u(T_1, \omega) + u(T_2, \omega)}.$$

Therefore, a candidate's expected vote equals

$$Ev_i(T_i | T_{-i}) = \int_\Omega \frac{u(T_i, \omega)}{u(T_1, \omega) + u(T_2, \omega)} dF(\omega).$$

Assume that each party's objective function is to maximize expected plurality, which is equivalent to maximizing the probability of winning in a large electorate[7]. Define the expected plurality for party 1 as

$$EPl_1 = Ev_1 - Ev_2 = \int_\Omega \frac{u(T_1, \omega) - u(T_2, \omega)}{u(T_1, \omega) + u(T_2, \omega)} dF(\omega),$$

and the expected plurality for party 2 as $EPl_2 = -EPl_1$.

Notice that this game is two-person, symmetric and zero-sum. It satisfies the

[6]See, e.g., Amemiya (1985), Chapter 9.
[7]See Ledyard (1984).

equivalence and interchangeability conditions, i.e., if $T_1^* \neq T_2^*$ in equilibrium, then both (T_1^*, T_1^*) and (T_2^*, T_2^*) are pure strategy equilibria as well.

Coughlin and Nitzan (1981) characterized an equilibrium when the policy set lies in Euclidean space.

Theorem 1 *(Coughlin, 1992, Theorem 6.3) If the policy space $X \subset R^m$ is compact, if voters vote probabilistically, and if $u(T)$ is concave in T, an alternative, $T^* \in X \subset R^m$, is an outcome of the electoral competition, if and only if $T^* \in argmax \int_\Omega \ln u(T, \omega) dF(\omega)$.*

We call $W = \int_\Omega \ln u(T, \omega) dF(\omega)$ the Nash social welfare function. In two party competition under plurality rule, the equilibrium policy outcome is the maximand of the Nash social welfare function.

2.3.2 Extension of Probabilistic Voting Results

Since we want to study the equilibrium tax structure, we need to extend the result to cover the case in which the policy belongs to a functional space. In this section we extend Theorem 1 to a functional space.

Lemma 2 *After tax consumption, $x(\omega, \omega_t)$, is nondecreasing in ω, where ω_t is his true type, and ω is his reported type.*

Proof: An individual's after tax consumption is $x(\omega, \omega_t) = \omega_t l(\omega) - \tau(\omega)$. He reports an optimal ω such that

$$x(\omega, \omega_t) + \nu(l(\omega), y) \geq x(\omega', \omega_t) + \nu(l(\omega'), y), \quad \forall \omega' \in \Omega.$$

Truthful revelation requires the above inequality holds for $\omega = \omega_t$, i.e.,

$$x(\omega_t, \omega_t) + \nu(l(\omega_t), y) \geq x(\omega', \omega_t) + \nu(l(\omega'), y), \quad \forall \omega' \in \Omega.$$

That is,

$$x(\omega_t, \omega_t) - x(\omega', \omega_t) \geq \nu(l(\omega'), y) - \nu(l(\omega_t), y).$$

If $\omega_t \geq \omega'$, by Lemma 1, we have $l(\omega_t) \geq l(\omega')$, and therefore, $\nu(l(\omega'), y) - \nu(l(\omega_t), y) \geq 0$. So

$$x(\omega_t, \omega_t) - x(\omega', \omega_t) \geq 0.$$

∎

Lemma 2 is used to put more structure on the revenue requirement function, as is shown in the following lemma.

Lemma 3 *$\tau(\omega)$ is of bounded variation.*

Proof: $\tau(\omega) = I(\omega) - x(\omega, \omega_t)$. From Lemma 1 and 2, we know that both $I(\omega)$ and $x(\omega, \omega_t)$ are nondecreasing in ω. So $\tau(\omega)$ is of bounded variation. ∎

Let $BV[a,b]$ denote the space of functions of bounded variation on $[a,b]$. Define $X_\tau = \{\tau :$ lower semicontinuous and of $BV\}$, and $X_I = \{I :$ nondecreasing$\}$. The policy space is therefore $X = \{(I \in X_I, \tau \in X_\tau) : I - \tau \geq 0; I.C.\}$, where $I.C.$ stands for the incentive compatibility constraint. To prove the existence of the electoral equilibrium, we need to show that X is compact.

Lemma 4 *The policy space X is compact.*

Proof: Since $l \in [0,1)$, we have $I \in [0,\bar{\omega})$. We know that I is nondecreasing. Therefore, I is of bounded variation, and variation norm bounded.

From Lemma 3, τ is of bounded variation. The feasibility constraint gives us $\tau \leq I$. From Assumption 8,

$$\tau(\omega) > -\int_\Omega \omega dF(\omega).$$

Therefore, τ is also variation norm bounded.

Let $M[a,b]$ be the set of all countably additive signed Borel measures on $[a,b]$. From Theorem 4.1 (Border 1991), the $\sigma(BV,M)$ topology and the topology of pointwise convergence coincide on the set $\{(I \in X_I, \tau \in X_\tau) : I - \tau \geq 0\}$.

Next, we show that adding the incentive compatibility constraint does not change pointwise convergence. The incentive compatibility constraint says

$$I_n(\omega) - \tau_n(\omega) + \nu(I_n(\omega)/\omega, y) \geq I_n(\omega') - \tau_n(\omega') + \nu(I_n(\omega')/\omega, y), \forall \omega' \in \Omega.$$

As $I_n(\omega) \to I(\omega)$, and $\tau_n(\omega) \to \tau(\omega)$, we have

$$I(\omega) - \tau(\omega) + \nu(I(\omega)/\omega, y) \geq I(\omega') - \tau(\omega') + \nu(I(\omega')/\omega, y), \forall \omega' \in \Omega.$$

So X is variation norm bounded and pointwise closed subset of BV, and therefore, from Theorem 4.1 (Border 1991), is $\sigma(BV, M)$-compact. ∎

Corollary 1 *In the policy space X, if voters vote probabilistically, and if $u(\cdot)$ is concave in (I, τ), then an equilibrium of the two party electoral competition exists; furthermore, (I^*, τ^*) is an equilibrium to the electoral competition if and only if*

$$I^*, \tau^* \in argmax \int_\Omega \ln u(I - \tau, I/\omega, y) dF(\omega).$$

Proof: Since $u(I - \tau, I/\omega, y)$ is concave in (I, τ), it follows that

$$EPl_i = \int_\Omega \frac{u(I_i - \tau_i, I_i/\omega, y) - u(I_{-i} - \tau_{-i}, I_{-i}/\omega, y)}{u(I_i - \tau_i, I_i/\omega, y) + u(I_{-i} - \tau_{-i}, I_{-i}/\omega, y)}$$

is concave in (I_i, τ_i), convex in (I_{-i}, τ_{-i}), and continuous in both (I_i, τ_i) and (I_{-i}, τ_{-i}). From Lemma 4, X is compact. Therefore, an electoral equilibrium exists.

Next, we show that $(I, \tau) \in X$ is an electoral equilibrium to the electoral game, if and only if it is a global maximum of $EPl_i((I_i, \tau_i), (I, \tau))$, given that $(I_{-i}, \tau_{-i}) = (I, \tau)$. This follows from the interchangeability condition for two-person, zero-sum games.

Let $W(I,\tau) = \int_\Omega \ln u(I-\tau, I/\omega, y)dF(\omega)$. We then show that $I^*, \tau^* \in argmaxW(I,\tau)$ is equivalent to $I^*, \tau^* \in argmaxEPl_i((I_i,\tau_i),(I,\tau))$, for $i = 1,2$. Since $\ln u(I - \tau, I/\omega, y)$ is a strictly monotone increasing concave function of $u(I-\tau, I/\omega, y)$, then $W(I,\tau)$ is concave in (I,τ). Therefore, every local maximum of $W(I,\tau)$ is also a global maximum. Similarly, since $EPl_i((I_i,\tau_i),(I,\tau))$ is concave in (I_i,τ_i), it follows that any of its local maxima are also a global maximum. So the first order conditions for the maximization problems are both necessary and sufficient. It suffices to show that the first order conditions of the two functions are equivalent. We prove this by using calculus of variation.

$$W(\tau+\epsilon h) = \int_\Omega \ln u(I-\tau-\epsilon h, I/\omega, \int_\Omega(\tau+\epsilon h)dF)dF(\omega).$$

Then,

$$\begin{aligned}\delta W(\tau;h) &= \frac{d}{d\epsilon}W(\tau+\epsilon h)|_{\epsilon=0} \\ &= \int_\Omega[-\frac{1}{u}+\int_\Omega \frac{u_3}{u}dF]hdF(\omega) \\ &= 0, \text{ for all } h.\end{aligned}$$

Therefore,

$$-\frac{1}{u}+\int_\Omega \frac{u_3}{u}dF = 0.$$

Similarly,

$$\begin{aligned}\delta EPl_1(\tau_1;h)|_{\tau_1=\tau;I_1=I} &= \frac{d}{d\epsilon}EPl_1(\tau+\epsilon h)|_{\epsilon=0;\tau_1=\tau;I_1=I} \\ &= \int_\Omega \frac{2u[-h+u_3\int_\Omega hdF]}{(2u)^2} \\ &= \int_\Omega [-\frac{1}{2u}+\int_\Omega \frac{u_3}{2u}dF]hdF(\omega) \\ &= 0, \text{ for all } h.\end{aligned}$$

Therefore,

$$-\frac{1}{u}+\int_\Omega \frac{u_3}{u}dF=0.$$

It follows that

$$\delta W(\tau;h)=2\cdot\delta EPl_1(\tau_1;h)|_{\tau_1=\tau;I_1=I},$$

so that $\delta W(\tau;h)\leq 0$ if and only if $\delta EPl_1(\tau_1;h)|_{\tau_1=\tau;I_1=I}\leq 0$. Similarly, we can prove that $\delta W(I;h)\leq 0$ if and only if $\delta EPl_1(I_1;h)|_{\tau_1=\tau;I_1=I}\leq 0$. ∎

Remark (Concavity): Notice that one of the critical assumptions for the characterization of the equilibrium in probabilistic voting is the concavity of the indirect utility function in the policy proposal which, in this case, is the tax function, τ. Let $V(\tau)$ denote the indirect utility function, then $V(\tau)$ is concave in τ, if and only if

$$V(\alpha\tau_1+(1-\alpha)\tau_2)\geq \alpha V(\tau_1)+(1-\alpha)V(\tau_2),$$

for $\alpha \in [0,1]$. An example of a utility function whose indirect utility function is concave in τ is a quasilinear utility function, $u(x,l,y) = I - \tau(I) + \beta \ln(1 - I/\omega) + (1-\beta)\ln y$ where $\beta \in [0,1]$. For a general utility function where the indirect utility function cannot be solved explicitly, the sufficiency proof of Proposition 6 checks the concavity of the indirect utility function in τ, i.e., Assumption 6 guarantees the concavity of the indirect utility function in τ.

Corollary 1 establishes that the equilibrium tax schedule under a two party plurality system with a single district can be obtained as if we are solving an optimal tax problem, with the exogenously given social welfare function taking the form of the Nash social welfare function.

So far, we have not assumed differentiability of the revenue requirement function or the income function. The next corollary establishes that we can restrict our attention to the subset of differentiable functions.

Corollary 2 *If one party's equilibrium policy proposals are differentiable functions, (τ_i, I_i), then it is an equilibrium for the other party to propose the same differentiable functions.*

Proof: It follows from the interchangeability conditions of the symmetric, two-person, zero-sum game. ∎

From here on, we can restrict ourselves to differentiable revenue requirement functions, τ, income functions, I, and income tax functions, $T(I)$.

2.4 Characterization of Equilibrium Tax Functions and the Optimality Conditions

The results of Section 2.3 suggests that in equilibrium the outcome of political processes is as if some particular social welfare function is maximized. In the case of two party plurality system under a single district, the equilibrium tax policy maximizes a Nash social welfare function. In this section, we start with a general optimal taxation model, and then characterize the equilibria of the two political institutions and the optimality conditions of these equilibria, which suggest that they are special cases of the optimal taxation model. The first type is a two party plurality system under a single district, which can be viewed as a simplified version of implementing the platform from a presidential election or the outcome of a simple majority rule/referendum. For comparison, we study the equilibrium policy outcome of a legislative game under a two party plurality system with multiple districts.

2.4.1 The General Case: Optimal Taxation with Public Good

We use the revelation principle to analyse the general equilibrium optimal taxation problem. The following analysis uses the first order approach to solve the optimization problem.

Given a revenue requirement function, $\tau(\omega)$, and an income function, $I(\omega)$, a

ω-person chooses to report his type, ω', to maximize his utility,

$$\max_{\omega'} u(I(\omega') - \tau(\omega'), \frac{I(\omega')}{\omega}, y).$$

The first order condition for this problem is

$$\frac{du}{d\omega'} = \frac{dI(\omega')}{d\omega'} - \frac{d\tau(\omega')}{d\omega'} + \frac{u_2}{\omega}\frac{dI(\omega')}{d\omega'} = 0.$$

Truthful revelation requires $\frac{du}{d\omega'}|_{\omega'=\omega} = 0$, i.e.,

$$\left.\frac{du}{d\omega'}\right|_{\omega'=\omega} = \frac{dI(\omega)}{d\omega} - \frac{d\tau(\omega)}{d\omega} + \frac{u_2}{\omega}\frac{dI(\omega)}{d\omega} = 0.$$

Using the shorthand, $I'(\omega)$, to stand for $\frac{dI(\omega)}{d\omega}$, and similarly for other variables, the incentive compatibility constraint becomes

$$I'(\omega) - \tau'(\omega) + \frac{u_2}{\omega}I'(\omega) = 0.$$

The optimal income tax problem is thus defined as

$$\max_{\tau,I} \quad \int_\Omega A(u(I(\omega) - \tau(\omega), \tfrac{I(\omega)}{\omega}, b\int_\Omega \tau(\omega)dF(\omega))dF(\omega) \quad (1Op)$$

$$s.t. \quad I'(\omega) - \tau'(\omega) + \tfrac{u_2}{\omega}I'(\omega) = 0 \quad (IC)$$

$$I(\omega) - \tau(\omega) \geq 0 \quad (F)$$

where $A(u(\omega))$ is some exogenously given, strictly increasing, concave and differentiable welfare function. Equation (IC) is the incentive compatibility constraint. Equation (F) is the feasibility constraint.

Proposition 6 *The optimal tax schedule, $T(I)$, satisfies Equation (2.1), (IC) and (F).*

Proof: This is a calculus of variations problem. Define the function J as

$$\begin{aligned} J &= \int_\Omega \{A[u(I(\omega) - \tau(\omega), \frac{I(\omega)}{\omega}, b\int_\Omega \tau(\omega)dF(\omega)]f(\omega) \\ &+\xi(\omega)[I'(\omega) - \tau'(\omega) + \frac{u_2}{\omega}I'(\omega)] + \theta(\omega)(I(\omega) - \tau(\omega))\}d\omega. \end{aligned}$$

Then,

$$\begin{aligned} J(\tau + \epsilon h) &= \int_\Omega \{A[u(I(\omega) - \tau(\omega) - \epsilon h, \frac{I(\omega)}{\omega}, b\int_\Omega [\tau(\omega) + \epsilon h]dF(\omega))]f(\omega) \\ &+\xi(\omega)[I'(\omega) - \tau'(\omega) - \epsilon h' + \frac{u_2}{\omega}I'(\omega)] \\ &+\theta(\omega)(I(\omega) - \tau(\omega) - \epsilon h)\}d\omega \end{aligned}$$

So,

$$\begin{aligned} \delta J(\tau, h) &= \frac{d}{d\epsilon}J(\tau + \epsilon h)|_{\epsilon=0} \\ &= \int_\Omega \{A^{'}[-h + u_3 b\int_\Omega hf(\omega)d\omega]f(\omega) + \xi(\omega)(-h') - \theta(\omega)h\}d\omega \\ &= \int_\Omega \{[-A^{'} + \int_\Omega bA^{'}u_3 f(\omega)d\omega]f(\omega) + \xi^{'} - \theta\}hd\omega \end{aligned}$$

$$= \; 0, \text{ for all } h,$$

it follows that

$$[-A' + \int_\Omega bA'u_3 f(\omega)d\omega]f(\omega) + \xi' - \theta(\omega) = 0, \text{ or}$$

$$bf\int_\Omega A'u_3 dF(\omega) - (A'f - \xi') - \theta(\omega) = 0.$$

Define the function G as

$$\begin{aligned} G &= A[u(I(\omega) - \tau(\omega), \frac{I(\omega)}{\omega}, b\int_\Omega \tau(\omega)dF(\omega)]f(\omega) \\ &+\xi(\omega)[I'(\omega) - \tau'(\omega) + \frac{u_2}{\omega}I'(\omega)] + \theta(\omega)(I(\omega) - \tau(\omega)). \end{aligned}$$

The Euler equation for I is

$$\frac{\partial G}{\partial I} = \frac{d(\frac{\partial G}{\partial I'})}{d\omega} \Longleftrightarrow$$

$$(1 + \frac{u_2}{\omega})(A'f - \xi') + \frac{\xi}{\omega^2}(\frac{I}{\omega}u_{22} + u_2) + \theta(\omega) = 0.$$

Combining the two necessary conditions, we have

$$bf\int_\Omega A'u_3 dF(\omega) + \frac{u_2}{\omega}(A'f - \xi') + \frac{\varepsilon^* u_2 \xi(\omega)}{\omega^2} = 0,$$

where $\varepsilon^* = 1 + lu_{22}/u_2$. From the inverse function theorem, we have $T' = \frac{\tau'}{I'} = 1 + \frac{u_2}{\omega}$.

Then we have

$$(1-T')(A'f-\xi') = bf\int_{\Omega} A'u_3 dF(\omega) + \varepsilon^*\xi(\omega)/\omega^2. \tag{2.1}$$

Notice that T is also on the righthand side of Equation (2.1). Equation (2.1), (IC) and (F) are the necessary conditions for a solution of the optimal income tax problem. To prove sufficiency, we need to check the concavity of G. Since G is linear in I' and τ', the Legendre and Weierstrass conditions are trivially satisfied. We only need to check the concavity of G in I and τ, which requires the matrix of the second partial derivatives with respect to I and τ to be negative semi-definite. Since both $A()$ and u are concave in I and τ, we can decompose the matrix as a sum of two matrices where one of them is negative definite. Then the sufficient conditions are verified if the other matrix, derived from the incentive compatibility and feasibility constraint, is concave in I and τ. Using Assumption 1 that $u()$ is C^3, we require the matrix

$$D = \frac{I'}{\omega}\begin{pmatrix} u_{211} + \frac{u_{222}}{\omega} & -u_{211} \\ -u_{211} & u_{211} \end{pmatrix}$$

to be negative semi-definite.

We get $I' > 0$ from Lemma 1. Thus the sufficiency condition is reduced to requiring $u_{211} \leq 0$ and $u_{222} \leq 0$, which are satisfied from Assumption 6. Thus, the first order approach used in obtaining the necessary conditions for the optimal income tax is valid. ∎

Interpretations for the optimal tax schedule using a general social welfare function can be found in Atkinson and Stiglitz (1980). Our result is different from Mirrlees due to the endogeneity of the public good and the additional feasibility constraint. The integral on the right-hand side of Equation (2.1) can be interpreted this way: suppose we reduce the utility of everyone by a marginal unit, then the gain in increased social welfare is $A^{'}u_3$. Therefore, the integral summarizes the net gain of the marginal reduction of utility. The net gain depends on the form of the social welfare function, $A()$, which, as we demonstrate in the later sections, is determined by the political institutions.

Having characterized the optimal income tax schedule, we proceed to analyse how the social welfare functions are endogenously determined by political processes and show that political institutions endogenously determine the weight of the social welfare function.

2.4.2 Two Party Plurality System Under a Single District

From Corollary 1, the equilibrium tax schedule for two party plurality system under a singel district is the solution to the following optimization problem,

$$\max_{\tau,I} \quad \int_{\Omega} \ln u(I(\omega) - \tau(\omega), \tfrac{I(\omega)}{\omega}, b\int_{\Omega} \tau(\omega)dF(\omega)dF(\omega)$$

$$s.t. \qquad I'(\omega) - \tau'(\omega) + \tfrac{u_2}{\omega}I'(\omega) = 0 \qquad (IC)$$

$$I(\omega) - \tau(\omega) \geq 0. \qquad (F)$$

Solving the above problem, we get the following proposition.

Proposition 7 *(a) The equilibrium tax schedule under the single district, two party plurality system satisfies (IC), (F) and the following equation:*

$$(1 - T')(f/u - \xi') = bf \int_{\Omega} u_3/udF(\omega) + \varepsilon^* \xi(\omega)/\omega^2.$$

(b) It is optimal if the welfare function is $\int_{\Omega} A(u)dF(\omega) = \int_{\Omega} \ln u dF(\omega)$.

Proof: Substituting $\ln u$ for $A(u)$ in Equation 2.1, we get the above result. ∎

The above result can be interpreted either as the equilibrium outcome of a single district two party competition, or as the outcome of a national election, where the winning party/candidate implements his platform. A more complicated political institution involves a legislature where each legislator is elected by plurality rule, and the final policy is the result of a legislative bargaining game.

2.4.3 Multiple Districts – Legislative Process

An alternative mechanism for deciding the income tax schedule in two party plurality systems is through the election of a legislative body. In each legislative district, the voting game determines each legislator's equilibrium platform and his/her objective function in the legislature. Suppose voters are sophisticated in the sense that they know their legislator is not going to be a dictator in the legislature, that the policy outcome is through a complicated process according to some legislative rule,

$\gamma(\cdot) : \{(I_j, \tau_j)\}_{j\in J} \to (I, \tau)$. Then the probability that a ω-person vote for the Incumbent from his district, given the Incumbent's platform, (I_j^I, τ_j^I), and the Challenger's platform, (I_j^C, τ_j^C), is

$$P((I_j^I, \tau_j^I), (I_j^C, \tau_j^C)) = \frac{u[\gamma((I_j^I, \tau_j^I), (I_{-j}, \tau_{-j}))]}{u[\gamma((I_j^I, \tau_j^I), (I_{-j}, \tau_{-j}))] + u[\gamma((I_j^C, \tau_j^C), (I_{-j}, \tau_{-j}))]}.$$

Then, applying Corollary 1 to each legislative district, maximizing expected plurality or the expected probability of winning is equivalent to maximizing the Nash social welfare function for the district in equilibrium, taking the legislative rules, $\gamma(\cdot)$, into consideration. Therefore, we get the following corollary for the equilibrium in each district.

Corollary 3 *In the voting game in district i, the equilibrium platform satisfies*

$$\begin{aligned} I_i^*, \tau_i^* \quad &\in argmax \textstyle\int_{\Omega_i} \ln u[\gamma((I_i, \tau_i), (I_{-i}, \tau_{-i}))] dF_i(\omega) \\ s.t. \qquad & I_i'(\omega) - \tau_i'(\omega) + \tfrac{u_2}{\omega} I_i'(\omega) = 0 \\ & I_i(\omega) - \tau_i(\omega) \geq 0 \end{aligned}$$

Although there are many different legislative processes, we consider a generalized version of the Baron-Ferejohn *random recognition* rule and model the legislative process as a *stochastic game*, $\Gamma^t = (S^t, \pi^t, \psi^t)$, where S^t is the set of *pure strategy n tuples*, where $\pi^t : S^t \to \mu(Z)$ is a *transition function* specifying for each $s^t \in S^t$ a probability distribution $\pi^t(s^t)$ on Z, the set of states that can be achieved in a game, and where $\psi^t : S^t \to X$ is an *outcome function* that specifies for each $s^t \in S^t$ an outcome $\psi^t(s^t) \in X$. Finally, we use $S = \prod_{t \in T} S^t$ to denote the collection of pure strategy n tuples, where $S^t = \prod_{i \in N} s_i^t$. Formally, $Z = R \cup P \cup V$ is the set of states. We use z to denote the possible states the game moves to. We use R to denote the *Recognition Game*, P to denote the *Proposal Game*, and V to denote the *Voting Game*.

At the beginning of period t, legislator j is recognized as a proposer with probability $p_j^t \in [0,1]$, $\sum_{j \in J} p_i^t = 1, \forall t$. Whoever is recognized proposes a tax schedule, (I_j^t, τ_j^t), then every legislator votes yes or no simultaneously. If, under m-majority rule, the number who say "yes" is greater than or equal to m, (I_j^t, τ_j^t) becomes the new status quo and the game ends; otherwise, the game proceeds to period $t+1$. If nothing gets passed forever, the payoff to the legislators is zero: $U_j(\phi) = 0$, for all $j \in J$.

In the legislative game, each legislator's objective function is to maximize the Nash social welfare function of his/her district, given the legislative rules, subject to the incentive compatibility constraint and the feasibility constraint, as stated in Corollary 3.

There are many equilibria to the stochastic game. The selection criteria we use is "simplicity" – we want to characterize the simplest equilibria involving no stage dominated strategies[8]. The simplest equilibrium can be described by an automaton of size 4, with one "rest" state (the Recognition Game), one "propose" state (the Proposal Game), and the "vote yes" and "vote no" state (the Voting Game), which gives us the simplest automaton. The resulting equilibria from the automaton are stationary equilibria[9]. The stationary equilibrium is characterized by a set of values $\{v_t\} \subseteq \mathcal{R}^n$ for each stage of the game, and a strategy profile $\sigma^* \in \Sigma$, such that

a) $\forall t$, σ^* is a Nash equilibrium with payoff function $G^t : \Sigma^t \to R^n$ defined by

$$\begin{aligned} G^t(\sigma^t; v) &= U(\psi^t(\sigma^t)) + \sum_{z \in Z} \pi^t(\sigma^t)(z) v^z \\ &= E_{\sigma^t}[U(\psi^t(s^t)) + \sum_{z \in Z} \pi^t(s^t)(z) v^z] \\ &= \sum_{s^t \in S^t} \sigma^t(s^t)[U(\psi^t(s^t)) + \sum_{z \in Z} \pi^t(s^t)(z) v^z]. \end{aligned}$$

b) $\forall t, v^t = U^t(\sigma^t; v)$.

We use the average payoff for each legislator's payoff for the entire stochastic game. So a legislator's payoff for the entire game is

$$U(\{I^t, \tau^t\}_t) = \lim_{N \to \infty} \sum_{t=1}^{N} U^t(\sigma^t; v).$$

Claim. *There exists a vector of continuation values,* $\overline{U} = (\overline{U}_1, \overline{U}_2, \cdots, \overline{U}_J)$, *where* $\overline{U}_i = \sum_{j \in J} p_j U_i(I_j, \tau_j)$, *representing the expected payoffs to player i at the beginning*

[8]See Baron and Kalai (1993) for an analysis of the simplest equilibrium in the majority rule divide-the-dollar game with a random recognition rule.

[9]For the existence and characterization of stationary equilibrium, see Sobel (1971).

of each stage game.

In the following proposition, we prove that one equilibrium strategy for legislator j is to vote yes with probability 1 if $U_j(I_i, \tau_i) \geq \overline{U}_j$, and to vote no otherwise. The simultaneous equilibrium strategy for any proposer is to maximize his own utility such that the "least expensive" $m-1$ members of the legislature would vote yes. Denote the set of legislators whose payoffs from the proposed tax schedule are greater than or equal to their continuation value as $M = \{k \in J : U_k(I_j, \tau_j) \geq \overline{U}_k\}$. Therefore, in equilibrium, proposer i proposes the tax schedule (I_i, τ_i) that maximizes the Nash social welfare of his own district subject to the constraint that at least $m-1$ other players also vote yes, and his proposal will be accepted. Baron (1993) characterizes similar equilibrium strategies with alternatives in the Euclidean space and presents a closed-form characterization of the equilibrium when the utility function is quadratic. Proposition 8 is a generalization of Baron's results when the set of alternatives lies in a functional space with generalized utility functions.

We introduce some more notations used in solving for the equilibrium tax schedule for the legislative game. We will use the indicator function,

$$\chi_i(\omega) = \begin{cases} 1 & \text{if } \omega \in \Omega_i, \\ 0 & \text{if } \omega \in \Omega - \Omega_i. \end{cases}$$

Proposition 8 *The following is a simplest subgame perfect stationary Nash equilibrium to the legislative game with stage undominated strategies:*

For $z \in P$ and $i = p$ (Proposer i):

$$
\begin{aligned}
I_i, \tau_i \quad & \in argmax \textstyle\int_{\Omega} \chi_i(\omega) \ln u(I_i(\omega) - \tau_i(\omega), \frac{I_i(\omega)}{\omega}, y) dF_i(\omega) \\
s.t. \quad & I_i'(\omega) - \tau_i'(\omega) + \tfrac{u_2}{\omega} I_i'(\omega) = 0 \\
& I_i(\omega) - \tau_i(\omega) \geq 0 \\
& |\{k \in J \backslash \{i\} : U_k(I_i, \tau_i) \geq \overline{U}_k\}| \geq m - 1
\end{aligned}
$$

For $z \in V$ and $j \in J\backslash\{i\}$ (Voter j):

$$
s_j(I_i, \tau_i) = \begin{cases} 1 & \textit{if } U_j(I_i, \tau_i) \geq \overline{U}_j, \\ 0 & \textit{otherwise.} \end{cases}
$$

Proof: We start by defining the strategy sets, transition functions and outcome functions for the game elements:

$$
\begin{aligned}
&\text{For } z \in R: && S_i^t = \{0\}, \forall i \in J, \\
&\text{(Recognition Game)} && \pi^t(s^t)(z) = 1, \text{if } z \in P \\
& && \psi^t(s^t) = \phi, \forall s^t \in S^t.
\end{aligned}
$$

The Recognition Game is indexed by $z \in R$. The order of recognition is randomly decided according to some exogenously given probabilities; therefore, the strategy set of each player is $\{0\}$. The game proceeds to the Proposal Game with probability 1, and the null outcome prevails.

$$\text{For } z \in P: \qquad S_i^t = \begin{cases} \{I_i, \tau_i\} & \text{if } i = p, \\ \{0\} & \text{if } i \in N - \{p\}, \end{cases}$$

$$\text{(Proposal Game)} \quad \pi^t(s^t)(z) = 1, \text{if } z \in V,$$

$$\psi^t(s^t) = \phi, \forall s^t \in S^t.$$

In the Proposal Game, we use p to denote the Proposer. The strategy set for the Proposer is the set of tax schedules $\{I_i, \tau_i\}$, while the strategy set for each voter is still $\{0\}$. The game proceeds to the Voting Game with probability one, and the null outcome prevails in this game.

$$\text{For } z \in V: \qquad S_i^t = \{0, 1\}, \forall i \in J,$$

$$\text{(Voting Game)} \quad \pi^t(s^t)(z) = 1, z \in R,$$

$$\psi^t(s^t) = \begin{cases} I_i^t, \tau_i^t & \text{if } \sum_{i \in J} S_i^t \geq m, \\ \phi & \text{otherwise.} \end{cases} \quad \forall s^t \in S^t.$$

In the Voting Game, each player can vote either no or yes (0 or 1) to the proposed tax schedule. If the new proposal, (I_i, τ_i), is accepted by at least m of the legislators, it becomes the new status quo; otherwise, the null outcome prevails for this period and the game moves to a new round starting from the Recognition game with probability 1.

The main steps to prove Proposition 3 follow the definition of stationary Nash equilibrium. We first specify the values associated with the equilibrium strategies, and then show that these values are self-generating. The third step is to show that the strategies specified in the proposition are subgame perfect Nash equilibria.

The values of the games are defined below. The interpretations of these values go back to the definitions of each game element above.

For $z \in R$: $v_i^t = v_i^{(R)}, \forall i \in R.$

(Recognition Game)

For $z \in P$: $v_i^t(I_i, \tau_i) = U_i(I_i, \tau_i), \text{for } i = p,$

(Proposal Game) $v_j^t(I_i, \tau_i) = \overline{U}_j, \text{for } j \in J - \{p\},$

where

$I_i, \tau_i \in argmax \int_\Omega \chi_i(\omega) \ln u(I_i(\omega) - \tau_i(\omega), \frac{I_i(\omega)}{\omega}, y_i) dF_i(\omega)$

$s.t. I_i^{'}(\omega) - \tau_i^{'}(\omega) + \frac{u_2}{\omega} I_i^{'}(\omega) = 0$

$I_i(\omega) - \tau_i(\omega) \geq 0$

$|\{k \in J \backslash \{i\} : U_k(I_i, \tau_i) \geq \overline{U}_k\}| \geq m - 1.$

For $z \in V$: $v_i^t = \alpha(|M|) U_j(I_i, \tau_i) + (1 - \alpha(|M|)) \overline{U}_j(I_i, \tau_i),$

(Voting Game) $\forall j \in J,$ where

$$\alpha(|M|) = \begin{cases} 1 & \text{if } |M| \geq m, \\ 0 & \text{otherwise.} \end{cases}$$

The next step is to verify that these values are self-generating, i.e., that they correspond to the payoffs under the equilibrium strategies. To do this, we plug the equilibrium strategies and other game elements into the definition of G, and show that they equal the corresponding values.

For $z \in R$: *(Recognition Game)*

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[U(\psi^t(s^t)) + \sum_{z\in Z} \pi^t(s^t)(z)v^z] \\ &= U(\phi) + \pi^t(s^t)(z) \cdot v^t \\ &= v^{(R)} = v^t. \end{aligned}$$

For $z \in P$: *(Proposal Game)*

For $i = p$ (Proposer i):

$$\begin{aligned} G_i^t(\sigma^t; v_i^t) &= E_{\sigma^t}[U(\psi^t(s^t)) + \sum_{z\in Z} \pi^t(s^t)(z)v^z] \\ &= U_i(\phi) + 1 \cdot U_i(I_i, \tau_i) \\ &= U_i(I_i, \tau_i) \\ &= v_i^t(I_i, \tau_i). \end{aligned}$$

For $j = J - \{p\}$ (Voter j):

$$\begin{aligned} G_j^t(\sigma^t; v_j^t) &= E_{\sigma^t}[U(\psi^t(s^t)) + \sum_{z\in Z} \pi^t(s^t)(z)v^z] \\ &= U_j(\phi) + 1 \cdot \overline{U}_j \\ &= \overline{U}_j \end{aligned}$$

$$= v_j^t(I_i, \tau_i).$$

For $z \in V$: *(Voting Game)*

$$\begin{aligned} G^t(\sigma^t, v^t) &= E_{\sigma^t}[U(\psi^t(s^t)) + \sum_{z \in Z} \pi^t(s^t)(z)v^z] \\ &= \alpha(|M|)U_j(I_i, \tau_i) + (1 - \alpha(|M|))(U_j(\phi) + \overline{U}_j(I_i, \tau_i)) \\ &= v^t. \end{aligned}$$

Next, we verify that the strategies specified in Proposition 3 are subgame perfect Nash equilibrium strategies. Since the strategies are history-independent, it suffices to show that for each game element no player will benefit from a unilateral one-shot deviation.

For $z \in P$, we want to show that tax proposal (I_i, τ_i) is the equilibrium strategy for Proposer i, where

$$\begin{aligned} I_i, \tau_i &\in argmax \int_\Omega \chi_i(\omega) \ln u(I_i(\omega) - \tau_i(\omega), \tfrac{I_i(\omega)}{\omega}, y_i) dF_i(\omega) \\ s.t. &\quad I_i'(\omega) - \tau_i'(\omega) + \tfrac{u_2}{\omega} I_i'(\omega) = 0 \\ &\quad I_i(\omega) - \tau_i(\omega) \geq 0 \\ &\quad |\{k \in J \backslash \{i\} : U_k(I_i, \tau_i) \geq \overline{U}_k\}| \geq m - 1. \end{aligned}$$

The corresponding payoff for Proposer i is

$$G_i^t(\sigma^t; v^t(I_i, \tau_i)) = U_i(I_i, \tau_i).$$

If the proposer defects to any other pure strategy $(I_i^0, \tau_i^0) \neq (I_i, \tau_i)$, $\forall i \in J$, there are two possible consequences:

(i) $U_i(I_i^0, \tau_i^0) \leq U_i(I_i, \tau_i)$:

in which case he is not better off by defection, so he will not defect in this case.

(ii) $U_i(I_i^0, \tau_i^0) > U_i(I_i, \tau_i)$:

in this case, if $|\{k \in J \backslash \{i\} : U_k(I_i^0, \tau_i^0) \geq \overline{U}_k\}| \geq m - 1$ still holds, $\forall j \neq i$, then

$$\begin{aligned} I_i, \tau_i \quad & \notin argmax \int_\Omega \chi_i(\omega) \ln u(I_i(\omega) - \tau_i(\omega), \tfrac{I_i(\omega)}{\omega}, y_i) dF_i(\omega) \\ s.t. \quad & I_i'(\omega) - \tau_i'(\omega) + \tfrac{u_2}{\omega} I_i'(\omega) = 0 \\ & I_i(\omega) - \tau_i(\omega) \geq 0 \\ & |\{k \in J \backslash \{i\} : U_k(I_i, \tau_i) \geq \overline{U}_k\}| \geq m - 1. \end{aligned}$$

but this contradicts the definition of (I_i, τ_i).

So the proposer has no positive incentive to defect unilaterally from his strategy specified in Proposition 3, which means that it is a Nash equilibrium for the Proposer. Since it is history independent, it is also a subgame perfect equilibrium.

For $z \in V$, we want to check if voters' strategies specified in the proposition are Nash equilbrium strategies. We consider three cases:

(1) When $|M| > m$, no voter is pivotal, so they have no positive incentive to defect from their equilibrium strategies.

(2) When $|M| = m$, any voter $i \in M$ is pivotal. Since $G_i(s_i = 0, s^*_{-i}) - G_i(s_i = 1, s^*_{-i}) = \overline{U}_i - U_i(I_i, \tau_i) \leq 0$, i has no positive incentive to defect from his equilibrium strategy.

(3) When $|M| = m - 1$, any voter $i \in J \backslash M$ is pivotal. Since $G_i(s_i = 1, s^*_{-i}) - G_i(s_i = 0, s^*_{-i}) = U_i(I_i, \tau_i) - \overline{U}_i \leq 0$, i has no positive incentive to defect either.

Therefore, the voter strategies specified in the proposition are Nash equilibrium strategies. They are subgame perfect, since they are history independent. ∎

We use a three district example to solve the stationary equilibrium tax schedule for the legislative game. It can be easily extended to the J district case. In the following legislative game, $J = 3$, $m = 2$, and $p_i = 1/3$, for $i = 1, 2, 3$. The problem in Proposition 8 reduces to

$$\max_{\tau_i, I_i} \int_\Omega \chi_i(\omega) \ln u(I_i(\omega) - \tau_i(\omega), \tfrac{I_i(\omega)}{\omega}, y_i) dF_i(\omega)$$

$$s.t. \qquad I_i'(\omega) - \tau_i'(\omega) + \tfrac{u_2}{\omega} I_i'(\omega) = 0 \qquad (IC_i)$$

$$I_i(\omega) - \tau_i(\omega) \geq 0 \qquad (F_i)$$

$$U_j(I_i, \tau_i) \geq \overline{U}_j \qquad (MA_i)$$

where j is i's coalition member. Equation (MA_i) can be expanded as

$$\int_\Omega \chi_j(\omega)\ln u(I_i(\omega)-\tau_i(\omega),\frac{I(\omega)}{\omega},y)dF_j(\omega)\geq$$

$$\frac{1}{2}\int_\Omega \chi_j(\omega)[\ln u(I_j(\omega)-\tau_j(\omega),\frac{I_j(\omega)}{\omega},y_j)+\ln u(I_k(\omega)-\tau_k(\omega),\frac{I_k(\omega)}{\omega},y_k)]dF_j(\omega).$$

Apart from the usual incentive compatibility constraint and balanced budget constraint, the tax schedule has to pass a majority of the legislature. The last constraint, (MA), requires the payoff to legislator j to be greater than or equal to his continuation value.

Proposition 9 *(a) The equilibrium tax schedule, (I_i,τ_i), for the legislative game under a random recognition rule in the three district case, satisfies (IC_i) ,(F_i) and the following equation:*

$$\begin{aligned}&(1-T_i^{'})[(\chi_i(\omega)f_i(\omega)+\lambda\chi_j(\omega)f_j(\omega))/u-\xi^{'}]\\ =\ & b\int_\Omega u_3[\chi_i(\omega)f_i(\omega)+\lambda\chi_j(\omega)f_j(\omega)]/udF(\omega)+\varepsilon^*\xi(\omega)/\omega^2,\end{aligned}$$

where $\lambda\geq 0$, i is the proposer and j is the legislator in the majority coalition with i.

(b) It is optimal if the welfare function is

$$\int_\Omega A(u)dF(\omega)=\int_\Omega[\chi_i(\omega)f_i(\omega)+\lambda\chi_j(\omega)f_j(\omega)]\ln u d\omega.$$

Proof: Define the function J as

$$\begin{aligned} J &= \int_\Omega \{[\chi_i(\omega)f_i(\omega) + \lambda\chi_j(\omega)f_j(\omega)] \ln u \\ &\quad + \xi(\omega)[I'(\omega) - \tau'(\omega) + \frac{u_2}{\omega} I_i'(\omega)] + \theta(\omega)(I_i(\omega) - \tau_i(\omega))\} d\omega. \end{aligned}$$

Let $g(I_i, \tau_i, I_{-i}, \tau_{-i}) = \int_\Omega \chi_j(\omega) \ln u(I_i(\omega) - \tau_i(\omega), \frac{I(\omega)}{\omega}, y) dF_j(\omega) - \frac{1}{2}\int_\Omega \chi_j(\omega)[\ln u(I_j(\omega) - \tau_j(\omega), \frac{I_j(\omega)}{\omega}, y_j) + \ln u(I_k(\omega) - \tau_k(\omega), \frac{I_k(\omega)}{\omega}, y_k)] dF_j(\omega)$. The complementary slackness condition requires

$$\begin{aligned} \lambda(\omega) g(I_i, \tau_i, I_{-i}, \tau_{-i}) = 0, &\quad \text{with } \lambda(\omega) \geq 0, \\ \theta(\omega)(I_i - \tau_i) = 0, &\quad \text{with } \theta(\omega) \geq 0. \end{aligned}$$

The rest of the proof is similar to that of Proposition 6, with $A(u) = [\chi_i(\omega)f_i(\omega) + \lambda\chi_j(\omega)f_j(\omega)] \ln u$. ∎

Notice the equilibrium tax schedule of the legislative process is different from that of the two candidate competition. The difference comes from the specific forms of the social welfare functions. Therefore, the welfare weight of individuals in districts whose legislators are not in the majority coalition is zero, while the welfare of individuals whose legislators are in the majority coalition is taken into account when solving for the equilibrium income tax schedule. This confirms our conjecture that the welfare weights of the optimal income tax schedule are endogenously determined by the political processes.

We have characterized the *ex post* equilibrium income tax schedule. One question is if the *ex ante* result is the same as the single district case. To see that this is usually not the case, consider the following situation. Suppose we have three districts, and the distribution of types are such that if 1 is the proposer, he will form a coalition with 2; if 2 or 3 is the proposer, they will form a coalition with each other. Let the probability of i being recognized be p_i. Then the *ex ante* equilibrium tax schedule will be

$$I, \tau \in argmax \int_{\Omega} [p_1(\chi_1(\omega)f_1(\omega) + \lambda_{12}\chi_2(\omega)f_2(\omega)) + p_2(\chi_2(\omega)f_2(\omega) + \lambda_{23}\chi_3(\omega)f_3(\omega)) + p_3(\chi_3(\omega)f_3(\omega) + \lambda_{32}\chi_2(\omega)f_2(\omega))] \ln u dF(\omega).$$

The *ex ante* result will be the same as the single district case if and only if

$$p_1(\chi_1(\omega)f_1(\omega) + \lambda_{12}\chi_2(\omega)f_2(\omega)) + p_2(\chi_2(\omega)f_2(\omega) + \lambda_{23}\chi_3(\omega)f_3(\omega)) + p_3(\chi_3(\omega)f_3(\omega) + \lambda_{32}\chi_2(\omega)f_2(\omega)) = 1.$$

One special case is when all districts are identical, i.e., when $\lambda_{ij} = 0$ for all $i \neq j$, then the single district case has the same outcome with the multiple district case.

When the districts are hetereogeneous, however, the outcome of the legislative process in multiple districts will usually be different from that of single district. We illustrate this with an example.

2.5 An Example

We will use a simplified economy to show the difference in income tax structures under different political institutions. Suppose wage rate, ω, is uniformly distributed in the interval $[1, 4]$. Individuals have quasilinear utility function of the form, $I - \tau + \ln(1 - I/\omega) + \ln y + e$, where y is the amount of public good produced, and e is the initial endowment.

Under a two-party, plurality system in a single district, the equilibrium tax structure, $\tau(\omega)$, maximizes the Nash social welfare function of the whole district subject to the incentive compatibility constraint and the feasibility constraint. This is a calculus of variation problem, which is set up as follows.

$$\max_{\tau(),I()} \quad \int_1^4 \ln[I - \tau + \ln(1 - I/\omega) + \ln(\int_1^4 \tau/3d\omega) + e]d\omega \tag{2.2}$$

$$s.t. \qquad I' - \tau' = \frac{I'}{\omega - I} \tag{2.3}$$

$$I - \tau \geq 0 \tag{2.4}$$

Define

$$\begin{aligned} J &= \int_1^4 \{\ln[I - \tau + \ln(1 - I/\omega) + \ln(\int_1^4 \tau/3d\omega) + e] \\ &\quad + \xi(\omega)(I' - \tau' - \frac{I'}{\omega - I}) + \theta(\omega)(I - \tau)\}d\omega. \end{aligned}$$

$$\begin{aligned}
\delta J(\tau,h) &= \frac{d}{d\epsilon}J(\tau+\epsilon h)|_{\epsilon=0} \\
&= \int_1^4 \frac{1}{u}\left[-h(\omega)+\frac{\int_1^4 h(\omega)d\omega}{\int_1^4 \tau(\omega)d\omega}\right]-\xi(\omega)h'(\omega)-\theta(\omega)hd\omega \\
&= \int_1^4 \left[-\frac{1}{u}+\frac{\int_1^4 \frac{1}{u}d\omega}{\int_1^4 \tau(\omega)d\omega}+\xi'(\omega)-\theta(\omega)\right]h(\omega)d\omega \\
&= 0, \text{ for all } h(\omega), \text{ which implies}
\end{aligned}$$

$$-\frac{1}{u}+\frac{\int_1^4 \frac{1}{u}d\omega}{\int_1^4 \tau(\omega)d\omega}+\xi'(\omega)-\theta(\omega)=0. \tag{2.5}$$

Define

$$F=\ln[I-\tau+\ln(1-I/\omega)+\ln(\int_1^4 \tau/3d\omega)+e]+\xi(\omega)(I'-\tau'-\frac{I'}{\omega-I})+\theta(\omega)(I-\tau).$$

Using Euler's equation, we get

$$\frac{\partial F}{\partial I}=\frac{d(\frac{\partial F}{\partial I'})}{d\omega} \Longleftrightarrow$$

$$(\omega-I)(\omega-I-1)(\frac{1}{u}-\xi'(\omega))-\xi(\omega)I'+\theta(\omega)(\omega-I)^2=0. \tag{2.6}$$

There is no analytical solution to the set of equations (Equation 2.3, 2.4, 2.5, 2.6), so we resort to numerical solutions. For simplicity of calculation, we normalize $\tau(1)=0$, and let $e=5.0$.

Figure 2.1 shows the equilibrium income function, $I(\omega)$, and revenue requirement function, $\tau(\omega)$. Both the income function and the revenue requirement function are monotone increasing in ω, but $I''(\omega)>0$ while $\tau'(\omega)$ starts from zero, increases, then decrease down to zero.

Figure 2.2 shows the income tax schedule. The marginal tax rate at both ends of income is zero. Tax is an increasing function of income.

The level of public goods provided in this case is 0.2892.

It is interesting to compare the outcome of single district case with that of the multiple district case. We consider the case when there are three districts, each with a uniform distribution of wage rates over the intervals, $[1,2)$, $[2,3)$ and $[3,4]$. Then there are eight cases of legislative coalition formation. We use the symbol, $\rightarrow$, to represent "propose to and form coalition with". The eight cases are $(1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 1)$, $(1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 2)$, $(1 \rightarrow 2, 2 \rightarrow 1, 3 \rightarrow 1)$, $(1 \rightarrow 2, 2 \rightarrow 1, 3 \rightarrow 2)$, $(1 \rightarrow 3, 2 \rightarrow 3, 3 \rightarrow 1)$, $(1 \rightarrow 3, 2 \rightarrow 3, 3 \rightarrow 2)$, $(1 \rightarrow 3, 2 \rightarrow 1, 3 \rightarrow 1)$, $(1 \rightarrow 3, 2 \rightarrow 1, 3 \rightarrow 2)$. As an example, the first case is set up as the following,

$$\max_{\tau_1, I_1} \quad \textstyle\int_1^2 \ln[I_1 - \tau_1 + \ln(1 - I_1/\omega) + \ln(\int_1^4 \tau_1/3d\omega) + e]d\omega$$

$$s.t. \quad I_1' - \tau_1' = \frac{I_1'}{\omega - I_1}$$

$$I_1 - \tau_1 \geq 0$$

$$\textstyle\int_2^3 \ln[I_1 - \tau_1 + \ln(1 - I_1/\omega) + \ln(\int_1^4 \tau_1/3d\omega) + e]d\omega \geq$$

$$\tfrac{1}{2}\{\textstyle\int_2^3 \ln[I_2 - \tau_2 + \ln(1 - I_2/\omega) + \ln(\int_1^4 \tau_2/3d\omega) + e]d\omega$$

$$+ \textstyle\int_2^3 \ln[I_3 - \tau_3 + \ln(1 - I_3/\omega) + \ln(\int_1^4 \tau_3/3d\omega) + e]d\omega\}$$

$$\max_{\tau_2, I_2} \quad \int_2^3 \ln[I_2 - \tau_2 + \ln(1 - I_2/\omega) + \ln(\int_1^4 \tau_2/3d\omega) + e]d\omega$$

$$s.t. \quad I_2^{'} - \tau_2^{'} = \frac{I_2^{'}}{\omega - I_2}$$

$$I_2 - \tau_2 \geq 0$$

$$\int_3^4 \ln[I_2 - \tau_2 + \ln(1 - I_2/\omega) + \ln(\int_1^4 \tau_2/3d\omega) + e]d\omega \geq$$

$$\tfrac{1}{2}\{\int_3^4 \ln[I_3 - \tau_3 + \ln(1 - I_3/\omega) + \ln(\int_1^4 \tau_3/3d\omega) + e]d\omega$$

$$+ \int_3^4 \ln[I_1 - \tau_1 + \ln(1 - I_1/\omega) + \ln(\int_1^4 \tau_1/3d\omega) + e]d\omega\}$$

$$\max_{\tau_3, I_3} \quad \int_3^4 \ln[I_3 - \tau_3 + \ln(1 - I_3/\omega) + \ln(\int_1^4 \tau_3/3d\omega) + e]d\omega$$

$$s.t. \quad I_3^{'} - \tau_3^{'} = \frac{I_3^{'}}{\omega - I_3}$$

$$I_3 - \tau_3 \geq 0$$

$$\int_1^2 \ln[I_3 - \tau_3 + \ln(1 - I_3/\omega) + \ln(\int_1^4 \tau_3/3d\omega) + e]d\omega \geq$$

$$\tfrac{1}{2}\{\int_1^2 \ln[I_1 - \tau_1 + \ln(1 - I_1/\omega) + \ln(\int_1^4 \tau_1/3d\omega) + e]d\omega$$

$$+ \int_1^2 \ln[I_2 - \tau_2 + \ln(1 - I_2/\omega) + \ln(\int_1^4 \tau_2/3d\omega)]d\omega\}$$

The equilibrium proposals of all three legislators can be calculated using numerical solutions. Figure 2.3 and 2.4 shows the numerical solutions to the three district case. It is interesting to observe that the equilibrium proposal of Legislator 2, the representative of the "middle productivity" district, coincides with the equilibrium proposal of the single district case. The public goods levels as outcomes of the three proposals are 0.3667, 0.2892, and 0.3272 respectively.

Although analytical solutions and comparative statics results are hard to obtain, we learned from the example that we can form some testable implications if the distribution of the wage rates are known and if we can parameterize the utility function somehow.

2.6 Conclusions

In this chapter we address two shortcomings of the optimal income taxation literature, i.e., exogenous social welfare functions and the neglect of institutional constraints. We characterize the optimal income tax schedule using a general equilibrium model with a public good entering consumers' utility functions. We show that the social welfare functions can be determined endogenously by political processes, i.e., electoral systems and the legislative process. We characterize the equilibrium tax schedules under the two party plurality system, including the single-district case and the multiple-district case. It is shown that under the two party plurality system, the equilibrium income tax is equivalent to an optimal tax schedule which puts equal weight over the whole population when there is a single district; when there are multiple districts, however, in the simplest subgame perfect stationary equilibrium to the legislative game, the equilibrium is equivalent to an optimal tax schedule which puts more welfare weight on the subsets of the population whose legislators are in the winning coalition of the legislature. Thus we have shown that the political processes endogenously determine the welfare weights of the optimal income taxation problem.

The characterizations of the equilibrium tax schedules in this chapter provide considerable insight into the factors influencing the equilibrium marginal tax rates under different political processes, and the way they interact. More general results are hard to obtain from these formulas. Given the distribution of the productivity levels, however, we can form some testable implications by parameterizing the utility functions to get the explicit equilibrium income tax schedules.

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Figure 2.1: Revenue Requirement Function and Income Function: Single District

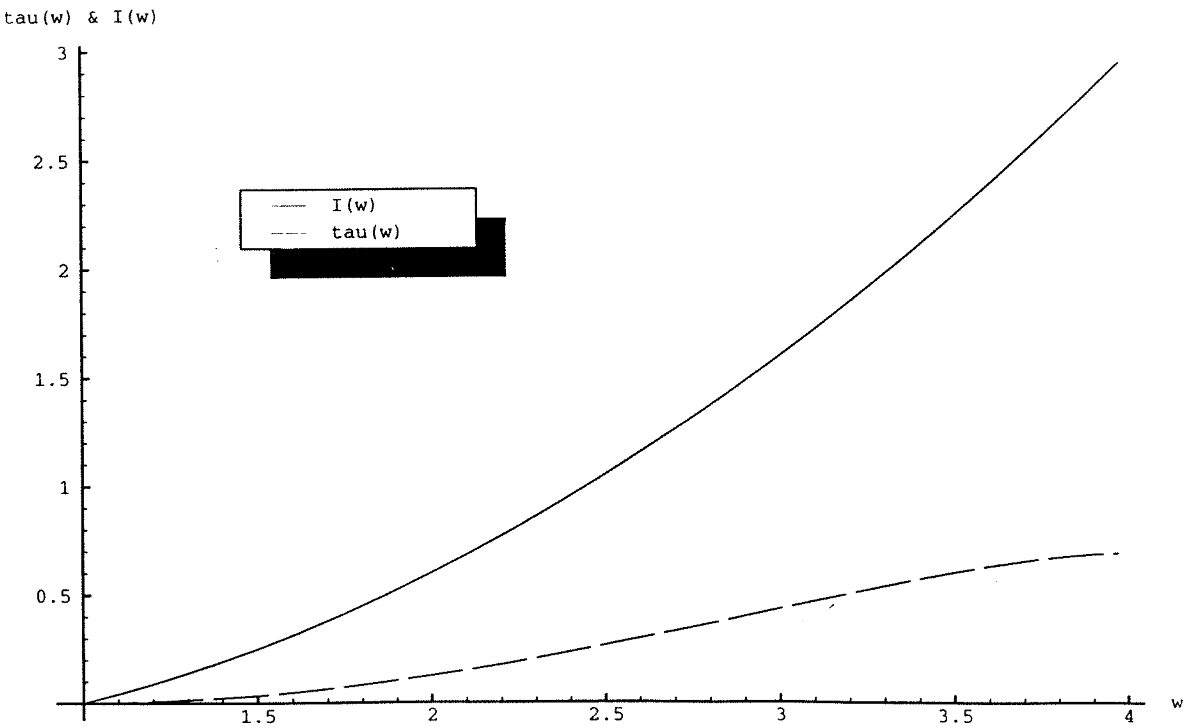

Figure 2.2: Income Tax Function: Single District

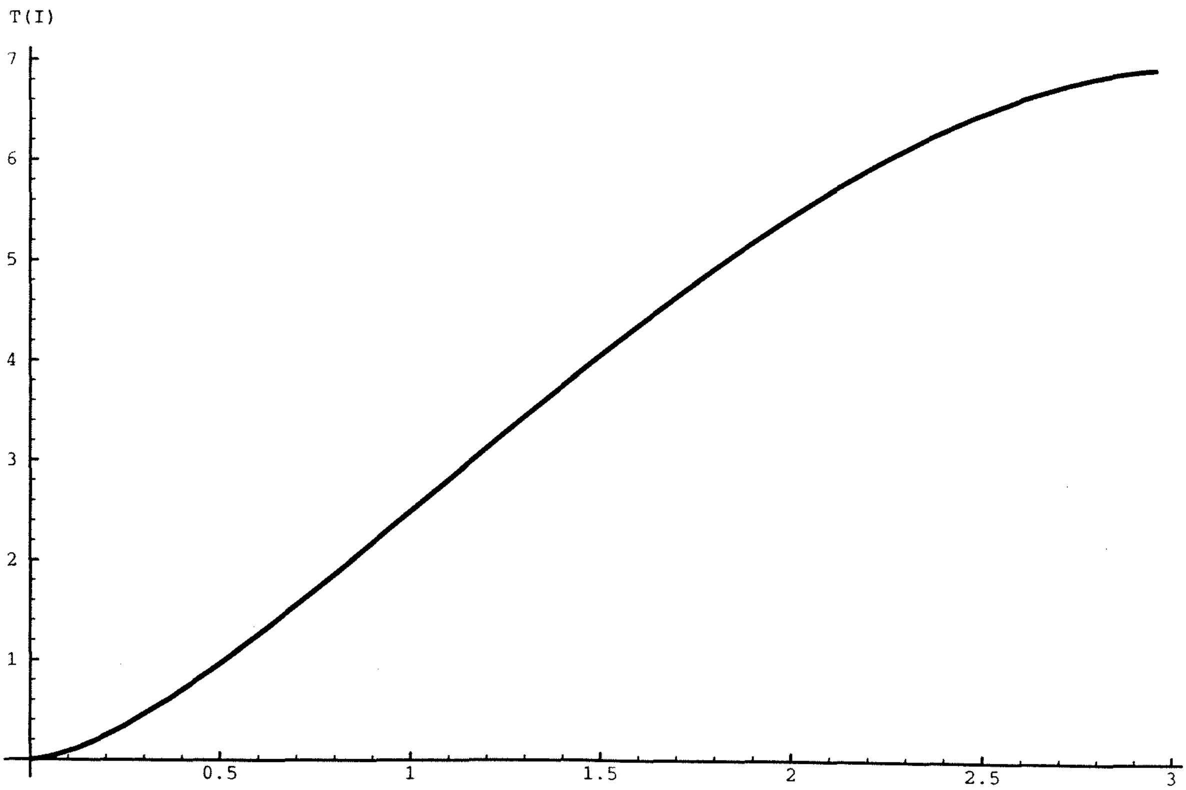

Figure 2.3: Revenue Requirement Functions and Income Functions: Three Districts

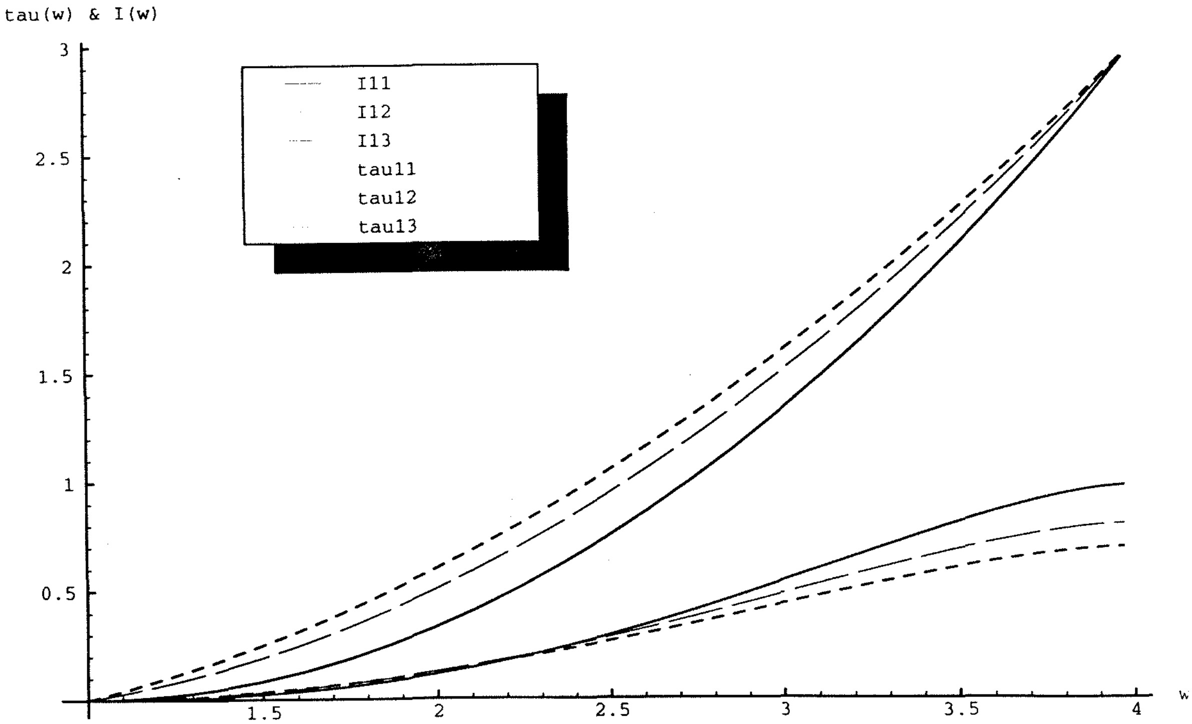

Figure 2.4: Income Tax Functions: Three Districts