THE EVAPORATION RATE OF LIQUID DROPLETS IN A HOT GAS

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SUMMARY

Calculations have been carried out in order to determine the rate of evaporation of a liquid droplet surrounded by hot gases. The present study represents an extension of earlier work by Penner on evaporation rates for isothermal droplets. Thus, allowance was made for temperature gradients within the droplet (a) by considering a droplet composed of an isothermal core and an isothermal shell and (b) by utilizing the actual temperature profile in the droplet as established as the result of a heat balance between thermal conduction within the droplet, convective heat transfer to the droplet, and cooling produced by evaporation at the droplet surface.

The results obtained for the shell model of the evaporating droplet were found to be in satisfactory agreement with the known data for evaporation of isothermal droplets, independently of the thickness chosen for the isothermal shells. On the other hand, the laborious conduction solution led to somewhat different results. The origin of the detailed deviations is not clear at this time and requires additional study.

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SYMBOLS

t	=	time in seconds
η	=	distance from center of drop to any point in cm.
r	=	radius of droplet, function of T and t, cm.
ε	=	thickness of isothermal shell, cm.
T	=	temperature, function of η and t, $^{\circ}K$
T_o	=	temperature of the droplet at time zero, $^{\circ}K$
T_1	=	droplet surface temperature, $^{\circ}K$
T_g	=	temperature of the hot gas, $^{\circ}K$
ΔT	=	$(T_g - T_1)$, $^{\circ}K$
b	=	$6000^{\circ}K$
m	=	mass rate of evaporation per unit area, function of T, gm/cm^2 sec
ρ	=	density of liquid, gm/cm^3
Q	=	rate of heat transfer to the entire droplet, cal/sec
λ	=	coefficient of heat conduction in the liquid, cal/cm-sec-$^{\circ}K$
k	=	thermal conductivity of hot gas, cal/cm sec $^{\circ}K$
h	=	surface coefficient of heat transfer, cal/cm^2 sec $^{\circ}K$
c	=	specific heat per unit mass, cal/gm $^{\circ}K$
ℓ	=	latent heat of evaporation, cal/gm
$(\ )_o$	=	quantity evaluated at time equal zero
D	=	λ/cmr, a nondimensional parameter
E	=	$k/\ell mr$, $^{\circ}K^{-1}$

SYMBOLS (Continued)

F	=	ET_1^2/b, a nondimensional parameter
G	=	$E \Delta T$, a nondimensional parameter
α	=	r/r_o
β	=	m/m_o
σ	=	T_1/T_o

I. INTRODUCTION

The rate of evaporation of droplets in hot gases is of obvious importance in heterogeneous combustion in rocket motors. If the vaporization rate is known, then the lifetime of droplets can be computed as a function of original size and motor conditions. Information of this sort is of interest in connection with the rational design of liquid-fuel injection systems.

The purpose of this thesis is to extend earlier work on evaporation rates of isothermal droplets in rocket engines (Reference 1). Penner treated the evaporation rate of a liquid droplet in a gas at constant temperature. Although the droplet temperature changed with time, it was assumed that the droplet remained isothermal during evaporation, i.e., that the thermal conduction coefficients of evaporating liquid droplets were, for all practical purposes, infinite.

We have considered two models for evaporating droplets. These are (a) the shell model of the evaporating droplet in which the droplet is divided into two parts, an inner core and an outer spherical shell. The inner sphere was assumed to be isothermal and to remain at the original temperature of the evaporating liquid; the outer shell was also assumed to be isothermal but its temperature was determined by making an appropriate heat balance equation. The thickness of the spherical shell was treated as a variable parameter and it was found that results substantially equivalent to those obtained for isothermal droplets are derived, independently of the thickness of the spherical shell.

The second temperature profile (b) was obtained from heat transfer equations set up by Tsien. The resulting model of the evaporating droplet will be referred to hereafter as the conduction model.

Of the evaporating droplets treated previously[1] by assuming isothermal conditions during evaporation, we have chosen for study with different temperature profiles, an aniline droplet with initial radius equal to 5×10^{-3} cm. The initial temperature was set equal to 300 ^{o}K. Radiant heat transfer to the droplet was neglected since it is known to be small in rocket motors[1].

II. THE SHELL MODEL OF THE EVAPORATING DROPLET

A. Outline of Theory

The concept of an isothermal droplet during evaporation in combustion chambers represents a reasonable first approximation. In practice one would expect that temperature gradients are set up during the life of the droplet. If the thermal conductivity is sufficiently small, then it is clear that steep temperature gradients may be set up near the surface of the droplet. However, the surface temperature is not necessarily higher than for an isothermal droplet because it is determined by a heat balance involving cooling by evaporation.

As a first approximation to a droplet with temperature gradients it was assumed that an evaporating droplet could be represented by an isothermal core surrounded by an isothermal shell. It was further assumed that no heat was transferred from the shell to the inner sphere and that the shell thickness was constant during evaporation. The shell thickness ε (cm) was set equal to 25%, 1%, and .001% of the original droplet radius.

In general the surface temperature will be determined by an energy balance between heat input into the droplet and absorption of energy by evaporation or by thermal conduction to the core of the spherical droplet.

The rate of heat transfer Q (cal/sec) to a sphere of radius r (cm) is

$$Q = 4\pi r^2 h \, \Delta T \tag{1}$$

where ΔT (°K) represents the temperature difference between the gases surrounding the liquid droplet and the surface temperature of the liquid droplet, and h (cal/cm^2 sec °K) is the over-all coefficient of heat transfer to a liquid droplet evaluated at the mean film temperature $T_f = T_1 + \frac{1}{2}(T_g - T_1)$, where T_g (°K) is the temperature of the hot gases and T_1 (°K) is the temperature of the surface of the liquid.

For the simplified evaporation model employed for the present studies, all of the energy input occurs into a shell of volume

$$\begin{aligned} V &= 4\pi r^3/3 - 4\pi \int_0^{r-\varepsilon} r^2 \, dr \\ &= (4\pi/3)[r^3 - (r-\varepsilon)^3] \\ &= (4\pi/3) r^3 [1 - (1 - \varepsilon/r)^3] \end{aligned} \tag{2}$$

From equation (2) it is seen that for

$$\begin{aligned} \varepsilon/r &<< 1 \\ V &\simeq 4\pi r^2 \varepsilon \end{aligned} \tag{2a}$$

and, for $\varepsilon/r = 1$ (isothermal droplet)

$$V = (4\pi/3)r^3 \tag{2b}$$

It is apparent from equations (1) and (2) that if heat transport to the spherical droplet were the only important physical process, then the temperature T_1 in the isothermal shell of thickness ε would rise at a rate

$$\left.\frac{dT}{dt}\right)_c = 3h\,\Delta T/r\,[1-(1-\varepsilon/r)^3]\,\rho c \tag{3}$$

where ρ (gm/cc) and c (cal/gm) represent, respectively, the density and heat capacity of the liquid in the isothermal shell. For

$$\varepsilon/r << 1$$

equation (3) becomes

$$\left.\frac{dT}{dt}\right)_c = h\,\Delta T/\varepsilon\rho c \tag{3a}$$

and for $\varepsilon/r = 1$ (isothermal droplet)

$$\left.\frac{dT}{dt}\right)_c = 3h\,\Delta T/r\rho c \tag{3b}$$

We assume that ε remains constant as the droplet radius r decreases during evaporation. Hence the rate of decrease of volume by evaporation of the isothermal shell of thickness ε is

$$-\frac{dV}{dt} = 4\pi r^2(-dr/dt) \tag{4}$$

The rate of absorption of heat during evaporation Q_e (cal/sec) is evidently

$$Q_e = (-dV/dt)(\rho \ell) \tag{5}$$

where ℓ (cal/gm) equals the heat of evaporation of the liquid in the shell of thickness ε . Since the total heat capacity of the shell is

$$V\rho c$$

the rate of decrease of temperature in the shell, if only evaporation occurred is

$$-(dT/dt)_e = (3\ell/cr)(-dr/dt) \tag{6}$$

The model of the evaporating droplet adopted in the present discussion requires allowance for another heat sink. Thus the assumption that ε is constant means that the inner surface of the isothermal shell must travel in the direction of the center of the sphere sufficiently rapidly to maintain ε constant. This travel of the inner surface of the outer shell means that some mass will be introduced from the colder core of the droplet into the outer shell. Since the shell must remain isothermal, heat must be added to this new mass to bring it up to the temperature of the shell. The surface area

$$4\pi(r-\varepsilon)^2$$

moves inward with a velocity (-dr/dt). Hence the energy absorbed per unit time in order to keep ε constant and to keep the shell isothermal is

$$4\pi(r-\varepsilon)^2(-dr/dt)\rho c\,(T_1-T_0)$$

In order to correct for this heat sink, we may say that of the total heat Q transferred to the droplet in unit time, only the amount

$$Q = 4\pi r^2 h\Delta T - 4\pi(r-\varepsilon)^2(-dr/dt)\rho c\,(T_1-T_0) \qquad \text{(1a)}$$

is effective in producing evaporation and heating the outer isothermal shell.

If equation (1a) is used in place of equation (1), equations (3) to (3b) become, respectively, for $0 \le \varepsilon/r \le 1$

$$(dT/dt)_c = 3h\Delta T/r[1-(1-\varepsilon/r)^3]\rho c$$
$$- 3(1-\varepsilon/r)^2(-dr/dt)(T_1-T_0)/r[1-(1-\varepsilon/r)^3] \qquad \text{(3)}$$

for $\varepsilon/r << 1$

$$(dT/dt)_c = h\Delta T/\varepsilon\rho c - (-dr/dt)(T_1-T_0)/\varepsilon \qquad \text{(3a)}$$

and for $\varepsilon/r = 1$

$$(dT/dt)_c = 3h\Delta T \qquad \text{(3b)}$$

For simplicity we consider the case in which heat transfer between the isothermal shells may be neglected. In this case

$$dT/dt = (dT/dt)_c + (dT/dt)_e$$

represents the rate of change of temperature in the isothermal shell with time. Using equations (3) to (3b) and (6) to (6b) we obtain the results

$$dT/dt = \{3/r[1-(1-\varepsilon/r)^3]\}\{(h\Delta T/c\rho)$$

$$-(1-\varepsilon/r)^2(-dr/dt)(T_1-T_0)-(\ell/c)(-dr/dt)\} \quad (7)$$

for $\varepsilon/r << 1$

$$dT/dt = (h\Delta T/\varepsilon\rho c)-(-dr/dt)[\ell/c\varepsilon + (T_1-T_0)/\varepsilon] \quad (7a)$$

and for $\varepsilon/r = 1$

$$dT/dt = 3h\Delta T/r\rho c - (3\ell/cr)(-dr/dt) \quad (7b)$$

In order to carry out approximate calculations for the rate of change of temperature T_1 and droplet radius r with time, we may assume that -dr/dt is given by the Knudsen equation with an evaporation coefficient α which we shall assume to be independent of temperature.* Thus

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* The assumption that α is independent of T_1 can be dropped by using recently developed theoretical methods for the calculation of α. The theoretical lower limit for h is k/r in a stagnant medium unless the spheres have diameters which are of the same order of magnitude as the mean free path in the fluid. Compare, for example, Ref. 1.

$$-dr/dt = (\alpha p/\rho)(m/2\pi RT)^{1/2} \tag{8}$$

where p is the saturated vapor pressure of the liquid at the temperature T_1 and R is the molar gas constant. The pairs of equations (7), (7a) or (7b) together with equation (8) can be solved by a simple iterative procedure.

Representative numerical values for aniline are the following: $c = 0.53$ cal/gm °K; $r^o = 5 \times 10^{-3}$ cm; $h = k/r = 4.30 \times 10^{-4}/r$ cal/cm^2 sec °K; $\Delta T = (3000 - T_1)$ °K; $\rho = 1.02$ gm/cc; $\ell = 129$ cal/gm; $-dr/dt = \text{antilog}_{10}\ (7.09 - 2600/T_1)$; $T_o = 300$ °K; $T_f = 1650$ °K.

For isothermal evaporation of aniline droplets equation (7b) becomes

$$\frac{dT}{dt} = 2.381 \times 10^{-3}(3000-T_1)/r^2 - (730/r)(-dr/dt) \tag{10}$$

Plots showing T_1 and r as a function of time if the initial radius is 5×10^{-3}, 1×10^{-2} or 5×10^{-2} cm have been given in Figs. 1 to 3 of Ref. 1.

B. Summary of Calculations

Since the pair of equations (7) and (8) could not be integrated readily to obtain $T \equiv T_1$ as an explicit function of time, an iteration procedure was used. First a value was assumed for the ratio ε/r. Then T_1 was set equal to T_0 or 300°K. Equations (7) and (8) were used to get the first definite value for dT_1/dt. Then an increment of time Δt was assumed. The new temperature T_1 was then equated to $T_0 + \Delta t(dT/dt)$. With this new value of T_1, dr/dt was computed and hence a new value of the radius. It was now possible to compute a second value for dT/dt. The entire process was repeated until the original radius had decreased by 50%.

The results of the computations are plotted for three values of ε/r in the attached Figure 1.

C. Discussion of Results

The smaller the value of ε/r chosen, the faster the shell temperature rises (Figure 1). However, the higher the temperature, the faster the evaporation rate. Therefore, although the temperature rises faster initially in the thinner shells, a pseudo-steady state temperature $T \equiv T_1$ is apparently reached at a slightly lower temperature than for an isothermally evaporating droplet. In general, reference to Figure 1 shows that the results obtained from the shell model are consistent with those obtained for the isothermal case. Hence it may be justified to conclude that calculation of evaporation rates, based on the isothermal approximation, are reasonable.

III. CONDUCTION MODEL OF THE EVAPORATING DROPLET

A. Outline of Theory

Although the shell model is a closer approach to a realistic description of an evaporating droplet than is the isothermal droplet, it is of obvious interest to consider a model with reasonable temperature gradients. This problem was set up by Tsien and its solution is discussed in the following paragraphs.

This model of the evaporating droplet, which will be referred to as the conduction model, involves a temperature distribution within the droplet which is approximated by a parabolic curve.

B. Summary of Calculations

The differential equation for the heat conduction is

$$\lambda \frac{1}{\eta^2} \frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial T}{\partial \eta}\right) = c\rho \frac{\partial T}{\partial t} \qquad (1)$$

We shall not try to solve the equation exactly. We shall assume T to be given by the following form

$$T(\eta, t) = A(t) + \eta^2 B(t) + \eta^4 C(t) \qquad (2)$$

where A(t), B(t) and C(t) are unknown functions of t, to be determined by the following three conditions

a) Equation (1) to be satisfied <u>in the mean</u>,

$$\lambda \int_0^{r} \frac{\partial}{\partial \eta}\left(\eta^2 \frac{\partial T}{\partial \eta}\right) d\eta = c\rho \int_0^{r} \eta^2 \frac{\partial T}{\partial t}\, d\eta \qquad (3)$$

b) Equation (1) to be satisfied at the center

$\eta = 0$ for all t.

c) At η = r, the heat balance is satisfied.*

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* This equation is

$$h \Delta T = \ell m + \lambda \left(\frac{\partial T}{\partial \eta} \right)_{\eta = r}$$

It can be gotten from our work done on the shell model using the shell model equation (7a)

$$\frac{\partial T}{\partial t} = \frac{h \Delta T}{\varepsilon \rho c} - \left(- \frac{dr}{dt} \right) \left[\frac{\ell}{c \varepsilon} + (T_1 - T_0)/\varepsilon \right]$$

The rate of heat loss from the spherical shell to the colder core is

$$-4\pi (r - \varepsilon)^2 \lambda \left(dT/dr \right)_{\eta = (r - \varepsilon)}$$

This term must replace our shell model term for heat absorbed by the incoming mass as its temperature is raised from T_0 to T_1. Our new conduction model neglecting the term for heating liquid from T_0 to T_1 can be expressed, for $\varepsilon/r \ll 1$, as

$$-\frac{\partial T}{\partial t} = \left[4\pi (r - \varepsilon)^2 \lambda / V \rho c \right] \left(\partial T / \partial r \right)_{r - \varepsilon} \simeq \lambda / \rho c \varepsilon \frac{\partial T}{\partial r}$$

Therefore equation (7a) for the shell model becomes for the conduction model

$$\frac{\partial T}{\partial t} = \frac{h \Delta T}{\rho c \varepsilon} - \left(- \frac{dr}{dt} \right) \left[\ell / c \varepsilon \right]$$

which can also be written as

$$\frac{h \Delta T}{\rho c} = \varepsilon \frac{\partial T}{\partial t} + \frac{\lambda}{\rho c} \frac{\partial T}{\partial r} - \ell/c \frac{dr}{dt}$$

If we now let ε approach zero we get the final form for condition c

$$h \Delta T = \ell m + \lambda \frac{\partial T}{\partial r}$$

Equation (3) gives

$$\lambda[2B(t) + 4r^2C(t)] = c\rho[\tfrac{1}{3}A'(t) + \tfrac{1}{5}r^2B'(t) + \tfrac{1}{7}r^4C'(t)] \tag{4}$$

where the primes mean differentiation with respect to the argument of the function.

The condition b gives

$$6\lambda B(t) = c\rho A'(t) \tag{5}$$

The condition c gives

$$h(T_g - T_1) = \ell m(T_1) + \lambda[2rB(t) + 4r^3C(t)] \tag{6}$$

T_1 is given by equation (2) as

$$T_1(t) = A(t) + r^2B(t) + r^4C(t) \tag{7}$$

And the equation for rate of evaporation is:

$$m(T_1) = -\rho\, dr/dt \tag{8}$$

The equations (4) to (8) are five equations for the five unknowns A(t), B(t), C(t), r(t), and T_1(t).

Now (4) and (5) give:

$$B'(t) = \frac{20\lambda C(t)}{c\rho} - \frac{5r^2C'(t)}{7} \tag{9}$$

Differentiating equation (7) and then eliminating A'(t) by equation (5):

$$\frac{dT_1}{dt} = \left(2r\frac{dr}{dt} + \frac{6\lambda}{c\rho}\right) B(t) + r^2 B'(t)$$

$$+ 4r^3 \frac{dr}{dt} C(t) + r^4 C'(t) \tag{10}$$

Equations (6), (8), (9), and (10) are now four equations for four unknowns, B(t), C(t), T_1(t), and r(t).

Equation (6) can be written as:

$$\frac{k(T_g - T_1)}{2\lambda r^2} - \frac{\ell m(T_1)}{2\lambda r} = B(t) + 2r^2 C(t) \tag{11}$$

Differentiating equation (11),

$$-\frac{k}{\lambda r^3}\frac{dr}{dt}(T_g - T_1) - \frac{k}{2\lambda r^2}\frac{dT_1}{dt} + \frac{\ell m(T_1)}{2\lambda r^2}\frac{dr}{dt}$$

$$-\frac{\ell m'(T_1)}{2\lambda r}\frac{dT_1}{dt} = B'(t) + 4r\frac{dr}{dt} C(t) + 2r^2 C'(t) \tag{12}$$

Using equation (9)

$$-\frac{k}{\lambda r^3}\frac{dr}{dt}(T_g - T_1) - \frac{k}{2\lambda r^2}\frac{dT_1}{dt} + \frac{\ell m(T_1)}{2\lambda r^2}\frac{dr}{dt} - \frac{\ell m'(T_1)}{2\lambda r}\frac{dT_1}{dt}$$

$$= \left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right) C(t) + \frac{9r^2 C'(t)}{7} \tag{13}$$

Equation (10) can be written as

$$\frac{1}{r^2}\frac{dT_1}{dt} = \left(\frac{2}{r}\frac{dr}{dt} + \frac{6\lambda}{c\rho r^2}\right)B(t) + B'(t) + 4r\frac{dr}{dt}C(t) + r^2C'(t) \tag{14}$$

Using equation (9)

$$\frac{1}{r^2}\frac{dT_1}{dt} = \left(\frac{2}{r}\frac{dr}{dt} + \frac{6\lambda}{c\rho r^2}\right)B(t) + \left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)C(t) + \frac{2r^2C'(t)}{7} \tag{15}$$

Using equation (11)

$$\frac{1}{r^2}\frac{dT_1}{dt} = \left(\frac{2}{r}\frac{dr}{dt} + \frac{6\lambda}{c\rho r^2}\right)\left\{\frac{k}{2\lambda r^2}(T_g - T_1) - \frac{\ell m(T_1)}{2\lambda r}\right\}$$
$$-\left(2r\frac{dr}{dt} + \frac{6\lambda}{c\rho}\right)2C(t) + \left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)C(t) + \frac{2r^2C'(t)}{7}$$

$$\frac{1}{2r^2}\frac{dT_1}{dt} = \left(r\frac{dr}{dt} + \frac{3\lambda}{c\rho}\right)\left\{\frac{k}{2\lambda r^4}(T_g - T_1) - \frac{\ell m(T_1)}{2\lambda r^3}\right\} + \frac{4\lambda}{c\rho}C(t) + \frac{r^2C'(t)}{7} \tag{16}$$

Now equations (8), (13) and (16) are three equations for three unknowns $C(t)$, $T_1(t)$ and $r(t)$.

We must now eliminate $C(t)$ remembering that $T_1(t)$ is a function of t.

Equation (16) can be written as:

$$C(t) = \frac{c\rho}{4\lambda}\left[\frac{1}{2r^2}\frac{dT_1}{dt} - \left(r\frac{dr}{dt} + \frac{3\lambda}{c\rho}\right)\left\{\frac{k}{2\lambda r^4}(T_g - T_1) - \frac{\ell m(T_1)}{2\lambda r^3}\right\} - \frac{r^2C'(t)}{7}\right] \tag{17}$$

Equation (13) can be written as:

$$C'(t) = \Big[-\frac{k}{\lambda r^3}\frac{dr}{dt}(T_g - T_1) - \frac{k}{2\lambda r^2}\frac{dT_1}{dt} + \frac{\ell m(T_1)}{2\lambda r^2}\frac{dr}{dt} - \frac{\ell m'(T_1)}{2\lambda r}\frac{dT_1}{dt}$$

$$- \left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right) C(t)\Big]\left[\frac{7}{9r^2}\right] \tag{18}$$

Combining equation (18) into (17) we get:

$$C(t) = \frac{c\rho/4\lambda}{1 - \frac{c\rho}{36\lambda}\left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)}\Big[\frac{1}{2r^2}\frac{dT_1}{dt}$$

$$- \left(r\frac{dr}{dt} + \frac{3\lambda}{c\rho}\right)\left(\frac{k(T_g - T_1)}{2\lambda r^4} - \frac{\ell m(T_1)}{2\lambda r^3}\right) \tag{19}$$

$$- \frac{1}{9}\Big\{ -\frac{k}{\lambda r^3}\frac{dr}{dt}(T_g - T_1) - \frac{k}{2\lambda r^2}\frac{dT_1}{dt}$$

$$+ \frac{\ell m(T_1)}{2\lambda r^2}\frac{dr}{dt} - \frac{\ell m'(T_1)}{2\lambda r}\frac{dT_1}{dt}\Big\}\Big]$$

$$C(t) = \frac{9}{4}\left(\frac{1}{\frac{4\lambda}{c\rho} - r\frac{dr}{dt}}\right)\left[\frac{1}{2r^2}\left(1 + \frac{k + lrm'(T_1)}{9\lambda}\right)\frac{dT_1}{dt}\right.$$

$$+ \frac{1}{2\lambda r^3}\left(\frac{8}{9}\,lrm(T_1) - \frac{7}{9}\,k(T_g - T_1)\right)\frac{dr}{dt}$$

$$\left. - \frac{3\lambda}{c\rho}\left(\frac{k(T_g - T_1)}{2\lambda r^4} - \frac{lrm(T_1)}{2\lambda r^4}\right)\right]$$

$$C(t) = \frac{1}{8\lambda r^4}\left(\frac{1}{\frac{4\lambda}{c\rho} - r\frac{dr}{dt}}\right)\left[r^2\left(9\lambda + k + lrm'(T_1)\right)\frac{dT_1}{dt}\right.$$

$$+ \left(8lrm(T_1) - 7k(T_g - T_1)\right)r\frac{dr}{dt}$$

$$\left. - \frac{27\lambda}{c\rho}\left(k(T_g - T_1) - lrm(T_1)\right)\right]$$

$$C(t) = \left(\frac{1}{\frac{32\lambda^2 r^4}{c\rho} - 8\lambda r^5\frac{dr}{dt}}\right)\left[r^2\left(9\lambda + k + lrm'(T_1)\right)\frac{dT_1}{dt}\right.$$

$$+ \left(8lrm(T_1) - 7k(T_g - T_1)\right)r\frac{dr}{dt}$$

$$\left. - \frac{27\lambda}{c\rho}\left(k(T_g - T_1) - lrm(T_1)\right)\right] \qquad (20)$$

Differentiating equation (20) we get:

$$C'(t) = -\left(\frac{1}{\frac{32\lambda^2 r^4}{c\rho} - 8\lambda r^5 \frac{dr}{dt}}\right)\left(\frac{128\lambda^2 r^3}{c\rho}\frac{dr}{dt} - 40\lambda r^4\left(\frac{dr}{dt}\right)^2\right.$$

$$\left.- 8\lambda r^5 \frac{d^2 r}{dt^2}\right)\Big[r^2(9\lambda + k + lr\,m'(T_1))\frac{dT_1}{dt}$$

$$+ \left(8lr\,m(T_1) - 7k(T_g - T_1)\right)r\frac{dr}{dt} - \frac{27\lambda}{c\rho}\left(k(T_g - T_1) - lr\,m(T_1)\right)\Big]$$

$$+ \left(\frac{1}{\frac{32\lambda^2 r^4}{c\rho} - 8\lambda r^5 \frac{dr}{dt}}\right)\Big[2r(9\lambda + k + lr\,m'(T_1))\frac{dr}{dt}\frac{dT_1}{dt}$$

$$+ r^2 l\,m'(T_1)\frac{dr}{dt}\frac{dT_1}{dt} + r^2 lr\,m''(T_1)\left(\frac{dT_1}{dt}\right)^2$$

$$+ r^2 lr\,m'(T_1)\frac{d^2T_1}{dt^2} + r^2(9\lambda + k)\frac{d^2T_1}{dt^2} + \left(8lr\,m(T_1)\right.$$

$$\left.- 7k(T_g - T_1)\right)\left(\frac{dr}{dt}\right)^2 + \left(8lr\,m(T_1) - 7k(T_g - T_1)\right)r\frac{d^2 r}{dt^2}$$

$$+ \left\{8l\,m(T_1)\frac{dr}{dt} + 8lr\,m'(T_1)\frac{dT_1}{dt} + 7k\frac{dT_1}{dt}\right\}r\frac{dr}{dt}$$

$$+ \frac{27\lambda k}{c\rho}\frac{dT_1}{dt} + \frac{27\lambda l\,m(T_1)}{c\rho} + \frac{27\lambda lr\,m'(T_1)}{c\rho}\frac{dT_1}{dt}\Big] \qquad (21)$$

Combining equations (17) and (18) we get:

$$C'(t) = \left\{ \frac{7/9\, r^2}{1 - \frac{1}{9}\left(\frac{c\rho}{4\lambda}\right)\left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)} \right\} \left\{ \left(\ell r\, m(T_1) - 2k\,(T_g - T_1) \right) \frac{dr}{dt} \right.$$

$$- (\ell r\, m'(T_1) + k)\, r\,\frac{dT_1}{dt} - \frac{1}{r}\left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)\left(\frac{c\rho}{4\lambda}\right) r^2 \lambda \frac{dT_1}{dt}$$

$$\left. - \left[r\frac{dr}{dt} + \frac{3\lambda}{c\rho} \right]\left[k\,(T_g - T_1) - \ell r\, m(T_1) \right] \right\} \left\{ \frac{1}{2\lambda r^3} \right\} \qquad \textbf{(22)}$$

From equations (21) and (22) we have eliminated C(t) and C'(t):

$$\left\{ \frac{7/9\, r^2}{1 - \frac{1}{9}\left(\frac{c\rho}{4\lambda}\right)\left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)} \right\} \left\{ (\ell r\, m - 2k\,\Delta T) \frac{dr}{dt} \right.$$

$$- (\ell r\, m' + k)\, r\,\frac{dT_1}{dt} - \frac{1}{r}\left(\frac{20\lambda}{c\rho} + 4r\frac{dr}{dt}\right)\left(\frac{c\rho}{4\lambda}\right)\left(r^2 \lambda \frac{dT_1}{dt} \right.$$

$$\left. - \left[r\frac{dr}{dt} + \frac{3\lambda}{c\rho} \right]\left[k\,\Delta T - \ell r\, m \right] \right\} \left\{ \frac{1}{2\lambda r^3} \right\} =$$

$$= -\left(\frac{1}{8\lambda r^4}\right)^2\left(\frac{1}{\frac{4\lambda}{c\rho} - r\frac{dr}{dt}}\right)^2\left(\frac{128\lambda^2 r^3}{c\rho}\frac{dr}{dt}\right.$$

$$\left. - 40\lambda r^4\left(\frac{dr}{dt}\right)^2 - 8\lambda r^5\frac{d^2r}{dt^2}\right)\left[r^2(9\lambda + k + lrm')\frac{dT_1}{dt}\right.$$

$$\left. + (8lrm - 7k\Delta T)r\frac{dr}{dt} - \frac{27\lambda}{c\rho}(k\Delta T - lrm)\right]$$

$$+ \left(\frac{1}{\frac{32\lambda^2 r^4}{c\rho} - 8\lambda r^5\frac{dr}{dt}}\right)\left[2r(9\lambda + k + lrm')\frac{dr}{dt}\frac{dT_1}{dt}\right.$$

$$+ r^2 lm'\frac{dr}{dt}\frac{dT_1}{dt} + r^2 lrm''\left(\frac{dT_1}{dt}\right)^2 + r^2(9\lambda + k + lrm')\frac{d^2T_1}{dt^2}$$

$$+ (8lrm - 7k\Delta T)\left(\frac{dr}{dt}\right)^2 + (8lrm - 7k\Delta T)r\frac{d^2r}{dt^2}$$

$$+ \left(8lm\frac{dr}{dt} + 8lrm'\frac{dT_1}{dt} + 7k\frac{dT_1}{dt}\right)r\frac{dr}{dt}$$

$$\left. + \frac{27\lambda k}{c\rho}\frac{dT_1}{dt} + \frac{27\lambda lm}{c\rho}\frac{dr}{dt} + \frac{27\lambda lrm'}{c\rho}\frac{dT_1}{dt}\right] \tag{23}$$

We now express ρ in terms of $-m/\frac{dr}{dt}$. We also express $\frac{d^2r}{dt^2}$ in terms of T_1, $\frac{dr}{dt}$, and $\frac{dT_1}{dt}$. We then divide equation (23) through by $(dr/dt)^3$ in the following manner:

From equation (8)

$$m = -\rho \frac{dr}{dt} \quad ; \quad \frac{1}{\rho} = -\frac{1}{m}\frac{dr}{dt}$$

Also:

$$\log_{10}\left(-\frac{dr}{dt}\right) = 7.09 - 2600/T_1$$

$$\ln\left(-\frac{dr}{dt}\right) = 2.303\,(7.09 - 2600/T_1)$$

$$\frac{dr}{dt} = -a\,e^{-6000/T_1} = -a\,e^{-b/T_1}$$

where "a" = $e^{16.35}$

"b" = 6000°K

$$\frac{d^2r}{dt^2} = -\frac{a\,b}{T_1^2}\,e^{-b/T_1}\,\frac{dT_1}{dt}$$

$$= \frac{b}{T_1^2}\,\frac{dr}{dt}\,\frac{dT_1}{dt}$$

$$= \frac{b}{T_1^2}\left(\frac{dr}{dt}\right)^2\frac{dT_1}{dr}$$

Making these substitutions in equation (23) we get:

$$\left(\frac{1}{2\lambda r^3}\right)\left(\frac{448\,\lambda^3 r^6}{cm}\right)\left(\frac{4\lambda}{cm}+r\right)\Big\{(lrm-2k\Delta T)-(lrm'+k)r\frac{dT_1}{dr}$$

$$+\frac{1}{r}\left(4r-\frac{20\lambda}{cm}\right)\left(\frac{cm}{4\lambda}\right)\left(r^2\lambda\frac{dT_1}{dr}-\left[r-\frac{3\lambda}{cm}\right]\left[k\Delta T-lrm\right]\right)\Big\}$$

$$+\left(-\frac{128\,\lambda^2 r^3}{cm}-40\lambda r^4-\frac{8\lambda r^5 b}{T_1^{\,2}}\frac{dT_1}{dr}\right)\Big[r^2(9\lambda+k+lrm')\frac{dT_1}{dr}$$

$$+r(8lrm-7k\Delta T)+\frac{27\lambda}{cm}(k\Delta T-lrm)\Big]$$

$$+8\lambda r^4\left(\frac{4\lambda}{cm}+r\right)\Big[2r(9\lambda+k+lrm')\frac{dT_1}{dr}+r^2lm'\frac{dT_1}{dr}$$

$$+r^3lm''\left(\frac{dT_1}{dr}\right)^2+\left(r^3lm'+r^2 9\lambda+r^2k\right)\left(\frac{b}{T_1^{\,2}}\left\{\frac{dT_1}{dr}\right\}^2+\frac{d^2T_1}{dr^2}\right)$$

$$+(8lrm-7k\Delta T)\left(1+\frac{rb}{T_1^{\,2}}\frac{dT_1}{dr}\right)+8lrm+8lr^2m'\frac{dT_1}{dr}$$

$$+7kr\frac{dT_1}{dr}-\frac{27\lambda}{cm}\left(k\frac{dT_1}{dr}+lm+lrm'\frac{dT_1}{dr}\right)\Big]=0 \qquad (24)$$

Further simplification can be made in this basic equation which we must solve, by using the following considerations.

For the particular problem with which we are working, it is found that λ equals k both numerically and dimensionally. Hence we can interchange λ and k at any point from here on.

The following symbols and notations have also proven to be useful.

$$m = a\rho e^{-b/T_1}$$

$$m' = \frac{dm}{dT_1} = \frac{d}{dT_1}\left(a\rho e^{-b/T_1}\right)$$

$$= \frac{ab\rho}{T_1^2} e^{-b/T_1} = \frac{bm}{T_1^2}$$

$$m'' = \frac{d}{dT_1}\left(\frac{bm}{T_1^2}\right) = -\frac{2bm}{T_1^3} + \frac{b^2 m}{T_1^4}$$

$$= m\left(\frac{b^2 - 2bT_1}{T_1^4}\right)$$

Define:

$$\alpha = \lambda / \lambda_0$$

$$\beta = m / m_0$$

$$\sigma = T_1 / T_0$$

Rearranging and simplifying

$$\left(224\frac{\lambda^2 r^4}{cm}\right)(1+4D)\Big\{ lrm(1-2G) - l\frac{r^2 m}{T_1^2}b\,(1+F)\frac{dT_1}{dr}$$

$$+ \frac{(1+5D)}{Dr}\left(r^2\lambda\frac{dT_1}{dr} - r\,[1-3D][G-1](lrm)\right)\Big\}$$

$$+\left(-128\lambda D r^4 - 40\lambda r^4 - 8\lambda\frac{r^5}{T_1^2}b\,\frac{dT_1}{dr}\right)\Big[r^2(9\lambda + k$$

$$+ lrm')\frac{dT_1}{dr} + lr^2 m(8-7G) + 27rDlrm(G-1)\Big]$$

$$+ 8\lambda r^5(4D+1)\Big[2r(9\lambda + k + lrm')\frac{dT_1}{dr}$$

$$+ r^2 lm'\frac{dT_1}{dr} + r^3 lm''\left(\frac{dT_1}{dr}\right)^2 + \left(r^3 lm' + kr^2\right.$$

$$\left. + r^2 9\lambda\right)\left(\frac{b}{T_1^2}\left\{\frac{dT_1}{dr}\right\}^2 + \frac{d^2T_1}{dr^2}\right) + lrm(8-7G)\left(1 + \frac{rb}{T_1^2}\frac{dT_1}{dr}\right)$$

$$+ 8lrm + 8lr^2 m'\frac{dT_1}{dr} + 7kr\frac{dT_1}{dr}$$

$$-\frac{27\lambda}{cm}\left(k\frac{dT_1}{dr} + lm + lrm'\frac{dT_1}{dr}\right)\Big] = 0$$

Dividing through by (1 + 4D) $(8\lambda r_o^6 \ell b \frac{m_o}{T_o})$ we get:

$$\frac{28 D\alpha^5 T_o}{r_o \ell b m_o}\left\{\ell r m(1-2G) - \frac{\ell r^2 m b}{T_1^2}(1+F)\frac{dT_1}{dr}\right.$$

$$\left. + \left(\frac{1-5D}{D}\right)\left(r\lambda\frac{dT_1}{dr} - \ell r m(1-3D)(G-1)\right)\right\}$$

$$+\left(\frac{1}{1+4D}\right)\left(-\frac{16\alpha^4 T_o D}{r^o \ell b m_o} - \frac{5\alpha^4 T_o}{r_o^2 \ell b m_o} - \frac{\alpha^5 T_o}{r_o \ell m_o T_1^2}\frac{dT_1}{dr}\right)\left[r^2(9\lambda\right.$$

$$\left. + k + \frac{\ell r m b}{T_1^2}\right)\frac{dT_1}{dr} + \ell r^2 m(8-7G) + 27r D\ell r m(G-1)\Big]$$

$$+\frac{\alpha^5 T_o}{r_o \ell b m_o}\left[2r\left(9\lambda + k + \frac{\ell r m b}{T_1^2}\right)\frac{dT_1}{dr} + \frac{\ell r^2 m b}{T_1^2}\frac{dT_1}{dr}\right.$$

$$+\ell r^3 m\left(\frac{b^2 - 2bT_1}{T_1^4}\right)\left(\frac{dT_1}{dr}\right)^2 + \left(\frac{\ell r^3 m b}{T_1^2} + 9\lambda r^2\right.$$

$$\left.+ kr^2\right)\left(\frac{b}{T_1^2}\left\{\frac{dT_1}{dr}\right\}^2 + \frac{d^2T_1}{dr^2}\right) + \ell r m(8-7G)\left(1 + \frac{rb}{T_1^2}\frac{dT_1}{dr}\right)$$

$$+ 8\ell r m + 8\ell\frac{r^2 m b}{T_1^2}\frac{dT_1}{dr} + 7kr\frac{dT_1}{dr}$$

$$\left.-27Dr\left(k\frac{dT_1}{dr} + \ell m + \ell\frac{r m b}{T_1^2}\frac{dT_1}{dr}\right)\right] = 0$$

Regrouping and letting $\lambda = k$ and dividing by $l r m$

$$\frac{\alpha^5 T_0}{r_0 l b m_0}\left(\frac{r^2 b}{T_1^2} + \frac{10\lambda r^2}{l r m}\right)\frac{d^2 T_1}{dr^2}$$

$$+\left[\left(\frac{1}{1+4D}\right)\left(\frac{-\alpha^5 T_0}{r_0 l m_0 T_1^2}\right)\left(10E + \frac{b}{T_1^2}\right)r^2 + \frac{\alpha^5 T_0}{r_0 l b m_0}\left\{r^2\left(\frac{b^2 - 2bT_1}{T_1^4}\right)\right.\right.$$

$$\left.\left. + \frac{b}{T_1^2}\left(\frac{r^2 b}{T_1^2} + \frac{10\lambda r^2}{l r m}\right)\right\}\right]\left(\frac{dT_1}{dr}\right)^2$$

$$+\left[\frac{28 D\alpha^4 T_0}{r_0 l b m_0}\left\{-\frac{r b(1+F)}{T_1^2} + \frac{(1-5D)\lambda}{D l m}\right\} + \left(\frac{1}{1+4D}\right)\left(\frac{-\alpha^5 T_0}{l r_0 m_0 T_1^2}\right)\cdot\right.$$

$$\cdot\Big(r(8-7G) + 27 r D(G-1)\Big) + \left(\frac{1}{1+4D}\right)\left(-\frac{16\alpha^4 T_0 D}{r_0^2 l b m_0}\right.$$

$$\left.-\frac{5\alpha^4 T_0}{r_0^2 l b m_0}\right)\left(r^2 10E + \frac{r^2 b}{T_1^2}\right) + \frac{\alpha^5 T_0}{r_0 l b m_0}\left(\frac{3rb}{T_1^2} + (8-7G)\frac{rb}{T_1^2}\right.$$

$$\left.\left. + \frac{8rb}{T_1^2} + \frac{20\lambda r}{l r m} + \frac{7kr}{l r m} - 27Dr\left(\frac{k}{l r m} + \frac{b}{T_1^2}\right)\right)\right]\frac{dT_1}{dr}$$

$$+\frac{28 D\alpha^5 T_0}{r_0 l b m_0}\left((1-2G) + \left(\frac{1-5D}{D}\right)(-1)(1-3D)(G-1)\right)$$

$$+\left(\frac{1}{1+4D}\right)\left(-\frac{16\alpha^4 D T_0}{r_0^2 l b m_0} - \frac{5\alpha^4 T_0}{r_0^2 l b m_0}\right)\Big(r(8-7G) + 27 r D(G-1)\Big)$$

$$+\frac{\alpha^5 T_0}{r_0 l b m_0}\Big((8-7G) + 8 - 27D\Big) = 0$$

Simplifying further: i.e. multiplying by $\dfrac{\ell r m \sigma^2}{\alpha^8 \beta}$

$$-(1+10F)\frac{d^2\sigma}{d\alpha^2} = \Bigg[\frac{T_1^{\,3}}{b\sigma}\left(\frac{2b^2}{T_1^{\,4}}\left(1-\frac{T_1}{b}+5F\right)\right)$$

$$-\left(\frac{1}{1+4D}\right)\left(\frac{b}{\sigma T_1}+\frac{10T_1E}{\sigma}\right)\Bigg]\left(\frac{d\sigma}{d\alpha}\right)^2 + \Bigg[\frac{28D}{\alpha}\Bigg(-(1+F)$$

$$+\frac{(1-5D)}{D}F\Bigg) + \left(\frac{1}{1+4D}\right)\left(\frac{-1}{\alpha}\right)\Big(8-7G+27D(G-1)\Big)$$

$$+\left(\frac{10F+1}{1+4D}\right)\left(-\frac{1}{\alpha}\right)\left(16D+5\right) + \Bigg(\frac{11}{\alpha}+\frac{(8-7G)}{\alpha}$$

$$+\frac{27F}{\alpha}-\frac{27D}{\alpha}(F+1)\Bigg)\Bigg]\frac{d\sigma}{d\alpha}$$

$$+\frac{28D}{\alpha^2}\frac{\sigma^2}{b}T_0\left((1-2G)-\frac{(1-5D)}{D}(1-3D)(G-1)\right)$$

$$+\left(\frac{16D+5}{1+4D}\right)\left(\frac{-\sigma^2 T_0}{\alpha^2 b}\right)\Big(8-7G+27D(G-1)\Big)$$

$$+\frac{\sigma^2 T_0}{\alpha^2 b}\Big(8-7G+8-27D\Big)$$

Finally -

$$\sigma'' = \frac{-1}{1+10F}\Bigg\{\Bigg[\frac{20}{\sigma^2}\frac{(1+10F)(4D)}{(1+4D)} + \frac{(20-2\sigma)}{\sigma^2}\Bigg](\sigma')^2$$

$$+\frac{\sigma'}{\alpha}\Bigg[-195DF - 55D + 55F - 7G + 19$$

$$+\left(\frac{1}{1+4D}\right)\left(-27DG + 11D - 160DF + 7G - 50F - 13\right)\Bigg]$$

$$+\frac{.05\sigma^2}{\alpha^2}\Bigg[420D^2(1-G) + 168DG - 223D$$

$$-35G + 44 - \frac{(16D+5)}{(1+4D)}\left(8 - 7G + 27D(G-1)\right)\Bigg]\Bigg\}$$

In order to solve this equation by iteration we need the following values for time equal zero. These values were taken from Penner's paper[1] with the exception of λ [2].

$k = 4.3 \times 10^{-4}$ cal/cm sec $^{\circ}$K

$\lambda = 4.3 \times 10^{-4}$ cal/cm sec $^{\circ}$K

$c = 0.53$ cal/gm $^{\circ}$K

ℓ = 129 cal/gm

ρ = 1.02 gm/cc

r_o = 5×10^{-3} cm

T_o = 300 ^oK

T_g = 3000 ^oK

m_o = 0.0263 gm/cm^2 sec

α = one

σ = one

β = $38.5\ e^{(16.35 - 20/\ \ )}$ = one

D = $8/\alpha\beta$ = 8

F = $0.5\ \sigma^2/\alpha\beta$ = 0.5

G = $10(10 - \sigma)/\alpha\beta$ = 90

We still need one more initial condition before we can begin the iterative procedure.

We now make the assumption that at time equal zero, our droplet is isothermal and hence from equation (2) we see that C(t) is also equal to zero. Hence we can set equation (20) equal to zero:

$$0 = \left(\frac{1}{\frac{32\lambda^2 r^4}{c\rho} - 8\lambda r^5 \frac{dr}{dt}} \right) \left[r^2 (9\lambda + k + \ell r\, m'(T_1)) \frac{dT_1}{dt} \right.$$

$$\left. + \left(8\ell r\, m(T_1) - 7k\Delta T\right) r \frac{dr}{dt} - \frac{27\lambda}{c\rho}\left(k\Delta T - \ell r m\right) \right] \quad (28)$$

$$0 = \left(\frac{1}{\frac{32\lambda^2 r^5}{c m r} + 8\lambda r^5} \right) \left[\left(9\lambda r^2 + k r^2 + \frac{l r^3 m b}{T_1^2} \right) \frac{dT_1}{dr} + 8 l r^2 m - 7 k \Delta T r + \frac{27\lambda r}{c m r} (k \Delta T - l r m) \right] \tag{29}$$

$$0 = \left(\frac{1}{8\lambda r^5} \right) \left(\frac{1}{1 + \frac{4\lambda}{c m r}} \right) \left[\left(9\lambda r^2 + k r^2 + \frac{l r^3 m b}{T_1^2} \right) \frac{dT_1}{dr} + 8 l r^2 m - 7 k \Delta T r + \frac{27\lambda r}{c m r} (k \Delta T - l r m) \right] \tag{30}$$

$$\left(\frac{dT_1}{dr} \right)_{t=0} = \frac{k \Delta T \left(7 - \frac{27\lambda}{c m r} \right) - l r m \left(8 - \frac{27\lambda}{c m r} \right)}{9\lambda r + k r + \frac{l r^2 m b}{T_1^2}} \tag{31}$$

$$\left(\frac{d\sigma}{d\alpha} \right)_{t=0} = \frac{k \Delta T (7 - 27 D) - l r m (8 - 27 D)}{\alpha T_0 \left(9\lambda + k + \frac{l r m b}{T_1^2} \right)} \tag{32}$$

This term is an initial condition for our equation.

$$\left(\frac{d\sigma}{d\alpha}\right)_{t=0} = \frac{k\,\Delta T\,(7-27D) - l\rho_0 m_0 \alpha\beta\,(8-27D)}{\alpha T_0\left(9\lambda + k + l\rho_0 m_0 \alpha\beta\, b/\sigma^2 T_0^2\right)}$$

But at t = o

$\alpha = \beta = \sigma = 1$; D = 8; E = .033; F = .5 G = 90

Thus:

$$\left(\frac{d\sigma}{d\alpha}\right)_{t=0} = \frac{(5\times 10^{-4})(3000-300)(7-27D) - \frac{k}{E}(8-27D)}{300\left(10\times 5\times 10^{-4} + k/F\right)}$$

$$= \frac{1.35(7-27D) - \frac{k}{E}(8-27D)}{1.5 + 300\,k/F}$$

$$= \frac{1.35(7-27\times 8) - \frac{5\times 10^{-4}}{.033}(8-27\times 8)}{1.5 + 300\times 5\times 10^{-4}/.5}$$

$$= \frac{-282 + 3.15}{1.5 + .3}$$

$$= -155$$

Our iterative procedure will make use of the Taylor expansion theorem whereby if $\Delta\alpha$ is a small quantity compared to α , then

$$\sigma(\alpha+\Delta\alpha) = \sigma(\alpha) + \Delta\alpha\,\sigma'(\alpha) + \frac{(\Delta\alpha)^2}{2}\sigma''(\alpha)$$

In like manner

$$\sigma'(\alpha+\Delta\alpha) = \sigma'(\alpha) + \Delta\alpha\,\sigma''(\alpha)$$

We can thus proceed as in the shell model case by solving for σ'' and σ' using the given values for time equal zero and from this new value of σ' we can find all the necessary values at $\sigma + \Delta\sigma$ so as to repeat the process until α has reached the value of 0.5 or less.

The results have been plotted on Figure 1 along with the other curves.

C. Discussion of Results

The results of calculations based on the conduction model indicate that a time of about .0040 sec is required for the radius to decrease to 50% of its original size (Figure 1). The surface temperature is seen to be consistently below the value found for both isothermal droplets and droplets represented by two isothermal shells. The points calculated give a monontone (Figure 1). Since the numerical work for the conduction model was checked repeatedly, the data given in Figure 1 should be considered as a valid solution for the problem under discussion.

REFERENCES

1. Penner, S. S.: "Evaporation of Liquid Droplets in a Rocket Motor", Progress Report No. 9-13, Project No. TU2-1, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, September 5, 1947.

2. McAdams, W.H., "Heat Transmission", McGraw Hill Publishing Co., New York, New York.

Figure 1 Evaporating Droplet Radius and Surface Temperature vs. Time.

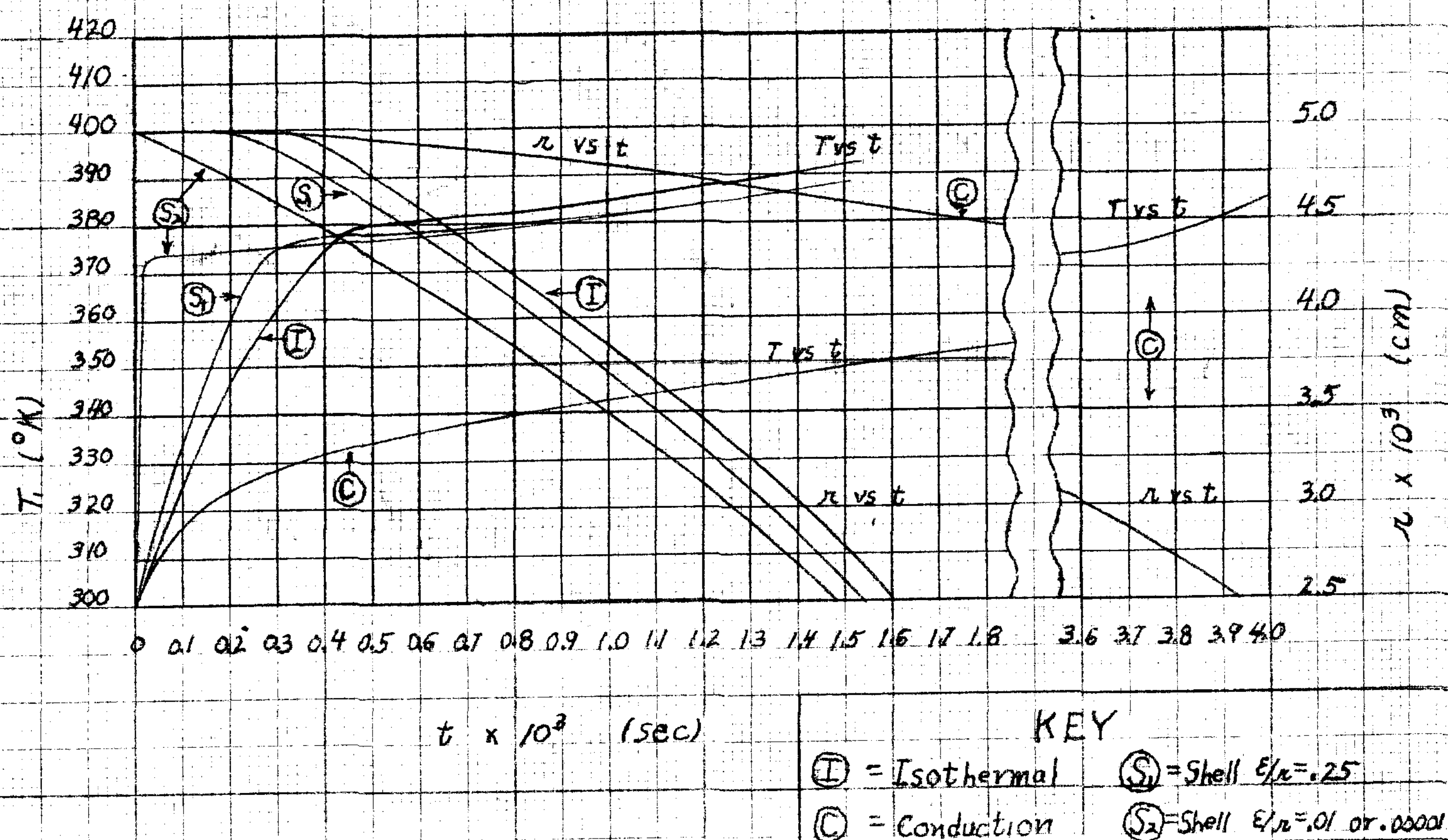

Figure 1 Evaporating Droplet Radius and Surface Temperature vs. Time.