ACCURATE MEASUREMENT OF THE DECLINATIONS OF RADIO SOURCES

Thesis by

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ABSTRACT

The two 90-foot steerable paraboloids of the Owens Valley Radio Observatory were used as a two-element interferometer at 960 Mc/s with various separations along a north-south baseline to measure accurately the declinations of a number of radio sources most of which were of small diameter. The measured values of declination are tabulated for 110 sources with right ascensions between 0 hours and 14 hours 10 minutes. The standard errors of the measured values range from 2.6 seconds of arc to 46 seconds of arc with an average of 13 seconds of arc. A discussion of the sources of error is included.

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I. PROBLEM

Since the identification of the intense radio source Taurus A with the Crab nebula by Bolton, Stanley, and Slee (1), an increasing number of discrete radio sources have been identified with optically observable objects. Such identifications are especially valuable to radio astronomy as they make it possible to obtain additional information about the physical nature of radio sources by optical means. Moreover the only known way to get a reliable measure of the distance of an extragalactic radio source (which is necessary, for example, in order to determine the radio luminosity function) is by measuring its optical red shift. Needless to say this can only be done for radio sources which have been identified with optical objects.

A prerequisite for the identification of a radio source is a sufficiently accurate radio position. In the case of an intense source located in the same region of the sky as such a striking and peculiar object as the Crab nebula, the identification is not too difficult to make even with an imperfect radio position. It more frequently happens, however, that the radio source is in a region of the sky where there is no outstanding optical object, but rather a multitude of weak, apparently commonplace objects. If one is to make an identification in such circumstances, the radio position must be known at least to such accuracy that there will be no more than one possible object within the error rectangle of the radio position. Indeed, since there is no reason to believe that there is an optically observable object associated with every radio source, the error rectangle should be so small that the probability that it would contain an optical object just by chance is very low.

Of the various published lists of the positions of radio sources there are three lists which because of the large number of sources they contain and relative accuracy of the measured pcsitions have been especially useful in attempting to identify radio sources. They are the third Cambridge survey (3C) (2), the survey by Mills, Slee, and Hill (MSH) (3, 4), and the list of accurate positions and fluxes by Elsmore, Ryle, and Leslie (ERL) (5). The 3C survey which was made at 159 Mc/s, lists 471 sources. The average quoted error of the measured positions is about ± 4. 8 seconds of time in right ascension and ± 4. 7 minutes of arc in declination. The MSH survey at 86 Mc/s contains over 200 sources between + 10 and - 50 degrees declination with standard errors averaging ± 4. 7 seconds of time and ± 10. 1 minutes of arc in right ascension and declination respectively. The ERL list at 178 Mc/s contains only 64 sources but quotes positions to considerably better accuracy. The standard errors are ± 1. 8 seconds of time in right ascension and ± 2. 1 minutes of arc declination.

A point worth noting in connection with the above mentioned lists is that in all three cases the accuracy of the measured declinations is substantially worse than the accuracy of the measured right ascensions (1 minute of arc is equivalent to 4 seconds of time at the celestial equator). In an effort to better the accuracy to which the declinations of radio sources are known, the two 90-foot steerable paraboloids of the Owens Valley Radio Observatory were employed as a two-element interferometer at 960 Mc/s on a north-south baseline. The resulting measured declinations, which are tabulated in table 4, have an average standard error of about ± 13 seconds of arc. Although it covers only about half of the observable sky, this table contains the declinations of 110 sources, 34 of which are also in the ERL list.

It is instructive to compute the average number of galaxies brighter than 20th magnitude that will lie by chance within an error rectangle of a given size. According to Holmberg (6) there are about 1350 such galaxies per square degree. Thus in the case of the ERL list there will be on the average slightly more than one such galaxy within the average standard error rectangle. The average standard error of the new declination measurements is almost an order of magnitude smaller than that of the ERL measurements. If the new declinations are combined with right ascensions of comparable accuracy (which have also been measured at the Owens Valley Radio Observatory with the same instrument), the probability of a galaxy brighter than 20th magnitude lying by chance within the average standard error rectangle is only 7 per cent.

The improved accuracy of the declinations of the new list should make possible identifications which have heretofore been impossible and, indeed, has already done so. In general, however, one might expect more new identifications to result from sources not contained in the ERL list since its accuracy is sufficiently good to have already made possible the identification of many of the sources it contains.

II. ADVANTAGES OF AN INTERFEROMETER

The instrument used for the declination measurements was a two-element interferometer consisting of the two 90-foot steerable paraboloids of the Owens Valley Radio Observatory operating at 960 Mc/s on a north-south baseline. The difference in arrival time at the two antennas of radiation being received was measured by means of a correlation receiver. From the arrival time difference it was possible to infer the direction from which the radiation was coming and hence the declination of the source that had emitted it.

There are several advantages in using an interferometer instead of a single antenna for position measurements. First, the greater resolution of the interferometer results in greater positional accuracy in the presence of noise. Positions are measured with a single antenna by sweeping the main beam of the antenna past the source and measuring the output of the receiver as a function of the position of the antenna. The positions of strong sources may be measured to a small fraction of the beamwidth of the antenna. The presence of noise on a weak signal, however, introduces an amplitude uncertainty which limits the accuracy attainable to a more moderate fraction of the beamwidth of the antenna. The presence of noise also limits the positional accuracy of an interferometer to a moderate fraction of the angular size of the interferometer lobes. These lobes, however, can easily be made much smaller than the beamwidth of any available single antenna simply by increasing the separation of the elements of the interferometer. Thus the interferometer can be made to yield superior positional accuracy on a weak signal.

A second advantage of an interferometer for position measurements, and one of the most important advantages, has to do with the mechanical difficulties involved in accurately knowing the direction an antenna is pointing. In order to measure positions with a single antenna, it is necessary to know the direction the antenna is pointing with an accuracy at least as high as that expected of the measured positions. The antennas used for radio astronomy are of necessity, however, large and heavy structures which cannot be regarded as being completely rigid. Not only is it difficult to make a steerable mount for such an antenna which is sufficiently precise to permit the desired degree of pointing accuracy, but the gravitational and wind loadings on the antenna itself produce flexures that are difficult to take into account. The same problems afflict the antennas used for the elements of an interferometer, of course. Antenna pointing errors, however, do not significantly affect the accuracy of interferometric position measurements since they have little influence on the relative arrival time of the radiation.

A third advantage of an interferometer over a single antenna is the ability of the interferometer to discriminate against background radiation. In addition to the discrete sources of radiation, there is radiation from extended regions of the sky produced by our own galaxy. The intensity of this extended background radiation is a slow function of position which typically changes by no more than a moderate amount within the beamwidth of an antenna. If an attempt is made, however, to measure the position of a discrete source which happens to lie in a region of extended background radiation with a single antenna, the variation of the background radiation within the beamwidth of the antenna can introduce significant errors in the measured position. There is no way in the case of a single antenna to separate the radiation coming from a discrete source from that of the extend background. An interferometer, on the other hand, tends not to see the extended background radiation. As will be seen in a later section, the response of an interferometer to extended radiation is reduced to nearly zero by a cancellation effect.

Lastly there is a fourth advantage of an interferometer over a single antenna which, although it does not eliminate any fundamental source of error, certainly facilitates one aspect of the computations involved in position measurements. That advantage is that the effects of atmospheric refraction can be completely ignored. It can be shown that for zenith angles of less than 75 degrees if one computes the atmospheric refraction on the basis of a plane parallel stratified atmosphere, the result will differ from the true refraction by less than 1 second of arc (7). If now the interferometer consists of two elements lying in a plane parallel to the planes of stratification, the existence of atmospheric refraction can produce no change in the observed quantity, the difference of the arrival times of the radiation at the two antennas, since the effect of the refraction is to delay the arrival of the radiation at the antennas by precisely the same amount in both cases (8). (Refraction will, however, cause the radiation to strike the antennas from a slightly different direction, and a small correction in the pointing of the antennas might be necessary to maximize the strength of the signal received.)

III. DESCRIPTION OF THE RECEIVER

The interferometer used for the declination measurements was of the correlation type. Figure 1 is a block diagram of the receiver. The crystal mixer of the superheterodyne receiver was connected by a short length of cable to the antenna feed without any preamplification at the signal frequency. The local oscillator frequency was 960 Mc/s and the center frequency of the IF amplifiers was 10 Mc/s with a bandwidth of about 4 Mc/s. No attempt was made to reject the image response response of the superheterodyne. Note that the IF amplifiers were split into two sections. The IF preamplifier was located at the prime focus of the paraboloid along with the mixer and amplified the signal sufficiently to allow it to be fed through a long connecting cable to the remainder of the receiver which was located in the laboratory building.

The phase lock system of the receiver developed by the author needs special mention since the observational techniques used for the north-south declination measurements depended upon the ease and precision of the lobe rotation made possible by the system. The phase lock system is shown schematically in figure 2. The local oscillator power for each half of the receiver was supplied by a separate klystron oscillator which was phase locked by a closed loop servo system to reference signals of a common, central origin. The high frequency reference signal power required by the system for a satisfactory lock was about six orders of magnitude weaker than the available local oscillator power required by the crystal mixers of

RECEIVER BLOCK DIAGRAM

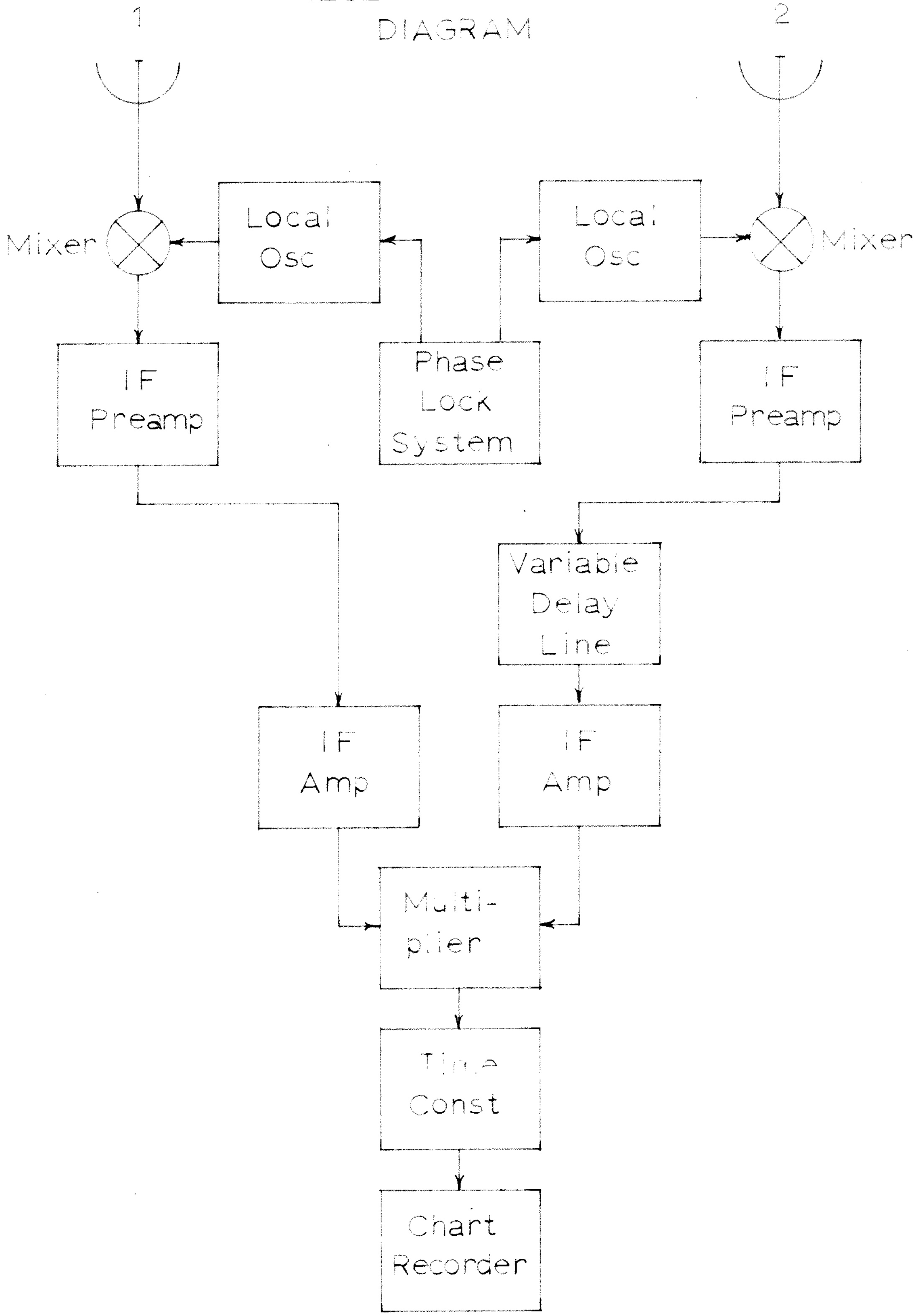


Figure 1

PHASE LOCK SYSTEM

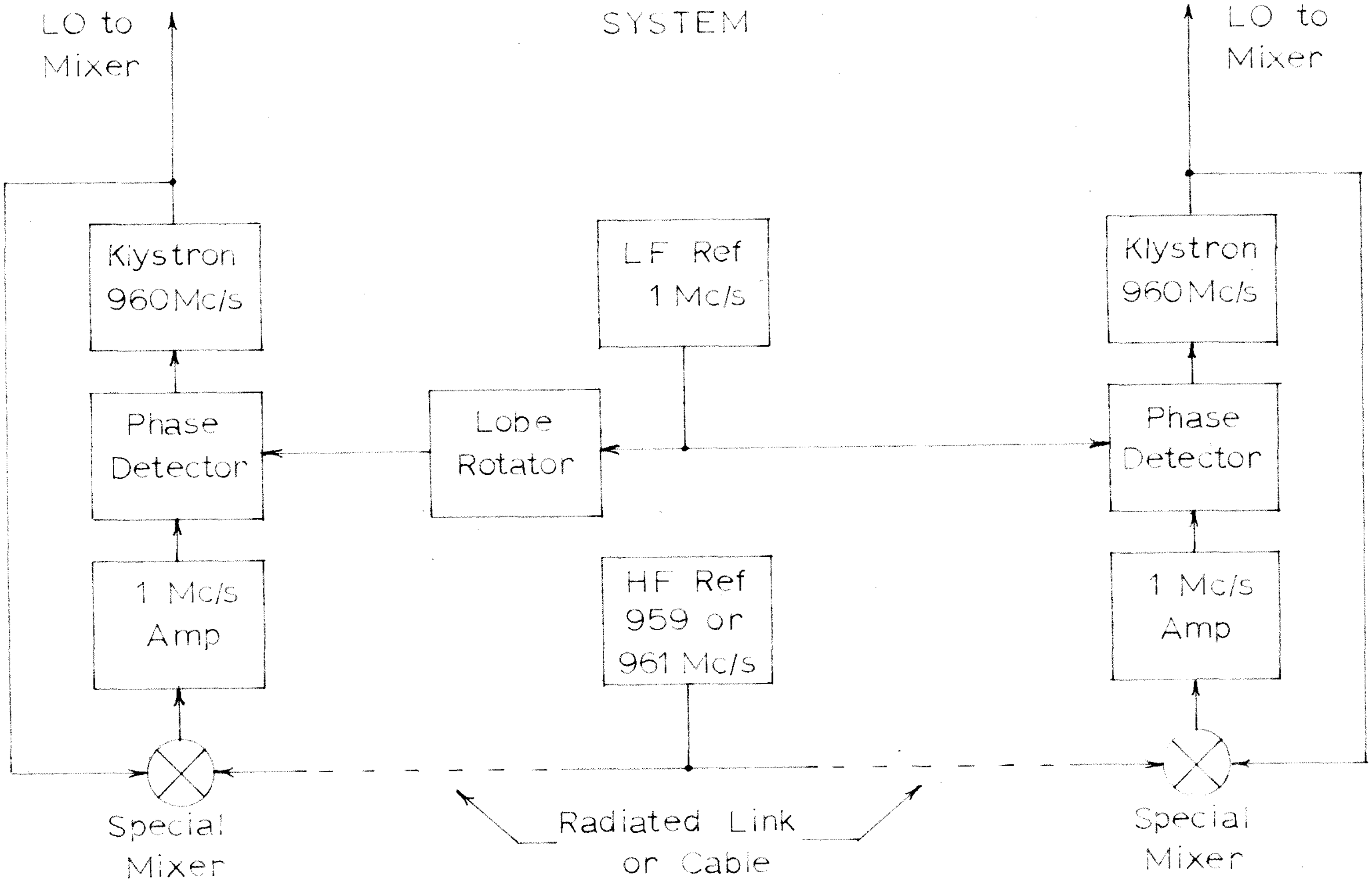


Figure 2

the superheterodyne receivers. The problem of getting phased local oscillator power to the two antennas when they were being used at large separations was thus greatly simplified. For the 200 and 400-foot spacings the high frequency reference signal was connected to the phase lock servos by means of low-loss coaxial cables. For the larger spacings, however, a different solution had to be found as there was insufficient low-loss coaxial cable available to permit a direct connection at large distances. The low power requirements of the phase lock system for the high frequency reference signal made it possible to transmit the signal to the two antennas from a central location by direct radiation using transmitting and receiving horns. It should be noted that two reference signals were required by the system: a high frequency reference which differed from the local oscillator frequency by 1 Mc/s, and a low frequency reference at 1 Mc/s. This dual frequency scheme circumvented some of the technical difficulties of phase detection and amplification which would otherwise have arisen, and, as will be seen, made possible a simple means of lobe rotation.

A sample of the output of the klystron which served as the local oscillator for the receiver was combined in a special mixer with the high frequency reference signal. Since the frequencies differed by 1 Mc/s, a 1 Mc/s beat was present in the output of the special mixer. The 1 Mc/s beat was amplified and then compared in a phase detector with the low frequency reference signal which was also 1 Mc/s. Any phase discrepancy between these two was sensed by the phase detector and applied as an error signal to the repeller of the klystron. Since phase relations are preserved in the heterodyne process and since the reference signals at the separate antennas had a definite phase relationship with each other, the two separate local oscillators also had such a definite phase relationship.

As it will appear later, it is desirable in north-south measurements to be able to rotate the lobes of the interferometer--that is to cause the two local oscillators to have very slightly different frequencies such that their relative phase is a slow, uniform function of time which is accurately known. This was conveniently accomplished in the phase lock system by rotating the phase of the 1 Mc/s reference signal sent to one antenna relative to the phase of that sent to the other. A special 4-plate phase-rotation capacitor fed by appropriate phasing lines was turned by a 1 RPM synchronous motor. An accurate timing mark was generated once each revolution. It was found that the phase rotation of the local oscillators produced in this fashion deviated from the desired linear rotation by only ± 0.9 degrees. An accurate measurement of the discrepancy yielded a calibration curve which made it possible to deduce the actual amount of phase rotation to within about one third of a degree.

IV. ANALYSIS OF THE RECEIVER OPERATION

In order to clarify the mathematical analysis of the receiver which is to follow, figure 3, a simplified block diagram of the receiver, shows only the essential logical functions. Since in position measurements only relative phases are important and not amplitudes, the existence of necessary amplifiers has been ignored. It is assumed for simplicity that the high frequency mixers of the superheterodyne receiver merely multiply the signal by the local oscillator which is assumed to have unity strength. As was mentioned above no attempt was made in the receiver to reject the image response of the superheterodyne. Indeed one would be at a loss to say which of the two responses of the receiver was signal and which was image since essentially equal amounts of noise power were received from the source by both responses. To avoid confusion the responses will be hereafter referred to as upper and lower responses rather than by the misleading terms "signal" and "image."

Let x, y, z be the electrical lengths of the cables connecting the mixer-IF filter combination to the antenna, local oscillator, and multiplier respectively for antenna number 1. Let $x + \Delta x$, $y + \Delta y$, $z + \Delta z$ be the corresponding cable lengths for antenna number 2. Let V be the amplitude of the signal, ω_{LO} be the angular frequency of the local oscillator, and ω_{IF} be the angular frequency to which the IF amplifier is tuned. Then $\omega_{LO} \pm \omega_{IF}$ are the angular frequencies of the upper and lower responses. Since the signal being received by the interferometer is of the nature of random noise, there is no

RECEIVER LOGIC

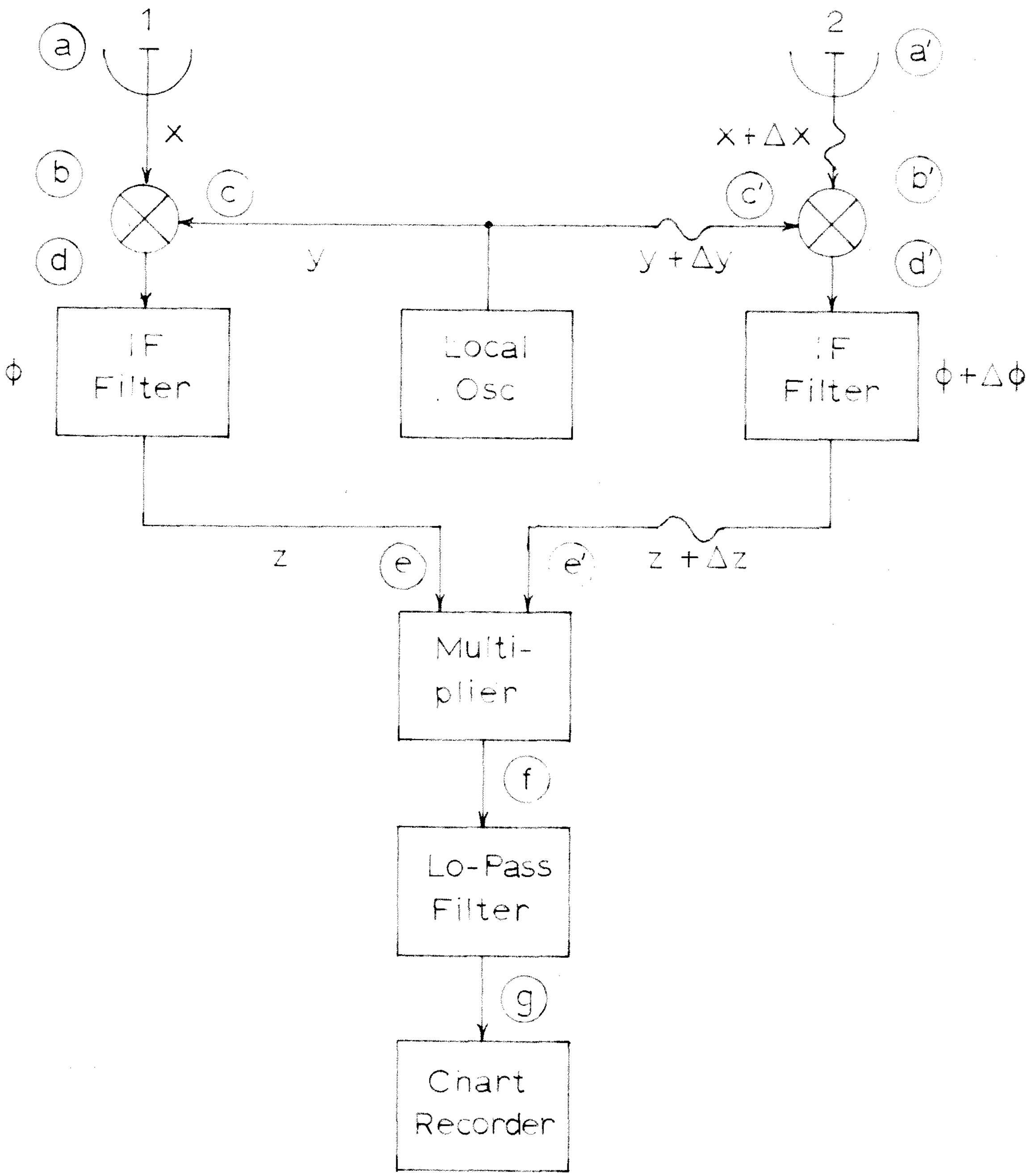


Figure 3

correlation between the upper and lower responses. The principle of superposition may therefore be applied and the effect of the upper and lower response will be worked out separately by carrying $\pm$ signs, and the combined effect will then be computed by taking the sum of the effects of both responses.

Writing out expressions for the signals at the various points of figure 3 designated by the letters, we have at (a):

$$V \sin \left[(\omega_{LO} \pm \omega_{IF})t\right] \tag{1}$$

at (b):

$$V \sin \left[(\omega_{LO} \pm \omega_{IF})(t - \frac{x}{c})\right] \tag{2}$$

at (c):

$$\cos \left[\omega_{LO}(t - \frac{y}{c})\right] \tag{3}$$

and at (d):

$$\frac{V}{2} \sin \left[(\omega_{LO} \pm \omega_{IF})(t - \frac{x}{c}) + \omega_{LO}(t - \frac{y}{c})\right] + \frac{V}{2} \sin \left[(\omega_{LO} \pm \omega_{IF})(t - \frac{x}{c}) - \omega_{LO}(t - \frac{y}{c})\right] \tag{4}$$

The IF bandpass filter rejects the first term of expression 4 and introduces a phase shift φ. We thus have at (e) after replacing t by $t + \frac{\varphi}{\omega_{IF}} - \frac{z}{c}$ and simplifying:

$$\frac{V}{2} \sin \left[\omega_{IF}(t - \frac{x}{c} - \frac{z}{c}) + \varphi \pm \omega_{LO}(\frac{y}{c} - \frac{x}{c})\right] \tag{5}$$

A similar calculation can be carried out for the primed letters. If we assume the signal reaching antenna number 2 is delayed by an amount τ compared to antenna number 1 due to geometrial considerations we find at (e') after making allowance for the different line lengths given above:

$$\pm \frac{V}{2} \sin \left[\omega_{IF}\left(t - \tau - \frac{x}{c} - \frac{\Delta x}{c} - \frac{z}{c} - \frac{\Delta z}{c}\right) + \varphi + \Delta\varphi \right.$$

$$\left. \pm \omega_{LO}\left(-\tau + \frac{y}{c} + \frac{\Delta y}{c} - \frac{x}{c} - \frac{\Delta x}{c}\right)\right] \qquad (6)$$

Taking the product of expressions 5 and 6 and employing a trigonometric identity gives as the output of the multiplier at (f):

$$\frac{V^2}{8} \cos \left[\omega_{IF}\left(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}\right) - \Delta\varphi \pm \omega_{LO}\left(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c}\right)\right]$$

$$- \frac{V^2}{8} \cos \left[\omega_{IF}\left(2t - \tau - \frac{2x}{c} - \frac{\Delta x}{c} - \frac{2z}{c} - \frac{\Delta z}{c}\right) + 2\varphi - \Delta\varphi \right.$$

$$\left. \pm \omega_{LO}\left(-\tau + \frac{2y}{c} + \frac{\Delta y}{c} - \frac{2x}{c} - \frac{\Delta x}{c}\right)\right] \qquad (7)$$

The low pass filter rejects the second term of expression 7 leaving at (g):

$$\frac{V^2}{8} \cos \left[\omega_{IF}\left(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}\right) - \Delta\varphi \pm \omega_{LO}\left(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c}\right)\right] \qquad (8)$$

Now taking the sum of the effects of the upper and lower responses as discussed above and employing another trigonometric identity we see that the deflection of the recorder is proportional to:

$$\frac{V^2}{4} \cos \left[\omega_{IF}\left(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}\right) - \Delta\varphi\right] \cos\left[\omega_{LO}\left(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c}\right)\right] \qquad (9)$$

It should be noted that in the derivation of expression 9 a discrete IF frequency ω_{IF} was assumed. Actually, of course, the IF amplifier responds to a band of frequencies so it is necessary to integrate expression 9 over the bandpass of the IF amplifier. This integration is difficult to do explicitly for reasonable forms of the IF frequency response except for a few simple cases. While numerical methods of integration are possible, it is doubtful that their use would be very enlightening since the exact form of the IF frequency response is not accurately known, and a qualitative approach gives results sufficiently good for the purpose at hand. Of the simple frequency response functions which can be easily integrated a Gaussian is perhaps the most realistic. A large number of cascaded tuned circuits tend toward a Gaussian response, and the receiver used had, indeed, a fair number of such cascaded circuits. If one makes the further simplifying assumption that the differential phase shift $\Delta\varphi$ is not a function of frequency, then the required integration yields a recorder deflection of the form:

$$\frac{V^2}{4} \exp\left[\frac{-(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c})^2 (BW)^2}{2}\right] \cos\left[\omega_{IF_o}\left(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}\right) - \Delta\varphi\right]$$

$$\cos\left[\omega_{LO}\left(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c}\right)\right] \qquad (10)$$

where ω_{IF_o} is the IF center frequency and BW is the bandwidth of the IF amplifiers. Note that the effect of the bandwidth integration is merely to multiply expression 9 by a Gaussian shaping function which depends on the bandwidth. If the form of the frequency response is not a Gaussian or if the differential phase shift is a function of frequency the shaping function may have a somewhat different shape. No matter what frequency response or phase shift is assumed, however, it should be noted that the last cosine factor of expression 9 will survive the integration unchanged as it is not a function of ω_{IF}. Thus $\cos[\omega_{LO}(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c})]$ will appear as a multiplicative factor in the expression for the recorder deflection irrespective of the bandwidth integration. The form of the observed recorder deflection agrees fairly well with expression 10 with one important exception: the nulls that should be produced by the first cosine factor of expression 10 are imperfect. A residual of a few percent of the maximum amplitude remains. The residual is due to a slight inequality of the upper and lower responses of the receiver and the effect of this inequality on the measured positions is discussed in the section on errors.

The delay of the signal reaching antenna number 2 relative to antenna number 1, τ, may be evaluated by referring to figure 4. The two antennas are separated by a baseline distance D, and the signal arrives from such a direction as to make an angle θ with the normal to the baseline. As indicated in the figure we may immediately write:

$$\tau = \frac{D}{c} \sin\theta \tag{11}$$

In order to express θ in terms of celestial coordinates one must employ a bit of spherical trigonometry. The relationships are indicated in figure 5. P is the north celestial pole, S is the position of the radio source in question, and A is the interferometer pole--that is,

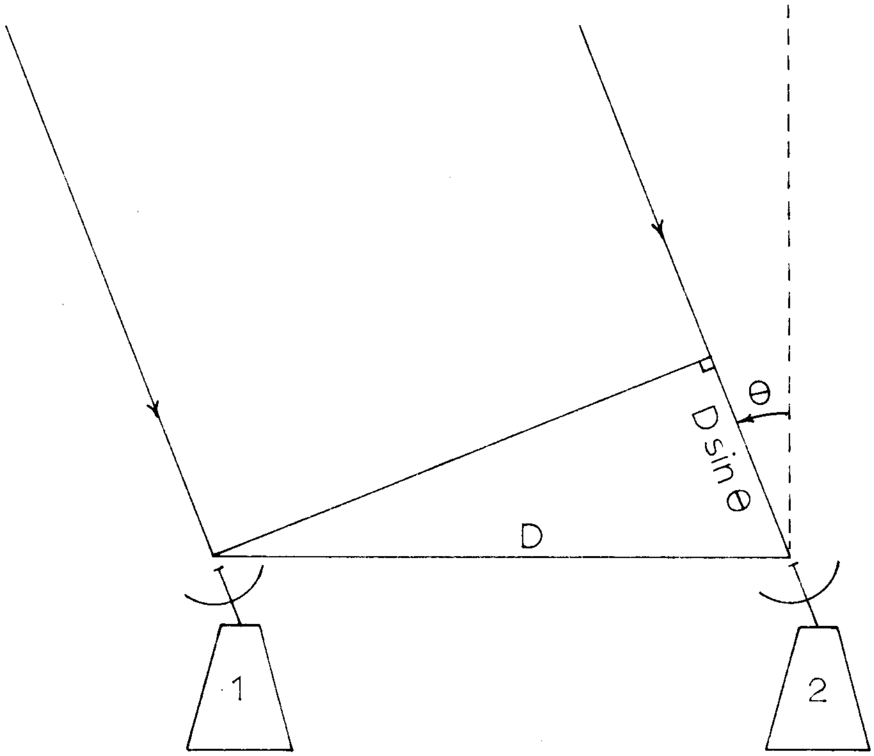


Figure 4

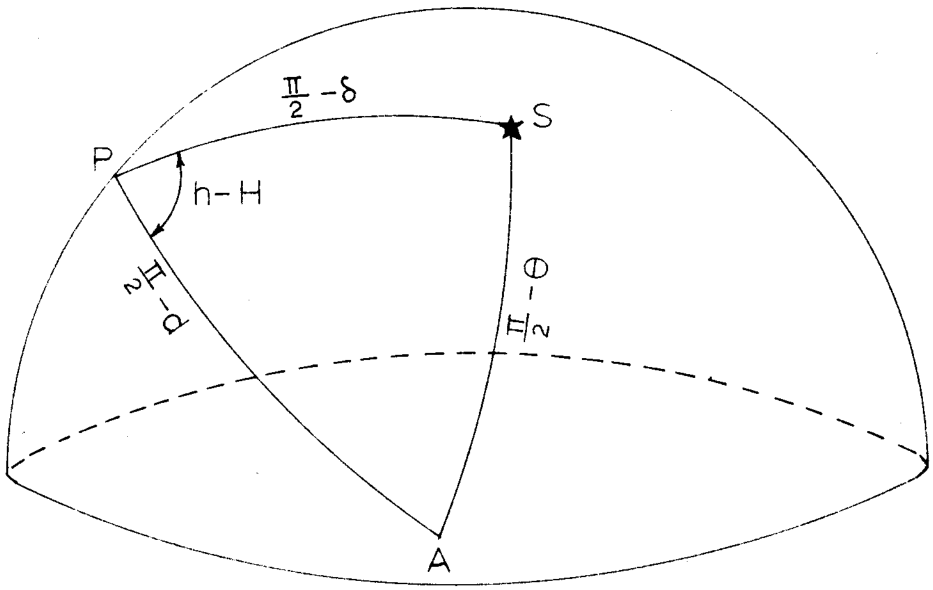


Figure 5

the intersection of the baseline from antenna number 2 to antenna number 1 with the celestial sphere. H and δ are the hour angle and declination of the source, respectively. Similarly h and d are the hour angle and declination of the interferometer pole. θ is as defined in figure 4. By the relations in a spherical triangle we may write:

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\left(\frac{\pi}{2} - d\right)\cos\left(\frac{\pi}{2} - \delta\right) + \sin\left(\frac{\pi}{2} - d\right)\sin\left(\frac{\pi}{2} - \delta\right)\cos(h - H) \qquad (12)$$

which simplifies to:

$$\sin\theta = \sin d \sin\delta + \cos d \cos\delta \cos(h-H) \qquad (13)$$

If in a north-south interferometer with a level baseline antenna number 1 is to the north of antenna number 2, h is π radians and d is the colatitude of the observatory. In practice, of course, one can not rely on the direction of the baseline being exactly lined up, so we shall write:

$$h' = h - \pi \qquad (14)$$

and:

$$\varphi' = \frac{\pi}{2} - d \qquad (15)$$

where h' is approximately zero, and φ' is approximately the latitude of the observatory. Substituting these values in equation 13 yields:

$$\sin\theta = \sin\delta\cos\varphi' - \cos\delta\sin\varphi'\cos(H-h') \qquad (16)$$

Of the three factors of expression 10 the last factor,

$\cos[\omega_{LO}(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c})]$, is a much faster function of τ than the other two since the local oscillator frequency, ω_{LO}, is about two orders of magnitude greater than either the IF center frequency, ω_{IF_o}, or the IF bandwidth. The last factor, then, is the one which contains the best positional information while the other two factors merely complicate the situation by effecting an undesirable attenuation of the signal for certain directions of arrival. This latter difficulty is easily got around, however, by the simple expedient of adjusting the differential IF cable length Δz by means of a variable delay line for each source observed such that the product of the first two factors of expression 10 (which both contain Δz) is always maximized. The correct adjustment of the delay line for a given source is easily computed from the known approximate position of the source. Note in particular that the last factor of expression 10, the one of interest, does not contain Δz and is therefore not affected by the adjustment of the delay line. Thus slight maladjustments of the delay line or for that matter phase shifts within the IF amplifiers do not affect the accuracy of the measured positions. This is a direct consequence of accepting equally both the upper and lower responses of the superheterodyne receiver. (The desired last factor is effectively separated out for phase measurement purposes both by maximizing the other factors and hence minimizing their variation and by measuring the phase by means of zero crossovers.)

If we substitute equations 11 and 16 in the last factor of expression 10 we have for the significant term of the recorder deflection:

$$\cos \left\{ \frac{\omega_{LO}}{c} \left[D \sin \delta \cos \varphi' - D \cos \delta \sin \varphi' \cos (H-h') + \Delta x - \Delta y \right] \right\} \tag{17}$$

We can now see why the type of interferometer used tends to discriminate against extended background radiation. The mathematical expressions which have been derived are for radiation coming from a discrete point on the celestial sphere. To evaluate the response of the interferometer to radiation coming from an extended region of the sky we must integrate our expressions over that region of the sky. If now the intensity of the background radiation is fairly constant over regions of the sky which are large compared to the spatial periodicity of expression 17 (approximately 20 minutes of arc for the smallest antenna spacing used), cancellation effects will reduce the integral nearly to zero.

V. REDUCTION OF THE OBSERVATIONS

The lobe rotator was used for the north-south declination measurements made at the 200 and 400-foot spacings of the antennas. The position of the midpoint of two successive zero crossovers (to minimize errors due to improper choice of zero level) of the chart record was measured relative to the once per revolution timing marks placed on the record by the lobe rotator. The measured value in terms of a fraction of a lobe was denoted by the letter η. (The successive zero crossovers were always paired in such a way that their midpoint corresponded to a positive maximum of expression 17.) After replacing the Δy of expression 17 by $\Delta y + 2\pi \frac{c}{\omega_{LO}} \eta$ and requiring the cosine to be +1, we have

$$\cos\left\{ \frac{\omega_{LO}}{c} \left[D \sin\delta \cos\varphi' - D\cos\delta \sin\varphi' \cos(H-h') + \Delta x - \Delta y \right] - 2\pi\eta \right\} = +1 \qquad (18)$$

which requires

$$\frac{\omega_{LO}}{c} \left[D \sin\delta \cos\varphi' - D\cos\delta \sin\varphi' \cos(H-h') + \Delta x - \Delta y \right] - 2\pi\eta = 2\pi N \qquad (19)$$

where N is an integer. If we now define $t_B = \frac{D}{c}$, $\nu_{LO} = \frac{\omega_{LO}}{2\pi}$ and $\Delta t = \frac{\Delta y - \Delta x}{c}$:

$$t_B\left[\sin\delta \cos\varphi' - \cos\delta \sin\varphi' \cos(H-h') \right] = \frac{N+\eta}{\nu_{LO}} + \Delta t \qquad (20)$$

The local oscillator frequency, ν_{LO}, was accurately measured for each observation and was kept nearly constant throughout the night.

Since the observations were all made quite close to meridian transit, we may ignore the higher order terms of the series expansion of the hour angle cosine and after rearranging write:

$$t_B \sin(\delta - \varphi') = \frac{N+\eta}{\nu_{LO}} + \Delta t - t_B \frac{(H-h')^2}{2} \cos\delta \sin\varphi' \qquad (21)$$

The $(H-h')^2$ factor of equation 21 was forced to be very small by restricting the observations to be within ten minutes of meridian transit. One may therefore solve equation 21 for a highly accurate value of $\sin(\delta-\varphi')$ by inserting an imperfect value of $\cos\delta$. Equation 21 is the working equation used for the determination of declinations. η, H, and ν_{LO} are the measured quantities. N is an integer which is chosen for each source on the basis of the already known approximate declination. Δt is a phase calibration term which is assumed to vary smoothly during the night as a result of equipment drifts and is determined empirically by observations of sources whose declinations are known (calibrators). t_B, φ', and h' are instrumental parameters having to do with the geometry of the interferometer. In order to minimize errors, measurements were made on several successive lobes of a record and the declination of the source was computed from the average of these measurements. (Since the hour angle, H, was different for each successive lobe measured, it was necessary to apply the last term of equation 21 separately to each measurement before taking the average.)

A somewhat different analysis is necessary for the declination measurements made at the 1600-foot north-south spacing. These measurements were originally undertaken for a different purpose and the lobe rotator was not used. They consisted of two observations of each source symmetrically spaced about meridian transit at an hour angle (usually $\frac{1}{2}$ to 1 hour) sufficiently large to allow the apparent motion of the sky due to the rotation of the earth to produce lobes of a reasonably short period (approximately four minutes). The lobe pattern of the north-south interferometer when the lobe rotator is not used, is symmetrical about meridian transit (or more precisely about $H = h'$). The total change in hour angle between corresponding zero crossovers before and after transit was measured by determining the total elapsed sidereal time between them. Let H_1 and H_2 be the hour angles of corresponding zero crossovers before and after transit. Then from the condition for a zero crossover of the recorder deflection we have:

$$\frac{\omega_{LO}}{c}\left[D \sin\delta \cos\varphi' - D \cos\delta \sin\varphi' \cos(H_1 - h') + \Delta x - \Delta y\right] = \left(M + \frac{1}{2}\right)\pi \tag{22}$$

$$\frac{\omega_{LO}}{c}\left[D \sin\delta \cos\varphi' - D \cos\delta \sin\varphi' \cos(H_2 - h') + \Delta x - \Delta y\right] = \left(M + \frac{1}{2}\right)\pi \tag{23}$$

where M is an integer. The angular frequency of the local oscillator, ω_{LO}, was held constant at the same measured value for both halves of the observation. If we assume that Δx and Δy do not change significantly between the two halves of the observation, and if we make the same substitutions made in equation 20, we have after manipulation:

$$t_B \sin(\delta - \varphi') = \frac{M + \frac{1}{2}}{2\nu_{LO}} + \Delta t + t_B\left[1 - \cos\left(\frac{H_2 - H_1}{2}\right)\right] \cos\delta \sin\varphi' \tag{24}$$

Equation 24 is the working equation which is solved for δ by an iterative procedure similar to that used for equation 21. As before several independent measurements of each record were averaged together, and the instrumental phase term, Δt, was determined empirically by observations of calibrators.

VI. CALIBRATION OF THE INSTRUMENT

No attempt was made to measure declinations on an absolute basis. The declinations were measured, rather, relative to the known declinations of certain sources which were used as calibrators. The accuracy of the measured declinations is thus directly dependent upon the accuracy of the assumed declinations of the calibrators.

The sources normally used as calibrators are those sources which have been identified with optical objects. The position of the center of the optical object is taken to be the position of the effective center of radio emission. While one expects a fair degree of correspondence between the radio and the optical positions of an identified object, the correspondence cannot in general be expected to be exact. For example, it is well known that in the case of certain identified double radio sources the radio and optical emissions cannot both be coming from the same region of thesky since the separation of the two components of the radio double is much greater than the apparent diameter of the optical object. It is reasonable to suppose, however, that if a radio source and its optical identification both have a rather small overall angular diameter, the radio and optical positions will correspond within some fraction of these diameters. Thus it would appear that small angular diameter should be used as a criterion for the selection of calibrators.

The sources used as calibrators for the declination measurements were all of small angular diameter. All but two of them are listed as "not resolved" by Maltby and Moffet (9). These two (3C 218 and 3C 274) are both halo-core type radio objects and are given less weight than the other calibrators. At the 200 and 400-foot antenna spacings, use was made of several sources as secondary calibrators. The declinations of these sources were measured very accurately at the 1600-foot spacing, and the values thus found were used for secondary calibration at the closer spacings. All calibrators are listed along with the assumed values of declination in the tables.

The calibration of the instrument involves the determination of four instrumental parameters: the h', t_B, φ', and Δt of equations 21 and 24. Of these four quantities, the first three are functions of the geometrical relationship of the antennas and are assumed to be constant for all observations made at the same spacing of the antennas. The fourth quantity, Δt, represents a phase drift of the receiving equipment and is assumed to be a slowly varying function of time during the observing night.

The quantity h', which represents an azimuth error of the baseline, was determined by careful surveying of the relative positions of the two antennas. The magnitude of h' was found to be less than 2 seconds of time for both the 200 and 400-foot spacings. For simplicity it was decided to assume the h' was zero for these spacings. The error in so doing is a function of the hour angle at which the observations made and is essentially zero for observations made at meridian transit. In so far as was possible the observations were indeed made at meridian transit, and in no case did the magnitude of the hour angle of an observation exceed 10 minutes of time. The error due to the assumption of h' equal zero was therefore no greater than

0.0015 of a lobe. This is much less than other errors, and hence the simplifying assumption does not significantly affect the accuracy of the measured declinations. The quantity h' does not appear in equation 24, the equation for the 1600-foot observations, and hence does not have to be evaluated for that spacing.

The quantity t_B, which is a measure of the spacing of the antennas, was determined for each spacing by finding that value of t_B which best fit the observations of the calibrators. In the case of the 200 and 400-foot spacings the determinations were based on the best fit to number of calibrators. Because of the small number of sources observed at the 1600-foot spacing, however, the determination of t_B for that spacing was based on two calibrators only: 3C 48 and 3C 71. Allowance was made in all cases for the time variation of Δt. Using subscripts to distinguish between the 200, 400, and 1600-foot spacings, the values which were determined are as follows: $t_{B\ 200} = 203.393 \pm 0.007$ nanosecond, $t_{B\ 400} = 406.661 \pm 0.014$ nanosecond, and $t_{B\ 1600} = 1626.798 \pm 0.050$ nanosecond.

As an additional check on the values determined for t_B, the 200 and 400-foot observations were intercompared by the following procedure. Thirty-five sources were chosen which were observed at both spacings and which were known by the angular size work of Maltby and Moffet to be unresolved "point" sources for these antenna spacings. The assumption was made that there would be no change in the apparent declinations of these sources between the 200 and 400-foot spacings. The unknown phase calibration term, Δt, was eliminated by considering only pairs of sources which were observed on the same night at nearly the same time and by assuming Δt to be the same for the two observations at each pair. This procedure yielded 131 simultaneous equations in two unknowns: the relation between the 200 and 400-foot values of t_B and also the relation between the 200 and 400-foot values of φ'. The equations were solved by a least squares procedure which gave the following results:

$$2\,t_{B\ 200} - t_{B\ 400} = 0.121 \pm 0.012 \text{ nsec} \tag{25}$$

and

$$\varphi'_{400} - \varphi'_{200} = 24'' \pm 14'' \tag{26}$$

The values of t_B for the 200 and 400-foot spacings given above, however, require:

$$2\,t_{B\ 200} - t_{B\ 400} = 0.125 \pm 0.016 \text{ nsec} \tag{27}$$

which is in good agreement with equation 25.

The quantity φ', is approximately equal to the latitude of the observatory but differs from the true latitude by the amount the baseline differs from level. In changing the spacing of the antennas, the south antenna remained fixed, and the north antenna was moved to various stations along the north-south track. The assumption was made that the north antenna was placed at the same height relative to the caisson heads at each station. This assumption combined with the known heights of the caisson heads gave the relative heights of the north antenna at the various stations within an estimated standard error of ± 0. 3 inch. The relative heights thus obtained were combined with the relation between the 200 and 400-foot values of φ' given in equation 26 with the known latitude of the observatory to give the following values for φ': $\varphi'_{200} = 37^{\circ}13'06'' \pm 38''$, $\varphi'_{400} = 37^{\circ}13'30'' \pm 29''$, $\varphi'_{1600} = 37^{\circ}13'40'' \pm 06''$.

The receiver phase calibration term, Δt, was determined by observations of the calibrators and was plotted as a function of sidereal time for each observing night. A simple smooth curve was drawn in each case so as to give a reasonably good fit with the experimental points determined by the calibrators. The curves were then used for the reduction of the observations: the appropriate value of Δt for each source being read from the curve for the time of the observation. Several of the Δt curves are reproduced in the appendix.

VII. DISCUSSION OF ERRORS

In discussing the sources of error in the measured positions, it is necessary to distinguish between three rather different types of errors: random errors, accidental errors, and systematic errors.

1. Random Errors

Random errors as the name suggests do not repeat in the same way for successive observations of the same source. The effect of random errors may therefore be reduced by averaging several independent observations of the source. In addition it is possible to make a good estimate of the magnitude of the random errors by noting the degree to which independent observations differ from one another. The magnitude of the random errors was evaluated empirically for the 200 and 400-foot observations as follows. Only sources for which three or more independent observations had been made at the same antenna spacing were considered. The deviations of the individual observations from the average were examined, and a standard deviation of a single observation was computed for each source. An attempt was then made to correlate these deviations with the intensities of the sources. The analysis was carried out independently for the 200 and 400-foot spacings, but the deviation expressed as a fraction of a lobe was essentially the same function of intensity for both spacings. The deviation versus intensity relation thus found is as follows:

$$\sigma = 10^{-3}\left[50 + \left(\frac{67}{I}\right)^2\right]^{1/2} \qquad (28)$$

where σ is the standard deviation per observation expressed as a fraction of a lobe, and I is the apparent intensity of the source expressed in units of 10^{-26} Watt meter^{-2} $(c/s)^{-1}$. The result is in good agreement with what one would expect on theoretical grounds. The first component of the standard deviation per observation is independent of intensity and represents equipmental instabilities and random inaccuracies in the phase calibration of the receiver. The second component of the standard deviation per observation is inversely proportional to intensity and represents the effect of signal to noise ratio.

In the case of the 1600-foot observations there were an insufficient number of different sources observed to carry out an error versus intensity analysis. The standard deviation per observation was therefore computed individually for each source on the basis of the root mean square of the deviations of the individual observations from the mean.

The standard deviation of the average of n independent observations, σ_n, was computed from the standard deviation per observation, σ, by the usual relation:

$$\sigma_n = \frac{1}{\sqrt{n}} \sigma \qquad (29)$$

2. Accidental Errors

Accidental errors are random in that they do not repeat in the same way for successive observations but differ sufficiently from the random errors discussed above to warrent special consideration. This kind of error which is attributable to such causes as a gross error on the part of the telescope operator occurs only infrequently, but when it does occur, it can be of a large magnitude. If an accidental error occurs in one of a number of independent observations of the same source, the affected observation sticks out like a sore thumb, and it is a simple matter to reject it. If an accidental error occurs in the only observation that has been made of a source, however, the situation is more serious as there is no way of knowing that the error has occurred. In such cases about all one can do is to estimate the probability of occurrence of a gross accidental error. This was done by considering only observations of sources for which three or more independent observations were made at the same antenna spacing. The total number of such observations for the 200 and 400-foot spacings was 450, and seven of these had to be rejected because of obvious gross errors. Thus it would appear that the measured position of a source which is computed on the basis of only one observation has about one chance in 64 of being completely erroneous.

3. Systematic Errors

Since systematic errors repeat in exactly the same way for each independent observation made under the same conditions, the averaging of a large number of such observations will not reduce the effect of systematic errors. Nor is there any way in which the magnitude of the systematic errors may be deduced by an analysis of the observational data. If at some time in the future a large number of the sources whose declinations have been measured are identified with optical objects a comparison of the optical and radio declinations will provide a measure of the errors. The best one can do for the present, however, is to make educated guesses as to the magnitudes of the systematic errors arising from various sources.

Five sources of systematic error are considered. They are:

a) uncertainty of calibrator declinations

b) wrong instrumental parameters

c) inequality of upper and lower responses

d) antenna pointing errors

e) confusion

a) The declinations of the sources were measured by a relative method and are hence dependent on the accuracy of the positions assumed for the calibrators. While it is felt that the optical identifications of the radio sources used as calibrators are all correct, there are still small but significant uncertainties in the correct radio positions. The origin of these uncertaintites is twofold. First, the optical positions of the sources are not always known to sufficient accuracy. In this connection it should be noted that the author is indebted for many of the optical positions used to Roger Griffin, who recently made accurate measurements using plates taken with the 48-inch Schmidt and the 200-inch Hale Telescope on Palomar Mountain. Second, there is no reason to believe that the radio and optical centers of these sources exactly coincide. It is reasonable to suppose, however, that the discrepancy is only a small fraction of the radio and/or optical diameter of the source. This means that for the small diameter calibrators chosen for these measurements the uncertainty of the radio position is typically of the order of one or two seconds of arc.

b) Systematic errors will also arise if the values adopted for the instrumental parameters t_B and φ' are incorrect. The estimated standard errors of the values adopted for these parameters at the various antenna spacings were given along with the values themselves in section VI. The magnitude of the effect of these errors on a measured value of declination depends on the declination of the source in question. The phase calibration function, Δt, was determined in such a way as to give the correct declinations for the calibrators on the basis of the adopted values of t_B and φ'. The error in a measured declination due to incorrect values of these parameters must therefore be small for a source observed at roughly the same time as a calibrator if the declinations of the source and the calibrator are approximately the same. If the declinations differ by a large amount, however, the error might be large. Although the calibrators do not, of course, all have the same declination, they are, as it happens, confined to a more limited range of declinations than are the sources whose declinations were measured. For example the error for declinations measured at the 400-foot spacing is less than 5 seconds of arc for declinations between +5 and +65 degrees but rises to about 10 seconds of arc for a declination of -5 degrees. For more southerly declinations the error increases rapidly reaching about 30 seconds of arc at a declination of -24 degrees.

c) The mathematical analysis of section IV showed that as a consequence of accepting both the upper and lower responses of the superheterodyne receiver changes in the phase shift of the IF amplifiers or in the effective length of the IF cables did not affect the measured positions. Since this result depends on the strict equality of the upper and lower responses, consideration should be given to errors resulting from deviations from equality. If we assume that the relative strengths of the upper and lower responses are $1 + \epsilon$ and $1 - \epsilon$ respectively, where $|\epsilon| < 1$, then we may write in place of expression 9:

$$(1+\epsilon)\frac{V^2}{8}\cos[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi + \omega_{LO}(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c})]$$

$$+ (1-\epsilon)\frac{V^2}{8}\cos[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi - \omega_{LO}(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c})] \qquad (30)$$

which simplifies to:

$$\frac{V^2}{8}\{\cos[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi]\cos[\omega_{LO}(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c})]$$

$$+ \epsilon\sin[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi]\sin[\omega_{LO}(\tau + \frac{\Delta x}{c} - \frac{\Delta y}{c})]\} \qquad (31)$$

This last expression indicates that the magnitude of ϵ may be determined observationally. If the amplitude of the lobes produced by a source is observed first with the IF delay line $(\frac{\Delta z}{c})$ adjusted for maximum amplitude, that is:

$$\cos[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi] = \pm 1 \qquad (32)$$

as in normal observations and then with the IF delay adjusted for minimum lobe amplitude:

$$\cos[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi] = 0 \quad (33)$$

the ratio of the two amplitudes observed will be the magnitude of ϵ. It was found that ϵ was less than 5%. With the assumption of ϵ small the resulting error in the measurement of τ and hence the declination of the source may be shown from expression 31 to be approximately:

$$\frac{\epsilon}{\omega_{LO}} \tan[\omega_{IF}(\tau + \frac{\Delta x}{c} + \frac{\Delta z}{c}) - \Delta\varphi] \quad (34)$$

Ideally the tangent factor of expression 34 is zero since the IF delay line is adjusted for each source so as to maximize a cosine of the same argument. Inaccuracies in the calibration of the delay line will introduce errors, however, It is estimated that the actual delay of the delay line deviated from the desired delay by no more than 3 nano-second. The error in the measured declinations due to this cause is thus no greater than 0. 0016 of a lobe. The corresponding angular error is of course a function of declination and of the separation of the elements of the interferometer, but is in all instances considerably less than other sources of error.

d) One might expect a phase shift and hence an error in the measured position to result if one of the paraboloidal antennas were incorrectly pointed so that the radiation was received not from the direction of the maximum response of the antenna but from a bit to one side of it. This effect was looked for experimentally and was found to be small, but detectable. For small deviations from the center of the beam, the phase shift was found to be proportional to the square of the deviation. The magnitude was such that a 5 minute of arc pointing error produced approximately 0. 001 lobe of phase shift. Since the two antennas used are almost identical, pointing errors arising from an imperfect knowledge of the position of the source would be present to the same degree in both antennas and the phase shifts thus produced would tend to cancel. A difference in pointing direction between the two antennas is required to produce an error in the measured positions. Such a difference could be caused by dial errors of the antenna setting circles, by misalignment of the polar axes, by non-perpendicularity of the polar and declination axes, or by operator carelessness. It is estimated that the differences in pointing of the two antennas did not exceed two or three minutes of arc. The errors due to pointing differences are thus less than 0. 001 of a lobe and are much smaller than errors due to other causes.

e) An important source of error in the measured positions, but one which is difficult to evaluate is confusion. By confusion is meant the inadvertant and often unwitting simultaneous observation of more than one radio source due to the close spatial proximity of the sources. The interpretation of the aggregate effect as being due to a single source results in error. One common form of confusion which frequently afflicts interferometric position measurements is of no importance for these measurements. That is the confusion produced by an intense source which is spatially some distance from the source being observed. The 90-foot paraboloids, which are the primary elements of the interferometer, are sufficiently directive to discriminate effectively against this type of confusion. Signals from sources substantially off the center of the beams of the paraboloids are severely attenuated, the attenuation exceeding 40 decibels for the sources more than a few degrees from the center of the beams. Thus even the most intense sources in the sky can cause no appreciable confusion if they are more than a few degrees from the source being observed. A second form of confusion, that caused by two or more sources of comparable intensity both lying within the main beam of the antennas is also of little consequence. The reason for this is that almost all the sources included in these declination measurements were also observed extensively by Maltby and Moffet (9) with the same instrument in the angular size program and are classified as to their multiplicity and complexity. Thus we are "put on our guard" in the interpretation of the measured positions for those sources determined by Maltby and Moffet to be multiple or complex. The type of confusion that presents the greatest problem to accurate positions is that in which the main beam of the antenna sees in addition to the principal source being observed one or more unrelated sources which are substantially weaker than the principal source--say by a factor of five or more. Such a situation could have escaped detection as being multiple or complex by the angular size program but could still significantly affect the measured positions--particularly in those cases where the position has otherwise been measured to a high degree of accuracy. It would be well beyond the scope of a primarily experimental paper to attempt a theoretical analysis of this problem, but some information as to its magnitude can be gleaned from an analysis of the experimental data. The type of confusion with which we are concerned here would not in general produce the same shift in apparent position at different spacings of the elements of the interferometer. A group of 67 sources was considered which were observed at both the 200 and 400-foot spacings and for which no significant phase shifts were predicted by the Maltby and Moffet results. The observed differences in the apparent declinations of these sources measured at the two spacings were compared with the corresponding composite standard errors predicted by equation 29. A total of eight sources had declination differences exceeding 1.65 times the composite standard error (90th percentile). They were 3C 23, 3C 27, 3C 55, 3C 75, 3C 86, 3C 234, 3C 265, and 3C 270. The Maltby and Moffet amplitude-spacing information for these eight sources was reexamined for any evidence of confusion. In four cases there were minor wiggles in the amplitude spacing plots which had been overlooked in the original work and which were of a sufficient amplitude to account for the observed discrepancies. These four cases have been designated "possibly confused" in table 4. On this basis one might suspect that of the order of 6% of the positions given in table 4 are in error due to confusion by amounts significantly greater than the quoted errors.

VIII. TABULAR DATA

The observations used for determining declinations were made in three groups. The first group of observations was made with the antennas separated by a distance of 200 feet along a north-south baseline on successive nights from the night of 1960 November 26/27 through the night of 1960 December 6/7 with the exception of the night of 1960 December 1/2 which was omitted due to high winds. During the ten nights of observation a total of 421 observations were made of 123 sources.

The second group of observations was made with the antennas separated by a distance of 400 feet along a north-south baseline on successive nights from the night of 1960 December 18/19 through the night of 1960 Decenber 23/24. During the six nights of observation a total of 173 observations were made of 90 sources.

The third and last group of observations was made with the antennas separated by a distance of 1600 feet along a north-south baseline on successive nights from the night of 1961 February 4/5 through the night of 1961 February 8/9. During the five nights of observation a total of 35 observations were made of 8 sources.

All observations were corrected for precession and all positions are tabulated in terms of 1950. 0 mean positions. The precession corrections included the higher order terms of nutation and aberration as computed from the Besselian day and star numbers. The corrections were computed, however, only for a mean day of each group of observations and applied to all observations of that group.

In section II it was shown that the effect of atmospheric refraction can be ignored for interferometric position measurements. Accordingly no corrections for atmospheric refraction have been applied to the observations. In the case of ionospheric refraction, however, the argument of section II does not apply since the ionospheric refraction cannot be treated on the basis of a plane parallel stratified model. Smith (10) gives as the formula for ionospheric refraction:

$$R = \frac{e^2 n}{2\pi m r \nu^2} \tan Z \sec^2 Z \qquad (35)$$

where e is the charge of the electron, m is its mass, n is the total number of electrons per unit area, r is the radius of the earth, Z is the zenith angle, and ν is the frequency of interest. If one uses Smith's value for n of 1.6×10^{13} electrons per square centimeter and sets $\nu = 960$ Mc/s, the refraction, R, is:

$$R = 0\overset{''}{.}22 \tan Z \sec^2 Z \qquad (36)$$

The largest zenith angle of any of the sources observed was 67 degrees. The corresponding ionospheric refraction is 3.4 seconds of arc. The magnitude of the refraction decreases very quickly with decreasing zenith angle. For the more moderate zenith angle of 40 degrees it is only 0.3 seconds of arc. Since the effect of ionospheric refraction is very much less than the quoted standard errors of the measured declinations, no corrections for this form of refraction have been made.

The numerical results of the north-south declination measurements are contained in four tables. Since the declinations measured at the 1600-foot spacing were used for secondary calibration purposes at the closer spacings, the 1600-foot data are given first in table 1. The sources used as calibrators at this spacing are listed along with the assumed declinations in the first part of table 1 and are followed in the second part by the measured values of declination. The errors quoted in table 1 are standard errors and are computed from the observed night to night repeatability of the measured declinations. These errors do not include any allowance for systematic effects.

The calibrators used for the 200 and 400-foot measurements are listed in table 2 along with the assumed declinations.

Table 3 contains the results of the 200 and 400-foot declination measurements. The errors quoted are the standard errors predicted by equation 29. These errors do not include any allowance for systematic effects. As an additional aid in determining the amount of credence to be assigned a particular measured value, the number of independent observations that have been averaged into that value is also given.

In order to put the data in a more generally useful form table 4 has been prepared. The declinations listed are appropriately weighted averages of the various measurements. All measurements for which the Maltby and Moffet (9) angular size results predict an excessive phase shift have been excluded from the averages. Because of the lack of reliable calibrators during the early evening portion of the 200 and 4oo-foot observations and because of the large equipmental drifts expected during this period, sources observed prior to 0 hours local sidereal time were omitted from table 4. Sources which were observed only once were also omitted. The quoted errors include allowances for all sources of error (including systematic errors) mentioned in section VII with the exception of errors due to confusion. As an aid in interpreting the listed declinations, the Maltby and Moffet source type classification has been given when available.

The declinations of two of the sources given in table 4 are in need of special comment. It will be noted that optical rather than radio declinations have been listed for 3C 48 and 3C 295. The assumed declinations of these two calibrators have entered so heavily into the computations that a meaningful radio position cannot be quoted for these two sources. On the basis of the good agreement with the other calibrators and on the basis of several good identifications that have already resulted from these measurements, however, one can conclude that the radio positions of 3C 48 and 3C 295 must agree with the quoted optical positions to within about 2 seconds of arc.

Although it is felt that the correct values of the integer N of equation 21 have been chosen for all the sources listed in the tables, it is possible that an error may have been made in one or two isolated cases. In such an eventuality the tabulated position would be in error by exactly one lobe. The size of the lobes is of course dependent on both the spacing of the antennas and on the declination of the source. For the 200-foot spacing the lobe size is given by:

$$(0^{\circ}\ 17'\ 36'')\ \text{secant}\ (\delta - \varphi')$$

The lobe size for the 400-foot spacing is exactly half of this.

TABLE 1

Declination Observations at the 1600-Foot North-South Antenna Spacing

Part 1: Assumed Declinations of Calibrators Used

Source	Declination (1950.0) °	'	"
3C 48	32	54	19.9
3C 71	-00	13	31.5
3C 171	54	12	49
3C 295	52	26	13.6

Part 2: Measured Declinations

Source	Declination (1950.0) °	'	"	Std Error "	# Obs
3C 48	32	54	20.0	0.53	3
3C 71	-00	13	31.7	0.50	4
3C 123	29	34	09.0	0.40	4
3C 147	49	49	38.9	0.40	4
3C 171	54	12	49.3	0.28	5
3C 196	48	22	05.7	0.34	5
3C 274	12	39	57.6	0.22	5
3C 295	52	26	13.6	0.14	5

TABLE 2

Calibrators Used for Observations at the 200 and 400-Foot North-South Spacings

Source	Declination (1950.0) °	'	"	Remarks
3C 433	24	51	36.3	optical (mean of double)
23-112	-12	23	56.3	optical
3C 48	32	54	19.9	optical
3C 71	-00	13	31.5	optical
3C 78	03	55	13	optical
3C 123	29	34	09	radio--1600 ft N-S
3C 147	49	49	39	radio--1600 ft N-S
3C 171	54	12	49	optical
3C 196	48	22	06	radio--1600 ft N-S
3C 212	14	21	27	radio--lunar occultation (Hazard) (11)
3C 218	-11	53	04.0	optical (mean of double)
3C 254	40	53	57	optical
3C 274	12	39	57.6	radio--1600 ft N-S
3C 295	52	26	13.6	optical

TABLE 3

Declinations Measured at the 200 and 400-Foot North-South Antenna Spacings

Source	Class	200-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs	400-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs
3C 433	N	24	51	38	3.4	7					
3C 436	N	27	56	47	8.4	4					
3C 438	N	37	46	28	5.0	4					
3C 441	N	29	14	08	13.4	2					
3C 442	S	13	35	40	18.7	3					
3C 444	(S)	-17	15	57	11.1	2					
3C 445	(E)	-02	20	58	8.1	5					
3C 446	U	-05	12	09	12.3	2					
CTA102	N	11	28	49	7.5	3					
3C 452	(U)	39	25	28	3.0	8					
3C 456	U	09	03	20	14.4	3	09	03	52	12.8	1
3C 459	N	03	48	52	15.5	1					
3C 461	*	58	32	48	3.1	7	58	32	48	2.8	2
23-1<u>12</u>	N						-12	23	50	13.1	2
3C 465	(H)	26	44	20	4.5	6					
3C 469	N	32	38	59	34	1					
CTA 1	*	71	55	14	26	1	71	54	44	26	1
3C 2	N	-00	21	17	11.3	3					
3C 5	N						00	35	01	12.6	2
00-2<u>9</u>	N						-29	44	42	10.3	5

TABLE 3 (Continued)

Source	Class	200-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs	400-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs
3C 10	*	63	51	53	3.6	6					
3C 15	N	-01	25	29	19.3	1	-01	25	39	6.5	2
3C 17	N	-02	23	53	10.8	2					
3C 18	N	09	46	34	16.1	1	09	46	51	8.4	1
3C 19	N	32	54	01	20	1	32	53	34	8.8	1
3C 20	N	51	47	11	5.3	3					
00-2<u>22</u>	(U)	-25	33	11	17.5	2	-25	32	55	6.4	5
3C 23	N	17	31	54	36	1	17	30	36	14	1
3C 26	N	-03	50	08	27	1	-03	49	45	16.3	1
3C 27	N	68	06	46	12.2	1	68	07	09	6.2	1
3C 28	N						26	08	36	13.8	1
3C 29	(U)	-01	38	30	8.3	3	-01	39	21	8.2	1
3C 32	(N)	-16	20	25	16.2	3	-16	20	17	12.2	1
3C 33	U	13	03	33	3.7	6	13	03	21	5.3	1
3C 38	U	-15	35	20	15.0	2	-15	35	21	12.0	1
3C 40	U	-01	37	11	10.6	4					
3C 41	(U)	32	57	05	18.1	1	32	57	47	8.1	1
3C 43	N	23	22	51	20.5	1	23	22	41	10.8	1
3C 46	(U)	37	38	44	36	1	37	38	24	21	1
3C 47	N	20	41	57	14.3	1	20	41	56	7.4	1
3C 48	N*	32	54	18	2.7	9	32	54	19	1.8	5
3C 54		43	17	19	25	1					

TABLE 3 (Continued)

Source	Class	200-Foot Spacing declination (1950.0) °	'	"	std error ±"	# Obs	400-Foot Spacing declination (1950.0) °	'	"	std error ±"	# Obs
3C 55	N	28	36	37	10.4	3	28	36	09	10.6	1
3C 58	*	64	35	25	6.1	2	64	35	23	4.5	1
3C 60		21	05	37	29	1					
3C 62	U	-13	13	26	17.6	2	-13	13	18	7.0	2
3C 63	N	-02	10	30	12.2	3	-02	10	34	6.7	2
3C 65	N	39	46	18	12.8	2					
3C 66	U	42	45	47	3.7	7	42	45	49	4.5	2
02-110	N	-19	44	47	13.9	3					
3C 71	N	-00	13	32	6.1	5	-00	13	34	4.5	3
3C 75	E	05	50	34	5.2	7	05	50	50	4.2	4
3C 78	N	03	55	17	5.2	6	03	55	15	2.7	5
3C 79	N	16	54	36	7.6	3	16	54	34	6.8	1
CTA 21	N	16	17	45	6.3	3					
3C 84	H	41	20	02	4.3	4	41	19	56	4.4	1
3C 86	U	55	10	55	10.9	1	55	10	23	5.7	1
3C 88	S	02	23	19	15.8	1	02	23	05	8.3	1
3C 89	U	-01	21	35	13.9	3	-01	21	16	7.9	2
03 -19	S	-14	38	09	15.7	3					
03-212	S						-27	53	12	10.1	2
3C 98	U	10	17	37	4.2	6	10	17	35	3.5	3
3C 103	E	42	52	17	8.8	2	42	52	16	6.8	1
3C 105	H	03	33	17	10.5	2	03	33	19	8.5	1

TABLE 3 (Continued)

Source	Class	200-Foot Spacing declination (1950.0)			std error	# Obs	400-Foot Spacing declination (1950.0)			std error	# Obs
		o	'	"	± "		o	'	"	± "	
04 -12	N	-12	19	43	29	1	-12	19	28	14.2	1
3C 109	N	11	04	29	15.3	1	11	04	33	7.9	1
04 -24	N						-21	03	25	12.4	2
3C 111	E	37	54	32	3.3	6	37	54	26	3.0	2
3C 119	N	41	32	03	7.5	2	41	31	54	5.3	1
04-112	S						-13	30	38	14.2	2
3C 123	N	29	34	06	3.4	5	29	34	13	3.8	1
04-218	N	-28	15	13	24	1	-28	14	42	6.1	4
3C 129	(U)	44	56	40	5.0	4					
3C 131	N	31	24	49	18.5	1	31	24	22	8.8	1
3C 132	N	22	44	50	15.4	1	22	44	39	9.0	1
3C 133	U	25	12	18	10.4	2	25	12	04	5.9	1
3C 134	E	38	01	59	5.2	3					
05 -13	N	-18	41	21	68	2	-18	41	38	22	3
3C 135	(U)	00	53	08	22	2					
3C 138	N	16	35	13	5.1	4					
3C 141	N	32	45	57	15.0	2					
3C 144	*	21	59	01	2.4	10	21	59	04	2.7	2
3C 145	*	-05	25	14	10.2	1	-05	25	16	5.1	1
3C 147	N	49	49	44	3.4	6	49	49	39	2.0	4
3C 153	N	48	04	53	6.5	5	48	04	54	5.0	2
3C 154	N	26	05	30	5.2	6	26	05	22	6.1	1

TABLE 3 (Continued)

Source	Class	200-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs	400-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs
3C 157	*						22	43	37	14.5	1
3C 158	N	14	33	23	17.0	2	14	33	49	13.2	1
3C 159	N	40	05	27	13.4	2	40	05	23	6.8	2
3C 161	N	-05	51	26	4.9	5	-05	51	22	5.4	1
3C 163	*	04	51	40	15.5	2	04	52	07	39	1
3C 166	N	21	25	08	10.4	4	21	25	00	6.2	3
3C 171	N	54	12	55	7.6	4	54	12	46	7.5	1
06-2<u>16</u>	N	-24	12	12	24	2	-24	12	46	9.5	3
3C 172	E	25	18	09	8.7	4	25	17	54	10.0	1
3C 175	N	11	51	33	9.5	4					
3C 178	N	-09	34	05	21	5					
3C 180	(S)	-01	58	49	16.2	2	-01	58	22	13.2	1
3C 184	N	70	08	51	34	1					
3C 187	U	02	07	41	18.9	3					
07-1<u>17</u>	U	-19	12	24	17.5	5	-19	12	01	21	1
3C 191	N	10	23	40	20.0	2					
3C 192	S	24	18	42	8.9	2	24	18	30	6.5	1
3C 195	N	-10	19	22	15.5	2					
3C 196	N	48	22	09	3.3	6	48	22	07	2.9	2
3C 198	H	06	06	27	16.0	3					
3C 202	N	17	11	06	9.0	6	17	10	53	10.4	1
3C 208	U	14	03	17	12.3	4					

TABLE 3 (Continued)

Source	Class	200-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs	400-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs
3C 212	N	14	21	26	10.9	4					
08-219	N	-25	43	13	14.9	3	-25	43	13	5.7	5
3C 216	N	43	05	54	8.3	3					
3C 218	H	-11	52	54	3.9	9	-11	53	00	2.6	5
3C 225	U	14	00	57	7.5	6					
3C 227	U	07	39	17	5.1	5					
3C 228	N	14	34	06	9.9	3					
3C 230	(U)	00	13	02	10.3	3	00	12	56	6.1	2
3C 234	(U)	29	01	38	4.6	7	29	01	26	3.4	3
3C 237	N	07	45	02	6.1	4	07	44	59	4.3	2
3C 238	(U)	06	39	34	8.6	4					
3C 243	U	06	42	43	15.1	6					
3C 245	N	12	19	18	5.8	9					
3C 249		-01	00	16	14.4	3					
3C 254	N	40	53	46	5.3	9	40	53	46	8.6	1
3C 261	N	30	23	46	16.5	4					
11 -18	N	-13	34	03	10.9	5					
3C 264	H	19	54	06	7.1	3	19	53	55	7.0	1
3C 265	N	31	51	07	10.5	3	31	50	21	8.2	1
3C 267	N	13	04	10	25	1	13	04	08	5.3	4
3C 270	E	06	06	39	4.7	4	06	06	25	2.4	4
M 84	N						13	09	37	6.9	1

TABLE 3 (Continued)

Source	Class	200-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs	400-Foot Spacing declination (1950.0) °	'	"	std error ± "	# Obs
3C 273	N	02	19	48	6.6	2	02	19	42	3.3	2
3C 274	H	12	39	50	4.1	4	12	39	57	1.9	5
3C 275	N						-04	41	35	5.5	4
Coma A	N						27	54	08	10.0	1
3C 278	S						-12	17	06	4.5	3
3C 283	N						-22	00	20	11.0	1
3C 286	N						30	45	55	4.2	1
3C 295	N						52	26	16	1.8	5

Note: the class designation refers to the angular size work of Maltby and Moffet:

N	not resolved at our spacings
S	simple, symmetrical source
E	equal double
U	unsymmetrical
( )	uncertain
*	galactic source

TABLE 3 (Concluded)

Maltby and Moffet list the following north-south diameters for the sources designated as galactic:

3C 461	$3'.8 \pm 0'.5$
CTA 1	$>20'$
3C 10	$7'.0 \pm 1'.0$
3C 48	$<0'.4$
3C 58	$2'.3 \pm 0'.3$
3C 144	$3'.7 \pm 0'.4$
3C 145	$4'.8 \pm 0'.4$
3C 157	$>20'$
3C 163	$>20'$

TABLE 4

Weighted Average of the Declinations Measured at the Various North-South Antenna Spacings

Source	Class	Declination (1950.0) °	'	"	Std Error ± "	Remarks
CTA 1	*	71	54	59	20	
3C 2	N	-01	21	17	14	
3C 5	N	00	35	01	15	
00 -29	N	-29	44	42	46	
3C 10	*	63	51	53	7	
3C 15	N	-01	25	38	11	
3C 17	N	-02	23	53	14	
3C 18	N	09	46	47	9	
3C 19	N	32	53	38	9	
3C 20	N	51	47	11	7	
00 -222	(U)	-25	32	57	35	
3C 23	N	17	30	46	14	possibly confused
3C 26	N	-03	49	51	17	
3C 27	N	68	07	04	9	
3C 29	(U)	-01	38	30	12	200-ft spacing only
3C 32	(N)	-16	20	20	22	
3C 33	U	13	03	33	6	200-ft spacing only
3C 38	U	-15	35	21	21	
3C 40	U	-01	37	11	14	200-ft spacing only
3C 43	N	23	22	43	11	
3C 46	(U)	37	38	29	19	

TABLE 4 (Continued)

Source	Class	Declination (1950.0)			Std Error	Remarks
		°	'	"	± "	
3C 47	N	20	41	56	8	
3C 48	N*	32	54	19.9		optical position
3C 55	N	28	36	23	9	possibly confused
3C 58	*	64	35	24	8	
3C 62	U	-13	13	19	17	
3C 63	N	-02	10	33	10	
3C 65	N	39	46	18	14	
3C 66	U	42	45	47	6	200-ft spacing only
02-110	N	-19	44	47	28	
3C 71	N	-00	13	33	9	
3C 75	E	05	50	44	9	
3C 78	N	03	55	15	7	
3C 79	N	16	54	35	7	
CTA 21	N	16	17	45	8	
3C 84	H	41	19	59	6	
3C 86	U	55	10	29	10	
3C 88	S	02	23	08	10	
3C 89	U	-01	21	21	11	
03 -19	S	-14	38	09	24	
03-212	S	-27	53	12	41	
3C 98	U	10	17	37	7	200-ft spacing only
3C 103	E	42	52	16	7	
3C 105	H	03	33	18	10	

TABLE 4 (Continued)

Source	Class	Declination (1950.0) °	'	"	Std Error ± "	Remarks
04 -12	N	-12	19	31	20	
3C 109	N	11	04	32	9	
04 -24	N	-21	03	25	29	
3C 111	E	37	54	29	5	
3C 119	N	41	31	57	7	
04-112	S	-13	30	38	22	
3C 123	N	29	34	09	2.9	
04-218	N	-28	14	44	42	
3C 129	(U)	44	56	40	7	
3C 131	N	31	24	27	9	
3C 132	N	22	44	42	9	
3C 134	E	38	01	59	7	
05 -13	N	-18	41	36	31	
3C 135	(U)	00	53	08	23	
3C 138	N	16	35	13	7	
3C 141	N	32	45	57	16	
3C 144	*	21	59	02	5	
3C 145	*	-05	25	16	12	
3C 147	N	49	49	39	2.6	
3C 153	N	48	04	54	6	
3C 154	N	26	05	27	6	
3C 158	N	14	33	39	11	
3C 159	N	40	05	24	8	

TABLE 4 (Continued)

Source	Class	Declination (1950.0) °	'	"	Std Error ± "	Remarks
3C 161	N	-05	51	24	12	
3C 166	N	21	25	02	7	
3C 171	N	54	12	49	3.1	
06-216	N	-24	12	41	33	
3C 172	E	25	18	03	8	
3C 175	N	11	51	33	11	
3C 178	N	-09	34	05	25	
3C 180	(S)	-01	58	33	14	
3C 187	U	02	07	41	20	
07-117	U	-19	12	14	27	
3C 191	N	10	23	40	21	
3C 192	S	24	18	34	7	
3C 195	N	-10	19	22	21	
3C 196	N	48	22	06	2.6	
3C 198	H	06	06	27	17	
3C 202	N	17	11	00	8	
3C 208	U	14	03	17	26	200-ft spacing only
3C 212	N	14	21	26	12	
08-219	N	-25	43	13	35	
3C 216	N	43	05	54	10	
3C 218	H	-11	52	58	15	
3C 225	U	14	00	57	9	
3C 227	U	07	39	17	8	

TABLE 4 (Continued)

Source	Class	Declination (1950.0) °	'	"	Std Error ± "	Remarks
3C 228	N	14	34	06	11	
3C 230	(U)	00	12	58	10	
3C 234	(U)	29	01	30	6	
3C 237	N	07	45	00	7	
3C 238	(U)	06	39	34	11	200-ft spacing only
3C 243	U	06	42	43	16	
3C 245	N	12	19	18	8	
3C 249		-01	00	16	17	
3C 254	N	40	53	46	7	
3C 261	N	30	23	46	17	
11 -18	N	-13	34	03	20	
3C 264	H	19	54	00	7	
3C 265	N	31	50	38	8	possibly confused
3C 267	N	13	04	08	7	
3C 270	E	06	06	28	6	possibly confused
3C 273	N	02	19	43	8	
3C 274	H	12	39	57	3.6	
3C 275	N	-04	41	35	12	
3C 278	S	-12	17	06	16	
3C 295	N	52	26	13.6		optical position

TABLE 4 (Concluded)

Note: the class designation refers to the angular size work of Maltby and Moffet:

N	not resolved at our spacings
S	simple, symmetrical source
E	equal double
U	unsymmetrical
( )	uncertain
*	galactic source

IX. CONCLUSIONS

The accuracy of the measured declinations is in most instances about an order of magnitude better than the accuracy of the best previously available radio declinations. The improvement in positional accuracy should make possible the identification of many more radio sources with optically observable objects. The problem of the positional accuracy necessary to effect the identification of an extragalactic radio source has been treated by Minkowski (12). His results indicate that a positional accuracy of ± 13 seconds of arc (the average standard error of the declinations of table 4) in both coordinates should lead to an identification in 60 to 80 percent of the cases. While right ascension measurements of about this accuracy now exist for many of the sources of table 4, it is still too early to say if such a large percentage of identifications will indeed be forthcoming. The limited experience to date, however, has been quite encouraging. The declination measurements have confirmed several tentative identifications and have suggested a number of others. In addition, four sources, 3C 26, 3C 171, 3C 196, and 3C 273, have been identified as a direct result of the declination measurements. In this connection it should be noted that these four identifications could be classed as "difficult" identifications. In each case the optical object is so faint that a plate taken with the 48-inch Schmidt is inadequate for the purpose of making an identification. A plate taken with the 200-inch telescope, however, shows the object in sufficient detail to permit an identification to be made.

In section I it was shown that the probability that a galaxy brighter than 20th magnitude will lie by chance within a ± 13 second of arc error rectangle is only 7 percent. In making a similar calculation for galaxies substantially fainter than this, it is necessary to make allowances for the red shift and the effects of cosmology since most of the galaxies which appear to be very faint are also quite distant. These allowances have been made by Sandage (13) in estimating the number of galaxies per square degree which are observable with the 200-inch telescope (red magnitude < 23). His estimate of 18 000 to 28 000 implies that on the average slightly more than one observable galaxy will lie by chance within a ± 13 second of arc error rectangle. The 1600-foot measurements indicate that somewhat better positional accuracy should be attainable in future measurements. A standard error of ± 4 seconds of arc is not an unreasonable expectation. The probability of an observable galaxy lying by chance within this error rectangle is only slightly more than 10 percent.

As a result of the experience gained during the course of the declination measurements, the author has formed some opinions relating to future measurements which he feels it would be worthwhile to set down here:

No difficulty was experienced in getting fairly good instrumental phase stability even at the largest antenna spacings available. It should be possible, however, to effect a considerable improvement in stability at all the spacings by improving the phase stability of the local oscillator or high frequency reference signal lines. It is suggested that in the future, low loss air dielectric coaxial cable buried to a depth of several feet be used for these lines. This should greatly reduce phase variations caused by changes of the temperature of the lines during the observing night.

While the problem of source resolution dictates the use of the smaller antenna spacings for position measurements of most radio sources, the 1600-foot north-south declination measurements demonstrated the value of using large antenna spacings for very precise position measurements of a few selected "point sources." It is suggested that in the future more extensive use be made of large antenna spacings for this purpose. Not only would a greater number of very precise positions result, but in addition a network of secondary calibrators would be established which would greatly facilitate calibration for measurements made at the closer spacings.

Although some attention was paid to relative phases (changes of apparent position with spacing) in the Maltby and Moffet (9) angular size work, the phases recorded were quite crude when compared to the accuracy of the present declination measurements. The accuracy of the Maltby and Moffet phases was in most cases adequate, however, for the simple two component model of a radio source that was assumed, but there is some indication (from observed changes in apparent declination with spacing) that a two component model is not always an adequate description of a radio source. The assumption of a more complicated source model would require, however, observations more extensive than those made by Maltby and Moffet. Not only would amplitude measurements have to be made at many more antenna spacings, but relative phase measurements would have to be made at all spacings to much higher accuracy. The declination measurements presented here indicate that the accuracy necessary for such phase measurements can be attained.

In the experimental and observational sciences accuracy of measurement is always something that one would like to improve. In order to make significant further improvement in the accuracy of radio position measurements, two important difficulties will have to be overcome. The first is the difficulty due to the uncertainty of the correct positions of the calibrators; the second is the difficulty due to confusion.

The first difficulty could be reduced if it were not necessary to rely on the observations of calibrators to determine the variation of the instrumental phase calibration term, Δt, during the observing night. This could be accomplished either by improving the instrumental phase stability to the point where Δt could be assumed constant during the entire night or by providing a means independent of any knowledge of the positions of radio sources for periodically measuring the variation of Δt. The requirement that observations of calibrators be used to evaluate the variation of Δt causes the measured position of a source to be strongly dependent on the assumed positions of the calibrators observed at roughly the same time as that source and only weakly dependent on the assumed positions of the other calibrators. If, however, the variation of Δt were known, the measured position of a source could be determined from an appropriately weighted average of all the calibrators. The effect of the uncertainties of the positions of the calibrators would be significantly reduced in this average.

In order to reduce the second difficulty, confusion, the angular resolving power of the 90-foot paraboloids would have to be increased. This could be done, however, in two different ways. The first way would be to increase the diameters of the paraboloids. Such an increase in size would pose major engineering problems and would certainly entail a large outlay of capital. Increasing the size of the paraboloids, therefore, could not be justified on the basis of improving the accuracy of position measurements alone. The other way to increase the resolution would be easier: make the position measurements at a higher frequency. Since the strength of radio sources decreases in general at higher frequencies, such a frequency change would result in a deterioration of the signal to noise ratio and therefore a possible reduction in positional accuracy. The present measurements indicate, however, that if a reasonable number of independent observations were averaged, a decrease of the signal to noise ratio by a factor of two or three could be tolerated without undue loss of accuracy. Future position measurements should therefore be made at a frequency somewhat higher than 960 Mc/s: 1390 Mc/s at the minimum, possibly as high as 2800 Mc/s.

X. APPENDIX

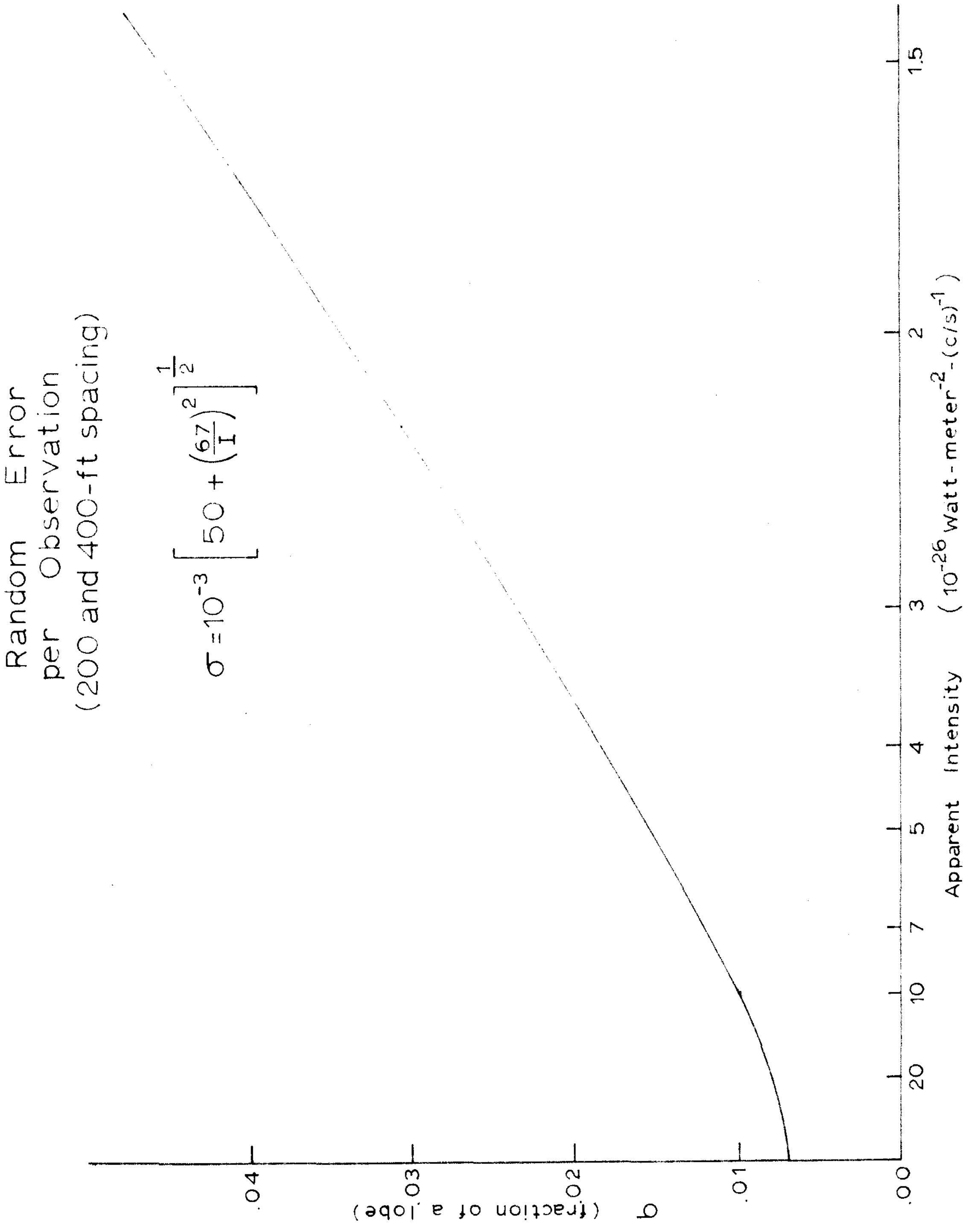
a. Definition of the baseline.

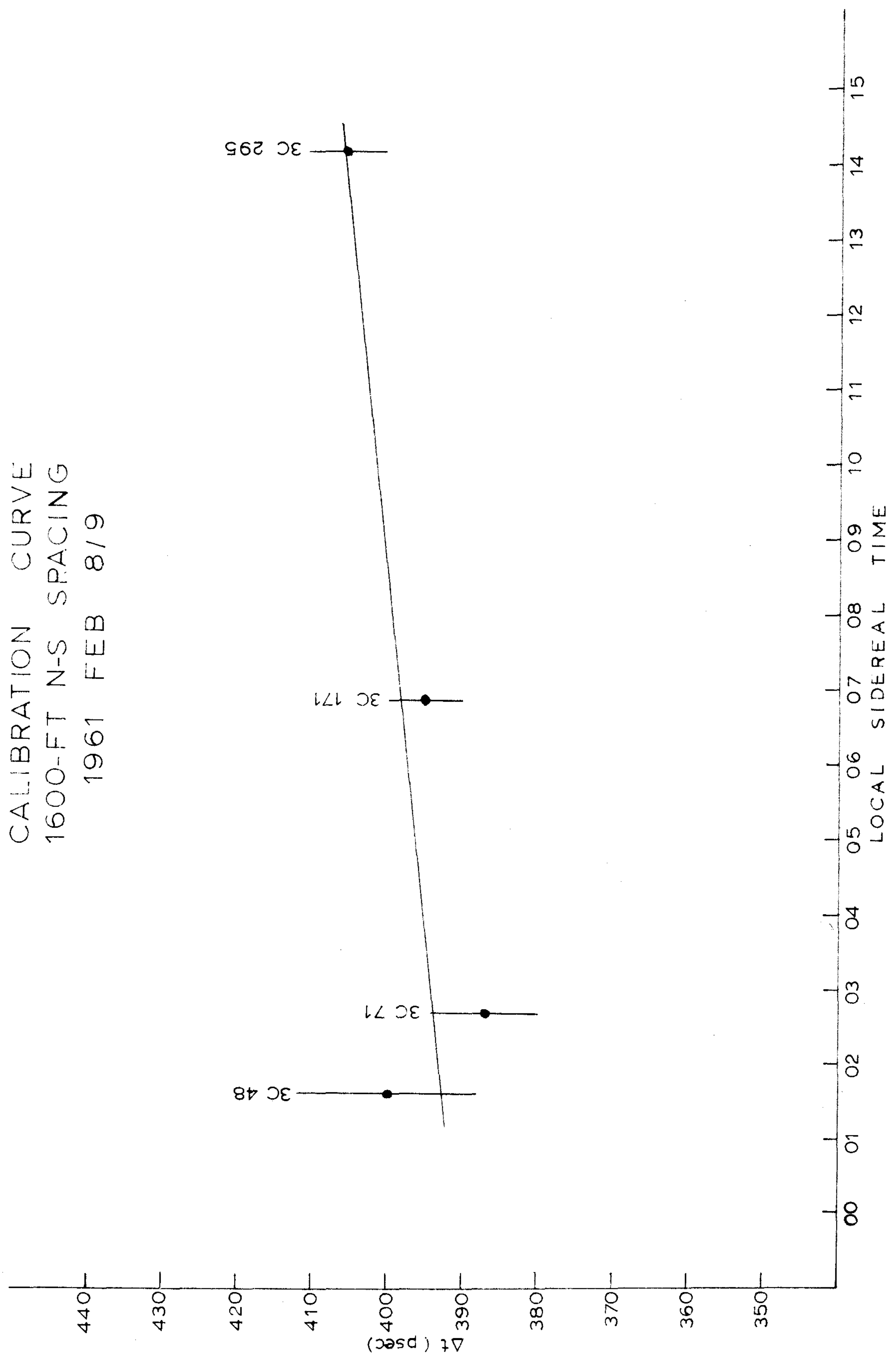
b. Plot of the random standard error per observation as a function of intensity.

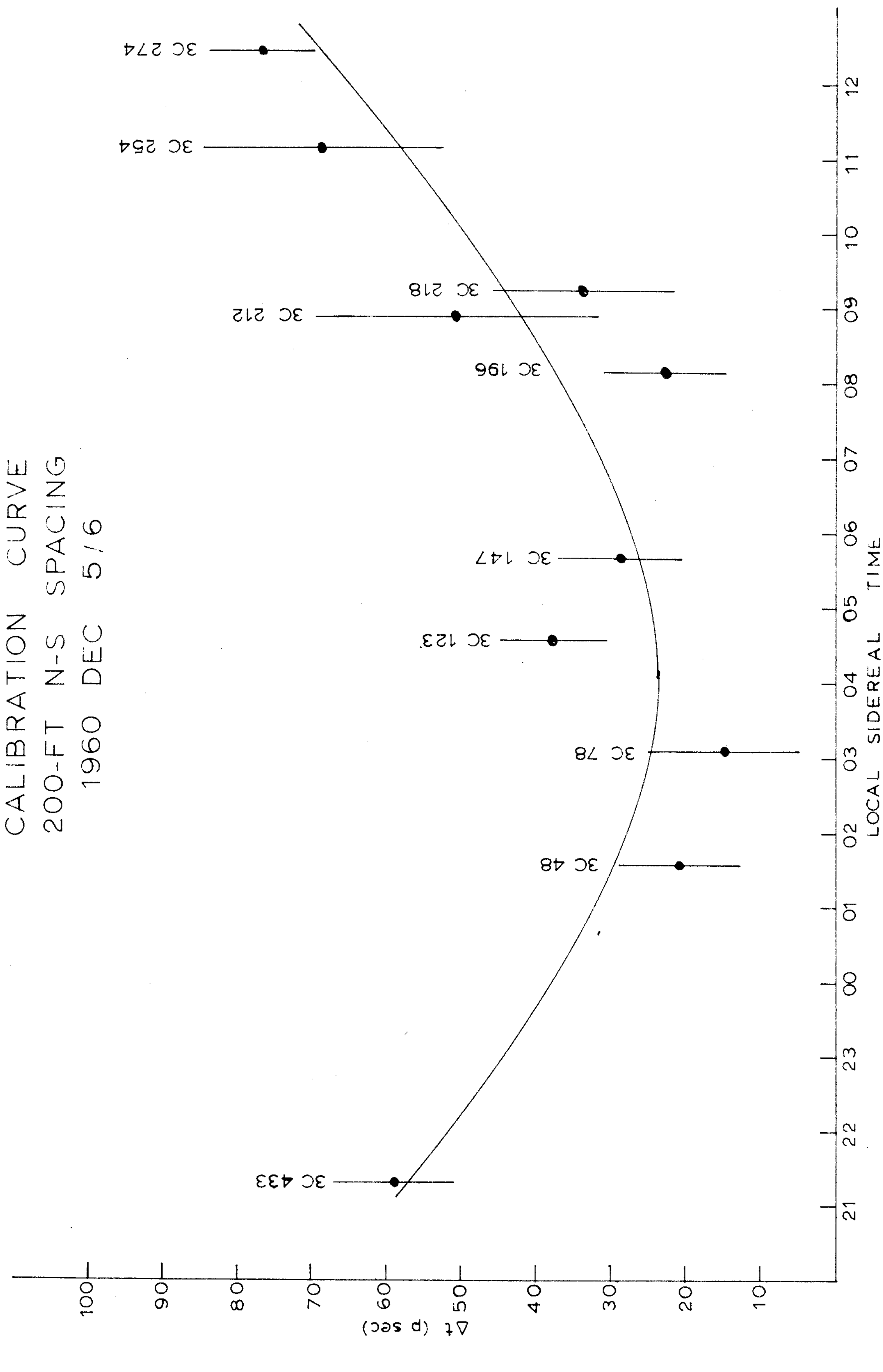
c. Typical plots of Δt as a function of time.

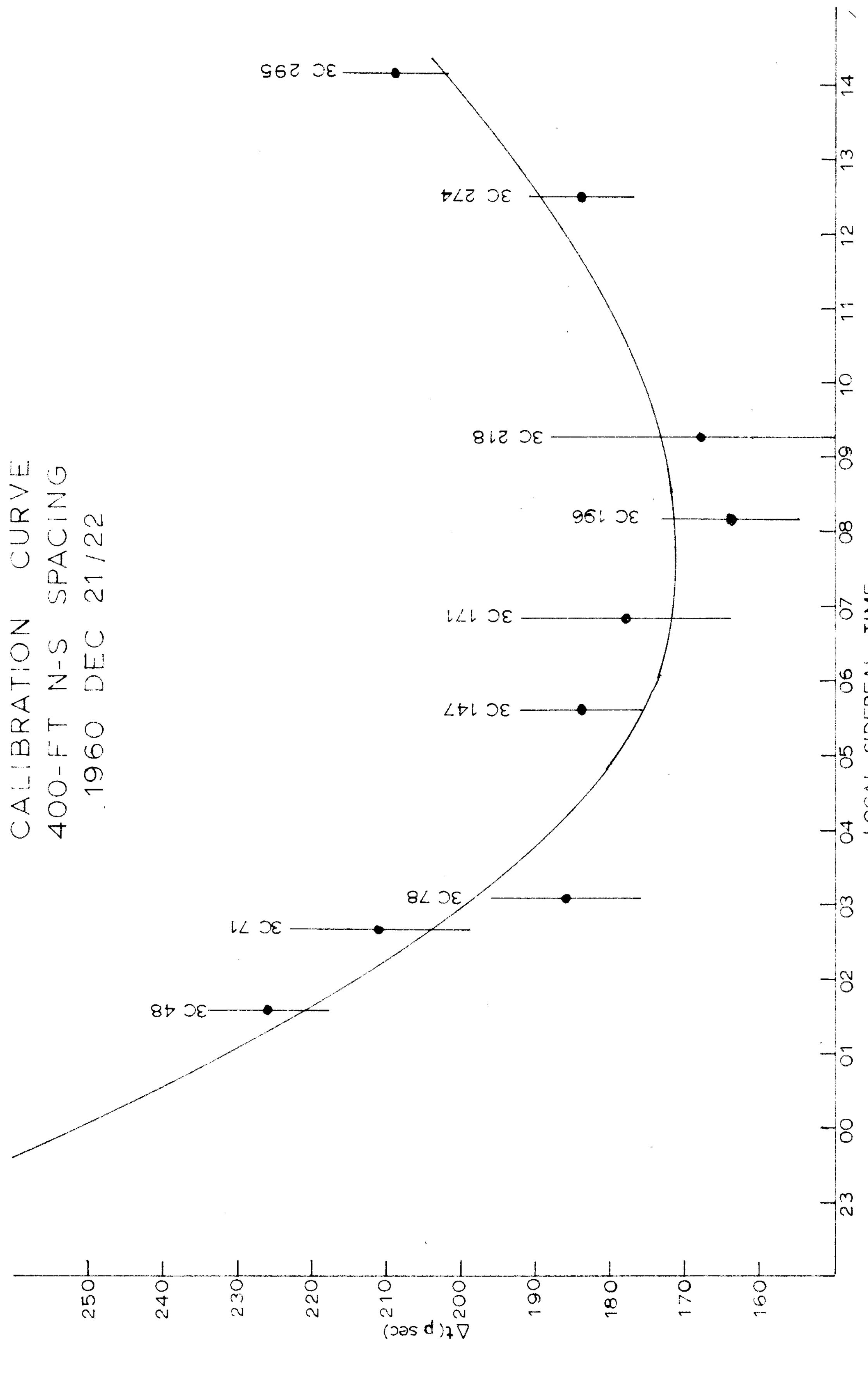
Definition of the Baseline

A question arises in the case of an interferometer with elements of large physical size as to just how the baseline is to be defined. To what point of a large element does one refer when measuring the length and direction of the baseline? For the interferometer used for the declination measurements the answer is clear. The elements of the interferometer were two 90-foot steerable paraboloidal antennas on equatorial mounts. The polar and declination axes of each mount intersected, and it is these points of intersection that are to be used as the effective points of the antennas. The proof is as follows: During the course of an observation clock drives cause the antennas to track the source being observed. Thus the orientation of an antenna does not change relative to the direction of arrival of the radiation it is receiving. The total distance from the source to any point on the moving structure of the antenna differs from the total distance travelled by the radiation reaching the feed horn by a constant amount independent of the apparent motion of the sky. Also, since the antennas are always pointed directly at the source being observed, this difference in distance is the same for all sources observed. The effective time of arrival of the radiation at the antenna may thus be referred to any point of the moving structure of the antenna by introducing a constant difference in the arrival time. The intersection of the polar and declination axes is not only a point on the moving structure of the antenna but is also fixed with respect to the ground. Thus, since the interferometer measures only the difference in arrival time of the radiation at the two antennas, the baseline may be conveniently defined as that line connecting the intersection of the polar and declination axes of one antenna with the intersection of the polar and declination axes of the other antenna.


Random Error
per Observation
(200 and 400-ft spacing)
$\sigma = 10^{-3}\left[50 + \left(\frac{67}{I}\right)^2\right]^{\frac{1}{2}}$
σ (fraction of a lobe)
.00
.01
.02
.03
.04
20
10
7
5
4
3
2
1.5
Apparent Intensity (10^{-26} Watt-meter^{-2}-(c/s)$^{-1}$)


CALIBRATION CURVE
1600-FT N-S SPACING
1961 FEB 8/9
3C 295
3C 171
3C 71
3C 48
LOCAL SIDEREAL TIME
00
01
02
03
04
05
06
07
08
09
10
11
12
13
14
15
Δt (μsec)
440
430
420
410
400
390
380
370
360
350


CALIBRATION CURVE
200-FT N-S SPACING
1960 DEC 5/6
3C 274
3C 254
3C 218
3C 212
3C 196
3C 147
3C 123
3C 78
3C 48
3C 433
Δt (p sec)
100
90
80
70
60
50
40
30
20
10
21
22
23
00
01
02
03
04
05
06
07
08
09
10
11
12
LOCAL SIDEREAL TIME



CALIBRATION CURVE
400-FT N-S SPACING
1960 DEC 21/22
3C 48
3C 71
3C 78
3C 147
3C 171
3C 196
3C 218
3C 274
3C 295
LOCAL SIDEREAL TIME
23
00
01
02
03
04
05
06
07
08
09
10
11
12
13
14
Δt(μ sec)
250
240
230
220
210
200
190
180
170
160

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