Appendix A

Announce-Listen: Inconsistency and Departures

Tables A.1 and A.2 are refinements to Tables 4.2 and 4.3, in that they consider the impact of process departures on the state of the system. These tables show the likelihood for each type of error that arises in the registry of a listener process: inconsistent state, false negatives and false positives. These probabilities are associated with states that appear in the listener's transition graph as shown in Figure 4.5.

Table A.1 examines inconsistencies arising when the current time, t, occurs after Δ, meaning $t \geq \Delta + \lfloor \frac{t}{T} \rfloor \cdot T$. Table A.2 accounts for inconsistencies when t occurs within Δ, meaning $t < \Delta + \lfloor \frac{t}{T} \rfloor \cdot T$.

The notation is summarized in Table 4.1. The abbreviation $n.a.$ is not applicable, meaning the state is not possible. Recall that $\overline{D}(t) = 1 - D(t)$ is defined as the probability that the announcer is alive at time t.

Table A.1 examines inconsistencies arising when the current time $t < \Delta + \lfloor \frac{t}{T} \rfloor \cdot T$, whereas Table A.2 accounts for inconsistencies when time $t \geq \Delta + \lfloor \frac{t}{T} \rfloor \cdot T$.

A.1 Arrivals

If we look at each registry entry and pretend that time begins at 0, the tables are accurate (Tables 4.2 and 4.3, and tables A.2 and A.1). When we look at global registry consistency, each entry may have a different start time. Thus, let us examine the impact of the arrival distribution on $Pr[Err(t)]$, the probability of error at time t.

Let $A(u)$ be the distribution for process arrivals into the Announce-Listen algorithm. If a process arrives into the system at time u, then for that process the new probability of error at time t is shifted in time becoming $Pr[E(t-u)]$.

Consider an example in the discrete realm. Process i $(0 \leq i \leq N)$ arrives at time i (relative to

Listener State		*Inconsistent State*		*False*	*False*
		ε	$Pr[Err = \varepsilon]$	$-$	$+$
a_n	*Alive*	0	$\overline{D}(t)(1-p)$	*n.a.*	$(1-p)(D(t) - D(nT))$
a_{n-1}	*Not Sure*	1	$\overline{D}(t)(1-p)p$	*n.a.*	$(1-p)[(D(nT) - D((n-1)T)) +$ $(1 - (D(nT) - D((n-1)T))) \times$ $p(D(t) - D(nT))]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_{n-k+1}	*Last Try*	$k-1$	$\overline{D}(t)(1-p)p^{k-1}$	*n.a.*	$(1-p)[D((n-k+2)T) - D((n-k+1)T)) +$ $(1 - D((n-k+2)T) - D((n-k+1)T))) \times$ $p(D((n-k+3)T) - D((n-k+2)T)) +$ $\vdots$ $D(nT) - D((n-1)T) +$ $(1 - (D(nT) - D((n-1)T))) \times$ $p^{k-1}(D(t) - D(nT))]$
a_{n-k}	*Departed*	0	X_t	$\overline{D}(t)p^k$	*n.a.*

Table A.1: **Inconsistency: After Δ.**

Listener State		*Inconsistent State*		*False*	*False*
		ε	$Pr[Err = \varepsilon]$	$-$	$+$
a_n	*Alive*	0	0	*n.a.*	*n.a.*
a_{n-1}	*Not Sure*	1	$\overline{D}(t)(1-p)$	*n.a.*	$(1-p)(D(t) - D((n-1)T))$
a_{n-2}	*Less Sure*	2	$\overline{D}(t)(1-p)p$	*n.a.*	$(1-p)[(D((n-1)T) - D((n-2)T)) +$ $(1 - (D((n-1)T) - D((n-2)T))) \times$ $p(D(t) - D((n-1)T))]$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_{n-k+1}	*Last Try*	$k-1$	$\overline{D}(t)(1-p)p^{k-2}$	*n.a.*	$(1-p)[D((n-k+2)T) - D((n-k+1)T)) +$ $(1 - (D((n-k+2)T) - D((n-k+1)T))) \times$ $p(D((n-k+3)T) - D((n-k+2)T)) +$ $\vdots$ $D((n-1)t) - D((n-2)T) +$ $(1 - (D((n-1)t) - D((n-2)T))) \times$ $p^{k-2}(D(t) - D((n-1)T))]$
a_{n-k}	*Departed*	0	Y_t	$\overline{D}(t)p^{k-1}$	*n.a.*

Table A.2: **Inconsistency: Within Δ.**

time 0 when the algorithm begins) and each stays through time A, the last arrival time.

$$\begin{aligned} Average\ Pr[Err(t)] &= Pr[Err(t)] \cdot u_0 + Pr[Err(t-1)] \cdot u_1 + \\ &\qquad Pr[Err(t-2)] \cdot u_2 + \ldots + Pr[Err(t-N-1)] \cdot u_N \end{aligned}$$

In the continuous domain, the average probability of error at time t becomes the integral

$$Average\ Pr[Err(t)] = \int_0^A A(u) \cdot Pr[Err(t-u)] du$$