THE EFFECT OF REYNOLDS NUMBER
ON THE FLOW FIELD ABOUT A 70° CONE

Thesis by
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SUMMARY

This investigation has undertaken to measure the possible Reynolds number effects in the flow field about a 70° finite cone in a supersonic flow. The principal means of observing such effects was by static pressure measurements at the surface of the cone.

Results indicate that such effects do exist when the free stream Mach number is low enough to produce subsonic flow over the face of the cone, but they are slight. At very low Reynolds numbers with a detached shock, some decrease in static pressure on the surface of the cone results, apparently due to increased boundary layer. Over the entire range of Reynolds numbers investigated with either attached or detached shock, an increase in static pressure on the cone face at some distance from the apex results from increased Reynolds number, apparently due to the shoulder disturbance.

The investigation was conducted at GALCIT as the basis for a thesis in partial fulfillment for the professional degree of Aeronautical Engineer.
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I. INTRODUCTION

The general objective of this investigation was to inquire into the possible effects of Reynolds number variations on the flow field about a finite 70° cone in supersonic flow.

The investigation was suggested in part by the fact that no systematic tests of Reynolds number effect on supersonic flow appears to have been conducted, and in part by B. W. Marschner's report on "An Investigation of Detached Shock Waves", Ref. 1, in which the possibility of viscous effects were noted. Such a possibility arose from the results concerning shock waves at Mach numbers just below theoretical detachment; from the data presented, these shocks appeared to behave as though attached.

Reynolds number effects were measured by observing the variation of the static pressure on the surface of the cone at various Reynolds numbers. A secondary gage consisted of observation of possible variation of shock configuration by means of a Schlieren apparatus.

The Reynolds number range used was between about $0.15 \times 10^6$ and $1.3 \times 10^6$, based on model diameter. Since the theoretical Mach number detachment values for the 70° cone is 1.633, the Mach numbers investigated were 1.64 and 1.72 to obtain data just below and above theoretical detachment.
II. EQUIPMENT

The wind tunnel used for this investigation was the GACKET Supersonic Wind Tunnel which is a closed single-return type, with 2.5" x 2.5" rectangular test section. The details of the tunnel are completely described in a GACKET publication, Ref. 2.

The nozzle blocks used for the two Mach numbers were both of the fixed type. The 1.64 nozzle was one of the standard machined steel removable nozzle blocks, as described in Ref. 2. For Mach number 1.72, a set of maple blocks originally designed to give 1.72 at a point considerably forward in the test section was available; these blocks were modified to give 1.72 at the model, which was mounted on a sting at the after end of the test section, by moving the throat portion downstream and inserting filler pieces forward. The resultant Mach number distribution at the model was only fair and not as uniform as the machined steel blocks. A Mach number survey along a line through the axis of the model for the two Mach numbers is shown in Figs. 1 and 2.

Pressure measuring apparatus consisted of mercury manometers; in the centerline survey a tetrabromomethane manometer was used to record as accurately as possible the relatively slight pressure variations.

The Schlieren apparatus utilized spark-gap photography with an exposure time of 1/500,000 second; the Schlieren system is described in the previously mentioned GACKET publication, Ref. 2.
The 70° cone models were made of brass, appropriately threaded to mount on the test section sting. Four cones were used, one for each location of the pressure orifices; detailed dimensions are given in Fig. 3. An apex angle of 70° was chosen in order to readily obtain an attached and detached shock and because theoretical calculations were available for such a cone in Kopal's report on "Supersonic Flow Around Cones", Ref. 3, which is based on the Taylor and MacColl solution for supersonic conical flow, Ref. 4. The 1/2 inch diameter was chosen to give as large a cone as could be accommodated in the tunnel, in order to permit a pressure orifice as close to the nose as possible. This maximum size was of course determined by blocking considerations, which had been investigated in the previously mentioned report by Marschner, Ref. 1.

III. PROCEDURE

The Reynolds number variation was obtained by changing the density of the air in the tunnel. Essentially, three Reynolds numbers were used = one with tunnel running under atmospheric conditions, one pressurized and one evacuated. In this way a Reynolds number variation of about 10 was obtained.

The atmospheric runs were conducted with the compressor discharge line vented to the atmosphere. The pressurized (high Reynolds number) runs were accomplished by:

1. Pressurizing settling tank by closing valve ahead of the test section, compressor intake vented to
atmosphere.

2. With compressors still running, and when desired pressure was reached, simultaneously opening valve to test section and closing atmospheric intake.

The evacuated (low Reynolds number) runs were conducted similarly, except that the settling tank was first evacuated.

Careful humidity control was found to be of considerable importance in achieving uniform results. In the normal atmospheric operation of the tunnel, humidity was automatically kept quite low since the air dryer was in the line at all times; relative humidity ran from 1 to 4.5%. For each pressure run, however, it was necessary to bring in, with only one pass through the dryer, a fresh batch of relatively moist air; the dryer could not be left in the line during the actual pressure run, since it was not constructed to withstand pressure. Hence the pressure runs had the highest humidity running from 8 to 13.5%. After each pressure run, recirculation of the tunnel air through the dryer was therefore necessary. The evacuated runs did not cause as much trouble, since no new air had to be brought in; however, the inevitable leakage in the system did allow the entry of some moist air, so that even after a low-pressure run recirculation was necessary. Relative humidity at the low pressures ran from 3 to 5.5%.

Research underway at GALCIT at the present time indicates that the above relative humidity values are all well within the limits at which condensation phenomena could be expected.
After the nozzle had been calibrated, the cones were tested in the tunnel. In order to establish zero angle of attack, each model was first tested at various angles of attack with the static pressure orifice located at the top and then at the bottom. An auxiliary plot then determined the angle setting for which the static pressures at the orifice were equal; this setting was used for zero angle of attack position. A typical plot for each Mach number is shown in Fig. 4.

Because the compressors in the tunnel were of a type that injected considerable oil into the air stream, the Schlieren pictures had to be taken as soon as possible after operation was commenced; otherwise, fogging of the glass windows prevented clear reproduction.

IV. REDUCTION AND PRESENTATION OF DATA

Pressure data taken consisted on the static pressures $P_x'$ at the orifice on the surface of the cone, together with the stagnation pressure $P_0$ in the tunnel settling tank. This data was then reduced in terms of the ratio $P_x' / P_0$.

Reynolds number was calculated from Mach number and observed settling tank stagnation temperature and pressure by use of the Reynolds number chart given in Fig. 10 of NACA Technical Note 1428, December, 1947.

The location of the pressure orifices was reduced to the dimensionless form $x'/s$, where $x'$ was the distance along the face of the cone from nose to orifice and $s$ the total slant height.
Plots showing the variation of pressure ratio $P_x'/P_o$ with Reynolds number for the various $x'/s$ locations at each Mach number are plotted in Figs. 5 to 11. Also included are cross-plots of $P_x'/P_o$ vs. $x'/s$ for the various Reynolds numbers.

Contact prints of a set of pictures for both Mach numbers are presented in Figs. 12 to 17, each set consisting of a picture at low, atmospheric, and high pressure. Two enlarged pictures, one with attached and one with detached shock waves, are presented in Figs. 18 and 19. To aid in differentiating between the attached and detached configuration a shock trace is presented in Fig. 20; this was obtained by projecting the negatives of the enlarged pictures successively on graph paper in a dark room, with maximum possible enlargement, and carefully tracing the shock on the paper.

V. DISCUSSION OF RESULTS

Prior to an analysis of the results, it is of interest to note the conditions existing on the surface of the cone as given by the Taylor-Maccoll theoretical solution, Ref. 4, which has been calculated by Kopal for various cone angles and Mach numbers, Ref. 3. From this, it is predicted that:

1. The shock for a $70^\circ$ cone becomes detached at Mach number 1.683.

2. At Mach numbers between 1.683 and 1.91 for a conical flow field the flow is subsonic over the surface of a $70^\circ$ cone, with uniform pressure.
As a consequence of the foregoing it can be further predicted that:

1. For Mach number 1.72:
   a. The shock will be attached.
   b. The flow on the surface of the cone will be subsonic, and hence viscous disturbances at the shoulder can be reflected upstream along the surface of the cone.
   c. The presence of the shoulder will produce a non-conical flow field, and hence the pressure will not be uniform.
   d. The pressure should decrease going from nose to shoulder due to accelerating subsonic flow over the face.
   e. The pressure should approach the proper conical flow value at the nose; for Mach number 1.72, Ref. 3, gives a $\frac{P_x'}{P_o}$ of 0.591.
   f. The included angle between the shock waves coming off opposite sides will be 128.4°.

2. For Mach number 1.64:
   a. The shock will be detached.
   b. Since the flow for Mach number 1.72 is subsonic on the cone surface it will also be for the lower Mach number of 1.64; hence viscous disturbances at the shoulder can be reflected upstream along the cone face.
   c. The pressure should decrease going from nose to shoulder due to the accelerating subsonic flow over the face.
   d. The pressure should approach that behind a normal shock at the nose; for Mach number 1.64, normal shock theory gives a $\frac{P_x'}{P_o}$ of 0.879.
Considering first the data concerning the theoretical detached shock at Mach number 1.64, it appears quite conclusively that the shock wave is actually detached, thus resolving the previously mentioned uncertainty in Marschner's report, Ref. 1, concerning the behaviour of shocks just below detachment. This conclusion can be reached by examining Fig. 5; it is apparent that the orifice pressures toward the nose are approaching the proper values that theory gives behind a normal shock. Further evidence can be obtained from Fig. 20, from which it is clear that the shock at Mach number 1.64 possesses the normal curved characteristics of a detached shock, although so close to the nose that it appears to touch.

Reynolds number effect, such as it was, is best shown in Fig. 6 for Mach number 1.64. Three essential features are noted:

1. **Flat portions.** The straight horizontal portions of the curves naturally indicate no Reynolds number effect.

2. **Gradual Change of slope.** There seems to be a tendency for the slope to increase at the greater $x'/s$ distances; the data for $x'/s = 0.5$ for example shows a definite Reynolds number effect. It can thus be concluded that the effect becomes noticeable at some distance from the apex. This can possibly be explained by the reflection of the disturbance at the shoulder upstream into the subsonic flow field on the surface of the cone. It is somewhat surprising that it extends as far forward as an $x'/s$ of 0.500".
3. Droop at low Reynolds number. Each curve tends to drop off at the low Reynolds numbers. Reynolds effects need not, of course, be linear so the presence of such a curvature is possible. That it appears at the low Reynolds number may be perhaps explained by the correspondingly thicker boundary layers over the surface of the cone.

It should be observed at this point that the $\frac{P_r}{P_o}$ scale used in the plots is considerably enlarged; such enlargement was considered advisable to show the data properly. The actual variations, therefore, are extremely small so that the Reynolds number effects are of minor magnitude, hardly more than the experiment scatter.

Considering now the attached shock data, Mach number 1.72, it can be noted from Fig. 20 that the configuration possesses the proper characteristics for an attached shock — i.e., straight waves coming off each side of the apex. The included angle measures 128°, while the Kopal report, Ref. 3, indicates that the included angle for the attached shock on the 70° cone at Mach number 1.72 should be 128.4°. Agreement is therefore excellent.

Reynolds number effects at Mach number 1.72 are shown in Figs. 7 to 10. Essentially it will be noted that at the three orifices nearest the nose, no effect is observed. At $x'/s = 0.5$ a definite slope develops, indicating that the shoulder disturbance extends at least this far forward on to the cone. This confirms the similar result obtained at Mach number 1.64. The generally greater effect at Mach
number 1.64 as compared to Mach number 1.72 may be explained by the reasonable assumption that the flow in the subsonic region is more sensitive to disturbances at the lower Mach number.

Fig. 11, the \( x'/s \) cross-plot at Mach number 1.72, reflects operational difficulties. The previously mentioned theoretical calculations, substantiated by the data of Marschner's report, Ref. 1, indicate that for the attached shock wave at this Mach number the \( x'/s \) data should be on a line intersecting the \( P_x'/P_o \) axis at a value of 0.581. The line should slope upwards to the left, reflecting the accelerating subsonic flow over the cone face. That the data fails to conform to expectations may be accounted for by the following considerations:

1. \( x'/s = 0.0436 \) - This model unfortunately sustained some damage to the nose, losing the sharp point, thus changing its conical characteristics.

2. \( x'/s = 0.5 \) - In order to give the proper slope, the points at this \( x'/s \) distance should be somewhat lower. Mach number 1.72 was obtained, as indicated previously, by using converted wooden blocks which did not give entirely satisfactory velocity distribution. Hence this undoubtedly had an adverse effect on the flow field.

However, it must be pointed out that neither of these effects would invalidate the data concerning Reynolds number effect, which was the primary purpose of this investigation.
VI. CONCLUSIONS

1. Reynolds number effects on the flow field about a finite 70° cone do exist when the free stream Mach number is low enough to give subsonic velocities over the face of the cone, but appear to be small.

2. The effects that were observed materialized in principally two ways:
   
a) At very low Reynolds numbers (0.15 x 10^6) with detached shock wave, the static pressure on the surface of the cone decreased slightly, apparently due to the increased boundary layer.

b) For the entire range of Reynolds numbers investigated, and for either attached or detached shock, the static pressure on the surface of the cone at some distance from the apex increased with increasing Reynolds number; this apparently was due to the disturbance at the shoulder.
REFERENCES


Fig. 1

Centerline Survey
Mach = 1.64
Shock Detached

Tunnel Reference Distance - Inches
FIG. 2

CENTERLINE SURVEY
Mach = 1.72

SHOCK ATTACHED

TUNNEL REFERENCE DISTANCE - INCHES
<table>
<thead>
<tr>
<th>Cone No.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>ORIFKE DIAM.</th>
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<tr>
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<td>.840</td>
<td>.006&quot; PIVOT DRILL</td>
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<td>.031</td>
<td>.803</td>
<td>#80 DRILL (.013&quot;)</td>
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<td>.061</td>
<td>.751</td>
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<tr>
<td>4</td>
<td>.218</td>
<td>.153</td>
<td>.591</td>
<td>&quot;</td>
</tr>
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</table>
TYPICAL DETERMINATION OF ZERO ANGLE OF ATTACK

Mach No. 1.64
$x'/s' = 0.2080$

Mach No. 1.72
$x'/s' = 0.2080$
Fig. 5

70° CONE PRESSURE DISTRIBUTION
Mach = 1.64

SHOCK DETACHED

\[ \triangle - R_N = 0.17 \times 10^6 \]
\[ \diamond - R_N = 0.43 \times 10^6 \]
\[ \forall - R_N = 1.27 \times 10^6 \]
FIG. 6

70° CONE
PRESSURE RATIO
vs.
REYNOLDS NUMBER
Mach = 1.64

SHOCK DETACHED

$\frac{p_x}{p_0}$

$\Delta - \frac{x}{h} = .0436$

$\circ - \frac{x}{h} = .1010$

$\square - \frac{x}{h} = .2020$

$\triangle - \frac{x}{h} = .5000$

$R_N \times 10^{-6}$
Fig. 7

70° Cone
Pressure Ratio vs Reynolds Number
Mach = 1.72
Shock Attached

\[ \frac{P_x}{P_0} \]

\[ \alpha - x^{1/3} = 0.436 \]

\[ R_N \times 10^{-6} \]
Fig. 8

70° CONE
PRESSURE RATIO
vs
REYNOLDS NUMBER
Mach = 1.72

SHOCK ATTACHED

\[ \frac{P_x'}{P_o} \]

\[ 0 - \frac{x}{c} = 0.1010 \]

\[ R_N \times 10^{-6} \]
Fig. 9

70° CONE
PRESSURE RATIO
vs
REYNOLDS NUMBER
Mach = 1.72

SHOCK ATTACHED

$\frac{P_x'}{P_0}$

$R_N \times 10^{-6}$

$X = 2020$
Fig. 10

70° Cone
Pressure Ratio
vs.
Reynolds Number
Mach = 1.72

Shock Attached

\[ \frac{P_x}{P_0} \]

\[ \Delta - \frac{x}{S} = .5000 \]

\[ R_N \times 10^{-6} \]
Fig. 11

70° CONE
PRESSURE DISTRIBUTION
Mach = 1.72

Shock Attached

$\Delta - R_N = 1.5 \times 10^6$  
$\Phi - R_N = 4.3 \times 10^6$  
$\Phi - R_N = 1.00 \times 10^6$
Fig. 12
Mach Number 1.72 (Shock Attached)
At Low Pressure (Low $P_N$)

Fig. 13
Mach Number 1.72 (Shock Attached)
At Atmospheric Pressure
Fig. 14
Mach Number 1.72 (Shock Attached)
At High Pressure (High \( R_N \))

Fig. 15
Mach Number 1.64 (Shock Detached)
At Low Pressure (Low \( R_N \))
Fig. 16
Mach Number 1.64 (Shock Detached)
At Atmospheric Pressure

Fig. 17
Mach Number 1.64 (Shock Detached)
At High Pressure (High $R_N$)
Fig. 19
Detached Shock Wave
Mach No. 1.64
70° CONE SHOCK WAVES

AT

Mach = 1.72 = ATTACHED
Mach = 1.64 = DETACHED