THE ASSIGNMENT PROBLEM:
THEORY AND EXPERIMENTS

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ABSTRACT

In this thesis I consider the problem of assigning a fixed and heterogeneous set of goods or services to a fixed set of individuals. I analyze this allocation problem with and without the use of monetary transfers to allocate good.

There are many applications in the literature associated with this problem. The usual approach to this problem has been to discuss the properties of individual mechanisms (variously called procedures, algorithms, or rules) to solve the problem, often ignoring the incentive properties. In this thesis I take a different approach, that is, to look at a large class of mechanisms and to determine the conditions necessary to induce mechanisms with desired optimality and incentive properties. This analytic technique is augmented by an experimental examination of some of the mechanisms that have been proposed to solve this problem. Mechanisms that use transfers and consider incentive properties exist in the literature, but mechanisms that do not use transfers do not. None of these mechanisms has been tested or compared. The thesis is divided into two chapters; in chapter I, I examine the class of nontransfer dominant and Nash strategy mechanisms, and in chapter II, I discuss the experimental tests of the known transfer mechanisms and of the nontransfer mechanisms discussed in chapter I.

In the first chapter of this thesis, I characterize the conditions necessary for a nontransfer mechanism to be implementable in dominant and Nash strategies. This characterization is an extension of the Gibbard-Satterthwaite theorem. One of the conditions, ordinality, explains a distinction that is observed in the mechanisms described in the literature, that is, the use of cardinal information when transfers are used, and the use of ordinal information when transfers are not used. In addition, I apply a little-known concept for strategic behavior, nonbossiness, which is a necessary condition for implementability.

In the second chapter of this thesis, I use experimental methods to explore some procedures that could be used to assign individuals to slots. I look at four mechanisms, two transfer mechanisms, a sealed-bid auction and a progressive auction, and two nontransfer mechanisms, a choice mechanism and a chit mechanism (which are also studied in part I of this thesis). The mechanisms were compared to their theoretical predictions and to each other. For the chit mechanism a genetic algorithm was used to compute the predicted outcome; since this is a new use for the technique, I discuss the methodology that I used.

The experimental results for the transfer auctions are similar to the results found for single and multiple unit auctions; that is, progressive auctions tend to be more efficient and extract higher revenue from the bidders. While the transfer mechanisms studied had the properties that they are efficient and extract surplus (in terms of revenue) from the bidders, nontransfer mechanisms retain most of the surplus for bidders but tend to be less efficient. The difference between the two classes of mechanisms was most apparent in a high-contention environment where the use of nontransfer mechanisms resulted in a much larger surplus to the individual bidders, and the transfer mechanisms resulted in slightly higher efficiencies (the differences in efficiencies were small in comparison to the differences in consumer surplus). In a low-contention environment the use of either a transfer or a nontransfer mechanism had little effect on either the efficiencies or the consumer surplus.

The results of this study are a first step to understanding the assignment problem and to understanding more difficult allocation problems with heterogeneous goods. Two simple results are evident from our results. In the low-contention environment the planner can choose among the mechanisms discussed and not be concerned about their relative merits, since there is little difference in the outcomes of these mechanisms; in the high-contention environment the planner must determine whether efficiency or consumer surplus is more important; if efficiency or revenue is most important then, the progressive auction is clearly superior, if consumer welfare is most important then the chit mechanism is superior.

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Chapter I: Dominant and Nash Strategy Mechanisms for the Assignment Problem

1. Introduction

Many allocation problems involve the assignment or matching of a set of individuals to a set of objects. This allocation problem appears in a variety of settings. A school administrator assigns office space to faculty members. A university computer group administers computer resources. A space agency schedules antenna time to spacecraft. Other allocation problems involve matching members of one group to members of another group. A marriage broker matches men with women. A community sports league matches players with teams. All these allocation problems have two properties in common: the task of matching one group to another and the institutional feature that money is not used.[1]

In the examples listed above there are two distinct categories of problems. One category, and the most studied, is the two-sided matching or marriage problem made famous by Gale and Shapley (1962); the other category is the one-sided matching problem. In two-sided matching problems, members of one group are to form partnerships with a member of another group and members of each group have preferences for members of the opposite group. A common example is the matching of sports teams and players. In one-sided matching problems, only one of the groups have preferences for members of the other group. An example of one-sided matching is the matching of office space and faculty members. Most of the research on matching problems has concentrated on the two-sided matching problem (Roth and Sotomayor (1989) provide an extensive review). The research on the one-sided matching problem has tended

[1] The marriage broker is not an outdated example; one only needs to look at the number of video matching services, which populate today's urban environment. The marriage broker also illustrates the distinction between paying money for matching services and paying money for the person or object being matched.

to propose rules and procedures and is generally found in the operations research literature.

In this paper, we examine the problem of assigning a fixed set of goods or services, which we will generically call slots, to a fixed set of agents. We consider the problem from the point of view of a planner who wishes to design a set of rules or procedures (usually called a mechanism), the outcomes of which satisfy certain criteria. We extend the literature that was begun by Gibbard (1973) and Satterthwaite (1975). Gibbard and Satterthwaite independently proved that a mechanism is manipulable if it is nondictatorial. A planner is interested in non-manipulable rules since many rules require knowledge of individual preferences or information. These preferences are privately known and it may not be in the best interest of individuals to truthfully reveal their private information (or preferences) to the planner. That is, agents may be able to manipulate the outcome by misrepresenting their preferences. Manipulation may result in unsatisfactory outcomes; so the planner would like to know what mechanisms are manipulable and to what extent.

The Gibbard-Satterthwaite theorem (hereafter G-S) establishes that a voting scheme (mechanism) must be either manipulable or dictatorial when all possible transitive orderings over the set of alternatives are allowed, and the set of alternatives is finite. Results by Barbera and Peleg (1990), Moreno and Walker (1990), Satterthwaite and Sonnenschein (1981), and Zhou (1990b) establish the Gibbard-Satterthwaite result when the domain of preferences is restricted. Barbera and Peleg show that a G-S result holds when preferences are required to be continuous, and the set of alternatives is a metric space (not necessarily a subset of $\Re^n$). Moreno and Walker add the restriction that some dimensions of the social decision do not affect all the participants. Zhou establishes a G-S-type result in economies with public goods when the set of admissible preferences are continuous and convex. For domain restrictions that typically satisfy economic models Satterthwaite and Sonnenschein (1981) (hereafter SS) provide a result similar to the G-S result. The SS environment requires a condition called *nonbossy* for a serial dictatorship to hold. The results of these papers indicate that there may not exist "satisfactory" mechanisms without the use of monetary transfers or other incentive tax schemes. Common incentive schemes that do not use money are waiting in line, inspection, and punishment.

Our environment differs from the previous environments in two ways: 1) by the restriction on the allocation space, and 2) by the restriction on the domain of preferences. In the problem I pose, allocations that a planner may make are constrained by two requirements: a feasibility constraint (at most one slot is assigned to each agent), and the institutional requirement (incentive taxes such as "money" or waiting in line cannot be used to allocate slots, and lotteries over the alternatives are not allowed). This problem will be referred to as the one-sided matching problem (also known as the assignment problem).[2]

The domain of preferences is restricted by the assumption that individuals are selfish and that their utility does not depend on the slots allocated to others. This restricted domain of preferences does not satisfy the conditions for the proof of the G-S result. For instance, Barbera and Peleg's proof of the Gibbard-Satterthwaite theorem requires that preferences having a single best alternative are not excluded from the domain; in our environment there are no preferences

[2] For a more detailed discussion of the assignment problem, see Chapter 8 of Roth and Sotomayor (1989), or Shapley and Shubik (1972).

that have a single best alternative and individuals are indifferent over large sets of outcomes. The matching environment differs from the environment constructed by Moreno and Walker, since allocations to one individual can affect another individual (*e.g.*, if person 1 is assigned slot A, then 2 cannot be assigned this slot).

In the environment discussed in this paper, there are two ways an agent can behave "strategically." The first is by manipulating the outcome of a social choice function and the second is by corrupting the outcome of a social choice function. A social choice function is manipulable if an agent can improve the outcome for himself by misrepresenting his true preferences, while a social choice function is corruptible if an agent can change the outcome to another agent without changing the outcome for himself. The ability of an agent to manipulate an outcome has been widely discussed and is the main condition that restricts the outcomes of the social choice functions of the papers cited above, while the ability of an agent to corrupt an outcome has received scant attention. Noncorruptible SCCs have been discussed by Ritz (1985) and Satterthwaite and Sonnenschein (1981), who call a noncorruptible mechanism nonbossy.[3]

The rules that have been proposed in the literature to match agents to goods can be classified by the message space and the mechanism (procedure, allocation rule, or algorithm) used to make the match. There are two basic types of message spaces: ordinal ranking and cardinal ranking. In a mechanism that uses an ordinal ranking message space, individuals are asked to submit a preference ranking over slots (*e.g.,* I like 1 better than 2, and 2 better than 3). In a mechanism that uses a cardinal ranking message space, individuals are

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[3] Ritz's definition of corruptible is more general than the SS definition of bossy; Ritz defines corruptible for choice correspondences; SS define bossy for direct mechanisms.

requested to choose from a subset $C \subset \Re^k_+$ (*e.g.*, 100 points must be divided among the different slots).

I classify the procedures found in the literature into three categories based on their operational characteristics—positional, chit, and choice.[4] Positional mechanisms are mechanisms wherein the message space is an agent's ordinal ranking over slots. Chit mechanisms have a cardinal message space. Choice mechanisms allow the individuals to choose from an available set of slots.

Positional mechanisms are discussed first. In a positional mechanism each agent submits a ranking over slots. The planner (or central coordinator) places a numerical value on each ranking and then determines the outcome by maximizing (or minimizing) an objective function defined on the numerical values. Various objective functions are possible. For instance, agents submit rankings[5] $r^i = (r_{i1}, \ldots, r_{ik})$, the most preferred good getting the highest number and so on, whereas the r's are chosen from a set of k specific weights $W = \{w_1, \ldots, w_k\}$. If $W = \{1, \ldots, k\}$, then this is similar to a Borda count. Given a submission of ranks, the assignment is determined by the allocation that maximizes the sum of weights. Gardenfors (1973) shows that assignments generated in this fashion satisfy conditions of neutrality, symmetry, unanimity, monotonicity, and Pareto optimality (see Gardenfors for definitions). In section 9 we show that for this class of mechanisms agents have incentives to misrepresent their ordinal rankings over slots.

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4 This is not an exhaustive classification of all possible mechanisms, only of those commonly found in the literature.

5 For some situations, when the number of slots is large, asking agents to submit rankings over all slots is impractical. Wilkonson (1972) suggests a solution where unranked slots are given a ranking one lower than the lowest ranked slot.

A second type of assignment is determined by the allocation that minimizes the worst case (*e.g.,* an assignment is made to make the agent with the lowest ranked slot as high as possible). This procedure was described by Proll (1972) and Wilson (1977). Hylland and Zeckhauser (1979) report on a similar procedure used by Harvard administrators in 1977 to assign students to housing. The administrators first assigned students to their first choice if possible; they then assigned the remaining students to their second choice, and so on (this is called a bottleneck procedure). Hylland and Zeckhauser (1979) report that the Harvard administrators believed they observed students acting strategically. If students believed their first choice was first among many others, they might list their less popular second choice as first.[6]

A second class of mechanisms includes chit mechanisms. A chit mechanism is one wherein the message space allows each person to allocate a certain number of points (or chits) to any of the items which he wishes. The only difference between chit and positional mechanisms is the message space. Operationally, chit mechanisms use "funny money" (sometimes called chits) instead of money (exchangeable currency) as the medium of exchange. A ***chit*** is a medium of exchange whose value is determined solely in the context of the given assignment problem (environment), and has no value for goods or services outside the assignment problem. An example of a chit mechanism is the implicit market mechanism of Hylland and Zeckhauser (1979).

A third class of mechanisms includes choice mechanisms. They are similar to positional mechanisms, since they only require an individual's rankings over slots. I consider them separately because they can be implemented by

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[6] See Hylland and Zeckhauser (1979), page 255, note 6.

procedures that require individuals to choose slots from an available set, so the entire ranking does not need to be obtained. Examples of choice mechanisms are the deferred-acceptance procedure and serial-dictatorship mechanisms. The deferred-acceptance mechanism is based on the Gale-Shapley algorithm used to solve the marriage problem.[7] For the serial dictatorship an ordering of agents is chosen, the first agent chooses her slot, the second agent chooses her slot of those remaining, and so on. These mechanisms will be discussed in more detail in a later section.

Except for the choice mechanisms, none of the literature on matching mechanisms explicitly considers incentive problems.[8] The only result in the matching environment is by Zhou (1990a) who proved that when the number of agents is greater than 3, there exists no mechanism that satisfies symmetry, *ex ante* Pareto optimality, and strategy-proofness. The results presented in this paper characterize the class of social choice rules that can be implemented when nonstrategic behavior (behavior that is both nonmanipulable and noncorruptible) is a condition; the properties of these rules are then discussed.

In this paper we present four basic results: 1) nonstrategic rules must be ordinal; that is, an individual's assignment from the social choice function does not change when his ordinal preferences do not change; 2) nonstrategic social choice functions must be choice mechanisms; 3) the allocation space is rich (in the sense of Dasgupta, Hammond, and Maskin (1979), hereafter DHM), and hence a social choice function is implementable in dominant strategies if and only if it is implementable in Nash strategies; 4) a subclass of nonstrategic social

[7] See Roth and Sotomayor (1989) and the classic reference Gale and Shapley (1962).

[8] Hylland and Zeckhauser made the assumption that when there are many agents, each agent's contribution is small and hence there is no incentive to be dishonest.

choice functions called serial dictators are Pareto optimal.

For the classes of mechanisms that we described above, the results imply that the only mechanisms that are implementable in dominant and Nash strategies are choice mechanisms that rely only on ordinal rankings. The class of mechanisms we call chits are not implementable since they rely on cardinal information.

This paper is divided into sections. In Section 2, we describe the formal model. In Section 3, strategic behavior is described. In Section 4, we show the equivalence of various notions of implementability for our environment. In Section 5, necessary conditions for dominant-strategy implementation are presented. In Section 6, the serial dictator is described and shown to characterize the class of nonstrategic rules. In Section 7, Nash implementation is presented. In Section 8, the optimality of implementable rules is presented. In Section 9, we discuss some of the results in the context of the categories of rules presented in the introduction. Finally, in Section 10, we make some concluding remarks.

2. Formal description of the model

The environment

The environment consists of n agents and k goods or services to be allocated, which we call slots. Let $N = \{1,\ldots,n\}$ index the set of agents, and let $K = \{1,\ldots,k\}$ index the set of slots. It is assumed that both N and K are finite and nonempty. Let $\mathcal{A}$ be the set of feasible, deterministic allocations of K to N, including the zero allocation where no agent receives a slot.

The set $\mathcal{A}$ consists of allocations wherein agents either receive or do not receive a slot. An allocation in $\mathcal{A}$ can be denoted by a feasible allocation matrix of zeros and ones. That is, $a \in \mathcal{A}$ is an $n \times k$ matrix consisting of at most a single 1 in each row and column, where an element $a_{ij} = 1$ if agent i is assigned slot j, and $a_{ij} = 0$, if he is not. We also define $a^i = (a_{i1},\ldots,a_{ik})$.

The elements of $\mathcal{A}$ must be feasible (*i.e.,* at most, one slot may be assigned to each agent). We provide the following definition:

DEFINITION 2.0. An allocation $x \in \mathcal{A}$ is ***weakly feasible (WF)*** if $\sum_j x_{ij} \leq 1$ $\forall i \in N$, $\sum_i x_{ij} \leq 1$ $\forall j \in K$, $\sum_i \sum_j x_{ij} \leq \min(k,n)$, and $x_{ij} \geq 0$. If $x \in \mathcal{A}$, then $x_{ij} \in \{0,1\}$. If $n = k$, this definition reduces to the requirement that $\sum_j x_{ij} \leq 1$, $\forall i \in N$, $\sum_i x_{ij} \leq 1$, $\forall j \in K$, and $x_{ij} \geq 0$.

Efficiency and monotonicity of preferences will imply that either all slots are allocated <u>or</u> every agent is allocated a slot. The following definition is used:

DEFINITION 2.1. An allocation $x \in \mathcal{A}$ is ***strictly feasible (SF)*** if $\sum_j x_{ij} \in \{0,1\}$ $\forall i \in N$, $\sum_i x_{ij} \in \{0,1\}$ $\forall j \in K$, $\sum_i \sum_j x_{ij} = \min(k,n)$, and $x_{ij} \geq 0$. If $x \in \mathcal{A}$, then $x_{ij} \in \{0,1\}$. If $n = k$, this definition reduces to the requirement that $\sum_j x_{ij} = 1$, $\forall i \in N$, $\sum_i x_{ij} = 1$ $\forall j \in K$, and $x_{ij} \geq 0$.

The preferences of each agent depend upon the slot allocated and the agent's type. An agent's type parameterizes the value he places on the goods being allocated. Let $\Theta^i \subset \Re^k$ be a set of possible types for agent i, $\forall i \in N$. Let $\Theta^N = \mathop{\times}_{i \in N} \Theta^i$. A $\theta \in \Theta^N$ will be called a profile. The number of agents and slots is fixed, so the feasible set is independent of the profile. Each agent i, of type θ^i, evaluates each outcome $x \in \mathcal{A}$ through a valuation function $U(x,\theta^i) = \sum_j x_{ij}\theta^i_j$. The quantity $U(x,\theta^i)$ represents the willingness to pay of agent i of type θ^i for outcome x.

Agents may be indifferent between distinct outcomes since they are selfish; that is, they care only about the slots allocated to them. When the outcome space is $\mathcal{A}$, and agents are selfish, there is no loss of generality in the linear description of utility since there are a finite number of slots. That is, when agents are selfish and the outcome space is $\mathcal{A}$, then for any utility function $\hat{U}(x)$, there is a θ^i such that $U(x,\theta^i) = \hat{U}(x)$.

Some of our results require that agents not be indifferent between slots. The following definition is used:

DEFINITION 2.2. A preference domain $\langle U, \Theta^N \rangle$ satisfies ***strict individual preferences (SIP)*** if $\forall i \in N$, $\forall \theta^i \in \Theta^i$, $U(m, \theta^i) \neq U(l, \theta^i)$ $\forall m \neq l \in K$; where $U(j, \theta^i) \equiv$ the utility to type θ^i of slot j. That is, each agent has strict preferences over slots. Given the definition of $U(\cdot)$, SIP holds if and only if $\theta^i_l \neq \theta^i_m$, $\forall m, l \in K$, $m \neq l$, $\forall i \in N$.

The planner

For every possible profile, the planner wishes to choose a single allocation from the set of feasible allocations; in addition, the planner requires these assignments to satisfy some criteria. That is, she wishes to implement a ***social choice function***[9] ***(SCF)***, $f: \Theta^N \mapsto \mathcal{A}$, that selects an outcome in $\mathcal{A}$ for every profile in Θ^N. Alternatively, we can describe a ***social choice correspondence (SCC)***, $f: \Theta^N \mapsto \mathcal{P}(\mathcal{A})$, which selects a nonempty subset of $\mathcal{A}$ for every profile in Θ^N, where $\mathcal{P}(\mathcal{A})$ denotes the power set of $\mathcal{A}$. A SCF is a single-valued SCC.

Given a SCF, the planner must then choose a procedure or device to obtain these allocations. For example, she may ask agents to place numerical values between 0 and 1 on each slot and then make the assignment that maximizes the sum of the valuations.[10] These procedures contain two parts, a message space and an outcome rule. The combination of message space and outcome rule is a game form, also called a mechanism. The planner chooses a mechanism to "implement" her choice of social choice rule. A SCF is implementable if there

[9] The literature often interchanges and confuses the terms social choice function, voting scheme, and mechanism. In this paper the term social choice function is used to describe the type of outcome or allocation that the planner may wish to obtain; a mechanism or voting scheme is a procedure or device to obtain allocations.

[10] This is the well-known assignment problem. We will discuss its incentive problems in a later section.

exists a game form (message space and outcome rule) such that the equilibria (under some appropriate solution concept) of this game corresponds to the outcomes of the social choice function.

We make two assumptions, as described by Palfrey (1990), about the planner's ability to implement a social choice rule with a mechanism: 1) the commitment assumption: The planner may commit to any feasible outcome rule, and he is committed to his choice; 2) the control assumption: The planner may choose any message space and the agents must communicate exactly one message from this message space and may not communicate with each other. We remark that if two different mechanisms can fully implement a SCC, each agent and the planner are indifferent between them.

Solution Concepts

There are a number of solution concepts that can be applied; in this paper we will be concerned with two solution concepts: dominant-strategy implementation and Nash implementation (Dasgupta, Hammond and Maskin (1979), and Maskin (1986) discuss these solution concepts in detail). The resulting mechanisms can be significantly different under the two solution concepts (*e.g.,* the divide-the-cake problem). Other solution concepts are virtual implementation (Abreu and Sen (1987)), and Bayesian implementation (Palfrey (1990)).

Dominant-strategy and Bayesian implementation are solution concepts that are consistent with the assumption of incomplete information, while Nash and virtual implementation require complete information (for the agents but not the planner). Dominant-strategy implementation is more robust than Bayesian implementation since it is prior-independent. Bayesian implementation requires the stronger condition that the information structure is common knowledge, the designer knows the common prior, and Bayesian rationality among all players is common knowledge. Dominant-strategy implementation tends to be more stable than Nash implementation since there are fewer equilibria, but there are fewer instances where dominant-strategy implementation is possible. All the solution concepts have multiplicity problems under certain conditions, but the problem is least difficult under dominant-strategy implementation (See Mookerhjee and Reichelstein (1989), and Ledyard (1986)).

More formally: let (g, S) denote a mechanism (or game form), where $g: S \mapsto \mathcal{A}$, $S = (S^1, \ldots, S^n)$ and S^i is the strategy space (or message space) for agent $i \in N$. Let $E_g: \Theta^N \mapsto S$ be an equilibrium correspondence.

A ***dominant-strategy equilibrium*** for profile $\theta \in \Theta^N$ of a mechanism (g, S) is an n-tuple of strategies $\tilde{s} = (\tilde{s}^1, \ldots, \tilde{s}^n) \in S$ such that $(\forall i \in N)(\forall s \in S)$ $(U(g(\tilde{s}^i, s^{-i}), \theta^i) \geq (g(s), \theta^i))$. Let $DE_g(\theta) \subset S$ be the dominant-strategy equilibria for profile θ of mechanism (g, S). A ***Nash equilibrium*** for profile $\theta \in \Theta^N$ of a mechanism (g, S) is an n-tuple of strategies $\tilde{s} = (\tilde{s}^1, \ldots, \tilde{s}^n) \in S$ such that $(\forall i \in N)(\forall s^i \in S^i)(U(g(\tilde{s}), \theta^i) \geq (g(s^i, \tilde{s}^{-i}), \theta^i))$. Let $NE_g(\theta) \subset S$ be the Nash equilibria for profile θ of mechanism (g, S).

There are a number of different notions of implementation. The strongest is full implementation, which is applied to general game forms. A weaker concept is truthful implementation, which is defined for direct mechanisms. A direct mechanism is a mechanism in which the strategy space S^i for each agent $i \in N$ is the set of possible types Θ^i. In direct mechanisms, agents report their types (not necessarily their true types) to the planner, and then the planner makes an assignment based on these reported types.

Implementation in dominant strategies and implementation in Nash strategies are now defined.

DEFINITION 2.3. A SCF $f: \Theta^N \mapsto \mathcal{A}$ is ***implementable in Nash strategies*** if $(\forall \theta \in \Theta^N)(\forall a \in f(\theta)) \exists (g, S)[g(NE_g(\theta)) \subset f(\theta)$ and $a \in g(NE_g(\theta))]$.

DEFINITION 2.4. A SCF $f: \Theta^N \mapsto \mathcal{A}$ is ***fully implementable in dominant strategies*** if there is a mechanism (g, S) such that $(\forall \theta \in \Theta^N)$ $[g(DE_g(\theta)) = f(\theta)]$.

DEFINITION 2.5. A SCF $f: \Theta^N \mapsto \mathcal{A}$ is ***truthfully implementable in dominant strategies*** if there is a direct mechanism $g: \Theta^N \mapsto \mathcal{A}$ such that $(\forall \theta \in \Theta^N)$ $[(\forall i \in N)(\forall \hat{\theta}^i \in \Theta^i)(U(g(\theta), \theta^i) \geq U(g(\hat{\theta}^i, \theta^{-i}), \theta^i)$) and $g(\theta) \in f(\theta)]$.

The simplest and most direct means of implementing a SCF is to ask agents to report their type, then calculate from this information the assignment using the SCF. This is a particular type of direct mechanism, where the outcome rule is the SCF to be implemented. This notion of implementation was used in the G-S theorem and the other results mentioned in the introduction.

3. *Strategic behavior*

In this section we simplify the notion of implementation and use the concept of strategy-proof SCFs. A strategy-proof SCF truthfully implements itself in dominant strategies. In the next section I show that this is not a restriction in this environment if we are interested in full implementation. A SCF $f:\Theta^N \mapsto \mathcal{A}$ is ***strategy-proof*** if

$$(\forall\theta \in \Theta^N)(\forall i \in N)(\forall\hat{\theta}^i \in \Theta^i)\ [U(f(\theta),\theta^i) \geq U(f(\hat{\theta}^i,\theta^{-i}),\theta^i)].$$

A SCF $f:\Theta^N \mapsto \mathcal{A}$ is ***manipulable*** if for some $i \in N$, $(\exists\,\theta \in \Theta^N)(\exists\,\hat{\theta}^i \in \Theta^i)$ such that $U(f(\hat{\theta}^i,\theta^{-i}),\theta^i) > U(f(\theta),\theta^i)$. In this case we say i manipulates f at θ with $\hat{\theta}^i$. If a SCF is strategy-proof, then an agent is not able to improve his allocation (be assigned a more preferred slot) by lying about his type to the planner. This restriction reduces the strategic options to the agent and hence the possible allocations. This form of strategic behavior has been studied quite extensively (Muller and Satterthwaite (1986) provide a good review).

A second form of strategic behavior, which has received very little attention[11], is the ability of an agent to change another agent's allocation without changing his own. Mechanisms with this property are labeled ***bossy***, by Satterthwaite and Sonnenschein (1981), and labeled ***corruptible*** by Ritz (1985). A rule is bossy (or corruptible) if an agent can maintain her allocation at the same time she causes changes in the allocations that other agents receive. Satterthwaite and Sonnenschein do not consider whether "nonbossiness is a reasonable or desirable condition to require of a mechanism". Although we also require a mechanism to be nonbossy, in our environment it is a reasonable requirement.

[11] I have found only two references in the literature to this concept, Satterthwaite and Sonnenschein (1981) and Ritz (1985).

In the context of this paper we refer to SCCs as being corruptible and to mechanisms as being bossy. We define these concepts formally:[12]

DEFINITION 3.1. The SCC $f\colon\Theta^N \mapsto \mathcal{A}$ is ***noncorruptible (NC)*** if $(\forall\theta\in\Theta^N)$ $(\forall i,j\in N)$ $(\forall\tilde{\theta}^j\in\Theta^j)$ $[f^j(\theta)=f^j(\theta^{-j},\tilde{\theta}^j)\Rightarrow f^i(\theta)=f^i(\theta^{-j},\tilde{\theta}^j)]$. The SCC $f\colon\Theta^N \mapsto \mathcal{A}$ is ***corruptible*** if $(\exists\,\theta\in\Theta^N)(\exists\, i,j\in N)(\exists\,\hat{\theta}^j\in\Theta^j)[f^j(\theta)=f^j(\theta^{-j},\hat{\theta}^j)$ $\Rightarrow f^i(\theta)\neq f^i(\theta^{-j},\hat{\theta}^j)]$.

DEFINITION 3.2. The mechanism (g,S) $g\colon S\mapsto\mathcal{A}$ is ***bossy*** if $(\exists\, s\in S)$ $(\exists\, i,j\in N$ and a $\hat{s}^j\in S^j)[g^j(s)=g^j(s^{-j},\hat{s}^j)\Rightarrow g^i(s^{-j},\hat{s}^j)\neq g^i(s^{-j},\hat{s}^j)]$. The mechanism (g,S) $g\colon S\mapsto\mathcal{A}$ is ***nonbossy*** if it is not bossy. An agent $i\in N$ is said to be bossy if she can change the outcome for some agent $j\in N$.

We combine the two notions of strategic behavior and say that a SCF is ***nonstrategic*** if it is both noncorruptible and strategy-proof. As an example to see the existence of noncorruptible, strategy-proof mechanisms, observe the following example:

EXAMPLE 3.3. Let $n=k=3$, and let $\Theta^i=\{\mathrm{A,B,C,D,E,F}\}$ $\forall i\in N$. Define the allocation rule:

$f^1(C,\theta^2,\theta^3)=1,\ f^2(C,\theta^2,\theta^3)=2,\ f^3(C,\theta^2,\theta^3)=3;$

$f^1(D,\theta^2,\theta^3)=1,\ f^2(D,\theta^2,\theta^3)=3,\ f^3(D,\theta^2,\theta^3)=2,\ \ \forall\theta^2,\theta^3,$

where $f^i()=j$ denotes that agent i receives slot j. Agent 1 is bossy, since by

[12] The definitions of noncorruptible and nonbossy vary slightly from SS and Ritz but are consistent with their usage in the matching environment.

changing his type from C to D, he changes the allocation to agents 2 and 3 but not to himself. The mechanism is strategy-proof for agent 1 since he always receives slot 1 (we make the assumption that when an agent is indifferent between being truthful and misrepresenting his type, he will be truthful), and for agents 2 and 3, since they cannot affect the outcome of the mechanism. Observe that the conditions of noncorruptibility and strategy-proofness are true for whatever meaning we give to the types θ^i as long as the individuals are selfish.

When a mechanism is strategy-proof and nonbossy, then an agent cannot improve his position directly by manipulating the outcome. But when a mechanism is strategy-proof and bossy, an agent may be able to improve his position indirectly by taking a "bribe" from the other agents. In the example above, agent 1 may be able to induce either agent 2 or 3 to pay him to choose in their favor.

If a SCF is corruptible, the planner's problem of predicting outcomes becomes more difficult. The planner must model (try to predict) the behavior of agents who may be able to bribe or coerce another agent. If a SCF is noncorruptible, then the task of predicting behavior, and hence the the outcome of a mechanism, is simpler.

4. *Implementation*

It is well known that if a SCC is implementable, then it is truthfully implementable; that is, there exists a direct mechanism such that truth-telling is an equilibrium. The converse is not always true. However, in this section, we will show that in the one-sided matching environment, if a SCC is truthfully implementable and noncorruptible, then it is fully implementable in dominant strategies. This equivalence allows us to restrict the planner's matching problem to direct mechanisms. An additional result is that a nonstrategic, fully implementable SCC must be single-valued, so we can restrict our attention to SCFs. We also show that full implementation is equivalent to strategy-proofness, when SCFs are noncorruptible, which allows us to restrict the problem even further to self-implementable SCCs.

LEMMA 4.1. If preferences satisfy SIP, then if a SCC is noncorruptible and fully implementable in dominant strategies, it is single-valued.

Proof. Suppose $f: \Theta^N \mapsto \mathcal{A}$ is not single-valued; then for $\theta \in \Theta^N$, let $a, b \in f(\theta)$, $a \neq b$. Let (g, S) fully implement f in dominant strategies, $\exists\, \tilde{s}, s \in S(\theta)$, such that $g(\tilde{s}) = b$, and $g(s) = a$. $\tilde{s}^1$ and s^1 are both dominant strategies for θ^1, so $U(g(\tilde{s}^1, s^{-1}), \theta^1) = U(g(s^1, s^{-1}), \theta^1)$. SIP implies that $g^1(\tilde{s}^1, s^{-1}) = g^1(s^1, s^{-1}) = a^1$; noncorruptibility implies that $g^i(\tilde{s}^1, s^{-1}) = g^i(s^1, s^{-1}) = a^i$, $\forall i \in N$; hence $g(\tilde{s}^1, s^{-1}) = g(s^1, s^{-1}) = a$. Similarly, $g(\tilde{s}^1, \tilde{s}^2, s^{-1,2}) = g(\tilde{s}^1, s^{-1}) = a$. Continuing iteratively, $g(\tilde{s}) = a$, which is a contradiction, so f is single-valued.

□.

PROPOSITION 4.2. If preferences satisfy SIP, then if a SCF is noncorruptible and truthfully implementable in dominant strategies, it is fully implementable in dominant strategies.

Proof. Let $f: \Theta^N \mapsto \mathcal{A}$ be truthfully implemented in dominant strategies by $g^*: \Theta^N \mapsto \mathcal{A}$. Define $f^*: \Theta^N \mapsto \mathcal{A}$ so that $\forall \theta \in \Theta^N$, $f^*(\theta) = g^*(E_{g^*}(\theta))$, where $E_{g^*}(\theta)$ is the set of dominant-strategy equilibria for preference profile θ in game form g^*. Now by construction, g^* fully implements f^*. Therefore, $f^*(\theta)$ must be a singleton $\forall \theta \in \Theta^N$. But $g^*(\theta) \in f(\theta)$; therefore, $f^*(\theta) \subseteq f(\theta), \forall \theta \in \Theta^N$. Since f is single-valued, $f^*(\theta) = f(\theta), \forall \theta \in \Theta^N$, so g^* fully implements f.

□.

We apply two well-known results. 1) If a SCF is strategy-proof, then it truthfully implements itself in dominant strategies. 2) If a SCF is fully implementable in dominant strategies, then it is truthfully implementable in dominant strategies.

PROPOSITION 4.3. If preferences satisfy SIP, and if a SCF is noncorruptible and strategy-proof then, it is fully implementable in dominant strategies.

Proof. Let $f: \Theta^N \mapsto \mathcal{A}$ be strategy-proof and noncorruptible. Since f is strategy-proof, it truthfully implements itself in dominant strategies. Since f is also noncorruptible, then by the previous lemma it is fully implementable in dominant strategies.

□.

PROPOSITION 4.4. If preferences satisfy SIP, and if a SCF is truthfully implementable in dominant strategies and noncorruptible, then it is strategy-proof.

Proof. Let $f: \Theta^N \mapsto \mathcal{A}$ be truthfully implementable in dominant strategies and noncorruptible. Since f is truthfully implementable, $\exists\, g: \Theta^N \mapsto \mathcal{A}$ such that $\forall \theta \in \Theta^N$, $\forall i \in N$, $\forall \hat{\theta}^i \in \Theta^i$, $U(g(\theta), \theta^i) > U(g(\hat{\theta}^i, \theta^{-i}), \theta^i)$, $g(\theta) \in f(\theta)$.

Since f is truthfully implementable and noncorruptible, it is fully implementable and hence it is single-valued, so $g(\theta) = f(\theta)$, $\forall \theta \in \Theta^N$. Therefore, $U(f(\theta), \theta^i) \geq U(f(\hat{\theta}^i, \theta^{-i}), \theta^i)$, so f is strategy-proof. □.

THEOREM 4.5. If preferences satisfy SIP, and if a SCF is noncorruptible, then strategy-proofness, truthful implementation, and full implementation in dominant strategies are all equivalent.

Proof: Apply the previous propositions. □.

In this section we have shown that there is no loss of generality by focussing on SCFs rather than on SCCs and on strategy-proofness rather than on full implementation in dominant strategies. The ability to reduce the class of feasible mechanisms (or implementable SCCs) represents a significant reduction in the complexity of the problem. This reduction has been achieved by making two assumptions. The first assumption is a restriction on the domain of preferences; we assume that individuals have strict preferences over slots. The second assumption is a restriction on the strategic behavior that is allowed; we enforce the requirement that SCFs are noncorruptible.

5. Dominant-strategy implementation (ordinality)

In this section we describe a necessary condition for implementation in dominant strategies. We first investigate a necessary condition in our environment for a SCF to be truthfully implementable in dominant strategies. From DHM we know that a SCF is truthfully implementable in dominant strategies if and only if it is satisfies independent person-by-person monotonicity (IPM) (Maskin (1986), and DHM (1979)). Some preliminary definitions are first provided.

DEFINITION[13] 5.1. A SCF $f: \Theta^N \mapsto \mathcal{A}$ satisfies ***independent person-by-person monotonicity (IPM)*** if $\forall \theta \in \Theta^N$, $\forall i \in N$, $\forall \overline{\theta}^i \in \Theta^i$, and $\forall \{a, \mathrm{b}\} \subseteq \mathcal{A}$ such that $a \in f(\theta)$, and $U(a, \overline{\theta}^i) > U(b, \overline{\theta}^i)$, it must be that $b \notin f(\theta^{-i}, \overline{\theta}^i)$.

DEFINITION 5.2. A ***rank function*** is a function $r(\theta^i) = (r_1(\theta^i), \ldots, r_k(\theta^i))$, $r: \Theta^i \mapsto \prod\{r_1, r_2, \ldots, \mathrm{r}_k\}$, such that $r_j(\theta^i) > r_l(\theta^i)$ if and only if $U(j, \theta^i) > U(l, \theta^i)$; where $U(j, \theta^i) \equiv$ the utility to type θ^i of slot j, and $\prod\{A\}$ is the set of all permutations of the set A.

DEFINITION 5.3. A SCF is ***ordinal*** if $\forall \theta \in \Theta^N$, $\forall i \in N$, $\forall \overline{\theta}^i \in \Theta^i$, such that $r(\theta^i) = r(\overline{\theta}^i)$, then $f(\theta) = f(\theta^{-i}, \overline{\theta}^i)$.

DEFINITION 5.4. A SCF is ***individually ordinal*** if $\forall \theta \in \Theta^N$, $\forall i \in N$, $\forall \overline{\theta}^i \in \Theta^i$, such that $r(\theta^i) = r(\overline{\theta}^i)$, then $f^i(\theta) = f^i(\theta^{-i}, \overline{\theta}^i)$, where f^i is the allocation to agent i.

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[13] This is not the definition of IPM that appears in DHM (1979), but it is the definition given in Laffont and Maskin (1982), and Maskin (1986).

A SCF is individually ordinal if an agent's assignment does not change when his ordinal preferences do not change. An SCF is ordinal if the group assignment does not change when an agent's ordinal preferences do not change. If a SCF is ordinal, then it is individually ordinal, but if a SCF is individually ordinal, it is not necessarily ordinal. If a SCF is individually ordinal but not ordinal, then an individual agent can affect the allocation of another agent without changing his own, so an ordinal SCF is noncorruptible. We now show this observation formally:

LEMMA 5.5. If a SCF is noncorruptible and individually ordinal, then it is ordinal.

Proof. Let f satisfy the hypothesis. Let $i \in N$, $\theta \in \Theta^N$, $\hat{\theta}^i \in \Theta^i$, and $r(\theta^i) = r(\hat{\theta}^i)$. Since f is individually ordinal, $f^i(\theta) = f^i(\theta^{-i}, \hat{\theta}^i)$. Since f is noncorruptible, $f^j(\theta) = f^j(\theta^{-i}, \hat{\theta}^i)\ \forall j \in N, j \neq i$. Therefore, $f(\theta) = f(\theta^{-i}, \hat{\theta}^i)$ and f is ordinal.
□.

To prove our result we show that if a SCF satisfies IPM, then it is individually ordinal. Our first result relies on the following lemma.

LEMMA 5.6. If preferences satisfy SIP, and if a SCF $f\colon \Theta^N \mapsto \mathcal{A}$ satisfies IPM, $a \in f(\theta)$, $c \in f(\overline{\theta}^i, \theta^{-i})$, and $a^i \neq c^i$, then $U(a, \overline{\theta}^i) < U(c, \overline{\theta}^i)$ and $U(c, \theta^i) < U(a, \theta^i)$.
(remark: if $a^i = c^i$, then equality holds).

Proof. Suppose our hypothesis holds; then

1) If $a \in f(\theta)$ and $U(a, \overline{\theta}^i) > U(c, \overline{\theta}^i)$, then IPM implies $c \notin f(\overline{\theta}^i, \theta^{-i})$.

2) If $c \in f(\overline{\theta}^i, \theta^{-i})$ and $U(c, \theta^i) > U(a, \theta^i)$, then IPM implies $a \notin f(\theta^i, \theta^{-i})$.

So 1) and 2) cannot both hold. This implies:

$$U(a, \overline{\theta}^i) \leq U(c, \overline{\theta}^i) \text{ and } U(c, \theta^i) \leq U(a, \theta^i). \tag{5.1}$$

But by individual strict preferences and $c^i \neq a^i$, it must be that $U(a, \overline{\theta}^i) < U(c, \overline{\theta}^i)$ and $U(c, \theta^i) < U(a, \theta^i)$.

□.

In the previous lemma if preferences do not satisfy SIP then $U(a, \overline{\theta}^i) \leq U(c, \overline{\theta}^i)$ and $U(c, \theta^i) \leq U(a, \theta^i)$.

The next two propositions establish that an ordinal condition is necessary for implementation in dominant strategies when the allocation space is $\mathcal{A}$.

PROPOSITION 5.7. If preferences satisfy SIP, $\mathcal{A}$ is the allocation space, and if a SCF satisfies IPM, then it is individually ordinal.

Proof. Let $f: \Theta^N \mapsto \mathcal{A}$ be a SCF. Suppose that the hypothesis holds but that f is not individually ordinal. Then for some $i \in N$, $\theta^i, \overline{\theta}^i \in \Theta^i$, where $r(\theta^i) = r(\overline{\theta}^i)$, there exists $a, b \in \mathcal{A}$ such that $a \in f(\theta)$, and $b \in f(\overline{\theta}^i, \theta^{-i})$. If $a^i \neq b^i$, then by lemma 5.6, $U(a, \overline{\theta}^i) < U(b, \overline{\theta}^i)$ and $U(b, \theta^i) < U(a, \theta^i)$. But this is a contradiction since θ^i, $\overline{\theta}^i$ have the same ordinal preferences over slots and a, b allocate a single slot to agent i. It must be that either a is preferred to b for both θ^i, $\overline{\theta}^i$, or b is preferred to a for both θ^i, $\overline{\theta}^i$.

□.

THEOREM 5.8. If preferences satisfy SIP, if $\mathcal{A}$ is the allocation space, and if a SCF can be truthfully implemented in dominant strategies, then the SCF is individually ordinal. In addition, if the SCF is noncorruptible then it is ordinal.

Proof. By DHM (theorem. 4.3.1), a SCF can be truthfully implemented in dominant strategies if and only if it is IPM. By applying the previous proposition, the first result follows.

For the second result, since the SCF is noncorruptible and individually ordinal, lemma 5.1 implies that it is ordinal.

□.

We have shown that an ordinal condition is necessary for IPM when preferences satisfy SIP, and when the allocation space is $\mathcal{A}$, but is it sufficient? The answer is no. We give an example of a SCF that is ordinal but cannot be truthfully implemented in dominant strategies.

EXAMPLE 5.9. Let $n = k = 3$, and $r{:}\Theta^i \mapsto \prod\{r_1, r_2, r_3\}$, be the rank function, where $r_1 = 1.0$, $r_2 = 0.5$, and $r_3 = 0$. Let $f(\theta) = \underset{x \in \mathcal{A}}{\text{argmax}} \sum_i \sum_j r_j(\theta^i) \cdot x_{ij}$; ties are resolved by giving the lower indexed agent his preferred slot, and x is strictly feasible. Let $U(x,\theta^i) = \sum_j x_{ij}\theta^i_j$, and $\theta^i = (\theta^i_1, \theta^i_2, \theta^i_3) \in \Re^3$.
Since f depends on θ only through the rank function r, f is ordinal.
We will show that truth is not a dominant strategy for some θ.
For our example we will define 2 types: type A, $\theta(\mathrm{A}) = (1.0, 0.5, 0)$ and type B $\theta(\mathrm{B}) = (0, 1.0, .5)$.
Two allocations of the SCF f are $f(\mathrm{A,A,B}) = (1,3,2)$, and $f(\mathrm{A,B,B}) = (1,2,3)$, where $f(i,j,k)$ is the allocation if agent 1 is type i, agent 2 is type j, and

agent 3 is type k, $f(\cdot) = (l, m, n)$ is the allocation of l to agent 1, m to agent 2, and n to agent 3.

When $\theta^{-2} = (1,3)$, a report of type A, by agent 2, gives him slot 3. A report of type B gives agent 2 slot 2. So when agent 2 is a type A, he is better off reporting a type B, which gives him his 2^{nd} ranked slot 2, instead of his 3^{rd} ranked slot 3.

□.

In summary, we have proven that if a SCF is truthfully implementable in dominant strategies, then it is individually ordinal, and if in addition, it is non-corruptible, then it is ordinal.

6. *Serial dictator*

In the previous sections we showed that if a social choice function is implementable in dominant strategies then it is ordinal and that there is no loss of generality in requiring strategy-proofness. Hence, we will restrict ourselves to finding strategy-proof ordinal mechanisms.

Satterthwaite and Sonnenschein (1981) established a Gibbard-Satterthwaite-type theorem in a classical economic environment (alternatives are a compact and convex subset of $\Re^l$). Their result relies on a differentiable allocation mechanism and a number of other technical conditions. The environment constructed for the assignment problem (without lotteries) lacks convexity, and we do not require a mechanism to be differentiable. Hence, the results of this paper do not fall into the class of environments established by Satterthwaite and Sonnenschein, although we obtain a similar result.

Satterthwaite and Sonnenschein (1981, p. 588) describe a serial dictatorship[14] as follows:

> Serial dictatorship means that the mechanism consists of one or more hierarchies of agents where the highest ranking agent in each hierarchy selects his allocation from a feasible set that is exogenously given, the second highest ranking agent selects his allocation from a feasible set that depends on the first agent's choice, the third highest ranking agent selects his allocation from a feasible set that depends on the first and second agents' choices, etc. Consequently, an agent who is high on a hierarchy is a dictator to those agents lower on that hierarchy in the sense that he can affect what is available to them to choose among and they can not affect him reciprocally. He is not, however, necessarily a dictator in the stronger senses of being able to choose any technologically feasible outcome for himself and being able to impose particular outcomes on the other agents.

[14] An early reference to a serial dictatorship is found in Luce and Raiffa (1957, p. 344), who observe that a serial dictatorship "is consistent with all of Arrow's conditions except nondictatorship".

The serial dictator is a member of the class of sequential-choice mechanisms. A sequential choice mechanism is a mechanism where there is an ordering (or hierarchy) of agents, which may depend on the profile. Given this ordering, each agent in turn is allowed to choose their slot from an option set, which is a nonempty set of slots presented to them. These option sets have the feature that each agent's decision affects only the option sets of those agents that are lower on the hierarchy. Associated with each serial dictator mechanism is an ordering $I(\theta) = \{i_1, \ldots, i_n\} \in \mathbb{P}(N)$, which may vary with θ, and where $\mathbb{P}(N)$ is the set of permutations of agents. This ordering indicates that the option set to agent i_1 is K, the option set to i_2 is a subset of K, and so on. That is, each agent has her turn to choose slots from a set of slots, whose elements are affected by those agents who are placed before her in the ordering.

The following definitions are used formally to define a sequential choice mechanism and the serial dictator:[15]

DEFINITION 6.1. For a given direct mechanism $x: \Theta^N \mapsto \mathcal{A}$, agent i ***affects*** agent j at $\theta \in \Theta^N$ if $(\exists \overline{\theta}^i \in \Theta^i) \ni x^j(\theta) \neq x^j(\overline{\theta}^i, \theta^{-i})$. We write this as $i A(\theta) j$.

DEFINITION 6.2. $o(i,\theta) = \{a \in \mathcal{A} \mid \exists \overline{\theta}^i \ni a = f(\theta^{-i}, \overline{\theta}^i)\}$.

Given $\theta^{-i} \in \Theta^{N-i}$, $o(i,\theta)$ is the set of agent i's options at profile θ that she can receive by deviating her messages. Note: $o(i,\theta)$ does not depend on θ^i (so $o(i,\theta) = o(i,\theta^{-i})$) and clearly $(\forall \theta \in \Theta^N)[f(\theta) \in o(i,\theta)]$.

[15] These definitions follow Satterthwaite and Sonnenschein (1981).

DEFINITION 6.3. For a given direct mechanism $x\colon \Theta^N \mapsto \mathcal{A}$, $A(\theta)$ is ***acyclic*** at $\theta \in \Theta^N$ if $\forall(i_1, i_2, \ldots, i_n) \in \mathbb{P}(N)$ $i_1 A(\theta) i_2, i_2 A(\theta) i_3, \ldots, i_{n-1} A(\theta) i_n \Rightarrow i_1 A(\theta) i_n$.

DEFINITION 6.4. A direct mechanism $x\colon \Theta^N \mapsto \mathcal{A}$ is a ***sequential choice mechanism*** if $\forall \theta \in \Theta^N$, $A(\theta)$ is acyclic.

DEFINITION 6.5. A direct mechanism $x\colon \Theta^N \mapsto \mathcal{A}$ is a ***serial dictator*** if $\forall \theta \in \Theta^N$, $A(\theta)$ is acyclic and $o(i_n, \theta) \subset o(i_{n-1}, \theta) \subset o(i_{n-2}, \theta) \subset \cdots o(i_1, \theta) \subset K$.

This definition of a serial dictator is broader than the description in the introduction. That description describes a ***simple serial dictator*** wherein an ordering $I = \{i_1, i_2, \ldots, i_n\}$ of agents is fixed and does not depend on the profile. Given this fixed ordering, agents choose their slots in turn: i_1 goes first, then i_2 chooses her slot from those remaining, and so on. The definition provided here (and also in SS) allows the ordering of agents to vary with $\theta \in \Theta^N$; we denote the ordering as $I(\theta)$.

We provide some further definitions:[16]

DEFINITION 6.6. For $\theta^i \in \Theta^i$, $x \in \mathcal{A}$, a $\hat{\theta}^i \in \Theta^i$ is a ***reshuffling of θ^i around x*** if $(\forall y \in \mathcal{A})[U(x, \theta^i) \geq U(y, \theta^i) \Leftrightarrow U(x, \hat{\theta}^i) \geq U(y, \hat{\theta}^i)]$. $r(\theta^i, x)$ denotes the set of all reshufflings of θ^i around x, and a $\hat{\theta}^i \in r(\theta^i, x)$ is a reshuffle of θ^i around x.

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[16] A number of concepts from Barbera (1983) are used; Barbera proved the Gibbard-Satterthwaite theorem by a pivotal-voter technique. A similar technique is used in this paper. A mechanism is ***pivotal*** at θ if $\exists \hat{\theta}^i \ni x(\theta^{-i}, \hat{\theta}^i) \neq x(\theta)$. Clearly, if i affects j at θ, then i is pivotal at θ.

A reshuffling of θ^i around x is another preference ordering such that x preserves the same ordinal rankings relative to all other alternatives. Observe: if SIP holds, $x, y \in \mathcal{A}$, and $\hat{\theta}^i \in r(\theta^i, x)$; then $[U(y,\theta^i) = U(x,\theta^i) \Leftrightarrow U(y,\hat{\theta}^i) = U(x,\hat{\theta}^i)]$. Some special preference relations will be used. Let ${}^x\theta^i$ be the preferences obtained from θ^i when x is ranked first, all other ordinal preferences remaining the same; and let ${}_x\theta^i$ be the preferences obtained from θ^i when x is ranked last, all other ordinal preferences remaining the same. Let ${}^x\Theta^i$ be the set of preferences that rank x first.

For a direct mechanism $f\colon \Theta^N \mapsto \mathcal{A}$, define $\mathcal{A}_f =$ range of f. If $Y \subseteq \mathcal{A}$, then Y^i is the set of slots $Y^i \subseteq K$ that are obtained by i in Y.

DEFINITION 6.7. For $Y \subseteq \mathcal{A}, \theta^i \in \Theta^i$, let

$$C(\theta^i, Y) = \{a \in Y \mid U(a,\theta^i) \geq U(b,\theta^i) \forall b \in Y\}$$

be the choice of agent θ^i in the set Y of allocations, and let

$$C^i(\theta^i, Y) = \{a^i \in Y^i \mid U(a,\theta^i) \geq U(b,\theta^i) \forall b \in Y\}$$

be the set of the best slots obtained by i in the set of allocations Y. If SIP holds, then $C^i(\theta^i, Y)$ is a singleton.

The main theorem is stated below:

THEOREM 1. If preferences satisfy SIP, $f\colon \Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible; then for all $\theta \in \Theta^N$, $A(\theta)$ is acyclic.

The result of Theorem 1. is that for each $\theta \in \Theta^N$, $A(\theta)$ is acyclic; this permits the hierarchies of serial dictators to vary as $\theta \in \Theta^N$. This is a similar result to SS. Before I prove the theorem I prove some lemmas that will be used in the proof.

LEMMA 0. For $f\colon \Theta^N \mapsto \mathcal{A}$, if f is noncorruptible and $(\forall \theta \in \Theta^N)(\forall i \in N)$ $(\forall \hat{\theta}^i \in \Theta^i)[f(\theta) \neq f(\theta^{-i}, \hat{\theta}^i) \Rightarrow f^i(\theta) \neq f^i(\theta^{-i}, \hat{\theta}^i)]$.

Proof. Suppose $f\colon \Theta^N \mapsto \mathcal{A}$ satisfies the hypothesis of the lemma. Then $f(\theta) \neq f(\theta^{-i}, \hat{\theta}^i)$ implies either a) $f^i(\theta) \neq f^i(\theta^{-i}, \hat{\theta}^i)$ or b) $f^j(\theta) \neq f^j(\theta^{-i}, \hat{\theta}^i)$ or both. If a) is true then the conclusion is true. If b) is true then noncorruptibility implies $f^i(\theta) \neq f^i(\theta^{-i}, \hat{\theta}^i)$ and the conclusion is true.

□.

LEMMA 1a. If preferences satisfy SIP and $f\colon \Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible, then $(\forall \theta \in \Theta^N)(\forall i \in N)[f(\theta) = C(o(i,\theta), \theta^i)]$.

Proof. Suppose not. Let $z = f(\theta)$ and $x \in C(o(i,\theta), \theta^i)$, $x \neq z$. By definition of $o(\cdot)$, $\exists \hat{\theta}^i \ni f(\hat{\theta}^i, \theta^{-i}) = x$ and $z = f(\theta) \in o(i, \theta)$, so $f(\hat{\theta}^i, \theta^{-i}) \neq f(\theta)$. Since f is noncorruptible, by lemma 0, it must be that $x^i \neq z^i$. Since $x, z \in o(i,\theta)$ and $x \in C(o(i,\theta), \theta^i)$, $\text{SIP} \Rightarrow U(x, \theta^i) > U(z, \theta^i) \Rightarrow U(f(\theta^{-i}, \hat{\theta}^i), \theta^i) > U(f(\theta), \theta^i)$. Hence f is manipulable at θ by i, a contradiction.

□.

The above lemma says that for every profile, the outcome must be the best option at that profile for each one of the agents. Without the noncorruptible condition the result of the lemma is $f(\theta) \in C(o(i,\theta), \theta^i)$, and if in addition SIP holds, then $f^i(\theta) = C^i(o(i,\theta), \theta^i)$.

LEMMA 1b. If preferences satisfy SIP, $f:\Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible, then $(\forall\theta \in \Theta^N)(\forall i \in N)[\hat{\theta}^i \in r(\theta^i, f(\theta)) \Rightarrow f(\theta^{-i}, \hat{\theta}^i) = f(\theta)]$.

Proof. Suppose not. Then for some $\theta \in \Theta^N$, $i \in N$, $\hat{\theta}^i \in r(\theta^i, f(\theta))$, $f(\theta^{-i}, \hat{\theta}^i) \neq f(\theta)$. Since f is noncorruptible, by lemma 0., it must be that $f^i(\theta^{-i}, \hat{\theta}^i) \neq f^i(\theta)$. By SIP, either

a) $U(f(\theta^{-i}, \hat{\theta}^i), \theta^i) > U(f(\theta), \theta^i)$, or

b) $U(f(\theta), \theta^i) > U(f(\theta^{-i}, \hat{\theta}^i), \theta^i)$.

By definition of $r()$ $\hat{\theta}^i \in r(\theta^i, f(\theta)) \Rightarrow [U(f(\theta), \theta^i) > U(y, \theta^i) \Leftrightarrow U(f(\theta), \hat{\theta}^i) > U(y, \hat{\theta}^i)$, $\forall y \in \mathcal{A},\ y^i \neq f^i(\theta)]$.

If a) is true, then a*) $U(f(\theta^{-i}, \hat{\theta}^i), \hat{\theta}^i) > U(f(\theta), \hat{\theta}^i)$.

If b) is true, then b*) $U(f(\theta), \hat{\theta}^i) > U(f(\theta^{-i}, \hat{\theta}^i), \hat{\theta}^i)$.

But a) and b*) contradict strategy-proofness, hence $f(\theta^{-i}, \hat{\theta}^i) = f(\theta)$.

□.

The above lemma states that no agent can change the outcome at a profile by changing his preferences to a reshuffle around this outcome. If f is corruptible then the conclusion of the above lemma is $f^i(\theta^{-i}, \hat{\theta}^i) = f^i(\theta)$.

LEMMA 1c. If preferences satisfy SIP, $f:\Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible; then $(\forall \in \Theta^N)(\forall i, j \in N)[\hat{\theta}^i \in r(\theta^i, f(\theta)) \Rightarrow o(j, \theta) = o(j, (\theta^{-i}, \hat{\theta}^i))]$.

Proof. Let $\hat{\theta} = (\theta^{-i}, \hat{\theta}^i)$. We will show that $o(j, \hat{\theta}) \subseteq o(j, \theta)$. We can show that $o(j, \theta) \subseteq o(j, \hat{\theta})$ by a similar argument, since $f(\theta) = f(\hat{\theta})$ by lemma (1b), and $\theta^i \in r(\hat{\theta}^i, f(\hat{\theta}))$ by the definition of $r(\cdot)$.

Suppose $o(j, \hat{\theta}) \not\subseteq o(j, \theta)$ and let $y \in o(j, \hat{\theta})$, $y \notin o(j, \theta)$, and let $x = f(\theta)$.

By lemma (1b), $f(\theta) = f(\hat{\theta}) = x$.

$x = f(\theta) \in o(j,\theta)$ by definition of $o(\cdot)$, so $x \neq y$.

By lemma (1a), $f(\theta) = C(o(j,\theta),\theta^j)$, and $x^j = f^j(\theta) = C^j(o(j,\theta),\theta^j)$.

Let ${}^y\theta^j$ denote the preferences obtained from θ^j by lifting y to first place, all other rankings remaining the same. Since $y \notin o(j,\theta)$, $x^j = f^j(\theta) = C^j(o(j,\theta), {}^y\theta^j)$.

By lemma (1a), $f^j(\theta^{-j}, {}^y\theta^j) = C^j(o(j,\theta^{-j}, {}^y\theta^j), {}^y\theta^j)$.

Also $C^j(o(j,\theta), {}^y\theta^j) = C^j(o(j,\theta^{-j}, {}^y\theta^j), {}^y\theta^j)$, since $o(j,\cdot)$ does not depend on θ^j.

So $f^j(\theta) = f^j(\theta^{-j}, {}^y\theta^j) = x$. But $y \in o(j,\hat{\theta}) \Rightarrow y = f(\hat{\theta}, {}^y\theta^j) = C(o(j,\hat{\theta}), {}^y\theta^j)$ by lemma (1a), and y is best for ${}^y\theta^j$. Expanding the arguments of $f(\cdot) \Rightarrow y = f(\theta^{-ij}, {}^y\theta^j, \hat{\theta}^i)$ and $x = f(\theta^{-ij}, {}^y\theta^j, \theta^i)$. Since $\hat{\theta}^i$ and θ^i maintain the same relative ordinal preference between x and y, i can either manipulate $(\theta^{-ij}, {}^y\theta^j, \hat{\theta}^i)$ or $(\theta^{-ij}, {}^y\theta^j, \theta^i)$, which is a contradiction to the strategy-proofness of f.

$\square$.

This lemma states that an agent cannot change the option set of any other agent by a reshuffling of his preferences.

Example with simple serial dictator:

Let $n = k = 5$ and the agent ordering is $\{1,2,3,4,5\}$ for each profile. Let the profile be such that the best slot for agent 1 is slot 1, the best slot for agent 2 is slot 2, and the ordinal preferences for agent 3 are $(1,2,3,4,5)$. The ordering assigns slot 1 to agent 1, slot 2 to agent 2, and slot 3 to agent 3; the option set for agent 4 is $\{4,5\}$. Suppose agent 3's preferences are reshuffled around the outcome $(2,1,3,5,4)$, then every agent's assignment remains the same. Let agent 3's preferences be changed to $(2,4,3,1,5)$, which is not a reshuffle around the outcome, then agent 3 is assigned slot 4 and the option set for agent 4 is $\{3,5\}$.

In the following lemmas if E is a relation, then $\sim E$ denotes not E; the cardinality of set A, is denoted by $|A|$.

LEMMA 1d (Asymmetry). If preferences satisfy SIP and $f\colon \Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible then $(\forall \theta \in \Theta^N)(\forall i, j \in N)$ $iA(\theta)j \Rightarrow$ either:

1. $o(i,\theta) = o(j,\theta)$ and $|o(j,\theta)| = 2$, or
2. $\sim jA(\theta)i$.

Proof. Suppose the hypothesis of the lemma holds, then $iA(\theta)j$ requires that $\exists \hat{\theta}^i \ni f^j(\theta^{-i}, \hat{\theta}^i) \neq f^j(\theta)$. Suppose that $jA(\theta)i$ then $\exists \hat{\theta}^j \ni f^i(\theta^{-j}, \hat{\theta}^j) \neq f^i(\theta)$. Let $y = f(\theta^{-i}, \hat{\theta}^i)$, $z = f(\theta^{-j}, \hat{\theta}^j)$ and $x = f(\theta)$. We will look at 2 mutually exclusive and exhaustive cases.

Case 1: $\forall \hat{\theta}^i, \hat{\theta}^j \ni f^j(\theta^{-i}, \hat{\theta}^i) \neq f^j(\theta)$ and $f^i(\theta^{-j}, \hat{\theta}^j) \neq f^i(\theta)$, $f(\theta^{-i}, \hat{\theta}^i) = f(\theta^{-j}, \hat{\theta}^j)$.

The definition of $o(\cdot)$ implies that $\{x,y\} = o(i,\theta)$ and $\{x,z\} = o(j,\theta)$. But $x = z$, so $\{x,y\} = o(i,\theta)$ and $\{x,y\} = o(j,\theta)$. Therefore $o(i,\theta) = o(j,\theta)$ and $|o(j,\theta)| = 2$.

Case 2: $\exists \hat{\theta}^i, \hat{\theta}^j \ni f^j(\theta^{-i}, \hat{\theta}^i) \neq f^j(\theta)$ and $f^i(\theta^{-j}, \hat{\theta}^j) \neq f^i(\theta)$, $f(\theta^{-i}, \hat{\theta}^i) \neq f(\theta^{-j}, \hat{\theta}^j)$.

The construction of $y, z,$ and x implies that $y \neq z$, $y^j \neq x^j$ and $z^i \neq x^i$. By lemma (1a), $x, y \in o(i,\theta)$ and $x, z \in o(j,\theta)$. $z^i \neq x^i \Rightarrow z \notin f(\theta)$ and if $U(z,\theta^i) > U(x,\theta^i)$ then $z \notin o(i,\theta^{-i})$.

Let $\acute{\theta}^i = {}_z\theta^i$ if $U(x,\theta^i) > U(z,\theta^i)$: put z last

$\acute{\theta}^i = {}^z\theta^i$ if $U(z,\theta^i) > U(x,\theta^i)$: put z first

Let $\acute{\theta}^j = {}_y\theta^j$ if $U(x,\theta^j) > U(y,\theta^j)$: put y last

$\acute{\theta}^j = {}^y\theta^j$ if $U(y,\theta^j) > U(x,\theta^j)$: put y first

For $k \neq i, j$, $\acute{\theta}^k = \theta^k$, so $\acute{\theta} = (\theta^{-ij}, \acute{\theta}^i, \acute{\theta}^j)$.

$\acute{\theta}^i$ and $\acute{\theta}^j$ are reshuffles of θ^i and θ^j about x.

Let $\ddot{\theta}^i = {}^y\acute{\theta}^i$ if $U(x,\theta^i) > U(z,\theta^i)$: put y first, z last

$\ddot{\theta}^i = {}^z({}^y\theta^i)$ if $U(z,\theta^i) > U(x,\theta^i)$: put z first, y second

Let $\ddot{\theta}^j = {}^z\acute{\theta}^j$ if $U(x,\theta^j) > U(y,\theta^j)$: z first, y last

$\ddot{\theta}^j = {}^y({}^z\theta^j)$ if $U(y,\theta^j) > U(x,\theta^j)$: y first, z second

Note that SIP and $y^j \neq x^j$, $z^i \neq x^i$ eliminate the equality in the above definitions.

We now show that $y = C(o(i,\acute{\theta}), \ddot{\theta}^i)$ and $y = f(\acute{\theta}^{-i}, \ddot{\theta}^i)$.

Since $\acute{\theta}^j$ is a reshuffle of θ^j around $x = f(\theta)$, then by lemma (1c), $o(i,\acute{\theta}) = o(i,\theta)$ and $y \in o(i,\acute{\theta})$. By construction $\ddot{\theta}^i$ ranks y first or second. If $\ddot{\theta}^i$ ranks y first then $y = C(o(i,\acute{\theta}), \ddot{\theta}^i)$. If $\ddot{\theta}^i$ ranks y second, $\ddot{\theta}^i$ ranks z first, but $z \notin o(i,\theta)$, so $z \notin o(i,\acute{\theta})$; and hence, $y = C(o(i,\acute{\theta}), \ddot{\theta}^i)$. By lemma (1a), $y = f(\acute{\theta}^{-i}, \ddot{\theta}^i)$.

By a similar argument, we can show $z = C(o(j,\acute{\theta}), \ddot{\theta}^j)$, and $z = f(\acute{\theta}^{-j}, \ddot{\theta}^j)$.

By construction, $\ddot{\theta}^j \in r(y, \acute{\theta}^j)$, and $\ddot{\theta}^i \in r(z, \acute{\theta}^i)$, so by lemma (1b), $y = f(\theta^{-ij}, \ddot{\theta}^j, \ddot{\theta}^i)$, and $z = f(\theta^{-ij}, \ddot{\theta}^i, \ddot{\theta}^j)$, which is a contradiction since $y \neq z$ and f is single-valued.

□.

In the previous lemma we showed that an agent i affects another agent j then agent j cannot affect agent i except in one instance. In that instance agent i and agent j both have the same option set (of size 2) and can affect each others outcome. This case describes the nature of the ordering that is induced by a sequential choice mechanism. When both agents can affect each other at a profile the relative ordering of agent i and j does not affect the outcome at that profile.

LEMMA 1e (Transitivity). If preferences satisfy SIP, $f\colon\Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible; then $(\forall\theta \in \Theta^N)(\forall i, j \in N)[iA(\theta)j \;\&\; jA(\theta)k \Rightarrow \sim kA(\theta)i]$.

Proof. Suppose $\exists i,j,k \in N \ni iA(\theta)j,\ jA(\theta)k$, and $kA(\theta)i$. Without loss of generality let $i=1$, $j=2$, and $k=3$. $1A(\theta)2 \Rightarrow \exists\hat{\theta}^1 \ni f^2(\theta^{-1},\hat{\theta}^1) \neq f^2(\theta)$, $2A(\theta)3 \Rightarrow \exists\hat{\theta}^2 \ni f^3(\theta^{-2},\hat{\theta}^2) \neq f^3(\theta)$, and $3A(\theta)1 \Rightarrow \exists\hat{\theta}^3 \ni f^1(\theta^{-3},\hat{\theta}^3) \neq f^1(\theta)$.

Let $y = f(\theta^{-1},\hat{\theta}^1)$, $z = f(\theta^{-2},\hat{\theta}^2)$, $w = f(\theta^{-3},\hat{\theta}^3)$, and $x = f(\theta)$; then $y^2 \neq x^2$, $z^3 \neq x^3$, and $w^1 \neq x^1$. Noncorruptibility implies $y^1 \neq x^1, z^2 \neq x^2, w^3 \neq x^3$. Asymmetry implies $y^3 \neq x^3$, $z^1 \neq x^1$, and $w^2 \neq x^2$.

By definition of $o(i,\cdot)$ $x,y \in o(1,\theta)$, $x,z \in (2,\theta)$, and $w,z \in o(3,\theta)$. $z \neq x \Rightarrow z \notin f(\theta)$, $y \neq x \Rightarrow y \notin f(\theta)$, $w \neq x \Rightarrow w \notin f(\theta)$.

Define ${}^{x:y}\theta^1$ to by the preferences defined by ${}^x({}^y\theta^1)$ if $U(x,\theta^1) > U(y,\theta^1)$, and by ${}^y({}^x\theta^1)$ if $U(y,\theta^1) > U(x,\theta^1)$. Define ${}_{x:y}\theta^1$ to by the preferences defined by ${}_x({}_y\theta^1)$ if $U(x,\theta^1) > U(y,\theta^1)$ and by ${}_y({}_x\theta^1)$ if $U(y,\theta^1) > U(x,\theta^1)$.

Let $\acute{\theta}^1 = {}_{w:z}\theta^1$ if $U(x,\theta^1) > U(w,\theta^1)$: put $(w\colon z)$ last

$\acute{\theta}^1 = {}_z({}^w\theta^1)$ if $U(w,\theta^1) > U(x,\theta^1)$: put w first, z last

Let $\acute{\theta}^2 = {}_{y:w}\theta^2$ if $U(x,\theta^2) > U(y,\theta^2)$: put $(y\colon w)$ last

$\acute{\theta}^2 = {}_w({}^y\theta^2)$ if $U(y,\theta^2) > U(x,\theta^2)$: put y first, w last

Let $\acute{\theta}^3 = {}_{z:y}\theta^3$ if $U(x,\theta^3) > U(z,\theta^3)$: put $(z{:}y)$ last

$\acute{\theta}^3 = {}_y({}^z\theta^3)$ if $U(z,\theta^3) > U(x,\theta^3)$: put z first, y last

For $k \neq 1,2,3$ $\acute{\theta}^k = \theta^k$, so $\acute{\theta} = (\theta^{-123}, \acute{\theta}^1, \acute{\theta}^2, \acute{\theta}^3)$. $\acute{\theta}^1, \acute{\theta}^2$, and $\acute{\theta}^3$ are reshuffles of $\theta^1, \theta^2, \theta^3$ about $x = f(\theta)$.

Let $\ddot{\theta}^1 = {}^y\acute{\theta}^1$ if $U(x,\theta^1) > U(w,\theta^1)$: put y first, $(w{:}z)$ last

$\ddot{\theta}^1 = {}_z({}^w({}^y\theta^1))$ if $U(w,\theta^1) > U(x,\theta^1)$: put w first, y second, z last

Let $\ddot{\theta}^2 = {}^z\acute{\theta}^2$ if $U(x,\theta^2) > U(y,\theta^2)$: z first, $(y{:}w)$ last

$\ddot{\theta}^2 = {}_w({}^y({}^z\theta^2))$ if $U(y,\theta^2) > U(x,\theta^2)$: y first, z second, w last

Let $\ddot{\theta}^3 = {}^w\acute{\theta}^3$ if $U(x,\theta^3) > U(z,\theta^3)$: put w first, $(z{:}y)$ last

$\ddot{\theta}^3 = {}_y({}^z({}^w\theta^3))$ if $U(z,\theta^3) > U(x,\theta^3)$: put z first, w second, y last

Since $\acute{\theta}^k$ are reshuffles for $k = 1,2,3$ by lemma (1c), $o(1,\acute{\theta}) = o(1,\theta)$, $o(2,\acute{\theta}) = o(2,\theta)$ and $o(3,\acute{\theta}) = o(3,\theta)$. Since $o(1,\acute{\theta}) = o(1,\theta)$, $C(o(1,\acute{\theta}),\ddot{\theta}^1) = C(o(1,\theta),\ddot{\theta}^1)$. By construction, $y \in o(1,\theta)$, and $\ddot{\theta}^1$ ranks y first or second. If $\ddot{\theta}^1$ ranks y first then $y = C(o(1,\acute{\theta}),\ddot{\theta}^1)$. If $\ddot{\theta}^1$ ranks y second, then $U(w,\theta^1) > U(x,\theta^1)$, and $x \in f(\theta)$, implies that $w \notin f(\theta)$, and $w \notin o(1,\theta)$. Hence, $y = C(o(1,\acute{\theta}),\ddot{\theta}^1)$. Similarly $w = C(o(3,\acute{\theta}),\ddot{\theta}^3)$ and $z = C(o(2,\acute{\theta}),\ddot{\theta}^2)$.

By lemma (1a), $y = f(\acute{\theta}^{-1},\ddot{\theta}^1)$, $z = f(\acute{\theta}^{-2},\ddot{\theta}^2)$ and $w = f(\acute{\theta}^{-3},\ddot{\theta}^3)$. But $\ddot{\theta}^1 \in r(w,\acute{\theta}^1)$, $\ddot{\theta}^1 \in r(z,\acute{\theta}^1)$, $\ddot{\theta}^2 \in r(w,\acute{\theta}^2)$ $\ddot{\theta}^2 \in r(y,\acute{\theta}^2)$, and $\ddot{\theta}^3 \in r(z,\acute{\theta}^3)$, $\ddot{\theta}^3 \in r(y,\acute{\theta}^3)$; hence, by lemma (1b), $y = f(\acute{\theta}^{-123},\ddot{\theta}^1,\ddot{\theta}^2,\ddot{\theta}^3)$, $z = f(\acute{\theta}^{-123},\ddot{\theta}^1,\ddot{\theta}^2,\ddot{\theta}^3)$ and $w = f(\acute{\theta}^{-123},\ddot{\theta}^1,\ddot{\theta}^2,\ddot{\theta}^3)$, which is a contradiction, since $y, z,$ and w are distinct.

If the $y, z,$ and w are not distinct, then the conditions of lemma (1d) are observed and the lemma still follows.

□.

Given the previous lemmas, the proof of the theorem is straightforward.

Proof of Theorem 1. Without loss of generality, suppose that $1A(\theta)2$, $2A(\theta)3, \ldots, n-1A(\theta)n$. By the transitivity of $A(\cdot)$, $1A(\theta)n$. By asymmetry, $1A(\theta)n \Rightarrow \sim nA(\theta)1$. Hence, $\sim nA(\theta)1$, and $A(\theta)$ is acyclic.

□.

In our environment we can give more structure to the acyclic relation of $A(\theta)$. Let $I(\theta)$ be an ordering induced by $A(\theta)$; that is, if $i_1A(\theta)i_2$, $i_2A(\theta)i_3$, ..., $i_{n-1}A(\theta)i_n \Rightarrow i_1A(\theta)i_n$, then $I(\theta) = \{i_1, i_2, \ldots, i_n\} \in \mathbb{P}(N)$. We first prove a simple lemma:

LEMMA 6.14. If preferences satisfy SIP, $f\colon \Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible, and if $iA(\theta)j$, $l = C^i(o(i,\theta),\theta^i)$, then $l \notin o(j,\theta)$.

Proof. By lemma (1a) and $l = C^i(o(i,\theta),\theta^i)$, $l = f^i(\theta)$. $iA(\theta)j \Rightarrow \sim jA(\theta)i$ $\Rightarrow l = f^i(\theta^{-j},\hat{\theta}^j)\ \forall \hat{\theta}^j$; hence $l \notin o(j,\theta)$.

□.

The result of the previous lemma is that if i affects j at θ, then there is no option open to j to obtain the slot that is the best of i's options at θ. We use this result in the following lemma.

LEMMA 6.15. If preferences satisfy SIP, $f\colon \Theta^N \mapsto \mathcal{A}$ is strategy-proof and noncorruptible, and if $A(\theta)$ is acyclic and induces the ordering $I(\theta) = \{i_1, i_2, \ldots, i_n\}$, and if $o(i_1,\theta) = K$, then $o(i_n,\theta) \subset o(i_{n-1},\theta) \subset \cdots \subset o(i_1,\theta)$.

Proof. Without loss of generality let $I(\theta) = \{1, 2, \ldots, n\}$, and $k_i = C^i(o(i,\theta), \theta^i)$; then $o(1,\theta) = K$ and $k_1 = C^1(o(1,\theta), \theta^1)$. $1A(\theta)2 \Rightarrow \exists\, \hat{\theta}^1 \ni f^2(\hat{\theta}^1, \theta^{-1}) \neq f^2(\theta) = k_2$. The previous lemma and $k_1 = f^1(\theta) \Rightarrow k_1 \notin o(2,\theta)$. Continuing iteratively: $k_1, k_2 \notin o(3,\theta)$, $k_1, k_2, k_3 \notin o(4,\theta)$, $\ldots$, $k_1, k_2, \ldots, k_{n-1} \notin o(n,\theta)$. This implies that $k_n \in o(n,\theta)$ is a singleton, and only k_{n-1} and k_n are possible elements of $o(n-1,\theta)$. By the definition $A(\cdot)$, $n-1A(\theta)n$ implies that $\exists\, \hat{\theta}^{n-1}$ such that $f^n(\theta) \neq f^n(\hat{\theta}^{n-1}, \theta^{-\{n-1\}})$. The definition of k_n implies that $\exists\, \hat{\theta}^{n-1}$ such that $[k_n \neq f^n(\hat{\theta}^{n-1}, \theta^{-\{n-1\}})]$. But $o(n, \hat{\theta}^{n-1}, \theta^{-\{n-1\}})$ is a singleton, so $[k_n \notin o(n-1, \hat{\theta}^{n-1}, \theta^{-\{n-1\}})]$. k_{n-1} and k_n are the only possible elements of $o(n-1,\theta)$, this implies that $k_n = f^{n-1}(\hat{\theta}^{n-1}, \theta^{-\{n-1\}})$, and $k_n \in o(n-1,\theta)$. Working backwards we have $o(j,\theta) \subset o(j-1,\theta), \forall j \in K$.

□.

The result of the previous lemma is that for a profile θ and an ordering $I(\theta)$ induced by a strategy-proof and noncorruptible mechanism, the options available to an agent is a subset of the options available to the agents ordered before her by $I(\theta)$. When the first agent is allowed to choose from the entire set of slots K this is exactly the serial dictatorship described in the introduction to this section.

7. Nash Implementation

We now investigate to see whether the use of Nash equilibria can expand the class of implementable SCFs. We do this by applying the result by DHM which states that if the environment is rich, then a SCF implementable in Nash strategies is truthfully implementable in dominant strategies. We show that our domain of preferences is rich[17] when preferences are not SIP. If preferences are SIP, then the domain is rich when the allocation space is $\mathcal{A}$.

DEFINITION 7.1. A class $\langle U, \Theta^i \rangle$ of utility functions is ***rich*** if $\forall$ pairs $\{\theta^i, \hat{\theta}^i\} \subset \Theta^i$, and $\forall \{a, b\} \subset \mathcal{A}$, such that

i) $U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\hat{\theta}^i) \geq U(b,\hat{\theta}^i)$, and

ii) $U(a,\theta^i) > U(b,\theta^i) \Rightarrow U(a,\hat{\theta}^i) > U(b,\hat{\theta}^i)$;

there exists a $\overline{\theta}^i \in \Theta^i$, such that $\forall\ c \in \mathcal{A}$,

a) $U(a,\theta^i) \geq U(c,\theta^i) \Rightarrow U(a,\overline{\theta}^i) \geq U(c,\overline{\theta}^i)$, and

b) $U(b,\hat{\theta}^i) \geq U(c,\hat{\theta}^i) \Rightarrow U(b,\overline{\theta}^i) \geq U(c,\overline{\theta}^i)$.

Substituting $\sum_j a_{ij}\theta^i_j$, for $U(a,\theta^i)$, lines i, ii, a, and b can be written:

i) $\sum_j (a_j - b_j)\theta_j \geq 0 \Rightarrow \sum_j (a_j - b_j)\hat{\theta}_j \geq 0$, and

ii) $\sum_j (a_j - b_j)\theta_j > 0 \Rightarrow \sum_j (a_j - b_j)\hat{\theta}_j > 0$;

there exists a $\overline{\theta} \in \Theta$, such that $\forall c \in \mathcal{A}$,

a) $\sum_j (a_j - c_j)\theta_j \geq 0 \Rightarrow \sum_j (a_j - c_j)\overline{\theta}_j \geq 0$, and

b) $\sum_j (b_j - c_j)\hat{\theta}_j \geq 0 \Rightarrow \sum_j (b_j - c_j)\overline{\theta}_j \geq 0$.

For convenience we have dropped the superscript i from θ, $\hat{\theta}$, and $\overline{\theta}$, and the subscript i from a, and b.

[17] Rich is also known as monotonically closed. DHM (1979) and Laffont and Maskin (1982) provide discussions of this concept.

PROPOSITION 7.2. If agents may be indifferent between slots and allocations are strictly feasible, then $\langle U, \Theta^N \rangle$ is rich.

Proof. Since agents are allowed to be indifferent over slots define $\overline{\theta}^i_j = \epsilon \ \forall j \in K$. Then $\sum_j (b_j - c_j)\overline{\theta}_j = \sum_j (b_j - c_j)\,\epsilon = 0$, since $\sum_j b_j = 1$, and $\sum_j c_j = 1$. So the right-hand side of a) and similarly b) holds $\forall\ a, b, c \in \mathcal{A}$. So the definition holds for all θ^i, $\overline{\theta}^i \in \Theta^i$ and $a, b \in \mathcal{A}$.

□.

PROPOSITION 7.3. If preferences satisfy SIP then the domain of preferences is rich.

Proof. Without loss of generality let $\theta_1 > \theta_2 > \cdots > \theta_k$.

i) $\sum_j (a_j - b_j)\theta_j \geq 0 \Rightarrow \sum_j (a_j - b_j)\hat{\theta}_j \geq 0$, and

ii) $\sum_j (a_j - b_j)\theta_j > 0 \Rightarrow \sum_j (a_j - b_j)\hat{\theta}_j > 0$.

i) and ii) are true for a_l and b_m such that $l \leq$ m and $\hat{\theta}_l > \hat{\theta}_m$.

Choose $\overline{\theta}$ such that $\overline{\theta}_l > \overline{\theta}_m$ and $\overline{\theta}_m > \overline{\theta}_j$, for$j \neq l$ or m.

Then for preferences to be rich, any $c_p \in \mathcal{A}$, where c_p assigns the p^{th} slot, must satisfy:

a) $a_l\theta_l - c_p\theta_p \geq 0 \Rightarrow a_l\overline{\theta}_l - c_p\overline{\theta}_p \geq 0$, and

b) $b_m\hat{\theta}_m - c_p\hat{\theta}_p \geq 0 \Rightarrow b_m\overline{\theta}_m - c_p\overline{\theta}_p \geq 0$.

Both the right-hand side and left-hand side of a) are true for $l \leq p$, and both the right-hand side and left-hand side of b) are true for $m \leq p$. Therefore, a) and b) hold for any $c \in \mathcal{A}$, so $\mathcal{A}$ is rich.

□.

When our domain of preferences is rich, we can apply the following result from DHM (theorem 7.2.3). If the domain of preferences $\langle U, \Theta^i \rangle$ is rich $\forall i \in N$, then if a SCF is implementable in Nash equilibrium, it is truthfully implementable in dominant strategies. This result follows since for rich domains and single-valued choice functions, monotonicity implies independent, weak monotonicity (IWM), which implies independent person-by-person monotonicity (IPM). If we also add the requirement that the SCF is noncorruptible, then it is fully implementable in dominant strategies. In addition, by applying DHM (theorem 7.1.1), if a SCF is truthfully implemented in dominant strategies, then it is truthfully implemented in Nash strategies. It is not necessarily true that a SCF that is fully implemented in dominant strategies is fully implemented in Nash strategies.

Therefore, the Nash solution concept does not allow us to implement more SCFs than the dominant-strategy solution concept. Furthermore, even if we use the Nash solution concept, we can fully implement only those SCFs that are ordinal. This does not imply that if we use a cardinal SCF, there are no Nash strategies, but that there are additional equilibria that do not result in the implementation of the SCF.

8. *Optimality of Implementable Rules*

In this section we discuss the optimality properties of implementable rules. A minimal requirement for optimality of a SCC is Pareto optimally (PO). If an allocation is not PO, then either there is an agent who can be made strictly better off by taking the surplus, or there are at least two agents who can be made strictly better off by trading. We will explore two notions of Pareto optimality.

DEFINITION 8.1. A SCC $f: \Theta^N \mapsto \mathcal{A}$ is ***Weak Pareto Optimal (WPO)*** if $\forall \theta \in \Theta^N$, and all $a \in f(\theta)$, there does not exist a $b \in \mathcal{A}$, such that $\forall i \in N$, $U(b, \theta^i) > U(a, \theta^i)$.

DEFINITION 8.2. A SCC $f: \Theta^N \mapsto \mathcal{A}$ is ***Strong Pareto Optimal*** (SPO) if $\forall \theta \in \Theta^N$, and all $a \in f(\theta)$, there does not exist a $b \in \mathcal{A}$, such that $\forall i \in N$, $U(b, \theta^i) \geq U(a, \theta^i)$ and for some $j \in N$, $U(b, \theta^j) > U(a, \theta^j)$.

Remark: SPO $\Rightarrow$ WPO.

The serial dictator can be readily seen to be strong Pareto optimal, since every agent is matched with her most preferred slot in the available set.

PROPOSITION 8.3. When preferences satisfy SIP, if a SCF is SPO, then it is strictly feasible.

Proof. If the allocation is not SF, then there exists a slot that is not allocated to an available agent. That agent can be made better off by being assigned that slot, since SIP implies that slots have strictly positive value. Hence the allocation is not SPO. Therefore, the proposition is satisfied.

□.

The above proposition allows us to restrict attention to the set of strictly feasible (SF) allocations if we want to restrict ourselves to PO allocations. A strictly feasible allocation may not be PO, but if it is not strictly feasible, then it is not PO.

In previous sections we restricted agents' preferences over slots to be strict. This is different from the restriction of strict preferences used in DHM and in much of the dominant-strategy literature. In most implementation papers preferences are strict over outcomes. In our case, agents are selfish and are indifferent between allocations that give them the same slot but give other agents different slots. Because of this indifference, many of the results of DHM and others are not applicable.

DHM show that when the preference domain is rich and consists of strict preferences, if a SCF satisfies citizen sovereignty (CS) and IPM, then the SCF is weak Pareto optimal and we obtain similar results, but for our environment we also require that the SCF be noncorruptible.

To prove our results we introduce three terms: IWM*, S-IWM*, and consumer sovereignty. IWM* and S-IWM* are concepts that are similar to the DHM concept of independent weak monotonicity, which we define for reference.

DEFINITION[18] **8.4.** A SCF $f: \Theta^N \mapsto \mathcal{A}$ satisfies ***independent weak monotonicity (IWM)*** if $\forall \theta \in \Theta^N$, $\forall C \subseteq N$, $\forall \bar{\theta}^C \in \prod_{i \in C} \Theta^i$, and $\forall \{a, b\} \subseteq \mathcal{A}$ such that $a \in f(\theta)$, and $(\forall i \in C)[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\bar{\theta}^i) > U(b,\bar{\theta}^i)]$; it must be that $b \notin f(\theta^{-C}, \bar{\theta}^C)$.

Our definitions of IWM* and S-IWM* do not involve coalitions and are used only to obtain intermediate results.

DEFINITION 8.5. A SCC $f: \Theta^N \mapsto \mathcal{A}$ satisfies ***IWM**** if $\forall \{\theta, \hat{\theta}\} \subset \Theta^N$, and $\forall \{a, b\} \subset \mathcal{A}$ such that:

1) $a \in f(\theta)$, and 2) $(\forall i \in N)[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\hat{\theta}^i) > U(b,\hat{\theta}^i)]$;

then $b \notin f(\hat{\theta})$.

DEFINITION 8.6. A SCC $f: \Theta^N \mapsto \mathcal{A}$ satisfies ***S-IWM**** if $\forall \{\theta, \hat{\theta}\} \subset \Theta^N$, and $\forall \{a, b\} \subset \mathcal{A}$ such that:

1) $a \in f(\theta)$, and 2) $(\forall i \in N)[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\hat{\theta}^i) \geq U(b,\hat{\theta}^i)]$;

then $b \notin f(\hat{\theta})$.

DEFINITION 8.7. A SCC $f: \Theta^N \mapsto \mathcal{A}$ satisfies ***citizen sovereignty (CS)*** if $\forall a \in \mathcal{A}^*$, $\exists \theta \in \Theta^N$, such that $a \in f(\theta)$, where $\mathcal{A}^* \equiv \{a \in \mathcal{A} \mid a \text{ is strictly feasible}\}$. That is, the mapping f is onto the set of strictly feasible allocations.

We first provide a lemma that is a similar to lemma 5.6.

[18] This is the definition of IWM that appears in DHM. Neither Laffont and Maskin (1982) or Maskin (1986) have definitions of IWM.

LEMMA 8.8. If preferences satisfy SIP, and if a SCF $f: \Theta^N \mapsto \mathcal{A}$ satisfies IPM, and is noncorruptible, $a \in f(\theta)$, $c \in f(\overline{\theta}^i, \theta^{-i})$, and $a \neq c$, then $U(a, \overline{\theta}^i) < U(c, \overline{\theta}^i)$ and $U(c, \theta^i) < U(a, \theta^i)$.

Proof: Suppose our hypothesis holds; then noncorruptibility and $a \neq c$ imply that $a^i \neq c^i$; lemma 5.6 is then applied to obtain the result.

□.

As in lemma 5.6 if preferences do not satisfy SIP in the previous lemma then $U(a, \overline{\theta}^i) \leq U(c, \overline{\theta}^i)$ and $U(c, \theta^i) \leq U(a, \theta^i)$. The difference between lemma 5.6 and the previous lemma is that in lemma 5.6, $a^i \neq c^i$ and noncorruptibility is not a condition. In the previous lemma $a \neq c$, and noncorruptibility implies $a^i \neq c^i$.

Weak Pareto optimality

LEMMA 8.9. If a SCF is noncorruptible and IPM, it is IWM*.

Proof. Suppose $f:\Theta^N \mapsto \mathcal{A}$ satisfies the hypothesis of the lemma; then for $\theta, \hat{\theta} \in \Theta^N$, $a \in \mathcal{A}$, such that, $a \in f(\theta)$, and $(\forall i \in N)(\forall b \in \mathcal{A}, b \neq a)$ $[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\hat{\theta}^i) > U(b,\hat{\theta}^i)]$, but $b \in f(\hat{\theta})$; we will show that this is a contradiction.

Suppose $c \in f(\hat{\theta}^1, \theta^{-1})$ some $c \in \mathcal{A}$. If $c \neq a$, then by noncorruptibility, SIP, IPM and lemma 8.6, $U(a,\hat{\theta}^1) \leq U(c,\hat{\theta}^1)$ and $U(c,\theta^1) \leq U(a,\theta^1)$. But $[U(a,\theta^1) \geq U(c,\theta^1) \Rightarrow U(a,\hat{\theta}^1) > U(c,\hat{\theta}^1)]$, so it must be that $c = a$, and $a \in f(\hat{\theta}^1, \theta^{-1})$.

Similarly, $a \in f(\hat{\theta}^1, \hat{\theta}^2, \theta^{-1,2})$, continuing iteratively, $a \in f(\hat{\theta})$. But f is a SCF, so it is single-valued; hence $b \notin f(\hat{\theta})$.

□.

LEMMA 8.10. If a SCF satisfies noncorruptible, IPM, and CS, it weak Pareto optimal (WPO).

Proof. Suppose that $f:\Theta^N \mapsto \mathcal{A}$ satisfies the hypothesis but that f is not WPO. Then there exist a $\overline{\theta} \in \Theta^N$ and a pair $\{a,b\} \subseteq \mathcal{A}$, such that $\forall i \in N$, $U(a,\overline{\theta}^i) > U(b,\overline{\theta}^i)$, but $b \in f(\overline{\theta})$. By CS, there exists a $\theta \in \Theta^N$ such that $a \in f(\theta)$. By the previous lemma f satisfies IWM*. Since $(\forall i \in N)$ $[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\overline{\theta}^i) \geq U(b,\overline{\theta}^i)]$ and $a \in f(\theta)$, IWM* implies that $b \notin f(\overline{\theta})$, a contradiction.

□.

Strong Pareto optimality

The next propositions show that the assumption of strict individual preferences yields a stronger notion of optimality.

LEMMA 8.11. If preferences satisfy SIP, then if a SCF is noncorruptible and satisfies IPM, it is S-IWM*.

Proof. Suppose that $f:\Theta^N \mapsto \mathcal{A}$ satisfies the hypothesis of the lemma; then for $\theta, \hat{\theta} \in \Theta^N$, $a \in \mathcal{A}$, such that $a \in f(\theta)$, and $(\forall i \in N)(\forall b \in \mathcal{A}, b \neq a)$ $[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow U(a,\hat{\theta}^i) \geq U(b,\hat{\theta}^i)]$, but $b \in f(\hat{\theta})$; we will show that this is a contradiction.

Suppose that $c \in f(\hat{\theta}^1, \theta^{-1})$, for some $c \in \mathcal{A}$. If $c \neq a$, then by noncorruptibility, SIP, IPM and lemma 8.6, $U(a,\hat{\theta}^1) < U(c,\hat{\theta}^1)$ and $U(c,\theta^1) < U(a,\theta^1)$. But $[U(a,\theta^1) \geq U(c,\theta^1) \Rightarrow U(a,\hat{\theta}^1) \geq U(c,\hat{\theta}^1)]$, so it must be that $c = a$, and $a \in f(\hat{\theta}^1, \theta^{-1})$.

Similarly, $a \in f(\hat{\theta}^1, \hat{\theta}^2, \theta^{-1,2})$, continuing iteratively, $a \in f(\hat{\theta})$. But f is a SCF, so it is single-valued; hence $b \notin f(\hat{\theta})$.

□.

PROPOSITION 8.12. If preferences satisfy SIP, then if a SCF is noncorruptible, satisfies IPM, and CS, it is strong Pareto optimal (SPO).

Proof. Suppose that $f:\Theta^N \mapsto \mathcal{A}$ satisfies the hypothesis but that f is not SPO. Then there exist a $\overline{\theta} \in \Theta^N$ and a pair $\{a,b\} \subseteq \mathcal{A}$, such that $\forall i \in N$, $U(a,\overline{\theta}^i) \geq U(b,\overline{\theta}^i)$, and $b \in f(\overline{\theta})$. By SIP, either $a = b$ or there exists a j such that $U(a,\overline{\theta}^j) > U(b,\overline{\theta}^j)$. By CS, there exists a $\theta \in \Theta^N$ such that $a \in f(\theta)$. By the previous lemma, f satisfies S-IWM*. Since $(\forall i \in N)[U(a,\theta^i) \geq U(b,\theta^i) \Rightarrow$

$U(a,\overline{\theta}^i) \geq U(b,\overline{\theta}^i)]$ and $a \in f(\theta)$, S-IWM* implies that $b \notin f(\overline{\theta})$, a contradiction.

□.

The following is an example wherein a mechanism satisfies SP and noncorruptibility but not CS.

EXAMPLE 8.13. Let $N = \{1,2,3\}$, $K = \{1,2,3\}$. Let f be the SCF that is implemented by the following mechanism:

Let $O^1(\cdot) = \{2,3\}$: the option set available to agent 1.

Let $O^2(\cdot) = \{2,3\}$: the option set available to agent 2.

Let $O^3(\cdot) = \{1\}$: the option set available to agent 3.

and $I(\theta) = \{1,2,3\}$ $\forall\theta$, the order that agents choose slots.

That is, agent 1 selects first from the set $\{2,3\}$, agent 2 selects second from the remainder of set $\{2,3\}$, and agent 3 receives slot 1. This mechanism satisfies SP, since I doesn't depend on θ, and is noncorruptible (it is also nonconstant), but it does not satisfy CS.

Let $\theta^1 = 1{>}2{>}3$, $\theta^2 = 1{>}2{>}3$, and $\theta^3 = 3{>}2{>}1$; then $a = f(\theta)$. But if $b = (1,2,3)$, then $U(b,\theta^i) > U(a,\theta^i)$ $\forall i$. So f is not Pareto optimal (strong or weak).

The results of this section indicate that if a SCF is strategy-proof and noncorruptible and satisfies citizen sovereignty, then it is Pareto optimal. If preferences satisfy SIP, then a nonstrategic SCF is strong Pareto optimal, and if preferences do not satisfy SIP, then a nonstrategic SCF is weak Pareto optimal.

9. *Positional Mechanisms*

In the introduction we described three categories of mechanisms that have been proposed in the literature to "solve" the one-sided matching problem. These were positional, chit, and choice mechanisms. In the previous sections we have proven some results that allow us to draw some conclusions about the ability to predict behavior in these mechanisms.

We can readily apply the results of the previous sections to determine the implementability in dominant and Nash-strategy equilibrium of chit and choice mechanisms. Since the outcome of chit mechanisms, as described in the introduction, can be affected by changing cardinal information, we cannot make dominant-strategy-equilibrium predictions, or Nash-strategy-equilibrium predictions. Since a necessary condition for implementation in dominant strategies is that the outcome can only be affected by a change in ordinal information, chit mechanisms are not dominant strategy mechanisms. Choice mechanisms have been shown to be implementable in dominant strategies and if the mechanism is a serial dictator, it is also Pareto optimal. We now discuss the last class of mechanisms—positional mechanisms.

positional mechanisms

We begin by formally describing a positional mechanism: a mechanism $x: S \mapsto \mathcal{A}$ is ***positional*** if there are strictly monotonic weighting functions $w^i: S^i \mapsto \{w_1, \ldots, w_n\}$, $w_i \in \Re$, $\forall i \in N$, and $x(s)$ maximizes the function $\sum_{ij} x_{ij} w^i(s^i)$, where ties are broken arbitrarily. If a SCF is implemented by positional mechanism, the SCF can be easily seen to be symmetric, and Pareto optimal, but it cannot be implemented in dominant strategies as the following proposition shows.

PROPOSITION 9.1. If a SCF can be implemented by a positional mechanism, then it is not implementable in dominant strategies.

Proof. Without loss of generality let $N = K$. Suppose that the SCF $f\colon \Theta^N \mapsto \mathcal{A}$ is implementable and let $\theta \in \Theta^N$, such that $\theta^i = \theta^j, \forall i,j \in N$, and $\theta_1^i > \theta_2^i > \cdots > \theta_n^i$; θ is an allowable profile in the matching environment. Since f is positional, any strictly feasible allocation $a \in \mathcal{A}$ will maximize $x(a,\theta) = \sum_{i,j} a_{ij} w(\theta_j^i)$, as long as $w(\cdot)$ is strictly monotonic. Without loss of generality let $a = f(\theta) = \{1,\ldots,n\}$; that is, agent i is assigned slot i, $a^i = i$. So $U(a,\theta^1) = \theta_1^1$, $U(a,\theta^2) = \theta_2^2$, and so on.

Let $\hat{\theta}^3$ be such that $\hat{\theta}_2^3 > \hat{\theta}_l^3$, $\forall l \neq 2$. Then $x(a,(\hat{\theta}^3,\theta^{-3}))$ is maximized by $a^3 = 2$, and $a^2 = 3$; this implies that $2A(\theta)3$.

Let $\hat{\theta}^2$ be such that $\hat{\theta}_3^2 > \hat{\theta}_l^2$, $\forall l \neq 3$. Then $x(a,(\hat{\theta}^2,\theta^{-2}))$ is maximized by $a^2 = 2$, and $a^3 = 3$; this implies that $3A(\theta)2$.

But since f is implementable, there is an affects relation $A(\theta)$ induced by f, and the asymmetry property of the affects relation $A(\theta)$ implies that $2A(\theta)3$ and $\sim 3A(\theta)2$, so f cannot be implementable.

□.

The only class of procedures with a dominant strategy prediction is the class of choice mechanisms. An example of one of these mechanisms is the serial dictator, which is Pareto optimal but not symmetric, unless there is a random selection of order.

Conclusion

There have been a number of procedures proposed in the literature to "solve" the one-sided matching problem. Almost all of these procedures assume an agent's behavior is nonstrategic. If we assume that an agent's behavior reflects his own best interest, strategic behavior is likely. The importance of understanding an agent's strategic behavior reflects the importance of the planner's ability to determine the outcome of an allocation mechanism. The more that can be said about an agent's strategic behavior, the more precise can be the planner's prediction of the outcome of an allocation mechanism.

Determining an agent's strategic behavior can be quite complex, but in the one-sided matching problem there does exist a class of SCCs whose outcomes are easily predicted. This is the class of noncorruptible and strategy-proof SCFs. In this paper we were able to show that the class of strategy-proof and non-corruptible SCFs does not exclude any SCCs that can be implemented in dominant strategies. But most importantly, we were able to characterize the class of implementable SCFs, that is, those SCFs that are strategy-proof and non-corruptible. We found that SCFs must rely only on ordinal information to be implementable and the Nash solution concept does not enlarge the class of implementable SCFs. We also found that the only implementable SCFs were sequential choice mechanisms, and that a particular member of this class, the serial dictator, was also Pareto optimal.

Chapter II: An Experimental Examination of the Assignment Problem

1.0 Introduction

We consider the problem of allocating a fixed set of goods or services, which we will generically call slots, to a fixed set of individuals or agents. We look at the problem from the point of view of a planner or institution designer, who wishes to design a mechanism that implements a social welfare maximum constrained by the feasibility constraint: at most one slot is assigned to each agent. Our formulation supposes that the planner himself attaches no value to any assignment. We consider this allocation problem with and without the use of monetary transfers to allocate slots.

This problem appears in a variety of settings; computer scheduling, the administration of office space, the assignment of students to dormitory rooms or courses, and the disbursement of social services. The problem encountered by the Jet Propulsion Laboratory in allocating antenna time on NASA's Deep Space Network (DSN) to spacecraft outside the earth's orbit motivated this project. The DSN problem is an example of the allocation of a set of services in fixed supply within a given time period to a group of agents, a scheduling problem. In its most abstract and generic form the scheduling problem can be modeled as an assignment or one-sided matching problem.

In the assignment or matching problem if the planner knows the values agents place on slots then he can optimally assign agents by solving an integer programming problem. However, true values are known only to the agents so that any mechanism, which the planner uses, must work with revealed rather than true valuations. Several allocation schemes (auctions) have been proposed to solve the coordination and incentive problems posed by the assignment problem when the planner is allowed to use transfers (money). There is little theoretical or experimental evidence to guide the planner in his choice of mechanisms when the planner is not allowed to use transfers .

In the assignment or matching environment, although the planner is interested in social welfare, agents wish to maximize their own surplus or utility. In many cases the agents may be able to influence the choice of mechanism by appealing to authority above the planner. In these instances the planner may be willing to reduce social welfare to increase consumer surplus. Options that are often put forth are the use of committee decision making, or some form of "funny money" market, or a procedure based on ordinal rankings. These procedures form a class of mechanisms we shall call ***nontransfer mechanisms***. In this paper we explore the use of some specific transfer and nontransfer mechanisms and their effect on social welfare and agent surplus in several assignment environments. Our results provide evidence that in some environments the absence of transfers does not significantly reduce social welfare but does increase significantly the agents' surplus or welfare. This observation helps explain the existence and persistence of inefficient nontransfer institutions.

We complete this section with some historical notes. In Section 2 we describe the problem formally. In Section 3 some mechanisms to solve the assignment problem are described. In Section 4 the experimental design is described. In Section 5 we describe the mechanisms tested and the specifics of their implementation in the experimental environment. In Section 6 we describe the predictions of the mechanisms in the experimental environment and the behavioral assumptions that were made. In Section 7 the results we observed in our experiments are presented. Section 8 contains a summary and some concluding remarks.

1.1 Historical Notes

The problem of assigning or matching demanders to items that are heterogeneous but related has a long and developed literature. The interest of economists in the assignment problem dates back to at least 1891 in Böhm-Bawerk's horse market[1] (see Shapley and Shubik (1972) for a discussion). The assignment market game was introduced by Shapley in 1955, who showed that a class of market games could be solved as an assignment problem. It later received a full treatment by Shapley and Shubik in 1972. They characterized the core of the assignment game and showed that the core has two distinguished points, the maximum buyer surplus and the maximum seller surplus.

Another type of matching problem is the marriage problem (see Gale and Shapley (1962), Halmos and Vaughn (1950), Roth and Sotomayor (1989)). The marriage problem involves finding stable pairings of two sets of players when they have differing ordinal preferences over players in the other set. Algorithms have been developed that map ordinal information into ***stable*** matchings.[2] These mechanisms have the property that no monetary transfers are made across agents or to the planner.

The assignment problem, which matches a group of buyers to a group of heterogeneous objects under the constraint of one buyer to an object, is an example of a general class of problems. This class includes single-unit auction models, where one good is matched to one of a set of potential buyers, and

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[1] Shapley and Shubik (1972) note that Cournot and Edgeworth also made observations on two-sided markets, and that John von Neumann and Oskar Morgenstern briefly treated some simple market games.

[2] Roth (1984) notes that the algorithm described by Gale and Shapley had been in place since 1951 to match medical interns to hospitals.

multiple-unit auction models, where many homogeneous goods are matched to a set of potential buyers. This class also includes models that are more general and allow complementarities among goods (see, *e. g.,* Rassenti, Smith and Bulfin (1982) and Banks, Ledyard and Porter (1989), hereafter RSB and BLP).

To date, there is very little empirical (experimental) evidence germane to the ability of such auctions to solve the assignment problem. Much evidence exists for single-unit and multiple-unit versions of the Vickrey and English Auctions for homogeneous goods (see Cox et al. (1982), McCabe et al. (1990) and Coppinger et al. (1980)). However, when the goods to be allocated are heterogeneous, the only evidence available is that of Rassenti et al. (1982), who present a combinatorial version of a "Vickrey" auction to allocate goods with severe complementarities (*e.g.* airline landing slots) and Banks et al. (1989), who use an English auction for multi-dimensional bundles of services (*e.g.* weight and volume in the Space Shuttle).[3] One purpose of this paper is to fill that gap for the standard assignment problem by providing some experimental evidence on the performance of a sealed-bid auction and a variant of the English auction.

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[3] Nalbantian and Schotter (1990) examine three two-sided matching mechanisms: private negotiations, English auctions, and a sealed bid with negotiating. The experiments were designed to study efficiency in the free-agent market for matching baseball players with teams, and all the mechanisms use monetary transfers.

2. Description of the Problem

In this section we describe the classic assignment problem as a planner's problem. We assume throughout that the planner's goal is to maximize the total welfare of the system. He is to accomplish his goal by assigning a set of slots to a group of agents. Each agent attempts to maximize her own utility (acquire the slot that is most valuable to him). Since some agents may place their highest value on the same slots, the planner needs to know the relative value of the slots to each agent. However, depending on the mechanism used, it may be in the agent's best interest to overstate or understate her relative preference for slots.

To overcome the problem of misrepresentation, mechanisms have been proposed that use monetary transfers to give incentives to agents to report their valuations honestly. In this paper we test two such mechanisms from the literature on the assignment problem. The transfer mechanisms we will investigate yield efficient allocations, but they transfer surplus from the agents to the planner. We also describe and test mechanisms that do not rely on transfers. One such mechanism elicits an agent's ordinal rankings over slots, and another requires the use of "funny money" or "chits."

Formal description of model

Our environment consists of n agents and k goods or services to be allocated, which we will call ***slots***. Let $N = \{1,\ldots,n\}$ index the set of agents, and let $K = \{1,\ldots,k\}$ index the set of slots. It is assumed that both N and K are finite and nonempty. Let $\mathcal{A}$ be the set of feasible deterministic, allocations of K to N, including the zero allocation, wherein no agent receives a slot. An element in $\mathcal{A}$ is an $n \times k$ matrix consisting of at most a single 1 in each row and column, where an element $a_{ij} = 1$, if agent i is assigned slot j, and $a_{ij} = 0$, if he is not. We also define $a^i = (a_{i1}, \ldots, a_{ik})$.

The preferences of each agent depend upon the slot allocated, any monetary payment, and the agent's type. An agent's type parameterizes the value he places on the goods being allocated. Let $\Theta^i \subset \Re^k$ be a set of possible types for agent i, $\forall i \in N$. Let $\Theta^N = \mathsf{X}_{i \in N} \Theta^i$. A $\theta \in \Theta^N$ will be called a ***profile***. The number of agents and slots is fixed, so the feasible set is independent of the profile. Each agent i, of type θ^i, evaluates each outcome $x \in \mathcal{A}$ (or assignment) through a valuation function $v(x, \theta^i) = \sum_j x_{ij} \theta^i_j$. The quantity $v(x, \theta^i)$ represents the willingness to pay of agent i of type θ^i for outcome x. The utility of agent i is quasi-linear and is given by $U(x, t, \theta^i) = v(x, \theta^i) + t^i$, where t^i is any monetary transfer to (or from) agent i.

We note that in the above definition agents may be indifferent between distinct outcomes since they are selfish; that is, they care only about the slots allocated to them. When the outcome space is $\mathcal{A}$, and agents are selfish, there is no loss of generality in the linear description of utility, since there are a finite number of slots. That is, when agents are selfish and the outcome space is $\mathcal{A}$, then for any utility function $\hat{U}(x)$, there is a θ^i such that $U(x, \theta^i) = \sum_j x_{ij} \theta^i_j = \hat{U}(x)$.

The planner's objective is to assign the agents in N to the slots in K such that total system welfare is maximized. We can describe this problem as follows:

$$\text{Given a profile } \theta \in \Theta^N, \quad \underset{x \in \mathcal{A}}{\text{Max}}\; W = \sum_{i \in N} \sum_{j \in K} \theta^i_j x_{ij}\ ;$$

$$\text{such that} \quad \text{A1)} \sum_{j \in K} x_{ij} \leq 1, \quad \forall i \in N;$$

$$\text{A2)} \sum_{i \in N} x_{ij} \leq 1, \quad \forall j \in K; \qquad \text{(A)}$$

$$\text{A3)}\ x_{ij} \in \{0,1\}, \quad \forall i \in N, \ \forall j \in K.$$

Koopmans and Beckmann (1957) were the first to consider this problem in an economic context. One of their results is that the problem (A), constrained by A1, A2 and $x_{ij} \in [0,1]$, must always have a solution of the form $x_{ij} \in \{0,1\}$, $\forall i \in N$, $\forall j \in K$. They also noted that there always exists a solution to the problem but that it is not necessarily unique. In addition, they found that there always exists a competitive equilibrium[4] set of prices $\{p_j \geq 0\}$ $j \in K$, which may not be unique. A further observation concerned the additive invariance of the parameter θ^i. That is, if a positive constant is added to every element in the vector θ^i, then the solution remains the same. If an allocation solves (A), then we say that it is ***outcome efficient***, and we call W the ***total (or social) welfare*** of the system.

[4] A competitive equilibrium price vector is simply a vector of prices, indexed by a slot number, such that there is no excess demand for any slot and each slot is allocated to an agent at these prices. That is, if we announce these prices, every individual will be satisfied buying the slot to which he has been optimally assigned.

3. Allocation Mechanisms

In the literature, two classes of mechanisms have been proposed to solve assignment and matching problems when each bidder is interested in acquiring at most one item. One class involves auction mechanisms and the second class involves direct revelation of ordinal preferences (or rankings). We discuss these mechanisms and introduce a third class that does not involve monetary transfers but does rely on a richer message space than rankings over slots. Formal descriptions of the mechanisms are given in the Appendix C.

3.1 Transfer mechanisms

Given the environment described above, several mechanisms are available to implement the outcome-efficient allocation in weakly dominant strategies. However, these mechanisms require that payment be made in transferable utility (monetary transfers with quasi-linear utility). These mechanisms are multi-object generalizations of the "second-price" auction first described by Vickrey (1961). In these mechanisms the allocation is outcome-efficient and the prices paid by each agent are the minimum market clearing prices. We shall call these the Vickrey prices.

The extension of Vickrey's model to the case of several buyers and sellers and heterogeneous goods was first proposed by Barr and Shaftel (1976), hereafter BS, who proposed a variation of a Dutch clock auction to obtain the optimal allocation and Vickrey prices.[5] For a single object, a Dutch clock auction is conducted by first setting an arbitrarily high asking price for the object and

[5] Thompson (1979) recognized that the Barr-Shaftel model is an assignment market game being solved for a special core point, the buyer surplus point. Thompson also extended the model to transportation market games and discussed their core points.

allowing the price to fall steadily by a fixed amount until an agent stops the clock. This act is an offer to buy the object at the current price, so the object is then sold to the agent who stopped the clock at the current price.

In the BS variation, a second-price Dutch clock is used. In the single-object version of the second-price Dutch clock auction, the price starts at an arbitrarily high value. As it is lowered, bidders submit bids (a bid is an offer to buy at the announced price). The price continues to fall until there are two bids on the object. No bidder knows if another bid has been placed. The object is sold to the bidder who entered a bid at the highest price, and he pays a price equal to the price at which the second bidder entered. The multiple-object generalization is described in Appendix C.

Leonard (1983) proposed a sealed-bid auction to obtain the optimal allocation and Vickrey prices, which we shall call the Vickrey-Leonard mechanism. The Vickrey-Leonard auction requires each bidder to submit a sealed bid listing his valuation of each of the items. The planner then determines the assignment by solving the assignment problem (A) using each bidder's submitted bids in place of her valuations. There are two ways to find the prices that the agents must pay. One way is to compute the impact of a second slot of similar type. This entails the solution of k additional assignment problems.[6] A computationally simpler solution is to find the minimum dual prices.[7] Given a profile $\theta = (\theta^1, \ldots, \theta^n)$, prices are determined by solving the dual program:

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[6] Prices can be computed directly by setting $p_j = (W_N^{K+j} - W_N^K)$, where $W_N^K \equiv$ largest sum of bids on slots $K = \{1, \ldots, k\}$ assigned to agents in N, and $W_N^{K+j} \equiv$ maximum of the sum of bids on slots $K \cup \{j\}$ to agents in N; that is, add another slot j and solve the assignment program and obtain W_N^{K+j}. This is how the price calculation was described to the subjects--see Appendix B for instructions.

[7] A similar approach was used by Rassenti, Smith, and Bulfin (1982).

$$\underset{p_j}{\text{Min}} \quad \sum_{j \in K} p_j$$

such that

$$w_i + p_j \geq \theta^i_j \qquad \forall j \in K, \forall i \in N$$

$$\sum_{j \in K} p_j + \sum_{i \in N} w_i = W$$

$$w_i, p_j \geq 0, \qquad \forall j \in K, \forall i \in N,$$

where w_i are slack variables.

For this sealed-bid auction, Leonard (1983) and Demange and Gale (1985) have shown that it is a dominant strategy for agents to reveal their true valuations; that is, they have shown there is no other strategy that provides a strictly higher payoff to agents. However, in some environments truthful revelation is not necessarily a strong dominant strategy, in the sense that there may be many bids which generate the same outcome for an agent, a fact that we make precise in the following theorem.

Theorem : When $n \leq k$, in the Vickrey-Leonard mechanism, it is a weakly dominant strategy to reveal, and all weakly dominant strategies differ by a constant on all slots; *i.e.*, $b^i_j = \theta^i_j + c$, $c \in \Re$.

proof. Recall that the optimal solution to the assignment problem does not change when a constant is added to any row of the valuation matrix (see, *e.g.*, Koopmans and Beckman (1957)). That is, suppose for some fixed $i^* \in N$, $\hat{\theta}^{i^*}_j = \theta^{i^*}_j + c$, $\forall j \in K$; then $\sum_i \sum_j \hat{\theta}^i_j x_{ij} = \sum_i \sum_j \theta^i_j x_{ij} + c$. The (x_{ij}) that maximizes $\sum_i \sum_j \hat{\theta}^i_j x_{ij}$ also maximizes $(\sum_i \sum_j \theta^i_j x_{ij} + c)$. So two vectors of bids that differ by a constant will assign an agent to the same slot, and since the price an agent pays is independent of his reported valuations, the price he pays is the same price when his bids differ by a constant. Since the allocation and the price of the slot are the same when an agent's bids differ by only a constant, he is indifferent between submitting the two bids.

□.

Demange, Gale and Sotomayor (1986), hereafter DGS, proposed two variations of an English auction to obtain the Vickrey prices.[8] For a single-object auction an English clock auction is conducted by first setting an arbitrarily low asking price for the object; each bidder then announces that he wants to buy the object at the announced price. If only one bidder demands the object, he is awarded it at the announced price and the auction ends. If more than one bidder demands the object, the price is increased by a fixed amount. The auction continues by increasing the price until only one bidder demands the object.

DGS proposed an "exact" and an "approximate" auction (both auctions are multiobject variations of the English clock auction), where bids are requests to buy an object at the announced price. The two variations differ in the procedure used to determine which objects will have their prices increased in the next step. The exact auction requires computation by a centralized algorithm at each step and produces the exact Vickrey prices. The approximate auction does not use a centralized algorithm and produces an outcome that is "close" to the Vickrey prices.

Both the exact and the approximate auctions begin with the planner's announcing a set of prices, one for each slot. Each agent submits a bid for each

[8] Leonard (1983) and, Demange, Gale and Sotomayor (1986) seem to be unaware of the work done by Barr and Shaftel (1976).

slot, which is a request to purchase the item at the announced price; selecting more than one slot implies that the agent is indifferent among the slots selected for assignment. After bids are submitted, the exact auction requires the prices to be increased in the largest, pure-overdemanded set.[9] A set of items is overdemanded if the number of bidders demanding only items in this set is greater than the number of items in the set. The largest pure-overdemanded set contains all overdemanded sets. The auction ends when there are no overdemanded sets. In the approximate auction a bidder is allowed to bid on only a single item, and is allowed to bid only if he does not have an outstanding bid on an item. If a bidder bids on an item, he cannot remove his bid; he is committed. A bidder can become uncommitted only if another bidder places a bid on that item, at which time the price of that item is increased. Both the exact and the approximate auctions are described in more detail in Appendix C.

Both DGS and BS rely on the implicit assumption that agents will act honestly,[10] that is, bid on all those items and only on those items that maximize his utility, given a set of prices. If this assumption is met, then the outcome-efficient allocation and Vickrey prices will be obtained in the BS Dutch auction and the DGS "exact" auction.

However, for the DGS "exact" auction process, honesty is not necessarily a dominant strategy. The following example shows that in the DGS exact auction honesty is not necessarily a dominant strategy.

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[9] This variation is due to Mo (1988).

[10] A bid $B \subset K$ is *honest* if $\forall l \in B,\ v_l - p_l = \max_{j \in K}(v_j - p_j)$; if $\forall j \in K\ (v_j - p_j) < 0$, then $B = \emptyset$ and the agent does not submit a bid, where p_j is the price of item j, and v_j is the value of slot j to the bidder.

Example:

Let the profile be: $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 3 & 4 & 0 \end{bmatrix}$, where rows indicate agents, columns indicate slots, and the element v_{ij} is the value of slot j to agent i. At the first iteration, prices and bids are:

t	prices:	net valuation:	honest bids:	overdemanded slots:
0	$p^0 = (0,0,0)$	$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 3 & 4 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	1
1	$p^1 = (1,0,0)$	$\begin{bmatrix} 0 & 0 & \underline{0} \\ \underline{3} & 2 & 0 \\ 2 & \underline{4} & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & \underline{1} \\ \underline{1} & 0 & 0 \\ 0 & \underline{1} & 0 \end{bmatrix}$	none.

At $t = 1$ the allocation $(3,1,2)$ can be made with the resulting net valuations $(0,3,4)$.

Suppose that at $t = 0$ and $t = 1$, agents 1 and 2 do not bid honestly but instead submit bids either strategically or in error (indicated by the superscript e), and that in the following iterations they submit bids honestly, and agent 3 submits bids honestly at all iterations:

t	prices:	net valuation:	bids:	overdemanded slots:
0	$p^0 = (0,0,0)$	$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 3 & 4 & 0 \end{bmatrix}$	$\begin{bmatrix} 0^e & 0 & 0 \\ 0^e & 1^e & 0 \\ 0 & 1 & 0 \end{bmatrix}$	2
1	$p^1 = (0,1,0)$	$\begin{bmatrix} 1 & -1 & 0 \\ 4 & 1 & 0 \\ 3 & 3 & 0 \end{bmatrix}$	$\begin{bmatrix} 0^e & 1^e & 0 \\ 0^e & 1^e & 0 \\ 1 & 1 & 0 \end{bmatrix}$	2
2	$p^2 = (0,2,0)$	$\begin{bmatrix} 1 & -2 & 0 \\ 4 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$	1
3	$p^3 = (1,2,0)$	$\begin{bmatrix} 0 & -2 & \underline{0} \\ \underline{3} & 0 & 0 \\ 2 & \underline{2} & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & \underline{1} \\ \underline{1} & 0 & 0 \\ 1 & \underline{1} & 0 \end{bmatrix}$	none.

The final allocation is $(3,1,2)$ with net valuations: $(0,3,2)$. The bidding behavior in the above example by agents 1 and 2 did not make them better off but made agent 3 worse off. This type of behavior is described as bossy or corruptible and is discussed in Olson (1991).

Now suppose that agent 3 responds dishonestly (indicated by the superscript d) to agents 1 and 2's errors at $t = 0$:

t	prices:	net valuation:	bids:
0	$p^0 = (0,0,0)$	$\begin{bmatrix} 1 & 0 & 0 \\ 4 & \underline{2} & 0 \\ \underline{3} & 4 & 0 \end{bmatrix}$	$\begin{bmatrix} 0^e & 0 & 0 \\ 0^e & \underline{1}^e & 0 \\ \underline{1}^d & 0^d & 0 \end{bmatrix}$.

An immediate allocation can be made. The final allocation is $(-,2,1)$ with net valuations: $(0,2,3)$. Hence, agent 3 is made better off by bidding on slot 1 when slot 2 has a higher net value, given the prices $(0,0,0)$.

If honesty were a dominant strategy, it would provide the best (or no worse) outcome for an agent independent of the bids the other agents submit before or after his bid.

We have presented a circumstance where it is not to an agent's advantage to bid honestly. It required that the agent take advantage of another agent's errors or strategic bids. We make this clear in the following theorem, where we show that honesty is a Nash equilibrium.

Theorem : In the DGS "exact" auction, it is a Nash equilibrium for each agent to select the slot(s) that maximize utility at each price announcement (truthful revelation); *i. e.*, given $(p_1^t, \ldots, p_k^t)$, agent i selects the set of slots $l \in K$, such that $(\theta_l^i - p_l^t) = \max_j \{\theta_j^i - p_j^t\}$, and $(\theta_l^i - p_l^t) \geq 0$.

Before we prove the theorem, we provide some notation and describe behavior that is implied by honest bidding. At each iteration t, an agent places a bid $b^t \subseteq K$, which is a selection of slots. At iteration $t+1$, $b_p^{t+1} \subseteq b^t$ is the set of slots that the agent bids for at iteration t and that also had a price increase at iteration $t+1$. At iteration $t+1$, $b_c^{t+1} \subseteq b^t$ is the set of slots that the agent bid for at iteration t and that did not have a price increase at iteration $t+1$. The following implications are implied by honest bidding:

1. If $b^t = \emptyset$ at t, then $b^{t+1} = \emptyset$ at $t+1$. If an agent does not bid for a slot at iteration t, she will not bid for a slot at iteration $t+1$.

2. If $b^t \subseteq K$ at t and $b_p^{t+1} = b^t$, then either $b^{t+1} = b^t \cup b$, or $b^{t+1} = \emptyset$, where $b \subseteq K \backslash b^t \cup \emptyset$. If an agent bids for a set of slots at iteration t and each slot has a price increase at iteration $t+1$, then either the agent will bid on the same slots and possibly an additional slot, or she will not bid on any slot.

3. If $b^t \subseteq K$ at t and $b_p^{t+1} \subset b^t$ (a proper subset), then $b^{t+1} = b_c^{t+1}$. If the price increases on some of the slots that the agent has bid for at iteration t, then he will drop his bids on those slots and keep his bids on the slots that did not have an increase in price.

We now provide the proof:

proof: In the DGS auction, if all agents are honest, then the vector of prices p^e that results from the auction is the vector of minimum core prices (or competitive equilibrium; see DGS). By definition of the core, an agent cannot increase his net value in any slot other than the slot assigned to him. So once the equilibrium prices are reached, there is no advantage for an agent to be dishonest. If $p^t \geq p^e$ and all agents are honest, the auction stops; if an agent is not honest, the auction will stop and he will be no better off at another slot. If the auction does not stop, then there will be a price increase on some slot, and the agent cannot be made better off, since p^e gives him his highest net value.

Also, by definition of the core, if $p_j^t < p_j^e$ at some iteration t and some slot j there will be overdemanded slots. If all agents are honest, prices will increase at $t+1$. So an auction will end with $p_j^t < p_j^e$ only if an agent is dishonest. But that would imply removing a bid from slot j or adding a bid to another slot. In either case the agent is worse off if the auction stops (since bidding on his highest net-valued slot is the honest bid).

□.

An agent cannot directly make himself better off by being dishonest; only if other agents bid dishonestly in the following iterations is it possible to be made better off by a dishonest bid. We note that honesty is not a weakly dominant strategy in our environment, and that there may be many Nash equilibria that do not correspond to the social optimum, though our experimental results indicate that 100% honesty is not necessary to obtain an outcome-efficient allocation.

Theoretically, all of these mechanisms (Leonard, BS, DGS) yield the same outcome (outcome-efficient assignment and Vickrey prices), assuming that bidders act honestly. In the experimental literature there has been much success with the use of progressive (English) auctions in obtaining efficient allocations (examples of this literature are Banks et al. (1989) and McCabe et al. (1990)) . This is not so true of their sealed-bid counterparts. The ability of the English auction to provide feedback to participants concerning where they stand and how to improve their current standing appears helpful. To test the viability of this assumption, we will investigate to determine whether a sealed-bid and a progressive auction (the "exact" auction of DGS) yield the same outcomes experimentally.

While the auction mechanisms have the advantage that the outcome is efficient, the amount of surplus that is extracted from the buyer can be large.

For example, let the profile be:

slot	1	2
Buyer 1	10	0
Buyer 2	9	0 .

The Vickrey prices are $p_1 = 9$, $p_2 = 0$, consumers' surplus = 1, and welfare = 10. We observe that ex ante, the random allocation mechanism yields a consumers' surplus = $9\frac{1}{2}$, and a welfare = $9\frac{1}{2}$.

The situation is different if the profile is:

slot	1	2
Buyer 1	10	0
Buyer 2	0	9 .

The Vickrey prices are $p_1 = 0$, $p_2 = 0$, consumers' surplus = 19, and welfare = 19. We observe that ex ante, the random allocation mechanism yields consumers' surplus = $9\frac{1}{2}$, and a welfare = $9\frac{1}{2}$. In general the less diverse[11] the profile, the more likely it is that the random mechanism will yield higher ex ante utility to the agents.

There are a number of possible ways one might provide individual agents with an increase in consumer welfare above the Vickrey allocation without drastically reducing social welfare. One possible mechanism is to assign the slots to individuals in some fashion (perhaps randomly) and then allow agents to trade among themselves. This type of aftermarket has been studied by Grether, Isaac and Plott (1981), and Rassenti, Smith and Bulfin (1982) in the context of allocating airline landing slots. These results indicate that the use of after-markets increased efficiency above the initial allocation. The advantage of

[11] A profile is diverse if it is possible to assign each agent her most preferred slot. A profile is less diverse than another profile if fewer agents can be assigned their most preferred slot.

allowing agents to trade among themselves is that all the surplus goes to the agents.

The disadvantage of a bilateral trade aftermarket is that it may be difficult for agents to coordinate. One possible solution to the coordination problem is to add a rebate rule to the VL auction that returns the money paid to the agents. With a rebate rule that returns the entire amount collected to the agents, the VL mechanism loses its dominant strategy property, since the net transfer to an agent depends upon his own bids. It is an open problem whether the ability to coordinate offsets the lack of honest revelation.

We will investigate a different approach, which is often used in practice (e.g. Chicago Business School assignment of students to interview slots, and computer scheduling), but which has received little attention in the theoretical literature and no attention in the experimental literature: the use of nontransfer mechanisms. We will discuss this approach in the next two sections.

3.2. Nontransfer Ordinal Mechanisms

The second class of mechanisms is a one-sided variation of the Gale-Shapley matching algorithm[12] used to solve the marriage problem. In the one-sided matching environment, this mechanism is equivalent to the serial dictator (SD) (see Luce and Raiffa (1957); Satterthwaite and Sonnenschein (1981), discussions of the serial dictator).

In the SD mechanism the planner chooses an ordering of the agents. Each agent in turn (according to the order chosen by the planner) selects her slot from the slots not chosen by the predecessors. The process continues until all slots

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[12] Also known as the deferred acceptance algorithm.

have been allocated or all agents have been assigned slots. This mechanism can also be implemented by asking agents to submit their rankings over slots before the planner randomly chooses an ordering, allowing the planner to allocate the slots using the agents' submitted rankings and the realized ordering. This random variation has the property that it is symmetric. In both variations of the SD mechanism it is a dominant strategy for an agent to reveal her rankings over slots. Under honest revelation the outcome is ex post Pareto efficient although it is not outcome efficient as the following example shows:

slot	1	2
Buyer 1	8	4
Buyer 2	10	5.

If the ordering is {1,2}, then the SD allocates slot 1 to agent 1 and slot 2 to agent 2 for a total welfare of 13, but the outcome-efficient allocation is slot 2 to agent 1 and slot 1 to agent 2 with a total welfare of 14.

When there is indifference of slots, an agent may have a number of strict rankings that he can truthfully report. The indeterminacy could have been avoided by using the following variation of the serial-dictator mechanism.[13] Allow agents to report indifference. When the next agent to be assigned a slot has several most preferred slots, slots will be assigned to those agents succeeding him until there is only one of his most preferred slots left, which is assigned to him.

However, unlike the transfer mechanisms discussed previously, the SD does have the property that none of the surplus goes to the planner. Thus, while the SD is less efficient than these transfer mechanisms, it can yield greater

[13] This variation was described by Zhou (1990a).

consumer surplus to the agents.

In both the transfer and nontransfer mechanisms described, honest revelation is a dominant strategy in the respective message spaces. A major difference between the two mechanisms is the ordinal message space of the nontransfer mechanism and the cardinal message space of the transfer mechanism. We now ask the question: Can we use a richer message space and get a more efficient nontransfer mechanism if we drop the requirement of dominant strategy implementation?

3.3. A chit mechanism

A ***chit*** is defined as a medium of exchange whose value is determined solely in the context of the given assignment problem (environment), and which has no value for goods or services outside the assignment problem. A chit mechanism has a message space that allows each person to allocate a certain number of points (or chits) to any of the items he wishes, from a predetermined chit budget; the amount of chits he retains does not affect his utility.[14] Any transfer mechanism can be employed by using chits instead of money as the medium of exchange. Instead of agents being allowed to pay for the slots they obtain with cash, they must use chits.

Little is known about the properties of this class of mechanisms. We will study it in the context of the sealed-bid VL mechanism. We use the VL mechanism for its known properties with cash transfers: efficiency and the existence of a competitive equilibrium. While the VL mechanism using chits does not have a dominant strategy equilibrium it does have Bayesian equilibria, which

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[14] An example of a chits mechanism is the implicit market mechanism of Hylland and Zeckhauser (1979).

depend on the distribution of types (or an agent's belief about the distribution). In general these equilibria do not elicit more information about an agent's relative valuations than just ordinal rankings. For some distributions of types it is a best response for an agent to place all her chits on her preferred slot or on her second preferred slot independent of her relative slot values.[15] In other cases, an agents optimal placement of chits is spread over more than one slot and depends her slot values. While it appears that there is little information conveyed in the bids, the dependence of the bids on the distribution of types and individual slot values may provide sufficient information to yield a more efficient assignment than a mechanism that uses rankings of slot values.

4. Experimental Design

The experimental design consists of two fixed factors: type of mechanism (sealed-bid (Vickrey-Leonard) auction, sealed-bid chit, progressive (DGS) auction, and serial dictator), and parameter set (high and low contention). We begin by discussing the parameters of the environment and then describe the payment conditions. We end this section with a summary list of the experiments we have conducted.

4.1. Parameters of the Environment

The environment under consideration consists of 6 slots, $K = \{1, 2, \ldots, 6\}$, which must be allocated to a set of agents. We consider an environment where there are 6 agents and an environment where there are 8 agents.

For our experiments we consider two distinct sets of parameters defining

[15] Appendix F contains a detailed discussion of the equilibria of the chit mechanism.

preferences. Preferences are induced using monetary payoffs for each slot provided to each agent (see Smith (1976)). Each participant could be assigned one of 10 possible payoff sheets that defined her type. An abbreviated list of payoffs is provided below in Table 1 (the complete listing of the payoffs used in our experiment can be found in Appendix A). For example, given the payoff list in Table 1, if an agent were provided with sheet 2 and assigned item 3, he would obtain a value of 800. At the beginning of each period each subject is assigned a payoff sheet that is drawn uniformly from the set with replacement, i.e., the fact that priors over types are uniform was given as ***common knowledge*** to the subjects.

Table 1:

An Example of a Payoff List

Payoff Sheet	**Item Number**					
Number	1	2	3	4	5	6
1	800	600	400	200	400	600
2	400	600	800	600	400	200
.						
.						
10	300	300	300	300	300	900

Given the payoff tables, priors, and number of agents, we can solve for the optimal assignments and the set of competitive equilibrium prices (***the core***). Let p_j denote the minimal dual prices in the core determined from (1), which we call the ***Vickrey prices***. If v_j^* is the value of slot j from the optimal assignment determined in (A), then the closer p_j is to v_j^*, the higher is the level of competition for the slot (more of the buyers' surplus is transferred to the planner). Competition for a slot is a function of both the profile and the number of agents wanting a slot allocation. In our experiments we created two alternative competitive environments based on the following ratio we call the ***contention index*** ($\mathbb{C}$):

$$\mathbb{C} = \mathbb{E}_L \frac{\sum_j p_j}{W^*} ,$$

where j indexes the slot, p_j is its Vickrey price, W^* is the outcome-efficient welfare for a profile of payoff sheets from the payoff list, and $\mathbb{E}_L$ is the expectation operator defined over the possible profiles from a given payoff list.[16] Notice that $\mathbb{C} \in [0,1]$. A realization of $c = 1$ implies that all the surplus in the system is paid out at the "competitive" equilibrium prices, and a realization $c = 0$ implies that the profiles are diverse and all the surplus is retained by the agents. For an example of $\mathbb{C} = 1$, let the profile list be such that:

slot	1	2
Buyer 1	10	0
Buyer 2	10	0 .

In this example the welfare = 10, and the Vickrey prices = (10,0); hence, $\mathbb{C} = 1$. For an example of $\mathbb{C} = 0$, let the profile list be such that:

slot	1	2
Buyer 1	10	0
Buyer 2	0	10 .

In this example the welfare = 20, and the Vickrey prices = (0,0); hence, $\mathbb{C} = 0$.

Varying $\mathbb{C}$ in the experiments provides us with a check on the robustness

[16] We could define this index for each slot by dividing the Vickrey price of the item by the value of the item determined in outcome-efficient allocation; we could also define contention as the mean of the contention of each slot. We chose the definition above for its relative simplicity.

of potential allocation mechanisms, so that we may explore the hypothesis that the surplus and efficiency of the tested mechanisms are sensitive to the expected contention in a particular environment. For our experiments two environments are considered: one with a "low-contention index" and one with "high-contention index." The low-contention environment utilizes six people and six items, with values such that, on average, there was contention for one or two of the items. In the high-contention environment, there are six items and eight agents and the values were such that almost all items would have a high-contention index. Figure 1 supplies a graph of the actual realization of contention levels used for the low-contention and high-contention treatments. Appendix A supplies the individual draws and associated core prices for each slot for the experiments we conducted.

Table 2 shows our 2x2 design. The number of experiments for each cell is listed. A summary list of each experiment we conducted is listed in Table 3. All of our experiments were conducted at the California Institute of Technology using graduate and undergraduate subjects. Each experimental session lasted for 20 periods where at the beginning of each period, each subject would be given a payoff sheet. All communication was done through computer terminals, and a history of prices and personal selections was provided by the software so that subjects could review past periods. Each experimental session consisted of only one allocation mechanism and one set of payoff parameters (high-contention or low-contention parameters). A partial set of subject instructions can be found in Appendix B.

Table 2

2x2 Design Factors
(number of experiments per cell)

Mechanism	*Environment Parameters* Low contention[17]	High Contention
Serial Dictator	1	2
Vickrey-Leonard	3	2
Demange-Progressive	2	2
Chit-VL	1	2

Given the environment defined above, the planner's objective is to design allocation mechanisms to assign slots to agents, which result in the maximum social welfare. We consider this design question next.

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17 In the low-contention environment we ran only 1 experiment in the serial-dictator and chit treatment. In these instances we had strong prior beliefs about subject behavior and the outcome of the experiment; after the observation was made, the deviation from this prior belief was not strong enough to persuade us that another experiment would provide additional information. This is a nontechnical use of sequential experimental design. While the author realizes the need for more development and rigorous design procedures their development here would distract from the emphasis of this study.

5. Implementation of Allocation Mechanisms Tested

There were four mechanisms tested: the Vickrey-Leonard sealed-bid auction, the DGS progressive auction, the serial dictator, and a chit implementation of the VL sealed-bid auction.

Sealed-bid auction

The implementation of the VL mechanism in our experimental environment was straightforward. At the beginning of a period each subject was given a payoff sheet (on her computer terminal) listing the value of each slot. Subjects submitted a sealed bid for each of the six items to be allocated (if no bid was entered for an item, it was assumed to be 0). Each subject's bid consisted of a vector of monetary bids $(b_1^i, \ldots, b_k^i)$ over the slots with the restriction that $b_j \in [0, 9999]$, $\forall j \in K$. The allocation is determined by solving the integer program described in (A), replacing θ_j^i with b_j^i (using bidder's submitted bids in place of their valuations).[18] The prices were determined by solving the dual program. Once the allocation and prices were determined, they were transmitted to the subjects, profits were then calculated and histories updated, after which a new period was started.

Progressive auction

The implementation of the DGS auction was more involved. The process proceeded as follows: First, at the beginning of a period (iteration $t = 0$) initial prices were set at zero for each slot. Given these prices individuals selected the slots they would like at those prices. Given the selections, an algorithm was used

[18] If there were ties in the bids to determine allocations, they were broken randomly. If a slot was not demanded in the auction, it was assigned randomly to those who were not previously assigned a slot.

to determine whether the process was to stop or which slots were overdemanded. The algorithm is a variation of the Ford-Fulkerson procedure (see, *e.g.,* Franklin (1980) and Gale (1960)).

At each iteration the algorithm returned either an assignment or a collection of overdemanded slots. For each overdemanded slot the price at the next iteration was increased by 50 francs. In a pilot we tried increments of 10 and 25 francs but found that each period took too long. The process stopped when there were no overdemanded slots; an assignment was then made. Those assigned to the slots paid the current price, except in an instance that is described below.

The implementation of the DGS auction process provided a number of difficulties. The first difficulty was the possibility of subjects using dishonest bids to manipulate the outcome. In order to contain this possibility, we imposed a commitment rule. If an agent selected a slot at iteration and the slot was not overdemanded, then he was ***committed*** to select that slot at the next iteration (*i.e.,* agents could not renege on selections if the price of those selections did not increase).

The second difficulty involved the inability of the mechanism to elicit bids when a subject's maximum net value was zero, or when there were more agents than slots and a price increase caused the number of bidders to drop to zero. This can be observed in the following examples.

Example:

Let the profile be: $\begin{bmatrix} 10 & 10 \\ 5 & 10 \\ 23 & 11 \end{bmatrix}$, where rows indicate agents, columns indicate slots, and the element v_{ij} is the value of slot j to agent i. The outcome-efficient assignment and the VL prices are either $(-,2,1)$ or $(2,-,1)$ and $p=(10,10)$. Consider the outcome when subjects do not bid on zero net valued slots:

t	prices:	net valuation:	bids:	overdemanded slots:
0	$p^0=(0,0)$	$\begin{bmatrix} 10 & 10 \\ 5 & 10 \\ 23 & 11 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	1, 2
t^*	$p^t=(9,9)$	$\begin{bmatrix} 1 & 1 \\ -4 & 1 \\ 14 & 2 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	1, 2
t^*+1	$p^t=(10,10)$	$\begin{bmatrix} 0 & 0 \\ -5 & 0 \\ \underline{13} & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \underline{1} & 0 \end{bmatrix}$	none.

The outcome assigns $(-,-,1)$ and $p=(10,10)$.

Suppose subjects do bid on zero net valued slots:

t	prices:	net valuation:	bids:	overdemanded slots:
0	$p^0 = (0,0)$	$\begin{bmatrix} 10 & 10 \\ 5 & 10 \\ 23 & 11 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	1, 2
10	$p^t = (10,10)$	$\begin{bmatrix} 0 & 0 \\ -6 & 0 \\ 13 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	1, 2
11	$p^t = (11,11)$	$\begin{bmatrix} -1 & -1 \\ -6 & -1 \\ \underline{2} & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \underline{1} & 0 \end{bmatrix}$	none.

The outcome assigns $(-,-,1)$ and $p = (11,11)$.

The second problem can be alleviated by adding an extra slot, so that $k = n$, and as as long as subjects bid when their maximum net value is zero:

10	$p^t = (10,10,0)$	$\begin{bmatrix} 0 & 0 & \underline{0} \\ -5 & \underline{0} & 0 \\ \underline{13} & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & \underline{1} \\ 0 & \underline{1} & 1 \\ \underline{1} & 0 & 0 \end{bmatrix}$	none

The outcome assigns $(3,2,1)$ and $p = (10,10,0)$. But if subjects do not bid on indifferent slots, then the problem remains. To alleviate this problem we used a back-up rule: if at the end of a period a slot is unassigned, then the object is randomly allocated between the last unassigned bidders who placed a bid on the slot at a previous iteration.

Chit process

The chit mechanism was implemented by applying the VL auction with chits used in place of monetary transfers. Each subject was given a budget of 1000 "chits" from which to make bids on the slots; chits that were not used had no value to the participants. The same algorithm used in the VL-money treatment was used in the VL-chit. Also, as in the VL-money, the procedures were computerized and subjects were provided information on the past history of their decisions, and market prices of all items were supplied as public information to participants.

Serial dictator mechanism

We also conducted a set of experiments using the random serial dictator. The process was implemented by asking subjects to submit a strict rank order (1 through 6) over slots. A randomly drawn ordering of subjects was chosen after they submitted their rankings, and a SD algorithm determined the outcome. Given the planner's ordering and the rankings by the agents, the algorithm selected the first person in the planner's ordering and allocated to him his highest ranked slot. The algorithm proceeded down the planner's ordering and allocated a slot to the next agent in the list according to his ranking so that his highest ranked unit, not already chosen by his predecessors, was allocated to him.

In the experiment, subjects were told that the ordering for each period would be random and would be revealed only after the period ended. At the beginning of each period, subjects would get individual payoff sheets displayed on their computer screen. Each subject, through his computer terminal, would submit a unique sealed bid of his preference ordering of the six slots. The algorithm would then determine the allocation and send payoff information to each participant.

Table 3 lists the experiments we have conducted along with pertinent information about each session.

Table 3. Experiment History

name	#	content	frc-conv	payoff	time	date
leo1	1	high	400	$7.8+8	1.6	8/29/90
leo2	3	low	800	$15.2	1.2hr	10/3/90
leo3	4	high	400	$9.2+5.0	1.5hr	10/4/90
leo4	8	low	800	$15.00	1.1hr	10/17/90
sd2	9	low	900	$13.25	45min	11/13/90
sd1	10	high	900	$10.45	45min	11/30/90
sd3	11	high	900	$10.00	35min	2/6/91
chit1	12	high	600	$16.38	1hr	11/14/90
chit2	13	low	900	$13.83	1hr	11/14/90
dem61	14	low	800	$18.67	1hr	2/7/91
dem62	15	low	800	$18.00	50min	2/20/91
dem81	16	high	300	$9.38+6	1.5hr	2/13/91
dem82	17	high	150	$16.00	1.5hr	2/18/91
chit3	18	high	600	$15.75	65min	3/15/91
leo5	19	low	800	$15.17	55min	1/15/91

Notes:

All experiments had 6 slots and 20 periods. High-contention experiments had 8 subjects; low-contention experiments had 6 subjects. The name describes the type of experiment: leo = Leonard–Vickrey sealed–bid, chitb = sealed–bid chit, sd = serial dictator, dem = Demange et al., progressive auction.

We also ran 2 pilot VL sealed-bid auctions (1 low-contention and 1 high-contention), one pilot English auction (low-contention), one pilot DGS auction (high-contention), and 4 chit experiments where chit budgets were not renewed after every period (2 low-contention and 2 high-contention).

6. Experimental Predictions

Both the VL and serial-dictator mechanisms have weakly dominant-strategy outcomes. We will assume that individuals play one of their weak-dominant strategies, or their strong dominant strategy if one exists. For the VL this poses no problem since any of the weakly dominant strategies will produce the same outcome. In our implementation of the serial dictator, a subject's indifference over slots and the requirement that submitted rankings be strict can distort the outcome as the following example shows.

Example:

Let values for agent 1 be $(10, 5)$, and for agent 2 be $(5, 5)$. We order the agents $(2, 1)$, and agent 1 truthfully reveals his preference ranking of the slots $(1, 2)$ (1 being the best). If agent 2 ranks the slots $(1, 2)$, then the outcome is $(2, 1)$ (agent 1 gets slot 2 and agent 2 gets slot 1) and the total welfare is 10. If agent 2 ranks the slots $(2, 1)$, then the outcome is $(1, 2)$ and the total welfare is 15. The predictions reported below are the expectation over all possible reported orderings of indifferent slots.

In our implementation of the serial dictator, there is no incentive for agents to misrepresent their ordinal rankings, when we allow any ranking to represent an agent's indifference truthfully. But an agent can manipulate the outcome; that is, he can affect the utility of another agent without affecting his own utility. This concept of manipulation is discussed in Olson (1991) and will be investigated in the section on individual behavior.

As a baseline measure we compare the efficiencies of our experiment with expected efficiencies for a random allocation, and the expected efficiencies for the random serial dictator. The random allocation was calculated as the average over all possible assignments. The SD was calculated by permuting over all possible subject orderings (tie-breaking rules) and assuming that all subjects revealed their rankings truthfully.

For the DGS progressive auction, we make the assumption that all agents are honest, which is a Nash equilibrium, and hence the outcome and prices are equivalent to the VL outcome, which yield the same predictions.

For the chit mechanism we computed a Bayesian Nash equilibrium. The computation of the equilibrium strategies was accomplished via a ***genetic algorithm*** (see Appendix F for the construction). For this initial prediction we restricted the set of strategies that a subject could use. The restricted strategies comprised distributing the chits over a set of slots. For example, if there were 3 slots, an agent could place all of his chits on one slot, divide them between any two, or three of the slots. Noninteger division of chits was truncated. We then computed the outcome, given these strategies. Since the outcome is not unique, given a submission of bids, the predictions are based on the expected outcome, given a submission of bids. The following example clarifies the point.

Example:

Let the values for agent 1 be $(10,0)$, for agent 2 be $(5,0)$, and the number of chits is 100. The best response for both agents is to bid 100 chits on slot 1. If the ties are broken so that the outcome is $(1,2)$, then the total welfare is 10. If the outcome is $(2,1)$, the total welfare is 15, and the expected outcome is 12.5.

The predicted efficiencies and consumer surplus are presented in Table 4 below. The table presents the predicted values for the VL and DGS auctions (theoretically they should have the same outcome), the expected serial dictator (expectation taken over all possible orderings of agents), the expected chit outcome (expectation taken over all possible allocations when the equilibrium bids of agents are the same), the random mechanism (over all possible strictly feasible allocations), and the realized serial dictator (expectation over all possible orderings of an agent's indifferent slots). The mean and standard deviation (σ) measures reported are the mean values of all means and standard deviations for the periods reported.

Table 4: Predictions

Efficiency	Low-contention Periods			High contention Periods		
	All	First10	Last10	All	First10	Last10
VL-DGS	1.00	1.00	1.00	1.00	1.00	1.00
Expected-SD	0.902	0.901	0.903	0.859	0.856	0.862
σ	(0.059)	(0.060)	(0.058)	(0.075)	(0.076)	(0.074)
Chit	0.926	0.922	0.930	0.898	0.902	0.895
σ	(0.035)	(0.032)	(0.029)	(0.047)	(0.044)	(0.049)
Random	0.754	0.748	0.759	0.615	0.612	0.618
σ	(0.095)	(0.10)	(0.092)	(0.082)	(0.083)	(0.082)
Realized-SD	0.912	0.924	0.901	0.831	0.806	0.856
σ	(0.046)	(0.040)	(0.051)	(0.025)	(0.039)	(0.013)
Consumer Surplus	All	First10	Last10	All	First10	Last10
VL-DGS[19]	0.871	0.854	0.887	0.227	0.231	0.224
Expected-SD	1.05	1.07	1.03	4.28	4.19	4.37
σ	(0.07)	(0.07)	(0.07)	(0.36)	(0.36)	(0.36)
Chit	1.07	1.09	1.06	4.44	4.39	4.49
σ	(0.04)	(0.04)	(0.03)	(0.23)	(0.21)	(0.26)
Random	0.880	0.889	0.872	3.09	3.03	3.15
σ	(0.012)	(0.012)	(0.013)	(0.58)	(0.56)	(0.60)
Realized-SD	1.06	1.09	1.03	4.15	3.94	4.37
σ	(0.05)	(0.05)	(0.05)	(0.13)	(0.18)	(0.06)
Revenue						
VL-DGS	490	550	430	3253	3270	3235
σ	(335)	(314)	(362)	(289)	(281)	(311)

19 The entries in the first row of the consumer surplus table are the percent of total welfare that goes to the agents in the outcome-efficient allocation and the Vickrey prices.

The models predict:

1. The random allocation yields uniformly lower efficiencies for both contention parameters. When contention is low the random allocation yields lower consumer surplus than the VL outcome, but when contention is high, the random allocation yields higher consumer surplus than the VL outcome. So when contention is high, agents may prefer a random allocation (ex ante) than an efficient pricing mechanism because of the amount of surplus transferred to the planner (via the prices paid to the planner).

2. The two nontransfer mechanisms, chit and SD, both yield lower efficiencies and higher consumer surplus with both contention parameters. When contention is high the difference is larger. The chit mechanism has slightly higher efficiencies and lower consumer surplus than the SD mechanism. The difference is small and for any realization of the mechanisms, the order may be reversed. In Appendix D a timeseries graph shows these values for each period and contention type.

7. Experimental Results

For each mechanism and environment, we measure two aspects of mechanism performance: efficiency and consumer surplus. The efficiency of the mechanism measures its overall performance relative to the optimal allocation (as defined in (A)); that is, it measures the ability of the mechanism to maximize total welfare. Consumer surplus measures the distribution of system surplus to the agents. These measurements are normalized by the outcome-efficient allocation and the Vickrey prices. For the transfer mechanisms (sealed-bid and English auction), the revenue to the planner is also measured; revenue measures the distribution of surplus to the planner (or seller). We measure how close the observed values are to the predicted values and how the observed values relate to each other. Also, to measure the viability of our behavioral assumptions, we investigate individual behavior.

7.1 Efficiency

Efficiency is measured as the total welfare observed in the experiment divided by the total welfare that would have been realized if the optimal allocation had been implemented. That is, for each period,

$$E = (\sum_{i,j} x_{ij} v_{ij}) / (\sum_{i,j} x^*_{ij} v_{ij}) ,$$

where (x^*_{ij}) is the optimal allocation, and x_{ij} is the allocation that was actually realized. We divide by the total welfare in order to normalize the data of each trial so that we can compare relative efficiencies across trials.

In Table 5 we display the average efficiencies achieved with the Vickrey-Leonard auction, the serial-dictator mechanism, the English auction, and the chit mechanism for low and high contention. Efficiencies are averaged using three different restrictions on an experiment session: (all periods, the first ten periods, and the last ten periods of a session) to see if "learning has occurred."[20] A probability level is presented to test the hypothesis that there is no difference between the first ten and the last ten periods. The t-test and the Lord-test are both performed.

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[20] We will discuss the issue of learning in the section on individual behavior.

Table 5a: Efficiencies

(observed relative to predicted VL outcome)

	Periods All	First10	Last10	t-test	Lord-test[21]
Mechanism					
Low Contention					
Vickrey-Leonard	0.97	0.97	0.97	0.97	0.005
Serial Dictator	0.92	0.94	0.90	0.51	0.2
Sealed-bid-chit	0.92	0.88	0.96	0.21	0.47
DGS Auction	0.95	0.92	0.98	0.34	0.34
High Contention					
Vickrey-Leonard	0.95	0.94	0.97	0.48	0.18
Serial Dictator	0.80	0.76	0.84	0.43	0.42
Sealed-bid-chit	0.89	0.89	0.90	0.92	0.03
DGS Auction	0.99	0.99	0.99	0.91	0.02

The numbers for the t-test are tail probabilities, so a larger number is evidence of no difference between the first and last ten periods. For the Lord test smaller numbers are evidence of a lack of difference between first and last periods.

[21] The Lord test uses the range of variation in place of the estimated variance; it is slightly more robust than the t-test and more likely to reject the null hypothesis. We note that if the t-test fails to reject the null hypothesis, then many of the most common nonparametric tests will also fail to reject the null hypothesis; similarly, if a nonparametric test rejects the null hypothesis, then the t-test will also reject the null hypothesis. So in these two cases, one test includes the other.

Table 5b: Efficiencies
(observed vs. predicted)

Experiments	1	2	3	Predicted
Mechanism				
Low Contention				
Vickrey-Leonard	0.97	0.97	0.97	1.00
σ	0.034	0.035	0.044	
Serial Dictator	0.92			0.912 ± 0.046
σ	0.058			
Chit	0.92			0.926 ± 0.035
σ	0.082			
DGS Auction	0.92	0.98		1.00
σ	0.15	0.046		
High Contention				
Vickrey-Leonard	0.95	0.96		1.00
σ	0.075	0.058		
Serial Dictator	0.81	0.79		0.831 ± 0.025
σ	0.088	0.13		
Chit	0.90	0.88		0.898 ± 0.047
σ	0.079	0.063		
DGS Auction	0.99	0.99		1.00
σ	0.032	0.036		

From Table 5 we can make a number of observations:

1. In all trials there is no significant difference between the first 10 and the last 10 periods, suggesting that if learning did take place among subjects, it did not significantly affect efficiency.

2. Except for the high-contention SD and low-contention DGS, all the observations appear to be very close to predicted efficiencies. In the high-contention SD there were individuals whose behavior differed from predicted behavior, these observations will be discussed in the section on individual behavior. In the low contention DGS some of the subject were confused in the first five periods, we call this the Chris effect; Chris is an undergraduate who helped run this particular experiment.

3. For both contention treatments the chit and SD mechanisms resulted in lower efficiencies than the use of transfers, but there is a smaller difference in the low-contention treatment.

4. The use of chits yielded a significantly higher efficiency than the serial-dictator mechanism for the high-contention treatment but no difference for the low-contention treatment.

To determine if the difference in efficiencies is significant we estimate the following model:

$$y_{ep} = \beta_{1m_e} + \beta_2 f(p) + \beta_3 c_{m_e p} + \epsilon_{ep},$$

where:

$p \equiv$ period number $\in \{1, \ldots, 20\}$.

$e \equiv$ experiment index.

$m_e \equiv$ mechanism used in experiment e, {VL, SD, chit, DGS}

$y_{ep} \equiv$ efficiency of experiment e and period p.

$f(p) \equiv$ a monotonic function of the period.

$c_{m_e p} \equiv$ contention of mechanism m_e

$\epsilon_{ep}, \equiv$ error term, $\mathbb{E}[\epsilon_{ep}] = 0, \quad \mathbb{E}[\epsilon_{ep}^2] = \sigma_\epsilon^2$.

$\beta_{1m_e} \equiv$ intercept, which varies over mechanism and captures the difference in behavior over mechanisms.

$\beta_1, \beta_2 \equiv$ coefficients of period variate and contention, respectively.

This is estimated as a fixed effects model since the mechanisms are not chosen randomly (it would be a random effects model if we were modeling individual behavior and wished to make imputations to the general population from which the individuals were randomly chosen). The model we are estimating is variously called time-series cross-section model, panel data model, or an analysis of covariance (see, *e.g.,* Judge et al. (1985), and Hsiao (1986)). In the following estimation we used:

$$f(p) = 1/p.$$

The group variables referenced below are the mechanisms tested:

Vickrey-Leonard $= 1$, serial dictator $= 2$, Chit $= 3$, DGS $= 4$.

Contention was the $\mathbb{C}$ index computed separately for each period.

Table 5c: Efficiencies
(fixed effects model)

Dependent variable: Efficiency

Observations	: 300	
Number of Groups	: 4	
Degrees of freedom	: 294	
Residual SS	: 1.184	
Std error of est	: 0.063	
Total SS (corrected)	: 1.335	
F	= 18.807	with 2, 294 degrees of freedom
P-value	= 0.000	

Variable	Coef.	Std. Coef.	Std. Error	t-Stat	P-Value
1/PERIOD	-0.093783	-0.306241	0.016830	-5.572492	0.000
CONTENT	-0.031547	-0.152608	0.011361	-2.776928	0.006

Group Number	Group Variable	Standard Error
1	0.993985	0.008353
2	0.873379	0.010862
3	0.936658	0.010862
4	0.998409	0.009325

F-statistic for equality of group variables :
F(3, 294) = 57.1668 P-value: 0.0000

The estimation indicates:

1. The period is significant with the first period averaging 9% lower than the last period.

2. Contention is significant with high contention averaging 3% higher than low-contention efficiencies for the same mechanism.

3. The model specification is significant though it accounts for only 10% of the variation.

4. The mechanisms are a significant effect and are ranked by efficiency generation:

$$\text{DGS} > \text{VL} > \text{Chit} > \text{SD}$$

5) The model was also estimated under various groupings, specifications of $f(p)$, and measurements of efficiency; and the results indicate that the specification is robust.

We conclude that the use of the SD and chit mechanisms resulted in lower efficiencies than the use of cash transfers. The Vickrey-Leonard and DGS auctions seem to result in efficiencies close to the theoretical prediction, and the loss of information from using a nontransfer mechanism did result in lower efficiencies. We can also conclude that in the low-contention treatment, the efficiencies from the chit experiment did not differ from the efficiencies of the SD mechanism, but with high contention, the chit mechanism did significantly better (in terms of efficiencies) than the SD mechanism.

7.2 Consumers' Surplus, Prices and Revenue

We measure relative consumers' surplus as the sum of the surplus realized by all agents divided by the sum of the surplus that would have been realized if the optimal allocation and the Vickrey prices had been implemented.[22] That is:

$$S = (\sum_{i,j} x_{ij} v_{ij} - \sum_j p_j)/(\sum_{i,j} x^*_{ij} v_{ij} - \sum_j p^*_j),$$

where (x^*_{ij}) is the optimal allocation and p^*_j are the Vickrey prices, and x_{ij} is the allocation and p_j are the prices that are actually realized. These are listed in Tables 6a and 6b below.

Table 6a: Consumers' Surplus
(observed relative to predicted VL outcome)

	Periods				
	All	First10	Last10	t-test	Lord-test
Mechanism					
Low Contention					
Vickrey-Leonard	1.00	1.00	1.00	0.95	0.0015
Serial Dictator	1.1	1.1	1.0	0.59	0.022
Sealed-bid-chit	1.1	1.0	1.1	0.60	0.018
DGS Auction	0.88	0.83	0.92	0.39	0.026
High Contention					
Vickrey-Leonard	1.5	1.5	1.4	0.88	0.037
Serial Dictator	4.0	3.8	4.3	0.78	0.15
Sealed-bid-chit	4.4	4.3	4.5	0.92	0.048
DGS Auction	1.0	0.89	1.2	0.66	0.081

[22] Prediction intervals are ± one standard deviation from the mean.

Table 6b: Consumers' Surplus

(observed vs. predicted)

Experiments	1	2	3	Predicted[23]	test
Mechanism					
Low Contention					
Vickrey-Leonard	1.03	1.00	0.99	1.00	
σ	0.12	0.07	0.11		
Serial Dictator	1.07			1.03 ± 0.05	
σ	0.14				
Chit	1.06			1.06 ± 0.03	
σ	0.12				
DGS Auction	0.87	0.89		1.00	
σ	0.19	0.13			
High Contention					
Vickrey-Leonard	1.33	1.59		1.00	
σ	1.09	0.83			
Serial Dictator	4.04	4.00		4.15 ± 0.13	
σ	1.73	1.83			
Chit	4.45	4.40		4.61 ± 0.17	
σ	1.62	1.72			
DGS Auction	1.1	0.97		1.00	
σ	0.62	0.62			

From Tables 6a and 6b we can observe that:

1. As with efficiencies there is no significant difference between the first 10 and the last 10 periods.

23 For the nontransfer mechanisms the prices are zero for all slots.

2. With low-contention, the relative consumers' surplus in the chit treatments differs little from the relative consumers' surplus in the SD mechanism, but it is slightly higher in the SD and chit treatments than in the VL and DGS treatment.

3. With high contention, the relative consumers' surplus from the chit mechanism is much larger than the relative consumers' surplus from the VL and DGS, and slightly higher than the SD mechanism.

4. In both the high-contention and low-contention treatment, the VL gives higher relative consumers' surplus than the DGS. The only way that relative consumers' surplus can be over 1 in the cash treatment is for participants to under-reveal.

To determine if the difference in consumers' surplus is significant, we estimate the following model:

$$y_{ep} = \beta_{1m_e} + \beta_2 f(p) + \beta_3 c_{m_e p} + \epsilon_{ep},$$

This is the same as the model estimated for efficiencies except that:

$y_{ep} \equiv$ relative consumers' surplus of experiment e and period p.

Table 6c: Consumers' Surplus
(fixed effects model)

Dependent variable: Consumers' Surplus

Observations	: 300	
Number of Groups	: 4	
Degrees of freedom	: 294	
Residual SS	: 347.914	
Std error of est	: 1.088	
Total SS (corrected)	: 526.790	
F	= 75.579	with 2,294 degrees of freedom
P-value	= 0.000	

Variable	Coef.	Std. Coef.	Std. Error	t-Stat	P-Value
PERIOD	-0.546571	-0.089847	0.288549	-1.894207	0.059
CONTENT	2.349955	0.572255	0.194780	12.064689	0.000

Group Number	GROUP Variable	Standard Error
1	0.379240	0.143208
2	1.820880	0.186236
3	2.091392	0.186236
4	-0.005672	0.159883

F-statistic for equality of GROUP variables :
$F(3, 294) = 61.9926$ P-value: 0.0000

The estimation indicates:

1. The period is significant with the first periods significantly lower than the last periods.

2. Contention is significant with low-contention consumers' surplus being higher than the high-contention consumer surplus for the same mechanism.

3. The model specification is significant though it accounts for only 30% of the variation.

4. The mechanisms have a significant effect and are ranked for surplus generation as follows:

$$\text{Chit} > \text{SD} > \text{VL} > \text{DGS}$$

5. The model was also estimated under various groupings, specifications of $f(p)$, and measurements of efficiency; and the results indicate that the specification is robust.

We conclude that the use of the serial-dictator and chit mechanisms resulted in higher surplus than the use of cash transfers when contention was high, but that when contention was low there was almost no difference. This shows that there is a strong difference in the relative behavior of mechanisms in these two environments. When contention is low and there is little disagreement over the assignment of slots, any of the mechanisms tested (except random) will give the agents the same level of consumer surplus. But when contention is high and there is much disagreement over the assignment of slots, the nontransfer mechanisms will give the agents more consumer surplus. This is evidence that may help explain the existence of inefficient nontransfer institutions.

7.4 Revenue

In Table 7 we present the revenue generated by the two transfer mechanisms: the Vickrey-Leonard sealed-bid auction and the DGS progressive auction. Even though revenue (or producer surplus) is total welfare less consumer surplus, we present this information for two reasons. One, the efficiency and consumer surplus measures may be confounding and the effect on revenue generation may not be apparent; that is, if one mechanism has both lower efficiency and lower consumer surplus than another mechanism, then the revenue from one mechanism could be either higher or lower than the revenue from the other mechanism. Two, in most auction studies, revenue generation is one of the central measures that is used and have generally observed that progressive auctions tend to generate more revenue than sealed-bid auctions (see, *e.g.,* Banks et. al (1989)).

Table 7: Revenue
(observed vs. predicted)

Mechanism\Experiments	1	2	3	Predicted
Low Contention				
Vickrey-Leonard	314	375	437	490
σ	(277)	(236)	(269)	(335)
DGS Auction	613	768		490
σ	(367)	(602)		
High Contention				
Vickrey-Leonard	2821	2658		3253
σ	(780)	(693)		(289)
DGS Auction	3098	3225		3253
σ	(731)	(727)		

From Table 7 we observe that in both the high-contention and low-contention environments, the DGS auction generated higher levels of revenue. In the low-contention environment the DGS auction generated higher than predicted levels of revenue and a very high variance in the second experiment. The high variance in the second DGS auction appears to be from the first 5 periods, where it appears that there was some confusion (or inexperience) on the part of one of the subjects. The results we find here are consistent with the the results found in other studies with multiple object allocation problems (see, *e.g.,* Banks et. al (1989)).

7.4 Individual Behavior

In the sections 7.1, 7.2, and 7.3, we observed instances where observed values differed from the predicted values. In this section, we look at individual behavior to see if the mechanisms are robust to individual deviations from predicted behavior and if the behavior assumptions we applied were appropriate.

We do not formally model an individual's "learning" nor formally model the events that determine an agent's behavior, but only inquire if we can measure the direction of the difference in our treatment effects. This means that if we can measure a difference in subjects' bids between the first and last periods of an experiment, then we cannot announce that we have found learning, but only that there is a difference in bidding.

Auction Behavior

Smith (1978) and Coppinger, Smith, and Titus (1980) report the results of experiments in which the second-price auction was compared to a first-price auction. It was found that many subjects "learned" their dominant strategy fairly rapidly, but that violations of single-period, dominant strategy behavior were common, especially in the "early" trials of an experiment session. Miller and Plott (1985) study a market in which buyers may purchase multiple units and pay the highest rejected bid for each unit. For their parameters (many units on the margin), the one-price auction is demand-revealing, and they find that after replication, bidders report their true valuation. They report high efficiencies but their solution relies on the restriction of no over-revelation, and the fact that bidders kept the same payoff sheets every period. Do subjects in multiobject auctions "learn" their dominant strategy as well as in single-object, single-unit auctions?

We do not restrict our subjects to bids below their values as do most other auction experiments. The rationale given by some experimentalists for this restriction is that if a subject makes substantial overbids, his profits may fall substantially below zero. The conjecture is that this subject may "sabotage" the experiment since there will then be only a small chance that he may obtain positive profits. Not being able to extract payment from the subjects allows them to be indifferent between a payoff of zero and any negative amount. We had very low priors on this happening so we chose to allowed bidding to be in $[0, 9999]$. We did not observe "sabotaging" to happen in any of our experimental trials.

Vickrey-Leonard sealed-bid auction:

In the VL auction we hypothesized that subjects would play a weakly dominant strategy, and when it existed, their strong dominant strategy. Subjects in the low-contention environment have a set of weakly dominant bidding strategies; each bid differs from the subject's slot valuations by the addition of a constant. In the high-contention environment, subjects have a unique, strong dominant strategy to bid their slot valuations. The lack of a larger set of weak strategies in the high-contention environment is the result of eight subjects and only six slots; this creates two implied slots that have zero value for all the subjects. For weakly dominant strategies to exist, the subjects must be able to place bids on all the slots, but in the high-contention case, they are permitted to place bids on only six of the slots. Because of this dichotomy we are not able to distinguish effects that arise because of the difference in contention and the difference in the class of available strategies.

In Appendix G.1 we display bidding behavior for each subject and each VL sealed-bid experiment. Each graph shows three series, which are based on the difference between a subject's bid and his slot values. The three series are:

1) $\max_j\{\text{bid}_j - \text{val}_j\}$,

2) $\text{mean}_j\{\text{bid}_j - \text{val}_j\}$, and

3) $\min_j\{\text{bid}_j - \text{val}_j\}$,

where $\text{bid}_j \equiv$ bid slot j, and $\text{val}_j \equiv$ value of slot j.

So high values on the graph indicate overbidding and low values indicate underbidding. The plotted variables were truncated so that all plots were in the range [-1000, 1000]; since this makes comparisons between subjects easier. The low-contention experiments were also adjusted by a constant for each period, depending on the valuations and bids, since in the low-contention environment subjects had many weakly dominant strategies that varied only by a constant. The adjustment was accomplished as follows:

Let $\text{bid}^*_j = \text{bid}_j - \min_j\{\text{bid}_j\}$, and let $\text{val}^*_j = \text{val}_j - \min_j\{\text{val}_j\}$. Then the three series are:

1) $\max_j\{\text{bid}^*_j - \text{val}^*_j\}$,

2) $\text{mean}_j\{\text{bid}^*_j - \text{val}^*_j\}$, and

3) $\min_j\{\text{bid}^*_j - \text{val}^*_j\}$.

From these displays we can observe that:

1. The high-contention environment has a higher variance of (bid − value) and more consistent overbidding. Often the overbidding occurs in the earlier periods and disappears in latter periods.

2. The low-contention environment has more consistent underbidding, and rarely is overbidding observed.

Table 8a: Vickrey-Leonard Sealed-bid
(bid-value summary statistics)[24]

High Contention	$\bar{x}$	$\bar{\sigma}$	min	max	$\tilde{x}$
Min (val-bid)	-167.1	660.3	-900.0	9099	-200
Mean (val-bid)	88.5	820.3	-416.6	9649	-50.0
Max (val-bid)	652.5	1983.8	-300.0	9999	1.0

Low Contention	$\bar{x}$	$\bar{\sigma}$	min	max	$\tilde{x}$
Min (val-bid)	-160.9	174.7	-600.0	0	-100
Mean (val-bid)	-51.0	93.7	-299.8	241.7	-16.7-
Max (val-bid)	57.9	142.4	0	800	0.0

High Contention (truncated)	$\bar{x}$	$\bar{\sigma}$	min	max	$\tilde{x}$
Min (val-bid)	-208.5	266.7	-900.0	1000	-----
Mean (val-bid)	2.3	300.0	-416.6	1000	-----
Max (val-bid)	190.0	382.3	-300.0	1000	-----

Table 8 shows that for high-contention environments the means and standard deviation of the bid value difference are much larger than for low-contention environments. The last part of the table contains the summary statistics for the high-contention environment when the bid – value observations are truncated above at 1000. This was done because a few very high differences above 1000 (there were 4 truncations in the minimum observations, 16 for the mean observations, and 51 for the maximum observations) skew the summary statistics except for the median. Summary statistics by experiment are presented in Appendix G.2.

[24] $\bar{x}$ is the mean value, $\bar{\sigma}$ is the standard deviation, and $\tilde{x}$ is the median.

The second measure of subjects' behavior is found by substituting an individual's bid with his valuations to determine if there is a gain, and hence if a subject's deviations were costing him. If deviations from truthful reporting do not cost the subject, then we cannot argue that it is in his best interest to play the dominant strategy. The descriptive statistics below indicate that on average the gain for truthful revelation was largest in the high-contention environment.

Table 8b: Vickrey-Leonard Sealed-bid
(net gain statistics)[25]

Experiment	contention	$\bar{x}$	$\tilde{x}$	$\bar{\sigma}$
1	high	82.4	61.6	288.9
3	low	30.6	26.0	63.9
4	high	53.3	37.6	188.4
8	low	29.2	30.0	80.9
19	low	16.7	22.3	84.9

In addition, an analysis of covariance was performed, wherein subjects were considered to be random effects, and type (payoff values) were considered to be a fixed effect, and time over periods was measured as (1/period). The following period-effect results are presented (the results for the entire analysis are presented in Appendix G):

[25] $\bar{\sigma}$ is the mean of the standard deviations for the subjects.

Table 8c: Vickrey-Leonard Sealed-bid
(period effect)

		Experiment type	
Variable	Low contention	High Contention	High Truncated
vbmin	-65.2 (0.08)	286 (0.09)	-14.4 (0.002)
vbmean	-32.0 (0.09)	786 (0.00)	131.0 (0.028)
vbmax	80.0 (0.004)	2010.0 (0.00)	213.0 (0.003)

A Hausman (1978) χ^2–test specification test was performed, and the null hypothesis of correct specification could not be rejected with probability $p = 0.99$ on all the models except vbmin for high truncation.

From the above model we observe:

1. The period effect is found to be significant in all cases, especially for vbmax.

2. In the high-contention treatment, each of the (bid − value) measurements is higher than average in the earlier periods (as observed from the positive coefficient).

3. In the low-contention treatment, vbmin and vbmax are lower than average in the earlier periods. This would indicate that subjects underbid more on their least favorable slots in the earlier periods.

Appendix G also presents variable statistics by periods 1-5, 6-10, 11-15, and 16-20. It is readily observed that in the low-contention treatment there is a small difference in the mean and variance of (bid − value) between the first and last periods, but in the high-contention treatment there is large difference between the first and last periods. Estimation is also performed to determine if the bidding measurements differed by individual payoff sheets. This initial test indicates that only the mean in the low-contention environment was affected by individual payoff sheets.

DGS Progressive Auction

To study individual behavior in the DGS progressive auction experiments we construct four measures of bidding behavior. In the description of the DGS experiments, a bid refers to a vector of zeros and ones, where each element corresponds to a slot, and an entry of one indicates a request for that slot; and net value is the subject's value for that slot minus the price for that slot. We say that a subject bids on a slot if there is a bid of one for that slot.

1. Strict honesty: A subject bids only on the slots (and all of the slots) that maximize his net value; he does not bid on any other slot. If the maximum net value is zero (the subject is indifferent between receiving the maximizing slot and not receiving a slot), then a bid of either zero or one is considered strictly honest.

2. Nonstrict honesty: At least one of the bids is on a slot that maximizes net value, and there are no bids on slots that have negative net value. A bid may be placed on a nonmaximizing slot, and there may be maximizing slots that do not receive a bid (if there is more than one maximizing slot).

3. Marginal honesty: There are no bids on slots that maximize net value and there are no bids on slots that have negative net value. If there are slots that have positive net value, there is at least one bid.

4. Dishonesty: There are bids on slots that have negative net value, or there are no bids when there is a slot with positive net value.

Measures 2, 3, and 4 are mutually exclusive and exhaustive; that is, a bid falls in one and only one of the three categories. Measure 1 is a subset of measure 2. The following table presents summary statistics of these four measures for the four DGS auction experiments. We also present a 5th measure of the average number of iterations per period.

Table 9a: DGS
(low-contention summary statistics)

Group EXP_1: DGS experiment 14
Group EXP_2: DGS experiment 15

Variable	Group	N	Mean	Std Dev	Minimum	Maximum
dem01	EXP_1	20	0.7049	0.1361	0.3670	1.0000
	EXP_2	20	0.7112	0.1325	0.3330	0.9500
dem02	EXP_1	20	0.9090	0.0833	0.6670	1.0000
	EXP_2	20	0.9505	0.0649	0.7500	1.0000
dem03	EXP_1	20	0.0586	0.0558	0.0000	0.2330
	EXP_2	20	0.0308	0.0450	0.0000	0.1330
dem04	EXP_1	20	0.0324	0.0602	0.0000	0.1940
	EXP_2	20	0.0188	0.0577	0.0000	0.2500
dem05	EXP_1	20	7.5000	4.3347	1.0000	18.0000
	EXP_2	20	7.5000	4.6848	1.0000	16.0000

DIFFERENCE OF MEANS TESTS

						Test of Variances		
Variable	Group	Variances	t	df	p>|t|	F'	df	p>F'
dem01	EXP_1	Equal	-0.15	38.0	0.884	1.06	19	0.907
dem02	EXP_1	Equal	-1.76	38.0	0.087	1.65	19	0.285
dem03	EXP_1	Equal	1.74	38.0	0.091	1.54	19	0.356
dem04	EXP_1	Equal	0.73	38.0	0.470	1.09	19	0.853
dem05	EXP_1	Equal	0.00	38.0	1.000	1.17	19	0.738

The critical t-value for a 5% overall significance level is 3.10; this value takes into account that there is more than one test being performed. From Table 9a, we cannot reject the hypothesis that the observations are from the same distribution.

Table 9b: DGS

(high-contention summary statistics)

Group EXP_1: DGS experiment 16
Group EXP_2: DGS experiment 17

Variable	Group	N	Mean	Std Dev	Minimum	Maximum
dem01	EXP_1	20	0.5963	0.0961	0.4090	0.7330
	EXP_2	20	0.5033	0.0946	0.2500	0.6450
dem02	EXP_1	20	0.8495	0.0609	0.7320	0.9510
	EXP_2	20	0.8714	0.0980	0.5000	0.9860
dem03	EXP_1	20	0.1045	0.0566	0.0000	0.2270
	EXP_2	20	0.1030	0.0556	0.0130	0.2500
dem04	EXP_1	20	0.0460	0.0417	0.0000	0.1880
	EXP_2	20	0.0259	0.0548	0.0000	0.2500
dem05	EXP_1	20	22.3500	6.2767	2.0000	33.0000
	EXP_2	20	19.0500	3.8997	4.0000	25.0000

DIFFERENCE OF MEANS TESTS

						Test of Variances		
Variable	Group	Variances	t	df	p>|t|	F'	df	p>F'
dem01	EXP_1	Equal	3.09	38.0	0.004	1.03	19	0.943
dem02	EXP_1	Equal	-0.85	38.0	0.401	2.59	19	0.044
dem03	EXP_1	Equal	0.09	38.0	0.931	1.04	19	0.939
dem04	EXP_1	Equal	1.31	38.0	0.199	1.72	19	0.244
dem05	EXP_1	Equal	2.00	38.0	0.053	2.59	19	0.044

The critical t-value for a 5% overall significance level is 3.10; this value takes into account that there is more than one test being performed. From Table 9a, we cannot reject the hypothesis that the observations are from the same distribution (dem01 is borderline).

Since there is no significant difference between experiments of the same type of contention, we can pool these observations to test the hypothesis that there is no difference between contention types. We present these results in Table 9c below.

Table 9c: DGS

(test between contention types)

Variable	Group	N	Mean	Std Dev	Minimum	Maximum
dem01	LOW	40	0.7080	0.1326	0.3330	1.0000
	HIGH	40	0.5498	0.1053	0.2500	0.7330
dem02	LOW	40	0.9297	0.0766	0.6670	1.0000
	HIGH	40	0.8604	0.0813	0.5000	0.9860
dem03	LOW	40	0.0447	0.0519	0.0000	0.2330
	HIGH	40	0.1037	0.0553	0.0000	0.2500
dem04	LOW	40	0.0256	0.0586	0.0000	0.2500
	HIGH	40	0.0359	0.0492	0.0000	0.2500
dem05	LOW	40	7.5000	4.4549	1.0000	18.0000
	HIGH	40	20.7000	5.4217	2.0000	33.0000

DIFFERENCE OF MEANS TESTS

Variable	Group	Variances	t	df	p>|t|	Test of Variances F'	df	p>F'
dem01	LOW	Equal	5.91	78.0	0.000	1.59	39	0.154
dem02	LOW	Equal	3.93	78.0	0.000	1.13	39	0.714
dem03	LOW	Equal	-4.92	78.0	0.000	1.14	39	0.693
dem04	LOW	Equal	-0.85	78.0	0.396	1.42	39	0.276
dem05	LOW	Equal	-11.90	78.0	0.000	1.48	39	0.224

The t–test rejects the null hypothesis that four of the measures are the same for the two treatments.

In Appendix H.1 we present graphs by period for the first four measures. There is a noticeable spike at period 15 in the high-contention treatment for the dishonest bidding measure, which occurs in both of the high-contention experiments. These spikes can be explained by observing the individual behavior. After period 10 in the high-contention experiments subjects began to sit out of the early iterations of a period. In most of the periods there was considerable contention for slots, and a lot of bidding in the early iterations, so that waiting did not affect the outcome. In period 15 there is less contention, and when two subjects sat out, there was an immediate (after 2 iterations) allocation omitting the subjects who sat out. In subsequent periods the subjects no longer sat out the early iterations.

Serial Dictator

For the serial-dictator experiments we count the number of completely truthful rankings, that is, the number of bids that are honestly ranked for each slot; we also count the number of bids that honestly rank the top 1 or 2 slots, the top 3 or 4 slots, and the top slot is not honestly ranked. We also measure the gain that would have been realized if had agents submitted truthful rankings, given that the other rankings remained the same.

Table 10: Serial Dictator
(truthful bids)

Experiment	truthful bids				net gain
	All	none	1 or 2	3 or 4	$\Delta\Pi$
SD1 (high)					
subject					
1	15	2	3	0	0
2	20	0	0	0	0
3	3	4	10	3	550
4	20	0	0	0	0
5	20	0	0	0	0
6	20	0	0	0	0
7	17	1	2	0	0
8	4	16	0	0	1400
SD2 (low)					
subject					
1	20	0	0	0	0
2	20	0	0	0	0
3	20	0	0	0	0
4	19	0	0	1	0
5	18	0	1	1	0
6	17	2	1	0	400
SD3 (high)					
subject					
1	20	0	0	0	0
2	20	0	0	0	0
3	0	6	14	0	1000
4	20	0	0	0	0
5	18	0	2	0	0
6	20	0	0	0	0
7	20	0	0	0	0
8	16	2	1	1	1100

In table 10a: above, 19 of 22 subjects appear to have reported honestly (12 reported truthfully in all periods, and 7 reported honestly in 15 or more periods). Two subjects appear to have ranked only their top slots. Subject 8 of experiment SD1 appears to have misreported in every period, and we have no explanation for this behavior. We checked the data to see if the subject was confused and submitted the rankings in inverse order, but this was not the case (*i.e.,* the inverse ordering appears just as unpredictable). It should be observed that even with such strong deviation from hypothesized behavior, the actual loss was not large. In the high-contention treatment it is less likely that a subject has an effect on the outcome than in the low-contention treatment. In the high-contention treatment when a subject is ordered 7th or 8th, he receives no slot; and when he is 6th he receives what is left from the previous 5 choices of the group. In these cases the submitted rankings of the 6th, 7th or 8th subjects have no effect on the outcome.

Chit mechanism

For the chit mechanism one obvious class of dominant strategies is to bid all of the chits. Bidding less can only decrease your chances of receiving a preferred slot. The first table gives the number of bids per experiment where subjects bid less than 1000 chits (their allotted budget).

Table 11: Chits
(<1000 bids)

Experiment	#<1000	#indiff	#bids	#viable	%<1000
Chit1 (high)	4	0	160	160	2.5
Chit2 (low)	21	27	120	99	21
Chit3 (high)	3	0	160	160	1.8

In Table 11, #<1000 is the number of bids that totaled less than 1000 chits and did not correspond to sheet types that were completely indifferent over slots, and this occurs only in the low-contention environment. The number of indifferent sheet types are listed in the column labeled #indiff. The total number of possible bids in an experiment is #bids; #viable = #bids − #indiff; and %<1000 = #<1000/#bids.

From Table 11 we can observe that in the low-contention environment 21% of the viable bids totaled less than a 1000. For the high-contention treatment, the number is approximately 2%. Eighteen of the 21 <1000 low-contention bids belonged to two of the subjects. We conjecture that since subjects do not lose large amounts because of errors (recall that in the low-contention environment a subject was guaranteed a slot) they did not spend the time to make sure there bidding was optimal.

8. Concluding Remarks

In this paper I considered the allocation problem of assigning (or matching) a set of slots to a set of agents. I experimentally tested four different mechanisms (sealed-bid auction, progressive auction, sealed-bid auction with chits, and serial dictator) in two different environments (low and high contention). I compared the observed outcomes of the four mechanisms against their predicted outcomes, among themselves, and looked at individual behavior.

Two of the mechanisms involved cash transfers (sealed-bid and progressive auction). As in single-object auctions, the progressive auction generated higher revenues and higher efficiencies than the sealed-bid auction, even though the predicted outcomes were identical. An examination of the individual subject data in the sealed-bid auction revealed that subjects tended to overbid in the high-contention environment, especially in the early periods, but that in the low-contention environment, underbidding was more prevalent. In the progressive auction subjects tended to bid "honestly," and deviations from honest behavior had little effect on the outcome.[26]

As predicted, the nontransfer mechanisms (chit and serial dictator) were less efficient than the transfer mechanisms but resulted in higher consumer surplus. The differences in efficiencies and consumer surplus between the transfer and nontransfer were small in the low-contention environment and larger in the high-contention environment. In particular, the difference between consumer surplus in the high-contention environment was dramatic (300% more surplus from the nontransfer mechanisms), while the difference between efficiencies in the high-contention environment was less dramatic (89% vs. 99%

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[26] Except in one instance and in that case the effected individuals quickly reverted to "honest" bidding.

efficiency).

In general, subjects tended to behave differently in the two different environments. There was more variance in behavior in the earlier periods and less variance in the later periods particularly in the high-contention environment. We conjectured that this behavioral difference was due to the amount of competition in the high-contention environment and the higher likelihood that a bad strategy would be punished with more severity.

Overall, the experiments provide some evidence that in certain environments the absence of transfers does not significantly reduce social welfare but does increase the agents' surplus. This helps explain the existence and persistence of inefficient nontransfer institutions. In addition the testing of the mechanisms in different environments underscores the effect of the environment on the outcome of a mechanism; and reinforces the need to perform experiments in different environments.

Appendix A.

Low-Contention Payoff List

Set of Values	Unit Number 1	2	3	4	5	6
1	800	600	400	200	400	600
2	400	600	800	600	400	200
3	400	200	400	600	800	600
4	850	350	350	850	350	350
5	750	400	400	750	400	400
6	900	300	300	300	300	300
7	300	300	900	300	300	300
8	500	500	500	500	500	500
9	550	550	550	550	550	550
10	300	300	300	300	300	900

High-Contention Payoff List

Set of Values	Unit Number 1	2	3	4	5	6
1	900	450	400	350	300	250
2	400	600	800	600	400	200
3	800	600	400	200	400	600
4	100	100	900	400	300	200
5	400	800	400	200	0	200
6	900	600	300	200	100	0
7	300	300	300	300	300	900
8	750	250	250	750	400	400
9	400	200	400	600	800	600
10	850	350	350	650	150	150

Low-Contention Payoff Lists

period 1
400 600 800 600 400 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200
550 550 550 550 550 550
400 600 800 600 400 200

period 2
900 300 300 300 300 300
900 300 300 300 300 300
400 600 800 600 400 200
800 600 400 200 400 600
300 300 900 300 300 300
800 600 400 200 400 600

period 3
500 500 500 500 500 500
800 600 400 200 400 600
300 300 300 300 300 900
300 300 300 300 300 900
300 300 300 300 300 900
750 400 400 750 400 400

period 4
750 400 400 750 400 400
400 200 400 600 800 600
900 300 300 300 300 300
500 500 500 500 500 500
550 550 550 550 550 550
750 400 400 750 400 400

period 5
400 200 400 600 800 600
900 300 300 300 300 300
750 400 400 750 400 400
750 400 400 750 400 400
800 600 400 200 400 600
850 350 350 850 350 350

period 6
500 500 500 500 500 500
500 500 500 500 500 500
750 400 400 750 400 400
850 350 350 850 350 350
850 350 350 850 350 350
750 400 400 750 400 400

period 7
900 300 300 300 300 300
550 550 550 550 550 550
400 600 800 600 400 200
750 400 400 750 400 400
900 300 300 300 300 300
900 300 300 300 300 300

period 8
500 500 500 500 500 500
800 600 400 200 400 600
850 350 350 850 350 350
800 600 400 200 400 600
750 400 400 750 400 400
800 600 400 200 400 600

period 9
300 300 900 300 300 300
550 550 550 550 550 550
400 200 400 600 800 600
850 350 350 850 350 350
900 300 300 300 300 300
500 500 500 500 500 500

period 10
850 350 350 850 350 350
800 600 400 200 400 600
400 600 800 600 400 200
550 550 550 550 550 550
800 600 400 200 400 600
900 300 300 300 300 300

period 11
850 350 350 850 350 350
750 400 400 750 400 400
850 350 350 850 350 350
750 400 400 750 400 400
500 500 500 500 500 500
550 550 550 550 550 550

period 12
550 550 550 550 550 550
400 600 800 600 400 200
750 400 400 750 400 400
500 500 500 500 500 500
500 500 500 500 500 500
800 600 400 200 400 600

period 13
500 500 500 500 500 500
550 550 550 550 550 550
850 350 350 850 350 350
900 300 300 300 300 300
400 200 400 600 800 600
900 300 300 300 300 300

period 14
900 300 300 300 300 300
400 600 800 600 400 200
800 600 400 200 400 600
300 300 900 300 300 300
800 600 400 200 400 600
400 200 400 600 800 600

period 15
550 550 550 550 550 550
400 600 800 600 400 200
750 400 400 750 400 400
800 600 400 200 400 600
850 350 350 850 350 350
900 300 300 300 300 300

Low-Contention cont.

period 16
800 600 400 200 400 600
500 500 500 500 500 500
850 350 350 850 350 350
800 600 400 200 400 600
750 400 400 750 400 400
800 600 400 200 400 600

period 17
850 350 350 850 350 350
300 300 900 300 300 300
550 550 550 550 550 550
400 200 400 600 800 600
900 300 300 300 300 300
500 500 500 500 500 500

period 18
900 300 300 300 300 300
850 350 350 850 350 350
800 600 400 200 400 600
400 600 800 600 400 200
550 550 550 550 550 550
800 600 400 200 400 600

period 19
750 400 400 750 400 400
850 350 350 850 350 350
750 400 400 750 400 400
850 350 350 850 350 350
750 400 400 750 400 400
550 550 550 550 550 550

period 20
400 600 800 600 400 200
550 550 550 550 550 550
750 400 400 750 400 400
500 500 500 500 500 500
500 500 500 500 500 500
800 600 400 200 400 600

High-Contention Payoff Lists

period 1
400 600 800 600 400 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200
400 800 400 200 0 200
400 600 800 600 400 200
100 100 900 400 300 200
900 600 300 200 100 0

period 2
900 450 400 350 300 250
900 450 400 350 300 250
400 600 800 600 400 200
800 600 400 200 400 600
100 100 900 400 300 200
800 600 400 200 400 600
400 800 400 200 0 200
400 800 400 200 0 200

period 3
900 600 300 200 100 0
800 600 400 200 400 600
300 300 300 300 300 900
300 300 300 300 300 900
300 300 300 300 300 900
750 250 250 750 400 400
900 600 300 200 100 0
100 100 900 400 300 200

period 4
750 250 250 750 400 400
400 200 400 600 800 600
900 450 400 350 300 250
900 600 300 200 100 0
400 800 400 200 0 200
750 250 250 750 400 400
900 450 400 350 300 250
750 250 250 750 400 400

period 5
400 200 400 600 800 600
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400
800 600 400 200 400 600
850 350 350 650 150 150
850 350 350 650 150 150
400 200 400 600 800 600

period 6
900 600 300 200 100 0
900 600 300 200 100 0
750 250 250 750 400 400
850 350 350 650 150 150
850 350 350 650 150 150
750 250 250 750 400 400
400 600 800 600 400 200
800 600 400 200 400 600

period 7
900 450 400 350 300 250
400 800 400 200 0 200
400 600 800 600 400 200
750 250 250 750 400 400
900 450 400 350 300 250
900 450 400 350 300 250
800 600 400 200 400 600
850 350 350 650 150 150

period 8
900 600 300 200 100 0
800 600 400 200 400 600
850 350 350 650 150 150
800 600 400 200 400 600
750 250 250 750 400 400
800 600 400 200 400 600
900 450 400 350 300 250
400 600 800 600 400 200

period 9
100 100 900 400 300 200
400 800 400 200 0 200
400 200 400 600 800 600
850 350 350 650 150 150
900 450 400 350 300 250
900 600 300 200 100 0
750 250 250 750 400 400
900 600 300 200 100 0

period 10
850 350 350 650 150 150
800 600 400 200 400 600
400 600 800 600 400 200
400 800 400 200 0 200
800 600 400 200 400 600
900 450 400 350 300 250
400 200 400 600 800 600
100 100 900 400 300 200

period 11
850 350 350 650 150 150
750 250 250 750 400 400
850 350 350 650 150 150
750 250 250 750 400 400
900 600 300 200 100 0
400 800 400 200 0 200
400 800 400 200 0 200
850 350 350 650 150 150

period 12
400 800 400 200 0 200
400 600 800 600 400 200
750 250 250 750 400 400
900 600 300 200 100 0
900 600 300 200 100 0
800 600 400 200 400 600
850 350 350 650 150 150
400 800 400 200 0 200

period 13
900 600 300 200 100 0
400 800 400 200 0 200
850 350 350 650 150 150
900 450 400 350 300 250
400 200 400 600 800 600
900 450 400 350 300 250
400 200 400 600 800 600
850 350 350 650 150 150

period 14
900 450 400 350 300 250
400 600 800 600 400 200
800 600 400 200 400 600
100 100 900 400 300 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200
900 600 300 200 100 0

period 15
400 800 400 200 0 200
400 600 800 600 400 200
750 250 250 750 400 400
800 600 400 200 400 600
850 350 350 650 150 150
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400

High-Contention cont.

period 16
400 800 400 200 0 200
400 600 800 600 400 200
100 100 900 400 300 200
900 600 300 200 100 0
400 600 800 600 400 200
800 600 400 200 400 600
400 200 400 600 800 600
400 600 800 600 400 200

period 17
100 100 900 400 300 200
800 600 400 200 400 600
400 800 400 200 0 200
400 800 400 200 0 200
900 450 400 350 300 250
900 450 400 350 300 250
400 600 800 600 400 200
800 600 400 200 400 600

period 18
300 300 300 300 300 900
750 250 250 750 400 400
900 600 300 200 100 0
100 100 900 400 300 200
900 600 300 200 100 0
800 600 400 200 400 600
300 300 300 300 300 900
300 300 300 300 300 900

period 19
400 800 400 200 0 200
750 250 250 750 400 400
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400
400 200 400 600 800 600
900 450 400 350 300 250
900 600 300 200 100 0

period 20
800 600 400 200 400 600
850 350 350 650 150 150
850 350 350 650 150 150
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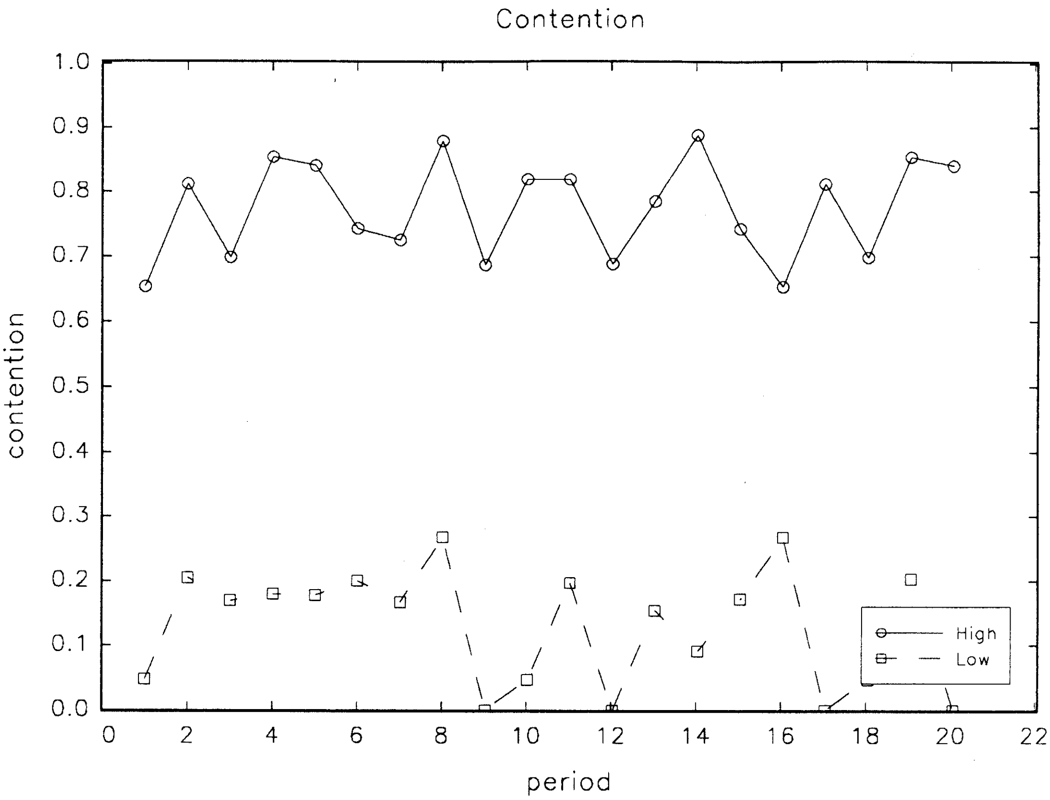
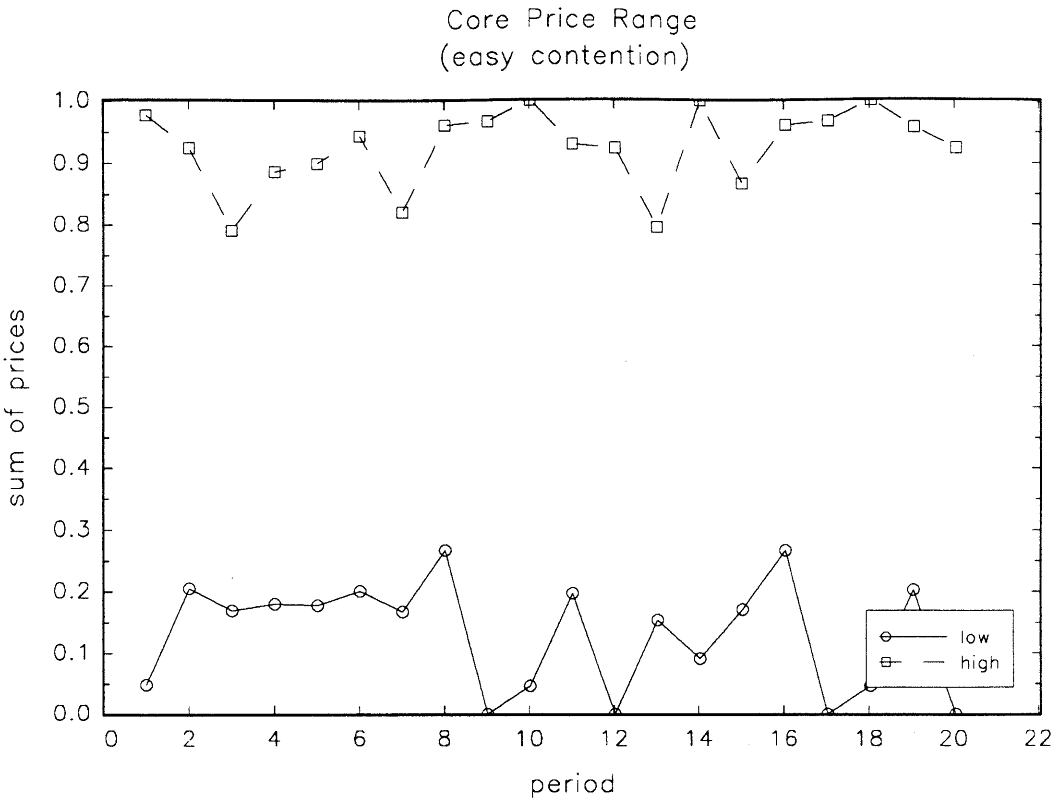
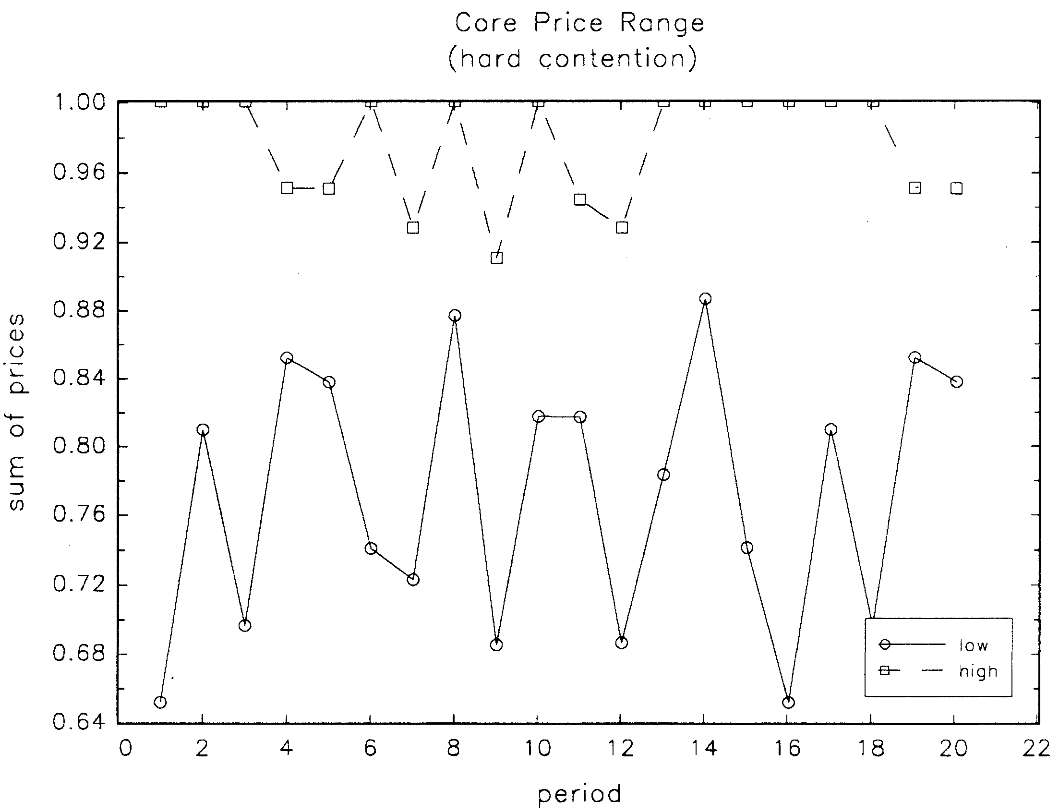
400 200 400 600 800 600
900 450 400 350 300 250
750 250 250 750 400 400
750 250 250 750 400 400


Contention
contention
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
2
4
6
8
10
12
14
16
18
20
22
period
High
Low


Core Price Range
(easy contention)
sum of prices
period
low
high
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
2
4
6
8
10
12
14
16
18
20
22


Core Price Range
(hard contention)
sum of prices
period
low
high
1.00
0.96
0.92
0.88
0.84
0.80
0.76
0.72
0.68
0.64
0
2
4
6
8
10
12
14
16
18
20
22

Appendix B.

INSTRUCTIONS FOR EXPERIMENT

You are about to participate in an experiment in which you will make decisions in a market. Your profits from the experiment will be in terms of francs. You can convert your franc earnings into U.S. dollars at a conversion rate of 600 francs to 1 U.S. dollar. Any profits you make in the experiment are yours to keep. You will be paid at the end of the experiment.

The experiment will be divided up into a series of "periods." At the beginning of each period you will be given redemption values on your terminal screen. The redemption values are the franc values to you of six different items. Your redemption values are known only to you, and you should not reveal them to any other participants. Your profit each period is equal to the redemption value of the unit you receive. For example, suppose the redemption value for the unit you bought is 700 francs. Then your profit for that period is 700 francs.

In our market you will be one of 8 participants to be assigned units. There will be six units, which will be numbered from one to six, allocated simultaneously each period. These units are not the same; that is, they do not necessarily have the same redemption values to a participant. They will be allocated through a procedure that will be described later.

In Table 1 you will find the ten possible sets of redemption values. The table lists the number of the unit and the corresponding value. The sheet has eleven rows. The first row, labeled unit, indicates the number of the unit being allocated (in the experiment there will be six units assigned, which will be referred to as units 1,2,...,6). The second through the eleventh row give the possible participants' redemption values.

For the first set of redemption values, unit 1 is worth 800 francs to the participant, whereas unit 4 is worth 200.

Each participant in the experiment is given one of the ten possible sets of redemption values at the beginning of each period. The sets of redemption values other participants happen to receive do not affect the redemption values you receive.

TABLE 1.

	Unit Number					
Set of Values	1	2	3	4	5	6
1	900	450	400	350	300	250
2	400	600	800	600	400	200
3	800	600	400	200	400	600
4	100	100	900	400	300	200
5	400	800	400	200	0	200
6	900	600	300	200	100	0
7	300	300	300	300	300	900
8	750	250	250	750	400	400
9	400	200	400	600	800	600
10	850	350	350	650	150	150

THE ALLOCATION PROCESS

Each period you will be given a budget of "chits" with which to bid. The chits themselves have no value to you and cannot be redeemed at the end of the experiment. They are of use only to make bids on the units. You may use only chits (not francs) to make bids. You need not use all your chits in a period. The chits will be good only for that period, after which any remaining chits will be forfeited. You will then receive a new budget for the next period.

Each period you will see a display on your terminal like the one shown below. The top row gives the number of the unit to be allocated. The third row, labeled value, gives your redemption values for that period. The second row indicates your bids on the corresponding units. You may enter a bid on the unit by selecting the correct box and typing in your bid. You must enter a bid that is greater than or equal to zero for each unit. In the following figure, the buyer has bid 0 for unit 1, 500 for unit 2, and 600 for unit 3. Notice that the amount of chits in your budget is given in the upper right corner.

Chits = 3980

Unit	1	2	3	4	5	6
Bid	0	500	600	199	205	22
Value	800	600	400	200	400	600

Once you have entered all of your bids and pressed the END key, you will be asked to confirm them. After checking them and making sure they are the bids you want, press the Y key to send the bids. A market program determines the recipient of each unit. <u>By this procedure you can receive at most one unit, and all six units will be assigned</u>.

THE ALLOCATION:

Once the bids are received from all the participants, the six units are allocated by the following method. It finds the combination of "assignments" for which the total of the winning bids for all six units is the greatest. That is, it gives units to buyers (recall, however, that one buyer can get at most one unit) so that the total of the bids of the buyers on the units they actually receive is the highest possible. An example with three participants and three units is given below (example 1).

Example 1: **Three Buyers and Three Units**

	Units and Bids		
Buyer	1	2	3
1	800	700	200
2	700	500	400
3	400	400	400

Here, buyer 1 has bid 800 chits for unit 1, 700 for unit 2, and 200 for unit 3. Buyer 2 has bid 700 chits for unit 1, 500 for unit 2, and 400 for unit 3. Buyer 3 has bid 400 chits for each unit.

The allocation is:

Buyer 1 receives unit 2.

Buyer 2 receives unit 1.

Buyer 3 receives unit 3.

The total of the winning bids from this assignment is 1800, and the total of winning bids from any other assignment is less than 1800. Notice from the example, that the buyer who bids the most on a unit does not necessarily receive that unit. If two or more assignments yield the same maximum total, the assignment is chosen randomly.

PRICES

In addition to allocating the slots, the market program computes a price for each slot. These would be the prices paid if money was being used; they will be subtracted from your chit budget each period, but they do not influence your profits. They are for your information only, and the prices of all 6 slots will be posted on the board each period. They are calculated as follows:

2) After the allocation is made, the program calculates the total of the bids of the buyers on those units that they are allocated.

3) The following total is calculated for unit 1. The program supposes that there was an extra unit 1 available and therefore a total of seven units to be sold. It then finds the combination of assignments for which the total amount bid for units received is the greatest possible (as in step 1). The total of the bids of the buyers on the units they would receive is calculated (as in step 2). Notice this is always greater than or equal to the amount in step 2 because there are more combinations available, and all of the combinations previously available are still available.

4) The difference between the two-bid total is calculated. This difference is the price charged for unit one.

5) Steps 3 and 4 are repeated for units 2-6. The example below works out the process for a case when there are two units to be sold and two buyers.

EXAMPLE 2:

	Units and Bids	
Buyer	1	2
1	1000	600
2	800	100

The combination of assignments where the total amount bid on the units is the greatest possible is the following: Buyer 1 receives unit 2 and buyer 2 receives unit 1. The total amount bid is 600 + 800 = 1400. If there were another unit 1 available, however, each buyer would receive a unit 1, and the total amount bid would be 100 + 800 = 1800. Therefore, the price charged for one unit is 1800 - 1400 = 400. If there was another unit 2 available, the allocation would be unchanged. Therefore, the price of unit 2 is zero.

After this process is completed, the terminal will indicate the unit you received. The redemption value of the unit you receive is your profit for the period. If you do not receive a unit, your profit is zero for the period.

You can press the **H** key at any time to see the history screen. The screen shows your redemption values for each unit during the past periods in the rows labelled values, and the bids you submitted on each unit during the past periods in the rows labelled bid. The units which you have already received and the payoffs you have earned are in the rows labelled payoffs, which are highlighted.

This concludes the instruction for the experiment. If you have any questions, please raise your hand and a monitor will answer your questions. We will have a practice period to help familiarize you with the experiment.

Appendix C: Formal description of the mechanisms.

Let $K \equiv$ the set of slots, and $N \equiv$ the set of agents.

VICKREY-LEONARD MECHANISM (Leonard 1983)

Message Space:

- Each agent submits a vector of monetary bids $b = (b_1, \ldots, b_k)$, one bid for each of k slots.
- $b_j \geq 0, \quad j \in K$.

Outcome Rule:

1) The allocation (x_{ij}) is determined by solving the assignment problem:

$$\text{Find } (x_{ij}) \text{ to } \underset{x \in \mathcal{A}}{\text{Maximize}}\ V = \sum_{i \in N} \sum_{j \in K} b_{ij} x_{ij}. \qquad \text{(VL)}$$

such that

VL1) $\sum_{j \in K} x_{ij} \leq 1, \quad \forall i \in N;$

VL2) $\sum_{i \in N} x_{ij} \leq 1, \quad \forall j \in K;$

VL3) $x_{ij} \in \{0, 1\}, \quad \forall i \in N,\ \forall j \in K,$

where b_{ij} is agent i's bid for slot j.

2) Prices for Vickrey-Leonard

Two methods to determine prices

2a) Dual Method

$$\text{Find } (p_j) \text{ to Minimize } \sum_{j \in K} p_j \qquad \text{(D)}$$

such that

D1) $w_i + p_j \geq b_{ij} \quad \forall j \in K, \quad \forall i \in N;$

D2) $\sum_{j \in K} p_j + \sum_{i \in N} w_i = W;$

D3) $p_j,\ w_i \geq 0, \quad \forall i \in N,\ \forall j \in K,$

where V is the total value of the optimal assignment.

2b) Direct Vickrey-Groves method:

Prices are determined by $p_j = V_N^{K+j} - V_N^K$, where V_N^K = maximum of bids on slots $K = \{1,\ldots,k\}$ assigned to agents $N = \{1,\ldots,n\}$, and V_N^{K+j} = maximum of bids on slots $K \cup \{j\}$ assigned to agents $N = \{1,\ldots,n\}$; that is, adding another slot of type j. The price charged to each bidder is equal to the cost he imposes on the rest of the bidders.

In the VL mechanism honest revelation is a weak dominant strategy. Honest revelation is a weak strategy since the optimal allocation is additive-invariant as noted in Section 2. In the VL transfer mechanism, the surplus goes to the planner (extracted via the payments).

BARR-SHAFTEL PROGRESSIVE AUCTION

(Barr and Shaftel 1976)

Message Space:

- At each step of the auction, each agent submits a request for a set of slots.
- A request $c \subset K \cup \emptyset$ is an offer to buy a set of slots at the announced prices.

Outcome Rule:

- At the start of the auction, the auctioneer announces a vector of prices (one price for each object) at arbitrarily high values.
- The prices are then lowered in any manner whatever—one at a time, all at once or at different rates. As the price of an object drops, requests are placed on that object. Agents do not know the number of requests on any slot.
- As soon as two bids on the same object have been made, the highest bidder is informed that he has been temporarily assigned that object at a price equal to the second highest bidder. The price of that object stops dropping at that time. No knowledge of objects assigned is provided to anyone but the highest bidder.
- If the bidder is temporarily assigned two or more objects, he must retract his bid on all but one of them. The prices on those objects with retracted bids will then start dropping once again until a new second bid is received.
- When there are exactly two bids on each object, the auction stops.

DEMANGE-GALE-SOTOMAYOR EXACT AUCTION

(Demange, Gale and Sotomayor 1986)

Message Space:

- At each step of the auction, each agent submits a request for a set of slots.
- A request $c \subset K \cup \emptyset$ is an offer to buy slots at the announced prices.

Outcome Rule:

- At the start of the auction, the auctioneer announces a vector of prices (one price for each slot) at arbitrarily low values.
- At each announcement of prices, agents submit requests.
- The auctioneer finds either an assignment <u>or</u> the largest pure-overdemanded set. (A set of slots is overdemanded if the number of bidders demanding only items in this set is greater than the number of items in the set).
- If no overdemanded set exists (then there is a feasible assignment), the auction ends and an assignment is made.
- If an overdemanded set exists, prices are raised to the next increment in the overdemanded set.
- The process continues until a feasible assignment is found (no overdemanded set exists).
- If a slot goes unrequested, it goes unassigned.

DEMANGE-GALE-SOTOMAYOR APPROXIMATE AUCTION

(Demange, Gale and Sotomayor 1986)

Message Space:

- At each step of the auction, each agent submits a request for <u>one</u> slot.
- A request $c \in K \cup \emptyset$ is an offer to buy a slot at the announced price.

Outcome Rule:

- At the start of the auction, the auctioneer announces a vector of prices (one price for each slot) at arbitrarily low values.
- At each announcement of prices, agents submit requests.

- The auctioneer finds either an assignment or the set of slots with more than one request.
- If there is a feasible assignment the auction ends and an assignment is made.
- If there are slots with more than one request, then the prices of those slots are raised to the next increment.
- Once an agent submits a request for a slot he can remove his request only if the price is increased (another agent submits a request for the same slot).
- The process continues until a feasible assignment is found (there is at most one request for each slot).
- If a slot goes unrequested, it goes unassigned.

SERIAL DICTATOR

Message Space:

- Each agent submits a vector of ranks $r = (r_1, \ldots, r_k)$, where $r \in \prod(K)$, where $\prod(K) \equiv$ set of permutations of $K = (1, \ldots, k)$ slots.

Outcome Rule:

- An ordering $f \in \prod(N)$ of agents is chosen by the planner and announced to the agents.
- The first agent in the ordering is assigned to her preferred slot.
- The second agent in the ordering is assigned to her preferred slot of those remaining.
- This continues with the second, third, and so on until all slots are assigned.

MULTIPLE-UNIT ENGLISH CLOCK AUCTION

Message Space:

- At each step of the auction, agents submit a request for a single slot, $c \in K \cup \emptyset$.

Outcome Rule:

- At $t = 0$, an initial price vector p_0 is announced by the auctioneer. Agents submit requests and if a feasible assignment exists, the auction ends.

- A feasible assignment is one where there is at most one agent requesting each slot.

- If no such assignment exists, prices are raised to the next increment in those slots with more than one bidder.

- The process continues until a feasible assignment is found.

MULTIPLE-UNIT ENGLISH OPEN AUCTION

Message Space:

- At each step of the auction, agents submit a bid for a single slot, $b_j \geq 0, \quad j \in K$.

Outcome Rule:

- At $t = 0$, an initial price vector p_0 is announced by the auctioneer.

- Agents submit bids and the highest bidder in each slot becomes committed to that slot.

- Agents are allowed to submit a bid on at most one slot. A bid is accepted and the bidder becomes committed if the bid is greater than the current bid on a slot.

- If no new bids are tendered for 30 seconds, then the auction ends and committed bidders are assigned to their slots.

GALE-SHAPLEY MECHANISM (Gale and Shapley 1962)

Message Space:

- Each agent submits a vector of ranks $r = (r_1, \ldots, r_k) \in \prod(K)$, where $\prod(K) \equiv$ set of permutations of $K = (1, \ldots, k)$ slots.

Outcome Rule:

- An ordering $f \in \prod(N)$ of agents is chosen by the planner and announced to the agents.

- The agent-optimal Gale-Shapley algorithm is applied, with agents as the proposers, and the ordering f as the rejector or tie-breaking rule.

- The algorithm:

1) Each agent is matched with their first-ranked slot.

2) In each slot all but the highest-ranked agent is rejected according to the ranking f.

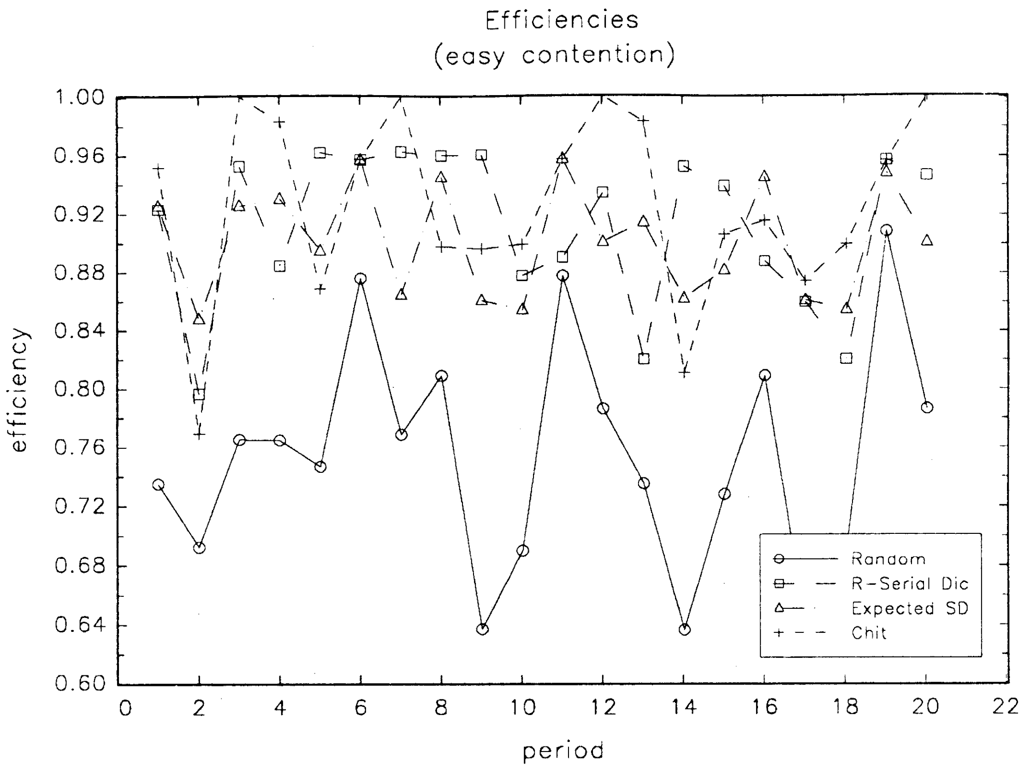
3) Each rejected agent is matched with their next-highest ranked slot.

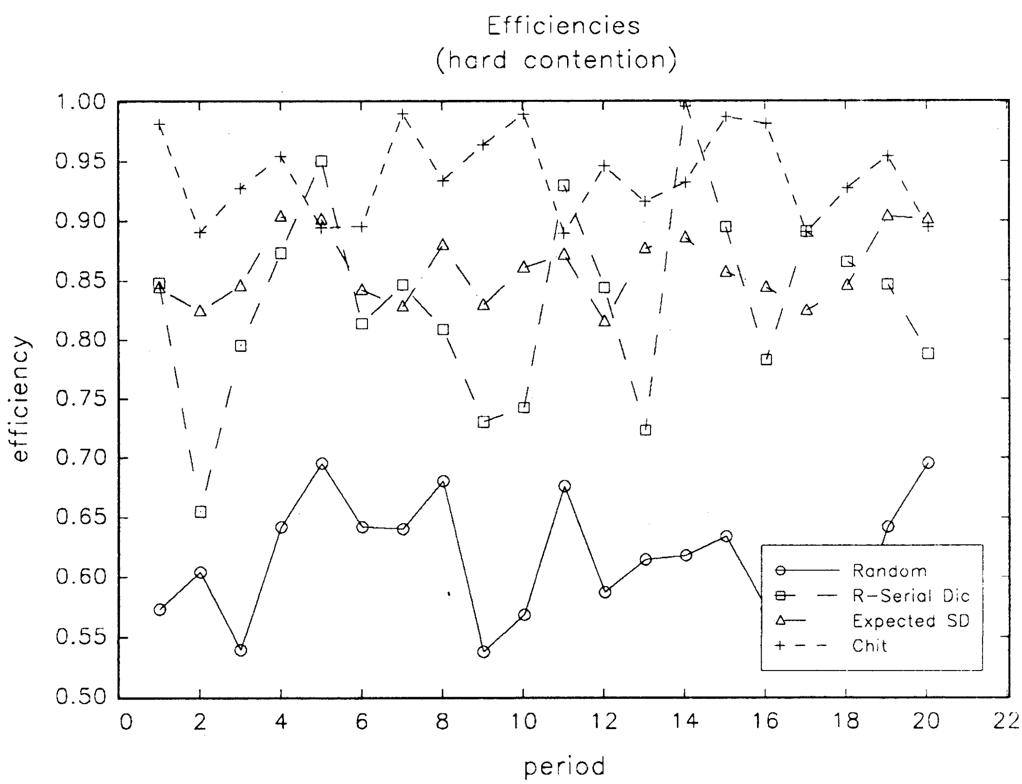
4) Steps 2 and 3 are repeated until there are no rejected agents.

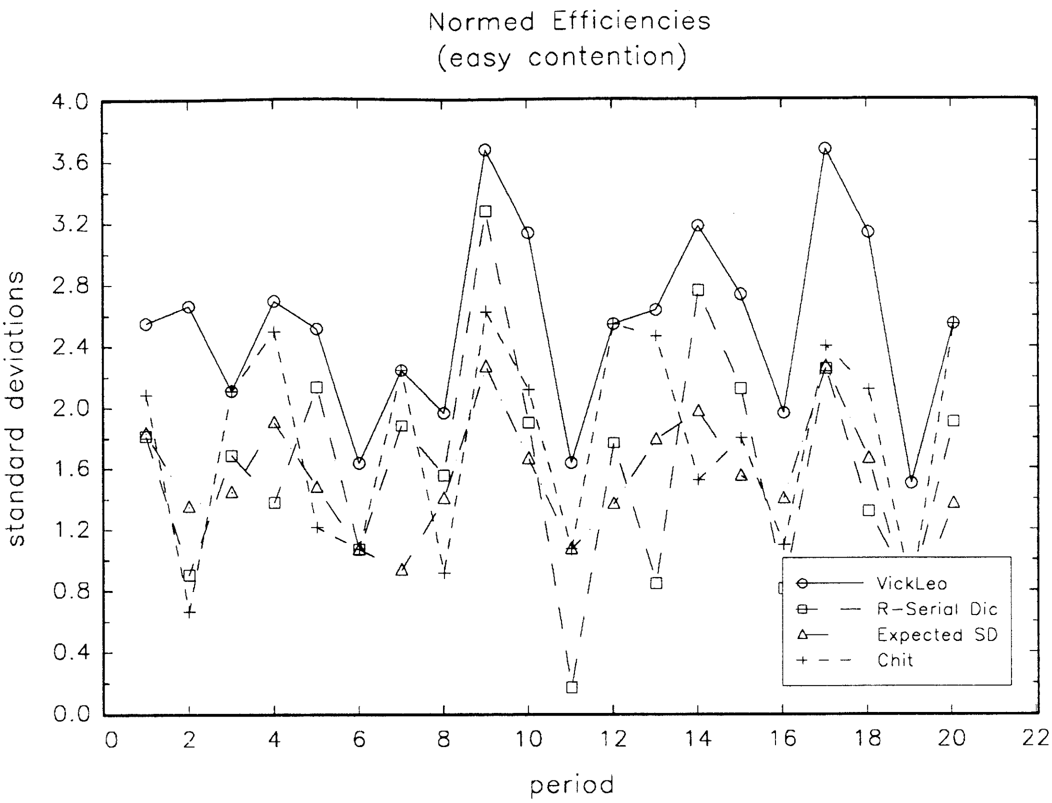
- If the number of agents > the number of slots, extra slots are added that are ranked lowest among all agents.

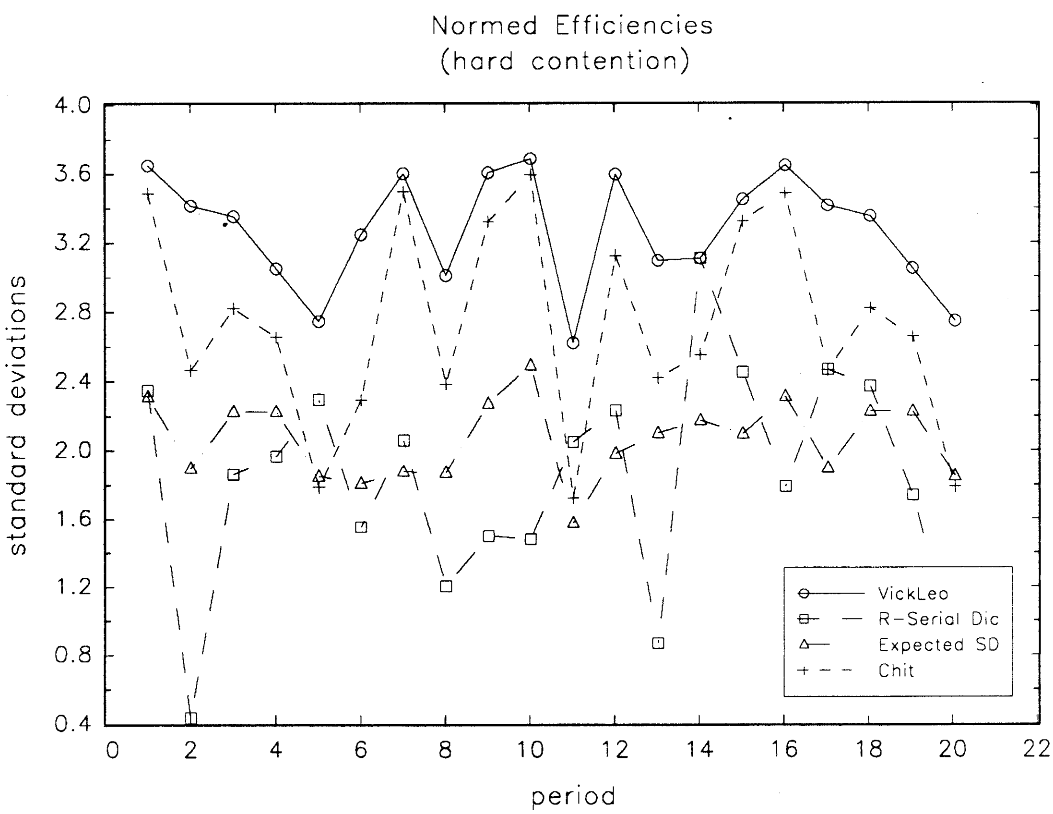
- Since the slots have the same preference ordering over agents, there is a unique outcome (for that ordering).

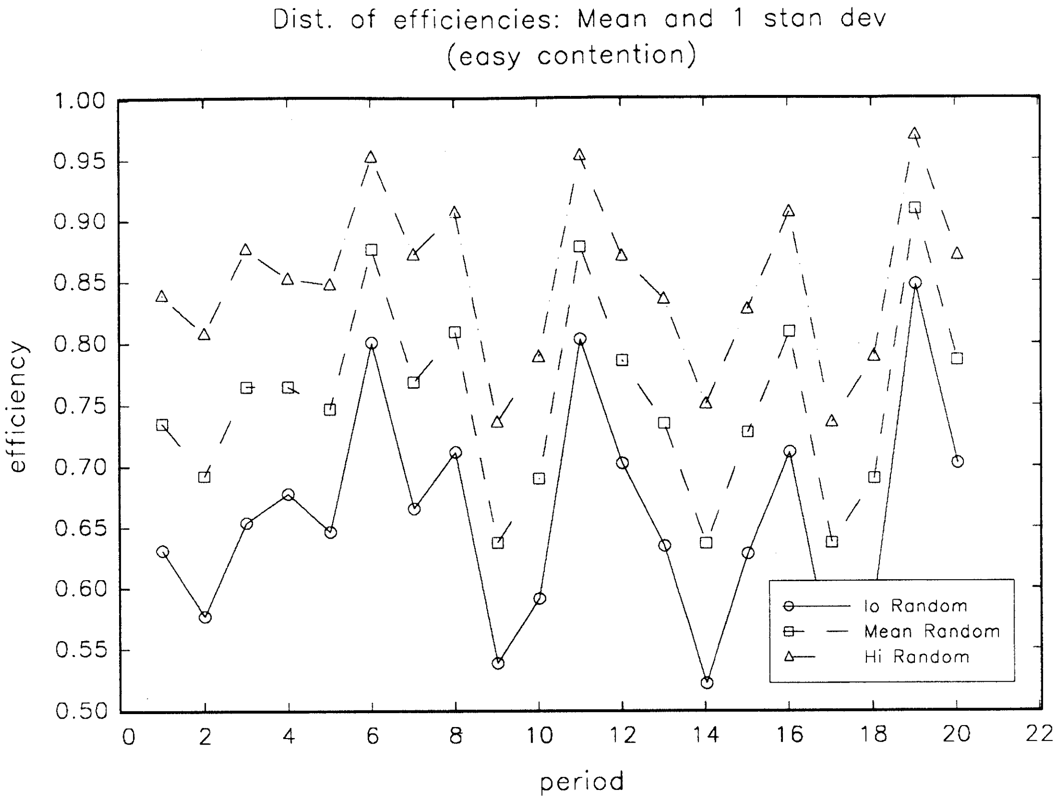
Appendix D. Predicted efficiencies--graphs

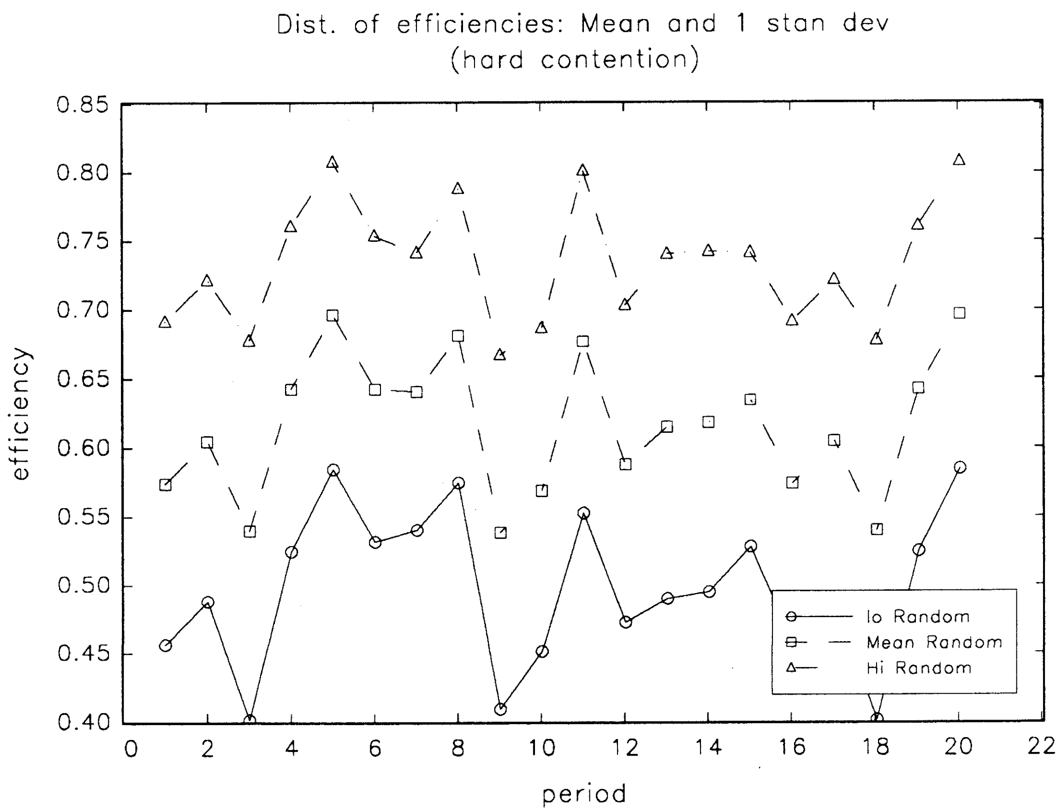




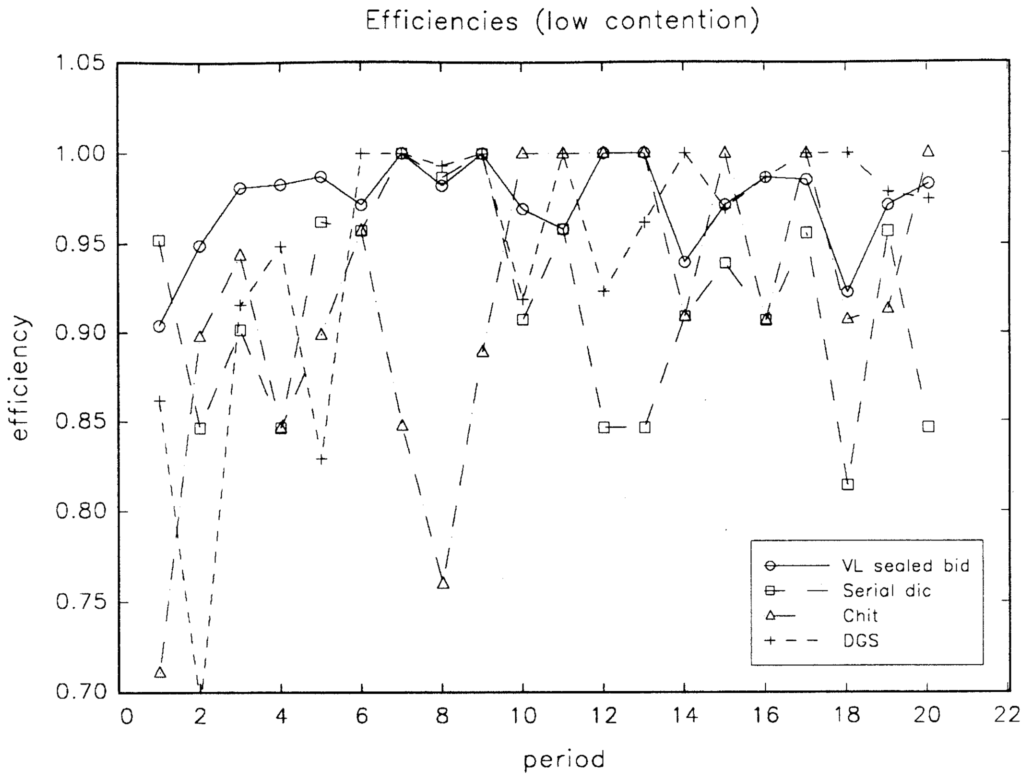

Normed Efficiencies
(easy contention)
standard deviations
period
VickLeo
R-Serial Dic
Expected SD
Chit

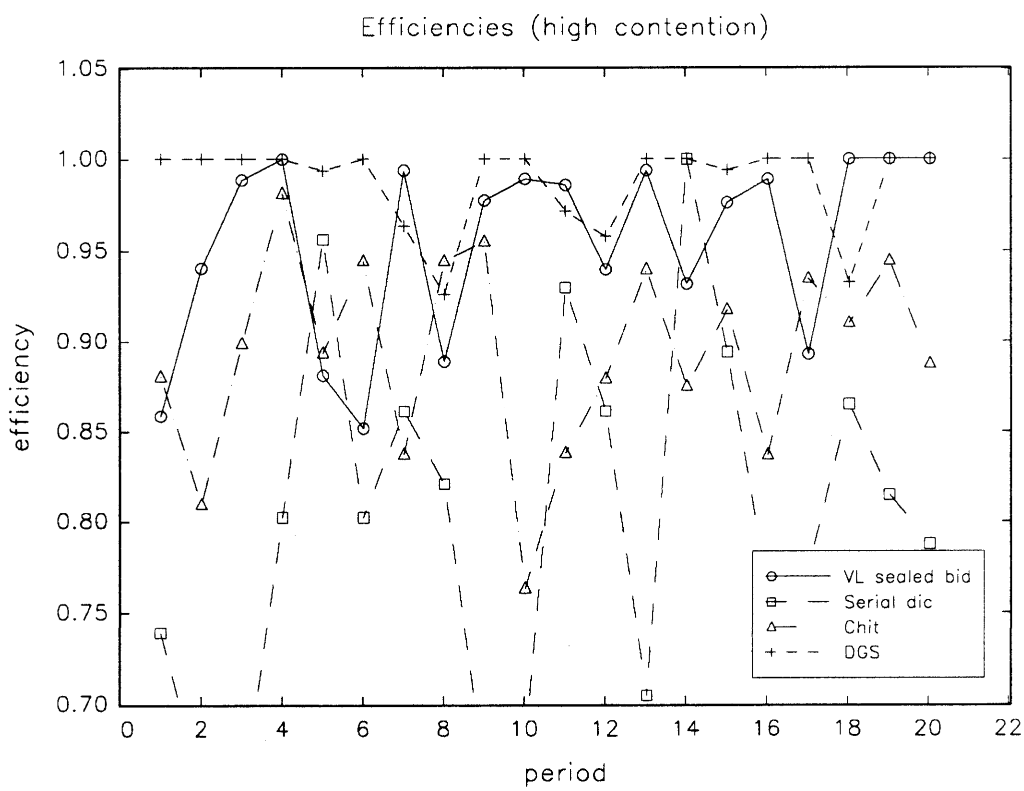

Normed Efficiencies
(hard contention)
standard deviations
period
VickLeo
R-Serial Dic
Expected SD
Chit

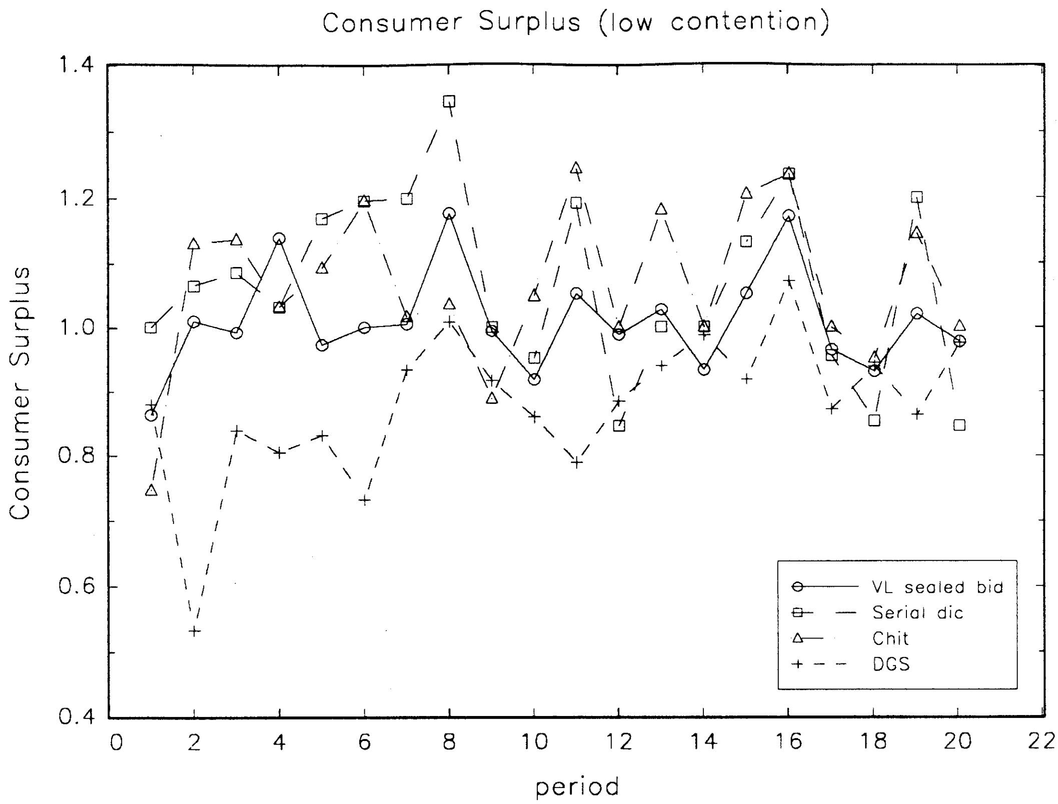

Dist. of efficiencies: Mean and 1 stan dev
(easy contention)
efficiency
period
lo Random
Mean Random
Hi Random


Dist. of efficiencies: Mean and 1 stan dev
(hard contention)
efficiency
period
lo Random
Mean Random
Hi Random

Appendix E. Observations--graphs






Consumer Surplus (low contention)
Consumer Surplus
period
1.4
1.2
1.0
0.8
0.6
0.4
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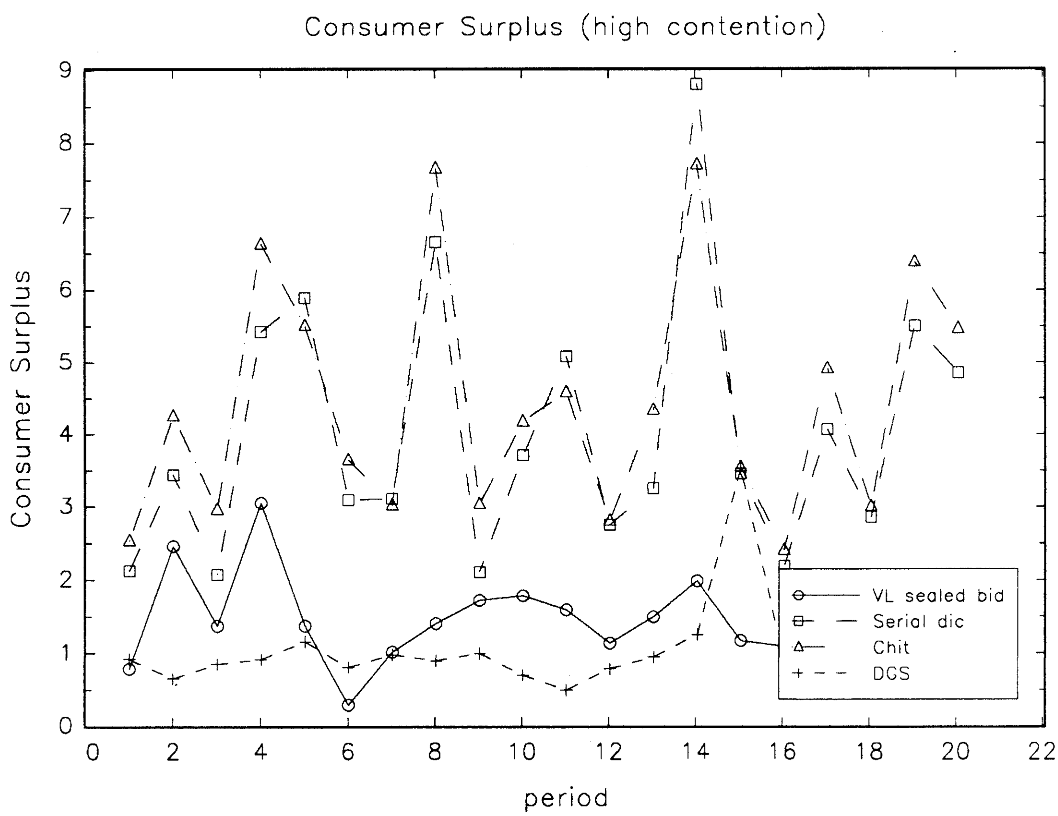
2
4
6
8
10
12
14
16
18
20
22
VL sealed bid
Serial dic
Chit
DGS


Consumer Surplus (high contention)
Consumer Surplus
period
9
8
7
6
5
4
3
2
1
0
0
2
4
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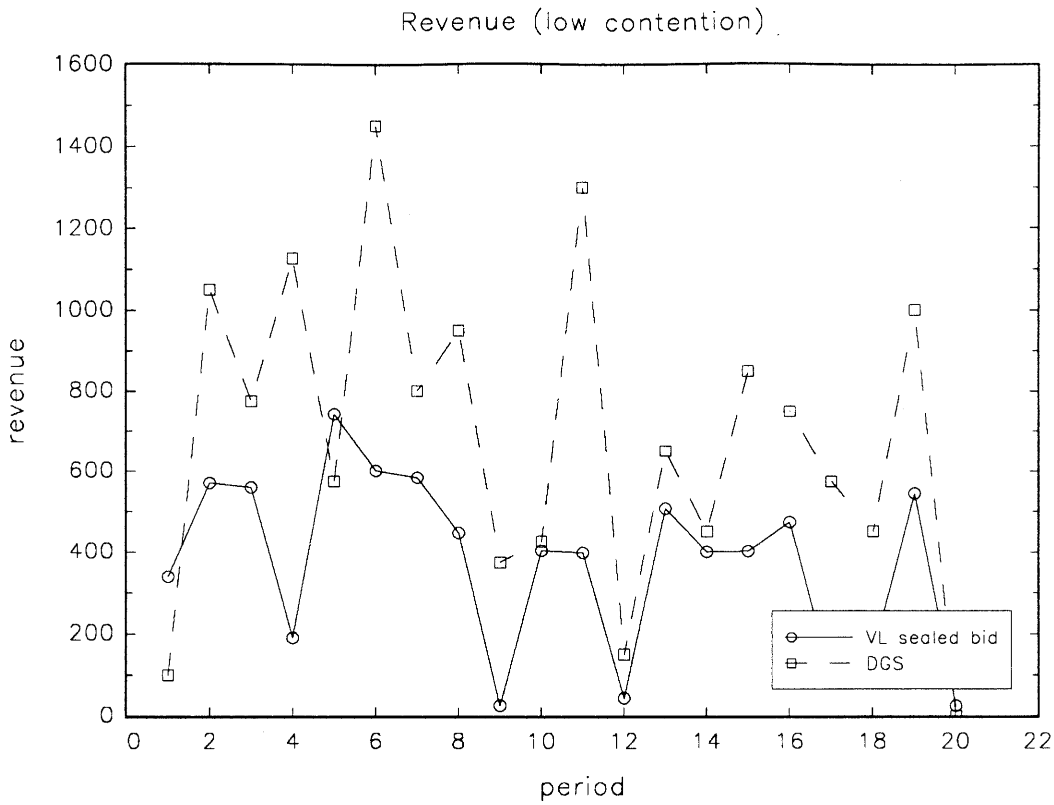
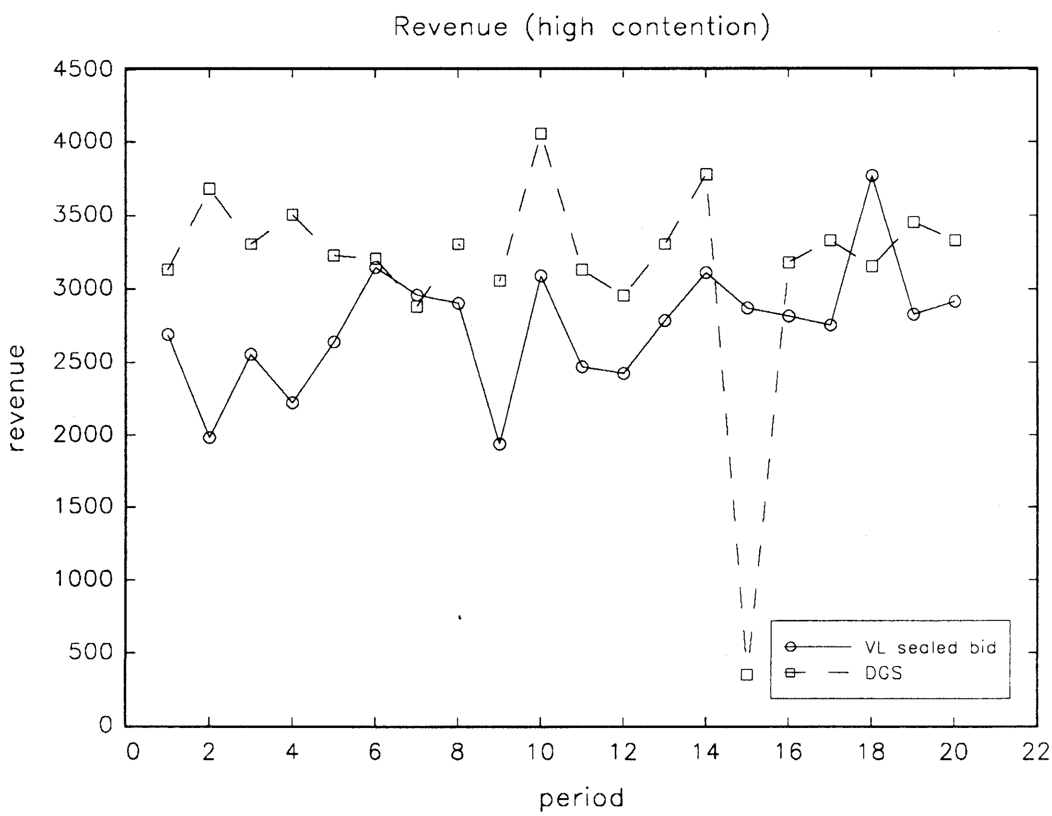
VL sealed bid
Serial dic
Chit
DGS


Revenue (low contention)
1600
1400
1200
1000
800
600
400
200
0
revenue
0
2
4
6
8
10
12
14
16
18
20
22
period
VL sealed bid
DGS


Revenue (high contention)
4500
4000
3500
3000
2500
2000
1500
1000
500
0
revenue
0
2
4
6
8
10
12
14
16
18
20
22
period
VL sealed bid
DGS

Appendix F: Equilibria for chit mechanism

In this section we discuss equilibria for the chit mechanism. We will discuss equilibria in 2 simple cases (1) $k = 2, n \geq 2$, and 2) $k = 3, n = 3$), and give some numerical examples. We will then discuss the equilibria of the chit mechanism in the experimental environment, including the use of a genetic algorithm to compute an equilibrium. We first present some facts associated with the implementation of the chit mechanism:

1. The chit mechanism uses the solution to the optimal assignment problem, where an agent submits a bid of chits or "funny money" to allocate slots; ties are broken randomly.

2. Chit bids are allowed to be from a discrete set with lower-bound 0 and upper bound an assigned and known chit budget. Every agent's budget is known to the planner, and each individual agent knows her own budget and the budgets of the other agents.

3. Chits do not affect an agent's valuation of a slot or her utility; chits affect only the probability of being assigned a particular slot.

4. An agent's preferences (her type) are randomly chosen with replacement from a given population of types.

Notation:

Let $K = \{1,\ldots,k\}$ be the set of slots to be allocated and let $N = \{1,\ldots,n\}$ be the set of agents. Let $B^i \equiv$ agent i's chit budget, and let $B^i = B$, $\forall i \in N$; this is not necessarily a simplifying condition. A bid is a vector $b \in \Re^k_+$ such that $\sum_j b_j \leq B$. Let $P_j(b^i) \equiv$ Probability that i wins slot j with bid b^i. If $n \geq k$, then $\sum_{j=1}^{k} P_j(b^i) = 1$, $\forall j \in K$, since the VL mechanism yields a strictly feasible outcome when all bids are nonnegative; that is, all slots are assigned if there are as many bidders as slots. Let $\Delta^{k\text{-}1}(B) \equiv \{b \in \Re^k_+ \mid \sum_j b_j = B\}$ the set of feasible bids where all chits are used. Let $U(b^i, \theta^i) \equiv$ Utility of a type $\theta^i \in \Theta^i$ with a bid of b^i; we will usually write $U(b)$ with θ^i and i implicit, specifically, $U(b) = \sum_{j \in K} \theta_j P_j(b)$.

There are two simple restrictions on bidding rules that are necessary for a bidding rule to be undominated:

C1. Bid all chits on the slots.

C2. Bid zero on the least preferred slot. If there are more bidders than slots, then there is an implied slot that has zero value to all agents.

We will discuss equilibria in 2 simple cases[1]: 1) $k = 2, n \geq 2$, and 2) $k = 3, n = 3$.

---

[1] For the trivial case of $k = 1$, $n > 1$ we observe: $U(b)=\theta_1 P_1(b)$; $b \in [0,1]$. Result: $b^i = B^i$ is a dominant strategy for all $i \in N$. The probability of winning the slot is:

$$P_1(b^i) = \text{Prob}(b^i > \max_{j \neq i}\{b^j\}) + \text{Prob}(b^i = \max_{j \neq i}\{b^j\}) \sum_{\substack{l=1 \\ l \neq i}}^{n-1} \text{Prob}(l \text{ bids tied with } b^i \mid b^i = \max_{j \neq i}\{b^j\}) \left(\frac{1}{l+1}\right)$$

$P_1(b^i)$ is maximized with the largest b possible, which is B^i, independent of the other agents' bids. In this case no information about the value of the slot is obtained from the agents and the resulting allocation is random.

Case n = k = 2

$$U(b) = \theta_1 \cdot P_1(b) + \theta_2 \cdot P_2(b),$$
$$= \theta_2 + (\theta_1 - \theta_2) \cdot P_1(b); \quad b \in \Delta^1(B);$$

result: if $\theta_1 > \theta_2$ bid so $P_1(b)$ is maximized, $b_1 = B^i$, $b_2 = 0$,

if $\theta_1 < \theta_2$ bid so $P_1(b)$ is minimized, $b_1 = 0$, $b_2 = B^i$,

if $\theta_1 = \theta_2$, the agent is indifferent between slots, so any feasible bid,

is a weakly dominant strategy for all $i \in N$.

The probability of winning slot 1, given that agent i uses the above strategy, is

$$P_1(b^i) = \text{Prob}(b_1^i > \max_{l \neq i}\{d^l = b_1^l - b_2^l\})$$

$$+ \text{Prob}(b_1^i = \max_{l \neq i}\{d^l\}) \sum_{\substack{m=1 \\ m \neq i}}^{n-1} \left\{ \text{Prob}(m\ d^m \text{ tied with } b_1^i \mid b_1^i = \max_{l \neq i}\{d^l\}) \cdot \left(\frac{1}{m+1}\right) \right\},$$

where $d^l = b_1^l - b_2^l$. $P_1(b^i)$ is maximized with the largest b_1^i possible, which is B^i, independent of the other agents' bids.

$P_2(b^i)$ is similar to P_1.

In this case the agents supply information on their rankings, and the allocation is the same as the serial dictator.

Case n = k = 3

$$U(b) = \theta_1 \cdot P_1(b) + \theta_2 \cdot P_2(b) + \theta_3 \cdot P_3(b) \quad ,$$
$$= \theta_3 + (\theta_1 - \theta_3) \cdot P_1(b) + (\theta_2 - \theta_3) \cdot P_2(b); \quad b \in \Delta^2(B).$$

Without loss of generality let $\theta_1 > \theta_2 > \theta_3$.

Given the observations made in C1 and C2, chits will be allocated between an agent's two highest valued slots. There are three resulting types of strategies:

S1. Bid all chits on the highest valued slot.

S2. Bid all chits on the second highest valued slot.

S3. Divide the chits between the highest valued and the second highest valued slots. There are three variations of the strategy:

S3-1. Bid more on the highest valued slot.

S3-2. Bid more on the second highest valued slot.

S3-3. Bid an equal amount on the highest valued slot and the second highest valued slot.

We will first describe the probability of being assigned a slot. We will restrict the strategies of agents $i = 2$ and 3 to strategy S1, and compute the probabilities of receiving a slot to agent $i = 1$.

Suppose agent 1 uses the strategy S1:

Then $b = (B, 0, 0)$,

$P_1(b) = \Pr(\text{no tie slot 1}) + \frac{1}{2}\Pr(\text{1 tie slot 1}) + \frac{1}{3}\Pr(\text{2 ties slot 1})$;

$P_1(b) = \Pr(\text{no tie slot 1}) + \frac{1}{2}\{\Pr(\text{bid slot 1 and 3}) + \Pr(\text{bid slot 1 and 2})\}$
$+ \frac{1}{3}\Pr(\text{2 ties slot 1})$;

$P_2(b) = \frac{1}{2}\Pr(\text{bid slot 1 and 3}) + \frac{1}{3}\Pr(\text{2 ties slot 1})$;

$P_3(b) = \frac{1}{2}\Pr(\text{bid slot 1 and 2}) + \frac{1}{3}\Pr(\text{2 ties slot 1})$,

where the probabilities on the right-hand side describe the placement of bids by agents 2 and 3; *e. g.*, $\Pr(\text{bid slot 1 and 2}) \equiv$ Probability that either agent 2 bids on slot 1 and 3 bids on slot 2 or agent 2 bids on slot 2 and 3 bids on slot 1. When agents $i = 2,3$, use strategy S1, then

$$P_1(b) = \text{Prob}(b_1^1 > \max_l\{b_1^l\}) + \tfrac{1}{2}\text{Pr}(b_1^1 = b_1^2, b_1^1 > b_1^3)$$
$$+ \tfrac{1}{2}\text{Pr}(b_1^1 = b_1^3, b_1^1 > b_1^2) + \tfrac{1}{3}\text{Pr}(b_1^1 = b_1^2, b_1^1 = b_1^3).$$

To determine this probability, denote the possible rankings of slots:

(0) : 1,2,3 $\equiv \theta_1 > \theta_2 > \theta_3$	(1) : 1,3,2
(2) : 2,1,3	(3) : 2,3,1
(4) : 3,1,2	(5) : 3,2,1 .

Define three different types by their highest valued slot; type A $\equiv$ {ranks 0 and 1}, B $\equiv$ {ranks 2 and 3}, C $\equiv$ {ranks 4 and 5}. Let $P_A = \text{Pr(type A)}$, $P_B = \text{Pr(type B)}$, and $P_C = \text{Pr(type C)}$. Given that agent 1 is type A we define the following notation:

$$P(2) = \text{Pr(both agents 2 and 3 are type A)} = P_A{}^2;$$

$$P(1) = \text{Pr(only one agent is type A)}$$
$$= \text{Pr(1 type A, one type C)} + \text{Pr(1 type A, 1 type B)}$$
$$= P(\text{A},\text{C}) + P(\text{A},\text{B})$$
$$= 2P_AP_C + 2P_AP_B;$$

$$P(0) = \text{Pr(no other agent is type A)}$$
$$= \text{Pr(one type B, one type C)} + \text{Pr(2 type B)} + \text{Pr(2 type C)}$$
$$= 2P_CP_B + P_B{}^2 + P_C{}^2.$$

$P_1(b) = P(0) + \frac{1}{2}P(1) + \frac{1}{3}P(2) = 2P_C P_B + P_B^2 + P_C^2 + P_A P_C + P_A P_B + \frac{1}{3}P_A^2;$

$P_2(b) = \frac{1}{2}P(\text{A},\text{C}) + \frac{1}{3}P(2) = P_A P_C + \frac{1}{3}{P_A}^2;$

$P_3(b) = \frac{1}{2}P(\text{A},\text{B}) + \frac{1}{3}P(2) = P_A P_B + \frac{1}{3}{P_A}^2.$

Suppose agent 1 uses the strategy $b^* = (0, B^i, 0)$, while agents 2 and 3 use the strategy S1, then:

$P_2(b^*) = \text{Pr(no tie slot 2)} + \frac{1}{2}\text{Pr(1 tie slot 2)} + \frac{1}{3}\text{Pr(2 tie slot 2)};$

$P_1(b^*) = \frac{1}{2}\text{Pr(1 tie slot 2, bid slot 3)} + \frac{1}{3}\text{Pr(2 tie slot 2)};$

$P_3(b^*) = \frac{1}{2}\text{Pr(1 tie slot 2, bid slot 1)} + \frac{1}{3}\text{Pr(2 tie slot 2)}.$

$P_1(b^*) = \frac{1}{2}P(B,C) + \frac{1}{3}P(2);$

$P_2(b^*) = P(0) + \frac{1}{2}P(1) + \frac{1}{3}P(2);$

$P_3(b^*) = \frac{1}{2}P(\text{A},\text{B}) + \frac{1}{3}P(2).$

$P(0) = P(\text{A},\text{B}) + P(\text{B},\text{C});$

$P(2) = {P_B}^2;$

$P(\text{B},\text{C}) = 2P_B P_C;$

$P(\text{B},\text{A}) = 2P_B P_A;$

$P(1) = 2P_B P_A + 2P_B P_C;$

$P(0) = 2P_C P_A + {P_A}^2 + {P_C}^2.$

$P_2(b^*) = P_B P_C + \frac{1}{3}{P_B}^2;$

$P_1(b^*) = 2P_C P_A + P^2{}_A + P^2{}_C + P_A P_B + P_C P_B + \frac{1}{3}{P_B}^2;$

$P_3(b^*) = P_B P_A + \frac{1}{3}{P_B}^2;$

Suppose agent 1 uses the strategy $b^\epsilon = (B^i - \epsilon, \epsilon, 0)$, while agents 2 and 3 use the strategy S1; then:

$$P_1(b^\epsilon) = \Pr(2 \text{ bids slot } 3) + \Pr(2 \text{ bids slot } 2) + \Pr(1 \text{ bid } 2,\ 2 \text{ bid } 3),$$
$$P_2(b^\epsilon) = \Pr(1 \text{ bid } 1,\ 1 \text{ bid } 3) + \Pr(2 \text{ bid } 1),$$
$$P_3(b^\epsilon) = \Pr(1 \text{ bid } 1,\ 1 \text{ bid } 2).$$

$$P_1(b^\epsilon) = {P_C}^2 + {P_B}^2 + 2P_BP_C$$
$$P_2(b^\epsilon) = 2P_CP_A + {P_A}^2$$
$$P_3(b^\epsilon) = 2P_BP_A$$

Given the probabilities computed above, we can find the utility to agent one of playing each one of the strategies b, b^*, b^ϵ.

$$\begin{aligned} U(b) &= \theta_3 + P_1(b)(\theta_1 - \theta_3) + P_2(b)(\theta_2 - \theta_3) \\ &= \theta_3 + (2P_CP_B + P_B^2 + P_C^2 + P_AP_C + P_AP_B + \tfrac{1}{3}P_A^2)(\theta_1 - \theta_3) \\ &+ (P_AP_C + \tfrac{1}{3}P_A^2)(\theta_2 - \theta_3). \end{aligned}$$

$$\begin{aligned} U(b^*) &= \theta_3 + P_1(b^*)(\theta_1 - \theta_3) + P_2(b^*)(\theta_2 - \theta_3) \\ &= \theta_3 + (P_BP_C + \tfrac{1}{3}P_B^2)(\theta_1 - \theta_3) \\ &+ \ (2P_CP_A + P_A^2 + P_C^2 + P_AP_B + P_CP_B + \tfrac{1}{3}P_B^2)\ (\theta_2 - \theta_3). \end{aligned}$$

$$\begin{aligned} U(b^\epsilon) &= \theta_3 + P_1(b^\epsilon)(\theta_1 - \theta_3) + P_2(b^\epsilon)(\theta_2 - \theta_3) \\ &= \theta_3 + (2P_CP_B + P_B^2 + P_C^2)(\theta_1 - \theta_3) + (2P_AP_C + P_A^2)(\theta_2 - \theta_3). \end{aligned}$$

We can describe the conditions necessary for S1 to be an equilibrium strategy. Without loss of generality let $\theta_3 = 0$.

$$U(b) > U(b^*)$$

$$(2P_C P_B + P_B^2 + P_C^2 + P_A P_C + P_A P_B + \tfrac{1}{3}P_A^2)\theta_1 + (P_A P_C + \tfrac{1}{3}P_A^2)\theta_2$$
$$> (P_B P_C + \tfrac{1}{3}P_B^2)\theta_1 + (2P_C P_A + P_A^2 + P_C^2 + P_A P_B + P_C P_B + \tfrac{1}{3}P_B^2)\theta_2$$

$$(P_C P_B + \tfrac{2}{3}P_B^2 + P_C^2 + P_A P_C + P_A P_B + \tfrac{1}{3}P_A^2)\theta_1$$
$$> (P_C P_A + \tfrac{2}{3}P_A^2 + P_C^2 + P_A P_B + P_C P_B + \tfrac{1}{3}P_B^2)\theta_2$$

$$\frac{(P_C P_B + \tfrac{2}{3}P_B^2 + P_C^2 + P_A P_C + P_A P_B + \tfrac{1}{3}P_A^2)}{(P_C P_A + \tfrac{2}{3}P_A^2 + P_C^2 + P_A P_B + P_C P_B + \tfrac{1}{3}P_B^2)} > \frac{\theta_2}{\theta_1}.$$

So $U(b) > U(b^*)$ is true for all θ_2 and θ_1, since $\theta_2 < \theta_1$, when

$$\tfrac{2}{3}P_B^2 + \tfrac{1}{3}P_A^2 \geq \tfrac{2}{3}P_A^2 + \tfrac{1}{3}P_B^2 \Rightarrow P_B^2 \geq P_A^2 \Rightarrow P_B \geq P_A.$$

Similarly, $U(b) > U(b^\epsilon)$ is true for all θ_2 and θ_1 when $P_B \geq \tfrac{1}{3}P_A$,

and $U(b^*) > U(b^\epsilon)$ is true for all θ_2 and θ_1 when $\tfrac{1}{3}P_B \geq P_A$.

When $P_A = P_B = P_C$, the strategy S1 is an equilibrium strategy. The above relationships indicate that in some circumstances it is optimal (in an expected utility sense) to bid on the second, most preferred slot.

Genetic algorithm for chit equilibria

To calculate the equilibria for the experimental environment, we used a genetic algorithm[2]. Genetic algorithms (GAs) are "search algorithms that are based on the mechanics of natural selection and natural genetics."[3] We use the GA as numerical method to compute the equilibria of the chit mechanism and not as a tool for studying "learning behavior."[4] The GA is designed to be effective in solving problems where more traditional methods (such as calculus-based methods) fail. The GA is robust to changes in its parameters and uses payoff (or objective function) information, not derivatives or other external knowledge[5]. Because of the use of payoff functions, GAs are easily adapted to find equilibria of games.

To compute equilibria using a GA, we used the following approach. Individual strategies were represented by a string of 6 digits (0,...,9), one for each slot. Each digit represented the percent of chits (relative to the sum of the digits) to be applied to its corresponding slot. For example, if $s = 402712$, the sum is 16 so the percent of chits placed on slot 1 is 4/16 or 25%, and on slot 6 is 2/16 or 12.5%. The total number of chits was set to 1000 (as in our experiments), so the corresponding bid is (250, 0, 125, 437, 62, 125), fractions are truncated so that the total may not be 1000. This truncation rule is arbitrary but meets the requirement that the sum of bids is no greater than 1000. In this representation all of the chits are allocated to the slots, except for 1 or 2 chits because of truncation. We will discuss this restriction on the strategy space

[2] See Goldberg (1989) for an introduction to genetic algorithms, and Andreoni and Miller (1990) for the use of a GA in an economic game.

[3] Goldberg (1989), p. 1.

[4] See Andreoni and Miller (1990) for an example of GAs used to study individual learning behavior .

[5] Goldberg (1989), p. 7.

below.

The GA was performed under both the low-contention and the high-contention experimental environments. Since there are 10 types (slot-valuation sheets) in each of our experimental environments, we created the same 10 types for the GA in each environment. Each type had a population of 20 strategies. Initial strategies were chosen at random by choosing a digit from the uniform distribution, Prob(digit $= d) = \frac{1}{10}$, $d \in \{0,\ldots,9\}$, with replacement. This population of strategies comprises a generation. The genetic operations of reproduction, crossover, and mutation are then applied to a generation to form a new generation. The process repeats itself for a specified number of generations.

To form a new generation of strategies, participants (six for the low-contention environment and eight for the high-contention environment) were chosen at random (uniformly) from the population of strategies with replacement. The strategies of these chosen participants were submitted to an assignment algorithm, and the optimal assignment was computed and the payoffs assigned to each participant (based on their type). Four hundred games were played, given a population of strategies.[6] If a strategy was chosen more than once (each strategy was expected to be chosen 12 times in the low-contention environment and 16 times in the high-contention environment), the average payoff was calculated and assigned to the strategy. If a strategy was not chosen in the 400, games it was assigned the average payoff for its type.

A new population was then constructed by first assigning a probability weight to each strategy. The weight was constructed by first removing strategies that had payoffs 1.5 standard deviations below the mean payoff to the strategies

[6] I initially set the number of games to 200, but found that in playing 400 games, there was faster convergence, since a better estimate of the performance of a strategy was obtained.

of its type (the standard deviation was calculated only from the strategies of the same type). The payoffs were then scaled as described by Goldberg (1989), page 79, and the resulting scaled payoffs are called the fitness of the strategy. The fitness was then used to construct the probability weights (weights for each type were constructed separately), where the probability that strategy s is chosen is the fitness of s divided by the sum of fitness for its type. Ten pairs of strategies were then chosen with replacement on the basis of these probability weights.

The process of constructing a new population from the fitness is called reproduction and is a genetic operator. Two other genetic operators, crossover and mutation, were then applied to the new pairs of strategies. A crossover was applied to a pair of strategies with a 60% probability. A crossover exchanges information between two strategies to create two new strategies and mimics innovation and mating. After the population of strategies completes the crossover step, individual digits undergo mutation. Mutation is a random selection of a new digit. If a digit is selected for mutation, then a new digit is chosen at random from $\{0,\ldots,9\}$. The probability of mutation was initially set to between 3 and 6% and was decreased after a specified number of generations. This completes a generation.

We varied the parameters (number of generations, mutation probability, generations between reduction in mutation probability, and number of games each generation) of the GA, and the results we obtained appear robust. We also artificially used an equilibrium outcome as an initial population to see if it would be stable. There were no variations from the initial population.

The construction of strategies by representing slots by a 0–9 digit is not the only construction available. In fact, there is a question about the restriction of the strategy space that should allowed. For example, the first representation of strategies we used consisted of a binary digit 0/1 for each slot; the allocation of chits was divided among those slots that were represented by a 1. A restriction that we used for both of these representations was to allocate the entire chit budget, which is a weakly dominant strategy for all types and a strong dominant strategy for most types. This relates to a third restriction which we have begun to test, the restriction to dominant strategies when they exist or to perturbations about dominant strategies when they exist. For instance, in the low-contention case, type 9 has a dominant strategy to bid all her chits on slot 6 (value 900), since she is guaranteed to receive a slot worth 300. We could restrict strategies of this type to the dominant strategy, or we could allow slight perturbations, perhaps in the range $[-50, +50]$.

The result of allowing a broader strategy space may be in equilibria that are more robust. The most evident result of restriction of the strategy space is to speed convergence. It is unknown whether a broader strategy space will allow us to find different or more robust equilibria, or whether it is worth the cost of a slower process. A faster process allows us to have a larger number of GA runs, since if there are more than one equilibria, it will usually take a number of runs to find them.

In the next two pages we present the chit equilibrium that resulted from the GA. In the low-contention environment, notice that a number of types have dominant strategies to bid their entire budget on one slot. Also in the low-contention environment, notice that there are two strategies that are entirely indifferent between any bid, since they are guaranteed to receive a slot and they value all of the slots the same.

In the high-contention environment, all strategies except for type #2 had an equilibrium strategy to bid all the chits on one of the top-valued slots. Where there are two top-valued slots (type #7), the equilibrium strategy is to bid all chits on the lowest-contention slot (slot 4). These strategies resulted from a number of different GA runs; in each of the runs the only strategy that differed was for type #2. All the strategies thay were computed for #2 were consistent in that they all had positive bids on the same slots, and the ranking of the slot values was the same; they differed only on the numerice quantity for the bids. In the low contention environment the results were even more consistent and there were no other equilibria that appeared.

Results of genetic algorithm

High contention

type#		slot 1	2	3	4	5	6	Comment
0	value:	900	450	400	350	300	250	
	bid 1:	1000	0	0	0	0	0	
1		400	600	800	600	400	200	
		0	0	1000	0	0	0	
2		800	600	400	200	400	600	not unique to
		0	500	0	125	125	250	perturbations
3		100	100	900	400	300	200	
		0	0	1000	0	0	0	
4		400	800	400	200	0	200	
		0	1000	0	0	0	0	
5		900	600	300	200	100	0	
		1000	0	0	0	0	0	
6		300	300	300	300	300	900	
		0	0	0	0	0	1000	
7		750	250	250	750	400	400	
		0	0	0	1000	0	0	
8		400	200	400	600	800	600	
		0	0	0	0	1000	0	
9		850	350	350	650	150	150	
		1000	0	0	0	0	0	
#top bids		3	1	2	1	1	1	does not include #2

Low contention

type#		slot 1	2	3	4	5	6	Comment
0	value:	800	600	400	200	400	600	
	bid 1:	1000	0	0	0	0	0	
1		400	600	800	600	400	200	
		0	0	1000	0	0	0	
2		400	200	400	600	800	600	
		0	0	0	0	1000	0	
3		850	350	350	850	350	350	
		0	0	0	1000	0	0	
4		750	400	400	750	400	400	
		0	0	0	1000	0	0	
5		900	300	300	300	300	300	
		1000	0	0	0	0	0	dominant strategy
6		300	300	900	300	300	300	
		0	0	1000	0	0	0	dominant strategy
7		500	500	500	500	500	500	
		?	?	?	?	?	?	all strategies indifferent
8		550	550	550	550	550	550	
		?	?	?	?	?	?	all strategies indifferent
9		300	300	300	300	300	900	
		0	0	0	0	0	1000	dominant strategy
#top bids		2	0	2	2	1	1	does not include #7,8

Numerical examples, $n = k = 3$:

example 1.

Let $P_A = P_B = P_C = \frac{1}{3}$:

$P(\mathrm{A,C}) = \frac{2}{9},\ \ P(\mathrm{A,B}) = \frac{2}{9},$

$P(0) = \frac{2}{9} + \frac{1}{9} + \frac{1}{9} = \frac{4}{9},\ \ P(1) = \frac{2}{9} + \frac{2}{9} = \frac{4}{9},\ \ P(2) = \frac{1}{9}.$

$P_1(b) = \frac{4}{9} + \frac{1}{2} \cdot \frac{4}{9} + \frac{1}{3} \cdot \frac{1}{9} = 0.70,$

$P_2(b) = \frac{1}{2} \cdot \frac{2}{9} + \frac{1}{3} \cdot \frac{1}{9} = 0.15,$

$P_3(b) = \frac{1}{2} \cdot \frac{2}{9} + \frac{1}{3} \cdot \frac{1}{9} = 0.15.$

$$\begin{aligned} U(b) &= \theta_3 + P_1(b)(\theta_1 - \theta_3) + P_2(b)(\theta_2 - \theta_3) \\ &= 0.7 \cdot \theta_1 + 0.15 \cdot \theta_2 + 0.15 \cdot \theta_3 \\ &= \theta_3 + 0.7 \cdot (\theta_1 - \theta_3) + 0.15 \cdot (\theta_2 - \theta_3). \end{aligned}$$

$P_1(b^*) = 0.15,\ \ P_2(b^*) = 0.70,\ \ P_1(b^*) = 0.15,$

$$\begin{aligned} U(b^*) &= \theta_3 + P_1 \cdot (\theta_1 - \theta_3) + P_2 \cdot (\theta_2 - \theta_3) \\ &= \theta_3 + 0.15\ (\theta_1 - \theta_3) + 0.7\ (\theta_2 - \theta_3). \end{aligned}$$

$U(b) > U(b^*)$:

$\theta_3 + 0.7 \cdot (\theta_1 - \theta_3) + 0.15 \cdot (\theta_2 - \theta_3) > \theta_3 + 0.15\ (\theta_1 - \theta_3) + 0.7\ (\theta_2 - \theta_3),$

$0.55 \cdot (\theta_1 - \theta_3) > 0.55 \cdot (\theta_2 - \theta_3),$ which is true since $(\theta_1 - \theta_3) > (\theta_2 - \theta_3).$

$P_1(b^\epsilon) = \frac{4}{9},\ \ P_2(b^\epsilon) = \frac{3}{9},\ \ P_3(b^\epsilon) = \frac{2}{9}.$

$U(b^\epsilon) = \theta_3 + 0.\bar{4}(\theta_1 - \theta_3) + 0.\bar{3}(\theta_2 - \theta_3),$

$U(b) > U(b^{\epsilon})$:

$\theta_3 + 0.7 \cdot (\theta_1 - \theta_3) + 0.15 \cdot (\theta_2 - \theta_3) > \theta_3 + 0.\bar{4}(\theta_1 - \theta_3) + 0.\bar{3}(\theta_2 - \theta_3),$

$0.2\bar{5} \cdot (\theta_1 - \theta_3) > 0.18\bar{3} \cdot (\theta_2 - \theta_3),$

$\frac{(\theta_1 - \theta_3)}{(\theta_2 - \theta_3)} > 0.71,$ which is always true since $(\theta_1 - \theta_3) > (\theta_2 - \theta_3)$.

example 2.

Let $P_A = 0.9, \;\; P_B = 0.05, \;\; P_C = 0.05$:

$P(0) = 0.01, \;\; P(1) = 0.18, \;\; P(2) = 0.81,$

$P_1(b) = 0.01 + \frac{1}{2} \cdot (0.18) + \frac{1}{3} \cdot (0.81) = 0.37,$

$P_2(b) = \frac{1}{2} \cdot (0.09) + \frac{1}{3} \cdot (0.81) = 0.315,$

$P_3(b) = \frac{1}{2} \cdot (0.09) + \frac{1}{3} \cdot (0.81) = 0.315.$

$U(b) = \theta_3 + P_1(b)(\theta_1 - \theta_3) + P_2(b)(\theta_2 - \theta_3),$

$\quad = \theta_3 + 0.37 \cdot (\theta_1 - \theta_3) + 0.315 \cdot (\theta_2 - \theta_3).$

$P(2) = 0.0025, P(1{:}0,3) = 0.005, \;\; P(1{:}1,0) = 0.09, \;\; P(1) = 0.095, P(0) = 0.9025.$

$P_1(b^*) = 0.00\bar{3}, \;\; P_2(b^*) = 0.9509\bar{3}, \;\; P_1(b^*) = 0.0458\bar{3}.$

$U(b^*) = \theta_3 + P_1 \cdot (\theta_1 - \theta_3) + P_2 \cdot (\theta_2 - \theta_3),$

$\quad = \theta_3 + 0.00\bar{3}(\theta_1 - \theta_3) + 0.9508\bar{3}(\theta_2 - \theta_3).$

$U(b) > U(b^*)$:

$\theta_3 + 0.37 \cdot (\theta_1 - \theta_3) + 0.315 \cdot (\theta_2 - \theta_3) > \theta_3 + 0.00\bar{3} \; (\theta_1 - \theta_3) + 0.9508\bar{3}(\theta_2 - \theta_3),$

$0.3\bar{6} \cdot (\theta_1 - \theta_3) > 0.6358\bar{3} \cdot (\theta_2 - \theta_3),$

$$\frac{(\theta_1 - \theta_3)}{(\theta_2 - \theta_3)} > \frac{0.63583\bar{3}}{0.36\bar{6}} = 1.73409.$$

So b is optimal only if $\dfrac{(\theta_1 - \theta_3)}{(\theta_2 - \theta_3)} > 1.73409$.

$$P_1(b^\epsilon) = 0.01, \;\; P_2(b^\epsilon) = 0.90, \;\; P_1(b^\epsilon) = 0.09.$$

$$\begin{aligned} U(b^\epsilon) &= \theta_3 + P_1 \cdot (\theta_1 - \theta_3) + P_2 \cdot (\theta_2 - \theta_3), \\ &= \theta_3 + 0.01(\theta_1 - \theta_3) + 0.90(\theta_2 - \theta_3). \end{aligned}$$

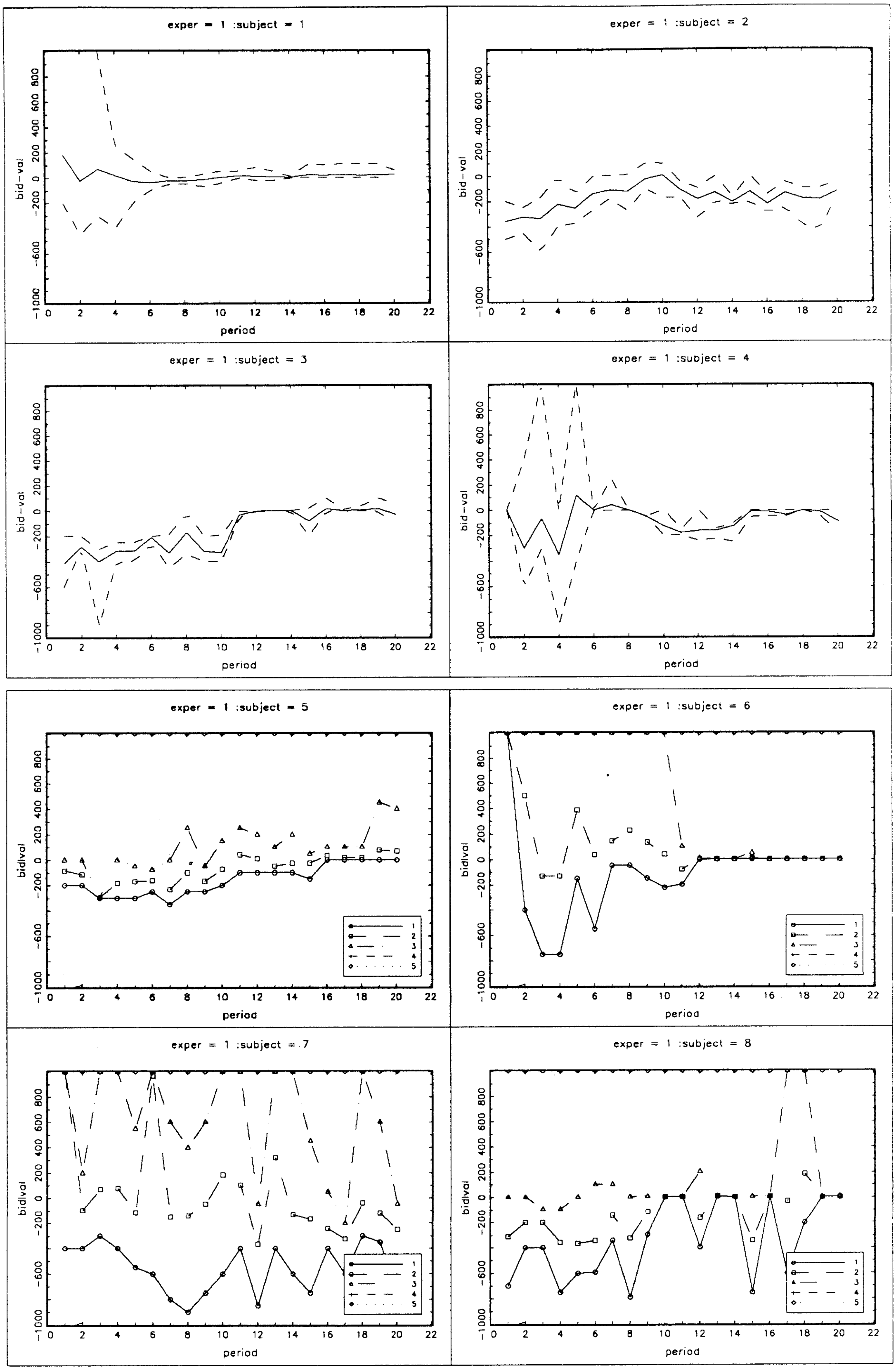
$U(b) > U(b^\epsilon)$:

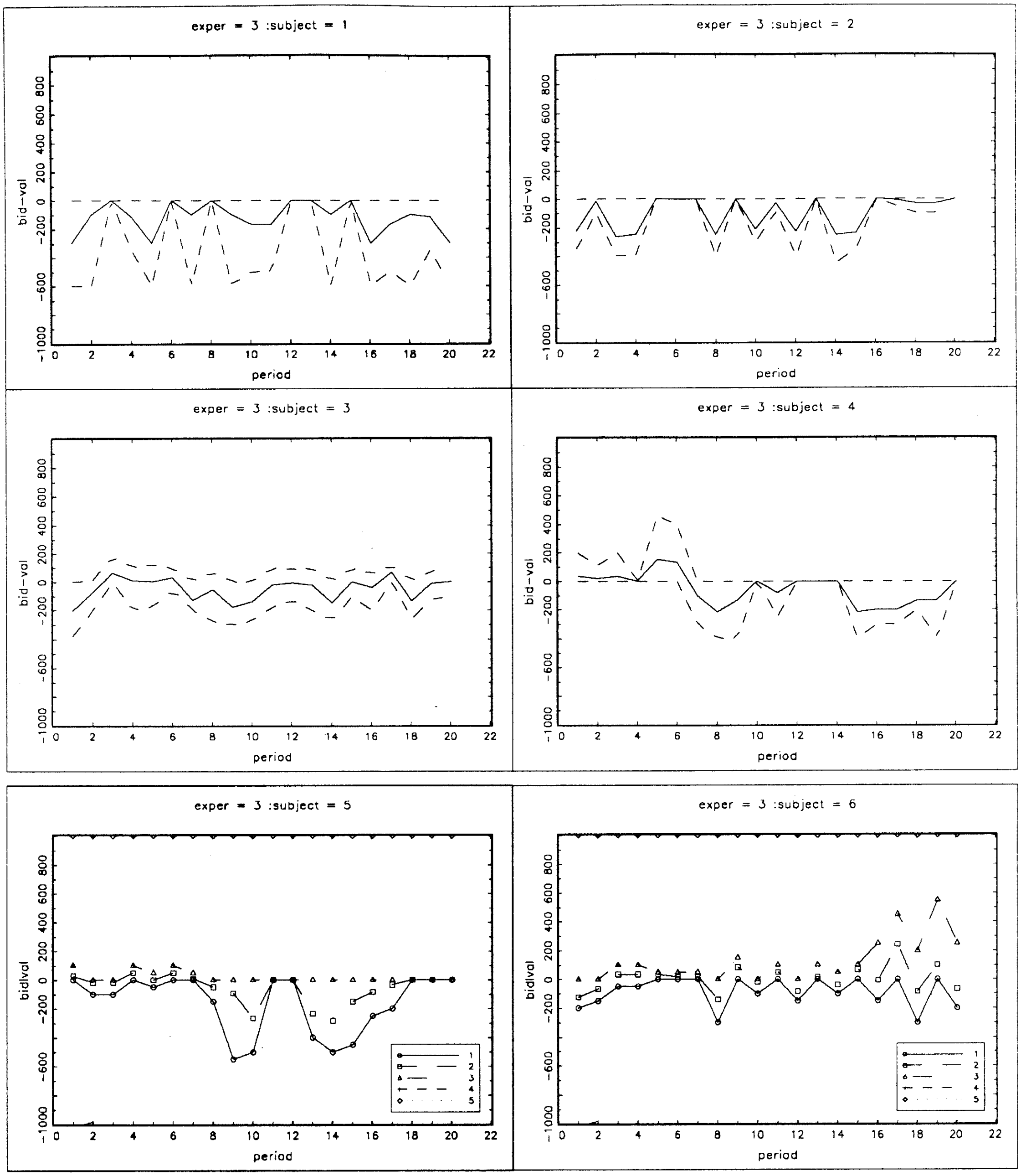
$$\theta_3 + 0.37 \cdot (\theta_1 - \theta_3) + 0.315 \cdot (\theta_2 - \theta_3) > \theta_3 + 0.01 \; (\theta_1 - \theta_3) + 0.90(\theta_2 - \theta_3),$$

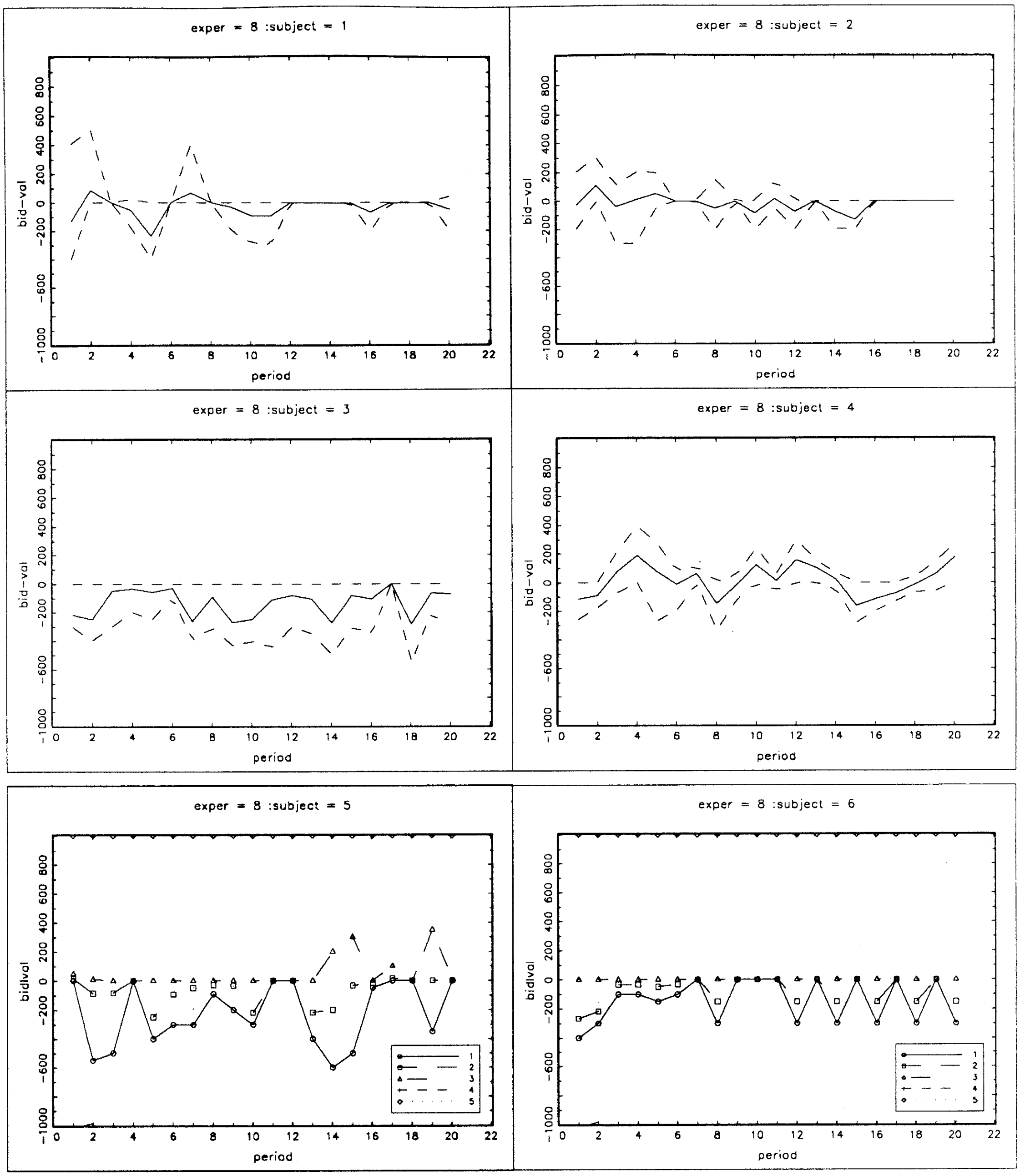
$$\frac{(\theta_1 - \theta_3)}{(\theta_2 - \theta_3)} > \frac{0.585}{0.36} = 1.625.$$

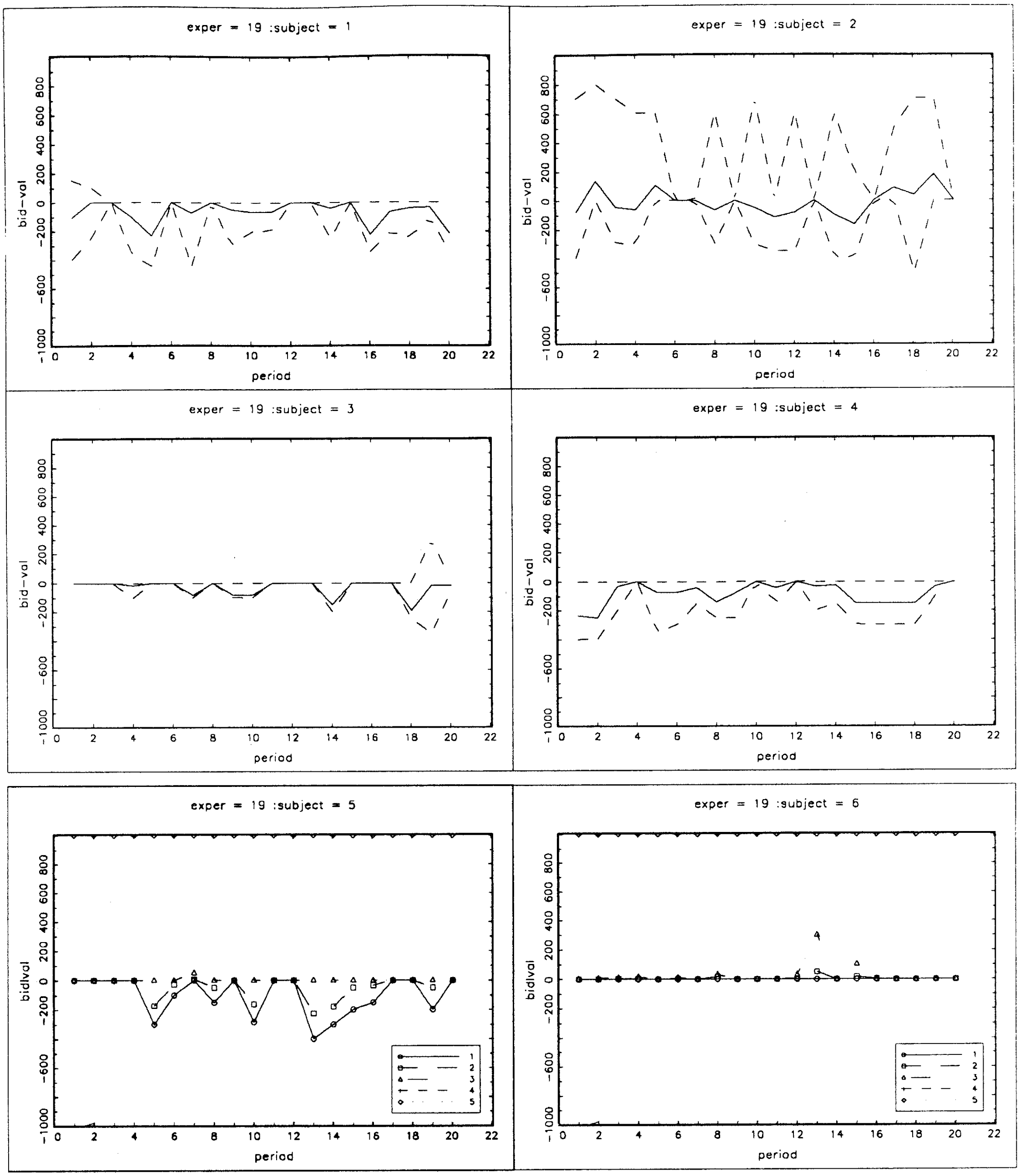
So b is optimal for the agent only if $\dfrac{(\theta_1 - \theta_3)}{(\theta_2 - \theta_3)} > 1.625$.

Appendix G.1. Individual behavior sealed-bid auction--graphs




exper = 3 :subject = 1
exper = 3 :subject = 2
exper = 3 :subject = 3
exper = 3 :subject = 4
exper = 3 :subject = 5
exper = 3 :subject = 6
bid-val
bidival
period
-1000
-600
-200
0
200
400
600
800
0
2
4
6
8
10
12
14
16
18
20
22
1
2
3
4
5


exper = 8 :subject = 1
exper = 8 :subject = 2
exper = 8 :subject = 3
exper = 8 :subject = 4
exper = 8 :subject = 5
exper = 8 :subject = 6
bid-val
bidlval
period


exper = 19 :subject = 1
exper = 19 :subject = 2
exper = 19 :subject = 3
exper = 19 :subject = 4
exper = 19 :subject = 5
exper = 19 :subject = 6
bid-val
bidival
period
-1000
-600
-200
0
200
400
600
800
2
4
6
8
10
12
14
16
18
20
22
1
2
3
4
5

Appendix G.2 Individual summary statistics for VL auction

All experiments:

Variable	Mean	Std Dev	Minimum	Maximum	Valid
PAYOFF	380.7632	310.5245	-1600.000	900.000	680.00
TRUTHPAY	426.2000	258.5429	0.000	900.000	680.00
TRUTHDIF	45.4368	146.7570	0.000	1600.000	680.00
VBMIN	-163.8368	470.0848	-900.000	9099.000	680.00
VBMEAN	14.6676	570.6582	-416.667	9649.000	680.00
VBMAX	337.7397	1395.6819	-300.000	9999.000	680.00

(Contention Low)

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-160.9028	174.7272	-600.000	0.000	360.00
VBMEAN	-50.9958	93.6785	-299.833	241.667	360.00
VBMAX	57.9250	142.3833	0.000	800.000	360.00
PAYOFF	575.9583	172.5558	100.000	900.000	360.00
TRUTHPAY	601.4500	157.3724	300.000	900.000	360.00
TRUTHDIF	25.4917	77.8262	0.000	500.000	360.00

(High contention)

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-167.1375	660.2900	-900.000	9099.000	320.00
VBMEAN	88.5391	820.3294	-416.667	9649.000	320.00
VBMAX	652.5313	1983.8441	-300.000	9999.000	320.00
PAYOFF	161.1688	283.4856	-1600.000	895.000	320.00
TRUTHPAY	229.0438	201.8817	0.000	895.000	320.00
TRUTHDIF	67.8750	195.1230	0.000	1600.000	320.00

(By Experiment)

==

exper eq 1

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-247.5375	271.8287	-900.000	1200.000	160.00
VBMEAN	-51.7479	251.8547	-416.667	1666.500	160.00
VBMAX	312.6313	912.5508	-300.000	9099.000	160.00
PAYOFF	148.0375	304.8587	-1600.000	895.000	160.00
TRUTHPAY	230.4438	210.5809	0.000	895.000	160.00
TRUTHDIF	82.4063	219.7484	0.000	1600.000	160.00

Exper eq 3

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-188.1917	197.9628	-599.000	0.000	120.00
VBMEAN	-62.6042	107.1294	-299.833	241.667	120.00
VBMAX	47.1333	97.5774	0.000	550.000	120.00
PAYOFF	586.0833	177.4733	200.000	900.000	120.00
TRUTHPAY	616.6917	159.1284	300.000	900.000	120.00
TRUTHDIF	30.6083	89.1370	0.000	500.000	120.00

exper eq 4

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-86.7375	887.5851	-900.000	9099.000	160.00
VBMEAN	228.8260	1116.7245	-381.333	9649.000	160.00
VBMAX	992.4313	2613.5725	-200.000	9999.000	160.00
PAYOFF	174.3000	260.6678	-1352.000	749.000	160.00
TRUTHPAY	227.6438	193.4438	0.000	796.000	160.00
TRUTHDIF	53.3438	166.3463	0.000	1352.000	160.00

exper eq 8

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-168.6833	166.5264	-600.000	0.000	120.00
VBMEAN	-51.1542	96.6341	-283.333	188.000	120.00
VBMAX	50.7417	107.3225	0.000	501.000	120.00
PAYOFF	575.4750	167.0253	100.000	900.000	120.00
TRUTHPAY	604.6500	148.1712	300.000	900.000	120.00
TRUTHDIF	29.1750	74.7656	0.000	300.000	120.00

exper eq 19

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-125.8333	152.0834	-500.000	0.000	120.00
VBMEAN	-39.2292	73.3766	-248.833	175.000	120.00
VBMAX	75.9000	199.0602	0.000	800.000	120.00
PAYOFF	566.3167	173.8830	200.000	900.000	120.00
TRUTHPAY	583.0083	163.8610	300.000	900.000	120.00
TRUTHDIF	16.6917	67.9640	0.000	400.000	120.00

By sheet index (Low contention)

shtindx eq 1

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-149.8788	189.3781	-599.000	0.000	33.00
VBMEAN	-33.5202	96.0956	-299.833	178.500	33.00
VBMAX	61.3939	144.6876	0.000	700.000	33.00
PAYOFF	537.8182	158.6698	200.000	900.000	33.00
TRUTHPAY	573.0909	132.7673	400.000	900.000	33.00
TRUTHDIF	35.2727	80.9345	0.000	350.000	33.00

shtindx eq 2

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-121.2381	178.8194	-597.000	0.000	21.00
VBMEAN	-43.8492	98.4283	-298.500	100.000	21.00
VBMAX	49.7619	136.0029	0.000	600.000	21.00
PAYOFF	596.3333	141.4826	350.000	800.000	21.00
TRUTHPAY	606.3333	130.1704	400.000	800.000	21.00
TRUTHDIF	10.0000	45.8258	0.000	210.000	21.00

shtindx eq 3

Variable

Mean Std Dev Minimum Maximum Valid

VBMIN	-204.7778	231.1708	-597.000	0.000	9.00
VBMEAN	-91.2593	112.3225	-298.500	0.333	9.00
VBMAX	0.2222	0.4410	0.000	1.000	9.00
PAYOFF	655.6667	148.7078	501.000	850.000	9.00
TRUTHPAY	655.6667	148.7078	501.000	850.000	9.00
TRUTHDIF	0.0000	0.0000	0.000	0.000	9.00

shtindx eq 4

Variable

Mean Std Dev Minimum Maximum Valid

VBMIN	-195.8000	159.2303	-550.000	0.000	45.00
VBMEAN	-74.7778	91.7295	-255.000	76.333	45.00
VBMAX	59.3778	144.0018	0.000	700.000	45.00
PAYOFF	627.3111	201.2411	200.000	900.000	45.00
TRUTHPAY	655.1556	168.8651	300.000	900.000	45.00
TRUTHDIF	27.8444	93.9797	0.000	400.000	45.00

shtindx eq 5

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-153.3833	176.1485	-600.000	0.000	60.00
VBMEAN	-37.5111	70.0996	-283.333	157.167	60.00
VBMAX	38.0167	92.5932	0.000	501.000	60.00
PAYOFF	533.7000	164.9302	100.000	899.000	60.00
TRUTHPAY	568.3667	148.8919	300.000	899.000	60.00
TRUTHDIF	34.6667	92.0010	0.000	500.000	60.00

shtindx eq 6

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-209.3333	175.8559	-597.000	0.000	39.00
VBMEAN	-78.0598	107.7966	-298.500	133.333	39.00
VBMAX	74.8462	173.8781	0.000	700.000	39.00
PAYOFF	635.7692	176.0150	300.000	850.000	39.00
TRUTHPAY	653.9487	173.7980	300.000	850.000	39.00
TRUTHDIF	18.1795	66.8013	0.000	350.000	39.00

shtindx eq 7

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-180.4889	172.0896	-599.000	0.000	45.00
VBMEAN	-71.9074	95.0623	-266.833	83.333	45.00
VBMAX	51.2444	140.9837	0.000	700.000	45.00
PAYOFF	636.6000	170.2225	300.000	900.000	45.00
TRUTHPAY	648.4000	163.6025	300.000	900.000	45.00
TRUTHDIF	11.8000	44.8825	0.000	200.000	45.00

shtindx eq 8

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-207.0833	206.0262	-550.000	0.000	12.00
VBMEAN	-80.0000	86.9182	-271.667	0.000	12.00
VBMAX	0.8333	2.8868	0.000	10.000	12.00
PAYOFF	599.8333	196.5622	300.000	900.000	12.00
TRUTHPAY	665.2500	121.0328	550.000	900.000	12.00
TRUTHDIF	65.4167	119.9897	0.000	300.000	12.00

shtindx eq 9

Variable

	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-122.1667	148.6959	-500.000	0.000	42.00
VBMEAN	-24.6111	95.2401	-275.000	153.333	42.00
VBMAX	94.8810	190.3243	0.000	800.000	42.00
PAYOFF	506.7619	140.2572	250.000	800.000	42.00
TRUTHPAY	527.7143	133.5042	300.000	800.000	42.00
TRUTHDIF	20.9524	55.1835	0.000	279.000	42.00

shtindx eq 10

Variable

	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-123.5926	171.6961	-595.000	0.000	54.00
VBMEAN	-30.0123	90.3325	-263.333	241.667	54.00
VBMAX	66.7963	146.7699	0.000	600.000	54.00
PAYOFF	537.0000	156.6922	300.000	850.000	54.00
TRUTHPAY	565.9815	152.3612	300.000	850.000	54.00
TRUTHDIF	28.9815	91.4113	0.000	485.000	54.00

By sheet index (High contention)
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shtindx eq 11

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-172.5250	174.9529	-600.000	1.000	40.00
VBMEAN	41.7667	309.7123	-327.333	1266.500	40.00
VBMAX	681.4000	1931.3904	-200.000	9099.000	40.00
PAYOFF	190.8250	400.6573	-1200.000	749.000	40.00
TRUTHPAY	286.4750	218.3016	0.000	749.000	40.00
TRUTHDIF	95.6500	276.4384	0.000	1200.000	40.00

shtindx eq 12

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-273.8438	246.0135	-900.000	2.000	32.00
VBMEAN	-64.1875	160.6272	-350.000	575.833	32.00
VBMAX	246.5313	601.2750	-185.000	2779.000	32.00
PAYOFF	153.3125	184.7999	-20.000	600.000	32.00
TRUTHPAY	189.0000	169.6558	0.000	600.000	32.00
TRUTHDIF	35.6875	63.7902	0.000	200.000	32.00

shtindx eq 13

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-175.8095	200.1959	-600.000	300.000	42.00
VBMEAN	119.6746	759.4356	-300.000	4666.167	42.00
VBMAX	582.5714	1627.3214	-100.000	9599.000	42.00
PAYOFF	136.3571	196.7648	-300.000	500.000	42.00
TRUTHPAY	190.0952	179.9111	0.000	573.000	42.00
TRUTHDIF	53.7381	145.0105	0.000	600.000	42.00

shtindx eq 14

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-265.3889	237.8816	-750.000	0.000	18.00
VBMEAN	-59.8426	130.4816	-250.000	225.833	18.00
VBMAX	258.3889	487.0777	-109.000	1400.000	18.00
PAYOFF	176.8333	264.1620	-449.000	800.000	18.00
TRUTHPAY	220.6111	209.1437	0.000	800.000	18.00
TRUTHDIF	43.7778	140.8623	0.000	600.000	18.00

shtindx eq 15

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-64.5882	984.0650	-850.000	5350.000	34.00
VBMEAN	300.4265	1295.8821	-366.667	6249.833	34.00
VBMAX	1033.9412	2628.9678	-190.000	9149.000	34.00
PAYOFF	55.5588	428.3433	-1600.000	497.000	34.00
TRUTHPAY	184.6176	191.8102	0.000	649.000	34.00
TRUTHDIF	129.0588	352.5876	0.000	1600.000	34.00

shtindx eq 16

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	53.3333	1566.2086	-700.000	9099.000	36.00
VBMEAN	265.6065	1637.6937	-416.667	9649.000	36.00
VBMAX	714.7778	2234.6621	-200.000	9999.000	36.00
PAYOFF	197.8333	225.5003	-200.000	696.000	36.00
TRUTHPAY	268.5556	218.8579	0.000	796.000	36.00
TRUTHDIF	70.7222	163.7679	0.000	800.000	36.00

shtindx eq 17

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-316.4167	243.5292	-750.000	0.000	12.00
VBMEAN	212.1944	950.2532	-358.333	2866.333	12.00
VBMAX	1477.0000	3596.5829	-200.000	9249.000	12.00
PAYOFF	332.5000	326.5735	-249.000	895.000	12.00
TRUTHPAY	407.4167	240.9200	100.000	895.000	12.00
TRUTHDIF	74.9167	146.7685	0.000	400.000	12.00

shtindx eq 18

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-169.8913	359.6489	-900.000	1600.000	46.00
VBMEAN	16.7065	534.2716	-400.000	2999.833	46.00
VBMAX	493.3913	1913.2696	-300.000	9199.000	46.00
PAYOFF	222.4348	217.6088	-27.000	800.000	46.00
TRUTHPAY	263.7826	205.6451	0.000	800.000	46.00
TRUTHDIF	41.3478	110.5821	0.000	600.000	46.00

shtindx eq 19

Mean	Std Dev	Minimum	Maximum	Valid	Variable
VBMIN	-209.0000	364.8068	-800.000	1200.000	26.00
VBMEAN	35.0320	420.4939	-381.333	1500.000	26.00
VBMAX	689.5000	1814.9992	-200.000	9099.000	26.00
PAYOFF	124.3846	310.4104	-878.000	745.000	26.00
TRUTHPAY	223.5000	200.1296	0.000	745.000	26.00
TRUTHDIF	99.1154	246.9503	0.000	1200.000	26.00

shtindx eq 20

Mean	Std Dev	Minimum	Maximum	Valid	Variable
VBMIN	-245.2059	281.8173	-900.000	200.000	34.00
VBMEAN	22.4902	394.4564	-349.000	1333.167	34.00
VBMAX	744.5000	2275.7007	-295.000	9249.000	34.00
PAYOFF	107.5882	159.8354	-125.000	596.000	34.00
TRUTHPAY	148.6176	147.3760	0.000	596.000	34.00
TRUTHDIF	41.0294	76.4875	0.000	275.000	34.00

Appendix G.3 Analysis of Covariance for VL auction

================ Low contention by subject ========================

---------------- GLS ERROR COMPONENTS RESULTS ----------------------

Dependent variable: VBMIN

```
Observations            :  360
Number of Groups        :  18
Degrees of freedom      :  349
Residual SS             :  8004446.701
Std error of est        :  151.444
Total SS (corrected)    :  8499879.406
F                       =  5.441        with 11,349 degrees of freedom
P-value                 =  0.000
Std. errors of error terms:
 Individual constant terms: 105.844
 White noise error        : 152.591
```

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-Value
CONSTANT	-128.45	-0.182106	36.188089	-3.549768	0.000
PERIOD	-65.90	-0.100518	37.974833	-1.735371	0.084
D2	35.41	0.034404	41.496244	0.853523	0.394
D3	4.07	0.008693	57.274231	0.071163	0.943
D4	-71.32	-0.167352	34.413821	-2.072477	0.039
D5	-3.50	-0.007031	32.381191	-0.108388	0.914
D6	-85.07	-0.181856	35.209334	-2.416199	0.016
D7	-28.73	-0.033451	34.091385	-0.843004	0.400
D8	-35.01	-0.072629	50.698363	-0.690628	0.490
D9	4.03	0.009159	34.497670	0.116917	0.907
D10	-0.18	-0.185159	33.032698	-0.005605	0.996

Group Number	Random Components
31	-210.864392
32	-12.419136
33	-13.737341
34	15.920589
35	-0.796971
36	72.598412
81	34.119915
82	60.123754
83	-140.668986
84	47.072023
85	-58.061607
86	16.146577
191	-54.170855
192	-16.290779
193	92.879196
194	-33.188654
195	51.162395
196	150.175785

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 158.5066
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 1.7064
P-value = 0.9981

---------------- GLS ERROR COMPONENTS RESULTS ----------------------

Dependent variable: VBMEAN

Observations	:	360
Number of Groups	:	18
Degrees of freedom	:	349
Residual SS	:	2429750.586
Std error of est	:	83.439
Total SS (corrected)	:	2609150.635
F	=	3.741 with 11,349 degrees of freedom
P-value	=	0.000

Std. errors of error terms:
Individual constant terms: 52.445
White noise error : 84.280

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-Value
CONSTANT	-27.80	-0.071152	19.001426	-1.463477	0.144
PERIOD	-35.58	-0.098002	20.921961	-1.701075	0.090
D2	-7.07	-0.012429	22.857708	-0.309724	0.757
D3	-38.32	-0.147566	31.526900	-1.215550	0.225
D4	-45.45	-0.192649	18.948060	-2.398983	0.017
D5	2.63	0.009515	17.832001	0.147569	0.883
D6	-49.25	-0.190097	19.391099	-2.540295	0.012
D7	-30.30	-0.063694	18.775509	-1.614058	0.107
D8	-31.52	-0.118074	27.918552	-1.129288	0.260
D9	2.67	0.010952	18.998048	0.140580	0.888
D10	-3.24	-3.249203	18.189483	-0.178631	0.858

Group Number	Random Components
31	-65.662690
32	-45.619217
33	3.334816
34	0.153149
35	-3.904560
36	48.777015
81	14.659179
82	32.071568
83	-72.495911
84	60.103229
85	-11.457626
86	-23.235174
191	-16.667029
192	34.327375
193	18.194185
194	-24.941312
195	3.058917
196	49.304101

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 85.1848
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 0.9190
P-value = 0.9999

---------------------- GLS ERROR COMPONENTS RESULTS ----------------

Dependent variable: VBMAX

Observations	:	360
Number of Groups	:	18
Degrees of freedom	:	349
Residual SS	:	4225199.817
Std error of est	:	110.030
Total SS (corrected)	:	4406599.336
F	=	1.888 with 11,349 degrees of freedom
P-value	=	0.040

Std. errors of error terms:
 Individual constant terms: 99.566
 White noise error : 110.451

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	60.95	0.120016	30.243064	2.015589	0.045
PERIOD	80.38	0.170228	27.591127	2.913562	0.004
D2	-23.03	-0.030998	30.159951	-0.763727	0.446
D3	-32.04	-0.094879	41.680008	-0.768886	0.442
D4	-22.46	-0.073117	25.032414	-0.897436	0.370
D5	-21.51	-0.059845	23.546417	-0.913766	0.361
D6	0.16	0.000500	25.599083	0.006585	0.995
D7	-38.80	-0.062685	24.785958	-1.565721	0.118
D8	-49.94	-0.143812	36.867271	-1.354736	0.176
D9	10.29	0.032421	25.084333	0.410239	0.682
D10	-24.77	-24.778965	24.023318	-1.031455	0.303

Group Number	Random Components
31	-52.651226
32	-55.001202
33	10.360827
34	5.248988
35	-33.419858
36	62.423792
81	12.168077
82	6.636152
83	-51.465261
84	54.659014
85	-3.839465
86	-58.568620
191	-40.292593
192	321.851372
193	-38.003988
194	-57.621209
195	-48.072124
196	-34.412644

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 494.5633
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 1.4409
P-value = 0.9991

================== High contention by subject ==========

---------------------- GLS ERROR COMPONENTS RESULTS ----------------

Dependent variable: VBMIN

Observations : 320
Number of Groups : 16
Degrees of freedom : 309
Residual SS : 129905239.426
Std error of est : 648.387
Total SS (corrected) : 133994691.159
F = 1.772 with 11,309 degrees of freedom
P-value = 0.058
Std. errors of error terms:
Individual constant terms: 158.676
White noise error : 653.928

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	-232.18	-0.078160	109.951793	-2.111739	0.036
PERIOD	286.26	0.130968	169.705389	1.686845	0.093
D2	-87.32	-0.045366	154.049544	-0.566882	0.571
D3	20.65	0.007319	141.900411	0.145579	0.884
D4	-26.14	-0.012302	181.645924	-0.143937	0.886
D5	129.50	0.063080	153.188561	0.845396	0.399
D6	201.64	0.059004	149.273125	1.350867	0.178
D7	-126.91	-0.067769	215.137344	-0.589947	0.556
D8	11.08	0.004659	141.350555	0.078448	0.938
D9	-19.71	-0.009280	163.306049	-0.120700	0.904
D10	-100.98	-100.985391	151.608099	-0.666095	0.506

Group Number	Random Components
11	41.381488
12	-70.324744
13	-40.706971
14	-12.036099
15	-11.932653
16	44.880858
17	-210.533720
18	-90.365941
41	-29.686375
42	-23.700370
43	34.286809
44	64.832715
45	205.983006
46	-21.919980
47	-15.560546
48	135.402547

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 0.7528
P-value: 0.3856

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 1.7258
P-value = 0.9980

---------------------- GLS ERROR COMPONENTS RESULTS ----------------

Dependent variable: VBMEAN

Observations	:	320
Number of Groups	:	16
Degrees of freedom	:	309
Residual SS	:	154702691.490
Std error of est	:	707.570
Total SS (corrected)	:	167456591.732
F	=	2.349 with 11,309 degrees of freedom
P-value	=	0.009

Std. errors of error terms:
Individual constant terms: 573.287
White noise error : 716.606

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	-78.53	-0.023647	180.667683	-0.434667	0.664
PERIOD	768.16	0.310119	185.229677	4.147087	0.000
D2	-49.10	-0.022698	169.797604	-0.289196	0.773
D3	137.17	0.043065	155.345481	0.883036	0.378
D4	53.73	0.022380	199.914914	0.268769	0.788
D5	232.46	0.100928	169.013479	1.375431	0.170
D6	125.42	0.032501	163.730970	0.766017	0.444
D7	86.01	0.040501	236.875839	0.363114	0.717
D8	-88.15	-0.032788	155.218068	-0.567946	0.570
D9	-39.45	-0.016388	178.974800	-0.220451	0.826
D10	-124.06	-124.064053	167.115720	-0.742384	0.458

Group Number	Random Components
11	-114.982839
12	-211.485156
13	-215.812286
14	-157.949042
15	-162.504842
16	65.582534
17	-59.945554
18	-186.243770
41	-212.164434
42	-164.333470
43	-60.425093
44	-39.954872
45	859.834086
46	-171.642291
47	-222.938646
48	1054.965892

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 126.1285
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 0.4373
P-value = 1.0000

---------------------- GLS ERROR COMPONENTS RESULTS ------------------

Dependent variable: VBMAX

Observations	: 320
Number of Groups	: 16
Degrees of freedom	: 309
Residual SS	: 700594639.164
Std error of est	: 1505.754
Total SS (corrected)	: 774915286.217
F	= 3.163 with 11,309 degrees of freedom
P-value	= 0.000

Std. errors of error terms:
Individual constant terms: 1831.087
White noise error : 1525.035

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	463.46	0.064876	511.316353	0.906425	0.365
PERIOD	2010.18	0.376730	394.187869	5.099569	0.000
D2	-371.61	-0.079805	361.731936	-1.027308	0.305
D3	7.06	0.001031	330.698134	0.021373	0.983
D4	-91.65	-0.017728	425.820197	-0.215243	0.830
D5	55.68	0.011234	360.082940	0.154633	0.877
D6	-361.23	-0.043471	348.616517	-1.036188	0.301
D7	441.59	0.096520	504.548040	0.875230	0.382
D8	-460.25	-0.079493	330.537129	-1.392447	0.165
D9	-146.17	-0.028184	381.040742	-0.383615	0.702
D10	-333.98	-333.989378	356.014020	-0.938135	0.349

Group Number	Random Components
11	-449.085323
12	-652.929629
13	-657.748002
14	-506.956624
15	-488.335517
16	23.686027
17	575.749522
18	-465.820472
41	-740.646871
42	-634.747569
43	-430.208602
44	-467.079513
45	3195.221783
46	-555.560734
47	-731.089433
48	2985.550403

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 432.2591
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 0.2298
P-value = 1.0000

==================== BY SHEET INDEX ================================

================== Low contention by sheet index ==================

----------------------- OLS DUMMY VARIABLE RESULTS -----------------

Dependent variable: VBMIN

```
Observations          :  360
Number of Groups      :  10
Degrees of freedom    :  349
Residual SS           :  10505614.025
Std error of est      :  173.499
Total SS (corrected)  :  10575031.962
F                     =  2.306      with 1,349 degrees of freedom
P-value               =  0.130
```

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	-66.16	-0.081021	43.573032	-1.518581	0.130

Group Number	Dummy Variable	Standard Error
1	-136.559336	31.450173
2	-111.472797	38.402899
3	-200.631386	57.897556
4	-184.029452	27.000235
5	-141.260560	23.778739
6	-200.910431	28.330402
7	-159.648734	29.279128
8	-194.551290	50.760290
9	-111.223797	27.724386
10	-114.892352	24.295451

F-statistic for equality of dummy variables :
F(9, 349) = 1.3852   P-value: 0.1932

---------------------- OLS DUMMY VARIABLE RESULTS ----------------

Dependent variable: VBMEAN

Observations : 360
Number of Groups : 10
Degrees of freedom : 349
Residual SS : 2957292.682
Std error of est : 92.052
Total SS (corrected) : 2977008.970
F = 2.327 with 1,349 degrees of freedom
P-value = 0.128

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	-35.26	-0.081381	23.118211	-1.525380	0.128

Group Number	Dummy Variable	Standard Error
1	-26.421762	16.686278
2	-38.644905	20.375133
3	-89.049488	30.718264
4	-68.504804	14.325308
5	-31.050426	12.616104
6	-73.570948	15.031045
7	-60.800897	15.534403
8	-73.321200	26.931499
9	-18.779240	14.709516
10	-25.375656	12.890252

F-statistic for equality of dummy variables :
F(9, 349) = 2.2085 P-value: 0.0211

---------------------- OLS DUMMY VARIABLE RESULTS ----------------

Dependent variable: VBMAX

Observations : 360
Number of Groups : 10
Degrees of freedom : 349
Residual SS : 7003445.350
Std error of est : 141.659
Total SS (corrected) : 7108467.024
F = 5.234 with 1,349 degrees of freedom
P-value = 0.023

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	81.387788	0.121549	35.576477	2.287685	0.023

Group Number	Dummy Variable	Standard Error
1	45.011075	25.678414
2	37.750627	31.355170
3	-4.877813	47.272153
4	44.900049	22.045132
5	23.105709	19.414847
6	64.486026	23.131185
7	25.611140	23.905800
8	-14.581020	41.444724
9	81.421270	22.636386
10	56.095039	19.836731

F-statistic for equality of dummy variables :
F(9, 349) = 1.0317 P-value: 0.4140

================== High contention by sheet index ==================

----------------------- OLS DUMMY VARIABLE RESULTS -----------------

----

Dependent variable: VBMIN

Observations : 320
Number of Groups : 10
Degrees of freedom : 309
Residual SS : 134656039.907
Std error of est : 660.136
Total SS (corrected) : 135908130.115
F = 2.873 with 1,309 degrees of freedom
P-value = 0.091

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	292.186805	0.095983	172.375869	1.695056	0.091

Group Number	Dummy Variable	Standard Error
11	-209.325890	106.610774
12	-335.544103	122.241973
13	-217.419733	104.777474
14	-308.867083	157.695676
15	-112.013598	116.618458
16	-16.023568	117.384906
17	-401.384015	197.047401
18	-223.982897	102.429565
19	-267.885211	134.043263
20	-295.045945	116.968407

F-statistic for equality of dummy variables :
F(9, 309) = 0.7943 P-value: 0.6219

OLS DUMMY VARIABLE RESULTS

Dependent variable: VBMEAN

Observations	:	320
Number of Groups	:	10
Degrees of freedom	:	309
Residual SS	:	201486264.302
Std error of est	:	807.502
Total SS (corrected)	:	210098223.424
F	=	13.207 with 1,309 degrees of freedom
P-value	=	0.000

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	766.291079	0.202460	210.856150	3.634189	0.000

Group Number	Dummy Variable	Standard Error
11	-54.747610	130.410001
12	-226.003259	149.530627
13	10.547387	128.167446
14	-173.868808	192.898828
15	176.048405	142.651748
16	83.710604	143.589294
17	-10.641538	241.035225
18	-125.154437	125.295401
19	-119.400698	163.966375
20	-108.220677	143.079817

F-statistic for equality of dummy variables :
F(9, 309) = 0.7365 P-value: 0.6754

---------------------- OLS DUMMY VARIABLE RESULTS ----------------

----

Dependent variable: VBMAX

Observations : 320
Number of Groups : 10
Degrees of freedom : 309
Residual SS : 1175058403.536
Std error of est : 1950.071
Total SS (corrected) : 1232427786.193
F = 15.086 with 1,309 degrees of freedom
P-value = 0.000

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	1977.802301	0.215754	509.205723	3.884093	0.000

Group Number	Dummy Variable	Standard Error
11	432.296043	314.932805
12	-171.116287	361.108041
13	300.913361	309.517161
14	-35.913521	465.839802
15	712.920526	344.495936
16	245.303333	346.760056
17	901.858912	582.086489
18	127.247240	302.581336
19	290.908061	395.969559
20	407.134383	345.529698

F-statistic for equality of dummy variables :
F(9, 309) = 0.6383 P-value : 0.7641

============ TRUNCATED VARIABLES: BY SUBJECT =====================

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------------- GLS ERROR COMPONENTS RESULTS --------------

Dependent variable: VBMIN

```
Observations          : 320
Number of Groups      : 16
Degrees of freedom    : 309
Residual SS           : 17279990.755
Std error of est      : 236.479
Total SS (corrected)  : 18248536.447
F                     = 5.387       with 11,309 degrees of freedom
P-value               = 0.000
Std. errors of error terms:
 Individual constant terms: 117.098
 White noise error        : 235.773
```

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	-146.11	-0.133281	47.638663	-3.067152	0.002
PERIOD	-144.88	-0.177793	61.903229	-2.340488	0.020
D2	-107.79	-0.151130	56.600618	-1.904400	0.058
D3	-10.79	-0.010293	51.875560	-0.208149	0.835
D4	-80.56	-0.101918	66.666989	-1.208454	0.228
D5	-35.51	-0.046752	56.329108	-0.630513	0.529
D6	-7.84	-0.006172	54.649676	-0.143501	0.886
D7	-81.98	-0.117358	78.989545	-1.037883	0.300
D8	3.70	0.004189	51.791462	0.071593	0.943
D9	-9.58	-0.012105	59.750329	-0.160465	0.873
D10	-111.79	-111.792581	55.707506	-2.006778	0.046

Group Number	Random Components
11	103.221630
12	-85.282845
13	-26.873781
14	17.980824
15	49.337123
16	76.663914
17	-279.773947
18	-121.828375
41	-4.877923
42	-16.278889
43	87.091029
44	133.342876
45	178.361200
46	-17.447328
47	17.997899
48	-111.633388

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 122.6052
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 6.0091
P-value = 0.8145

---------------------- GLS ERROR COMPONENTS RESULTS ----------------

Dependent variable: VBMEAN

Observations	:	320
Number of Groups	:	16
Degrees of freedom	:	309
Residual SS	:	15967254.503
Std error of est	:	227.319
Total SS (corrected)	:	16715162.361
F	=	1.316 with 11,309 degrees of freedom
P-value	=	0.214

Std. errors of error terms:
Individual constant terms: 283.247
White noise error : 230.034

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	36.84	0.035114	78.735408	0.467931	0.640
PERIOD	131.51	0.167808	59.509364	2.209966	0.028
D2	-110.37	-0.161396	54.611989	-2.021149	0.044
D3	-1.81	-0.001799	49.925175	-0.036294	0.971
D4	-66.57	-0.087670	64.287173	-1.035547	0.301
D5	-66.33	-0.091128	54.363158	-1.220281	0.223
D6	-77.57	-0.063562	52.630712	-1.474000	0.141
D7	-15.55	-0.023141	76.172909	-0.204150	0.838
D8	-92.26	-0.108502	49.901555	-1.849008	0.065
D9	-27.75	-0.036437	57.525600	-0.482502	0.630
D10	-109.26	-109.263513	53.748703	-2.032859	0.043

Group Number	Random Components
11	2.641768
12	-169.029291
13	-152.679493
14	-78.524075
15	-51.027282
16	95.842366
17	21.679731
18	-141.280675
41	-98.861051
42	-124.237628
43	8.103828
44	41.141112
45	614.068863
46	-126.700567
47	-118.344720
48	277.207322

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 497.4950
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 0.2801
P-value = 1.0000

---------------------- GLS ERROR COMPONENTS RESULTS ----------------

----
Dependent variable: VBMAX

Observations : 320
Number of Groups : 16
Degrees of freedom : 309
Residual SS : 23456995.193
Std error of est : 275.522
Total SS (corrected) : 25227427.873
F = 2.483 with 11,309 degrees of freedom
P-value = 0.005
Std. errors of error terms:
Individual constant terms: 379.310
White noise error : 278.859

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
CONSTANT	220.36	0.170960	103.402833	2.131136	0.034
PERIOD	213.63	0.221845	72.128610	2.961889	0.003
D2	-104.71	-0.124629	66.202977	-1.581790	0.115
D3	-5.24	-0.004236	60.514890	-0.086637	0.931
D4	-0.35	-0.000378	77.929699	-0.004526	0.996
D5	-107.34	-0.120025	65.901854	-1.628885	0.104
D6	-95.25	-0.063521	63.796082	-1.493150	0.136
D7	-120.59	-0.146042	92.337687	-1.305968	0.193
D8	-145.52	-0.139272	60.489195	-2.405745	0.017
D9	41.88	0.044751	69.728525	0.600730	0.548
D10	-133.32	-133.328892	65.156323	-2.046292	0.042

Group Number	Random Components
11	13.685775
12	-242.685790
13	-255.092611
14	-90.784929
15	-55.538979
16	293.754882
17	396.618040
18	-75.175078
41	-212.175700
42	-221.746325
43	-72.836627
44	-23.739790
45	645.425312
46	-149.589821
47	-245.513838
48	295.395658

Lagrange Multiplier Test for Error Components Model

Null hypothesis: Individual error components do not exist.

Chi-squared statistic (1): 646.3336
P-value: 0.0000

Hausman (1978) Chi-Squared Specification Test

Null hypothesis: Error components model is the correct specification.

Chi-squared statistic (10) = 0.1100
P-value = 1.0000

============= TRUNCATED VARIABLES : BY SHEET INDEX =================

---------------- OLS DUMMY VARIABLE RESULTS --------------

this is from file tsbdtnc2.out

Dependent variable: VBMIN

Observations	:	320	
Number of Groups	:	10	
Degrees of freedom	:	309	
Residual SS	:	21817143.649	
Std error of est	:	265.717	
Total SS (corrected)	:	22120376.542	
F	=	4.295	with 1,309 degrees of freedom
P-value	=	0.039	

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	-143.79	-0.117082	69.384505	-2.072375	0.039

Group Number	Dummy Variable	Standard Error
11	-154.414565	42.912827
12	-243.479819	49.204676
13	-155.332340	42.174889
14	-243.992428	63.475453
15	-169.190485	46.941106
16	-137.507021	47.249616
17	-274.602585	79.315257
18	-156.315274	41.229812
19	-187.713761	53.954916
20	-220.678635	47.081967

F-statistic for equality of dummy variables :
F(9, 309) = 0.8035 P-value: 0.6133

OLS DUMMY VARIABLE RESULTS

Dependent variable: VBMEAN

Observations	:	320
Number of Groups	:	10
Degrees of freedom	:	309
Residual SS	:	28074094.913
Std error of est	:	301.421
Total SS (corrected)	:	28309340.099
F	=	2.589 with 1,309 degrees of freedom
P-value	=	0.109

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	126.64	0.091158	78.707557	1.609114	0.109

Group Number	Dummy Variable	Standard Error
11	16.744356	48.678934
12	-90.931737	55.816206
13	14.348830	47.841842
14	-78.688373	72.004518
15	19.271738	53.248485
16	-9.331513	53.598449
17	-7.926626	89.972684
18	-57.457062	46.769776
19	-22.536825	61.204726
20	-14.789626	53.408273

F-statistic for equality of dummy variables :
F(9, 309) = 0.5485 P-value: 0.8384

------------------------ OLS DUMMY VARIABLE RESULTS ----------------

Dependent variable: VBMAX

Observations	:	320
Number of Groups	:	10
Degrees of freedom	:	309
Residual SS	:	45115090.506
Std error of est	:	382.104
Total SS (corrected)	:	45753822.360
F	=	4.375 with 1,309 degrees of freedom
P-value	=	0.037

Var	Coef.	Std. Coef.	Std. Error	t-Stat	P-value
PERIOD	208.69	0.118153	99.775627	2.091595	0.037

Group Number	Dummy Variable	Standard Error
11	192.915488	61.709083
12	117.181413	70.756827
13	190.970984	60.647922
14	185.668553	91.278350
15	178.421231	67.501791
16	148.629507	67.945431
17	53.979954	114.056150
18	63.018041	59.288891
19	254.057475	77.587720
20	126.608344	67.704350

F-statistic for equality of dummy variables :
F(9, 309) = 0.7457 P-value: 0.6668

Appendix G.4 Individual behavior by period divisions

---------------------------- Low contention -----------------------

PERIOD 1 to 5

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-175.4667	179.8640	-599.000	0.000	90.00
VBMEAN	-49.8426	107.0428	-299.833	188.000	90.00
VBMAX	93.8444	180.6452	0.000	800.000	90.00
PAYOFF	542.1556	178.5770	100.000	900.000	90.00
TRUTHPAY	580.4778	156.0742	300.000	900.000	90.00
TRUTHDIF	38.3222	88.3086	0.000	400.000	90.00

PERIOD 6 to 10

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-158.3889	170.7468	-599.000	0.000	90.00
VBMEAN	-53.2056	84.6268	-271.667	133.333	90.00
VBMAX	37.8556	116.3402	0.000	700.000	90.00
PAYOFF	576.3444	181.1273	200.000	900.000	90.00
TRUTHPAY	592.0222	169.5073	300.000	900.000	90.00
TRUTHDIF	15.6778	60.4226	0.000	400.000	90.00

PERIOD 11 to 15

Variable	Mean	Std Dev	Minimum	Maximum	# Valid
VBMIN	-163.0667	180.6967	-600.000	0.000	90.00
VBMEAN	-57.4778	89.4872	-283.333	157.167	90.00
VBMAX	42.1667	108.2595	0.000	600.000	90.00
PAYOFF	585.4667	162.0807	300.000	850.000	90.00
TRUTHPAY	611.2667	145.9788	300.000	850.000	90.00
TRUTHDIF	25.8000	82.8256	0.000	500.000	90.00

PERIOD 16 to 20

Variable	Mean	Std Dev	Minimum	Maximum	# Valid
VBMIN	-146.6889	168.9991	-597.000	0.000	90.00
VBMEAN	-43.4574	93.0829	-298.500	241.667	90.00
VBMAX	57.8333	148.0688	0.000	700.000	90.00
PAYOFF	599.8667	165.2683	300.000	900.000	90.00
TRUTHPAY	622.0333	156.3231	300.000	900.000	90.00
TRUTHDIF	22.1667	76.4680	0.000	485.000	90.00

------------------------- High contention -----------------------

PERIOD 1 to 5

Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-211.7500	1107.8233	-900.000	9099.000	80.00
VBMEAN	296.4167	1408.3965	-416.667	9649.000	80.00
VBMAX	1397.7000	3046.3494	-300.000	9999.000	80.00
PAYOFF	195.0250	367.1203	-1600.000	895.000	80.00
TRUTHPAY	326.6250	222.7397	0.000	895.000	80.00
TRUTHDIF	131.6000	267.1279	0.000	1600.000	80.00

PERIOD 6 to 10

Variable	Mean	Std Dev	Minimum	Maximum	# Valid
VBMIN	-185.9125	664.4671	-900.000	5350.000	80.00
VBMEAN	74.4521	764.2088	-343.833	6249.833	80.00
VBMAX	750.1375	2055.7620	-200.000	9249.000	80.00
PAYOFF	152.6875	312.9644	-1352.000	749.000	80.00
TRUTHPAY	231.4000	205.2678	0.000	749.000	80.00
TRUTHDIF	78.7125	222.8717	0.000	1352.000	80.00

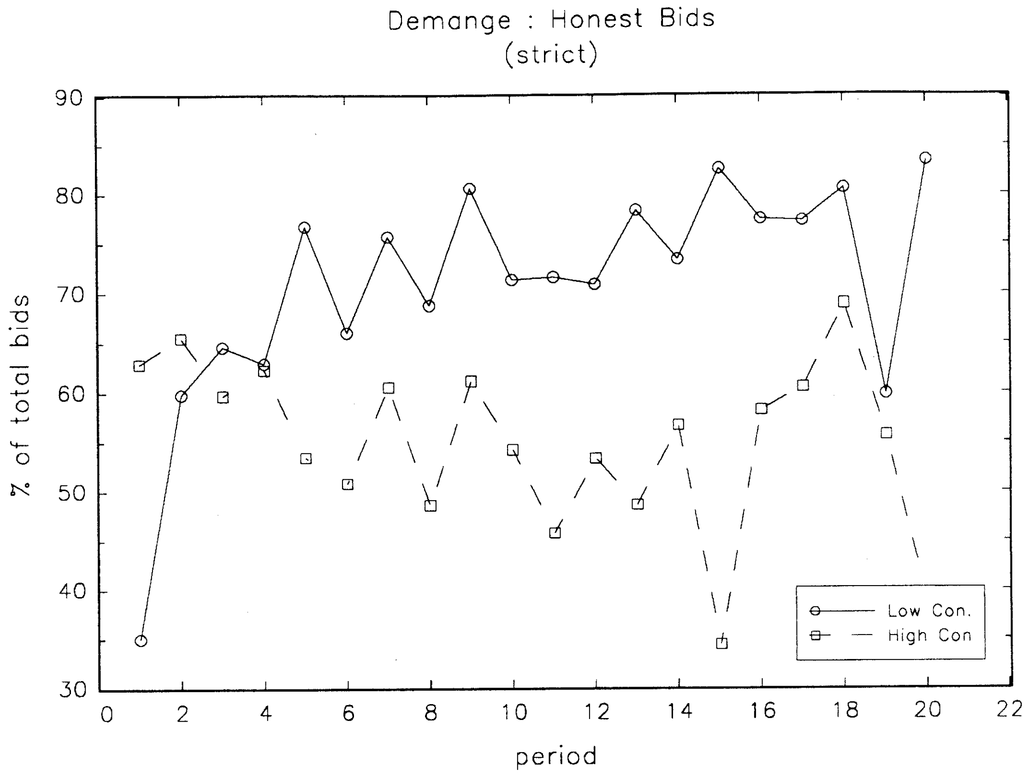
PERIOD 11 to 15

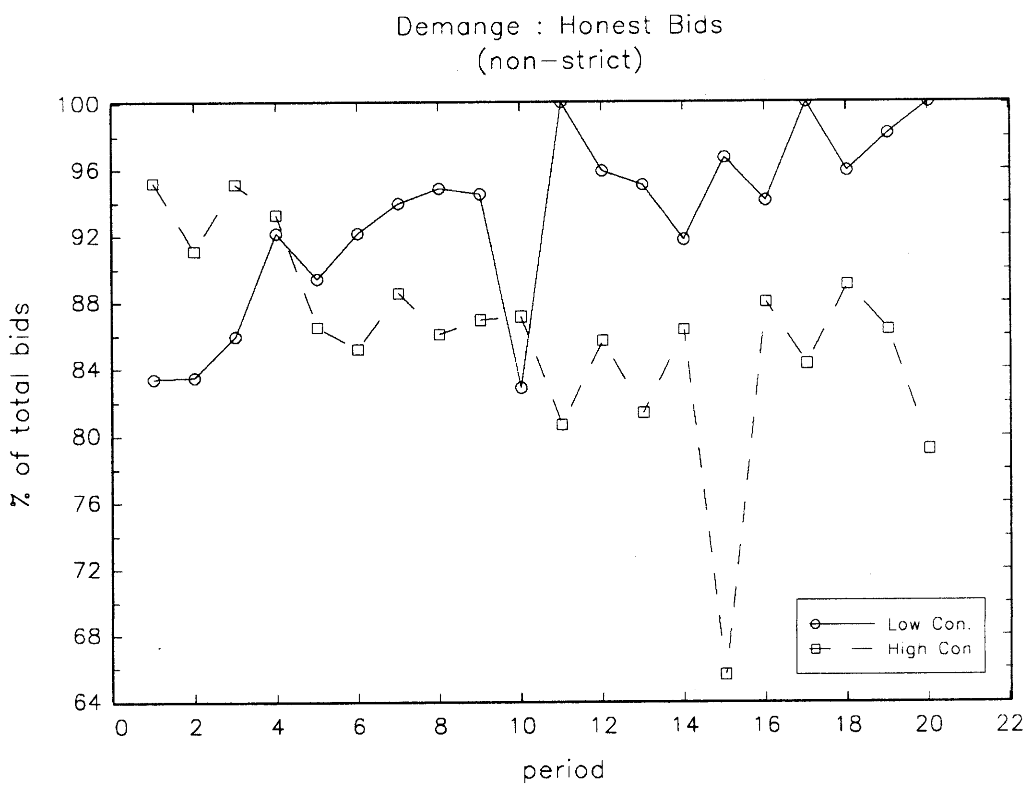
Variable	Mean	Std Dev	Minimum	Maximum	# Valid
VBMIN	-144.3375	220.2144	-850.000	400.000	80.00
VBMEAN	-12.5250	219.5355	-366.667	983.333	80.00
VBMAX	196.9625	524.7299	-162.000	2300.000	80.00
PAYOFF	153.3500	189.8666	-200.000	600.000	80.00
TRUTHPAY	181.8000	170.5538	0.000	600.000	80.00
TRUTHDIF	28.4500	56.0314	0.000	210.000	80.00

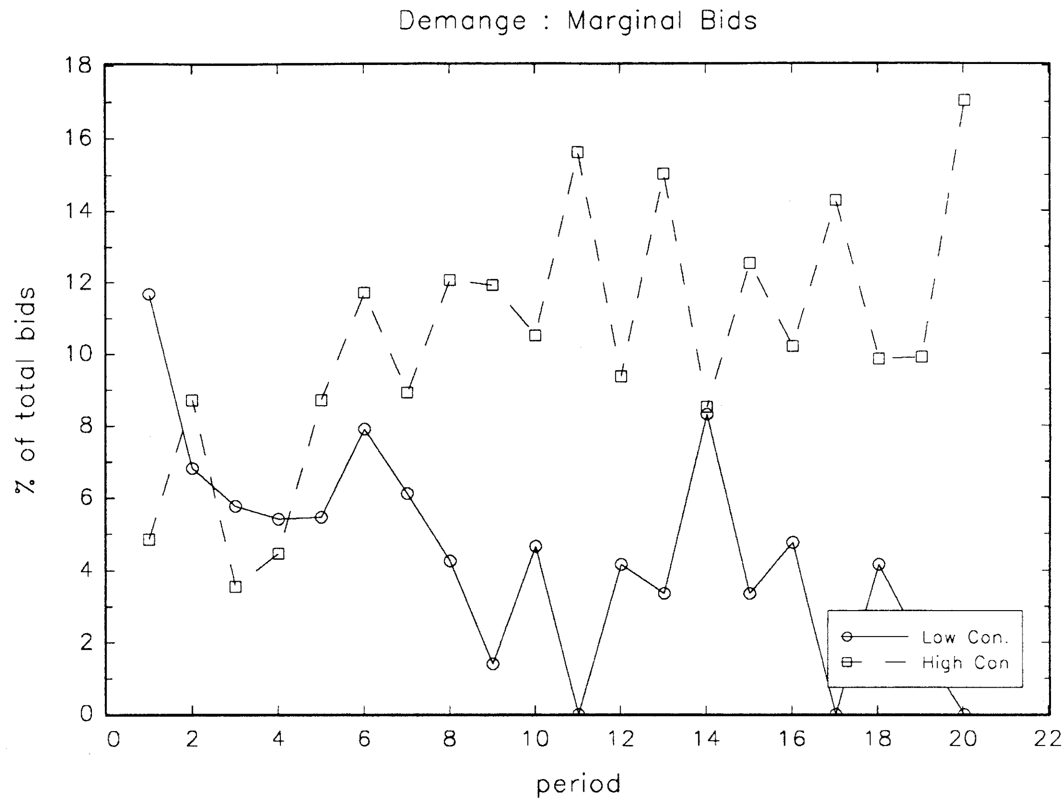
PERIOD 16 to 20

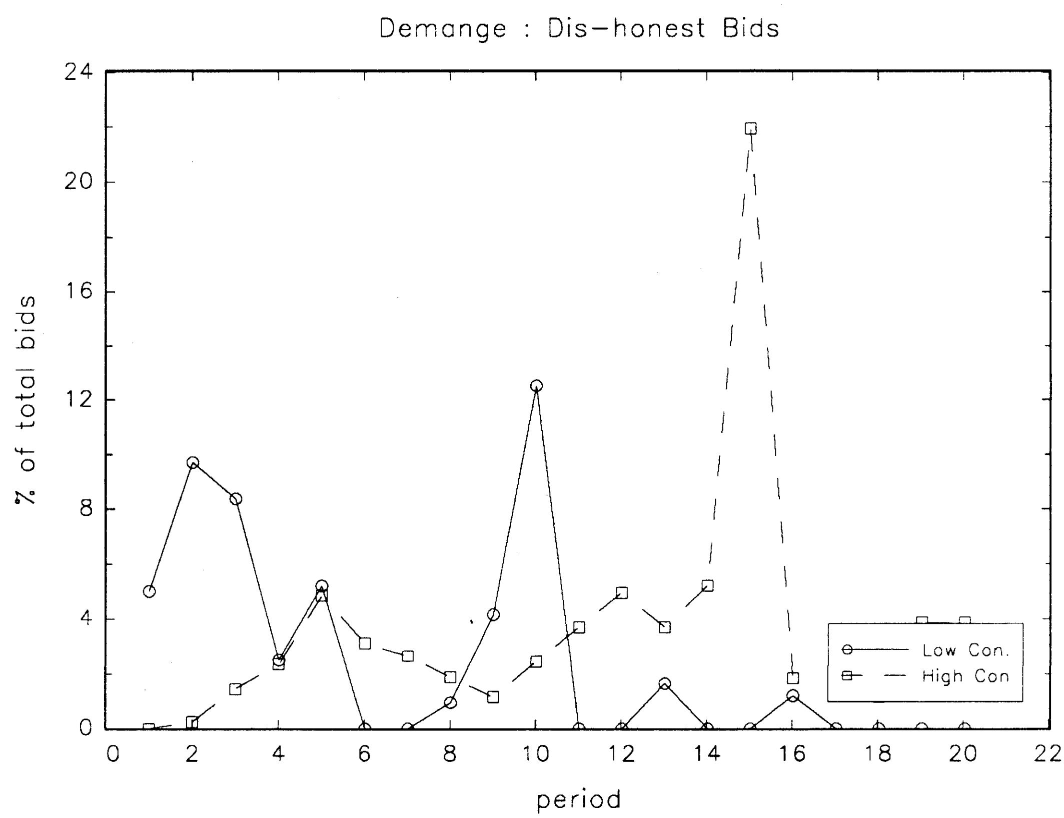
Variable	Mean	Std Dev	Minimum	Maximum	Valid
VBMIN	-126.5500	196.5641	-900.000	100.000	80.00
VBMEAN	-4.1875	196.2467	-325.000	1266.500	80.00
VBMAX	265.3250	1084.5790	-200.000	9099.000	80.00
PAYOFF	143.6125	232.6523	-1200.000	600.000	80.00
TRUTHPAY	176.3500	170.3407	0.000	600.000	80.00
TRUTHDIF	32.7375	150.0455	0.000	1200.000	80.00

Appendix H. Progressive individual behavior--graphs






Demange : Marginal Bids
% of total bids
period
0
2
4
6
8
10
12
14
16
18
20
22
Low Con.
High Con


Demange : Dis-honest Bids
% of total bids
period
0
4
8
12
16
20
24
Low Con.
High Con

Appendix I. Algorithm for overdemanded set

file demagalg.c --

```
/*
	these routines find either an assignment or
	an overdemanded set give a matrix of bids
*/
#include <stdio.h>
#include <mem.h>

#define mem0i(a,b)   memset(a,0,(b)*sizeof(int));
#define MAXN 20
#define PINC 1                          /* price increment    */
#define RINT  register int
#define DEBUG 0

int cnt=0;                              /* demage if this is first time */


demagepkn(K, N, q, s, assign,price,incrmnt)
int K,N,q[][MAXN],s[],assign[],price[],incrmnt;
/*
 * the same as demagep  except assumes k=n
 * and that q = 1 when max val-p = 0
 * hence simplier
 */
{
RINT j, ovrd;

	if (cnt==0) {
		/* first time thtrough at the begin of period */
		cnt=1;
		mem0i(assign,N+1);
		mem0i(price,K+1);
		}

	ovrd=demage(K, N, q, s, assign);

	if (ovrd==1) {                                          /* overdemanded*/
			for (j=1;j<=K;j++)
				if(s[j-1] == 1) {
					price[j-1] +=incrmnt;
				}
		return(1);
		}
	else {		/* else  assigned */
		cnt=0;
		return(0);
		}
}
```

```
int
demage(K,N,q,s,ass_p)
int K,N,q[][MAXN],s[],ass_p[];
/* Demange algorthim to find and assignment or an overdemanded set
 *      via Gale --
 *      -- turns it into a pure overdemanded set via Mo
        -- ala molson (in a manner of speaking)
 *              N       = number of players
 *              K       = number of slots
                q[][] is the qualification matrix
                        q[i][j] = 1 if player i bids for slot j
                        q[i][j] = 0 otherwise
                s returns overdemanded slots
                ass_p returns  slots  subjects are assigned to
                slots = [0,...,K-1]   = -1 if not assigned

 */
{
int i,j;
int p[MAXN];
int bids[MAXN];
int unass_bid;

/*      let bids[i] = 1 if player i has a bid
                bids[i] = 0 otherwise
                let s[1,...,K] = 0, and p[1,...,N] = 0
                s[] is for slots and p[] is for players
                                s[j]=1 ==> slot j is assigned.
                                s[j]=0 ==> slot j is not assigned.
                                similar for p[j]
*/

        mem0i(s,K+1);
        mem0i(bids,N+1);
        mem0i(p,N+1);

        for (i=1;i<=N;i++)
                for (j=1;j<=K;j++) {
                        if (q[i][j] > 0) bids[i-1] =1;
                        }

        flowass(N,K,q,s,ass_p) ;

        unass_bid=0;
        for (i=0;i<N;i++)
                if ( bids[i]>0 && ass_p[i]<0 ) {
                        unass_bid=i+1;
                        break;
                        }

        if (unass_bid==0 ) {
                return(0);
```

```
                } /* end if unass=0 */
        else {
                fndovrd(N,K,s,p,q,unass_bid);  /* overdemaned */
                return(1);
                }

        /* no unassigned bidder so assignment is possible
                returned in ass_p
                return(0);
                */

} /* end demage */


fndovrd(N,K,s,p,q,unass_bid)
int N,K,s[],p[],q[][MAXN],unass_bid;
{
/*6*/
RINT i,j,u;
int numbids[MAXN] ;
int unass[MAXN];

        /* this is hopefully the corrected version that gets
                the minimum pure overdemanded set
                for only one unassigned bidder that can not
                be re-assigned
                */

        mem0i(s,K+1);
        for (j=1;j<=K;j++)
                if (q[unass_bid][j]==1) s[j-1]=1 ;
        return(0);

}/* end 6*/



file flowass.c -----------------------------------------------------

#include <stdio.h>

#define MAXN 20
#define RINT  register int
#define DEBUG 0
#define maxV  (3*MAXN)

int val[maxV];
int dad[maxV];
int id;
int V;
int size[maxV][maxV];
int flow[maxV][maxV];
int savef[maxV][maxV];
```

```
int
flowass(n,k,q,s,assign)
int n,k;
int q[][MAXN];
int s[], assign[];
{
int i,j;

        V=n+k+2;
        makeflow(n,k,q);
        assflow();

        for (i=1;i<=n;i++) {
                assign[i-1] = -1;
                for (j=n+2;j<V;j++)
                        if (savef[i+1][j]==1) assign[i-1]=j-n-2;
                }

        for (i=n+2;i<V;i++) s[i-n-1]=savef[i][V];
}


int
makeflow(n,k,mq)
int n,k;
int mq[][MAXN];
{
int i,j;

int bids[MAXN];  /* if a subject placed a bid */

        for (i=0;i<=n+2;i++) bids[i]=0;
        for (i=1;i<=n;i++)
                for (j=1;j<=k;j++) {
                        if ( mq[i][j]>0 ) {
                                bids[i]=1;
                                break;
                                }
                }

        for (i=1;i<=V;i++)
                for (j=1;j<=V;j++) {
                        flow[i][j]=0;
                        savef[i][j]=0;
                        size[i][j]=0;
                        }

        for (j=2;j<=n+1;j++)      /* from source */
                size[1][j] = bids[j-1];

        for (i=2;i<=n+1;i++)
                for (j=2;j<=k+1;j++)
                        if (mq[i-1][j-1]) {
```

```
                    size[i][n+j] = 1;
                    }

    for (i=k+2;i<=n+k+1;i++)     /* to sink */
        size[i][V] = 1;

}

int
assflow()
{
int listmpfs();
    int y,x;

    for (;;) {
        if (!listmpfs(1,V)) break;
        y = V; x = dad[V] ;
        while (x !=0) {
            flow[x][y] = flow[x][y]+1;
            flow[y][x] = -flow[x][y];
            y = x; x = dad[y];
            }
        }
}




int
visit(k)
int k;
{
    int t,tmax,m;

    val[k] = ++id;
    if (k==V) return(V);
    tmax=k;

    for (t=1; t<=V; t++)
        if (size[k][t] != 0)
            if (val[t] == 0) {
                m=visit(t);
                dad[t]=k;
                if ( m>tmax ) tmax=m;
                if ( tmax==V) return(V);
                }
    return(tmax);
}

int
listmpfs(a,b)
int a,b;
{
    int i,k,mv;
```

```
for (i=1;i<=V;i++) {
        flow[i][1]=0;
        flow[V][i]=0;
        }

for (i=1;i<=V;i++)
        for (k=1;k<=V;k++) {
                size[i][k]-=flow[i][k];
                savef[i][k]+=flow[i][k];
                flow[i][k]=0;
                }

for (k=0; k<= V; k++){ val[k]=0;  dad[k]=0; }
id=0;
mv=visit(1) ;
if (mv==V) return(1);
else return(0);

}
```

BIBLIOGRAPHY

Abreu, Dilip and Arunava Sen (1987). "Virtual Implementation in Nash Equilibrium." Mimeo, Universidad Autónoma de Barcelona, Barcelona, forthcoming in *Econometrica.*

Andreoni, James and John H. Miller (1990). "Auctions with Adaptive Artificially Intelligent Agents." Working paper, University of Wisconsin.

Banks, J., J. Ledyard, and D. Porter (1989). "Allocating Uncertain and Unresponsive Resources: An Experimental Approach." *Rand Journal of Economics*, 20, 1-25.

Barbera, S. (1983). "Strategy-Proofness and Pivotal Voters: A Direct Proof of the Gibbard-Satterthwaite Theorem." *International Economic Review*, 24, 413-417.

Barbera, S. and Peleg, B. (1990). "Strategy-Proof Voting Schemes with Continuous Preferences." *Social Choice and Welfare*, 7, 31-38.

Barr, J. L. and T. L. Shaftel (1976). "Solution Properties of Deterministic Auctions." *Journal of Financial and Quantitative Analysis*, 287-311.

Border, Kim C. (1989). "Implementation of Reduced Form Auctions: A Geometric Approach." Mimeo, California Institute of Technology, to appear in *Econometrica.*

Coppinger, Vicki M., Smith, Vernon L., and Titus, Jon A. (1980). "Incentives and Behavior in English, Dutch, and Sealed-Bid Auctions." *Economic Inquiry*, 18, 1-22.

Cox, J. C., Roberson, B. and Vernon L. Smith (1982). "Theory and Behavior of Single Object Auctions," in *Research in Experimental Economics, Vol. 2.*, ed., V. L. Smith. JAI Press, Greenwich, Conn.

Crawford, Vincent P., and Knoer, Elsie M. (1981). "Job Matching with Heterogeneous Firms and Workers." *Econometrica*, 49, 437-450.

Dantzig, G. B. (1963). *Linear Programming and Extensions.* Princeton University Press, Princeton, N. J.

Dasgupta, P., P. Hammond, and E. Maskin (1979). "The Implementation of Social Choice Rules." *Review of Economic Studies*, 44, 185-216.

Demange, Gabrielle, and David Gale (1985). "The Strategy Structure of Two-sided Matching Markets." *Econometrica*, 53, 873-883.

Demange, Gabrielle, David Gale, and Marilda Sotomayor (1986). "Multi-Item Auctions." *Journal of Political Economy*, 94, 863-872.

Engelbrecht-Wiggans, R. and R. J. Weber (1979). "An Example of a Multi-object Auction Game." *Management Science*, 25, 1272-1277.

Engelbrecht-Wiggans, Richard (1980). "Auctions and Bidding Models: A Survey." *Management Science*, 26, 119-142.

Francis, N. D. and D. I. Fleming (1986). "Optimum Allocation of Places to Students in a National University System." *Bit*, 25, 307-317.

Franklin, Joel (1980). *Methods of Mathematical Economics*. Springer-Verlag, New York.

Gale, David (1960). *The Theory of Linear Economic Models*. McGraw-Hill, New York.

Gale, David (1968). "Optimal Assignments in an Ordered Set: Application of Matroid Theory." *Journal of Combinatorial Theory*, 5, 176-180.

Gale, David (1987). "College Course Assignments and Optimal Lotteries." University of California at Berkeley.

Gale, David and L.S. Shapley (1962). "College Admissions and the Stability of Marriage." *American Mathematical Monthly*, 69, 9-15.

Gardenfors, Peter (1973). "Assignment Problem Based on Ordinal Preferences." *Management Science*, 20, 331-340.

Gibbard, A. (1973). "Manipulation of Voting Schemes: A General Result." *Econometrica*, 41, 587-602.

Goldberg, David (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley, New York.

Green, Jerry, and Laffont, Jean-Jacques (1977). "Characterization of Satisfactory Mechanisms for the Revelation of Preferences for Public Goods." *Econometrica*, 45, 427-438.

Green, Jerry, and Laffont, Jean-Jacques (1978). "Satisfactory Mechanisms for Environments with Consumption Lower Bounds." *Journal of Economic Theory* 19, 359-375.

Grether, D., M. Isaac, and C. Plott (1979). "Alternative Methods of Allocating Airport Slots: Performance and Evaluation." CAB Report, Polynomics Research Laboratories, Inc.

Grether, D., M. Isaac, and C. Plott (1981). "The Allocation of Landing Rights by Unanimity by Competitors." *American Economic Review*, 71, 166-171.

Halmos, P. R. and H. E. Vaughn (1950). "The Marriage Problem." *American Journal of Mathematics*, 72, 214-215.

Harrison G. W. and K. A. McCabe (1987). "Stability and Distortion in Resource Matching: An Experimental Study of the Marriage Problem." Discussion Paper No. 87-11, University of Arizona.

Hausch, D. B. (1986). "Multi-Object Auctions: Sequential vs. Simultaneous Sales." *Management Science*, 32.

Hausman, J. A. (1978). "Specification Tests in Econometrics." *Econometrica*, 46, 1251-1272.

Holmström, Bengt (1979). "Groves' Scheme on Restricted Domains," *Econometrica*, 47, 1137-1144.

Holmström, Bengt and R. B. Myerson (1983). "Efficient and Durable Decision Rules with Incomplete Information." *Econometrica*, 51, 1799-1819.

Hsiao, Cheng (1986). *Analysis of Panel Data*. Cambridge University Press, New York.

Hylland, Aanund and Richard Zeckhauser (1979). Efficient Allocation of Individuals to Positions." *Journal of Political Economy*, 87, 293-324.

Judge, G. G., R. C. Hill, W. E. Griffiths, H. Lutkepohl and T. C. Lee (1985). *The Theory and Practice of Econometrics*. Wiley, New York.

Koopmans, T. C. and Beckmann, M. J. (1957). "Assignment Problems and the Location of Economic Activities." *Econometrica*, 25, 53-76.

Laffont, J. J., and E. Maskin (1982). "Nash and Dominant Strategy Implementation in Economic Environments." *Journal of Mathematical Economics*, 10, 17-47.

Ledyard, John O. (1986). "The Scope and the Hypothesis of Bayesian Equilibrium." *Journal of Economic Theory*, 39, 59-82.

Ledyard, John O. and Thomas R. Palfrey (1990). "On the Optimality of Lottery Drafts: Characterization of Interim Efficiency in a Public Goods Problem." Social Science Working Paper, 717, California Institute of Technology.

Leob, Martin (1977) "Alternative Versions of the Demand-Revealing Process." *Public Choice*, 29, Special Supplement to Spring, 15-26.

Leonard, H. B. (1983). "Elicitation of Honest Preferences for the Assignment of Individuals to Positions." *Journal of Political Economy*, 91, 461-479.

Luce, R. D. and Raiffa, H. (1957). *Games and Decisions*. Wiley, New York.

Maskin E. (1986). "The Theory of Implementation in Nash Equilibrium: a Survey," in *Social Goals and Social Organizations: Volume in Memory of Elisha Pazner*, ed., L. Hurwicz. Cambridge University Press, New York.

Maskin, E. and J. Riley (1984). "Optimal Auctions with Risk Averse Buyers." *Econometrica*, 52, 1473-1518.

Matthews, S. A. (1984). "On the Implementability of Reduced Form Auctions." *Econometrica*, 52, 1519-1522.

McAfee, R. Preston and John McMillan (1987). "Auctions and Bidding." *Journal of Economic Literature*, 25, 699-738.

McCabe, K. A. (1989). "Fiat Money as a Store of Value in an Experimental Market." *Journal of Economic Behavior and Organization*, 12, 215-232.

McCabe, Kevin A., Rassenti, Stephen J., and Vernon L. Smith (1990). "Auction Institutional Design: Theory and Behavior of Simultaneous Multiple Unit Generalizations of the Dutch and English Auctions." *American Economic Review*, 80, 1276-1283.

Miller, Gary J., and Charles R. Plott (1985) "Revenue Generating Properties of Sealed-Bid Auctions: An Experimental Analysis of One Price and Discriminative Processes," in *Research in Experimental Economics*, vol. 3, ed., Vernon L. Smith. JAI Press, Greenwich, Connecticut, 159-183.

Mo, Jie-ping (1988). "On Pure and Minimal Overdemanded Sets: A Note on Demange, Gale, and Sotomayor." Discussion Paper No. 8808, Institute of Economics, Academia Sinica, Taipei, Taiwan.

Mookherjee, Dilip and Stefan Reichelstein (1989). "Dominant Strategy Implementation of Bayesian Incentive Compatible Allocation Rules." Mimeo, Graduate School of Business, Stanford University.

Moreno, D. and M. Walker (1990). "Nonmanipulable Voting Schemes When Participants' Interests Are Partially Decomposable." Forthcoming in *Social Choice and Welfare.*

Moulin, Hervé (1990). "Fair Division under joint ownership: Recent results and open Problems." *Social Choice and Welfare*, 7, 149-170.

Muller, Eitan and Mark A. Satterthwaite (1986). "Strategy-proofness: the Existence of Dominant-strategy Mechanisms," in *Social Goals and Social Organizations: Volume in Memory of Elisha Pazner*, ed., L. Hurwicz. Cambridge University Press, New York.

Nalbantian, Haig and Andrew Schotter (1990). "Matching and Efficiency in the Baseball Free-Agent System: An Experimental Examination." *Economic Research Report*, New York University.

Noussair, C. and D. Porter (1990). "Priority Auctions." Working Paper, Caltech.

Olson, Mark (1990). "Dominant and Nash Strategy Mechanisms without Transfers in the Assignment Problem." Mimeo, Caltech.

Olson, Mark (1991). "An Experimental Examination of the Assignment Problem." Mimeo, Caltech.

Olson, Mark and D. Porter (1989). "Resource Allocation in Complex Systems: Auctions, Priority, and Scheduling." Economic Research Series, Jet Propulsion Laboratory.

Palfrey, T. R., (1980) "Multiple-object, Discriminatory Auctions with Bidding Constraints: A Game-Theoretic Analysis." *Management Science*, 26, 935-946.

Palfrey, Thomas R. (1985). "Buyer Behavior and the Welfare Effects of Bundling by a Multiproduct Monopolist: A Laboratory Investigation." in *Research in Experimental Economics*, vol. 3, ed., Vernon L. Smith. JAI Press, Greenwich, Connecticut, 73-104.

Palfrey, Thomas R. (1990). "Implementation in Bayesian Equilibrium: The Multiple Equilibria Problem in Mechanism Design." Mimeo, Caltech, Pasadena, Ca.

Proll, L. G. (1972). "A Simple Method of Assigning Projects to Students." *Operational Research Quarterly*, 23, 195-201.

Rassenti, S. J., V. L. Smith, and R. L. Bulfin (1982). "A Combinatorial Auction Mechanism of Airport Time Slot Allocation." *The Bell Journal of Economics*, 13, 402-417.

Ritz, Zvi (1985). "Restricted Domains, Arrow Social Welfare Functions and Noncorruptible and Nonmanipulable Social Choice correspondences: The Case of Private and Public Alternatives." *Journal of Economic Theory*, 35, 1-18.

Roth, Alvin E. (1982). "The Economics of Matching: Stability and Incentives." *Mathematics of Operations Research*, 7, 617-628.

Roth, Alvin E. (1984). "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory." *Journal of Political Economy*, 92, 991-1016.

Roth, Alvin E. and Marilda A. Oliveira Sotomayor (1989). *Two Sided Matching: A study on Game-Theoretic Modeling and Analysis*. Forthcoming, Cambridge University Press, New York.

Satterthwaite, Mark A. (1975). "Strategy-proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions." *Journal of Economic Theory*, 10, 187-217.

Satterthwaite, Mark A. and Hugo Sonnenschein (1981). "Strategy-Proof Allocation Mechanisms at Differentiable Points." *Review of Economic Studies*, 53, 587-597.

Shapley, L. S. (1955). *Markets as Cooperative Games*. The Rand Corporation, P-629.

Shapley, L. S., and M. Shubik (1972). "The Assignment Game I: The Core." *International Journal of Game Theory*, 1, 111-130.

Shapley, L.S. and H. E. Scarf (1974). "On cores and Indivisibility." *Journal of Mathematical Economics*, vol 1, 23-37.

Smith, Vernon (1976). "Experimental Economics: Induced Value Theory." *American Economic Review*, 66, 274-79.

Smith, Vernon L. (1978) "Relevance of Laboratory Experiments to Testing Resource Allocation Theory," in *Evaluation of Econometric Models*, eds., Jan Kmenta and James Ramsey. The Academic Press, New York.

Thompson, G. L. (1979). "Pareto Optimal Deterministic Models for Bid and Offer Auctions." *Methods of Operations Research*, 35, 517-530.

Thompson, G. L. (1980). "Computing the Core of a Market Game." in *Extremal Methods and Systems Analysis*, ed., A. V.. Fiacco and K. O. Kortanek, Springer-Verlag, New York, 312-334.

Thompson, G. L. (1981). "Auctions and Market Games." in *Essays in Game Theory and Mathematical Economics, in Honor of Oskar Morgenstern*, ed., Martin Shubik. Bibliographisches Institut, Zurich, 181-196.

Thompson, William (1979). "Maximin Strategies and Elicitation of Preferences," in *Aggregation and Revelation of Preferences*, ed., Jean-Jacques Laffont. North-Holland Publishing, Amsterdam, 245-268.

Vickrey, W. (1976). "Auctions, Markets, and Optimal allocation." in *Bidding and Auctioning for Procurement and Allocation*, Proceeding of a Conference at the Center for Applied Economics. New York University Press, New York, 13-20.

Vickrey, William (1961). "Counter Speculation, Auctions and Competitive Sealed Tenders." *Journal of Finance*, 16, 8-37.

Wilkonson (1972). "Assigning Projects to Students, Discussion of Proll's Paper." *Operational Research Quarterly*, 23, 579-580.

Wilson, L. B. (1977). "Assignment Using Choice Lists." *Operational Research Quarterly*, 28, 569-578.

Zhou, Lin (1990a). "On a Conjecture by Gale about One-Sided Matching Problems." *Journal of Economic Theory*, 52, 123-135.

Zhou, Lin (1990b). "Impossibility of Strategy-Proof mechanisms in Economies with Pure Public Goods." *The Review of Economic Studies*, 58, 107-120.