EFFECT OF RIVET SPACING ON STIFFENED THIN SHEET UNDER COMPRESSION

Thesis by

W. Lavern Howland

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I SUMMARY

This thesis discusses an experimental investigation of the stress distribution across a stiffened thin sheet panel under compression. The effect of bending stresses and curvature of the sheet on extensometer readings is also dealt with. An attempt is made to determine a rational design criteria for rivet spacing on stiffened thin sheet under compression. Design curves of rivet spacing against thickness are developed. The effect of changing the rivet spacing or size on the ultimate load of a compression panel is discussed in view of some experiments.

II ACKNOWLEDGEMENT

The author wishes to express his appreciation to Drs. E. E. Sechler and von Karman for their helpful suggestions and criticisms during the preparation of this thesis.

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III INTRODUCTION

The introduction of "stressed skin" construction into airplanes, created a large number of new structural problems. One of these is the determination of the effect of sheet attachments on the behavior of the sheet and built up structures when under compression loads.

This problem divides itself into several parts,

a.) Effect of various types of fastenings,
b.) Effect of attachment spacing on the sheet,
c.) Effect of spacing on the reinforcement, i.e. stiffness or corrugation.
d.) Effect of spacing on ultimate load of compression panels.

Upon looking through the literature the author found a great deal of information on the behavior and design of riveted joints. Large numbers of formulas and charts giving the strength of riveted joints with which the designer of "stressed skin" construction is confronted, can be found. However, no sound design criteria could be found on the attachment spacing of thin sheet to reinforcement where the unit was under compression.

The author became interested in this problem when Lt. C. B. Hutchins found that there was a variation of strength of compression panels with a variation in attachment methods. It is obvious that there must be some variation, for if we consider the two limiting conditions, line support or attachment between sheet and reinforcement, and no support or attachments, we see that there must be a change in the compression strength.

Lt. Hutchins found out that for the same spacing between attachments that a sheet stiffener combination would carry more load if spot welded, than if it was riveted, and more load if it was riveted than if it was bolted. Also, that as the attachment spacing increased the failing load of similar panels decreased. The author felt that due to the increasing use of semi-manoque construction, where thin sheet panels are under compression, this effect should be studied further.

IV INVESTIGATION OF STRESS DISTRIBUTION ACROSS A STIFFENED THIN SHEET PANEL UNDER COMPRESSIVE LOADING

The first problem undertaken was the determination of the stress distribution on a stiffened sheet. To the author's knowledge this had never been done before experimentally, and there were several reasons for interest in this problem. First, to see physically the meaning of Drs. von Kármán and E. E. Sechler's conception of effective width, second to determine the stress distribution near the attachments, and third to find out if the stress distribution changed with different attachments or different attachment spacing. At the present time rivets are by far the most universal means of attachment, therefore throughout all of this investigation rivets were used.

Test Apparatus

The testing was done in the special compression machine in the structures laboratory of G.A.L.C.I.T., which has been described in Reference I. See Fig. 1. The panels were tested with as near a fixed end condition as could be obtained by clamping them rigidly with heavy structural steel angles to the testing head. See Fig. 4. For the dimensions of the panels used see the bottom of Fig. 20. The ends were clamped in order to reduce the number of unknown conditions. The lateral deflection of the middle of the panel was checked at low loads in an attempt to eliminate as much eccentricity of loading as possible. The deflection being measured with an Ames Dial gage reading 1/100 of a millimeter per division. By adjusting the clamping bolts on both sides of the panel it was possible to eliminate nearly all lateral deflections up to reasonably high loads. In the preliminary tests, the longitudinal deflection of the whole panel was measured on each side of the sheet by means of similar Ames Dial gages, which may be seen attached to the V-groove supports in Figs. 2, 4, and 5. Adjustments were made so that the test heads were parallel at all times.

It was decided to measure the strain across the panel with Huggenberger Tensometers. Two types of extensometers were used, both having a two centimeter gage or base length. One type measured a strain of 1.5×10^{-4} inches/inch per division and the other measured 4×10^{-5} inches/inch per division. A support was made to hold one of these near the upper end of the back side of panel in any chosen position across the sheet. See Fig. 6. One extensometer was attached directly to the stiffener near the bottom of the panel. Fig. 5. Preliminary runs with the first panel disclosed that the vertical distribution of strain along the stiffener was not constant, thus indicating non-uniform deflection at the ends.

Apparently the stiffener was a little bit shorter at the bottom than the sheet, thus the sheet was putting load into the stiffener by means of shearing forces. The panel was removed and special care was taken with the second panel to insure uniform deflection across the whole width of the panel.

Load was applied by means of a hydraulic jack up to about 80% or the failure value of the panel, (the load is measured by accurate calibrated pressure gages on the hydraulic jack) and the strain at various stations across the panel as indicated by the extensometer was recorded at numerous increments of load. Curves of strain against load were then plotted for the various stations and these were faired to eliminate experimental scatter. Values were taken from these deflection-load curves to plot the distribution of stress across the panel. This plot gave very surprising results for the curves were nothing like the ones expected. The first explanation of the peculiar behavior was that the sheet upon going into the wave state at low loads, might cause an error in the strain readings. To check the effect of bending stresses on the strain readings of the extensometers, a piece of sheet was mounted in a vice as shown in Fig. 7 with balanced extensometers attached. The strip was bent first one direction and then the other, and it was found that for rather small curvatures of the sheet the bending stresses are of considerable magnitude.

After this simple experiment, the author is certain that any theory which tries to give the failing load of a thin sheet under compression must somehow take into account the combined stresses. The first attempt to do this was by M. Yamamoto and K. Kendo of Tokyo Imperial University. The strains measured on a thin sheet panel by the author show that M. Yamamoto and Kendo's theory does not give the same type of total stresses as measured. The probable reason for this is that they make the assumption of a single wave while actually near the edges there are multiple waves.

The experiment of the two extensometers on the piece of sheet disclosed that there was a difference in deflection between the tension and the compression side. One would normally expect that the two deflections would be equal and of opposite sign in the case of pure bending as was applied. Simple reasoning presented the idea that the difference in readings on the two sides of the sheet was due to curvature shortening. Thus, if readings were taken on both sides of the sheet for various curvatures, a correction might be obtained which if subtracted from the mean value of the two strain readings would give the actual strain at the neutral axis. This correction is equal to the difference between the mean value and one of the readings when measurements are made on the strip with pure bending movements applied to it. In other words the shortening due to curvature is one half the absolute difference between the strain readings on the two sides of the sheet. The correction curve of Fig. 18 was plotted from tests on the strip and was used to correct the readings which were thereafter made. It is undoubtedly possible to calculate this correction but the author believed that it was much easier to make the experiment. The reason it is not necessary to measure the radius of curvature is that we can make the assumption that the radius of curvature is proportional to the bending stress which in turn is proportional to the difference between the deflection on the two sides of the sheet. Therefore we can plot a curve of curvature correction against difference between the strain readings, this being done in Fig. 18.

In order to get the compressive stress distribution on the panel, it was necessary to apply two corrections to the readings. First, the readings of the two extensometers on each side of the panel (they were placed accurately opposite each other) had to be averaged, this eliminating deflection due to bending stress. Second, the correction for curvature, discussed above, had to be applied. Upon application of these two corrections, very reasonable looking curves were obtained. See Fig. 19 and 20.

It was impossible to make simultaneous readings across the whole panel, so only the stiffener stress was measured clear up to failure. Using this curve as a basis, and by referring to the stress-distribution curves at lower loads, the load-deflection curves for all other stations were extrapolated to failure, see Fig. 19. The distribution of stress for the ultimate load was drawn from these extrapolations and is shown in Fig. 20.

Discussion of Results

The curves obtained are quite interesting. From them it is seen that upon application of a 1000 pound load the stress in the sheet was nearly uniform and equal to the stress in the stiffener. It might be mentioned at this point that the stress in the stiffener was nearly the same as in the sheet at the center line of the rivets. This load of 1000 pounds was slightly below where the sheet buckling was noticeable (buckled at 1200 pounds approximately). As the load was increased the stress in the center of the sheet remained constant but the stress near the stiffener and the V-grooves increased. This effect is also clearly seen on Fig. 19. In Reference II, Drs. von Karman and E. E. Sechler predicted exactly this type of strain distribution.

The idea of "effective width" can be seen quite clearly from the curves, for as the load is increased on the panel, more and more load is carried near the stiffener and the V-grooves. The effect of the rivet is shown from the fact that the sheet near the rivets or stiffener carries more load than does the sheet near the V-grooves. One would expect this because of the bending restraint applied to the sheet by the rivet heads. Thus, it could be said that the "effective width" near the stiffener is larger than that by the V-grooves. The maximum combined stresses in the sheet occurred at about 3/8" to 1/2" from the center line of the stiffener and the V-grooves. At this point the direct compressive stress is high, and the bending stresses are large, indicating that yielding would probably start in this region.

V THEORY OF BUCKLING BETWEEN RIVETS

Upon the study of the stress distribution of the stiffened panel (see Fig. 20) and the appearance of actual buckling between rivets (see Figs. 11, 12, 13, 14, 15, 16, and 17) on test panels, the author decided that an expression could be derived to explain the buckling between rivets. Since the stress near the center line of the rivets is quite uniform (at reasonably high loads, not at ultimate) we can assume as a just approximation that a little element between the rivets has no support from adjoining elements. See Fig. 21.

Tests help confirm this assumption because on test panels it has been observed that the buckling between rivets will extend from 17 to 20 times the thickness of the sheet on each side of the center line of the rivets. A section through the center line of the rivets will look like the bottom part of Fig. 21.

The equation for the curve which the element W will form, assuming fixed ends, is found in nearly any elasticity text as,

$$y = \frac{y_m}{2}\left(1 - \cos\frac{2\pi x}{L}\right) \qquad (1)$$

The maximum stress at any point of such an element is equal to

$$\sigma_t = \sigma_c + \sigma_B \qquad (2)$$

where

σ_t = total stress
σ_c = compressive stress
σ_B = stress due to bending

The compressive stress σ_c is a maximum when it is equal to

$$\sigma_{EULER} = 4\pi^2 E\left(\frac{\rho}{L}\right)^2 \qquad \text{(fixed end)}$$

where

E = Young's modulus
ρ = radius of gyration
L = length.

Thus we get

$$\sigma_t = \sigma_E + \sigma_B = 4\pi^2 E\left(\frac{\rho}{L}\right)^2 + \sigma_B \qquad (2a)$$

Now we know that

$$\rho = \sqrt{\frac{I}{A}}$$

I for thin sheet

$$= \frac{wt^3}{12} \qquad A = wt \qquad \therefore \rho = \sqrt{\frac{wt^3}{12wt}} = \sqrt{\frac{t^2}{12}}$$

or $$\sigma_E = \frac{4}{12}\pi^2 E\frac{t^2}{L^2} = \frac{\pi^2}{3}E\frac{t^2}{L^2}$$

This equation is plotted up in Fig. 22. σ_E is plotted against L

for different thicknesses. These curves will be discussed later.

The bending stress at any point is equal to $\sigma_B = \frac{Mc}{I}$

but $M = -EI\frac{d^2y}{dx^2}$

$$\frac{d^2y}{dx^2} = 2y_m\left(\frac{\pi}{L}\right)^2\cos\frac{2\pi x}{L}$$

or $M = -2EIy_m\left(\frac{\pi}{L}\right)^2\cos\frac{2\pi x}{L}$

The movement is a maximum when $\cos\frac{2\pi x}{L} = -1$ which is when $x = \frac{L}{2}$ i.e. at the center.

We then get $M = 2EIy_m\left(\frac{\pi}{L}\right)^2$ or the maximum bending stress is equal to

$$\sigma_B = \frac{2EI}{I}y_m\left(\frac{\pi}{L}\right)^2 c = 2Ec\,y_m\left(\frac{\pi}{L}\right)^2$$

but $c = t/2$

$$\therefore\ \sigma_B = \frac{\pi^2}{L^2}E\,t\,y_m \qquad (4)$$

Now we must determine y_m.

The shortening of the column due to bending is equal to

$$\lambda_B = \tfrac{1}{2}\int_0^L\left(\frac{dy}{dx}\right)^2 dx$$

$$= \frac{1}{2}\,\frac{y_m^2}{4}\int_0^L\left(\frac{4\pi^2}{L^2}\right)\sin^2\frac{2\pi x}{L}\,dx = \frac{1}{4}\,\frac{\pi^2}{L}y_m^2$$

or we get that

$$y_m = \sqrt{\frac{4\lambda_B L}{\pi^2}} = \frac{2}{\pi}\sqrt{\lambda_B L} \qquad (5)$$

Substituting this in equation (4) we get

$$\sigma_B = \frac{\pi^2}{L^2}Et\,\frac{2}{\pi}\sqrt{\lambda_B L} = \frac{2Et}{L^2}\sqrt{\lambda_B L} \qquad (6)$$

Substituting (3) and (6) into equation (2a)

$$\sigma_t = \frac{\pi^2}{3}E\frac{t^2}{L^2} + 2\frac{Et}{L}\sqrt{\lambda_B L} \qquad (7)$$

Solving for λ_B

$$\lambda_B = \frac{1}{L}\left[\frac{\sigma_t L^2}{2\pi E t} - \frac{\pi t}{6}\right]^2 \qquad (8)$$

The total change in length of the element is equal to

$$\lambda_t = \lambda_B + \lambda_c \qquad (9)$$

where

λ_B = shortening due to bending

λ_c = shortening due to compressive stress.

$$\lambda_c = \frac{P/A}{E}L = \frac{\frac{4\pi^2EI}{L^2wt}}{E}L \qquad I = \frac{wt^3}{12}$$

$$\lambda_c = \frac{4\pi^2wt^3}{12L^2wt}L = \frac{\pi^2t^2}{3L} \qquad (10)$$

Thus the total shortening of the element is equal to

$$\lambda_t = \frac{1}{L}\left[\frac{\sigma_t L^2}{2\pi E t} - \frac{\pi t}{6}\right]^2 + \frac{\pi^2 t^2}{3L} \qquad (11)$$

This equation is plotted up on Fig 23., λ_t is plotted against L Since the two rivets are attached to some type of a stiffener (corrugation also considered as a type of stiffener), then in order for a shortening of an amount λ_t to take place between the rivets, the stiffener must go to a stress equal to

$$\sigma_{STIFF.} = \frac{\lambda_t E}{L} \qquad (12)$$

Or as a final result we get,

$$\sigma_{STIFF.} = \frac{E}{L^2}\left[\left(\frac{\sigma_t L^2}{2\pi E t} - \frac{\pi t}{6}\right)^2 + \frac{\pi^2 t^2}{3}\right] \qquad (13)$$

From this expression we see that for agiven material of a certain sheet thickness and a limiting value of the maximum total stress σ_t that for various rivet spacings L, the value of the stiffener stress will vary as shown in Figs. 24, 25, 26, 27, 28, and 29. This means that if the designer does not want permanent set in the sheet between rivets, he must hold the rivet spacing to a certain value depending upon the design value for σ_{STIFF} and the properties and thickness of the sheet.

VI EXPERIMENTAL STUDY OF BUCKLING BETWEEN RIVETS

After deriving an expression for the buckling between rivets, it was found to be desirable to check the equation with experimental points. The problem is then, to determine experimentally, how large a deflection between rivets can be applied without resulting in permanent set in the sheet.

Accurate stress-strain curves were made on two thicknesses of 17 ST Dural (Figs. 30, 31). On both thicknesses it was found that the proportional limit is very near to 20,000 lbs. per sq. in. Thus, the assumption was made that if the maximum stress in the sheet between the rivets passed 20,000 lbs. per sq. in. there would be a permanent set upon unloading to zero stress. Since the object of these experiments is to measure the deflection between rivets at which permanent set begins, it can be seen from the stress-strain curves Figs. 30, 31, that it is desirable to make the tests with the load being applied across the grain of the material, i.e., the larger the permanent set for a given stress above the proportional limit the greater the accuracy.

The test set up consisted of a jig which had a sliding head and a fixed head in the same plane. A rivet would hold one end of the sheet to the fixed head and another rivet would fasten the other end of the sheet to the movable head. See Figs. 8 and 9. Compression load could be applied to the unit by a small testing machine. This applied load forces the sheet between the two rivets to buckle.

After running a few tests it was decided to put clamps on the sheet directly behind the rivets. The reason for doing this was that on a regular panel most of the compression load on the sheet between the rivets is not applied by the rivets. See Fig. 10. These small clamps had the effect of loading the sheet similar to the stress distribution found on a stiffened panel. Fig. 20. The desired width of test specimen was found to be three times the rivet spacing on each side of the rivets. On Fig. 32 is seen the effect on the failing load of changing the width of specimen, notice that it approaches a value asymptotically. Any thing less than three times the rivet spacing effected the behavior of the sheet between the rivets.

The test procedure was as follows; the load on the movable head was increased until there was a certain deflection between rivets. Both normal and longitudinal deflections were measured with Ames Dial gages reading 1/100 m.m. per division. The load was then removed and any permanent set either normal to the sheet or longitudinally was recorded. See Fig. 33. In Figs. 8 and 9, can be seen the electric vibrator which was used to reduce static friction of the sliding head. On large spacings the normal deflection gage was held off of the sheet during loading because of the undesirable support it added to the sheet. Until permanent set started the two instruments would always come back to zero upon unloading. After obtaining the deflection between rivets which causes permanent set, it is necessary to multiply it by E and divide by the rivet spacing in order to get the stiffener stress which would correspond to the same deflection or strain between the rivets. This stress is exactly what the theory should give when a certain allowable stress, thickness, E, and rivet spacing is substituted in the equation derived. i.e.

$$(\text{Stiffener stress} = \sigma_{STIFF.} = \frac{E}{L^2}\left[\left(\frac{\sigma_t L^2}{2\pi E t} - \frac{\pi t}{6}\right)^2 + \frac{\pi^2 t^2}{3}\right]$$

The results from doing this on two different sheet thicknesses for various rivet spacings, are seen in Figs. 34 and 35.

It is seen from these two curves, Figs. 34 and 35, that the agreement with the theory, for these two thicknesses of sheet, is quite good, and is especially good for the .016" sheet. Since there is quite a sudden drop off in the critical buckling stress of thin sheet between rivets, the designer must be careful about the rivet spacing that is used, or else the skin on the airplane will look like a washboard after the first flight. This is known to have happened in some cases, no failures occurred, just permanent buckles appeared between the rivets.

The curve on Fig. 36 is of considerable interest from a design standpoint and also from agreement with the above theory. One should notice that the sheet thickness is .025" and the rivet spacing is 1" on this particular panel. On Fig. 26, we see that this is the correct rivet spacing for no permanent buckles up to a stiffener stress of 20,000 lbs. per sq. in. However, on Fig. 22 we see that for .25" sheet with 1" rivet spacing that the maximum compression stress it will stand with out dropping its load is 22,000 lbs. per sq. in. In other words, when the compression stress between the rivets reached 22,000 lbs. per sq. in. the sheet would no longer take any more load. Going back to Fig. 36 we see that when the stiffener stress reached 22,000 lbs. per sq. in. the sheet began to drop its load. This factor is quite important because sometimes designers assume stresses to be carried by the sheet near the stiffener in excess of that allowed by the Euler length between rivets. Of course the curves on Fig. 22 are probably somewhat conservative because they assume an unsupported strip (see section V Theory of Buckling Between Rivets, for derivation', but on the other hand this is partly compensated by the fact that an end fixity of four is assumed which never really exist, and also there is undoubtedly some eccentricity of loading which is not considered.

VII DESIGN CRITERIA FOR RIVET SPACING

The selection of rivet spacing on parts of an airplane is determined by consideration of several factors. First, if the rivets are transmitting load they must be close enough together so that there is enough shear or bearing area. Second, if they are not transmitting any appreciable load then they should be as far apart as the other factors will allow, because of the labor cost of putting in rivets.

Another factor is the size of wrinkles, both from the passenger standpoint (psychological) and aerodynamic effect.

This paper presents a new factor which the author feels is quite important and that is the possibility of permanent buckles between the rivets. Surely the designer does not want the rivet spacing on the top surface of the wing such that when the airplane is subjected to applied loads, that there will be permanent buckles between the rivets (see Figs. 11, 12, 13, 14, 15, 16, and 17). These pictures show actual buckles between rivets which have been left in the sheet after the test panels had been taken just to failure and no further. In order to be sure that permanent set will not take place, the maximum sheet stress should be set equal to the proportional limit. For the development of design curves let us also put the stiffener stress equal to the proportional limit. Besides simplification of the calculations, one of the reasons for doing this is that, we are interested in not having permanent set under the action of applied loads on the structure, since for these loads the stiffener stress is probably near the proportional limit. If we were chiefly interested in behavior at failure we would undoubtedly substitute in the stress at failure (probably near yield point).

Substituting in $\sigma_{P.L.}$ (stress at proportional limit) in equation 13, we get;

$$\sigma_{P.L.} = \frac{E}{L^2}\left[\left(\frac{\sigma_{P.L.} L^2}{2\pi E t} - \frac{\pi t}{6}\right)^2 + \frac{\pi^2 t^2}{3}\right]$$

or

$$\sigma_{P.L.} = \frac{\sigma_{P.L.}^2 L^2}{4\pi^2 E t^2} - \frac{\sigma_{P.L.}}{6} + \frac{\pi^2 t^2 E}{36 L^2} + \frac{\pi^2 t^2 E}{3 L^2}$$

$$\sigma_{P.L.} = \frac{\sigma_{PL}^2 L^2}{4\pi^2 E t^2} - \frac{\sigma_{PL}}{6} + \frac{13}{36}\,\frac{\pi^2 t^2 E}{L^2}$$

$$\frac{7}{6}\,\sigma_{PL}\,\frac{t^2}{L^2} = \frac{\sigma_{PL}^2}{4\pi^2 E} + \frac{13}{36}\,\frac{\pi^2 t^4}{L^4}\,E$$

or

$$\frac{13}{36}\,\frac{\pi^2 E t^4}{L^4} - \frac{7}{6}\,\sigma_{PL}\,\frac{t^2}{L^2} + \frac{\sigma_{PL}^2}{4\pi^2 E} = 0$$

Solving for $\frac{t^2}{L^2}$ we get,

$$\frac{t^2}{L^2} = \frac{7/6\,\sigma_{PL} \pm \sqrt{\left(\frac{7}{6}\right)^2 \sigma_{PL}^2 - \frac{13}{36}\sigma_{PL}^2}}{\frac{26}{36}\pi^2 E}$$

or finally

$$\frac{t^2}{L^2} = \frac{\sigma_{PL}}{E}\left[\frac{42 \pm 36}{26\pi^2}\right] = \frac{\sigma_{PL}}{E} \times \frac{78}{26\pi^2} \quad \text{or} \quad \frac{\sigma_{PL}}{E} \times \frac{6}{26\pi^2}$$

$$\frac{t^2}{L^2} = .304\,\frac{\sigma_{PL}}{E} \qquad \text{or} \qquad \frac{t^2}{L^2} = .0234\,\frac{\sigma_{PL}}{E}$$

$$L/t = 1.814\sqrt{\frac{E}{\sigma_{PL}}} \qquad \text{or} \qquad L/t = 6.535\sqrt{\frac{E}{\sigma_{PL}}}$$

Table I

σ_{PL}	20,000 $\#/\square$	30,000 $\#/\square$	35,000 $\#/\square$	40,000 $\#/\square$
Small spacing	41.4	33.8	30.8	29.2
Large spacing	149	121.5	111.1	105.2

$E = 10.4 \times 10^6$

We can now plot up sheet thickness against rivet spacing for different proportional limits. This has been done in Figs. 37 and 38.

It is very interesting that for $\sigma_{PL} = 20,000\ \#/\square$ we get a rivet spacing of 41.4 times the thickness, because a standard practice among a good many designers during the past few years was to use 40 times the thickness. This rule was purely an arbitrary one which probably originated from shop practice.

If the designer is interested in going to large rivet spacing he should keep in mind two factors; first, whereas the stiffener stress for buckling between rivets goes up for large rivet spacing the load carried by the sheet goes down. Also the normal deflection between the rivets becomes larger, for the size of the normal deflection see Fig. 39.

VIII EFFECT OF RIVET SPACING ON THE FAILING LOAD OF STIFFENED PANELS

One of the difficulties of trying to determine the effect of rivet spacing on the ultimate load of a stiffened panel (other than experimental) is that a change of spacing affects both the sheet and the stiffener, and different stiffeners act entirely unsimilar. The reason for this is that as the rivet spacing is changed the type of failure may also change. For example, a certain sheet-stiffener combination may fail as a Euler column when it has small rivet spacing. As the spacing is increased the type of failure may change to local buckling of the stiffener, still more increase, may change it to rolling of a flange, and at a very large spacing the stiffener may fail in torsion. Thus we see that the problem of ultimate load is a function of the particular stiffener. The results from several series of tests are shown on Fig. 40. The author has failed to find any correlation except that the failing load decreases with an increase in rivet spacing. (Even this has exceptions.)

There is one case where the stiffener is probably not affected a great deal by a change in rivet spacing and that is for corrugation. However, even in this case, the depth of the sheet waves may produce forces on the corrugation which will assist failure. Tests have also been made on corrugation panels. See Fig. 41. On these tests there seems to be some correlation between the two curves for a skin thickness of .040", because the two curves are quite similar. It is very interesting to note that the curve for a skin thickness of .025" goes down and then starts up again. The bottom of the curve occurs at the same rivet spacing as does the bottom of the curve of stiffener stress for buckling between rivets against rivet spacing. See Fig. 26. However, it must be remembered that the failing load of the sheet goes down with an increase in spacing but the strain required to make the sheet start dropping its load is much larger.

The author believes that a great deal more work should be done along this line. Especially, more work at larger rivet spacing and with commonly used extruded sections. Large rivet spacing is desirable from a cost standpoint but the strength of the panel and permanent buckles between rivets must be kept in mind.

IX EFFECT OF RIVET DIAMETER ON THE FAILING LOAD OF STIFFENED PANELS

It was believed at the start of this investigation that not only the spacing but also the size of the rivets would affect the failing load. Thus, a series of tests were run to determine the influence of rivet size.

All the panels tested were made as nearly alike as possible except for the rivet spacing and the size of the rivets. The type of stiffener used and the dimensions of the panels are shown on Fig. 42. These tests were conducted with a great deal of care in order that these two factors would be the only variables. The ends of the panels were clamped very rigidly, (see Fig. 4) and each panel was lined up in the machine as carefully as possible.

The results of these tests are seen in Figs. 42, 43, 44, 45, and 46. Since it was felt that 5/32" rivets and 1/16" rivets were somewhat out of the practical range on the thickness sheet used, the larger rivet spacings for these sizes were not tested. The author regretted that 5/32" rivets at larger spacing than 1.5" were not tested because of the upward trend of the curve. The curve for the 5/32" rivets is higher at all points for three reasons; first, because the actual length of sheet between rivets is shorter, second, because the larger head increases the load carrying capacity of the sheet near the center line of the panel, and third, the larger rivet gives the stiffener more support in torsion which was the critical type of failure for this stiffener.

The curve for the 1/16" rivets is lower for reasons opposite to those for 5/32" rivets, plus the fact that, except for the ½" rivet spacing, actual rivet failures helped cause failure of the panels. For all practical purposes there is little difference between the failing load for 1/8" rivets and that for 3/32" rivets on these particular panels.

The following conclusions can be drawn from these and other tests;

a.) Rivet failures reduce the ultimate load of a panel by putting the sheet load into the stiffener, which helps cause local buckling or failure of the stiffener.

b.) The ultimate load carried by a stiffened panel usually increases with an increase in rivet size.

c. The increase in ultimate load is small if the increase in rivet size does not change the conditions from rivet failure to non-rivet failure.

(It might be mentioned at this point that by rivet failure is meant that the rivet fails in combined tension and shear.)

Observation as to possibility of rivet failure in compression panels

Sheet Thickness

Rivet size	.032"	.040"	.051"
1/16"	Fail if spacing $> \frac{1}{2}$"	No tests	No tests
3/32"	Fail occasionally if spacing $> 1\frac{1}{2}$"	Usually fail if R. S. $> \frac{1}{2}$"	No tests
1/8"	Usually no failure	Failure if R. S. $>$ 1"	Usually fail if R. S. $>$ 1"

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I "The Ultimate Compressive Strength of Thin Sheet Panels," by Dr. E. E. Sechler, G.A.L.C.I.T. Publication No. 27

II "The Strength of Thin Plates in Compression" by Drs. von Kármán, E. E. Sechler and Donnel, Applied Mechanics, 1932

III "Riveting in Metal Airplane Construction" by Wilhelm Pleines. Part I, II, III, IV, Luftfahrtforschung, Vol. VII No. 1, April 30, 1930

IV "The Relative Strength of Stiffener and Sheet Attachment Methods" by C. B. Hutchins. Thesis at California Institute of Technology, 1934

V "Buckling and Failure of Thin Rectangular Plates in Compression" by Yamamoto and Kondo, Tokyo Imperial University, Report No. 119 (v. 10, no. 1)

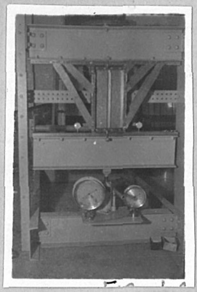

Fig.1.

Testing Machine for Panels.

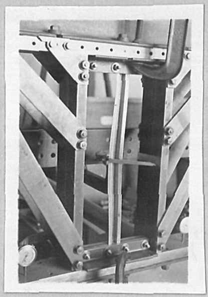

Fig.2.

Failure of Stiffener Alone
Torsion Failure.

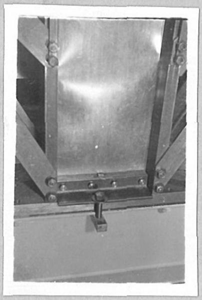

Fig.3.

Failure of Sheet Alone
Shows Method of Clamping.

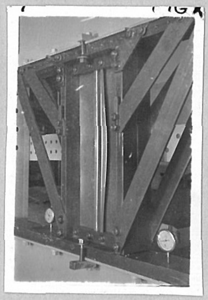

Fig.4.

Failure of Sheet and Stiffener
No Rivets.

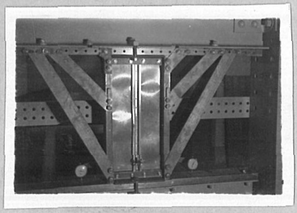

Fig.5.

Typical Test Specimen After Failure.

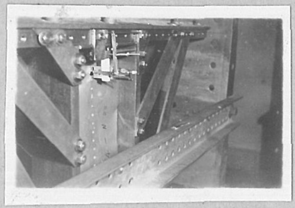

Fig.6.

Extensometer on Panel
Shows Method of Attachment.

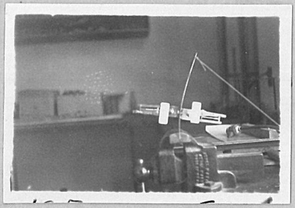

Fig.7.

Test for Curvature Correction.

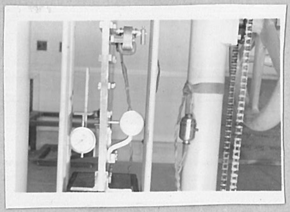

Fig.8.

Apparatus for Study of Buckling
Between Rivets.

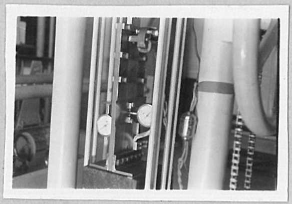

Fig.9.

Apparatus for Study of Buckling
Between Rivets.

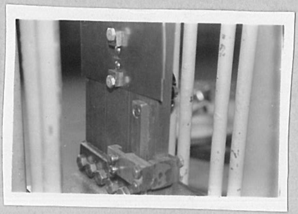

Fig.10.

Method of Loading Sheet for Buckling
Between Rivets.

Fig.11.

Buckling Between Rivets on a
Test Panel. (At Failure)

Fig.12.

Buckling Between Rivets on a
Test Panel. (At Failure)

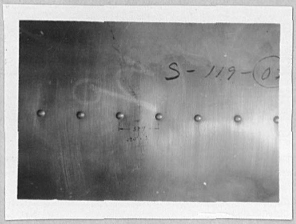

Fig.13.

Permanent Set Between Rivets,
Panel Just Taken to Failure.

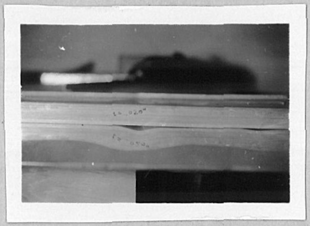

Fig.14.

Side View of Permanent Buckles
Between Rivets. Skin Thickness .020"
Panel Just Taken to Failure.

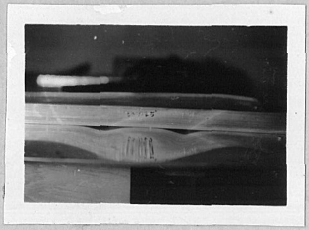

Fig.15.

Side View of Permanent Buckles
Between Rivets. Skin Thickness .025
Panel Just Taken to Failure.

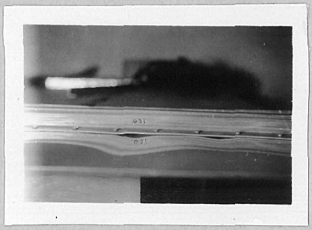

Fig.16.

Side View of Permanent Buckles
Between Rivets. Skin Thickness .032 .

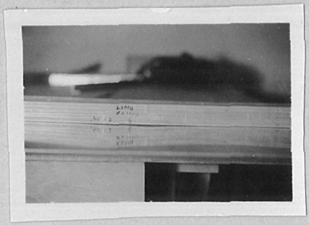

Fig.17.

Side View of Rivet Failure and
Permanent Buckle Between Rivets.
Skin Thickness .040 .

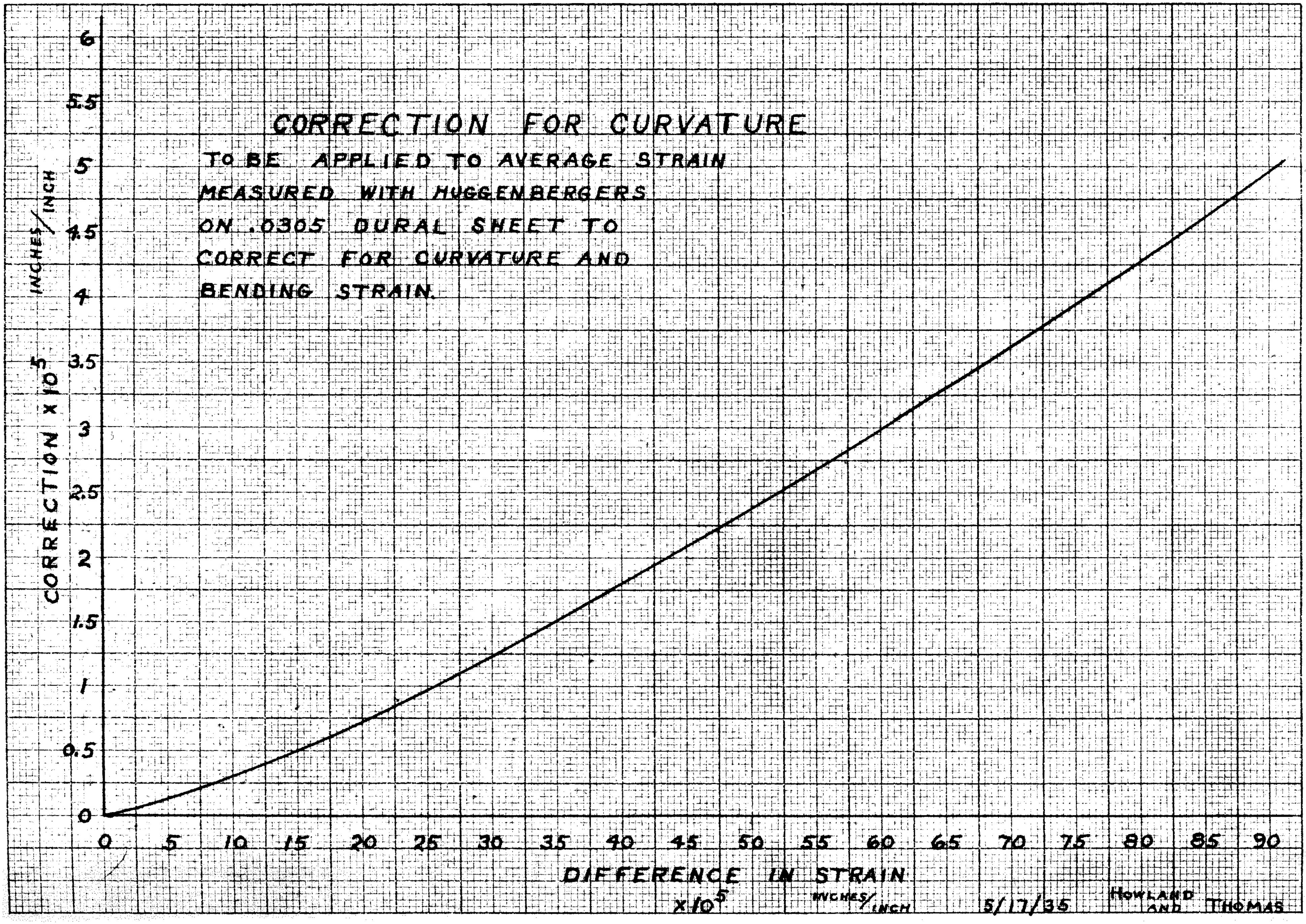

FIG. 18

CORRECTION FOR CURVATURE

TO BE APPLIED TO AVERAGE STRAIN
MEASURED WITH HUGGENBERGERS
ON .0305 DURAL SHEET TO
CORRECT FOR CURVATURE AND
BENDING STRAIN.

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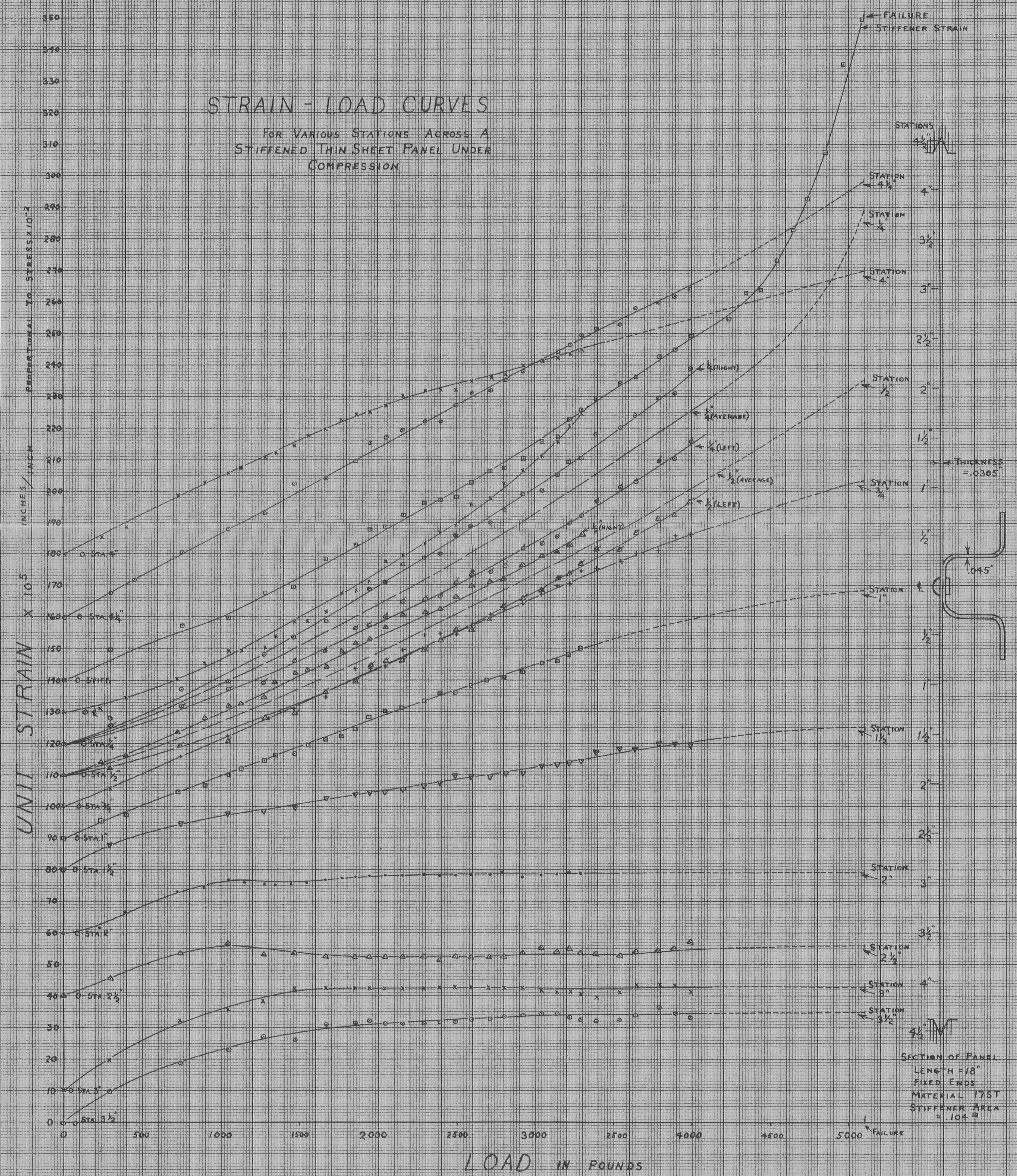
Howland - WL - 1936

Fig. 19.



STRAIN - LOAD CURVES

For Various Stations Across A
Stiffened Thin Sheet Panel Under
Compression

PANEL No. 3715 4/19/35 HOWLAND - THOMAS

By W. L. Howland and C. F. Thomas
October 1936

STRESS DISTRIBUTION ACROSS A THIN SHEET STIFFENED PANEL
UNDER COMPRESSION LOADS

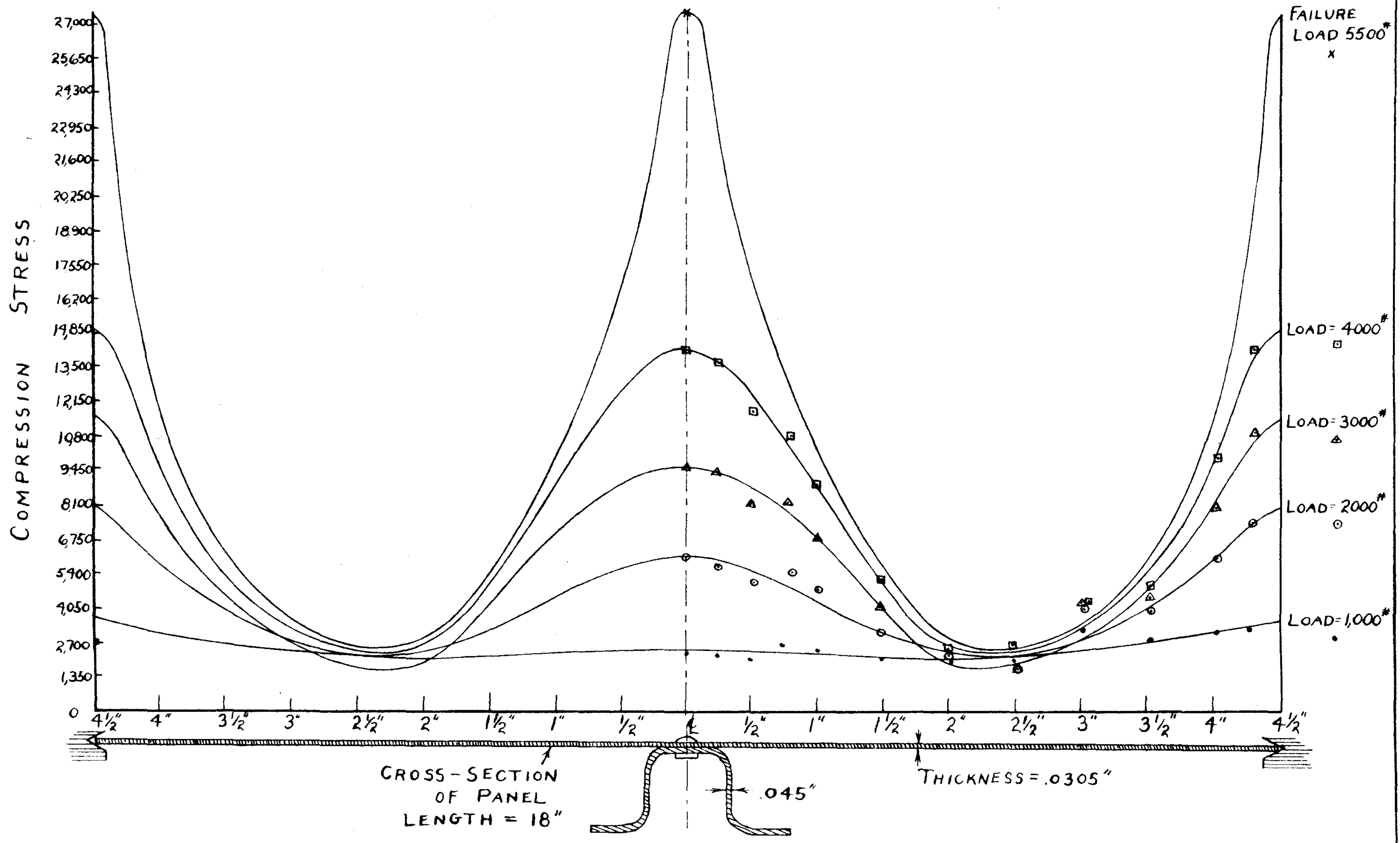


FIG. 20.

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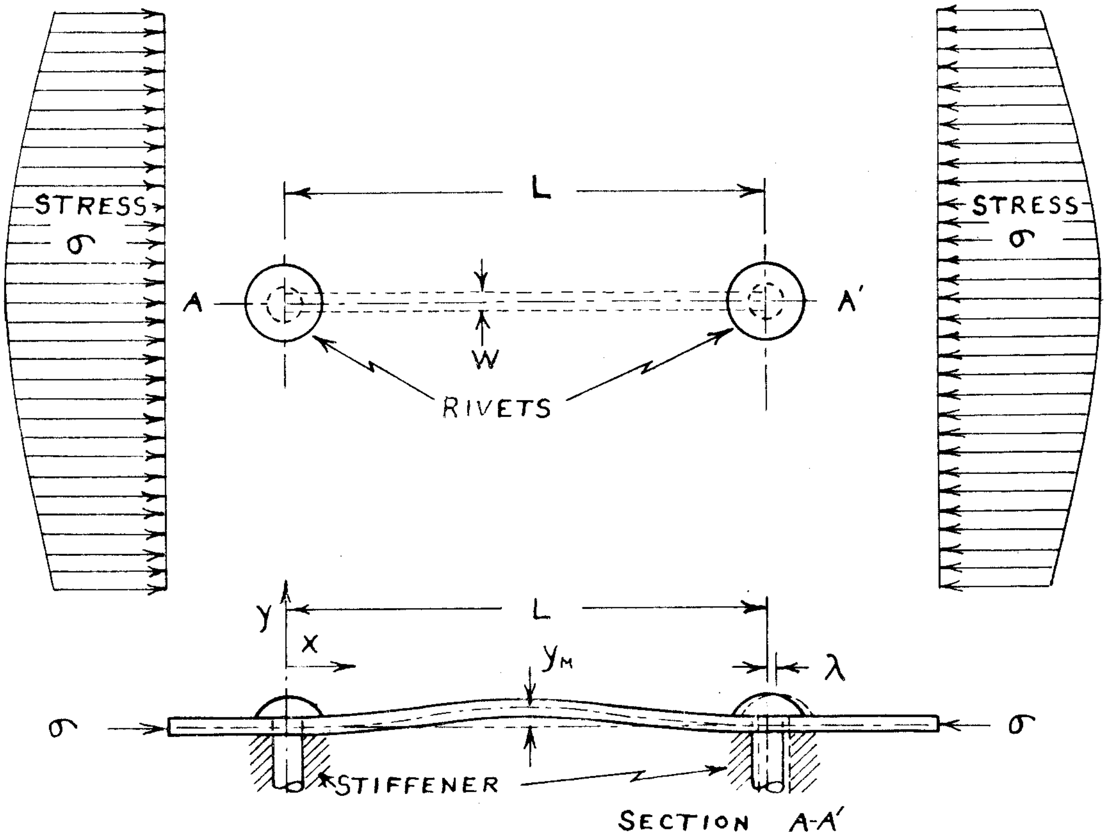
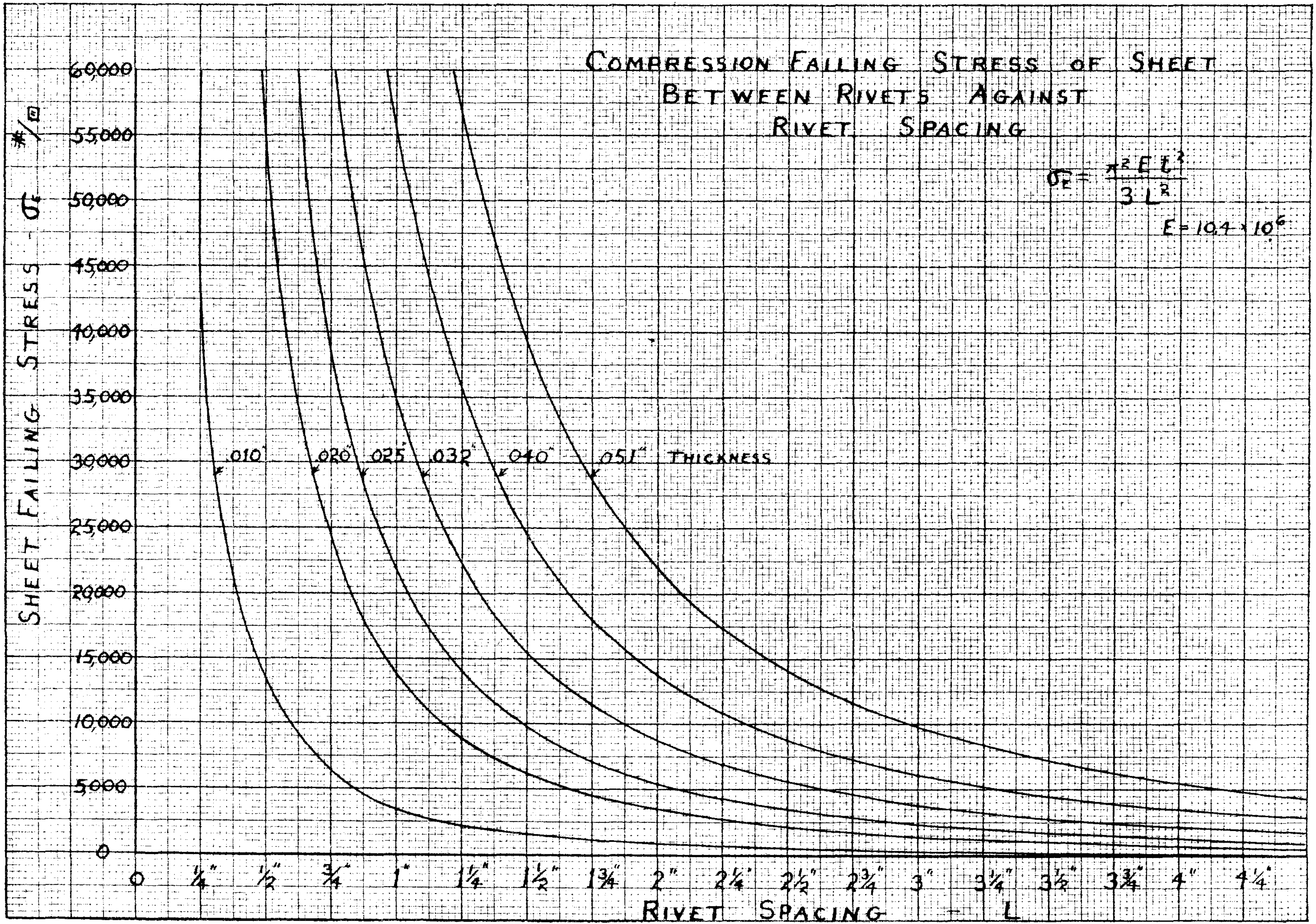
FIG. 21.



STRESS
σ
L
A
A'
W
RIVETS
STRESS
σ
y
L
x
y_M
λ
σ
σ
STIFFENER
SECTION A-A'



COMPRESSION FAILING STRESS OF SHEET
BETWEEN RIVETS AGAINST
RIVET SPACING

FIG 22.

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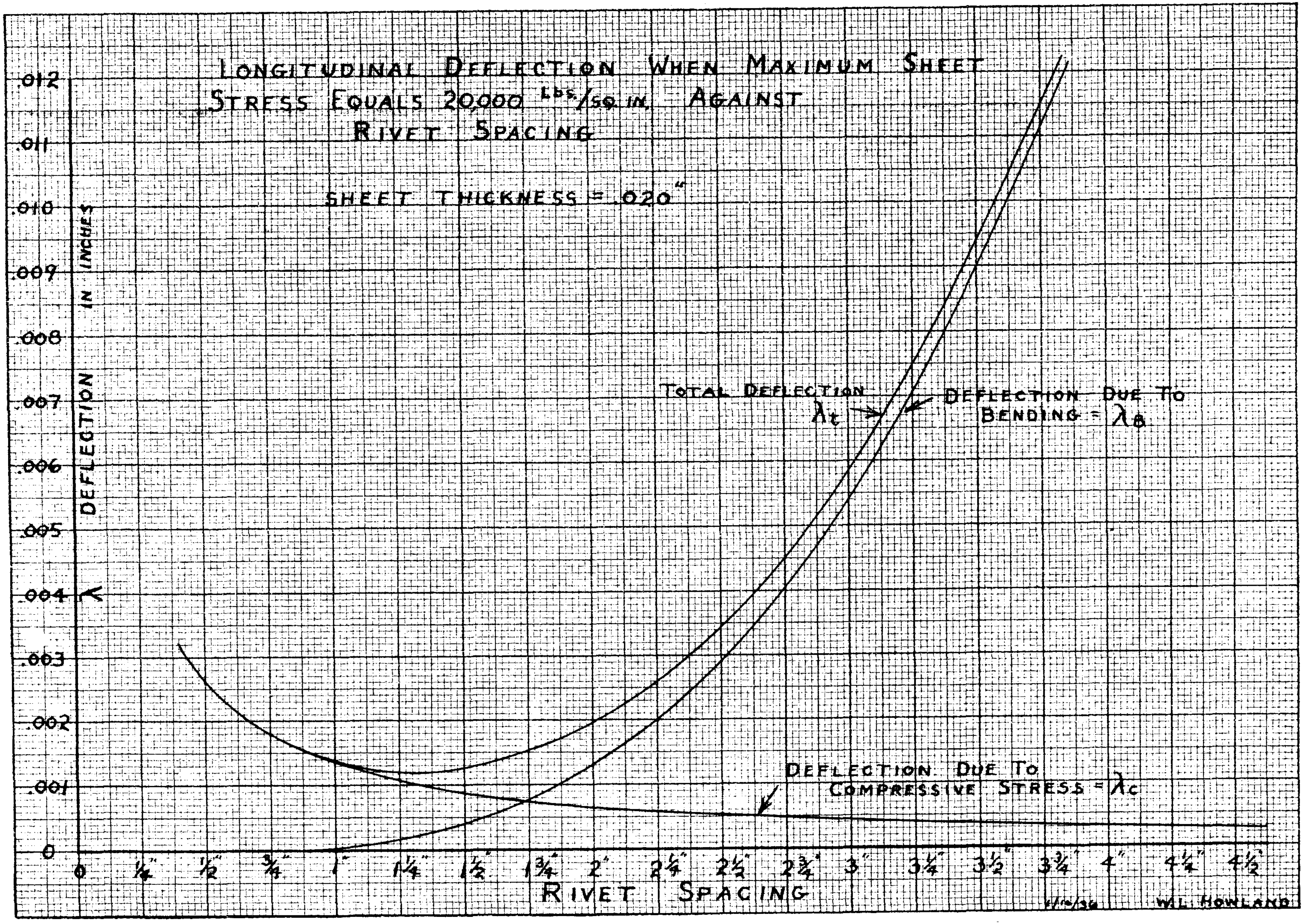
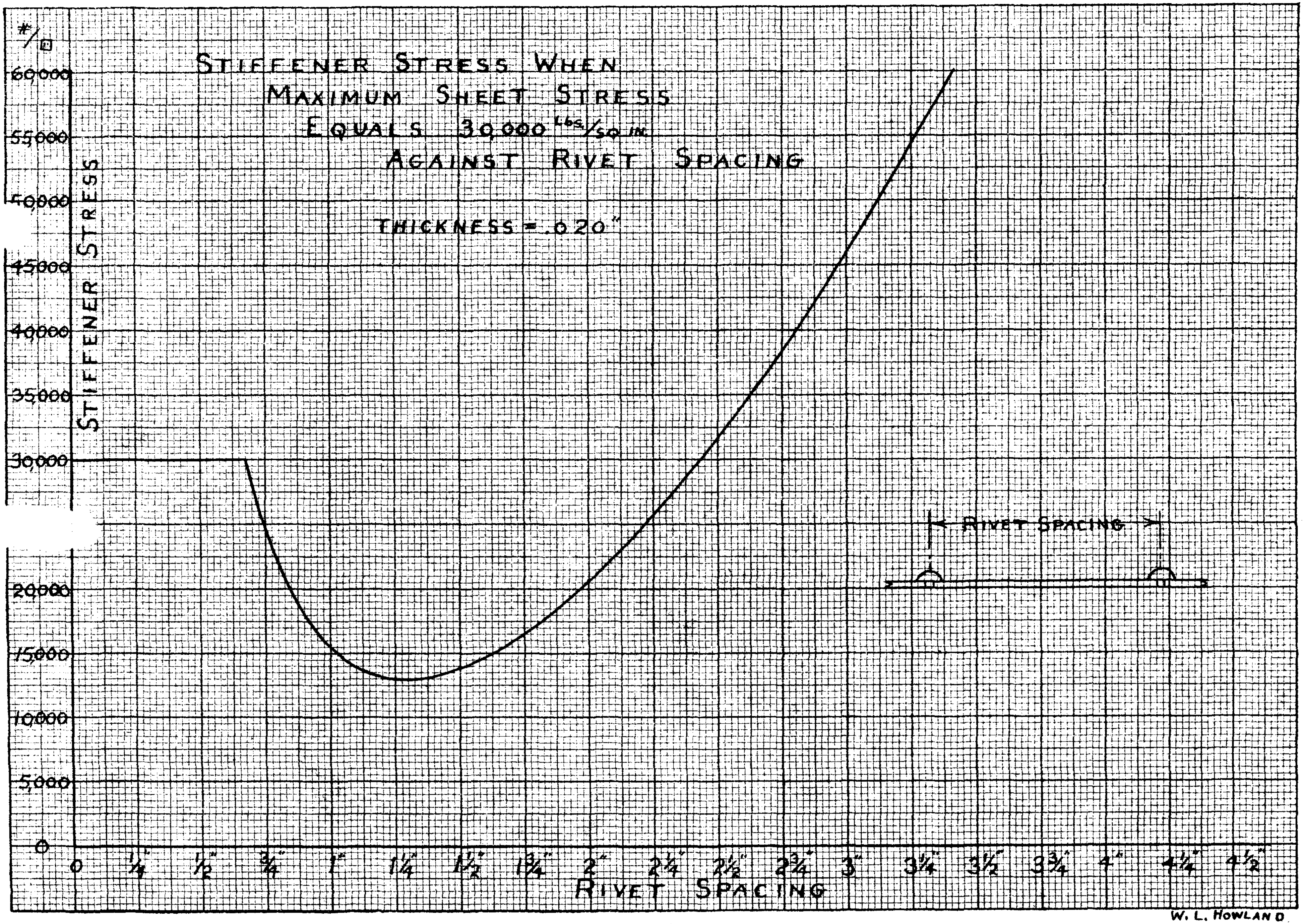

LONGITUDINAL DEFLECTION WHEN MAXIMUM SHEET STRESS EQUALS 20,000 LBS./SQ. IN. AGAINST RIVET SPACING
SHEET THICKNESS = .020"
DEFLECTION IN INCHES
λ
.012
.011
.010
.009
.008
.007
.006
.005
.004
.003
.002
.001
0
0
1/4"
1/2"
3/4"
1"
1 1/4"
1 1/2"
1 3/4"
2"
2 1/4"
2 1/2"
2 3/4"
3"
3 1/4"
3 1/2"
3 3/4"
4"
4 1/4"
4 1/2"
RIVET SPACING
TOTAL DEFLECTION λ_t
DEFLECTION DUE TO BENDING = λ_B
DEFLECTION DUE TO COMPRESSIVE STRESS = λ_c
W.L. HOWLAND

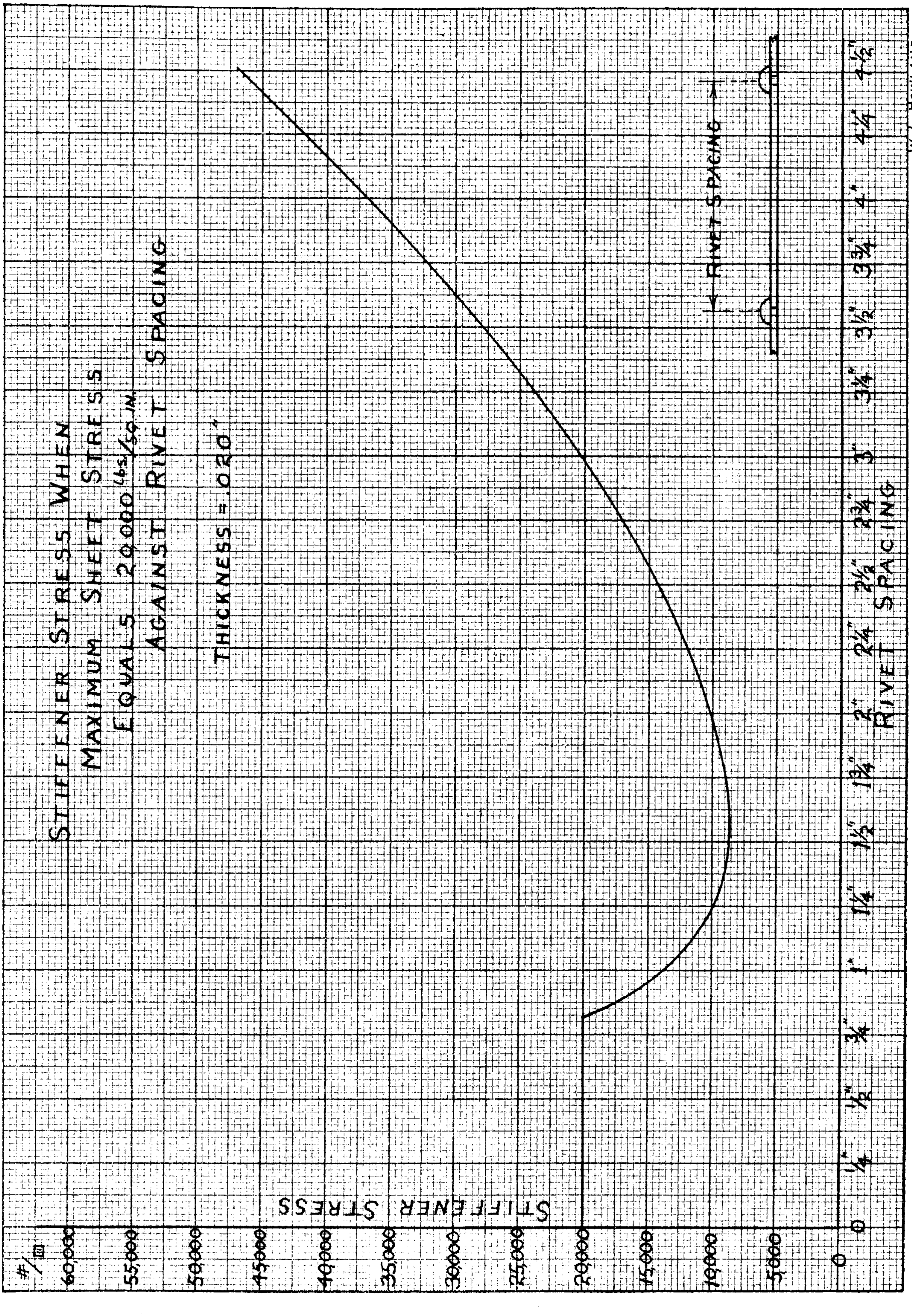
FIG. 23.

FIG. 24.



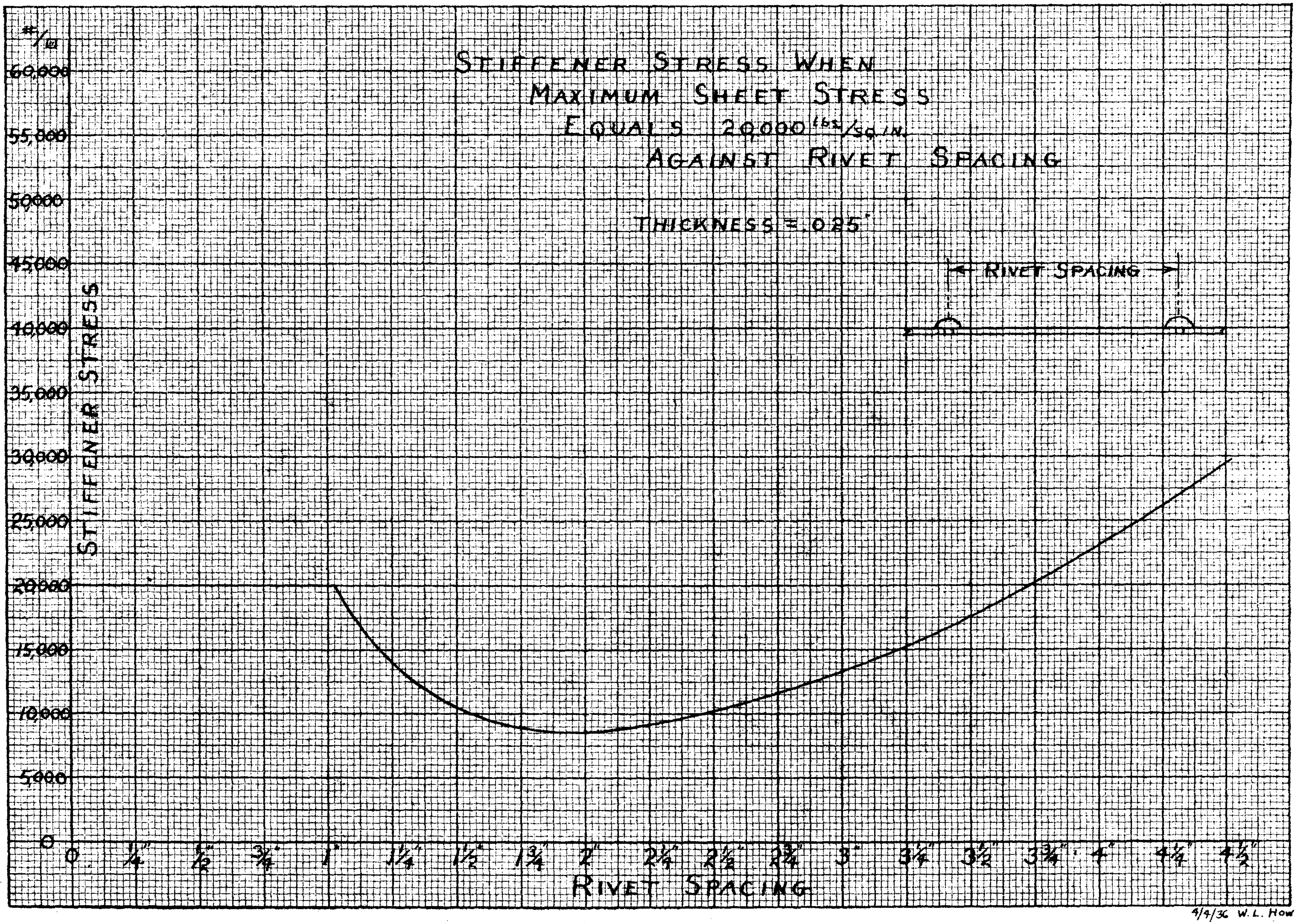
W. L. HOWLAND

FIG. 25.



STIFFENER STRESS WHEN
MAXIMUM SHEET STRESS
EQUALS 20,000 LBS/SQ. IN.
AGAINST RIVET SPACING

THICKNESS = .020″


STIFFENER STRESS WHEN
MAXIMUM SHEET STRESS
EQUALS 20,000 lbs/sq.in.
AGAINST RIVET SPACING

THICKNESS = .025"

FIG. 26.

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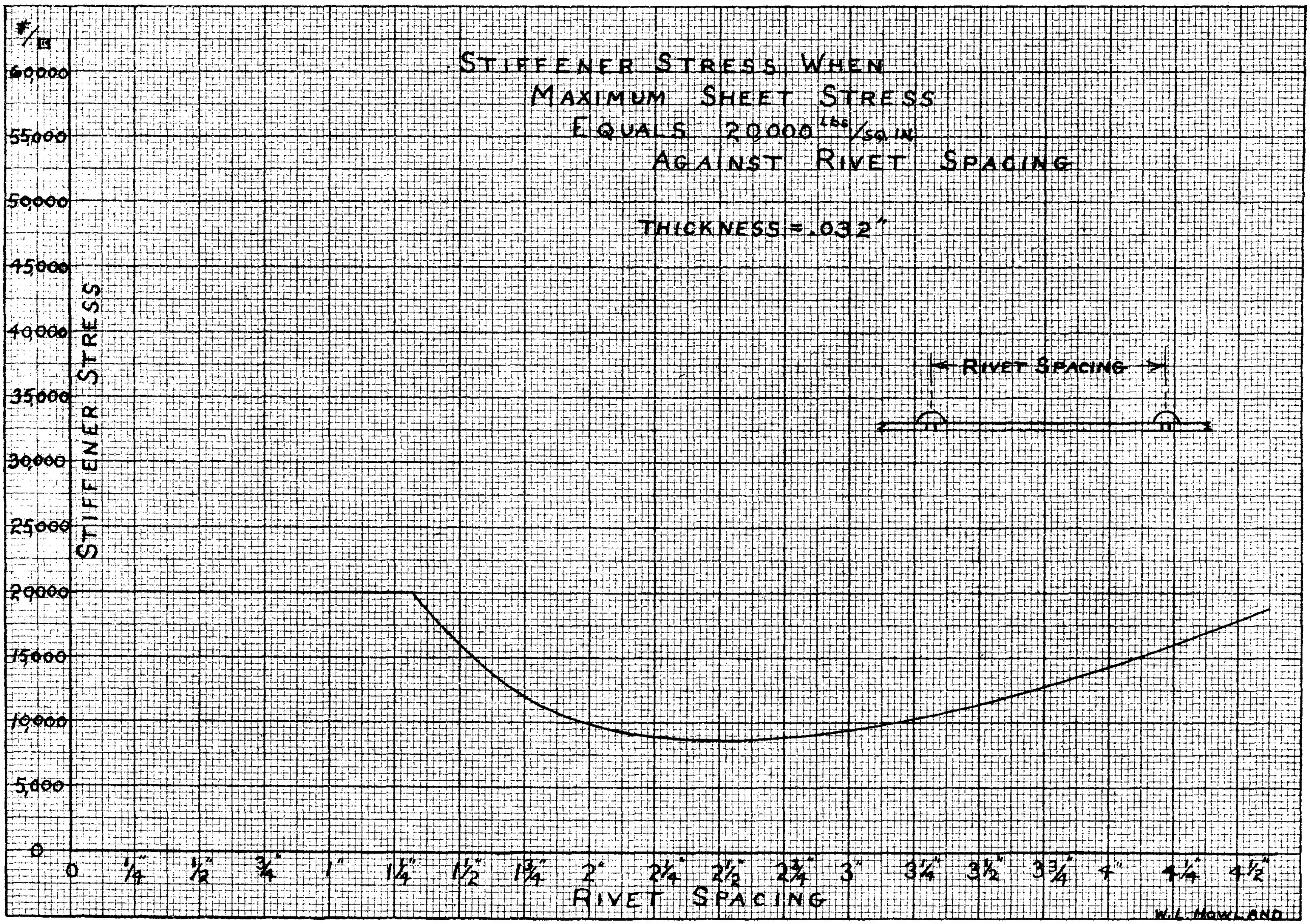
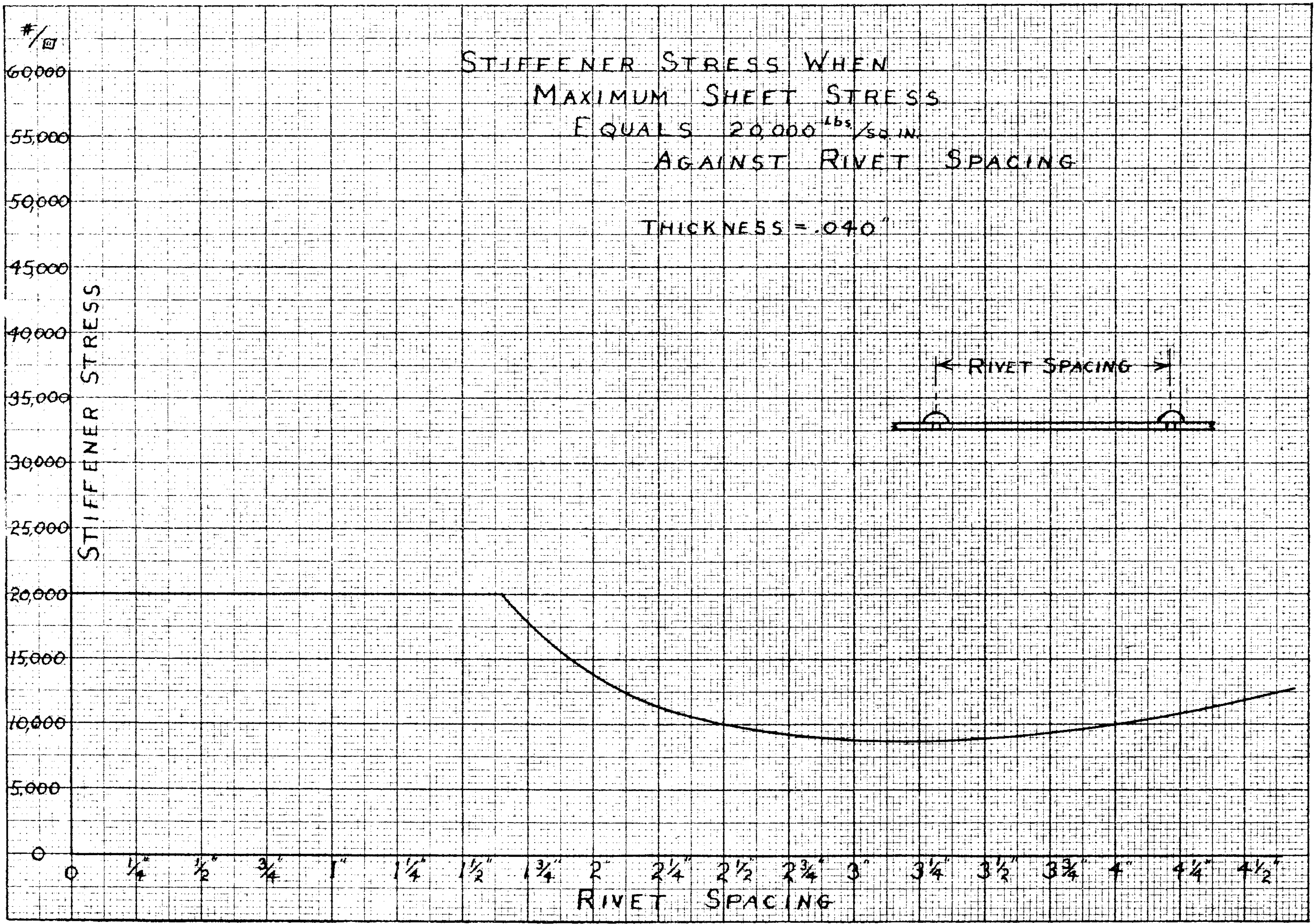

STIFFENER STRESS WHEN
MAXIMUM SHEET STRESS
EQUALS 20000 LBS/SQ. IN.
AGAINST RIVET SPACING
THICKNESS = .032"
RIVET SPACING
#/□"
60000
55000
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
STIFFENER STRESS
0
1/4"
1/2"
3/4"
1"
1 1/4"
1 1/2"
1 3/4"
2"
2 1/4"
2 1/2"
2 3/4"
3"
3 1/4"
3 1/2"
3 3/4"
4"
4 1/4"
4 1/2"
RIVET SPACING
W.L. HOWLAND

FIG. 27.



W. L. HOWLAND

FIG. 28.

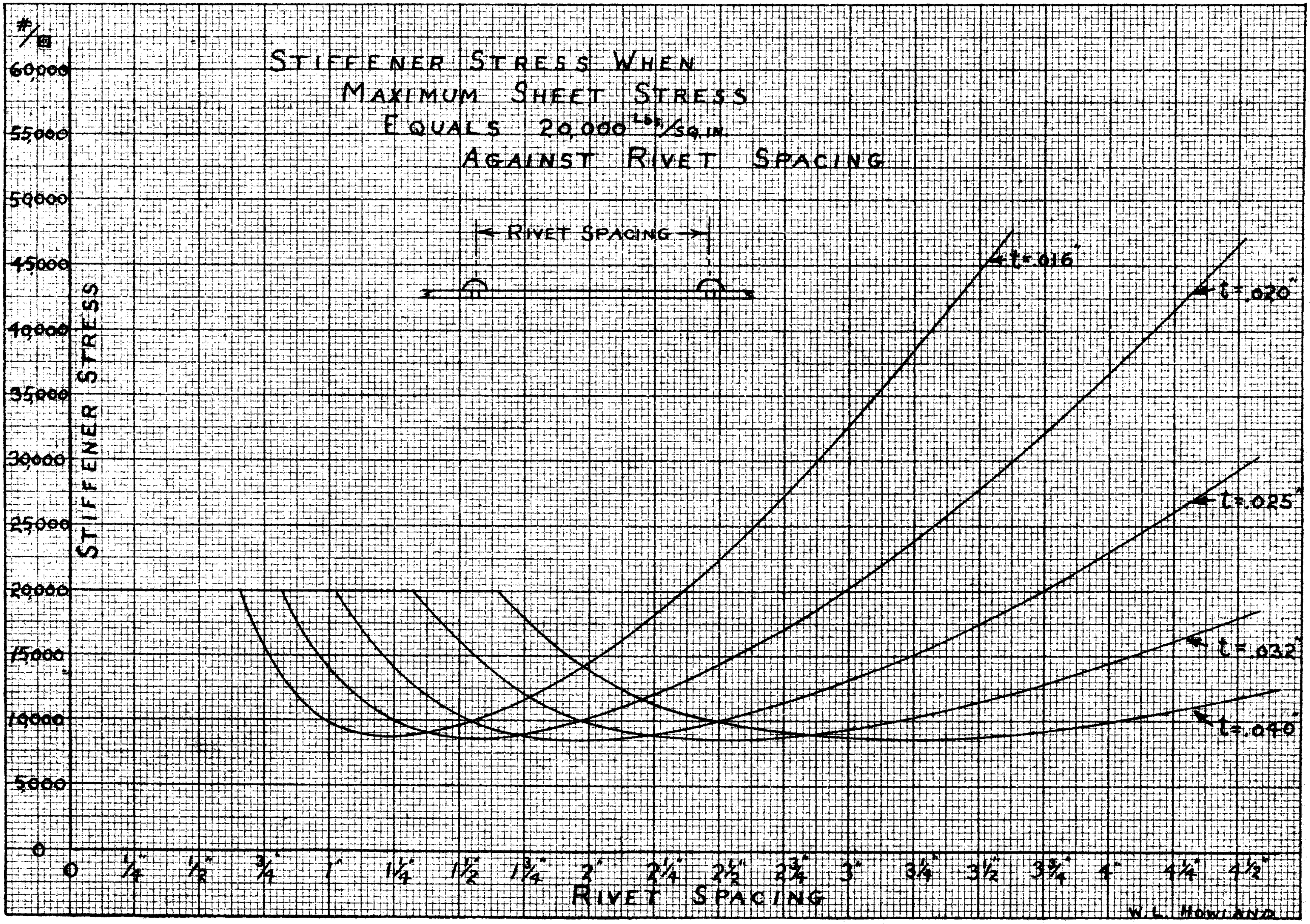

STIFFENER STRESS WHEN
MAXIMUM SHEET STRESS
EQUALS 20,000 LBS/SQ.IN.
AGAINST RIVET SPACING
← RIVET SPACING →
#/□
60,000
55,000
50000
45000
40,000
35,000
30000
25000
20,000
15,000
10000
5000
0
STIFFENER STRESS
0
1/4"
1/2"
3/4"
1"
1 1/4"
1 1/2"
1 3/4"
2"
2 1/4"
2 1/2"
2 3/4"
3"
3 1/4"
3 1/2"
3 3/4"
4"
4 1/4"
4 1/2"
RIVET SPACING
t=.016"
t=.020"
t=.025"
t=.032"
t=.040"
W.L. HOWLAND

FIG. 29.

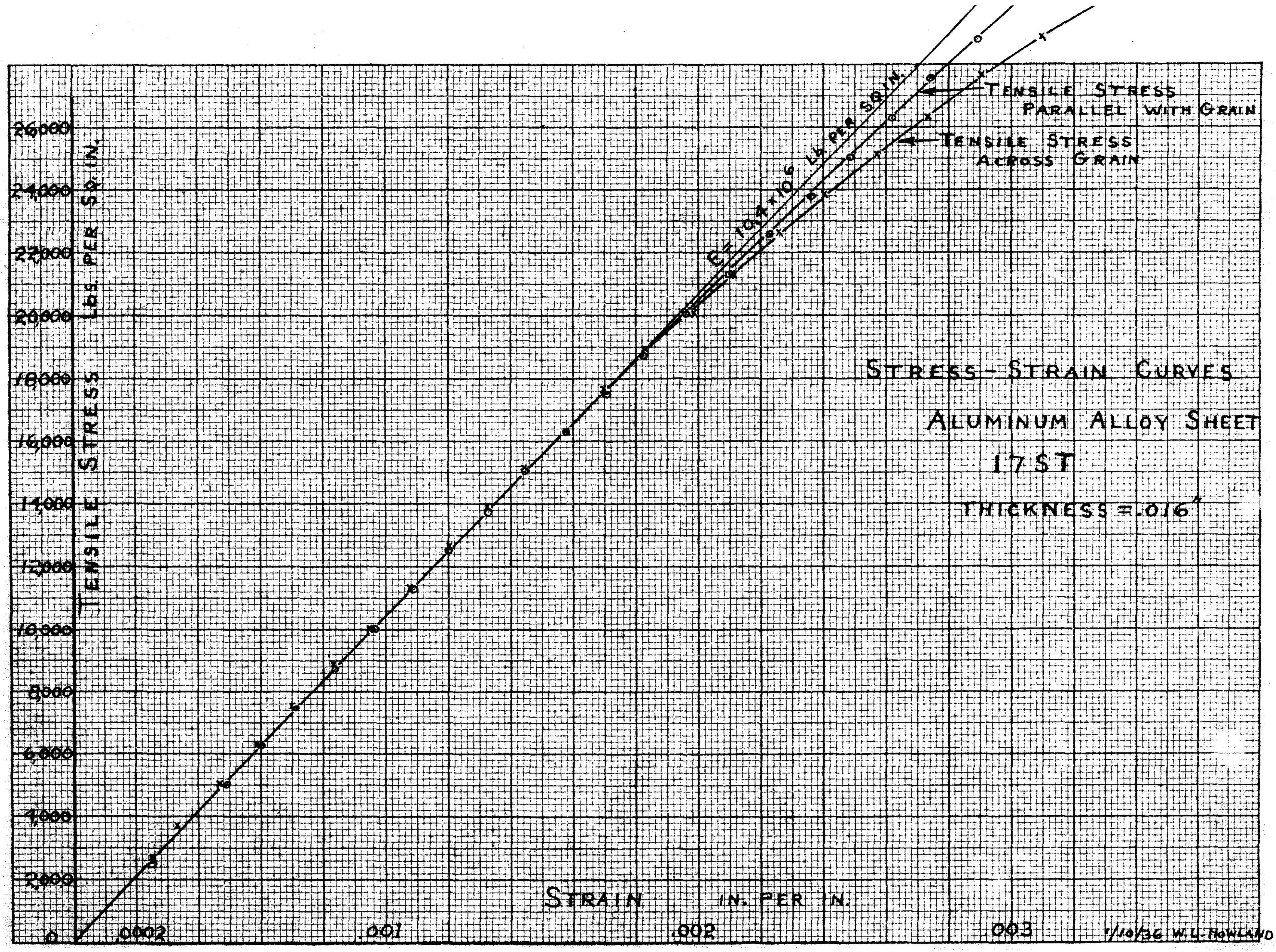

Tensile Stress Parallel with Grain
Tensile Stress Across Grain
E = 10.4 × 10⁶ Lb per sq in.
Stress-Strain Curves
Aluminum Alloy Sheet
17 ST
Thickness = .016"
Tensile Stress Lbs per sq in.
26,000
24,000
22,000
20,000
18,000
16,000
14,000
12,000
10,000
8,000
6,000
4,000
2,000
0
.0002
.001
.002
.003
Strain in. per in.
1/10/36 W.L. Howland

FIG. 30.

FIG. 31.

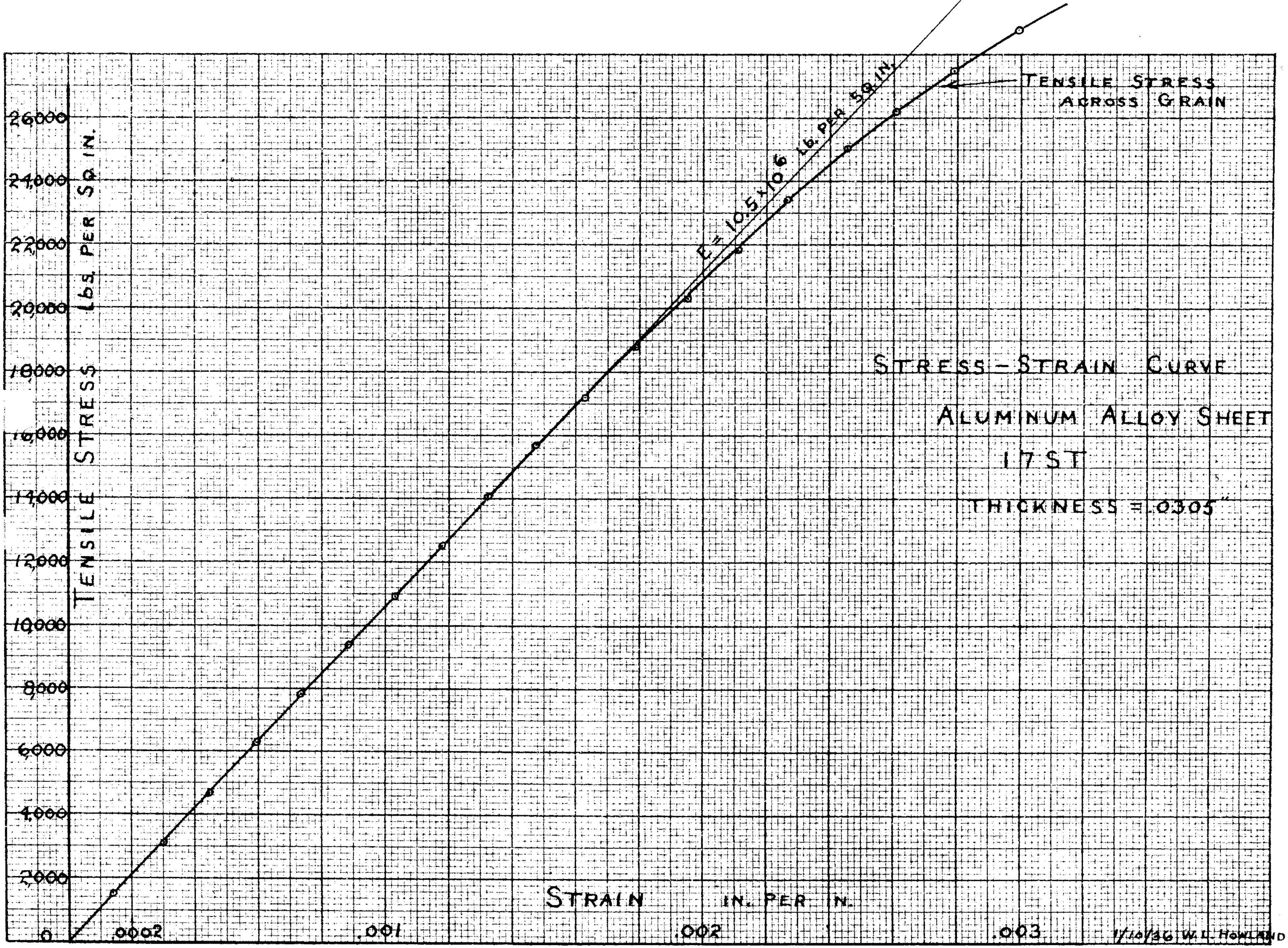

STRESS-STRAIN CURVE
ALUMINUM ALLOY SHEET
17ST
THICKNESS = .0305"
TENSILE STRESS ACROSS GRAIN
E = 10.5×10⁶ Lb. PER SQ. IN.
TENSILE STRESS Lbs. PER SQ. IN.
STRAIN IN. PER IN.
0
2000
4000
6000
8000
10,000
12,000
14,000
16,000
18000
20,000
22,000
24,000
26000
.0002
.001
.002
.003
1/10/36 W.L. HOWLAND

FIG. 32.

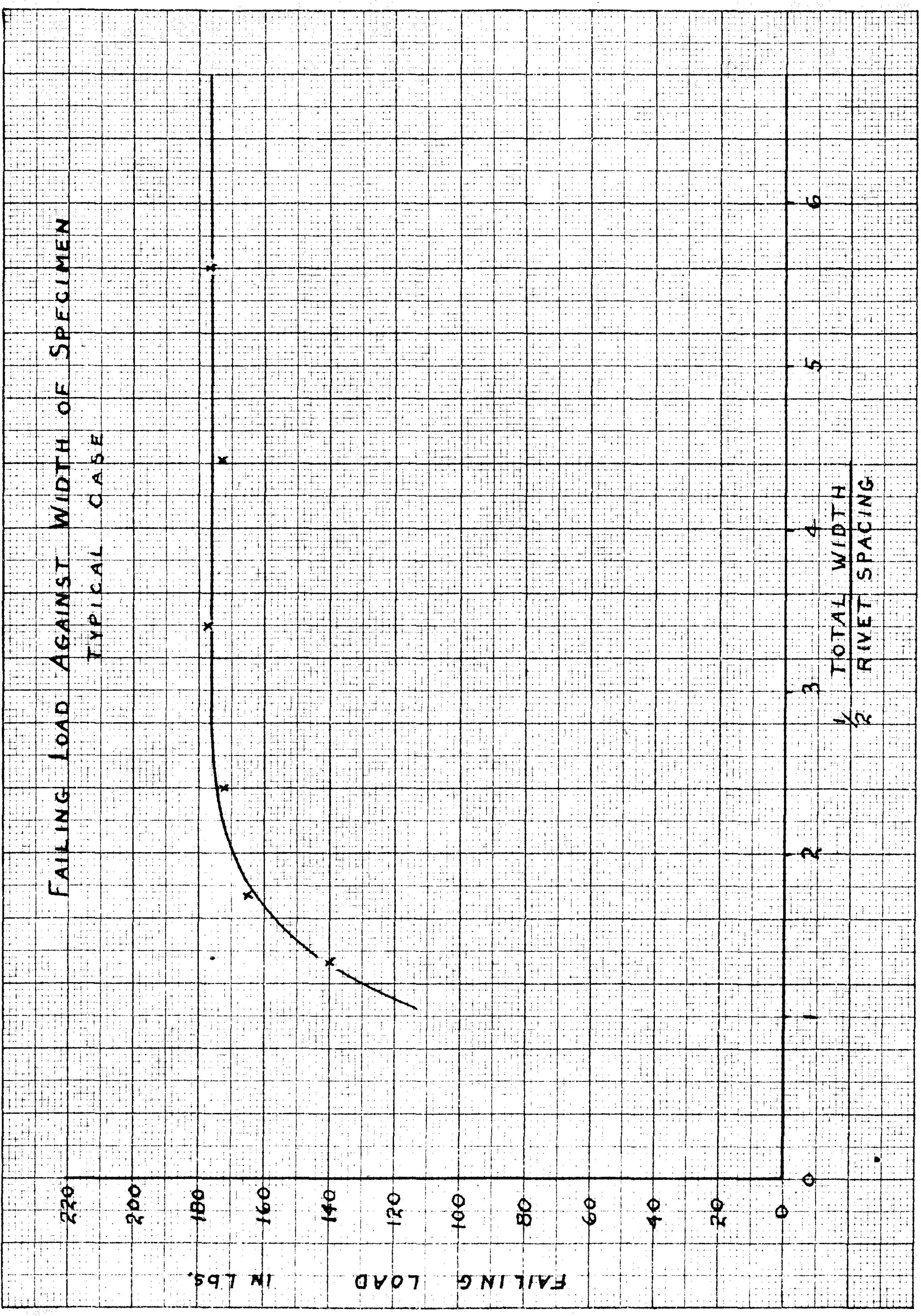
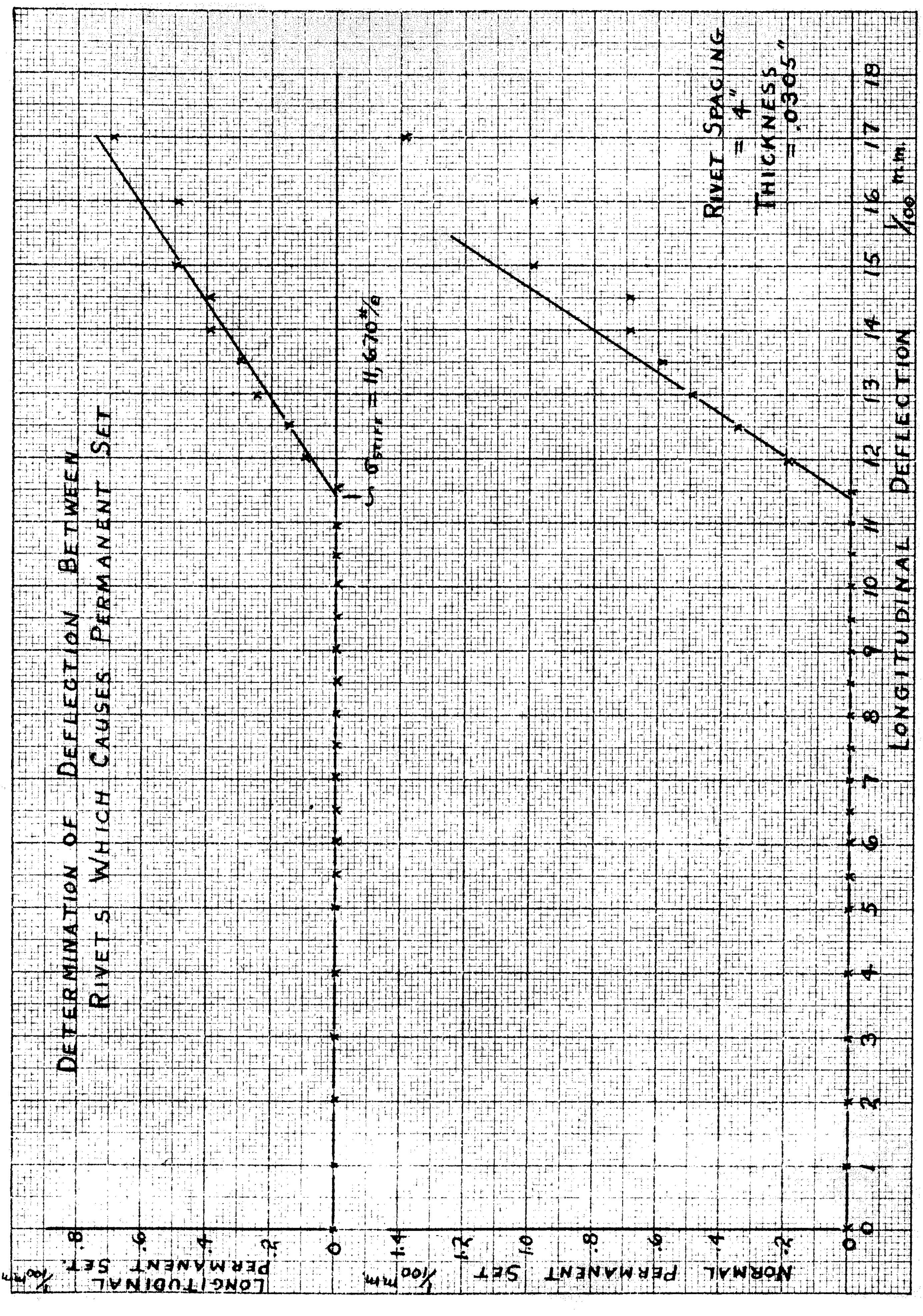
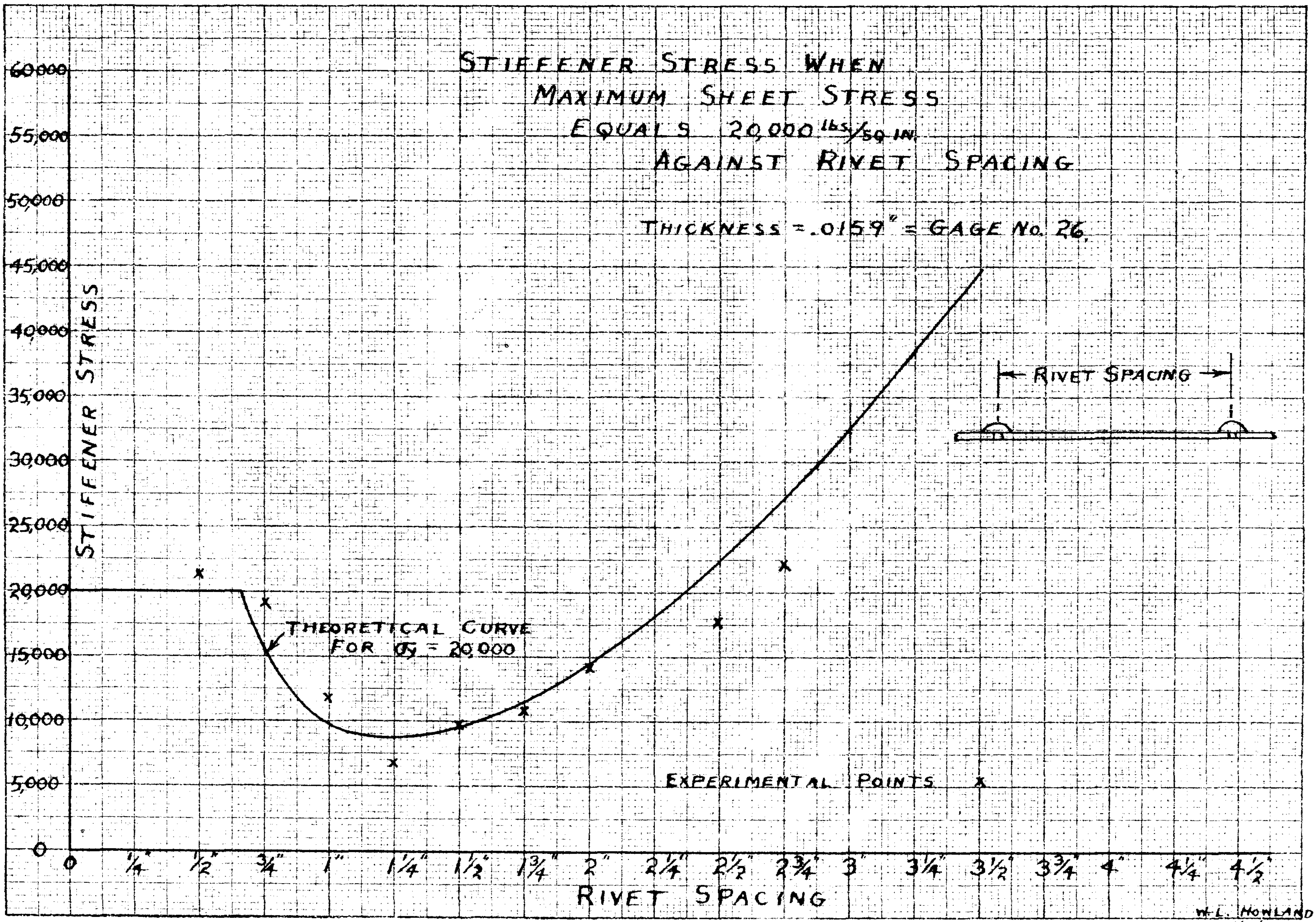

FAILING LOAD AGAINST WIDTH OF SPECIMEN
TYPICAL CASE
FAILING LOAD IN LBS.
0
20
40
60
80
100
120
140
160
180
200
220
0
1
2
3
4
5
6
$\frac{1}{2}$ $\frac{\text{TOTAL WIDTH}}{\text{RIVET SPACING}}$

FIG. 33.



DETERMINATION OF DEFLECTION BETWEEN RIVETS WHICH CAUSES PERMANENT SET



STIFFENER STRESS WHEN
MAXIMUM SHEET STRESS
EQUALS 20,000 lbs/sq in.
AGAINST RIVET SPACING

THICKNESS = .0159" = GAGE No. 26.

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1/10/36

FIG. 34.

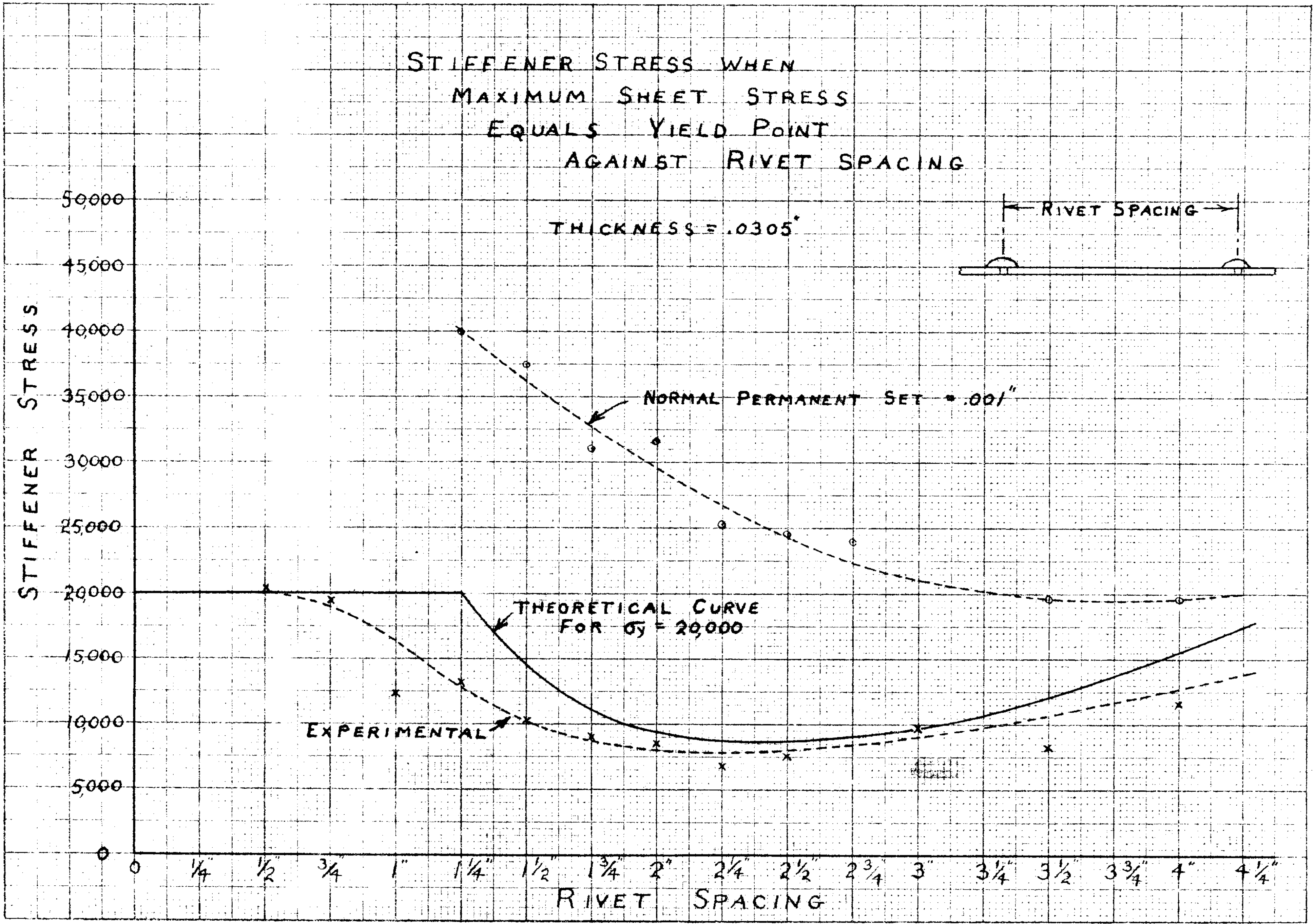

FIG. 35

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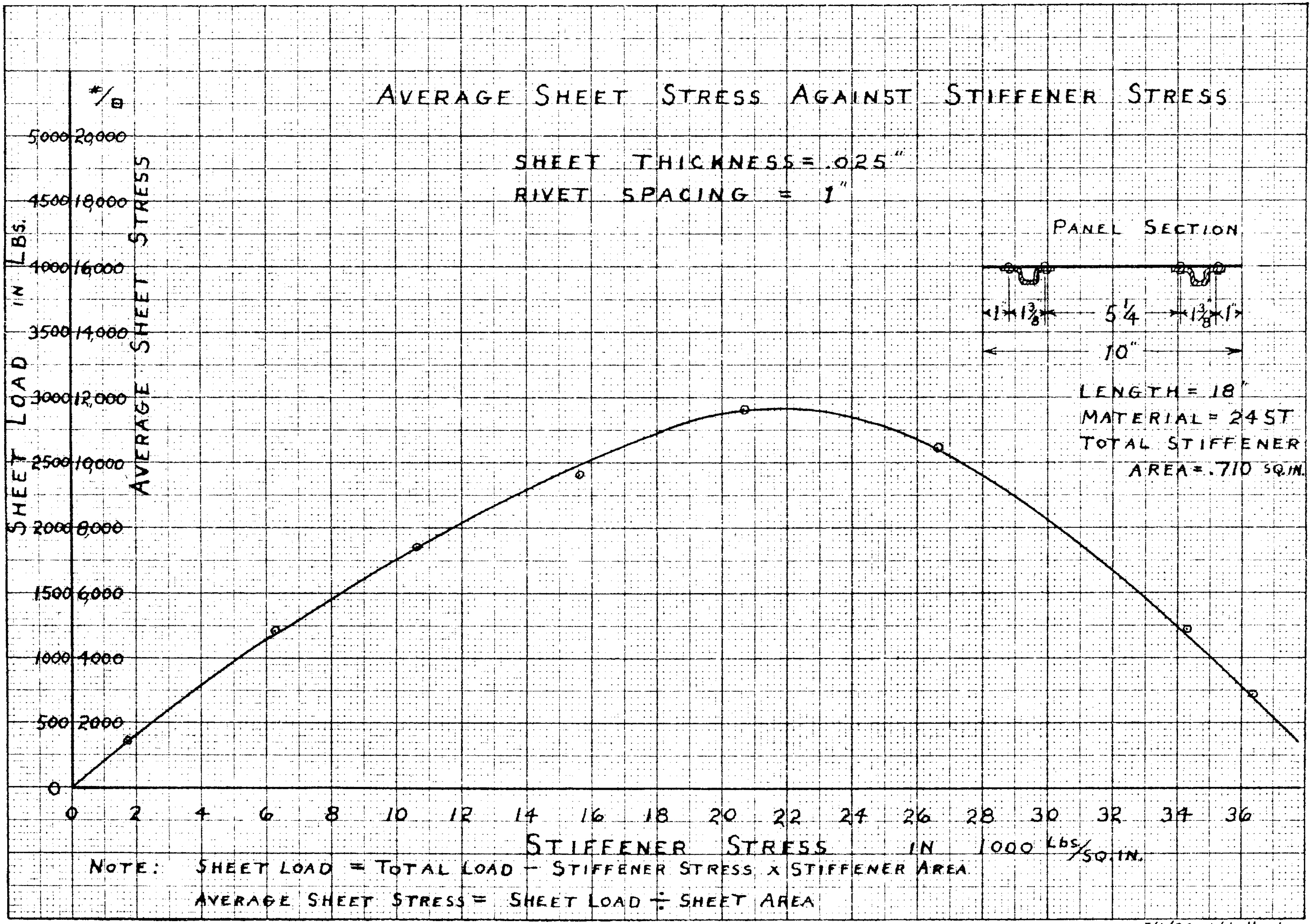
AVERAGE SHEET STRESS AGAINST STIFFENER STRESS

SHEET THICKNESS = .025"
RIVET SPACING = 1"

NOTE: SHEET LOAD = TOTAL LOAD − STIFFENER STRESS x STIFFENER AREA
AVERAGE SHEET STRESS = SHEET LOAD ÷ SHEET AREA

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FIG. 30

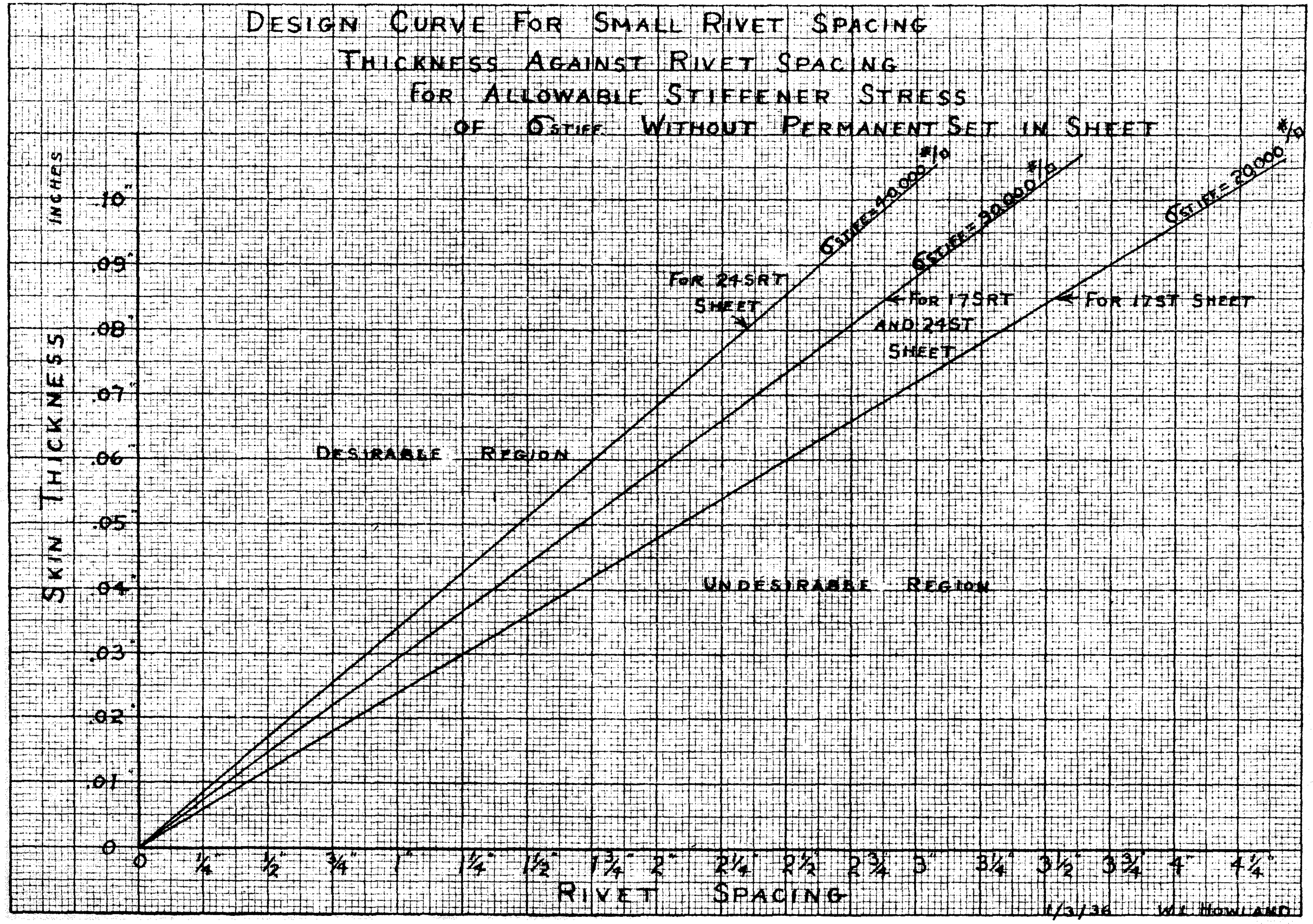
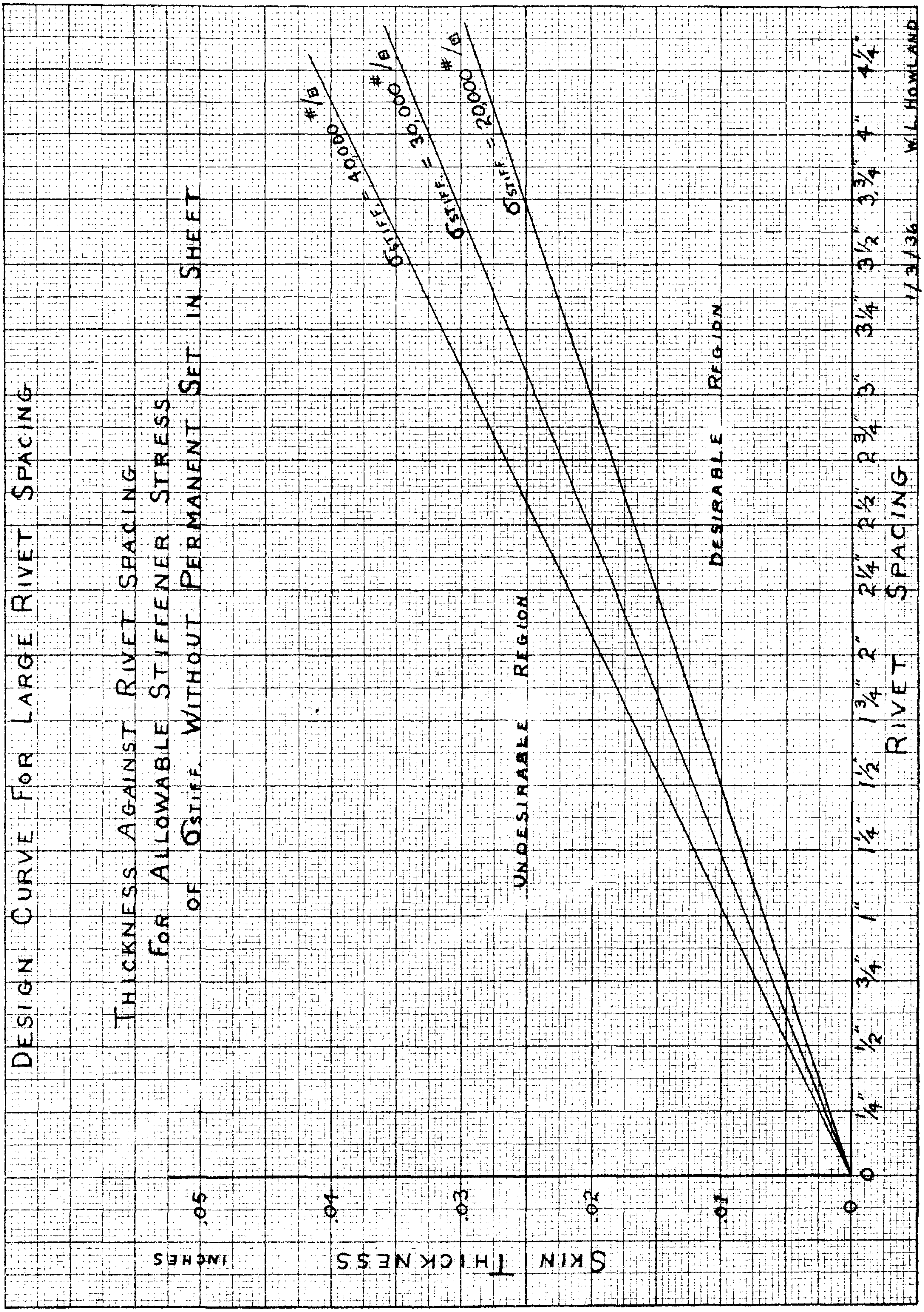


FIG. 37.

DESIGN CURVE FOR SMALL RIVET SPACING
THICKNESS AGAINST RIVET SPACING
FOR ALLOWABLE STIFFENER STRESS
OF σ_{STIFF} WITHOUT PERMANENT SET IN SHEET

Fig. 38.



DESIGN CURVE FOR LARGE RIVET SPACING

THICKNESS AGAINST RIVET SPACING
FOR ALLOWABLE STIFFENER STRESS
OF $\sigma_{STIFF.}$ WITHOUT PERMANENT SET IN SHEET

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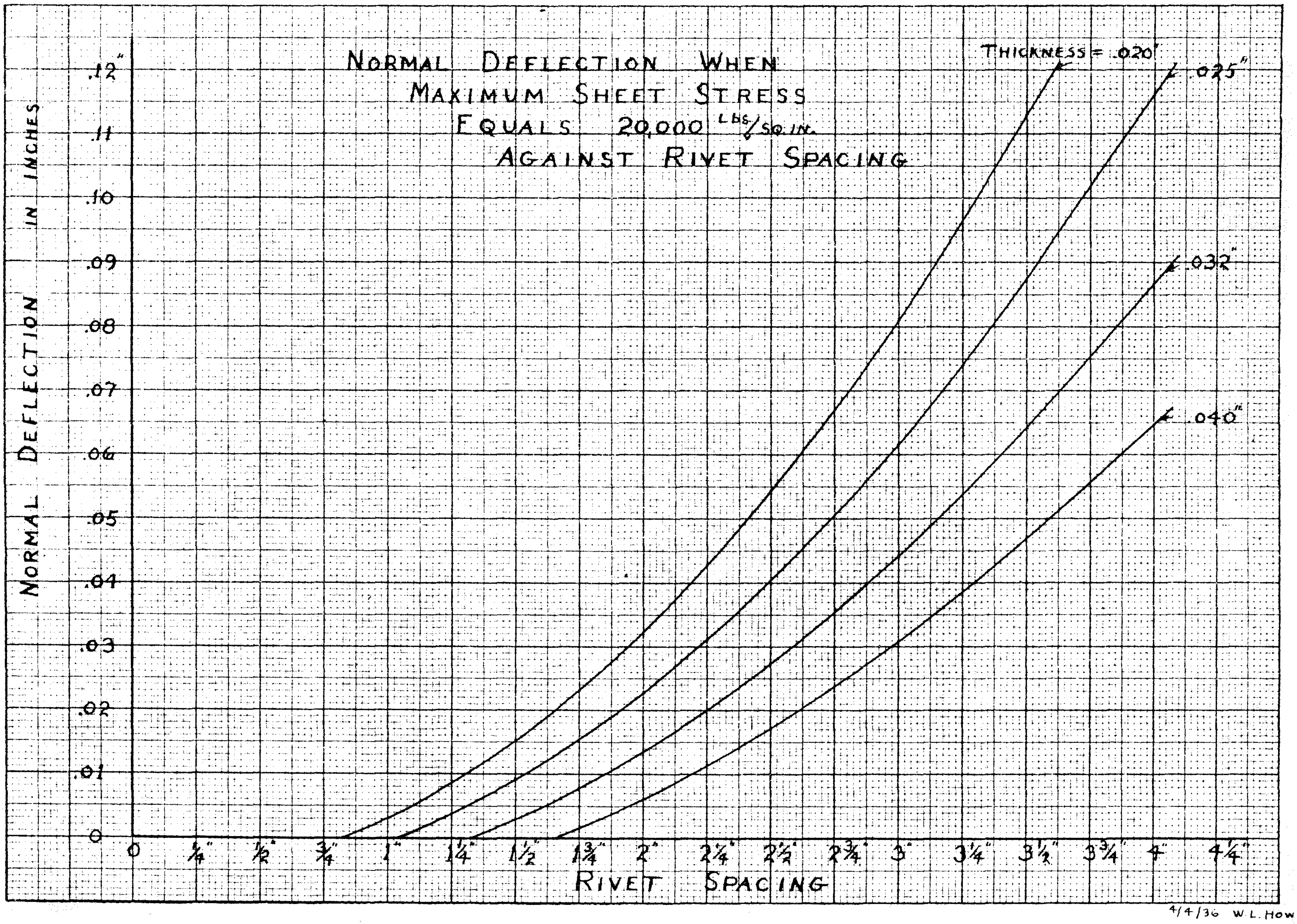

FIG. 39.

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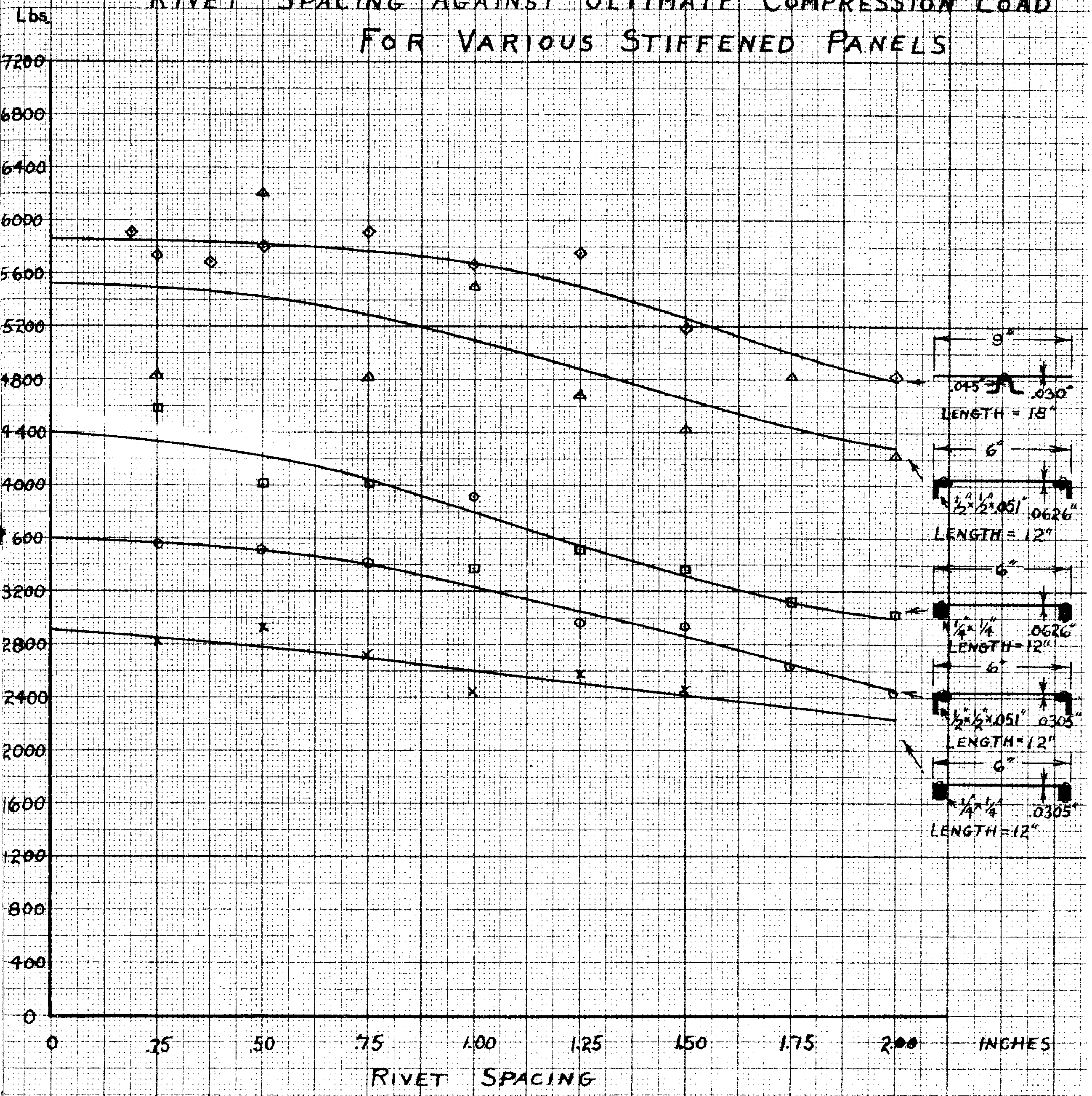
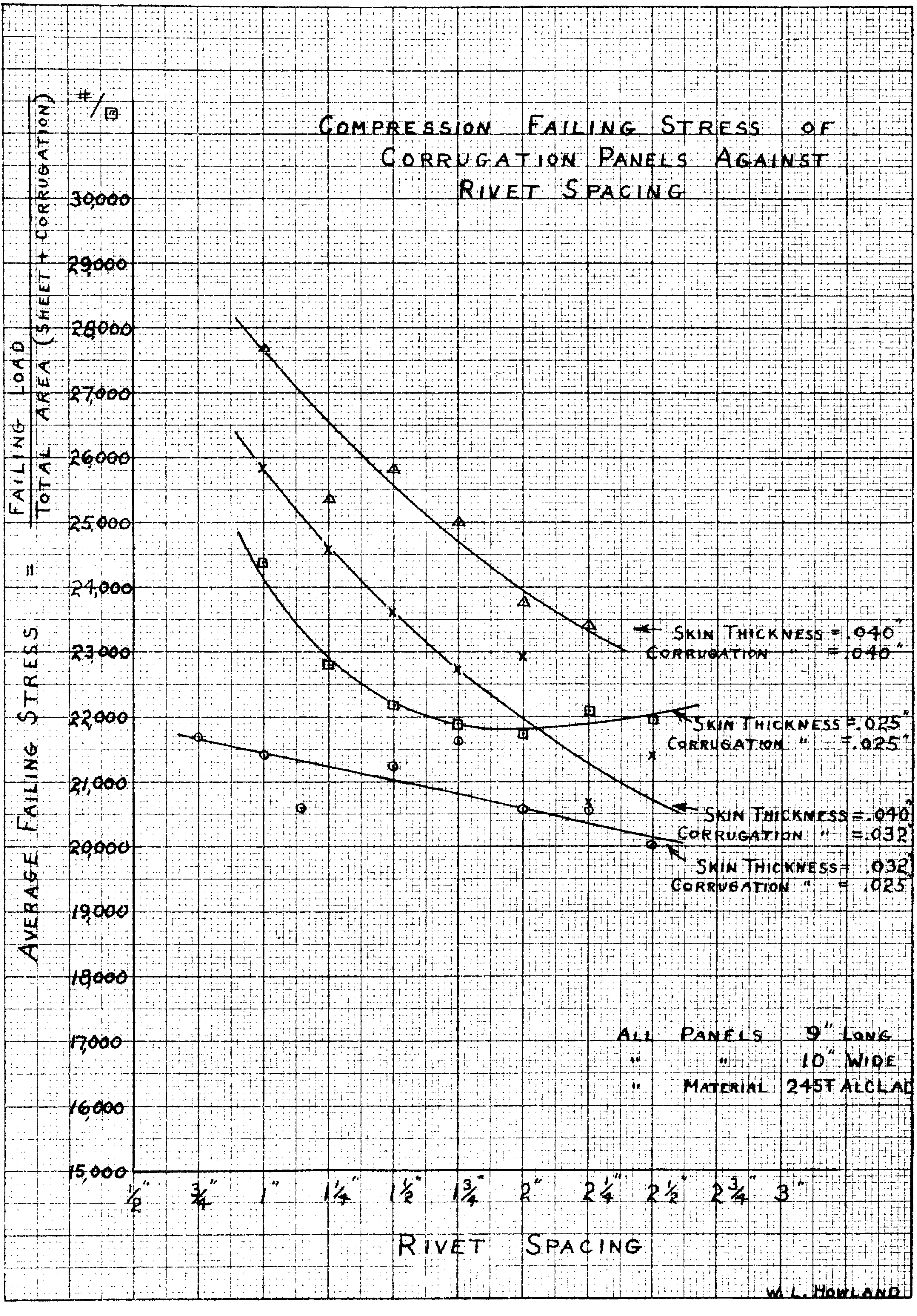
FIG. 40.

RIVET SPACING AGAINST ULTIMATE COMPRESSION LOAD FOR VARIOUS STIFFENED PANELS



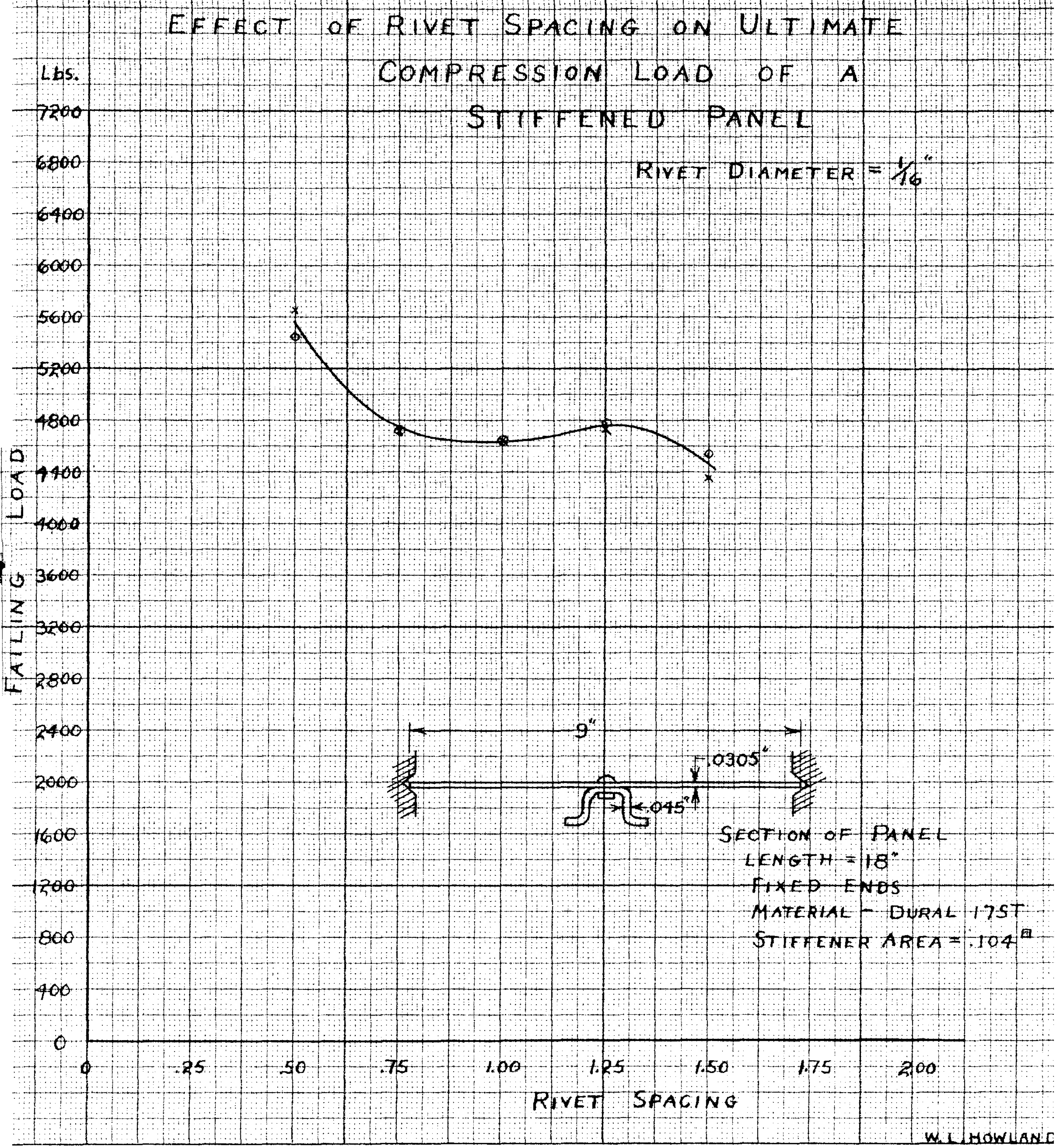
W. L. HOWLAND
October 1935
GALCIT

FIG. 41.



COMPRESSION FAILING STRESS OF CORRUGATION PANELS AGAINST RIVET SPACING

FIG. 42



EFFECT OF RIVET SPACING ON ULTIMATE COMPRESSION LOAD OF A STIFFENED PANEL

RIVET DIAMETER = $\frac{1}{16}$"

SECTION OF PANEL
LENGTH = 18"
FIXED ENDS
MATERIAL - DURAL 17ST
STIFFENER AREA = .104$^{□}$

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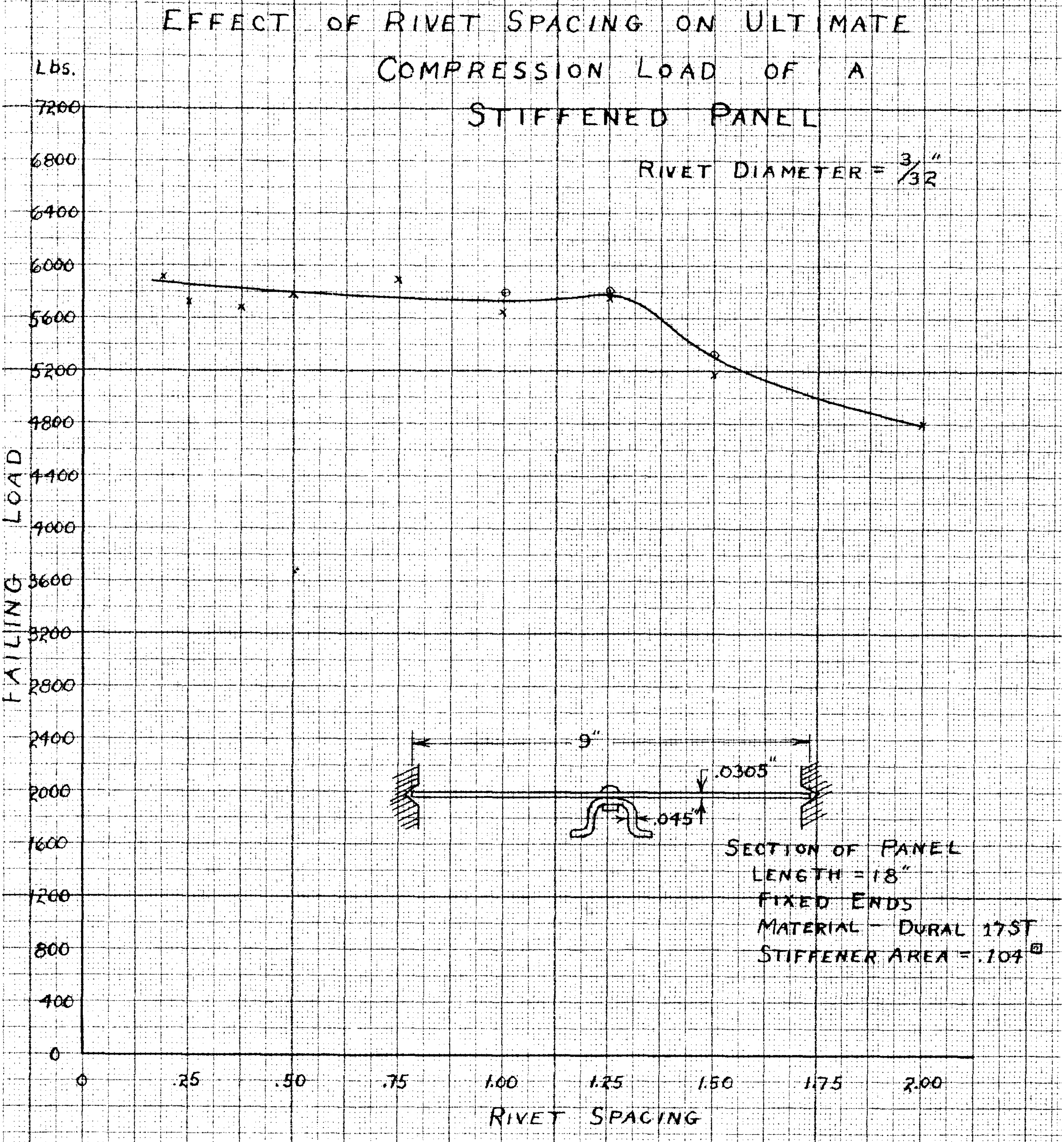
9/27/35

FIG. 43



EFFECT OF RIVET SPACING ON ULTIMATE COMPRESSION LOAD OF A STIFFENED PANEL

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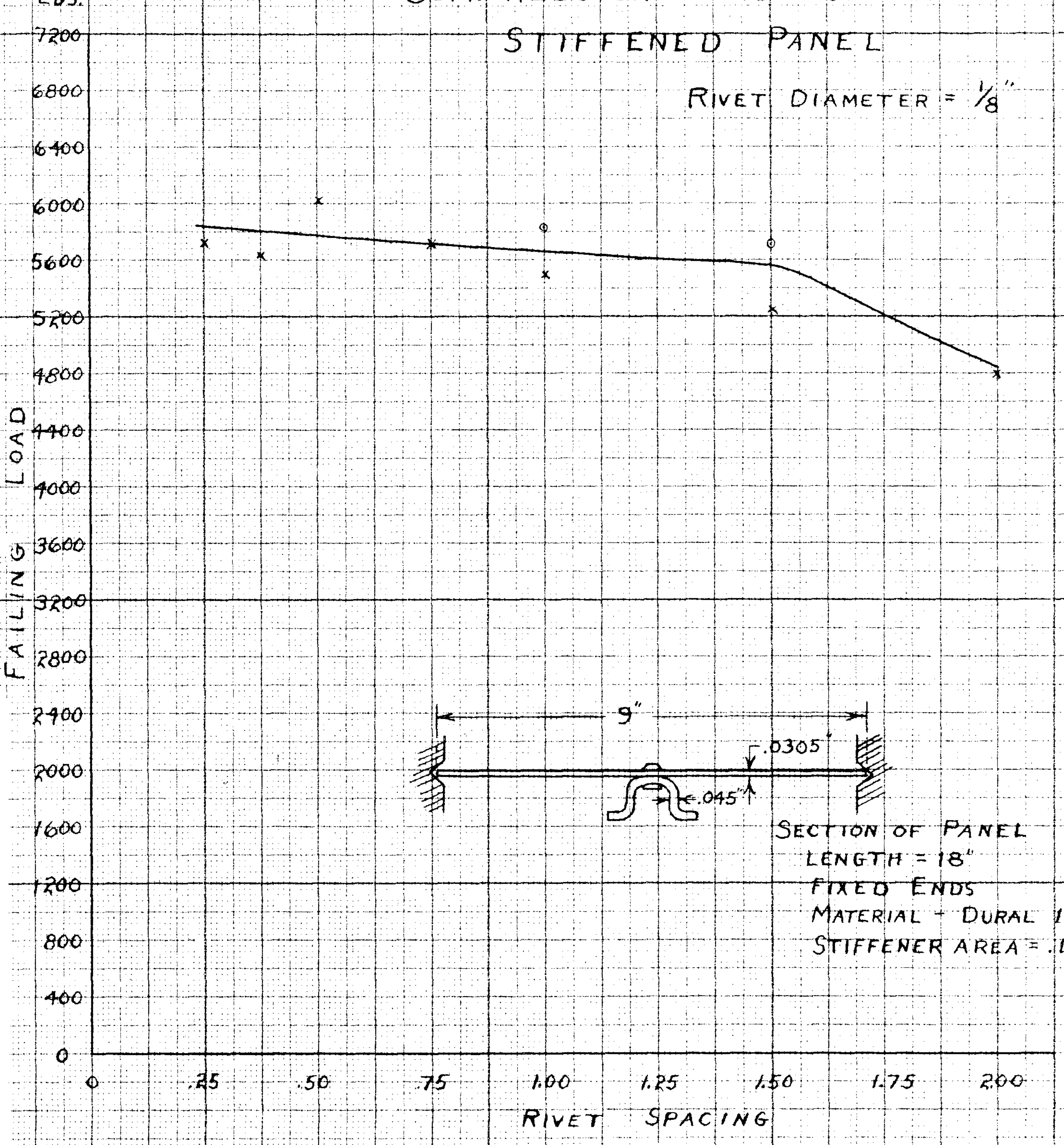
9/27/35

FIG. 44.

EFFECT OF RIVET SPACING ON ULTIMATE COMPRESSION LOAD OF A STIFFENED PANEL

RIVET DIAMETER = 1/8"



SECTION OF PANEL
LENGTH = 18"
FIXED ENDS
MATERIAL - DURAL 17ST
STIFFENER AREA = .104□"

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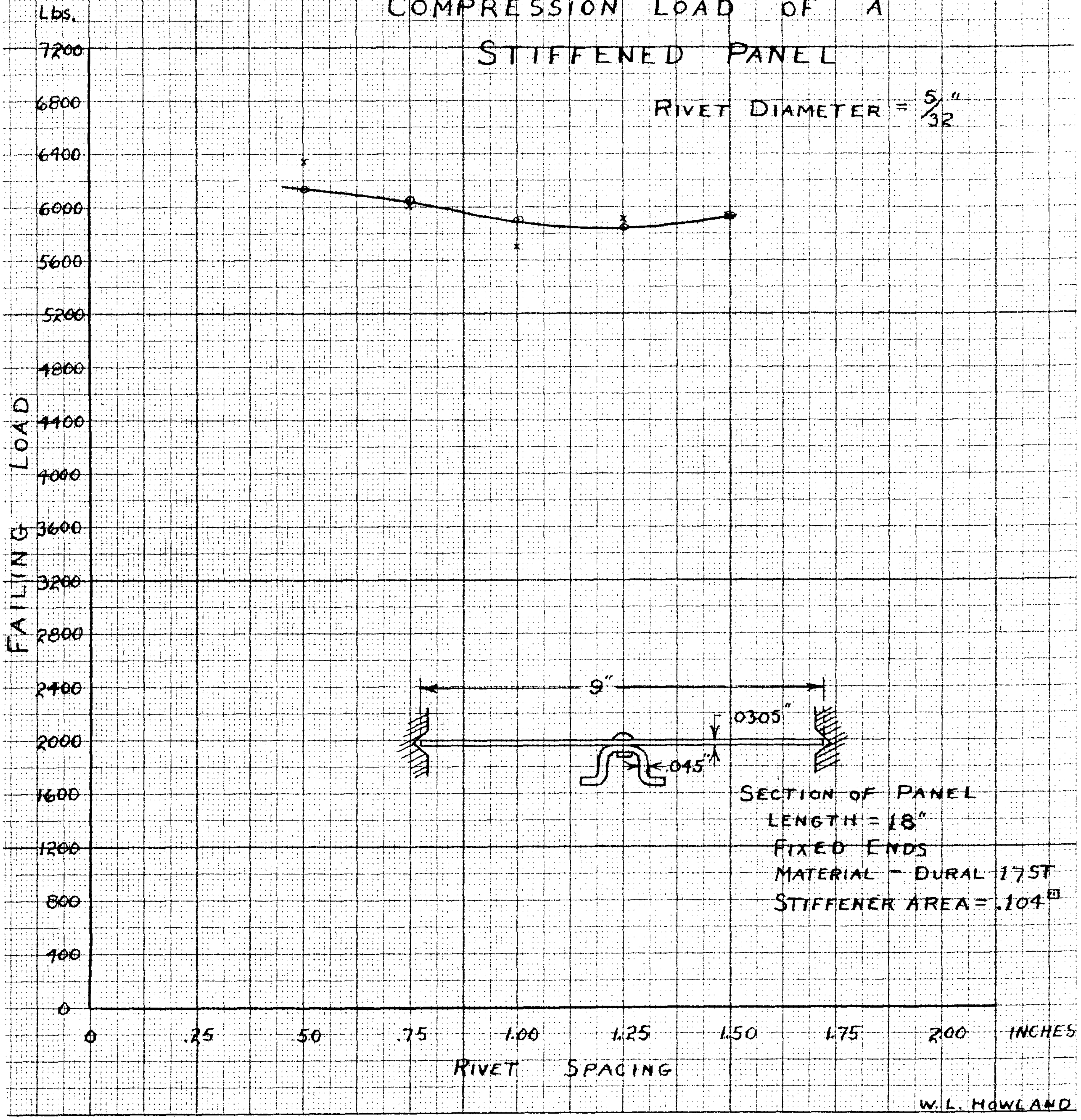
9/27/35

FIG. 45.

EFFECT OF RIVET SPACING ON ULTIMATE COMPRESSION LOAD OF A STIFFENED PANEL

RIVET DIAMETER = $\frac{5}{32}''$



SECTION OF PANEL
LENGTH = 18"
FIXED ENDS
MATERIAL - DURAL 17ST
STIFFENER AREA = .104□"

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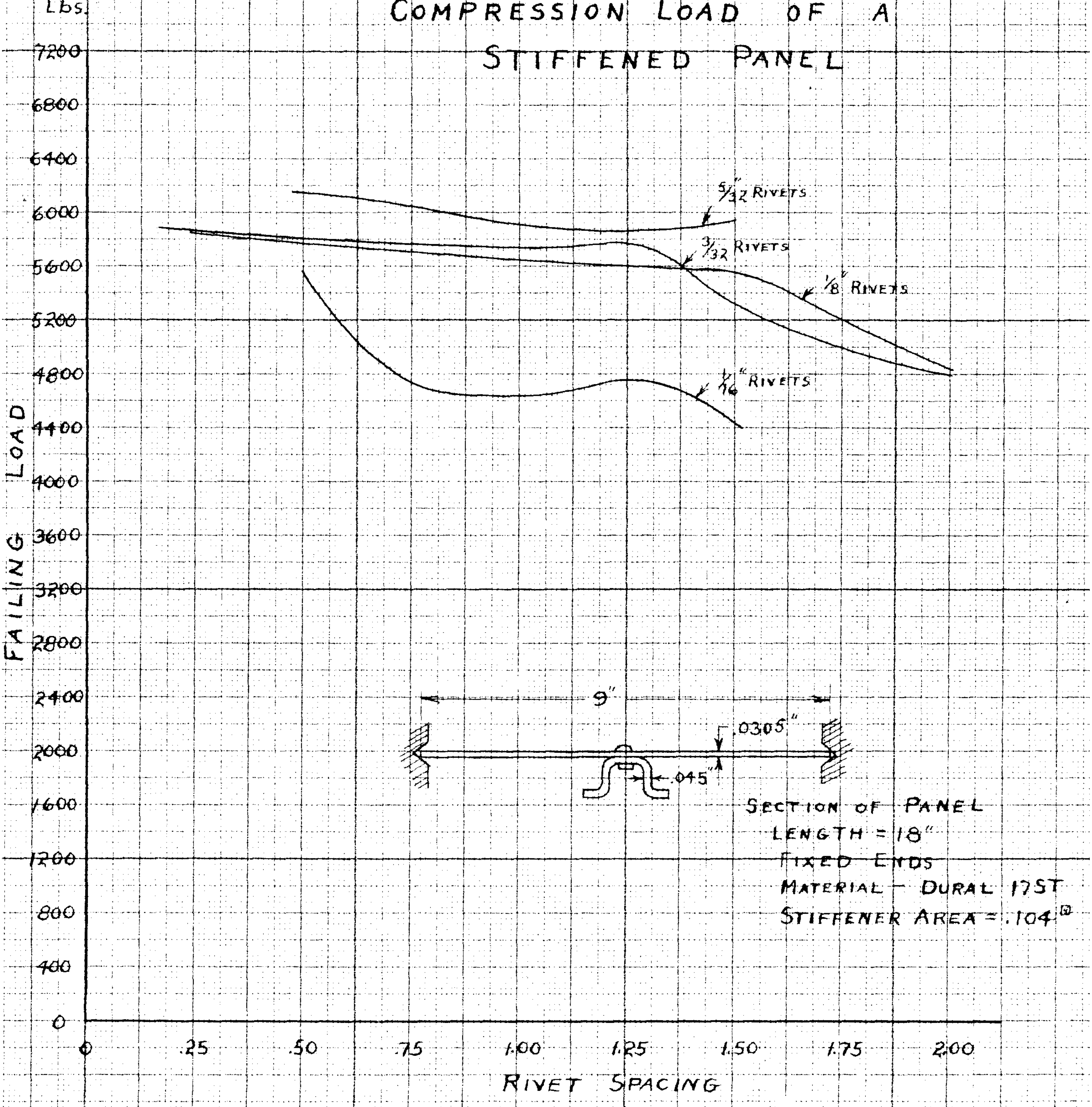
9/27/35

FIG. 46.

EFFECT OF RIVET DIAMETER ON ULTIMATE COMPRESSION LOAD OF A STIFFENED PANEL



SECTION OF PANEL
LENGTH = 18"
FIXED ENDS
MATERIAL - DURAL 17ST
STIFFENER AREA = .104 □"

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9/27/35

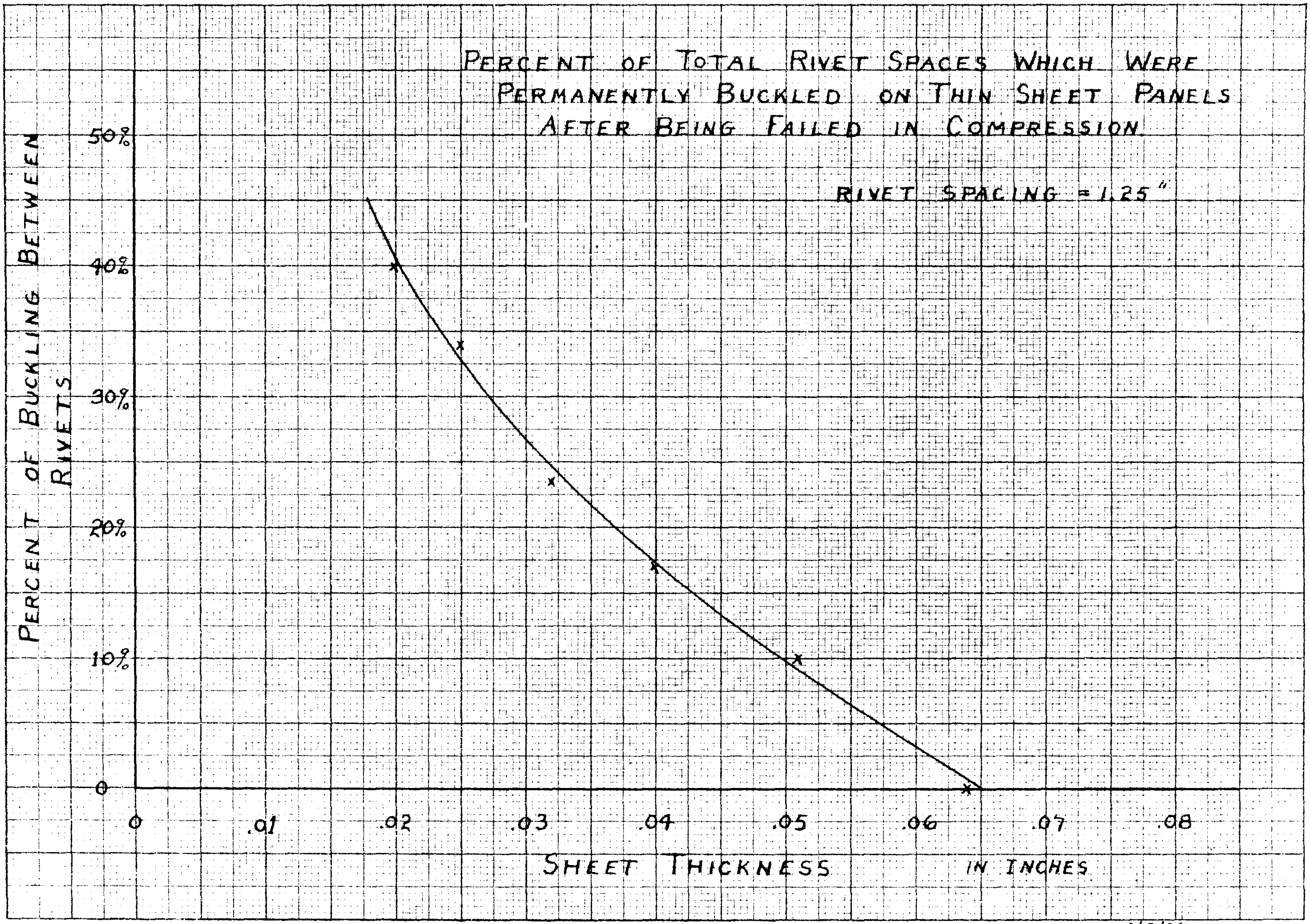

FIG. 47.

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