AN EXPERIMENTAL INVESTIGATION OF FLAME STABILIZATION IN A HEATED TURBULENT BOUNDARY LAYER

Thesis by

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ABSTRACT

For a number of years the significant parameters governing flame stabilization in moving streams have been known. In high speed applications the chemical time delay plays a fundamental role. For the low speed problems the normal flame speed and quenching distance govern stabilization. Unfortunately the transition region between the two groups of problems has not been investigated. Also the actual relation between these parameters and the properties of the combustible mixture has not been established.

To investigate these fundamental questions an experiment was set up to study flame stabilization in heated turbulent boundary layers. For wall temperatures above about 1700°F. the chemical time delay, represented by the length of the heated flame holder wall required for stabilization, was found to be a systematic and reproducible variable. A rational explanation was made for the transition from the low speed stabilization mechanism known to be applicable in unheated turbulent boundary layers and heated laminar boundary layers to the ignition mechanism applicable in heated turbulent boundary layers.

An attempt was made to relate the observed stabilization measurements to a theoretical solution based on ignition in a laminar sub-layer. The present methods of solution for such problems were found to be inadequate. A similarity solution yielded an interesting result which agreed fairly well with experiments.

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SYMBOLS

A	symbol for reactant in simple decomposition reaction
B	symbol for product in simple decomposition reaction
c_p	specific heat at constant pressure
c_w	heat capacity of flame holder wall per unit mass
D	diameter of flame holder
D_{ij}	binary diffusion coefficient
D_{TW}	diameter of trip wire
$\mathfrak{D}_I$	first Damkohler number $\left(\frac{X}{U\tau}\right)$
d	length of recirculation zone
E	activation energy
i	direct current through graphite glow bar
K	thermal conductivity
N_x	Nusselt number (equation 22)
P_o	free stream static pressure
Q	heat transfer into the boundary layer from the flame holder wall up to the point of flame attachment, per unit width and time
q_F	heat transfer from hot combustion gases to the flame holder wall per unit area and time
q_G	heat transfer by radiation from glow bar to flame holder per unit area of flame holder wall and time
q_R	heat transfer from flame holder wall by radiation per unit area and time
q_w	heat transfer out of the flame holder wall due to its drop in temperature per unit area and time
q_x	heat transfer into the boundary layer from the flame holder wall per unit area and time

R	gas constant
$\mathcal{R}$	Reynolds number $\left(\frac{Ux}{\nu}\right)$
$\mathcal{R}_\theta$	Reynolds number based on momentum thickness
r_G	resistance of the graphite glow bar per unit length
S_c	Schmidt number $\left(\frac{\nu}{D_{ij}}\right)$
s_F	normal flame speed
T	temperature
T_F	adiabatic flame temperature
T_0	free stream temperature
T_w	flame holder wall temperature
T_{ws}	minimum wall temperature required for stabilization
t	time
t_1	chemical time delay associated with bluff body flame stabilization
t_2	chemical time delay associated with boundary layer stabilization
t_3	pre-exponential factor
U	velocity parallel to the flat plate
U_0	free stream velocity
U_τ	friction velocity $\left(\sqrt{\frac{\tau_w}{\rho}}\right)$
V	velocity normal to the flat plate
w	thickness of the flame holder wall
x	coordinate along the flat plate
x_F	position of flame attachment
y	coordinate normal to the flat plate
z	dummy variable of integration

α	emissivity of the flame holder wall
δ_c	convective boundary layer thickness
η	dimensionless coordinate (equation 9)
θ	momentum thickness
κ	species concentration
μ	absolute viscosity
ν	kinematic viscosity $\left(\frac{\mu}{\rho}\right)$
ρ	density
ρ_o	free stream density
ρ_w	density of flame holder wall
σ	Prandtl number $\left(\frac{\mu}{C_p K}\right)$
τ_w	wall shearing stress
ϕ	fuel-air ratio, fraction of stoichiometric
Ω	dimensionless temperature (equation 9)

I. INTRODUCTION

In terms of practical applications flame stabilization is one of the most important branches of the combustion field. Because of the complex nature of the stabilization problems very little knowledge of the basic mechanisms has been available. As a result the designer of a combustion apparatus must rely upon his experience or at best on semi-theoretical, empirical relations. Recently some progress has been made in obtaining a rational basis for the understanding of some of the simpler stabilization problems.

For many years the study of flame stabilization in moving streams was restricted to low speed flows. The basic research tool was the simple bunsen burner with the stability limits, flashback and blowoff, the subject of most interest. A good summary of the experimental results, and the attempts to correlate them, has been given by Lewis and von Elbe (1). In particular a mechanism explaining flashback was postulated in which the normal burning velocity was equated to the flow velocity at a distance from the burner wall called the quenching distance. It should be noted that there are other methods of defining the quenching distance that give somewhat different values, e.g., the minimum separation which will allow a flame to propagate between parallel plates. Good correlation of the above mechanism with experiment was found. Attempts to develop a theory from the basic equations for conservation of momentum, energy, and chemical species were less successful. A thermal theory was developed by Lewis and von Elbe (2) and some agreement with experiment was noted. The discrepancies were attributed by the authors to differential diffusion.

With the advent of the continuous flow, air-breathing aircraft engine, it became necessary to stabilize flames in high speed flows. For this purpose can burners and bluff body flame holders were developed. It soon became apparent that the stability limits of the bunsen burner type problem were not applicable to the high speed applications. In particular the basic mechanism seemed to be different.

Since the bluff body flame stabilizer was relatively simple it became an important research tool. A significant parameter for correlating the data on blowoff limits was the chemical time delay given by,

$$t_1 = \frac{d}{U_0} \qquad (1)$$

where d is the length of the recirculation zone behind the bluff body and U_0 is the free stream velocity. This discovery led Marble and Zukoski (3) to hypothesize that the basic stabilization mechanism was one of continuous ignition. The hot gases in the recirculation zone continuously ignite the unburned gases in the mixing region. Since the fluid mechanics of flow behind a bluff body are not well understood even in an isothermal problem without combustion, detailed studies of the ignition mechanism were out of the question.

In order to obtain a better insight into the stabilization mechanisms simpler flow systems have been studied. Combustion in the turbulent mixing region between a cold combustible mixture and a hot inert gas has been investigated experimentally by Wright and Becker (4). Unfortunately stability problems made quantitative measurements difficult. Also the presence of both an initial and a propagating flame complicated the understanding of the mechanism. The problem of stabilization in a laminar boundary layer heated by a constant temperature flat plate has been investigated experimentally by Zeimer and Cambel (5). These authors obtained a reasonably good correlation between their experiments and the bunsen burner type mechanism previously discussed for wall temperatures between $1500^{o}F$. and $2000^{o}F$.

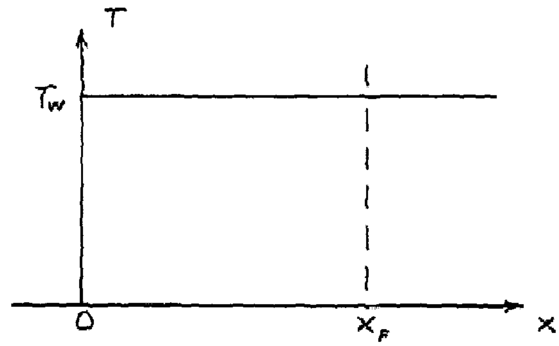
Several theoretical solutions to flame stabilization problems in high speed streams have been obtained. These assume a continuous ignition mechanism and solve boundary layer type conservation equations including chemical reaction. A solution for stabilization in a laminar mixing region has been given by Marble and Adamson (6). The conservation equations for momentum, energy, and chemical species including chemical reaction were solved by a perturbation procedure. The corresponding problem in the laminar boundary layer has been solved by Dooley (7) using an iteration technique. Unfortunately no true comparison between the theoretical work and appropriate experiments has been possible. In the mixing region problem the experiments were carried out in the turbulent regime while the theory is applicable in the laminar regime. Some qualitative agreement between the laminar theory and the turbulent experiments was noted but quantitative comparisons were out of the question. In the case of the heated laminar boundary layer the basic mechanism observed experimentally seemed to be different from that used in the theoretical solution.

In order to obtain a little more insight into the problems of stabilization in high speed streams an experiment was set up to investigate flame stabilization in heated turbulent boundary layers. Since a turbulent boundary layer is formed by transition, either natural or induced, of a laminar boundary layer, heating from the origin of the velocity boundary layer was not practical. Instead the wall temperature was maintained in the form of a step function. The idealized experiment is illustrated in figure 1. Two possible mechanisms governing the stabilization are the bunsen burner type, illustrated in figure 2a, and the continuous ignition type, illustrated in figure 2b.

It should also be noted that flame stabilization in heated boundary layers has been proposed by Tsien (8) for use in ramjet combustion chambers and in turbojet afterburners. The turbulent boundary layer over a step function in temperature studied here closely simulates such a practical application. The principal advantage of this type of flame holder would be the reduction in pressure drop through the combustor. Use of such a flame holder has been retarded by the lack of a suitable method of heating a wall of the combustion chamber or a section of an airfoil flame holder within the chamber. However, with the mixture temperatures in high Mach number ramjets in the range of ignition the heat addition problem should be much simpler if present at all.

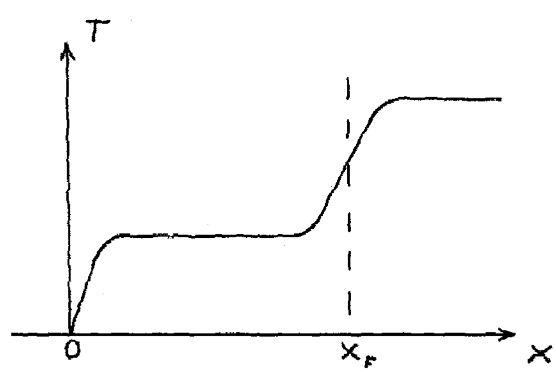
II. THE EXPERIMENT

The desired experiment would have a combustible mixture flowing over a flat plate with a wall temperature distribution in the form of a step function, A flame would be stabilized in the boundary layer some distance downstream from the step. This is illustrated in figure 1 and also in sketch 1. Ideally the flame position and wall temperature distribution would not be dependent on time. Unfortunately the strong temperature gradients in the region of flame attachment preclude any such steady state experiment. No practical experimental apparatus could be expected to maintain a constant wall temperature through this region. For a steady state experiment the type of temperature distribution that could be expected is illustrated in sketch 2.



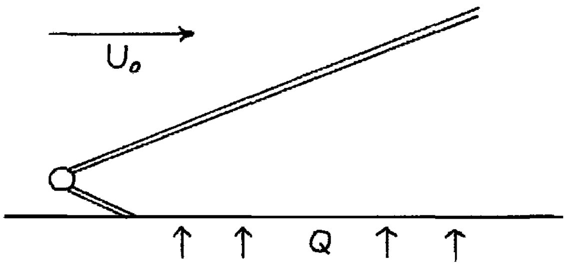
Sketch 1.

Ahead of the flame attachment point the heat transfer is from the wall to the flow, behind this point the hot combustion gases transfer heat to the flame holder wall.



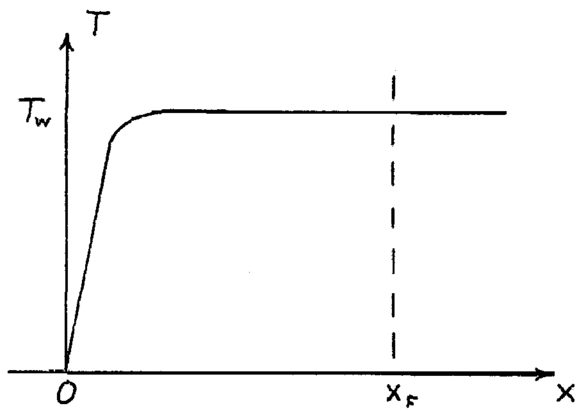
Sketch 2.

A transient experiment seems to be answer to this problem. With the desired temperature distribution established, the flame is allowed to stabilize in the boundary layer. The necessary measurements must be made before an appreciable change in the wall temperature takes place. Whether such an experiment is possible depends upon the heat capacity of the flame holder wall. If the characteristic time associated with a change in the wall temperature is large compared with the characteristic time in the stabilization problem significant measurements are possible.



Sketch 3.

Two methods of heating the flame holder wall to the desired temperature distribution seem applicable. In the first the wall is heated to the desired temperature with cold flow passing over it. After ignition the flame is stabilized somewhere along the heated wall. Behind the point of stabilization the wall temperature increases. Alternatively the propagating flame may be stabilized on an auxiliary flame holder ahead of the test section during the heating process. The hot combustion gases aid in heating the flame holder wall. After the wall temperature reaches the desired distribution the auxiliary flame holder is removed. The



Sketch 4.

propagating flame again stabilizes somewhere along the heated wall. Upstream of the point of stabilization the wall is cooled by the cold combustible mixture passing over it.

The latter method was selected since it required the least power input and seemed to provide the best temperature distribution. Also the problems of ignition were not present. This heating process is illustrated in sketch 3. The expected temperature distribution after the removal of the auxiliary flame holder is shown in sketch 4.

A. Experimental Apparatus

The experiments were carried out in a standard low speed combustion tunnel. The test section had a length of 12 inches and an area of 28.3 square inches in either a square or a circular geometry. The air was preheated so that the liquid fuel could be completely vaporized. A plenum chamber, screens, and a smoothly convergent section assured sufficient mixing and a good velocity distribution at the entrance to the test section. A schematic diagram of the flow system is given in figure 3.

A cylindrical flame holder parallel to the flow was used to eliminate edge effects and simplify heating. The flame holder was a length of stainless steel pipe with an outer diameter of 1.825 inches and was cantilevered from the downstream end. With an ogive nose a flat plate boundary layer was formed on the flame holder wall through the test section. A diagram of the flame holder is shown in figure 4a.

To study flame stabilization in a heated boundary layer it was necessary to heat a section of the flame holder wall to temperatures of from 1500°F to 2000°F. During the heating an auxiliary flame holder was required. This was a section of water cooled, 3/8 inch diameter tubing which encircled the flame holder and could be moved along it to any position desired. The auxiliary flame holder acted as a bluff body stabilizer. The hot combustion gases heated the flame holder to about 1500°F. Radiative heat losses kept this temperature from being higher. To increase the wall temperature to the desired level a graphite glow bar coaxial with the flame holder was used. The radiation from the glow bar heated the flame holder wall. Details of the heater are shown in figure 4b.

Photographs of the apparatus are shown in figures 5, 6, 7, and 8. Figure 5 shows a view of the plenum chamber, convergent nozzle, and test section. Figure 6 shows the flame holder ready to be moved into position for a study of a turbulent boundary layer. Figure 7 is a closeup of the test section. The round collar encircling the flame holder is the water cooled auxiliary flame holder. Parts of the radiation heater are shown in figure 8.

A detailed description of the particular parts of the experimental apparatus will now be given.

1. Air Supply and Heat Exchanger

Air was supplied at a nominal pressure of 65 psia. by two reciprocating compressors. The maximum capacity of the flow system was 0.23 lb./sec. The flow rate was regulated by a sonic throat valve located upstream of the fuel injector and heat exchanger. This valve allowed the mass flow of air to be held constant during changes in the fuel injection rate, mixture temperature, and combustion chamber static pressure.

The air was heated in a shell and tube type heat exchanger, the hot air supply was furnished by a turbojet can burner. Two butterfly valves fixed the combustion chamber mixture temperature by shunting a fraction of the air around the heat exchanger. The vapor bulb sensing element for the control unit was located in the air supply line downstream from the fuel injection point. A minimum temperature of 610°R. was required to vaporize the fuel. A maximum temperature of 860°R. was obtainable for the range of mass flows considered.

2. Fuel Supply

The fuel used was commercial paint thinner (Standard of California Thinner No. 200). This fuel was composed of 94.5 per cent saturated hydrocarbons, 5.0 per cent aromatics, and 0.5 per cent olefins. The average molecular weight was 93. The stoichiometric ratio, fuel to air by weight, was 0.0674. A complete summary of the properties of this fuel type has been given by Haddock (9). The liquid fuel was pressurized with nitrogen and injected at 100 psia.

3. Plenum Chamber and Test Section

The plenum chamber was a pipe section 16 inches in diameter and 5 feet long. An 8 inch diameter perforated baffle was used at the upstream entrance of the chamber to break up the high speed air stream. Six 200-mesh calming screens were used to reduce the turbulence level.

A smoothly convergent nozzle section 18 inches in length reduced the flow from an area of 250 square inches to an area of 28.3 square inches in either a round or a square geometry. In both geometries total pressure traverses indicated flat velocity profiles at the entrance to the test section.

The round test section was a 12 inch length of pyrex glass pipe with a diameter of 6 inches and a thickness of 1/2 inch. Due to the curvature of the glass, it was impossible to take schlieren photographs with this configuration. Also breakage was high. For these reasons a water cooled, square duct was constructed with the same cross sectional area. Two sides were fitted with vycore glass windows 12 inches in length. Experiments showed that the change in duct geometry did not noticeably affect the flame stabilization measurements. All results reported here were obtained while using the square duct. The duct used in these experiments extended 6 inches beyond the end of the test section. Other duct lengths were tried and did not affect the experimental results provided they did not terminate over the test section.

4. Flame Holder

The flame holder was a 5 foot length of 310 stainless steel pipe with an outer diameter of 1.825 inches. The flame holder was made up of sections which were interchangeable. Except in the test section the wall thickness was 3/8 inch. The flame holder was parallel to the flow, cantilevered from the downstream end. The supports were cooled by a spray of water. An ogive nose was used on the upstream end. When a turbulent boundary layer was desired an additional section was inserted with a trip wire wrapped around it. Various other methods of tripping the boundary layer were tried but the circular wire was found to give the most consistent results. This is in agreement with the conclusions of Preston (10). Various sizes of trip wire were used to change the boundary layer thickness at the test section.

A ten inch length of the flame holder wall was heated by radiation from a one inch diameter graphite glow bar coaxial with the flame holder. The flame holder wall in the test section had a thickness of 0.25 inch. The direct current for heating was supplied by a motor-generator set with a maximum output of 1500 amps. at 10 volts. The current was transmitted to the heater through a 3/4 inch diameter steel electrode coaxial with the flame holder and a 1/2 inch diameter transverse electrode. Both electrodes were cooled with nitrogen, the nitrogen also serving to prevent oxidation of the glow bar. Some reaction between the nitrogen and the graphite did take place requiring replacement of the glow bar after about 50 hours of operation. The flame holder was used to complete the electrical circuit to ground. The outer wall temperature of the flame holder could be raised to a maximum of 2000°F. Above this temperature excessive scaling occurred, adversely affecting the boundary layer.

The auxiliary flame holder encircled the heated flame holder with a clearance of less than 1/16 inch. It was made of 3/8 inch diameter steel tubing and was water cooled. A 3/16 inch diameter auxiliary flame holder was also used and the change did not affect the flame stabilization measurements.

5. Measurements

The mass flow of air was measured by a sharp edged orifice installed upstream of the heat exchanger. The installation of the orifice and the associated pressure measurements were made in accordance with standard ASME practices for such devices. The velocity in the test section was calculated from a knowledge of the temperature, pressure, and fuel-air ratio. The maximum error in the velocity was estimated to be $\pm$ 3 per cent. Rates of fuel flow were measured with a Fisher and Porter flowrator calibrated on the fuel used. The maximum error in the fuel-air ratio was estimated to be $\pm$ 3 per cent. The static pressure in the test chamber was measured by means of a mercury manometer. Temperatures under $900^{\circ}F$. were measured by means of chromel-alumel thermocouples and a Brown automatic potentiometer. The temperature of the flame holder was measured with an optical pyrometer. The emissivity of the flame holder was estimated to be 0.86 (11). Absorption by the flame was assumed to have a negligible effect. The total error in wall temperature measurement was estimated to be $\pm 5^{\circ}F$. The position of the point of flame attachment was measured visually using scribed lines on the vycor glass. The accuracy was $\pm$ 0.1 inch.

6. Boundary Layer Studies

A standard NACA type total head pitot tube was used to traverse the boundary layer. The effect of the curvature of the flame holder wall was found to have a negligible effect on the measurements.

7. Schlieren System

The schlieren system was of the conventional double mirror, single pass type. Two 9 inch diameter concave mirrors were used with a BH-6 lamp for illumination. For single exposures a spark was used with a duration of less than 7 microseconds. For high speed motion pictures continuous illumination was provided, and a Fastax camera was used at a speed of one or two thousand frames per second. All pictures were taken with a single, horizontal knife edge.

B. Boundary Layer Investigation

An investigation of the turbulent boundary layer over the test section was carried out, profiles were measured in isothermal flow without either combustion or heat transfer. Various trip wire sizes and mixture temperatures were considered. The trip wires encircled the flame holder 6 inches upstream of the test section. Measurements at the same longitudinal station but at different angular positions indicated that the boundary layer was axially symmetric. A set of profiles measured at $x = 0$ for different free stream velocities are shown in figure 9. A comparison is made with the experimental results of Klebanoff and Diehl (12). The deviation of the experimental points near the wall can be attributed to wall interference with the probe.

Measurements at different longitudinal positions indicated similar profiles and some growth. The growth was insufficient to provide an adequate means of determining the friction velocity through the integral momentum equation,

$$\frac{d\theta}{dx} = \left(\frac{U_\tau}{U_o}\right)^2 \qquad (2)$$

Two other methods of determining the friction velocity were used. The empirical equation of Squire and Young (13) based on the measured momentum thickness was used in the form,

$$\frac{U_o}{U_\tau} = 5.890 \log_{10} (4.075 R_\theta) \qquad (3)$$

The friction velocity was also determined by fitting the law of the wall (14),

$$\frac{U}{U_\tau} = 5.75 \log_{10} \left(\frac{y U_\tau}{\nu}\right) + 5.10 \qquad (4)$$

to the experimental profiles. The results by the two methods agreed surprisingly well. For a typical example (T_o = 200°F., D_{TW} = 0.0201 in., U_o = 100 ft./sec.) the empirical equation gave $U_o/U_\tau = 20.4$ while fitting the law of the wall gave $U_o/U_\tau = 20.8$. A detailed study of the methods available for determining the wall shearing stress in a turbulent boundary layer has been given by Klebanoff (15). The experimentally determined ratios of free stream velocity to friction velocity for the various combinations of trip wires and free stream temperatures considered are plotted in figure 10. The values are the average of the two methods discussed above. Since the measurements were carried out at $X = 0$ these values of the friction velocity are valid only at that point. However the change in friction velocity along the length of the test section as determined from equation 2 was less than 10 per cent. Within this approximation the values determined above are valid for the whole test region.

C. Experimental Procedure

In order to stabilize flames in the turbulent boundary layer both the convective heat transfer from the hot combustion gases and the radiative heat transfer from the graphite glow bar were required. The amount of power necessary to produce a given wall temperature depended upon the free stream conditions. Ordinarily about 4000 watts were required to allow a flame to be stabilized in a turbulent boundary layer.

Several typical temperature distributions are shown in figure 11. The distributions for different temperature levels were similar. At the same level various combinations of velocity, fuel-air ratio, and power input did not change the distribution appreciably. The temperature referred to as the wall temperature was measured at $x = 3.20$ inches. The decrease in temperature at the downstream end of the heated section was due to the nitrogen used to cool the inner electrode. Due to this decrease, the test section for qualitative experiments was restricted to the first six inches of the heated region.

When the flame holder wall temperature reached the desired level the auxiliary flame holder was moved to its downstream position. This position was sufficiently far downstream so that the flow over the heated section was not appreciably affected. If the temperature level was too low the flame did not stabilize in the boundary layer. If the temperature was high enough the flame would stabilize in the boundary layer somewhere on the heated section. The sequence of operations is illustrated in figure 12. Ahead of the flame position cooling took place; as a result the flame attachment point would gradually move downstream. A typical example of the transient problem with a turbulent boundary layer is given in figure 13 and figure 14. Figure 13 gives the position of the flame as a function of time. In figure 14 the wall temperature is plotted as a function of time for various positions on the wall. Due to the complicated interaction of time dependent heat transfer between the fluid and the flame holder, the transient problem with combustion was not studied in detail.

As previously discussed the condition that the initial point of flame attachment corresponded to that in a steady state problem depended upon whether the characteristic time associated with the combustion problem was small compared with the characteristic time associated with the transient heat transfer problem. The characteristic time associated with the combustion problem can be estimated by dividing the length of heated wall required for stabilization by the free stream velocity. A reasonable value of this parameter would be 0.005 sec. The characteristic time associated with the heat transfer problem can be estimated from the dependence of the wall temperature on time. A typical experimental plot is presented in figure 16. The time associated with a temperature change of 10^{o}F.

is about 0.5 sec. This temperature change would correspond to a change in reaction rate of 10 per cent. The ratio of the two estimated times is 100; therefore the required condition apparently was satisfied. The consistency of the data and the small scatter also favored this assumption. If the initial flame attachment position was not the response to the initial temperature distribution then the variations in timing and initial stabilization would have caused a far greater scatter.

In the remainder of the discussion only the initial flame position in the transient experiment will be considered. It will be assumed that this position corresponded to the flame position in the idealized steady state problem.

D. Heat Transfer into a Turbulent Boundary Layer

Before considering the combustion problem in the turbulent boundary layer with a step function temperature distribution, the corresponding heat transfer problem was studied. This study allowed a comparison to be made with the experiments of another investigator. It also provided the basis for a theoretical study of the combustion problem.

An experimental investigation of this problem has been carried out by Johnson (16). The temperature changes in his experiment were so small that the flow could be considered incompressible. Johnson measured the convection boundary layer thickness defined by,

$$\delta_c = \int_0^\infty \frac{U}{U_o}\left(\frac{T-T_o}{T_w-T_o}\right) dy \qquad (5)$$

A set of his results are presented in figure 15. The convective boundary layer thickness is related to the heat transfer rate at the wall through the heat flux equation,

$$\frac{d\,\delta_c}{d\,x} = \frac{q_x}{\rho c_p U_0 (T_w - T_0)} \qquad (6)$$

Johnson made no attempt to obtain an analytic solution to the problem.

If the important features of the heat transfer mechanism are assumed to take place within the laminar sub-layer of the turbulent boundary layer, the heat transfer rate at the wall can be related to the wall shearing stress through a laminar theory. Near the wall the parallel component of the velocity may be linearized. If in addition the wall shearing stress is assumed constant, then the velocity components are,

$$U = \frac{U_\tau^2 y}{\nu}, \quad V = 0 \qquad (7)$$

Although the assumption of constant wall shearing stress violates the conservation of momentum, the approximation should be good in the case of the laminar sub-layer of a well developed turbulent boundary layer.

The problem of a thermal boundary layer over a wall with a step function wall temperature distribution requires the solution of the energy equation. For incompressible flow and with the above assumptions on the velocity the energy equation with the appropriate boundary conditions is,

$$\frac{U_\tau^2 y}{\nu} \frac{\partial T}{\partial x} = \frac{\nu}{\sigma} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

$$\begin{aligned} T &= T_0 \, ; \quad y = \infty \\ & \qquad\quad y = 0 \, , \; x < 0 \\ T &= T_w \, ; \quad y = 0 \, , \; x > 0 \end{aligned}$$

Due to the type of boundary conditions a similarity solution seems applicable. The appropriate non-dimensional variables are,

$$\Omega = \frac{T_w - T}{T_w - T_0} \, , \qquad \eta = y \sqrt[3]{\frac{U_\tau^2 \sigma}{\nu^2 x}} \tag{9}$$

Using these variables only the quarter plane $y > 0$, $x > 0$ is considered. The energy equation in terms of these variables is,

$$-\frac{\eta^3}{3} \frac{d\Omega}{d\eta} = \frac{d^2\Omega}{d\eta^2} \tag{10}$$

$$\begin{aligned} \Omega &= 0 \, ; \quad \eta = 0 \\ \Omega &= 1 \, ; \quad \eta = \infty \end{aligned}$$

This complete linear differential equation may be solved directly by two integrations. The solution in terms of the original variables is,

$$\frac{T_w - T}{T_w - T_0} = 0.537 \int_0^{y \sqrt[3]{\frac{U_\tau^2 \sigma}{\nu^2 x}}} e^{-z^3/9} \, dz \tag{11}$$

And the rate of heat transfer at the wall is,

$$q_x = 0.537 \; K \, (T_w - T_0) \sqrt[3]{\frac{U_\tau^2 \sigma}{\nu^2 x}} \tag{12}$$

This result can also be obtained by the more general method given by

Lighthill (17).

The theoretical result applicable to the experiments of Johnson can be found using equation 12 and equation 6. The result is,

$$\frac{\delta_c}{x} = 0.806 \frac{1}{\sigma^{2/3}} \frac{U_\tau}{U_0} \sqrt[3]{\frac{\nu}{U_\tau x}} \qquad (13)$$

This theoretical relation is compared with the experiments in figure 15. As might be expected the theoretical curve is low. This can be attributed to the very restricted applicability of a laminar theory. Actually the term laminar sub-layer is somewhat misleading. The turbulent transfer will be identically zero only at the wall. However the agreement with experiment must be considered good when compared with most theories of turbulent boundary layer phenomenon. Apparently the region near the wall where viscous transfer predominates, governs the heat transfer mechanism.

The rate of heat transfer in the present experiment without combustion was measured directly. This was done by using the transient method applicable to the combustion problem. The wall temperature distribution was first established with a flame stabilized on the auxiliary flame holder. The auxiliary flame holder was then removed and the wall allowed to cool. The rate at which the wall cooled was used to determine the heat transfer into the boundary layer.

The heat balance before the removal of the auxiliary flame holder may be set up in a straight forward manner. Without any heat addition from the glow bar the heat balance may be written in terms of heat transfer per unit length of the flame holder as,

$$q_{F1} = q_{R1} \tag{14}$$

q_F = heat transfer from hot combustion gases to the flame holder

q_R = heat transfer from the flame holder wall by radiation

The equilibrium wall temperature with no heat addition is denoted by T_{w1} The radiative heat transfer can be calculated by the relation,

$$q_{R1} \left\langle \frac{btu.}{ft.^2 sec.} \right\rangle = 0.171 \times 10^{-8} \alpha \left(T_{w1} \langle {}^\circ R \rangle \right)^4 \tag{15}$$

Using equation 14 the heat transfer from the combustion products may be calculated. With power input into the glow bar electrode the heat balance may be written,

$$q_{F2} = q_{R2} - q_G \tag{16}$$

q_G = radiative heat transfer from the glow bar per unit area of the flame holder wall

The heat transfer from the glow bar may be calculated from a knowledge of the power loss in it by,

$$q_G \left\langle \frac{btu.}{ft.^2 sec.} \right\rangle = 3.412 \frac{r_G \left\langle \frac{ohm}{ft.} \right\rangle \left(i \langle amp \rangle \right)^2}{\pi \; D \langle ft. \rangle} \tag{17}$$

The two heat transfer rates from the combustion gases are related by,

$$\frac{q_{F2}}{q_{F1}} = \frac{T_F - T_{w2}}{T_F - T_{w1}} \tag{18}$$

And the radiative heat loss can be calculated from an equation similar to equation 15. The above relations are redundant in that the unknown, the heat transfer from the combustion products, may be calculated in two independent ways. This offers a check on the method and on the experimental observations.

After the removal of the auxiliary flame holder the heat transfer problem is transient in nature. However after the initial transient response a quasi steady state heat balance should be applicable in the form,

$$q_x = q_G + q_w - q_{R2} \tag{19}$$

q_w = heat transfer from the flame holder wall due to the drop in wall temperature

q_x = heat transfer to the turbulent boundary layer

In terms of the properties of the stainless steel flame holder wall, the heat transfer from the wall should be given to a good approximation by the relation,

$$q_w = w \rho_w c_w \frac{dT_{w2}}{dt} \tag{20}$$

Combining the above relations the heat transfer into the boundary layer is given by,

$$q_x = 3.412 \frac{r_G i^2}{\pi D} + w \rho_w c_w \frac{dT_{w2}}{dt} - 0.171 \times 10^8 \alpha T_{w2}^4 \tag{21}$$

The variation of wall temperature with time for a particular experiment is given in figure 16. The curve is from equation 21 with q_x chosen to give the best fit to the data. The initial transient response between the heat balances in equations 16 and 19 can clearly be seen in the first two seconds. The required unknowns and the calculated results for this particular example are:

$$\alpha = 0.86$$

$$r_G = 5.25 \times 10^{-3} \text{ ohm/ft.}$$

$$D = 0.152 \text{ ft.}$$

$$w = 0.0298 \text{ ft.}$$

$$\rho_w = 501 \text{ lb./ft}^3.$$

$$c_w = 0.12 \text{ btu./lb.}^\circ\text{F.}$$

$$T_{w1} = 2098^\circ\text{R.}$$

$$T_{w2} = 2205^\circ\text{R.}$$

$$T_o = 760^\circ\text{R.}$$

$$T_F = 4210^\circ\text{R.}$$

$$x = 3.2 \text{ in.}$$

$$U_o = 100 \text{ ft./sec.}$$

$$U_o/U_r = 18.0$$

$$i = 550 \text{ amp.}$$

$$\frac{dT_{w2}}{dt} = 12^\circ\text{F./sec.}$$

q_{R1} = 2.84 x 10^4 btu./$ft.^2$ hr. (equation 15)

q_{F1} = 2.84 x 10^4 btu./$ft.^2$ hr. (equation 14)

q_{F2} = 2.70 x 10^4 btu./$ft.^2$ hr. (equation 18)

q_{R2} = 3.47 x 10^4 btu./$ft.^2$ hr. (equation 15)

q_G = 1.13 x 10^4 btu./$ft.^2$ hr. (equation 17)

q_{F2} = 2.34 x 10^4 btu./$ft.^2$ hr. (equation 16)

q_x = 3.25 x 10^4 btu./$ft.^2$ hr. (equation 21)

Using the above method the heat transfer rate was obtained at different positions on the flame holder wall for a given set of free stream conditions. A set of experimentally determined Nusselt numbers are plotted as a function of position in figure 17. The local Nusselt number is defined by,

$$N_x = \frac{x\, q_x}{K_o (T_w - T_o)} \qquad (22)$$

If the Howarth transformation (18) is used the equations for heat transfer in a compressible boundary layer may be transformed into those for an incompressible boundary layer. The required assumptions are

$$\mu \rho = \mu_o \rho_o \,, \quad c_p = c_{p_o} \,, \quad \sigma = \sigma_o \qquad (23)$$

The transformation is

$$X = \bar{X}, \quad U = \bar{U}, \quad y = \int_0^{\bar{y}} \frac{\rho}{\rho_o} dy, \quad \bar{V} = -\frac{\rho_o}{\rho}\left(-v + \int_0^{\bar{y}} \frac{\partial}{\partial x}\left(\frac{\rho}{\rho_o}\right) d\bar{y}\right) \qquad (24)$$

where the barred quantities are in the compressible plane. Using this transformation and the method of the previous section the rate of heat transfer at the wall is,

$$q_x = 0.537 K_o (T_w - T_o) \sqrt[3]{\frac{U_{r_o}^2 \sigma_o}{\nu_o^2 X}} \qquad (25)$$

The subscript "o" refers to isothermal conditions corresponding to the free stream. The best agreement with experiment is obtained if the viscosity law is fitted so that equation 23 agrees with the experimental values for the wall temperature. When this is done the above result agrees, within a constant factor, with that of Liepmann (19) who obtained a more general solution to the compressible problem using integral techniques.

Combining equation 25 with equation 22 the theoretical solution for the Nusselt number is obtained,

$$N_x = 0.537 \sigma_o^{\frac{1}{3}} \left(\frac{U_{r_o} X}{\nu_o} \right)^{\frac{2}{3}} \qquad (26)$$

This result is also plotted in figure 17. For a direct comparison with the work of Johnson the convection boundary layer thickness is required. The appropriate experimental and theoretical curves are shown in figure 18. The experimental curve is obtained by integrating the experimental results as indicated in equation 6. A comparison of figure 18 with figure 16 indicates that the extension of the sub-layer theory to the compressible regime does not appreciably affect the results. Again the agreement between theory and experiment seems satisfactory.

III. TURBULENT BOUNDARY LAYER FLAME STABILIZATION

The quantitative study of flame stabilization in a turbulent boundary layer consisted of a determination of the point of flame attachment just after the removal of the auxiliary flame holder. Measurements were made for various values of the independent parameters. The independent parameters considered were the wall temperature, the free stream velocity, the fuel-air ratio, the trip wire diameter, and the free stream temperature. The measurements were carried out either visually or by studying spark schlieren photographs. In order to better understand the stabilization mechanism high speed schlieren motion pictures were taken.

A. Experimental Measurements and Observations

Probably the most interesting independent parameter in the stabilization problem was the flame holder wall temperature. With all other variables fixed the dependence of the flame attachment position on wall temperature was found. For a sufficiently high wall temperature the flame would stabilize on the wall at a definite and repeatable position. As the wall temperature was decreased the length of heated wall upstream of the attachment point increased in a continuous manner. At a definite wall temperature this continuous relation ended abruptly. Below this stabilization limit the flame would not stabilize in the boundary layer. An illustration of this dependence is given by the schlieren photographs in figure 19. Figure 19c was taken below the stability limit so that no flame stabilized in the boundary layer. It is interesting to note that in all cases when the boundary layer was turbulent the propagating flame was also turbulent.

A plot of the flame attachment position versus wall temperature for various free stream velocities and a stoichiometric fuel-air ratio is given in figure 22. The stability limit is indicated with a dashed line. In the region to the right of this line a flame may be stabilized in the boundary layer. The scatter in the data was quite low. This can be attributed largely to the stability of the flame atachment point. Although the propagating flame oscillated considerably the point of attachment did not vary a visible amount.

The minimum temperature required to stabilize a flame in the boundary layer had a definite measurable value. These values are plotted as a function of velocity in figure 23 for a stoichiometric fuel-air ratio. In figure 24 the stability limit is plotted against fuel-air ratio for various velocities. Although the variation is not large, there seems to be a minimum stabilization temperature at about $\phi = 1.20$. Such a minimum stabilization temperature would be expected at or near a stoichiometric fuel-air ratio since the temperature rise due to combustion would be a maximum. However if differential diffusion was involved the actual concentration near the flame holder wall might be different from that of the free stream. Since the average fuel molecular weight is approximately three times larger than the average molecular weight of air this shift can be explained from a consideration of differential molecular diffusion. The oxygen molecules would diffuse toward the wall more rapidly than the fuel molecules; therefore if combustion was occurring the mixture ratio would be less than in the free stream. This type of molecular diffusion would only be of importance in a laminar regime. This indicates that the mechanism governing stabilization in a turbulent boundary layer is a laminar phenomenon and hence is confined to the region very near the wall.

Any solution for the dependence of the flame position on wall temperature can be expected to be complicated due to the presence of the Arrhenius reaction rate term. For this reason the dependence of the flame position on free stream velocity at constant wall temperature is important. In particular one might expect that the dependence would be given by one of two similarity parameters, either the Reynolds number,

$$\mathcal{R} = \frac{U_0 x_F}{\nu_0} \tag{27}$$

a significant parameter in boundary layer problems, or the first Damkohler number,

$$\mathcal{D}_I = \frac{x_F}{t_1 U_0} \tag{28}$$

a significant parameter in flow problems with chemical reaction. The dependence of the flame attachment position on free stream velocity is given in figure 25 for a stoichiometric fuel-air ratio. The existence of the stability limit eliminates any similarity. Schlieren photographs illustrating this dependence are presented in figure 20. At low velocities as in figure 20b the turbulence in the flame seems to be damped out as it propagates into the free stream. Instead of constant wall temperature, the temperature difference between the wall temperature and the minimum wall temperature required for stabilization may be taken as the significant temperature variable. In figure 26 the flame position is plotted against velocity for constant values of this temperature difference. The curves exhibit some degree of similarity. For plots requiring a constant temperature variable, a constant value of $T_w - T_{ws}$ would seem to be the most logical choice. An arbitrary value of 50°F. has been selected.

The dependence of flame position on fuel-air ratio is given in figure 27 for various free stream velocities and $T_w - T_{ws} = 50°F$. This is also illustrated in the schlieren photographs presented in figure 21. Particularly interesting is the shift of the minimum stabilization distance to rich mixture ratios. This can also be explained in terms of differential molecular diffusion as was the minimum in the stabilization limit wall temperatures. The plot in figure 27 is an even better demonstration that the mechanism governing stabilization in a turbulent boundary layer is a laminar phenomenon and hence is confined to the region very near the flame holder wall.

By changing the trip wire diameter the thickness of the boundary layer over the test section was varied. Trip wires with diameters of 0.0101 inch and 0.0406 inch were considered in addition to the 0.0201 inch trip wire previously discussed. The effect of the change in boundary layer thickness was investigated for a stoichiometric mixture. In figure 28a and figure 28b plots are presented of flame position versus wall temperature for various free stream velocities. In figure 29 the minimum wall temperatures required for stabilization are plotted against velocity for all three trip wires. Within the experimental error this plot seems to be independent of boundary layer thickness over the range considered. In figure 30 the flame position is plotted against velocity for $T_w - T_{ws} = 50^{\circ}F.$ for the three trip wires. Thinner boundary layers apparently require a longer length of heated wall for stabilization.

Free stream temperatures of 200°F. and 400°F. were investigated in addition to the 300°F. temperature previously discussed. In figures 31a and 31b plots are presented of flame position versus wall temperature for various free stream velocities and a stoichiometric fuel-air ratio. In figure 32 the wall temperature required for stabilization is plotted against velocity for all three free stream temperatures. In figure 33 the flame position is plotted against velocity for $T_w - T_{wt} = 50^{\circ}F.$ for the three mixture temperatures. An increase in the mixture temperature decreased both the wall temperature required for stabilization and the length of heated wall required.

To obtain a better understanding of the turbulent stability mechanism high speed schlieren motion pictures were taken. The stability of the point of flame attachment was clearly demonstrated; although there was considerable oscillation of the flame in the free stream, the point of attachment was defined to within $\pm$ 0.05 inch. The details of initial stabilization were studied to determine whether the mixture was ignited by the wall at the point of attachment or whether the flame propagated upstream through the boundary layer from some other ignition point. A set of frames illustrating this initial ignition are shown in figure 34. The film speed was 1000 frames per second. The schlieren effect with a single knife edge made the flame visible above the flame holder only. The measurement of time commenced when the auxiliary flame holder was removed. The delay before the first appearance of a flame can be attributed to the build up of a boundary layer over the test section. It appears that the mixture first ignited near x = 5 inches (t = 0.182 through t = 0.189 sec.) and a flame propagated into the mixture. The point of attachment then moved upstream to x = 2.0 inches (t = 0.187 through t = 0.195 sec.) which was the measured flame position. The high speed films were studied to determine the velocity with which the flame propagated upstream through the turbulent boundary layer during oscillations and the initial stabilization period. The range of velocities was found to be between 25 and 100 ft./sec.

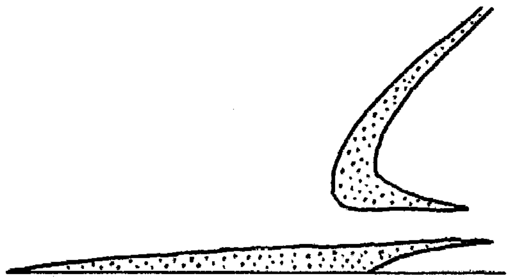
B. Stabilization Mechanism

In an isothermal laminar boundary layer a flame should stabilize by a mechanism similar to that governing flashback in bunsen burners. This bunsen burner mechanism requires that the flame speed shall equal the flow

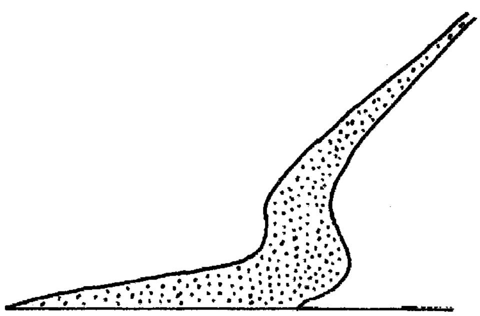
Sketch 5.

velocity at a distance from the wall called the quenching distance. The cool wall acts as a heat sink that quenches the end of the flame. The validity of the mechanism in the isothermal laminar boundary layer has been demonstrated by Hottel, Toong, and Martin (20).

In order to understand what the stabilization mechanism might be in a heated boundary layer consider what change might be expected in the bunsen burner mechanism as the wall temperature is increased. For the purposes of this intuitive discussion assume that the wall velocity gradient increases with the wall temperature to keep the propagating flame stabilized at the same point. As the temperature increases reaction begins to occur near the flame holder wall; this is illustrated in sketch 6 with the two regions of strong chemical reaction shaded. The end of the propagating flame is still quenched by the wall. However the heat released through chemical reaction near the wall provides thermal shielding to the heat transfer process in the quenching region, i. e. the temperature distribution in the region between the end of the flame and the wall may no longer be approximated by a linear relation, the heat loss from the

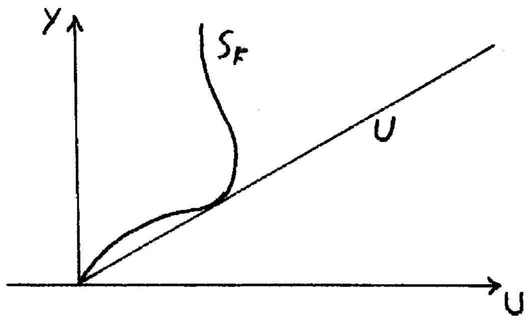


Sketch 6 .



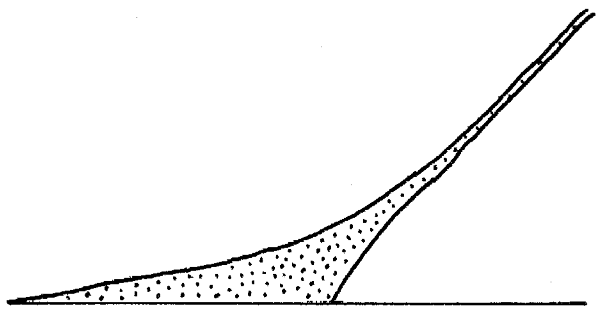
Sketch 7.

end of the flame to the wall is reduced. As the wall temperature is increased further the two regions of chemical reaction will join; this is illustrated in sketch 7. The high wall temperature and the thermal shielding by the reaction near the wall prevent true quenching. However the stabilization mechanism may still be governed by an equality of the flame speed and the flow velocity. Such an equality is illustrated in sketch 8 even though the flame is not quenched.



Sketch 8.

Now consider what might happen if the wall temperature and wall velocity gradient were increased even more. The expected form of the flame is illustrated in sketch 9 and in figure 2b. The flame is anchored to the wall, the flame thickness depends on the distance from the wall. The important heat transfer is now from the heated plate into the combustible mixture. The heat ignites the mixture on the plate and the flame propagates into the mixture. The chemical reaction in the region near the wall completely shields the rest of the propagating flame, no quenching occurs. The flow velocity is greater than the normal flame speed all through the boundary layer, no remnant of the bunsen



Sketch 9.

burner mechanism remains. This new mechanism will be referred to as a continuous ignition mechanism.

Data on flame stabilization in heated laminar boundary layers have been obtained by Zeimer and Cambel (5). A comparison was made by these authors with a bunsen burner mechanism; good agreement was obtained. Although some of the empirical extrapolations used in the comparison might be questioned, there is no indication that the choice of stabilization mechanism was invalid. Apparently the picture of the mechanism given in either sketch 6 or in sketch 7 is applicable in the heated laminar boundary layer.

The above reasoning on the stability mechanism can be applied in the turbulent regime directly. Although it is not really necessary for the qualitative discussion, it will be assumed that the mechanism governing stabilization in turbulent boundary layers originates within the laminar sub-layer. This seems to be a reasonable restriction for both a bunsen burner mechanism and a continuous ignition mechanism. The equality of velocities required in the former can only occur, for a reasonable free stream velocity, at a velocity found in the laminar sub-layer. In the latter the temperatures that are sufficiently high to produce chemical reaction are found in the laminar sub-layer until the propagating flame is formed.

The present experiments can be evaluated in terms of the above discussion without going into the details of an exact solution. Some features of the two mechanisms can be deduced and compared with the observed features of stabilization in turbulent boundary layers.

First the continuous ignition mechanism will be considered. With the flame anchored on the plate as illustrated in sketch 9 the visible point of flame attachment should be stable; this was observed experimentally. The characteristic length in the problem would be the length of heated wall required for stabilization. Experimentally this was found to be an important and reproducible variable.

If, instead, the bunsen burner mechanism were applicable the important parameters would be the quenching distance, the normal flame speed, and the wall velocity gradient. However along the heated wall in the well developed turbulent boundary layer none of these quantities vary significantly. Therefore the length of heated wall up to the point of stabilization would not be a significant variable. If stabilization occurred anywhere along the heated surface the flame would be expected to propagate upstream through the boundary layer to the end of the heated section; the change in quenching distance with wall temperature would prevent a further movement. This property of the bunsen burner mechanism has also been discussed by Toong (21). The bunsen burner mechanism requires that the flame speed and flow velocity be equal at some point. Since this equality occurs away from the wall its position along the wall may fluctuate.

The observed systematic dependence of the flame attachment position on the independent variables and the stability of the attachment point indicate the validity of a continuous ignition mechanism in turbulent boundary layer stabilization. The question now arises as to why the mechanism applicable in heated laminar boundary layer stabilization does not apply in heated turbulent boundary layer at stabilization. The answer can be seen in the discussion relating sketch 9 to sketch 8. One of the key variables was the wall velocity gradient. But the most significant change that occurs when a boundary layer becomes turbulent is the increase in the wall shearing stress, and this is directly proportional to the velocity gradient at the wall. Therefore the change in mechanism follows directly from the above discussion.

A further verification of the applicability of a continuous ignition mechanism in turbulent boundary layer flame stabilization would be a demonstration that the point of initial ignition was the same as the point of stabilization. High speed motion pictures indicated that initial ignition did occur on the heated section but at a point downstream from the final point of attachment. For the particular example illustrated in figure 34 the point of initial ignition was near $x = 5.0$ inches while the flame stabilized at $x = 2.0$ inches. This change of position can be attributed to the presence of the propagating flame. Once a propagating flame is established the flow field is somewhat altered; a resultant change in the point of ignition would be expected. Certainly the range of velocities at which the point of attachment moved upstream through the boundary layer indicated some sort of ignition phenomenon. The velocities, 25 to 75 ft./sec., were sufficiently large so as to preclude normal flame propagation through the boundary layer.

It seems reasonable to conclude that while the bunsen burner mechanism is applicable in laminar boundary layers, the continuous ignition mechanism governs stabilization in heated turbulent boundary layers. This is not to predict that the correspondence is generally valid. In a very thick turbulent boundary layer with a low wall temperature a bunsen burner mechanism would certainly be applicable. Also, if the wall temperature was sufficiently high an ignition mechanism might be applicable in the laminar boundary layer problem.

In general it might be said that the mechanism which gives the minimum stabilization distance is the applicable mechanism in a particular case. However in some cases an interaction between the two mechanisms might occur. A heated wall might cause a dilution in the combustible mixture near the wall affecting the quenching distance in such a manner that a flame would not stabilize anywhere in a flat plate boundary layer.

C. Heat Addition Required for Stabilization

When considering the practicality of boundary layer flame stabilization the heat input required is an important parameter. Unfortunately this could not be measured directly in the present experiments. However by integrating equation 25 up to the point of flame attachment the heat addition into the turbulent boundary layer can be calculated if the presence of combustion is neglected. Since the combustion in the boundary layer should have an appreciable effect on the heat transfer rate only near the point of flame attachment, this approximation should provide a satisfactory estimate. In figure 35 the calculated heat transfer rates are plotted against the reciprocal of the wall temperature for various free stream velocities and a stoichiometric mixture ratio. This plot indicates that the heat transfer rate at the stability limit may have an exponential dependence on the wall temperature. In figure 36 the heat transfer rate at the stability limit is plotted for the various trip wires and free stream temperatures considered. An empirical equation is plotted for comparison in the form,

$$Q\left\langle\frac{\text{btu.}}{\text{ft. hr.}}\right\rangle = 2.22 \times 10^{15}\ e^{-\frac{33,600}{T\langle {}^\circ K\rangle}} \qquad (29)$$

The dependence of the heat required for stabilization on an Arrhenius relation provides an opportunity for determining the activation energy of the fuel used. The activation energy of 66,800 calories is fairly close to the best available estimate for the fuel used which is 40,000 to 50,000 calories. The dependence on an Arrhenius rate law tends to substantiate the validity of a continuous ignition mechanism. Exactly how this dependence comes about is certainly not clear. However the heat transfer into the boundary layer would seem to be more fundamental to the continuous ignition stabilization mechanism illustrated in sketch 9 than to the bunsen burner mechanism illustrated in sketch 5.

D. Comparison with Theory

The development of theoretical solutions to combustion problems has been retarded by the complexity of the chemical kinetics. For a hydrocarbon flame the exact kinetics are unknown. Even when a reaction such as a simple decomposition is considered, analytic solutions are difficult due to the presence of the Arrhenius reaction rate term. Much of the effort in theoretical combustion work has been directed toward laminar flame propagation. Considerable success has been realized in the solution of problems with simplified kinetics and in the few cases where the exact kinetics are known. A good review of this work has been given by Penner (22).

Another group of problems that seem amenable to theoretical analysis are chemically reacting boundary layer flows. Solutions of the boundary layer equations with chemical reaction have been carried out for the laminar mixing region (6) and the flat plate laminar boundary layer (7), (23) using iteration (7) and perturbation (6), (23) techniques. None of these theories have been compared quantitatively with an appropriate experiment. Toong (24) carried out experiments on ignition in a laminar boundary layer; he correlated his results with an appropriate similarity parameter, but not with the ignition solution he had previously published (23).

The continuous ignition mechanism found applicable in the present experiments invites a comparison with an ignition theory. However the limitations of such a comparison must be recognized. The boundary layer assumption must be applicable, therefore the solution must be limited to the region where the strong thermal gradients are normal to the wall. This precludes a solution that includes the development of the propagating flame. As a result an arbitrary definition of the flame attachment point must be made. The definition generally accepted (6), (7) is the point where the heat transfer rate at the wall is zero. The reasoning behind this choice is that once sufficient heat has been released to balance the heat transfer from the wall, the exponential dependence of the reaction rate on the temperature will cause the propagating flame to form in a short distance. Another limitation necessary to keep the problem from being intractable requires the effect of the propagating flame on the flow field in the ignition region to be neglected. The importance of this effect has been discussed in connection with figure 34.

A theory applicable to the turbulent boundary layer must be restricted to the laminar sub-layer. If turbulence is considered, the problem seems intractable since neither the turbulent boundary layer nor the one-dimensional turbulent flame are understood. That such an approach might be fruitful has been indicated by the success in predicting heat transfer rates with a laminar sub-layer theory as indicated previously. The applicable energy relation for the heat transfer problem is equation 8. The applicable diffusion equations are similar. To include combustion, terms must be added to provide for production of heat and chemical species. The Howarth transformation may still be applied under the same assumptions as in the heat transfer problem. The additional assumption of a simple decomposition reaction ($A \rightarrow B$) is made. For this case only two equations are required since $K_A + K_B = 1$ throughout the flow field. The applicable equations and boundary conditions in the incompressible plane are,

$$\frac{U_\tau^2 y}{\nu}\frac{\partial T}{\partial x} = \frac{\nu}{\sigma}\frac{\partial^2 T}{\partial y^2} + \frac{T_F - T_0}{\mathcal{X}_3} K_A \, e^{-\frac{E}{RT}} \qquad (30)$$

$$\frac{U_T^2 y}{\nu}\frac{\partial K_A}{\partial x} = \frac{\nu}{S_c}\frac{\partial^2 K_A}{\partial y^2} - \frac{1}{\tau_3} K_A\, e^{-\frac{E}{RT}} \tag{31}$$

at
$$y = 0, \quad x > 0; \quad T = T_w, \quad \frac{\partial K_A}{\partial y} = 0$$
$$y = 0, \quad x < 0; \quad T = T_o, \quad K_A = 1$$
$$y = \infty; \quad T = T_o, \quad K_A = 1$$

For a more detailed description of the assumptions involved in deriving the above equations the work of Marble and Adamson (6) and Dooley (7) should be consulted. The arbitrary definition of the point of flame attachment is,

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = 0 \qquad \text{when} \qquad x = x_F \tag{32}$$

The solution of these equations would provide a prediction of the point of flame attachment observed in the experiments.

In order to illustrate the difficulties of solution the method applied successfully to the heat transfer problem will be tried. Again introduce the variables given in equation 9 after substitution into equations 30, 31, and 32 the resultant equations defined for $x > 0$ and $y > 0$ are,

$$-\frac{\eta^3}{3}\frac{\partial \Omega}{\partial \eta} + x\eta\frac{\partial \Omega}{\partial x} = \frac{\partial^2 \Omega}{\partial \eta^2} - \frac{T_F - T_o}{T_w - T_o}\frac{\sigma}{\nu \tau_3}\left(\frac{x\nu^2}{\sigma U_T^2}\right)^{\frac{2}{3}} K_A\, e^{-\frac{E}{R(T_w - \Omega[T_w - T_o])}} \tag{33}$$

$$-\frac{\eta^3}{3}\frac{\partial K_A}{\partial \eta} + x\eta\frac{\partial K_A}{\partial x} = \frac{\sigma}{S_c}\frac{\partial^2 K_A}{\partial \eta^2} - \frac{\sigma}{\nu k_3}\left(\frac{x\nu^2}{\sigma U_\tau^2}\right)^{\frac{2}{3}} K_A \, e^{-\frac{E}{R(T_w - \Omega[T_w - T_0])}} \tag{34}$$

$$\left.\frac{\partial \Omega}{\partial \eta}\right|_{\eta=0} = 0, \quad x = x_F \tag{35}$$

$$\text{at} \quad y = 0;\ \Omega = 0, \quad \frac{\partial K_A}{\partial y} = 0$$
$$y = \infty;\ \Omega = 1, \quad K_A = 1$$

The dependence of the reaction term on x eliminates the possibility of solutions that are locally similar. Perturbation methods may be used expanding the dependent variables in powers of a dimensionless streamwise coordinate. These methods do not appear to be convergent in the region of interest. Apparently the change in the concentration of the reactant near the wall is too great.

Without going into a detailed solution the dependence of the flame attachment position on friction velocity with other variables held constant can be obtained. Assume two different experiments are being carried out with only the friction velocity different, the first with U_{τ_1} and the second with U_{τ_2}. Let a coordinate system for the second experiment be defined in terms of the first by

$$x_2 = \frac{U_{\tau_1}}{U_{\tau_2}} x_1, \quad y_2 = y_1 \tag{36}$$

The dependent variables in the two experiments are assumed to be related by

$$T_1(x_1, y_1) = T_2(x_2, y_2), \quad K_{A_1}(x_1, y_1) = K_{A_2}(x_2, y_2) \tag{37}$$

That these variables satisfy the conservation equations with the correct friction velocities can be seen by direct substitution into equations 30 and 31. When this is done for the energy equation, the result, is

$$\frac{U_{\tau_1}^2 y_1}{\nu} \frac{\partial T_1}{\partial x_1} = \frac{\nu}{\sigma} \frac{\partial^2 T_1}{\partial y_1^2} + \frac{T_F - T_0}{t_3} K_{A_1} e^{-\frac{E}{RT_1}} \tag{38}$$

$$\frac{U_{\tau_2}^2 y_2}{\nu} \frac{\partial T_2}{\partial x_2} = \frac{\nu}{\sigma} \frac{\partial^2 T_2}{\partial y_2^2} + \frac{T_F - T_0}{t_3} K_{A_2} e^{-\frac{E}{RT_2}} \tag{39}$$

The result for the diffusion equation is the same. The boundary conditions remain unchanged. Using equation 32 the flame attachment positions in the two experiments can be related,

$$\frac{\partial T_1}{\partial y_1}(x_{F_1}, 0) = \frac{\partial T_2}{\partial y_2}(x_{F_2}, 0) = 0 \tag{40}$$

The dependence of the point of flame attachment on friction velocity must be of the form

$$X_{F_2} = \frac{U_{\tau_2}^2}{U_{\tau_1}^2} X_{F_1} \tag{41}$$

which may also be written as

$$X_F \sim U_\tau^2 \tag{42}$$

Since the essential features of this analysis involve only the convective term in the conservation equations it is easy to see that the result given in equation 42 is valid for any order reaction. This result also implies that an observed stability limit should be independent of friction velocity since this variable is related to the streamwise coordinate only through a change of scale.

In figure 37 the experimentally determined stabilization position is plotted against the friction velocity for a constant wall temperature of 1900°F. and the various trip wires considered. In figure 38 the same correlation is made using $T_w - T_{ws} = 50°F.$ instead of constant wall temperature. Both cases are correlated with the relation

$$X_F \sim U_\tau^3 \qquad (43)$$

This compares with the second power dependence derived above. The agreement might be better than this indicates since the origin of the x-axis is somewhat arbitrary. If the origin was moved upstream one inch the agreement with a second power law would be considerably improved.

Two other methods of solution for problems of this type presented in the literature recently should be mentioned. The work of Fay and Riddell (25) on stagnation point heat transfer with dissociation would be applicable except the equations are slightly different. In the boundary layer problem after suitable non-dimensionalization the chemical reaction term is still dependent on the streamwise coordinate as shown in equations 33 and 34. In stagnation point flows the reaction term is independent of this variable. As a result the principle of local similarity does not seem applicable in flat plate boundary layer flows. The work of Lees (26) on chemically reacting boundary layers is particularly interesting. However the application of his methods to this problem introduce such a degree of similarity that no characteristic distance may be defined. No solution for a flame attachment point is possible. The existance of finite reaction rates is the essential reason that this method is not applicable. The same reason that the wall concentration is such an important parameter.

The possibility of obtaining a solution to the ignition problem remains. It is the mathematical problem of solving the set of differential equations, such as equations 33 and 34, with the correct boundary conditions. However the limitations of any such solution must be considered as previously discussed.

IV. CONCLUDING REMARKS

Flames were successfully stabilized in heated turbulent boundary layers. The point of attachment was found to be a well defined measurable quantity. Data was obtained for its dependence on wall temperature, free stream velocity, fuel-air ratio, boundary layer thickness, and free stream temperature. A stability limit was also found: A wall temperature below which stabilization was not possible under the given conditions.

The observed features of the stabilization indicated that the governing mechanism originated in the laminar sub-layer of the turbulent boundary layer. It was essentially a wall temperature governed phenomenon. These observations led to the conclusion that the stabilization was governed by a continuous ignition type of mechanism. The flat plate served as an ignition source. This conclusion was further substantiated by the dependence of the rate of heat transfer into the boundary layer at the stability limit on the wall temperature according to an arrhenius rate law. The activation energy agreed fairly well with the estimated value for the fuel used.

The validity of the continuous ignition mechanism in turbulent flow does not contradict the validity of the bunsen burner mechanism in laminar flow as found by Ziemer and Cambel (5). The change in mechanism can be attributed to the large increase in the wall velocity gradient in turbulent flow.

The good agreement between the experimental values of the heat transfer into the turbulent boundary layer and a laminar sub-layer theory indicated a possibility of obtaining a theoretical solution to the stabilization problem. Unfortunately the present methods of solution for such problems do not seem applicable. The expected dependence of the flame attachment position on friction velocity at constant wall temperature can be obtained from a similarity argument. Some agreement with experiment was noted.

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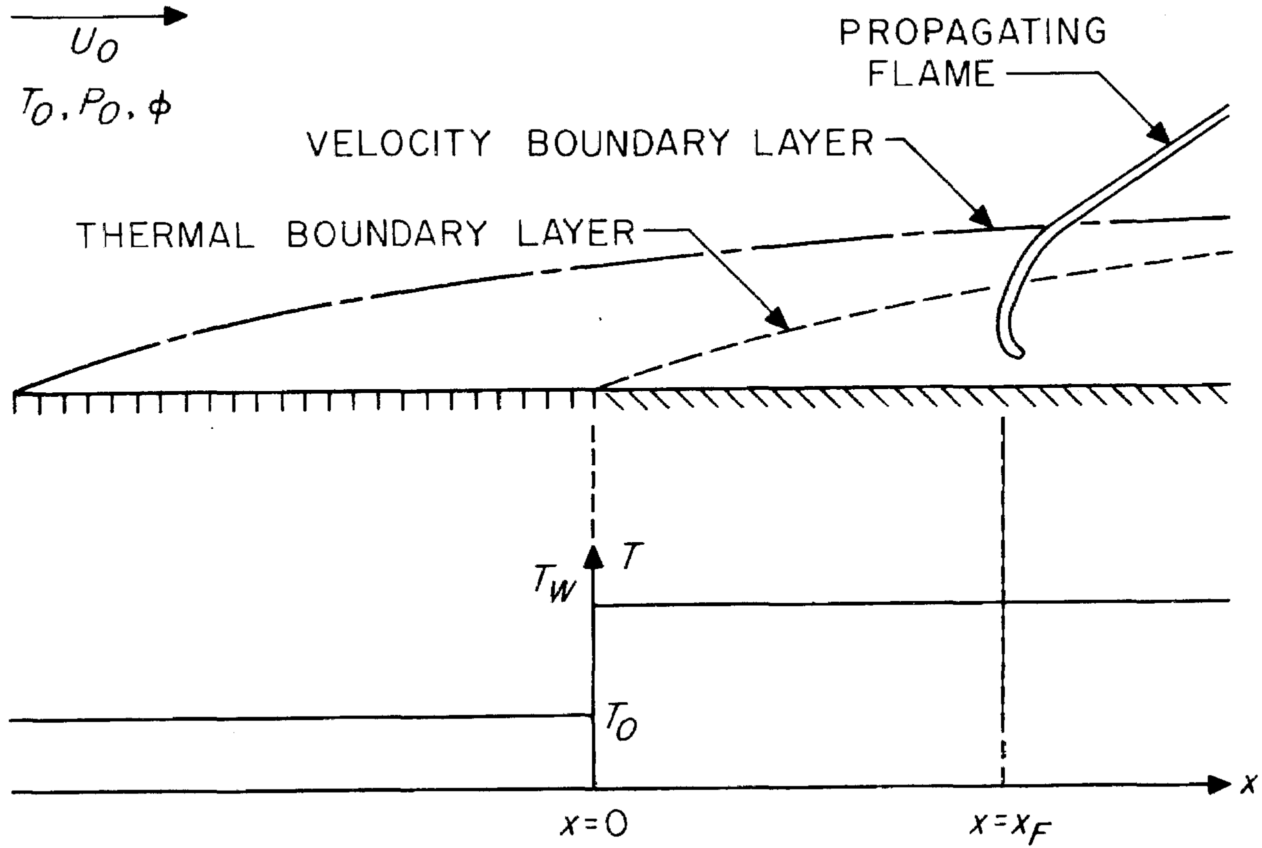


Figure 1. Sketch of a Flame Stabilized in the Boundary Layer with the Idealized Temperature Distribution.

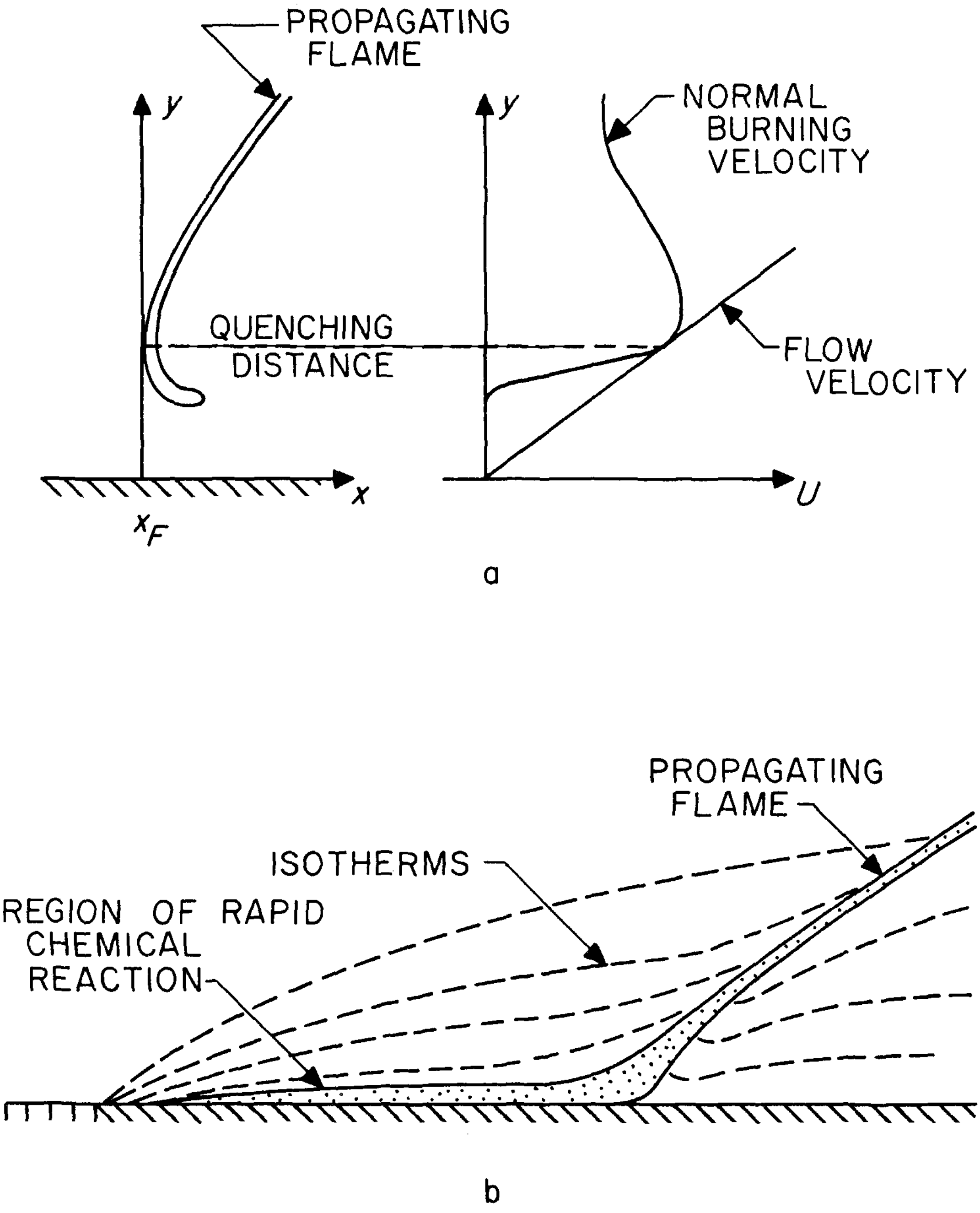


Figure 2. Illustration of the Alternative Stabilization Mechanisms; a. Bunsen Burner, b. Continuous Ignition.

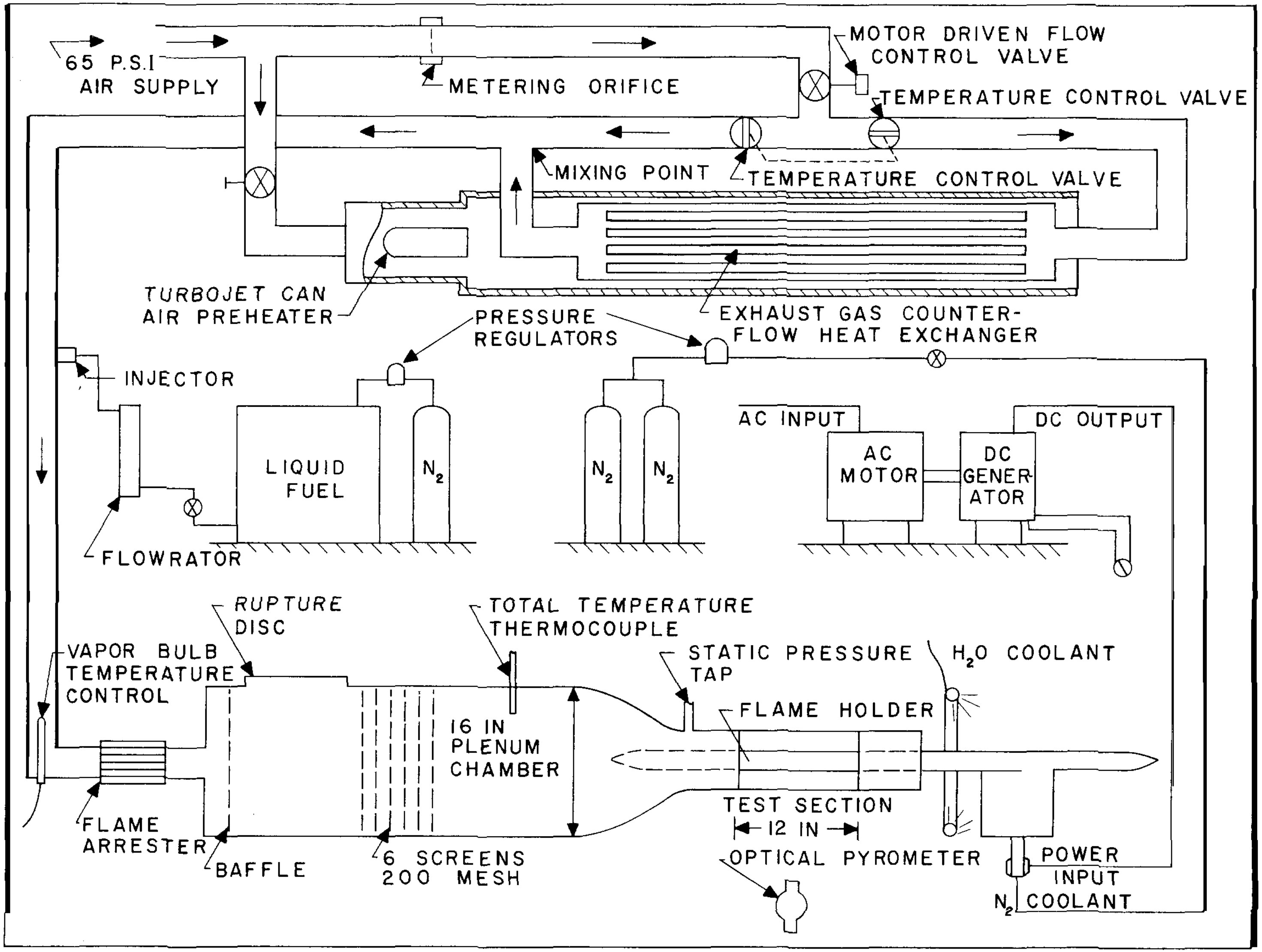


Figure 3. Schematic Diagram of the Flow System.

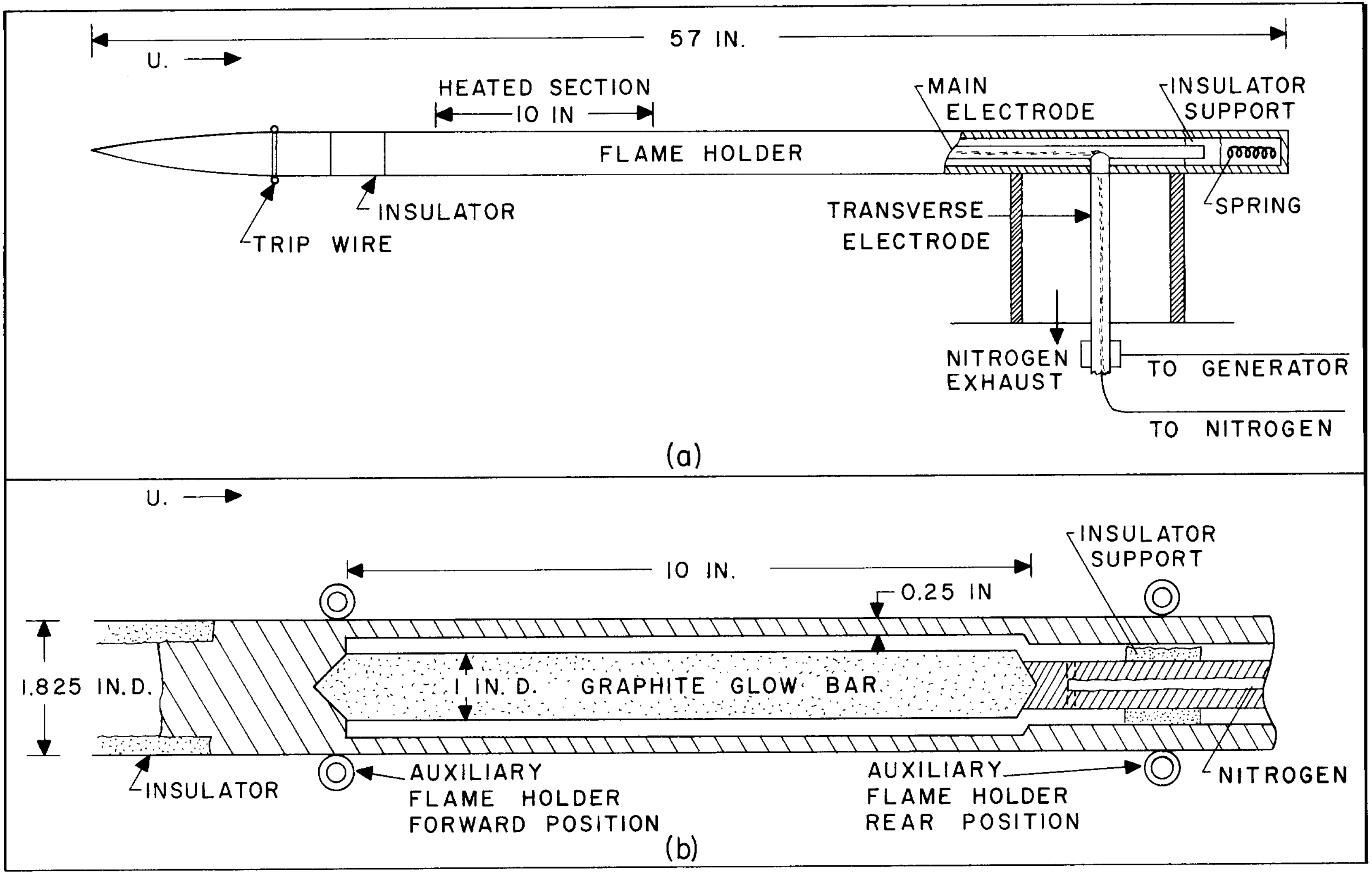


Figure 4. a) Diagram of Flameholder; b) Diagram of Heater.

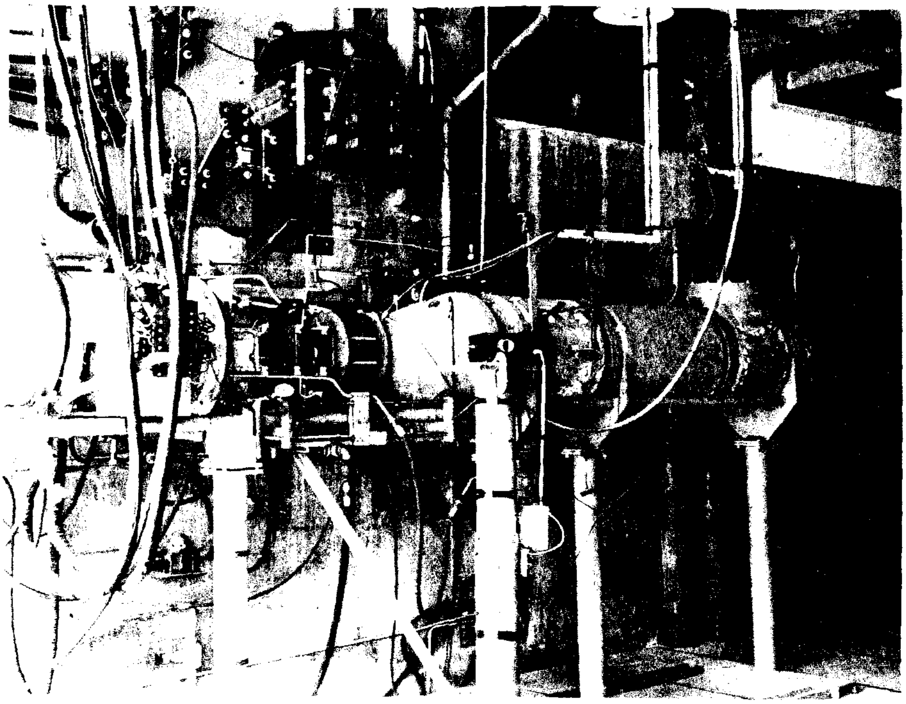


Figure 5: General View of Plenum Chamber, Convergent Nozzle, and Test Section.

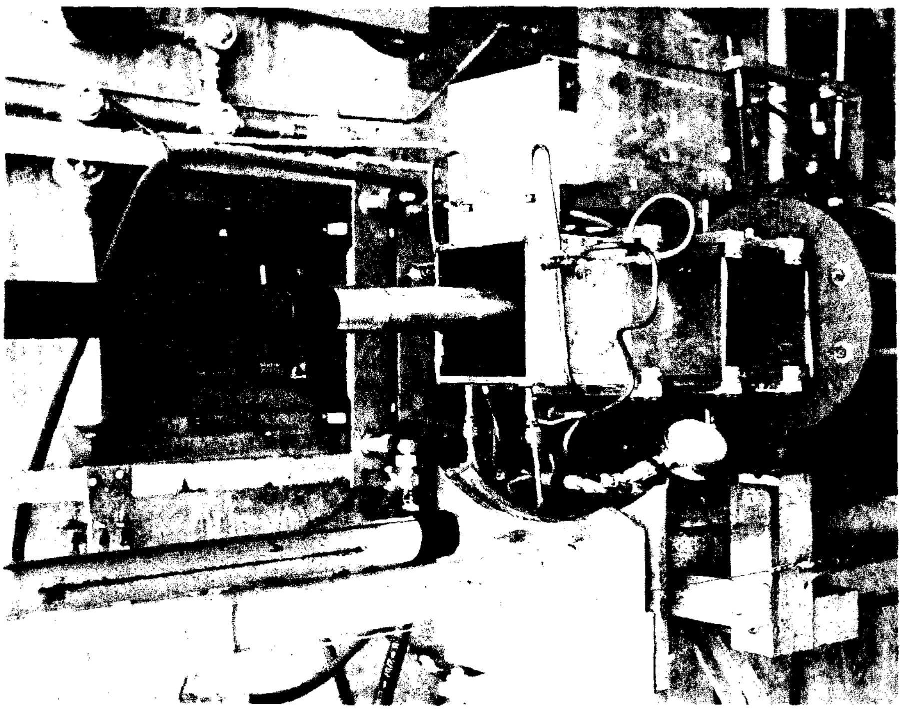


Figure 6. View of Flame Holder Withdrawn from Combustion Chamber, Note Trip Wire on Aluminum Nose Piece.

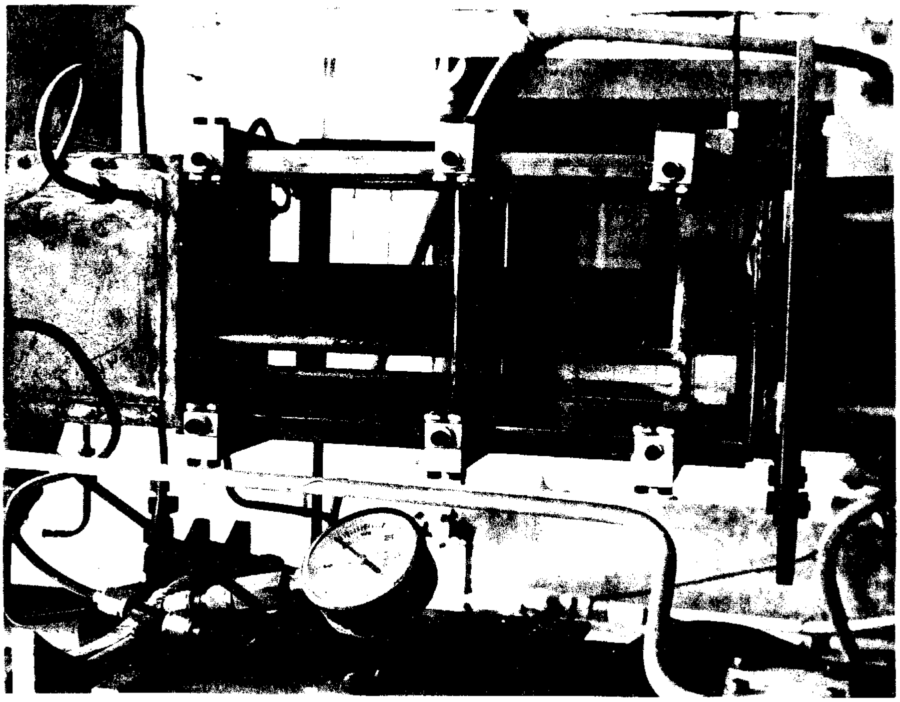


Figure 7. Side View of the Test Section

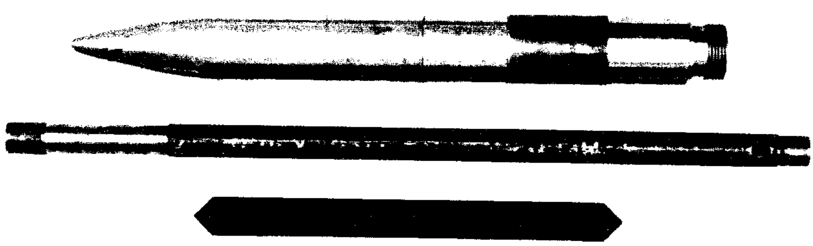


Figure 8. View of a. Nose Section of the Flame Holder with Tripwire, b. Main Electrode, c. Graphite Glow Bar Heater.

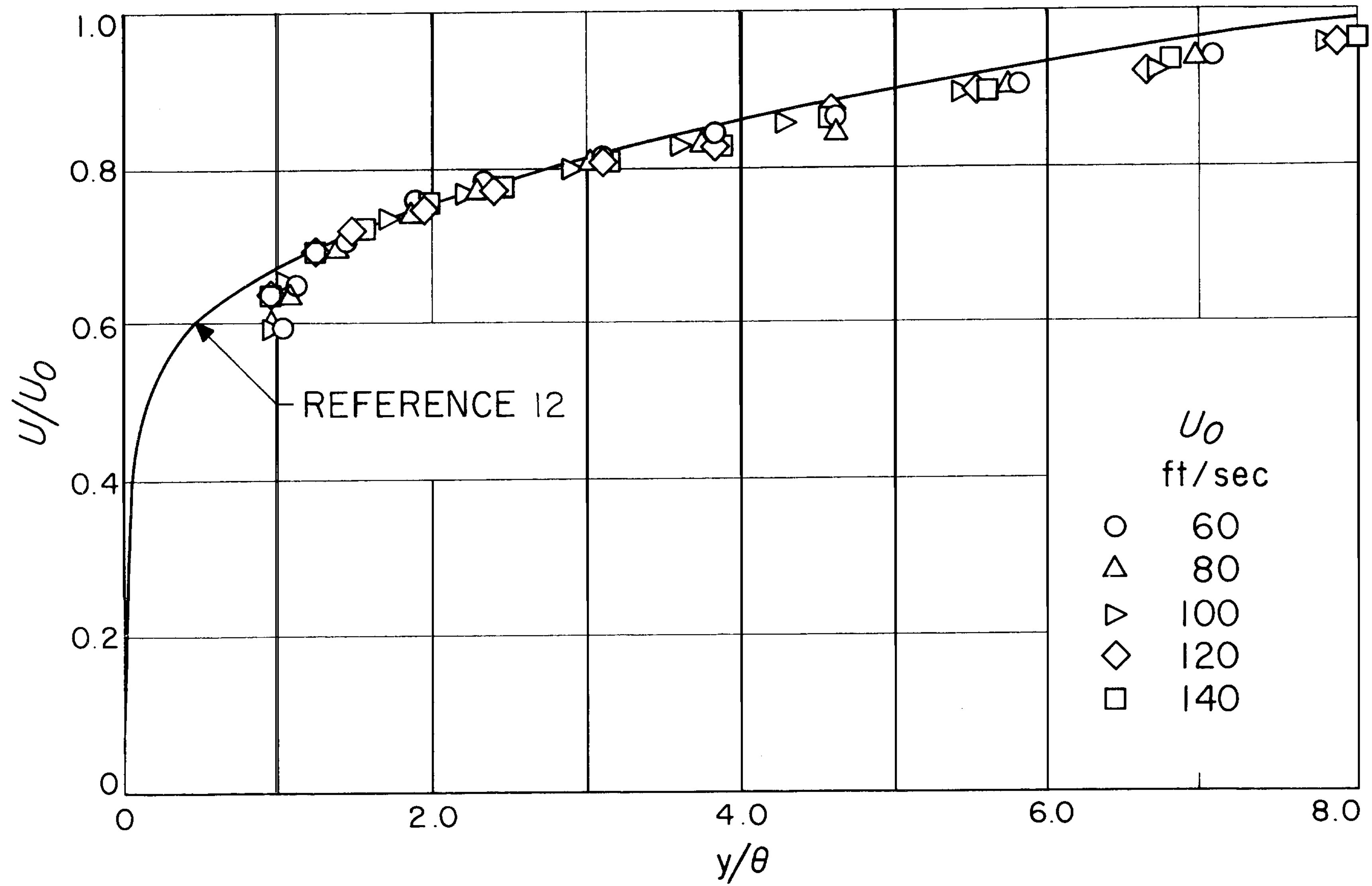


Figure 9. Non-Dimensional Velocity Distributions for Various Free Stream Velocities; T_o = 200°F, D_{TW} = .0201 in.

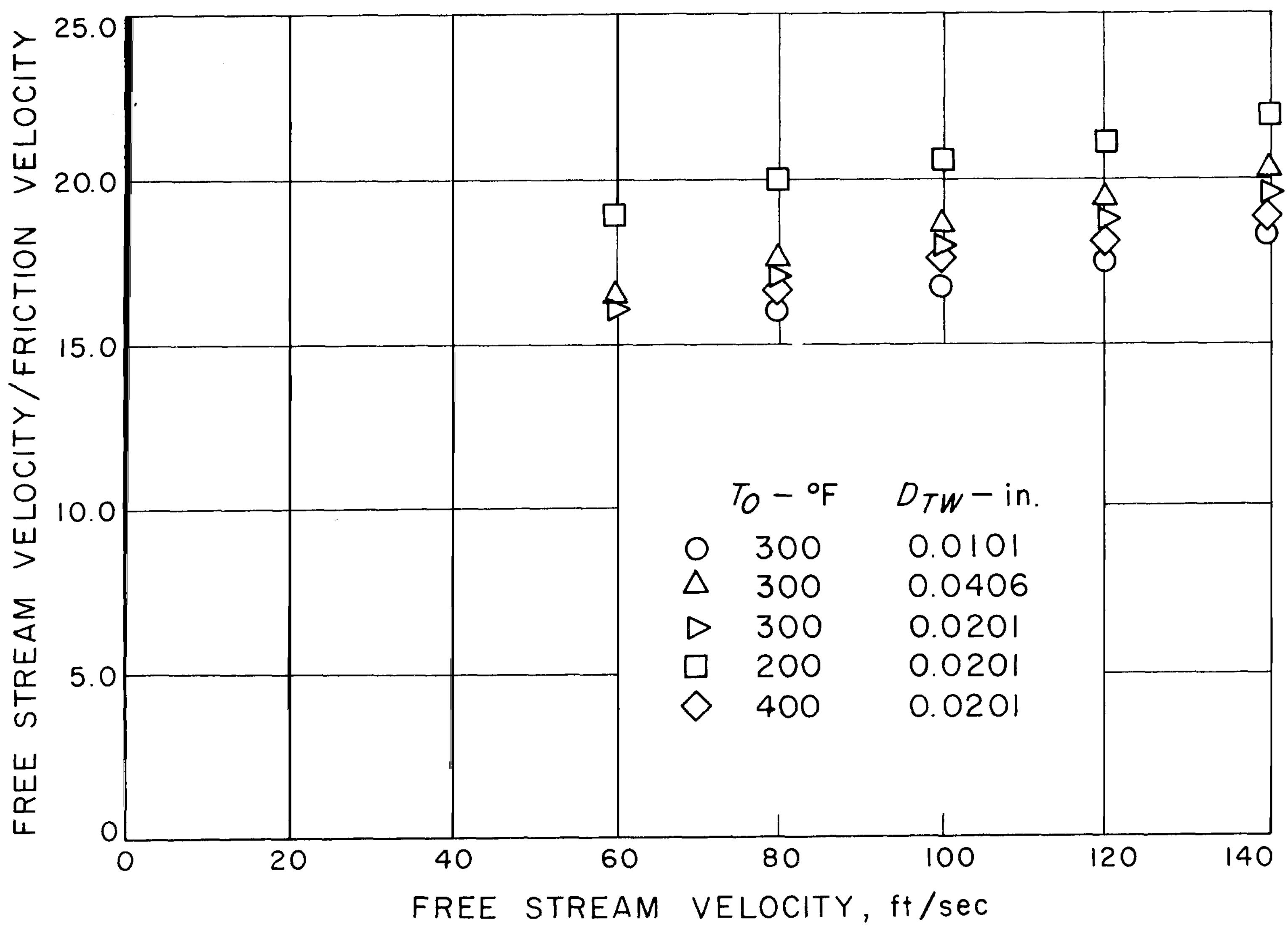


Figure 10. Dependence of Ratio of Free Stream Velocity to Friction Velocity on Free Stream Velocity.

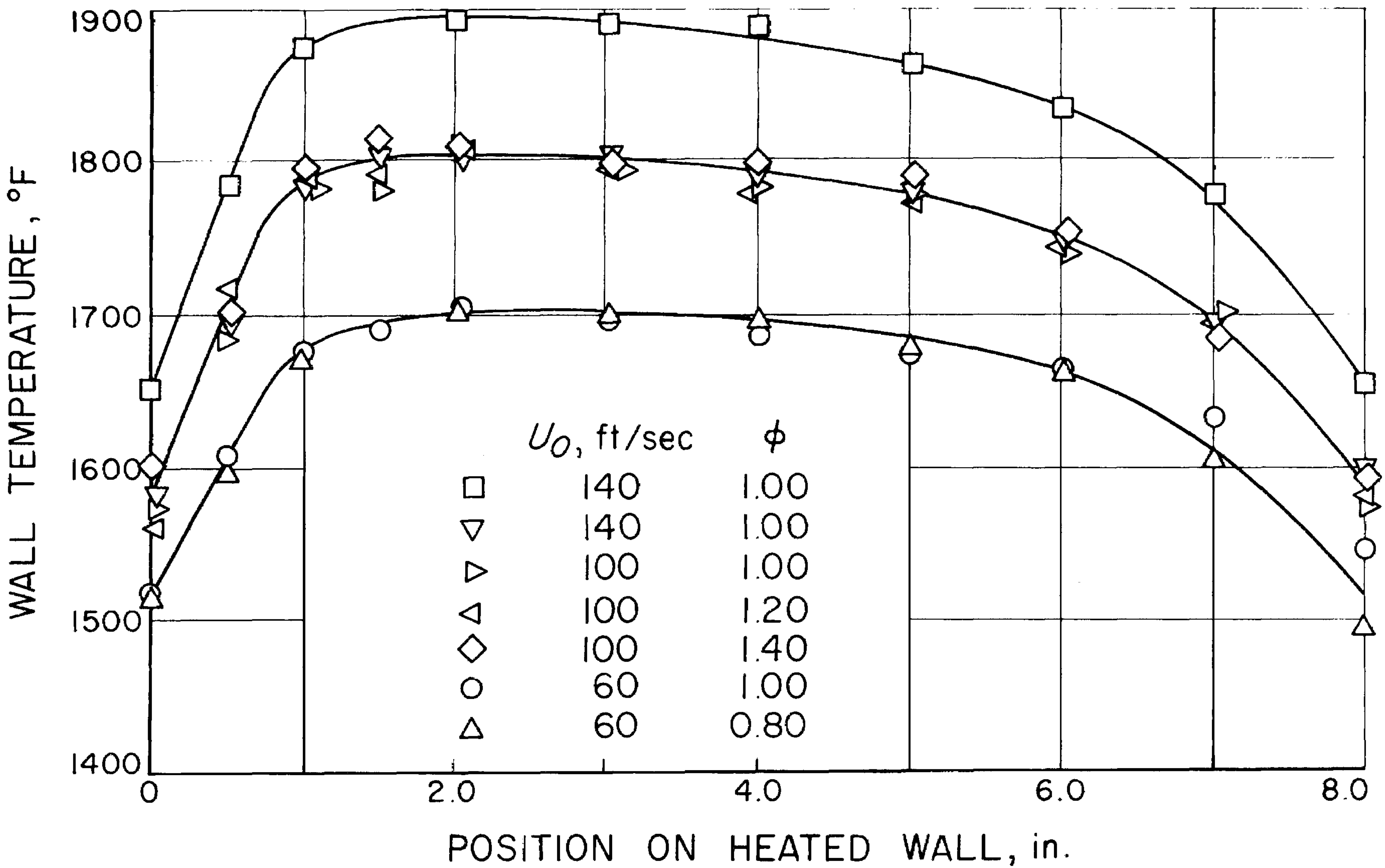


Figure 11. Wall Temperature Distributions under Various Heating Conditions; T_o = 300°F.

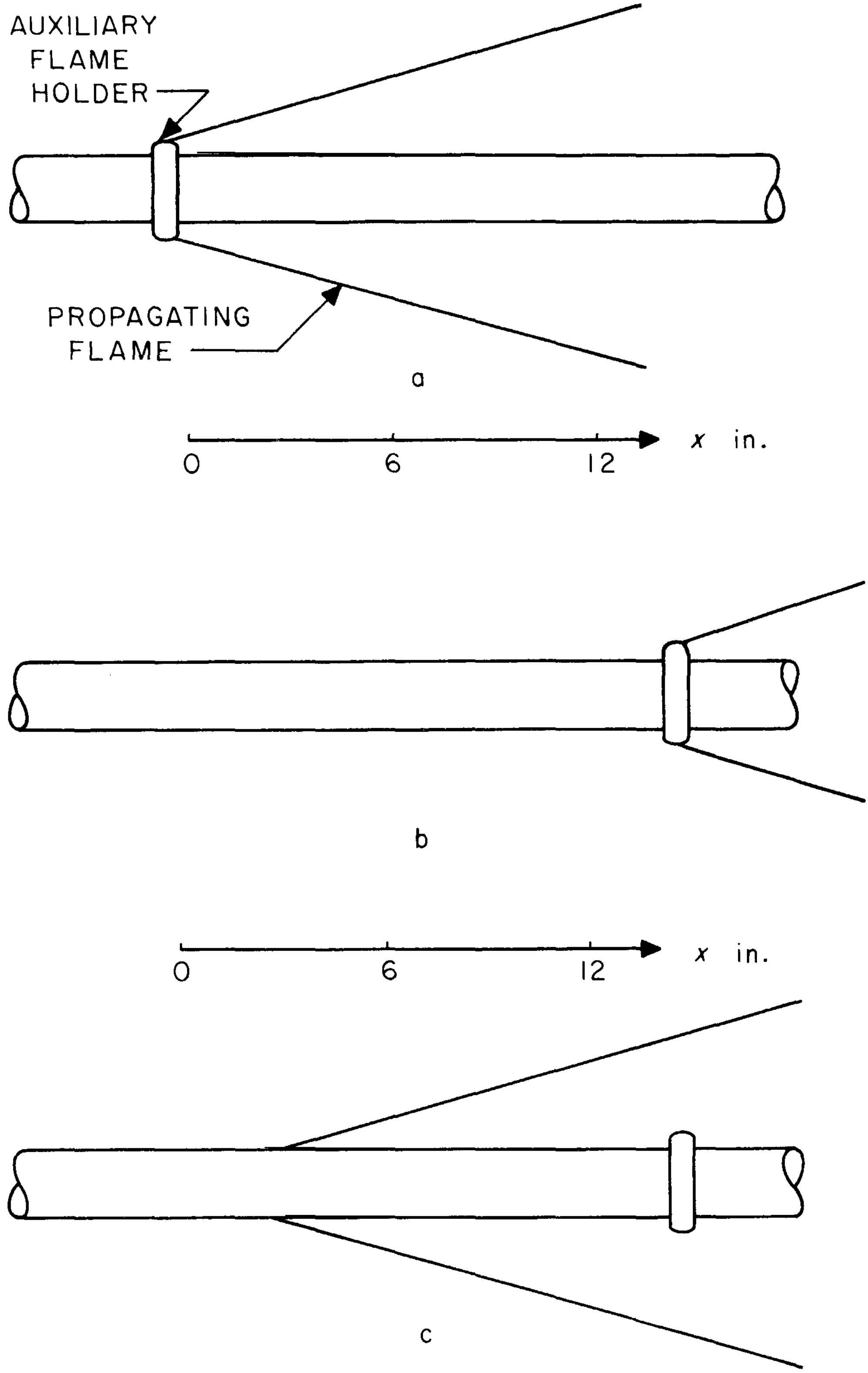


Figure 12. Illustration of Experimental Procedure; a. Heating, b. No Boundary Layer Stabilization, c. Boundary Layer Stabilization.

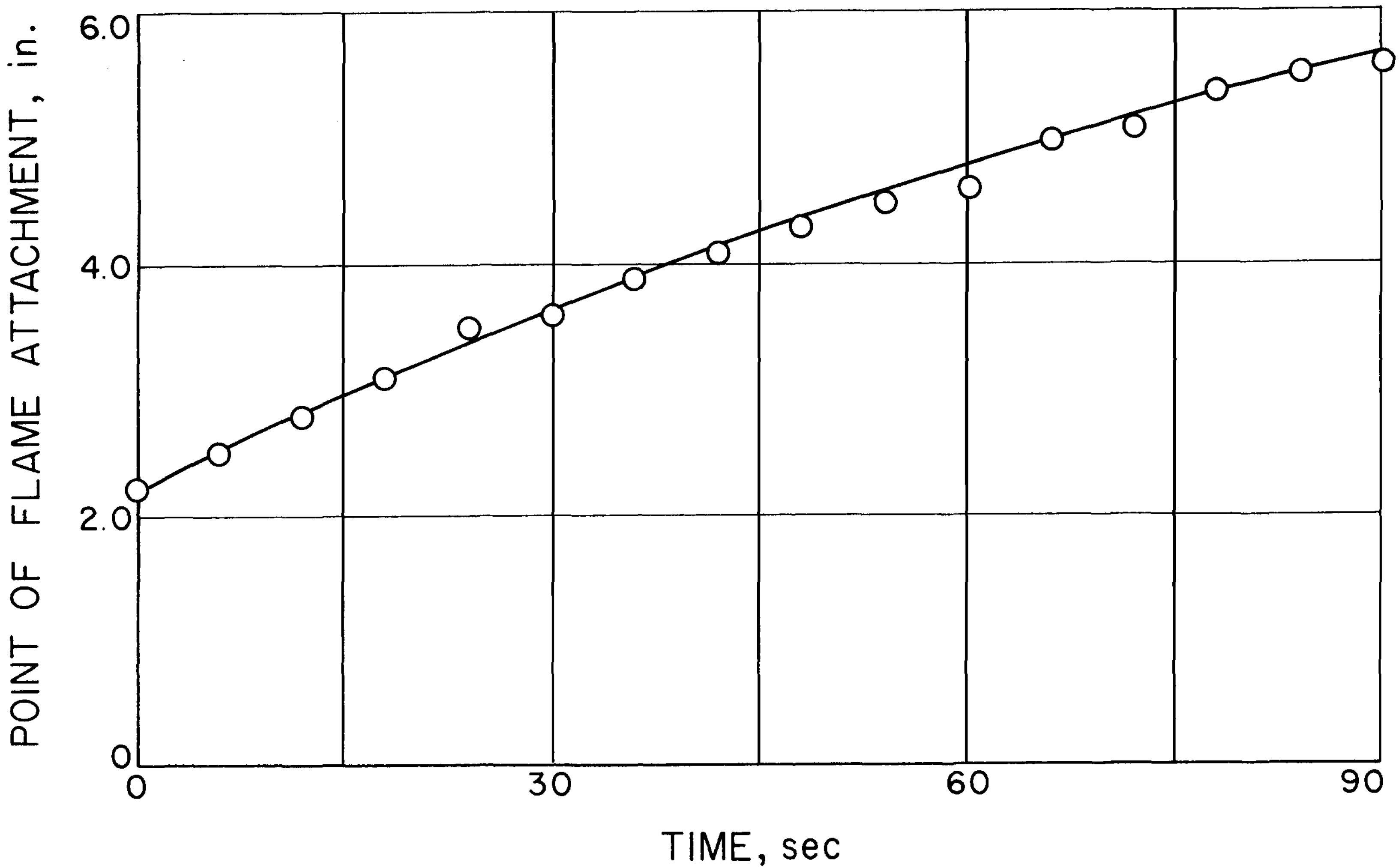


Figure 13. Dependence of Flame Attachment Position on Time; U_o = 100 ft/sec, ϕ = 1.00, T_w = 1831°F, T_o = 200°F, D_{TW} = 0.0201 in.

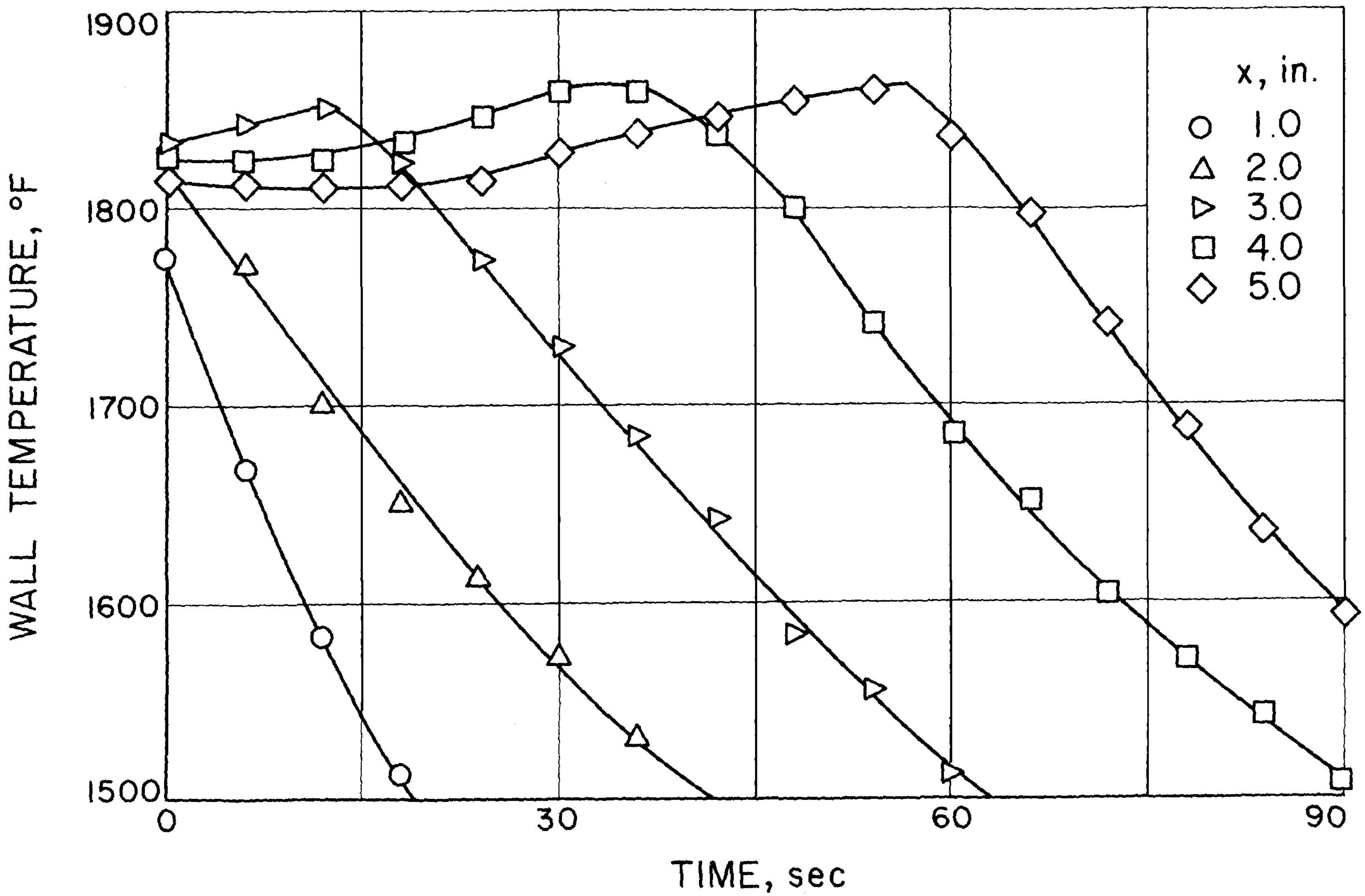


Figure 14. Dependence of Wall Temperature on Time at Various Wall Positions; U_a = 100 ft/sec, ϕ = 1.00, T_w = 1831°F, T_o = 300°F, D_{TW} = 0.0201 in.

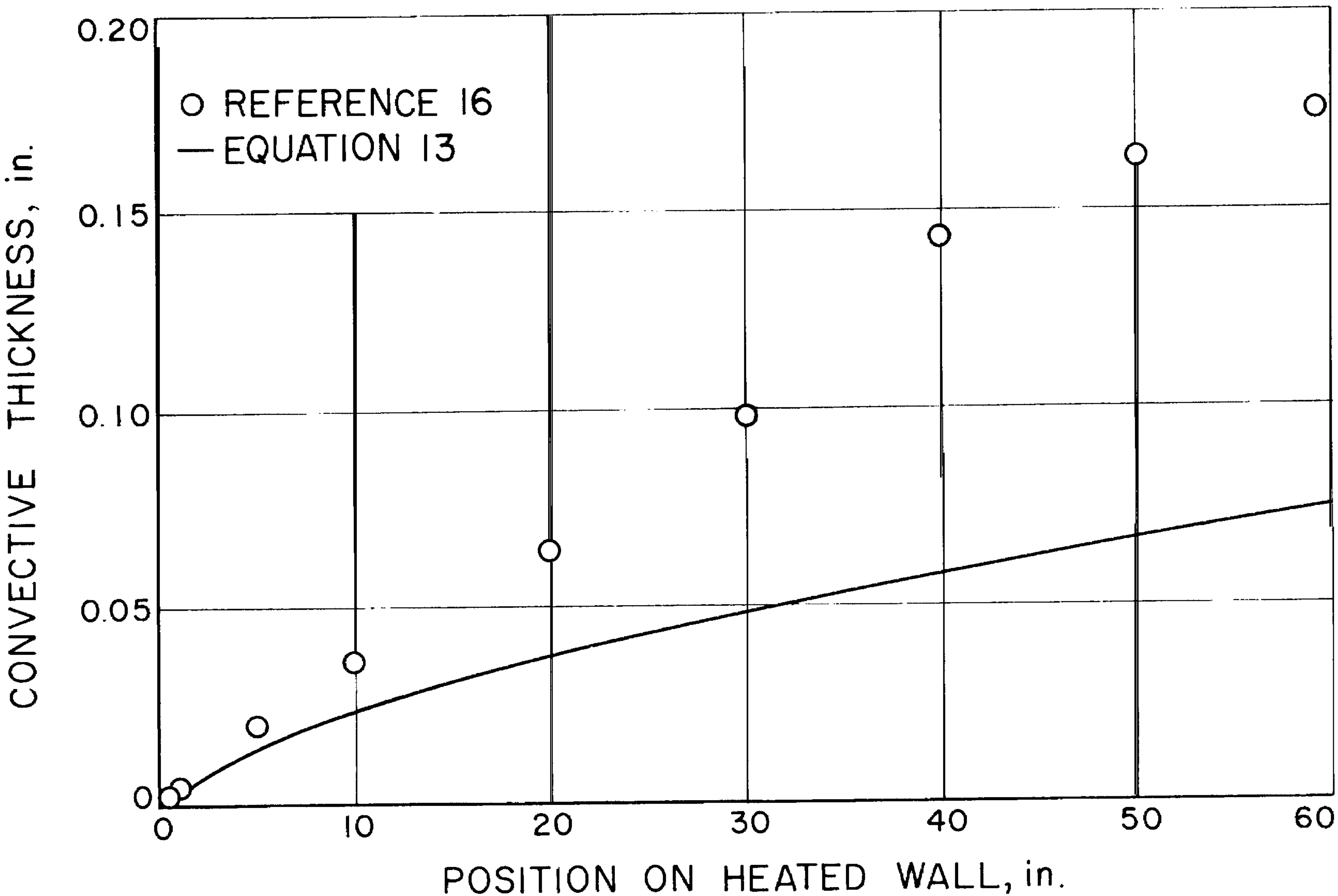


Figure 15. Dependence of Convective Thickness on Position, From (16); U_o = 25 ft/sec, U_o/U_τ = 26.2, $T_w - T_o$ = 27°F.

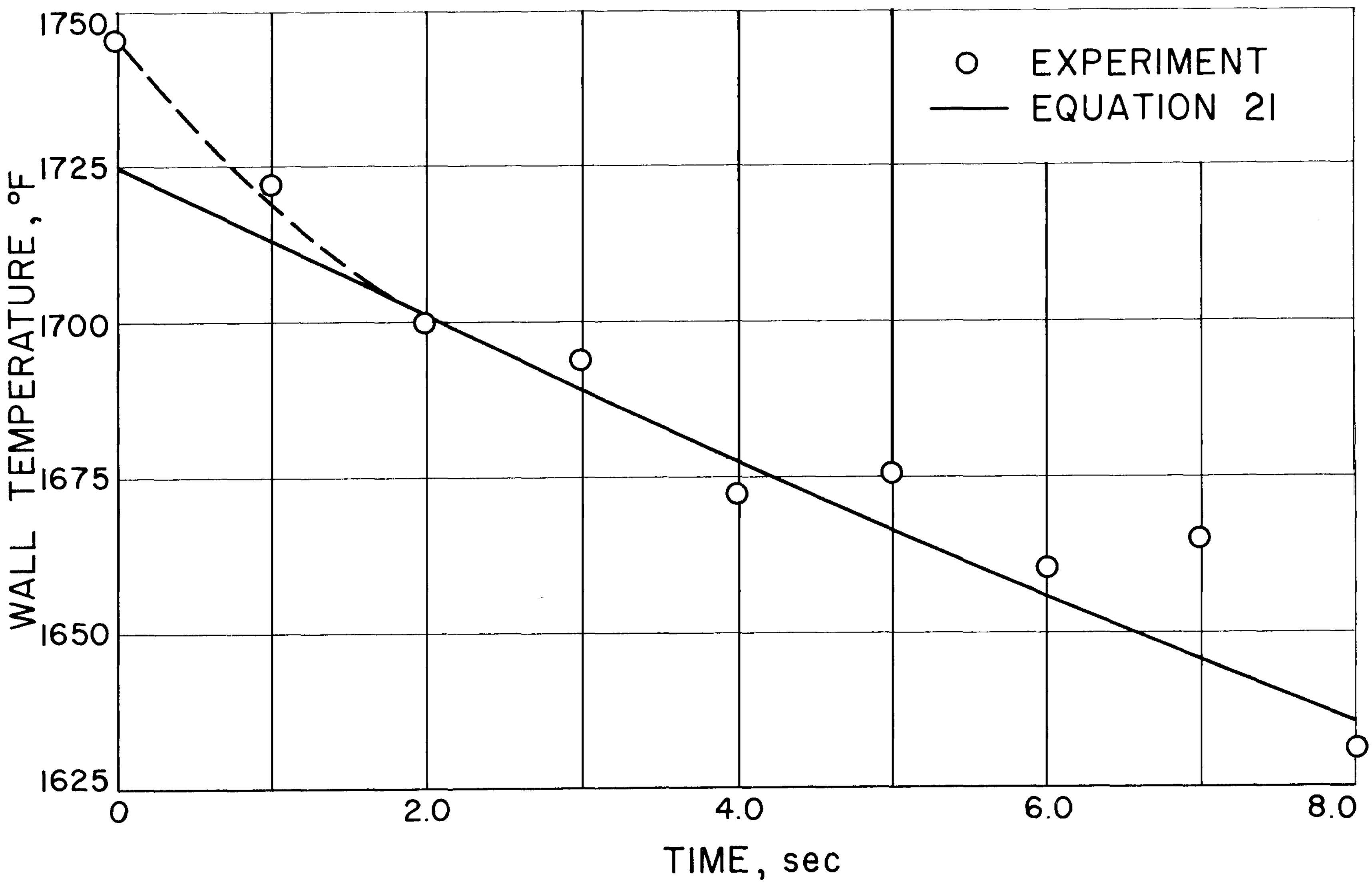


Figure 16. Dependence of Wall Temperature on Time; $X = 3.2$ in., $U_o = 100$ ft/sec.

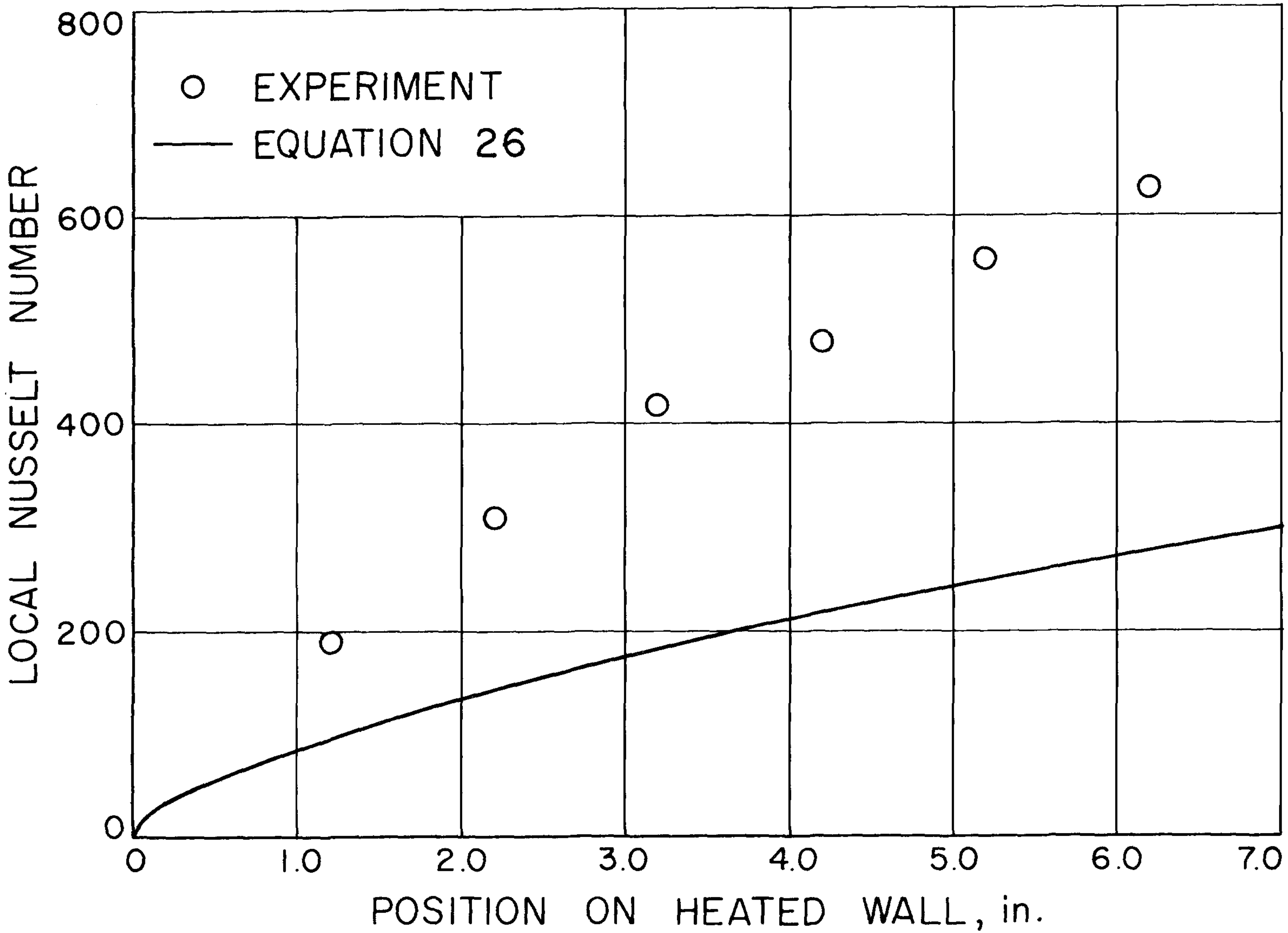


Figure 17. Dependence of Local Nusselt Number on Position; $U_o = 100$ ft/sec, $U_o/U_\tau = 18.0$, $T_w - T_o = 1445^\circ$F, $T_o = 300^\circ$F.

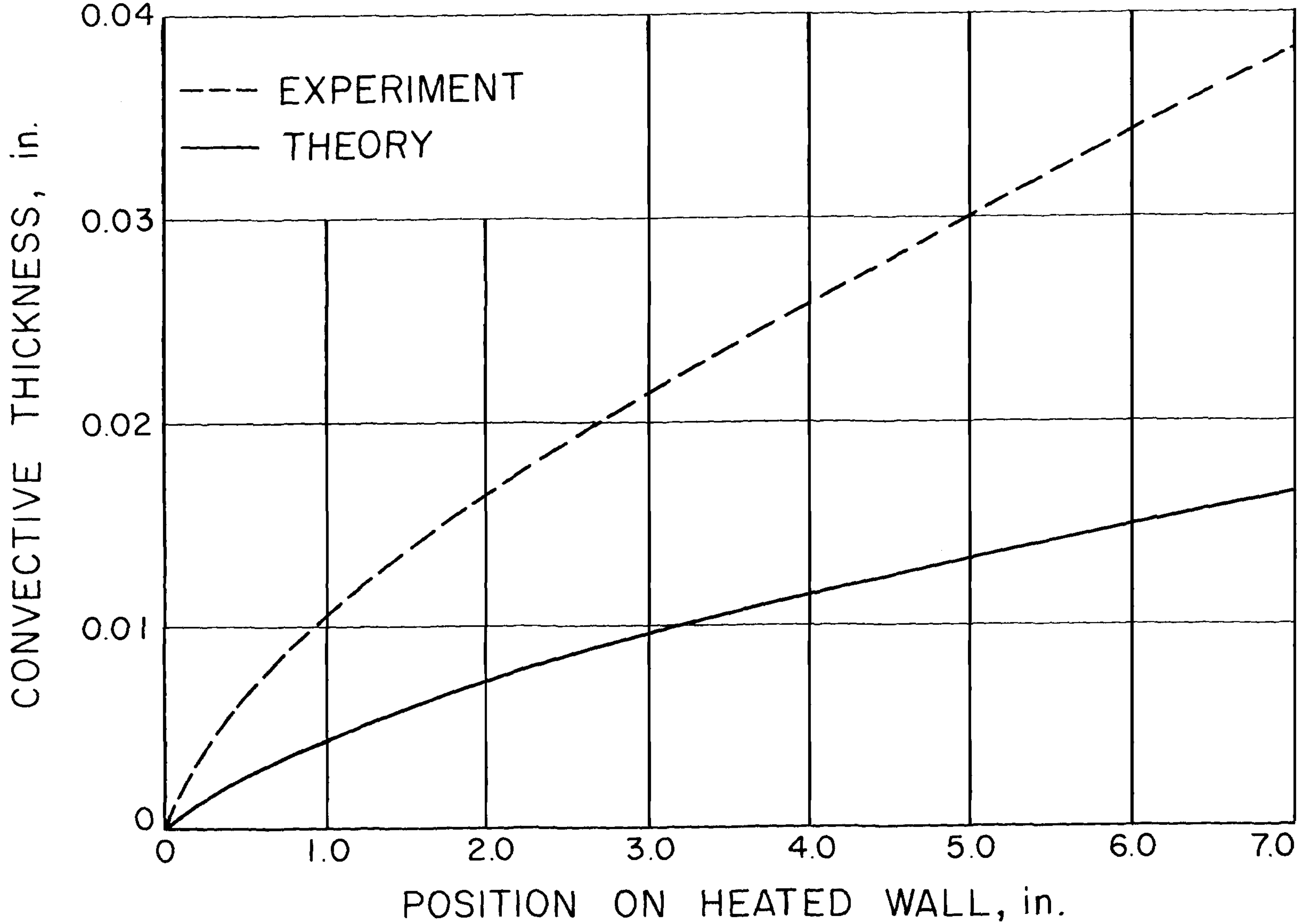
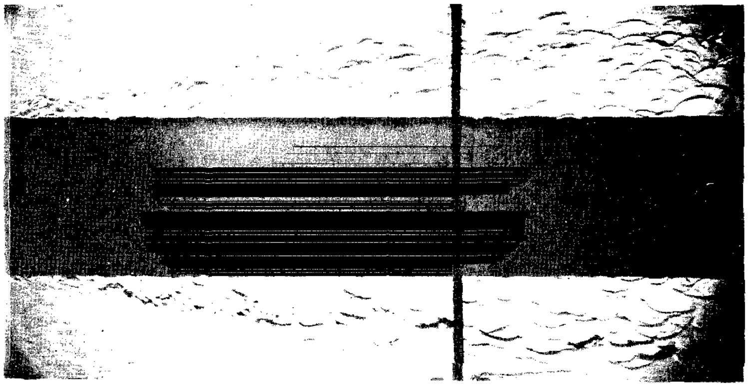
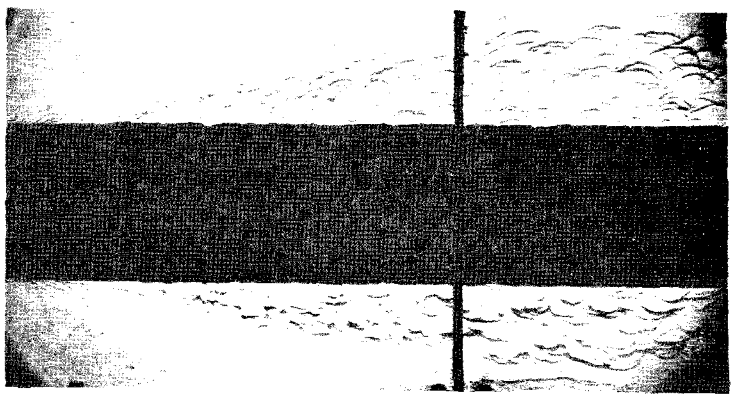


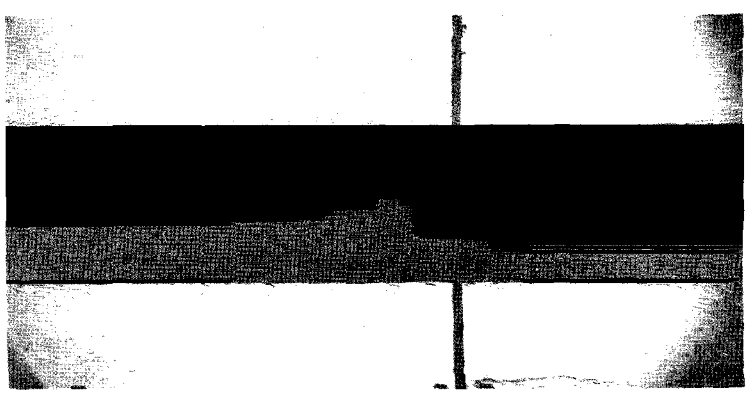
Figure 18. Dependence of Convective Thickness on Position; $U_o = 100$ ft/sec, $U_o/U_\tau = 18.0$, $T_w - T_o = 1445^\circ F$, $T_o = 300^\circ F$.



(a)

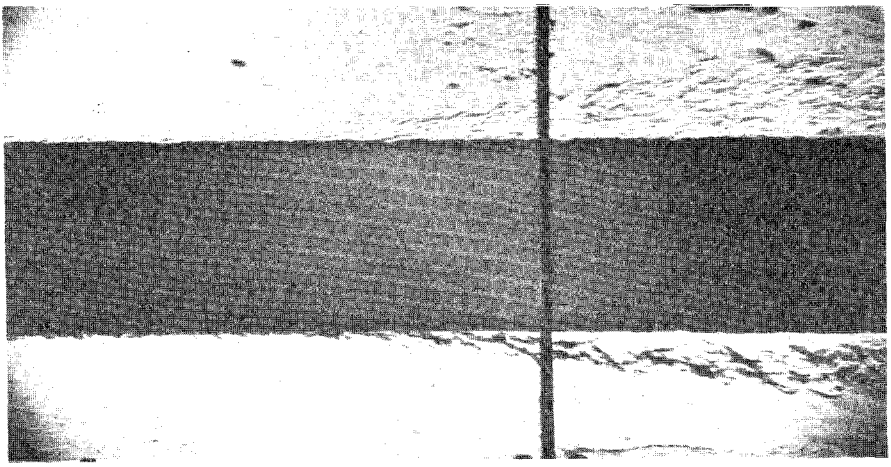


(b)

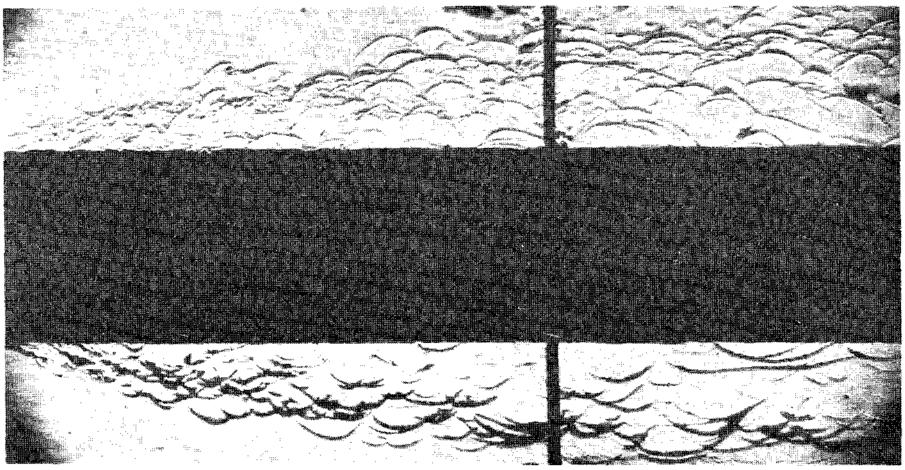


(c)

Figure 19. Turbulent Boundary Layer; D_{TW} = 0.0[illegible]01 in., T_o = 300°F., ϕ = 1.00, U_o = 85 ft./s[illegible]c.; a) T_w = 19[illegible]0°F., b) T_w = 1771°F., c) T_w = 17[illegible]5°F.

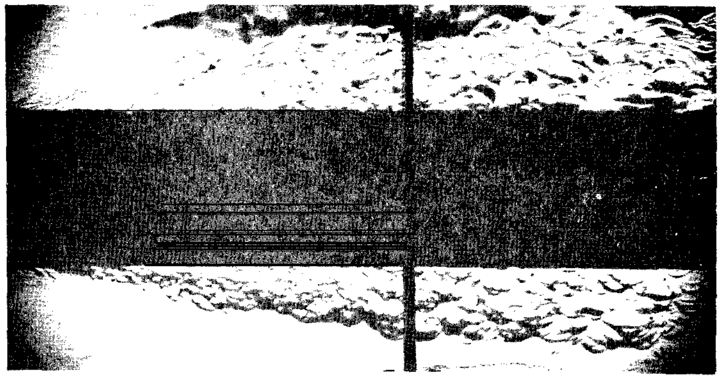


(a)

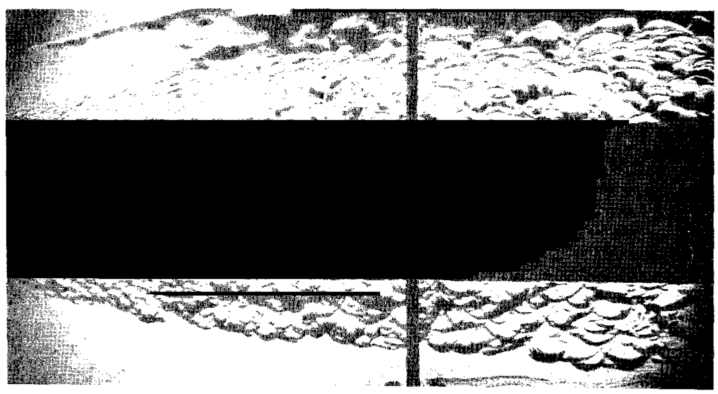


(b)

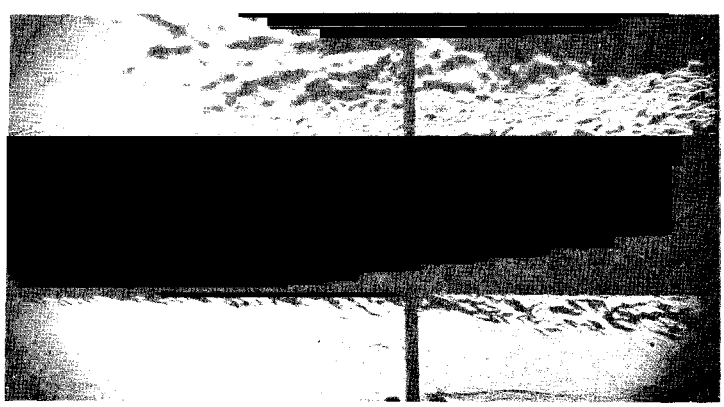
Figure 20. Turbulent Boundary Layer; $D_{Tw} = 0.0201$ in., $T_o = 300^{\circ}F.$, $\phi = 1.00$;
a) $U_o = 150$ ft./sec., $T_w = 1990^{\circ}F.$,
b) $U_o = 60$ ft./sec., $T_w = 1759^{\circ}F.$



(a)



(b)



(c)

Figure 21. Turbulent Boundary Layer; D_{rw} = 0.0201 in., T_o = 300°F., U_o = 100 ft./sec., T_w = 1870°F.
a) ϕ = 1.50, b) ϕ = 1.25, c) ϕ = 0.80.

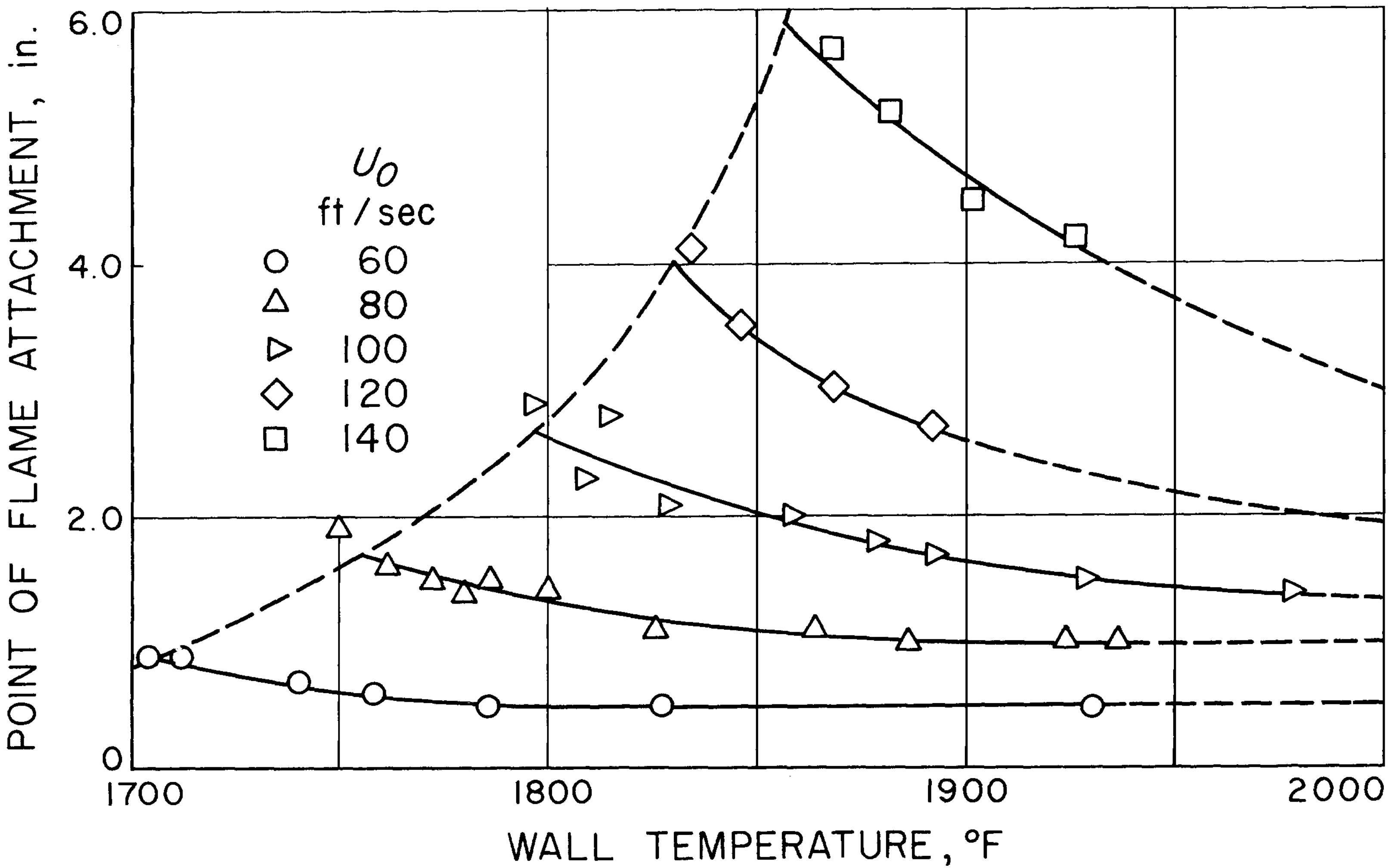


Figure 22. Dependence of Flame Attachment Position on Wall Temperature for Various Free Stream Velocities; $\phi = 1.00$, $T_0 = 300^{\circ}F$, $D_{TW} = 0.0201$ in.

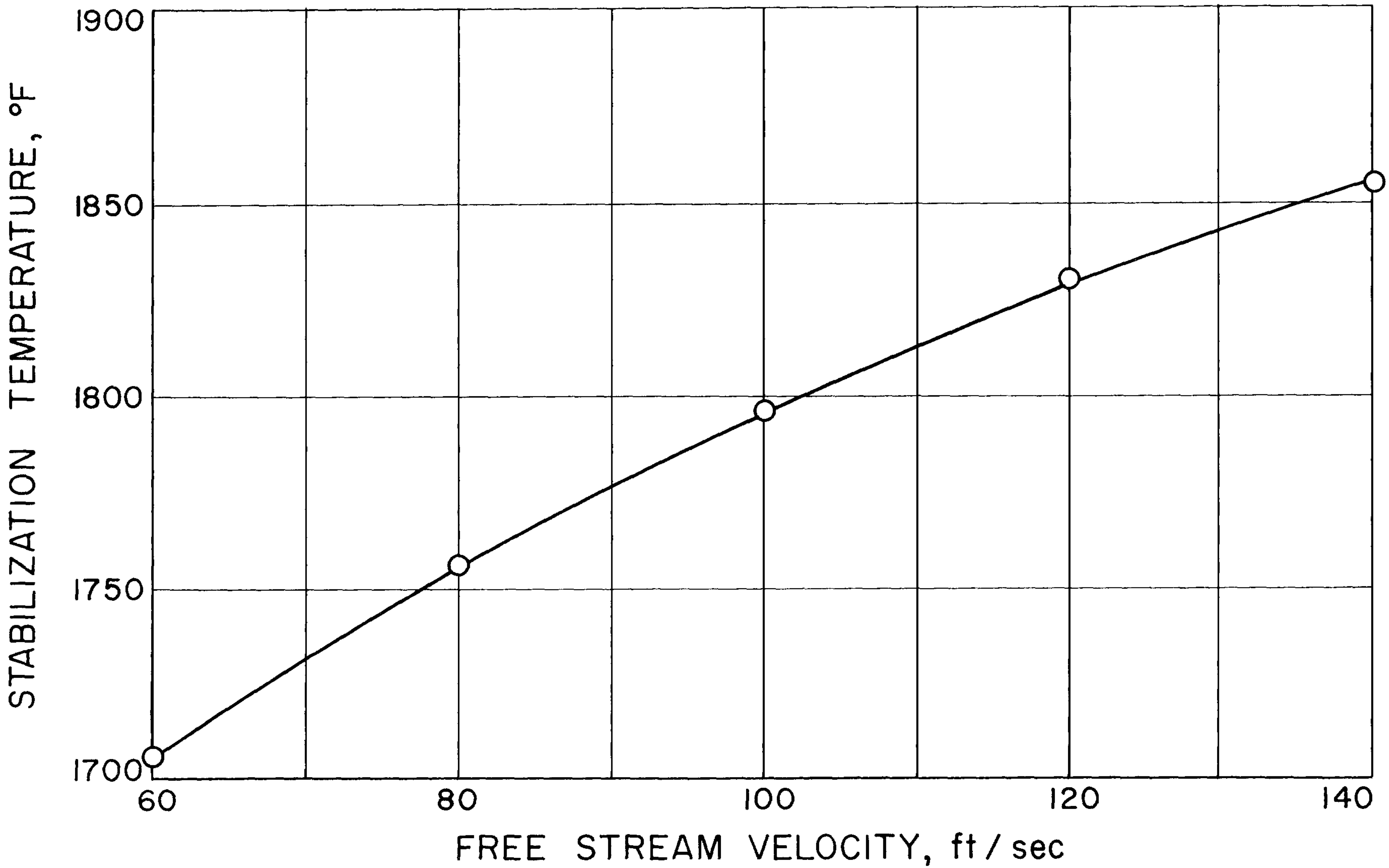


Figure 23. Dependence of Minimum Wall Temperature for Stabilization on Free Stream Velocity; ϕ = 1.00, T_o = 300°F, D_{TW} = 0.0201 in.

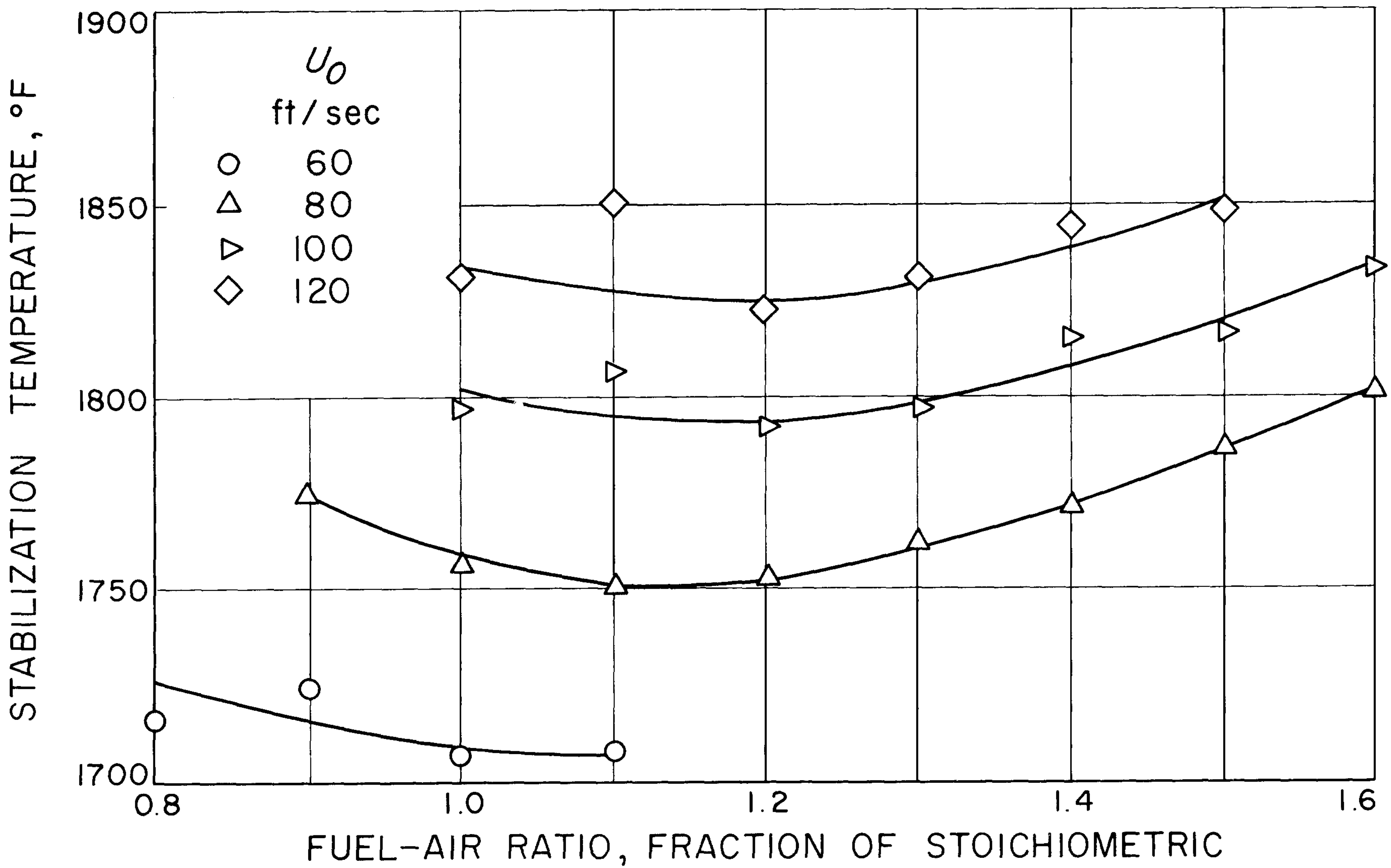


Figure 24. Dependence of Minimum Wall Temperature for Stabilization on Fuel-Air Ratio For Various Free Stream Velocities; $T_o = 300^oF$, $D_{TW} = 0.0201$ in.

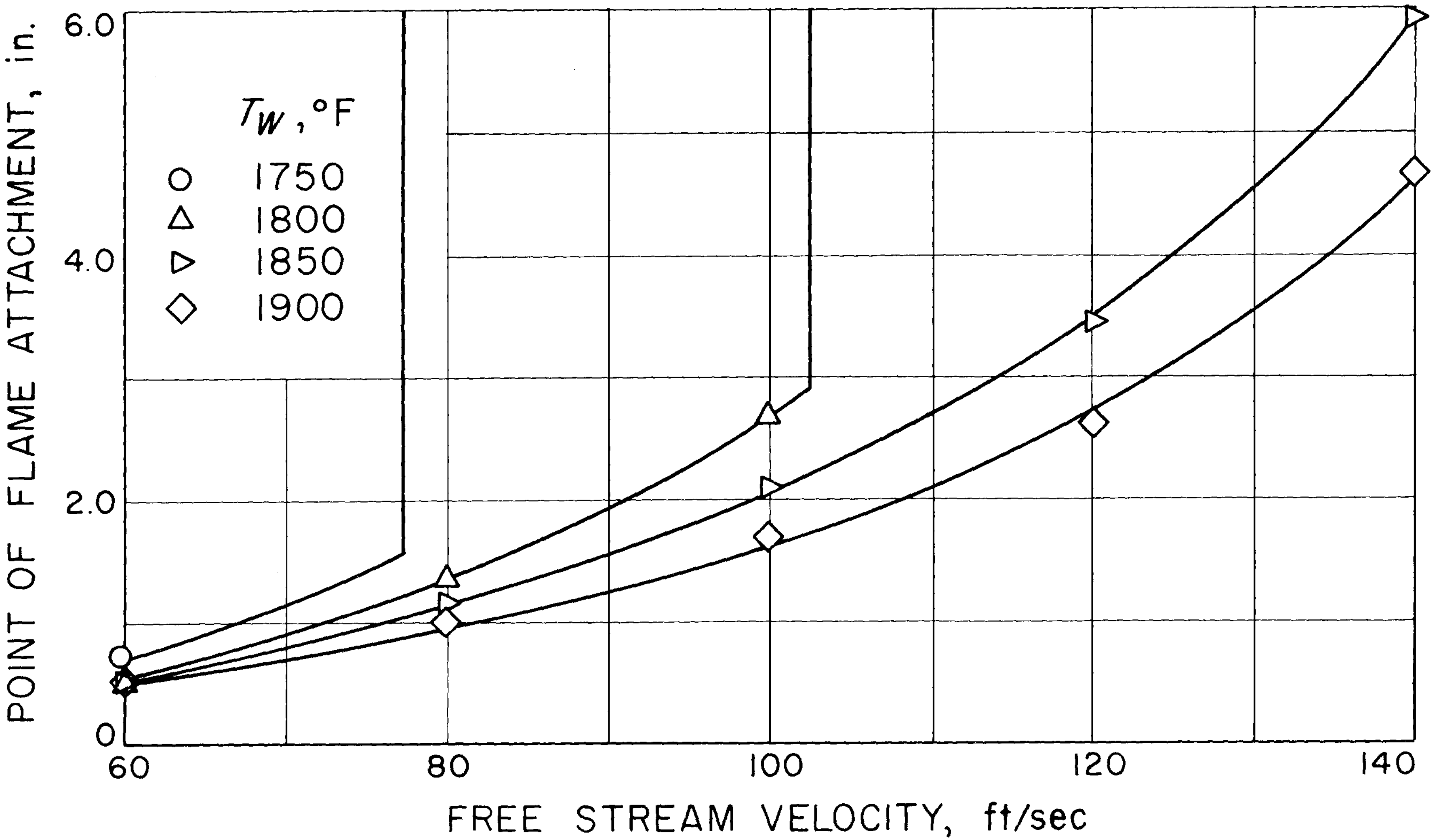


Figure 25. Dependence of Flame Attachment Position on Free Stream Velocity for Various Wall Temperatures; $\phi = 1.00$, $T_0 = 300^\circ F$, $D_{TW} = 0.0201$ in.

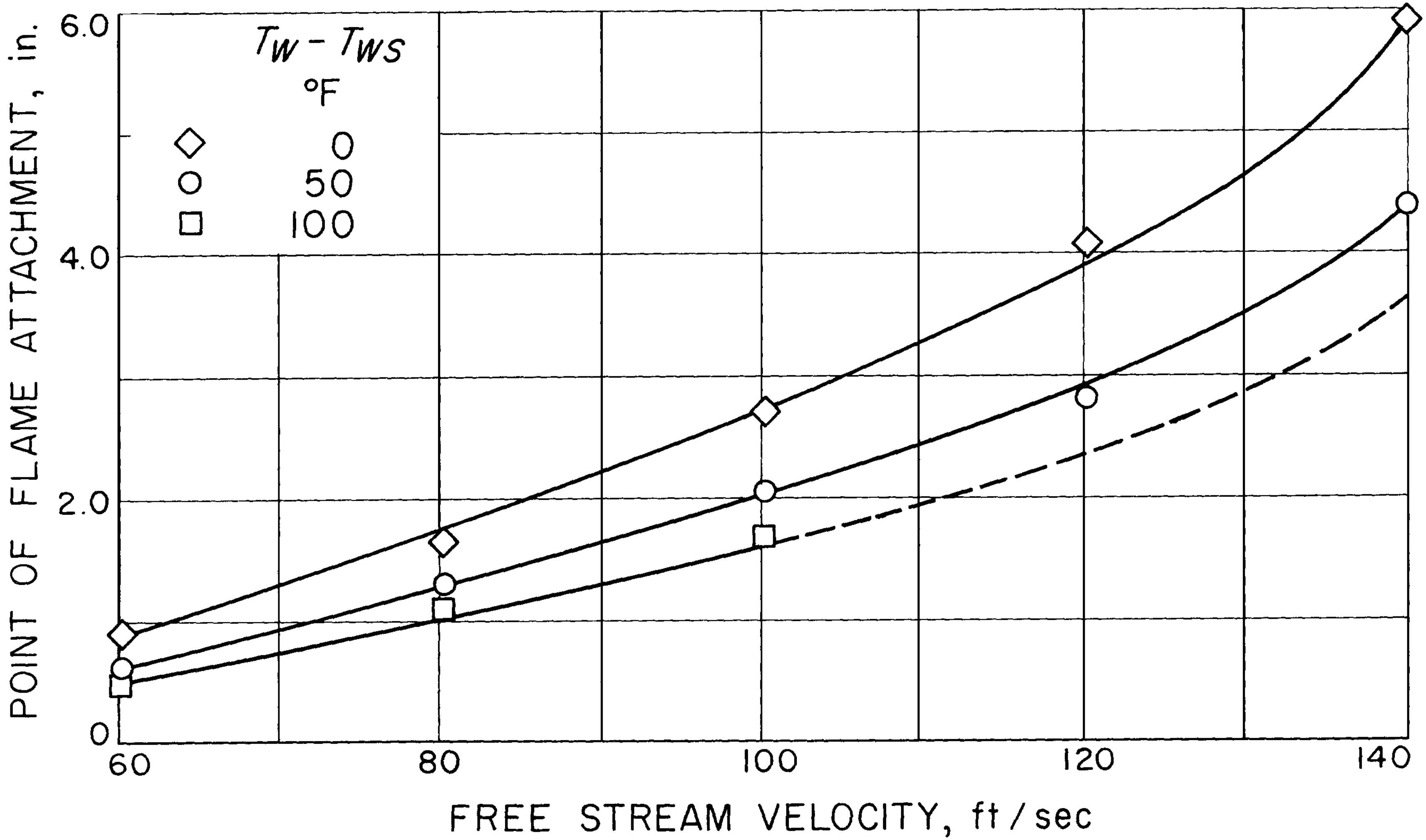


Figure 26. Dependence of Flame Attachment Position on Free Stream Velocity For Values of the Parameter $T_w - T_{ws}$; $\phi = 1.00$, $T_o = 300^{\circ}F$, $D_{TW} = 0.0201$ in.

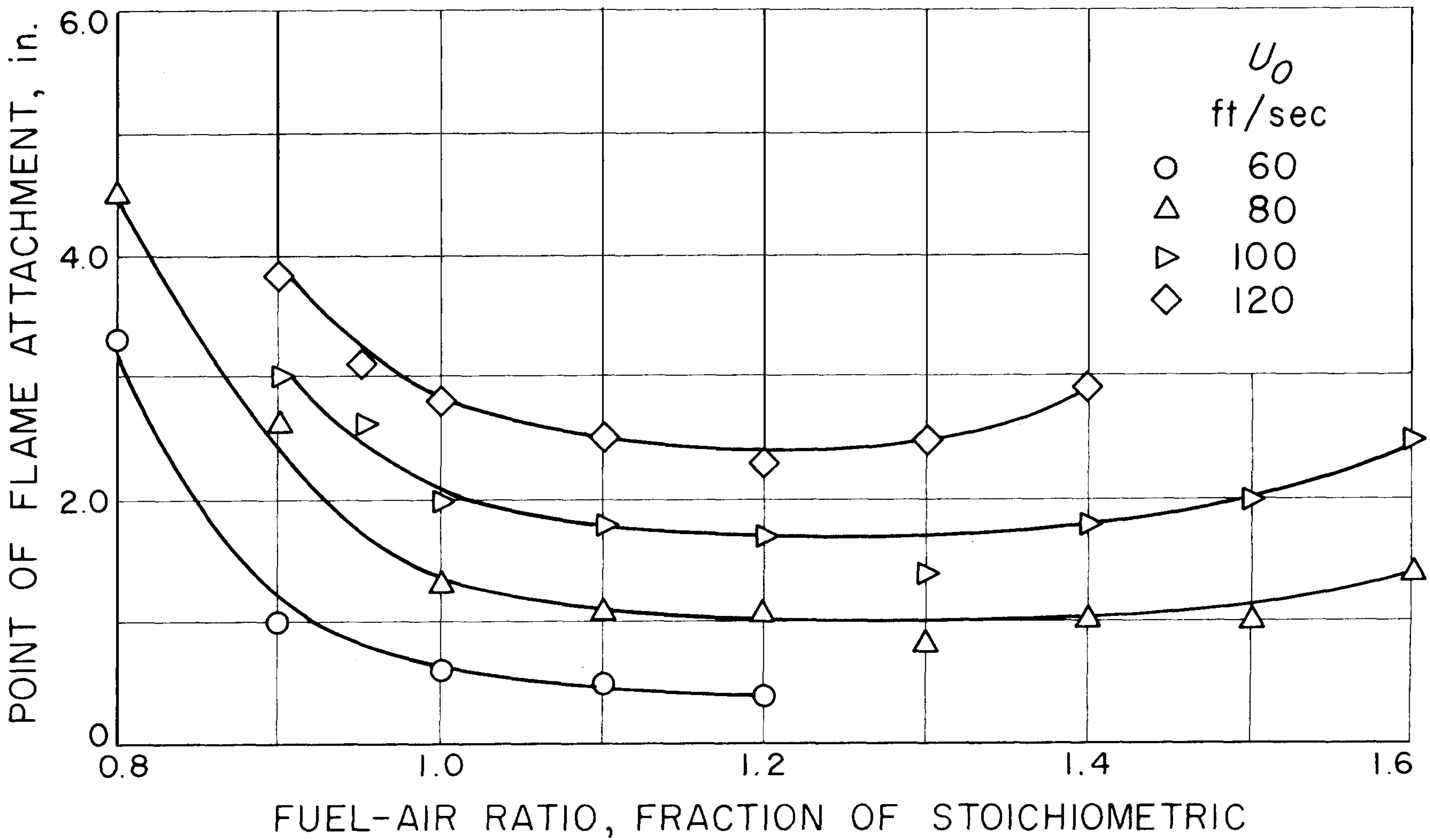


Figure 27. Dependence of Flame Attachment Position on Fuel-Air Ratio for Various Free Stream Velocities; $T_w - T_{ws} = 50^{o}F$, $T_o = 300^{o}F$, $D_{Tw} = 0.0201$ in.

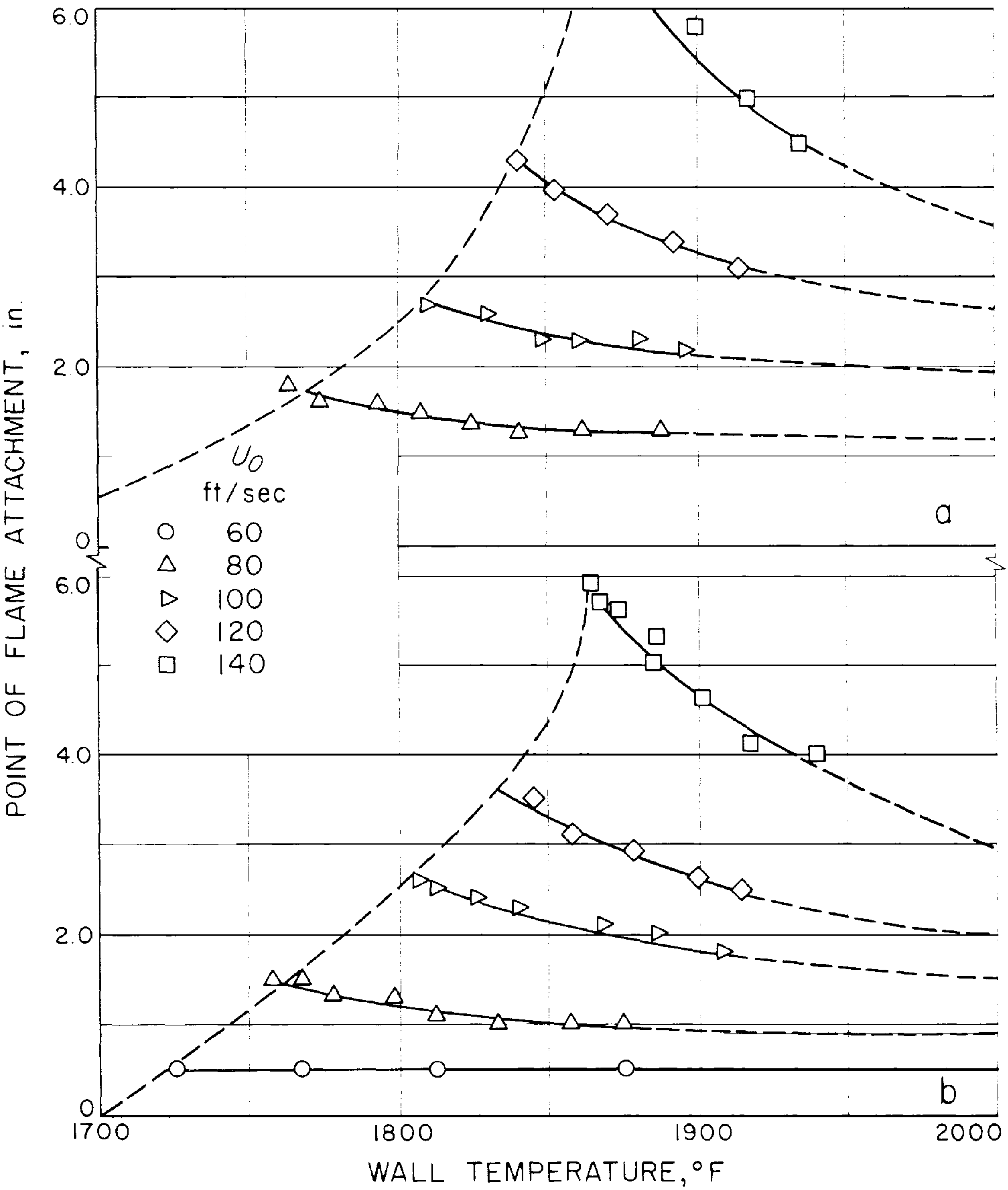


Figure 28. Dependence of Flame Attachment Position on Wall Temperature for Various Free Stream Velocities; $\phi = 1.00$, $T_0 = 300^{\circ}F$; a. $D_{TW} = 0.0101$ in., b. $D_{TW} = 0.0406$ in.

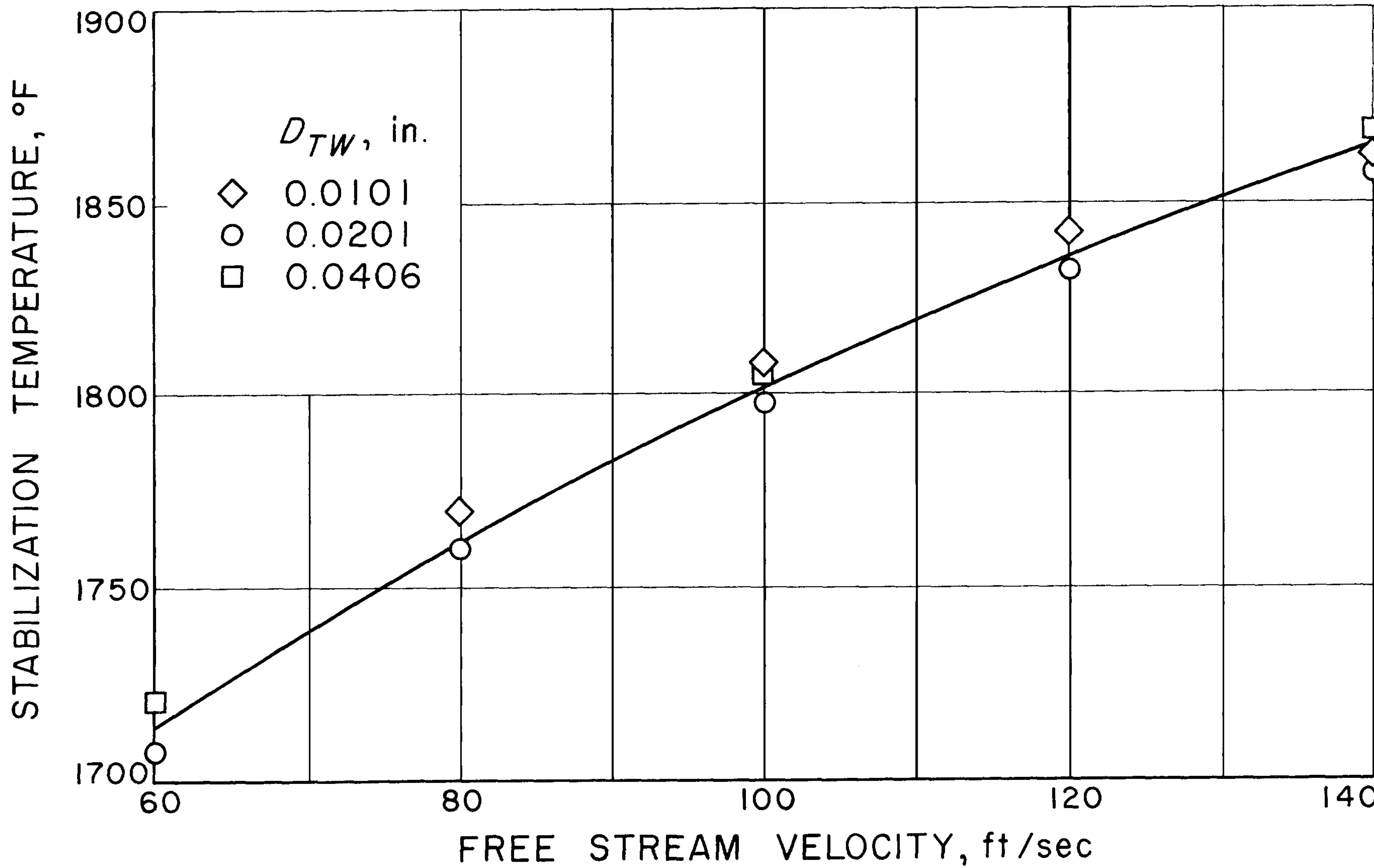


Figure 29. Dependence of Minimum Wall Temperature for Stabilization on Free Stream Velocity for Various Trip Wires; ϕ = 1.00, T_o = 300°F.

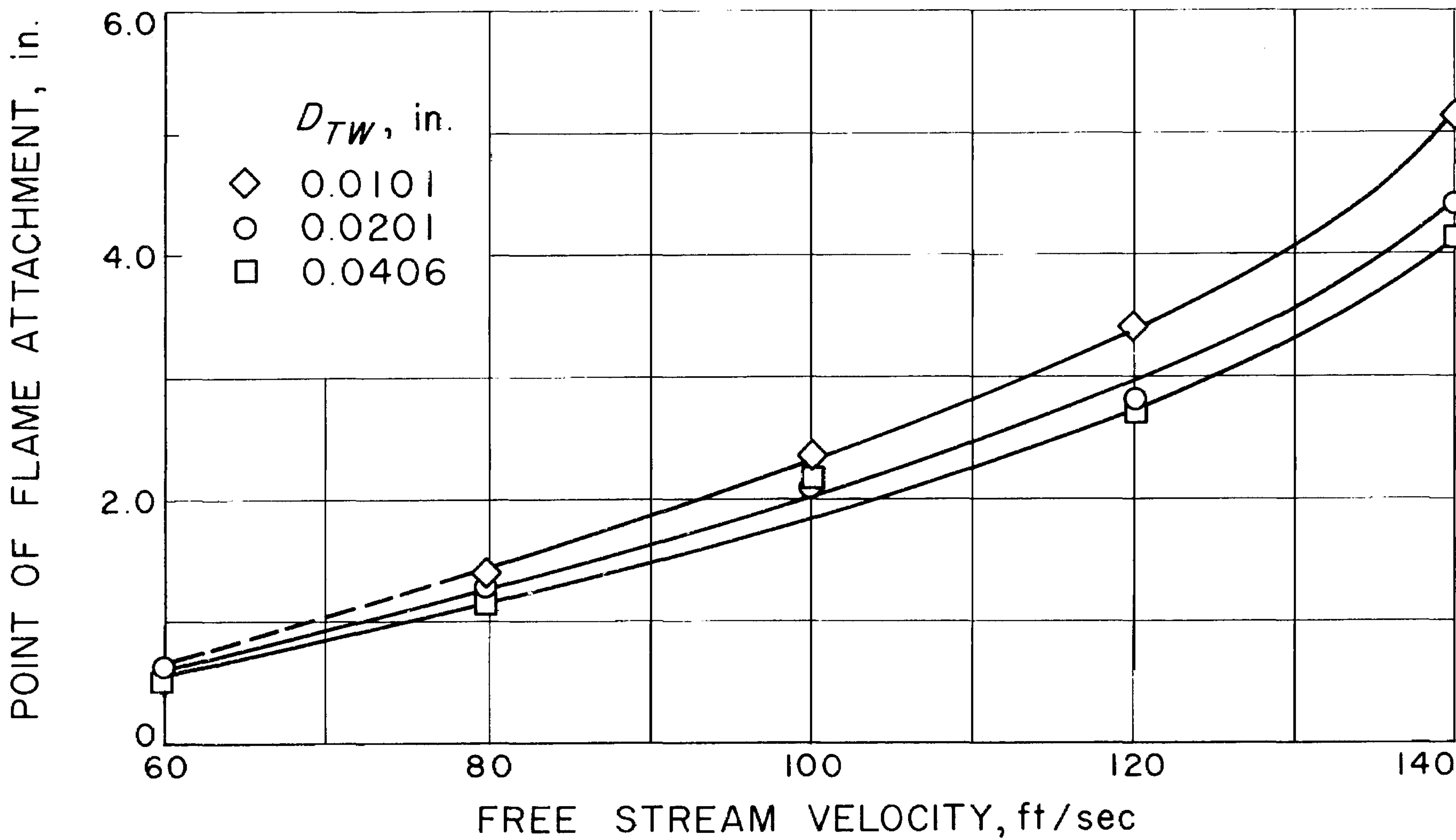


Figure 30. Dependence of Flame Attachment Position on Free Stream Velocity for Various Trip Wires; $\phi = 1.00$, $T_o = 300^oF$, $T_w - T_{w_1} = 50^oF$.

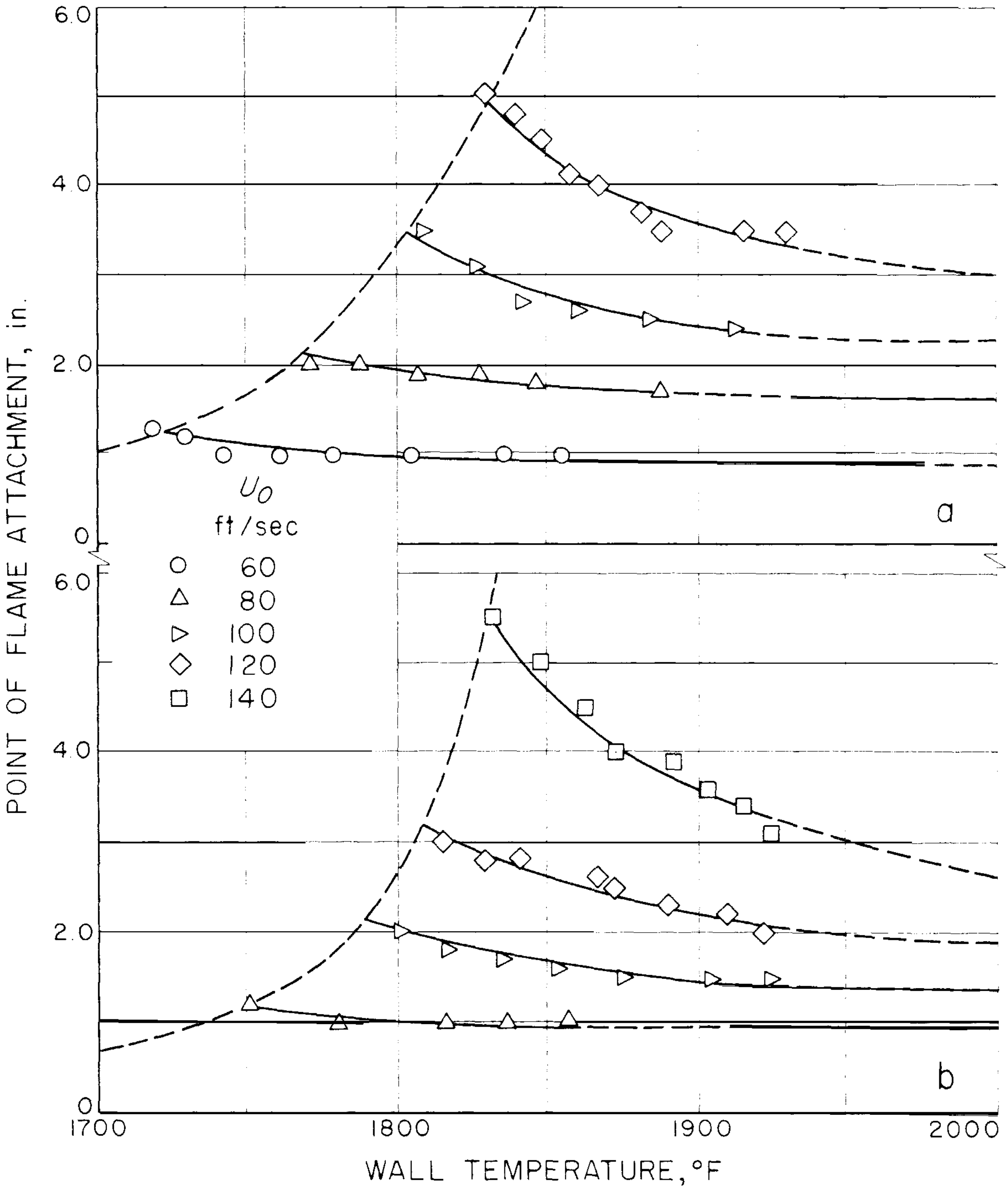


Figure 31. Dependence of Flame Attachment Position on Wall Temperature for Various Free Stream Velocities; ϕ = 1.00, D_{TW} = 0.0201 in.; a. $\overline{T_0}$ = 200°F, b. $\overline{T_0}$ = 400°F.

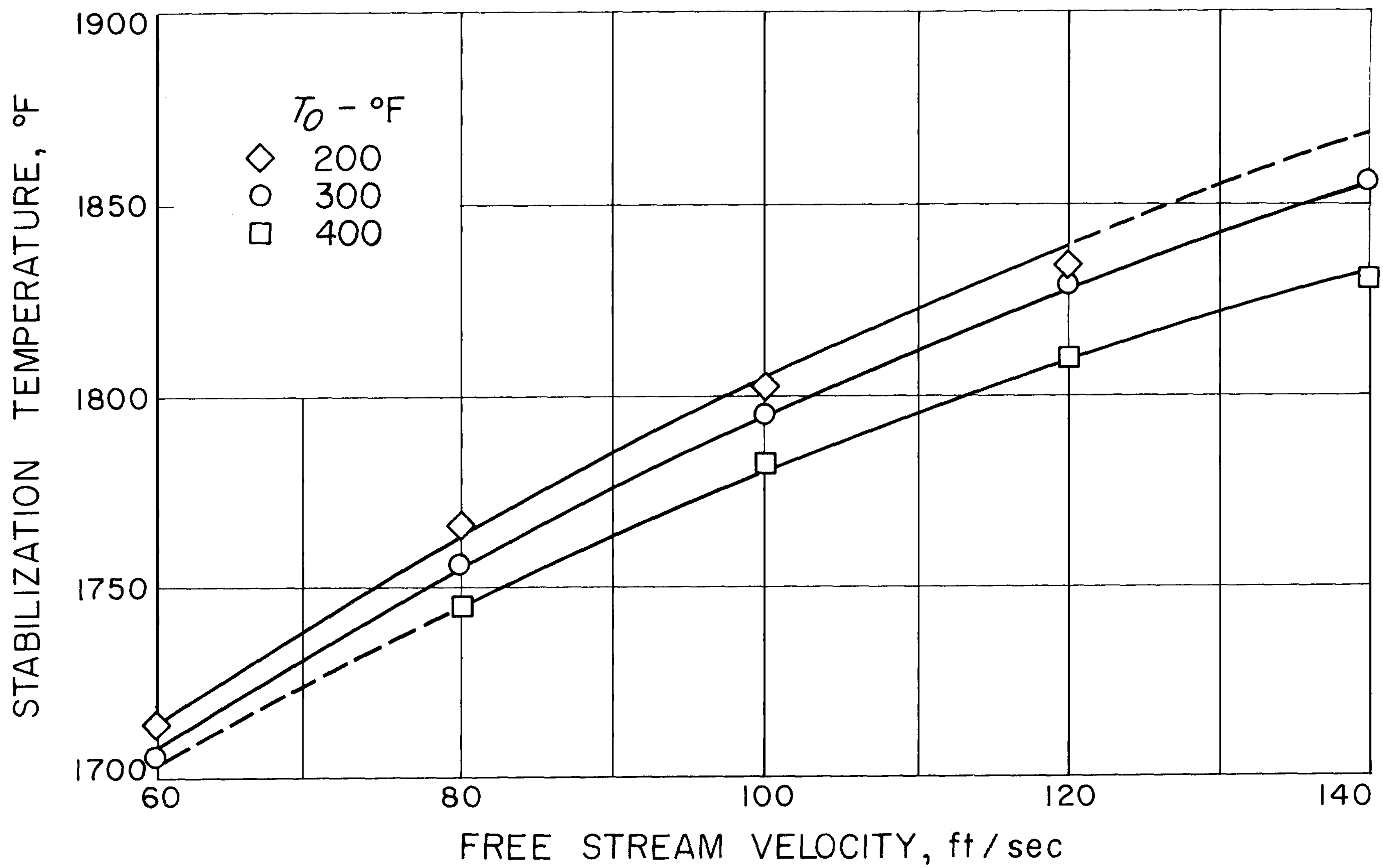


Figure 32. Dependence of Minimum Wall Temperature for Stabilization on Free Stream Velocity for Various Free Stream Temperatures; $\phi = 1.00$, $D_{TW} = 0.0201$ in.

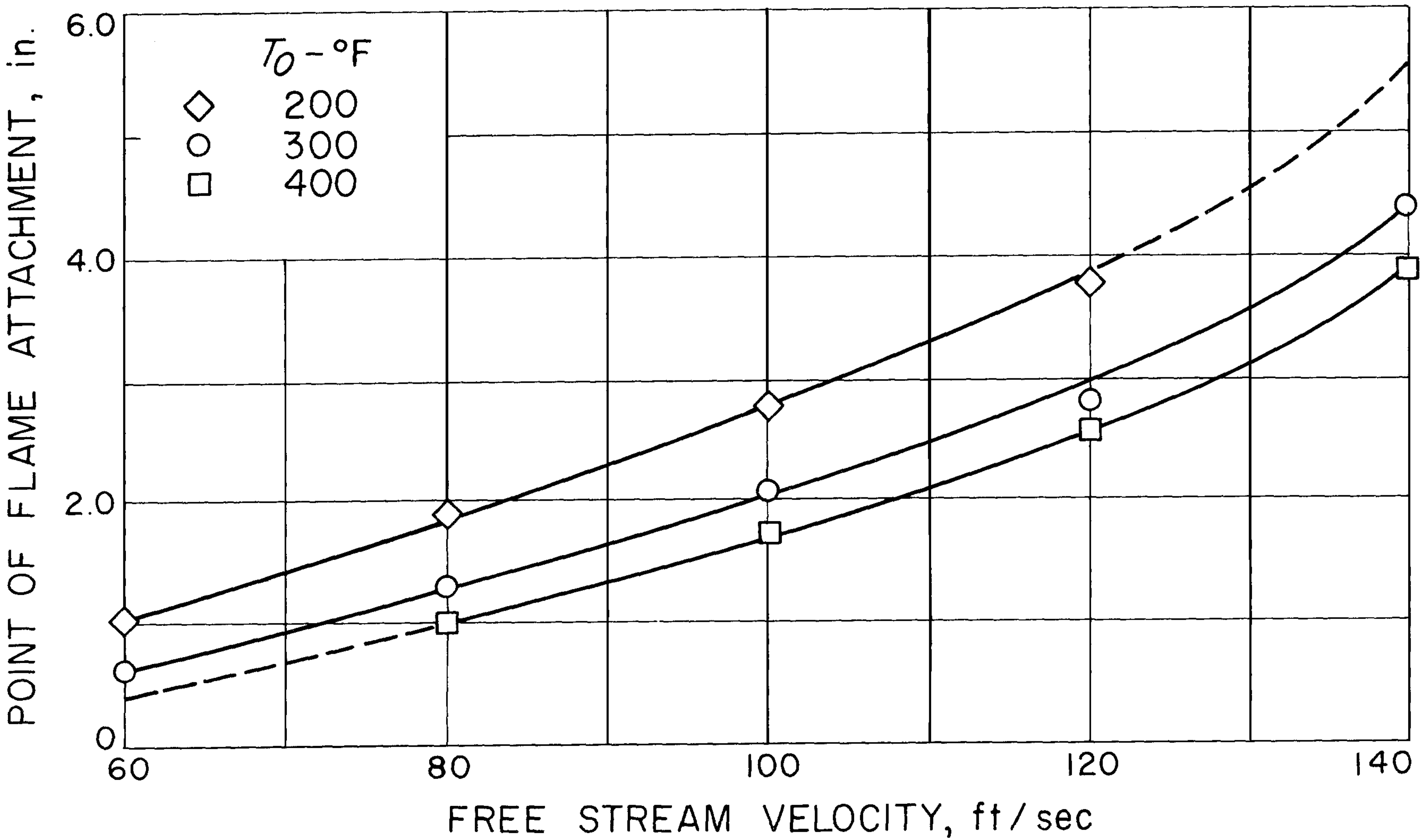


Figure 33. Dependence of Flame Attachment Position on Free Stream Velocity For Various Free Stream Temperatures; $\phi = 1.00$, $D_{TW} = 0.0201$ in., $T_w - T_{ws} = 50^{\circ}$F.

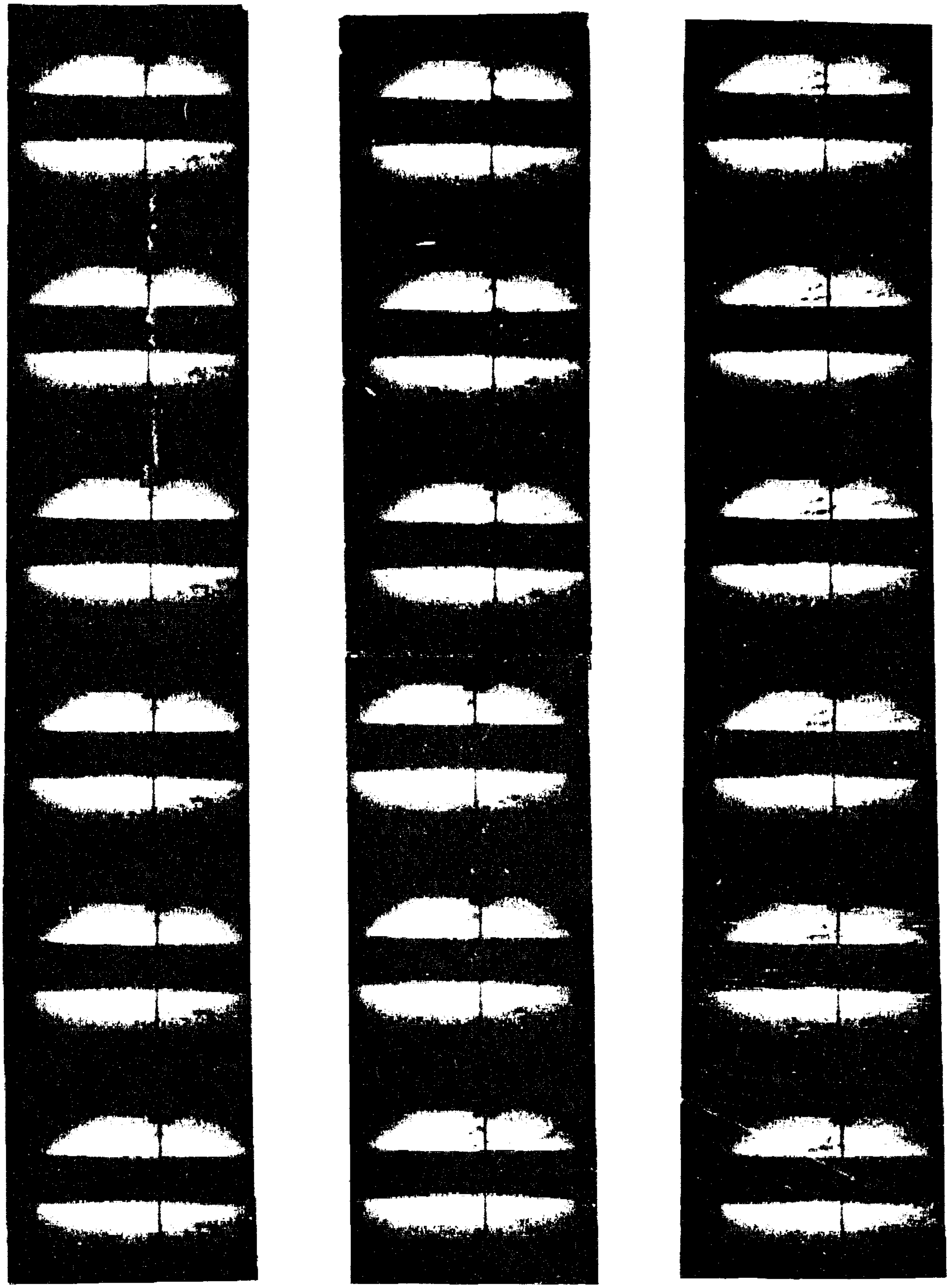


Figure 34. High Speed Motion Picture Showing Initial Ignition in a Turbulent Boundary Layer, Film Speed 1000 F.P.S., t = 0.180 - 0.198 sec., D_{TW} = 0.0201 in., T_o = 300°F., U_o = 100 ft./sec., T_w = 1875°F., ϕ = 1.00.

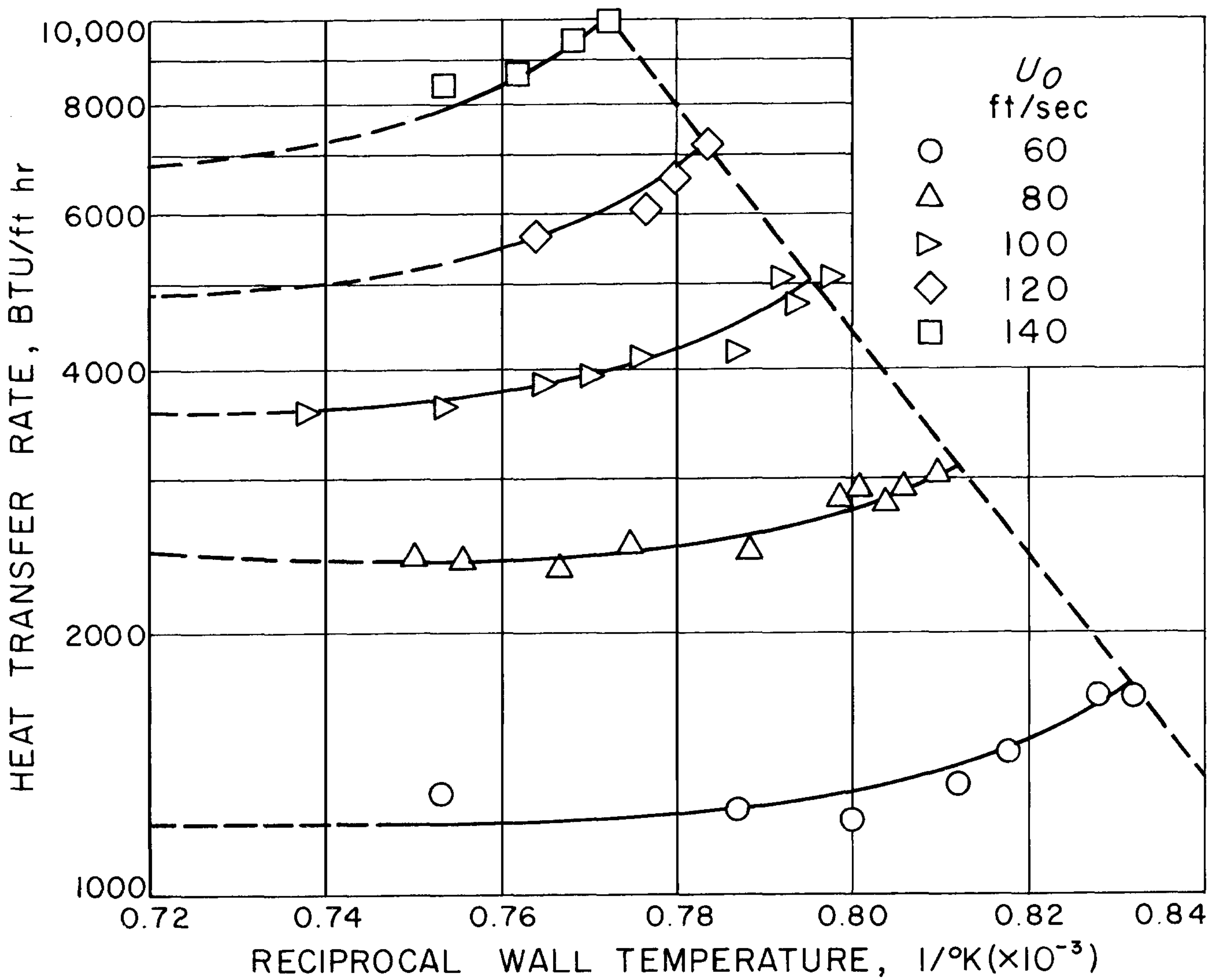


Figure 35. Dependence of Heat Transfer for Stabilization on Reciprocal Wall Temperature for Various Free Stream Velocities; $\phi = 1.00$, $T_o = 300°F$, $D_{TW} = 0.0201$ in.

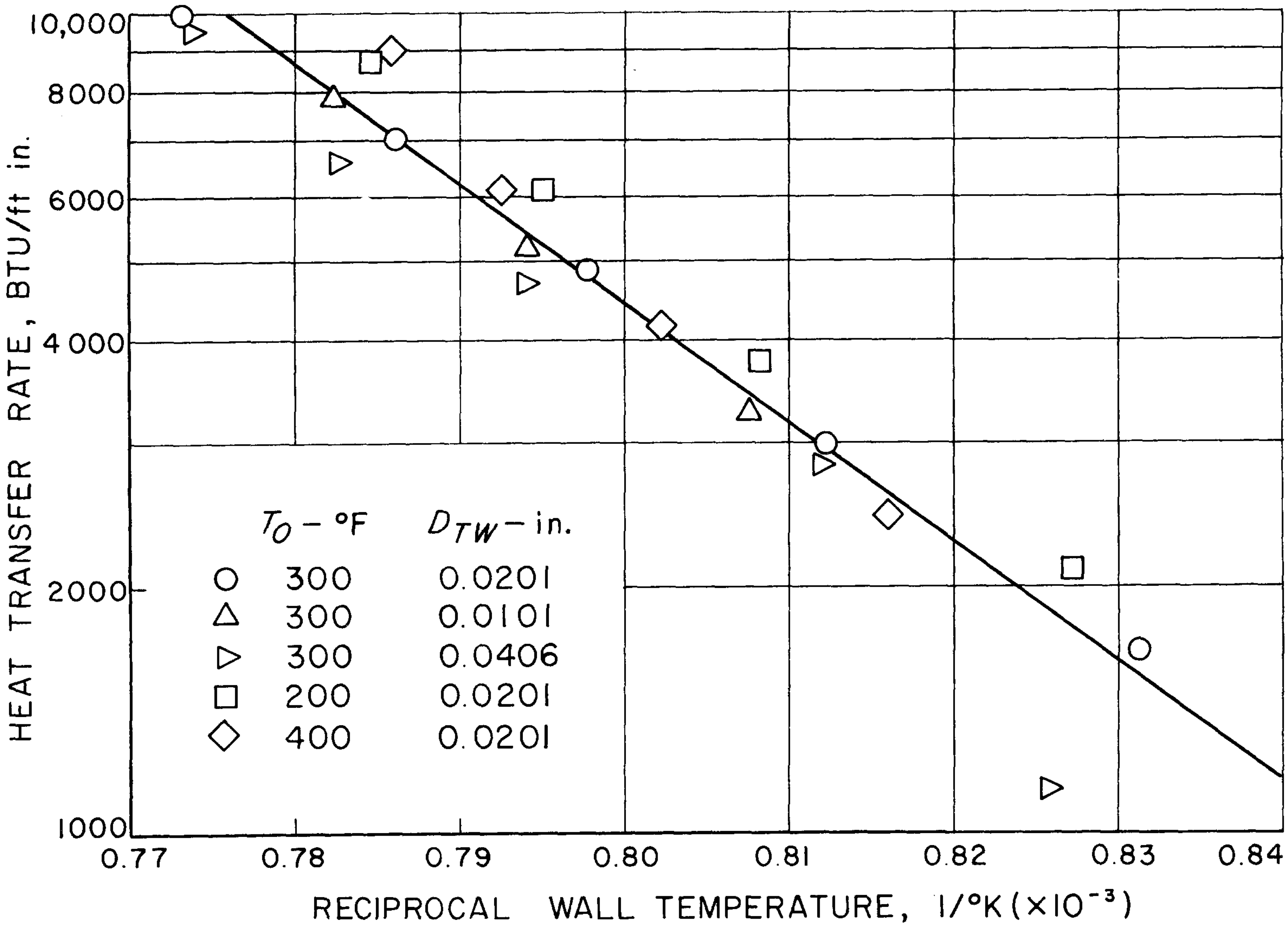


Figure 36. Dependence of Heat Transfer for Stabilization on Reciprocal Wall Temperature at Stability Limit; $\phi = 1.00$.

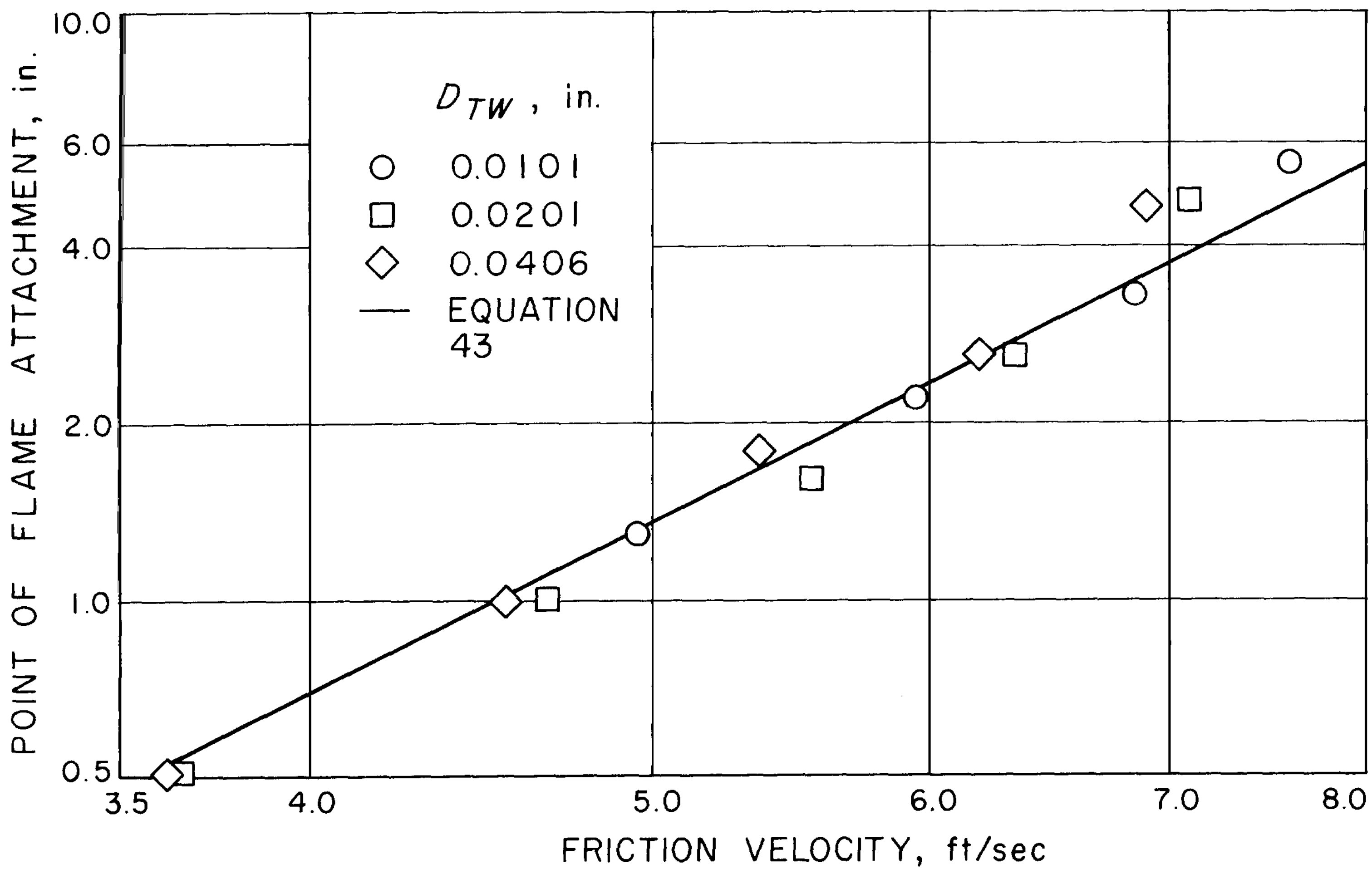


Figure 37. Dependence of Flame Attachment Position on Friction Velocity for Various Trip Wires; $\phi = 1.00$, $T_o = 300^{o}F$, $T_w = 1900^{o}F$.

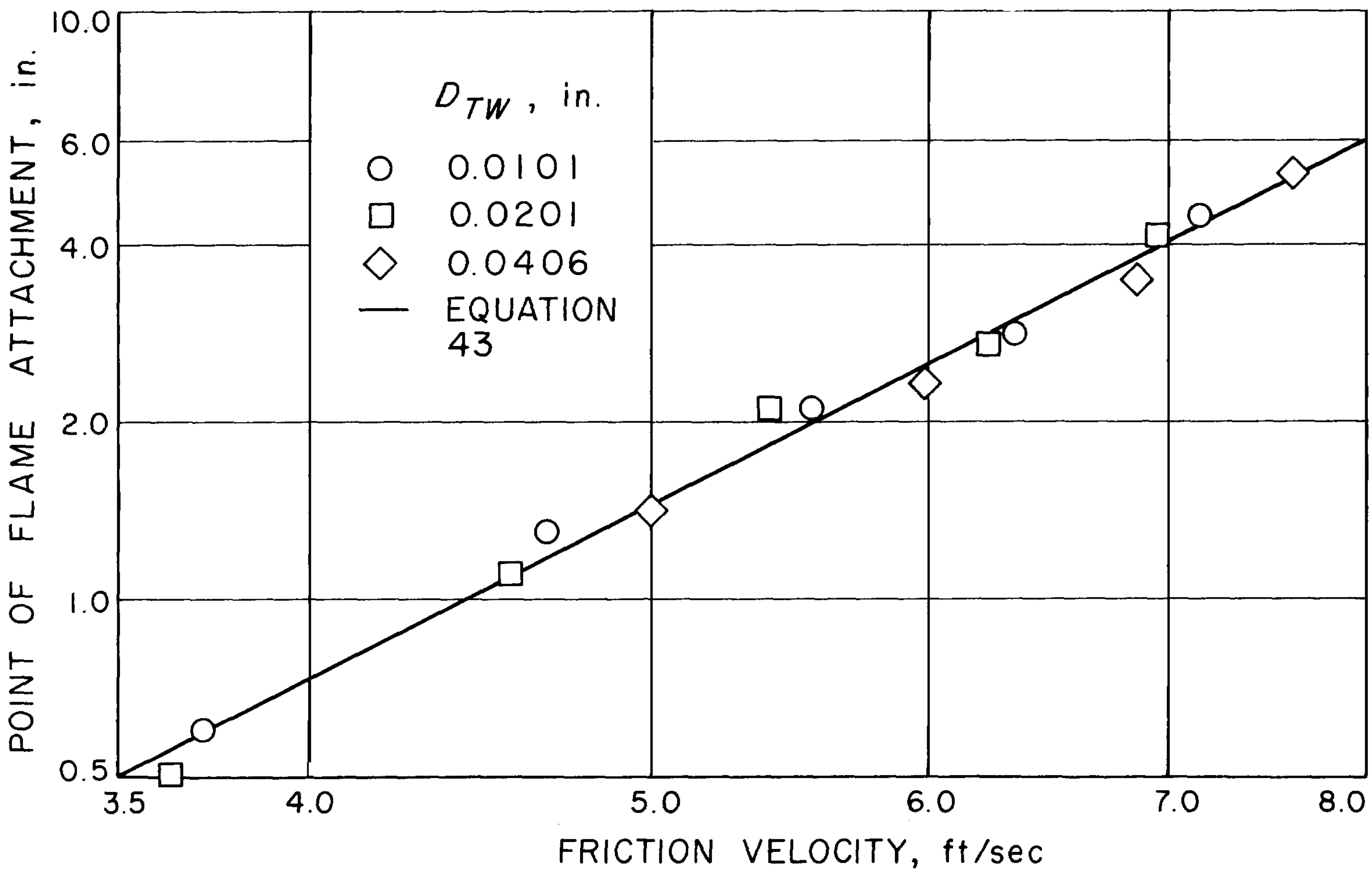


Figure 38. Dependence of Flame Attachment Position on Friction Velocity for Various Trip Wires; ϕ = 1.00, T_o = 300°F, $T_w - T_{ws}$ = 50°F.