ANALYSIS OF THE AIRLINE SYSTEMS PERFORMANCE BY VARYING INPUTS, VIZ. JET AIRCRAFT, ROUTES AND SCHEDULING

Thesis by

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TO MY PARENTS

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ABSTRACT

This is a systems analysis study of the commercial airlines. In essence, it provides a systematic procedure for determining a combination of decisions regarding the aircraft, routes and scheduling which maximizes overall effectiveness.

The model treats the aircraft, route and the scheduling as basic inputs, passenger-mile or ton-mile as basic outputs, revenue and cost as the monetary output and input respectively, to the airline system. Out of the numerous and diverse management objectives, the profit is chosen as a performance parameter, however noting that the return on investment would be a more realistic parameter. As a simplification, a multiple relationship between the direct and the indirect operating cost is used to eliminate indirect operating cost, and the attention is focused on the direct operating cost. The core idea of the profit computation is the deterministic approach to the costs and the probabilistic approach to the revenues.

Four major steps can be observed, namely:

1. Collection and systematic presentation of the relevant data about the aircraft and routes, taking into account the operational and technical constraints. This system reduces the number of possible alternatives, thus simplifying decision analysis. The pictorial representation of this system appears on the next sheet.

2. A set of equations for the direct operating cost computation is developed, employing a technique of multiple regression analysis, by observing U. S. domestic experience of major airlines using

B707-320B, B727-100 C/QC, DC9-30 aircraft. (All data is obtained from CAB-Wash.) DOC is further divided into flying, maintenance and depreciation. With the help of these equations, the direct operating cost could be predicted knowing the characteristics of the aircraft (number of engines, operating empty weight, fuel consumption), routes (average stage length) and scheduling (utilization).

3. The probabilistic revenue calculations are carried out using a simplified Monte Carlo simulation and expected value approach. The impact of alternative modes of transportation and the competition on the route has been given proper weightage. Even though calculations are done for three station problems, an extension is possible for complicated route networks.

4. By judging the profit performance, a feedback system is proposed.

I. INTRODUCTION

GROWTH

In recent years, a spectacular development and growth have been observed in the aviation business all over the world. The present trend promises that this activity will continue to flourish in the years ahead. The significant factors that account for this expansion can be identified and attempts are being made by different agencies in the aviation world to put forth a mathematical model that incorporates these factors. Notably among them are

CAB projection model:

$$\Delta \log Q = 0.085 - 1.28 \Delta \log F + 1.16 \log Y - 0.04 \log T$$

where
- Q = per capita revenue passenger miles
- F = fare or revenue per passenger mile deflated by consumer price index
- Y = per capita income deflated by consumer price index
- T = trend variable

FAA projection model:

$$\log Q = 1.0 \log Y - 1.5 \log F - 0.6 \log S$$

where
- Q = per capita RPM
- Y = excess income
- F = average fare deflated by consumer index
- S = speed

These models are based on data from the United States and are obtained by employing multiple regression techniques. Yet another model* based on the time series analysis is as follows

$$\log x = 6.5496 - 0.3157 \log P - 0.7613 \log y + 0.1825\, t$$

where x = yearly passenger demand

t = time

p = price in real terms

y = income in real terms

COMPLEXITIES

The aviation transportation business is uniquely complex, and the problems encountered are so diverse and deep that efforts are made to systematize, to analyze, to simulate and to computerize different situations faced by the management. To get some rough idea of which organizations are involved, the sketch below may be of interest

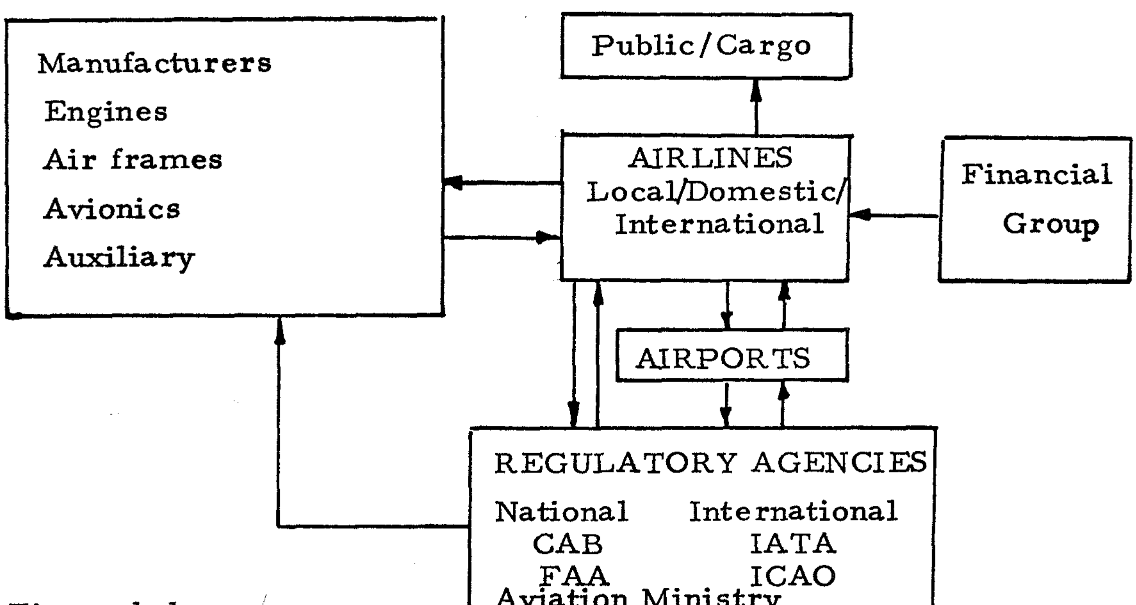

Figure 1-1

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*Adopted from "The International Airlines Industry" by Straszheim.

PROBLEM DEFINITION

By isolating the airlines segment from the total picture and treating it as a complete system, further analysis of its performance can be carried out. Searching for the inputs for our system, three basic ones can be identified and examined in detail, even though there exist many more.

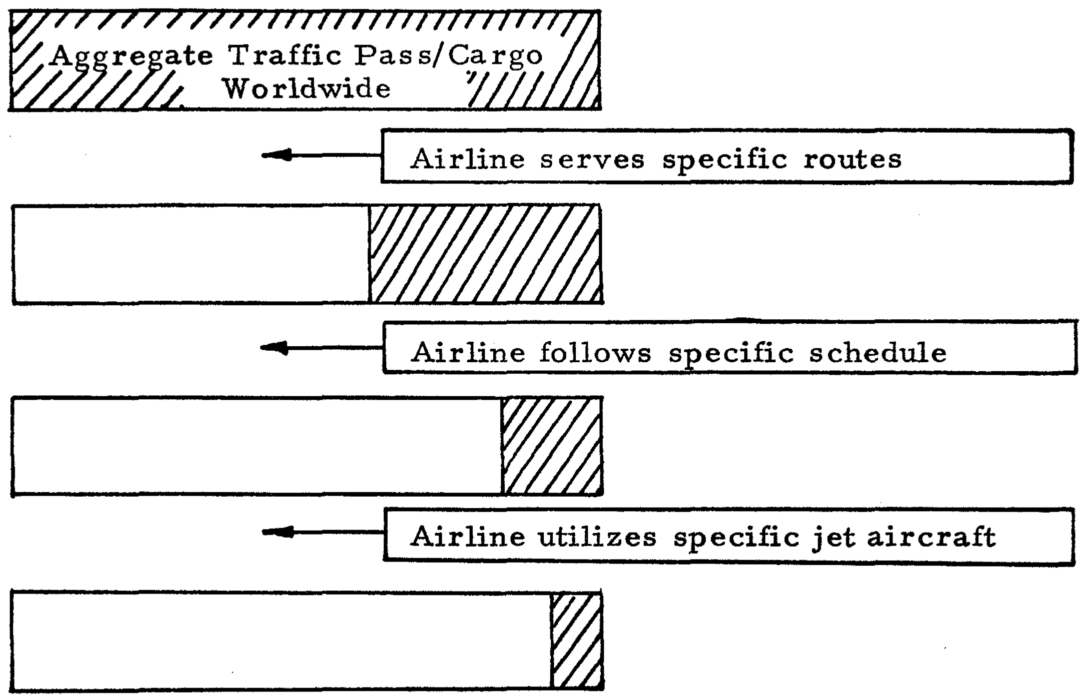


Figure 1-2

This picture depicts how the market share is narrowed down as the three basic inputs go into effect. We label them as basic inputs for the reason that they constitute the operation of an air carrier. If we define an element of operation of an airline as transporting a certain capacity by a certain type of aircraft (Jet aircraft: Input) from point A to point B (Route: Input) at particular time

(Scheduling:Input).

If we define the unit output of the airline system as one passenger mile or one ton mile, the fundamental picture of system input-output would be complete. The two characteristics of the output are noteworthy.

1. With the well-known definition of load factor, the ratio of passenger miles to seat miles, the non-revenue production can be observed.

i.e., Waste = (1-load factor) total output; in seat miles

2. The output is generated more or less as a 'quantum', or as discrete levels of production.

So we summarize

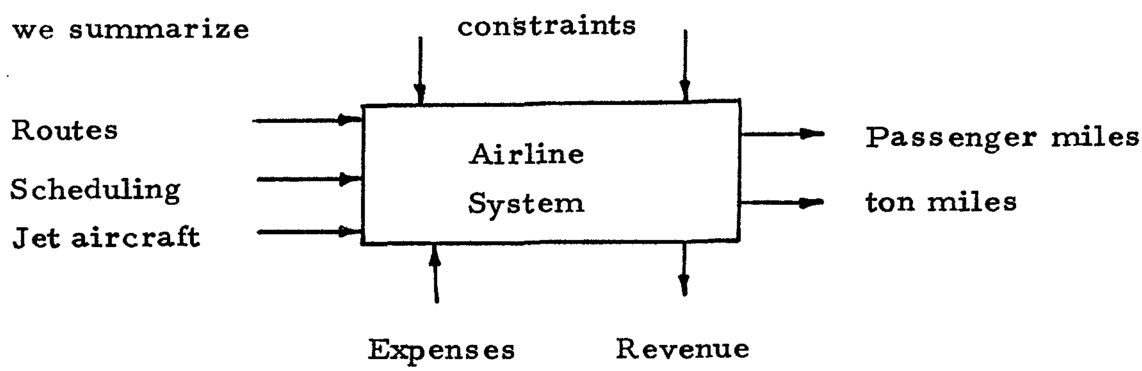


Figure 1-3

The logical question that arises next is how can one measure the system's performance. This question can be answered from the other direction by asking what would be the objectives of the management to improve the system's performance. These objectives can be classified under three heads; however it should be noted that the items listed are interlinked.

Economic objectives:

1. Profit per seat mile should rise
2. Cost per seat mile should drop
3. Load factor should rise
4. Return on investment should go up

Substantial objectives:

1. Serve more passengers and more miles and hence their product should grow
2. Similarly for cargo
3. As means to achieve 1. and 2.
    a. Increase jet capacity
    b. Expand route structure
    c. Multiply frequency
    d. Capture more market
4. Utilization factor of the jet aircraft should rise

Organization objectives:

1. Passenger (ton) miles or total production per employee should go up
2. Improvement in wages, salaries, and working conditions
3. More dividends to the shareholders, etc.

Though the system appears to be multiobjective, many of these, if not all, would be met by a single objective of raising net profit. But,

Profit = Revenue - Cost

In essence, raising net profit means maximizing revenue and minimizing cost.

Basic Model Equation:

If we denote:

R = variable input routes

S = variable input scheduling

J = variable input jet aircraft

Parentheses denote functional relation

$$\overline{\text{Net profit}}\ (R, S, J) = \overline{\text{Revenue}}\ (R, S, J) - \overline{\text{Cost}}\ (R, S, J)$$

It is extremely difficult to comprehend the intricacies involved, and almost impossible to formulate an exact mathematical relation as shown above. Suppose there exists such a 'hypothetical' model, then one can think of the solution shown in Table 1-1.

$$\overline{P}\{R, S, J\} = \overline{R}\{R, S, J\} - \overline{C}\{R, S, J\}$$

The problem would be very hard if the other combinations of two or three variations (simultaneous) are considered. In the real world problem, such situations occur frequently.

Interdependency of R, S, J, the input variables:

It should be noted here that all the strategies mentioned below may not be feasible.

Strategy in scheduling (J = const, R = const, S = variable) is feasible

Table 1-1

strategy in	constant inputs	solutions
scheduling	J R	$\frac{\partial \bar{R}}{\partial S} = 0 \quad \frac{\partial^2 \bar{R}}{\partial S^2} < 0$ $\frac{\partial \bar{C}}{\partial S} = 0 \quad \frac{\partial^2 \bar{C}}{\partial S^2} > 0$
purchase	S R	$\frac{\partial \bar{R}}{\partial J} = 0 \quad \frac{\partial^2 \bar{R}}{\partial J^2} < 0$ $\frac{\partial \bar{C}}{\partial J} = 0 \quad \frac{\partial^2 \bar{C}}{\partial J^2} > 0$
operation and planning	J S	$\frac{\partial \bar{R}}{\partial R} = 0 \quad \frac{\partial^2 \bar{R}}{\partial R^2} < 0$ $\frac{\partial \bar{C}}{\partial R} = 0 \quad \frac{\partial^2 \bar{C}}{\partial R^2} > 0$

The notation above indicates $\bar{R}$ = max and $\bar{C}$ = min. with respect to the variable involved.

Strategy in purchase (S = const, R = const, J = variable) is feasible except J is confined to certain ranges. In other words, variable R dictates the choice of J.

Strategy in operations and planning (J = const, S = const, R = variable) is not feasible, since variation in R implies variation in S. Besides that, J dictates variation in R.

II. ECONOMICS

Now we have reached a point where we can investigate the revenue function, $\overline{R}$, and the cost function, $\overline{C}$, thoroughly. Between the two, $\overline{C}$ is more complex in nature. As in any other big enterprise, the expenses incurred in the airline operation are diverse. We get a better notion of operating expenses on examining 'income statements' of the airlines. The items can be classified under heads of direct operating cost (DOC) and indirect operating cost (IDOC).

DOC + IDOC = TOC   Total operating expenses

Table 2.1

DOC	IDOC
1. Flying operations	1. Maintenance-indirect
2. Maintenance-direct	2. Passenger service
3. Depreciation of flight equipment	3. Aircraft traffic service
	4. Promotion and sales
	5. General and administrative
	6. Amortization of development and preoperating expenses
	7. Depreciation, other than flight equipment

On scrutinizing the statistics, one finds that the DOC bears an almost constant ratio with the TOC. This ratio varies between airlines but it is between 1.8 to 2.2 (= TOC/DOC). Fig. 2.1 shows

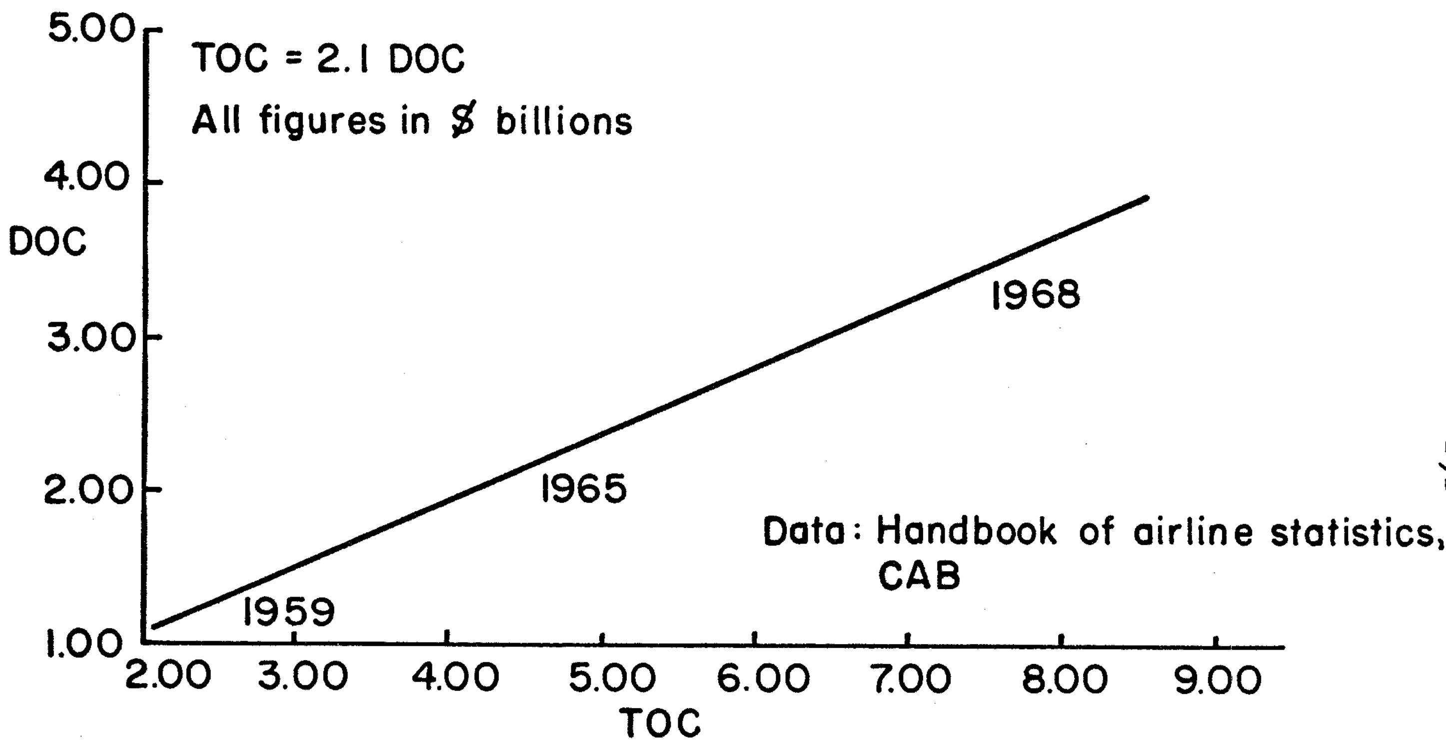


FIG. 2-1 DIRECT VS. TOTAL OPERATING COST FOR TOTAL CERTIFICATED ROUTE CARRIERS

a graph of DOC against TOC for the total certificated U. S. route carriers. The relationship obtained from the graph is

$$TOC = 2.1\ DOC$$

After years of experience, every management of the airline gets a fairly good idea of this relationship. McDonnell Douglas Company (commercial aircraft division) states that the DOC is 210% of the TOC in their literature. There are often inherent difficulties in estimating indirect operating cost, however this relation facilitates their estimation.

$$\bar{C} \equiv TOC = 2.1\ DOC$$

Further Breakup of DOC

Table 2.2

A	Flying operations	Nomenclature
	1. Crew expenses	DOC1
	2. Fuel & oil expenses	DOC2
	3. Insurance	DOC3
	4. Other	DOC4
B	Maintenance direct flight equipment	
	1. Direct maintenance - airframe & others	DOC5
	2. Direct maintenance - engine	DOC6
	3. Maintenance burden	DOC7
C	Depreciation of flight equipment	
	1. Depreciation - airframe & others	DOC8
	2. Depreciation - engine	DOC9
	3. Obsolescence & deterioration	DOC10
	4. Rentals	DOC11

This completes the anatomy of the direct operating cost.

$$\therefore \quad \text{Total DOC} = \sum_{i=1}^{11} DOC_i$$

Analysis of the Revenue Function, $\overline{R}$

From the income statements of the air carriers, the following are the revenue items.

Table 2. 3

Operating Revenue	Nomenclature
Transport	
1. Passenger. Firstclass - scheduled	Rv1
2. Passenger. Coach/economy class - scheduled	Rv2
3. Freight - scheduled	Rv3
4. Express - mail (U. S. -foreign. /priority - no priority)	Rv4
5. Excess baggage - scheduled	Rv5
6. Passenger - charter	Rv6
7. Freight - charter	Rv7
8. Other transport	Rv8
Non transport	
9. Subsidy	Rv9
10. Incidental revenue	Rv10

The revenue items 1, 2, 3 account for about 87% of the total revenue earned by the airlines. Fig. 2-2 depicts this fact distinctly. The graph is drawn from the data for the total certificated U. S. route carriers.

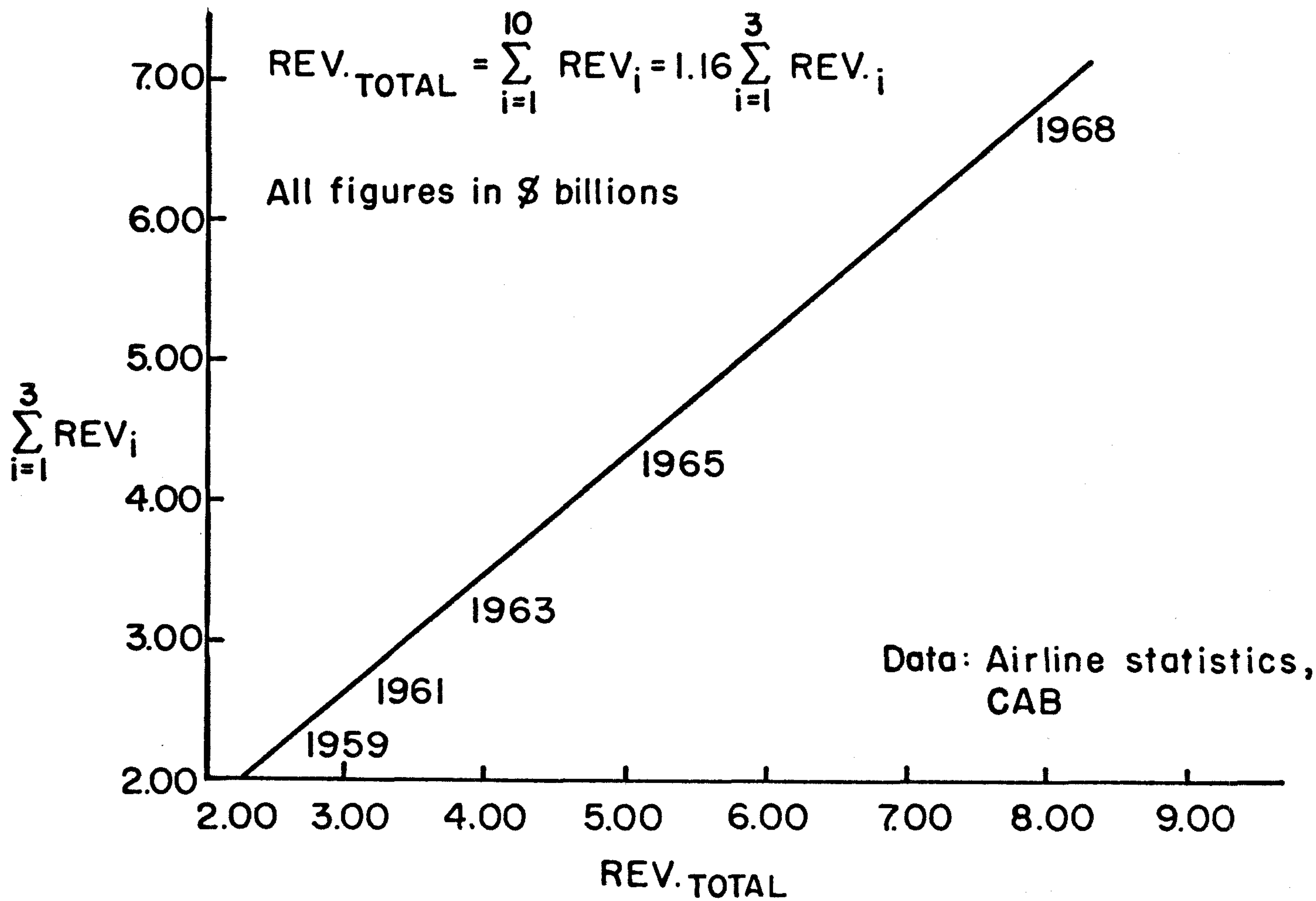


FIG. 2-2 BREAK UP OF REVENUE FOR TOTAL CERTIFICATED ROUTE CARRIERS

$$\overline{R} \equiv \text{Revenue}_{\text{total}} = 1.16 \sum_{i=1}^{3} Rv_i$$

The whole idea behind this analysis of $\overline{R}$ and $\overline{C}$ is to narrow down the vast area covered by revenue and cost while keeping precision and realities. This simplifies our model equation to a great extent. Finally we end up with a modified version of the model.

$$\overline{P}\{R, S, J\} = 1.16 \sum_{i=1}^{3} Rv_i \{R, S, J\} - 2.10 \sum_{i=1}^{11} DOC_i\{R, S, J\}$$

III. BASIC CONCEPTS IN THE ROUTE NETWORK ANALYSIS: INPUT R

Route network analysis is essentially dependent upon the geometrical figure which can be called a linear graph consisting of lines and points. A line corresponds to a route or branch. A point corresponds to a node or airport.

A route pattern, G, is a composite concept of $\{N, R, \partial^+, \partial^-\}$ where

(a) N, a set of airports $= \{n_1, n_2, \ldots . n_m\}$

(b) R, a set of branches $= \{b_1, b_2, \ldots . b_n\}$

(c) An incident relation (∂^+, ∂^-)

The intuitive interpretation of the function ∂^+ and ∂^- is that the nodes n_α and n_β are the two end points of a branch, b_k, or more precisely, b_k starts from n_α and ends at n_β if and only if

$$\partial^+ b_k = n_\alpha \qquad\qquad \partial^- b_k = n_\beta$$

The incident relation, ∂^+ or ∂^-, is single-valued, but its inverse δ^+ or δ^- is often multivalued. To define δ^+ and δ^-,

$\delta^+ n_\alpha$ contains b_k if and only if $\partial^+ bk = n_\alpha$

$\delta^- n_\alpha$ contains b_k if and only if $\partial^- b_k = n_\alpha$

Some immediate outcomes:

1) $$\sum_{\alpha=1}^{m} \delta^+ n_\alpha = \sum_{\alpha=1}^{m} \delta^- n_\alpha = R$$

sum of all possible routes; set R

2) Isolated node: $\delta^{+}n_{\alpha} = \delta^{-}n_{\alpha} = \phi$, empty set

From a air carrier's reference, a new airport or a new market to be sought can be called an isolated node prior to the introduction of service.

Two other concepts, which have no relevance in our problem, are given for the sake of completion.

3) Self loop: $\partial^{+}b_{k} = \partial^{-}b_{k}$

4) Isomorphic route pattern: $G_1 = \{N_1 \;\; R_1 \;\; \partial_1^{+} \;\; \partial_1^{-}\}$

$$G_2 = \{N_2 \;\; R_2 \;\; \partial_2^{+} \;\; \partial_2^{-}\}$$

are said to be isomorphic if there is one to one correspondence between N_1 and N_2 as well as between R_1 and R_2 and if, under this correspondence, ∂_1^{+} and ∂_1^{-} are isomorphic with ∂_2^{+} and ∂_2^{-}.

Sometimes an air carrier might think that the reoriented route pattern is entirely different that the one they had previously, but it can be reduced to original pattern if the two graphs are isomorphic.

Incidence Matrix $\overline{D}_{k}^{\alpha}$

The notion of an incidence matrix is very significant. In essence, it is a mathematical representation of the route pattern. Let us define the matrix $\overline{D}_{k}^{\alpha}$

Rows: α ... represents an airport, $\alpha = 1, 2, \ldots m$

Columns: k ... represents a branch, $k = 1, 2, \ldots n$

Element of the D_{k}^{α} (k^{th} column, α^{th} row)

= +1 ... denotes the starting point of branch k from airport a

= -1 ... denotes terminating point of branch k at airport a

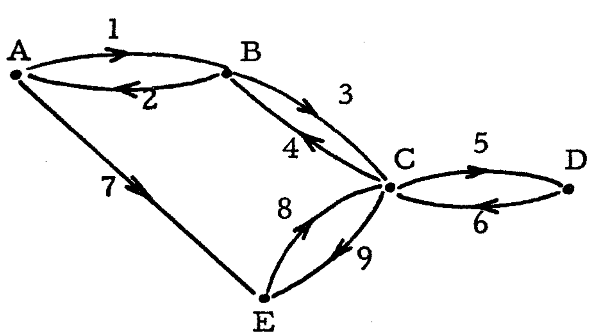
= 0 otherwise

It will be clear from the above that for a fixed k (finite route segment), the number of non-null $\overline{D}_k^a$ is either 0 or 2. It is zero if and only if it is a self loop, otherwise one of the non-null $\overline{D}_k^a$ is equal to +1 and the other to -1.

$$\therefore \sum_{a=1}^{m} \overline{D}_k^a = 0 \qquad \begin{array}{l}\text{For a fixed k}\\ \text{Relation is valid for all k}\end{array}$$

Generally the isolated node, a row vector with all of its elements zero, are excluded from the incidence matrix. Their existence is implied and will be shown whenever necessary.

To clarify the concepts mentioned above, let us take an example. Consider the following route pattern:



$a = \{A, B, C, D, E\}$

$k = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Fig. 3-1

a \ k	1	2	3	4	5	6	7	8	9
A	1	-1	0	0	0	0	1	0	0
B	-1	1	1	-1	0	0	0	0	0
C	0	0	-1	1	1	-1	0	-1	1
D	0	0	0	0	-1	1	0	0	0
E	0	0	0	0	0	0	-1	1	-1

$= \bar{D}_k^a$

$\partial^+ 1 = A$ $\quad \partial^+ 2 = B$ $\quad \partial^+ 3 = B$ $\quad \partial^+ 4 = C$ $\quad \partial^+ 5 = C$

$\partial^- 1 = B$ $\quad \partial^- 2 = A$ $\quad \partial^- 3 = C$ $\quad \partial^- 4 = B$ $\quad \partial^- 5 = D$

$\partial^+ 6 = D$ $\quad \partial^+ 7 = A$ $\quad \partial^+ 8 = E$ $\quad \partial^+ 9 = C$

$\partial^- 6 = C$ $\quad \partial^- 7 = E$ $\quad \partial^- 8 = C$ $\quad \partial^- 9 = E$

Inverse Relations:

$\delta^+ A = \{1, 7\}$ $\quad \delta^+ B = \{2, 3\}$ $\quad \delta^+ C = \{4, 5, 9\}$

$\delta^- A = \{2\}$ $\quad \delta^- B = \{1, 4\}$ $\quad \delta^- C = \{3, 6, 8\}$

$\delta^+ D = \{6\}$ $\quad \delta^+ E = \{8\}$

$\delta^- D = \{5\}$ $\quad \delta^- E = \{7, 9\}$

In the total airlines' route network, an operation is comprised of a subset of nodes and correspondingly consistent subset of branches. This leads to a very elementary algorithm of forming an operation. Stretching the example mentioned before, some operations can be formed as follows.

1. Specify origin. A
2. Look for $\delta^{+}A = \{1, 7\}$
3. Look for $\partial^{-}1 = B$

   $\partial^{-}7 = E$
4. Look for $\delta^{+}B = \{2, 3\}$

   $\delta^{+}E = \{8\}$
5. Look for $\partial^{-}2 = A$ operation one

   $\partial^{-}3 = C$

   $\partial^{-}8 = C$
6. Look for $\delta^{+}C = \{4, 5, 9\}$
7. Look for $\partial^{-}4 = B$ Repeat steps 4 and 5

   $\therefore$ operation two

   $\partial^{-}5 = D$

   $\partial^{-}9 = E$ Repeat steps 4, 5, 6, 7

   $\therefore$ operation three
8. Look for $\delta^{+}D = \{6\}$
9. Look for $\partial^{-}6 = C$ Repeat steps 6 and 7

   operation four

The above method yields some sample operations, although many more are possible. Every node or airport has its own characteristics. This is also true for branches or routes. We will compile this information in a matrix form and place it alongside our incidence matrix $\overline{D}_k^{a}$. Node characteristic parameters will be arranged as column vectors and will be placed parallel to the nodes column. Similarly branch characteristic parameters will be put in the row vectors and will be laid below. The information vital to the node is as follows:

Known T_{GM} .... GMT time reference

L_R .... Runway length, ft

$0.9\ L_R$ .... 10% reduced runway length

Takeoff performance is based on a 10% reduction of the field length

E .... Elevation above sea level, ft

t_m .... mean temperature, ° C (annual)

t_{DF} .... mean temp. I quarter December-February

t_{MM} .... mean temp. II quarter March-May

t_{JA} .... mean temp. III quarter June-August

t_{SN} .... mean temp. IV quarter September-November

F_L .... landing fee $

unknown: Sc ... station cost ... $ Parameter to be evaluated.

Therefore the format will take a shape similar to

$$\underbrace{\{Sc\}}_{\text{unknown}}\ \underbrace{\{F_L\ t_{SN}\ t_{JA}\ t_{MM}\ t_{DF}\ t_m\ E\ .9L_R\ L_R\ T_{GM}\}}_{\text{known}} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_M \end{Bmatrix}$$

Now turning our attention to the branch characteristic parameters, we will find that there are more unknown parameters than known. The vital parameters are as shown on the following page.

Known: d .... distance, miles

V_w .... Wind velocity - average, knots

+ ve in direction of flight

- ve in opposition to flight

Unknown: V_J .... Cruise speed of jet plane miles/hr.

T_B .... Block time, hrs.

P_L .... Payload (lbs) $\equiv$ Passenger capacity

F_B .... Fuel burned

F_R .... Fuel reserve

DOC .... Direct operating cost \$/Trip

R_{ALF} .... Revenue, $\sum_1^3 Rv_i$ at average load factor

R_{BLF} .... Revenue, $\sum_1^3 Rv_i$ at breakeven load factor

The Format will appear as

$$\{b_1, b_2, \ldots\ldots\ldots b_n\}$$

$\left\{\begin{matrix} d \\ V_w \end{matrix}\right\}$ known

$\left\{\begin{matrix} V_J \\ \vdots \\ \cdot \; \vdots \\ R_{BLF} \end{matrix}\right\}$ all unknown

As mentioned before, the main idea in carrying out this analysis is to investigate changes in $\bar{R}$ and $\bar{C}$ due to changes in route pattern. Now we will interpret partial differentiation with respect to R, $\frac{\partial}{\partial R}$, as a change in the incidence matrix. Obviously two kinds of changes can be expected.

(a) Add (or deduct) one or more branches, that is a column vector. Operational meaning of this change is to add or deduct a jet route between the airports that are being served.

It should be noted here that deduction of a two column vector may result in the isolation of a node.

(b1) Add a new node (or airport) in routes. This itself does not suffice, since the existence of an isolated node is already implied. It should be followed by the addition of at least two column vectors with non-null $\overline{D}_k^\alpha$. As no node is a sink or a source of jet planes, the non-null elements should be +1 and -1.

(b2) Remove an existing node in the routes.

Removal of a row vector will automatically cancel all column vectors which have non-null $\overline{D}^\alpha$ with the removed row vector.

The operational meaning of this change amounts to stopping all the operations from that airport.

One can incorporate both of these changes simultaneously.

In the concluding part of the network analysis, a typical transportation problem is mentioned. In any real-world network problem, a geometrical graph and the physical phenomena superposed thereupon should be considered together.

Flow in any branch would be taken as flights per unit time. It will be denoted by ξ^k. Flights per unit time in branch k.

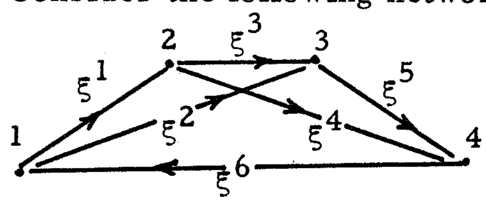
The continuity relation should be satisfied.

$$\sum_{k=1}^{n} \overline{D}_k^\alpha \, \xi^k = 0 \qquad \alpha = 1, 2 \ldots M \quad .$$

This condition implies that the number of arrivals must be equal to the number of departures for any airport.

The capacity of a node, i.e. the capacity to handle the flow is also limited.

Consider the following network:



Four nodes.

Branches as shown by arrows.

ξ represents flight/time unit time may be a day.

Let us define φ as a cost function, which strongly depends upon ξ. This assumption is very valid if the direct operating costs are concerned.

$\therefore$ let $\varphi_k(\xi^k)$ be the cost of transportation, say the D.O.C., of branch k, which is a function of flight frequency on that route.

$$\text{Total operation cost } \overline{Z} = \sum_{k=1}^{6} \varphi_k(\xi^k)$$

We want to minimize this and solve the set of values for $\{\xi^1, \xi^2 \ldots \xi^6\}$; subject to the condition,

$$\sum_{k=1}^{6} \overline{D}_k^{\alpha} \xi^k = 0 \qquad \alpha = 1, 2, 3, 4.$$

non-stagnation condition

$\overline{D}_k^{\alpha}$ =

α \ k	1	2	3	4	5	6
1	1	1	0	0	0	-1
2	-1	0	1	1	0	0
3	0	-1	-1	0	1	0
4	0	0	0	-1	-1	1

Introducing Lagrange's multipliers η_α, $\alpha = 1, \ldots 4$, the joint function will take form,

$$F(\xi^k, \eta_\alpha) = \sum_{k=1}^{6} \varphi_k(\xi^k) - \sum_{\alpha=1}^{4} \eta_\alpha \sum_{k=1}^{6} \bar{D}_k^\alpha \xi^k$$

hence the solution of the problem is

$$\frac{\partial F}{\partial \xi^k} = \varphi_k'(\xi^k) - \sum_{\alpha=1}^{4} \eta_\alpha \bar{D}_k^\alpha = 0$$

$$\frac{\partial F}{\partial \eta_\alpha} = \sum_{k=1}^{6} D_k^\alpha \xi^k = 0$$

$$\therefore \quad \boxed{\begin{array}{lll} \sum_{k=1}^{6} \bar{D}_k^\alpha \xi^k = 0 & & \alpha = 1, \ldots 4 \\ \sum_{\alpha=1}^{4} \eta_\alpha \bar{D}_k^\alpha = \varphi_k'(\xi^k) & & k = 1 \ldots 6 \,. \end{array}}$$

The fundamental assumption that underlies the analysis made so far is far from reality. The free market, free entry, and the airline's choice of changing, extending, modifying or reducing their route structure simply does not exist.

Entry into a new route system is not always based on economic considerations. On international operations, the rights of entry are obtained through inter-governmental negotiations which are highly influenced by political considerations. The prime example of this is that many routes, operated at low profit or at loss, are used as a link to the former or present colonies of a nation. Thus, bilateral agreements and bargaining reflect upon the political and economic motives, vested interest of the different countries, diplomatic moves, foreign aid policy and so forth. Route grants often result in

(a) Determination of the city pairs to be served.

Sometimes (b) Restrictions on the amount of capacity.

(c) Restrictions on the schedule frequency.

(d) Restrictions on the departure times.

(e) Restrictions on the type of equipment.

One of the adverse outcomes of such bilateral agreements is the entry of two carriers into a market which can support only one. Such cut-throat competition eventually leads to the attitude, shown by many nations, of protecting carriers under a national flag.

The well-known Bermuda agreement of 'Five Freedoms of Air' is as follows:

(1) To fly across the territory of a foreignnation without landing.

(2) To land for the purposes other than traffic.

(3) To set down in a foreign country traffic coming from the country of the airline's nationality.

(4) To pick up in a foreign country traffic destined for the country of the airline's nationality.

(5) To carry traffic from a point of origin in one foreign country to a point of destination in another foreign country.

In the U. S., the CAB is the highest authority in route-grants. The interested airlines submit applications, CAB officials examine those and finally either grant or reject the route award. Once an airline gets a new award, starts operating, and then finds service uneconomical, it is very hard to cancel such routes. CAB policy has often resulted in excessive entry into many markets.

In many other countries, domestic routes are being served by government owned air-carriers which operate almost monopolistically.

This brief discussion indicates how it may be impossible to establish a route structure which satisfies the computer oriented maximum profit pattern.

IV. THE JET-AIRCRAFT AND COST ANALYSIS

This is probably the most significant input and, at the same time, the most complex in nature, to the airline system. The prices of the new jet equipment are sky-rocketing every year, consequently airlines have become highly capital-sensitive industries. In turn, the decisions made about this input, J, may well affect the very survival of the airline under intense competition.

At the outset, one point should be brought to notice that, although the discussion in this paper is concentrated on jet equipment, however, it is not 'the best choice' under all circumstances in comparison with turbo-prop equipment. The motivation is twofold: First, to limit the boundaries of the thesis, and secondly, the popular trend towards jet equipment, considering factors like high productivity and low unit costs, is beyond question.

A wide variety is observed in the jet aircraft available on the market. They differ in accommodation, dimensions, range of operations, weights, unit costs and prices, to name just a few variables. To ask a question like - which is the best jet aircraft - would be highly misleading. Instead, an appropriate inquiry would be - which jet aircraft is best suited for a particular airline for a particular operation.

As far as costs are concerned, the figures in the tables on the following page may lead to fallacious conclusions.

Table 4-1 Costs of Boeing 707-300 C

	1969		1968		1967		1966	
	TOE	TOE/RPM	TOE	TOE/RPM	TOE	TOE/RPM	TOE	TOE/RPM
Pan American -Atlantic	775.81	2.635	706.56	2.519	643.67	1.890	--	--
TWA-Atlantic	810.03	2.188	699.01	1.993	796.91	NA	831.44	NA

TOE ... Total Operating Expenses
RPM ... Revenue Pass-miles

The total operating expenses are given as $/block hour and the TOE/RPM figures are in cents/block hour.

The important point to note is that it is the same type of aircraft, Boeing 707 - 300C, turbofan, four engines, same type of operation, viz; north Atlantic, being used by two different airlines, resulting in a wide variation in their costs.

Table 4-2 Costs experienced by TWA
(Domestic Operations)

	1969		1968		1967		1966	
$/BH and cents/BH	TOE	TOE/RPM	TOE	TOE/RPM	TOE	TOE/RPM	TOE	TOE/RPM
B-707-100 Turbo Jet, 4 Engines	810.59	3.491	802.62	3.503	868.90	3.467	725.46	2.905
CV-880 Turbo Jet, 4 Engines	819.58	4.161	691.75	3.414	753.26	3.402	717.98	3.169

Source - CAB - Aircraft operating costs and performance reports.

The above table reveals that considerable differences in cost exist for an airline, using two different aircraft, having the same broad characteristics.

In conclusion, the statement could be made that -

$$\text{Cost of a Jet Aircraft} = \text{Function}\begin{pmatrix}\text{Inherent design \& Engineering} \\ \text{Factors, How is it being used}\end{pmatrix}$$

Incidentally, time also plays an important role in the cost function. It embodies two diverse effects:

(a) As time passes, airlines gain more experience on the aircraft, hence costs decline.

(b) General inflationary effects in labor costs, materials, etc. tend to increase cost.

The aircraft manufacturers often generate cost curves when they introduce a new model into the market. Some small airlines are not able to, or cannot afford to, analyze an aircraft from the cost point of view, hence they expect manufacturers to provide such details. The aircraft manufacturers put out their results based on certain economic and operational assumptions. Standard methods have been devised to work out such a cost analysis - to name one, the Boeing-Lockheed method. These methods take into account FAR and other regulations.

It is quite possible that there exists a significant difference between costs experienced by an airline and the figures given by the aircraft manufacturer. The airlines may have altogether different labor costs, maintenance structure, utilization or stage length which justifies such discrepancies. To cite an example, the case of the Boeing 747 would be interesting.

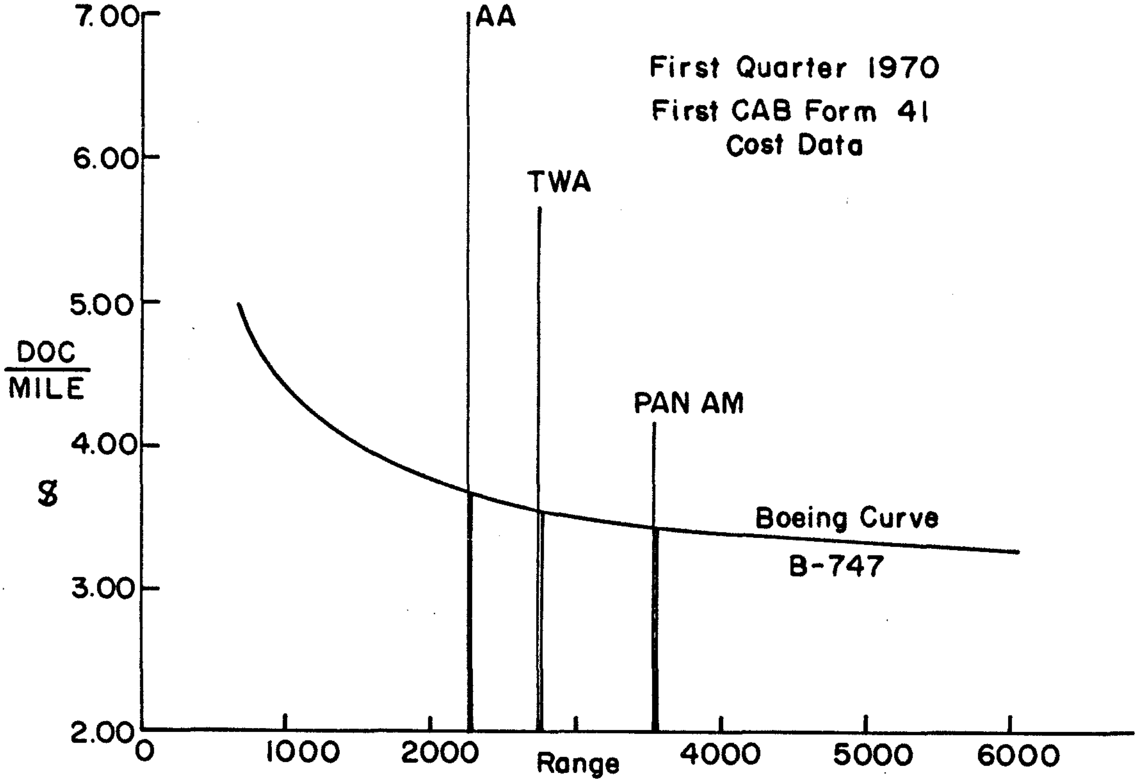
Table 4-3

Economic Assumptions for Boeing 747 Cost Analysis

Boeing 747 - Passenger configuration-International operations

Basic Price - 1969 Dollars	$ 18.7 million
Engine Price	$ 0.725 million
Spares allowance (10-40%)	$ 2.830 million
Cost Factors: (At 2500 N Miles Utilization:	
Crew	$ 203/BH
Fuel	11¢ per U.S. gallon
Oil	13¢ per engine B.H.
Insurance (2% of first cost)	91 $/BH
Maintenance	$331/flight hr.
Maintenance	$453/flight
Depreciation	$432/BH
Economic life	12 years to zero residual.

Source - Boeing Company



DIRECT OPERATING COST PER MILE

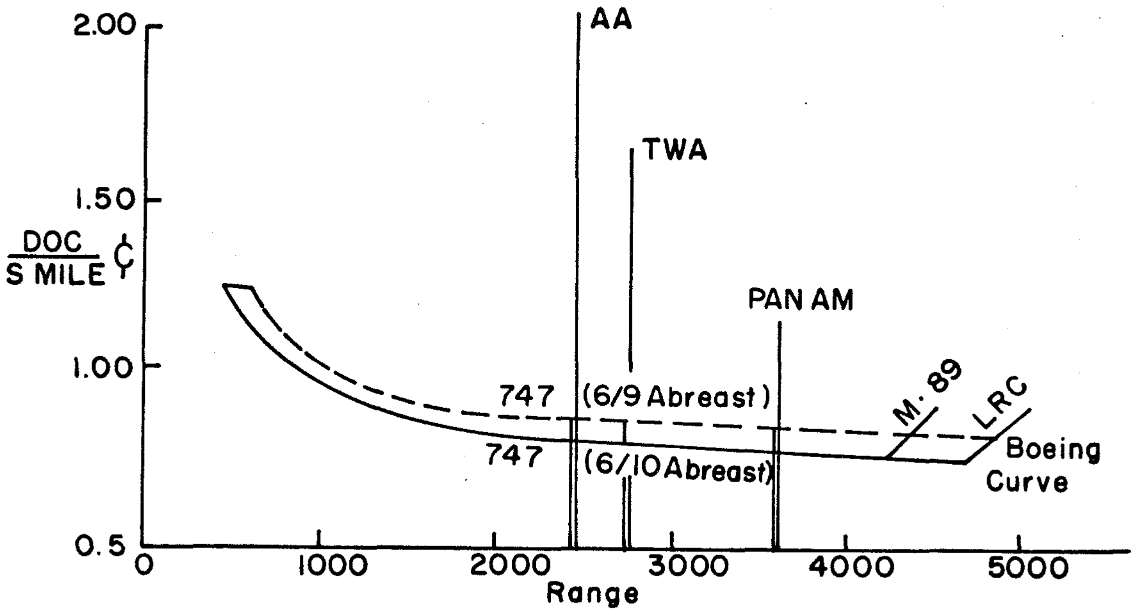


FIG. 4-1 DIRECT OPERATING COST PER SEAT MILE

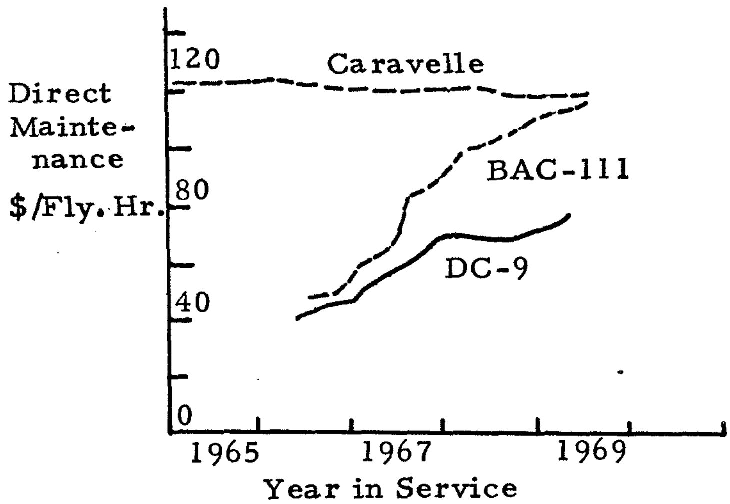
Just after the introduction of the B-747, the CAB received the first Form 41 which revealed the cost data for the first quarter of 1970. With a very limited experience in hours flown and number of jet aircraft in the fleet, cost figures were very high compared to the predicted values.

Table 4-4
Boeing 747 Cost Data (First Quarter 1970)

	PANAM	TWA	AA
Fleet Size	3	2	1
Hours Flown	1680	514	297
Utilization BH	7.57	8.13	11.18
Average stage length NM	3523	2753	2471
Pass. load factor (%)	56.97	56.57	67.79
Flying operations $/BH	843.85	769.85	637.11
Maintenance $/BH	424.11	774.83	436.58
Depreciation & Rentals $/BH	599.65	834.97	1877.82
Total Direct Operating Cost $/BH	1867.61	2379.65	2951.51
Cost per Rev. Mile $	4.13	5.68	7.13
Cost per seat mile ¢	1.15	1.66	2.02

The last two items from the above table have been plotted alongside the manufacturer's predicted values (Fig. 4-1). Discrepancy is partly due to the fact that the structure of the airline operations are different than Boeing's assumed form.

An appropriate last remark about the cost variation would be the effect of 'brand-loyalty'. All airlines show a propensity towards a particular aircraft manufacturer. There is a good deal of evidence to show that this loyalty helps to keep costs, especially maintenance, low. The spare parts, auxiliary equipment, sometimes pilot training facilities, cockpit simulators, etc. are some of the items which account for additional cost, if the brand is changed. The following figure depicts the direct maintenance costs for the French built Caravelle, the British built BAC-111 and the American Douglas DC-9, as experienced by U. S. operators.



Twin Jets
U. S. Operators

Source:
ATA/CAB

Form 41 Reports

Figure 4-2

The high cost for the SE-210 Caravelle and BAC-111 is a composite effect of brand loyalty and engineering design factors. A close scrutiny of the fleet of U. S. operators like United, TWA, Eastern, American Continental, Delta reveals that United is the only airline operating 20 Caravelles in their total jet fleet of 390 and American Airlines is the only one operating 27 BAC-111 in their total fleet of 247! (1969 Data - IATA). Some illogical conclusions may be drawn by looking at such graphs.

There is a wide variety of jet aircraft available on the market for purchase. They generally go as a basic version plus some modifications at the option of the airlines. It would be an incredibly long list if an attempt were made to list all the modified models. The table below shows a list of basic versions of the jet aircraft.

Table 4-5*

Manufacturer	4 Engine		3 Engine		2 Engine	
	Turbo Fan	Turbo Jet	Turbo Fan	Turbo Jet	Turbo Fan	Turbo Jet
Boeing	747-B	747	727-100		737-100	
	747-C		727-100C		737-200	
	747-F		727-100C/QC		737-200C	
	707-120B	707-120	727-200			
		707-220				
	707-320B	707-320				
	707-320C					
		707-420				
	720B	720				
Douglas	DC-8-43	DC-8-10	DC-10-10		DC-9-15	
	DC-8-55	DC-8-20	DC-10-20		DC-9-21	
	DC-8-55F	DC-8-30	DC-10-30		DC-9-32	
	DC-8-61				DC-9-41	
	DC-8-61F					
	DC-8-62					
	DC-8-63					
	DC-8-63F					
Convair	CV-990	CV-880				
British Aircraft Corp.					BAC-111-200	
					BAC-111-400	
Sud Aviation						SE-210 Caravelle

* See Notes on next page

Notes for Table 4-5:

1. Some more British built jet aircraft - De Havilland Comet 4, HS Trident, Vickers VC-10, Super VC-10
2. This list excludes Russian built jet aircraft.
3. L-1011 will be introduced in 1972-73.

There is a bulk of information and data associated with every aircraft which is sought by the airline management before purchases are made. This information can be classified into 3 categories:

(A) Information relevant to operations of the aircraft;

(B) Information relevant to revenue or productivity;

(C) Information relevant to cost.

This is subdivided into 1) Initial cost
2) Direct Operations cost
3) Depreciation

(A) Information about Operations: - Out of a long list of items that come under this heading, a few are given as examples.

1) Systems: Hydraulic - Fuel - Flight controls - Engines - Thrust Reversers - Landing Gears - Air Conditioning and Pressurization - Electric Power - Avionics-Electronics - Brakes and Antiskid-Auxiliary Power Unit - Precision Approach and Landing System, etc.

2) Dimensions: Lengh, Wing span, Tail height, Wheel base, etc.

This information is important from the point of view of Hanger Design and Servicing Facilities.

3) Field Service and Technical Support

Aircraft Spares Support

4) Training Facility for - Flight Crew,

- Maintenance Personnel,

- Operations Staff.

5) Ramp - Dimensions and height of the doors.

6) Cargo Handling: Orientation and Dimensions of cargo doors, their heights.

7) Aircraft maneuverability on the airports: Turning radius, wheel base, etc.

8) Terminal Servicing: Servicing Facilities. Terminal operations. Time schedule is usually given for turnaround station and enroute station. It usually varies from 20 to 60 minutes.

9) Environmental & Safety: Noise level contours at take-off. Sound level inside the plane. Jet blast temperature contours for safety.

10) Weights: This is very important piece of information.

Some basic definitions:

(a) Maximum Ramp Weight = Maximum taxi weight = Maximum take-off weight + taxi fuel weight + runup fuel weight.

(b) Maximum landing weight = weight at touchdown = maximum takeoff weight - fuel burned.

(c) Operating empty weight = weight of (structures + powerplant + systems + unusable fuel + personnel + equipment, etc.)

(d) Operating empty weight + fuel + payload = maximum ramp weight.

(e) Zero fuel weight = maximum ramp weight - usuable fuel.

(f) Maximum structural payload = maximum design payload of (Passengers, their baggage and cargo).

(g) Usable Fuel = Fuel burned in (flight + taxi + runup + reserve).

11) Takeoff runway length data, Fig. 4-3. Takeoff runway lengths are plotted against takeoff weights. Higher airport elevation and higher temperature require longer runway length.

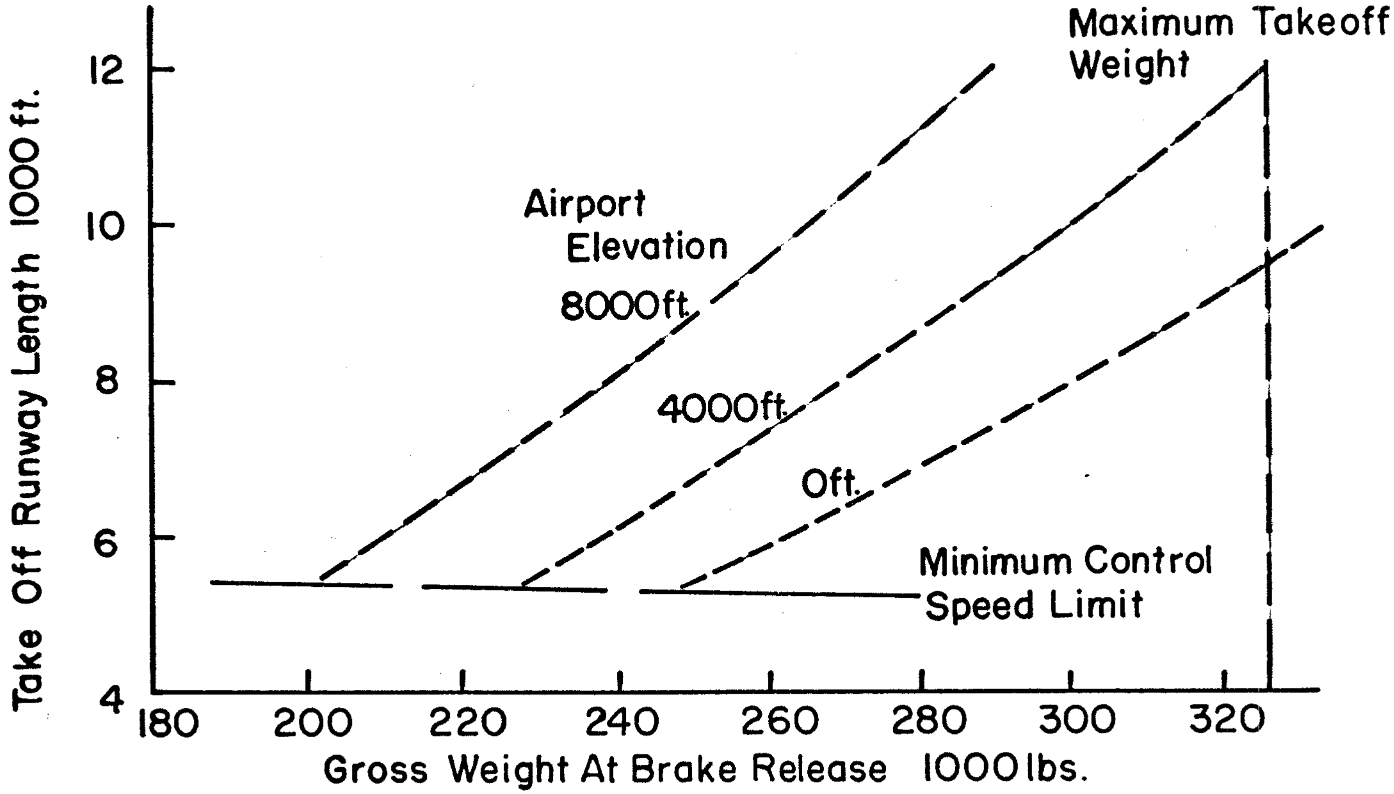
12) Landing runway length data, Fig. 4-4. Landing runway lengths are plotted against landing weights. Higher airport elevation and wet conditions require longer runway length.

13) Approach speed: Straight linear relation exists between approach speeds and landing weights.

(B) Information about Revenue or Productivity:

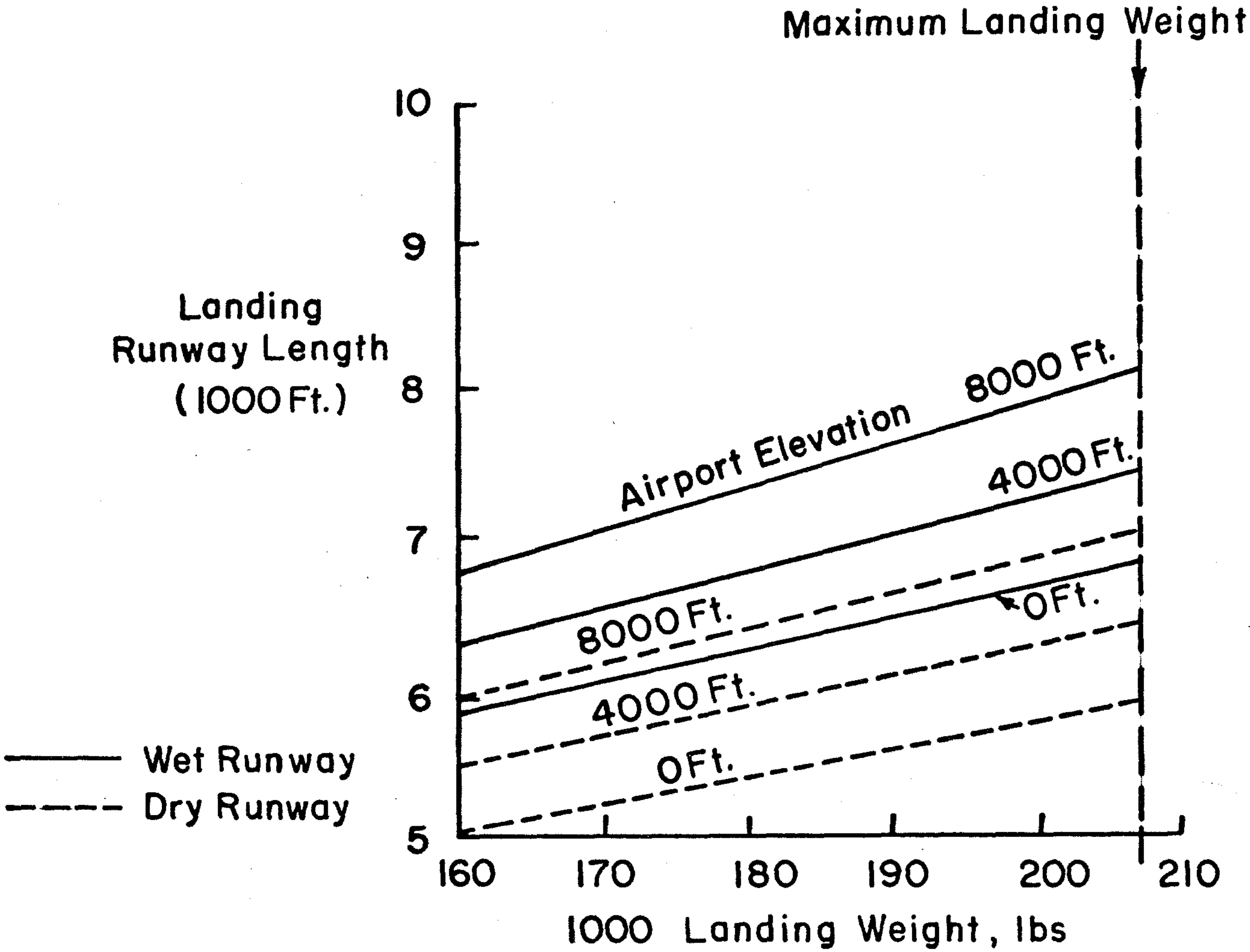
1) Range - Payload characteristics: Fig. 4-5 shows this characteristic for Boeing 707-320B. Beyond a certain range, any expansion in the range is achieved by the reduction of payload and by admitting an equivalent amount of fuel. The graph also depicts the characteristics for different takeoff weights and the point for 141 passengers + baggage payload.

2) Seating capacity: This depends upon the jet aircraft under consideration and the pitch of the seats. Information about first class, coach/economy class accomodation seem relevant. The Pitch and the interior arrangements



Standard Day, 14° Flaps JT3D-7 Engines Zero Runway Gradient
Take Off Thrust =19000 lb. Model 707-320 B Advanced
Source: Boeing Co.

FIG. 4-3 TAKE OFF RUNWAY LENGTH



Source: Boeing Co.

FIG. 4-4 MODEL-BOEING 707-320 B (ADVANCED)

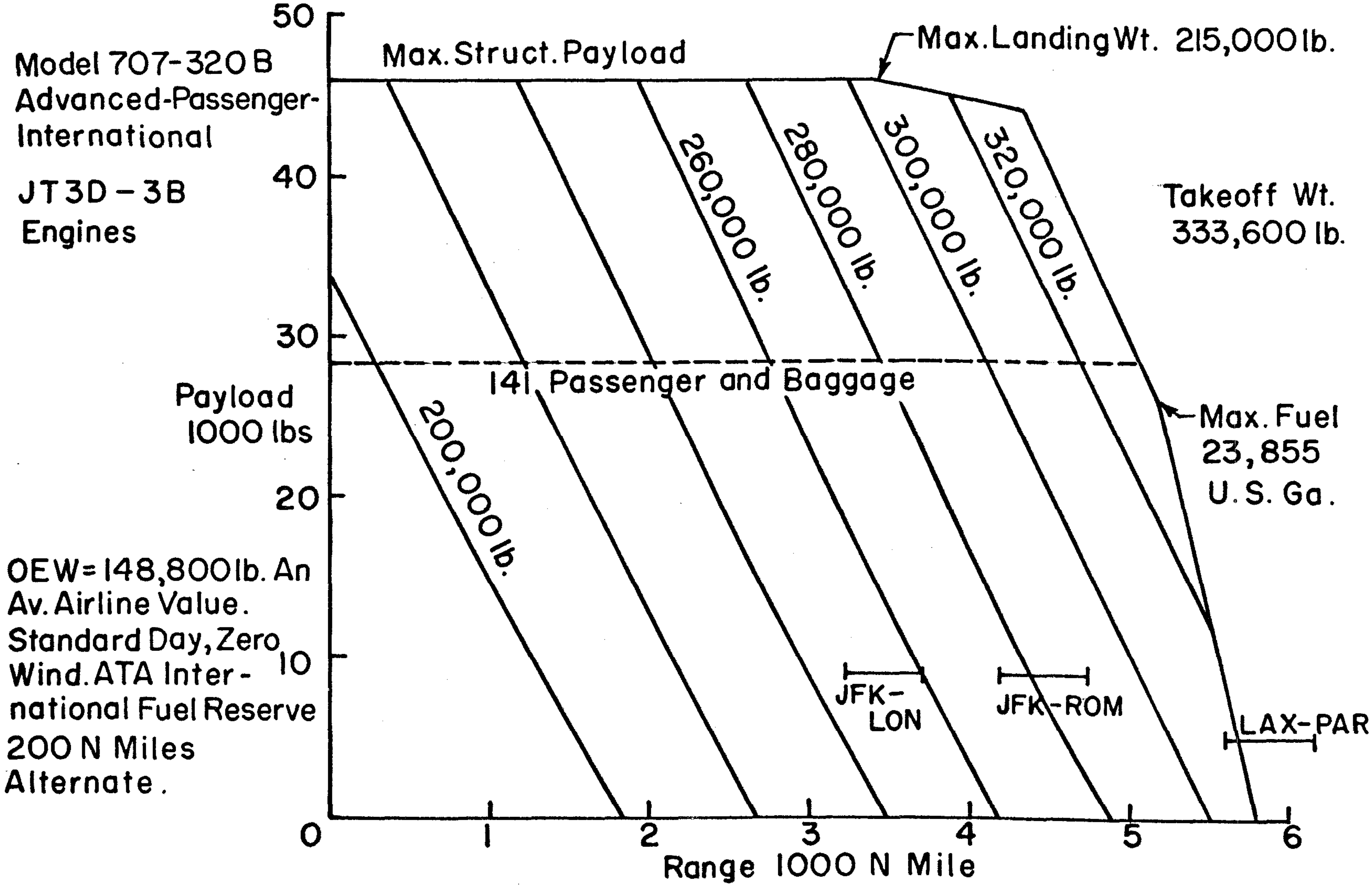


FIG. 4-5 PAYLOAD/RANGE FOR LONG RANGE STEP CLIMB CRUISE
Source: Boeing Co.

are at the air carrier's option.

3) Cargo: Belly cargo + upper compartment cargo. Number pallets/containers, size, volume, weight, etc.

(C) Information relevant to cost.

This is one of the major areas of the study. The variation in the cost of the aircraft is so wide that it is almost impossible to suggest a generalized formula of cost. It varies amongst the air carriers, it varies over the years, and it often deviates from the predicted values provided by the aircraft manufacturers.

(i) Depreciation: This is probably the most complex item in cost computations. As a rule, the aircraft depreciation goes like this - '12 years to 10% residual value - straight line method'. Sometimes airframe depreciation and powerplant depreciation follow different systems of computations. However, it could be said that management policy determines the method of computing depreciation. More about this appears under direct operating cost.

(ii) Initial or startup cost:

These costs are soaring rapidly for advanced jet equipment and wide-body jets. The startup cost consists of the following items:

(a) Complete aircraft price

(b) Airframe spares

(c) Powerplant spares

(d) Spare engines

(e) Workshop equipment. (Line and dock, engine overhaul, test equipment)

(f) Ramp equipment

(g) Hangar and building facilities

(h) Flight and ground crew training

(i) Delivery flight

(j) Insurance, customs duty, etc.

(iii) Direct Operating cost:

Let us recapitulate three relations developed previously

$$\sum TOC = 2.10 \sum DOC$$

$$\text{Cost} = \text{Function} \begin{pmatrix} \text{Design and engineering factors,} \\ \text{How is jet aircraft being used} \end{pmatrix}$$

A desirable relation

$$DOC = \text{Function (Jet aircraft, routes, scheduling)}$$

The last two relations are almost identical, since the use of the aircraft is reflected in the routes on which it is deployed and its scheduling. This sort of relation is developed by examining the experience of different airlines over the past few years. The purpose of such a formulation is four-fold:

1) To predict the direct operating cost, with prior knowledge of jet aircraft, routes and scheduling.

2) To assess the total cost of the jet planes of 'our' airline in comparison with the industry average.

3) To determine comparative profitability of the various equipment.

4) To provide a good guideline in the purchase policy of the jet equipment.

Boundaries of the cost-relationship:

This model will be derived from the past experience of some of the air carriers of the United States of America. The time span of experience is also limited to 4 years, 1966 to 1969. The costs of a very limited number of jet aircraft, one representing each class, are incorporated in the formulation.

Hence, the following drawbacks should be well understood.

(A) It cannot be successfully applied in other countries where cost structure and money systems are different.

(B) A model derived from a 4 year experience may not truly represent the phenomena.

(C) The cost and performance data are taken for three jet aircraft. For a more refined model, a large number of jet types should be taken into account.

(D) Obviously there are many elements which directly contribute towards cost, but only limited parameters are considered in the formulation.

Referring to Table 2-2 which shows the breakup of the direct operating cost, we can write

$$\text{DOC} = \text{Flying Operations Cost (F)}, + \text{Maintenance Cost (M)}, + \text{Depreciation Cost (D)} .$$

$$\therefore \quad \text{DOC} = F + M + D .$$

F = cost of (crew + fuel & oil + insurance)

M = cost of maintenance of (Airframe + Powerplant)

D = cost of depreciation of (Airframe + Powerplant) .

This is a rather simplified version of the cost breakdown. Three independent relations will be developed.

Not let us search for the independent variables or parameters which will represent the inputs, J, R and S. Though there are many alternatives, the following parameters seem best.

Input J ---	characteristics of the jet plane	1) Weight-operating empty. 2) Number of engines. 3) Fuel consumption per block hour running.
Input R ---	characteristics of the route structure	1) Average stage length.
Input S ---	characteristics of the scheduling	1) Utilization of aircraft per day.

The model equation will be derived relating these parameters to the DOC.

Some attempt is made to investigate the effects of maintenance on the cost structure. The impact of good maintenance during the previous period would be observed during the present period -- e. g. a reduction in present maintenance costs and a reduction in oil and fuel consumption in the power plants, hence lowering flying operations cost. Theoretically, it should also have some impact on the depreciation -- good maintenance should bring down depreciation. Some of these hypotheses could well be tested by taking into account the impact of the maintenance of the previous year. The tacit assumption in this type of analysis is that the quality of maintenance is wholly judged by the amount of money spent on it -- which is only a half truth.

Table 4-6

Mathematical Formulation of the Cost Model - Econometric Approach

Variable	Notation	Nature	Unit
Flying Operations Cost	F	Endogenous	\$/Block hr.
Maintenance Cost	M	Endogenous	\$/BH
Depreciation	D	Endogenous	\$/BH
Number of Engines	E	Exogenous	Number
Operating Empty Weight	W	Exogenous	lbs.
Fuel consumption	C	Exogenous	$\frac{\text{gallons}}{\text{BH}}$
Average Stage Length	L	Exogenous	(Statute) miles
Daily Utilization	U	Exogenous	$\frac{\text{Hours}}{\text{Day}}$
Maintenance of previous year	M_{-1}	lagged Endogenous	\$/BH
Stochastic Variable	u	--	--
DOC per available ton-mile	$\frac{DOC}{ATM}$	Endogenous	¢/BH
DOC per available seat-mile	$\frac{DOC}{ASM}$	Endogenous	¢/BH

Assumptions: 1) Linear relations exist.

2) Intercept terms are present.

3) Stochastic terms are present.

The last two items, $\frac{DOC}{ATM}$ and $\frac{DOC}{ASM}$ represent unit costs. The operating empty weight gives an aggregate impact of the complexities of the airframe, the systems, and their weight. Of all the parameters, the OE weight aptly represents the magnitude and structural complexity of the airframe. The utilization is an indirect outcome of the scheduling.

The following linear relations could be expected:

$$F = \alpha_0 + \alpha_1 W + \alpha_2 C + \alpha_3 M + u_1 \quad \ldots\ldots (1.1)$$

$$M = \beta_0 + \beta_1 E + \beta_2 W + \beta_3 U + \beta_4 L + B_5 M_{-1} + u_2 \quad \ldots\ldots (1.2)$$

$$D = \gamma_0 + \gamma_1 E + \gamma_2 W + \gamma_3 M_{-1} + u_3 \quad \ldots\ldots (1.3)$$

$$\frac{DOC}{ATM} = \delta_0 + \delta_1 E + \delta_2 W + \delta_3 U + \delta_4 L + \delta_5 C + u_4 \quad \ldots\ldots (1.4)$$

$$\frac{DOC}{ASM} = \rho_0 + \rho_1 E + \rho_2 W + \rho_3 U + \rho_4 L + \rho_5 C + u_5 \quad \ldots\ldots (1.5)$$

Substituting (1.2) into Eq. (1.1) and simplifying

$$F = (\alpha_0 + \alpha_3 \beta_0) + \beta_1 \alpha_3 E + (\alpha_1 + \alpha_3 \beta_2)W + \alpha_2 C + \alpha_3 \beta_5 M_{-1} + (\alpha_3 \beta_3)U + (\alpha_3 \beta_4)L + (u_1 + \alpha_3 u_2) \quad \ldots\ldots (1.6)$$

$$\frac{DOC}{ATM} = \frac{DOC}{Rev.\ TM} \cdot \text{Load Factor} \quad \ldots\ldots (1.7)$$

$$\frac{DOC}{ASM} = \frac{DOC}{Rev.\ PS} \cdot \text{Load Factor} \quad \ldots\ldots (1.8)$$

Data:

Complete Data are given in Appendix . Data are taken from publications, entitled - "Aircraft Operating Costs and Performance Report," published by the Civil Aeronautics Board - Washington. The time span of the data is chosen as 1965-69. The air carriers had gained enough experience about the jet aircraft mentioned on the following page and, during those years, airlines had stabilized their cost structure. The data refer to only three aircraft, one representing each class. They are the Boeing 707-320B, the Boeing 727-100 C/QC and the DC-9-30, with 4, 3 and 2 engines respectively. Data refer to the experience of many domestic and international carriers of the U. S. A total of 40 sets of data are taken into account.

The parameters of the functional relations have been estimated using 'Multiple Regression Source Program' on IBM-360-75 computer. The computer printouts are summarized below:

$$F = \underset{(53.432)}{-57.488E} + \underset{(0.0014)}{0.0013\ W} - \underset{(5.030)}{17.983\ U} + \underset{(0.015)}{0.022\ L} + \underset{(0.055)}{0.207\ C} + \underset{(0.085)}{0.207\ M_{-1}} + 173.895$$

$$R = 0.975$$

$$M = \underset{(62.054)}{-22.601\ E} + \underset{(0.0016)}{0.0011\ W} - \underset{(5.861)}{19.339\ U} + \underset{(0.0158)}{0.0124\ L} + \underset{(0.0967)}{0.784\ M_{-1}} + 156.725$$

$$R = 0.874$$

$$D = \underset{(35.217)}{-61.845\ E} + \underset{(0.00076)}{0.002\ W} - \underset{(0.074)}{0.200\ M_{-1}} + 124.196$$

$$R = 0.794$$

$$\frac{DOC}{ATM} = \underset{(2.855)}{2.652\ E} - \underset{(0.00007)}{0.0001\ W} - \underset{(0.269)}{1.321\ U} + \underset{(0.0007)}{0.00014\ L} + \underset{(0.00288)}{0.0058\ C} + 17.541$$

$$R = 0.91$$

$$\frac{DOC}{ASM} = \underset{(0.365)}{-0.0036\ E} - \underset{(0.00001)}{0.000006\ W} - \underset{(0.0344)}{0.165\ U} - \underset{(0.000099)}{0.00003L} + \underset{(0.00037)}{0.0008\ C} + 2.423$$

$$R = 0.884$$

The figures in the brackets represent the estimated standard deviations of the coefficients. R is a correlation coefficient.

The next few pages show these relations in graphical form for different average and group average values.

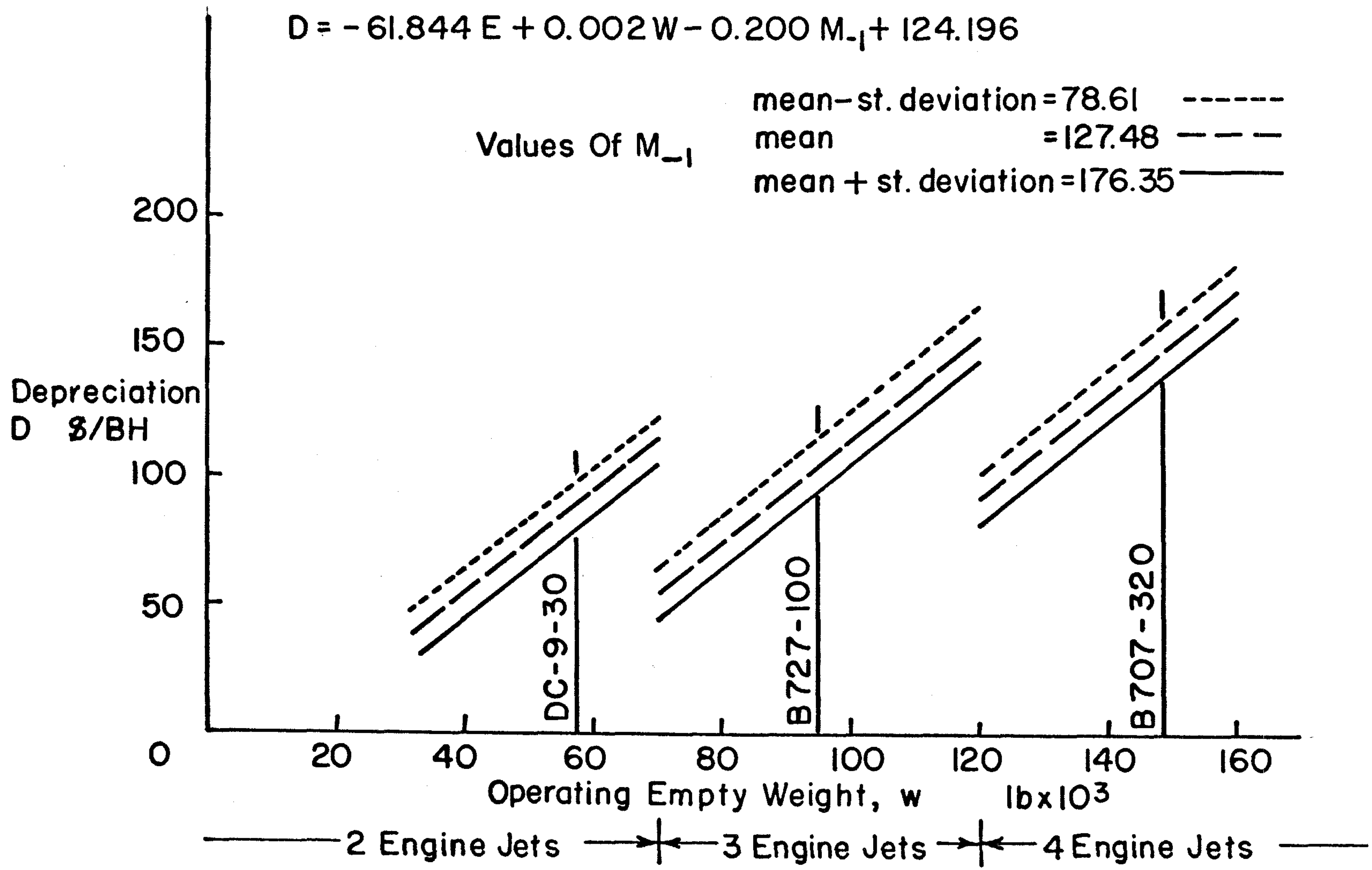
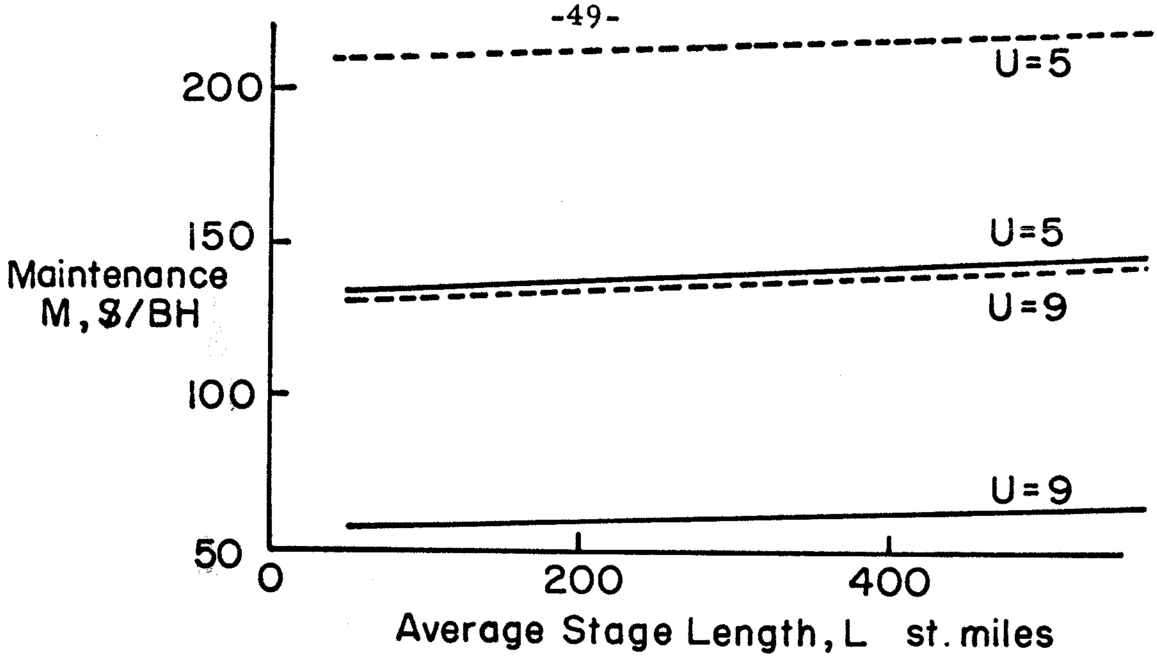


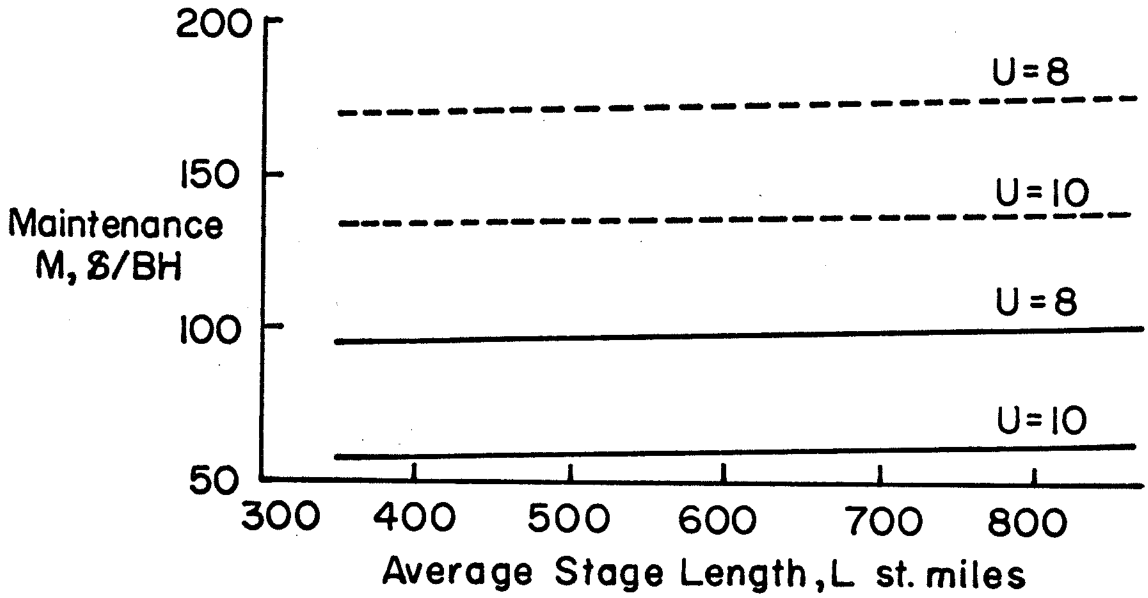
FIG. 4-6 DEPRECIATION VS. OE WEIGHT



(a) DOUGLAS DC-9-30

$$M = -22.601E + 0.001W - 19.339U + 0.012L + 0.784M_{-1} + 156.75$$

Values Of M-1 mean − st. deviation = 78.61 ———
mean + st. deviation = 176.35 – – –



(b) BOEING 727-100 C/QC

FIG. 4-7 MAINTENANCE VS. STAGE LENGTH

$$M = -22.601E + 0.001W - 19.339U + 0.012L + 0.784M_{-1} + 156.725$$

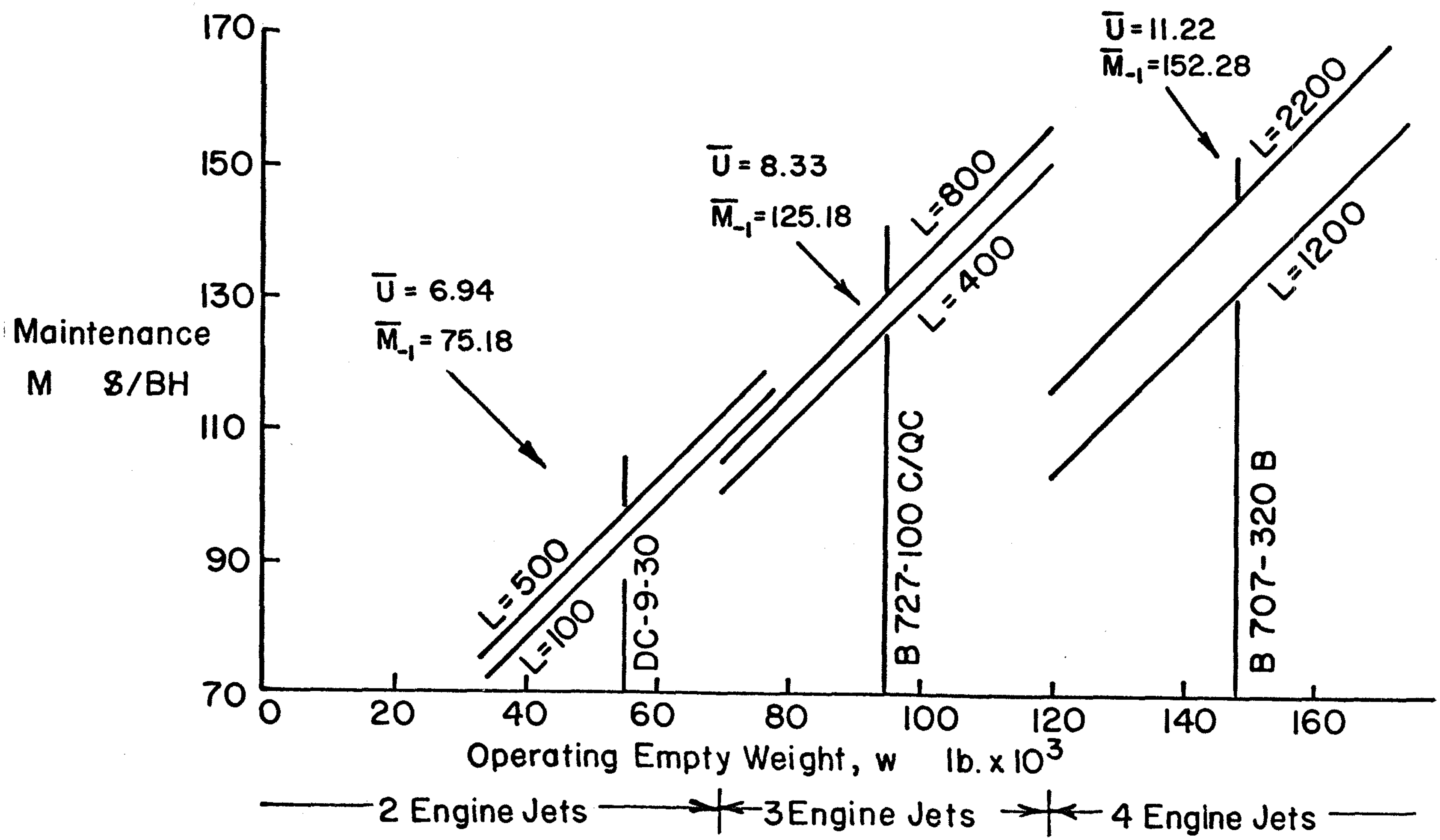
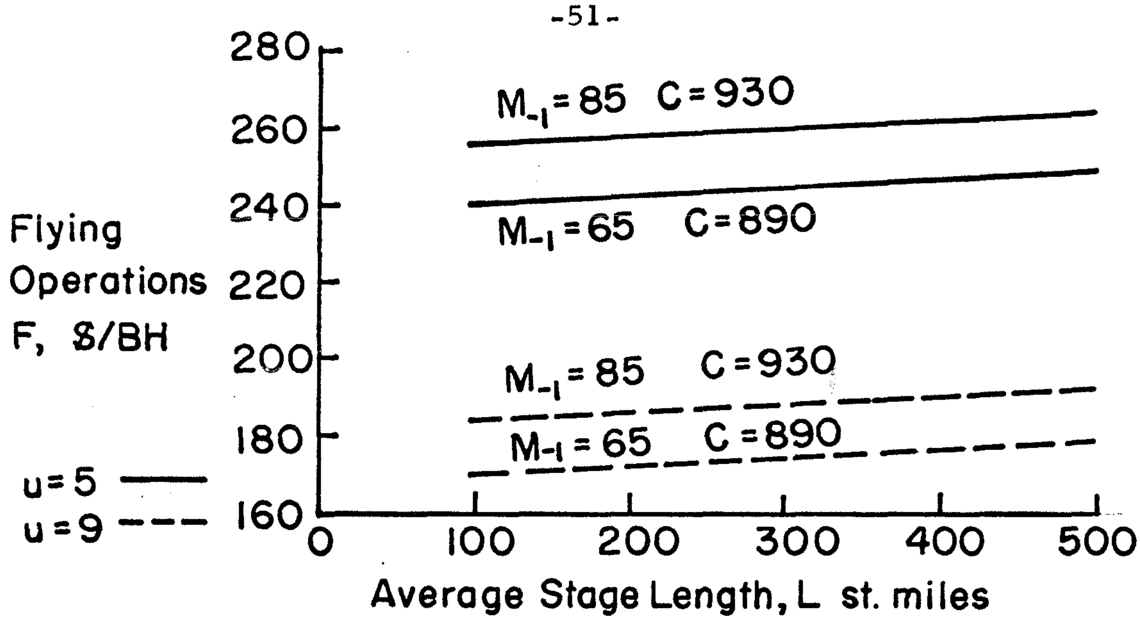
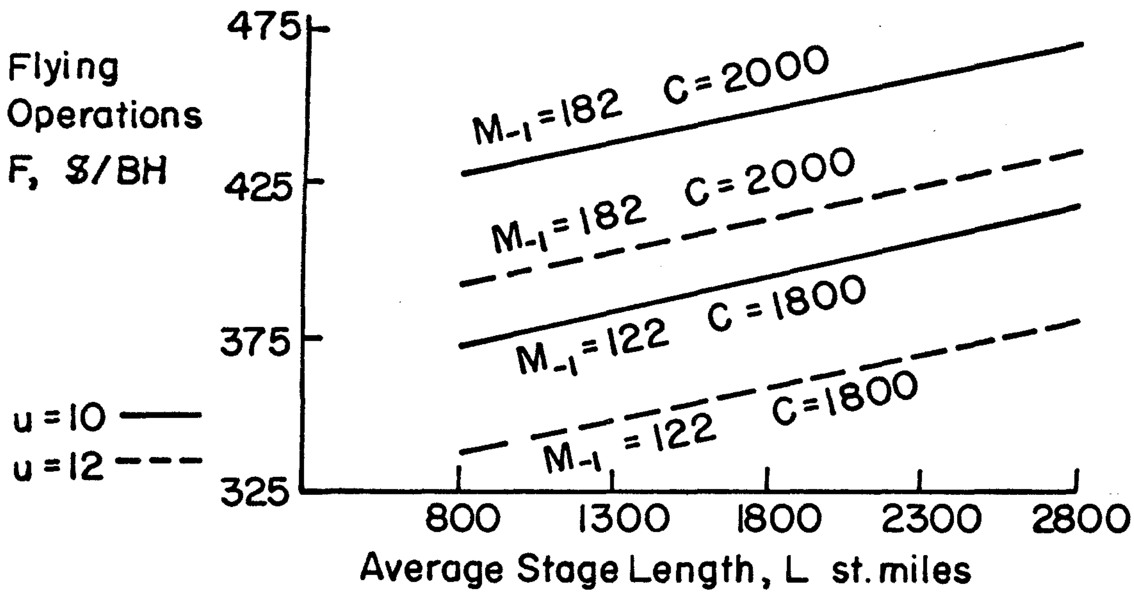


FIG. 4-8 MAINTENANCE VS. OE WEIGHT



(a) DOUGLAS DC-9-30

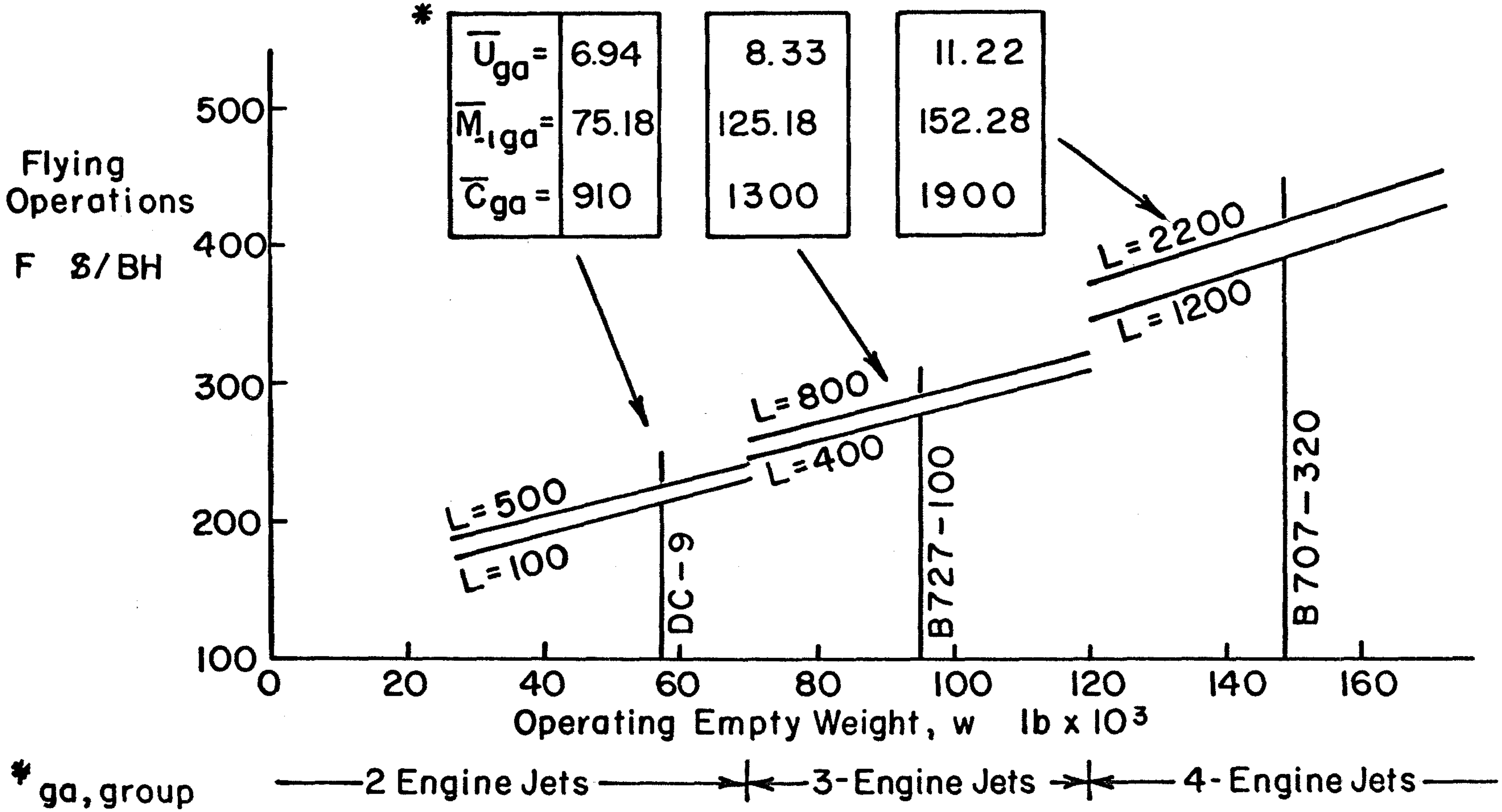
$$F = -57.488E + 0.0013W - 17.983U + 0.022L + 0.207C + 0.207M_{-1} + 173.89$$



(b) BOEING 707-320B

FIG. 4-9 FLYING OPERATIONS VS. STAGE LENGTH

$$F = -57.488E + 0.0013W - 17.983U + 0.022L + 0.207C + 0.207M_{-1} + 173.89$$



* ga, group average of data

FIG. 4-10 FLYING OPERATIONS VS. OE WEIGHT

V. INPUT S - SCHEDULING

This is the last input to the airline system under consideration. In order of sequence, it should appear third, next to Routes and Jet aircraft. The scheduling is a very general concept in the realm of airlines, and it may mean the scheduling of aircraft or flight crew or maintenance personnel. However, in our case Input S specifically refers to the scheduling of aircraft only which amounts to the determination of arrival and departure times, frequency, etc.

Once decisions are taken regarding the routes to be served, and the jet plane to be operated, the question of scheduling follows immediately. Items that go into making the schedules are --

(i) Existing traffic estimate. Market survey.

(ii) Expert opinion about the traffic growth for short range/long range planning.

(iii) Seasonal fluctuations.

(iv) Cyclical fluctuations.

(v) Summer charter peaks.

(vi) The position of such traffic fluctuations in relation to the delivery cycle of the new equipment. To meet these varying demands is one of the toughest problems faced by the airlines.

Some of the fundamental difficulties in tailoring schedules are --

(A) An indivisible unit of capacity is a plane-load of seat-miles, not the individual seat-mile. To match this quantum of capacity to the demand is not easy. This problem became extremely severe when the wide-body jets were put into service.

(B) Every airline treats each of their schedules as a distinct and specialized product having its own appeal for the passengers in that market. The passenger appeal and psychological factors are far beyond mathematical formulation. Passengers do not prefer 100% or very high load factors, they do not want to change planes, they do not like delays in schedules.

(C) The competition on the same route poses many problems for individual airlines. If the airline has to offer an effective competition, it has to provide at least approximately the same schedule convenience as that of other competing carriers, which inevitably leads to offering the same numerical frequency at approximately the same time! An airline, operating at low frequency, even at convenient times, does not stand out under severe competition. The next logical consequence is overcapacity on that route and hence detrimental low load factors.

(D) In the past, the route structure was not well spread as it is today. The airlines used to get origin-destination traffic and in addition to this, what is termed as 'beyond point' traffic. With the overgrowth in commercial aviation, the non-stop services have almost crippled the beyond point market.

(E) In the past, a single airport complex used to serve the entire metropolitan area; however, today we see the development of satellite airports with traffic being diverted from main to satellite airports. The phenomena, though healthy, from other points of view, poses difficulties for every airline in arranging sound schedules and in meeting the demand.

The travelling public derives considerable benefits from increased schedules, but the airlines suffer badly.

Some of the constraints in arranging schedules are:

1. For the short haul market, the schedule times must be very convenient to the passengers. Off hours and night departures are barred. Contrary to this, for the long haul market, off hour schedules are tolerable.

2. In a major airline network, the spares and minor/major overhaul facilities are located at very few places. Therefore, a particular aircraft has to return to its base after a certain number of flying hours.

3. Sometimes an airline has to undertake a non-revenue flight in order to reach a point of origin for a new flight.

4. The interdependent schedules, the schedules for the connected flights, and joint schedules for two airlines as part of a pooling arrangement are examples of the difficult situations which may arise.

The revenue is a very important function of the schedule. To arrange an optimum schedule, maximizing the number of passengers or, more precisely, maximizing the revenue is a difficult task. An optimum schedule may be thought of as one at the peak of the demand; however, the question remains unanswered as to what determines 'the peak'. In a monopolistic market, probably an airline has more 'say' in the determination of the peak, while in a competitive market, the travelling public has more voice.

In cost calculations, as mentioned before, utilization appears predominantly which, in fact, is an immediate outcome of scheduling. The average utilization of 3600 block hours per year does not mean daily utilization of 10 block hours; it may be 13 or 16 hours. Some of the major overhauls keep the jet aircraft idle for 15 days. The overhaul cycles have been extended up to 6000 $\sim$ 9000 flying hours for modern jet equipment.

VI. REVENUE ANALYSIS

We are entering into a complex realm of analysis which is far more arbitrary than anything discussed so far. The economic, political factors, passenger psychology, amongst many others, play a conspicuous role in the revenue considerations. This highly non-mathematical problem could be investigated by assigning arbitrary values and noting the trends. These trends can be more refined by market research and deeper analysis using past experience. Therefore, the following analysis should be read keeping well in mind its inherent limitations.

We have already stated -

$$\text{Total Revenue} = \sum Rv_i = 1.16 \sum_{1}^{3} Rv_i$$

where Rv_1 .... revenue from scheduled first class passengers.

Rv_2 .... revenue from scheduled coach class passengers.

Rv_3 .... revenue from scheduled freight.

We seek a relation,

$$\text{Revenue} = Rv\,(R, J, S)$$

In other words, for a given set of the route, the aircraft and the scheduling, we should be able to compute revenue numerically.

$Rv(R)$: For a given route, from airport A to airport B, the revenue goes directly with the number of passengers going from A to B or B to A. How many people do go from A to B? An expert market analyst may answer this question by giving three 'estimates'. The most likely number of travellers (most likely estimates) is, say, 600, but sometimes as high as 690 (optimistic estimate) and occasionally as low as 480 (pessimistic estimates). On obtaining these three estimates, there are two distinct ways to deal with them.

(A) Expected Value Approach:

This has been adopted from the PERT technique -- Program Evaluation and Review Technique -- of operations research. In this approach, 3 estimates; opti: for everything goes exceptionally well; pessi: under most adverse condition, most likely: most realistic; of most intuitively meaningful quantities are obtained and then converted into the estimate of expected value and variance.

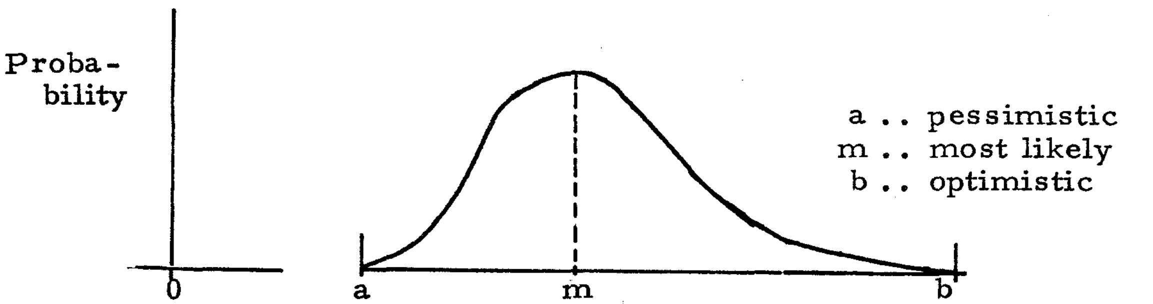


Fig. 6-1

Model of probability distribution for estimating the expected number of passengers

Two assumptions are made in such conversion --

1) Standard deviation or square root of variance is equal to the one-sixth of the range of reasonably possible market values.

i. e. $$\sigma^2 = \left[\frac{1}{6}(b-a)\right]^2$$

Rationale of the assumption: The tails of many probability distribution, such as normal distribution are considered to lie at about 3 standard deviations from the mean, so that there would be a spread of about 6 standard deviations between the tails.

2) The distribution is approximately β, with 'm' as mode, 'a' as a lower bound and 'b' as an upper bound.

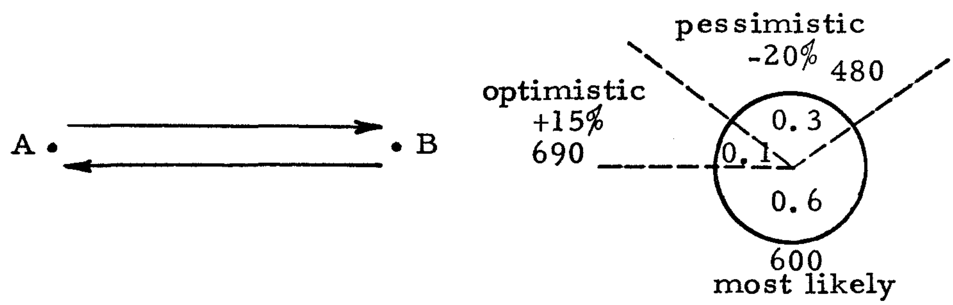
$\therefore$ Ne = expected number of travellers

$$= \frac{1}{3}\left[2\,m + \frac{1}{2}\,(a + b)\right]$$

This is a weighted arithmetic mean of the mode m and the mid-range $\frac{a+b}{2}$, where mode is carrying $\frac{2}{3}$ of the entire weight.

(B) Monte Carlo Approach:

Suppose we attribute 3 distinct probabilities to three estimates. We denote it as follows:



The figures in the circular segments represent respective probabilities whose sum is obviously unity.

In most simplified versions of Monte Carlo simulation, these probabilities are converted into Monte-Carlo numbers. Suppose we decide to use two digit random numbers (from 00 to 99). Simulation could be carried out by first converting probabilities to 'accumulated' probabilities, then choosing appropriate Monte Carlo numbers. The table on the following page is self-explanatory.

Table 6-1

Event	Probability	Accumulated Probability	Monte-Carlo numbers
optimistic (690)	0.1	0.1	00 - 09
most likely (600)	0.6	0.7	10 - 69
pessimistic (480)	0.3	1.0	70 - 99

Every time a random number of two digits is drawn and its position amongst Monte-Carlo numbers is determined, the nature of the event is determined. One thing should be noted, however, that this simulation will yield satisfactory results for a large number of random number drawings. In other words, the system is best for infinite trials.

These 600 likely passengers, going from A to B and reverse, are distributed over a period of one day or 24 hours. Here we assume that the whole phenomena is repetitive or cyclic in nature and has a period of 24 hours. We also assume that the indivisible unit of time is one hour, hence we will not split hours into minutes.

A more realistic set of steps for investigation would be:

(a) How many people desire to go AB?

(b) How many people really go AB?

(c) How many people travel by other modes of transportation?

(d) Assuming the rest go by air, how many go by the competitors' airline?

(e) What is the first coach-class breakup of the captured market?

The analysis is done by following the framework given below.

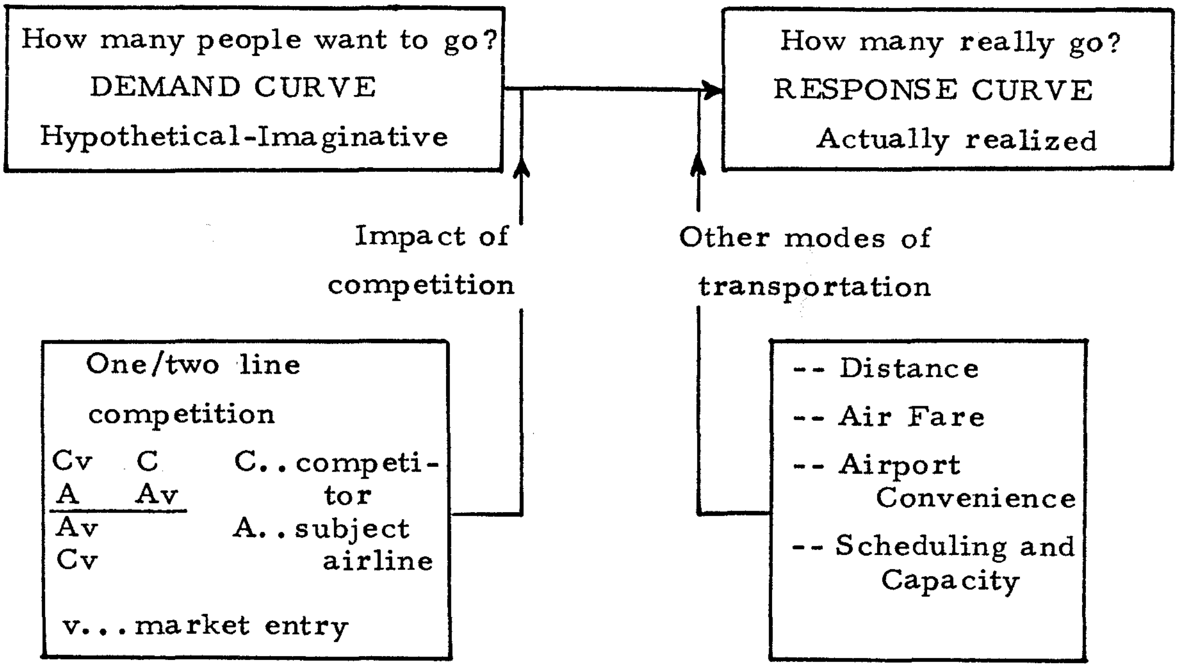


Figure 6-2

Simple arithmetic might suggest 4 frequencies of operation of an aircraft with an accomodation of 150 to serve the above market, AB, of 600 daily travellers. The fallacy of this logic lies in the fact that these 600 potential travellers are distributed (i. e. their preference for departure varies in time) over 24 hours and the 4 schedules at the pre-determined times may not necessarily meet this demand. It should be remembered at this point that the quantum of accomodation of the aircraft is 150 which cannot be further divided. This situation leads to the phenomena of 'Falling average load-factors' as shown in Fig. 6-3. The first frequency (or schedule) can be arranged by looking at the peak of demand so as to yield 150 travellers at that specific time, giving 100 percent load factor. The second schedule may not necessarily yield another 150, but a little less, thus reducing the average of two load factors. This reduction sharpens as we go higher on the frequency scale. But the airlines, for the sake of their passengers, do not prefer 100 percent load factor, but somewhere around 0.6.

$$\therefore \quad \text{Frequency} \times \underset{\text{per aircraft}}{\text{seat capacity}} \geq 600$$

also

$$\frac{\text{passenger miles}}{\text{frequency} \times \text{seats} \times \text{miles}} = \text{load factor} \leq 1.00$$

$$= 0.6$$

$$\therefore \quad \text{frequency} \times \text{seats per aircraft} = \frac{600}{.6} = 1000$$

If seat accomodation of the aircraft is 150, the desired schedule frequency would be 6 or 7.

Figures 6-4, 6-5 and 6-6 follow directly from 6-3. Fig. 6-4 shows the percent market share for air carriers (out of 600) goes up with higher frequency scale and finally flattens out. This again is an outcome of a falling average load factor. Assuming a $20 fare on route AB, Fig. 6-5 shows the trend of total revenue against frequencies. Fig. 6-6 depicts the rapid fall of revenue yield per schedule as schedules go up. In other words, revenue does not go in proportion with the schedules, once again the effect of the falling average load factor.

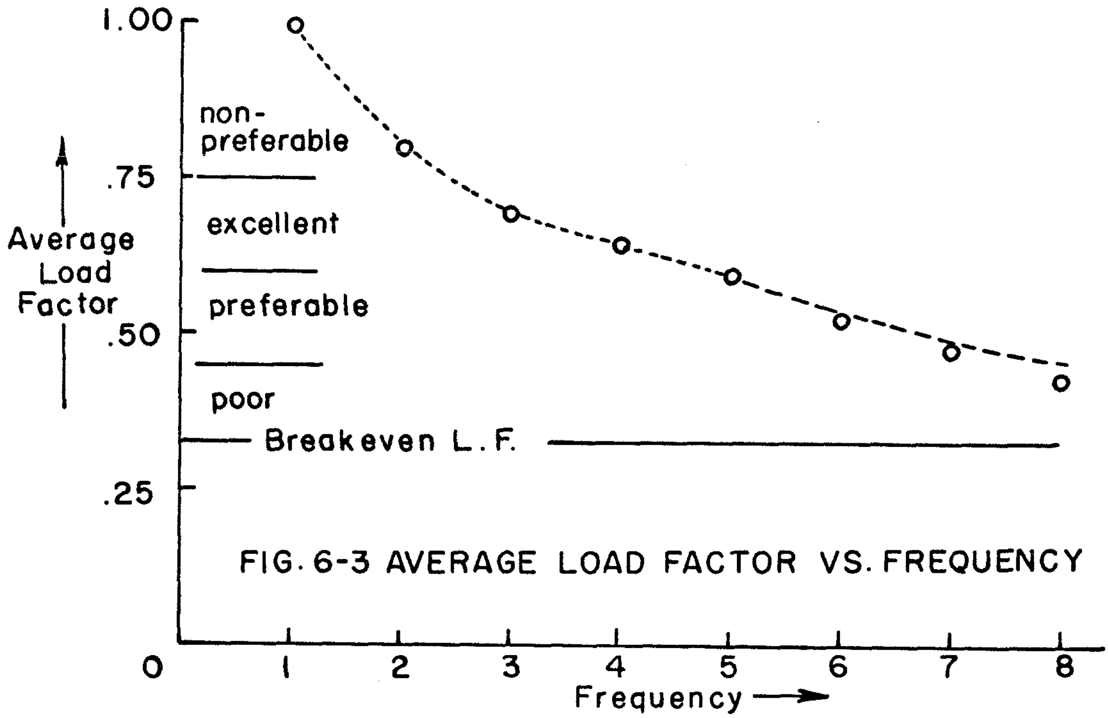


FIG. 6-3 AVERAGE LOAD FACTOR VS. FREQUENCY

These graphs are not continuous, since frequency scale is an integer one, the dotted line is just meant to show the "trend"

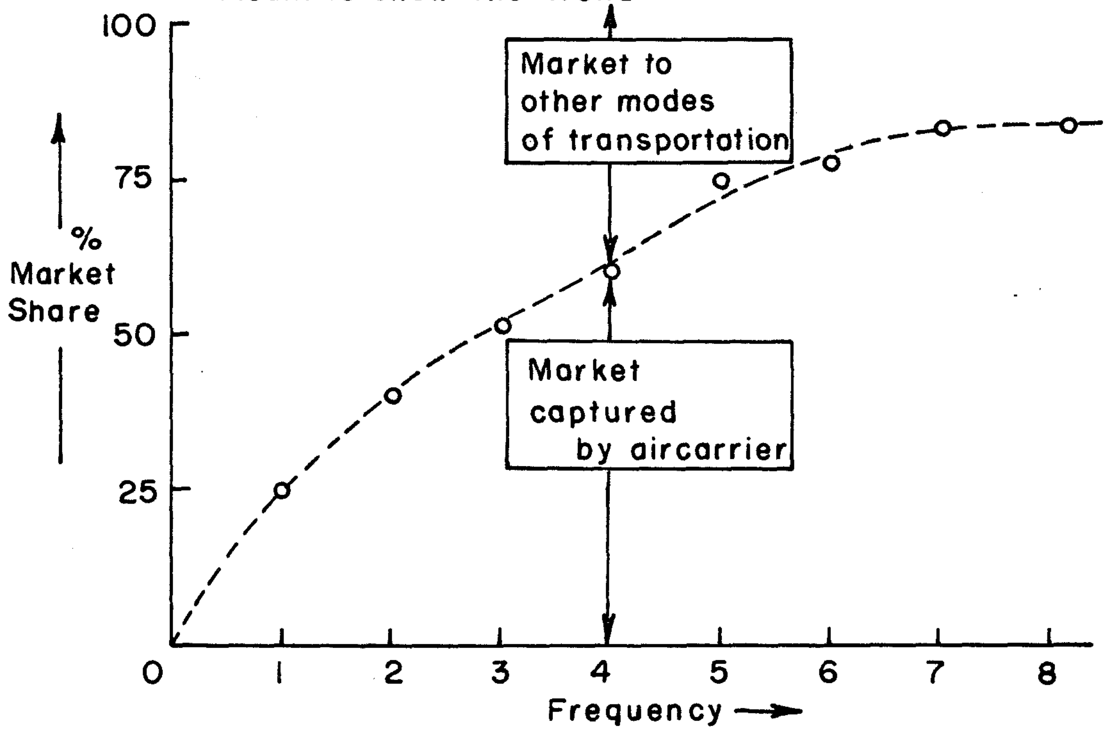


FIG. 6-4 % MARKET SHARE VS. FREQUENCY

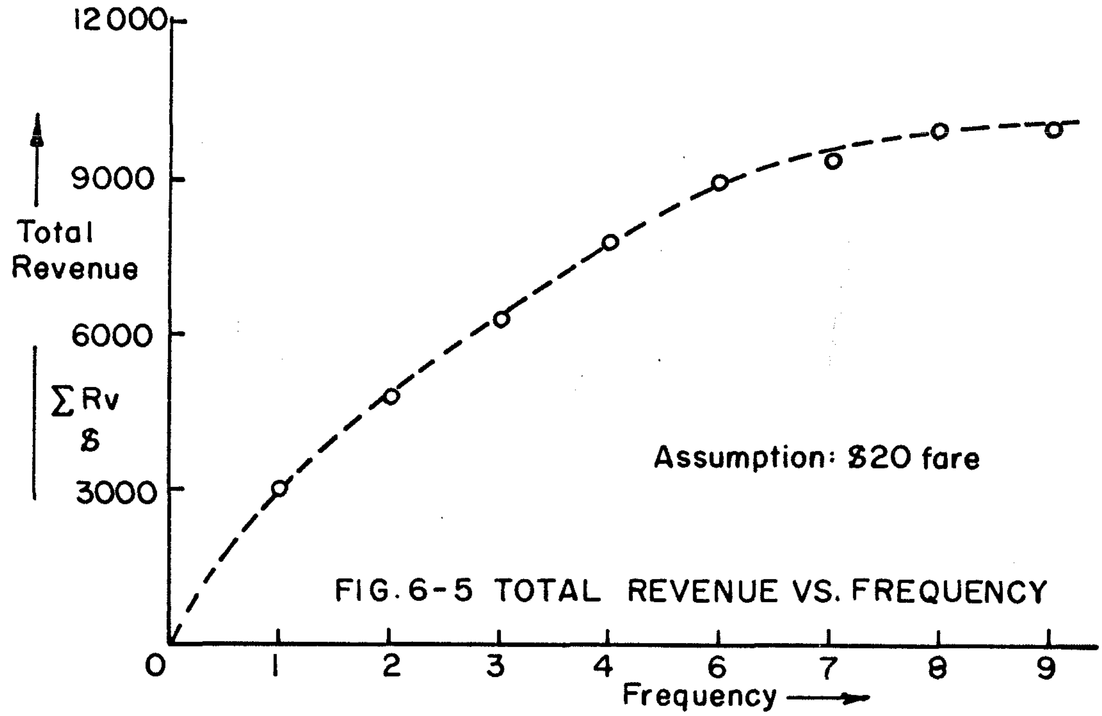


FIG. 6-5 TOTAL REVENUE VS. FREQUENCY

These graphs are not continuous, since the frequency scale is an integer one, the dotted line is just meant to show the "trend"

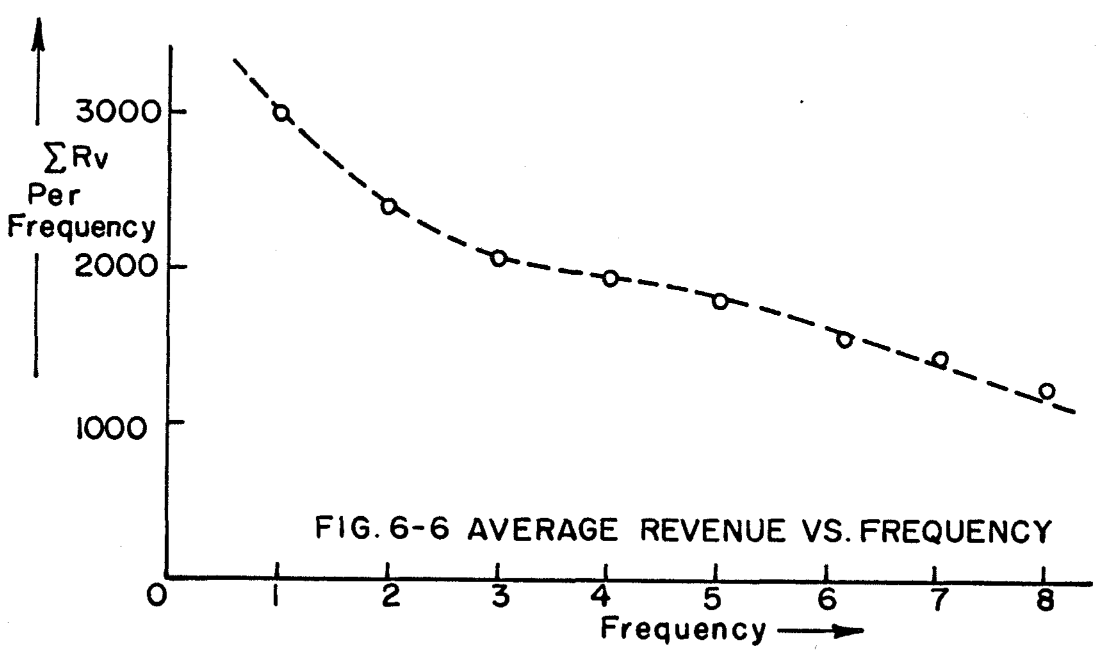


FIG. 6-6 AVERAGE REVENUE VS. FREQUENCY

Other modes of transportation (OMOT)

Suppose that route AB has been served by air and all other modes of transportation grouped together. Then the number of potential travellers between AB, going by OMOT will mainly depend upon:

(1) Distance: The air transport becomes more meaningful as the distance between A and B goes up. This behavior or preference of travellers for air transport and its variation with distance is shown in Figs. 6-7 and 6-8. Both are obviously asymptotic at very large distance. Figure 6-8 with its inverse exponential distance-market variation accounts quite well at zero distance, while the rectangular hyperbola does not. The numerical figures, though arbitrary, very well reflect the passengers' psychological preference.

(2) Airfare: The variation of the number of travellers with the amount of fare is what an economist would call a 'Demand Curve'. (Not to be confused with Demand/Response curves mentioned earlier.) Shown in Fig. 6-9 it is a characteristic of a particular city pair. It also depends upon the population, per capita income, disposable money, business activities, cost of OMOT and similar things. The numerical figures in the graph make the above demand curve 'elastic' since the elasticity coefficient exceeds unity. Its well known definition is the ratio of percent change in passenger volume to the percent change in fare.

(3) Airport convenience: The airports located far away from the cities and the congestion and delay of either aircraft or ground transportation to and from the airport will certainly divert a large volume

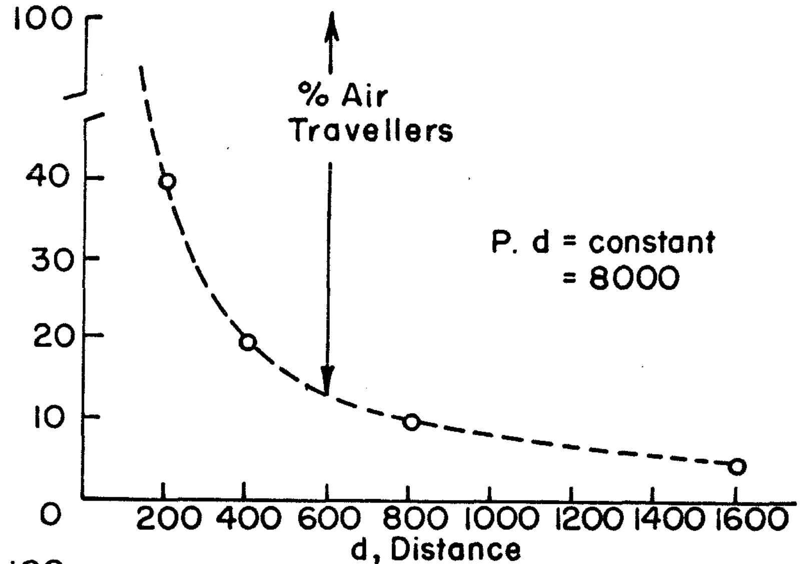


FIG. 6-7

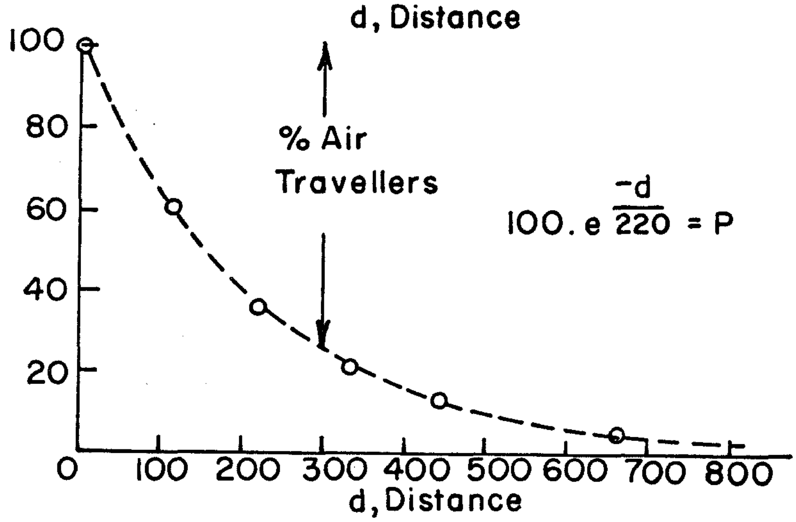


FIG. 6-8

EFFECT OF DISTANCE ON AIR MARKET

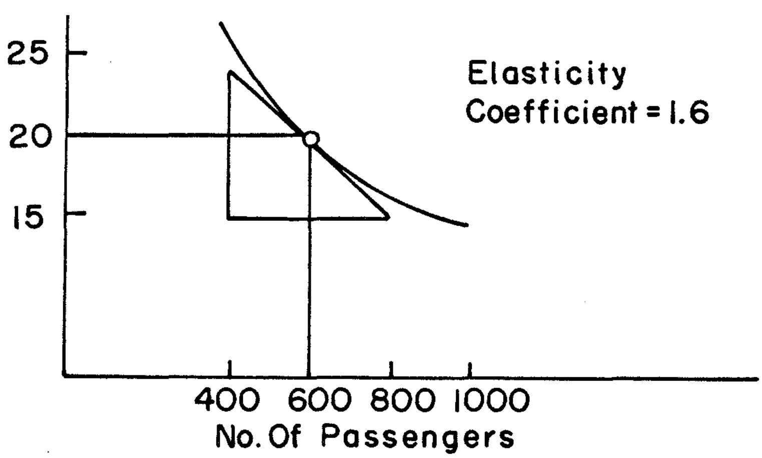


FIG. 6-9

EFFECT OF AIR FARE ON AIR MARKET

of potential passengers to the OMOT. We may here introduce a multiplying factor, Ac, assuming values from 0 to 1.0 and representing the percent reduction due to airport convenience.

(4) Scheduling and Capacity offered: The demand curve is a hypothetical curve and the area under the curve is proportional to the total number of travellers who would like to go AB or BA. The curve represents their distribution during the hours of the day, moreover the number of passengers whose preference for departure lies between hour H and H+1, where H is an integer between 0 and 23, will be given by --

$$N_H^{H+1} = \frac{\int_H^{H+1} D\,dt}{\oint_{24\text{ hours}} D\,dt} \cdot N_{Total} \cdot \qquad \text{D is a function of time, t.}$$

where D is the demand curve. One hour will be treated as a unit of time. The equation suggests that the number of passengers is proportional to the fraction of areas.

To give a typical example, suppose on the route, AB, some 400 miles long, negligible travelling occurs during 9:00 P.M. to 7:00 A.M. and it builds up between 7:00 A.M. to 8:00 A.M., continues at peak level until 10:00, drops to half at 11:00 A.M. and retains that level during the whole afternoon until 5:00 P.M., reaching the same level peak again between 6:00 to 8:00 P.M. and the traffic level dies down at 9:00 P.M. This is not what actually happens, but it represents the traveler's preference; hence the label - hypothetical!

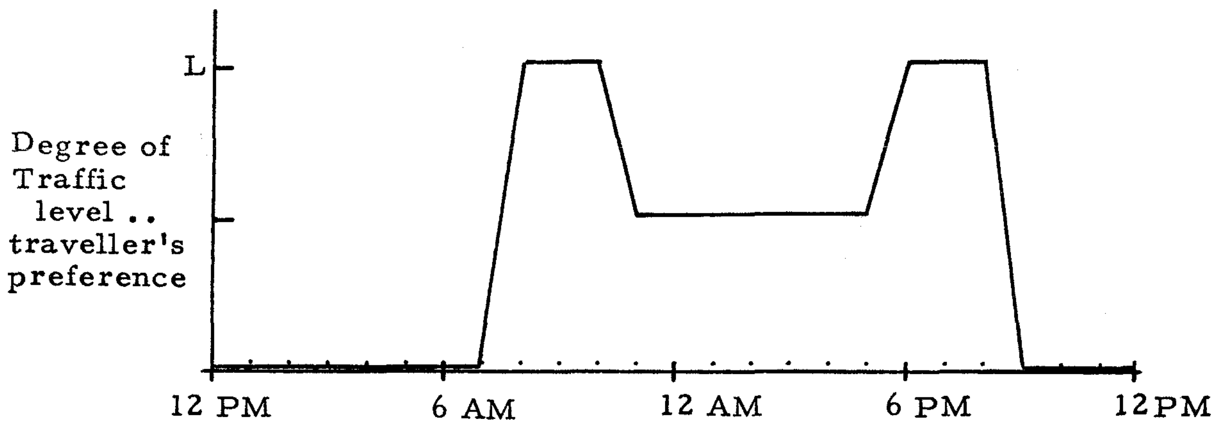


Fig. 6-10 Typical Demand Curve

$$\oint_{24\ \text{hours}} D\,dt = \text{Total area under the curve} = \frac{19}{2} L$$

Now we would like to devise a mechanism to transform this Demand Curve (D) to the Response Curve (R), and such transformation should account for competition and the OMOT.

On examining carefully, it will be found that there are <u>three</u> types of probabilistic variations. Let us represent each such variation on a 3-axis system of coordinates.

On X-axis: Hourly variation. Response curve or the distribution of passengers along the hours of the day

On Y-axis: Probabilities of the 'total' number of travellers.

On Z-axis: Subsequent variations in the Response curve due to 'switch-over' probabilities.

Switch-over probabilities describe the passengers' tendency to change from one airline to another under competition. A typical switch-over probability matrix for 3-airline competition may look as follows:

From Airline i	To Airline j = 1	j = 2	j = 3	Sum of prob.
i = 1	0.57	0.23	0.20	1.00
i = 2	0.41	0.43	0.16	1.00
i = 3	0.17	0.50	0.33	1.00

Suppose such switch-over takes place once a week, the first row in the matrix would suggest that out of the total number of passengers last week with the airline 1, 57% of those would continue to do so this week, 23% of those would switch-over to airline 2 and 20% to airline 3. The entries in the second and third row also represent such 'From-to switch over' for the airlines 2 and 3, respectively. These entries can also be interpreted as probabilities of switch-over whose sum is unity, shown in the adjoining column. This kind of considera-tion also holds good even if 'a particular' passenger would not travel every week, but he influences somebody else's choice of the airline.

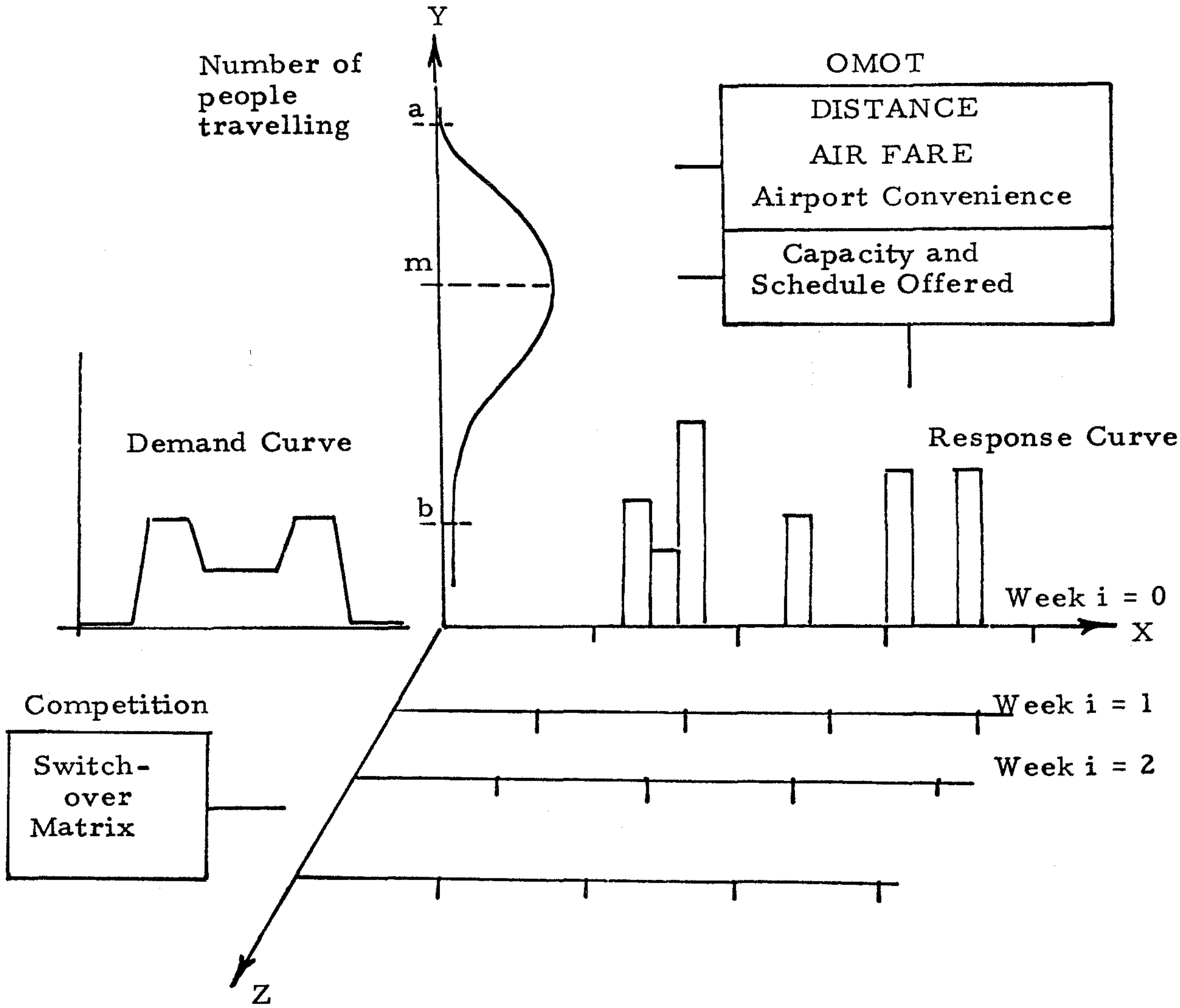


Figure 6-11

Three Dimensional Distribution of the Market

The essential details of the market are represented on the three axis-system. The Y-axis total traffic volume probabilities are affected by the distance of the route, airfare and airport convenience. The capacity and the schedules offered by the airlines have impact on both the Y-axis and the X-axis hourly variation of the market. The competition has influence on the Z-axis.

Y-axis: A city pair, A and B, 400 miles apart and having a total daily flux of 1000 passengers each way, will serve the purpose of illustration. Referring to Fig. 6-7 which isolates the impact of distance, there would be 20% reduction in the market, lowering the figure to 800 which may go by air. Assigning an arbitrary value of 0.77 to the airport convenience factor, Ac, the market may further drop to 600. Finally, referring to the airfare vs. passengers demand curve, Fig. 6-9, the airfare of $20 would stabilize the market at 600; however, past experience would show the fluctuations with optimistic and pessimistic boundaries of 690 and 480 respectively. We may have a continuous or discrete probability variation in the total volume of passengers.

X-axis: Now an attempt would be made to devise a fairly reasonable and logically consistent mechanism to transform the demand curve, D, into a response curve, R. Such a transformation is a direct outcome of the schedules being arranged by the airlines.

Suppose after exhaustive efforts of market research in people's preference for travel times, we come out with the demand curve shown in Fig. 6-10. We also know,

Total number of passengers, $N_{Total} \quad \alpha \quad \oint_{24 \text{ hours}} D \, dt$

$\oint$ represents integration over complete cycle, 0^{th} hour and 24^{th} hour are the same.

The number of people who prefer to go between hours H and (H+1), H being an integer, is given by

$$N_H^{H+1} = \frac{\int_H^{H+1} D\,dt}{\oint_{24} D\,dt} \cdot N_{Total} . \qquad 0 \le H \le 23$$

Any single schedule would perturb this demand curve. The resulting curve can be called the response curve, R. This perturbation should be such that the area under the curve remains unchanged, i. e.

$$\oint_{24\ hours} D\,dt = \oint_{24\ hours} R\,dt$$

It seems highly logical to believe that a schedule would not perturb the entire demand curve, but the part of it representing precisely the neighboring period of time at which this particular schedule is arranged. Let us call this perturbed area as an 'area of influence', denoted by T, expressed in terms of hours.

T depends upon two factors,

d, the distance of the journey, the longer the distance, the larger the T and

f, the total number of schedule frequencies offered by all airlines.

The larger the f, the smaller the T.

$$\therefore \qquad T = \kappa_1 \frac{d}{f} .$$

where κ_1 is a constant, having units of $\frac{\text{hours}}{\text{mile}}$. A typical value of κ_1 would be around 0.005 .

When the airline arranges a particular schedule at a particular time - hereafter referred as a reference schedule and the reference time -- the people whose preference for the departure lies in the period -- influence area -- adjoining the reference time, are compelled to go on the reference schedule. Therefore, the demand curve forms a 'bulge' near the reference time indicating more people are travelling at that hour.

It seems rational to say that the amount of 'bulge' or 'swelling' depends upon the original area, the influence area or the period T; it also depends upon a peculiar factor, $\bar{t}$ which is explained a little later.

$$\therefore \int_{H_{ref}}^{(H_{ref}+1)} R\,dt \;\alpha\; \int_{H_{ref}}^{(H_{ref}+1)} D\,dt, \quad \text{original area}$$

$$\alpha \quad T, \quad \text{influence period}.$$

$$\alpha \quad \bar{t}$$

$$\therefore \int_{H_{ref}}^{H_{ref}+1} R\,dt = \kappa_2 \cdot \left[\int_{H_{ref}}^{H_{ref}+1} D\,dt\right] \cdot T \cdot \bar{t}$$

where H_{ref} .... reference time

κ_2.......... proportionality constant, having units of $\frac{1}{\text{hour}}$, takes on values between 0.5 to 3.0 .

The factor, $\bar{t}$, whose values are dependent on the 'time lapse' between the reference time and the next adjacent schedule time. The logic of its presence in the above equation is as follows -- the amount of bulge at the reference time will be influenced by the next adjacent bulge at the adjacent schedule time. The asymptotic variation of $\bar{t}$

with the time lapse between the reference time and the next (nearest) schedule time is shown below.

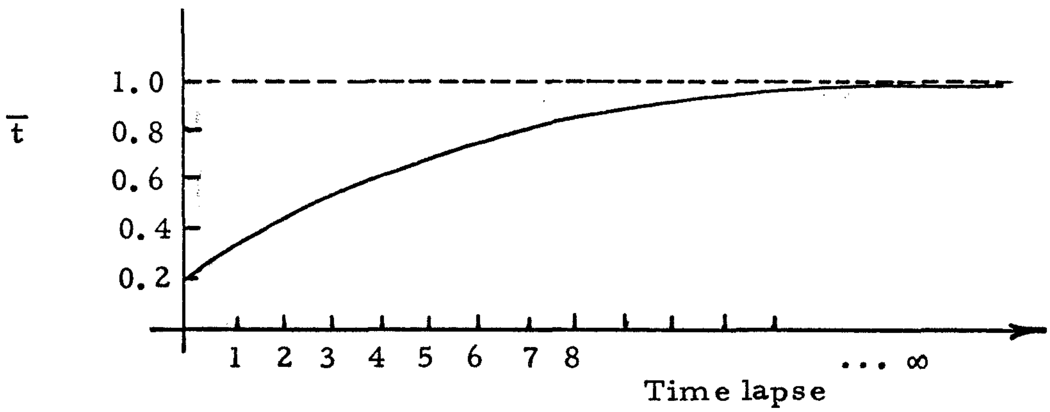


Figure 6-12 $(H_{ref} - H_{adj})$

The factor $\bar{t}$ brings about the percent reduction in the bulge at reference time due to an adjoining bulge. The infinite time lapse is interpreted as the existence of a single frequence -- or the next adjacent frequency is at infinite time away.

Substituting for T, we obtain the bulge relation in its final form,

$$\int_{H_{ref}}^{(H_{ref}+1)} R \,.\, dt = K_2 \cdot \left[\int_{H_{ref}}^{(H_{ref}+1)} D\, dt \right] \cdot K_1 \frac{d}{f} \cdot \bar{t}$$

Thus, this systematic 'twisting' of the demand curve into the response curve involves the following steps --

1) Construct the demand curve.

2) Looking at optimistic, pessimistic and the most likely estimates of the traffic volume, obtain expected values.

3) Introduce the schedule at the particular time, call it reference time.

4) Assign arbitrary but appropriate value to κ_1 and κ_2 -- from experienced expert opinion.

5) Determine T and $\bar{t}$.

6) Figure out response curve.

7) Distribute the remaining area in proportion to original demand curve area.

8) Introduce next schedule -- Repeat steps 3 to 6, modifying original calculations.

Illustration:

Given: Demand curve, shown in Fig. 6-10

City pair, A and B, 600 miles apart

Traffic estimates: optimistic: 1200

most likely: 900

pessimistic: 700

Suppose two flights, first scheduled between 9:00 to 10:00 A. M. and the other between 2:00 P. M. to 3:00 P. M. are to be arranged.

(a) $\oint_{24} D\,dt = \frac{19}{2} L \qquad H_{ref} = 9$

expected No. $Ne = \frac{1}{3}\left[(2\times 900) + \frac{1}{2}(1200 + 700)\right] = 917$

$\int_{9\,AM}^{10\,AM} D \cdot dt = L \qquad d = 600 \qquad f = 2$

Let $\kappa_1 = 0.0066$

$\therefore \quad T = 0.0066\left(\frac{600}{2}\right) = 1.98 \cong 2.00$ Influence period .

$H_{ref} - H_{adj} = (9\text{ AM} - 2\text{ PM}) = 5$ hours, time lapse .

$\therefore \qquad \bar{t} = 0.8 \qquad\qquad$ Let $\kappa_2 = 1.3$.

$$\int_{9\,AM}^{10\,AM} R\,dt = 1.3\,(L)\,.\,(2)\,(0.8 = 2.08\,L$$

$$\cong 2.00\,L$$

(b) $\quad H_{ref} = 2 \qquad\qquad \int_{2\,PM}^{3\,PM} D\,dt = \frac{1}{2}\,L$

$d = 600 \quad f = 2 \qquad\qquad$ Let $\kappa_1 = 0.0066$.

$T = 2.00 \qquad \bar{t} = 0.8$ as before

Let $\kappa_2 = 1.00$

$$\int_{2\,PM}^{3\,PM} R\,dt = 1.0\,(\tfrac{1}{2}\,L)\,(2)\,(0.8) = 0.8\,L$$

Thus, due to the first schedule (9 AM to 10 AM), the perturbed period is 8 AM to 11 AM → influence period is 2 hours, one hour on each side. The bulge is twice, i.e. area is doubled. Due to second schedule (2 PM to 3 PM), perturbed period is 1 PM to 4 PM.

Bulge is 60% more.

The rest of the Demand Curve remains unaffected.

Traffic may go by OMOT.

Thus response curve may look as follows:

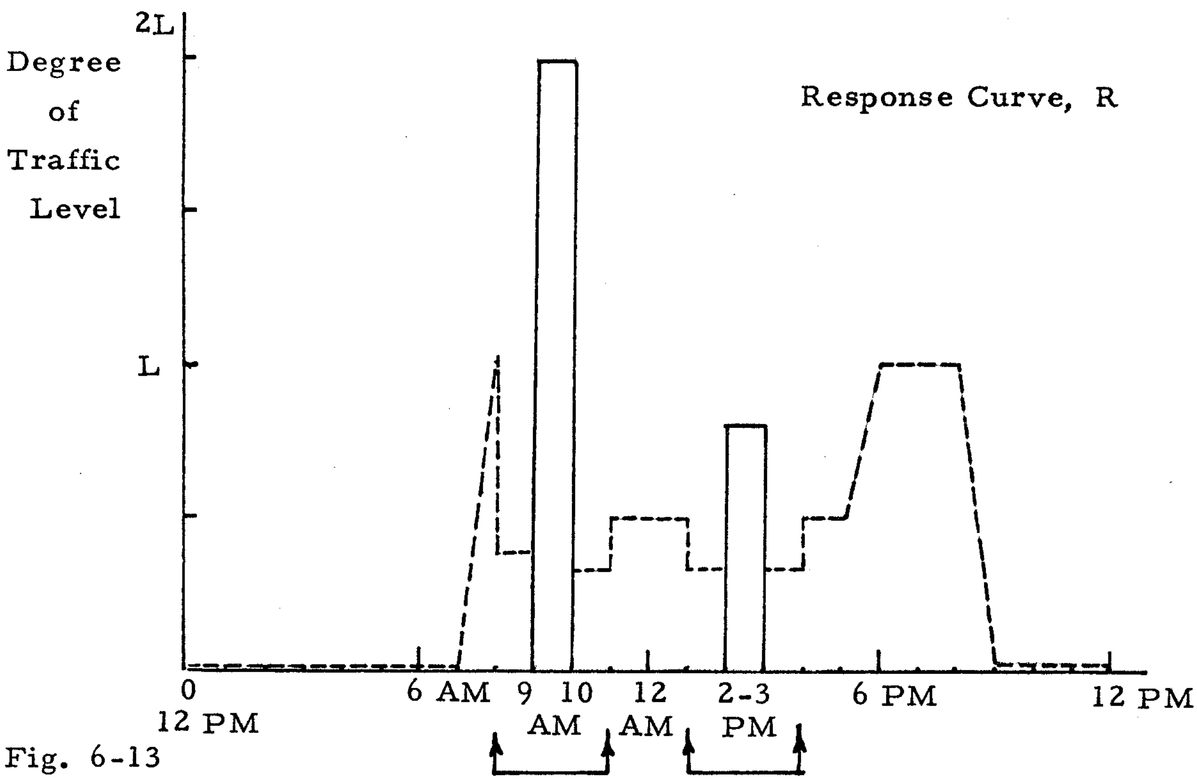


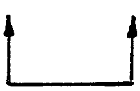
Fig. 6-13

Compare Fig. 6-10 and Fig. 6-13. Area under the curve for both figures is the same, viz. $\frac{19}{2}$ L.

expected number of passengers for first schedule $= N_9^{10} = \dfrac{2\,L}{\frac{19}{2}\,L}\ (917)$

$= 193$

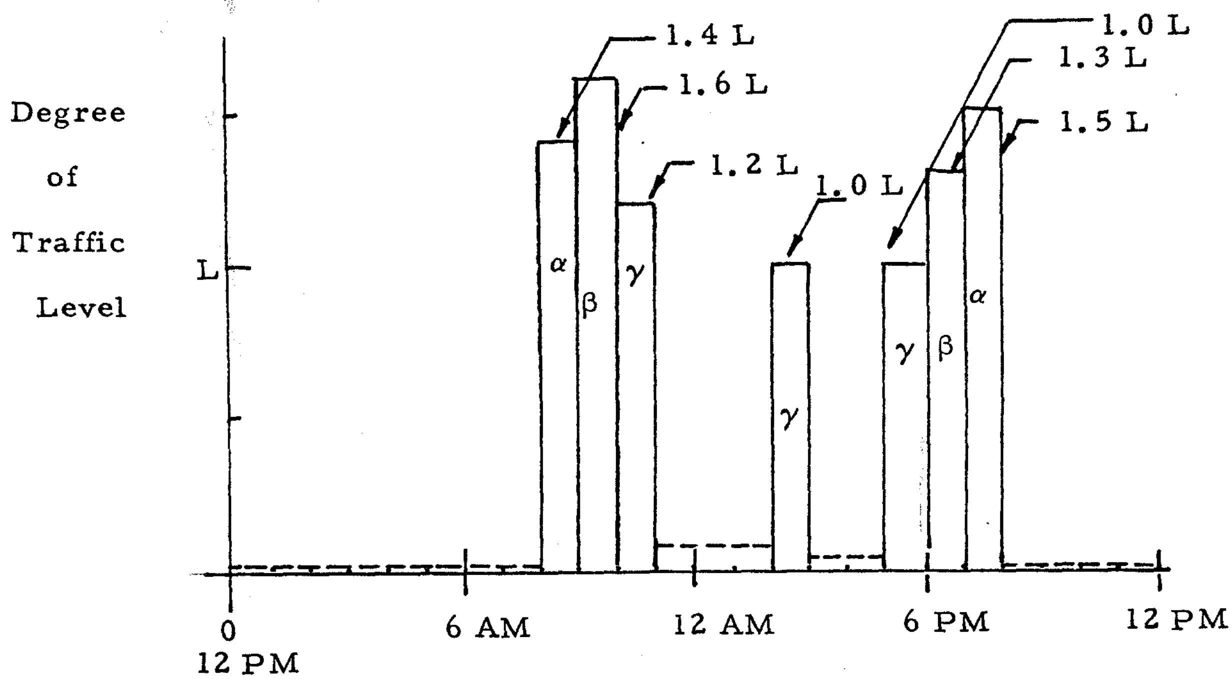
also $\quad N_2^3 = \dfrac{\frac{8}{10}\,L}{\frac{19}{2}\,L}\ (917) = 77$.



This indicates the influence period.

Z-axis: Switch-over probability.

The variation along the Z-axis principally depicts the impact of competition in the airline market. Assume the switch-over computations are done once a week. Suppose we are dealing with the same illustrative problem and 3 airlines are serving the route AB. The final response curve may appear as follows.



Response curve at week$_{i = 0}$

Fig. 6-14 Area under the curve: $\oint_{24} R \cdot dt = \frac{19}{2} L$

expected airlines market $= \dfrac{(1.4 + 1.6 + 1.2 + 1.0 + 1.0 + 1.3 + 1.5)L}{\frac{19}{2} L} \times 917$

$= \dfrac{9.0}{9.5} \times 917 = 867$

Market lost to OMOT $= \dfrac{0.5}{9.5} \times 917 = 50$

optimistic $= 1200 \times \frac{9.0}{9.5} = 1140.$ Similarly --

most likely estimate = 853. Pessimistic = 664.

Hereafter, the dotted portion of the graph is neglected.

$$\text{Expected share for airline } \alpha = 867 \times \frac{2.9\,L}{9.0\,L} = 279$$

$$\text{airline } \beta = 867 \times \frac{2.9\,L}{9.0\,L} = 279$$

$$\text{airline } \gamma = 867 \times \frac{3.2\,L}{9.0\,L} = 309$$

Suppose that the switch-over matrix appears as follows:

From airline i	To airline j: α	β	γ	Sum of prob.	$S_i(t_0)$	$S_i(t_1)$	$S_i(t_2)$	$S_i(t_3)$ ..	$S_i(t_n)$
α	0.57	0.23	0.20	1.00	0.322	0.376	0.414	0.425	0.426
β	0.41	0.43	0.16	1.00	0.322	0.390	0.371	0.362	0.360
γ	0.17	0.50	0.33	1.00	0.356	0.234	0.215	0.213	0.214

$S_i(t_0)$ represents original market-shares at $\text{week}_{j=0}$

$S_i(t_1)$, $S_i(t_2)$ ... $S_i(t_n)$ are respective market shares for week j = 1, 2, ... n, for the airlines α, β and γ .

There will be two distinct approaches to calculate the market shares.

A) Expected Value Approach:

The expected total market for the airline is 867.

The initial market shares for the airline α, β and γ at week j = 0 are 0.322, 0.322 and 0.356 respectively.

The first iteration will look like --

$$S_1(t_1) = 0.57\, S_1(t_0) + 0.41\, S_2(t_0) + 0.17\, S_3(t_0)$$

$$S_2(t_1) = 0.23\, S_1(t_0) + 0.43\, S_2(t_0) + 0.50\, S_3(t_0)$$

$$S_3(t_1) = 0.20\, S_1(t_0) + 0.16\, S_2(t_0) + 0.33\, S_3(t_0)$$

where $S_1(t_1) + S_2(t_1) + S_3(t_1) = 1.00$

if expressed in proportions.

$\therefore \quad S_1(t_1) = 0.376 \quad S_2(t_1) = 0.390 \quad S_2(t_1) = 0.234$

The second iteration for week $j = 2$ would be the repetition of the above set of equations; similarly for the 3rd, 4th...nth iteration. But this kind of repeated iteration reveals to us that this process eventually leads to the stable values for the market share.

The condition for such equilibrium would be

$$S_i(t_{n+1}) = S_i(t_n)$$

In other words, it would be the solution of the set of following simultaneous equations:

$$S_1 = 0.57\, S_1 + 0.41\, S_2 + 0.17\, S_3$$

$$S_2 = 0.23\, S_1 + 0.43\, S_2 + 0.50\, S_3$$

$$S_3 = 0.20\, S_1 + 0.16\, S_2 + 0.33\, S_3$$

$$S_1 + S_2 + S_3 = 1.$$

There are 3 unknowns and 4 equations, but the solution obtained from any three will satisfy the fourth.

The solution or stable values of market shares are

Airline	α	=	0.426
	β	=	0.360
	γ	=	0.214

The successive market shares for three airlines are shown in Fig. 6-16.

In the expected value approach, all the proportions, $S_i(t_n)$, if multiplied by the total expected traffic of 867, would yield the respective market share. Their schedule-wise distribution can also be easily determined.

B) Monte Carlo Approach:

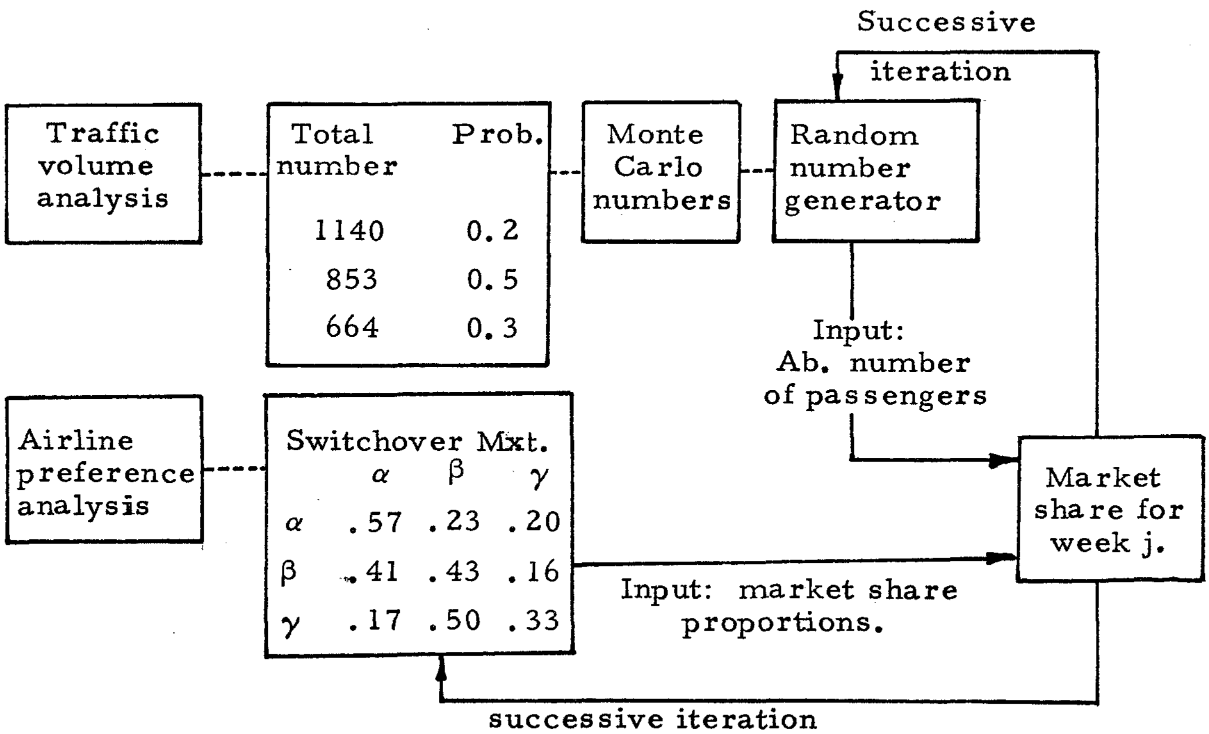


Figure 6-15

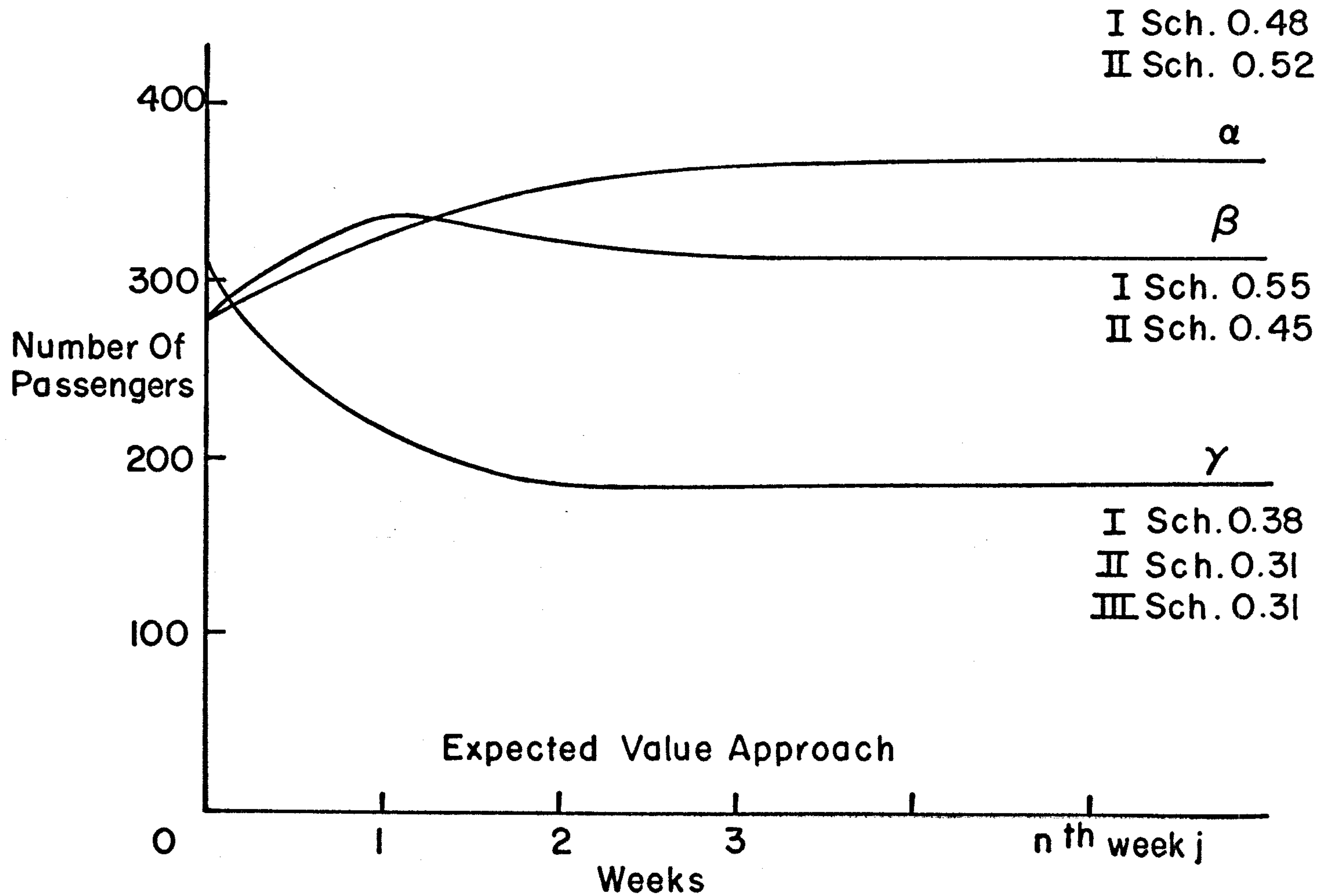


FIG. 6 – 16 MARKET-SHARE FLUCTUATIONS FOR 3 AIRLINES UNDER COMPETITION

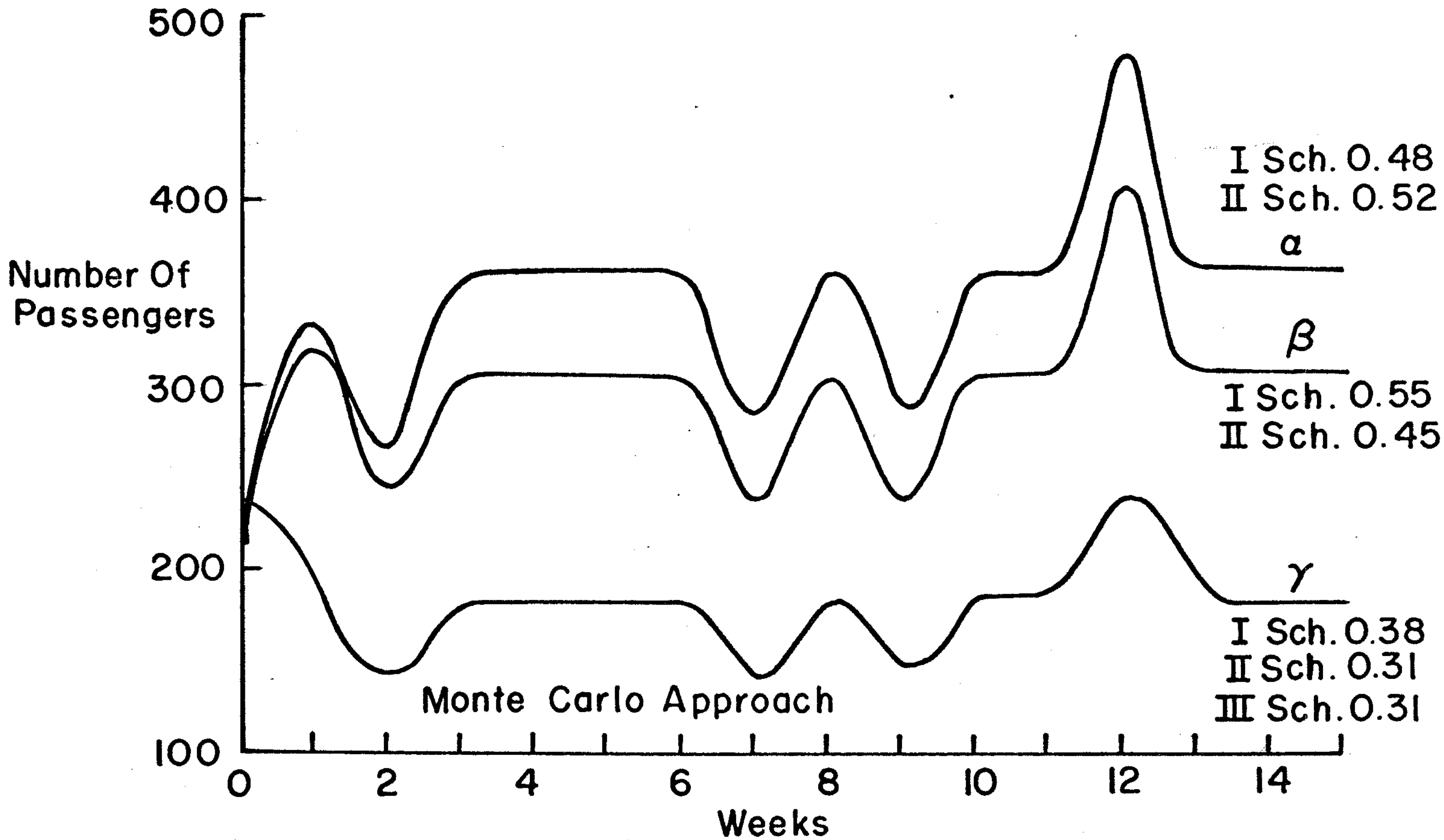


FIG. 6 - 17 MARKET-SHARE FLUCTUATIONS FOR 3 AIRLINES UNDER COMPETITION

It is essentially the same type of iteration, except that the total number of passengers is not equal to the expected value; however, it is determined at each iteration by the random number generator and according to the Monte Carlo numbers. In other words, the switch-over computations fix the percent proportion of the market share, but the absolute number of passengers is predicted with the help of the Monte Carlo simulation. Figure 6-17 will indicate market share tendencies. It has been plotted with the following set of random numbers -- 84, 27, 81, 55, 63, 31, 53, 76, 25, 78, 65, 46, 16, 20, 38 and 68.

At the end of the discussion on revenue analysis, some appropriate remarks will be made.

1. New entry into the market by the airline would be indicated by the zero market share at the week j = 0.

2. One amazing characteristic of the response curve is its 'rigidity' to change of its shape. Following a specific schedule for a long period of time, passengers' response gets almost frozen with it, so that any airline, newly entered into the market or the old operator, intending to change its schedule, may find the initial response of passengers very poor. Consequently, it becomes almost inevitable for the airline management to arrange their new schedule near the already existing traffic peaks -- or the same as the already existing schedules. Thus, the rigidity or inflexibility of the response curve contributes greatly towards a much-discussed issue -- Why do the competing airlines have coincident schedules?

3. It may be wondered as to how we get the entries of the switch-over matrix. Admittedly, it is not an easy task. The switch-over matrix we were discussing so far is essentially a static matrix. More realistic, though complex, is the dynamic switch-over probability matrix, the entries of which would change according to the time, advertisement strategies, introduction of novel service, new type of aircraft, other lucrative concessions, courtesy of employees and many other things. But we will not dwell on such topics because it would shift us from our main purpose.

4. This approach of transforming a demand curve into a response curve almost eliminates the basic riddle as to who, airline or passengers, determines the peak of traffic curve.

5. Two real-world features are missing in the above discussion. The combined problem of arranging a schedule between A and B, and back from B to A is more complicated. It involves two distinct demand curves, terminal service time of the aircraft, GMT-time reference and many more things.

The problem dealing with three stations, A, B and C would have six probability distributions of traffic volume. For a network of reasonable size, the data becomes so vast that we have to turn to the computer for solutions.

6. It may be recalled that we aimed at formulating a relationship of the type Rv = Rv (S, J, R). We did not achieve this exactly, however, given the specific route, aircraft and a particular schedule, we are now able to calculate revenue as shown in the prolonged discussion.

VII. "ANALYSIS OF THE AIRLINE SYSTEMS PERFORMANCE BY VARYING INPUTS" -- SYSTEMS APPROACH

Here we consolidate the entire system methodically and in turn sum up whatever has been stated from the beginning.

In order to integrate the system, we recapitulate all the ideas, concepts and isolated elements mentioned so far, locate them in proper position, then try to show their interrelationships, interdependence, sequence and information flow, thus leading to the objective of the systems analysis and concluding with the feedback paths, if any. Figure 7-1 depicts the complete system in detail.

From the definition of an element of airline operation, we have chosen aircraft, routes and the scheduling as the inputs and have concentrated on the performance parameter of profit. Then we go on collecting relevant, essential, if not complete information about the inputs. For the aircraft we try to gather data about the type, number of engines, average speed, fuel consumption, operating empty weight and the terminal service time. For the routes, we have constructed a node-branch matrix, thus assuring a 'finite' route structure and parallel to it, there appears the information columns for time reference, runway length, elevation, temperature, landing fee and the station cost, also information rows for the distance and the wind velocity. The scheduling block appears in the lower left corner of the diagram.

Next we distinguish four regions in the systems diagram. They are as follows:

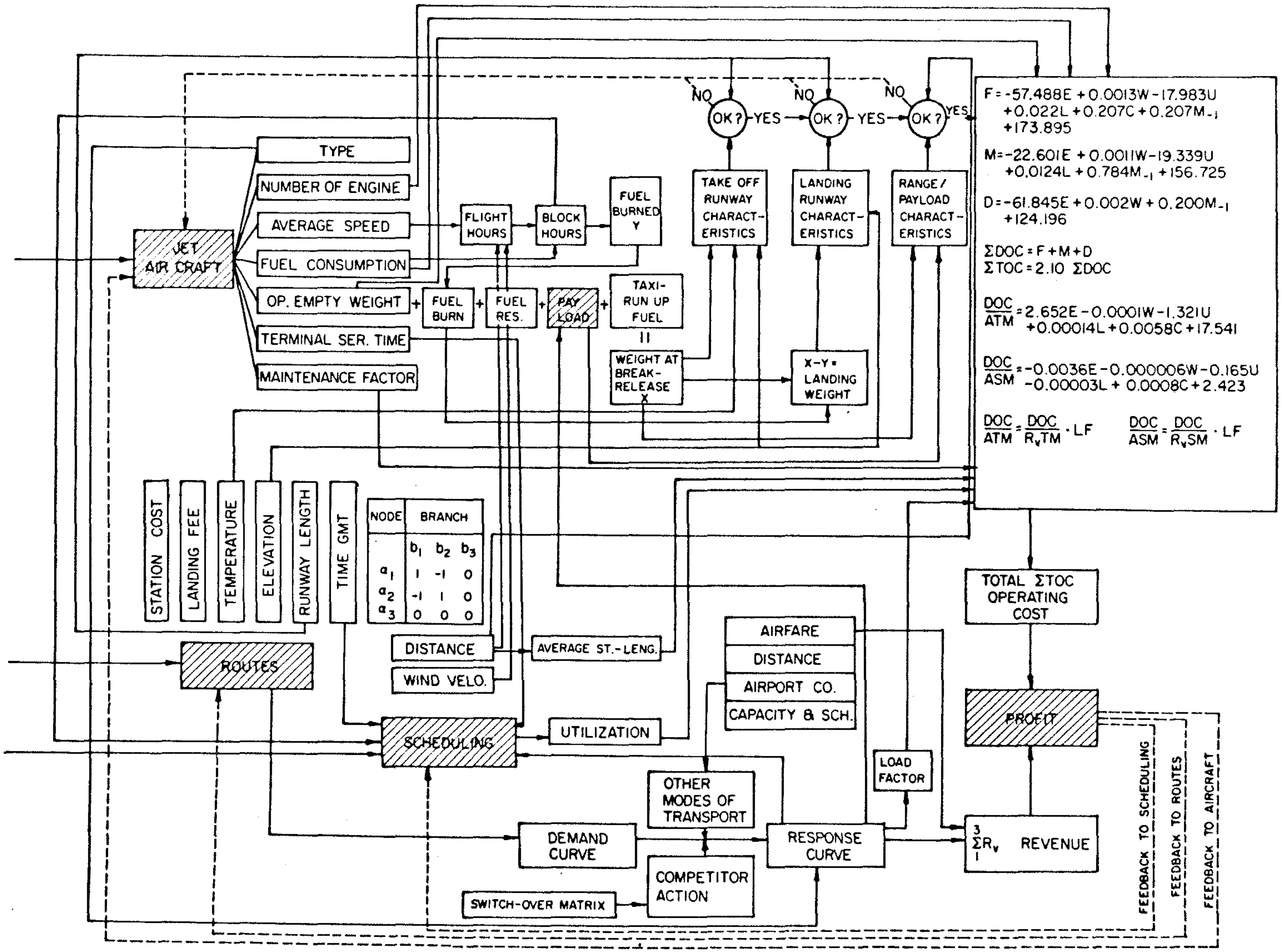

JET AIR CRAFT
TYPE
NUMBER OF ENGINE
AVERAGE SPEED
FUEL CONSUMPTION
OP. EMPTY WEIGHT
TERMINAL SER. TIME
MAINTENANCE FACTOR
FLIGHT HOURS
BLOCK HOURS
FUEL BURNED Y
FUEL BURN
FUEL RES.
PAY LOAD
TAXI-RUN UP FUEL
WEIGHT AT BREAK-RELEASE X
X-Y= LANDING WEIGHT
TAKE OFF RUNWAY CHARACT-ERISTICS
LANDING RUNWAY CHARACT-ERISTICS
RANGE/PAYLOAD CHARACT-ERISTICS
OK?
YES
NO
F=-57.488E+0.0013W-17.983U +0.022L+0.207C+0.207M-1 +173.895
M=-22.601E+0.0011W-19.339U +0.0124L+0.784M-1 +156.725
D=-61.845E+0.002W+0.200M-1 +124.196
ΣDOC=F+M+D
ΣTOC=2.10 ΣDOC
DOC/ATM=2.652E-0.0001W-1.321U +0.00014L+0.0058C+17.541
DOC/ASM=-0.0036E-0.000006W-0.165U -0.00003L+0.0008C+2.423
DOC/ATM=DOC/RvTM·LF
DOC/ASM=DOC/RvSM·LF
STATION COST
LANDING FEE
TEMPERATURE
ELEVATION
RUNWAY LENGTH
TIME GMT
NODE
BRANCH
b1 b2 b3
a1 1 -1 0
a2 -1 1 0
a3 0 0 0
DISTANCE
WIND VELO.
AVERAGE ST.-LENG.
ROUTES
SCHEDULING
UTILIZATION
AIRFARE
DISTANCE
AIRPORT CO.
CAPACITY & SCH.
OTHER MODES OF TRANSPORT
DEMAND CURVE
COMPETITOR ACTION
SWITCH-OVER MATRIX
RESPONSE CURVE
LOAD FACTOR
TOTAL ΣTOC OPERATING COST
PROFIT
ΣRv REVENUE
FEEDBACK TO SCHEDULING
FEEDBACK TO ROUTES
FEEDBACK TO AIRCRAFT

Fig. 7-1

1) Technical and operational constraint - Top middle region.

2) Cost computation -- Top right location.

3) Revenue calculations -- Bottom middle position.

4) Performance parameter and feedback - Bottom right corner.

In the region of technical and operational constraints, the prime consideration is given to the takeoff-landing runway characteristics and the range/payload relationship. All the cost computations will be done on the basis of the multiple linear regression equations we developed before. In the area of revenues, the conversion of the demand curve into the response curve, giving due weight to the inter-airline competition and other modes of transportation, occupies a major role. The diagram is concluded with profits and the feedback loops. If the arrows are followed correctly, the information flow seems logical and the diagram becomes self-explanatory.

VIII. PROFIT ANALYSIS

Here we explore the basic objective of the study, profit maximization or loss minimization by way of choosing the right set of inputs. It is not intended to seek the best solution; however, an attempt will be made to analyze two alternative solutions and by applying the sole criteria of profit, one solution is selected and the other is rejected. The word 'solution' implies the specification of three inputs, $J = J_1$, $R = R_1$ and $S = S_1$. The initial or setup cost has been excluded from cost computations.

Instead of tackling a complex problem involving many variables and their probabilistic variation, a step-by-step approach is followed. All the jet planes considered in the problems are of 'all tourist' configuration, thus assuring a uniform level of fare to all passengers.

Figure 8-1 shows the break-even characteristics of a particular set of inputs.

→ J = Boeing 707-320-B.

With E = 4, operating empty weight, w = 148, 800 lb.

Fuel consumption, C. (Average of B707 data) = 1948 gal/BH.

Previous maintenance cost, M_{-1} (average of B707 data)

= 164.22 $/BH

All tourist configuration, accomodation = 189

→ R = Route A to B and back. 3000 st. miles apart.

∴ L = average stage length = 3000 st. miles.

→ S = One round trip every day.

∴ u = utilization = 6 + 6 = 12 block hours per day.

Repeating the direct operating cost equations,

$$F = -57.488\,E + 0.0013\,W - 17.983\,U + 0.022\,L + 0.207\,C + 0.207\,M_{-1} + 173.895$$

$$M = -22.601\,E + 0.0011\,W - 19.339\,U + 0.0124\,L + 0.784\,M_{-1} + 156.725$$

$$D = -61.845\,E + 0.002\,W - 0.200\,M_{-1} + 124.196$$

also, $$\frac{DOC}{BH} = F + M + D$$

$$\therefore \quad \frac{DOC}{BH} = -141.934\,E + 0.0044\,W + 0.0344\,L - 37.322\,U + 0.207\,C + 0.791\,M_{-1} + 454.816\,.$$

$$TOC = \frac{DOC}{BH} \cdot \frac{BH}{Day} \cdot \quad (2.1) \quad \ldots\ldots \text{ in \$/Day}\,.$$

This total operating cost in $/Day is equated to the product of fare and $\frac{\text{passengers}}{\text{day}}$, i.e. total revenue per day. The resulting curve, a rectangular hyperbola, represents the breakeven characteristic. Suppose the fare is $150 for this route, AB, 3000 miles long (this case is similar to Los Angeles - New York route), then the breakeven number of passengers is 61, yielding the load-factor of 32.5%, as shown on the lower x-scale. The area on the right top side of the curve stands as a profit region while the lower left side of the curve is a loss region.

For the same type of route AB with L = 3000, and a similar schedule with U= 12, the total operating cost of a Boeing 707-320 B with E = 2 and W = 148, 800, will depend upon the values of C and M_{-1}. Different airlines will have different values of M_{-1} and C. The fuel consumption, C, also depends upon weather conditions, takeoff procedures, taxi delays, etc. Figure 8-2 shows the sensitivity of the break-even curve to variations in C and M_{-1}. The C average $\overline{C}$, for Boeing 707 data is 1948 gallons per block hour, with a standard deviation of 79.33. Corresponding figures for M_{-1} are $\overline{M}_{-1}$ = 164.22 and σ = 30.02. The values of total operating cost for nine possible combinations of $\overline{C}$, $\overline{C}$ ± standard deviation and $\overline{M}_{-1}$, $\overline{M}_{-1}$ ± standard deviation are given in the Table near Fig. 8-2. The nine graphs have been drawn corresponding to each of 9 values of the TOC. For the fare level of $150, the break-even number of passengers varies from 58 to 64, as shown in the graph. This graph, hence airline experience, suggests that the average break-even number of 61 is fairly uniform for this kind of operation for the airline industry. It shows a very minor variation.

Figure 8-3 shows a probabilistic Monte Carlo simulation for revenue and cost, considering a one year period. This approach seems more realistic since cost and revenue vary over time. Usually revenue variations are more than those for cost, which is obvious from the graph. This graph has been established by drawing random numbers, one number for each five-day period. The Y-axis scale between $15, 000 to $20, 000 has been expanded for convenience. The dark

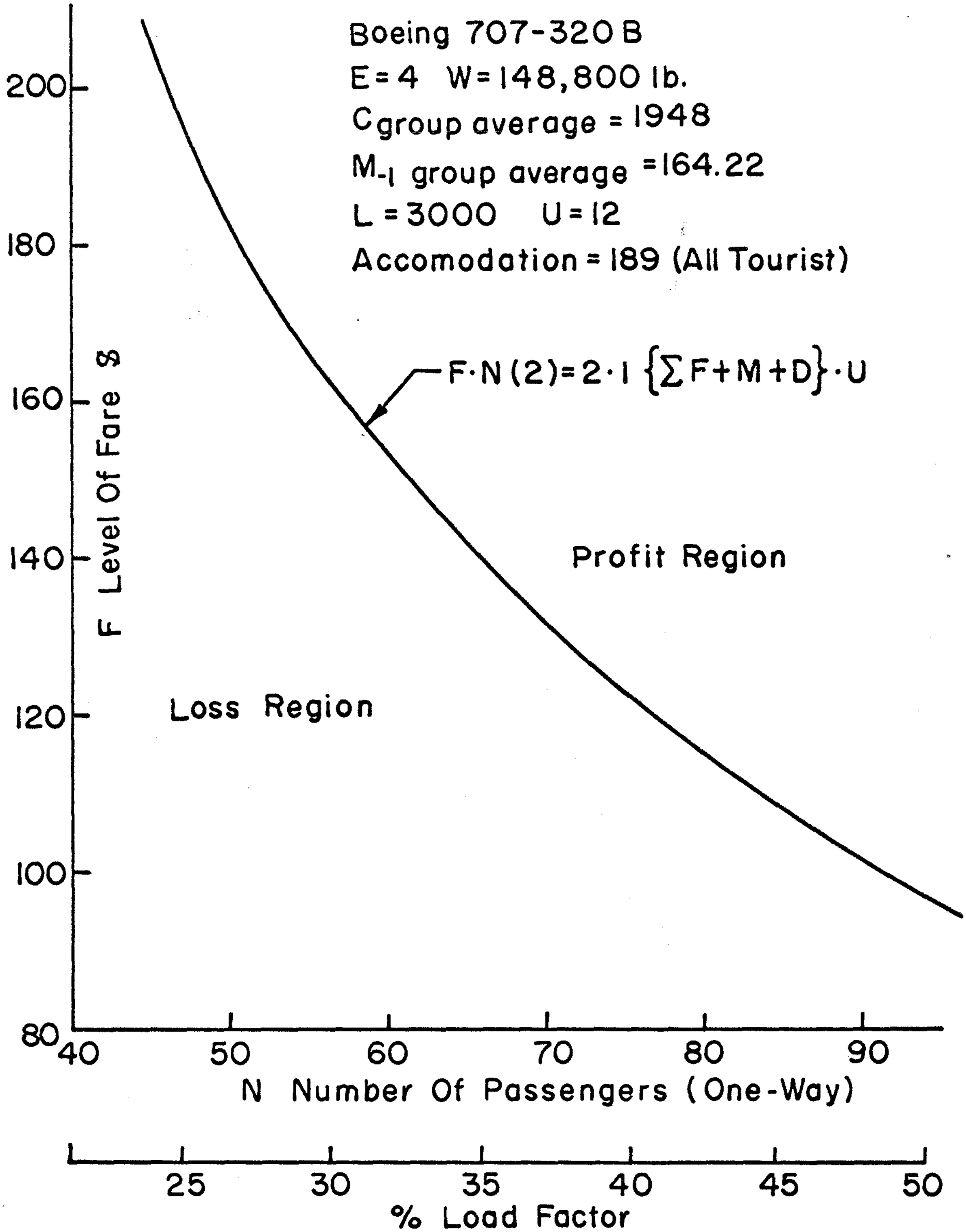
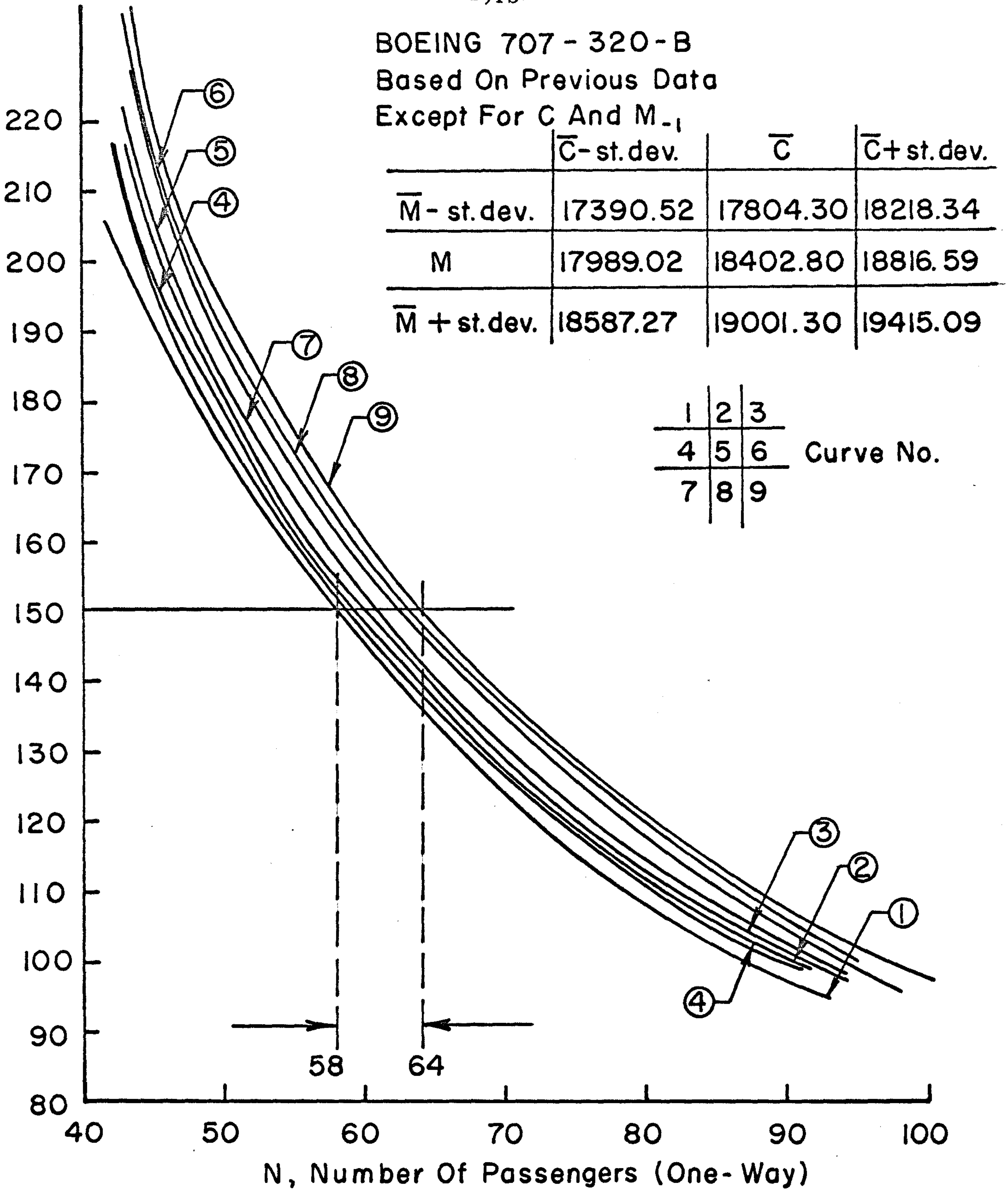


FIG. 8-1 BREAK-EVEN CURVE - RECTANGULAR HYPERBOLA



	$\overline{C}$ - st. dev.	$\overline{C}$	$\overline{C}$ + st. dev.
$\overline{M}$ - st. dev.	17390.52	17804.30	18218.34
M	17989.02	18402.80	18816.59
$\overline{M}$ + st. dev.	18587.27	19001.30	19415.09

FIG. 8-2 SENSITIVITY ANALYSIS

PROBABILISTIC REVENUE VARIATION

PASSENGERS	PROBABILITY	MONTE CARLO NUMBER	REVENUE
30	0.15	0-14	9,000
50	0.20	15-34	15,000
70	0.10	35-44	21,000
95	0.40	45-84	28,500
110	0.08	85-92	33,000
130	0.05	93-97	39,000
160	0.02	98-99	48,000

PROBABILISTIC COST VARIATION

TOTAL COST

	$\bar{C}$ - St. devi	$\bar{C}$	$\bar{C}$ + St. devi
$\bar{M}$ - St. devi	17,390.52	17,804.30	18,218.34
$\bar{M}$	17,989.02	18,402.80	18,816.59
$\bar{M}$ + St. devi	18,587.27	19,001.90	19,415.09

PROBABILITY

0.08	0.07	0.02
0.10	0.50	0.09
0.03	0.06	0.05

MONTE CARLO NUMBER

0-7	8-14	15-16
17-26	27-76	77-85
86-88	89-94	95-99

---- REVENUE CURVE

—— COST CURVE

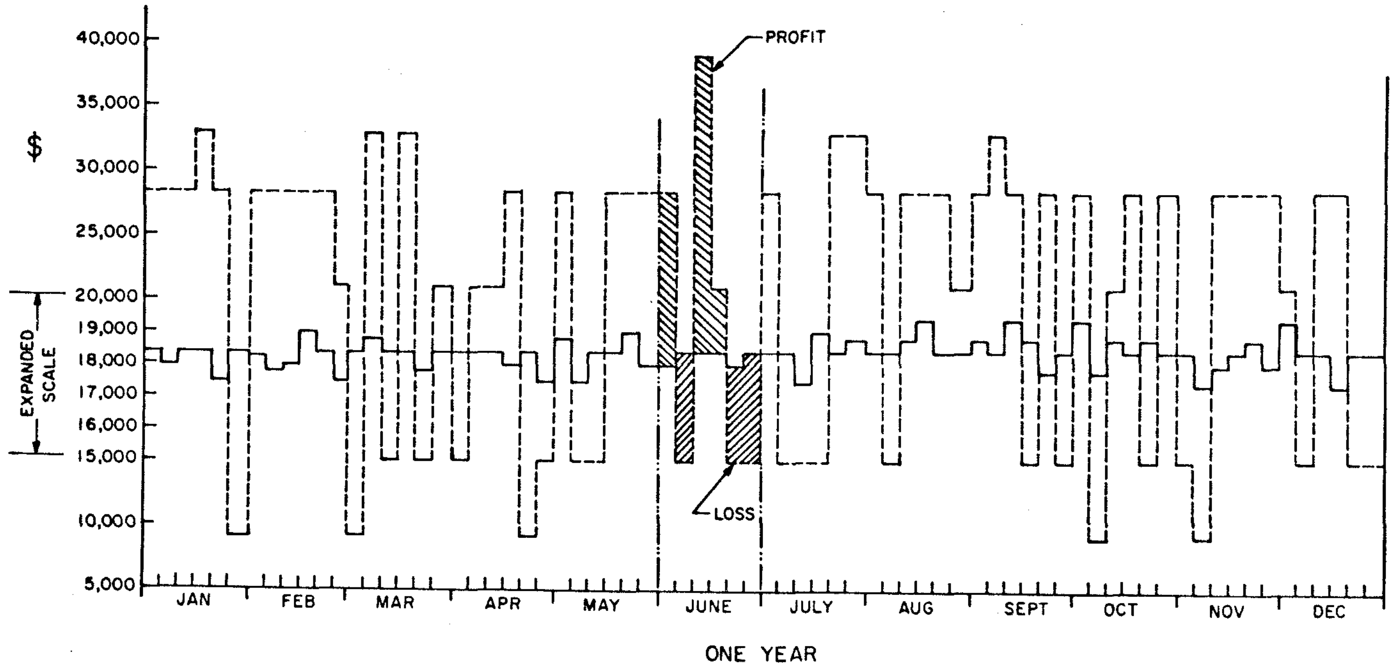


Fig. 8-3 PROBABILISTIC REVENUE - COST SIMULATION FOR ONE YEAR

B707-320B JET

ASSUMPTION: FARE $150.00

black line shows cost variation, while the dotted line depicts revenue variation. For the month of June, profit and loss are shown by cross hatched portions respectively. The chosen probability figures are arbitrary but reasonable. The calculaticns are carried out for the same market situation, i. e. route AB, 3000 miles, one round trip schedule per day and Boeing 707-320B service.

Figures 8-4 and 8-5 represent another management decision. Suppose there is a route PQ, 1000 miles long and two planes are available, one a Boeing 707-320B and the other a Boeing 727-100 C/QC. It takes 2 block hours for both the aircraft to go from P to Q or Q to P. Therefore, one round trip amounts to 4 block hours of daily utilization. It is possible to make a maximum of 3 round trips every day.

Problem: Which aircraft should be used?

How many round trip schedules should be arranged?

Decision criteria: Profit .

There are 6 possible combinations, arising out of 2 aircraft and 3 or less schedules. The market situation is shown on Fig. 8-5 which takes into account the concept of falling average load factor. It is very important to treat B707 and B727 curves independently and unrelated, because there is an inherent weakness if one tries to associate them in any way.

TOC's are calculated in the usual way for U = 4 (one schedule), 8 (2 schedule), 12 (3 schedule), block hours. L = 1000 st. miles.

For B727-100 C/QC: $\bar{C}$ = 1319.54 gal/BH

$\bar{M}_{-1}$ = 125.19 \$/BH .

Corresponding figures for B707 were given previously.

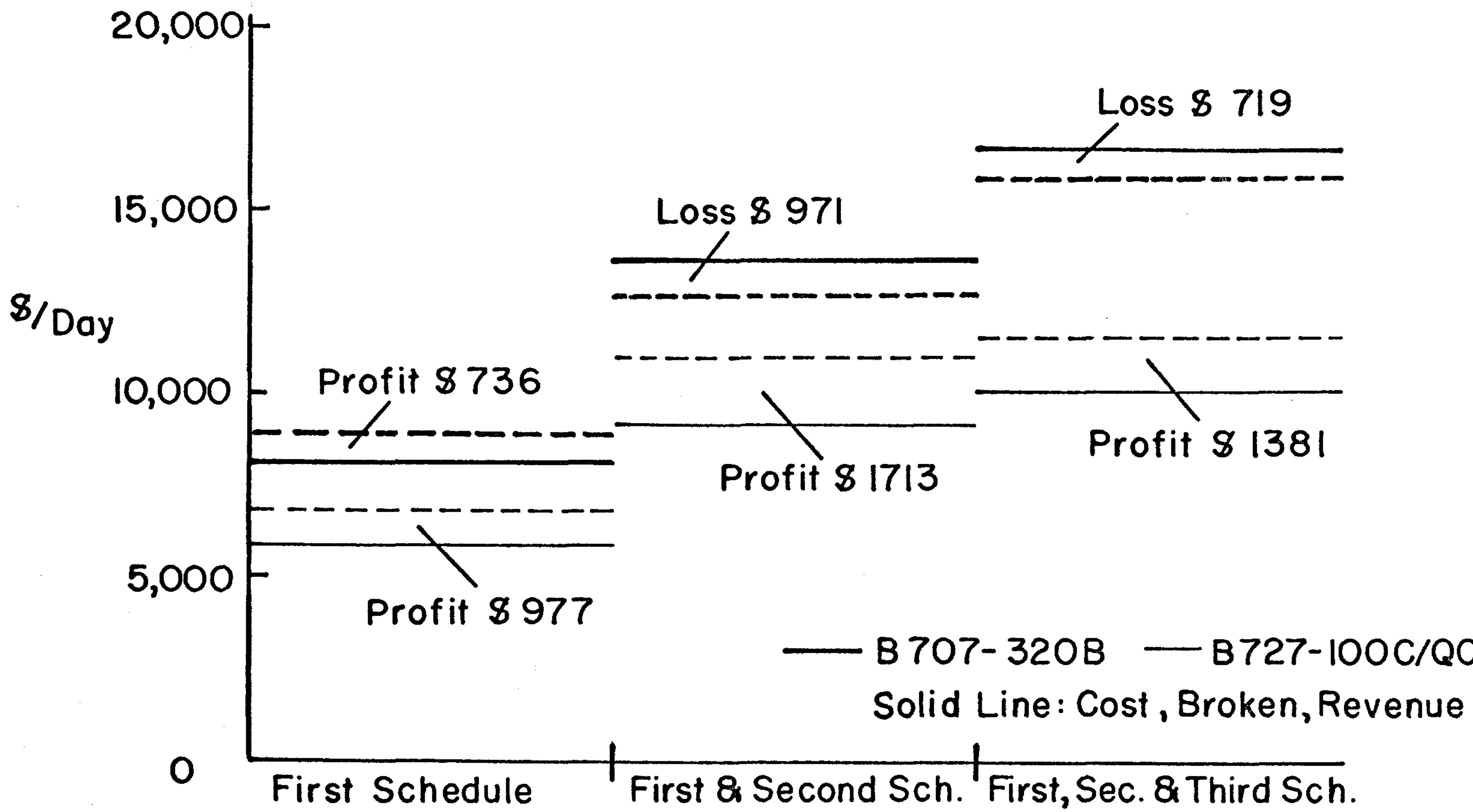


FIG. 8-4 AIRCRAFT DECISION

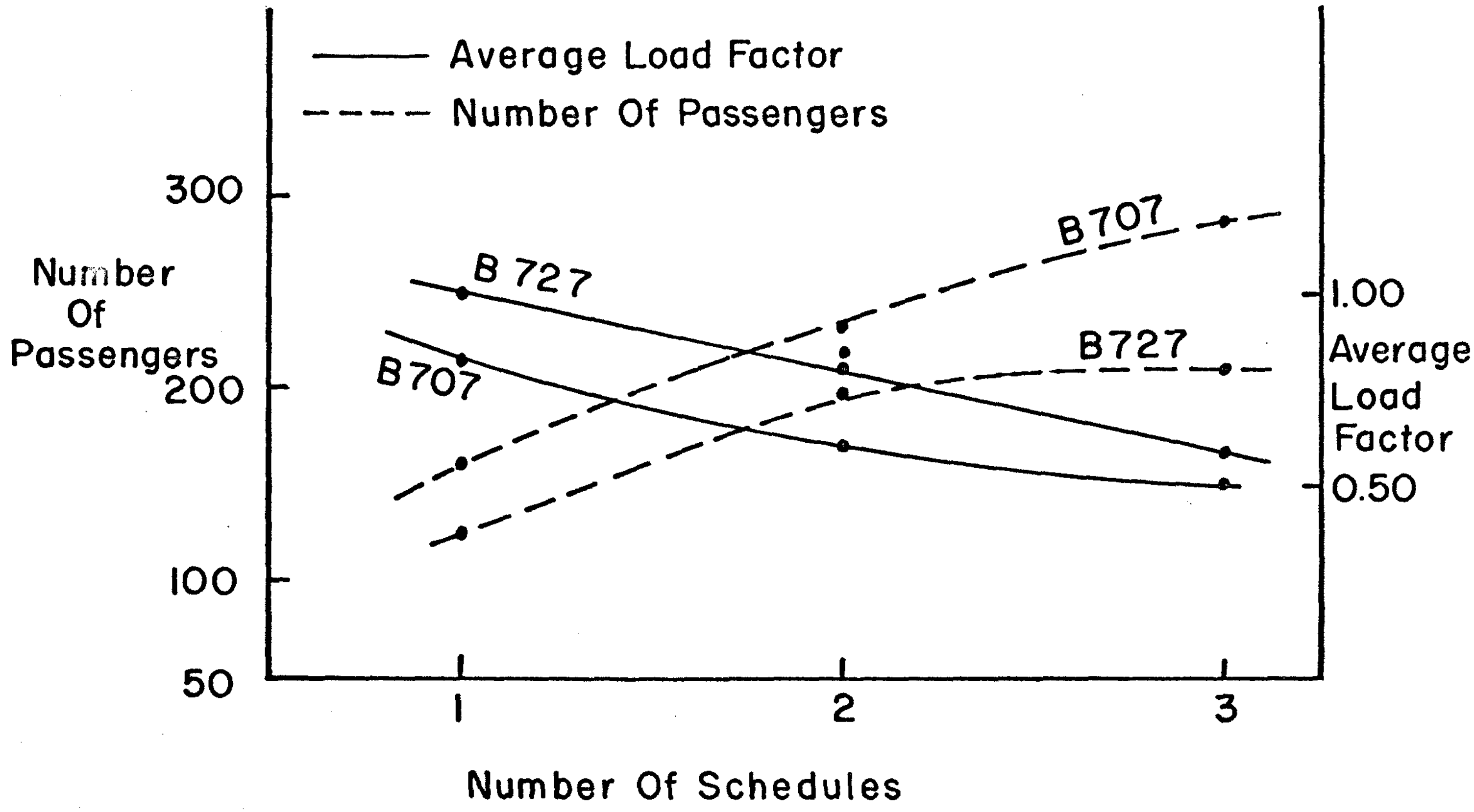


FIG. 8-5 AIRCRAFT DECISION

Figure 8-4 shows profit (or loss) for six alternatives. Judging from the figures, the best solution would be:

J = B727-100 C/Q C, S = 2 round trip/day .

Profit = \$1713 per day .

One interesting feature in Fig. 8-4 should be noted.

We know DOC/block hour = Linear function (block hour, u)

also TOC = DOC/B.H. x B.H. x 2.1

$\therefore$ TOC = Function (u^2)

This quadratic relationship is apparent in Fig. 8-4. The cost levels, shown by the solid line, go parabolic. (u^2) has a negative coefficient.

Thus the decision about the aircraft and the schedule are taken simultaneously. The example is an oversimplified version of a real world problem. Its solution depends upon the availability of Fig. 8-5. A simple route decision problem is investigated next. Suppose an aircraft of type DC-9-30 is available and the choice is to be made between two routes, AB and MN. The characteristics of the routes are as follows: AB, 700 miles long $\therefore$ L = 700 .

Fare \$38.00 Two schedules - round trips - are possible.

Block hour time for one trip is 1.75 hr.

Two round trips amount to utilization of 7 hrs.

MN, 400 miles L = 400

3 round trips possible. Block time for one-way = 1 hr.

Fare \$20.00 trip $\therefore$ u = 6 hrs.

Figure 8-6 shows TOC, \$/day for two different routes. \$20 and \$38 fare lines are also shown. Their corresponding intersections with the

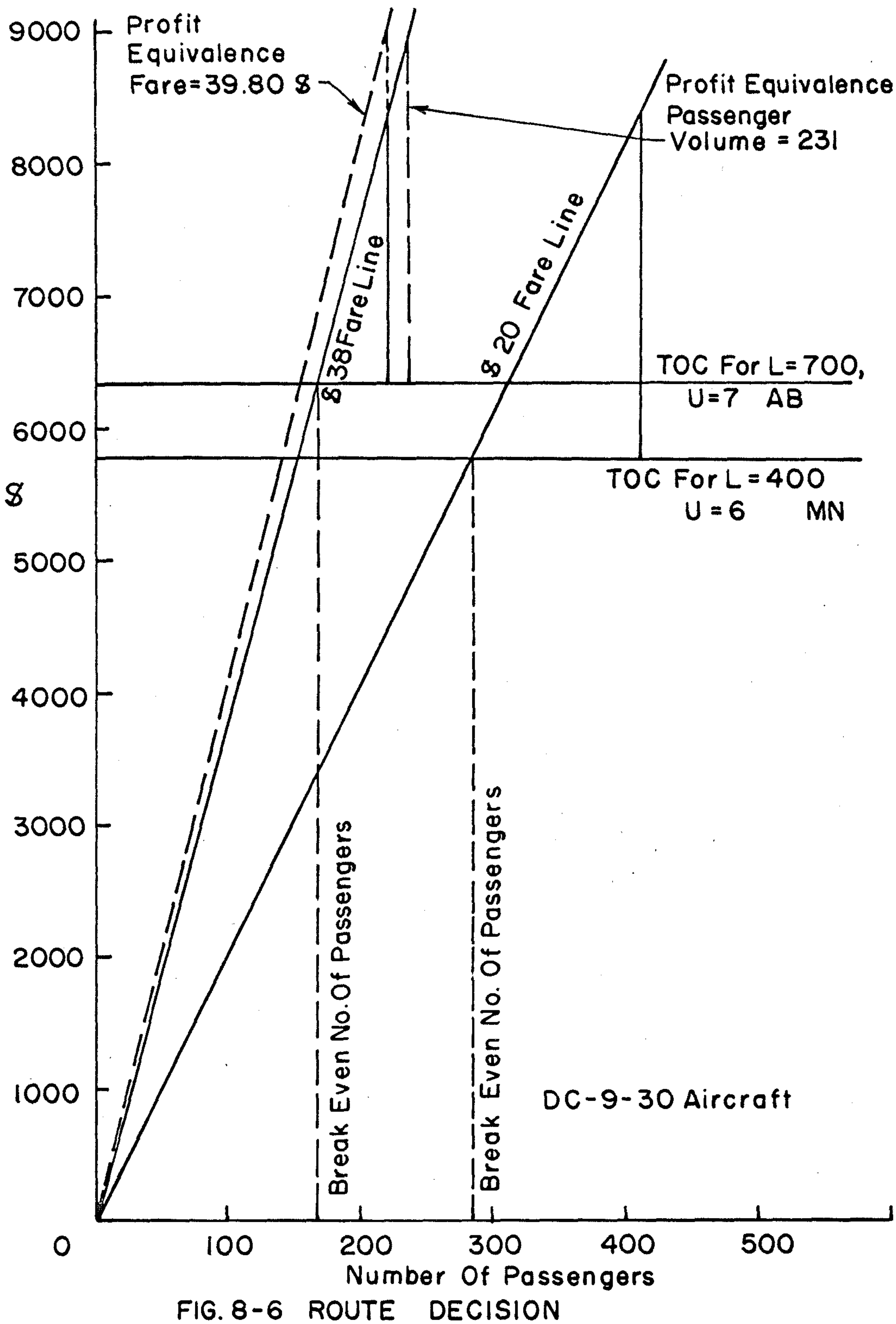


FIG. 8-6 ROUTE DECISION

TOC lines yield the break-even number of passengers. The DC-9-30 with all tourist configuration has an accomodation of 115. Average break-even load factors for the two routes can be calculated as follows:

AB: Av. load factor break-even $= \frac{282}{115 \times 6} = 40.9\%.$

MN: Av. load factor break-even $= \frac{168}{115 \times 4} = 36.5\%$

Since the criterion for route selection is profit, the profit equivalence of route MN and AB can be given. Suppose the expected number of passengers for route AB is 220 and for MN, it is 410. Solid vertical lines indicate the profit.

$$\text{Profit}_{MN} > \text{Profit}_{AB}.$$

$\therefore$ solution J = DC-9-30 R = MN S = 3 round trips .

In order to have the same amount of profit for route AB, either fare should be raised or passenger volume should go up. The profit equivalence may be stated as follows:

$$\text{Profit}_{MN} \begin{array}{l} N = 410 \\ F = \$20 \end{array} \equiv \text{Profit}_{AB} \begin{array}{l} N' = 231 \\ F = \$38 \end{array}$$

$$\equiv \text{Profit}_{AB} \begin{array}{l} N = 220 \\ F' = \$39.80 \end{array}$$

Here again the inputs R and S are selected on the basis of profit for a given input J.

The problems dealt with so far are essentially 'one input' decision situations where other inputs have been specified. Finally, an example will be taken, comprising of elements of real-world situation, to show how one could select three inputs on the basis of profit. It will also indicate some inherent weaknesses of such optimal solutions.

Let us consider a four-station (A, B, C, and D) situation for which it is evident that there will be 12 possible branches. Assumption (1): The existing markets between airport D and B (and reverse) as well as between D and C (and reverse) are considered very poor.

Therefore, we, the decision makers of the airline, which we will call 'Apollo', are not interested in operating services DB-D and DC-D. This eliminates 4 branches. The station D is near to station A, the operating headquarters of Apollo. The service between D and A will be taken as a feeder line to hub A and further services from A towards B and C.

The map then looks as follows:

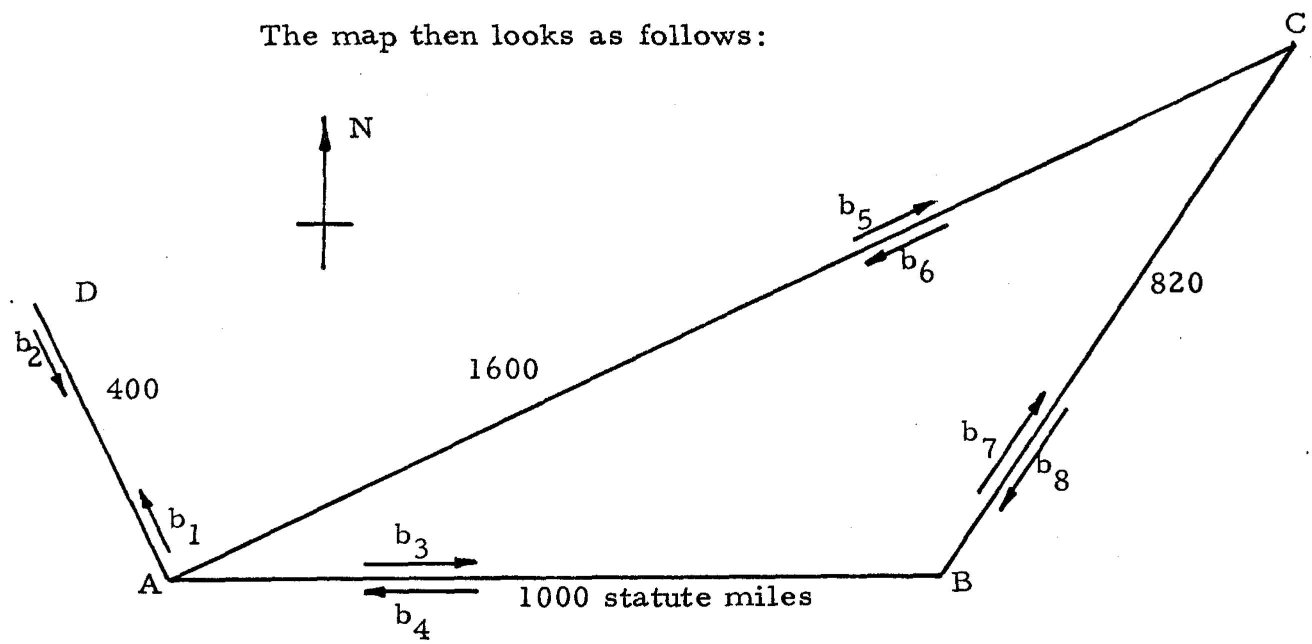


Fig. 8-7

Legal constraint: The airline Apollo will not have a right of operating a service between B and C exclusively.

Thus there are two possible route structures or route inputs as shown in the table below.

Input	Description	Route		Description	Route
R_1	Feederline connecting D and A	AD/DA	AND	Acquire rights for route AC-A and also serve C via B	AC/CA; ABC/CBA

OR

Input	Description	Route		Description	Route
R_2	Feederline connecting D and A	AD/DA	AND	Acquire rights for route AB-A and also serve B via C	AB/BA; ACB/BCA

Let us suppose that the route matrix, along with station and branch characteristics is found to be as follows.

temp.	elevation	runway length	time GMT	nodes	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
					Branches							
St. day	2000	8000	0	A	1	-1	1	-1	1	-1	0	0
St. day	4000	8000	+1	B	0	0	-1	1	0	0	1	-1
St. day +15°C	4000	7000	+1	C	0	0	0	0	-1	1	-1	1
St. day	0	5300	0	D	-1	1	0	0	0	0	0	0
		Distance (st. m.)			400	400	1000	1000	1600	1600	820	820
		Wind velocity										

Let us suppose the following jet planes are available for service,

J	DC-9-30	B727-100 c/Qc	B707-320B
Number	1	1	2
with the characteristics given below:			
Engine Type	JT 8D - 7	JT 8D - 7	JT 3D - 3B
Number of Engines	2	3	4
Operating e. weight lb.	56855	94500	148800
Maxi. Takeoff wt. lb.	108000	163000	333600
Maxi. Landing wt. lb.	99000	140500	215000
*Fuel Consumption gal/BH	912	1319.54	1948
*Average airborne speed MPH	380	440	480
*M_{-1} $/BH	75.18	125.19	164.22
Accomodation (All Tourist Con.)	115	125	189
Terminal Service Time Hours	0.50	0.50	1.00
Takeoff runway Landing runway Payload/Range	characteristics are shown in Appendix		

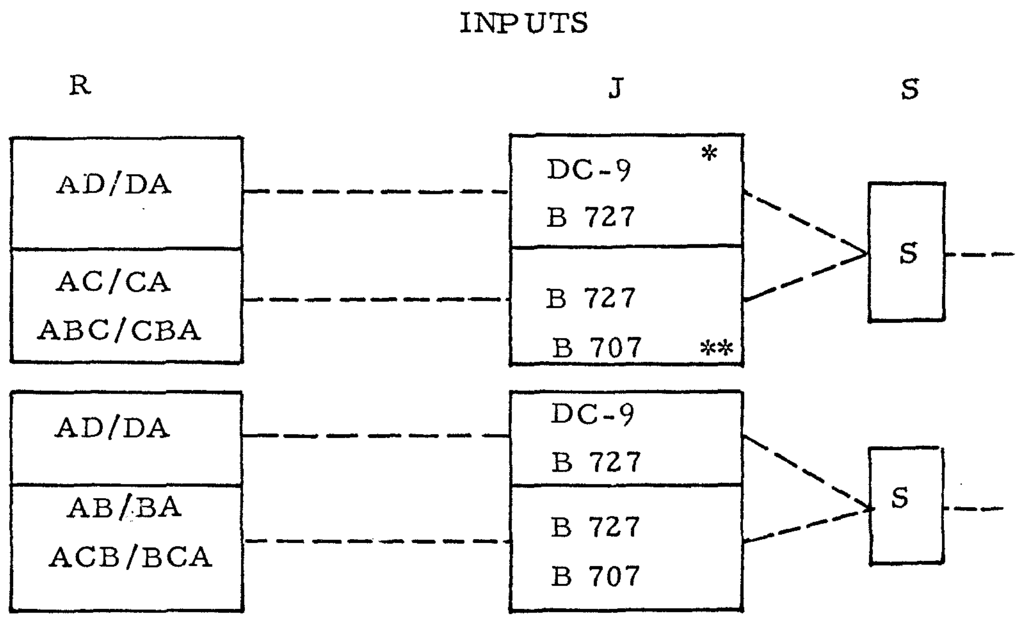
* These are industry average figures.

Assumption (2): It is possible to drop altogether one of the types of aircraft in the final decision about input J.

This situation is similar to lease operation. Airline Apollo will lease some aircraft for service.

Assumption (3): Another airline, called Gemini, offers competition on route BC-B.

Decisions to take:



* B707 is excluded from consideration at the airport D because runway length is too short.

** DC-9 is excluded from consideration at triangle ABC network for range/payload constraint.

Criteria for Decision : PROFIT .

Airfares:

This analysis, for the sake of simplicity, is carried out considering all tourist market (economy class) and all tourist class configuration jets.

Assumption (3): Payload consists of only passengers and their allowable baggage.

$$\therefore \quad \text{Total Revenue} = \sum_{i=1}^{3} Rv_i = \sum_{i=2} Rv_i = \sum Rv_2$$

Airfare Table: (Oneway tourist class fares)

Journey	Fare, $
AD/DA	20
AB/BA	56
AC/CA	75
BC/CB	45

Market structure:

A suitable traffic flow is visualized in this four station network. It would be reasonable to assume a uniform level of traffic in both the directions. We will approach the AD/DA market with a Monte Carlo technique and triangle ABC with an expected value technique.

Let us suppose that after giving due weight to the airfare, distances (both of which have been specified) and airport convenience, the marketing division comes up with three estimates of 'from-to' traffic flow. These figures are so far not affected by scheduling and capacity.

'From-to' tendencies:

	Most likely				Pessimistic				Optimistic			
From \ To	A	B	C	D								
A	0	230	200	180	0	190	160	150	0	290	260	240
B	230	0	250	40	190	0	200	20	290	0	300	70
C	200	250	0	30	160	200	0	20	260	300	0	50
D	180	40	30	0	150	20	20	0	240	70	50	0

Now the market is split into two, one on route AD/DA and the other on triangle ABC. In order to apply Monte Carlo techniques to market AD/DA, corresponding probability figures are given. Also note that the traffic on DB/BD and DC/CD has been diverted via A.

AD/DA Market:

Passengers -- Revenue purpose	180	150	240
Probability	0.5	0.3	0.2
Passengers -- Accomodation purpose	250	190	360

ΔABC Market: (Modified)

	Most Likely			Pessimistic			Optimistic		
From \ To	A	B	C	A	B	C	A	B	C
A	0	270	230	0	210	180	0	360	310
B	270	0	250	210	0	200	360	0	300
C	230	250	0	180	200	0	310	300	0

The expected values, Ne, have been evaluated using $Ne = \frac{1}{3}(2m + \frac{a+b}{2})$

ΔABC traffic flow expected values.

From \ To	A	B	C
A	0	275	235
B	275	0	250
C	235	250	0

This market is further divided according to proposed schemes of route structure.

R_2: AB/BA — 275

For accomodation and Revenue purpose.

ACB/BCA

	A	B	C
A	0	0	235
B	0	0	$\chi_1(250)$
C	235	$\chi_1(250)$	0

Assumption (4): No passenger would prefer to go from A to B via C. $\chi_1((250)$ is an indication of competition on route BC and χ_1 represents % market share for airline Apollo. More explanation is given later.

R_1: AC/CA

$|(1-P)(235)|$

For accomodation purpose only.

ABC/CBA

	A	B	C
A	0	275+P(235)	0
B	275+P(235)	0	$\chi_1(250)$+P(235)
C	0	$\chi_1(250)$+P(235)	0

Assumption (5): P% of passengers may prefer to go from A to C via B. This percentage varies according to the schedules and capacity offered.

For revenue purpose,

	A	B	C
A	0	275	235
B	275	0	$\chi_1(250)$
C	235	$\chi_1(250)$	0

Dynamic Switchover matrix:

As previously noted, airline Gemini had been operating on route BC/CB for quite some time and airline Apollo will introduce its new service. The matrix below is labelled as a dynamic one, because its elements vary with time.

From \ To	Apollo	Gemini
Apollo	$(1 - 0.70\, e^{-w/52})$	$0.70\, e^{-w/52}$
Gemini	$0.40\, e^{-w/52}$	$(1 - 0.40\, e^{-w/52})$

where w .... is the number of weeks.

Iterations are carried out for w = 0, 5, 10, 15 ...... 50 . These tendencies are illustrated in Fig. 8-8. The equations of iteration appear as follows

$$\chi_1' = (1 - 0.70\, e^{-w/52})\chi_1 + (0.40\, e^{-w/52})\chi_2$$

$$\chi_2' = (0.70\, e^{-w/52})\chi_1 + (1 - 0.40\, e^{-w/52})\chi_2$$

and $\chi_1 + \chi_2 = \chi_1' + \chi_2' = 1.00$

For stable values (equilibrium), $\chi_1' = x_1$

$\chi_2' = x_2$

$\therefore$ Stable solution $\chi_1 = 0.36$

$\chi_2 = 0.64$ reached at the end of 15th week.

The different stages of iteration appear as shown in the table on the next page.

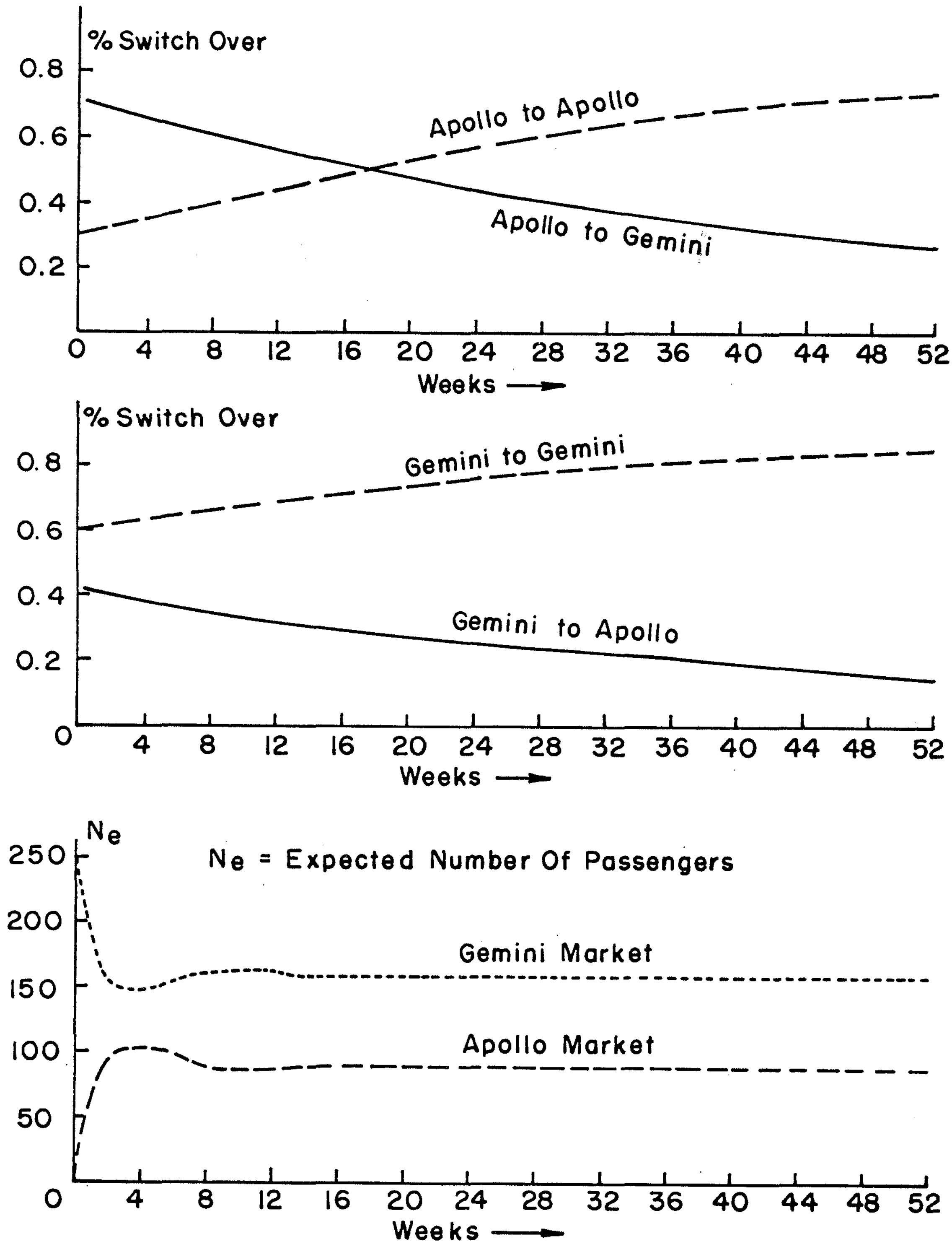


FIG. 8-8 ROUTE BC/CB COMPETITION

ω_i	χ_1	χ_2	Ne	x_1	x_2
0	0	1.00		0	250
1-5	0.40	0.60	250	100	150
6-10	0.36	0.64		90	160
11-15	0.35	0.65		88	162
16-20	0.36	0.64		90	160
	:	:		:	
...52	0.36	0.64		90	160

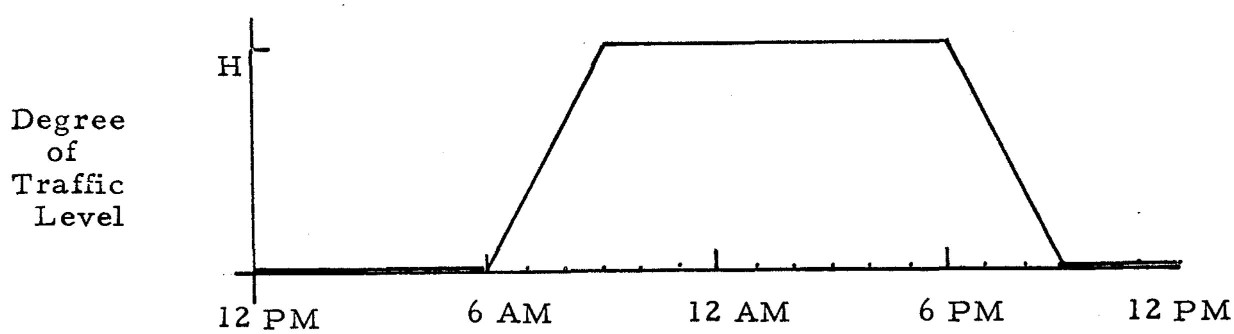
The last two columns show the split of the expected number of passengers on route BC/CB.

$$x_1 = \chi_1(Ne) \qquad \text{and} \qquad x_1 + x_2 = Ne \,.$$
$$x_2 = \chi_2(Ne)$$

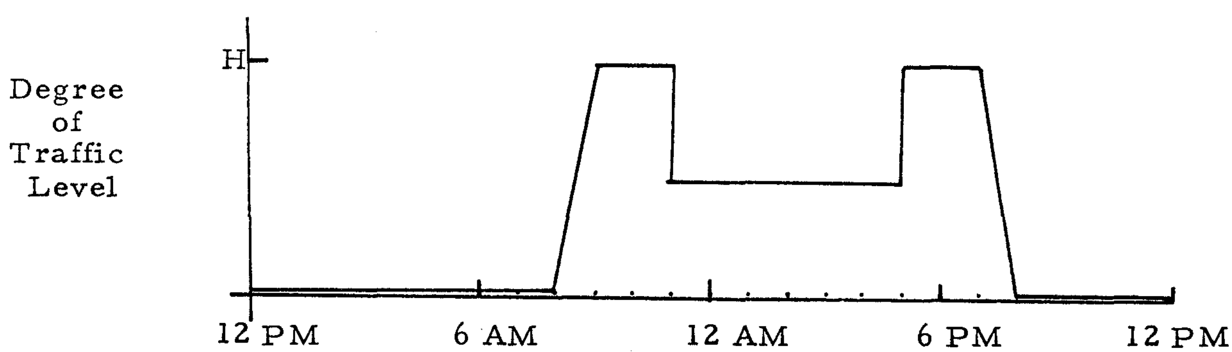
Demand curves:

The following demand curves are assumed:

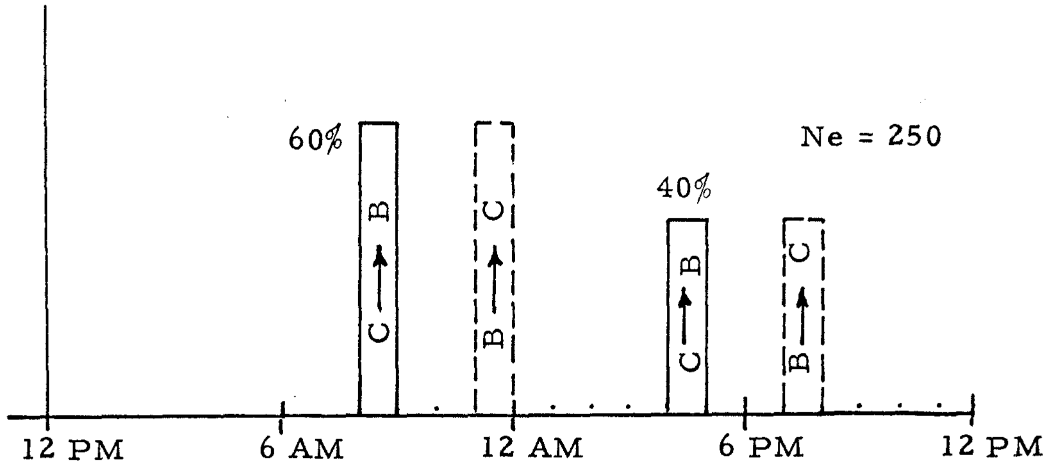
(1) AD/DA market: constant demand pattern.



(2) AB/BA; AC/CA: Double peak demand pattern



(3) BC/CB: Predetermined Response Curve. Gemini Airline.



Three constraints in arranging schedules exist -- in specifying input S, namely:

(a) Since the traffic DB/BD and DC/CD is diverted via A, there should be reasonable connections between AD/DA schedule and further ABC schedules.

(b) If B727 is put into service on route DA/AD as well as ABC, schedule should be appropriately adjusted.

(c) Any schedule on route BC should take into account the already existing traffic peaks.

The block hours required to complete each leg of the network for different aircraft are given below. The table also shows the expected upper limit of utilization.

Aircraft	expected utilization	AD	AB	AC	BC
DC-9-30	U < 8 hours	1.25	--	--	--
B 727-100 c/Qc	U < 11	1.00	2.50	3.75	2.00
B 707-320 B	U < 14	--	2.25	3.25	1.75

With this table, the long list of information, data assumptions, and constraints is over.

Now we specify three of the inputs, J, R and S and they will form a set. The cost and expected revenue of each set can be calculated. It may appear that there is an infinite number of such combinations and permutations out of these inputs. But assumptions and constraints limit considerably the possible combinations.

Only six of such sets are given here, however, noting that this is not the entire list of possible sets. The cost and revenue have been evaluated for each of the six sets, and one set (specified values of inputs R, J and S) is selected on the basis of profit.

The following notation is used:

Two B707 aircraft are available. They are distinguished by ^{1}B707 and ^{2}B707.

S = Round trip schedule

U = Block hour utilization

T = Terminal servicing time, hours .

6 sets of inputs (R, S, J)

Set	R	J	S
1.	AD/DA	B727	S = 3 U = 6 T = 2.5
	AC/CA	^{1}B707	S = 2 U = 13 T = 3
	ABC/CBA	^{2}B707	S = 1 U = 8 T = 3
2.	AD/DA	DC-9	S = 3 U = 7.5 T = 2,5
	AC/CA	^{1}B707	S = 2 U = 13 T = 3
	ABC/CBA	B727	S = 1 U = 9 T = 1.5
		^{2}B707	S = 1 U = 8 T = 3
3.	AD/DA	DC-9	S = 3 U = 7.5 T = 2.5
	AC/CA	B727	S = 1 U = 7.5 T = 0.5
	ABC/CBA	^{1}B707	S = 1 U = 8 T = 3
		^{2}B707	S = 1 U = 8 T = 3
4.	AD/DA	B727	S = 3 U = 6 T = 2.5
	AB/BA	B727	S = 1 U = 5 T = 0.5
		^{1}B707	S = 2 U = 9 T = 3
	ACB/BCA	^{2}B707	S = 1 U = 10 T = 3

Contd:

Set	R	J	S
	AD/DA	B727	S = 3 U = 6 T = 2.5
5.	AB/BA	B727	S = 1 U = 5 T = 0.5
		^{1}B707	S = 1 U = 4.5 T = 1
	ACB/BCA	^{1}B707	S = 1 U = 10 T = 3
		^{2}B707	S = 1 U = 10 T = 3
	AD/DA	DC-9	S = 3 U = 7.5 T = 2.5
6.	AB/BA	B727	S = 2 U = 10 T = 1.5
		^{1}B707	S = 1 U = 4.5 T = 1
	ACB/BCA	^{1}B707	S = 1 U = 10 T = 3
		^{2}B707	S = 1 U = 10 T = 3

Cost Computation: (Summary)

$$\frac{DOC}{BH} = -141.934\, E + 0.0044\, W + 0.0344\, L - 37.322\, U + 0.207\, C + 0.791\, M_{-1} + 454.816$$

$$TOC = \frac{DOC}{BH} \cdot \frac{BH}{Day} \quad . \; (2.10) \ldots\ldots \$/Day .$$

Aircraft	E	W lb	C gal/BH	M_{-1} $/BH		SET 1	2	3	4	5	6
DC-9-30	2	56, 855	912	75. 18	L_{Mile}	--	400	400	--	--	400
					U_{Hr}	--	7. 5	7. 5	--	--	7. 5
B727-100 c/Qc	3	94, 500	1319.54	125.19	L	400	910	1600	550	550	1000
					U	6	9	7. 5	11	11	10
[1]B707-320 B	4	148,800	1948	164.22	L	1600	1600	910	1000	1140	1140
					U	13	13	8	9	14. 5	14.5
[2]B707-320 B	4	148, 800	1948	164. 22	L	910	910	910	1210	1210	1210
					U	8	8	8	10	10	10

TOTAL COSTS OF SIX SETS:

Set	TOC/Day $	Annual TOC $
1	38817	14 168 300
2	47206	17 230 200
3	42813	15 627 000
4	40053	14 619 700
5	42883	15 652 400
6	49449	18 049 000

Revenue computation:

Market AD/DA: All the six sets mentioned before offer overcapacity on route AD/DA. The Monte Carlo simulation is carried out on a time basis of one week using the following random numbers -- 22, 19 16, 78, 03, 93, 23, 15, 58, 57, 48, 61, 36, 18, 88, 09, 12, 85, 38, 53, 40, 02, 95, 35, 26, 77, 46, 37, 61, 93, 21, 95, 97, 69, 04, 70, 85, 21, 15, 03, 87, 98, 10, 47, 22, 67, 27, 33, 13, 36, 17, 43. This amounts to a one-year simulation.

$$\text{Revenue} = \{147 \text{ days} \times (2 \times 150) \times \$20 + 147 \text{ days} \times (2 \times 180) \times \$20 + 71 \text{ days} \times (2 \times 240) \times \$20\} \ (0.80)$$

$$= \$\ 2\ 097\ 600$$

Factor (0.80) is arbitrary, but is a reasonable percent reduction due to scheduling.

Market BC/CB: Here again, all the six sets overrun in capacity. The market share of Apollo in competition with Gemini varies as 90 passengers for 295 days, 88 passengers for 35 days and 100 for 35 days. Here percent reduction due to scheduling is taken as zero.

$$\text{Revenue} = 2.\{(90 \times 295) + (88 \times 35) + (100 \times 35)\}\ \$45 \times 1.00$$

$$= \$\ 2\ 981\ 700$$

These two contributions are the same for all 6 sets.

Set	Route market	Revenue
1	AD/DA	2 097 600
	AC/CA P = 0	
	(2x235) $ 75 (365 days) (0.95)	
	↓ Capacity = 380	
	AB(C)/(C)BA	20 742 400
	(2x189) $56 (365 days) (1.00)	
	Ne = 275, Capacity = 189	
	∴ under capacity	
	(A) BC/(CB)A	2 981 700
	Total	$ 25 821 700

Set	Route Market	Revenue
2	AD/DA	2 097 600
	AC/CA P = 0	
	(2x235) $75 (365 days) (0.95)	
	↓ Capacity = 380	
	AB(C)/(C)BA	22 340 700
	(2x275) $56 (365) (0.90)	
	↓ Capacity = 314	
	(A)BC/CB(A)	2 981 700
	Total	$ 27 420 000
3	AD/DA	2 097 600
	AC/CA P = 0.47	
	(2x125) (365 days) $75 (1.00)	
	Ne = 235 Capacity = 125 47% traffic diverted via B	
	AB(C)/(C)BA	22 080 700
	(2x275) $56 (365 days) (0.90)	
	↓ Capacity = 380	
	(2x110) $75 (365 days) (0.85)	
	(A)BC/CB(A)	2 981 700
	Total	$ 27 160 000

Set	Route Market	Revenue
4	AD/DA	2 097 600
	AB/BA	
	(2x275) $56 (365 days) (0.95)	
	↓ Capacity = 505	
	AC(B)/(B)CA	22 259 500
	(2x189) $75 (365 days) (1.00)	
	Ne = 235 Capacity = 189 excess 45 diverted via B	
	(2x45) $75 (365 days) (0.50)	
	(A)CB/BC(A)	2 981 700
	Total	$ 27 388 800
5	AD/DA	2 097 600
	AB/BA	
	(2x275) $56 (365) (0.90)	
	↓ Capacity = 315	
	AC(B)/(B)CA	22 340 700
	(2x235) $75 (365 days) (0.95)	
	↓ Capacity = 380	
	(A)CB/BC(A	2 981 700
	Total	$ 27 420 000

Set	Route Market	Revenue
6	AD/DA	2 097 600
	AB/BA	
	(2x275) \$56 (365) (0.96)	
	↓ Capacity = 440	
	AC(B)/(B)CA	22 902 800
	(2x235) \$75 (365 days) (0.95)	
	↓ Capacity = 380	
	(A)CB/BC(A)	2 981 700
	Total	\$ 27 982 100

Final Decision Table

Set	Annual Revenue \$	Annual Cost \$	Profit \$	
1	25, 821, 700	14, 168, 300	11, 653, 400	← suggested solution criteria -- minimum cost
2	27, 420, 000	17, 230, 200	10, 189, 800	
3	27, 160, 000	15, 627, 000	11, 533, 000	
4	27, 338, 800	14, 619, 700	12, 719, 100	← suggested solution criteria -- maximum profit
5	27, 420, 000	15, 652, 400	11, 767, 600	
6	27, 982, 100	18, 049, 000	9, 933, 100	← suggested solution criteria -- maximum revenue

Solution: SET 4:

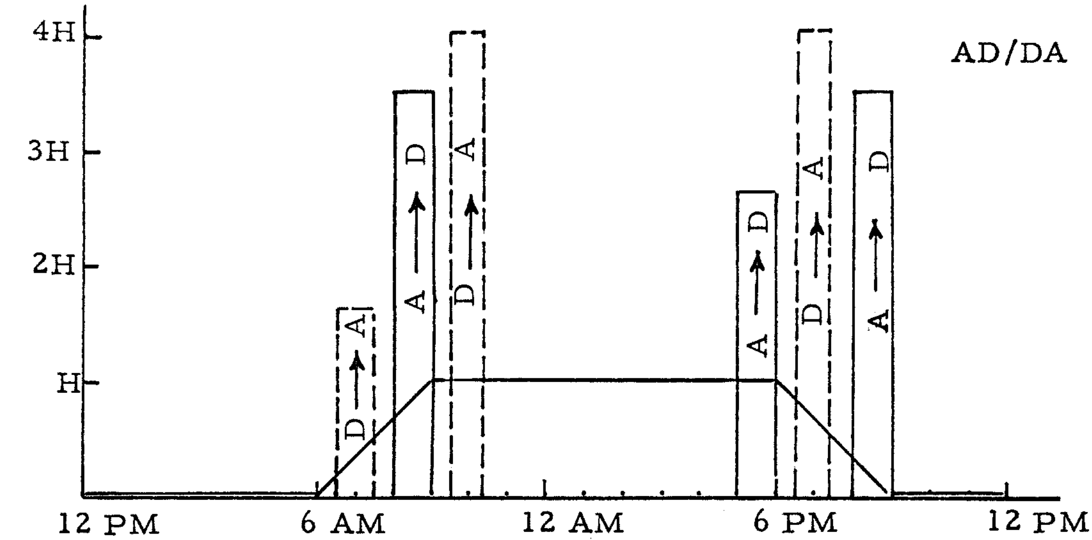
Schedule -- (All times in local time)

Time	Station		Aircraft		
				B727	
0	A	Departure	8:30 AM	5:30 PM	8:30 PM
0	D	Arrival	9:30 AM	6:30 PM	9:30 PM
0	D	Departure	7:00 AM	10:00 AM	7:00 PM
0	A	Arrival	8:00 AM	11:00 AM	8:00 PM

Time	Station		Aircraft		
			B707	B727	B707
0	A	Departure	8:30 AM	11:30 AM	4:00 PM
+1	B	Arrival	11:45 AM	3:00 PM	7:15 PM
+1	B	Departure	12:45 PM	3:30 PM	8:15 PM
0	A	Arrival	2:00 PM	5:00 PM	9:30 PM

Time	Station		Aircraft	
			B707	B707
0	A	Arrival		10:00 PM
		Departure	9:00 AM	
+1	C	Arrival	1:15 PM	6:45 PM
		Departure	2:15 PM	7:45 PM
+1	B	Arrival	4:00 PM	
		Departure		5:00 PM

Expected Response Curves:



$\oint_{24}$ D dt = 12 H $\oint$ R dt = 9.6 H   OMOT = 2.4 H

Ne = 150, 180, 240

e.g. For Ne = 180, Load factor at 10:00 AM DA flight

$$= 180\ (0.80)\ \left(\frac{4H}{9.6H}\right) \Big/ 125 = 44\%$$

Thin graph shows original demand curve.

80% reduction due to scheduling.

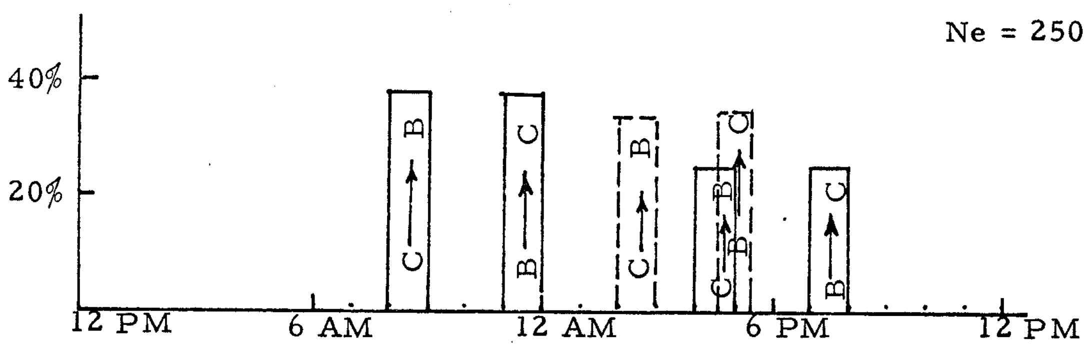
——— Gemini

- - - - - Apollo

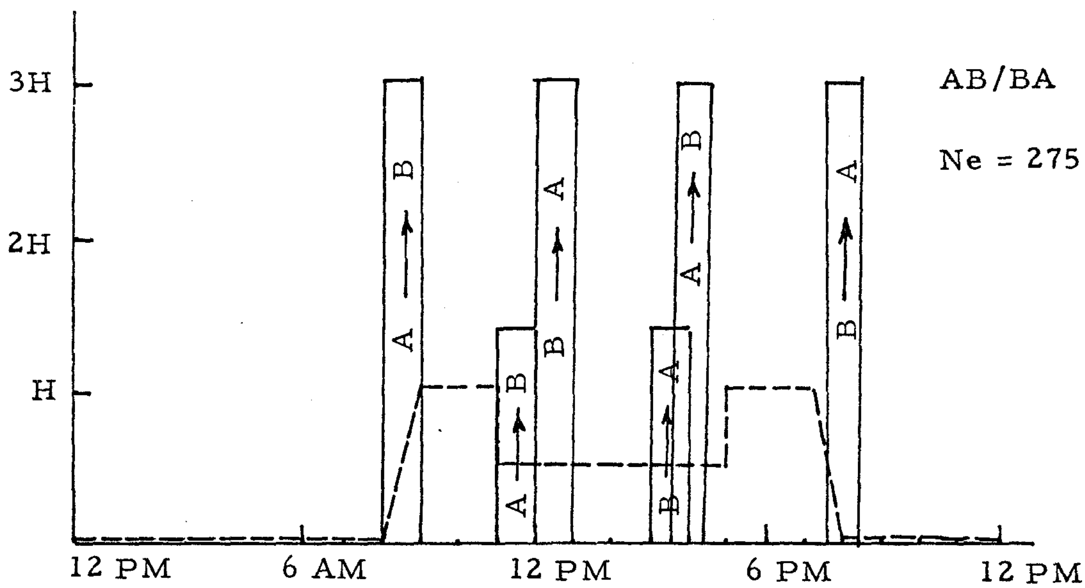
At Equilibrium Stage

BC/CB

Ne = 250



$\oint_{24} D\,dt = 8H$ $\oint_{R} R\,dt = 7.6$ OMOT = 0.4 H



Broken line represents original demand curve. Similarly, response curve for AC/CA can also be shown.

IX. CONCLUDING REMARKS

Our objective of the study was to derive a relationship of cost and revenue as a function of three inputs, aircraft, routes, and scheduling so that the decision maker would be able to evaluate different combinations of inputs and choose the right one. The right solution has been defined as one which maximizes profit.

After analyzing the hard facts of real world situations encountered by the airlines, the study did not unveil a functional relationship; however, it yielded a methodology. In a true sense of mathematics, it may be that such a well defined formula simply does not exist, but this is not altogether discouraging. The extensive treatment given here reveals a systematic way to carry out such an analysis and reach logical and rational solutions. Its usefulness could be ruined if one tries to apply this technique without modifying or adjusting many of the parameters that appear in the analysis.

One of the interesting questions that may come up at the end would be about its applicability. Can a decision maker really optimize the profit? What meaning does the word optimization have for such huge and complex systems?

Optimization may have some definite meaning in 'closed systems'. Everyone should be aware that the airline is by no means a closed system. Several external factors have considerable influence on the system's performance. Relatively speaking, the revenues are affected by the external factors more severely than the costs. Any attempt to optimize profit could be easily upset by some adverse variation in external environment. This dyanamic nature of the system compels one to adopt strategies so as to optimize performance for a short period of time during which outside parameters are looked upon as unchanged. This leads to the concept of step-wise optimization.

Another major drawback of this maxi-mini approach would be the conflicting performance parameters of the same system. To cite an example, the return on investment and passenger comfort/public image are not necessarily complimentary. Moreover, they may suggest two opposite sets of inputs as the best solutions. This apparent inconsistency sheds some light on sub-optimization. A system may gain somewhere, but it has to lose at another place. Therefore, the most appropriate job for the decision maker is certainly not to seek a maximization or minimization of a certain parameter, but to search for best trade-offs of different objectives which are suitable and compatible to the organization's policies.

Commenting on the sensitivity of each input towards profit, one might as well admit the vast difficulties in isolating the impact of variation of each input on the performance of the system. Costs, in general, are particularly sensitive for all of three inputs, while revenues are mainly sensitive towards routes and scheduling. In addition to this bottleneck, input variables are not solely independent. The perturbation in input R inevitably involves variation in S. It would not be possible to state a hard and fast rule to demonstrate sensitivity of each input on profit, however it could easily be shown for a particular input by actual calculations.

Can the decision maker bring about effective changes in inputs on the basis of the feedback? For a complex system such as an airline, there will be unavoidable lags in time between specifying inputs (R, S, J) and registering the first profit signal. This lag can extend for months. More important is the inflexibility of the decision maker to undertake corrective steps. To illustrate, if a feedback suggests change in input J, the aircraft, there may not be enough room to maneuver for the decision maker once purchase of the aircraft has been completed. Some legal restrictions make it impossible to terminate the service in one of the routes to vary input R. Such inflexibility behaves as a constraint on the system. Thus the theoretician's notion of feedback and corresponding changes in the inputs poses a formidable task for the management.

The advanced technology is bringing cheaper 'per seat-mile cost' aircraft but, unfortunately, these advantages are being wiped out due to rising costs of material and labor, excessive competition and over-capacity. The state of the economy and the introduction of a new aircraft can also badly hurt the fragile balance between costs and revenues. More broad understanding and cooperation amongst agencies involved in air transportation system, proper matching between demand and supply and vigilant management would solve many, if not all, problems. Looking ahead in the future, the potential of the airline industry seems unlimited, growth and popularity of air travel -- business and pleasure, passengers and cargo -- appear incredible. More fitting description would be -- 'the sky is the limit' !

APPENDIX

```
//ARUNA   JOB    (98728,ANK,AE),'ARUN'
//        EXEC   FORTG
//FORT    DD     *
      DIMENSION   X(40,9)
      COMMON/REG001/BETAZ,BETA(25)
      COMMON/REG002/SB(25)
      COMMON/REG003/SY,PHI,V(25)
      COMMON/REG004/XBAR(25),SIGMA(25)
    5 READ(5,500)N,M
      DO10I=1,M
   10 READ(5,501) (X(I,J),J=1,N)
      F1=0
      F2=0
      CALL REGRES(N,M,X,F1,F2,0,4HTEST)
      GO TO 5
  500 FORMAT(2I5)
  501 FORMAT(8F10.0)
      END
//DATA    DD     *
```

Notation:

Pan American	P
Delta	D
Northeast	N
Airwest	A
North Central	C
Ozark	O
Brannif	B
TWA	T
United	U
1969	9
1968	8
1967	7
1966	6
1965	5

E 4	40 W	M_{-1}	D	
2.00	56855.00	95.98	103.21	D 9
2.00	56855.00	93.32	106.31	D 8
2.00	56855.00	114.57	108.02	E 9
2.00	56855.00	45.54	115.50	E 8
2.00	56855.00	105.01	104.75	N 9
2.00	56855.00	76.00	101.32	N 8
2.00	56855.00	24.69	117.01	A 9
2.00	56855.00	62.92	79.02	C 9
2.00	56855.00	58.63	87.98	O 9
3.00	94500.00	127.46	100.68	B 9
3.00	94500.00	93.90	89.59	B 8
3.00	94500.00	103.37	103.84	B 7
3.00	94500.00	174.35	103.79	E 9
3.00	94500.00	82.99	95.22	E 8
3.00	94500.00	66.09	123.10	T 9
3.00	94500.00	48.42	119.88	T 8
3.00	94500.00	162.12	91.64	U 9
3.00	94500.00	164.93	75.72	U 8
3.00	94500.00	187.89	64.56	U 7
3.00	94500.00	165.50	160.14	P 9
4.00	142780.0	134.67	148.48	T 9
4.00	142780.0	115.25	129.84	T 8

E	W	M_{-1}	D	
4.00	142780.0	74.22	165.12	T 7
4.00	142780.0	93.94	162.31	T 6
4.00	148800.00	116.12	137.06	T 9
4.00	148800.00	118.25	128.43	T 8
4.00	148800.00	101.14	158.51	T 7
4.00	148800.00	119.39	164.69	T 6
4.00	148800.00	188.53	1[illegible].94	P 9
4.00	148800.00	175.21	1[illegible].44	P 8
4.00	148800.00	192.24	137.15	P 7
4.00	146800.00	172.77	141.11	P 6
4.00	148800.00	182.61	153.41	P 9
4.00	148800.00	179.05	137.74	P 8
4.00	148800.00	187.36	139.92	P 7
4.00	148800.00	169.43	137.76	P 6
4.00	148800.00	175.71	144.87	P 9
4.00	148800.00	177.00	133.51	P 8
4.00	148800.00	194.83	139.49	P 7
4.00	148800.00	177.85	144.11	P 6

E 6	40 W	U	L	C	DOC/ATM
2.00	56855.00	8.17	280.00	950.00	12.14
2.00	56855.00	7.92	270.00	963.00	12.02
2.00	56855.00	6.78	310.00	908.00	13.57
2.00	56855.00	5.64	277.00	889.00	14.80
2.00	56855.00	6.75	322.00	904.00	13.65
2.00	56855.00	6.92	332.00	913.00	10.47
2.00	56855.00	7.06	268.00	885.00	10.39
2.00	56855.00	6.50	158.00	900.00	12.62
2.00	56855.00	6.73	195.00	896.00	11.74
3.00	94500.00	8.95	427.00	1285.00	10.76
3.00	94500.00	8.86	434.00	1343.00	10.59
3.00	94500.00	9.16	466.00	1356.00	9.68
3.00	94500.00	8.97	585.00	1372.00	11.8
3.00	94500.00	8.51	477.00	1353.00	12.93
3.00	94500.00	8.29	677.00	1342.00	13.03
3.00	94500.00	8.09	611.00	1361.00	11.29
3.00	94500.00	8.50	583.00	1243.00	12.09
3.00	94500.00	8.47	580.00	1239.00	11.50
3.00	94500.00	8.15	563.00	1258.00	11.10
3.00	94500.00	5.69	544.00	1366.00	16.95
4.00	142780.0	10.71	1317.00	1851.00	7.68
4.00	142780.0	10.76	1262.00	1859.00	7.47
4.00	142780.0	10.46	1204.00	1721.00	7.58
4.00	142780.0	9.67	1133.00	1781.00	7.45
4.00	148800.00	11.85	2299.00	1885.00	7.12
4.00	148800.00	11.94	2307.00	1909.00	6.66
4.00	148800.00	11.99	2318.00	1925.00	7.29
4.00	148800.00	11.28	2081.00	1905.00	6.92
4.00	148800.00	10.79	1777.00	1922.00	9.13
4.00	148800.00	11.39	1825.00	1922.00	8.42
4.00	148800.00	11.67	1735.00	1914.00	8.11
4.00	148800.00	11.52	1650.00	1788.00	8.14
4.00	148800.00	10.79	2214.00	2027.00	8.73
4.00	148800.00	11.39	2172.00	2057.00	8.14
4.00	148800.00	11.62	2309.00	2115.00	8.26
4.00	148800.00	11.53	2219.00	2065.00	8.40
4.00	148800.00	10.75	1268.00	1953.00	9.33
4.00	148800.00	11.28	1286.00	1974.00	8.61
4.00	148800.00	11.52	1605.00	1893.00	8.32
4.00	148800.00	11.50	2059.00	1912.00	8.58

6 40

E	W	U	L	C	DOC/ASM
2.00	56855.00	8.17	280.00	950.00	1.52
2.00	56855.00	7.92	270.00	963.00	1.55
2.00	56855.00	6.78	310.00	908.00	1.77
2.00	56855.00	5.64	277.00	889.00	1.87
2.00	56855.00	6.75	322.00	904.00	1.91
2.00	56855.00	6.92	332.00	913.00	1.45
2.00	56855.00	7.06	268.00	885.00	1.46
2.00	56855.00	6.50	158.00	900.00	1.78
2.00	56855.00	6.73	195.00	896.00	1.42
3.00	94500.00	8.95	427.00	1285.00	1.54
3.00	94500.00	8.86	434.00	1343.00	1.46
3.00	94500.00	9.16	466.00	1356.00	1.29
3.00	94500.00	8.97	585.00	1372.00	1.53
3.00	94500.00	8.51	477.00	1353.00	1.69
3.00	94500.00	8.29	677.00	1342.00	1.71
3.00	94500.00	8.09	611.00	1361.00	1.47
3.00	94500.00	8.50	583.00	1243.00	1.57
3.00	94500.00	8.47	580.00	1239.00	1.49
3.00	94500.00	8.15	563.00	1258.00	1.44
3.00	94500.00	5.69	544.00	1366.00	2.18
4.00	142780.0	10.71	1317.00	1851.00	1.11
4.00	142780.0	10.76	1262.00	1859.00	1.08
4.00	142780.0	10.46	1204.00	1721.00	1.10
4.00	142780.0	9.67	1133.00	1781.00	1.08
4.00	148800.00	11.85	2299.00	1885.00	1.02
4.00	148800.00	11.94	2307.00	1909.00	0.97
4.00	148800.00	11.99	2318.00	1925.00	1.05
4.00	148800.00	11.28	2081.00	1905.00	1.00
4.00	148800.00	10.79	1777.00	1922.00	1.34
4.00	148800.00	11.39	1825.00	1922.00	1.23
4.00	148800.00	11.67	1735.00	1914.00	1.19
4.00	148800.00	11.52	1650.00	1788.00	1.20
4.00	148800.00	10.79	2214.00	2027.00	1.28
4.00	148800.00	11.39	2172.00	2057.00	1.20
4.00	148800.00	11.62	2309.00	2115.00	1.23
4.00	148800.00	11.53	2219.00	2065.00	1.26
4.00	148800.00	10.75	1268.00	1953.00	1.37
4.00	148800.00	11.28	1286.00	1974.00	1.26
4.00	148800.00	11.52	1605.00	1893.00	1.23
4.00	148800.00	11.50	2059.00	1912.00	1.27

E 6 40	W	U	L	M_{-1}	M
2.00	56855.00	8.17	280.00	95.98	93.91
2.00	56855.00	7.92	270.00	93.32	95.98
2.00	56855.00	6.78	310.00	114.57	119.82
2.00	56855.00	5.64	277.00	45.54	114.57
2.00	56855.00	6.75	322.00	105.01	165.80
2.00	56855.00	6.92	332.00	76.00	105.01
2.00	56855.00	7.06	268.00	24.69	55.01
2.00	56855.00	6.50	158.00	62.92	106.26
2.00	56855.00	6.73	195.00	58.60	62.19
3.00	94500.00	8.95	427.00	127.46	113.45
3.00	94500.00	8.86	434.00	93.90	127.46
3.00	94500.00	9.16	466.00	103.37	93.90
3.00	94500.00	8.97	585.00	174.35	160.29
3.00	94500.00	8.51	477.00	82.99	174.35
3.00	94500.00	8.29	677.00	66.09	131.50
3.00	94500.00	8.09	611.00	48.42	66.09
3.00	94500.00	8.50	583.00	162.12	158.22
3.00	94500.00	8.47	580.00	164.93	162.12
3.00	94500.00	8.15	563.00	187.89	164.93

3.00	94500.00	5.69	544.00	165.50	234.12
4.00	142780.0	10.71	1317.00	134.67	118.40
4.00	142780.0	10.76	1262.00	115.25	134.67
4.00	142780.0	10.46	1204.00	74.22	115.25
4.00	142780.0	9.67	1133.00	93.94	74.22
4.00	148800.00	11.85	2299.00	116.12	123.31
4.00	148800.00	11.94	2307.00	118.25	116.12
4.00	148800.00	11.99	2318.00	101.14	118.25
4.00	148800.00	11.28	2081.00	119.39	101.14
4.00	148800.00	10.79	1777.00	188.53	202.83
4.00	148800.00	11.39	1825.00	175.21	188.53
4.00	148800.00	11.67	1735.00	192.24	175.21
4.00	148800.00	11.52	1650.00	172.77	192.24
4.00	148800.00	10.79	2214.00	182.61	208.84
4.00	148800.00	11.39	2172.00	179.05	182.61
4.00	148800.00	11.62	2309.00	187.36	179.05
4.00	148800.00	11.53	2219.00	169.43	187.36
4.00	148800.00	10.75	1268.00	175.71	200.50
4.00	148800.00	11.28	1286.00	177.00	175.71
4.00	148800.00	11.52	1605.00	194.83	177.00
4.00	148800.00	11.50	2059.00	177.85	194.83

E 8	40 W	P	U	L	C	M_{-1}	F
2.00	56855.00	4.00	8.17	280.00	950.00	95.98	205.40
2.00	56855.00	4.00	7.92	270.00	963.00	93.32	207.07
2.00	56855.00	4.00	6.78	310.00	908.00	114.57	229.40
2.00	56855.00	4.00	5.64	277.00	889.00	45.54	218.91
2.00	56855.00	4.00	6.75	322.00	904.00	105.01	232.73
2.00	56855.00	4.00	6.92	332.00	913.00	76.00	196.57
2.00	56855.00	4.00	7.06	268.00	885.00	24.69	222.05
2.00	56855.00	4.00	6.50	158.00	900.00	62.92	223.09
2.00	56855.00	4.00	6.73	195.00	896.00	58.60	217.84
3.00	94500.00	4.50	8.95	427.00	1285.00	127.46	285.91
3.00	94500.00	4.50	8.86	434.00	1343.00	93.90	261.66
3.00	94500.00	4.50	9.16	466.00	1356.00	103.37	244.05
3.00	94500.00	4.50	8.97	585.00	1372.00	174.35	277.13
3.00	94500.00	4.50	8.51	477.00	1353.00	82.99	273.85
3.00	94500.00	4.50	8.29	677.00	1342.00	66.09	329.49
3.00	94500.00	4.50	8.09	611.00	1361.00	48.42	306.36
3.00	94500.00	4.50	8.50	583.00	1243.00	162.12	297.05
3.00	94500.00	4.50	8.47	580.00	1236.00	164.93	284.57
3.00	94500.00	4.50	8.15	563.00	1258.00	187.98	269.61
3.00	94500.00	4.50	5.69	544.00	1366.00	165.50	353.03
4.00	142780.0	5.00	10.71	1317.00	1851.00	134.67	384.19
4.00	142780.0	5.00	10.76	1262.00	1859.00	115.25	364.39
4.00	142780.0	5.00	10.46	1204.00	1721.00	74.22	345.92
4.00	142780.0	5.00	9.67	1133.00	1781.00	93.94	323.82
4.00	148800.00	5.00	11.85	2299.00	1885.00	116.12	402.93
4.00	148800.00	5.00	11.94	2307.00	1909.00	118.25	384.01
4.00	148800.00	5.00	11.99	2318.00	1925.00	101.14	405.19
4.00	148800.00	5.00	11.28	2081.00	1905.00	119.39	360.40
4.00	148800.00	5.00	10.79	1777.00	1922.00	188.53	428.53
4.00	148800.00	5.00	11.39	1825.00	1922.00	175.21	404.48
4.00	148800.00	5.00	11.67	1735.00	1914.00	192.24	386.59
4.00	148800.00	5.00	11.52	1650.00	1788.00	172.77	369.95
4.00	148800.00	5.00	10.79	2214.00	2027.00	182.61	460.02
4.00	148800.00	5.00	11.39	2172.00	2057.00	179.05	447.89
4.00	148800.00	5.000	11.62	2309.00	2115.00	187.36	454.79
4.00	148800.00	5.00	11.53	2219.00	2065.00	169.43	429.29
4.00	148800.00	5.00	10.75	1268.00	1953.00	175.71	441.35
4.00	148800.00	5.00	11.28	1286.00	1974.00	177.00	420.46

4.00	148800.00	5.00	11.52	1605.00	1893.00	194.83	411.6
4.00	148800.00	5.00	11.50	2059.00	1912.00	177.65	436.3