

THE EFFECT OF WHEEL UNBALANCE ON VEHICLE DYNAMICS

Thesis by  
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In Partial Fulfillment of the Requirements  
For the Degree of  
Mechanical Engineer

California Institute of Technology  
Pasadena, California

1956

## ACKNOWLEDGMENTS

The writer wishes to thank Professor Harold Lurie for the helpful discussions and encouragement given during the work. Thanks are also due to Professor Kyropoulos and Professor Hudson for their interest in the project. Further thanks are due to Mr. Don Laird and Mr. Fred MacDonald for their assistance in the project.

## ABSTRACT

Some unbalance either inherent or induced is always to be expected in automobile wheels. It may be inherent due to imperfect balancing of the wheel or it may be induced by the uneven wear of the tires. While in motion this unbalance simultaneously sets up two vibrations in the wheel, one a linear vibration normal to the road and the other an angular vibration about the king pin. The frequency of these vibrations depends on the forward speed of the vehicle and when it coincides with the natural frequency of vibration of the wheels the well known phenomenon of resonance occurs.

The research reported in this thesis is a systematic investigation of this phenomenon in one front wheel of an automobile with the other three wheels in true balance. Known amounts of unbalance (both static and dynamic) were introduced into the left front wheel and road tests were conducted over a range of speeds covering resonance. Vibration pickups on the wheels recorded both vertical and angular displacement as a function of speed. Response curves have been plotted with the amount and type of unbalance as parameters. Also an attempt has been made to establish some bounds for the amount of unbalance that are noticeable to the passengers while traveling through the range of resonance.

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## INTRODUCTION

Some unbalance either inherent or induced is always to be expected in automobile wheels. It may be inherent due to imperfect balancing or it may be induced by uneven wear of the tires. This unbalance sets up a centrifugal force in the plane of the wheel when the vehicle is in motion. The frequency of this force depends on the forward speed of the vehicle and when it coincides with the natural frequency of vibration of the wheels the well known phenomenon of resonance occurs. So long as the driving speeds of automobiles remained those corresponding to below the natural frequency of the suspension system, the effect of these vibrations was not serious.

The advent of the high speed automobile in recent times has increased the driving speed through the resonance range, and the resulting vibrations are being increasingly felt by the passengers. In order to eliminate these unpleasant and sometimes dangerous vibrations, it is desirable to know the magnitudes of these oscillations as well as the amount and type of unbalance that induce these oscillations. Very little quantitative data of this nature exists in the literature. The primary objective of this project was therefore to measure the amplitudes of these vibrations for various exciting frequencies and for varying amounts of static and dynamic unbalance on the wheels. Also an effort was made to establish some bounds for the amount of unbalance that would not be felt by the passengers.

For the purpose of this investigation the study was limited to the left front wheel, the other three wheels being maintained in balance. There is always some coupling between the various parts of an automobile, but the 'independent suspension' of the front wheels enables one to study their characteristics separately for certain kinds of motion (Reference 1). In the unbalance problem, provided the other three wheels are balanced, the motion of one unbalanced front wheel can be regarded as that of an isolated system to first order accuracy.

For known amounts of unbalance on the left front wheel, measurements were made on the amplitudes of linear vibration in the vertical plane as well as the angular vibrations in the horizontal plane. Since these two modes of vibration are coupled the measurement of their amplitudes had to be made simultaneously. The tests were conducted on a 1954 Chevrolet 4 door sedan, model 210, equipped with 6.7 x 15 tires.

The results presented here may be of interest to those in the field of balancing generally and wheel balancing in particular.

Because there is some confusion about the concepts of balancing as related to automobile wheels it will be useful to define these concepts precisely.

(a) Static balance exists when the centre of mass of the system lies on the axis of rotation.

(b) Dynamic balance exists if in addition to (a) the axis of rotation is a principal axis of the wheel.

It follows from the above definitions that if a wheel is in dynamic

balance, then it is also in static balance. However, a wheel can be in static balance without necessarily being in dynamic balance.

For the sake of brevity vertical oscillation of the wheel normal to the road will be referred to as 'bounce', and the angular oscillation of the wheel about the king pin as 'wobble'.

## INSTRUMENTATION

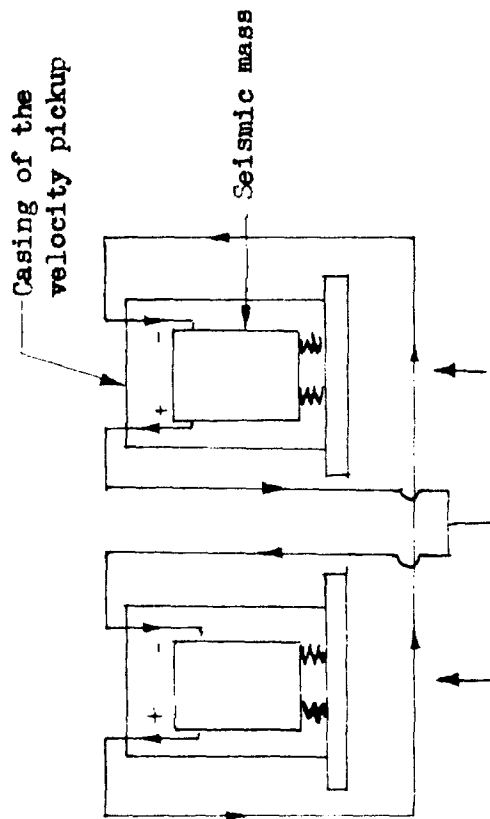
Although a variety of instruments by different manufacturers was available for making the vibration measurements, certain special requirements expected of the instrument limited the choice to a few. Since the project involved making measurements in the field, the instruments carried had to be simple and of rugged construction and require as little power as possible for operation. Also the instrument was expected to measure adequate amplitudes at fairly low frequencies. These requirements dictated the use of the Consolidated Engineering Co. type 4-102 Velocity Pickup. This is a self-generating unit of coil and magnet type, fluid damped, seismic instrument with a natural frequency of 9 c.p.s. and an approximately flat frequency response for all double amplitudes up to at least 0.25". One such pickup was used for measuring the amplitude in bounce. For measuring wobble two of these pickups were connected in series opposed as shown in Fig. 1. This arrangement was devised to record only angular motion. The signals generated by the pickups from linear lateral motion of the wheel due to king pin clearance were thus automatically cancelled electrically.

The output of the pickups was fed into a Consolidated Engineering Co. Vibration Meter. An integrating circuit incorporated in the meter converted the velocity signal to displacement which was read on a graduated scale. The vibration meter was used as a check on the oscillograph record. The output from the vibration meter was fed into a Brush two channel recorder-amplifier combination. The

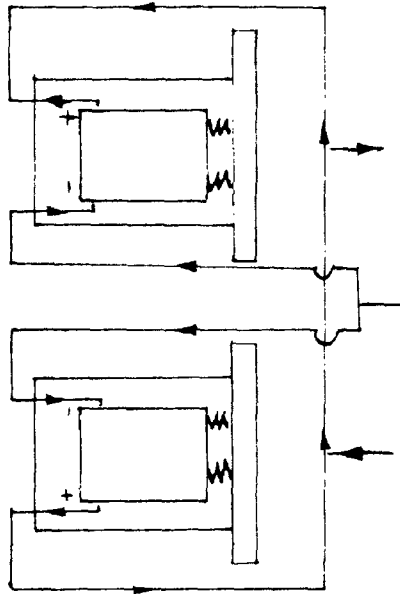


Brush recorder-amplifier combination had an over-all frequency response which was flat from 0.5-100 c.p.s. Both the bounce and wobble were simultaneously recorded on the two channels of the Brush recorder.

Since the frequency of the exciting force depended on the forward speed of the car a means had to be devised to measure the automobile speed accurately. To accomplish this a fifth wheel was used to calibrate the speedometer. The arrangement is shown in Fig. 2 . The fifth wheel is coupled to a generator which produces a voltage directly proportional to the velocity and can be read on a voltmeter. The speedometer on the car thus calibrated, served to indicate the speeds. The calibration curve is shown in Fig. 3. Also a curve of frequency vs. speed is given in Fig. 4, The schematic diagram of the instrument layout is shown in Fig. 5. The electrical connections of the two horizontal pickups to measure angular motion only are indicated in Fig. 1 . A major portion of the work involved a careful calibration of the pickups, and this is described in detail in Appendix I. The serial numbers of all the instruments used are listed in Appendix II.



a) Pickups move in the same direction



b) Pickups move in the opposite direction

Fig. 1: Diagram illustrating the series opposed connection of the velocity pickups.

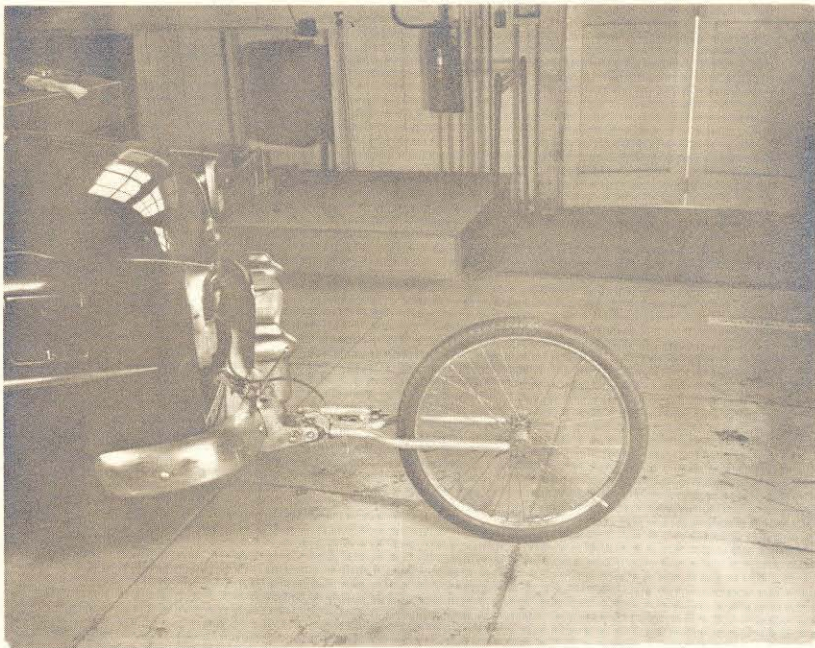
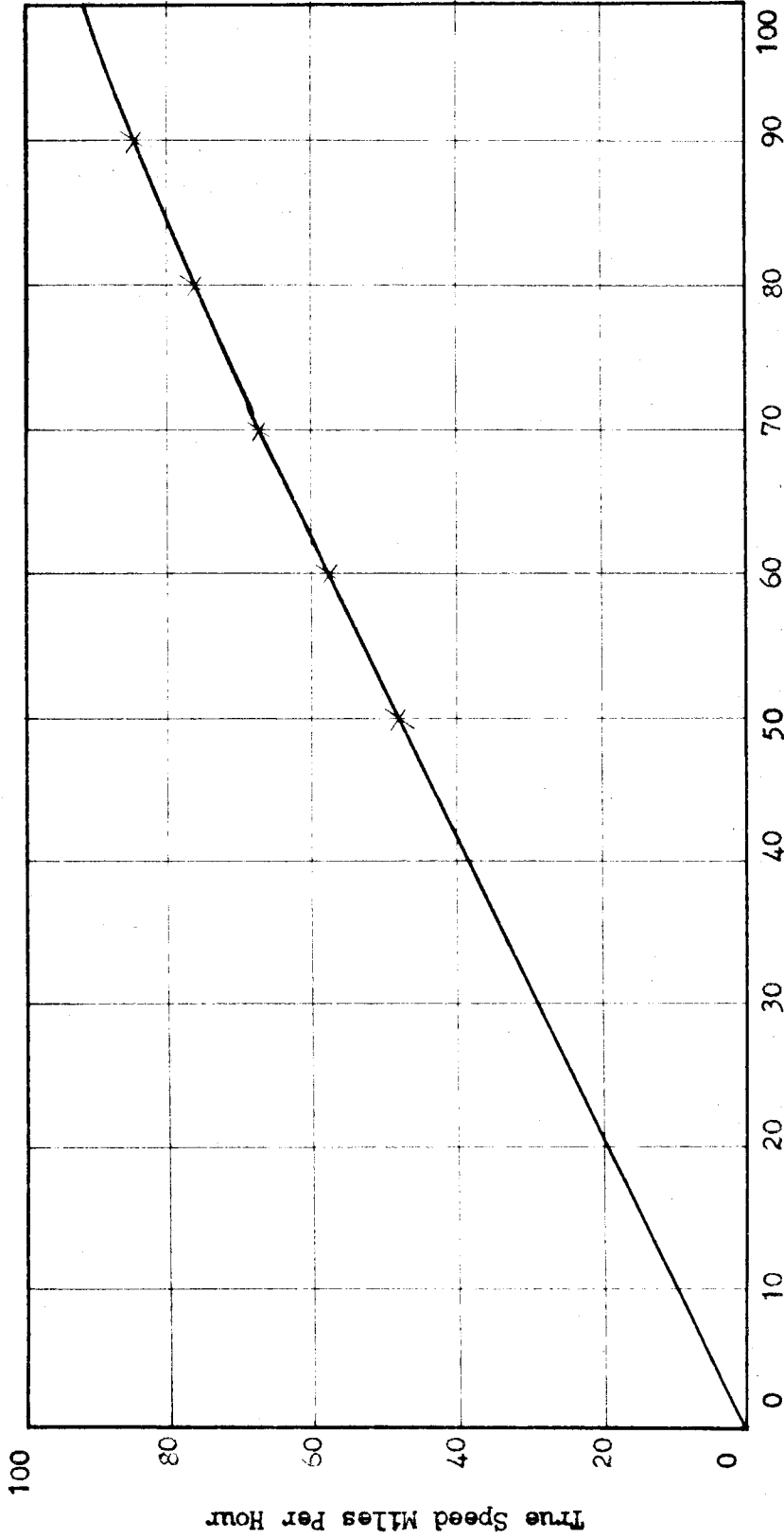


Fig. 2: The fifth wheel attachment for calibration of speedometer.



Speedometer Reading Miles Per Hour

Fig. 3: Speedometer Calibration Curve.

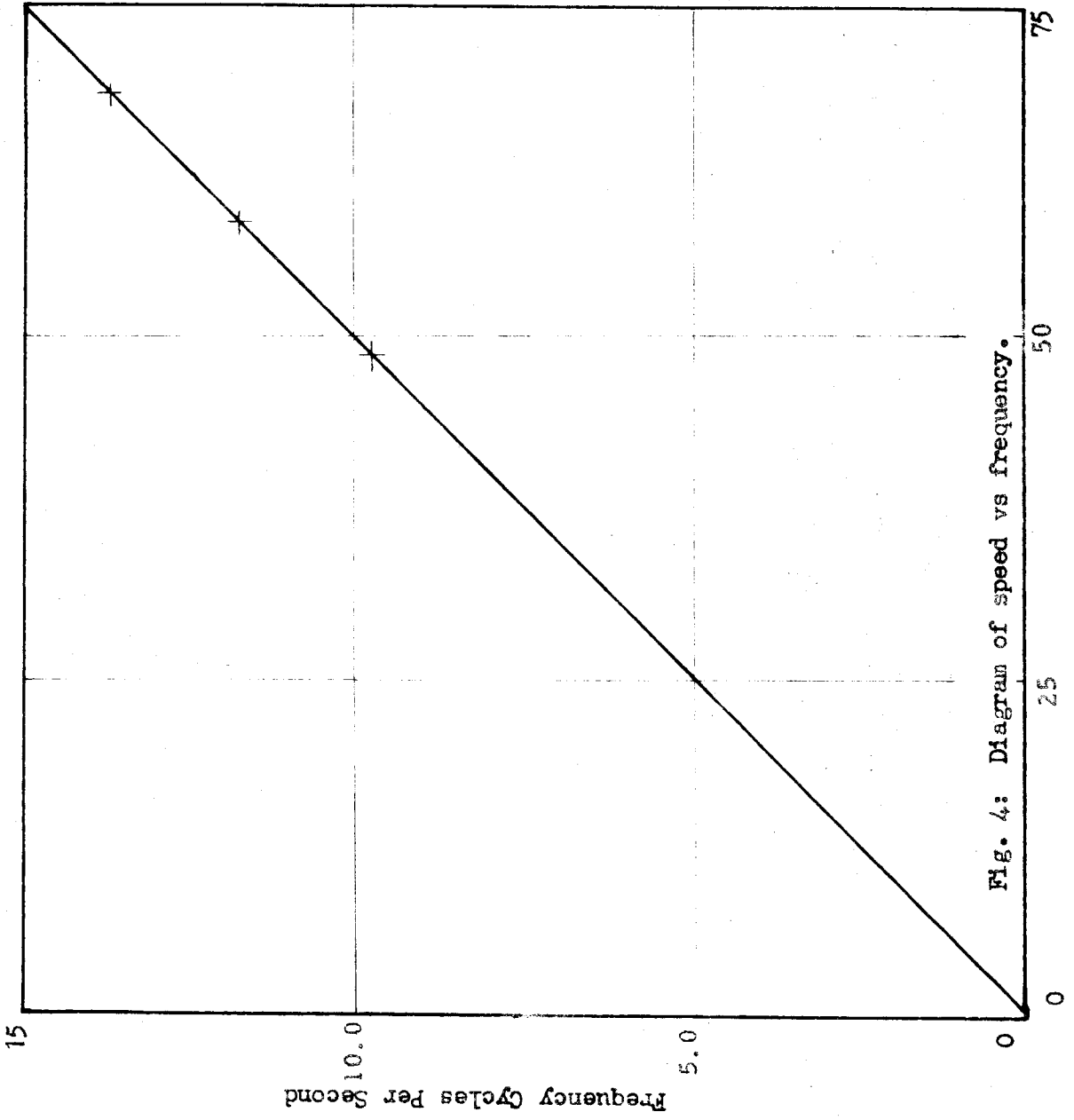


Fig. 4: Diagram of speed vs frequency.

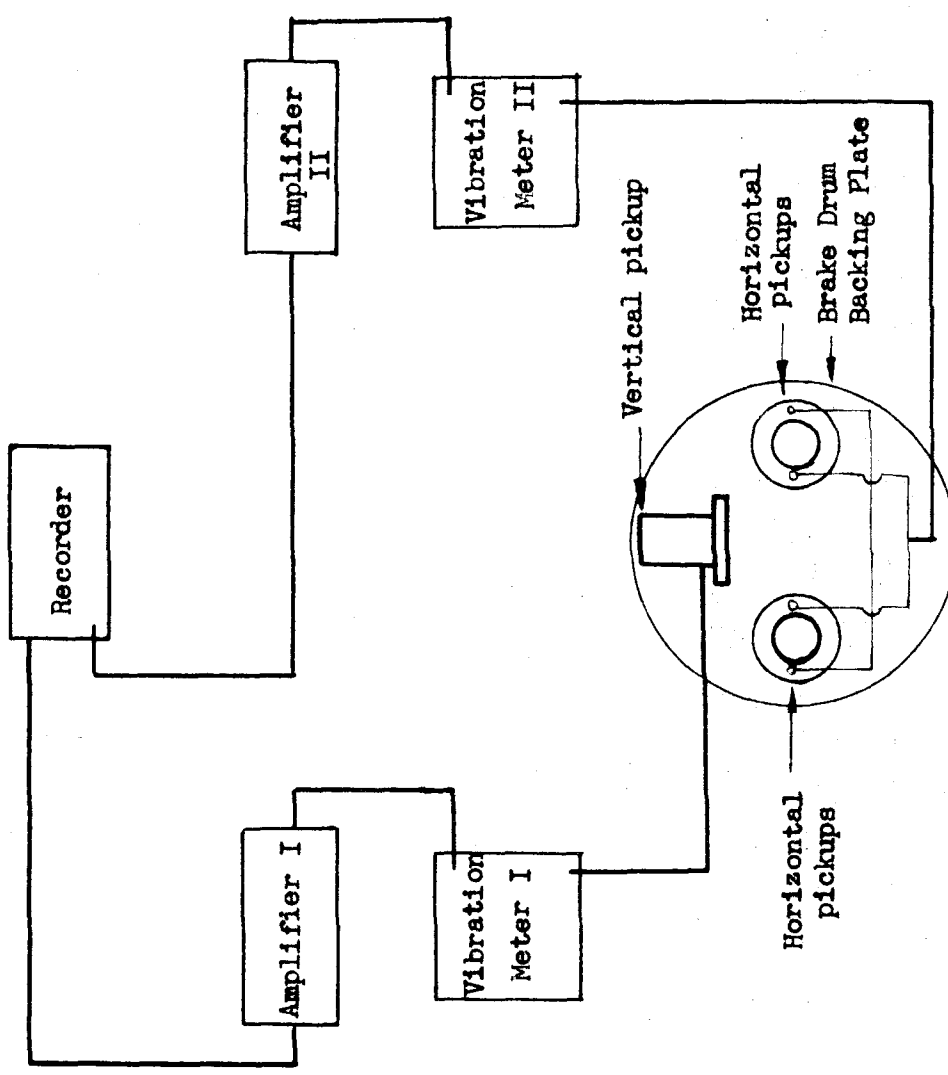


Fig. 5: Simplified schematic diagram of the instruments.

### PROCEDURE.

The interpretation of measurements depended to a great extent on prior precise balancing of the wheels. In order to achieve such balance the tires were skimmed first on a Mabco tire skimming machine to remove any out-of-round. The wheels and the drums were then balanced separately both statically and dynamically on a Westinghouse dynetric balancer. The balanced wheels were mounted on the car.

The velocity pickups were mounted on the brake drum backing plate of the left frontwheel (Fig. 6). A schematic diagram of the arrangement of the pickups is shown in Fig. 7. The rear seat of the automobile was removed and a platform erected on which the amplifiers, vibration meter and oscillograph were securely fastened (Fig. 8). The power supply was located in the trunk of the car (Fig. 9).

A reasonably smooth and flat test track was selected on U.S. Highway 66 to conduct these measurements. Throughout the tests the tire pressure was maintained constant at 24 psi.

Initial exploratory tests with unbalance on one front wheel indicated that the resonance peak was quite sharp. Outside this resonance range, moderately large unbalance in the wheel could not be felt by the observers in the automobile. Hence all tests were limited to speeds between 60 and 75 m.p.h. as this speed range clearly included the region of interest.

Test runs were made first with static unbalance on the left front wheel with a single unbalance weight on the outer rim

and the wheel response recorded as a function of speed. The magnitude of the unbalanced weights was increased in increments from one ounce to five ounces in steps of one ounce. With each unbalance weight the wheel response was recorded at frequencies corresponding to 60, 65, 70, 75 miles per hour.

In a second series of tests the same amounts of unbalance weight as in the first were equally divided between the inner and outer rims.

In the final series of tests dynamic unbalance only was studied by putting equal amounts of unbalanced weights on each of the inside and outside rims at  $180^{\circ}$  out of phase.

Throughout the tests, the speed at which vibration in the automobile first became noticeable to the two observers was noted for each unbalance condition. The measurements were always made on the inside lane of the same test strip of the highway as close to the centre line as possible in order to maintain similar road conditions. Typical oscillograph records of the wheel response to various unbalance conditions are shown in Figs. 25-28 in Appendix IV.



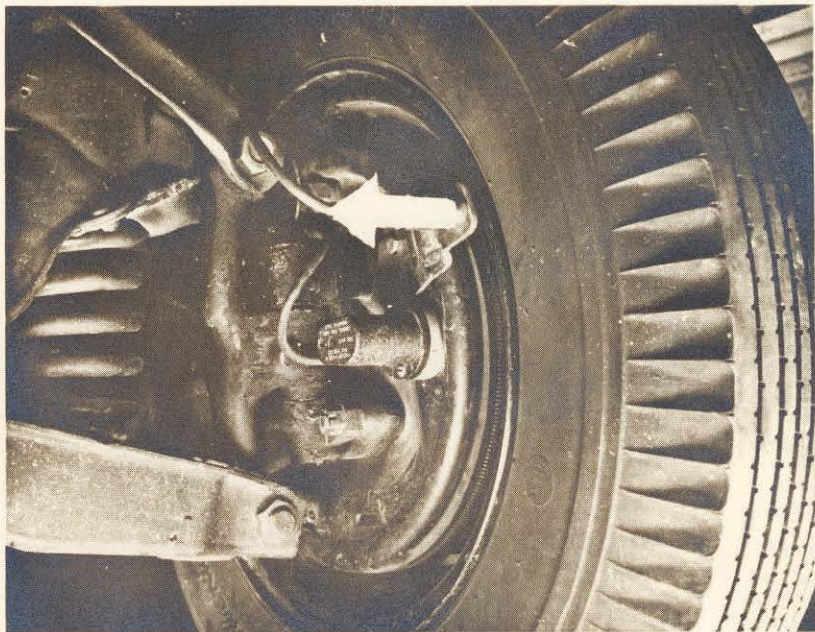
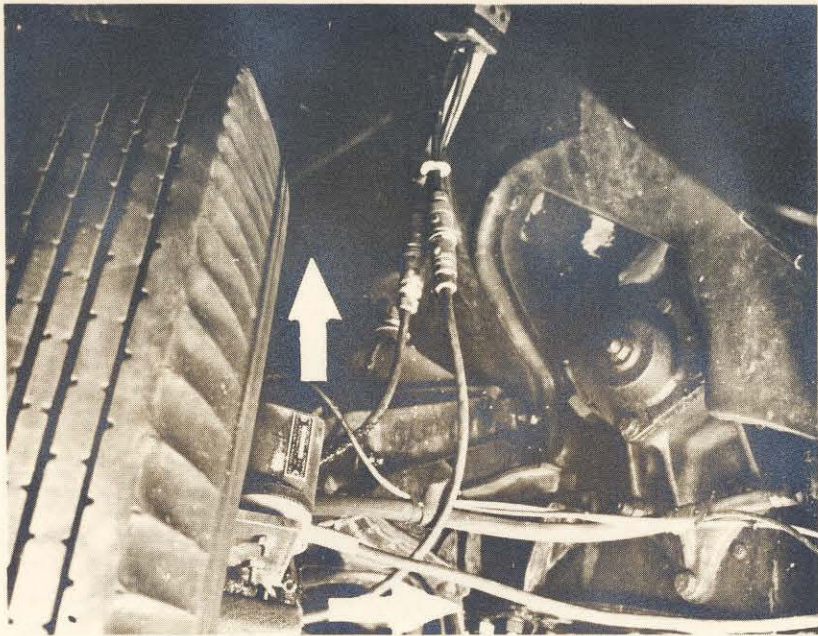


Fig. 6: Pickups for measuring bounce and wobble. Note the two horizontal pickups, one to the left of the king pin (top) and the other to the right of the king pin (bottom). Both pickups equidistant from the king pin.

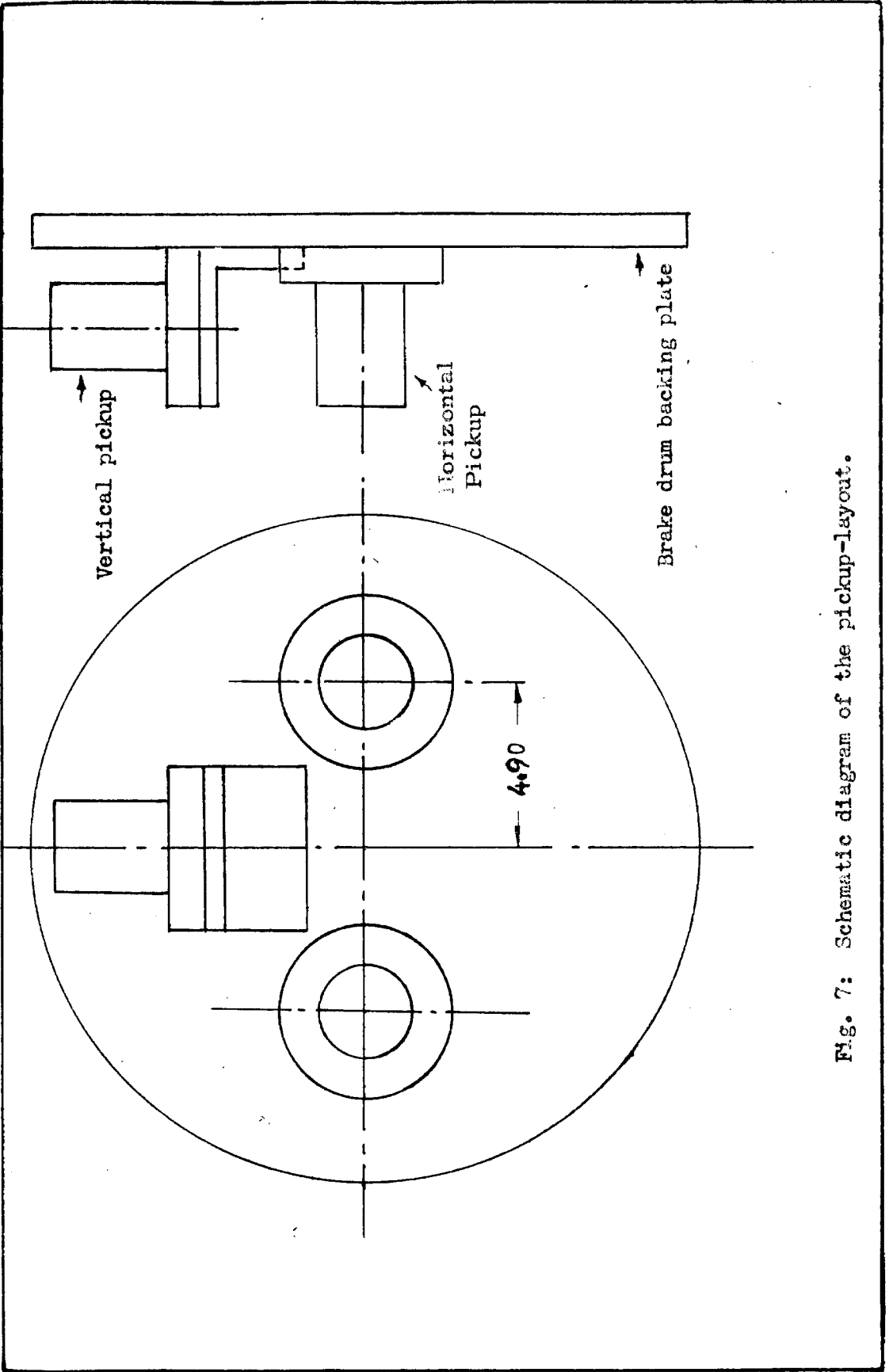
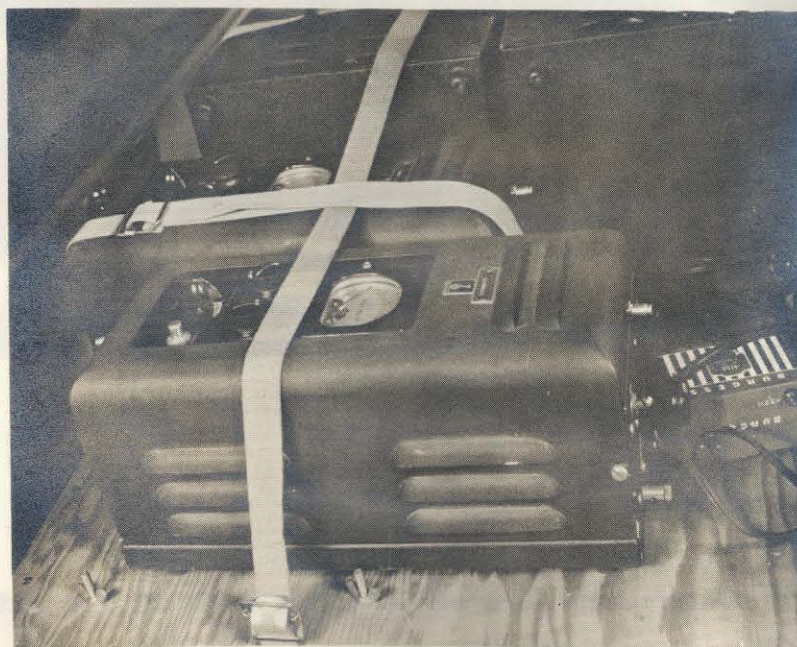


Fig. 7: Schematic diagram of the pickup-layout.



(a) Vibration meters (right). Recorder (left).



(b) Amplifiers (foreground).

Fig. 8: Showing the mounting of instrument on the table.

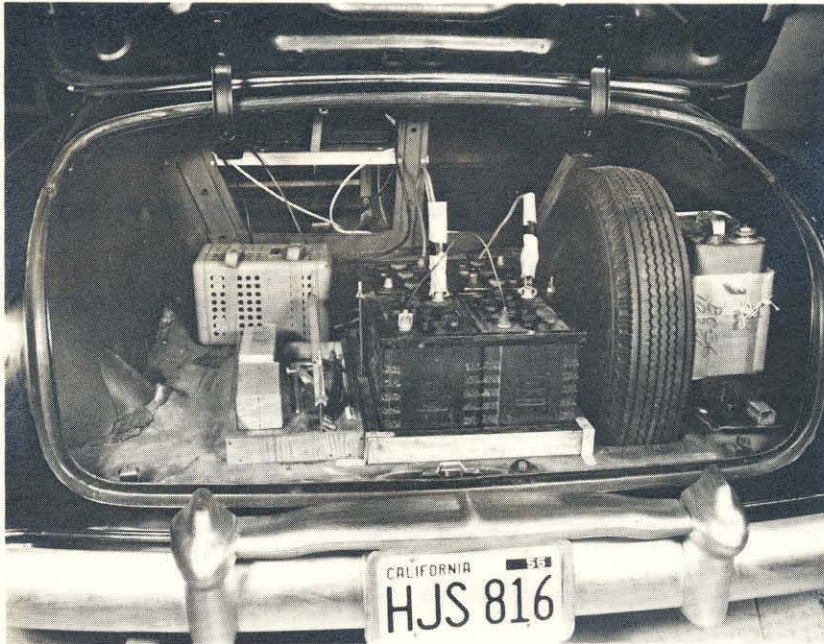


Fig. 9: The power supply in the trunk compartment.

## RESULTS AND CONCLUSIONS

The response diagrams of the wheel (double amplitude vs. speed) for various classes and magnitudes of unbalance are given in Fig. 10-15. The case of static unbalance and dynamic unbalance only are considered separately. Further, static unbalance has been considered under two headings:

- (a) with a single unbalance weight on the outside rim,
- (b) with the unbalance weight of case (a) divided equally between the inside and outside rims (in phase).

Response curves for both bounce and wobble are given for all the above three cases of unbalance. The units adopted for static unbalance are oz-in., for dynamic unbalance oz-in.-in. The mean radius to the centre of mass of each unbalance weight is 8 in., hence static unbalance is measured by multiplying the unbalance weight by 8 in. The normal distance between weights on the two rims is 6 in. So when one weight is attached to each rim and diametrically opposite to each other ( $180^{\circ}$  out of phase), the dynamic unbalance is measured by multiplying one unbalance weight by 8 in. x 6 in. A curve passing through the speeds at which vibrations become noticeable to an observer in the car is also marked on the response curves.

From the results it is clear that a well defined resonance range (13.5-14.5 c.p.s. or 67-72 miles per hour) exists for the front wheel, for all kinds of unbalances. For static unbalance resonance occurs at a forcing frequency of 13.5 c.p.s. (67 miles

per hour). For dynamic unbalance only resonance occurs at a slightly higher frequency of 14.5 c.p.s. (72 miles per hour).

Except for the short time intervals between the tests (extending over one and one half months), both static unbalance and dynamic unbalance measurements were made under exactly similar conditions. The increase in the resonance frequency (7-1/2%) for dynamic unbalance only cannot be attributed to a change in the elastic properties of the tire alone. This discrepancy still lacks adequate explanation.

It is also apparent from the results, that unbalance in the wheel not only excites the vertical oscillations of the wheel, but also angular motion of the wheel about the king pin. Both these oscillations occur with the same frequency--they are coupled. This can be explained on the basis of gyroscopic theory. An account of the gyroscopic coupling in the frontwheel is given in the Appendix III.

For static unbalance, peak amplitudes are always attained at a frequency of 13.5 c.p.s. (67 miles per hour). For dynamic unbalance only, these occur at a frequency of 14.5 c.p.s. (72 miles per hour). Since the response of the wheel was recorded at discrete frequencies, it should be noted that the peak amplitudes are estimated maxima. For a more exact determination, it is suggested that a continuous record be made of the wheel response over the entire speed range. Then the maximum amplitude obtained will serve as an upper bound for fitting the curve through the observed points.

A static unbalance of 40-oz-in. on the outside rim of the wheel gave rise to a maximum double amplitude in bounce of 0.22" and a maximum double amplitude in wobble of  $0.75^{\circ}$ . When the same unbalance was equally divided between the two rims (in phase) the amplitudes were reduced to 0.15" and  $0.65^{\circ}$ , respectively. Dividing the unbalance equally between the rims brings about better balance conditions, as might be expected.

Unbalance in the wheel not only sets up a centrifugal force in the vertical plane, but it also gives rise to a couple. The vibrations that are setup in the wheel are the result of both the force and the couple. When the unbalance weights are divided between the rims, the same force is maintained, but the couple is reduced, with the result the amplitudes of vibrations setup are reduced.

With dynamic unbalance only of 384 oz-in.-in. ( $8-8-180^{\circ}$ ) the maximum amplitude in bounce is 0.11" and the maximum amplitude in wobble is  $1.35^{\circ}$ . It is interesting to compare this to static unbalance with a single weight on the outside rim. For instance, 16-oz-in. static unbalance produces a maximum bounce of 0.11", and a maximum wobble of  $0.48^{\circ}$ . Thus in the bounce mode, dynamic unbalance only of 384 oz-in.-in and a static unbalance of 16-oz are statically equivalent whereas the wobble mode in the case of dynamic unbalance only is magnified nearly four times.

In the case of dynamic unbalance only, the resultant force vanishes, but the couple is considerable. Hence the wobble modes are magnified to a greater extent.

With each unbalance weight the amplitude of vibration increases with increase in speed up to resonance and decreases with high speeds. It is also observed that as the amount of unbalance is increased, vibrations begin to appear at decreasing speeds.

A static unbalance of 8-oz-in. on the outside rim is still unnoticeable to an observer, and vibration begins to be felt at about 16-oz-in. Whereas when the weights are divided equally between the rims, vibrations appear to be felt at 24 oz-in. With dynamic balance only vibrations begin to appear in the steering gear with 288-oz-in-in. unbalance. The speeds at which vibrations appear have been marked on the response diagram, Figs. 10, 12, 14.

Owing to the brief duration of the tests, it was difficult to estimate the speeds at which the disturbing effects of vibration began to disappear once the peak had been passed. For an estimate of this the test stretch has to be very long in order that persistence of feeling may be overcome. It must be emphasized that these tests have been conducted on a single automobile. An investigation of this nature should be carried out on a large scale and a statistical mean of both the response of the wheel to unbalance, as well as the physiological response of the observers to these vibrations should be recorded and computed. This investigation may be considered only as a first step towards such a goal.

Although it is only a preliminary investigation, the qualitative and quantitative data presented may be of interest to those in the field of wheel balancing generally. The following



conclusions are drawn from this investigation.

- (1) There is a clearly defined neighborhood of resonance in the frequency range of 13.5-14.5 c.p.s. (67-72 miles per hour).
- (2) For static unbalance peak amplitudes are attained at resonance at a frequency of 13.5 c.p.s. (67 miles per hour). For dynamic unbalance peak amplitudes are attained at resonance at a frequency of 14.5 c.p.s. (72 miles per hour).
- (3) Unbalance excites two modes of vibration, bounce and wobble, simultaneously. Both bounce and wobble resonate at the same frequency.
- (4) The amplitudes of vibration increase as the amount of unbalance increases for the same speed.
- (5) As the amount of unbalance is increased, the vibrations begin to appear at decreasing speeds.
- (6) The threshold of vibration consciousness with static unbalance is about 16-oz-in., and with dynamic unbalance only, 288-oz-in.-in.

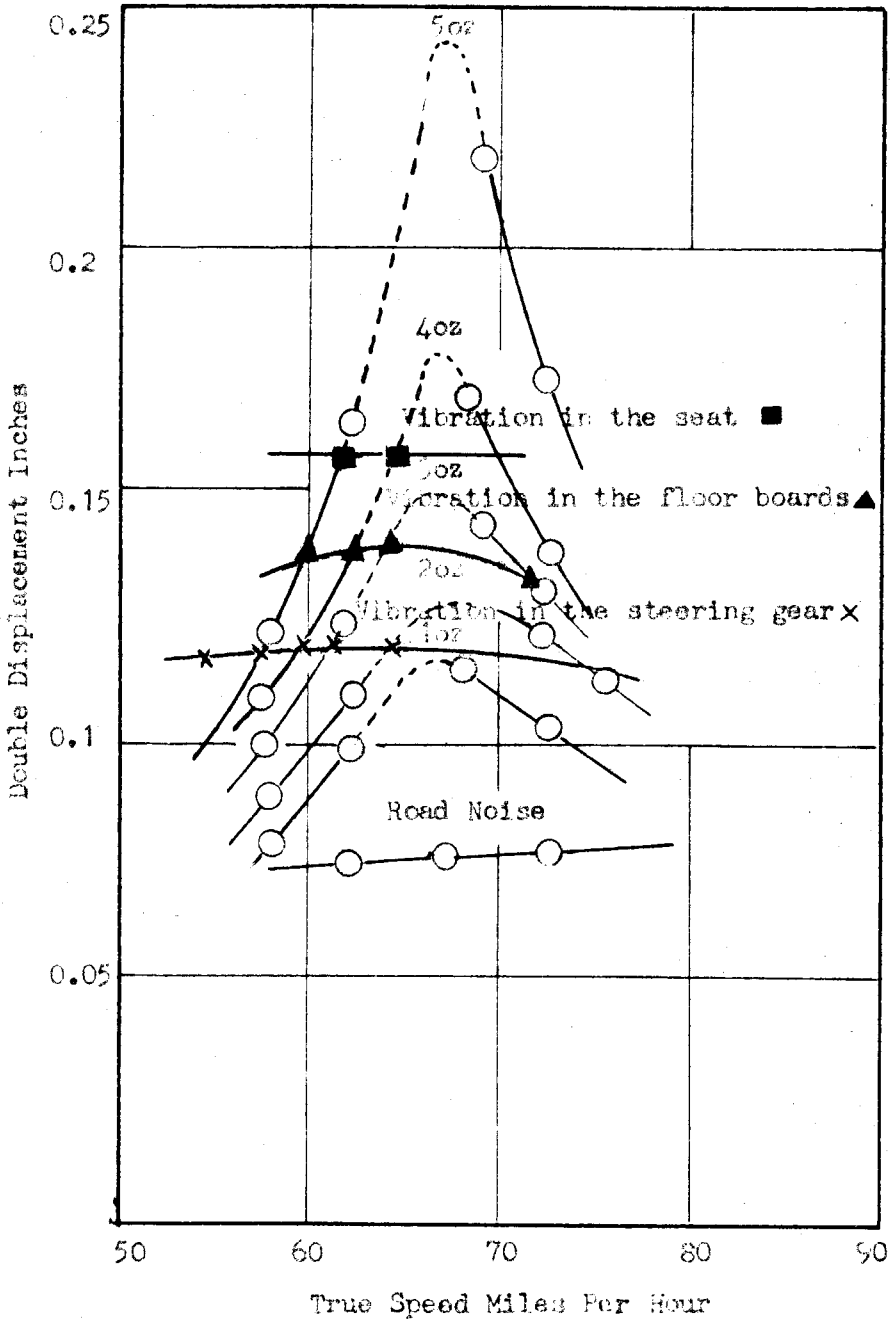


Fig. 10: 'Bounce' response to static unbalance.  
Single unbalance weight on the outside rim.

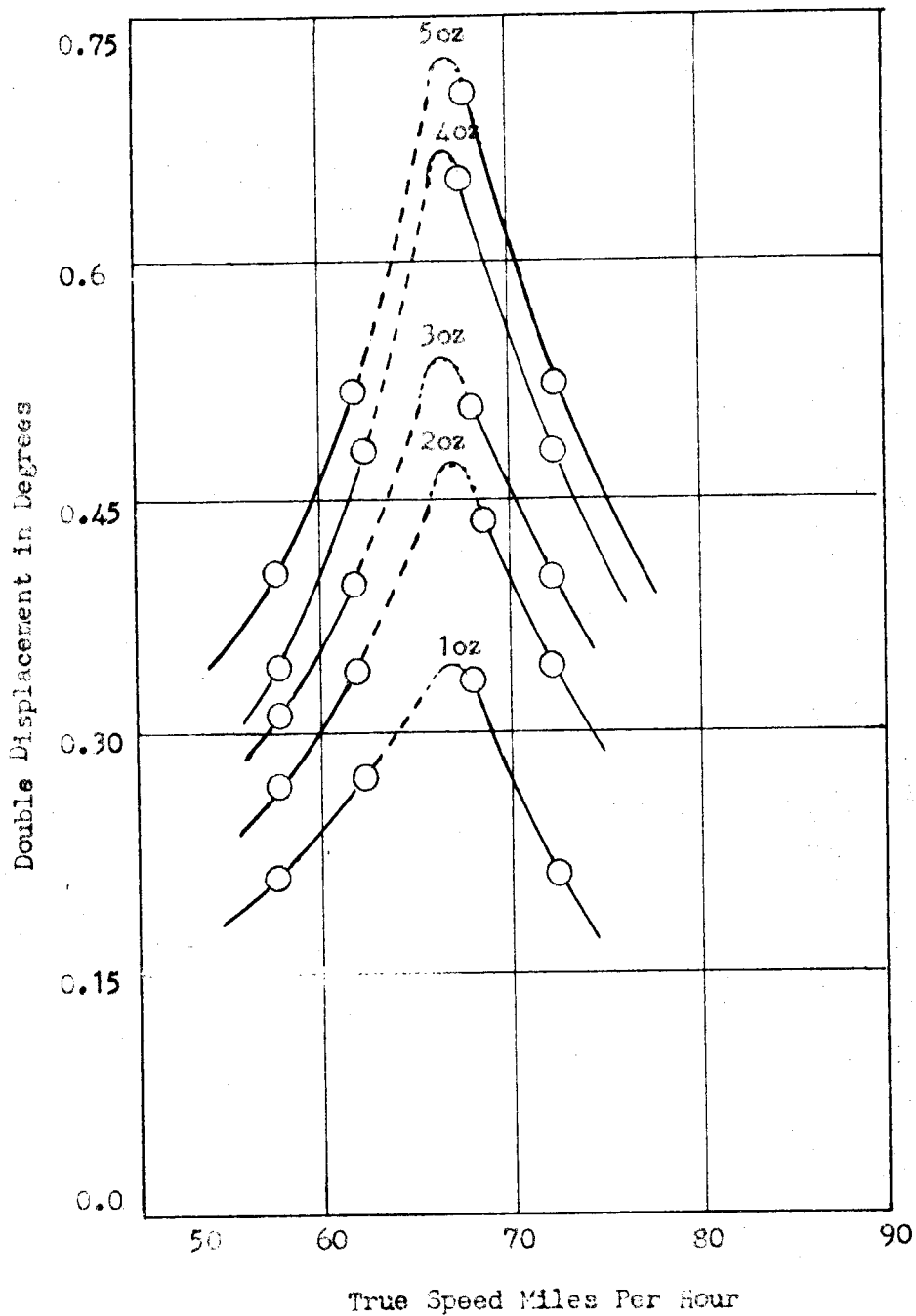


Fig. 11: 'Wobble' response to static unbalance.  
Single unbalance weight on the outside rim.

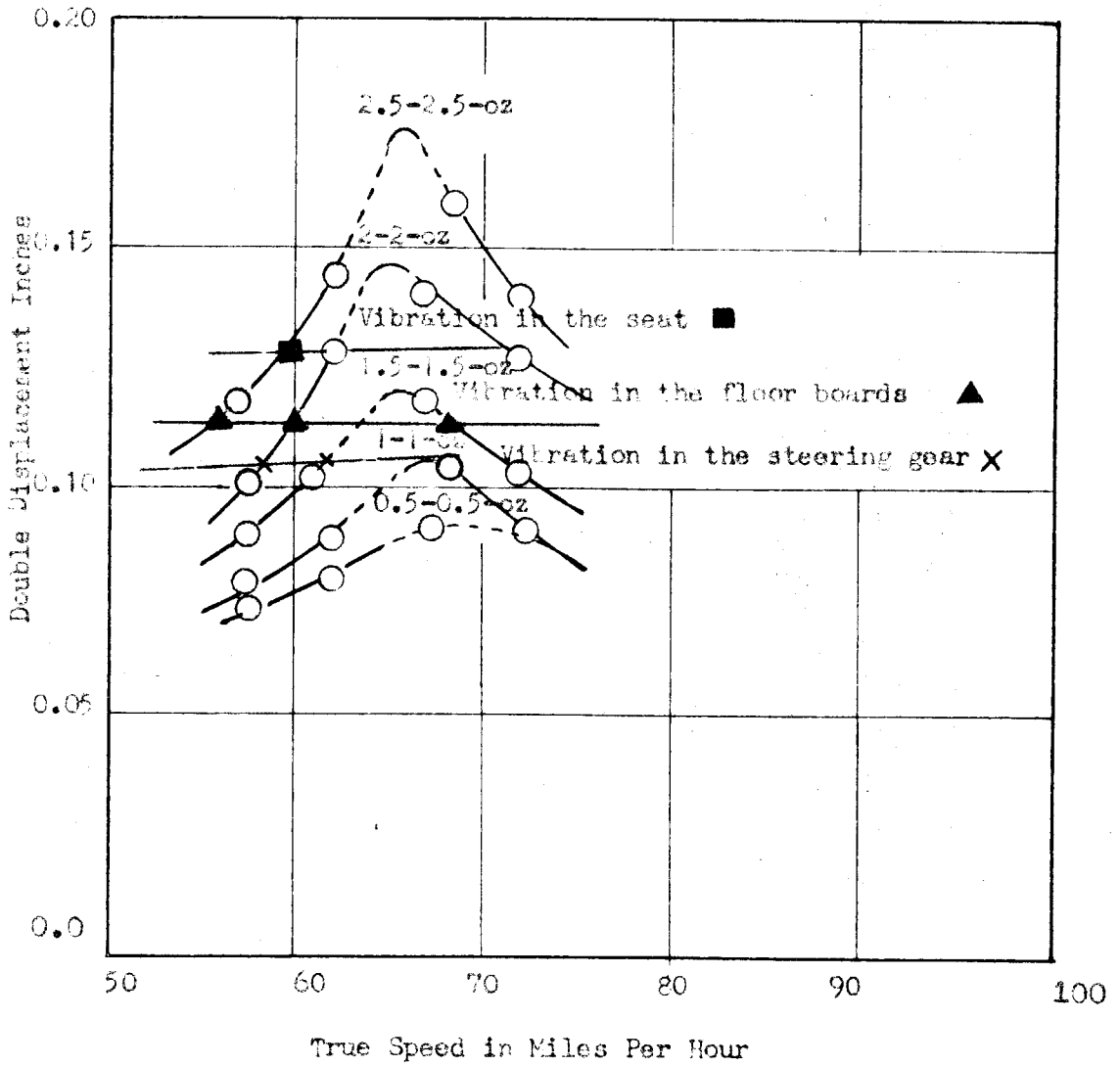


Fig. 12: 'Bounce' response to static unbalance. Unbalance weight equally divided between the inside and outside rims (in phase).

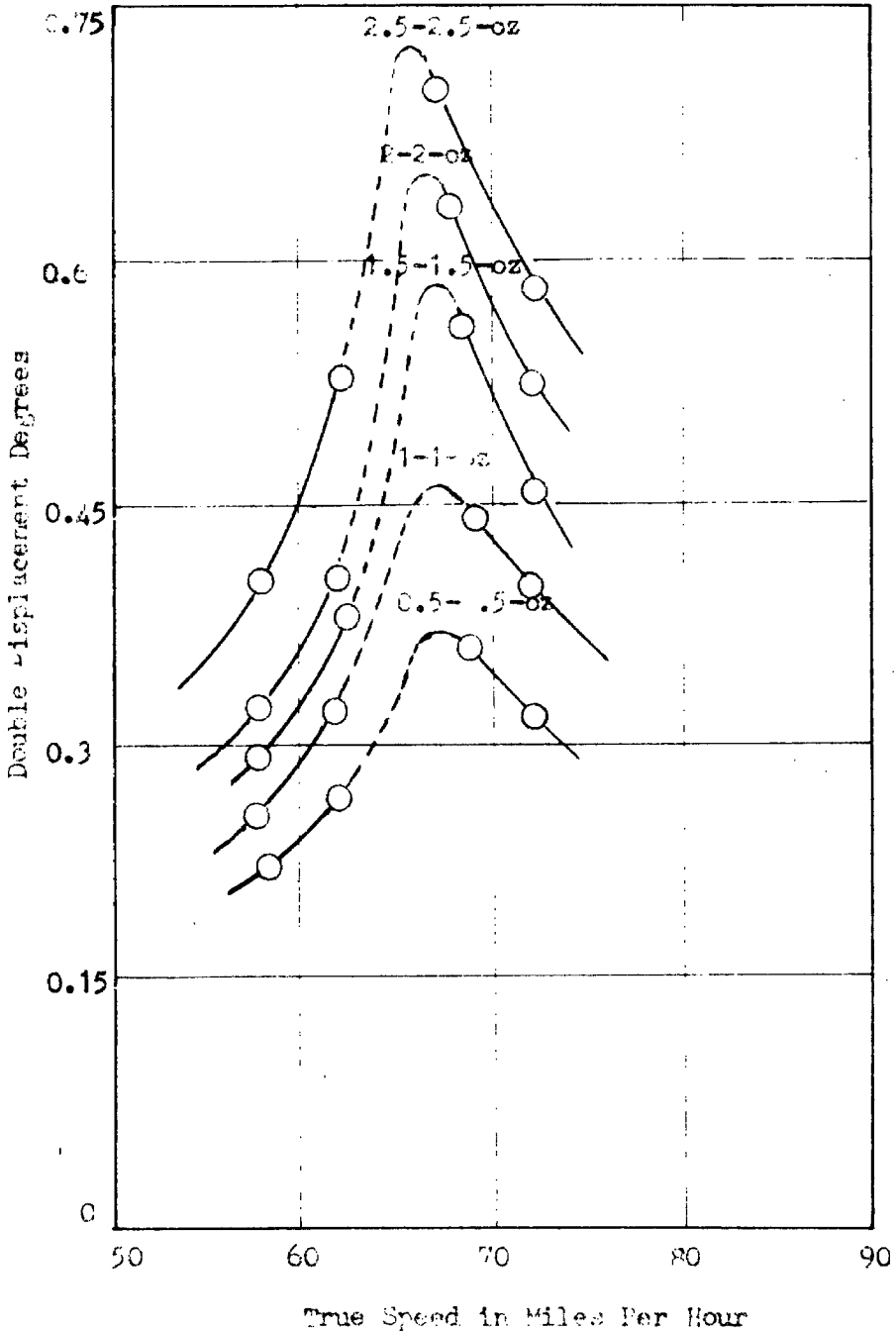


Fig. 13: 'Wobble' response to static unbalance. Unbalance weight equally divided between the inside and outside rims (in phase).

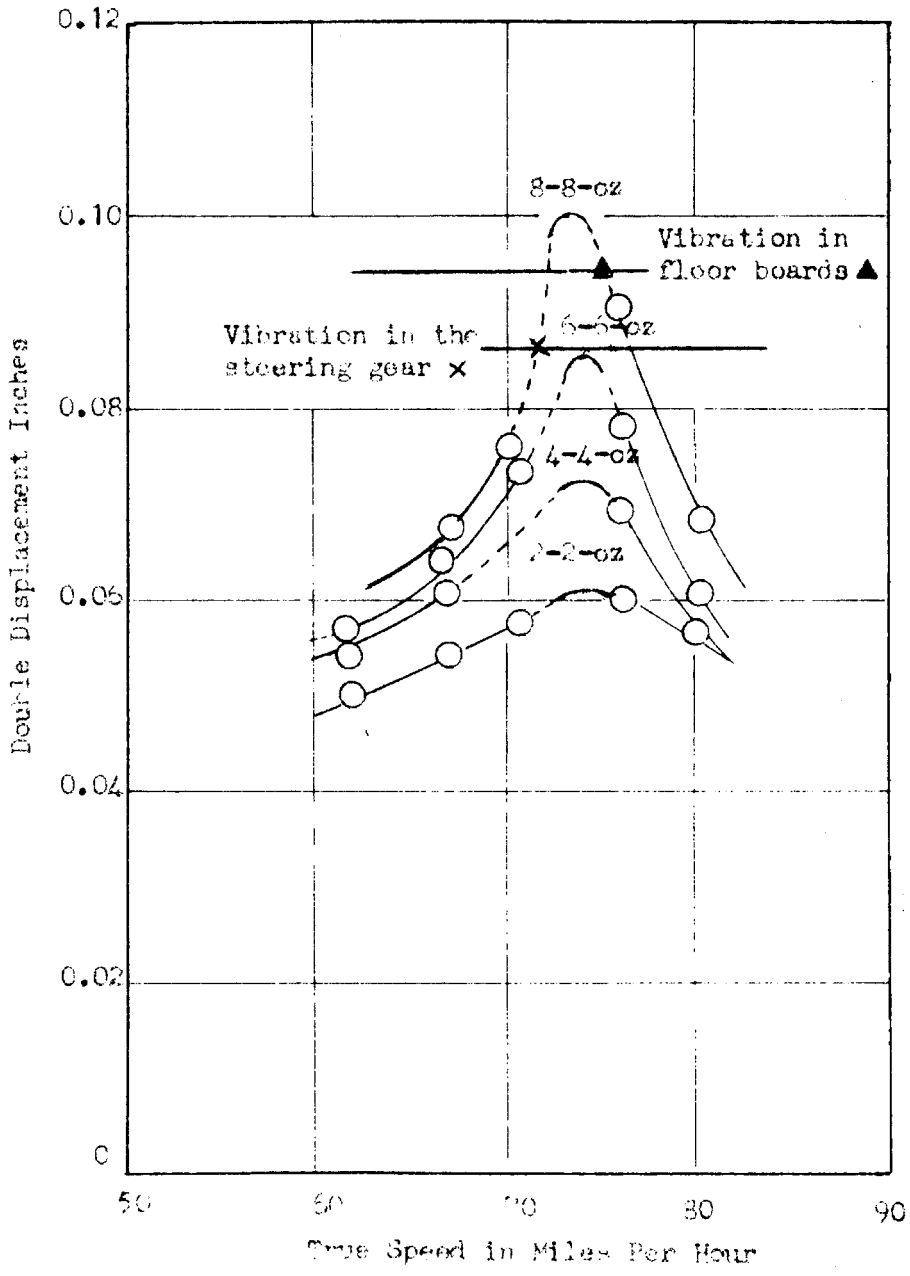


Fig. 14: 'Source' response to dynamic unbalance only.

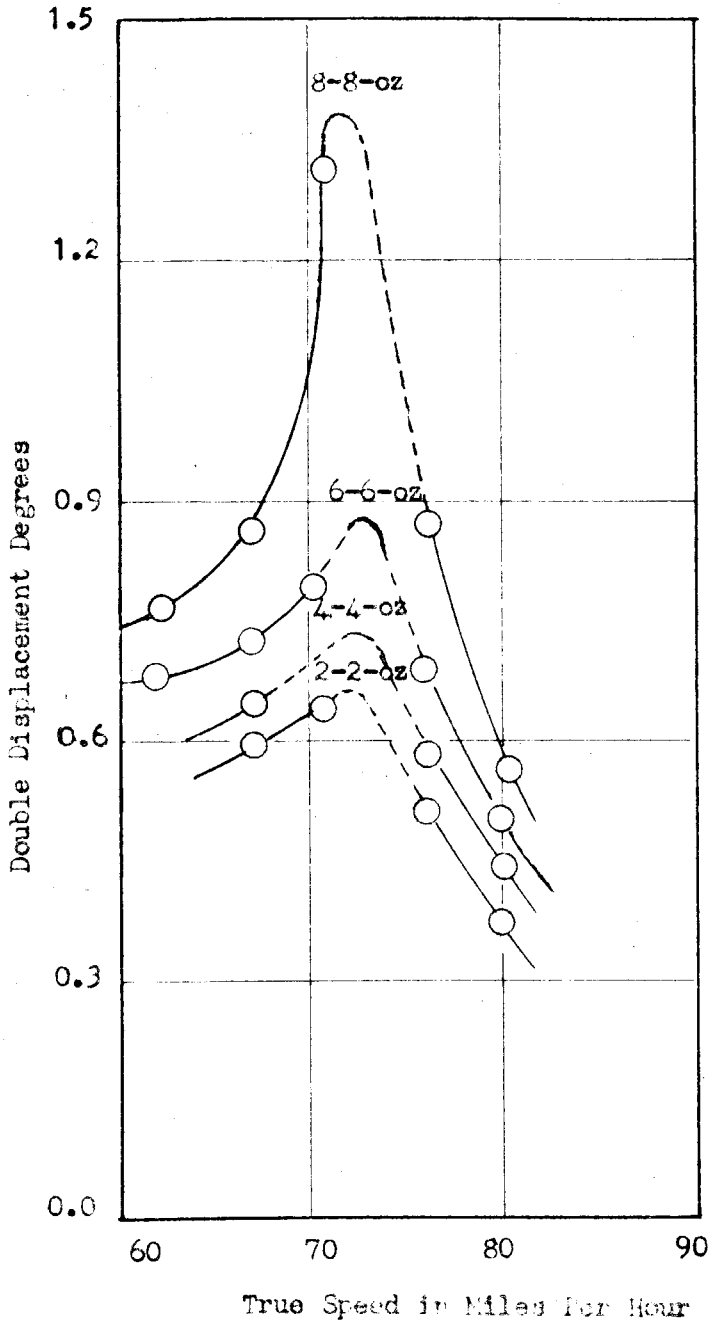


Fig. 15: 'Wobble' response to dynamic unbalance only.

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2. E. Marquard, Schwingungsdynamik des schnellen strassenfahrzeugs (Vibration Dynamics of the Fast Road-Vehicle), Chapter 10, Verlag W. Girardel-Essen 1952.



APPENDIX I

Instruments

A. Instruments used for vibration measurements.

Description	Manufacturer	Model and serial No.	Dynamic Lab No.
1. Two channel direct inking oscillograph	Brush Development Co.	BL-202 # 3027	180
2. Amplifier	Brush Development Co.	BL-905 # 1381	163
3. Amplifier	Brush Development Co.	BL-905 # 1382	164
4. Velocity pickup	Consolidated Eng. Corp.	4-102-A # 287E	202
5. Velocity pickup	Consolidated Eng. Corp.	4-102 # 280E	203
6. Vibration meter	Consolidated Eng. Corp.	1-110B # 807D	129
7. Vibration meter	Consolidated Eng. Corp.	1-110B # 9325	245
8. Inverter	ATR Mfg. Co. Inc.	12HSF-type 12 #1152296	191

B. Instruments used in the laboratory for the calibration of the velocity pickups.

Description	Manufacturer	Model and serial No.	Dynamic Lab No.
1. Vibration calibrator-exciter	MB Mfg. Co.	C1ES # 183	229
2. Power supply	MB Mfg. Co.	T112031 # 184	230
3. Capacitor box	MB Mfg. Co.	B1 # 111	231

Description	Manufacturer	Model and serial No.	Dynamic Lab No.
4. Telemicroscope	Gaertner Sci. Corp.	M204	239
5. Universal counter and timer	Berkeley	5510	243

APPENDIX II. CALIBRATION OF VELOCITY PICKUPS.

The calibrator consists of a magnetically driven mechanical oscillator which produces a vertical sinusoidal motion of the table of adjustable amplitude and frequency. The oscillator was first calibrated against a Berkeley universal counter.

The velocity pickup was rigidly attached to the calibrator table and the table was set in motion. The electrical output of the pickup was connected to a vibration meter. An integrating circuit in the meter converted the voltage to read double displacement directly on the dial. This integrated signal was also connected to a Brush oscillograph in order that the wave form could be observed.

The amplitudes of table motion were observed optically through a telescope. A small piece of #320 grit emery cloth was cemented to the edge of the vibration pickup and a beam of light was directed normally to the plane of the emery cloth. The reflected light from a single emery particle was observed as a vertical line of finite length when the table was in motion. Selecting a single reflecting source for brightness and smallness, the displacement was read in a telescope by noting the extremes of the line on the graduated scale in the eyepiece.

In order to convert the displacements observed in the telescope to actual displacement, the telescope was calibrated by means of a dial gage. The calibrator table on which the pickup is mounted was raised by statically loading it and observing the displacement in the dial gage as well as in the microscope scale.

The vibration meter was calibrated for both (0-150) and (0-500) meter factors for frequencies from 7 to 20 c.p.s. At each frequency the amplitudes were varied from zero to maximum. At the same time the table motion was recorded in the Brush oscillograph and the oscillograph readings were similarly calibrated against the optically observed displacements. All oscillograph readings were conducted at (1) meter factor and (full) gain. The calibration curves are shown in Figs. 16-19.

The method of calibrating the two pickups for measuring wobble was exactly similar except that the orientation of the two pickups on two opposite sides of the calibrator table was horizontal. The outputs of the two pickups were connected in series opposed in a junction box and the combined output was connected to a second vibration meter. The output of the two vibration meters was, in turn, connected to the second channel of the Brush recorder.

The schematic arrangement of the setup is shown in Fig. 1 and the calibration curves are given in Figs. 20-23.

The reason that the meter reading is approximately twice the observed displacement in Figs. 20 and 21 is because of the fact that the two pickups are connected in series. Hence their respective signals are cumulative.

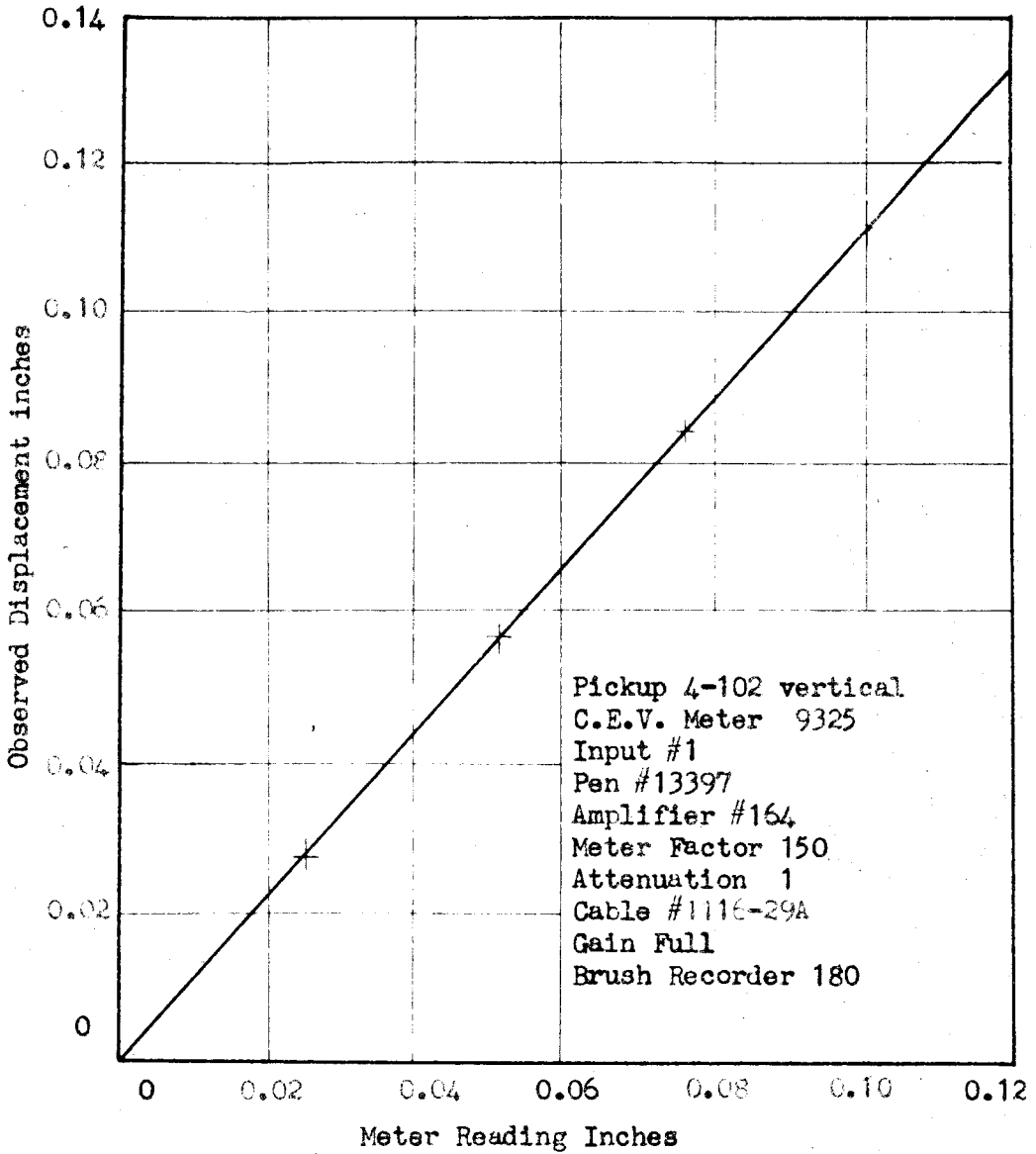


Fig. 16: Vibration Meter I Calibration Curve (Range 0-.15 inches)

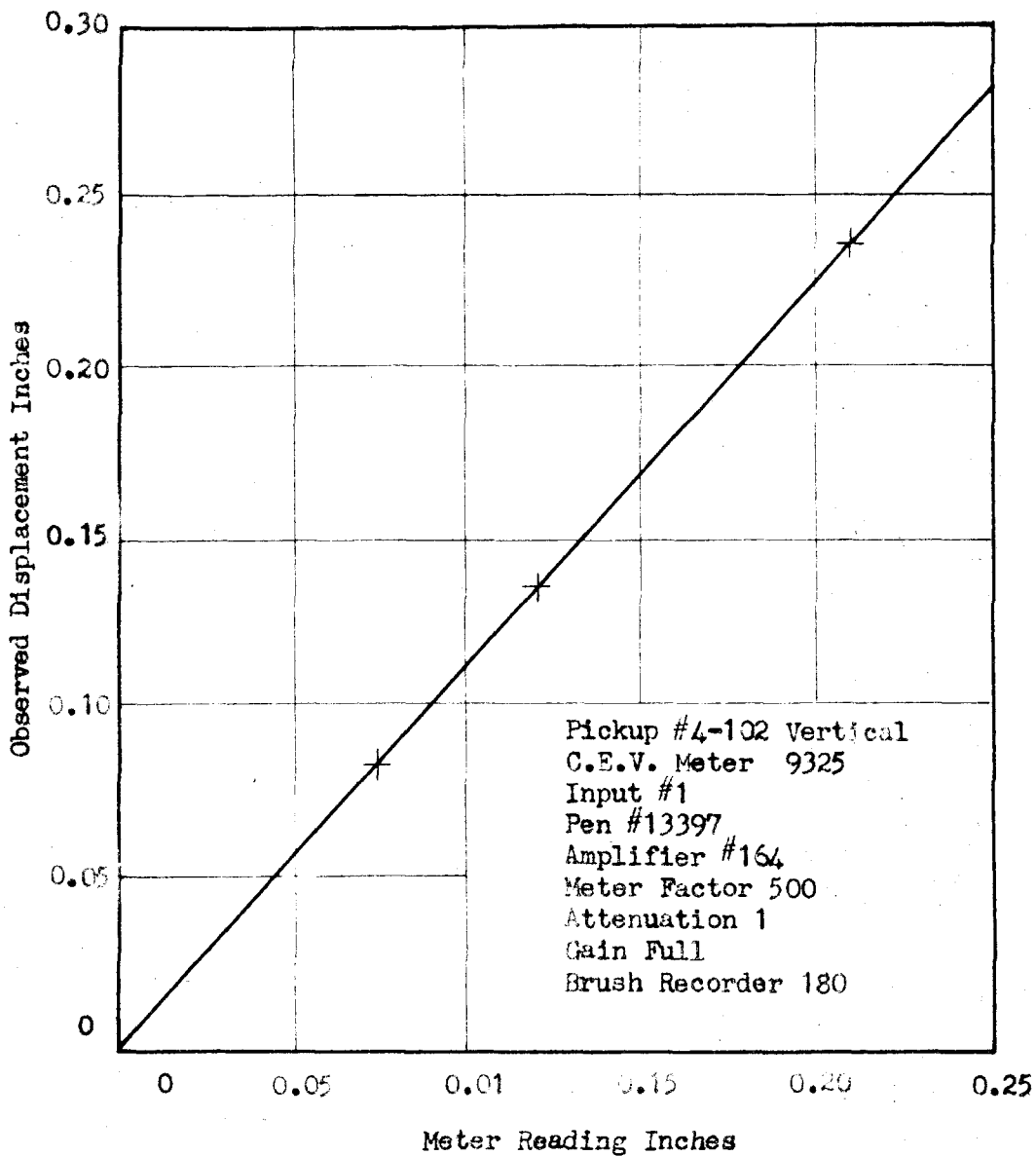


Fig. 17: Vibration Meter I Calibration Curve  
(Range 0-0.5 inches)

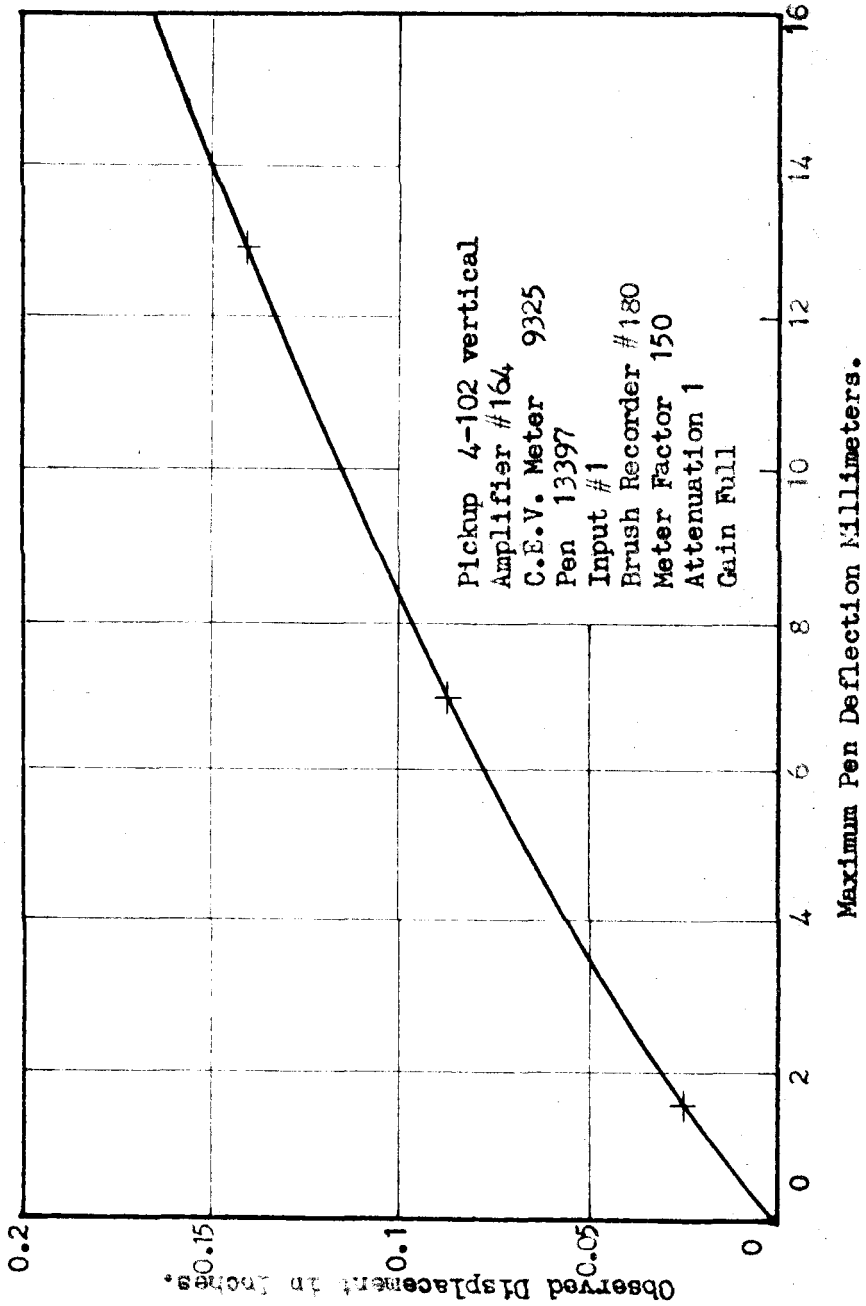


Fig. 18: Brush Recorder---(Channel I) Calibration Curve.

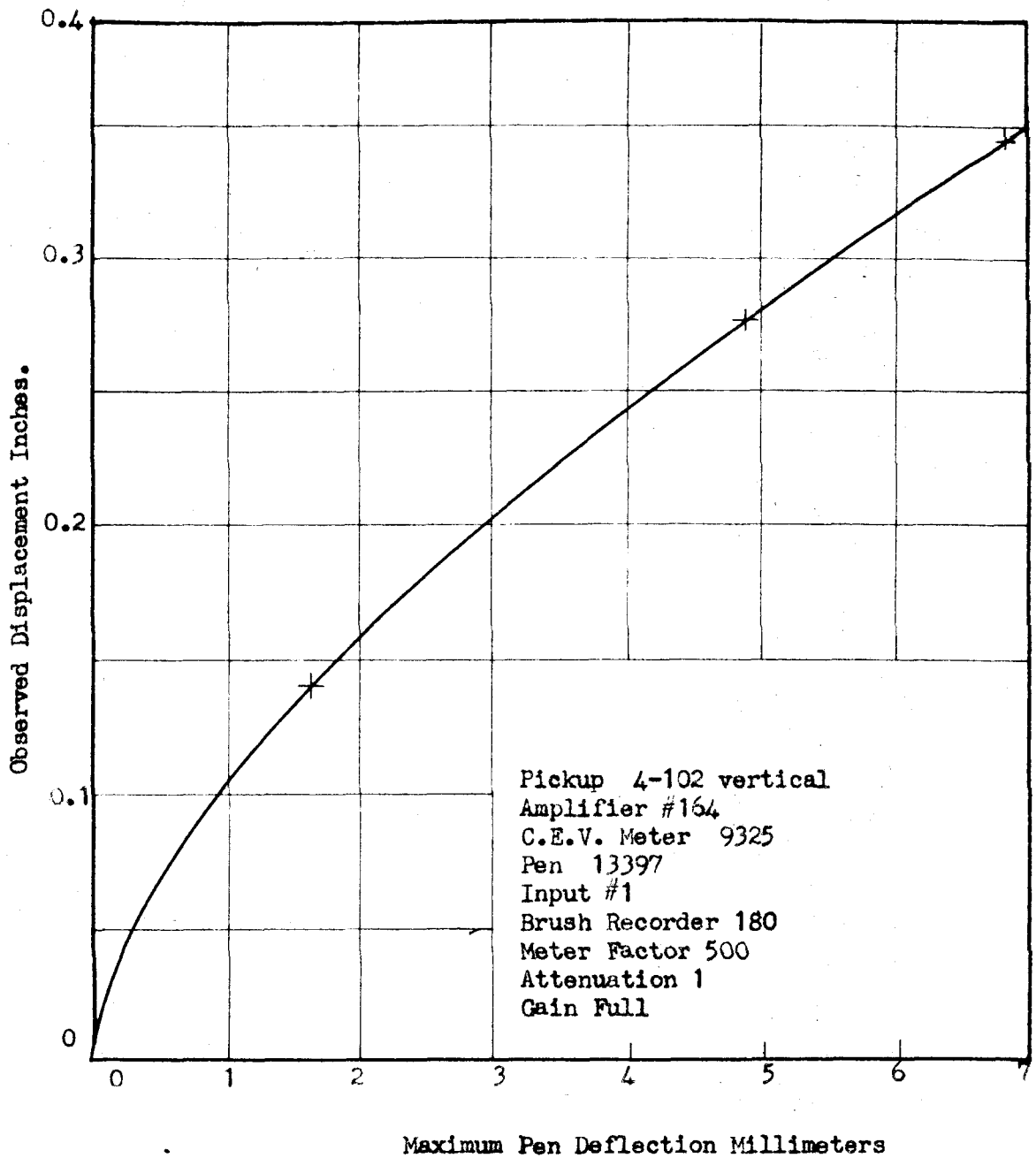


Fig. 19: Brush Recorder (Channel I) Calibration Curve.



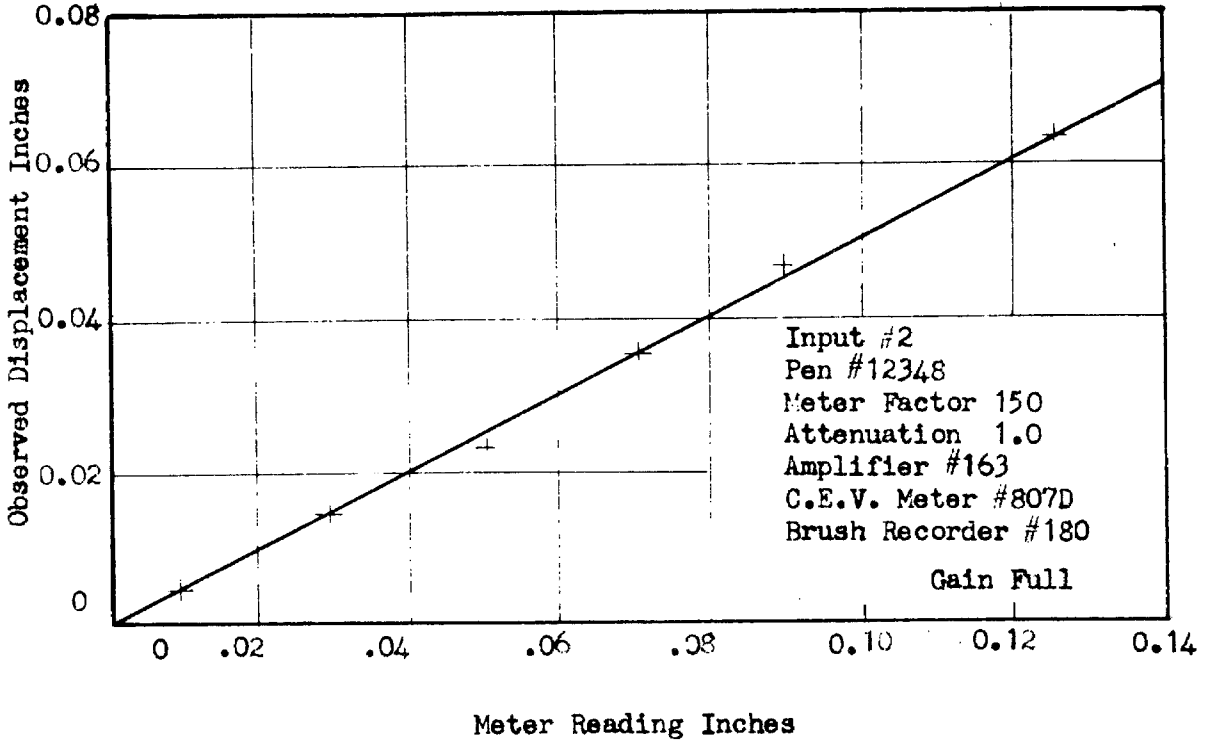


Fig. 20: Vibration Meter II—calibration curve (Range 0-.15 inches)

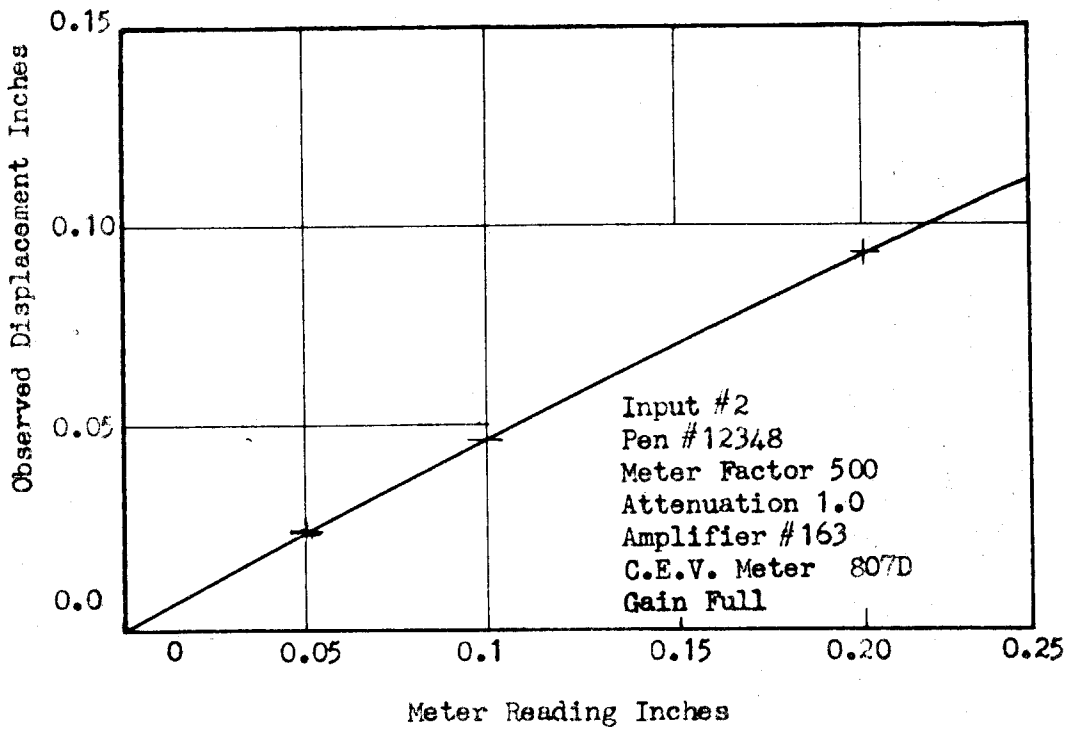


Fig. 21: Vibration meter II--calibration curve (Range 0-.5 inches)

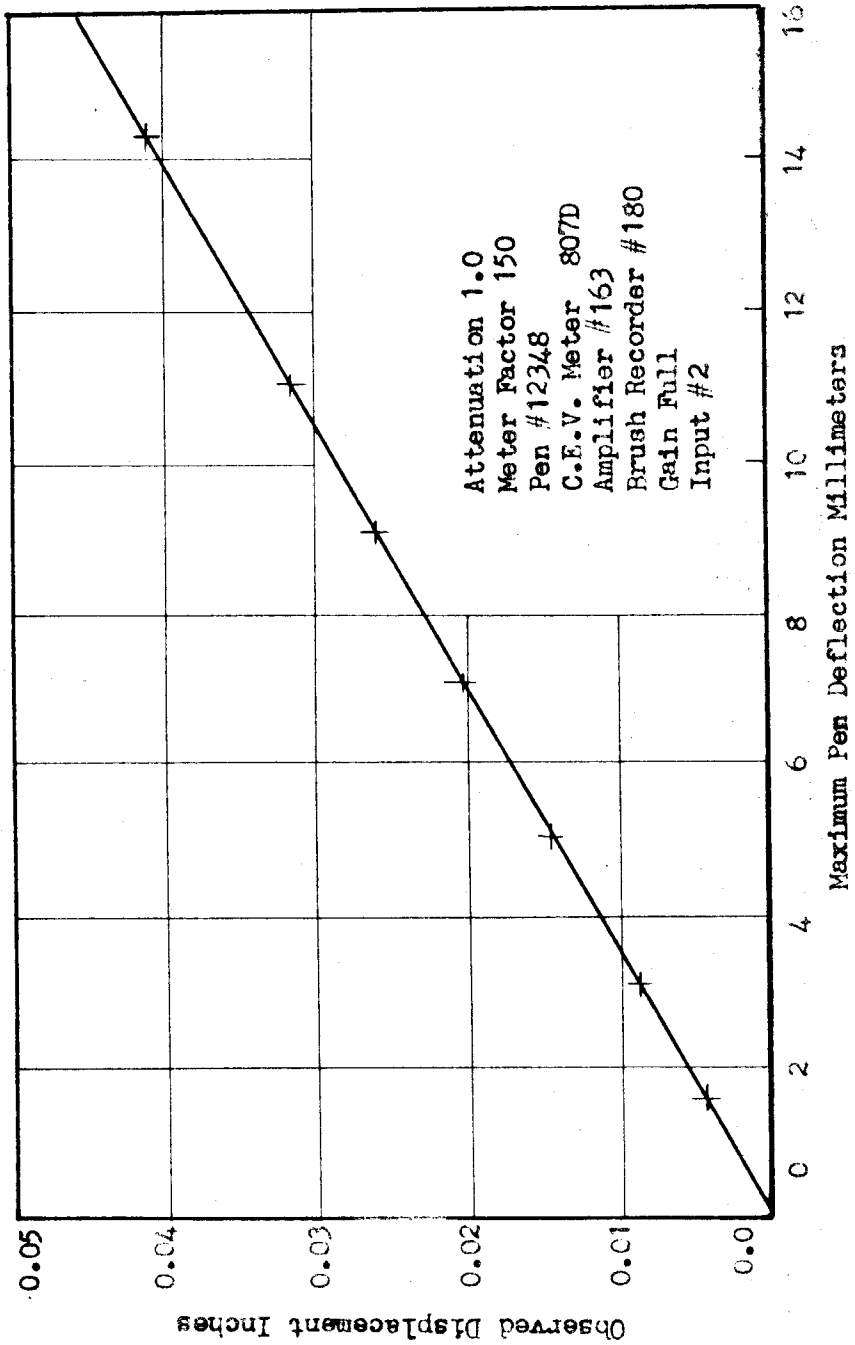


Fig. 22: Brush recorder (Channel II)---calibration curve.

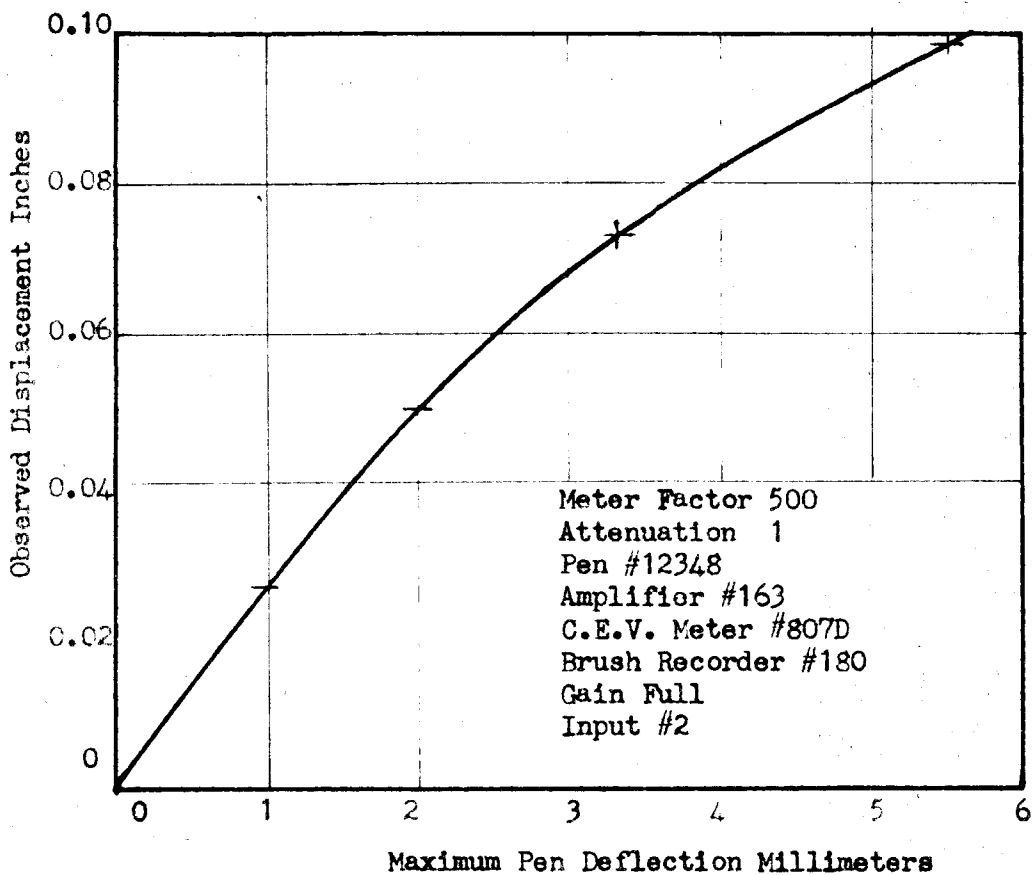


Fig. 23: Brush recorder (Channel II)—Calibration curve

### APPENDIX III

#### Gyroscopic Coupling.

Let the fig. 2<sup>4</sup> represent an automobile wheel. The spin vector points outward. Suppose an impulse is imparted to the wheel in the upward direction tending to push the spin vector upward, there results a wobble about the king pin in a direction opposite to the change of angular momentum  $dB$  so that the wheel deviates from its path towards the right. This is because the displacement of the spin axis in the impulse direction can only take place if it were forced by an external counterclock wise torque in a plane normal to the direction. Since this external moment is lacking the spin axis escapes in the direction opposite to  $dB$ . This effect is referred to as Gyroscopic effect.

Wobbling in turn effects a change in the spin direction and this time in the horizontal plane which causes a deviation in the plane of the vertical oscillation. Thus wobble has a reaction on the vertical motion and vice versa and the two motions wobble and bounce are said to be coupled.

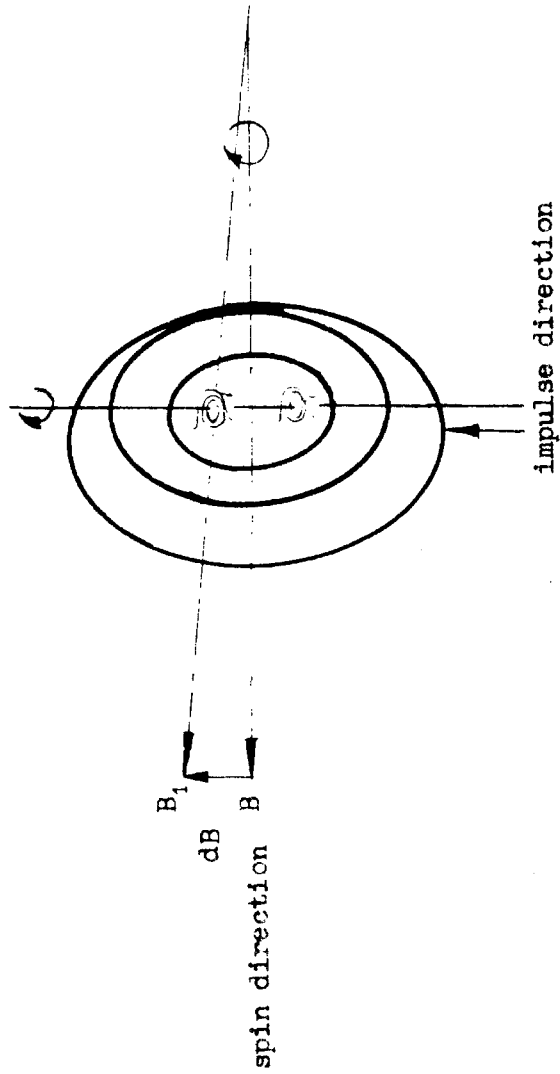


Fig. 24: Gyroscopic coupling in the front wheel.

#### APPENDIX IV

Typical oscillographic records obtained for bounce and wobble with various kinds of unbalances are shown in Figs 25-28. The procedure for reducing the Brush record reading to double amplitude in bounce and wobble respectively is indicated for one frequency of Fig 27.

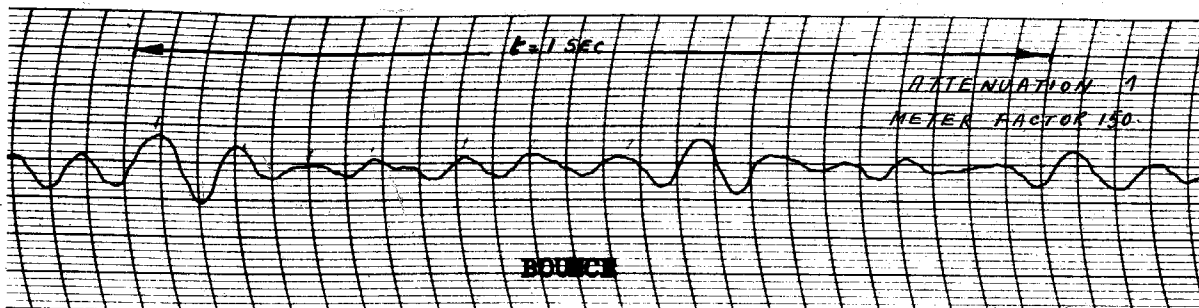
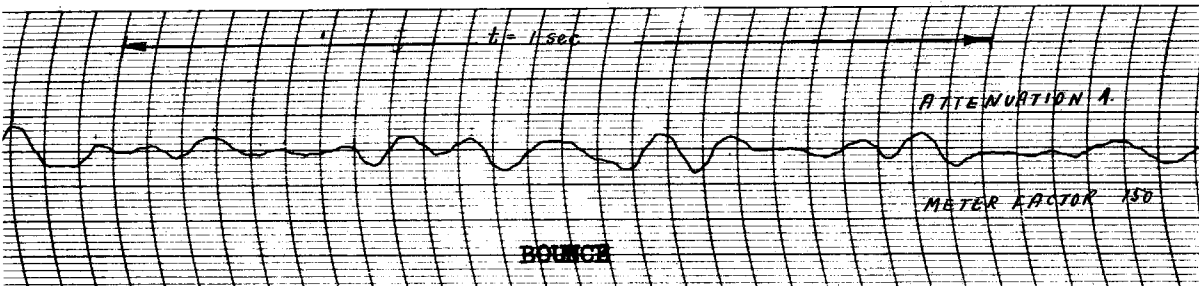
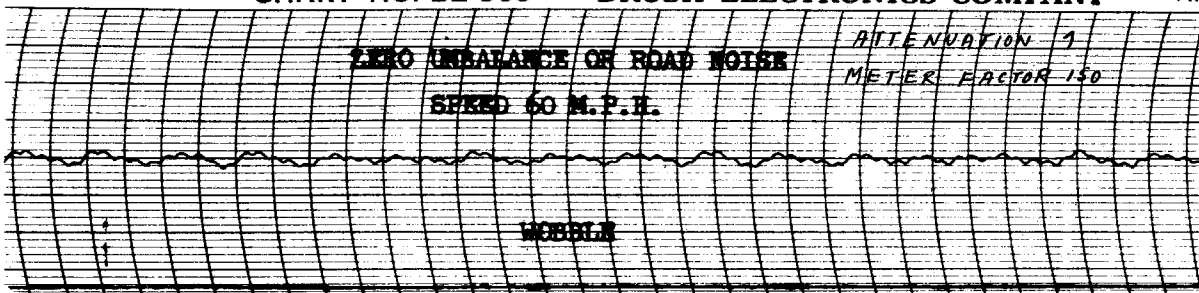


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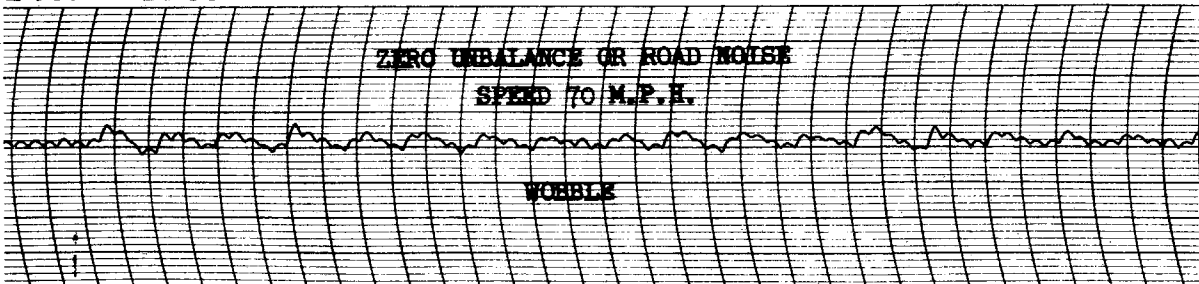


Figure 25.



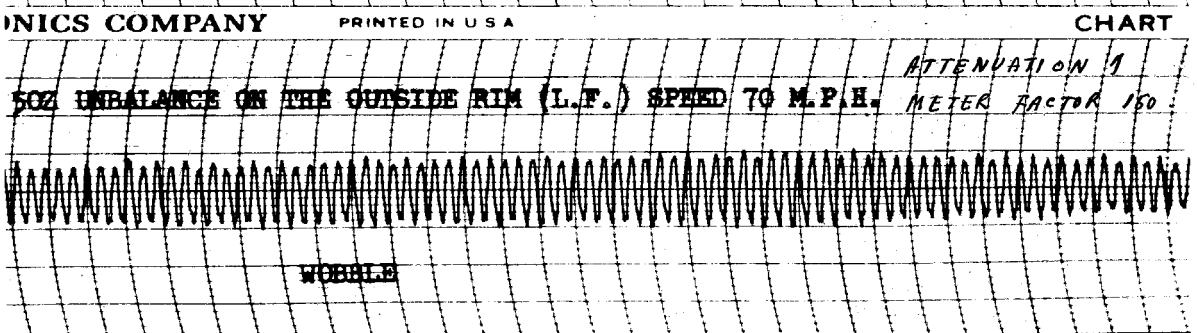
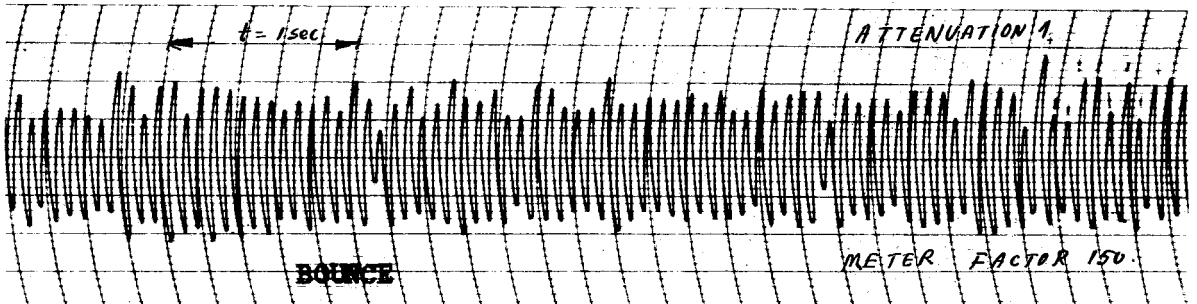
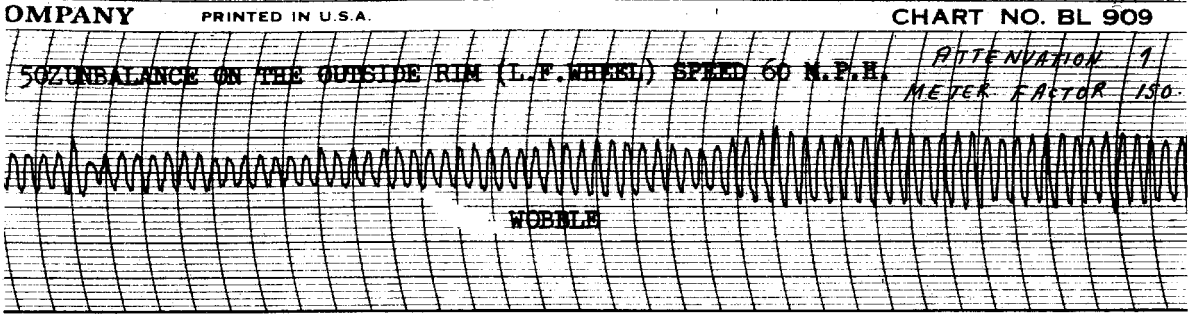
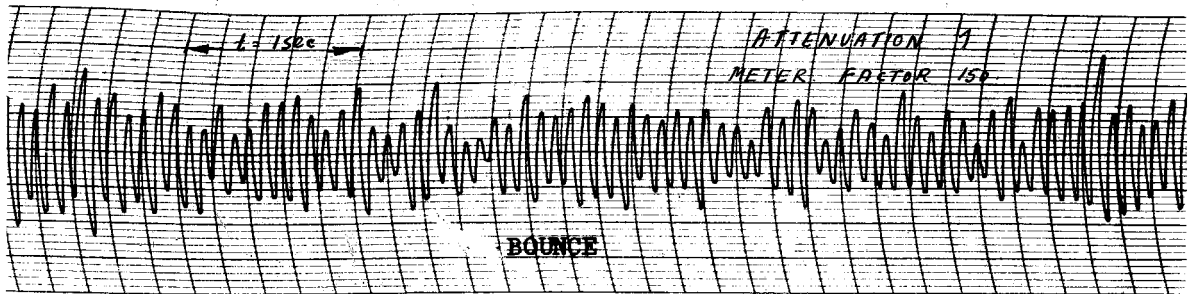


Figure 26.



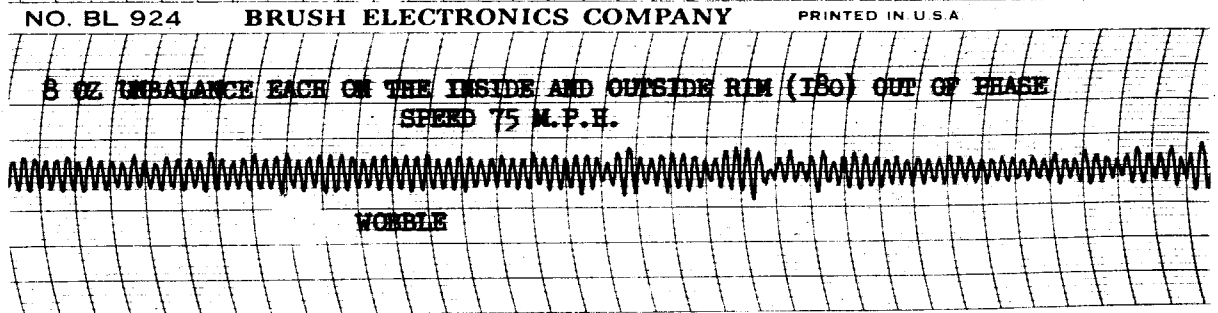
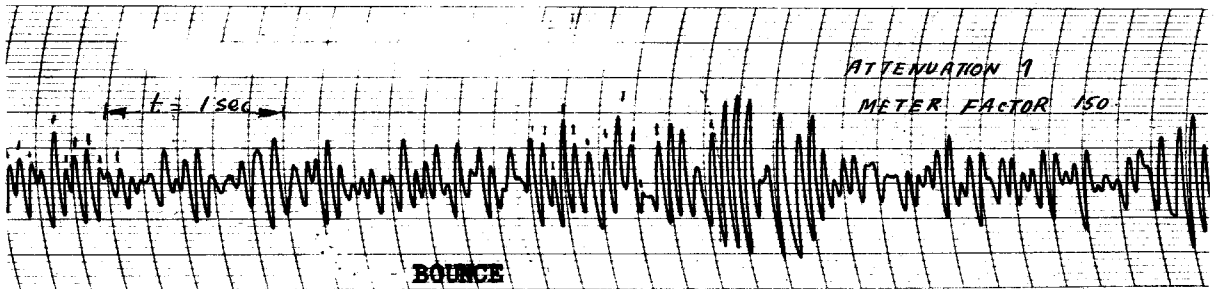
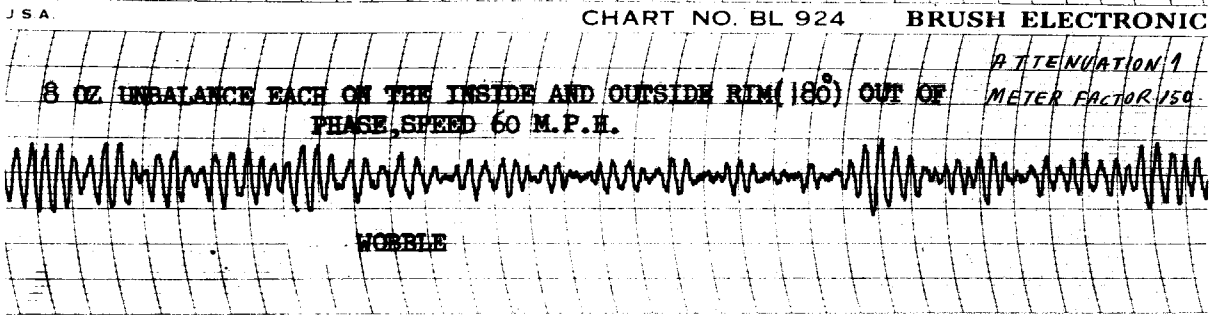
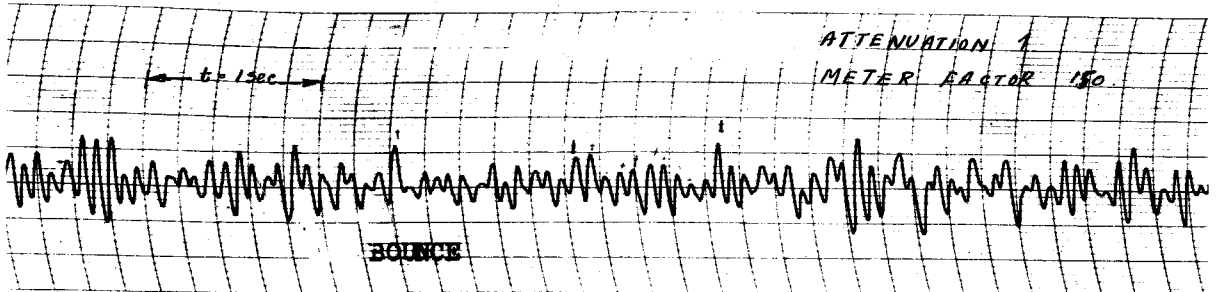


Figure 28.

An example of reducing the Brush record into double displacement is shown for the case of static unbalance of 2.5 oz each on the inside and outside rims (in phase) at a speed of 60 m.p.h. (Fig. 27, top).

Although the test track is assumed perfectly smooth and flat there are always irregularities on the road as seen from the Road noise diagram Fig. 25. Hence to compute the double amplitude judgement was used to select uniform intervals over the trace and a statistical average of the pen deflections at these intervals (judiciously selected on the record) was used as a criterion. For instance, in the upper record (Fig. 27) the maximum pen deflections in mm for the interval I are 9.0, 8.5, 7.5, 8.0, 8.0, 11.0, 7.0, 10.5, 7.5, 7.5 which average to 8.4. For the interval II maximum pen deflections in mm 12, 6, 8, 8, 9, 12, 6, 7, 7, 9 which average to 8.3. The mean for these two intervals is 8.35 mm. From the Brush record calibration curve (Fig. 18) the double displacement corresponding to 8.35 mm pen deflection is 0.098 in.

In the case of wobble, pronounced beats of very low frequency occur. The criterion adopted in reducing the oscillographic record to double displacement was to average out the pen deflection in the neighborhood of the maxima. Again judgement was used to select a uniform trace on the record for this purpose. For the interval III on the wobble trace of Fig. 27 the pen deflections in mm are 6.0, 5.5, 5.5, 4.0, 6.0, 4.0, 4.5, 5.0, 5.0, 5.0 which averages to 5.05. At a neighboring interval IV the

pen deflections are 5.0, 5.0, 5.0, 5.0, 5.0, 6.0, 5.0, 5.0, 5.0 which average to 5.1. The mean of these is 5.075.

From the calibration curve (Fig.22 ) the double displacement corresponding to 5.075 is .03 in. This represents the arc displacement  $r\theta$  where  $r$  is the distance of one horizontal pickup from the king pin center line, and  $\theta$  is the angular displacement. The distance  $r$  was 4.9 in. Therefore double displacement in degrees

$$= \left[ \frac{0.03}{4.9} \times \frac{180}{\pi} \right] = 0.354 \text{ degrees.}$$