MESON THEORETICAL ORIGINS OF THE NON-STATIC TWO NUCLEON POTENTIAL

Thesis by

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ABSTRACT

We discuss the origins, in meson theory, of a nuclear potential which will adequately describe the interactions of nucleons up to energies of approximately 150 Mev. Our Hamiltonian is obtained from a non-relativistic reduction of the relativistic pseudo-scalar theory. The coefficients of all terms in the Hamiltonian are treated as parameters whose values are determined from experiment. In computing the actual potential we follow precisely the philosophy of Breuckner and Watson and Gartenhaus. The resulting potential is an expansion in powers of the coupling constant and the ratio of the velocity of the nucleons to that of light. Only the second and fourth order terms in the coupling constant and the zeroth and first order term in $(v/c)_{nucleon}$ will be retained. All pieces of the potential are evaluated numerically with a smooth momentum space cutoff. We also consider a modified type of relativistic theory where pair terms are supressed. A crude comparison is made with experiment. It is found that the potential is in qualitative agreement with the experimental data on nucleon-nucleon scattering, the splitting of the levels in He_5, and the observed magnetic moment of the deuteron.

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I. INTRODUCTION

In this thesis we propose to discuss the origins in meson theory, of a nuclear potential, which will adequately describe the interactions of nucleons up to energies of approximately 150 Mev. A complete understanding of the nucleon-nucleon potential depends upon a correct description of the interaction of nucleons with π-mesons and with the host of recently discovered metastable hyperons and K particles. Such a description is of course not available. Our attitude will be to forget the heavier mesons and consider, within the framework of the Yukawa theory, only the π-meson nucleon interaction. The Yukawa theory itself was originally conceived to explain the force between nucleons, the meson playing a role with respect to the nucleon which is analogous to that played by the photon with respect to the electron. The exchange of photons gives rise to the electromagnetic forces between charged particles while the exchange of mesons gives rise to the shorter range nuclear force.

The low energy (up to about 20 Mev) properties of the two nucleon potential have been adequately described in terms of a fixed source meson theory. (We will only touch on these lightly and refer the reader to the numerous articles on the subject.) The potential at moderate energies is not so easily understood. The fine structure splittings of the levels in the nucleus have prompted many people to propose that a velocity dependent spin orbit term be included in the two nucleon potential. Recently Signell and Marshak[1] have shown that a good fit of the unpolarized and polarized two nucleon elastic scattering data up to 150 Mev may be obtained by adding an empirical charge independent short range attractive spin orbit potential to the Gartenhaus potential.[2] The so-called Gartenhaus potential has a plausible meson theoretic basis. More precisely Gartenhaus used the non-relativistic, P wave extended source Hamiltonian. He calculated in perturbation theory keeping terms to fourth order in the coupling constant. However, he omitted the so-called "ladder corrections" dropping them with the aid of "Bruckner and Watson's" argument.[3] These terms lead, as is well known, to an unbound deuteron.

The Gartenhaus meson theoretic potential gives a good fit to all of the low energy two nucleon data. This is both encouraging and surprising since the use of perturbation theory in meson theory calculations has dubious validity. Presumably the inclusion of a renormalized coupling constant $f^2 \approx 0.09$ and a cutoff energy $\omega_{max} \approx 6\mu$,[4] which are determined from real meson nucleon scattering and photoproduction at low energies, takes into account certain higher order effects. The hope, of course, is that the higher order effects which are not included in this manner modify only the high energy, short range, behavior of the potential. The higher the order of the graph the shorter the range of the resulting potential. In so far as we are concerned with large distances, the order of the meson compton wavelength, the fourth order potential should be a reasonable approximation to the actual potential.

It is interesting to investigate the (first order) nucleon velocity dependent terms, in meson theory to see if a plausible explanation for the spin orbit term can be given. We will follow precisely the philosophy of Brueckner and Watson (B.W.) and Gartenhaus (G) in deriving these terms. Our Hamiltonian will be a non-relativistic reduction of the (PS) (PS) theory. The potential itself will then be computed, using non-relativistic perturbation theory. As in the case of G we drop all so-called "ladder terms." The (PS) (PS) theory in this manner leads to a spin orbit potential of the type postulated by Signell and Marshak (S.M.). Our results will be an expansion in powers of the coupling constant and the ratio of the velocity of the nucleons to that of light. We shall assume that $(v/c)_{nucleon}$ μ/M. We shall keep only the second and fourth order terms in the coupling constant. Only the zeroth order term in μ/M for the instantaneous potential and the first order term in μ/M for the L•S potential will be retained.

Other treatments of this problem have been given: Sato, Itabashi and Sato,[5] Klein,[6] and Marshak and Okubo.[7] All have used a modified type of (PS) (PV) theory. These treatments only include a computation of the L•S pieces of the potential generated by the additional terms in the coupling Hamiltonian. By additional we mean terms other than the usual $\sigma\cdot\nabla\phi$ coupling used by Gartenhaus. In addition it may be pointed out that these earlier treatments concerned themselves only with cutoff independent Hamiltonians. As a result the final potentials in position space are extremely singular at the origin. In order to avoid this singularity the potentials are crudely cut to zero at an arbitrary cutoff distance.

We have, in addition to computing the L•S pieces of the potential, computed the corrections to the Gartenhaus static potential. We have done this for both the cutoff independent and cutoff dependent Hamiltonians. For the case of the cutoff independent results our analytic forms for the potential are consistent with the results of the previous investigators.

In sections I we give a precise definition of the potential in terms of the S matrix scattering element for free particle scattering. In section II, we reduce the relativistic γ_5 theory to non-relativistic form. In section III, we compute the actual matrix elements, and potentials predicted by the non-relativistic theory. Section IV contains a brief discussion of a modified relativistic theory and section V concludes with a discussion of the relationship of the computed potentials to the available experimental data.

II. DEFINITION OF A POTENTIAL

We begin by mentioning first a few of the conventions to be used in this thesis. We use the natural system of units: $\hbar = c = 1$. The dot product of two four vectors k_μ is $k_\mu \cdot k_\mu' = -\mathbf{k}\cdot\mathbf{k}' + k_4 k_4$. The Dirac γ matrices are given by

$$\vec{\gamma} = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix} \qquad \gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \gamma_5 = i\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Slashed four vectors are defined by $a_\mu \gamma_\mu = \not{a}$.

In order to define a potential we begin with the formal relation for the S matrix.

$$S_{fi} = \delta_{fi} - 2\pi i\delta(E_i - E_f)R_{fi} \tag{2.1}$$

$$R_{fi} = \langle \phi_f | V | \Psi_i^+ \rangle \tag{2.2}$$

$$\Psi_i^+ = \phi_i + 1/(E - H_o + i\epsilon)\, V \Psi_i^+ \tag{2.3}$$

where V in field theory is an integral over space of annihilation and creation operators. For the case of the relativistic γ_5 form of meson theory, $V = g\int \bar{\psi}\, \gamma_5 \psi \phi\, d^3x$. For the fixed source meson theory $V = (f/\mu)\int \sigma\cdot\nabla\phi\rho(r)d^3r$,[4] where $\phi = \sum_{i=1}^{3} \tau_i\phi_i$. Formula 2.2 allows us to define an equivalent non-relativistic potential for any order in the coupling constant via a direct comparison with the R matrix as derived from field theoretic considerations. The potential is expanded as a power series in the coupling constant:

$$V = V^{(2)} + V^{(4)} + \text{---} \tag{2.4}$$

and similarly R may be expanded

$$R = R^{(2)} + R^{(4)} + \text{---} \tag{2.5}$$

since

$$R_{fi} = \langle \phi_f | V | \phi_i \rangle + \frac{\langle \phi_f | V | \phi_n \rangle \langle \phi_n | V | \phi_i \rangle}{E_i - E_n - i\epsilon} + \cdots \tag{2.6}$$

then to fourth order

$$R_{fi}^{(2)} = \langle f | V^{(2)} | i \rangle \tag{2.7}$$

$$R_{fi}^{(4)} = \langle f | V^{(4)} | i \rangle + \frac{\langle f | V^{(2)} | n \rangle \langle n | V^{(2)} | i \rangle}{E_i - E_n} \tag{2.8}$$

This means that the two diagrams (first and second Born Approximation (B.A.)) (fig. 1) must correspond to the fourth order R matrix. Equation 2.8 allows us to trivially define the so-called fourth order potential. $V^{(2)}$ is calculated in second order and is precisely defined only on the energy shell. However, it is possible from this point of view to include in $V^{(2)}$ certain non-static pieces which depend on the nucleon velocities and which vanish on the energy shell. In so far as we are concerned only with the scattering of two real free nucleons on the energy shell, the S matrix is still unambiguously duplicated. However if the potential is to be included in a Schroedinger equation where higher order effects are implicitly taken into account, the treatment of these off energy shell pieces of the matrix elements is important. In either case iteration of the second order potential must be subtracted from the fourth order R matrix.

We calculate in momentum space. Since V is defined conventionally in position space, we need only transform to coordinate space by superposing solutions in momentum space. There is some difficulty here. It is connected to our original hypothesis that we use only matrix elements on the energy shell. In so far as the potential is used to analyze real free particle scattering, there is no difficulty. We are merely inverting the process of superposition and going back to momentum space. This process will pick out those fourier components

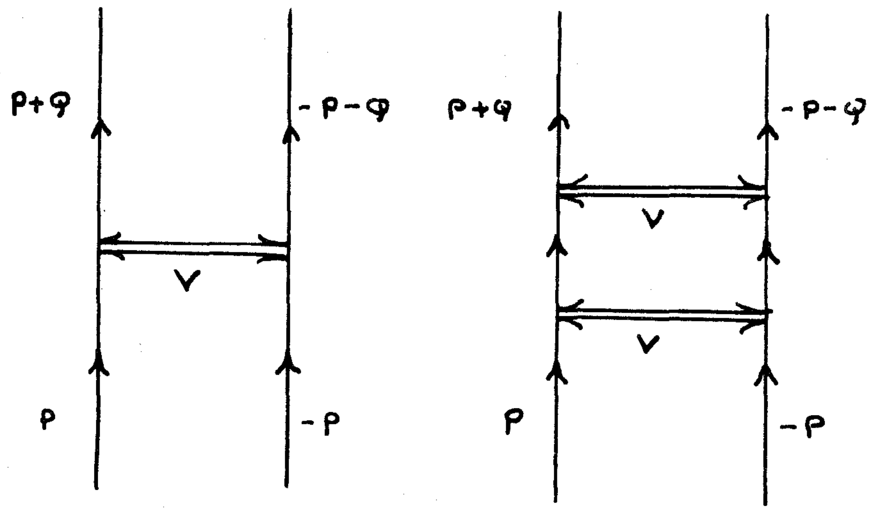
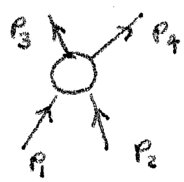


Figure 1

First and second B.A. for two nucleon scattering.

of $V(r)$ which correspond to scattering on the energy shell. In the general case we fix the energy at some value and sum over all momenta to obtain the position space $V(r)$. This is the situation which exists in a bound state. As long as we satisfy the free particle conservation laws our potential is exact. However in integrating over all momentum transfers for a fixed energy we go off the shell. This introduces an error for a bound state roughly of the order of the binding energy/nucleon mass.

With this in mind we consider the scattering process



In the C.M. there are two independent three momenta $p = p_1 = -p_2$, $Q = p_3 - p_1$. Let us call the matrix in position space, $\mathbb{T}_{fi}$.

$$\mathbb{T}_{fi} = 1/(2\pi)^3 \int \phi_f^*(r') \, e^{-iQ/2\cdot(r'-r)} \, e^{ip\cdot(r'+r)} \, R(Q,p) d^3Q d^3p \phi_i(r) \, d^3r \, d^3r' \tag{2.9}$$

where

$$\langle \phi_f(r) | \phi_i(r) \rangle = 1.$$

If R is only a function of the momentum transfer Q then $V(r)$ is seen to be simply

$$V(r) = 1/(2\pi)^3 \int e^{+iQ\cdot r} R(Q) d^3Q \tag{2.10}$$

From this potential must, of course, be subtracted the iterated second order contributions. For velocity dependent potentials the extension is trivial and will be considered in the specific case.

III. FOLDY WOUTHUYSEN TRANSFORMATION OF (PS,PS) THEORY

The so-called Foldy Wouthuysen (F.W.) Transformation serves as a convenient starting point for the reduction of the Dirac equation to a non-relativistic two component form. The transformation has the advantage that it may be carried out to an arbitrary order in M^{-1}, the mass of the nucleon, always keeping the transformed wave functions normalized to unity.[8]

In order to apply the Foldy-Wouthuysen transformation to the Dirac equation we write it as

$$H\psi = (\beta Mc^2 + O + \mathcal{E})\psi = i\hbar(\partial\psi/\partial t) \qquad (3.1)$$

where the odd operator O and the even operator $\mathcal{E}$ are explicitly separated. By odd operator we mean an operator which couples the upper and lower components of the four component spinor ψ. The new Hamiltonian H' is defined as

$$H' = e^{iS}He^{-iS} - ie^{iS}(\partial/\partial t)e^{-iS} \qquad (3.2)$$

where we choose $S = -1/2M\ \beta O$.

Expanding 3.2 in powers of $^1/M$ we easily find that

$$e^{iS}He^{-iS} = H + i[S,H] + i^2/2!\ [S,[S,H]] + i^3/3!\ [S,[S,[S,H]]] + \cdots \qquad (3.3)$$

and

$$e^{iS}i(\partial/\partial t)e^{-iS} = (\partial S/\partial t) + (i/2)[S,\dot{S}] + (i^2/3!)[S[S,\dot{S}]] + \cdots \qquad (3.4)$$

For the case of the relativistic pseudo-scalar theory with pseudo-scalar coupling

$$H = \alpha\cdot p + \beta M + g\beta\gamma_5\,\tau\cdot\phi \qquad (3.5)$$

and

$$S = (-i/2M\ \beta[\alpha\cdot p + g\beta\gamma_5\,\tau\cdot\phi] \qquad (3.6)$$

We find, to order g^2/M^3, that the transformed Hamiltonian becomes:

$$H' = M + p^2/2M + g/2M\, \sigma\cdot\nabla\phi + (g^2\phi^2/2M) + (g^2\dot{\phi}^2/8M^3) + g^2/4M^2 \, \boldsymbol{\tau}\cdot(\phi \times \dot{\phi}) + g/8M^2[\sigma\cdot p, \dot{\phi}] - 1/8M^3[(\sigma\cdot p - ig\phi)(\sigma\cdot p + ig\phi)]^2 \quad (3.7)$$

Most of the terms in equation 3.7 may be given simple interpretations. The first two terms in H' are, respectively, the mass and kinetic energy of the non-relativistic nucleon. The third term is the usual pseudo-vector coupling term of the static model, where g/2M is to be identified with f/μ, f being the pseudo vector coupling constant. $(f^2/4\pi \approx 0.09)$. The fourth term gives rise to an extremely strong S wave scattering. This term is in sharp disagreement with experiment and its presence has been noted for a long time. The presence of the ϕ^2 term in the reduction is a manifestation of the fact that the matrix γ_5 has large matrix elements, the order of 1, between nucleon anti-nucleon states. Thus in higher order than the lowest the anti-nucleon intermediate states completely dominate the pseudo-scalar theory.[9,10] It has been argued that a more exact solution of the field equations results in a self damping of the S wave pions. In some cases this S wave damping is so strong that S wave effects may be neglected altogether. If the only term in the static interaction Hamiltonian is of the form ϕ^2 then it is possible to solve exactly for the normal modes of the meson field variables. The scattering is reduced, in this case, from its value in Born Approximation (B.A.) by approximately a factor of one hundred.[11] B.A. does not take into account properly the diminuation of the wave function near the origin due to the repulsive potential simulated by the ϕ^2 term.

The fifth term is also an S wave term but a much smaller one. The sixth term is an effective charge exchange S wave scattering term. The seventh term is a rather interesting one. Combined with the second one it gives an effective vertex of the form $\omega_{meson}\, \sigma\cdot(\vec{v}_{meson} - \vec{v}_{nucleon}/2)$. The reduced theory must be Galilean invariant. As a result one might expect a term of the form $\sigma\cdot(\vec{v}_{meson} - \vec{v}_{nucleon})$ to appear at the vertex. This statement has been made previously.[14] However this is not the case. It is possible to show that the coefficient of $\vec{v}_{nucleon}$ may be any number at all, (See appendix 4.) and still leave this theory Galilean invariant.

The last term in the Hamiltonian may be broken down into six pieces, all of which, owing to the factor $^1/M^3$, are rather small and will not be considered in this thesis. However we list them and categorize them briefly. The six terms are:

(a) p^4

(b) $p^2\phi^2 + \phi^2 p^2 = (p_i^{\,2} + p_f^{\,2})\,\phi^2$

(c) $[\sigma\cdot\nabla\, \tau_i \phi_i][\sigma\cdot\nabla\, \tau_i \phi_i]$

(d) ϕ^4

(e) $p^2\sigma\cdot\nabla\phi + (\sigma\cdot\nabla\phi)p^2$

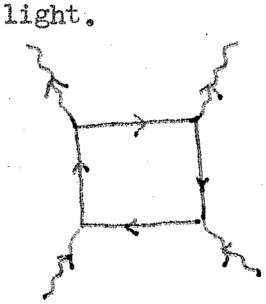
(f) $\phi^2\sigma\cdot\nabla\phi + (\sigma\cdot\nabla\phi)\phi^2$

(a) is a simple kinetic energy correction. (b) couples S-wave mesons and in fourth order is non-velocity dependent. (c) is a non-velocity dependent term which couples S, P, and D wave mesons. (d) is a four meson-vertex term. (e) is a v^2/c^2 correction to the static potential and (f) is a term which will not contribute until at least sixth order.

Since in the calculation of the potential, we shall use low order perturbation theory, we must regard the coefficients of all the terms in the reduced Hamiltonian as subject to an effective renormalization by higher order interactions. We cannot calculate the relative renormalization, (for the various terms of H'), but we shall adjust the coefficients so that they agree with the experimental pion-nucleon scattering. Thus, the coefficients of the terms in H' are regarded as experimentally measured quantities.

There has been and still is some question as to the existence of a direct meson-meson interaction term in the Hamiltonian. Apparently the relativistic γ_5 meson theory contains only the masses of the particles and the coupling constant as parameters. The adjusted coefficients in the reduced Hamiltonian are, we feel, manifestations of higher order corrections to perturbation theory and should in principle be calculable. However, some people believe that there is still another arbitrary parameter in the pseudo-scalar meson theory. When the renormalization program was first carried through for quantum electrodynamics, it was shown[12] that after renormalization of the mass and charge, everything was finite in perturbation theory. Immediately, people began to look at all other theories of the coupling of particles to find out whether they possessed the same remarkable property. It was found however that almost all theories were violently unrenormalizable. Gradient theories, or non-linear coupling schemes, or even the simple point Beta-Decay interaction are all unrenormalizable. Only a very limited number of theories lead to cutoff independent results; among these electrodynamics and the coupling of a pseudo-scalar meson to nucleons via a direct coupling. The feeling was and still is that perhaps the criterion of renormalizability acts as a selector on possible theories. Perhaps nature has chosen to use only theories which admit to renormalization and give finite answers in this especially simple manner. Of course even a gradient theory is finite if we put a cutoff on it and then say that the cutoff is real in the sense that our whole theory breaks down at small distances and must be replaced by something new. However, it is not necessary to take this point of view. For this reason it is interesting to investigate the consequences of the pseudo-scalar theory, which is precisely what we are attempting to do.

It must be pointed out that even after the renormalization program was carried out, the so-called "renormalizable" theories still contained one primitive divergence. The divergence arose from a fourth order diagram contributing to the meson-meson scattering and to the scattering of light off light.



For the case of electrodynamics this divergence may be eliminated. For the case of meson theory however, it exists and is cutoff dependent. If we believe firmly in the renormalization program we would then say that there is a direct meson meson term of the form $\lambda \phi^4$ in the coupling Hamiltonian. Thus the divergent diagram would just renormalize this parameter and theory would again be finite. This ϕ^4 term if it exists would also contribute to the nuclear force and possibly, if it were strong enough, to the L•S force. However, we omit entirely any such term in the Hamiltonian.

We stress again the fact that the importance of the F.W. transformation lies in the fact that it allows us to start with a non-relativistic (PS) (PS) theory and modify it, in what we consider a reasonable way to fit experiment. In addition it permits us to construct a potential in complete analogy with the B.W. and G. potential. There of course remains the question of how well the modified non-relativistic (PS) (PS) theory reflects the character of the γ_5 theory. We have investigated this central question by computing a potential with a modified relativistic theory, and comparing the results with those of the non-relativistic theory. By "modified relativistic theory" we mean that we take the γ_5 theory and modify it by relativistically suppressing pair terms. This is more difficult in a covariant theory since nucleons and anti-nucleons are treated on an equal footing.

In section V we will discuss this "modified relativistic theory," and give the potential computed within its framework. By comparing these results with those of the non-relativistic theory it will be possible to show that in the fourth order nuclear force problem, except for the explicit suppression of pair terms, the two theories are identical with each other and with the γ_5 theory. More explicitly we outline the procedure to be followed. For the purposes of comparison we suppress pair terms completely by dropping $2Mg^2\phi^2$ from the reduced Hamiltonian and then compute the second and fourth order, energy shell, scattering matrix elements for two nucleons. We then expand the answer in power of μ/M and retain the term of relative order one (velocity independent) and μ/M (velocity dependent). We then compute the same matrix element using the "modified" relativistic theory with the "pair term" dropped (if the ϕ^2 term is included the "modified" theory is precisely the γ_5 theory). The relativistic matrix element was integrated over the fourth component of the free virtual meson's momentum, and expanded to the appropriate order in μ/M. In both cases the coefficient of $\tau_1 \cdot \tau_2$ was the same although the total answer was different. Since the pair terms in both cases can only lead to terms proportional to a constant in iso-spin space (the separation into constant and $\tau_1 \cdot \tau_2$ is unique), this comparison will indicate that except for the explicit suppression of pair terms the non-relativistic approach is the same as the γ_5 theory.

We therefore write in place of equation 3.7

$$H' + M + p^2/2M + 2M\alpha(f/\mu)^2\phi^2 + (f/\mu)\sigma\cdot\nabla\phi + \gamma(f/\mu)^2\,\tau\cdot\phi \times \dot{\phi} + \beta\,(f/4M\mu)[\sigma\cdot p, \dot{\phi}]_+ + \epsilon/2M(f/\mu)^2\dot{\phi}^2 - \delta/8M^3[(\sigma\cdot p - ig\phi)(\sigma\cdot p + ig\phi)]^2 \quad (3.8)$$

where α, β, γ, δ, and ϵ, are empirical damping factors to be determined and we have used $(f/\mu) = g/2M$, the static coupling constant $f^2/4\pi \approx 0.09$.

Drell, Friedman and Zachariasen[13] do a fixed source analysis of the S-wave pion nucleon interaction. This analysis is constructed along the lines of the usual Chew-Low formalism. They assume only a bilinear S-wave interaction of the form $\lambda_0\phi\cdot\phi + \lambda\,\tau\cdot\phi \times \dot{\phi}$, which is added to the usual $(f/\mu)\sigma\cdot\nabla\phi$. They then determine the values for the renormalized coupling parameters λ_0 and λ, so that their theoretical expressions give agreement with the S-wave pion nucleon phase shifts up to 100 Mev pion kinetic energy. They find that the S-wave scattering data give $\alpha \approx 0.02$ and $\gamma \gtrsim 0.5$. The term β is not too well known experimentally. However it is known that $\beta \lesssim 1$ is not in violent disagreement with the S-wave pion production cross sections in nucleon-nucleon collisions. Gell-Mann and Rosenfeld[14] have made a very crude estimate of the coefficient β. They consider the ratio of S-wave to P-wave meson production in the reaction

$$p + p \rightarrow \pi + D.$$

If one considers this process as taking place only in lowest order perturbation theory then the only term in the Hamiltonian (equation 3.8) contributing to the S wave cross section is

$$\beta(f/4M)\,[\sigma\cdot p, \dot{\phi}]_+$$

A comparison of the experimental and theoretical values for the S-wave to P-wave ratio indicates that $\beta \gtrsim 1$. Since the terms ϵ and δ do not give any significant contributions unless ϵ and δ are significantly larger than one we shall neglect them.* In addition we should of course include rescattering effects for P wave mesons just as the Low equation does. We shall neglect all rescatterings except in so

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* Klein has included $\delta \approx M/\mu$ in an attempt to include the effective rescattering from the (3/2, 3/2) resonance. We ignore this. We also do not complete our Hamiltonian so as to make it relativistically covarient because we feel that all the empirical corrections made to the F.W. result reflect only the failure of perturbation theory to treat the γ_5 theory correctly.

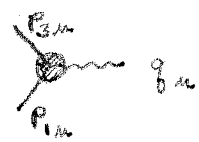
far as they are included in the renormalized coupling constant. It is therefore to be expected that there will be rescattering corrections to our results. Breuckner and Watson have investigated this P-wave rescattering for the static potential and find that its effects are small. A more sophisticated treatment of the nuclear force problem, perhaps from the point of view of dispersion theory would of course, in some approximate way, include these effects.

IV. THE NON-RELATIVISTIC POTENTIALS

We now write the second and fourth order potentials as

$$V = (f^2/4\pi) \left[A^{(2)} + \mu/M \, B^{(2)} \right] + (f^2/4\pi)\,\Delta + f^4/(4\pi)^2 \left[A^{(4)} + \mu/M \, B^{(4)} + \mu/M \, F \, \underline{L} \cdot \underline{S} \right] \qquad (4.1)$$

where A, B, and F are function of the relative distance between the two nucleons and where we have anticipated the result that the spin orbit term is at least fourth order in f and first order in μ/M. The fact that the general second order vertex cannot contribute to the velocity dependent force may be understood rather simply. Consider the most general relativistic second order vertex.



The matrix element under consideration summed to all orders in the radiative corrections must, for the case of pseudoscalar mesons with pseudo-scalar coupling, be a pseudo-scalar. With this in mind we first recall that whenever $\not{p}_1$ or $\not{p}_3$ occurs in one of our matrix elements, it is possible, using the commutation relations, to slide them over to the right or to the left side respectively of the entire group of Dirac matrices, so that they operate on the free nucleon spinors. Here they yield a factor M. The only invariant for the single vertex is $(p_3 - p_1)_\mu^2 = q_\mu^2$. Thus the only form our matrix element can take is

$$M_{fi} = F(q_\mu^2)\gamma_5$$

The matrix element of γ_5, as may easily be shown by direct substitution, depends only on the three momentum transfer, never on the sum of the two momenta. This implies (see section II) that the resulting potential is, in position space, a function of the relative coordinate only. The term $B^{(4)}$ will be neglected since it is a correction to the static potential of order $f^2/4\pi \cdot \mu/M$. We feel that there are enough uncertainties in the static potential itself to justify the neglect of $B^{(4)}$. Since $f^2/4\pi \approx \mu/M$, we would expect to include $B^{(2)}$ in the final result. Corrections of this form can come only from the term $(\sigma \cdot p, \dot{\phi})_+$ in the Hamiltonian. It will become evident that $B^{(2)}$ will vanish because of the symmetry properties of the diagrams. In the same spirit we keep only the largest velocity-dependent piece. As we have shown in the appendix, the only form such a potential (linear in the nucleon velocity and not vanishing for real free particle scattering) can have is L•S. The term $f^2/4\pi\, \Delta$, on the other hand, depends on the nucleon velocities as well as their positions. It is the term which gives rise to the controversial "ladder terms," whose origins and properties we will discuss in some detail in what follows.

As we have already pointed out, iterations of the second order energy shell potential must be subtracted from the fourth order matrix in defining an appropriate $V^{(4)}$. In addition, if one calculates and includes in $V^{(2)}$ certain non-static pieces which depend on the nucleon velocities and which vanish on the energy shell (these are designated by $f^2/4\pi\, \Delta$ in equation 4.1) then, of course, the iteration of these terms must be subtracted from the fourth order potentials. The momentum dependence of Δ is just such as to cancel the usual pole present as $E_i - E_n$ in equation 2.8 and yield a non-singular function.

These iterated terms are the so-called "ladder terms." B.W. have already noticed that if these terms are grouped with the "ordinary" fourth order pieces then the resulting static potential gives an unbound deuteron and in general disagrees with experiment. On the other hand, if the term $f^2/4\pi\,\Delta$ is kept in second order, then for wave function of the potential $V-\Delta$ with a phenomenological hard core, the addition of Δ shifts things so slightly that it may be neglected. Their conclusion is, that the retention of this term in fourth order seriously overestimates it, since its origin in the fourth order potential may be traced to the predominance of high momentum components in the wave function for the two nucleons. These high momentum components are in turn a consequence of the singularity in $V^{(2)}$ static which in coordinate space goes like $1/r^3$. These iterated terms are for these reasons omitted from the Gartenhaus potential. The same situation prevails with the L•S pieces. $f^2/4\pi\,\Delta$ when iterated leads to a fourth order L•S potential. To be consistent with the B.W. approach we shall drop this L•S piece from the fourth order potential and neglect the effect of $f^2/4\pi$ on the scattering since it vanishes on the energy shell and is presumably a small correction for a wave function satisfying the Schroedinger equation for $V - f^2/4\pi\,\Delta$. At energies of about 150 Mev the neglect of $f^2/4\pi\,\Delta$ should be reinvestigated since the previous results hold only for the deuteron. However the actual quantitative effect of $f^2/4\pi\,\Delta$ on the 150 Mev scattering will be much more sensitive to the inner parts of the potential than the low energy results were. As a result such an estimate would have little real significance. However as we shall see, this method of treating the iterated pieces of Δ leads to certain definite predictions about the shape of the L·S potential which would be invalidated if we neglected to treat them in the prescribed manner.

We now evaluate V of equation 4.1. Figure 2 is the only diagram arising in the calculation of the second order potentials. Figures 3 and 4 contain all the diagrams which are included in our calculation of the complete fourth order potentials. A number of things should be pointed out about these diagrams. In the first place they are non-relativistic diagrams so that the time ordering of internal events is important. The arrows always point in the direction of increasing time. To the diagrams listed must be added all the diagrams which are obtained by interchanging the role of the two nucleons, the so-called reflected diagrams. These may be obtained from the ones written by means of the simple substitution.

$$\tau_1 \to \tau_2$$
$$\sigma_1 \to \sigma_2$$
$$k \to -k$$
$$k' \to -k'$$

Thirdly all diagrams are multi-representational. By this we mean that at each single meson vertex we can place a $(\sigma \cdot \nabla\phi)$ or a $(\sigma \cdot p, \dot{\phi})_+$. Similarly at all double vertices we may place a $(\tau \cdot \phi \times \dot{\phi})$ or a ϕ^2.

Let us consider first the static limit of these diagrams. The two diagrams for the second order nuclear potential (fig. 1) and its reflection yield the same contribution to the energy. They give for the potential using the conventional rules for the static Hamiltonian

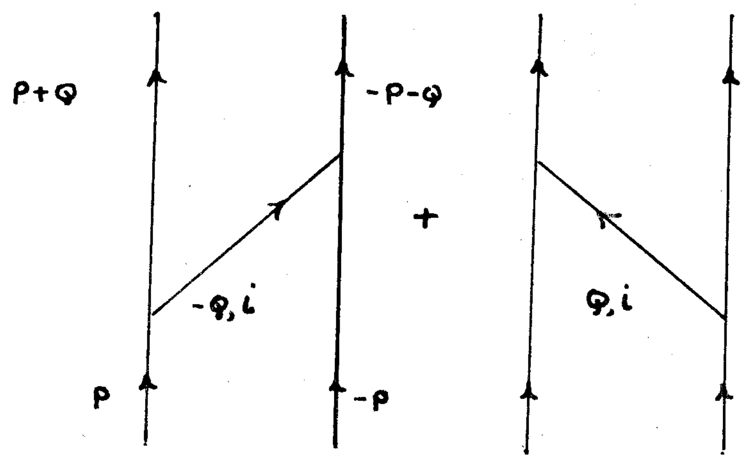


Figure 2

Non-relativistic second order nucleon force diagrams. Arrows point in the direction of increasing time.

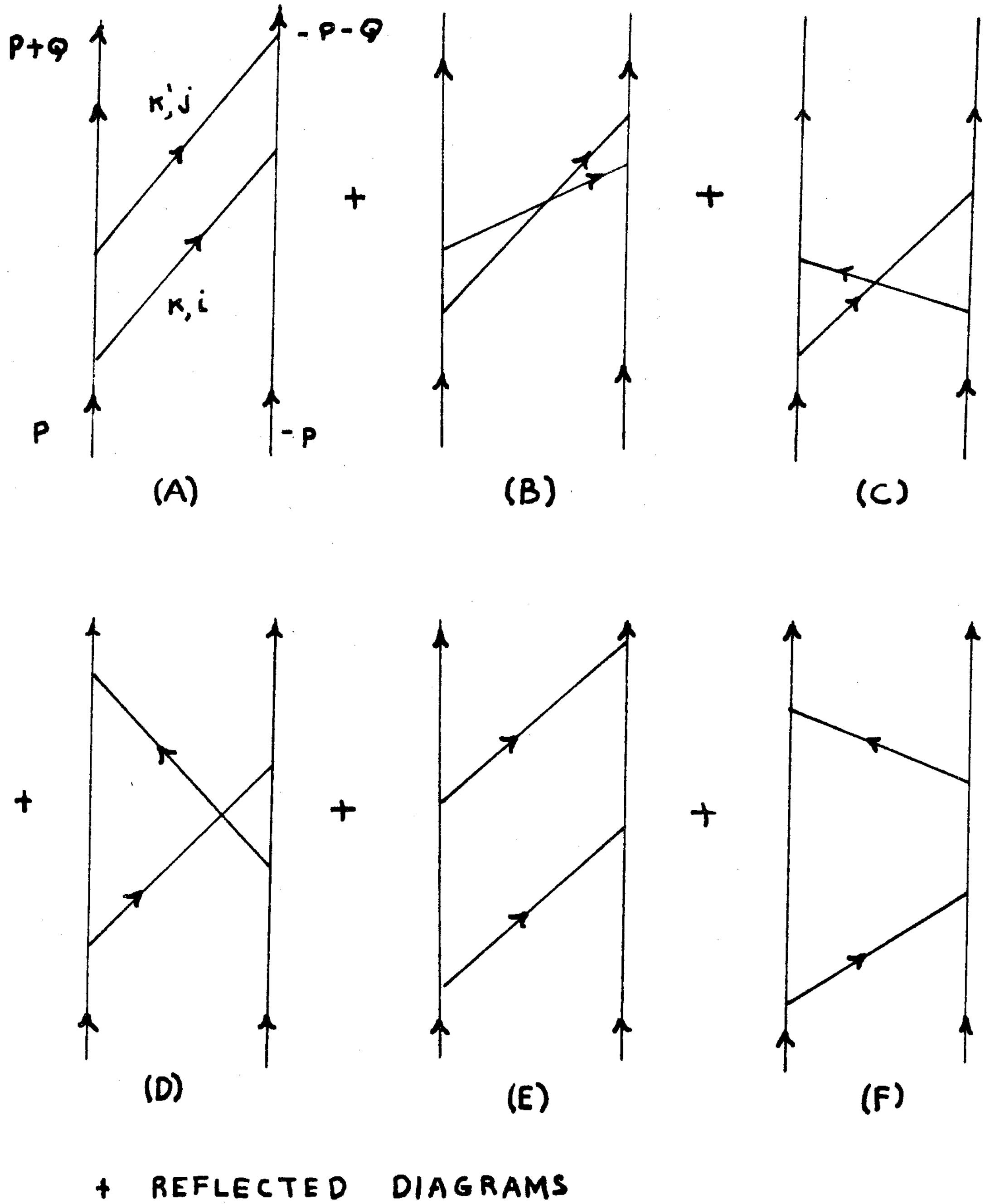


Figure 3

"Single action" fourth order non-relativistic nuclear force diagram. All vertices may contain a $\sigma\cdot\nabla$ or a $[\sigma\cdot p,\dot{\phi}]_+$ interaction.

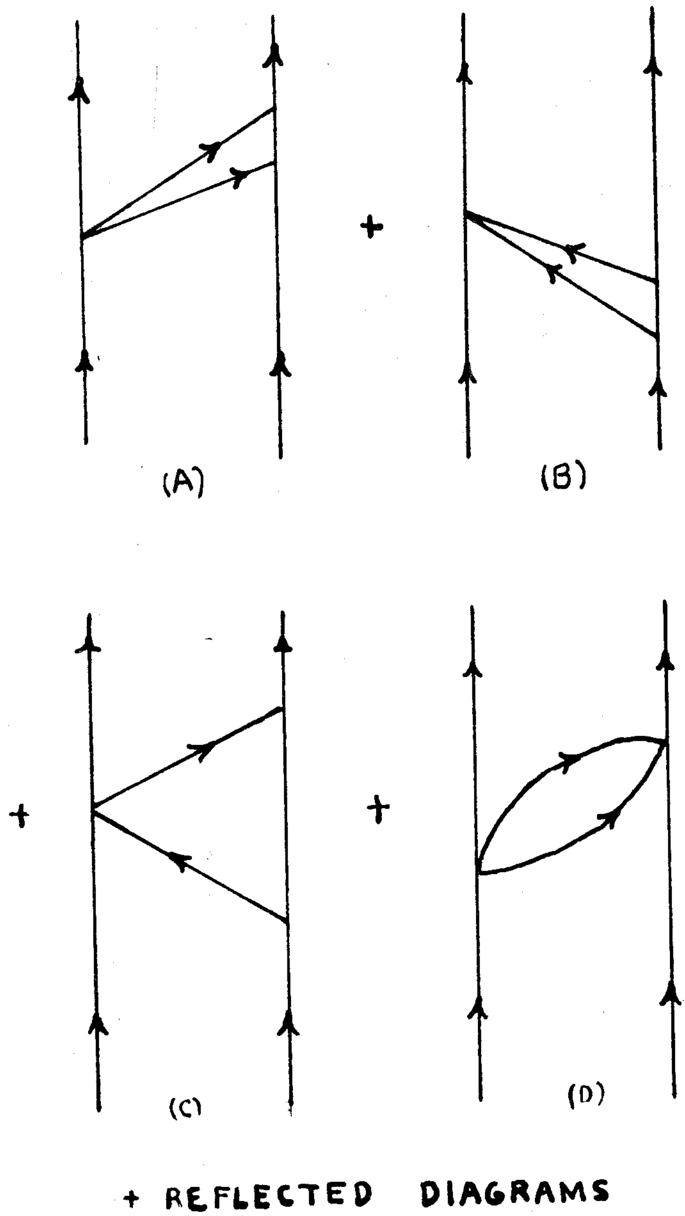


Figure 4

"Double action" fourth order non-relativistic nuclear force diagrams. All vertices may contain a ϕ^2 or a $\tau \cdot \phi \times \dot{\phi}$ interaction.

$$f^2/4\pi\, A^{(2)} = V^{(2)}_{B.W} = -(f/\mu)^2\, 1/(2\pi)^3$$

$$\int [d^3k(\sigma_1.k)(\sigma_2.k)e^{ik\cdot r}] \,/\, \omega^2\, \cdot v^2(k) \qquad (4.2)$$

For the sake of clarity we have in this case and will in future formula rename the pieces of the potential. The subnames will indicate where the separate pieces come from. By B.W. we indicate the pieces already written down by Brueckner and Watson. The cutoff factor $v^2(k)$ appears in all calculations in the static model and is defined as the transform of the nuclear source distribution. That is to say

$$v^2(k) = \int \rho(r)e^{ik\cdot r}\, d^3r \qquad (4.3)$$

where $\rho(r)$ is the distribution in space of the nucleon. It is to be pointed out that this cutoff is important even if the integral converges without it. Although the lowest order relativistic matrix element does not contain this same cutoff explicitly, it contains one implicitly. The dynamics of the nuclear recoil produces an effective cutoff in the relativistic theory. In the static theory where the nucleons are treated as bolted down we put the cutoff in by hand. This cutoff is of the order of one for $k \approx M$ and zero for $k > M$, and prevents the transfer of mesons of momentum the order of the compton wavelength of the nucleon. We regard a smooth momentum space cutoff as a necessary, if unexplained, part of the theory and therefore a careful comparison with experiment requires the use of such a smooth cutoff. In analogy with the Chew theory the results, (of using such a cutoff in k space), should not depend strongly on the form of the cutoff but may depend on the maximum momentum allowed by it.[15] This was Gartenhaus's contribution. He evaluated the static potential second, and fourth order, (which we shall write down shortly) using a gaussian cutoff. The integrals cannot be done analytically, but must be done numerically.

Continuing with the evaluation of the potentials we find for the sum of all diagrams of figures 3 and 4 plus their reflected diagrams

$$(f^2/4\pi)^2 A^{(4)} = V^{(4)}_{B.W} + V^{(4)}_{\dot{\phi}} + V^{(4)}_{\phi^2} \qquad (4.4)$$

where

$$V^{(4)}_{B.W} = -(f/\mu)^4\ 1/(2\pi)^6 \int d^3k\ d^3k'\ e^{i(k+k')\cdot r}\ v^2(k)v^2(k')\ x$$

$$\left\{ [3(k'\cdot k)^2 + 2\,\mathcal{T}_1\cdot\mathcal{T}_2\ \sigma_1\cdot(k'xk)\sigma_2\cdot(k'xk)]/\omega^3\omega'^2 + [2\,\mathcal{T}_1\cdot\mathcal{T}_2(k'\cdot k)^2 + 3\sigma_1\cdot(k'xk)\sigma_2\cdot(k'xk)]/\omega^3\omega'(\omega+\omega') \right\} \qquad (4.5)$$

and

$$V_{\dot{\phi}} = -(f/\mu)^4\ 2\,\mathcal{T}_1\cdot\mathcal{T}_2/(2\pi)^6 \int d^3k\ d^3k'\ e^{i(k+k')\cdot r}\ v^2(k)v^2(k')\ x$$

$$\left\{ [2\gamma(k'\cdot k) - \gamma^2\omega\omega']/\omega\omega'(\omega+\omega') + \gamma^2/2\omega' \right\} \qquad (4.6)$$

$$V_{\phi^2} = +(f/\mu)^4\ 1/(2\pi)^6 \int d^3k\ d^3k'\ e^{+i(k+k')\cdot r}\ v^2(k)v^2(k')\ x$$

$$\left\{ [12M\alpha\,k'\cdot k]/\omega^2\omega'^2 + [24M^2\alpha^2]/\omega'\omega'(\omega+\omega') \right\} \qquad (4.7)$$

These integrals are the results of straightforward application of perturbation theory. We have used a number of identities of the matrices to simplify the expressions. Since $\mathcal{T}_j^{\,2} = 1$, and $\mathcal{T}_i\mathcal{T}_j = i\mathcal{T}_k$ (i, j, k in cyclic order) it is clear that

$$\mathcal{T}_j\,\mathcal{T}_i\,\mathcal{T}'_i\,\mathcal{T}'_j = 3 + 2\,\mathcal{T}\cdot\mathcal{T}' \qquad (4.8)$$

$$\mathcal{T}_j\,\mathcal{T}_i\,\mathcal{T}'_j\,\mathcal{T}'_i = 3 - 2\,\mathcal{T}\cdot\mathcal{T}' \qquad (4.9)$$

We have also made use of the fact that the integrals are symmetric under the interchange of k and k' to drop terms of the form $i(k'\cdot k)(\sigma_1+\sigma_2)\cdot(k'\times k)$ in the integrand.

Now let us consider in somewhat more detail the first order nucleon velocity dependent corrections to the complete fourth order potentials. Consider for the moment the two second order diagrams. If the scattering takes place for two nucleons off the energy shell (i.e. $p_f^2 \neq p_i^2$), then there is a correction to equation 4.2 which has been designated as $f^2/4\pi\,\Delta$ and is :

$$\langle p_f|(f^2/4\pi)\,\Delta|p_i\rangle = -(f/\mu)^2\,[\tau_1\cdot\tau_2/(2\pi)^3]$$

$$\int [d^3k(\sigma_1\cdot k)(\sigma_2\cdot k)/\omega]\,[1/(\omega+\eta/2M) - 1/\omega]\,e^{ik\cdot r} \quad (4.10)$$

where

$$\eta = p_f^2 - p_i^2$$

We expand the denominators and hold on to terms up to order $(1/M^2)$. This implies that

$$\langle p_f|(f^2/4\pi)\,\Delta\,|p_i\rangle \approx f^2/4\pi(\Delta_1 + \Delta_2) \quad (4.11)$$

where

$$(f^2/4\pi)\,\Delta_1 = +(f/\mu)^2\,[\tau_1\cdot\tau_2/(2\pi)^3]$$

$$\int [d^3k(\sigma_1\cdot k\;\sigma_2\cdot k)/\omega^2]\,(\eta/2M\omega)\,e^{ik\cdot r} \quad (4.12)$$

$$(f^2/4\pi)\,\Delta_2 = (f/\mu)^2\int[\tau_1\cdot\tau_2/(2\pi)^3]$$

$$\int [d^3k(\sigma_1\cdot k)(\sigma_2\cdot k)/\omega^2]\,(\eta/2M\omega)^2\,e^{ik\cdot r} \quad (4.13)$$

It is true that we hope to ultimately retain only terms of order μ/M relative to the static piece. However it is also true that iterations of these terms, in the fourth order diagram, will have denominators of the form $\hbar/2M$. Thus terms of order $^1/M^2$ in the low order potentials will contribute to the required order of magnitude. Of course, in keeping with our previous arguments, we are going to drop these terms. However we give the results of these iterations for the sake of completeness, since other methods for handling the so-called ladder terms may in the future prove fruitful.

The iteration of $f^2/4\pi\,\Delta_1$, leads to the famous "ladder" piece of B.W. As we shall see, when we evaluate the piece in coordinate space, it is repulsive in the isotopic singlet state, enough repulsive so as to change the slightly attractive ^3S, well to a repulsion and thus forego the possibility of a bound deuteron. The iteration of Δ_1, (see fig. 3, diagrams e, f) leads to

$$V_{I\Delta_1} = +(f/\mu)^4\, 1/(2\pi)^6\,(3-2\,\tau_1\cdot\tau_2) \int [d^3k\; d^3k'\; e^{i(k+k')\cdot r}] \times$$

$$[(\sigma_1\cdot k)(\sigma_2\cdot k)(\sigma_1\cdot k')(\sigma_2\cdot k')\,/\omega^3\omega'^2]\; v^2(k)v^2(k') \qquad (4.14)$$

Iterations of the piece Δ_2 leads in the same manner to a velocity dependent potential which is of the form

$$V_{I\Delta_2 L\cdot S} = +(f/\mu)^4\, 1/(2\pi)^6\,(3-2\tau_1\cdot\tau_2/2M)$$

$$\int [d^3k\; d^3k'\, e^{+i(k+k')\cdot r}\; v^2(k)v^2(k')]\;\omega^4\omega'^3 \times$$

$$[i(\sigma_1+\sigma_2)\cdot(k'\times k)p\cdot k \times (2\omega'+\omega)/\omega^4\omega'^3] \qquad (4.15)$$

The basic second order diagram (fig. 2) may also be modified by placing the anti-commutator $[\sigma\cdot p, \dot{\phi}]_+$ at one of the two vertices where a $(\sigma\cdot\nabla)$ now operates. The result of this is a term which is anti-symmetric under the interchange of the two nucleons and as a result vanishes when the reflected diagram is added.

We redefine as in the case of the static potential the sum of all the L·S pieces

$$(f^2/4\pi)^2 \mu/M \; F \quad L\cdot S = V^x_{L\cdot S} + V^o_{L\cdot S} + V^{\phi^2}_{L\cdot S} + V^{\dot\phi}_{L\cdot S} \qquad (4.16)$$

All uncrossed fourth order diagrams which have two nucleons in the intermediate state are to be neglected, since they are iterations of the lower order diagrams. As a result we need to consider only the single uncrossed diagram (fig. 3, diagram **A**). As will always be the case all velocity dependent pieces can come from only one of two places. That is to say, they can be generated by placing a $(\sigma\cdot p,\dot\phi)_+$ at any one of the four vertices or by expanding the denominators which contain the nucleon kinetic energies. For the case of figure 3, diagram **A** the anti-commutator piece vanishes because of its reflection symmetries. The recoil piece however is perfectly finite and gives

$$V^o_{L\cdot S} = (f/\mu)^4 \, 1/(2\pi)^6 \, (3-2\tau_1\cdot\tau_2)/(2M) \int d^3k \, d^3k' \qquad (4.17)$$
$$v^2(k)v^2(k') \, e^{i(k+k')\cdot r} \times [i(\sigma_1+\sigma_2)\cdot(k' \times k)(p\cdot k)(k'\cdot k)/\omega^3\omega'^3]$$

A word of explanation should be added here. Actually the integral for $V^o_{L\cdot S}$ contains three terms. If we consider the numerator alone then these are of the form

(a) $(k'\cdot k)^2 \, (p\cdot k)$

(b) $(\sigma_1\cdot k' \times k) \, (\sigma_2\cdot k' \times k) \; p\cdot k$

(c) $(k'\cdot k) \, (\sigma_1 + \sigma_2)\cdot(k' \times k) \, (p\cdot k)$

If we keep in mind the fact that $(\vec{p}_i + \vec{p}_f)\cdot(\vec{p}_i - \vec{p}_f) = 0$ the angular integration eliminates the velocity dependent parts arising from (a) and (b). Of course the integration over the momentum transfer for the case (c) should be performed while retaining the subsidiary condition $p \cdot Q = 0$

where

$$p = p_i + p_f$$

$$Q = p_f - p_i$$

However, the integrals cannot be done analytically if we impose this condition. Thus for the case of (c) where the answer is finite we assume that we can integrate over all values of Q, the momentum transfer. This is in keeping with the discussion at the end of section I, and re-emphasizes the fact that off the energy shell the potential is not explicitly defined.

Let us now consider the L·S pieces arising from the six crossed diagrams (fig. 3) and their reflections. The pieces from the recoil denominators give nothing since they have the wrong symmetry. Unlike the uncrossed diagrams however, the Galilean piece gives a finite contribution. The sum of these three diagrams yields

$$V^{x}_{L\cdot S} = +(f/\mu)\ 1/(2\pi)^6\ [\beta(3+2\,\mathcal{T}_1\cdot\mathcal{T}_2)/(2M)]\int d^3k\ d^3k'\ v^2(k)v^2(k') \tag{4.18}$$
$$e^{i(k+k')\cdot r}\ \times\ [i(\sigma_1+\sigma_2)\cdot(k'\times k)(p\cdot k') + i(\sigma_1+\sigma_2)\cdot(p\times k')(k'\cdot k)/\omega^2\omega'^2]$$

We still have to consider all the diagrams with a double action at each vertex. Figure 4, diagram D, does not contribute to the L·S potential since it is not possible to expand any denominators or to add a "Galilean" vertex. Thus we look only at the remaining diagrams of figure 4.

Initially we place a $(\mathcal{T}\cdot\phi\times\dot{\phi})$ at the double vertex in question, and add a "Galilean" vertex at one of the other two vertices. The sum of these three diagrams plus their reflections yields

$$V^{\dot{\phi}}_{(GAL)(L\cdot S)} = 1/(2\pi)^6\ (1/M)\ (f/\mu)^4\ \gamma\beta\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \int d^3k\ d^3k'$$

$$e^{+i(k+k')\cdot r}\ v^2(k)v^2(k')\ \times\ [i(\sigma_1+\sigma_2)(k'\times p)/\omega'^2] \qquad (4.19)$$

This term has a δ function behavior in the relative coordinate. These δ function pieces are peculiar for a number of reasons. For the case of no cutoff in momentum space it is certainly legitimate to neglect these terms since they contribute to the potential only at one point and would presumably, in this way of looking at things, be lumped in with a phenomenological hard core. When a smooth momentum space cutoff is used this is not the case. These terms are spread out and presumably contribute to the potential. However two things should be pointed out about them. On the one hand the dependence of the potentials generated by these delta function pieces is extremely sensitive to the maximum cutoff momentum allowed. By increasing the value slightly above the value we shall set it at later on, we can make this term have no effect in the region of interest. However it is also true that by decreasing the value of the cutoff slightly these terms become very important. Thus there is a serious question here. We believe that they should be omitted for the reason that they do not appear in the relativistic γ_5 form of the theory. We suspect although we cannot prove it, that these terms reflect the inadequacies of the Foldy Transformation in treating precisely the very singular parts of the interaction energy of two nucleons. It must be remembered that in performing the reduction of the Hamiltonian the meson field variable ϕ was treated as a classical potential and not as a second quantized operator. Since the delta function pieces are all proportional to $\tau_1 \cdot \tau_2$ they must appear in the modified relativistic squared theory. Here we know that the fields ϕ are being treated exactly correctly. The absence of such terms leads us to the conclusion that these terms should be omitted even if a smooth momentum space cutoff is used.

The expansion of the denominators yields

$$V^{\dot{\phi}}_{L\cdot S(Recoil)} = +(f/\mu)^4 \; 1/M \, [2\tau_1 \cdot \tau_2 \gamma/(2\pi)^6 \int d^3k \; d^3k' \; v^2(k) v^2(k')$$

$$e^{i(k+k')\cdot r} \times [i(\sigma_1+\sigma_2)\cdot(k' \times k)(p\cdot k)/\omega^2\omega'^2] \qquad (4.20)$$

The ϕ^2 terms for this problem, have only academic interest in so far as our actual numerical results are concerned, since we have chosen the parameter so small. However since it is a parameter we do include the results. In this case both the "Galilean" vertex and the recoil terms contribute to the resulting potential. We have

$$V^{\phi^2}_{L\cdot S} = +f/\mu)^4 \; 1/(2\pi)^6 \int d^3k \; d^3k' \; e^{i(k+k')\cdot r} \; v^2(k) v^2(k')$$

$$[i(\sigma_1+\sigma_2)\cdot(k' \times k)(p\cdot k)6\alpha(\omega^2+\omega'^2+\omega\omega')/\omega^3\omega'^3(\omega+\omega')] -$$

$$[6\alpha\beta \; i(\sigma_1+\sigma_2)\cdot(k' \times p)/\omega\omega'(\omega+\omega')] \qquad (4.21)$$

The parameter β is included to show the separation into Galilean and recoil parts of this piece of the potential.

If we now assume that $v^2(k) = 1$, the evaluation of all the integrals using the identities in appendix II is quite straightforward and yields

$$V^{(2)}_{B\cdot W} = \mu(f^2/4\pi)\,\tau_1\cdot\tau_2[\sigma_1\cdot\sigma_2 + (3+3x+x^2/x^2)S_{12}](e^{-x}/x) \quad (4.22)$$

$$V^{(4)}_{B\cdot W} = -\mu(f^2/4\pi)^2\cdot(1/x^3)(2/\pi)\quad [(4+4x+x^2/x)\; e^{-x}K_1(x) +$$

$$(2+2x+x^2)K_o(x)e^{-x}](3-2\,\tau_1\cdot\tau_2) + [(23+4x^2)K_o(2x) + (23+12x^2/x)K_1(2x)]$$

$$\cdot\,\tau_1\cdot\tau_2 - 2(\sigma_1\cdot\sigma_2)[6K_o(2x) + (6+4x^2/x)K_1(2x)] +$$

$$2/3(\sigma_1\cdot\sigma_2)(3-2\,\tau_1\cdot\tau_2)[K_o(x)e^{-x}(1+x) + (2+2x+x^2/x)K_1(x)e^{-x}] +$$

$$1/3\ S_{12}[36K_o(2x) + (45+12x^2/x)K_1(2x) - (3-2\,\tau_1\cdot\tau_2)\cdot$$

$$((1+x)K_o(x)e^{-x} + (5+5x+x^2/x)K_1(x)e^{-x})] \quad (4.23)$$

$$V^{(4)}_{\phi} = (1/\pi)(f^2/4\pi)^2\mu(\tau_1\cdot\tau_2)\gamma(1/x^4)[(10+\gamma)K_1(2x)+8x^2K_1(2x)-(20+2\gamma)xK_1'(2x)] \quad (4.24)$$

$$V^{(4)}_{\phi^2} = -12M\alpha(f^2/4\pi)^2[x+1)^2/x^4]e^{-2x}-(48M^2\alpha^2/\mu)(f^2/4\pi)^2[K_1(2x)/\pi x^2] \quad (4.25)$$

$$V^{(4)}_{I\Delta_1} = +\mu(f^2/4\pi)^2\ 1/x^3\ 2/\pi\ (3-2\,\tau_1\cdot\tau_2)[\,[K_o(x)e^{-x}(1+x+x^2) -$$

$$[(1+x)/x]K_1(x)e^{-x} + 2/3\ \sigma_1\cdot\sigma_2[K_o(x)e^{-x}(1+x) + (2+2x+x^2/x)K_1(x)e^{-x}]\,] \quad (4.26)$$

$$V^{X}_{L\cdot S} = -\beta\mu(3+2\,\tau_1\cdot\tau_2)(\mu/M)(f^2/4\pi)^2[\,[(x+1)(x^2+2x+2)-(x+1)^2]/x^6]e^{-2x} \quad (4.27)$$

$$V^{o}_{L\cdot S} = \mu(3-2\,\tau_1\cdot\tau_2)(\mu/M)(f^2/4\pi)^2\ 4/\pi^2\ 1/x^4[xK_o(x) + 2K_1(2x)]^2 \quad (4.28)$$

$$V^{\phi}_{L\cdot S} = -4\,\tau_1\cdot\tau_2\ \mu(\mu/M)(f^2/4\pi)^2\ e^{-2x}[(x+1)^2/x^6] \quad (4.29)$$

$$V^{\phi^2}_{L\cdot S} = (24\mu\alpha/\pi)(1-\beta/2)(f^2/4\pi)^2[3K_1(2x) + 2xK_o(2x)/x^4] \quad (4.30)$$

$$V^{I\Delta_2}_{L\cdot S} = -V^{o}_{L\cdot S} - \mu(3-2\,\tau_1\cdot\tau_2)(\mu/M)(f^2/4\pi)^2e^{-2x}[(x+1)(x^2+3x+x)/x^6] \quad (4.31)$$

where $S_{12} = 3(\sigma_1\cdot r\ \sigma_2\cdot r/r^2) - \sigma_1\cdot\sigma_2$ and $x = \mu r$

The terms labeled B.W. have been computed by B.W. and are included here for reference purposes only. These potentials have also been integrated by G. numerically, using a smooth momentum space cutoff.

It is to be pointed out that some of the additional terms in the Hamiltonian contribute to the static potential as well as to the L·S potential. With the present choice of α all terms involving it could just as well be set equal to zero. There remain however three terms, arising from the $(\tau \cdot \phi \times \dot{\phi})$ in the interaction Hamiltonian which do contribute significantly to the G. potential. These terms were not included by G. These additional pieces should of course be included in a more detailed investigation of the actual effect of this particular potential on the scattering. Such an investigation would amount to a solution of the coupled Schroedinger equations.

Both the static and the L·S piece coming from the $\tau \cdot \phi \times \dot{\phi}$ interaction have a term which has a delta function singularity in the relative coordinate. Owing to their extremely singular behavior and to the fact that they do not occur in the relativistic matrix element we shall omit them (for the case of a unity cutoff and for the case of a smooth momentum space cutoff).

Since we regard a smooth momentum-space cutoff as a necessary part of the theory, a careful comparison with experiment requires recalculation of the potential in configuration space with such a cutoff. For this reason we have programed all the additional pieces of the potential for numerical evaluation on a computer, using a smooth cutoff in momentum space. These pieces include all of the L·S potential for $\alpha = 0$ and β and γ equal to one. They also include the V_ϕ pieces of the fourth order static potential. Using a square cutoff Gartenhaus found oscillations in the potential arising from the discontinuous nature of the cutoff. He eventually used a cutoff of the form

$$v(k) = \exp(-k^2/k_M^2) \tag{4.32}$$

with

$$\omega_M = (k_M^2 + \mu^2)^{1/2} = 6\mu \tag{4.33}$$

For this reason we have used the same cutoff function in evaluating all additional parts of the two nucleon potential.

We include here, in figure 5 a plot of the V_ϕ piece of the potentials in the iso-triplet state. For the purpose of comparison we plot the L•S potential simultaneously with and without a smooth momentum space cutoff (figs. 6 and 7). In making these plots with and without cutoffs α was set equal to zero and β and γ were set equal to one. As we had expected the cutoff considerably modified the behavior of the potentials at small distances. Tables I and II are numerical tabulations of the L•S potentials for two separate values of the maximum momentum squared. Tables III and IV are the corresponding tabulations for the V_ϕ potentials.

Before going on to a brief discussion of the relativistic theory and then a comparison with the available experimental data, we stress again the extreme simplicity of this calculation. Our Hamiltonian was chosen on the basis of a non-relativistic reduction of the (PS) (PS) theory. The S-wave interactions were then suppressed in the non-relativistic theory and the potential calculated with this Hamiltonian as the starting point. V has been chosen to duplicate the

Figure 5

Plot in the isotopic triplet state ($\tau_1 \cdot \tau_2 = +1$) of the static potential coming from the $\tau \cdot \phi \times \dot{\phi}$ term in the Hamiltonian. The dotted curve shown for comparison purposes is a plot of the Gartenhaus static potential in the spin triplet orbital odd states. (For example the ^3P_o state.) If $\Upsilon = 1$ the resulting potential is the sum of these two curves. The fact that these two terms tend to cancel one another is an important feature since the ^3P_o Gartenhaus phase shift is much too large to agree with experiment (see section VI).

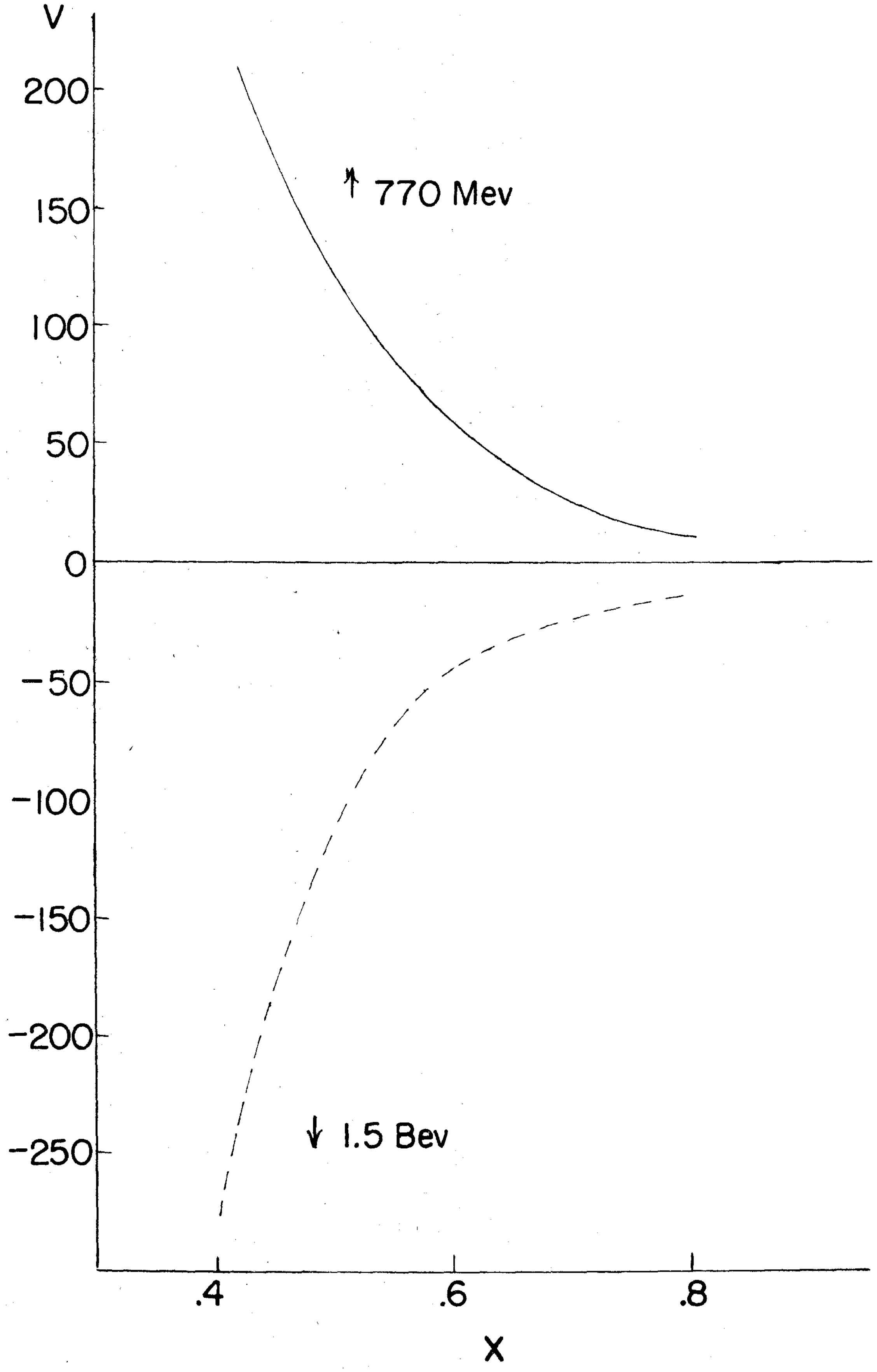

V
200
150
100
50
0
−50
−100
−150
−200
−250
↑ 770 Mev
↓ 1.5 Bev
.4
.6
.8
X

Figure 6

Plot in the iso-singlet state ($\tau_1 \cdot \tau_2 = -3$) of the static potential coming from the $\tau \cdot \phi \times \dot{\phi}$ term in the Hamiltonian. The dotted curve marked G is the Gartenhaus static potential in the spin-triplet orbital even states. (For example the $^3S_1$ state.) If $\gamma = 1$ the resulting potential is the sum of these two curves. The curve marked GT is the corresponding phenomenological potential of Gamel and Thaler. The sum of the two curves looks similar to the GT curve. However with the present choice of γ it would appear that the potential would be attractive all the way, which is no doubt undesirable. If on the other hand γ were changed to some smaller value in the neighborhood of 0.7 the curve might indeed give reasonable answers for the deuteron ground state.

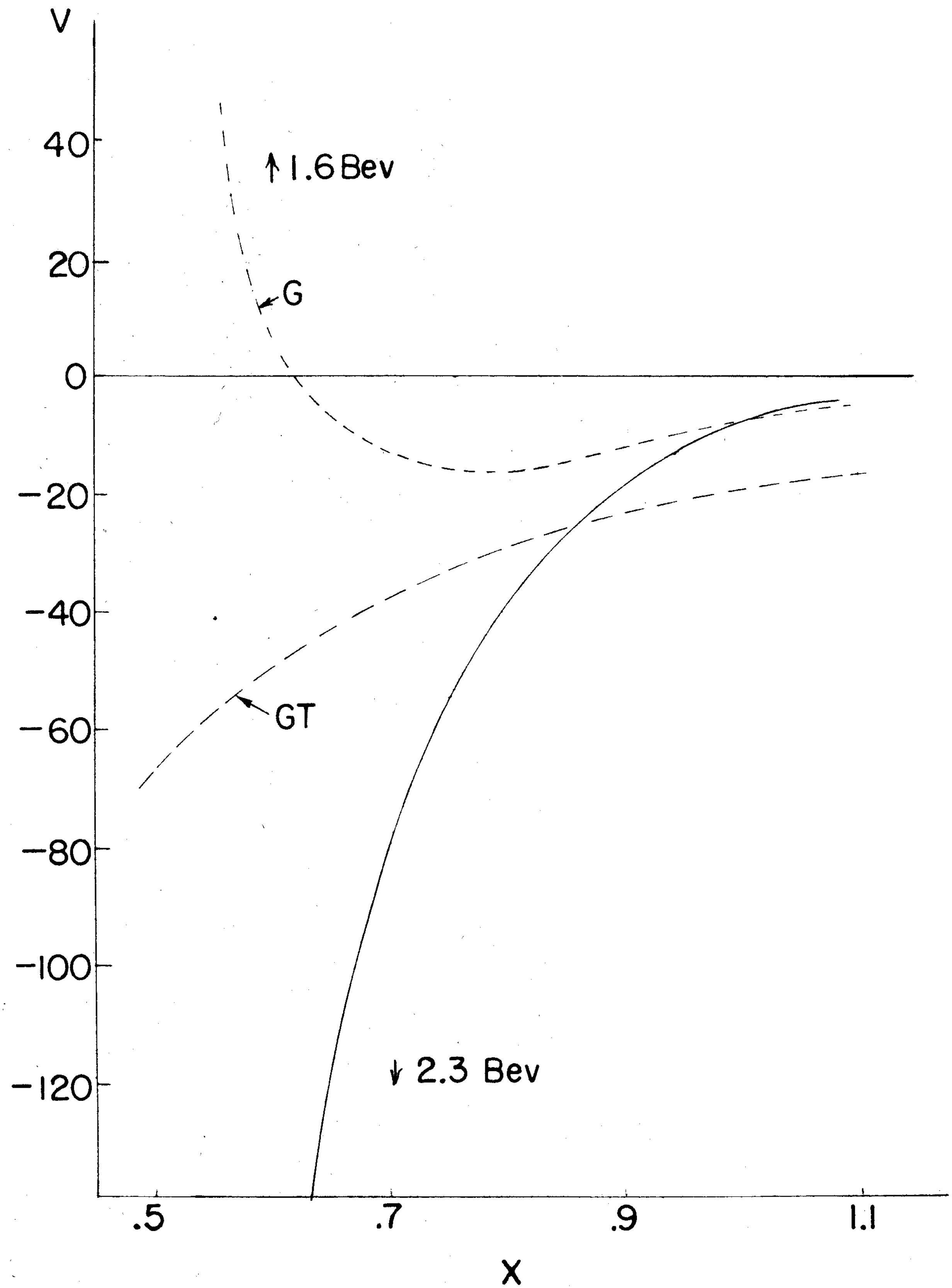

V
40
20
0
−20
−40
−60
−80
−100
−120
↑ 1.6 Bev
G
GT
↓ 2.3 Bev
.5
.7
.9
1.1
X

Figure 7

Plot of the coefficient of L•S in the iso-triplet state. The unlabeled dotted curve is a plot of "units cutoff" ($v^2(k) = 1$) and the solid curve for a gaussian cutoff. The ordinate is in Mev and $f^2/4\pi$ was chosen as 0.1. The curve labeled S.M. is a plot of the phenomenological Signell and Marshak L•S potential in the iso-triplet state.

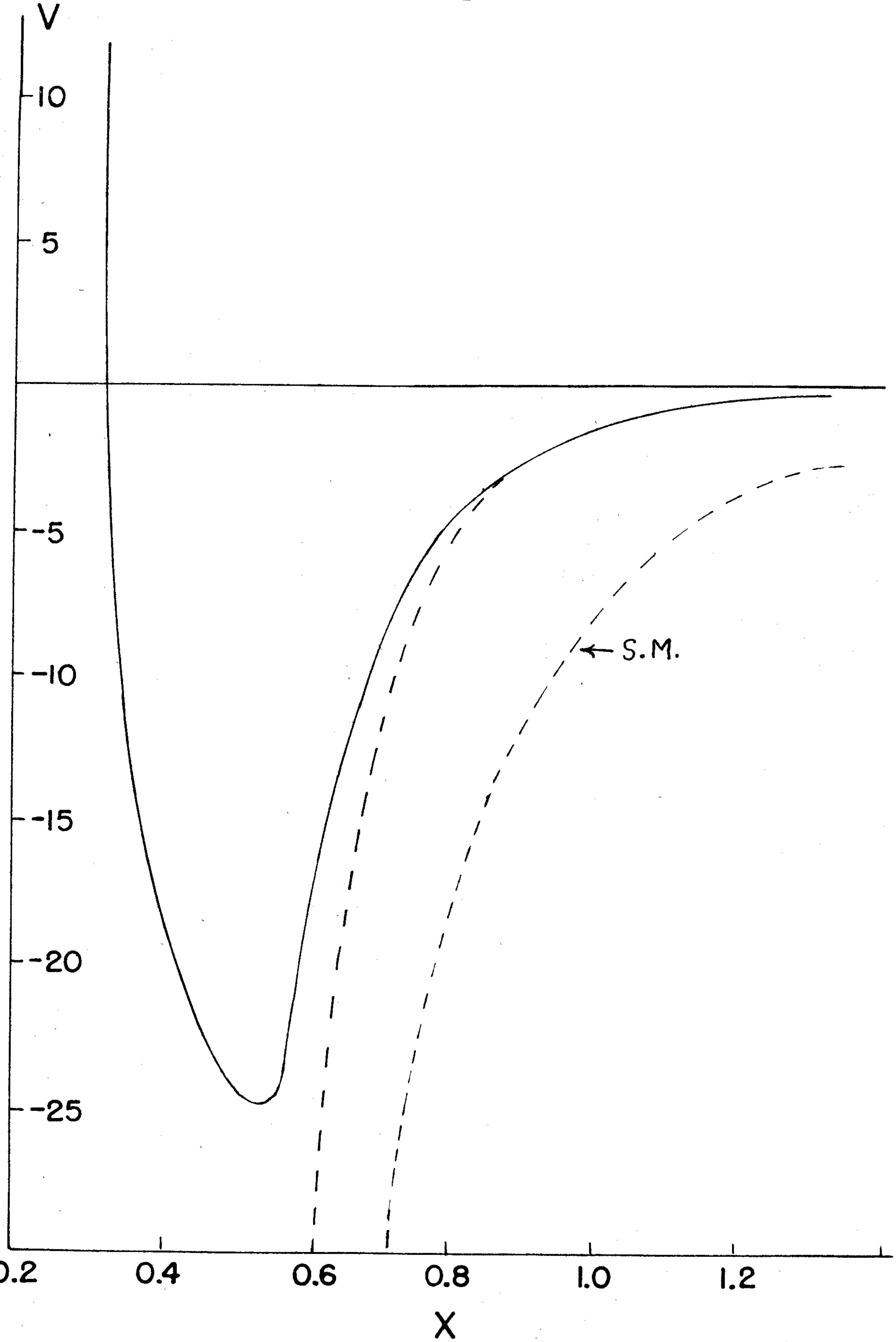

V
10
5
-5
-10
-15
-20
-25
S.M.
0.2
0.4
0.6
0.8
1.0
1.2
X

Figure 8

Plot of the coefficient of L·S in the iso-singlet state. The dotted curve is a plot for "unity cutoff" ($v^2(k) = 1$) and the solid curve for a gaussian cutoff. The ordinate is in Mev and $f^2/4\pi$ was chosen as 0.1.

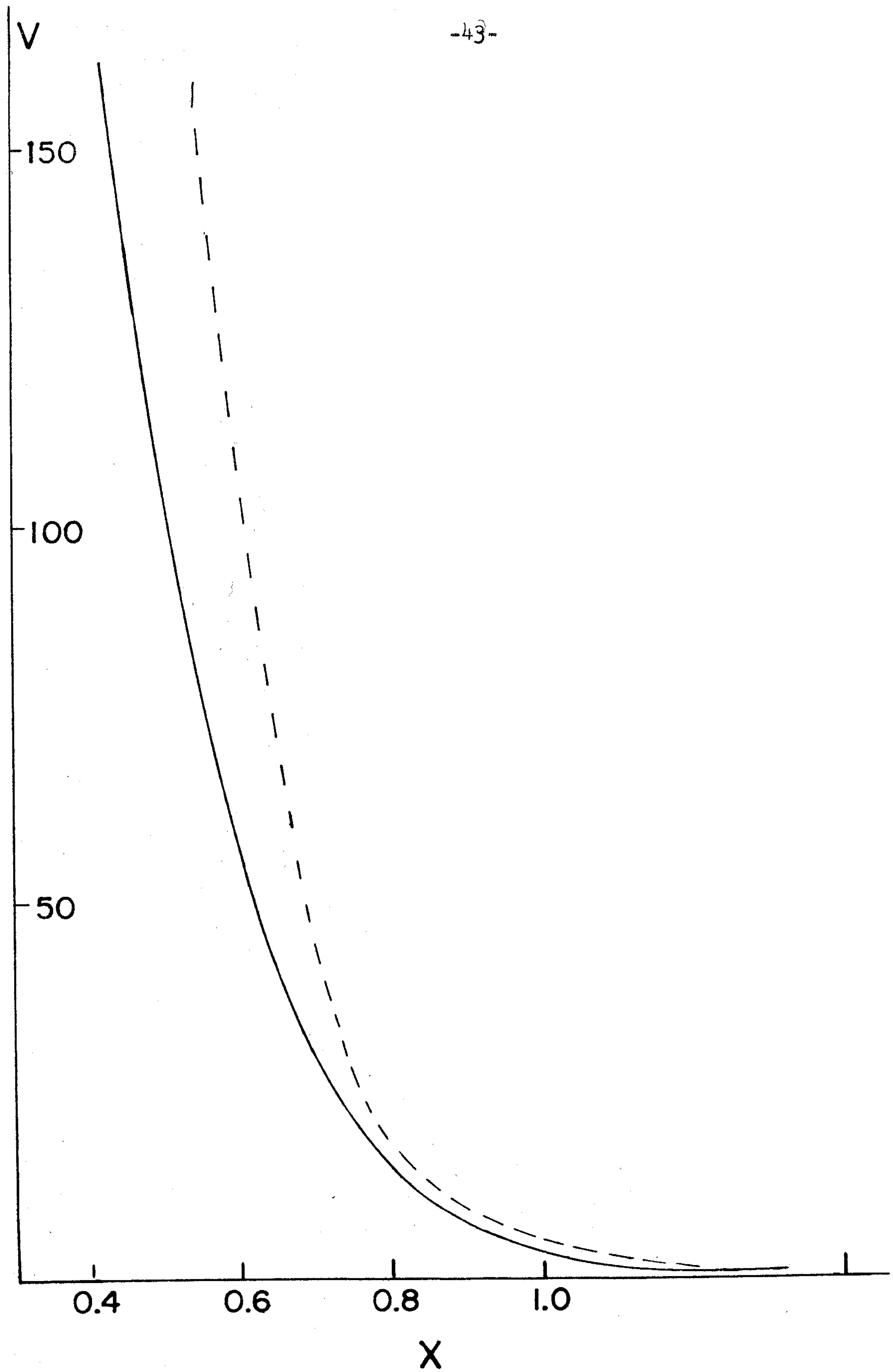

V
150
100
50
0.4
0.6
0.8
1.0
X

Table I

Triplet	Singlet	J
0.4294E 03	0.4324E 03	1
0.3253E 03	0.4139E 03	2
0.1560E 03	0.3595E 03	3
0.3051E 02	0.2709E 03	4
-0.1981E 02	0.1745E 03	5
-0.2447E 02	0.9817E 02	6
-0.1599E 02	0.5026E 02	7
-0.8468E 01	0.2458E 02	8
-0.4167E 01	0.1198E 02	9
-0.2047E 01	0.5980E 01	10
-0.1039E 01	0.3097E 01	11
-0.5508E 00	0.1667E 01	12
-0.3045E-00	0.9310E 00	13
-0.1747E-00	0.5366E 00	14
-0.1033E-00	0.3181E-00	15
-0.6280E-01	0.1932E-00	16
-0.3905E-01	0.1198E-00	17
-0.2477E-01	0.7570E-01	18
-0.1599E-01	0.4864E-01	19
-0.1048E-01	0.3170E-01	20
-0.6958E-02	0.2094E-01	21
-0.4678E-02	0.1399E-01	22
-0.3181E-02	0.9456E-02	23
-0.2182E-02	0.6449E-02	24
-0.1512E-02	0.4439E-02	25

where $x \equiv \mu r = 0.02 + (J-1) \times 0.1$, and the cutoff momentum squared is fixed at $k_M{}^2 = 35\mu^2$. The E indicates the power of ten multiplying the number preceding it.

Table II

Singlet	Triplet	J
0.6933E 03	0.6885E 03	1
0.6600E 03	0.5008E 03	2
0.5592E 03	0.2106E 03	3
0.3995E 03	0.1871E 02	4
0.2377E 03	-0.4003E 02	5
0.1221E 03	-0.3452E 02	6
0.5766E 02	-0.1905E 02	7
0.2669E 02	-0.9176E 01	8
0.1257E 02	-0.4302E 01	9
0.6130E 01	-0.2040E 01	10
0.3135E 01	-0.1015E 01	11
0.1687E 01	-0.5447E 00	12
0.9415E 00	-0.3050E-00	13
0.5381E 00	-0.1716E-00	14
0.3175E-00	-0.1002E-00	15
0.1941E-00	-0.6256E-01	16
0.1205E-00	-0.3948E-01	17
0.7536E-01	-0.2421E-01	18
0.4825E-01	-0.1539E-01	19
0.3181E-01	-0.1053E-01	20
0.2105E-01	-0.7109E-02	21
0.1391E-01	-0.4616E-02	22
0.9262E-02	-0.2969E-02	23
0.6380E-02	-0.2123E-02	24
0.4397E-02	-0.1500E-02	25

where $x \equiv \mu r = 0.02 + (J-1) \times 0.1$, and the cutoff momentum squared is fixed at $k_M^2 = 41\mu^2$.

Table III

Triplet	Singlet	J
0.7693E 03	-0.2308E 04	1
0.6992E 03	-0.2098E 04	2
0.5487E 03	-0.1646E 04	3
0.3674E 03	-0.1102E 04	4
0.2113E 03	-0.6339E 03	5
0.1077E 03	-0.3232E 03	6
0.5123E 02	-0.1537E 03	7
0.2402E 02	-0.7205E 02	8
0.1159E 02	-0.3476E 02	9
0.5881E 01	-0.1764E 02	10
0.3149E 01	-0.9448E 01	11
0.1769E 01	-0.5308E 01	12
0.1035E 01	-0.3104E 01	13
0.6250E 00	-0.1875E 01	14
0.3878E-00	-0.1163E 01	15
0.2461E-00	-0.7383E 00	16
0.1591E-00	-0.4774E-00	17
0.1045E-00	-0.3135E-00	18
0.6953E-01	-0.2086E-00	19
0.4675E-01	-0.1403E-00	20
0.3167E-01	-0.9502E-01	21
0.2157E-01	-0.6471E-01	22
0.1472E-01	-0.4416E-01	23
0.1002E-01	-0.3006E-01	24
0.6766E-02	-0.2030E-01	25

where $x \equiv \mu r = 0.02 + (J-1) \times 0.1$, and the cutoff momentum squared is fixed at $k_M{}^2 = 35\mu^2$.

Table IV

Triplet	Singlet	J
0.1145E 04	-0.3436E 04	1
0.1024E 04	-0.3073E 04	2
0.7710E 03	-0.2313E 04	3
0.4820E 03	-0.1446E 04	4
0.2543E 03	-0.7628E 03	5
0.1189E 03	-0.3568E 03	6
0.5283E 02	-0.1585E 03	7
0.2378E 02	-0.7133E 02	8
0.1128E 02	-0.3385E 02	9
0.5712E 01	-0.1714E 02	10
0.3065E 01	-0.9196E 01	11
0.1727E 01	-0.5181E 01	12
0.1013E 01	-0.3038E 01	13
0.6128E 00	-0.1838E 01	14
0.3806E-00	-0.1142E 01	15
0.2418E-00	-0.7255E 00	16
0.1564E-00	-0.4693E-00	17
0.1027E-00	-0.3080E-00	18
0.6830E-01	-0.2049E-00	19
0.4588E-01	-0.1376E-00	20
0.3100E-01	-0.9300E-01	21
0.2104E-01	-0.6312E-01	22
0.1430E-01	-0.4289E-01	23
0.9651E-02	-0.2895E-01	24
0.6437E-02	-0.1931E-01	25

where $x \equiv \mu r = 0.02 + (J-1) \times 0.1$, and the cutoff momentum squared is fixed at $k_M^2 = 41\mu^2$.

fourth order S matrix, (except for the "ladder" terms) for real free nucleons. All higher order rescattering effects were neglected. A number of fourth order velocity dependent non L·S pieces of order $(v/c)^2$ were not included since they vanish on the energy shell and thus would not appear in our definition of the potential until order f^6.

V. THE RELATIVISTIC SQUARED APPROXIMATION

We intend to calculate all matrix elements in this section within the framework of a modified relativistic theory, first suggested to us by Dr. M. Gell-Mann. By doing this we hope to show that except for the explicit suppression of pair terms the reduced "Foldy Hamiltonian" (equation 3.7) is an accurate reflection of the local γ_5 theory. The local γ_5 theory of course must be changed radically in order that it look like the static model or like experiment. Presumably the higher order effects in the relativistic theory may do this for you so that in modifying the relativistic theory we hope that we are in some way simulating higher order effects in the perturbation theory.

The nucleon field operator satisfies the equation

$$(i\partial\!\!\!/ - M - ig\gamma_5\phi) \qquad = 0 \qquad (5.1)$$

and the meson field satisfies the usual Klein Gordon equation

$$[\Box^2 - \mu^2 + g\bar{\psi}\gamma_5\psi]\phi = 0 \qquad (5.2)$$

We multiply 5.1 on the left by

$$(i\partial\!\!\!/ - M + ig\gamma_5\phi)$$

This leads to the equation

$$[\Box^2 - M^2 - g^2\phi^2 - g\gamma_\mu\gamma_5\partial_\mu\phi] = 0 \qquad (5.3)$$

The ϕ^2 term in the pseudo-scalar theory is the embarassing one. This term we believe damps itself out. In the case of the static model we have already stated that the problem of a ϕ^2 coupling may be solved exactly and shown to damp out. In the relativistic theory the recoil of the nucleon couples S-waves to P waves, P-waves to D waves, etc., and makes the problem insolvable. Therefore it is only a supposition that the relativistic theory behaves like the non-relativistic theory does. In any case we throw out the ϕ^2 term and calculate with the following chopped up equation for the nucleon field operator

$$\left[\Box^2 - M^2 - g\gamma_\mu\gamma_5\partial_\mu\phi\right]\psi = 0 \tag{5.4}$$

The Feynman rules for this approximate theory are given by using a $ig\gamma_5 \not{k}$ at each vertex and a boson propagator for each nucleon. A slight question of normalization arises which may be resolved in the following manner. Consider the matrix element for the transition of a nucleon under the action of a potential, $g\gamma_5\phi$. Then

$$R_{fi} = \langle \phi_f | g\gamma_5\phi | \psi_i^+ \rangle \tag{5.5}$$

Now

$$(g\gamma_5\phi)\psi_i^+ = (i\not{\partial} - M)\psi_i^+$$

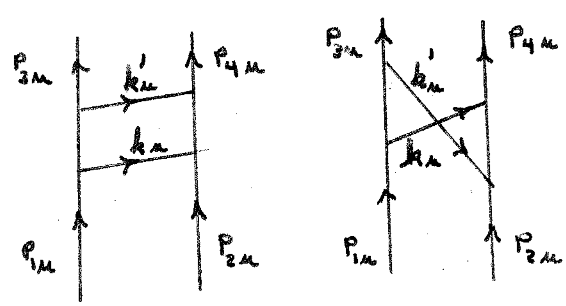
This implies that

$$R_{fi} = \langle \phi_f | (i\not{\partial} - M) | \psi_i^+ \rangle \tag{5.6}$$

If we now assume that $\langle \phi_f | \phi_i \rangle \equiv 0$ then it follows that

$$R_{fi} = 1/2M \langle \phi_f | g\gamma_5\gamma_\mu \partial_\mu \phi | \psi_i^+ \rangle \tag{5.7}$$

In the relativistic calculation we consider only the two Feynman diagrams,



The matrix element for these two diagrams may be written down, using the rules for the diagrams

$$R^x_{fi} = -\, i/(2\pi)^4\, g^4/(2M)^2\, M^2/E^2\, (3+2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \times$$

$$\int \frac{d^4\mathbf{k}\;\bar{u}(p_{3\mu})\,\mathbf{k}'\gamma_5\,\mathbf{k}\gamma_5\,u(p_{1\mu})}{[(p_{1\mu}-\mathbf{k}_\mu)^2 - M^2][(p_{2\mu}+\mathbf{k}_\mu)^2 - M^2]} \times$$

$$[\bar{u}(p_{4\mu})\,\mathbf{k}\gamma_5\,\mathbf{k}'\gamma_5\,u(p_{2\mu})] \,/\, [\mathbf{k}_\mu^2 - \mu^2][\mathbf{k}_\mu'^2 - \mu^2] \qquad (5.8)$$

$$R^o_{fi} = -\, i/(2\pi)^4 g^4/(2M)^2\, (3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(M^2/E^2) \times$$

$$\int [d^4\mathbf{k}\;\bar{u}(p_{3\mu})\,\mathbf{k}'\gamma_5\,\mathbf{k}\gamma_5\,u(p_{1\mu})] \,/\, [(p_{1\mu}-\mathbf{k}_\mu)^2 - M^2][(p_{2\mu}+\mathbf{k}_\mu')^2 - M^2] \times$$

$$[\bar{u}(p_{4\mu})\,\mathbf{k}'\gamma_5\,\mathbf{k}\gamma_5\,u(p_{2\mu})] \,/\, [\mathbf{k}_\mu^2 - \mu^2][\mathbf{k}_\mu'^2 - \mu^2] \qquad (5.9)$$

k, k', Q, and p are defined to be respectively the three momentum parts of $\mathbf{k}_\mu$ and $\mathbf{k}_\mu'$, the momentum transfer and the momentum of particle one in the C.M. In order to take the static limit we perform the integration over $\mathbf{k}_4$ and hold on to terms of order $g^4/(2M)^4 = (f/\mu)^4$. If we then multiply by $e^{iQ\cdot r}$ integrate over d^3Q and change variables to k and k' in place of Q and k we find for the fourth order static potentials

$$V^{(4)x} = -\, g^4/2(2M)^4\; 1/(2\pi)^6\, (3+2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \int d^3k\, d^3k'\, e^{i(k+k')\cdot r} \times$$

$$[\sigma_1\cdot(k'\times k)\;\sigma_2\cdot(k'\times k) + (k'\cdot k)^2][1/\omega^3\omega'(\omega+\omega') + 1/\omega^2\omega\omega'^2]$$

$$[\omega\omega' - 2k'\cdot k] \,/\, \omega+\omega' \qquad (5.10)$$

$$V^o_4 = +\, g^4/2(2M)^4\; 1/(2\pi)^4\, (3-2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \int d^3k\, d^3k'\, e^{i(k+k')\cdot r} \times$$

$$[(k'\cdot k)^2 - (\sigma_1\cdot k'\times k)(\sigma_2\cdot k'\times k)]\,[1/\omega^3\omega'(\omega+\omega') + 1/\omega^2\omega'^2] +$$

$$[\omega\omega' - 2k'\cdot k] \,/\, (\omega+\omega')(\omega\omega') \qquad (5.11)$$

The potential from the uncrossed diagram has had the iterated second energy shell order potential taken out of it. (See section II). It is easy to see that the sum of these two terms agrees with the static potential derived from the non-relativistic Hamiltonian if $\alpha = 0$, $\beta = 1$, $\gamma = 1$. Of course, this potential agrees with the terms from the non-relativistic Hamiltonian including the pieces $V_{I\Delta_1}$ which we have discarded using the B.W. argument.

We now consider the (v/c) corrections to the static potential which come from the crossed and the uncrossed diagrams. The crossed diagram in practice is less difficult to handle. It contains no iterations of the lower order potential, and none of the denominators vanish in the limit of infinitely heavy nucleons. Since this is true it is necessary to hold on to terms of order (μ/M). In the uncrossed diagram this is not the case, since we must take into consideration the fact that some denominators are of order μ/M.

Upon reduction to two component spinors the (v/c) corrections to the numerator becomes

$$N = -k_4/2M \quad [(\sigma_1 \cdot Q)(\sigma_1 \cdot k) - k_\mu^2][(\sigma_2 \cdot (-p-Q))(\sigma_2 \cdot Q) - (\sigma_2 \cdot Q)(\sigma_2 \cdot -p] +$$

$$[(\sigma_2 \cdot k)(\sigma_2 \cdot Q) - k_\mu^2][(\sigma_1 \cdot Q)(\sigma_1 \cdot p) - (\sigma_1 \cdot (p+Q))(\sigma_1 \cdot Q)] \qquad (5.11)$$

The entire L·S piece will, for the crossed diagram come from this correction to the numerator. Although it is true that recoil corrections to the propagator do modify the fourth order static potential, these modifications depend only on the momentum transfer and not on the total velocity. As a result they are not considered here. When considering the (v/c) correction to the numerator the mass of the nucleon in the propagators may be assumed to be infinite. Additional corrections to the propagators, owing to the finite mass of the nucleons will give higher order corrections in (v/c) and are of no interest here.

We close the contour in the upper half plane and pick up residues at $k_4 = 0$, $k_4 = -\omega$, $k_4 = -\omega'$. The result of these integrations for the matrix element is

$$R^x_{fi} = 1/(2\pi)^3\ (f/\mu)^4\ (3+2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)/2M \int d^3k$$

$$i(\sigma_1+\sigma_2)\cdot(Q \times p)(k'\cdot k)/\omega^2\omega'^2 \qquad (5.12)$$

The potential follows directly and is

$$V^x_{L\cdot S} = 1/(2\pi)^6\ (f/\mu)^4\ (3+2\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)/2M \int d^3k\ e^{i(k+k')\cdot r}/\omega^2\omega'^2 \times$$

$$i(\sigma_1+\sigma_2)\cdot(Q \times p)(k'\cdot k) \qquad (5.13)$$

The uncrossed diagram is slightly more difficult to handle but nevertheless straightforward. We must hold on to all terms of order (v^2/c^2), since the denominators will vanish to 0 (v/c). On the one hand we keep in mind the fact that $u^*u = E/M$. This means that

$$u(p) = \quad (E+M/2M) \begin{pmatrix} 1 \\ \sigma\cdot p/E+M \end{pmatrix} \qquad (5.14)$$

which implies that

$$u(p) \approx \quad (1+p^2/8M^2) \begin{pmatrix} 1 \\ \sigma\cdot p/2M \end{pmatrix} \qquad (5.15)$$

The total numerator after some algebra becomes,

$$N^0 = (1-p^2/2M^2) \Big\{ D_1D_2 + k_4/2M[(D_1 + k_4^{\ 2})A_2 + (D_2 + k_4^{\ 2})A_1]$$

$$+ k_4^{\ 2}/4M^2\ A_1A_2 - 1/4M^2[D_1B_2 + B_1D_2] \Big\} \qquad (5.16)$$

where

$$A_1 = [(\sigma_1\cdot p_3)(\sigma_1\cdot Q) - (\sigma_1\cdot Q)(\sigma_1\cdot p_1)]$$
$$B_1 = [(\sigma_1\cdot p_3)(\sigma_1\cdot k')(\sigma_1\cdot p_1)]$$
$$C_1 = [(\sigma_1\cdot p_3)(\sigma_1\cdot p_1)]$$
$$D_1 = [(\sigma_1\cdot k')(\sigma_1\cdot k)]$$

The corresponding quantities with $p_1 \to p_2$, $p_3 \to p_4$, $Q \to -Q$, and $\sigma_1 \to \sigma_2$ we call A_2, B_2, C_2, and D_2. If we now integrate over the fourth component and close our contour in the upper half plane we pick up residues at $k_4 = -\sqrt{k^2 + \mu^2}$, $k_4 = -\sqrt{k'^2 + \mu^2}$, $k_4 \approx \alpha_1 - \alpha_1^2/4M$, and $k_4 \approx -2M$, (where $\alpha_1 = [(p_1\cdot k/M) - (k^2/2M)]$). The resulting potential in momentum space is found easily to be

$$V_4^o = 1/(2\pi)^6\ (f/\mu)^4\ (3-2\tau_1\cdot\tau_2)/2M \int d^3k\ d^3k'\ v^2(k)v^2(k')\ e^{i(k+k'\cdot r)}\ \times$$
$$\Big\{ [i(\sigma_1+\sigma_2)\cdot Q \times p(k'\cdot k)/2\omega^2\omega'^2] - [i(\sigma_1+\sigma_2)\cdot(k' \times k)(p\cdot k)/\omega^2\omega'^2] +$$
$$i(\sigma_1+\sigma_2)\cdot k' \times k(\omega^2+\omega'^2)/\omega^4\omega'^4 +$$
$$(\sigma_1\cdot k)(\sigma_1\cdot k')(\sigma_2\cdot k)(\sigma_2\cdot k')(1 - 3/2\ p^2/M^2)/[\omega^2\omega'^2(k^2+2p\cdot k/2M)]\Big\} \quad (5.17)$$

Two points are quickly ascertained. The term which blows up like $E_i - E_n$, agrees precisely with the iterated second order energy shell potential as was to be expected since the neglect of ϕ^2 contributes nothing to the pole term in fourth. Combining this potential with the term from the uncrossed diagram, it is easily shown that the coefficient of the $\tau_1\cdot\tau_2$ term agrees with the result of the non-relativistic theory.

VI. DISCUSSION AND COMPARISON WITH EXPERIMENT

Nucleon scattering data up to 150 Mev give a reasonable indication of the type of L•S potential needed to fit the data. The data have been fitted quite well by S.M. They have added to the Gartenhaus potential the following isotopic spin independent, phenomenological potential.

$$V_{L\cdot S}(r) = (V_o)/r/r_o \; d/d(r/r_o) e^{-r/r_o}/r/r_o \; L\cdot S \qquad r > r_c$$

$$= V_{L\cdot S}(r)\Big|_{r = r_c} \qquad r < r_c \qquad (6.1)$$

with $r_o = 1.07 \times 10^{-13}$, $r_c = 1/M$, and $V_o = 30$ Mev.

The experimental data at these moderate energies indicate quite conclusively that the potential for the triplet odd states should be attractive. The situation with respect to triplet even states is not conclusive. The addition of a spin orbit potential to the straight Gartenhaus potential immensely improves the agreement with experiment. The major reason for this improvement is that the very large Gartenhaus ^3P_o phase shift has been brought down from 57.3^o to 15.5^o. This is due to the fact that the spin orbit potential supplies a large repulsive core in the ^3P_o state which wipes out the effect of the otherwise deep attractive well in that state (Ref. 2, fig. 1). S.M. finds on the other hand that they could just as well have chosen zero for the potential in the triplet even state.

The theoretical iso-triplet potential in agreement with S.M. is attractive except at short distances where the cutoff adds a repulsive core. However the theoretical potential is considerably smaller than the phenomenological potential of S.M.(fig. 7). However, it is possible, by changing the coupling constant slightly (from 0.10 to 0.12) and increasing the maximum cutoff slightly, to considerably improve the apparent overall agreement. The question of agreement though is not a simple one. In the first place it is evident that the spin orbit potential is not unique. Definite predictions can be made only on the basis of a numerical solution of the coupled Schroedinger equation. Thus one potential may be as acceptable as another one which looks quite different. In support of this statement we point out that any L•S potential derived from meson theory will involve the exchange of at least two mesons. As a result it will always have the asymptotic form of an algebraic function times $e^{-2\mu r}$. The original S.M. potential did not have this behavior. However in a recent issue of Phys. Rev. Let.[16], Signell et al. have shown that a change in the range of their original potential to agree with the theoretical range of $1/2\mu$, results in an improved agreement with the P-P scattering data.

Secondly there remains the open question of how sensitive the predictions at these energies are to change in the static potential presented here.

Otsuki[17] has argued that it is possible by modifying the core of the B.W. and G. potential, to get agreement with the observed data on nucleon-nucleon scattering without the use of a spin orbit potential. The pieces of the static potential arising from the $\tau \cdot \phi \times \dot{\phi}$ term in the Hamiltonian (3.8), (figures 5, 6) considerably modify the Gartenhaus potential and as a result present a number of interesting possibilities in connection with the point we have made. In the iso-triplet state these extra pieces provide a repulsive potential which automatically cuts down on the ^3P_o phase. Thus the L•S potential, for the purpose of cutting down the ^3P_o phase shift does not, apriori, have to be as big as the phenomenological L•S potential of S.M. The iso-triplet state is not of interest in the deuteron since only the ^3S_o and ^3D_o states are involved. However the additional iso-singlet does modify the central potential for the deuteron. In this state it is attractive and may actually improve the predictions of the low energy part of the nuclear potential. G. was forced to increase his coupling constant slightly to get agreement with the observed binding energy of the deuteron.

The L•S piece in the isotopic singlet state, on the other hand, is strongly repulsive in contrast with the results of S.M. This is a definite prediction of the particular way in which we have chosen the pieces of the fourth order potential. If the B.W. argument has been ignored, then the potential would have been attractive in both isotopic states. The fact that it is repulsive, at present, does not seem to be in obvious contradiction with the experimental results on two nucleon scattering.

Another and more qualitative comparison with experiment may be found in the shell model theory of spin orbit splitting in the nucleus.

Calculations to date in this area are very approximate since the nucleon wave functions are not well known. If we write the potential energy of interaction as

$$V = \sum_{\text{pairs } i,j} V_{L\cdot S}(r_i - r_j)(\sigma_i + \sigma_j)\cdot(r_i - r_j) x (p_i - p_j)/4 \qquad (6.2)$$

Then the first order energy splitting for a single particle outside a closed shell is

$$\Delta E = \sum_{\phi} \langle (\phi(1)\psi(2) - \psi(1)\phi(2) | V_{L\cdot S}(r_1 - r_2)(\sigma_1 + \sigma_2)\cdot(r_1 - r_2)/4$$

$$\times (p_1 - p_2) | \phi(1)\psi(2) \rangle \qquad (6.3)$$

The ϕ are the wave functions of the particles within the closed shell. The sum over ϕ includes the sum over all the distinct possibilities for ϕ. That is to say, spin up, spin down, neutron, proton, and various orbital states. ψ is the wave function of the single particle outside the closed shell. In writing equation 6.3 down we have assumed that the spin orbit force acts only between the single particle outside the shell and other particles interior to the shell. We omit all shell-shell interactions, assuming that they do not affect the splitting of the single particle level.

The integral is commonly broken down into a so-called direct integral and an exchange term.

$$\langle f|V|i\rangle_{\text{direct}} = \sum_{\phi} \langle \phi(1)\psi(2)| V_{L\cdot S}(r)\ r \times p|\phi(1)\psi(2)\rangle \cdot \vec{S} \qquad (6.4)$$

$$\langle f|V|i\rangle_{\text{exchange}} = \sum_{\phi} \langle \phi(1)\psi(2)| V_{L\cdot S}(r)\ r \times p|\phi(2)\psi(1)\rangle \cdot \vec{S} \qquad (6.5)$$

The term proportional to $\tau_1 \cdot \tau_2$ in the direct integration sums to zero if we assume that the closed shell has no net isotopic spin. The sum over wave functions in iso-space, if all states of isotopic spin are equally probable, merely takes the trace of τ which is zero.

Blanchard and Avery[18] give an explicit expression for the splitting due to this spin orbit perturbation. For a single particle outside a closed shell they find in a straightforward manner that

$$E_{\ell+1/2} - E_{\ell-1/2} = [V_1(2A-B) - V_2(3B)]/4 \qquad (6.6)$$

Where $V_{L\cdot S}$ is written as

$$V_{L\cdot S}(r) = [V_1 + V_2 \, \tau_1 \cdot \tau_2] J(r) \qquad (6.7)$$

and

$$A = [2(2k+1)(2\ell+1)]/(4\pi)^2 \int [f(r_1)]^2 [g(r_1)]^2 \, J(r)$$
$$[1 - (r_1/r_2)\,\mu] \, d^3r_1 \, d^3r_2 \qquad (6.8)$$

$$B = [2(2k+1)(2\ell+1)]/[\ell(\ell+1)(4\pi)^2] \int f(r_1)\, g(r_2)\, J(r) \times$$
$$\Big\{ \ell(\ell+1)[1-(r_1/r_2)\mu] P_k(\mu) \times P_\ell(\mu) + (1-\mu^2)(1-(r_1/r_2)\mu) P_k{}'(\mu)\, P_\ell{}'(\mu) -$$
$$r_1/r_2\,(1-\mu^2) P_k(\mu)\, P_\ell{}'(\mu) + r_1 r_2 (1-\mu^2) P_k(\mu)\, P_\ell{}'(\mu) \times$$
$$[(1/r_2)(\partial/\partial r_2) - (1/r_1)(\partial/\partial r_1)] \Big\} f(r_2)\, g(r_1) \qquad (6.9)$$

where k and ℓ are the orbital quantum numbers associated with the radial wave functions for f and g respectively and μ is the cosine of the angle between r_1 and r_2.

We choose to evaluate these integrals numerically for only the simplest case. This of course would be He^5. The splitting of the single P state neutron is known to be 3.5 Mev, with the 3/2 state lying lower than the 1/2 state. The properties of the alpha particle are fairly well known. A central harmonic oscillator potential of known strength can be used to describe the average interaction of the particles within the shells. In the heavier nuclei the concept of a definite single particle state is no longer valid and the shell model itself loses some of its validity.

In any case we re-emphasize the fact that our computation was undertaken to ascertain only qualitative results. We hoped to see if the splitting could be reconciled with our potential with a reasonable zero cutoff in coordinate space and a reasonable radius of the alpha particle. We chose harmonic oscillator wave functions for the nucleus taking

$$f = (\alpha/\sqrt{\pi})^{3/2}\sqrt{4\pi}\, e^{-\alpha^2 r^2/2} \tag{6.10}$$

for the nucleons in the closed shell and

$$g = (2\alpha)^2/\sqrt{3}\ \sqrt{\alpha/2\sqrt{\pi}}\ r\, e^{-\alpha^2 r^2/2} \tag{6.11}$$

for the nucleons in the P state. For the purpose of this crude estimate we approximated our two nucleon spin orbit potential by

$$\begin{aligned} V_{L\cdot S} &= (V_1 + V_2\, \tau_1\cdot\tau_2)\, e^{-2\mu r}/(\mu r)^5 \qquad && \text{for } r > r_c \\ V_{L\cdot S} &= 0 && \text{for } r < r_c \end{aligned} \tag{6.12}$$

The resultant splitting is, since B = -A.

$$(E_{3/2} - E_{1/2}) = [V_1 + V_2]\ 0.75A \tag{6.13}$$

where

$$V_1 + V_2 \simeq 20 \text{ Mev}$$

-A is plotted in figure 8 for various values of the cutoff radius, r_c, and the size of the nucleus $1/\alpha$. It is encouraging, although not terribly significant, that agreement with the observed splitting of 3.5 Mev corresponds to the choice $1/\alpha = 1.3/\mu$ and $r_c = 0.3/\mu$, a result not incompatible with the approximately known radius of the alpha particle $1/\alpha \approx 1.2/\mu$ and the cutoff which S.M. used in their calculations of the two nucleon data.

It is well known that the introduction of such a spin orbit potential into the two nucleon potential will change the magnetic moment of the deuteron. The spin orbit potential

$$V_{L\cdot S} = V(r)(s \times r)\cdot(p_1-p_2)/2 \qquad (6.14)$$

introduces an additional coupling to an externally applied electromagnetic field. Taking particle one as the proton and coupling in the E.M. field by making the usual replacement $p \rightarrow p - eA$ we find that the additional electromagnetic energy introduced by $V_{L\cdot S}$ is

$$\Delta\mu_{E.M.} = - e/2\ V(r)(s \times r)\cdot A \qquad (6.15)$$

The contribution of this term to the magnetic moment of the deuteron is the component of

$$(\Delta\vec{\mu})_{L\cdot S} = - e/8\ V(r)(s \times r)\times r \qquad (6.16)$$

in the direction of $\vec{J}$ the total angular momentum of the deuteron and is,

$$(\Delta\mu)_{L\cdot S} = - e/16 \langle | V(r)[(s\cdot r)^2 - r^2(s\cdot J)] | \rangle \qquad (6.17)$$

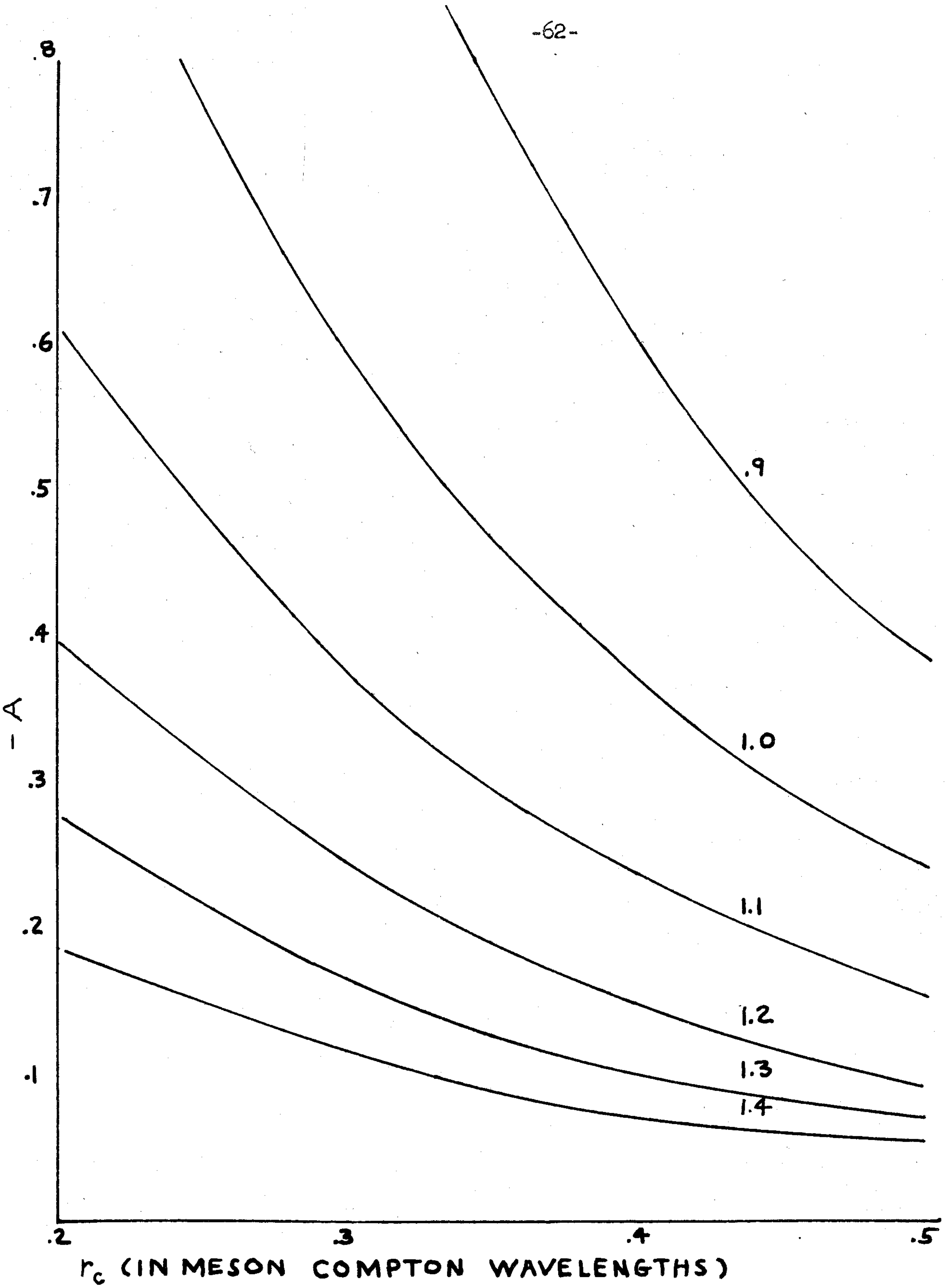

.8
.7
.6
.5
.4
.3
.2
.1
- A
.9
1.0
1.1
1.2
1.3
1.4
.2
.3
.4
.5
r_c (IN MESON COMPTON WAVELENGTHS)

FIG 9

If we now assume that the D-state part of the deuteron wave function is small in the region where r^2V is most important, Feshback[19] shows that

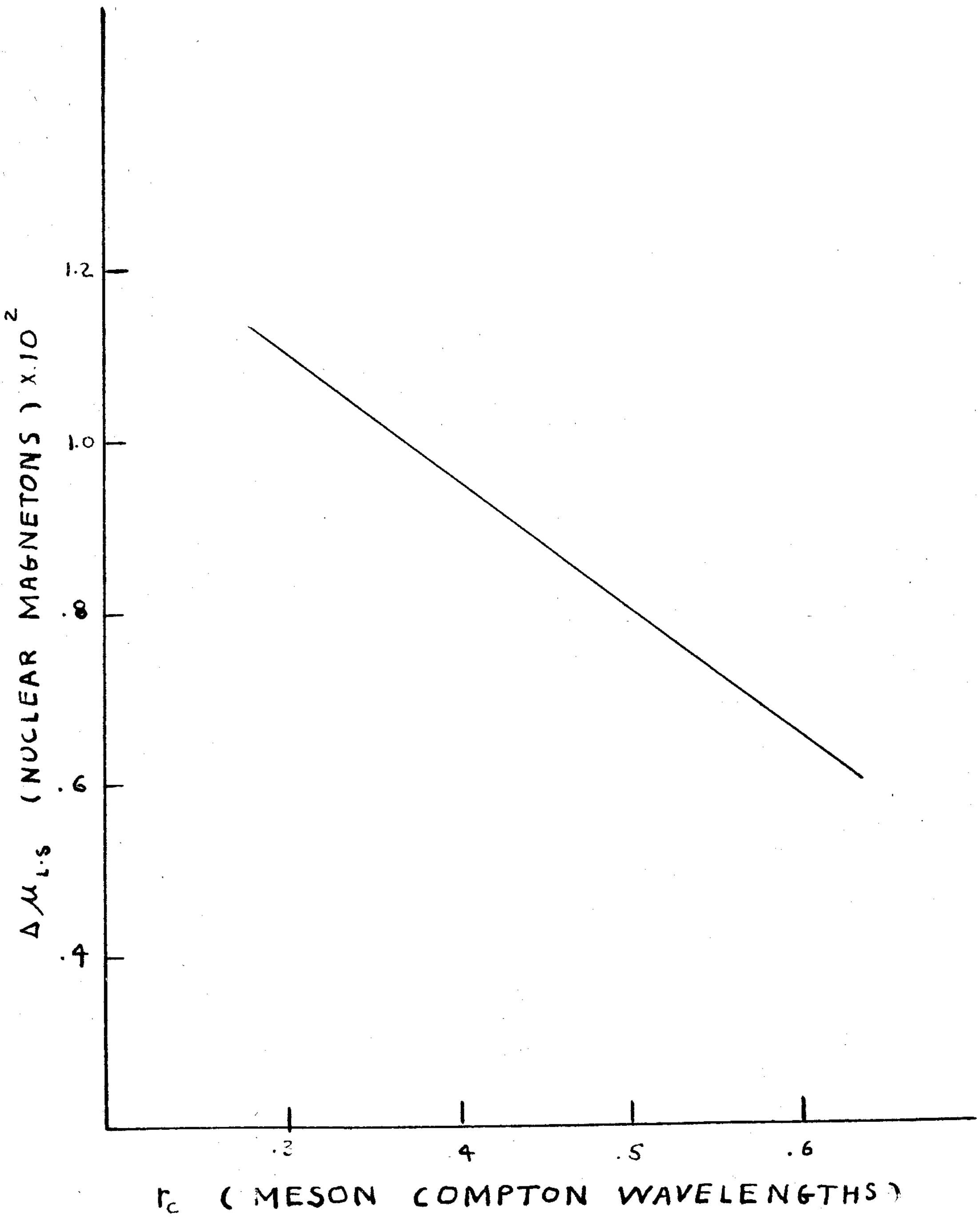
$$(\Delta\mu)_{L\cdot S} = e/12 \langle s | r^2 V | s \rangle \qquad (6.18)$$

Since we are considering the deuteron the appropriate L·S potential is the isotopic singlet. The intrinsically negative potential proposed by S.M. causes a shift in the deuterons magnetic moment of approximately

$$\langle \Delta\mu \rangle_{L\cdot S} = -0.056 \quad \text{Nuclear magnetons} \qquad (6.19)$$

This is an undesirable shift since it precludes the possibility of matching the experimental results even if the probability for finding the deuteron in a D state were reduced to zero (see Feshback).

Detailed calculations taking into account explicit meson and relativistic effects indicate that these corrections are of the wrong sign to compensate for the excessive negative shift. On the other hand our potential in the state is strongly repulsive and gives a shift of the opposite sign. Approximating our potential as in the nuclear force case and performing the integral 6.18, we find a positive shift $(\Delta\mu)_{L\cdot S} \cdot (\Delta\mu)_{L\cdot S}$ is plotted in figure 10 as a function of the zero cutoff in momentum space. To get a reasonable value of the shift one should pick a value of r_c of about $0.5/\mu$ which is rather large. However so many effects remain uncalculated and are apparently of the right sign to compensate for this apparent discrepancy that we do not consider this as an objection to the potential. We do however consider it interesting that a positive potential in the iso-singlet



1.2
1.0
.8
.6
.4
$\Delta\mu_{L\cdot S}$ (NUCLEAR MAGNETONS) x.10^2
.3
.4
.5
.6
r_c (MESON COMPTON WAVELENGTHS)

FIG 10

state seems to be more in line with the observed magnetic moment of the deuteron, a very accurately measured quantity.

APPENDIX I

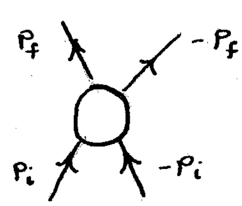
Possible Forms of Interaction

In this discussion we follow closely the procedure outlined by Marshak and Okubo[7] in their recent article. We inquire into the most general form of the two nucleon potentials. If we believe that our system has certain symmetries under transformation in space and isotopic spin space, then our potential is severely limited. As a result we can show that there are, neglecting the isotopic dependence, only five independent forms that the potential can take.

First we note that the strong interactions are charge independent. This implies that the potential is a scalar under rotations in isotopic spin space; and thus can be expressed as a linear combination of 1 and $\tau_1 \cdot \tau_2$. One and two refer to the two nucleons. This determines the form in isospin space and we need not consider this dependence further.

It is certainly true that space is isotropic. This implies that the vectors r_1 and r_2 can enter only in the combination $\vec{r}_1 - \vec{r}_2$. If the potential is to be Galilean invariant the p_1 and p_2 must appear in the form $(p_1 - p_2)$. Thus we can consider scattering in momentum space in the C.M. system.

If we consider the event



There exists two independent momenta

$$Q = p_f - p_i \qquad \text{(AI.1)}$$

$$p' = p_f + p_i \qquad \text{(AI.2)}$$

Now we know that V must be a scalar under reflections and rotations in space, since our interaction presumably conserves parity and angular momentum. Our interaction is also invariant under time reversal. This implies that

$$V(\sigma_1,\sigma_2,Q,p') = V^T(-\sigma_1,-\sigma_2,Q,-p') \qquad \text{(AI.3)}$$

Since $\sigma_1 \to -\sigma_1$, $\sigma_2 \to -\sigma_2$, $p_i \to -p_f$, $p_f \to -p_i$ under time reversal. T means reverse all the operators involved. Continuing we point out that the two nucleons are identical. This implies that the potential must be symmetric under the interchange of $1 \to 2$. This implies

$$V(\sigma_1,\sigma_2,Q,p') = V(\sigma_2,\sigma_1,-Q,-p') \qquad \text{(AI.4)}$$

Owing to the character of the σ operators it is always possible to write the potential as a term which is independent of σ_1 and σ_2, plus another term linear in σ_1 and σ_2, and plus a third term bilinear in σ_1 and σ_2, of the form $(\sigma_1 \cdot A)(\sigma_2 \cdot B)$.

The term independent of σ_1 and σ_2 we call V_1. V_1 is a function of Q^2 and p'^2, $(Q \times p')^2$ and $Q \cdot p'$. However, if we are interested in determining the potential only on the energy shell, then $Q \cdot p = p_i^{\,2} - p_f^{\,2} = 0$ and $(Q \times p')^2 = Q^2 p'^2$. For calculations involving the matrix element of the potential off the energy shell the $(Q \times p)^2$ and $Q \cdot p$ dependence should be considered. This question of on and off energy shell potential essentially reflects the arbitrariness in determining a potential for use in a Schroedinger equation. For our present purposes we consider all functions to be functions of Q^2 and

p'^2 only.

The term linear in σ_1 and σ_2 must be symmetric and thus must be of the form $(\sigma_1+\sigma_2)\cdot(Q \times p')$ if it is to satisfy all our requirements.

The term bilinear in σ_1 and σ_2 can be one of the following four forms.

$$(\sigma_1\cdot p')(\sigma_2\cdot p') \qquad \sigma_1\cdot\sigma_2$$
$$(\sigma_1\cdot Q)(\sigma_2\cdot Q) \qquad (\sigma_1\cdot Q \times p')(\sigma_2\cdot Q \times p') \qquad \text{(AI.5)}$$

However on the energy shell $Q\cdot p = 0$. Hence

$$\sigma_1\cdot\sigma_2 = [(\sigma_1\cdot p')(\sigma_2\cdot p')]/p'^2 + [(\sigma_1\cdot Q)(\sigma_2\cdot Q)]/Q^2 +$$
$$[\sigma_1\cdot(Q \times p')\ \sigma_2\cdot(Q \times p')]/\ Q^2 p'^2 \qquad \text{(AI.6)}$$

since p, Q and Q x p form an orthogonal coordinate system. Thus we may get rid of one of our four forms and the potential must be of the form

$$V = (A+B\,\tau_1\cdot\tau_2)[V_1(Q^2,p'^2) + V_2(Q^2,p'^2)L\cdot S + V_3(Q^2,p'^2)\sigma_1\cdot\sigma_2 +$$
$$V_4(Q^2,p'^2)(\sigma_1\cdot p')(\sigma_2\cdot p') + V_5(Q^2,p'^2)(\sigma_1\cdot Q)(\sigma_2\cdot Q)] \qquad \text{(AI.7)}$$

APPENDIX II

Useful Integrals

We consider the evaluation of certain integrals which appear in the body of the thesis.

1. For the functions

$$M_n(r) = \int e^{i(k+k)\cdot r} (k'\cdot k)^n \, d\Omega_k \, d\Omega_{k'} \qquad \text{(AII.1)}$$

we have, using the fact that,

$$e^{ik'\cdot r} = \sum_{\ell=0}^{\infty} (-i)^{\ell} (2\ell+1) P_{\ell}(\cos\Theta)(1/r \, \partial/\partial r)^{\ell} \sin kr/kr \qquad \text{(AII.2)}$$

and

$$P_m(x) = P_m(\cos\Theta)P_m(\cos\Theta') + \sum_{n=1}^{m} 2(m-n)!/(m+n)! \, P_m^{\,n}(\cos\Theta)$$

$$P_m^{\,n}(\cos\Theta') \cos(n\phi - n\phi') \qquad \text{(AII.3)}$$

where x is the cosine of the angle between Θ and Θ'. In addition $(k'\cdot k)^n$ may be expanded as a series of Legendre polynomials

$$(k'\cdot k) = \sum_m a_m P_m(x) \qquad \text{(AII.4)}$$

From this it immediately follows that

$$M_n(r) = (4\pi)^2 \sum_m a_m \left[(r/k)^m (1/r \, d/dr)^m \sin kr/kr\right]$$

$$\left[(r/k')^m (1/r \, d/dr)^m \sin k'r/k'r\right] \qquad \text{(AII.5)}$$

2. For integrals of the form

$$N(r) = \int e^{i(k+k')\cdot r} \sigma_1\cdot(k' \times k) \, \sigma_2\cdot(k' \times k) \, d\Omega_k \, d\Omega_{k'} \qquad \text{(AII.6)}$$

It is easy to perform the integration by differentiation with respect to r and subsequent contraction of the indices. Taking N(r) as an example we find

$$N(r) = \sigma_1^{\ i}\ \sigma_2^{\ j}\ \epsilon_{ijk}\ \epsilon_{\ell mn} [\partial/\partial x_i\ \ \partial/\partial x_m \int e^{ik\cdot r}\ d\Omega_{\underline{k}}]$$

$$[\partial/\partial x_k\ \ \partial/\partial x_n \int e^{ik'\cdot r}\ d\Omega_{k'}] \qquad \text{(AII.7)}$$

3. For integrals over the magnitudes of k the following relations are useful

$$K_n(\mu r) = \Gamma(n+1/2)/\pi^{1/2}\ (2\mu/r)^n \int_0^\infty \cos kr\ dk/(k^2+\mu^2)^{n+1/2} \qquad \text{(AII.8)}$$

where $K_n(\mu r)$ are the Hankel functions of imaginary arguments. In addition the following integral is useful.

$$F(k_2 r) = \int \sin k_1 r/(k_1^{\ 2}-k_2^{\ 2})k_1 dk_1 = \pi/2 \cos k_2 r \qquad \text{for } r > 0 \quad \text{(AII.9)}$$

Using this identity and the K function integral it is possible to evaluate, for example, integrals of the form,

$$A_1(r) = \int \sin k_1 r\ \sin k_2 r/\omega_1\omega_2(\omega_1+\omega_2)\ \ k_1 k_2\ dk_1\ dk_2 \qquad \text{(AII.10)}$$

$$A_2(r) = \int \sin k_2 r\ \sin k_1 r/\omega_1^{\ 2}\omega_2^{\ 2}(\omega_1+\omega_2)\ k_1 k_2\ dk_1\ dk_2 \qquad \text{(AII.11)}$$

For the A_1 case we multiply numerator and denominator by $(\omega_1-\omega_2)$.

$$A_1(r) = \int_0^\infty\int_0^\infty (1/\omega_2 - 1/\omega_1)\ (\sin k_1 r\ \sin k_2 r)/(k_1^{\ 2}-k_2^{\ 2})dk_1\ dk_2$$

$$= 2 \int_0^\infty \sin k_2 r/\omega_2\ F(k_2 r)\ k_2 dk_2$$

$$= \pi \int_0^\infty \sin k_2 r\ \cos k_2 r/\omega_2\ \ k_2 dk_2 = \frac{\pi\mu}{2} K_1(2\mu r) \qquad \text{(AII.12)}$$

On the other hand, A_2, the integral which appears in the L·S piece arising from the ϕ^2 term, is easily evaluated

$$A_2(r) = \iint \sin kr \sin k'r/\omega^2\omega'^2(\omega+\omega') \quad kk' \, dk \, dk' \qquad \text{(AII.13)}$$

Now the integral

$$\int k' \sin k'r \, dk'/\omega'^2(k^2-k'^2) = \pi/2 \, [-\cos kr/\omega^2 + e^{-\mu r}/\omega^2] \qquad \text{(AII.14)}$$

This implies using the K function identities that

$$A_2(r) = \pi/2\mu \, (\partial/\partial x)x \, K_1(2x) + e^{-x}/\mu \, (\partial/\partial x)x(\partial/\partial x)K_0(2x) \qquad \text{(AII.15)}$$

where

$$x = \mu r.$$

APPENDIX III

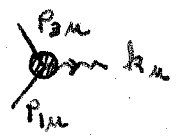
Comparison of Complete Squared Theory and Linear γ_5 Theory

With the rules given in section III, it is possible to calculate the matrix element for any given process using the complete squared theory. The question may obtrude itself on the reader's mind as to the complete equivalence of the theory and the local γ_5 form of the theory. Since we are dealing, in the former case, with a second order equation, and since we are still using free four component Dirac spinors as our unperturbed wave functions it would seem that we have doubled the degrees of freedom in the problem. Although it is indeed true that all solutions of the linear Dirac equation are solutions of the squared equation the reverse is of course not true. There will undoubtedly be solutions of the second order equations which are not solutions of the first order one. Evidently some sort of subsidiary condition will be needed in the calculation of the most general matrix element. It is not our desired aim to understand the nature of these conditions nor the intricacies of defining a consistent field theory using the complete squared equation. However we will indeed be able to show that for the case of the nuclear force problem, at least to sixth order, the two theories are explicitly equivalent. As a matter of fact it will become almost evident that as long as the diagram under consideration does not involve a completely closed nucleon loop that there is no ambiguity. The fact that the nucleon line is at one point free provides a sort of initial condition which gives us the correct solution.

For the purpose of comparison we will deal exclusively with one half of the nuclear force diagrams. That is to say we cut the diagrams down the middle (see fig. 2) and look at a single nucleon line. Thus, first order for the single nucleon line contributes to the second order nuclear force; second order to the fourth order force and third order to the sixth order force.

We denote the quantity proportional to the linear γ_5 theory matrix element by (PS) and the quantity proportional to the squared theory by $(PS)^2$.

The first order single nucleon line is trivial



$$(PS) \sim g\, \bar{u}(p_{3\mu})\gamma_5\, u(p_{1\mu})\mathcal{T}_i \qquad (AIII.1)$$

$$(PS)^2 \sim (g/2M)\, \bar{u}(p_{3\mu})\gamma_5\, \not{k}\, u(p_{1\mu})\, \mathcal{T}_i \qquad (AIII.2)$$

since

$$\not{k} = \not{p}_1 - \not{p}_3 \qquad (AIII.3)$$

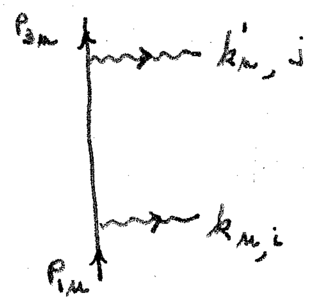
by anti-commuting $\not{p}_3$ through the γ_5 and noting that

$$\not{p}\, u(p) = M\, u(p) \qquad (AIII.4)$$

we find that

$$(PS)_1 = (PS)_1{}^2 \qquad (AIII.5)$$

This is precisely the demonstration of the equivalence of pseudo-scalar and pseudo-vector in lowest order. The second order diagram is slightly more difficult but nonetheless reduces trivially. For the (PS) theory we have the diagram

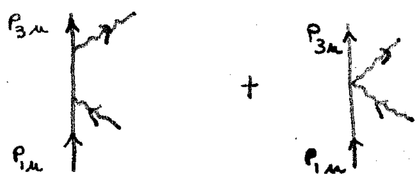


and the corresponding matrix element

$$(PS)_2 \sim g^2 \, \bar{u}(p_{3\mu})\gamma_5 \, (1/(\not p_1 + \not k - M))\gamma_5 \, u(p_{1\mu}) \, \tau_j \, \tau_i$$

$$= g^2 \, \bar{u}(p_{3\mu}) \not k \, u(p_{1\mu})/(k_\mu^2 + 2p_{1\mu}\cdot k_\mu) \, \tau_j \, \tau_i \qquad \text{(AIII.6)}$$

On the other hand the $(PS)^2$ theory has two diagrams



This implies that

$$(PS)^2_2 \sim (g^2/2M) \, [\bar{u}(p_{3\mu}) \, (-\not k' \not k/(k^2 + 2p_{1\mu}\cdot k_\mu) + 1)u(p_{1\mu})] \, \tau_j \, \tau_i \qquad \text{(AIII.7)}$$

since

$$\not k' = \not k - \not q$$

and

$$\not q = \not p_3 - \not p_1$$

We find using some simple γ algebra that

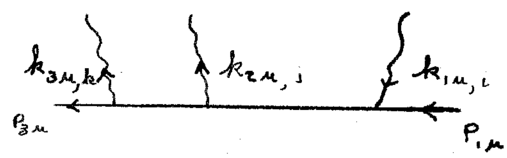
$$-\not k' \not k + k_\mu^2 + 2p_{1\mu}\cdot k_\mu = +2M \not k \qquad \text{(AIII.8)}$$

This implies that

$$(PS)^2_2 \sim g^2 \, \bar{u}(p_{3\mu}) \not k \, u(p_{1\mu})/(k_\mu^2 + 2p_{1\mu}\cdot k_\mu) = (PS)_2 \qquad \text{(AIII.9)}$$

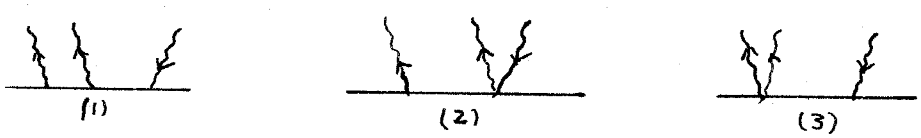
Of course there is a corresponding series of diagrams for the crossed case. The third order diagram is much the same. We outline it here briefly. We consider only one given time ordering for the emission and absorption of the three mesons. The time orderings are given a unique isotopic dependence and thus must agree one for one in both theories.

We consider for the γ_5 theory the diagram



$$(PS)_4 \sim g^3 \frac{\bar{u}(p_{3\mu})\gamma_5 \not{k}_3 \gamma_5 \not{k}_1 u(p_{1\mu})}{[k_{3\mu}^2+2p_{3\mu}\cdot k_{3\mu}][k_{1\mu}^2+2p_{1\mu}\cdot k_{1\mu}]} \tau_k, \tau_j \tau_i \quad \text{(AIII.10)}$$

For the squared theory we consider the three diagrams



We label the matrix elements in the squared theory for this case by their figure numbers

$$(1) \sim g^3/2M\, \bar{u}(p_{3\mu})\gamma_5 \not{k}_3 \gamma_5 \not{k}_1 (1/A_3A_1)\, u(p_{1\mu}) \; \tau_k \tau_j \tau_i$$

$$(2) \sim -g^3/2M\, \bar{u}(p_{3\mu})\gamma_5 \not{k}_3 A_1 (1/A_3A_1)\, u(p_{1\mu}) \; \tau_k \tau_j \tau_i$$

$$(3) \sim +g^3/2M\, \bar{u}(p_{3\mu})\gamma_5 \not{k}_1 A_3 (1/A_1A_3)\, u(p_{1\mu}) \; \tau_k \tau_j \tau_i \qquad \text{(AIII.11)}$$

where

$$A_1 = k_{1\mu}^2 + 2p_{1\mu}\cdot k_{1\mu}$$

$$A_3 = k_{3\mu}^2 + 2p_{1\mu} \cdot k_{3\mu}$$

and noting that

$$\gamma_5 \not{k}_3 \gamma_5 \not{k}_2 \gamma_5 \not{k}_1 = [\not{k}_3 (\not{p}_{1\mu} - \not{p}_{3\mu}) \gamma_5 \not{k}_1 + \not{k}_3 (\not{k}_1 - \not{k}_3) \gamma_5 \not{k}_1]$$

$$= 2M \not{k}_3 \gamma_5 \not{k}_1 + A_1 \gamma_5 \not{k}_3 - A_3 \gamma_5 \not{k}_1 \qquad \text{(AIII.12)}$$

Using this fact and combining (1)+(2)+(3) we find

$$(PS)_4^2 \sim g^3 \, \bar{u}(p_{3\mu}) \not{k}_3 \gamma_5 \not{k}_1 \, (1/A_3 A_1) \, u(p_{1\mu}) \gamma_k \gamma_k \gamma_i = (PS)_4 \qquad \text{(AIII.13)}$$

APPENDIX IV

Galilean Invariance

In this appendix we give a brief discussion of the lack of restrictions imposed on the coefficients of the Hamiltonian 3.7 by the requirement that the theory be Galilean invariant. We use in this section a number of arguments suggested by several discussions with Dr. R. P. Feynman.

Our Foldy reduced theory to a given order in $(v/c)_{nucleon}$ must be a Galilean invariant since it originates from a relativistically invariant theory. This implies that under the transformation;

$$x' = x - vt \tag{AIV1}$$

$$t' = t \tag{AIV2}$$

$$\psi' = \psi \times \exp(imvx' - imv^2/2\, t') \tag{AIV3}$$

and the requirement that c be allowed to approach to infinity that the Hamilton equations of motion are the same in the new system as in the old. We assume that the meson field operator ϕ transforms like a scalar.

Suppose we now rewrite the Hamiltonian 3.7 with the c's and $\hbar$'s reinstated. It is:

$$H' = Mc^2 + (p^2/2M) + (g/2Mc)\, \sigma\cdot\nabla\phi + (g^2\phi^2/2Mc^2) + (\hbar^2 g^2 \dot{\phi}^2/8M^3c^6) + g^2\hbar\, (\boldsymbol{\tau}\cdot\phi \times \dot{\phi}/4M^2c^4) - (1/8M^3c^6)\, [(\sigma\cdot p - ig\phi)(\sigma\cdot p + ig\phi)]^2 \tag{AIV4}$$

Suppose we now scale the quantity ϕ. We relabel the quantity $g\phi/c$ as ϕ. ϕ now has the dimensions of momentum and no more scaling can be done. If c is now allowed to go to infinity only the first four terms remain. The fact that Mc^2 is infinite merely readjusts our zero of energy. Under the transformation AIV1 - AIV 3

$$(\partial/\partial x')_{t'} = (\partial/\partial x)_t \tag{AIV5}$$

$$(\partial/\partial t)_x = (\partial/\partial t')_{x'} - v\cdot(\partial/\partial x')_{t'} \tag{AIV6}$$

and the first four terms may easily be shown to be Galilean invariant. The argument that the theory be Galilean invariant does not give us a simple relation between the $[\sigma\cdot p, \dot{\phi}]_+$ term and the $(\sigma\cdot\nabla\phi)$ term.

The essential physical point involved here is that we are coupling relativistic mesons to static nucleons. The $[\sigma\cdot p, \dot{\phi}]_+$ term is always a small correction to the entire Hamiltonian. If the mesons energy $\dot{\phi}$ is the order of Mc^2 then the term $[\sigma\cdot p, \dot{\phi}]_+$ is the order of $v_{nucleon}$. However the term $\sigma\cdot\nabla\phi$ is of the order of c and is still dominant. At low meson energies the $[\sigma\cdot p, \dot{\phi}]_+$ term is the order of $\mu^2/M^2 \sim (v/c)^2_{nucleon}$; and Galilean invariance can not give us information about this term.

We can see all this in a second way. Our mesons can be relativistic. Therefore our coupling should be of the form $\sigma\cdot(\vec{v}_{meson} - \vec{v}_{nucleon})/(1 - \vec{v}_{meson}\cdot\vec{v}_{nucleon}/c^2)$ since we must use the relativistic addition of velocities. Now $(v/c)_{nucleon} \ll 1$. Therefore the coupling may be expanded and approximated by

$$\sigma\cdot[\vec{v}_{meson} - \vec{v}_{nucleon}(1 - \vec{v}^2_{meson}/c^2)] \tag{AIV7}$$

Thus within the framework of this simple physical picture the coefficient of the $v_{nucleon}$ term may have any value between zero and one. Only in the limit of non-relativistic nucleons is the constant one.

We can tie this argument down in still a third way. Suppose we consider the relativistic invariant theory

$$[\alpha\cdot p + \beta M + a\beta\gamma_5 \boldsymbol{\tau}\cdot\phi + b\beta\gamma_\mu\gamma_5\partial_\mu \not\phi + c\beta\not\phi^2 + d\beta\gamma_\mu \boldsymbol{\tau}\cdot\phi \times \partial_\mu \phi]\,\Psi = i(\partial/\partial t)\,\Psi \qquad \text{(AIV8)}$$

Lorentz covariance of course puts no restrictions at all on the magnitudes of the coefficients a, b, c, d. With theory as a starting point it is then possible to show that after Foldy reduction, that the four arbitrary parameters a, b, c, d, introduce enough flexibility into the coefficients of the four terms $\sigma\cdot\nabla\not\phi$, $\not\phi^2$, $(\boldsymbol{\tau}\cdot\phi \times \dot{\phi})$ and $[\sigma\cdot p, \dot{\not\phi}]_+$ in the reduced Hamiltonian so as to allow fixing them at perfectly arbitrary values.

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