AN INK RECORDING OSCILLOGRAPH FOR STEADY STATE A.C. PHENOMENA IN THE AUDIO RANGE

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Summary

An oscillograph has been designed and constructed which will reproduce in ink on paper tape the wave form of steady state a.c. voltages or currents in the frequency range 50 to 25,000 cycles. The tracing is done at a speed of approximately 1 cycle in 3 seconds, and represents a recording of the instantaneous voltage or current progressively at different phase positions in the cycle.

The principle of the device is that of the Rosa curve tracer, but operation has been made automatic by the introduction of several new features, including an electrical phase shifter, a new type of electronic switch, and a recording milliammeter.

The sources of error in the apparatus are analyzed and are shown to be reasonably small in magnitude and the error capable of further reduction.

An appendix describes the use of the instrument as an harmonic analyzer.

It is an easily understood and well known fact that by repeatedly establishing momentary connection between a voltage measuring device and a steady a.c. voltage source at the same phase position during each cycle, a reading of the measuring device approximately proportional to the instantaneous value of the a.c. voltage will be obtained. Such repeated contact at the same phase position can be obtained if switching is accomplished by a contact-making device coupled mechanically to the voltage source or to a machine running synchronously with it. The circuit of such a device is shown in figure 1.

If the phase position at which the contact is made be changed by known increments, and voltage readings taken, repeatedly, until a total change in phase of 360° has been accomplished, a graph of wave form of the a.c. voltage may be obtained by plotting voltmeter readings vs. phase angle.

This is essentially the principle of the Rosa curve tracer, as described by Edward B. Rosa in 1898.[1)] Rosa succeeded in making his instrument semi-automatic by providing a mechanical arrangement which advanced a sheet of coordinate paper along the "phase angle" axis as the phase shift was accomplished by orienting the brush of the contact making device. A stylus moving along the "voltage" axis of the coordinate paper was connected to the slider of a potentiometer voltage measuring device. When the slider had been moved to obtain a balance between the instantaneous value of the a.c. source at any given phase position, the stylus was depressed and left a mark on the coordinate paper. A series of such operations at different phase positions yielded enough points to allow plotting of an accurate curve of wave form.

N. E. Bonn published material on a closely similar device in 1928.[2]

W. F. Westendorp described in 1935[3] a device using the same method of recording, but provided with an electronic rather than a mechanical switch, and an electrical phase shifter for advancing the phase position of the switching impulse. The operation of this device was confined to 60 cycles.

C. Nebel published, in 1934[4], details of a similar device using electronic switching. This device is fully automatic, but abandons mechanical recording for photographic recording at frequencies low in comparison to the frequency of the a.c. source. The important feature of Nebel's device is that the recording film is advanced at a constant rate, while the phase position of the switching operation is being advanced at a rate proportional to the supposedly constant difference between the frequency of the a.c. source and that of the oscillator-controlled switching operation. If the advance in phase is truly linear, a true record of the wave form of the a.c. source may be automatically obtained.

The device developed jointly by the author and by Dr. F. W. Maxstadt of the California Institute had its inception in the work of Mr. Fred Kroger, who prior to 1917 constructed a device in which the phase advance was obtained through operating the contact-making switch at a constant speed with a d.c. motor. Such a speed of operation was chosen that an advance of a few degrees in phase position of the switching operation with respect to the a.c. source was made between switching operations. As a voltage measuring device a syphon recorder was used. The tape was run at a uniform speed and a record of the wave form of the a.c. source was thus traced automatically.

The device as Mr. Kroger originally developed it had one serious fault, however; as the speed of the d.c. motor was in no way dependent on the frequency of the a.c. source, changes either in motor speed or frequency of source would change the rate of phase advance of the switching operation. A satisfactory ratio between recording frequency and original is probably 10^{-2} to 5×10^{-5} depending considerably upon the original frequency.

It is easily seen that the frequency of recording is the difference between the frequency of the a.c. source and that of the switching operation. Thus, assuming a 50 cycle a.c. source and the recording frequency to be 1/2 cycle per second, if the frequency of the switching operation is held constant a change of frequency of the source of 1% will give 100% change in the frequency of recording. If the frequency of the switching operation is not constant, changes in it will cause similarly large changes in the frequency of recording.

Such a source of variation of recording frequency introduces grave difficulties at 50 cycles, where a constancy of frequency of the a.c. source and of the switching operation each to within 0.01% is needed to assure a constancy of recording frequency to within 2%. This is surely not possible when the source is a small, independent a.c. generator. Moreover, the difficulty increases with the rise in frequency of the a.c. source and hence of the switching operation, for were the a.c. source to have a frequency of 5,000 cycles, this frequency and the frequency of the switching operation would have to be constant to within 0.0001% to keep a recording frequency of 1/2 cycle per second constant to within 2%.

Increasing the recording frequency as in the device of Nebel of course mitigates this difficulty, but such an increase of recording frequency necessitates photographic rather than mechanical recording and results in the loss of the chief advantage of such a device. A far better solution is to make the frequency of the switching operation directly dependent on that of the a.c. source.

Maxstadt accomplished this in 1933 by operating the switching device with an induction motor connected to the same a.c. source which furnished the wave it was desired to record. The slip of the motor which was not carrying a load furnished a constant difference between the frequency of the a.c. source and that of the switching operation, which difference was the recording frequency.

Maxstadt also provided the device with a d.c. amplifier which made it sensitive to very small voltages. The device now took the form shown in figure 2.

The switch is seen to be attached to the induction motor and makes contact once a revolution, when the condenser is charged to the instantaneous voltage of the source at that phase position. The condenser voltage, which remains virtually constant between switching operations, forms the input of the d.c. amplifier whose output actuates a modified syphon recorder. The syphon recorder makes a record on a uniformly moving strip of paper. In practice contact may be made actually only once each 2, 3 or n cycles, depending on the number of poles of the induction motor, without seriously affecting the operation of the device.

This device is very satisfactory for power work, and is capable of responding to a few millivolts while taking only a few hundred watts from the actual source being studied. It has three limitations, however; first, sufficient power to drive an induction motor must be available at exactly the same frequency as that of the a.c. source; second, if the frequency is greatly changed, a new induction motor must be supplied, a real difficulty for high frequencies; third, mechanical contacts fail to provide, at high frequencies, a switching operation accurately timed enough and of sufficiently short duration. For instance, at 5,000 cycles 5 electrical degrees correspond to approximately 3 microseconds, and if the recording were to be accurate to within 5 degrees on the phase displacement axis, the entire switching operation would have to take place within 5 microseconds and at a timing accurate to within such an amount. Consistent mechanical operation with such accuracy is not to be expected, although Maxstadt has obtained some detail at frequencies as high as 8,000 cycles, using silver contacts.

It was the author's purpose in continuing the work on a device operating on the above described principles to develop an oscillograph meeting the following criteria:

(1) That it should record mechanically and automatically on paper.

(2) That it should be very sensitive and cause no appreciable drain on the a.c. source to be recorded.

(3) That it should operate satisfactorily over a wide range of frequencies, say from 50 to 25,000 cycles.

(4) That it should be reasonably accurate and constant in calibration over the entire frequency range.

Such a device would, of course, be subject to the important limitation that the a.c. source must be truly periodic.

The implication of the criteria above will be seen to be:

From (1), that the recording must be slow, and hence, as has been explained, the frequency of the switching operation must be directly related to the frequency of the a.c. source.

From (2), that in synchronizing the switching operation with the frequency of the source, amplifiers must be used.

From (3), that mechanical switches must be abandoned.

From (4), that as far as possible, complicated circuits which could change calibration with frequency must be avoided, and everything intervening between the a.c. source and the recorder must be as simple as possible.

The author believes that the oscillograph to be described meets these requirements.

As has been explained, the frequency of recording is the difference between the frequency of the switching operation and the frequency of the a.c. source. If an amplified input having the frequency f_s of the a.c. source is applied to a phase shifter which operates uniformly to cause a complete 360° phase shift to take place with a frequency of f_r times per second, the output frequency of the phase shifter will be $f_o = (f_s \pm f_r)$, which is a suitable frequency for the switching operation if f_r is to be the frequency of recording.

Looking at the matter in another way, if the switching operation is synchronized with the output of such a phase shifter, then the switching operation will occur at different phase positions along the a.c. source as the phase shifter is operated, and after 360° of operation the original phase position of the switching operation with respect to the a.c. source will have been regained and another cycle of operation will commence.

Thus it is seen that by synchronizing the switching operation with the output of a phase shifter whose input has the frequency of the a.c. source, satisfactory operation will be assured.

A suitable phase shifter can be constructed using the circuit shown in figure 3. Here R_1 is a center tapped resistance. R_2 is a variable resistance and C a condenser. The vector diagram for this circuit, assuming no current is drawn at the output, is as shown in figure 4.

V_r and V_c are necessarily at right angles, and have a constant vector sum, V_{ab}, the input voltage. Thus the triangle formed by V_r, V_c, and V_{ab} can be inscribed in a semi-circle. Hence the output voltage, V_{cd}, is constant in magnitude.

It is seen the $\tan \theta/2 = V_c/V_r = R/X_c$ or, $R = X_c \tan \theta/2$.

If R is a variable resistance constructed in such a manner that a uniform rotation of a contact arm causes R to vary as the tangent of $\phi/2$, where ϕ is the mechanical angle the contact arm makes with the position where R is zero, and if R is made of the proper magnitude, we may say:

$$R = X_c \tan \phi/2 = X_c \tan \theta/2$$

$$\text{or, } \phi = \theta$$

Thus under these conditions a mechanical rotation of the contact arm through an angle θ causes an electric phase shift of θ degrees.

While the simple circuit shown above will allow phase shifts of only 180°, a circuit consisting essentially of two such phase shifting devices, which is shown schematically in figure 5 and the operating details of which are shown in figure 6, allows any phase shift without interruption of output. In the actual oscillograph this arrangement is used, and a condenser decade box is provided so that X_c may be made the same for all frequencies. In the oscillograph the phase shifter and the paper on which the wave is recorded are driven by mechanically coupled devices, assuring that a given advance in phase angle shall correspond to a definite linear advance on the record obtained.

As has been stated, it is necessary that the output of the phase shifter be unloaded. This may be accomplished by connecting it to the grid of a vacuum tube amplifier whose output may then control the switching operation.

It is also necessary that the input to the phase shifter be sinusoidal, since this was assumed in drawing the vector diagram. As the a.c. source will not in general be sinusoidal, merely supplying an amplifier from the source and using its output as an input to the phase shifter is not suitable. A simple resonant filter in the amplifier will, however, provide a sufficiently sinusoidal output to assure quite accurate operation of the phase shifter.

The switch which is closed once a cycle to establish contact between the a.c. source and the voltage measuring device cannot be of a mechanical nature, as has already been explained. It is possible that a switch using a gaseous tube of some nature could be devised, but the device most readily adapted to the purpose seems to be the high vacuum tube. Two possible types of circuit seem to suggest themselves, and will be discussed and compared.

The first type is not really a true switch at all, but rather a sort of combination switch and amplifier. If a triode is connected as shown in figure 7a and biased below cutoff so that normally no current will flow, and if a trigger voltage impulse is applied between terminals aa each cycle such that the grid potential will be momentarily driven well above cutoff, then the average plate current flowing through meter A will be roughly proportional to the sum of the voltage applied to aa and the instantaneous voltage of the a.c. source at the time when the trigger voltage is applied. A modification of this circuit is shown in figure 7b, in which a multi-electrode tube is used and the trigger voltage which allows plate current to flow momentarily is applied to another electrode than that to which the a.c. source is applied. The circuit in figure 7b is essentially that used by Nebel and Westendorp in the devices previously mentioned.

Such a circuit has one serious objection; the plate current is dependent on the magnitude and duration of the trigger voltage impulse as well as on the instantaneous voltage of the a.c. source. This feature is objectionable if the oscillograph using the device is to be adjusted for various frequencies, since it may not be possible to keep the magnitude of the trigger voltage or its electrical angular duration with respect to the original wave constant with frequency. For this reason the above mentioned switch, or substitute for a switch, was considered unsatisfactory.

The switch actually used was based on an idea of Kroger's and was developed by the author. It is a true switch, and, by using it, the effect of magnitude and duration of the trigger impulse on the oscillograph's calibration are eliminated.

In this switch the vacuum tubes utilized are connected as shown in figure 8a. The operation is effectively the same as would be that of the mechanical relay shown in figure 8b, where R represents an appreciable but unavoidable resistance due to the use of vacuum tubes.

Referring to figure 8a, the operation is as follows: The tubes are normally biased with the grids considerably negative, so that voltages of the order of 1 volt or less applied between terminals T1 and T2 will cause no current to flow. When, however, an impulse is applied to terminals aa such that the grids are momentarily driven positive, terminals T1 and T2 appear to be connected by a resistance. This resistance is found to be practically constant with voltage between T1 and T2 up to at least 1 volt, and with voltages of the grids between about 2.5 volts and 15 volts positive. While this constancy is desirable, it should of course be remembered that variations in the resistance of the switch have no influence on the calibration of the oscillograph, since it is important only that the switch provide an electrical path which will allow sufficient current to flow that the condenser may attain the same potential as the source in the period the switch is closed.

Figure 9 shows a curve for the plate current of a single type 75 tube with a constant positive grid voltage of 10 volts and a variable plate voltage. Figure 10 shows Ip vs. Ep as calculated for two tubes arranged as in figure 6a from data obtained for one tube. To obtain the curve for two tubes one merely takes the values for such a curve as figure 9 and subtracts values of Ip for negative values of Ep from values of Ip for corresponding positive plate voltages. The curve obtained must of course be symmetrical about the origin.

For a 75 tube the "plate resistance" under such conditions, or the resistance of the switch may be seen to be from 1,000 to 1,200 ohms, depending on the grid voltage. Similar curves for other tubes show resistances as follow:

Tube type	Approx. Resistance, two tubes
85	3,000
56	1,300
75	1,200
38, screen voltage 4 1/2 volts	700
38, screen voltage 22 1/2 volts	600
38, screen tied to plate	500
24a, screen tied to plate	500

In general, high mu tubes and screen grid tubes with screen tied to plate seemed superior to low mu tubes.

There is a certain net current circulating around the tube circuit with no plate voltage. If the tubes are unbalanced this will cause net voltage across the switch when there is no net current through the switch as a whole. Unsteadiness in this voltage due to changes in "trigger voltage" may introduce variations into the input of the d.c.

amplifier and hence spoil the voltage measurements. For this reason it is desirable to keep this circulating current as small as possible. For various tubes this unwanted current with zero plate voltage and a grid voltage of plus 5 volts is:

Tube	Measured Plate Milliamperes Plate voltage zero; grid voltage +5.0
38, screen voltage 22 1/2 volts	1.40
35, screen tied to plate	.42
24a, screen tied to plate	.32
38, screen voltage 4 1/2 volts	.22
56	.10
75	.08
85	.03

It is seen here that the screen grid tubes appear at a disadvantage. It also seems likely that greater variations of manufacture may exist in the more complicated screen grid tubes.

It was felt that the 75, or its 2 1/2 volt equivalent, the 2A6, and the 24a were the tubes best adapted for switching purposes, and oscillographs embodying each worked satisfactorily.

As the tubes in the switch must be biased negative most of the time, either batteries or some self bias device must be provided. Since the grids are driven positive during application of the trigger impulse, a grid leak shunted by a large capacitance and inserted in place of the C batteries shown in figure 8a will provide a negative grid bias. The bias provided, however, differs with the characteristics of the tubes and with the amplitude and duration of the trigger voltage, and it was thought best in practice to assure an adequate bias by the use of C batteries. As the grid current flows against the e.m.f. of the C battery its life will be long.

As a means of providing a trigger voltage impulse synchronized with the output of the phase shifting device, a relaxation oscillator using a type 885 gaseous discharge tube was used. This relaxation oscillator is simpler than the multivibrator type, and is satisfactory for the range of frequencies covered. The fundamental circuit is shown in figure 11. The condenser C is charged through the resistance circuit marked R_1, which acts as a resistance. When a certain potential has been reached, depending on the voltage of battery B, the C bias of the tube, the resistance R_1, and the capacitance of C, the tube becomes conducting, and the condenser discharges through the transformer primary, which has an inductance L. The oscillation can persist only while current is flowing in one direction, or during a half cycle. One would expect the wave forms of the current and voltage in the transformer primary to be as shown in figure 12 and tests with a cathode ray oscillo-scope prove this to be true.

The switch will be "closed" during only the quarter cycle when the voltage is positive. Knowing the desired period of closure, Δt, it is thus easy to find a relation between inductance and capacitance which will give this length of closure. Neglecting resistance in the oscillating circuit, the relation is:

$$\Delta t = \frac{1}{8\pi LC}$$

In designing the oscillograph, a suitable value of C was empirically obtained to make the voltage output and the natural frequency of the relaxation oscillator correct, and the correct value of L was obtained from the above relation.

It has been shown that during the period when the switch is closed the resistance is practically constant. By connecting the switch in series with a 1 1/2 volt cell and a resistance, an oscilloscope image of the current through the resistance during the switching operation was obtained. At a frequency of switching of 50 times a second this had an essentially block wave shape, as would be expected. At high frequencies of operation of the switch, say 5000 operations a second, the oscilloscope indicated a current in the resistance more of the form of the voltage wave shown in figure 12. This may have been because of stray capacitance and inductance effects.

By applying the amplified output of the pause shifter to the grid of the 885 tube, the operation of the switch can be made to have the same frequency as the phase shifter output, and to bear a constant phase relation to it, provided the natural frequency of the relaxation oscillator is adjusted to be near to that of the phase shifter output.

The only remaining distinctive feature of the oscillograph is the recorder as finally used. The design, due to Maxstadt, consists of a moving coil in a radial field, similar to the well known dynamic loud speaker, coupled to a lever system of light weight and of a type which gives straight line motion to the recording pen.

The complete schematic circuit of the oscillograph as actually constructed is shown in figure 15. The instrument was provided with two plate voltage supplies designed to operate on 110 v., 50-60 cycle a.c., rectifiers supplying all d.c. voltages necessary except the bias for the switching tubes.

In the operation of the oscillograph certain errors are necessarily present, some inherent and some due to exingencies of construction. A complete analysis of these errors has been made, and their magnitude for the construction as actually found most practical has been computed. A detailed analysis of the practical operation of the oscillograph follows.

The generalized circuit of the oscillograph as it has been described is shown in figure 13.

A perfectly steady a.c. voltage e_x of fundamental frequency f is applied between terminals 1 and 2. Switch s, having a resistance R is periodically closed for a short time, Δ t, with a frequency of (f ± F), where F is a low frequency of the order of magnitude of 1/2 cycle per second. Thus there is across condenser C a voltage e'_x having approximately the wave form of e_x, but having a frequency of F. The voltage on condenser C forms the input of a d.c. amplifier whose output drives a recording milliammeter, so that the deflection of the recording pen is proportional to e'_x. A paper strip, P, passing under the pen with a speed proportional to F will have recorded on it a curve representing the wave form of e'_x and hence, approximately, of e_x.

Assuming that the d.c. amplifier and recorder functions perfectly, the deviation of this curve from a true representation of the wave form e_x arises from several causes:

1. Since between two successive closings of switch S, the phase position of the switching has advanced $360(\frac{F}{f})$ degrees with respect to e_x, any variation of e_x within an angular interval $360(\frac{F}{f})$, can be recorded only as a net rise of fall.

The switching does not truly occur with a frequency of $(f \pm F)$, but rather, the phase position of the switching operation with respect to e_x is periodically advanced by incremental angles $\Delta\theta$ such that $\Delta\theta = \frac{360^\circ}{n}$ where n is an integer and represents the relatively small number of contacts on a phase shifting device. Thus variations in e_x occuring in intervals less than $\Delta\theta$ can be recorded only as a net rise of fall.

2. Since switch S is closed for a finite period of time, Δt, the value of e_x throughout an angular interval $360f\Delta t$ can be recorded only as some value ranging from the maximum to the minimum value of e_x in that interval.

3. Since the switch S has an appreciable resistance, R, it may be that during closure, the condenser **C** does not attain as nearly to the potential e_x during that time as is desired. (S must be closed for as long periods as permitted by other requirements.)

4. Since the condenser **C**, the input to the d.c. amplifier and the switch S in an open position, all have finite leakage resistances (represented in total by r) the condenser **C** loses some of its charge between closures of the switch S.

Examining these points one at a time we find:

1. That while $360(\frac{F}{f})$ represents a very small angular displacement if f is high, for a low value, say f = 60 cycles per second, F being, as stated earlier, 1 cycle per second, this represents a displacement of 6 electrical degrees with respect to the 60 cycle wave. Thus if it is desired to record at a rate of one cycle per second, variations occurring in e_x within angular intervals of less than 6^o cannot be reproduced in detail and will be shown only as a net rise or fall.

The recorder as used actually operates at a speed of approximately 1/3 cycle per second, hence the angle $360(\frac{F}{f})$ represents an electrical displacement of 2^o which permits somewhat finer detail in the record. More will be said about this later.

The phase shifting device used has 96 contacts, hence $\Delta\theta = 3.76^o$, and variations occurring in angular intervals less than 3.76^o cannot be recorded in detail but only as a net rise or fall. Thus the 2^o interval determined by the speed F of recording is not the limiting factor but rather the 3.76^o angle $\Delta\theta$ just analyzed.

2. Since the size of $\Delta\theta$ prevents detailed recording of variations in an angular interval less than 3.76°, the interval Δt through which switch S is closed, was made such that $360f\Delta t = 3.76°$ approximately, so that the switch will be closed throughout an angle of 3.76 with respect to e_x, hence Δt will be as long as possible without causing additional "loss of definition". It has already been suggested (3) above, that the switch must remain closed the greatest possible length of time. Thus the wave form of e_x would be recorded as a series of average values over angular increments of 3.76°, were there no other sources of error than those included in (1) through (2).

3. The size of the condenser C is determined by the fact that (1) it should be as small as possible, (2) it must be large enough so that no appreciable a.c. impulse appears across it on switching, since such impulses are amplified by the d.c. amplifier and may drive the final tube beyond cutoff, causing distortion. It was found empirically that C had to be at least .05 mf when the a.c. source had a frequency of 50 cycles.

In calculations relating to the charging of the condenser during switching, a value for the resistance of the switch of 1,000 ohms, which lies between the values for the 75 and the 24A tubes will be used.

The equation for the charging of a condenser, as during the interval Δt when the switch S is closed, is known to be of the form

$$e = A\,\varepsilon^{-\frac{1}{ck}t} + B$$

If we measure t from the time of closing the switch, when $e_x' = e_{xo}'$, evaluation of the constants gives

$$\frac{e_x' - e_x}{e_{xo}' - e_x} = \mathcal{E}^{-\frac{1}{ck}t} = B$$

Now $(e_{xo}' - e_x)$ is the difference of potential between e_x and the condenser at the instant of closing the switch. Let us call:

$$(e_{xo}' - e_x) = \Delta e_1$$

Also, $(e_x' - e_x)$ is the difference of potential between e_x and the condenser at any time during the interval from $t = 0$ to $t = \Delta t$; in particular when $t = \Delta t$, Let us say

$$\text{at } t = \Delta t, \qquad (e_x' - e_x) = \Delta e_2$$

Then for the switching operation, Δe_1 is the initial difference of potential between e_x and the condenser it is desired to charge, and Δe_2 is the final difference.

On page 3, section (2) it was shown that for our particular phase shifter, it was wise to make $360f\Delta t = 3.76$, or $\Delta t = \frac{3.76}{360f}$ assuming f, the frequency of e_x, to be 60 cycles, $\Delta t = 1.74 \times 10^{-4}$ seconds. We know that R = 1,000 ohms and $c = .05 \times 10^{-6}$ farads, hence we can evaluate B in equation (VI) for $t = \Delta t$, and obtain

$$B = \frac{e_2}{e_1} = \mathcal{E}^{-\frac{1.74 \times 10^{-4}}{.05 \times 10^{-6} \times 10^{-6}}} = \mathcal{E}^{-3.48} = 0.03$$

This means that at the end of the interval during which S is closed, Δe has been reduced from its initial value, Δe_1, to $0.03\Delta e_1$. That is, the fidelity, or lack of it due to R only, is within 3% of Δe_1 which in turn is small compared to the crest value of e_x.

Now if the phase shifter is run at such a speed that the contact arm touches a new button each switching operation, which is as fast as it can be run without introducing error, as is explained in section 1, page 2, each switching operation will take place at an angular distance 3.76^o further along the cycle of e_x than the preceding switching operation. Now if we assume, for example, that e_x is a sine wave with an amplitude of 1 volt, the <u>greatest amount</u> that e_x can charge in an interval of 3.76^o can be shown to be .065 volts. This is really the Δe_1 we have used before. Hence, after the switching operation, the discrepancy between the voltage of the condenser and the voltage e_x, which is Δe_2, cannot be greater than (.03)(.065) volts, or .0019 volts. Thus the maximum voltage error due to the resistance of the switch, R, is .0019 volts, or 0.19% of the peak value of e_x.

For higher frequencies than 60 cycles, B will of course be larger, and Δe_2 will be larger. But it may be shown by the following argument that this does not impair the accuracy of the device:

Referring again to 60 cycles, call $\Delta t = \Delta t_1$; at 6,000 cycles $\Delta t = \frac{\Delta t_1}{100}$. For 60 cycles

$$B_1 = \mathcal{E}^{-\frac{1}{CR}\Delta t_1}$$

For 6,000 cycles

$$B_2 = \varepsilon^{-\frac{1}{CR}\frac{\Delta t_1}{100}}$$

At 60 cycles, after one switching operation, Δe_1 is reduced to

$$B_1 \Delta e_1 = \Delta e_2$$

But at 6,000 cycles, after one switching operation, Δe_1 is reduced to

$$B_2 \Delta e_1 = \Delta e_2'$$

After two successive switching operations, it may be seen to be further reduced to

$$B_2 (B_2 \Delta e_1) = B_2^2 \Delta e_1$$

After 100 switching operations, it may be seen that e_1 has been reduced to

$$B_2^{100}(\Delta e_1) = \varepsilon^{-\frac{1}{CR}\frac{100\Delta t_1}{100}} = B_1 \Delta e_1$$

Hence it may be seen that closing the switch for a period of $\frac{\Delta t_1}{100}$ seconds a hundred times in succession is just as effective as closing it once for a period of Δt_1 seconds. The presumption has been, of course, that a 6,000 cycle unknown will have 100 times as many switchings per button on the phase shifter as a 60 cycle wave, i.e., 100 x 1 = 100.

Thus the unknown e_x has remained constant during these 100 switchings. Similar ratios hold for other frequencies above 60.

Actual measurement of average conductance of the switch when operating normally at 50 cycles gave a conductance of 3.3×10^{-6} mhos. The calculated average conductance would be (for a tube whose plate resistance is 1200 ohms)

$$\left(\frac{1}{1200}\right) \frac{3.76^{\circ}}{360^{\circ}} = 8.72 \times 10^{-6} \text{ mhos}$$

This is over twice the measured conductance, but is of the same order of magnitude. The significance is that the effective length of switching is shorter than was calculated. The effect of this on the operation of the oscillograph is to increase B somewhat, and hence the condenser C will not charge up as completely in one cycle as was calculated. The oscillograph will operate satisfactorily despite this, however, since the error is still very small.

4. Leakage resistance, represented by r, will merely tend to discharge the condenser C during the period when the switch S is open. The actual leakage resistance of the grid of the d.c. amplifier to ground is so high that it cannot easily be measured, but is of the order of several megohms, and its effect on the operation of the oscillograph is negligible. Exceptionally high resistance condensers should be used and switching tubes of high plate leakage resistance may be selected.

5. Nothing has been said thus far regarding the fidelity of the complete apparatus on other than sine waves. In order to determine the

highest harmonic of any given fundamental that can be satisfactorily recorded, assume that the switching (or the phase shifting, whichever is the limiting element) must be done once for every 30 degrees of the harmonic, or 12 times per cycle of the harmonic. It then becomes a matter of arithmetic to determine the harmonic.

There are 96 buttons on the phase shifter. That should be made the limiting element.

Thus $\frac{96}{12} = 8$ or harmonics up to the 8th can be recorded with good definition.

In order to record up to the 19th harmonic, a phase shifter of 228 buttons will be required and a slowing up of the recording rate to $\frac{228}{60} = 3.8$ seconds per cycle instead of 1.6 seconds as in the present model (assuming a 60 cycle unknown). For higher frequencies of the unknown the recorder speed does not enter but only the number of buttons on the phase shifter as will be seen if it is recalled that with a perfectly steady unknown the value of e_x is constant until the phase shifter moves to a new button.

6. There are two other disturbing effects not yet mentioned. Frequently an unknown has transient characteristics. For example a belt driven alternator with a bad splice in the belt or a direct driven alternator with unsymmetrical pole shapes or an oscillator with B-voltage pulsations will put out waves which this type of oscillograph cannot record well. The effect of such an input is shown in the oscillogram in figure 17.

7. The recorder does not actually give a true reproduction of the input of the d.c. amplifier. The actual action of the recorder may, however, be analyzed.

Figure 14 represents the recorder. We have an N turn coil which may be displaced in the direction x, in a radial magnetic field such that

$$\frac{\partial \phi_F}{\partial x} = B_o$$

where ϕ_F is the flux from the field linking the coil.

It may be seen that $B_o = \int_{Start}^{Finish} B\,dl$, where B is the magnetic field strength and dl is an elementary length of the conductor in the coil, and the integration is carried out from one end of the coil to the other.

Hence, if a current i flows in the coil, the force on the coil due to the current in the field will be

$$F = 10^7 \int_{Start}^{Finish} i\,B\,dl = i\,B_o \times 10^7 \text{ dynes}$$

The coil is assumed to have a leakage inductance L and a resistance R, and to be connected to a voltage source of voltage e through a resistance r. The mass of the coil and attached levers is taken as m, and a restoring spring of $F_r = -kx$ dynes and a damping force of $F_d = -u\frac{dx}{dt}$ dynes are assumed.

We may write the equation relating the voltage e and other variables:

$$e = (10^{-8}) \frac{\partial \phi_F}{\partial x} \frac{dx}{dt} + i(R + r) + L \frac{di}{dt}$$

$$= 10^{-8} B_o \frac{dx}{dt} + i(R + r) + L \frac{di}{dt} \qquad (1)$$

The motion of the coil is given by the differential equation

$$m \frac{d^2x}{dt^2} = -kx - u \frac{dx}{dt} + i \quad (10^7) \qquad (2)$$

Solution

Letting p represent $\frac{d}{dt}$, etc.

$$e = 10^{-8} B_o px + i(R + r) + Lpi \qquad (1a)$$

$$mp^2x = -kx - upx + i(B_o (10^7) \qquad (2a)$$

Solving (2a) for i

$$i = \frac{mp^2x + kx + upx}{B_o (10^7)}$$

Substituting this into (1a)

$$e = 10^{-8} B_o px + \frac{10^{-7}}{B_o} (R + r + Lp)(mp^2x + (k + up)x)$$

$$e = \frac{10^{-8}}{B_o} \Big[(B^2{}_o px + 10(R + r)mp^2x + 10(R + r)kx + 10(R + r)upx + 10 \, mLp^3x + 10 \, kLpx + 10 \, uLp^2x)\Big]$$

$$e = \frac{10^{-8}}{B_o} \Big[10mLp^3 + 10((R + r)m + uL)p^2 + (Bo^2 + 10(R + r)u + 10kL)p + 10(R + r)k \; x\Big]$$

If we assume e to be sinusoidal, we can write this as a vector equation, replacing p by jw, and obtain for r.m.s. value

$$\overline{E} = \frac{10^{-8}}{B_o}\Big[10(R + r)k - 10w^2\ ((R + r)m + wL) + jw\ (Bo^2 + 10(R + r)u + kL) - w^2 mL\ \overline{x}\Big]$$

Rearranging this

$$\overline{x} = \frac{(B_o)(10^8)\ \overline{E}}{10\ (R+r)k-w^2((R+r)m+wL + jw\ \big(Bo^2+10(R+r)u+10kL\big)-w^2 mL} \qquad (3)$$

It is here convenient to substitute for constants in terms of values for one turn of the coil, assuming all turns to have the same constants. We know that if the actual bulk of the coil remains constant

$$L = \ell_1 N^2$$

$$R = r_1 N^2$$

$$B_o = \beta_1 N$$

where ℓ_1, r_1, β_1 are values for one turn.

Hence the equation becomes

$$\overline{x} = \frac{10^8\,\beta_1\,\overline{E}}{\Big[10\Big\{(r_1+\frac{r}{N^2})k-w\ ((r+\frac{r}{N^2})m+w\ell_1)\Big\}+jw\ \Big\{\beta_1^{\,2}+10(r_1+\frac{r}{N^2})u+10k\ell_1-w^2 m\,\ell_1\Big\}\Big]\ N} \qquad (4)$$

If the recorder is to operate in the plate circuit of a vacuum tube, r must be of the order of 10,000 ohms. Then if w = 0 (d.c. voltage),

$$x = \frac{10^7 \beta_1 E}{N(r_1 + \frac{r}{N^2})k} = \frac{10^7 \beta_1 E}{(Nr_1 + \frac{r_1}{N})k}$$

Thus, for a given current, we obtain the maximum deflection when the quantity $Nr_1 + \frac{10,000}{N}$ has a minimum value. This is when

$$\frac{1}{N}\left(Nr + \frac{10,000}{N}\right) = 0 = r \quad \frac{10,000}{N^2}$$

$$\text{or} \quad N^2 r_1 = 10,000 = R$$

This, as we might expect, is the condition that the resistance of the coil and of the power source be equal. It would mean, however, that the coil should have a d.c. resistance of 10,000 Ω , which is hardly practical.

It should be noted that this condition is desirable because it makes the real term in the denominator of (4) as large as possible compared with the imaginary term, and hence avoids phase displacement between $\overline{E}$ and $\overline{x}$, and tends to avoid amplitude distortion.

It may be seen in general that the sensitivity of the instrument to d.c. voltages may be kept constant by decreasing both β_1 and k proportionally, k being the spring constant.

Moreover, this decreases the imaginary term, which is desirable. The true effect of such changes cannot be appreciated, however, unless the magnitudes of the terms are known, and it may be seen that if $w^2 > 10k$ the sign of the real term is reversed, a disastrous condition since a (+) voltage then records as a (-) deflection.

In all, the problem of analysis is very complicated, and general conclusions drawn from equation (4) are of doubtful value.

It is of some interest to substitute into the equation the approximate constants of the present recorder. These were obtained as:

$m = 5$ grams

$k = 6{,}000$ dynes/cm

$N = 1{,}800$ turns

$R = N^2 r_1 = 600$, hence $r_1 = 1.85 \times 10^{-4}$

$r = 10{,}000, \quad \frac{r}{N^2} = 3.08 \times 10^{-3}$

u was not readily obtained. However, it may be noted that the recorder action shows the damping to be critical or somewhat greater than critical. Assuming critical damping,

$$u = 2/km = 2/(6000)(5)$$

$$= 344 \text{ dynes/cm/second}$$

At the low frequencies involved, it may be considered that

$$\ell_1 = 0$$

For a steady d.c. voltage, equation (5) holds, and

$$x = \frac{10\,\beta_1 E}{N(r_1 + \frac{r}{N^2})k} = \frac{10^7 \beta_1}{(1800)(1.85 \times 10^{-4} + 3.08 \times 10^{-3})(6000)}$$

$$= \frac{10\,\beta_1 E}{(1800)(3.26)(10^{-3})(6000)} = .280 \times 10^3\,\beta_1\,E$$

But it is found for the recorder that a current of .001 ampere produces a deflection x of about .4 cm. A current of .001 ampere corresponds to a voltage of

$$E = RI = (10{,}600)(.001) = 10.6 \text{ volts.}$$

Hence
$$.4 = (0.28)\ (10.6)\ (10^3)\ \beta_1$$

or
$$\beta_1 = \frac{(.4)}{(.280)(10.6)(10^3)} = 1.35 \times 10^{-4}$$

Substituting these values in equation (4)

$$\bar{x} = \frac{(10^8)(1.35)(10^{-4})E}{\left[10\left\{(3.26)(10^{-3})(6{,}000) - w^2\ (3.26)(10^{-3})5\right\} + jw\left\{(1.35)^2(10^{-8})(10)(3.26)(10^{-3})\right\}\right]1800}$$

$$= \frac{13{,}500\ E}{1800\left[19.6 - w^2\ 16.3\ (10^{-2}) + jw\ 3.26\ (10^{-2})(344)\right]}$$

$$\bar{x} = \frac{E}{26 - w^2\ .0022 + jw\ 1.5}$$

Tracing at the recording rate of 1 cycle in 3 seconds, or 1/3 cycle per second, we may calculate the amplitude and phase displacement of the recordings of various harmonic components of the wave fed to the recorder, assuming equal E for all harmonics, and a displacement for the d.c. component with respect to the fundamental. Table in figure 16 gives the results of such a calculation.

It is important to note that, despite all the errors enumerated, the worst of which lie in the operation of the recorder, we may expect in the higher frequency ranges more accurate recording than with a mechanical oscillograph. For instance, at 5,000 cycles all harmonics up to the eighth will be recorded with reasonable accuracy. The eighth harmonic will be recorded with 0.74 of its true amplitude with respect to the fundamental, and with its true phase position to within a degree, considering the errors in the recorder alone. None of the other errors described are comparable in magnitude, hence we may take this as the overall accuracy. When it is remembered that the frequency of this eighth harmonic is 40,000 cycles and that mechanical oscillographs give extremely bad results at frequencies above 5,000 or, in special cases, 10,000 cycles, the advantages of the oscillograph here presented can be appreciated.

Some Results and Conclusions

As the analysis of errors in various parts of the apparatus shows, perhaps the most serious errors were introduced in the action of the recorder. These errors though small were of such magnitude that a perfect check between oscillograms obtained by this apparatus and oscillograms obtained by other methods could not be expected. However, for numerous wave shapes and various frequencies the wave recorded was checked with the visual image on a cathode ray oscilloscope, and the visual check was uniformly good. The group of cuts shown in figures 20, 17, 18, 19 show actual records for various phenomena as described.

It may be seen that the errors present in the machine are theoretically open to any degree of reduction. As constructed, the machine admirably fulfills what the author takes to be its main purpose; quickly and inexpensively giving a permanent record showing the general characteristics of any wave form.

Such a record is of exceptional value in adjusting various circuits or machinery when it is desired to observe the effect of circuit changes on the wave form. For this purpose, oscilloscopes provide no permanent record and hence are of small value, while the use of photographically recording oscillographs means expense and delay.

A similar example of the device's utility lies in testing the fidelity of radio sets. Using this oscillograph it would be a simple matter to apply an arbitrary input to the set and to take records of both the input and output which could be compared by superposition.

Another field of application lies in laboratories of educational institutions, where the use of the oscillograph would enable students to obtain at a small cost and delay records of such phenomena as the wave form of transformer exciting currents. Such records would be valuable for study, and could be included in notes and reports.

A further application which might be cited is that of keeping a permanent and continuous record of wave form at a negligible cost. Such a record might be of advantage in certain power work, especially in view of the possible extended use of inverters in the near future.

In all, it is believed that the device has unique advantages and fields of application, and that it may be used, not to replace, but to supplement, mechanical and cathode ray oscillographs and oscilloscopes, and to amplify in many ways the field of oscillography.

Appendix

The Instrument as an Harmonic Analyzer

From the mathematical development of the Fischer-Hinnen or "Selected Ordinate Method" of harmonic analysis[5)] it can be shown that the average value of m ordinates taken 360/m degrees apart on a periodic curve represents the sum of the values of the mth harmonic and all harmonics multiples of the mth at any of the points at which one of the ordinates is taken.

If averages of m such equally spaced ordinates are taken progressively along the cycle, it is seen that the plot obtained will be that of the mth harmonic and all harmonics multiples of the mth. Thus, if a device which will do this averaging automatically is available, a sort of harmonic analysis may be obtained. It will not be possible to obtain a true harmonic analysis, for it is seen that higher harmonics will be included along with the mth.

This is particularly objectionable in the case of even harmonics, since every other harmonic is even. Thus, the plot of the average of two ordinates per cycle will contain the second and all other even harmonics. In common a.c. work, however, even harmonics are conspicuous by their absence, hence presenting no difficulties.

In the case of odd harmonics, if we assume there are no appreciable harmonics higher than the seventh the plot of the averages of 3, 5, and 7 ordinates per cycle will accurately represent the 3rd, 5th, and 7th harmonics. It is to be noted that the fundamental cannot be obtained directly, but its magnitude can be calculated, if desired, from a knowledge of the r.m.s. value of the entire wave and the amplitudes of the harmonics.

In case there are higher harmonics the plot of the average of 3 ordinates will contain the 9th, the 15th, the 21st, etc. harmonics, and the harmonics can no longer be exactly separated by this method. Nevertheless, the harmonic amplitude can be estimated by visual inspection of the plots obtained.

With certain modifications, the oscillograph described presents a means of automatically averaging ordinates of a wave to obtain harmonics in the manner described above. For instance, if the switch is closed once in an angular interval of 360° (once a cycle) the voltage of the first harmonic (the fundamental) and of all harmonics multiples of it (all harmonics) at the instant of making contact appear across the condenser at the input of the d.c. amplifier. If the switch is closed at angular intervals of 180°, or twice a cycle, the average of the fundamental at these two instants must obviously be zero, while the voltages of the second and all other even harmonics must be the same at the two times of closure. Hence the average voltage applied to the condenser during these two switching operations is the sum of the voltages of all even harmonics.

We may show that the switching several times a cycle actually results in an averaging of the voltage "ordinates" at the times of closing.

Suppose the circuit is as shown in figure 13. Consider that the switch is closed m times a cycle for short, equal intervals of Δt seconds. Let:

e_{xn} = voltage of a.c. source during the nth interval when the switch is closed (considered a constant voltage)

e = voltage across the condenser

e_n = voltage across the condenser at beginning of nth closure of the switch

e_n' = voltage across the condenser at the end of nth closure of the switch

e_a = average voltage during the cycle

e_{na} = average voltage in the interval between the (n - 1)th and the nth closures of the switch

t_n = time of the nth closure of the switch

f = frequency of the a.c. source

Since most of the time the switch is open, we may neglect variations during the period of closure in obtaining e_a. Then the average voltage during the cycle is, substantially

$$e_a = 1/m \sum_{n=1}^{m} e_{na} \qquad (1)$$

When the switch is open, during the interval from t_{n-1} to t_n,

$$e = e'_{n-1}(1 - \mathcal{E}^{-t/rc}) \qquad (2)$$

measuring time from the time t_{n-1}.

The length of this interval is:

$$t_n - t_{n-1} = 1/mf \tag{3}$$

The voltage at the end of this interval is, then,

$$e_n = e'_{n-1}\,(1 - \mathcal{E}^{-1/mfrc}) \tag{4}$$

Defining a factor α ,

$$\alpha = (1 - \mathcal{E}^{-1/mfrc}) \tag{5}$$

We can say,

$$e_n = \alpha\, e'_{n-1} \tag{6}$$

The average voltage during the interval is

$$\begin{aligned} e &= mf \int_0^{1/mf} e\, dt \\ &= mf \int_0^{1/mf} e'_{n-1}\,(1 - \mathcal{E}^{-t/rc})\, dt \\ &= mfe'_{n-1} \left\{ 1/mf + r_2C\,(\mathcal{E}^{-1/mfrc} - 1) \right\} \\ &= e'_{n-1}\,(1 - \alpha\, rcmf) \end{aligned} \tag{7}$$

If we define

$$\beta = (1 - \alpha\, rcmf) \tag{8}$$

We can write this

$$e_{na} = \beta e'_{n-1}$$

If r $\gg$ R, we can say that during the nth closure of the switch, measuring time from the beginning of the nth closure,

$$e = e_n + (e_{nx} - e_n)(1 - \mathcal{E}^{-t/RC}) \qquad (10)$$

At the end of the period of closure, $t = \quad t$ and we obtain,

$$e'_n = e_n + (e_{nx} - e_n)(1 - \mathcal{E}^{-\Delta t/RC}) \qquad (11)$$

Defining

$$\gamma = (1 - \mathcal{E}^{-\Delta t/RC}) \qquad (12)$$

We obtain

$$e'_n = e_n(1 - \gamma) + \gamma e_{nx} \qquad (13)$$

From (6) and (13)

$$e'_n = \alpha e'_{n-1}(1 - \gamma) + \gamma e_{nx} \qquad (14)$$

From (9) and (14)

$$e_{na} = \beta e'_n = \beta \left\{ \alpha e'_{n-1} (1 - \gamma) + \gamma e_{nx} \right\} \qquad (15)$$

Now, because of the periodicity of the input,

$$e_n = e_{m+n} \qquad (16)$$

Hence

$$\sum_{n=1}^{m} e'_n = \sum_{n=1}^{m} e'_{n-1} \tag{17}$$

From (14) and (17)

$$\sum_{n=1}^{m} e'_n = \alpha(1 - \gamma) \sum_{n=1}^{m} e'_{n-1} + \gamma \sum_{n=1}^{m} e_{nx}$$

$$\sum_{n=1}^{m} e'_n = \frac{\gamma}{1 - \alpha(1 - \gamma)} \sum_{n=1}^{m} e_{nx} \tag{18}$$

From (1), (9), and (18),

$$e_a = \frac{1}{m} \frac{\gamma \beta}{(1 - \alpha(1 - \gamma))} \sum_{n=1}^{m} e_{nx} \tag{19}$$

But if we define the average of all values of e_{nx} throughout a cycle,

$$e_{xa} = 1/m \sum_{n=1}^{m} e_{nx} \tag{20}$$

Then

$$e_a = \frac{\beta \gamma}{1 - \alpha(1 - \gamma)} e_{xa} \tag{21}$$

This shows that the average voltage across the condenser is proportional to the average of the voltages at closure of the switch, and gives the constant of proportionality. If the d.c. amplifier is linear, the average current through the recorder, determining its deflection, will be proportional to e_a and hence to e_{xa}. Thus the deflection of the recorder will be proportional to the average of the voltage "ordinates" as is required in the previous development.

It is to be noted that in deriving (21) R_1 has been assumed to be constant, i.e., that the resistance of the switch does not vary with the voltage across the switch. As has been shown, this can be very nearly realized in practice with the electronic switch previously described.

As to the actual values of α, β, and γ, which are necessary in order that we may determine e_a in terms of e_{xa}, we may note that the voltage fall due to leakage through R_2 during the time the switch is open is very small, we may take $\alpha = 1$ and $\beta = 1$. Hence, very nearly,

$$e_a = \frac{\gamma}{1 - (1 - \gamma)} e_{xa} = e_{xa} \qquad (22)$$

We see that as long as α and β are nearly equal to 1, the value of γ does not matter, and $e_a = e_{xa}$. Thus the actual value of the resistance of the switch does not matter as long as it is constant with voltage.

A maximum to the allowable value of the resistance is set, however, by the necessity that the charge on the condenser change as the switching operation progresses along the cycle. On the other hand, it is not desirable that the resistance of the switch be too low, for that would cause undue fluctuations of voltage across the condenser, and would also lead to transient drains of considerable magnitude on the source. The actual resistance achieved in practice with an electronic switch, which is from 600 to 1,200 ohms, seems to be a happy medium.

Switching at n equally spaced intervals per cycle may be realized in several ways. Maxstadt, using the induction motor driven switch previously described, provided a friction gear arrangement which enabled him to drive the commutator approximately 3 and 5 times synchronous speed. Many satisfactory harmonic traces were obtained by this means.

The electronic switch as previously described may be used to advantage in this type of operation. In order that it may be made to "close" m times a cycle at equally spaced intervals a voltage of frequency m times that of the source, or, an mth harmonic of the source, is necessary to control the operation of the gaseous tube which supplies the switching impulse. This mth harmonic voltage may be applied to the phase shifter rather than the voltage of fundamental frequency which is used for ordinary oscillographic operation, and operation will then be such that one complete revolution of the phase shifter will represent an advance of 360/m degrees along the fundamental of the a.c. source, or of 360° along its mth harmonic, which is being traced.

There are two ways in which such a voltage may be obtained; by use of a tuned filter or by use of an oscillator.

In using a tuned filter to obtain an harmonic voltage, the a.c. source cannot be presumed to contain a strong mth harmonic and it is advantageous to apply it to a distorting circuit, such as a vacuum tube amplifier run at cutoff. The output of the distorting circuit then can form the input of the filter circuit, which may be tuned to any harmonic of the intial unknown. If the tuned filter is sufficiently sharp, only the mth harmonic of the distorted voltage will be in the output, which is as desired.

While a simple parallel resonant circuit with as low resistance as feasible will separate the fundamental from the harmonics and, under favorable conditions, individual lower order harmonics from all others, the <u>percentage difference in frequency</u> between harmonics, which is a criterion for ease of separating, decreases with an increase in the harmonic number.

For instance, the 11th harmonic has a frequency only 10% higher than the 10th harmonic. If it is desired to use a filter embodying a single parallel tuned circuit which will attenuate the 11th harmonic 10 times as much as the 10th harmonic, a simple calculation shows that the inductance used must have a Q, or ratio of reactance to resistance, of over 60. Such an inductance is about as good as can be designed for the audio range using ordinary care. Moreover, there is no assurance that the 11th harmonic will not be stronger than the 10th, and if a ratio of intensities in the output of ten to one is desired it may be necessary to use an even better inductance. Thus it is seen that obtaining pure harmonic voltages by using a tuned filter is difficult in the case of the 10th harmonic, and is increasingly difficult in the case of higher harmonics.

The alternative is to use a local oscillator to supply the harmonic voltages. By introducing into the grid circuit of the oscillator a voltage obtained by amplifying and distorting the a.c. source, the oscillator can be tied into synchronism with any harmonic of the source. This provides a fairly satisfactory means of supplying the harmonic voltage. However, for harmonics of the order of the 10th and higher the oscillator tends to slip out of synchromism with the a.c. source. Moreover, in operation at the higher harmonics the fundamental component of the a.c. source tends to modulate the oscillator output. In the brief experimental work so far undertaken of that particular detail it was found impossible to secure operation for harmonics of 50 cycles higher than the 17th. Neither the simple tuned circuit nor the local oscillator seems to be suitable for supplying voltages at higher frequencies.

It should be pointed out that the oscillograph circuit as shown in figure 15 can be used, without change, in harmonic analysis as described above. Its capabilities are limited only by the sharpness (selectivity) of the tuned filter circuit which supplies the phase shifter.

Figures [illegible], 17, 18, 19 show harmonic traces obtained using a circuit the same as that shown in figure 15 except that an oscillator rather than a tuned circuit was used to supply the voltage to operate the phase shifter.

In summarizing this method of harmonic analysis, the main points to be brought out are these:

The method is not exact for, save in special cases, the precise magnitude of the fundamental and of individual harmonics cannot be obtained. The uncertainties of the method increase with the number of harmonics present.

Practical difficulties become very great for harmonics higher than the 10th or 11th based on fundamentals of 50 or 60 cycles.

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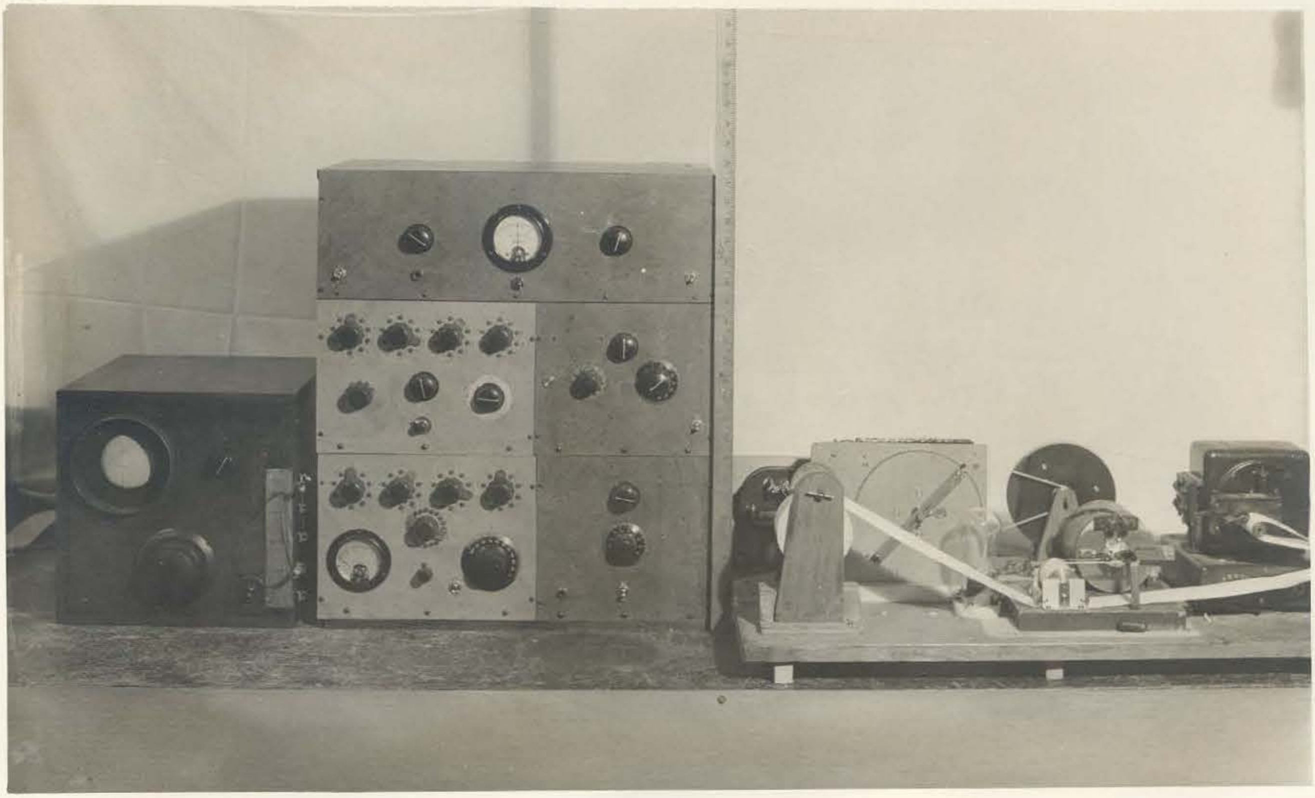
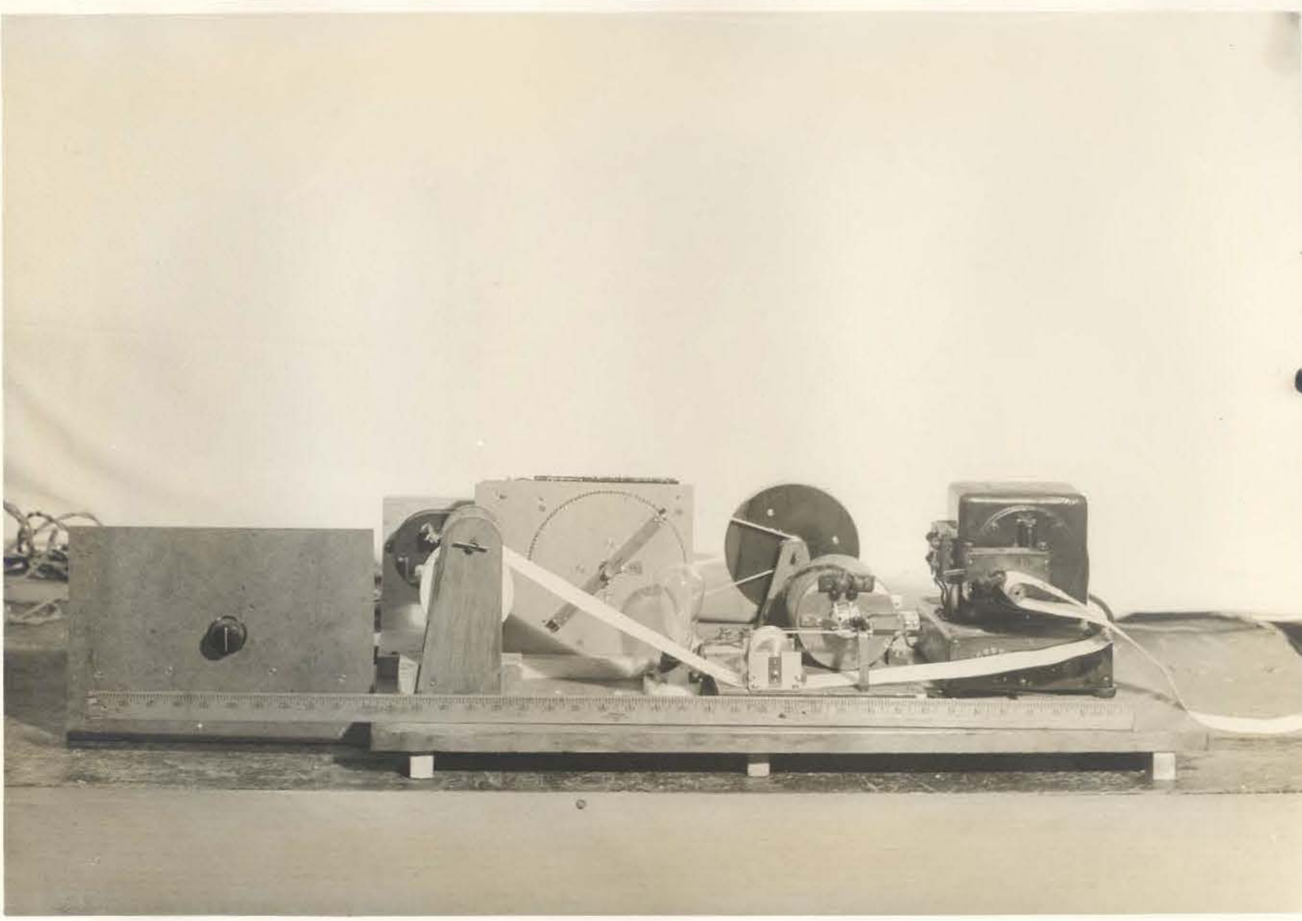


50 Cycle to 25000 Cycle Oscillograph with Paper Tape Ink Recorder
Checking Oscilloscope on Left.



50 or 60 Cycle Oscillograph with Paper Tape Ink Recorder
Meter Stick for Size

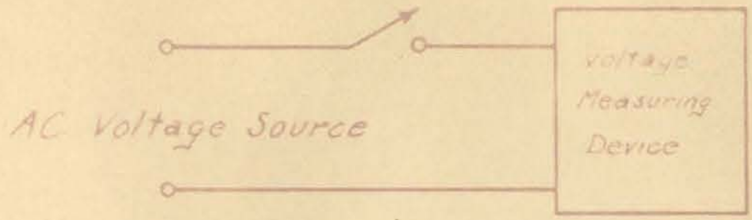


Figure 1

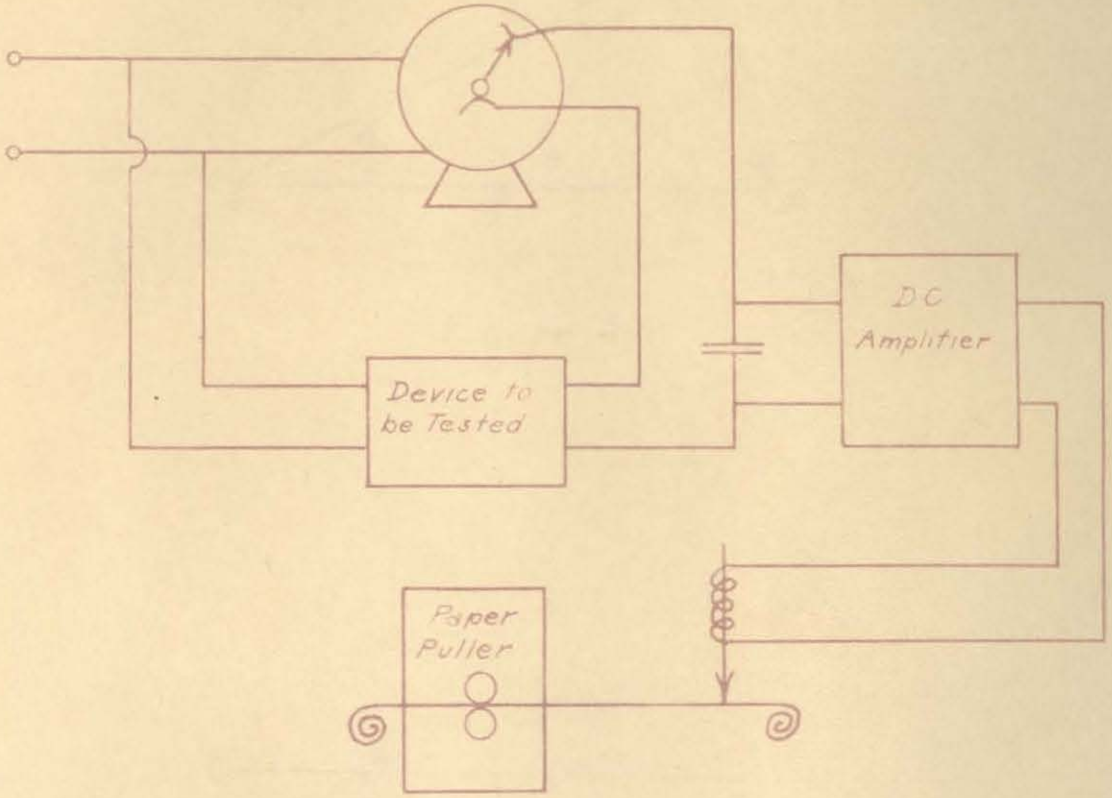


Figure 2

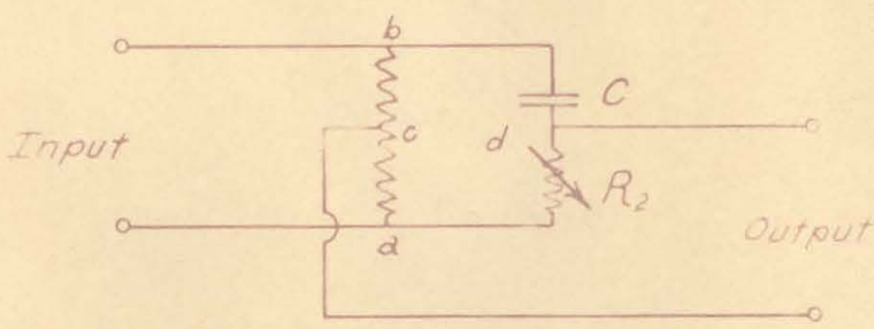


Figure 3

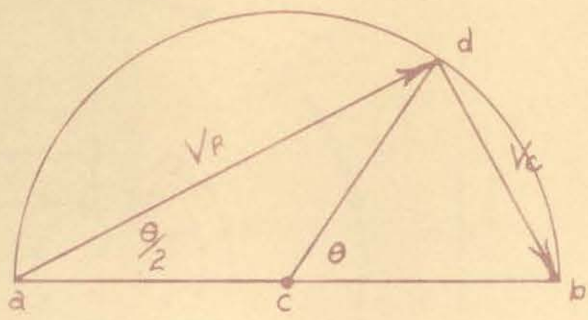


Figure 4

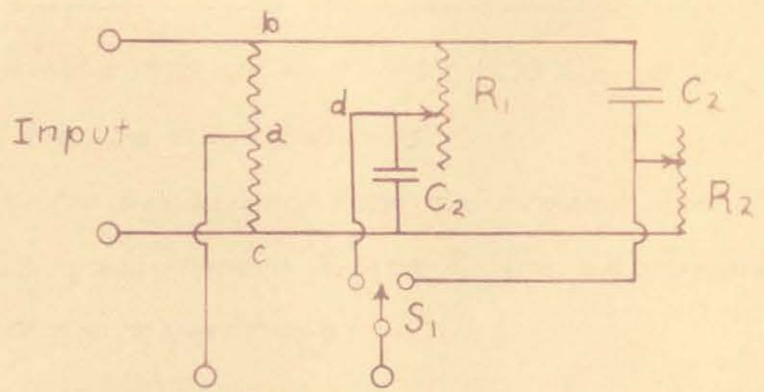
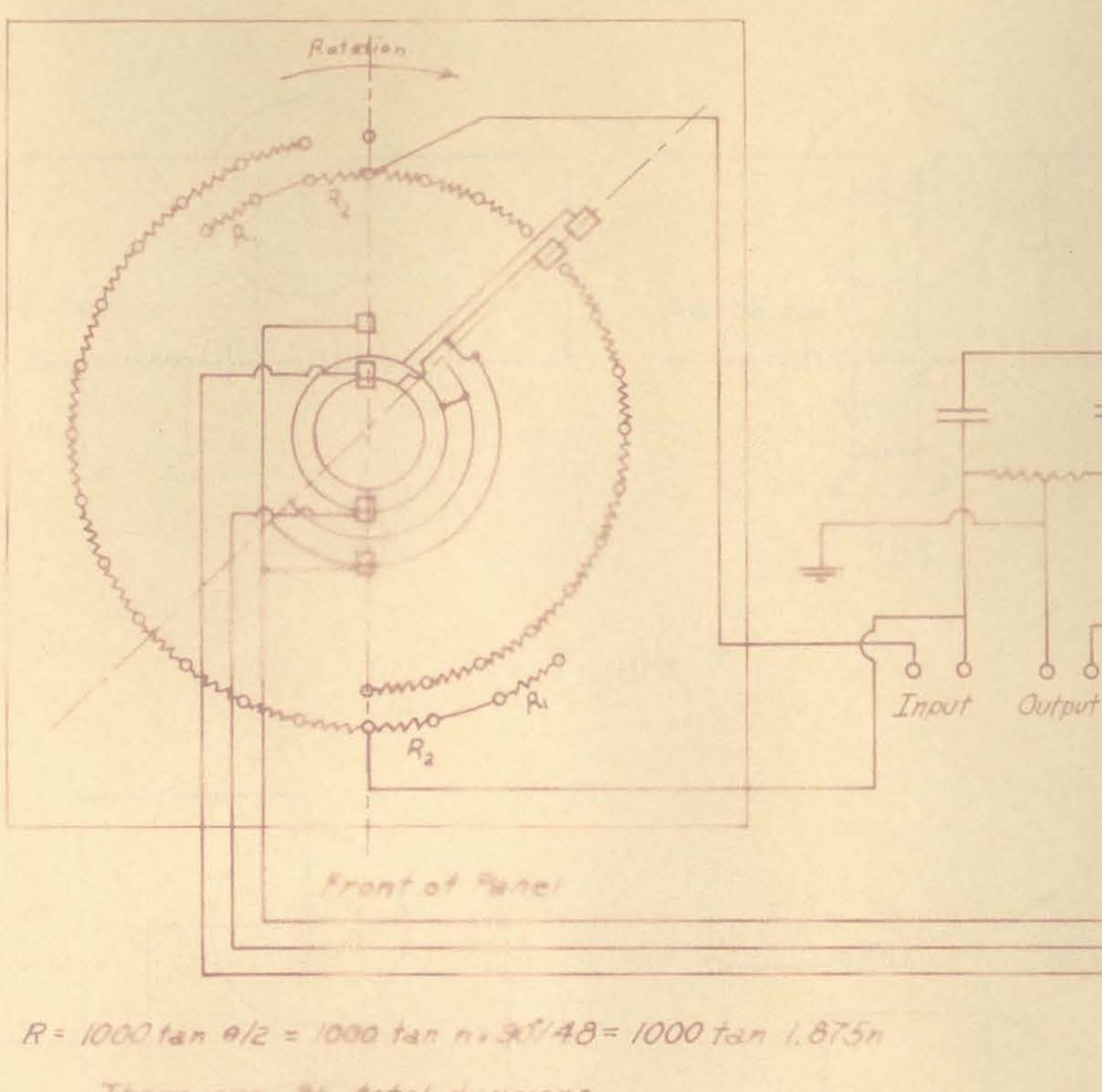


Figure 5

PHASE SHIFTER



$R = 1000 \tan \theta/2 = 1000 \tan n \cdot 90°/48 = 1000 \tan 1.875n$

There are 96 total divisions

R is the total resistance from start to any point

Initiating resistances R_1 and R_2 are approximately 10,000 and 200 ohms respectively

Figure 6

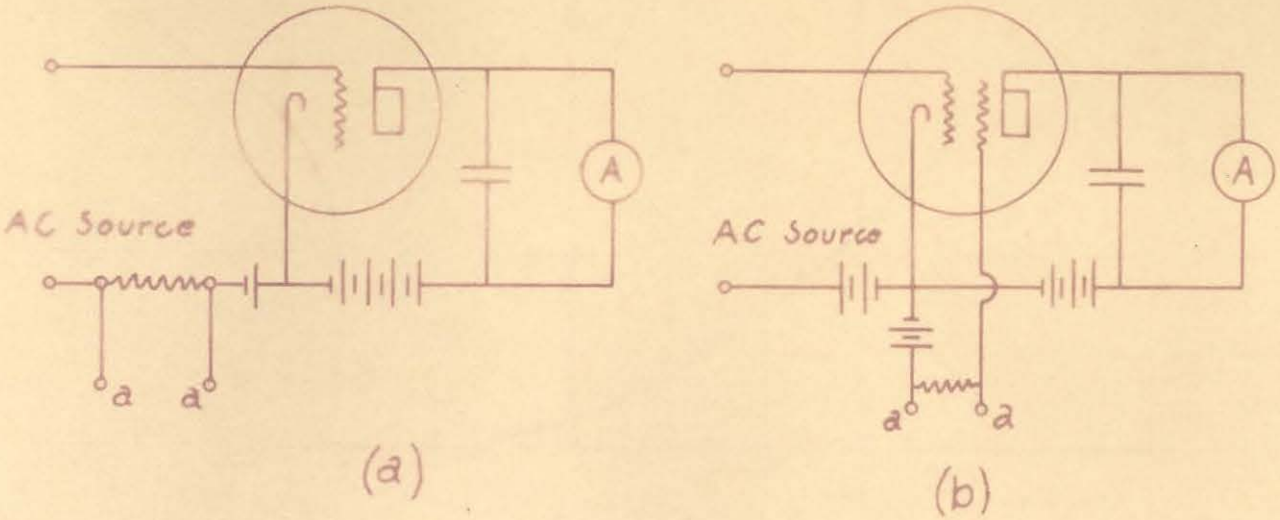


Figure 7

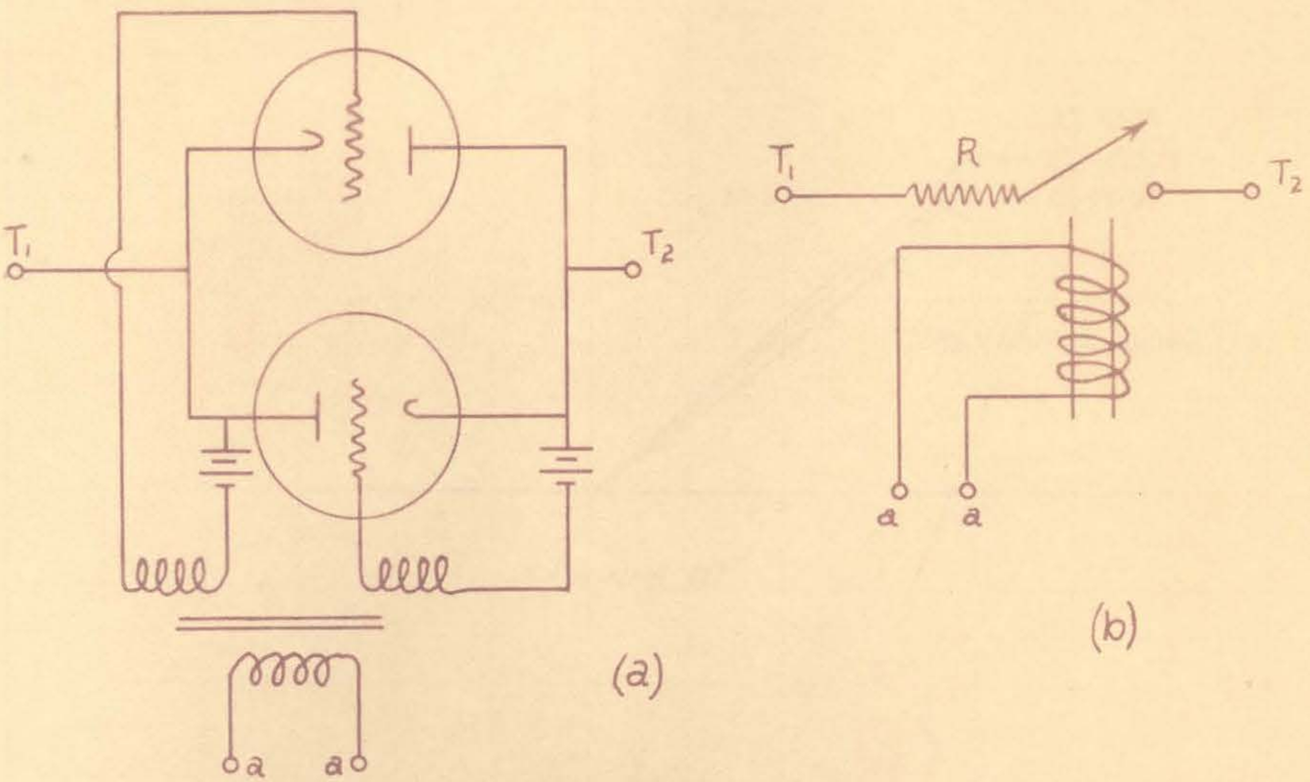


Figure 8

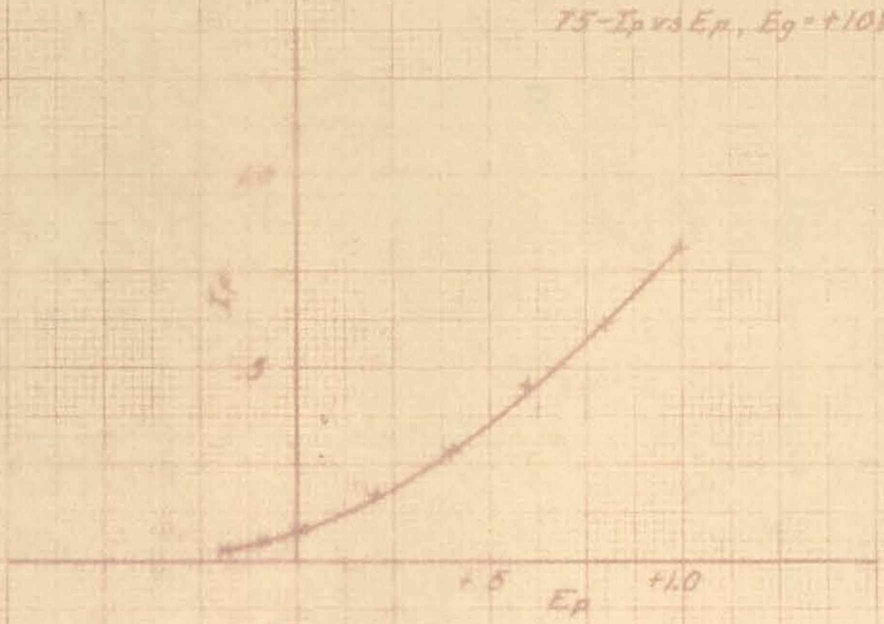


Figure 9

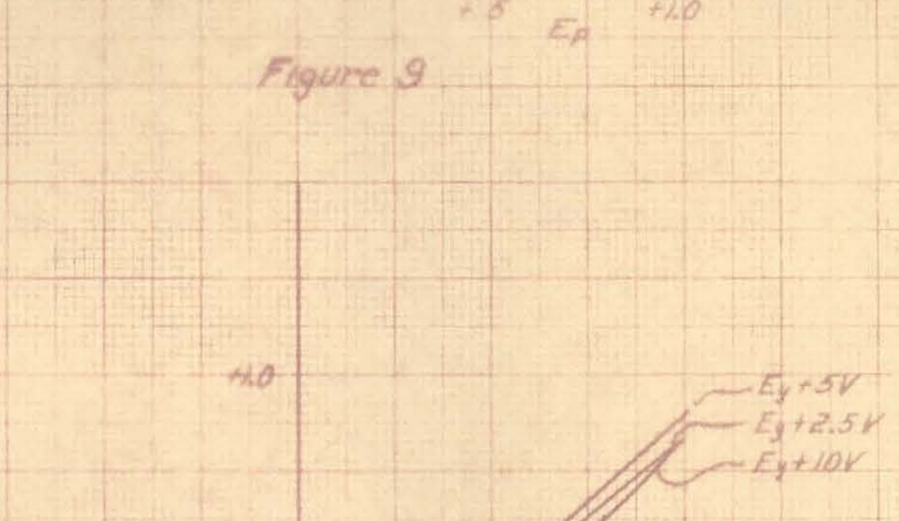


Figure 10

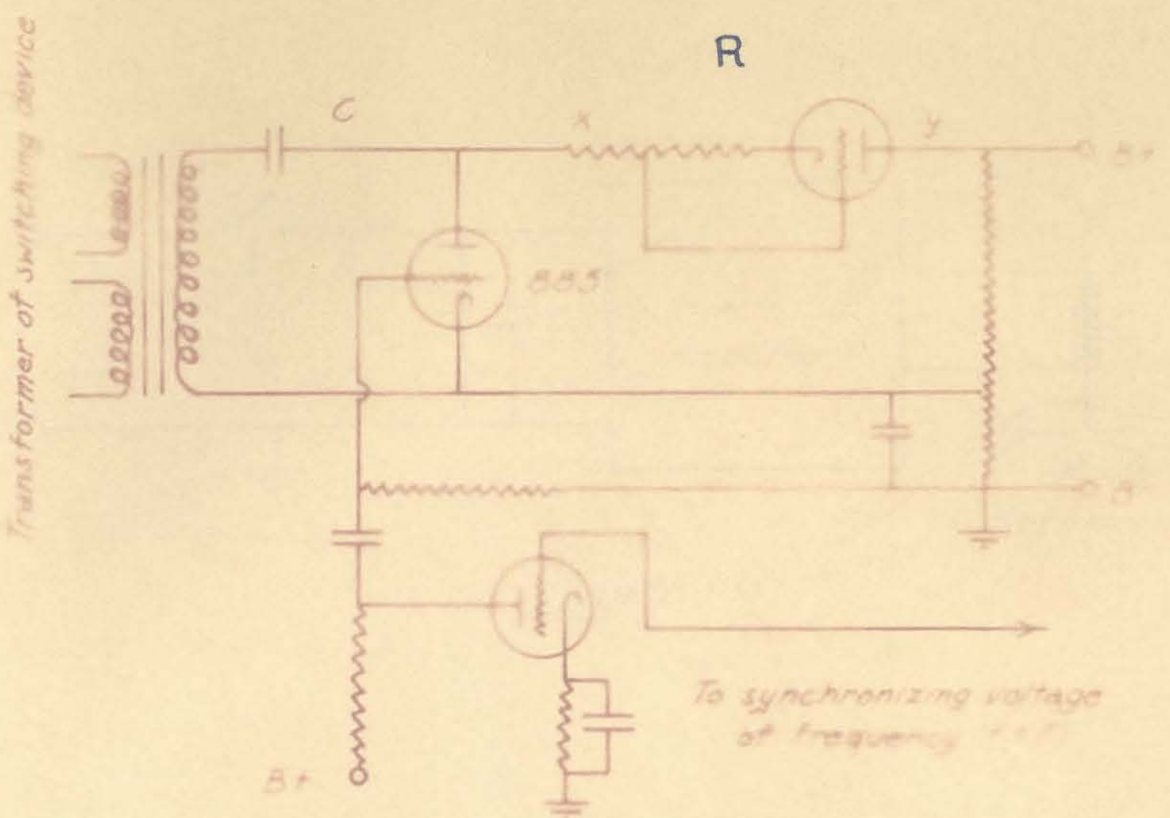


Figure 11

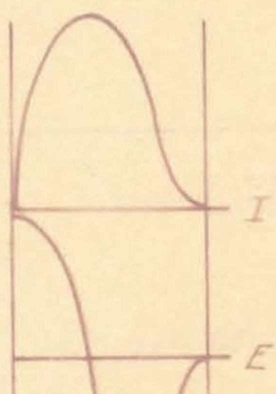


Figure 12

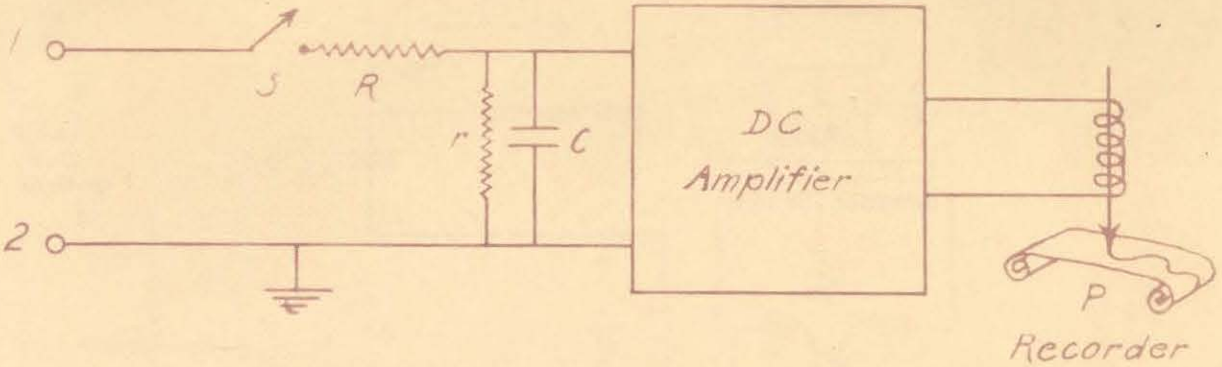


Figure 13

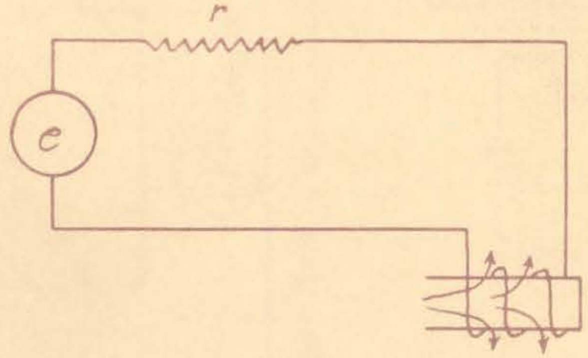


Figure 14

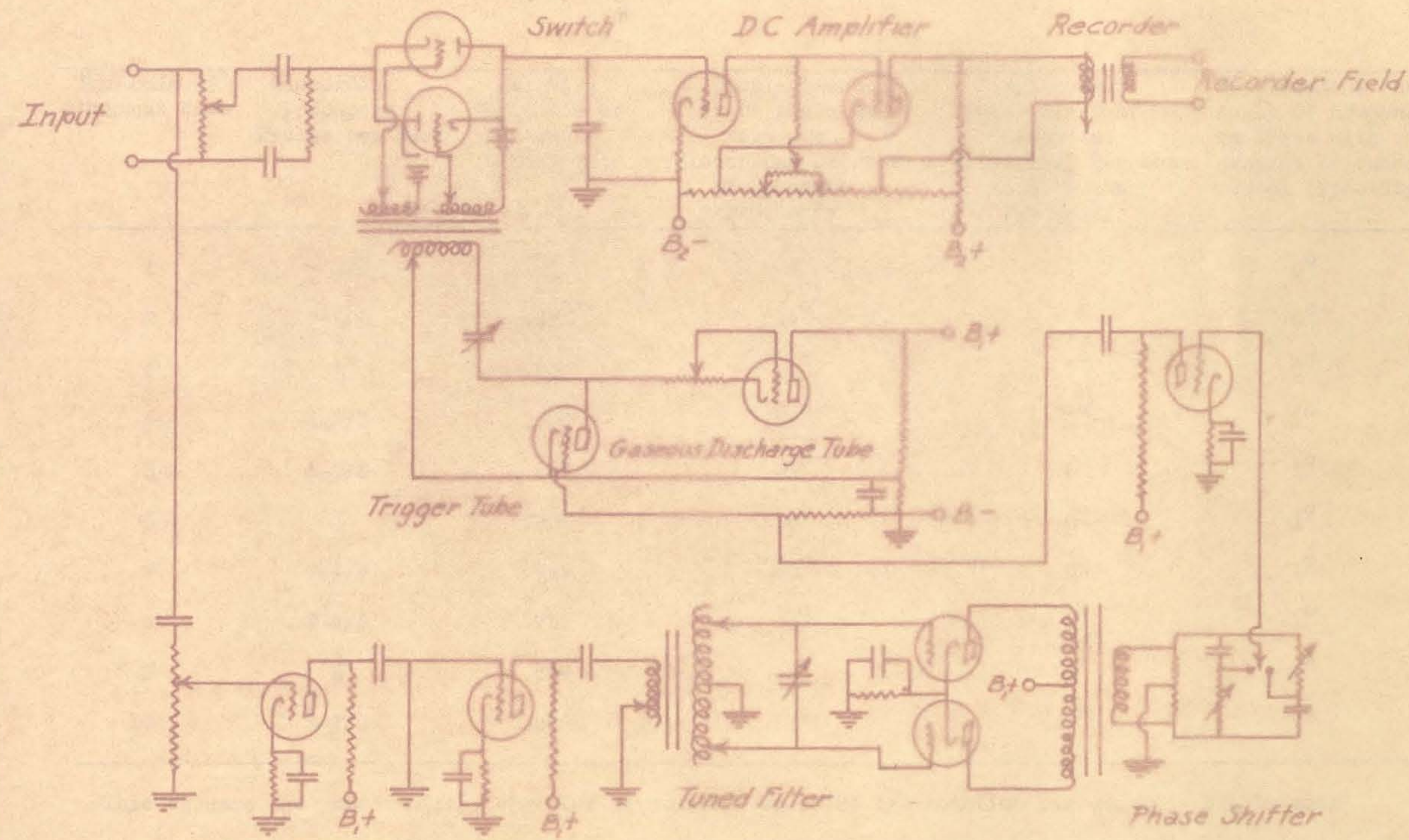
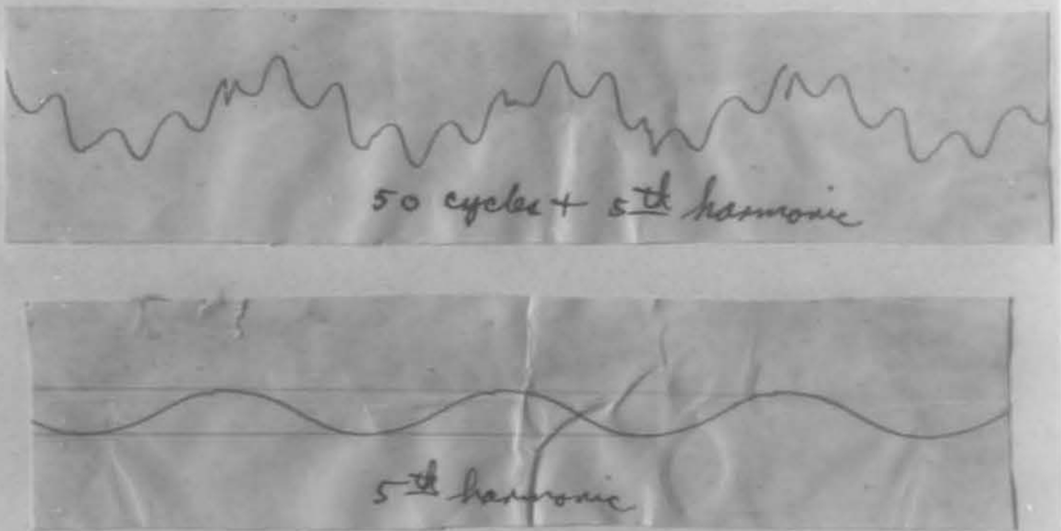

Input
Switch
DC Amplifier
Recorder
Recorder Field
B_2-
B_2+
B_1+
Gaseous Discharge Tube
Trigger Tube
B_1-
B_1+
B_1+
B_1+
Tuned Filter
B+
Phase Shifter

Fig. 15

Harmonic of Unknown Wave	*Recording Frequency Cycles per Sec.	Relative Amplitude of Response (Fidelity)	Displacement Phase Angle of Response, Electrical Degrees at Harmonic Frequency	Displacement Phase Angle of Response, Electrical Degrees at Recording Frequency	Displacement Phase Angle of Response on Trace with Respect to Fundamental (Fidelity)
1	1/3	.993	7°	7°	0°
2	2/3	.973	14°	7°	0°
3	1	.938	20°	7°	0°
4	1-1/3	.900	26°	6°	1°
5	1-2/3	.867	31°	6°	1°
6	2	.807	36°	6°	1°
7	2-1/3	.765	40°	6°	1°
8	2-2/3	.735	45°	6°	1°
9	3	.688	48°	5°	2°
10	3-1/3	.668	51°	5°	2°

*This assumes the phase shifter operates at constant speed of 1/3 rotation per second. The tracing frequency for any harmonic is then 1/3 times harmonic number, of course irrespective of the actual frequency of the unknown.

Fig. 16

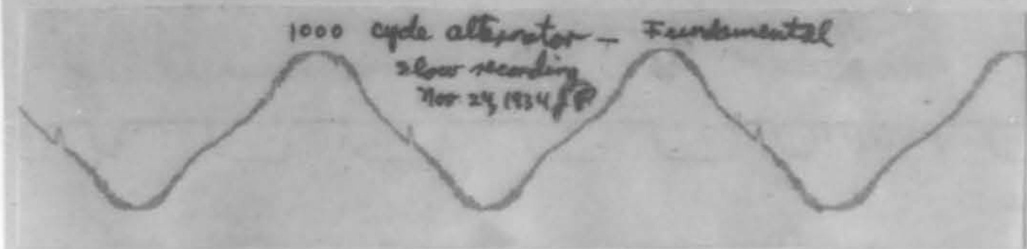


Harmonic records obtained by use of the oscillograph.

50 cycle sine wave with fifth harmonic added by oscillator.

Jumps in trace due to faulty operation of early phase shifter.

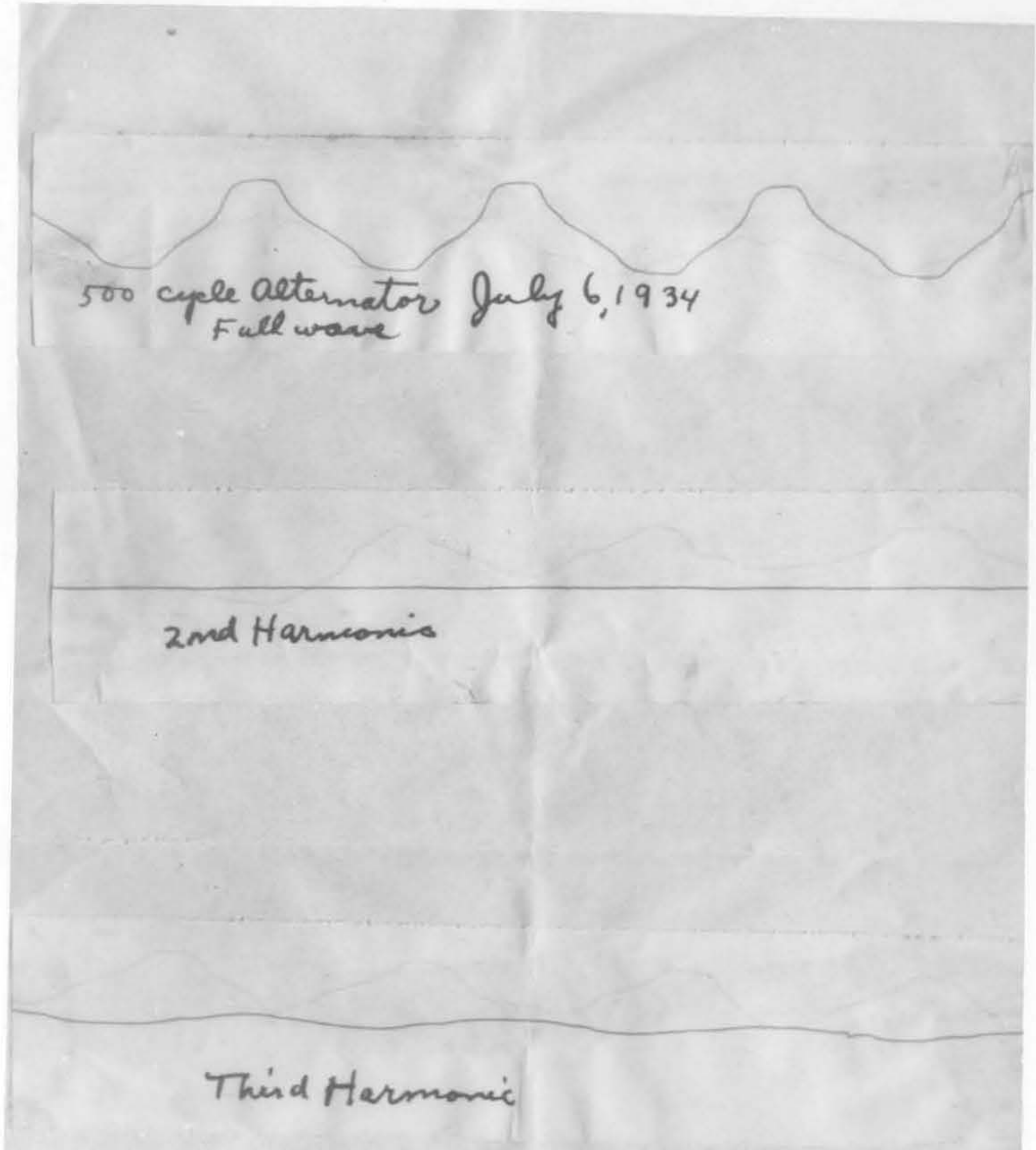
Harmonics to scale.



Oscillogram showing effect of unsteady voltage

from belt driven alternator.

Fig. 17



Harmonic traces obtained with oscillograph. Harmonics to scale.

Fig. 18

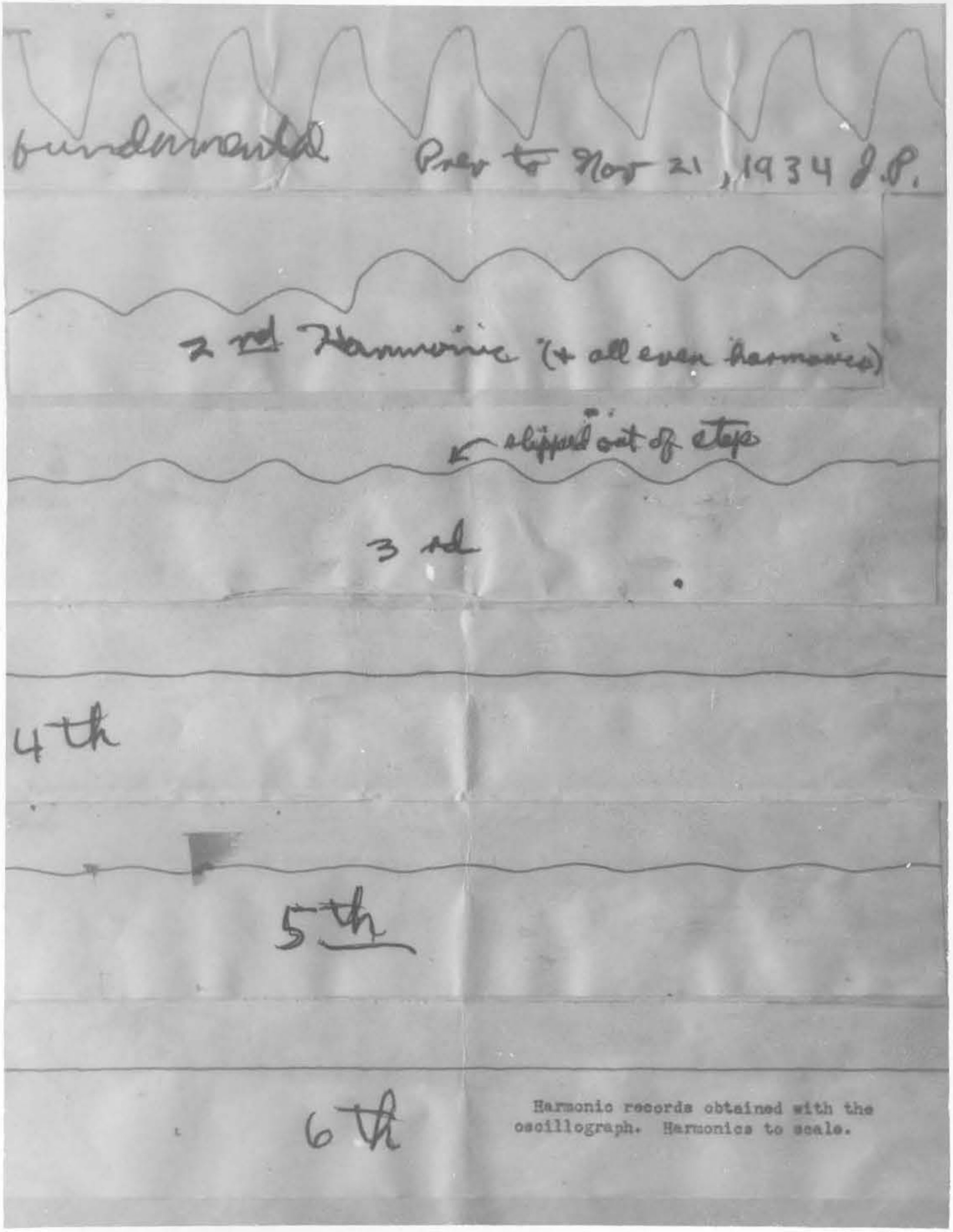
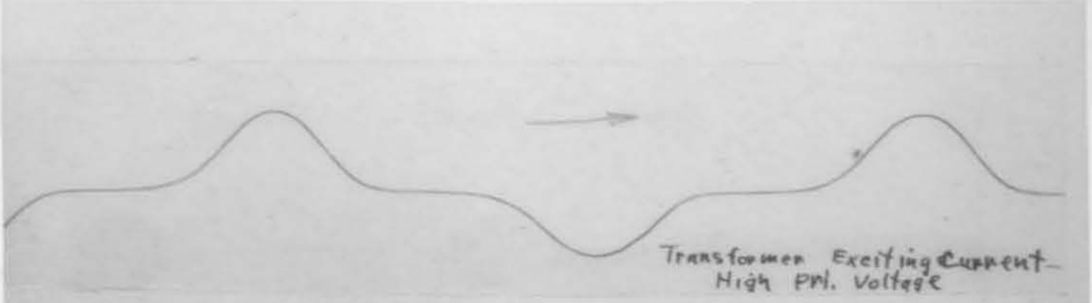
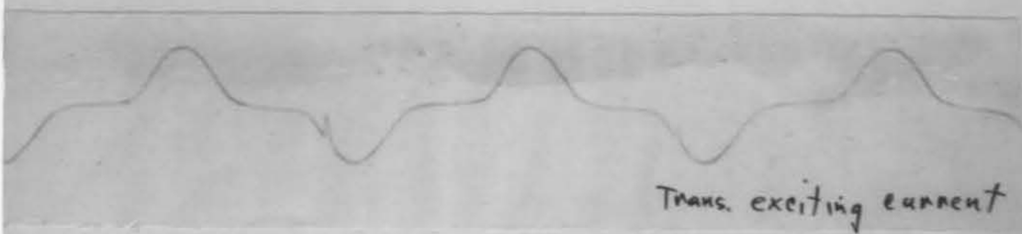

fundamental
Prev to Nov 21, 1934 J.B.
2 nd Harmonic (+ all even harmonics)
slipped out of step
3 rd
4 th
5 th
6 th
Harmonic records obtained with the oscillograph. Harmonics to scale.

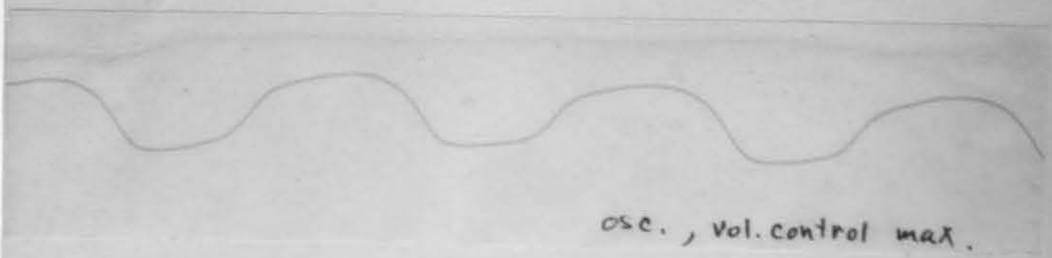
Fig. 19



Transformer wave--slight non-linearity in amplifier



Transformer wave



Oscillator wave, 1,500 cycles.

Fig. 20