A STUDY OF DISTORTIONS IN A CLOUD CHAMBER

Thesis by

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In Partial Fulfillment of the Requirements

for the Degree of

Doctor of Philosophy

California Institute of Technology

Pasadena, California

1956

ACKNOWLEDGEMENT

It is a pleasure to acknowledge the aid and encouragement given me by Professors C. D. Anderson, R. B. Leighton and E. W. Cowan. The concept of making direct measurements of distortions in cloud chambers was originated by Professor Leighton, and many of the suggestions for carrying out the work were provided by him.

The help of B. N Locanthi in designing the necessary electronic equipment and the assistance of P. F. Fuselier in the fabrication of mechanical components were greatly appreciated.

Finally, I wish to express my appreciation for the unfailing encouragement and help given me by my wife during my graduate study at the Institute.

ABSTRACT

Measurements of the distortions of the tracks in a cloud chamber were made using a multiple exposure technique. It was determined that the chamber, when used in a magnetic field of 8000 gauss, was capable of yielding measurements of the momentum of ionizing particles up to approximately 5 Bev/c. This corresponds to an uncertainty in the motion of the droplets forming a track of 0.005 cm. The operation of the chamber in different thermal conditions was studied, and it was concluded that the chamber should be operated with a top temperature several hundredths of a degree C higher than that of the bottom. An analysis of the motion of the gas in the chamber was made and compared with the experimental values. The behavior of tracks near the chamber walls was studied.

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I. INTRODUCTION

One of the most important uses of a Wilson cloud chamber is to measure the momenta of ionizing particles. This is done by measuring the curvature of the photographic image of the track formed upon expansion in a magnetic field. The momentum is proportional to the radius of curvature, and the limits of accuracy and the upper limits of measurable momentum are set by the errors in the measured curvature. The errors may be caused by the measuring techniques, by the photographic techniques, by multiple scattering, or by the failure of the cloud chamber to give tracks which are faithful reproductions of the path of the ionizing particles. This last effect is the most important source of error and is the subject of this thesis. Full discussion of the other phenomena may be found in references 1 and 2.

Due to the finite rate of growth of the droplets around the ions formed by the incoming particle it is necessary to wait for a time after expansion for these droplets to grow large enough to be photographed. Typical time intervals between expansion and photography are from fifty to one hundred milliseconds and during this period the droplets move from their original positions in space causing an erroneous indication of the track of the ionizing particle. The droplet motions are caused by those motions of the body of gas in the chamber which exist both before and after expansion, and by the action of gravity upon the droplets. The initial, pre-expansion, convection currents are due to the non-uniform thermal condition of the chamber and are present for extremely small temperature differentials. The movement of the gas after expansion is caused by the conduction of heat from the warm walls of the chamber into the cool volume of gas which results in a non-uniform density. Gravity acting upon this variable-density gas then causes non-uniform movements of the gas throughout the chamber.

To minimize the errors caused by the pre-expansion convection currents very elaborate schemes for temperature control are used. It is necessary to hold the temperature differentials throughout the chamber to a few hundredths of a degree centigrade for maximum accuracy, and the difficulty of this is great. It requires both precise measurement and control of temperature, usually in the presence of a large heat source (the magnet which is used with the 48" chamber at the Institute consumes 80 kw of power).

The effects of the post-expansion currents may be minimized by using as short a time interval between expansion and photography as possible, but the lower limit of the delay is set by the photographic techniques available. A practical lower limit of this interval is between 50 and 100 milliseconds.

These combined effects have been studied previously only by the method of measuring the curvature of tracks without a magnetic field.[3] When this is done for a sufficiently large sample, the probable error in curvature may be computed, and a corresponding momentum error evaluated for a given applied magnetic field. The method is open to criticism on two grounds: first, it assumes that the thermal conditions are the same for both no-field and full-field operation; and second, it does not readily yield information on the actual distortion of a track as a function of the thermal condition. The first is the more serious, as the distortions are very critically dependent upon the exact thermal condition of the chamber, and unless the magnetic field can be removed without disturbing both the total consumption and the distribution of the power dissipated near the chamber the thermal conditions are almost certain to change. Some laboratories remove the field by reversing the current in half of the windings and this is a much more acceptable method than that of merely removing the power source completely. The second objection is still valid.

For these reasons a new approach to the problem was attempted. What is desired is a time-displacement history for each droplet in a track. A device using multiple exposure photography of each event which gives a picture of the track at three different times was constructed. It is described fully in the next section. Given this information it would be possible to extrapolate the position of all points on the track back to the time of expansion, giving a true picture of the path of the ionizing particle through the chamber. If these time-displacement functions were constant from exposure to exposure, all data could then be converted from a single determination of the function throughout the chamber. If not, a statistical estimate of the errors involved may be made, yielding a value for the maximum measurable momentum.

With this technique it is possible to measure directly the effect of thermal gradients in the chamber and then to vary the conditions until the optimum operating conditions are obtained.

The two general situations which were examined are the chamber in which the top is definitely warmer than the bottom, and the chamber in which the top is either at the same or at a slightly lower temperature than the bottom. The former is a chamber with a stable atmosphere, the latter with an unstable atmosphere. The stable case should have no pre-expansion convection currents, and except near the walls the post-expansion currents should be uniform, causing little or no distortions.

The unstable atmosphere is expected to have a continuous circulation, resulting in severe pre-expansion distortion. The case of a truly uniform temperature is expected to be unstable, with any disturbance dying out very slowly. For these reasons, cloud chambers are typically operated with a stable atmosphere, but it is desirable to know how poorly a very slightly unstable chamber performs.

An analytical treatment of these problems is difficult, especially for the case of the unstable atmosphere where both conduction and convection determine the interior temperature distribution of the gas, and has not yet been given. In section III some simple approximate analyses are given in order to help evaluate and explain the experimental data, but due to the complexity of the equations involved, a complete analysis seems impossible.

II. DESCRIPTION OF APPARATUS

A rectangular cloud chamber designed by R. B. Leighton of the Institute with dimensions of 18 x 18 x 9 cm depth, filled with argon and 65% by volume alcohol and water mixture to a total gauge pressure of 24 cm of mercury was used. The expansion was caused by a movable rear piston covered with black velvet, and was controlled by Geiger counters in a standard manner. The illumination was from the side and from a General Electric FT- 422 high voltage flash tube. Photography was on 70 mm Eastman Linograph Pan film, using a stereoscopic system of two f/4.5 10" lenses stopped down to f/11, with a reduction of 3. The film was examined and measured on an optical comparator and on a reprojection system. The appearance of each droplet was that of a set of three images, the darkest being the latest in the time sequence, due to the droplet size increasing with time. This made it possible to determine the direction of motion easily.

A block diagram of the multiflash unit is shown in Figure 1. It consisted of a 2000 volt power supply to charge the condensers used in the flash system, a switching circuit to connect the various condensers to the lamp in sequence, and a timing unit to control the switching unit and to provide accurately timed pulses to flash the tube. While the relatively

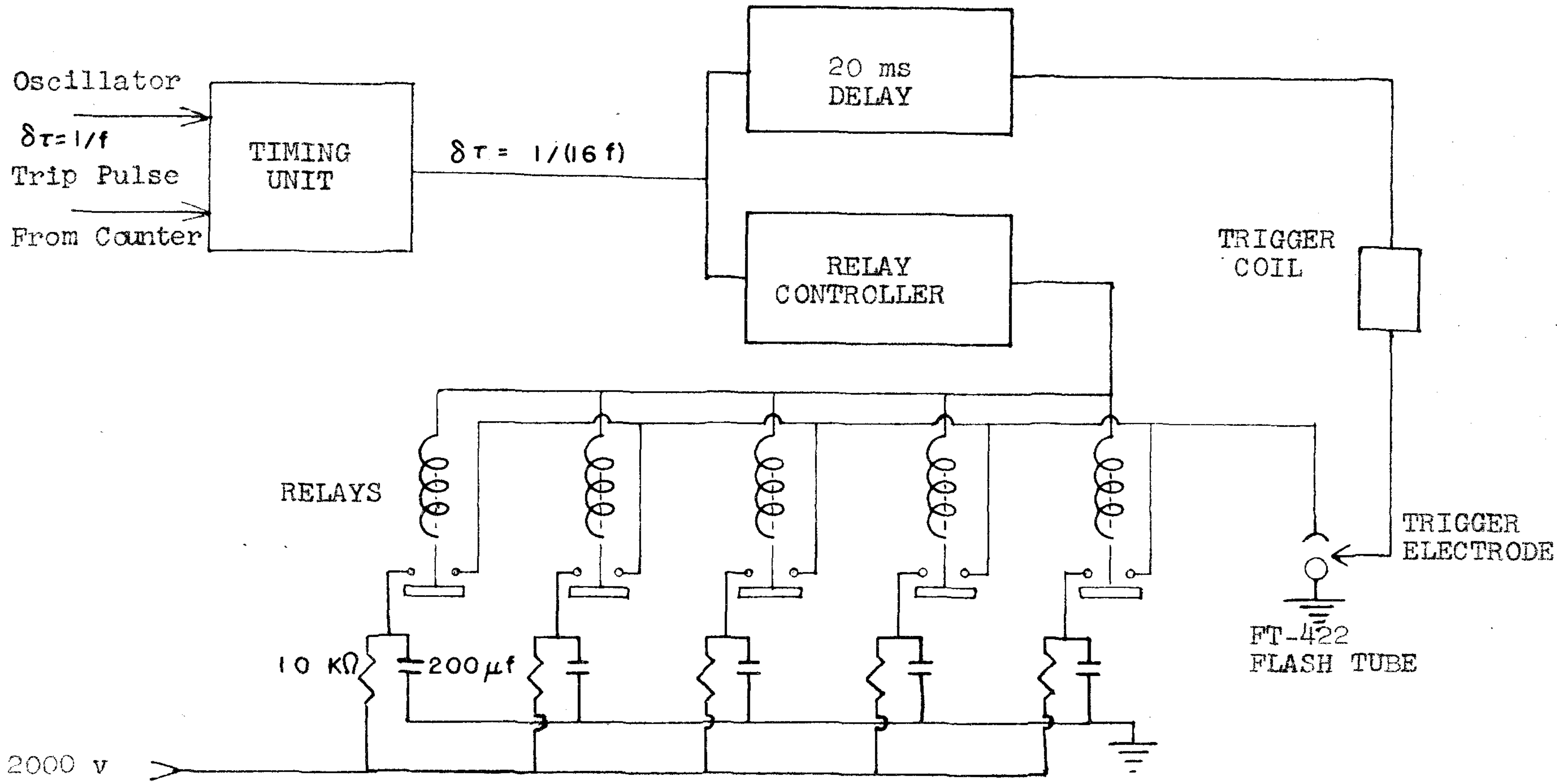

Figure 1. Multiflash unit.

crude relays switch the high voltage, the actual firing was electronically controlled.

The relays used were rewound and re-insulated 24 volt, 200 ampere aircraft contactors. They proved adequate to carry the rather high peak currents of the flash discharge which reach 250-300 amperes. The relay coils were wound to operate in a thyratron circuit in which a condenser was discharged through the tube and the relay coil. Out of six relays fabricated four have survived, two failing due to high voltage breakdown. The insulation used was polystyrene initially, with one unit rebuilt using Teflon. In general the performance of the relays has been adequate.

Since the expansion is counter controlled and occurs at random times, a method of synchronizing the flashes with the expansion was needed. For this purpose a special timing circuit was devised, using a free-running oscillator as a timing standard. For this, any commercial unit with proper frequency range is satisfactory, a Hewlett-Packard Model 200-C audio oscillator being used on this occasion. The oscillator signal was applied through a gating tube to a scale of sixteen frequency divider. When the signal initiating the sequence is applied to the gating tube, the next positive cycle from the oscillator starts the frequency divider operating. The pre-set condition of this divider is such that the first pulse is transmitted to the output as is each sixteenth cycle thereafter. This scheme results in an average time delay between the initiating pulse from the counter unit and the first output pulse of one-thirty-second of the flash spacing. Other possible solutions to the timing problem would require a scheme such as a ringing oscillator started from a pulse. At the low frequencies corresponding to the flash spacing, such a device is difficult, and the problem of measuring the actual spacing is greatly simplified with a continuously running oscillator. The timing unit also contains the necessary circuitry to stop the sequence of output pulses after a sufficient number have been generated and to reset the scaling circuit for the next sequence. The period from the expansion to the first light flash is also adjustable.

The series of output pulses from the timer is applied to a set of 2D21 thyratrons which control the high-voltage relays. The thyratrons are arranged so that they fire in sequence upon receiving the output pulses from the timing circuit, causing each relay to operate in sequence, connecting the charged condenser associated with it across the flash tube. Each relay stays closed during the approximate interval from 15 to 25 milliseconds after its associated thyratron is fired. The flash tube is fired by applying the output of an ignition coil to a trigger electrode connected to the tube. This ignition coil is triggered by a pulse from the timing unit which is delayed by twenty milliseconds from the pulse operating a thyratron. Thus the tube is flashed at a time when the relay contacts are firmly closed, minimizing the arcing and sticking of contacts. The variation of the time between the pulse initiating the timing sequence and the first flash is, on the average, one-thirty-second of the flash spacing, and succeeding pulses are exactly spaced an amount equal to one-sixteenth of the input oscillator pulse spacing. Since the initiating pulse is usually delayed after expansion by an amount of fifty to one hundred milliseconds, and the flash spacing is of the same magnitude, the uncertainty in the time interval between the expansion and the first flash is a few milliseconds out of approximately one hundred milliseconds.

The multi-flash unit was originally built to have a capacity of five flashes. Each flash was of 400 watt-second energy content. Experience showed that this was too many to give good photography, causing a generally poor quality picture, and the experiment was performed with three flashes. This was a sufficient number to show any non-linearities of droplet motion.

The time intervals between flashes were chosen experimentally for ease in interpreting the pictures. In general, initial delays between initiation of expansion and the first flash of sixty to eighty milliseconds, and flash spacing of fifty to one hundred fifty milliseconds were used. The longer spacings gave images with more separation on the film and hence were more easily interpreted, but the background "fog" in the chamber increased rapidly so that a compromise time was chosen. Most of the pictures were taken with an initial delay of eighty milliseconds and a flash spacing of one hundred thirty three milliseconds. The chamber had a time of expansion of 10 milliseconds, thus making the effective interval between expansion and the first flash 70 milliseconds.

The flash tube finally selected was the General Electric FT-422. This proved to be a rugged and reliable tube, several other types failing rapidly under the rather severe duty cycle. The FT-422 has a large gas volume and large electrodes, which probably accounts for its superior performance. It does not produce a beam as well-collimated as do smaller diameter tubes however, and this caused some deterioration in photography.

In order to control the temperatures with precision, the chamber was installed in a copper box whose temperature was stabilized by circulating water through tubes soldered to the surface of the box. The chamber was supported on wooden blocks and was in a very stable thermal condition. The temperature of the cooling water varied slowly, and the thermal gradients over the chamber were nearly independent of this change of ambient temperature. The change in ambient as measured on the wall of the box has not exceeded 5° C. This long term stability has been of the greatest importance and the measurement of thermal conditions was greatly facilitated by this constancy.

The temperature variations over the chamber were measured by means of a set of copper-constantan thermocouples. The copper-constantan wires are twisted together, soldered, then soldered into a small brass tube. A plastic tubing was forced over the brass sleeve for insulation and the sleeve and tubing inserted into the holes in the chamber wall. Another method used was to insert the thermocouples into copper blocks and fasten these blocks to the surface of the chamber. In general the first method gave more reproducible temperature readings.

The thermocouples were manufactured in pairs and used to measure the temperature differential between corners, that is, along the edges of the chamber, as shown in Figure 2. A tap switch with silver-plated contacts was used in conjunction with a Leeds and Northrop Model 2430 galvanometer to indicate the output from each thermocouple pair. This was so arranged that only the copper leads are switched, and the extraneous thermal electromotive forces from this source were experimentally determined to be insignificant. The overall sensitivity of the system was approximately 0.002° C per millimeter of galvanometer deflection. An estimate of the accuracy and general performance may be made by means of a "closure" check which consists of adding up the thermal drops around a closed path. The closures were generally better than 0.02° C for a four-sided path, implying a random error of perhaps 0.01° C for each thermocouple pair.

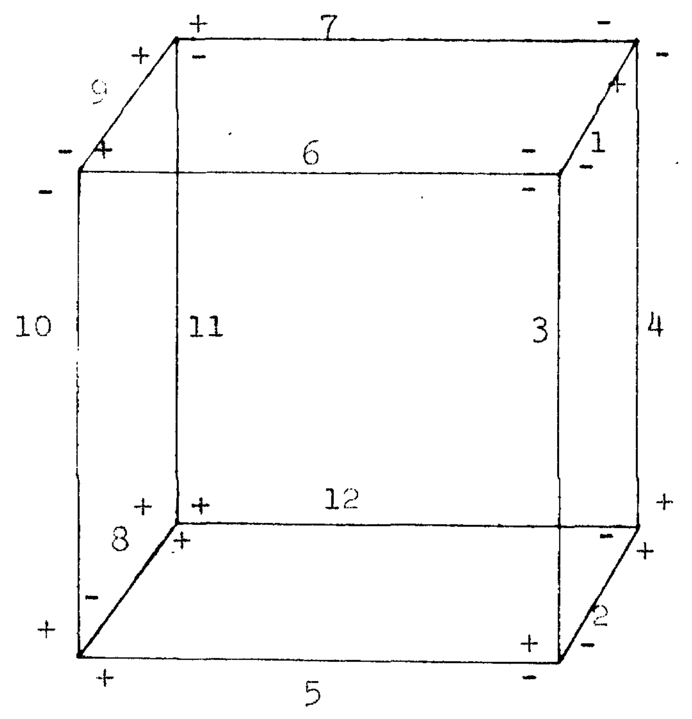

Figure 2. Thermocouple locations. The positive readings indicate that the bottom, the left, and the rear are warm. A positive differential means that the temperature increases along an edge from the minus to the plus sign. For example, the entry in Table I for run M-3 along the upper right edge indicates that the front of the chamber is warmer than the rear.

An automatic temperature recording system was constructed which periodically sampled the thermocouple readings and photographed them on thirty-five millimeter film. After preliminary operation of this unit it was discovered that the thermal gradients were constant within the accuracy of measurements for periods up to several hours. Accordingly the automatic feature was discontinued, and manual observations at less frequent intervals substituted. This large degree of stability was caused by the good thermal isolation of the chamber and by the lack of any large nearby heat source, such as a magnet.

To control the temperature of the chamber, a set of electrical heating elements was used. By varying the applied power, the gradients could be established as desired. The elements consisted of a series of one watt Ohmite carbon resistors, imbedded in Saureisen cement in an aluminum or bakelite block which was cemented to the chamber walls. The applied voltages were adjusted by a group of small Variacs until the desired thermal conditions prevail. Some difficulty was encountered in obtaining a given temperature distribution, particularly for the nearly isothermal chamber. An additional factor was that when the chamber is very nearly isothermal the alcohol water vapor tended to condense on all walls of the chamber, including the front plate. This was aggravated by the existence of the camera well which tends to cool the front plate of the chamber very slightly. By adjusting the flow of cooling water to the camera well, this tendency was greatly reduced but it remained a problem throughout the experiment, causing several experimental runs to be wasted.

III. EXPERIMENTAL RESULTS

1. General

The chamber was studied in two general conditions. In the first case the chamber was operated with a definitely stable atmosphere, the bottom being cooler than the top by an amount 0.10 - 0.15° C. Table I gives the detailed temperatures recorded during the three separate runs.

In the second case the chamber was operated with the top and bottom at nearly the same temperature. Table II gives the measured temperatures for the three runs in this condition. It had been found experimentally that the chamber atmosphere actually was unstable for conditions in which the top was 0.02 - 0.04° C warmer than the bottom. The gas was so lightly damped so as to be effectively unstable, the slightest disturbance requiring a considerable time to die out.

During a run lasting several hours it was estimated that the various temperatures varied not more than 0.01° C per reading for readings less than 0.05°C, and by not more than 0.02 - 0.03° C for readings greater than 0.05° C. The accuracy of measurement, estimated by means of closures, was of the same magnitude as the variations.

This evaluation of the general accuracy of the tempera-

TABLE I

TEMPERATURE MEASUREMENTS ON STABLE CHAMBER

IN DEGREES CENTIGRADE

Thermo-	Run		
couple	A-6	M-1	M-2
1	-.04	-.02	-.03
2	-.05	-.03	-.05
3	-.15	-.15	-.14
4	-.15	-.11	-.12
5	+.02	0	+.02
6	+.01	0	+.02
7	+.03	+.01	+.02
8	-.05	-.03	-.05
9	-.03	-.02	-.03
10	-.15	-.12	-.12
11	-.15	-.13	-.14
12	+.01	0	0

TABLE II

TEMPERATURE MEASUREMENTS ON UNSTABLE CHAMBER IN DEGREES CENTIGRADE

Thermo-	Run		
couple	M-3	M-4	M-5
1	-.02	-.02	-.03
2	-.02	-.03	-.04
3	-.01	0	+.01
4	+.01	+.02	-.01
5	+.02	+.03	+.02
6	0	+.02	+.01
7	+.01	+.02	+.01
8	-.03	-.03	-.03
9	-.01	-.01	-.01
10	0	+.01	+.01
11	+.01	+.01	0
12	0	+.01	0

ture measurements indicates that a limit of temperature control of about 0.01° C exists for all cloud chambers which use this method for measurement. In general the accuracies will be less, as the usual operating environment is much less favorable. For example, heat is conducted along the leads into the thermocouple if the leads pass through regions of higher temperatures than the chamber. If this temperature difference is large, the thermocouple reading will be in error due to the local heating at only one junction. In the present experiment the entire apparatus was at nearly the same temperature, eliminating this error.

The two cases will now be treated separately.

2. The Chamber Operated With a Stable Atmosphere

The displacements of the droplets between the first and second and between the second and third flashes were measured. The interval between expansion and the final flash was 70 milliseconds, and the interval between flashes was 133 milliseconds. A large number of the total displacements between the first and last flashes are plotted on the chamber rear plate in Figure 3. These displacements are from a total of about 100 pictures, and from several tracks in each picture, and give a clear indication of the predominately vertical motion of the droplets at regions further than about 1 cm from the chamber wall. It is estimated that 90% of all the displacements were within 30° of the vertical,

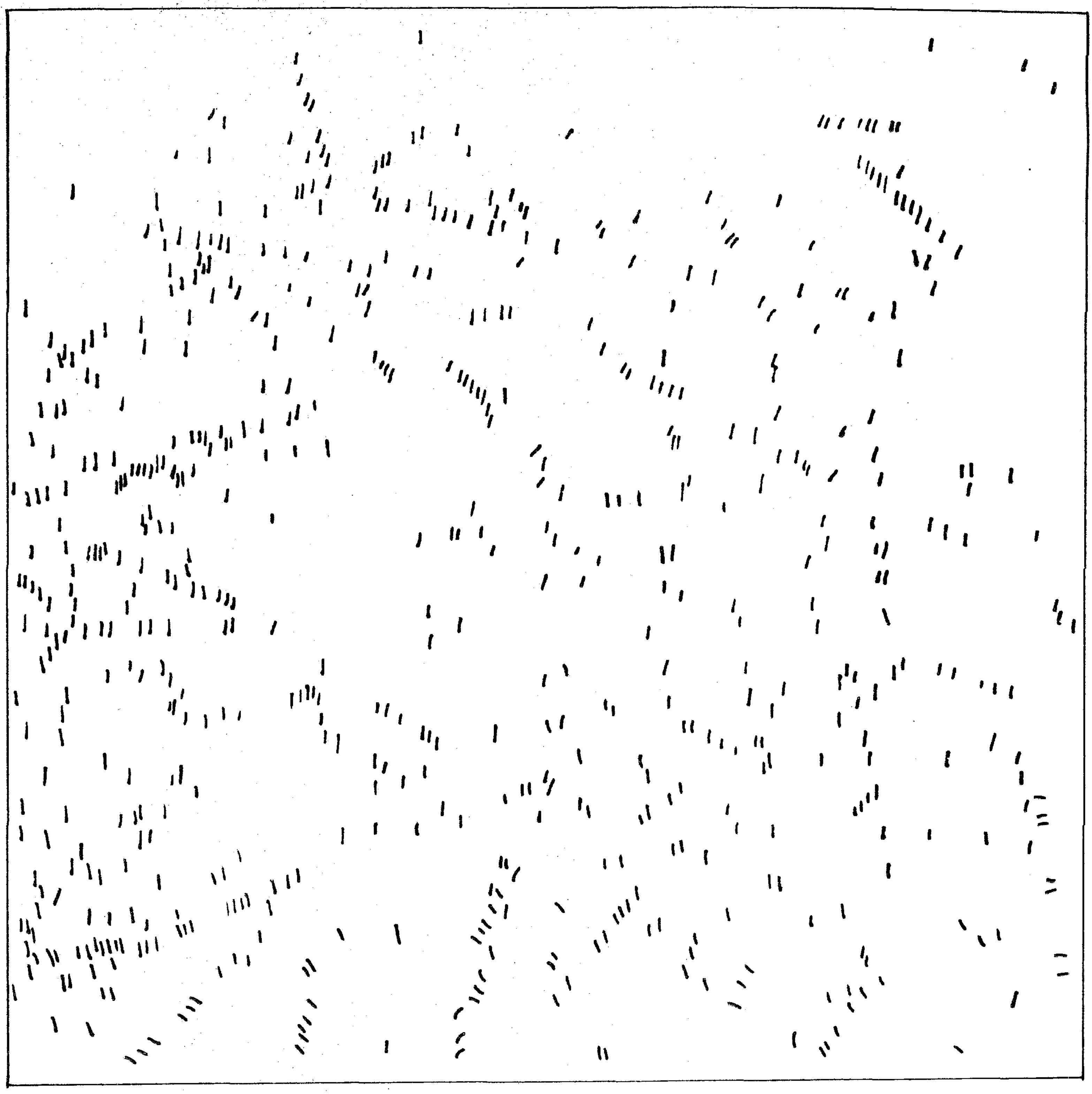

Figure 3. Displacements of droplets during 266 millisecond interval between first and third flashes, projected on rear of chamber, for a chamber with stable atmosphere.

and, excluding the lower left hand region of the chamber, 90% of all other displacements were within 15° of the vertical. This indicates that the peak horizontal displacements were about 1/3 the peak vertical displacements. The front-rear displacements were found to be approximately zero. The magnitudes of the vertical displacements between the first and second flashes ranged from 0.3 - 1.0 mm; the displacements between the second and third flashes ranged from 0.5 - 1.5 mm.

For distances of 1 cm or more from the walls of the chamber there was no connection between displacement and location in the chamber, except for the lower left hand corner of the chamber. This region had a consistent motion inclined at about 45° from the vertical, and it was not possible to eliminate it. The chamber was dismantled and reassembled in an effort to reduce this motion, but this gave no improvement, and it must be concluded that the source of this local distortion remains unknown.

In order to evaluate the uncertainty in momentum measurements caused by the falling droplets an estimate of the fall between expansion and photography was made by computing the time-displacement curve and evaluating it at the time of the first flash. A modified form of Stokes' law suitable for small droplets falling through a stationary body of gas was used to indicate the probable law of fall. This law may be written [4,5,6]

$$\dot{y} = \frac{2g}{9n}(d)(r^2 + br) \qquad (1)$$

where $\dot{y}$ = rate of fall

g = acceleration of gravity

d = droplet density = 0.9 gm cm^{-3}

b = mean free path of gas = 10^{-5} cm

n = viscosity of gas = 2 x 10^{-4} poise

r = droplet radius.

Hazen[7] and Barrett and Germain[8] have measured the rate of growth of droplets by applying this law, and have obtained the general result that

$$r^2 = Kt \qquad (2)$$

No values of K for argon were available, but an estimate made from theoretical considerations and from the rates of growth in heavy gases yielded

$$K = 4 \times 10^{-6} \text{ cm}^2 \text{ sec}^{-1} \qquad (3)$$

Substituting this in (1) and integrating results in

$$y = \frac{2gd}{9n} \quad \frac{K}{2}t^2 + \frac{2}{3} b \sqrt{K} t^{3/2} \qquad (4)$$

$$y = 2t^2 + 0.013t^{3/2} \text{ cm} \tag{5}$$

where y and t are measured from the time of completion of expansion. For the times of interest the second term is negligible, resulting in

$$y = 2t^2 \text{ cm} \tag{6}$$

The observed displacements should fit this law, but did not. In particular the predicted values of the displacements were:

expansion to first flash displacement	y_o	$= 0.01$ cm
first to second flash displacement	$y_1 - y_o$	$= 0.07$ cm
second to third flash displacement	$y_2 - y_1$	$= 0.12$ cm

(7)

Experimentally the measured displacements were found to fit a $t^{3/2}$ curve much better than they fitted the predicted t^2 curve. Using the $t^{3/2}$ variation as a guide the mean vertical displacement at the time of the first flash was determined to be

$$y_o = 0.02 \pm 0.005 \text{ cm} \tag{8}$$

This represents an average over the entire chamber and over many pictures. The average over droplets in a single track a few centimeters long gave a standard deviation of only $\pm$ 0.002 cm.

The mean value for horizontal left-right displacements was of course different, being zero. The standard deviation was approximately 0.005 cm, both for the average over the entire chamber and for the average over a given track. Given the uncertainties in the true location of a droplet at the time of expansion the maximum detectable radius of curvature R of a track s cm long may be computed. If the track were straight except for the uncertainties in the locations of the three droplets, the square of the standard deviation of the curvature $\frac{1}{R}$ is

$$\sigma^2\left(\frac{1}{R}\right)=\left(\frac{8}{s^2}\right)^2\overline{\left(\frac{y_1+y_2}{2}-y_3\right)^2} \tag{9}$$

where the displacements of the three droplets used in establishing the curvature are y_1, y_2, y_3, each with mean zero and (equal) standard deviations σ_D. This yields

$$\sigma^2\left(\frac{1}{R}\right)=\left(\frac{8}{s^2}\right)^2\left(\sigma_D^2 \times \frac{3}{2}\right) \tag{10}$$

and the corresponding maximum radius is

$$R_{max} = \frac{s^2}{8\sqrt{1.5}\,\sigma_D} \qquad (11)$$

Numerically for a track 10 cm long with

$$\sigma_D = 0.005 \text{ cm},$$
$$R = 20 \text{ meters}. \qquad (12)$$

With a magnetic field B of 8000 gauss, as used in the 48" cloud chamber at the Institute, the maximum measurable momentum becomes

$$pc_{max} = 300\, B\, R_{max} \quad \text{eV}$$
$$= 5 \text{ Bev} \qquad (13)$$

The estimate above is valid for tracks of moderate length, say 10 cm in this chamber, in regions of the chamber not closer than 1 cm to the walls. It is approximately the same for both horizontal and vertical tracks in the chamber. Using a larger chamber and doubling the track length will probably not increase the maximum measurable momentum by a factor of 4, since the deviation of the droplet motion tends to increase with the separation of the droplets concerned. For example, the deviation quoted above for relatively short tracks, say 5 cm, is smaller than that for the larger total space in the chamber by a factor of about 1/2. Consequently, the deviations over a larger chamber may be expected to increase, and doubling the track length will give only a portion of the increased accuracy expected. Based on the above calculation, a total increase in the maximum measurable momentum might be a factor of 2 instead of 4, due to the combined effect of an increase in 4 of s^2 and a decrease of 2 in $1/\sigma_D$. This is only a linear projection of the behavior of a small chamber to that of a large chamber, and consequently is only approximate.

The behavior of tracks near the chamber walls is of great practical importance, as there are usually rather abrupt changes in curvature and position which make momentum measurements uncertain. A typical picture of a track near a wall is shown in Figure 4.

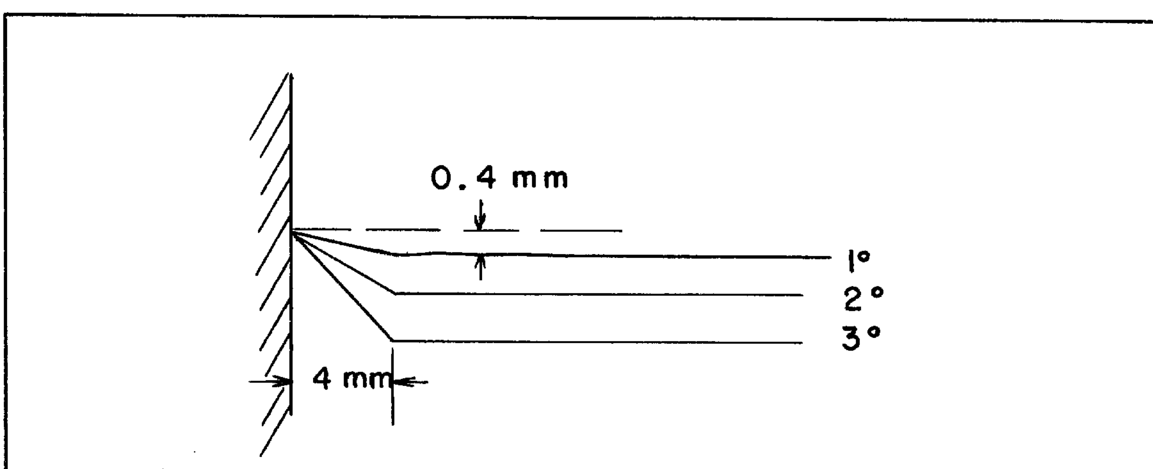

Figure 4. Trace of displacements near edge of chamber.

In general the set of three traces passes through the chamber wall in a point. If this point were the original position of the track at the time of expansion, the distance marked 0.4 mm should correspond to the fall between expansion and photography, measured above to be about 0.2 mm. The difference is more than that expected and is probably due to the apparent point of intersection actually being slightly away from the wall and being carried up rapidly by the thermal convection currents formed close to the wall. Support for this is given by a picture in which the first and second traces intercept at a point at or very near to the wall, but in which the third trace reaches the wall at a substantially higher point yet.

The magnitude of the upward currents near the walls may be determined approximately by considering the region to be the edge of a semi-infinite body of thermal diffusivity k in which the initial uniform temperature T_o is suddenly raised an amount ΔT_o at the edge.

The temperature distribution at time t and distance x from the edge is given by[9]

$$T = \Delta T_o \operatorname{erfc} \frac{x}{2\sqrt{kt}} + T_o \qquad (14)$$

where

$$\operatorname{erfc} x = 1 - \operatorname{erf} x \qquad (15)$$

and erf x is the usual error function.

The cloud chamber vapor has a value of $k = 0.50$, approximately, and for a time $t = 0.07$ sec values of $\Delta T = T - T_o$ are given in Table III

TABLE III

Thermal Distribution Near Chamber Wall After Expansion

	$\Delta T / \Delta T_o$	
x cm	t = .07 sec	t = .20 sec
0.1	0.7	0.8
0.2	0.4	0.7
0.3	0.2	0.5
0.4	0.1	0.4
0.5	0.06	0.3
1.0	0.00	0.02

Let us replace this exact distribution with a simple square step model. The temperature is assumed to be $T = T_o + \Delta T_o$ for $0 < x < 0.2$ cm, and to be $T = T_o$ for $x > 0.2$ cm. The density of the gas varies in just the opposite fashion, giving

$$d = d_o - \Delta d_o \text{ for } 0 < x < 0.2 \text{ cm}$$

and $$d = d_o \text{ for } x > 0.2 \text{ cm}$$

The equation governing the velocity of rise in this thin heated film of gas is the Stokes-Navier equation. For the case at hand it may be written as

$$n \frac{\partial^2 v}{\partial x^2} = gd + \frac{\partial p}{\partial y} + d_o \frac{\partial v}{\partial t} \tag{16}$$

where v = vertical velocity in the y direction

and $\frac{\partial p}{\partial y}$ = pressure gradient and is given approximately by $- gd_o$.

This gives

$$n \frac{\partial^2 v}{\partial x^2} = - g \Delta d_o + d_o \frac{\partial v}{\partial t} \tag{17}$$

subject to the boundary condition that $v = o$ at $x = 0$ and $x = 0.2$ cm.

The solution of (17) may be written down in terms of error functions. For times less than $t = .2$ sec the result for points 0.5 mm from the wall is

$$v = Kt \tag{18}$$

where

$$K = \frac{9}{4} \quad \frac{\Delta d_o}{d_o}$$

For the expansion used in this chamber, $\frac{\Delta d_o}{d_o} = .06$, giving

$$K = 15 \text{ cm sec}^{-2}$$

This is a value about four times as large as the expected rate of fall of the droplets in the still portion of the gas, explaining why the droplets rise so rapidly near the edge. It indicates also that extreme care must be exercised in using that portion of tracks passing through the chamber wall for curvature measurements.

3. The Chamber Operated With an Unstable Atmosphere.

The chamber in this case was operated under the conditions shown in Table II with the resulting displacement shown in Figure 5. The anticipated general circular motion of the gas is evident, as well as the superposed downward fall after expansion. The displacements at the time of expansion are rather difficult to predict because there exists no simple displacement-time relationship valid over the entire chamber.

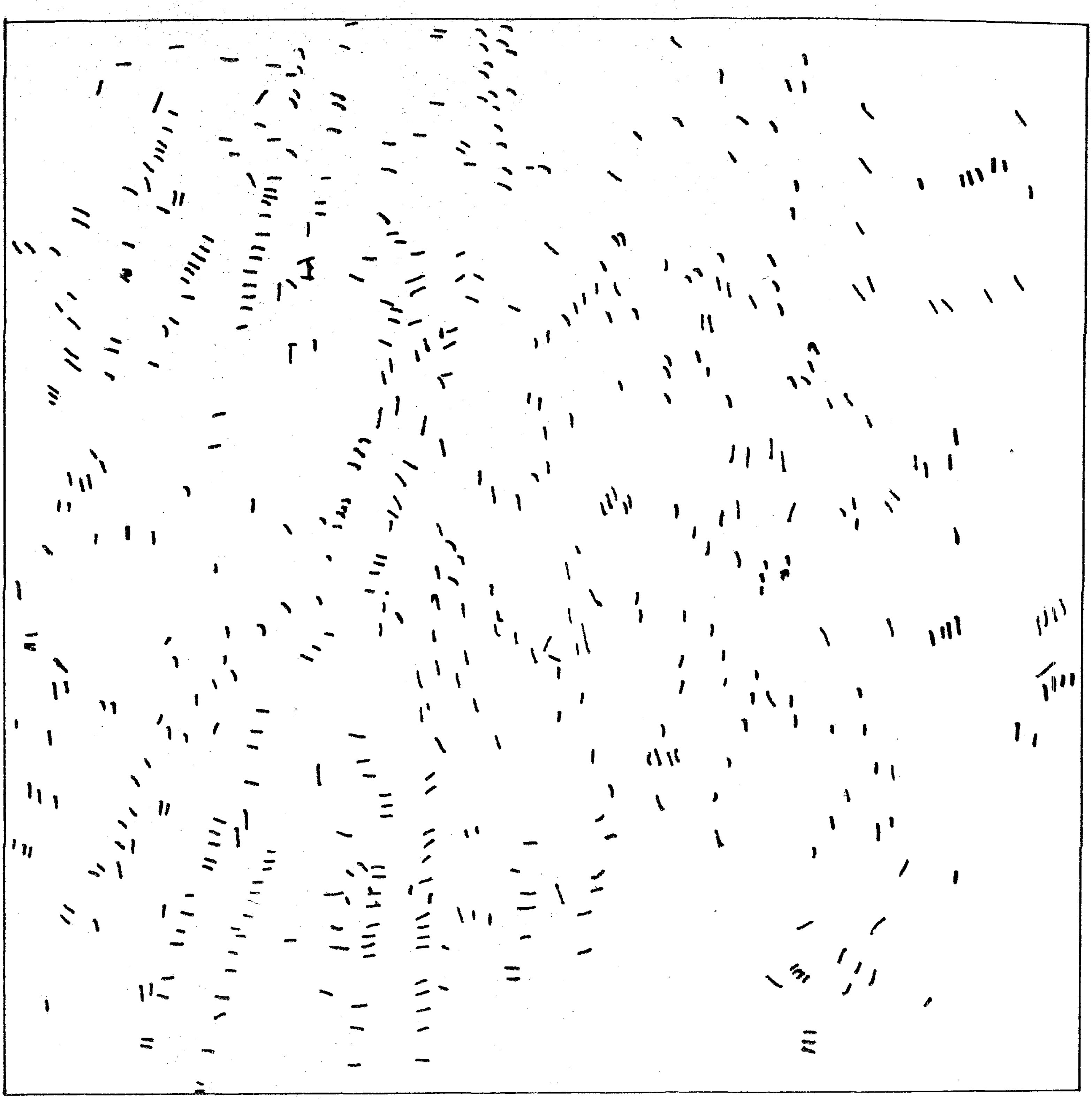

Figure 5. Displacements of droplets during 266 millisecond interval between first and third flashes, projected on rear of chamber, for a chamber with unstable atmosphere.

(For the stable case the displacement-time equation was independent of the location except very near the walls.) A very approximate analysis will be given to aid in interpretation of the photographed displacements.

The problem is to determine the motion of a non-uniform compressible viscous gas under the influence of gravity. The non-uniformity is due to the temperature distribution within the gas while the temperature distribution is itself influenced by the motion. There exists no general treatment of this problem in the literature pertaining to gaseous hydrodynamics. However, a similar problem is of interest in the phenomena of gaseous diffusion,[10] and a partial solution is obtained using methods applicable to the analysis of thermal diffusion columns.

The first consideration is to the relative importance of convection currents and conduction in determining the temperature distribution throughout the chamber. No point in the chamber is more than 4.5 cm from the front or rear plate, and hence the chamber has a characteristic thermal time constant for conduction from the front and rear plate, given by

$$T = \frac{4b^2}{k\pi^2} \tag{20}$$

where b = half-depth of chamber = 4.5 cm

k = thermal diffusivity = 0.5 cm^2 sec^{-1}

Numerically T = 16 sec.

If now the gas circulates from one side of the chamber to the other in about this same length of time the conduction and convection effects will be approximately equal. Since the path is semi-circular in shape and is about $\pi \cdot 9$ cm in length, the circulation velocity would have to be about 1.8 cm sec^{-1} in order that the two effects be approximately equal. The velocity is anticipated to be substantially less than this so that for a first approximation it may be assumed that the interior temperature of the chamber is established by conduction of heat. This represents a very great simplification of the problem as the heat distribution is now linear over the chamber.

We now proceed to find the velocity profile along the horizontal line through the center of the chamber. Construct a rectangular coordinate system through the center of the chamber with y positive upward and x positive to the right. Consider now the region near $y = z = 0$ as a semi-infinite region, neglecting the depth effects and the vertical variation of quantities of interest. The Stokes-Navier equation for the y-velocity v may be written

$$n \frac{d^2 v}{d\,x^2} = \frac{\partial p}{\partial y} + gd \qquad (21)$$

where $\frac{dp}{dy}$ is the pressure gradient, dg represents the body forces due to gravity, and n is the viscosity.

The temperature, assumed determined by conduction, varies uniformly along the x-axis, and for a left-right thermal difference of ΔT_o,

$$T = T_o + \frac{\Delta T_o}{2a} x \qquad (22)$$

where 2a is the chamber width and T_o the temperature at the center. Along the line y = 0 the pressure is constant, there being no x-velocity of the gas. The density varies inversely as the temperature, giving

$$d = d_o + \frac{\Delta d_o x}{2a} \qquad (23)$$

$$= d_o - \frac{\Delta T_o x}{2aT_o} \qquad (24)$$

where d_o is the density of the center.

The vertical pressure gradient is approximately $-gd_o$, giving finally

$$n \frac{d^2 v}{dx^2} = -d_o g \left(\frac{\Delta T_o}{T_o} \right) \left(\frac{x}{2a} \right) \qquad (25)$$

The boundary conditions are such that v vanishes at the chamber walls. The solution is

$$v = \frac{d_o g \Delta T_o x(a^2-x^2)}{12naT_o} \tag{26}$$

For the values for this chamber ($d_o = 10^{-3}$ g cm^{-3}, ΔT_o = 0.01° C, $n = 2 \times 10^{-4}$ poise, $T = 300^\circ$ K, $a = 9$ cm) $v_{max} = 0.4$ cm sec^{-1}.

This is sufficiently less than the 1.8 cm sec^{-1} previously computed for equal effects so that the assumption of a predominately conductive thermal equilibrium is reasonable. It is notable that as long as the chamber is unstable the amount of convection is set by a lateral thermal gradient rather than by the vertical gradient. Superposing this initial velocity upon that due to Stokes' law the vertical displacements along $y = 0$ are predicted to be

$$y = 2t^2 + 3 \cdot 10^{-3} (81 - x^2)xt \tag{27}$$

Again as in the previous case this is not expected to be valid close to the chamber walls due to the local heating after expansion.

Numerically the observed displacement between the first and third flashes is predicted to be

$$y_2 - y_o = 0.22 + 8.0 \cdot 10^{-4} (81 - x^2)x \text{ cm} \tag{28}$$

The second term is smaller than the first except at the maxima at $x = 3\sqrt{3}$ cm. At this point the two terms are nearly equal. Consequently the convection current effect is not large enough to reverse the apparent motion of droplets during the interval between the first and third flashes. This is in general agreement with the observations. The thermal gradient across the chamber was in the direction to cause the convection currents to rise on the left, making the net displacement smaller. This was observed to be the case, but the displacements were too erratic to give a detailed check of the predictions. For this reason also no attempt to evaluate the displacements at the time of the first flash or to compute the deviations of such displacements was made. It is clear from examining Figure 5 that the motion is far less predictable than for the stable case, and that the corresponding momentum measurements would be less accurate. Since this experiment was carried out on a nearly neutral chamber, it indicates that the required degree of temperature gradient from top to bottom for true stability may be higher than commonly thought.

IV. CONCLUSIONS

The following conclusions may be drawn:

1) The multiple exposure method provides a fairly convenient method for estimating distortions of tracks as a direct function of the thermal conditions, giving estimates of maximum detectable momentum in approximate agreement with those obtained by other methods. A track 10 cm long in an 8000 gauss field may yield a maximum momentum of about 5 Bev/c.

2) The droplets fall about 0.02 ± 0.005 cm during an 0.07 sec interval between expansion and photography, for distances further than 1 cm from the walls.

3) Near the walls the droplets are both computed and observed to rise sharply, throwing doubt on the momentum measurements using these portions of a track.

4) A neutral chamber, or one thought to be slightly stable on the basis of temperature measurements, is really quite unstable and causes larger errors in measurement than does a stable chamber.

5) A chamber should be operated with a substantial top to bottom gradient, the top being warmer than the bottom.

6) Temperatures on the test chamber may be measured and controlled to an accuracy of $0.01 - 0.02^{\circ}$ C for periods up to several hours. Under more severe environmental conditions a larger variation may be expected.

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