THE MEASUREMENT OF DIELECTRIC LOSSES AT A FREQUENCY OF ONE HUNDRED MEGACYCLES PER SECOND

A Thesis by

Bernard M. Oliver

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DEFINITORY NOTE

The term "Dissipation Factor" has been here used throughout to denote that quantity which, when multiplied by the current drawn by a given sample capacity gives the in-phase, or loss, component of the total current. It is thus the tangent of the "loss angle" and for small losses is therefore equal to the power factor of the dielectric.

It has been called to the author's attention that this definition is not in accord with general usage. To obtain the quantity which is usually called the "Dissipation Factor", the quantity here used should, in each case, be multiplied by the dielectric constant of the material to which the given dissipation factor pertains.

1. INTRODUCTION

The growth of the radio art today is principally in the direction of the utilization of higher and higher frequencies. The cry of "Lebensraum!" long familiar in the communication world is heard as often today as ever before. Room for more communication channels! Room for higher fidelity transmission by wide band frequency modulation! Room for high definition television - the monster whose frequency spectrum extends from zero to many megacycles per second! Room for these, and for many other radio uses: room to be found only in the ultra high frequencies, for above 100 megacycles the air is literally free.

This expansion into the shorter wavelengths has already gone far. Limited to frequencies below the highest then in common use, radio of 1925 could have found room on the air for but one modern television channel, and if that, then nothing else. By 1930 this number would have risen to three. Today four regular channels (each six megacycles wide) and many more experimental channels have been set aside for television and the "space" they take is not seriously missed.

However, the end is by no means yet in sight.

Today's dreams are of thousands of megacycles: centimeter waves guided like fluid in pipes, or radiated into space with optical directivity, as desired.

At the high frequencies even now in use, effects negligible at low frequencies may dominate design. One such effect which becomes of engineering importance is the "loss" present in all dielectrics. At very low frequencies a dielectric may be described in terms of its breakdown strength, dielectric constant, and leakage resistivity. At high frequencies the apparent resistivity may be much less than at low frequencies. The difference is not due to increased conductivity, but arises from the energy lost by the rapid rotations of the dielectric molecules in the polarizing field. The measurement of this dielectric loss at a very high frequency and in many dielectrics has been the object of the investigation to be here described.

This investigation of dielectric losses at a frequency of one hundred megacycles per second was begun by Mc Rae[1)] in 1935-36, at which time the author assisted with some of the work and became familiar with the problem. Mc Rae was successful in establishing the feasibility of the basic method employed here, but very few actual results were obtained. The project was then abandoned, and when the author returned to the California Institute of

Technology in 1937, it was his intention to investigate as a research subject the propagation of 41 megacycle radiation over the Mount Palomar - Pasadena path and to correlate these observations with atmospheric conditions. This work was carried on with Doll in 1937-38, and the results have been described by him[2]. However, by the end of 1938, it was apparent that no significant developments could be expected from a continuation of the measurements, and it was decided to return to the original problem of dielectric loss measurement: to bring this project to some degree of completion. This was done during the spring and summer of 1939.

Many refinements of the original method employed by Mc Rae were introduced, and the mechanical accuracy of the apparatus used was brought to a point where accurate determinations of the losses in the newer ultra-low-loss dielectrics were possible.

2. THE METHOD

2.1 Fundamental Principles of all Methods

In all the many ways by which dielectric losses can be measured, but two fundamental principles are employed: either the energy lost in the dielectric is measured directly, or else the effect of this energy drain on some associated electrical circuit is determined.

Methods by which the energy loss is measured directly usually involve calorimetric determinations of the rate of production of heat in the sample, and have been employed successfully by Debye, and others[3)], using liquid samples. Because the liquid in this case may serve as its own thermometer, the method is particularly applicable. However, since in all dielectrics, the losses increase very nearly as the square of the polarizing field, it is apparent that if anything more than mere relative values of loss are desired, it is necessary to know accurately the polarizing field present with this method. This fact, plus the difficulty of making accurate calorimetric measurements in the case of solid dielectrics and particularly those for which the loss is small, make the method impractical for measuring just those materials which are of engineering importance.

In the second class of methods: those in which the effect of the loss on an associated electric circuit is measured, the fact that the loss in the dielectric increases with the square of the polarizing field merely means that at any given frequency this loss may be represented as a resistance independent of the applied voltage, and either in shunt or in series with the capacitance of the dielectric, depending upon which is more convenient. The loss thus simply introduces another circuit element in the form of a resistance.

At audio and low radio frequencies this equivalent loss resistance associated with the dielectric may be rather accurately measured by bridge methods. With increasing frequency, however, the difficulties of making accurate bridge measurements become greater and greater until at frequencies well below 100 megacycles the method becomes hopeless.

Another method which suggests itself of measuring the equivalent resistance is to let this resistance be as nearly as possible the only loss present in a resonant circuit of some form. Since the sharpness of the resonance curve of any form of singly resonant circuit is determined solely and uniquely by the ratio of the resistance to reactance in the circuit, it is evident that by measuring the shape of the resonance curve of a given circuit with and without the sample present, a very direct indication of the loss in the sample may be obtained. This is essentially the method which has been used in the present work. It avoids the serious drawbacks of the heating method with its heat losses and the necessity for accurate voltage determination, and of the bridge methods with their uncontrollable effects of stray capacity and shielding defects, without having, as far as the author has been able to determine, any serious fundamental limitations of its own.

2.2 The Present Method

Briefly the method may be described as follows. A transmission line consisting of two coaxial conductors is short circuited at both ends, one of the shorting rings being movable and the other fixed. The line is supplied with a constant voltage at the desired high frequency by a very slight electromagnetic coupling at the fixed end. At a point one quarter wavelength down the line from this fixed end, the sample is introduced in such a fashion that it bridges the gap between the inner and outer conductors and thus has applied to it the entire line voltage, which is a maximum at that point.

The length of the remaining section of line is then adjusted by means of the movable short circuiting ring until the line is brought into resonance, as indicated by some detector designed to read either the voltage appearing at the center of the line or the current at the movable end.

The short circuited transmission line is thus the resonant circuit of this method and has the advantages of having very low losses, and of being self shielding. Furthermore, except for the sample and sample holder capacity and loss it is entirely a distributed constant system and is treated as such in the analysis, whereas any coil-condenser system would, although treated as a lumped constant circuit, actually have distributed constants of indeterminable magnitude as well.

Since variation of the frequency in order to cover the resonance range would require a calibrated oscillator of fantastic precision, the oscillator was allowed to stabilize itself at some fixed frequency and the line length was varied to pass through resonance. In the present arrangement the detector device was arranged to read the voltage at the center of the line, and this quantity was recorded as a function of the line length. This represents a departure from Mc Rae's method in which a thermocouple loop and galvanometer was used to measure the current at the movable end of the line. These two methods both have their respective advantages and disadvantages to be discussed later. The theory for the case in which the current at the movable end is the quantity read has already been given by Mc Rae[1]. The theory for the case in which the voltage at the quarter wavelength point is the quantity read is given in the Appendices and will only be summarized here.

The assumptions upon which the theory has been developed are few, but important. It has been assumed:

a). That no higher order modes of propagation exist in the line. In spite of the fact that the present line has a ratio of diameter to wavelength about three times that used by Mc Rae, the present ratio of about 1:40 is well under that required for the higher modes to become real[4]. The assumption is therefore justified.

b). That the sample and sample holder and detector may be treated as lumped constants placed across the line at the point indicated. This assumption is justifiable on the grounds that the dimensions of these objects are very short compared with a wavelength, and that therefore the only effect of the field modifications they produce is the introduction of some additional admittance and susceptance across the line.

c). That certain higher order terms which arise in expansions in the analysis may be neglected.

These cases are treated as they arise and proved negligible in every case. (See Appendix I).

Assumptions a) and b) are important because they permit of a simple analysis based on four terminal network theory rather than a much more complicated solution using field theory. Assumption c) is important because it permits the solution to be reduced to a very much simpler form than could otherwise be obtained.

2.3 Summary of Theory of Method

The first step in the analysis is the derivation of the relation between the voltage measured at the sample by the detector and the driving voltage, the line length and the sample loss.

If we define K_T such that

$$e_2 = K_T e_3$$

where e_2 = voltage across line at sample

e_3 = driving voltage supplied by coupling circuit,

K_T, the "transfer constant", can be shown (See Appendix I) to be very closely given by

$$K_T = \frac{1}{B\Delta\ell + j(A + \frac{Z_o}{R_s})} \qquad (20)$$

in which

A = a term representing all losses present in the line without a sample

$$B = \frac{\beta}{\sin^2 \beta l_o}$$

β = phase constant of line in radians per unit length.

l_o = length at resonance of section of line between sample and movable shorted end.

Δl = actual line length minus resonant length.

Z_o = characteristic impedance of line.

R_s = desired equivalent shunting resistance of sample.

j = operator $\sqrt{-1}$

Now it would be possible to determine directly the value of Δl which makes

$$B\Delta l = (A + \frac{Z_o}{R_s})$$

and hence reduces K_T to $\frac{1}{\sqrt{2}}$ times its maximum value. Thus, if the value of Δl which reduced the voltage at the center to 70.71% of its maximum value were determined both with the sample in place and removed, R_s could be calculated from these data very simply. However, such a determination rests upon the accuracy of only three readings of length and since these may be subject to slight statistical variations, the method is not very accurate. This fact was pointed out by Mc Rae.

Another method of obtaining R_s which involves as many points as desired is given in Appendix II. There it is shown that

$$\sqrt{\left(\frac{e_r}{e}\right)^2 - 1} \equiv \frac{B\Delta\ell}{A + \frac{Z_o}{R_s}} = (\text{constant}) \times \Delta\ell$$

where e_r = voltage at sample at resonance

e = voltage at sample in general.

Thus if $\sqrt{\left(\frac{e_r}{e}\right)^2 - 1}$ be plotted as a function of $\Delta\ell$ (or of ℓ itself) for all the observed values of e, the points should lie on a straight line, which can therefore be drawn in by visual least-square weighting. The slope of this line, m, is then given by

$$m_s = \frac{B}{A + \frac{Z_o}{R_s}}$$

if a sample is present, and by

$$m_o = \frac{B}{A}$$

if no sample is present. From these two equations it is easy to show that

$$R_s = \frac{z_o}{B} \frac{m_o m_s}{m_o - m_s} \quad . \tag{24}$$

This method of determining R_s, while more laborious than the simple method first mentioned, has two important advantages. The fact that a great number of points are used in the determination minimizes the chance of error in the result due to the slight random error which may be present in any single observation. Furthermore, the fact that over the range of variation of line length used in measuring any sample, these points should lie on a straight line when plotted as described, permits of an immediate check on the operation of the entire apparatus. In other words, if all the observed points, when plotted, obey the required straight line relationships, an entire class of errors: namely all those which would cause departure of the points from linearity, and which otherwise might be considered important, are automatically proved negligible by the experiment itself. These sources of error and many others will be discussed at some length in a later section.

Having determined the magnitude of R_s for any given sample, this may be translated into the effective resistivity and/or dissipation factor of the dielectric material itself provided the dimensions and/or capacity of the sample are/is known. This may be done by the relations:

$$\rho = R_s \frac{A_s}{d_s}$$

and

$$D.F. = \frac{1}{\omega C_s R_s}$$

in which

A_s = area of sample (perpendicular to field)*

d_s = thickness of sample (in direction of field)*

ω = angular frequency of applied field

C_s = sample capacity.

Both the effective resistivity and the dissipation factor have been calculated for all the dielectrics which have been tested. The value of the sample capacity C was in each case determined by two methods, the mechanism of which will be more apparent when the procedure of taking readings is discussed but which should be described here.

The first method, which was always used merely for a check or the results of the second, consisted of

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*A thin sample with two parallel, or nearly parallel plane faces is assumed.

varying the spacing of the sample-holding condenser till, without the sample present, resonance in the line was obtained at the same line length as with the sample present. From the two spacings of the sample condenser plates with and without the sample present it is possible to calculate an approximate value of the capacity of the sample. In Appendix (III) this capacity is shown to be given approximately by:

$$C_s \cong \frac{A_o}{4\pi}\left[\frac{1}{d_o} - \left(1 - \frac{A_s}{A_o}\right)\frac{1}{d_s}\right] \text{ cm.} \qquad (29a)$$

where A_o = area of sample condenser plates

A_s = area of sample

d_o = plate separation without sample

d_s = plate separation with sample.

This, for the constants of the actual apparatus, reduces to the convenient expression:

$$C_s \cong \left[\frac{1}{d_o} - \left(1 - \frac{A_s}{A_o}\right)\frac{1}{d_s}\right] \ \mu\mu\text{fds.} \qquad (30)$$

In the second method the spacing of the sample condenser plates is left equal to the thickness of the sample and the line length required to again establish resonance without the sample present is then, as shown in Appendix (III), given by

$$C_s = \left[\frac{10^{12}}{\omega Z_o}(\cot \beta \ell_o - \cot \beta \ell_o') + \frac{1}{d_s}\left(\frac{A_s}{A_o}\right)\right] \mu\mu fds \tag{32a}$$

in which

ℓ_o = length of line from sample to movable end with sample present

ℓ_o' = length of line from sample to movable end with sample absent.

This second method has been taken to be the more accurate of the two for reasons to be discussed later. The two methods gave in every case results in close agreement.

2.4 Sensitivity of Method

Obviously, there will be a maximum value of the equivalent sample resistivity, R_s, which can be measured to some specified degree of accuracy. Neglecting all other sources of error, it may be shown (See Appendix IV) that this limit is rather sharply established by the losses in the line itself and the accuracy with which the slopes m_o, and m_s, of the plots of the observed data can be determined. This relationship is given by the expression:

$$[R_s]_{max} = \frac{Z_o}{A}\left(\frac{\varepsilon_{max}}{2\left|\frac{\delta m_o}{m_o}\right|_{max}}\right) \text{ ohms.} \tag{38b}$$

In which

Z_o = characteristic impedance of line

A = a term proportion to the line losses.

$\varepsilon_{max} \times 100$ = maximum permissible percentage error in R_s

$\left|\frac{\delta m_o}{m_o}\right|_{max} \times 100$ = maximum possible percentage error in determining m_o (and m_s).

From this equation it is apparent that the sensitivity of the method, i.e., its ability to measure accurately very low loss samples is directly proportional to the quantity

$$\frac{Z_o}{A}\ .$$

Now as given in Appendix I,

$$A = \frac{\alpha \ell_o}{\sin^2 \beta \ell_o} + \alpha \ell_3 + \frac{Z_o}{R\ell} \qquad (18)$$

where:

α = attenuation constant of line

β = phase constant.

ℓ_o, ℓ_3 = line section lengths.

$R\ell$ = equivalent resistance present at center of line by all other losses besides the sample and those represented by α.

Therefore

$$\frac{Z_o}{A} = \frac{1}{\frac{\alpha}{Z_o}\left(\frac{\ell_o}{\sin^2 \beta\ell_o} + \ell_3\right) + \frac{1}{R\ell}}$$

and we see that in order to make $\frac{Z_o}{A}$ as large as possible, $\frac{Z_o}{\alpha}$ must be maximized and all "extraneous" losses due to support insulation and measuring apparatus (proportional to $\frac{1}{R\ell}$) must be minimized. Terman[5)] has shown that $\frac{Z_o}{\alpha}$ is a maximum when

$$\frac{b}{a} = 9.2$$

where

a = outer radius of inner conductor.

b = inner radius of outer conductor.

The conductor radius ratio of 9.2:1 is inconvenient for mechanical reasons. Fortunately, the maximum of $\frac{Z_o}{\alpha}$ at this ratio of b to a is a very broad one, and the ratio of $\frac{b}{a}$ = 4.8 which was actually used gives a value of $\frac{Z_o}{\alpha}$ only 12% below the maximum value. In view of the fact that the line losses were only about one half of the total loss without a sample, hardly more than a 6% gain could have been expected, had the optimum ratio of radii been used.

It is interesting to rewrite the sensitivity equation by redefining our terms slightly. Thus we have

$$|\varepsilon| \leq \frac{2\ R_s \left|\frac{\delta mo}{mo}\right|_{max}}{\frac{Z_o}{A}} \qquad (38c)$$

in which

R_s = equivalent resistance of loss in any sample.

$\varepsilon \times 100$ = maximum probable error of method (due to inaccuracies in measuring mo, m_s, only).

Substituting the values in this equation pertaining to the apparatus used one finds that the maximum probable error is approximately given by

$$|\varepsilon| \ \tilde{\leq}\ \frac{R_s}{4 \times 10^6}$$

Thus an R_s of 40,000ω could be measured with a probable error under 1%, an R_s of 400,000ω with an error under 10% etc., the maximum probable error being proportional to R_s.

At low values of R_s, such as presented by samples with high losses, this error due to the fundamental sensitivity limitation of the method becomes negligible and is masked by other stray errors such as are inherent in any experimental procedure. Various sources of these stray errors will be discussed in the succeeding section but it should be mentioned here that the values in the columns headed "Maximum Probable Error" in the tabulation of results were computed by allowing 5% in addition to 100 x ε, for the probable maximum error due to all these other sources. (The Maximum Probable Errors listed for different samples of equal loss may not be the same, because different sized samples were used). In view of the consistency of the results obtained, this value of 5% is believed to be conservative, but since in engineering applications, it is seldom necessary to know the dielectric loss to any greater degree of accuracy, the great amount of analysis necessary in order to be certain of a closer tolerance was felt to be unwarranted.

2.5 Sources of Error

As has been mentioned previously, all sources of error present in the apparatus can be divided into two classes: those which produce errors detectable in the data, and those which produce errors for which no check exists. The latter class we must prove negligible. The former class is shown to be negligible by the data, but must be considered, since as potential sources of error, the effects involved impose certain requirements on the design and mechanical accuracy of the equipment.

2.51 Sources of Detectable Errors

To be detectable, the error from any source must cause the experimental points taken to depart from a straight line relation when plotted as described in Section 2.3 and Appendix II, or the error must be such that it may be found by repeating the run. The principal sources of error for which this is true are discussed below, one by one.

a) Inaccuracies in measuring the small changes in line length required to pass through resonance could result in almost any kind of a non-linear plot, from a random scattering of the points above and below their proper positions, to a cyclic shift resulting in a wavy line instead of a straight one. Since in some of the runs the variation in line length from one point to the next was only 0.1 mm, it was imperative that the mechanism used be able to establish this interval with an accuracy of at least 10% or 0.01 mm.

This is the same order of magnitude as the expansion which can take place in the copper of the line with large changes in room temperature but this effect is largely balanced out as will be explained later.

b) Non-linearity in the response of the device used as a detector (here a vacuum tube voltmeter) would cause the line passing through the observed points when plotted, to have a different slope for points near resonance than for points far from resonance. A marked S-curvature would therefore result in place of a straight line. The linearity of the detector was checked by independent means, thereby eliminating the possibility of this error compensating some other.

c) Extraneous coupling to the detector, would produce a curvature in the plot of the data because of the change of phase of the voltage at the center of the line with respect to the stray coupled voltage as resonance is crossed. That no such coupling was present is evidenced by the fact that the meter gave no deflection with the line decoupled from the feeding circuit and detuned.

d) Any changes in the losses present in the line which might occur during a run would of course distort the data and be evident in the plotted results. Such changes could only occur due to changes in the line temperature or variation in the contact resistance of the sliding end short circuiting annulus. Care was taken to eliminate both of these sources, using methods to be described.

e) Since, particularly with low loss samples, the readings are very violently dependent upon frequency, it is essential that the frequency of the source oscillator remain constant. With a line length variation of only 0.007% between successive points with certain samples, the frequency variations should be held below 0.0005%. This represents a stability in the oscillator such that the drift not exceed two parts in ten million. After a sufficient warm-up period, the oscillator used was found to possess the required stability, not with respect to an absolute frequency value but with respect to the resonant frequency of the line at any instant. What is meant by this is explained in the section describing the oscillator. Fortunately frequency drift is particularly easy to check, simply by repeating the run. This was done in all cases when believed warranted.

f) Two other sources of error involve the voltage supplied to the line and have been analyzed by

Mc Rae[1]. One is the reaction on the coupling circuit produced as the line is tuned through resonance. This effect is proved negligible if the current in the feeding circuit remains constant throughout the run. The reaction of the line on the coupling circuit might in turn produce a reaction on the oscillator and thus a frequency shift as a function of line length about resonance, even though no change be noticeable in the magnitude of the current in the coupling circuit. This is especially likely in view of the high frequency stability required. However, in the set-up actually used, both the coupling to the oscillator and the coupling between the feeding circuit and line were so slight that no effect from this source was noticed.

As a matter of fact, no trouble was experienced from any of the above mentioned sources - nor any others, such as stray fields from the oscillator. Provided the proper precautions were observed in taking the readings, the points all were found to line on a straight line within the limits of observational error.

One effect did make itself manifest, an effect which was very confusing. It was found that if a run were taken with the feeding-circuit-to-line coupling reduced so that the voltage at resonance at the center of the line was only about 5 volts peak value, it was impossible to take data which would plot up to be a straight line. Instead, a characteristic, broken about the resonant point, would be obtained such that the slope was greater for line lengths longer than the resonant length, than it was for shorter line lengths. Tests were made which showed that this effect was a function only of the voltage at the center of the line at resonance and not of the degree of coupling to the line (ordinarily less for low voltages) or of the load resistance in the detector circuit. Such a test run is given in Section 4.2. While not definitely proved, it may be that this effect was due to the electron transit time in the diode detector tube causing the device to present a reactance dependent upon the applied voltage. The effect was avoided by operating the line so that the voltage at resonance was about 50 volts peak value.

2.52 Sources of Non-Detectable Errors

Most of the possible errors in the equipment used belong in the previous or detectable class. There are a few, however, for which no check exists. These are listed below.

a) Any change in the losses present in the line which occurs between the time the sample run is taken and the time the no sample run is taken would not show up in the results. Furthermore if this same change in losses occurred each time the experiment was repeated no indication of this error could ever be obtained. Temperature changes could of course bring about such a change in the line losses, but this same change could not be expected to occur each time. Only one alteration was made in the line between each run with a sample and the succeeding run without a sample. This was to adjust the spacing of the sample holding condenser till its capacity was returned to the same value as with the sample present. By so doing, the current distribution in the line was made very very nearly the same as with the sample present, and as a direct result of this the line losses should remain the same. The only place where any change in the current distribution might occur is on the surface of the condenser plates themselves, and this should represent a completely negligible change in loss.

b) If the contact of the sample with the plates of the sample holding condenser is not complete the apparent equivalent resistance and the apparent capacity (as measured) will be greater and less than their true values respectively, due to the effect of the air capacity thereby placed in series with the sample. The observed resistivity and dielectric constant will therefore be in error. The errors in the two are reciprocal, however, and no error is introduced in the observed dissipation factor.

This source of error was eliminated as far as possible by a specially designed sample holder.

c) Conversely anything which causes an error in the capacity determination alone will cause a similar error in the dielectric constant and dissipation factor, but no error in the resistivity. The method of capacity measurement has been described and is quite accurate, except for possible changes in the resonant length of the line and in the apparent spacing of sample condenser plates due to mechanical forces arising from the compression of the sample. The rigidity of the apparatus used was great enough so that not more than a 2% error should have been thus introduced.

We see that the undetectable sources of error are comparatively few and small, and the method can be expected to be quite accurate within the limits previously given.

Let us now consider some of the salient features of the apparatus actually used.

3. DESCRIPTION OF THE APPARATUS

The following section is a description of the apparatus actually used in carrying out the method described in the previous section.

Before considering the special details of each component let us look at the arrangement of the assembly as a whole in order that the function and location of each part may be more easily remembered. Figure 1 is a block diagram of the entire apparatus which, as well as showing the functional relationships of the various parts, is a fairly accurate plan view of the room arrangement actually used. It should be noted that all high frequency sources and leads are completely shielded, and at all points where leads cross the boundary between these shielded regions and the outside world, filters have been used. Because of the very high frequencies involved this shielding and filtering presented no special problems. Almost any conductor looks opaque to a hundred megacycles.

3.1 The Resonant Line

The coaxial line used consisted of two copper pipes about 6'6" long, the inner having an outer diameter of 5/8" and the outer an inner diameter of 3". Both had a wall thickness of 1/16". They were held in a coaxial

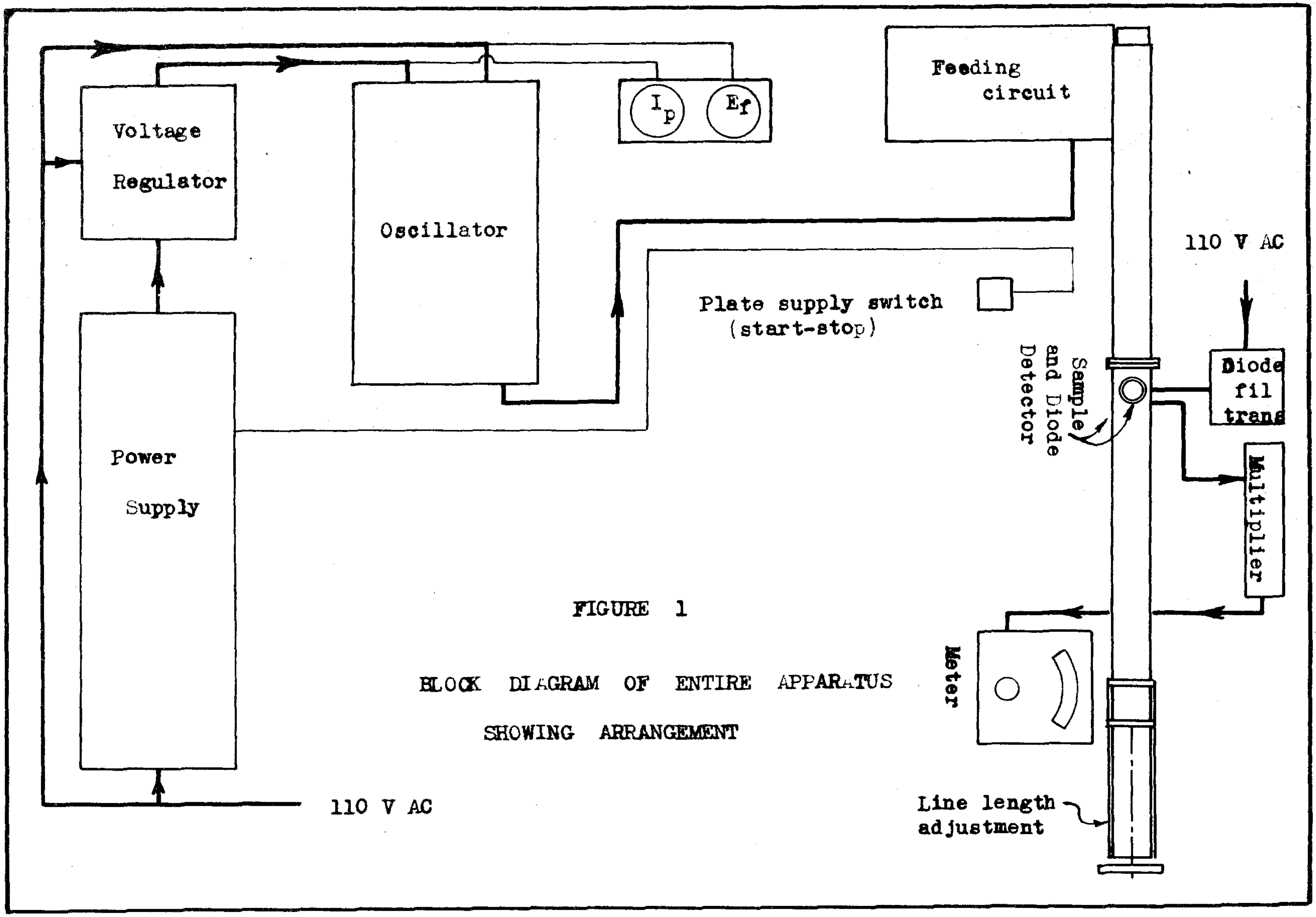


FIGURE 1

BLOCK DIAGRAM OF ENTIRE APPARATUS SHOWING ARRANGEMENT

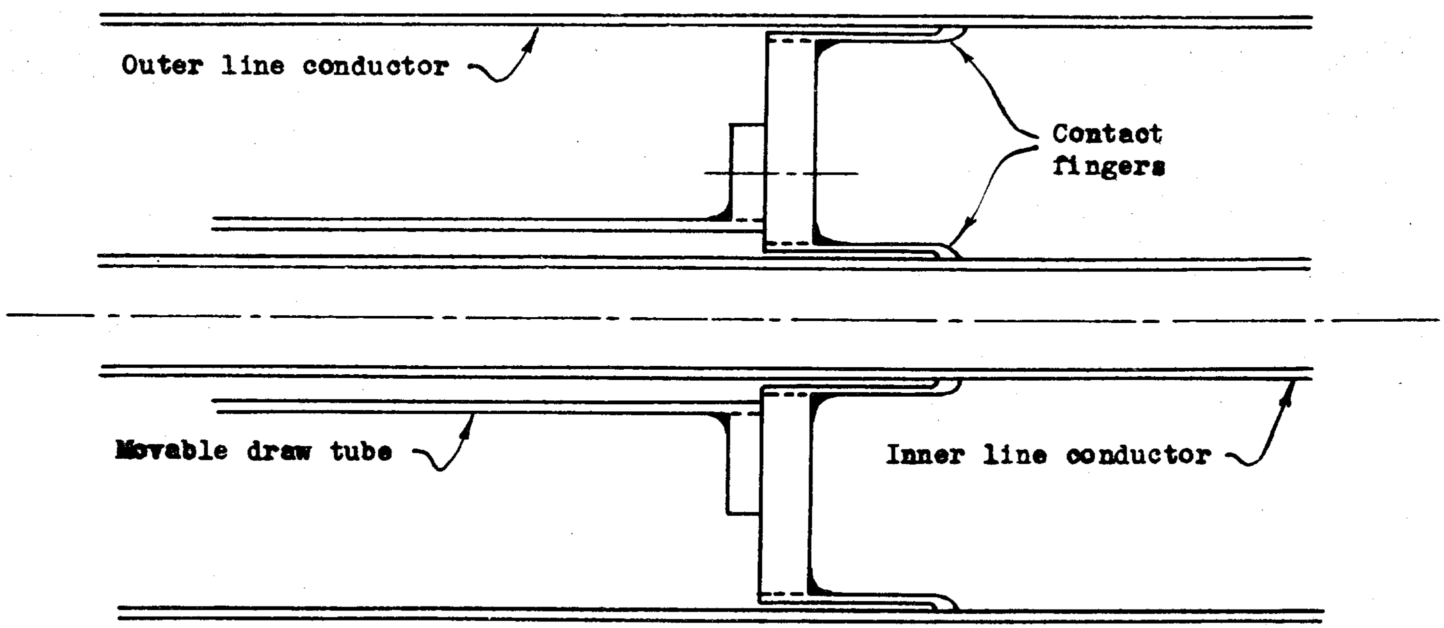


FIGURE 2

SHORT - CIRCUITING ANNULUS IN PLACE IN LINE position, at each end by the short circuiting annuli, and in the center of the line by a Victron bridge (The tests show polystyrene would be better).

The short circuiting rings were made from annular discs of brass 1/4" thick. At the inner and outer cylindrical boundaries, short spring brass cylinders were soldered. These cylinders were turned thin except at the free ends, and then slotted to form a number of spring fingers lying on circles of nearly the same diameters as those of the line conductors. (See Fig. 2) Finally the entire piece was silver plated to increase the surface conductivity.

Both of the short circuiting rings were movable although one (at the input end) was adjusted till the sample and peak voltmeter were at a point truly one quarter wavelength from the sending end, and then left fixed at that adjustment.

The length of the line was adjusted by sliding in and out the short-circuiting ring forming the "receiving" end of the line. To do this accurately, the ring was mounted on one end of a brass tube with flanges on each end (See Fig. 3). The other end of this tube was bolted to the nut of a special lead screw mounted in a cradle which was securely clamped to the end of the outer line conductor.

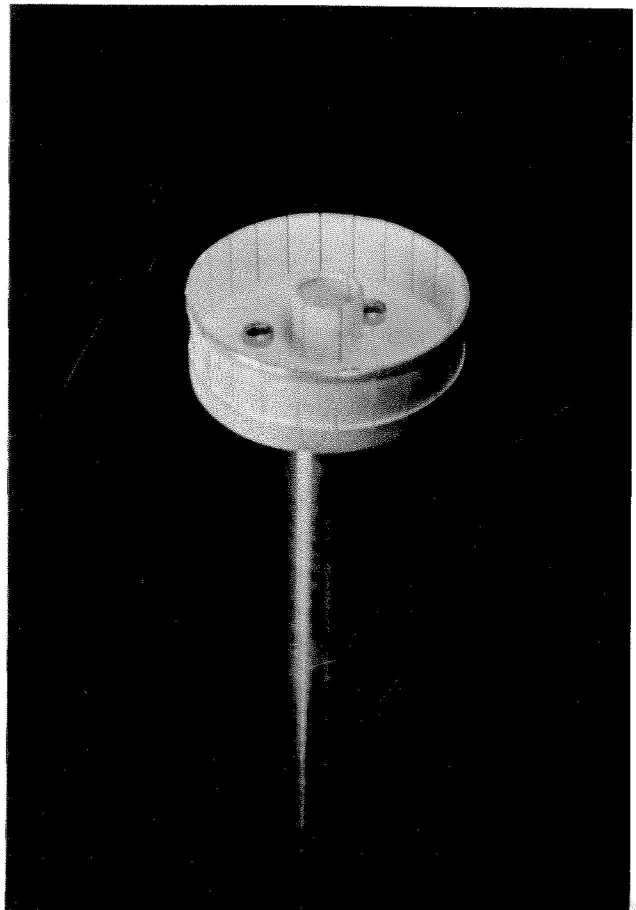


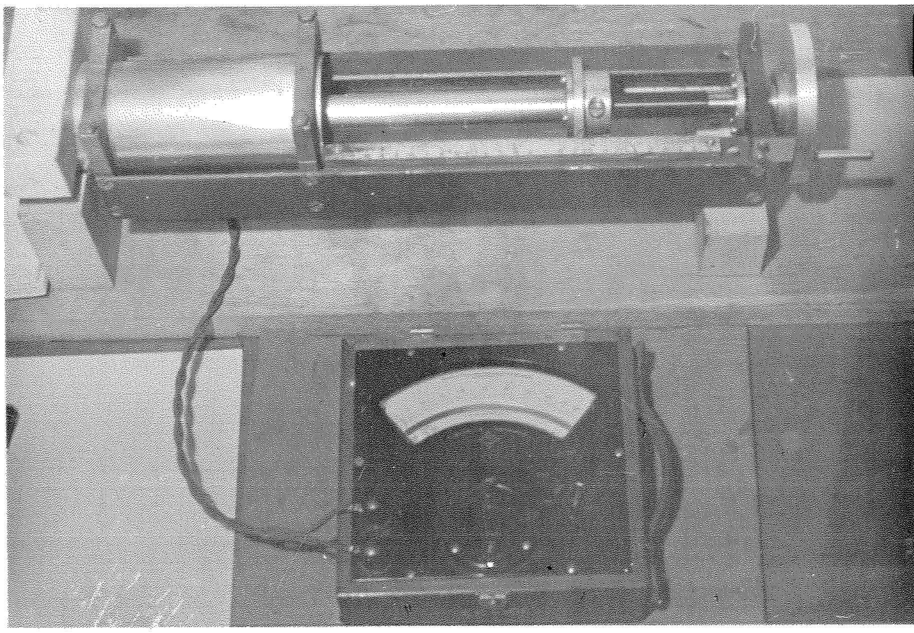
FIGURE 3

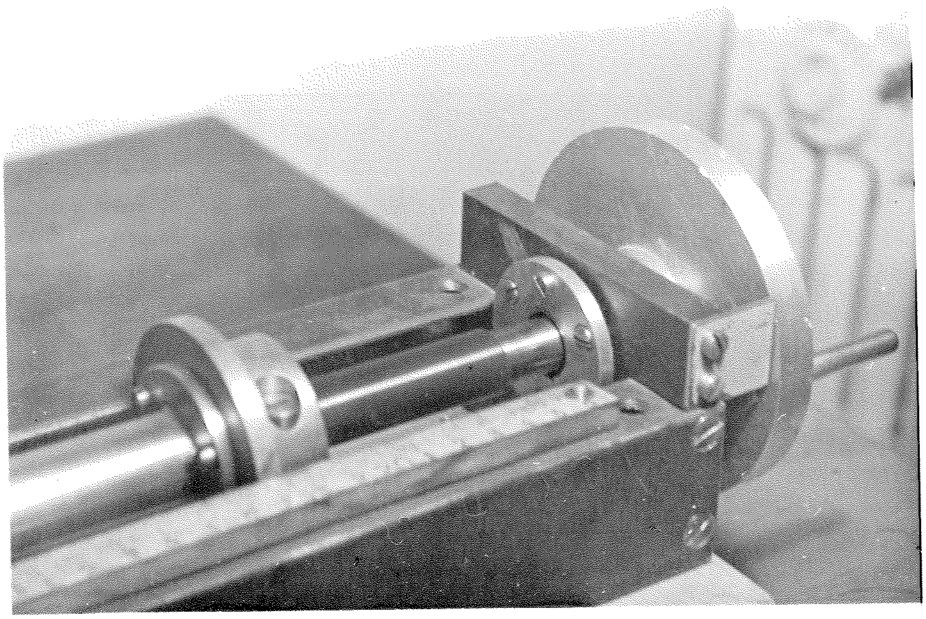
SHORT CIRCUITING RING

MOUNTED ON DRAW TUBE

(See Figures 4a and 4b). The pitch of this lead screw was 1 millimeter, so that a dial on the end marked off in twenty divisions permitted the adjustment of the line length in steps of 0.05 mm. By careful turning of the screw on an accurate lathe, followed by long and tedious lapping of the screw with carborundum dust, then white lead, an accuracy of 0.005 mm over the required range of about 10 mm was exceeded at all points along the screw. In order to cover the required range of line lengths used, two flanged tubes were made having lengths differing by twenty centimeters. This arrangement shortened the lead screw length necessary, and thereby considerably facilitated its construction. The change from one driving tube to the other could be brought about in a few minutes and was seldom necessary. The thrust of the lead screw was transmitted directly to the cradle by a collar near one end. This collar was turned at the same time as the screw itself so that the two would be accurately coaxial in order to avoid cyclic error which would be otherwise introduced if the plate against which this collar bore were not exactly plane and perpendicular to the axis of the screw.

As one of the first steps in the construction of the line, the copper pipe which was to form the outer conductor was cut into two pieces one about 3' long, and to





FIGURES 4

LEAD SCREW IN CRADLE

the two new ends thus formed were soldered brass rings which were then turned so that their exposed faces were plane, and perpendicular to axis of the tube. This formed a very accurate flange joint separable at any time by removing six screws. Access to the sample holder and peak voltmeter assembly inside the line was thereby facilitated. The joint was placed close to the sample holder on the input end side so that the currents across it were very small. The entire line was clamped firmly in three wooden yokes mounted on a heavy board. Figure 5 shows the complete line and its mounting.

3.2 The Sample Holding Condenser

Because of the increased diameter of outer conductor used in the present line over that used by McRae, it became feasible to employ a sample holding condenser so constructed that the plates of the condenser and the flat disc samples used were all contained inside the line itself. This has several advantages, among which are a decreased loss due to the elimination of side connections, and increased accuracy due to the fact that the voltage actually impressed upon the sample is definitely the potential difference between the inside surface of the outer conductor and the outside surface of the inner conductor.

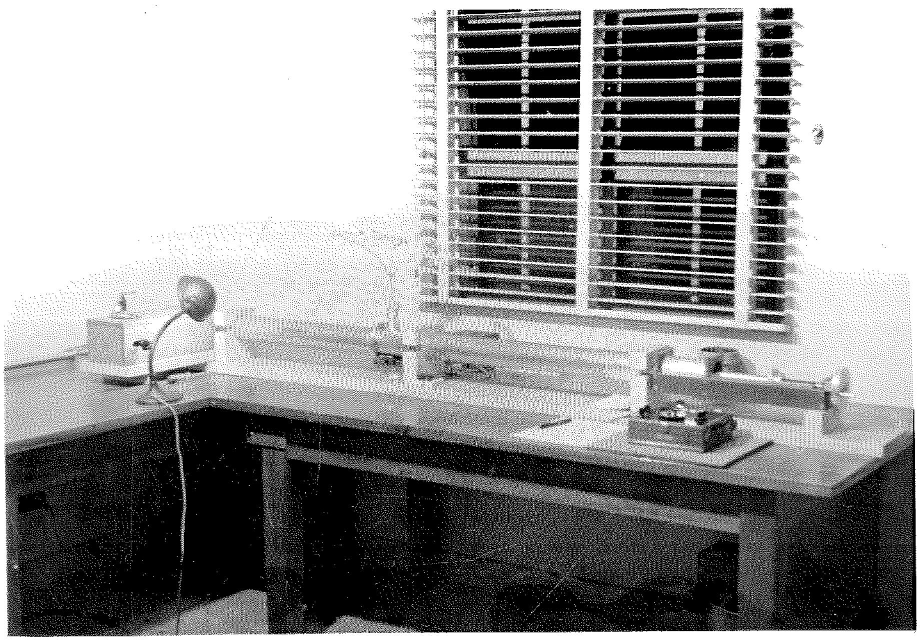


FIGURE 5

COMPLETE LINE

Figure 6 is a section through the sample condenser assembly as mounted in the line. The upper plate of the condenser is formed by the capped end of a hollow cylinder threaded and recessed to screw snugly into a brass sleeve soldered in a hole in the line. This brass sleeve was undercut in back of the internal threads and then slotted in the same fashion as the short circuiting ring cylinders. After silver plating both parts, the fingers thus provided a tight, elastic, fit of the threads. The lower plate is a disk shaped electrode mounted on a boss on the inner conductor. A special feature of this electrode was the flexible construction which permitted it to align itself so as to lie flat against the lower face of the sample when the upper face was flat against the upper plate and the two faces were not parallel. This was accomplished by recessing the back of the plate and soldering to the rim thus formed a tightly stretched sheet of copper foil. To the center of this disc of foil was also attached the small disc with a threaded hole which screwed on the stud on the boss. This stud was allowed to penetrate and contact the back of the plate itself so that a firm point of support was established, about which the plate could be rocked by distorting the foil backing. High frequency currents to the plate must flow along the outside surface which accordingly was silver plated. Fig. 7

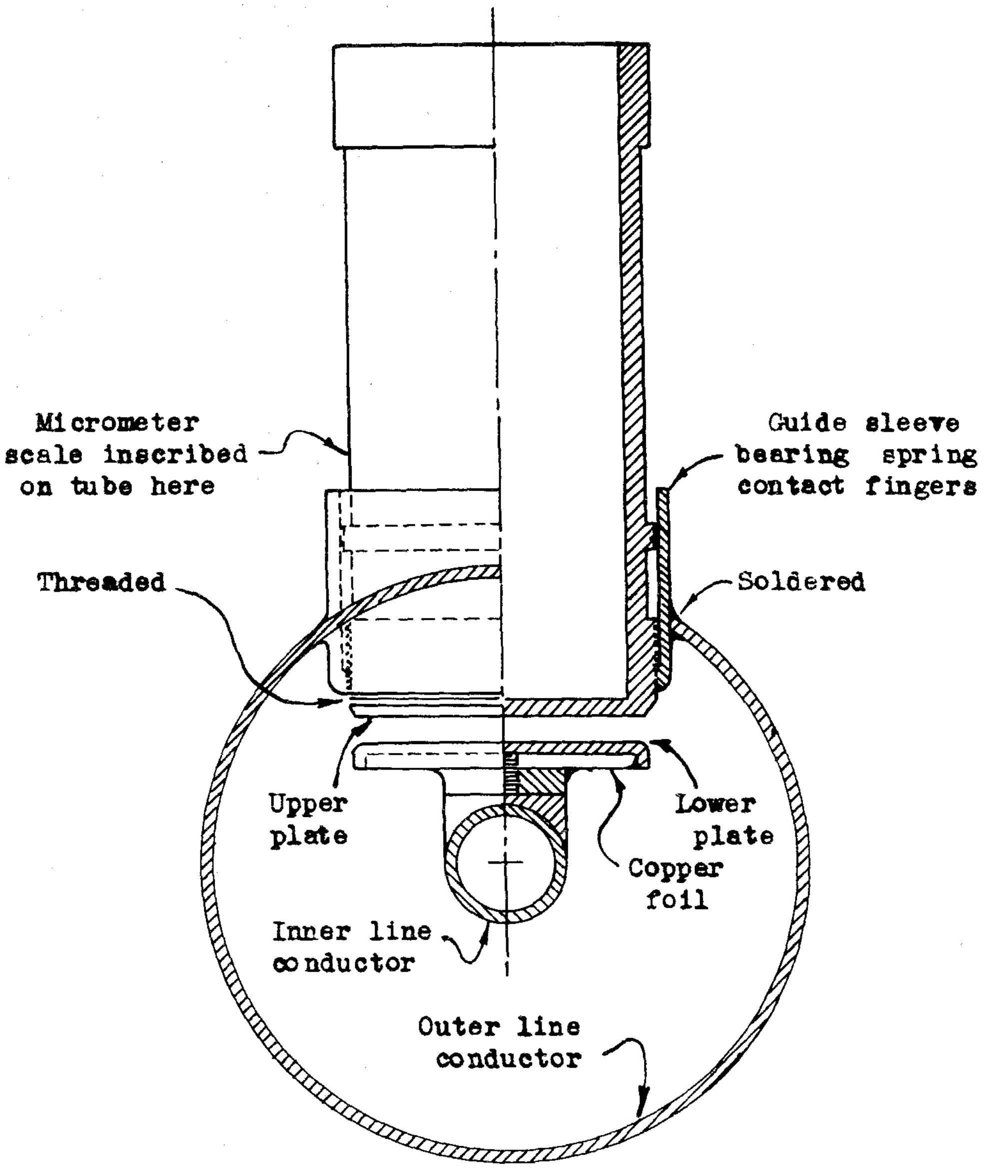


FIGURE 6

SECTION THROUGH SAMPLE HOLDING CONDENSER

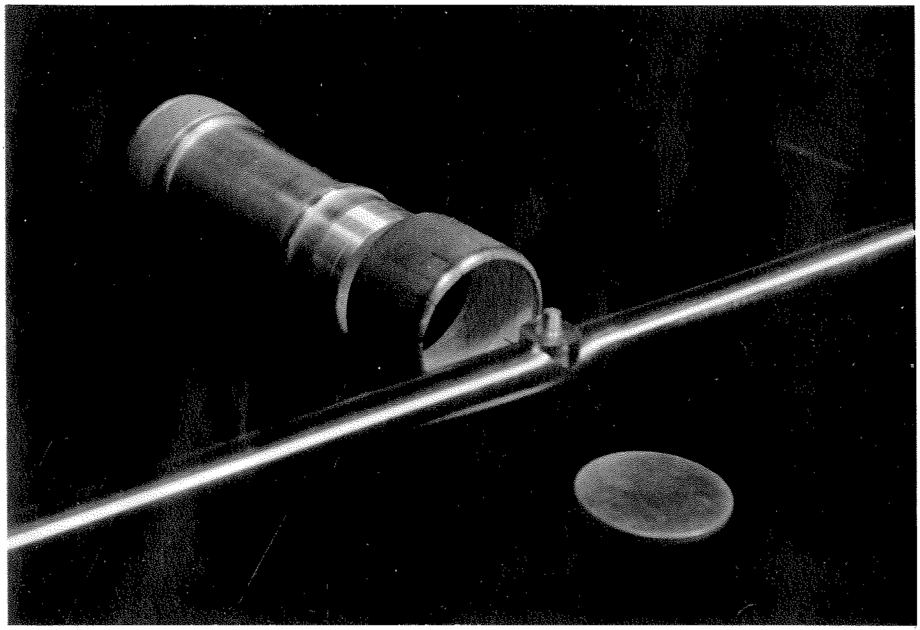


FIGURE 7

COMPONENTS OF SAMPLE CONDENSER

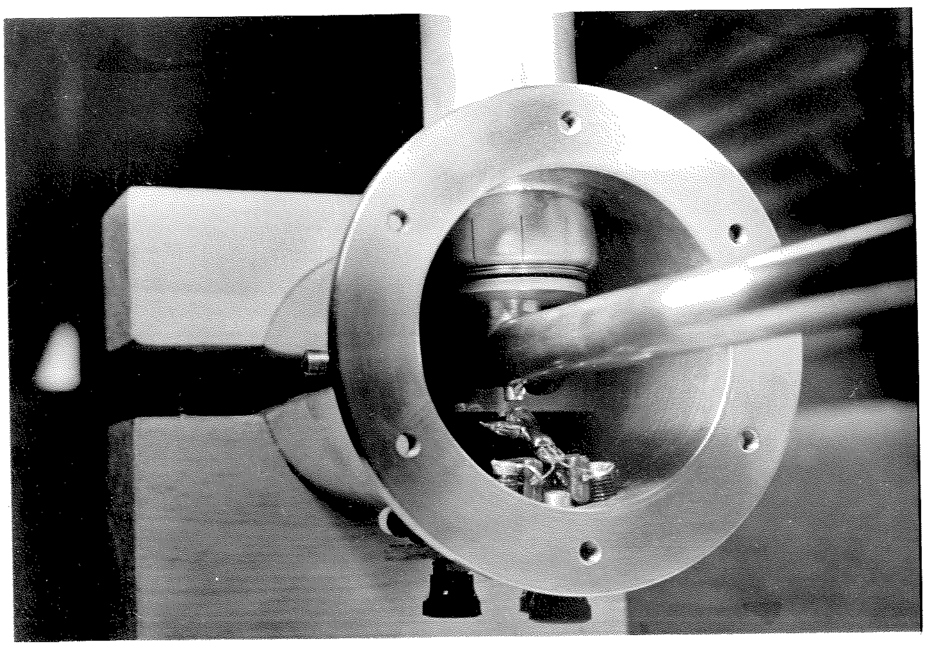


FIGURE 8

SAMPLE CONDENSER AND VOLTMETER

shows the components of the sample condenser before assembly and Figure 8, the complete condenser in place.

3.3 The Diode Detector

Also visible in Figure 8 is the diode detector used to measure the voltage at the sample. This detector consists of a special small diode with its plate connected to the inner conductor just below the sample condenser, and its cathode connected through a small capacity to the inner surface of the outer conductor. Special features of this diode were the absence of internal support insulation and an extremely small element size and spacing. The cathode and heater leads were brought out through filter sections to the supply transformer, and a high resistance voltmeter was connected between the external cathode lead and the line. With a time constant in the load circuit long compared with the period of the frequency being measured, and with a voltmeter resistance high compared with the internal resistance of the diode, the voltage measured by the device is very nearly the peak impressed voltage. As a result of this, the effective shunting admittance across the line due to the voltmeter is twice the actual D.C. admittance of the meter. In parallel with this, of course, are the admittance due to the capacity and dielectric loss of the diode itself.

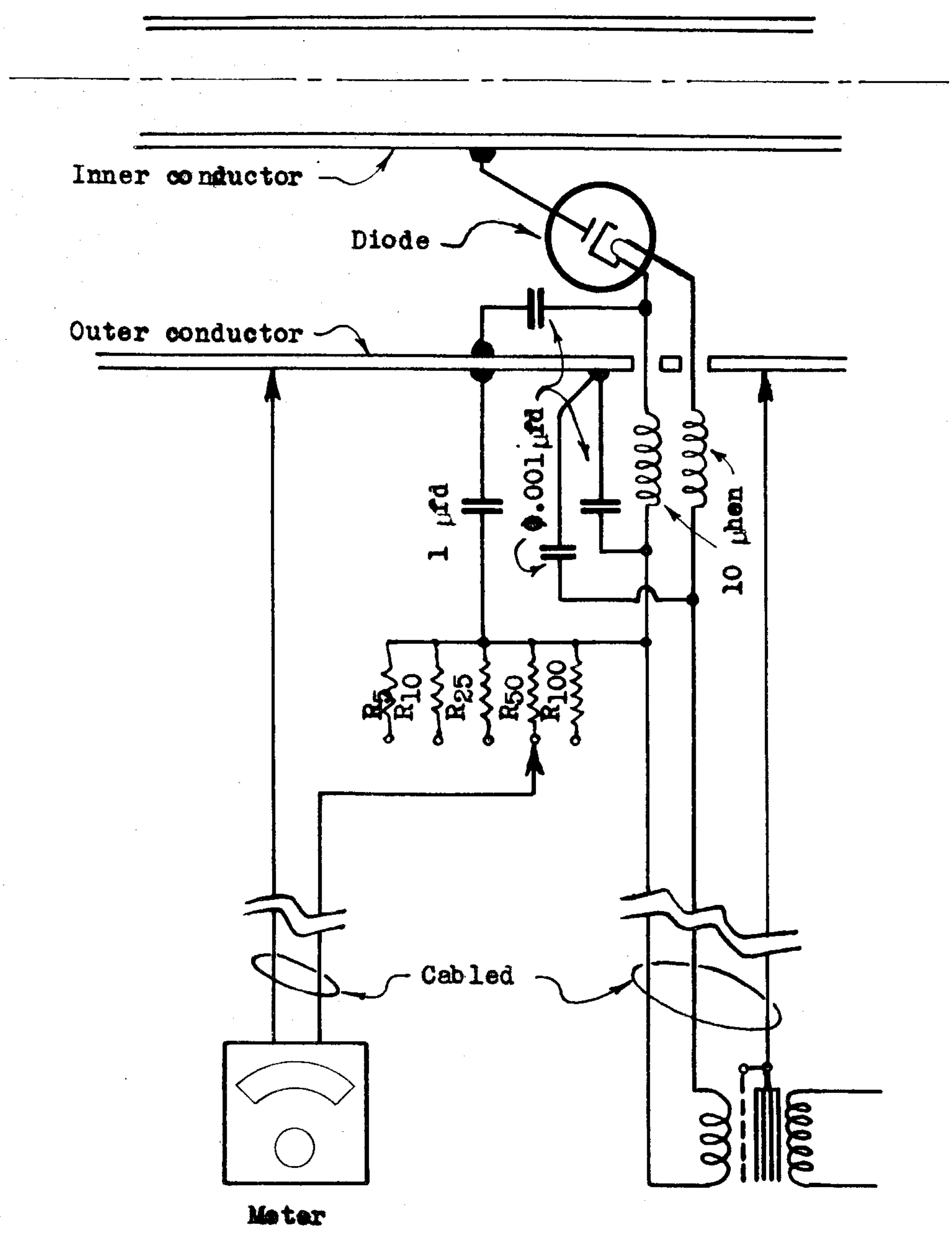


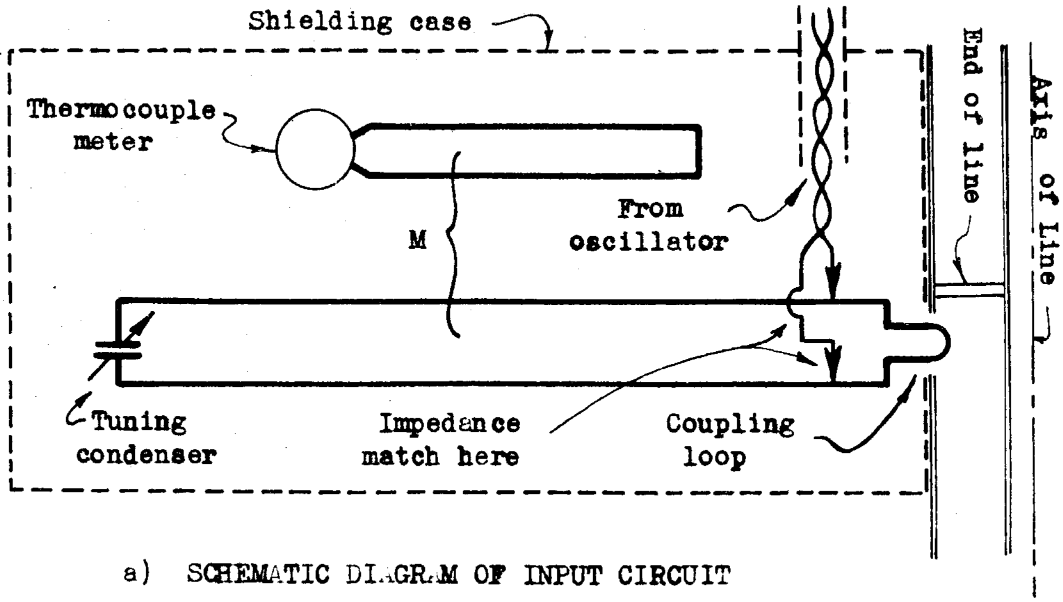
FIGURE 9

SCHEMATIC DIAGRAM OF VACUUM TUBE VOLTMETER

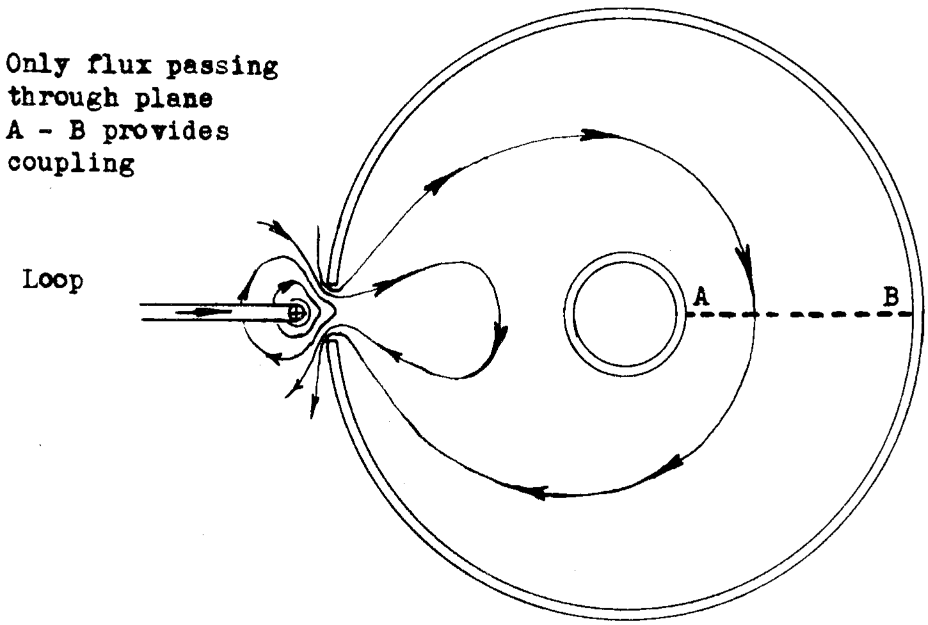
A complete schematic diagram of the detector is given in Figure 9. The meter actually used was so sensitive that the shunting effect of the admittance of its multiplier resistances was negligible. The whole device was checked for linearity by varying the current in the feeding circuit, and for magnitude of reading (relatively unimportant) at low frequencies, and found to be entirely satisfactory.

3.4 The Input Coupling Circuit

Input voltage to the line was secured by means of a very slight magnetic coupling to a tuned circuit driven by the oscillator. To achieve this coupling a slot was cut in the outer conductor of the line right at the short circuited input end, a slot 1/4" wide and about 2" long to allow for length adjustment. One finger of the short circuiting ring at that end was removed and the gap thus formed was rotated so as to coincide with the slot. A second slot in a long metal box was placed opposite the slot in the line. Inside this box, which served as a shield, was mounted a resonant circuit consisting of a pair of parallel copper tubes terminated in a variable tuning capacity at one end and a loop of heavy wire at the other. See Figures 10 and 11. A shielded twisted pair from a one turn pickup coil, loosely coupled to the oscillator, was



a) SCHEMATIC DIAGRAM OF INPUT CIRCUIT



b) SHOWING COUPLING ACTION OF LOOP

FIGURE 10

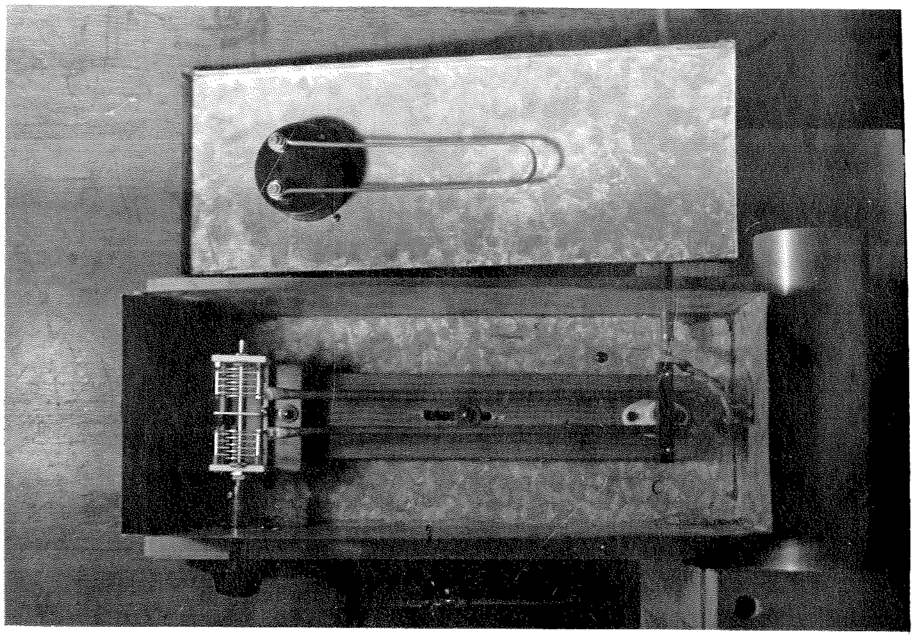


FIGURE 11

FEEDING CIRCUIT WITH LID REMOVED

connected to this loop at two points so chosen that the twisted pair was properly terminated. By tuning the circuit to resonance, a large current could be obtained in the loop end. By introducing this loop into the slot in the line, a small part of the flux due to the circulating current was allowed to link the inner conductor and thus induce a voltage effectively in series with the short circuited end of the line. As a result of the low losses in the line, a surprisingly small amount of coupling was sufficient. With no sample present, it was found that a voltage in excess of 100 volts could be obtained at the center of the line at resonance, with the end of the loop not even fully inside the slot.

Resonance in this tuned feeding circuit was indicated by a thermocouple meter mounted in the lid of the shield box and inductively coupled to the tuned circuit (See Figures 10 and 11). This meter also served as a check on the constancy of input to the line and on the reaction of the line on the feeding circuits. It was found that this reaction was just noticeable with very high loss samples requiring close coupling to the line to secure enough input, but this reaction was not apparent in the recorded dates.

3.5 The Frequency Source

The frequency source used in the present work, i.e., the oscillator and its associated power supply and regulator did not differ materially from that used and described by McRae[1]. The same pieces of equipment were used and only minor modifications in the wiring and arrangement of parts were made. As a result, only a brief description will be given here as a convenient reference.

3.51 The Oscillator

A schematic diagram of the oscillator is given in Figure 12. The circuit consists of two Western Electric Type 304A triodes in a conventional self biasing push-pull connection. The resonant circuit to which the plates are connected is tuned to present a slightly inductive reactance to the tubes, at the frequency of oscillation. Energy is thus transferred into the grid circuit by the grid to plate capacity of the tubes and drives the quarter wavelength resonant line to which the grids are connected. This resonant line is the primary frequency controlling element of the system and accounts for the freedom from trouble due to frequency drifts arising from temperature changes. The required frequency stability of the oscillator was seen in Section 2.51 (e) to be on the order of 0.0005%. It may have occurred to the reader that since copper expands

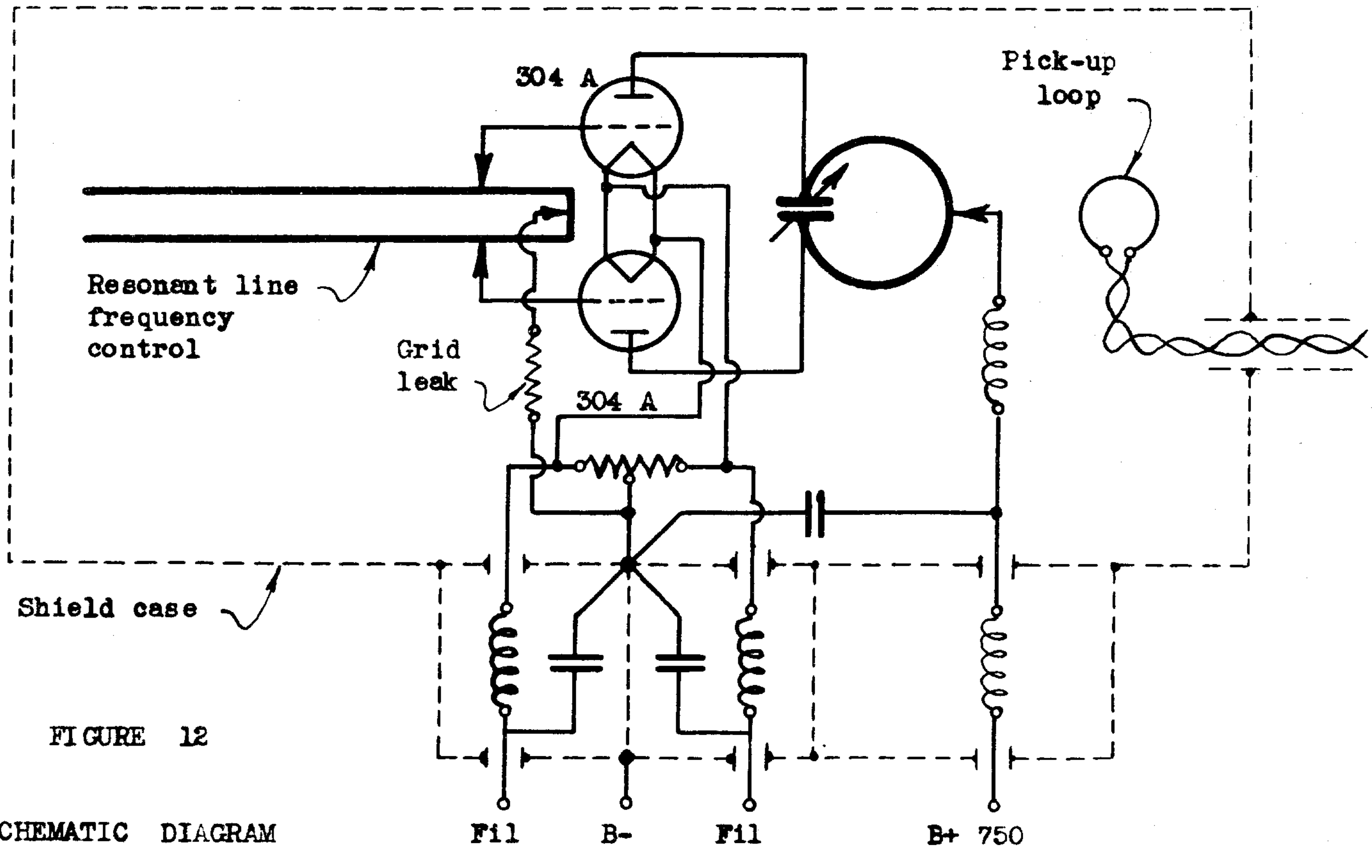


FIGURE 12

SCHEMATIC DIAGRAM

OF

OSCILLATOR

0.0016% per degree centigrade, it would be necessary with a constant oscillator frequency to hold the ambient temperature variation to within 0.3° C. in order to avoid errors due to change in the line length with temperature. However, the stability requirement of 0.0005% is fixed solely by the sharpness of the line resonance, and it is therefore only the frequency of the oscillator relative to the resonant frequency of the line which must be held this closely. The absolute frequency of the oscillator can vary much more than 0.0005% provided the resonant frequency of the line vary isochronously. By making the frequency controlling resonant line of the oscillator also out of copper, this compensatory action was achieved. Such compensation is only exact if the time constants as well as the magnitude of the two counter-acting effects are equal. Actually the time constants were not the same and it was therefore necessary to avoid sudden temperature changes. Figure 13 shows the complete oscillator with the shielding case removed.

3.52 The Power Supply and Regulator

The plates of the oscillator tubes were supplied with 750 volts from the output of a voltage regulator. The input of this regulator in turn was supplied by a conventional design full wave mercury vapor type rectifier and

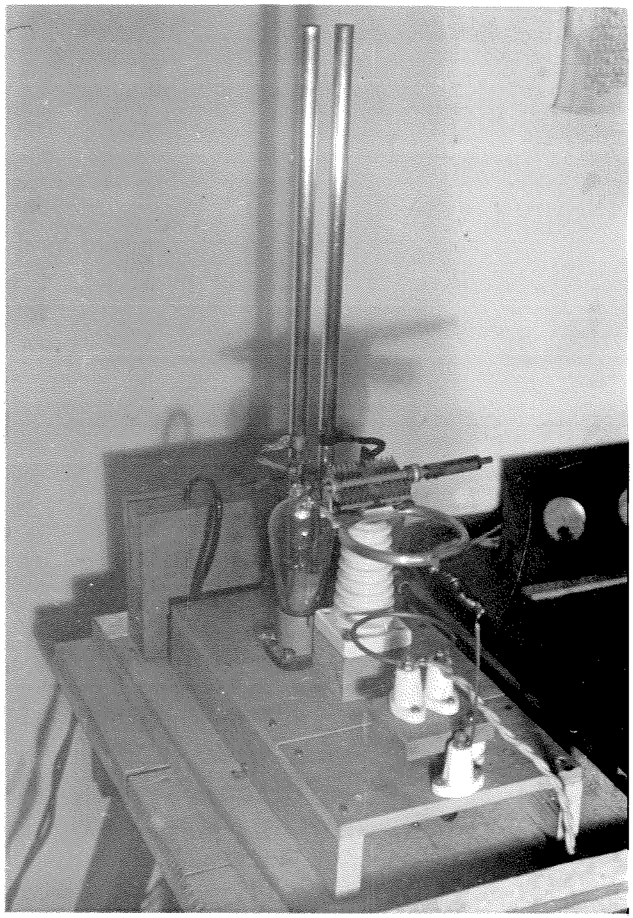


FIGURE 13

COMPLETE OSCILLATOR

WITH SHIELD REMOVED

filter. This voltage of this supply was adjustable and was normally set to deliver 850 volts at full load to the regulator.

The voltage regulator was absolutely essential to prevent changes in the oscillator frequency with changes in the A.C. line voltage. The regulator also served as an additional filter on the power supply so that the hum present in the output was less than 0.01 volt. The regulation when properly adjusted was within 1 volt over long periods of time.

A circuit diagram of the power supply and regulator is given in Figure 14.

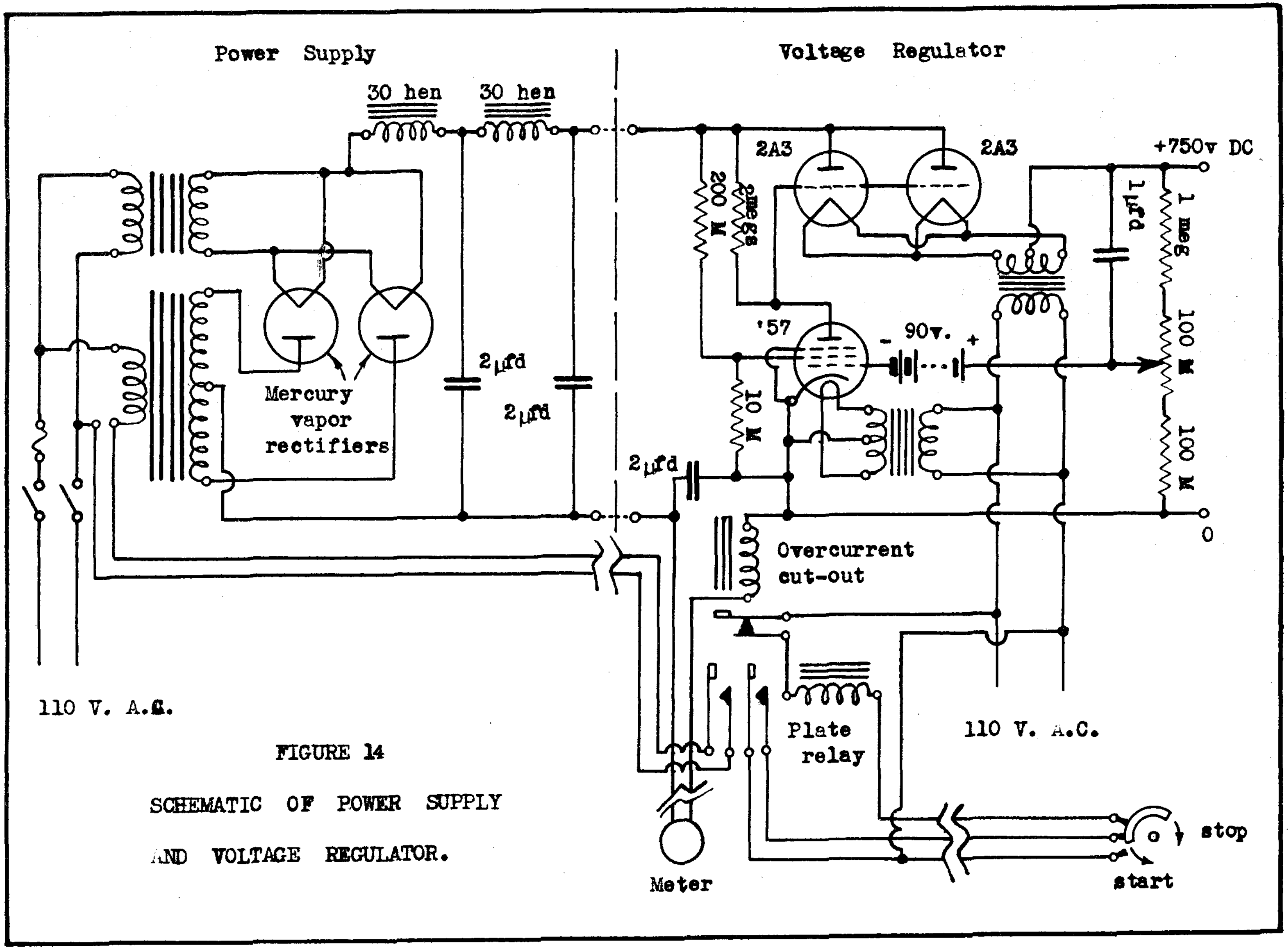


FIGURE 14

SCHEMATIC OF POWER SUPPLY AND VOLTAGE REGULATOR.

4. PRELIMINARY TESTS AND ADJUSTMENTS

4.1 Tests of Components

Before any attempt was made to use the apparatus as a whole, many preliminary measurements and adjustments were made on the individual components to insure their proper working. The line length controlling mechanism and its lead screw were checked in place by means of a micrometer indicator. The diode detector circuit and associated meter were checked as to magnitude of readings, and more especially as to the linearity of deflection, by means of the laboratory standard equipment. With the oscillator frequency roughly adjusted, and the device drawing its normal plate current, the voltage regulator was adjusted to operate in the middle portion of its control range with the line voltage at its average value. The oscillator frequency was then adjusted by varying the length of the resonant line in the grid circuit, until the proper frequency was obtained with the shielding cover in place. The frequency was measured with a Lecher Wire system and the result obtained was in close agreement with the line length required (after allowing for the diode, and Victron bridge, and other stray capacity at the line center).

With the oscillator in operation but with no coupling to the line (which was detuned), a deflection was first noticeable on the diode voltmeter meter. A slight revision of the circuit, to the final form given, and the inclusion of more filtering in certain leads than was originally used succeeded in eliminating this effect.

4.2 Resonance Tests

After all these adjustments had been made, the feeding circuit loop was brought near enough to the coupling slot to give the desired rectified voltage at resonance, and a series of runs was made to test the symmetry of the line resonance under various operating conditions. The results of these test runs were extremely gratifying; for with the exception of those runs taken with an input to the line so slight that the voltage at resonance was on the order of only 5 volts, the plots of the rectified data ($\sqrt{(e/e_r)^2 - 1}$ as a function of Δl) showed no visible consistent departure from strict linearity. A sample resonance run "curve" is shown in Fig. 15.

In Fig. 16, are given three curves representing a controlled test of the effect at low resonance voltages mentioned in Section 2.51. All three curves were taken using the 5 volt multiplier resistance (R_5 Fig. 9) in the diode voltmeter circuit. As a result, the apparent line

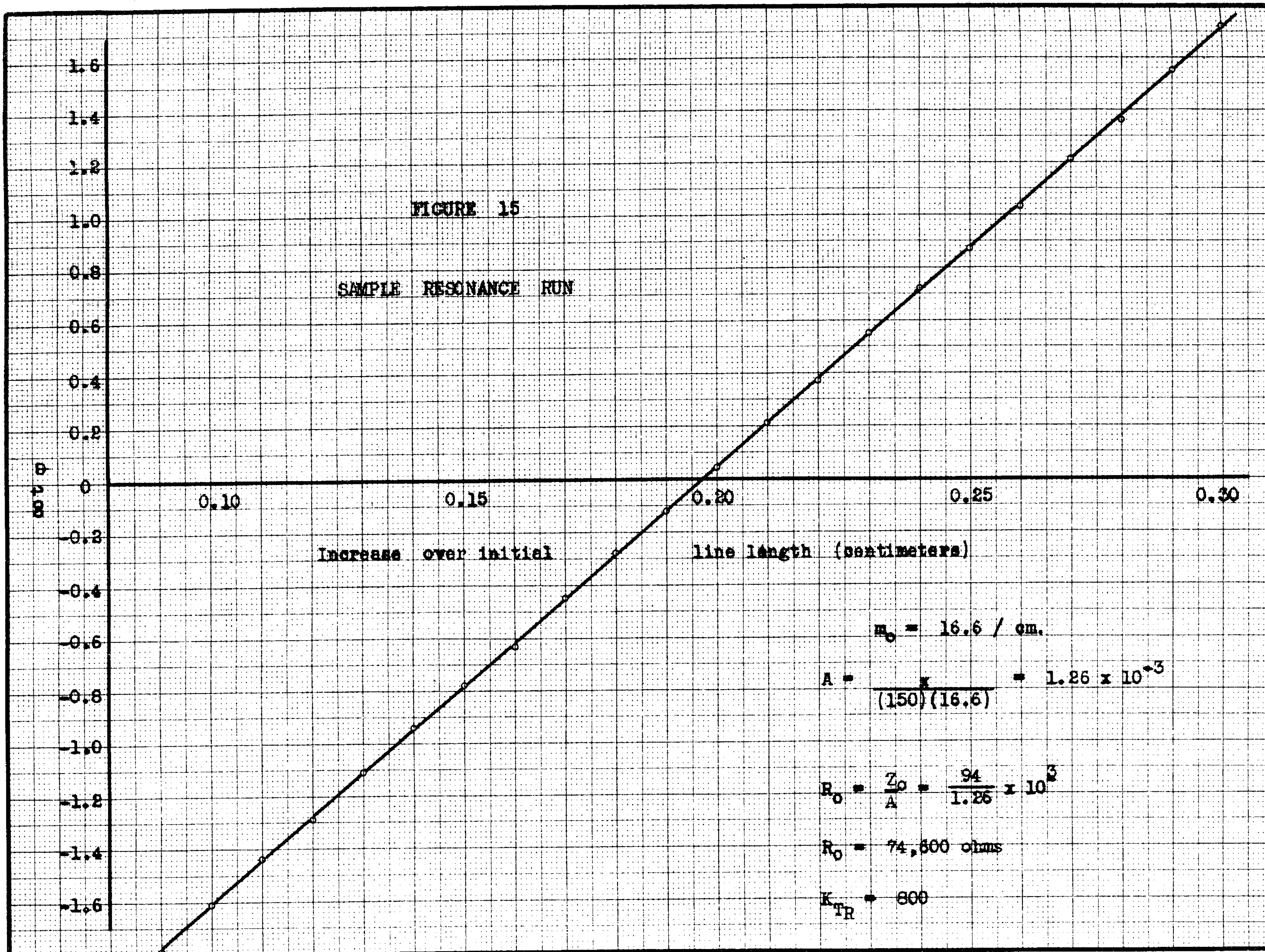


FIGURE 15
SAMPLE RESONANCE RUN
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6
0.10
0.15
0.20
0.25
0.30
Increase over initial line length (centimeters)
m_o = 16.6 / cm.
A = $\frac{x}{(150)(16.6)}$ = 1.26×10^{-3}
$R_o = \frac{Z_o}{A} = \frac{94}{1.26} \times 10^3$
R_o = 74,600 ohms
K_{T_R} = 800

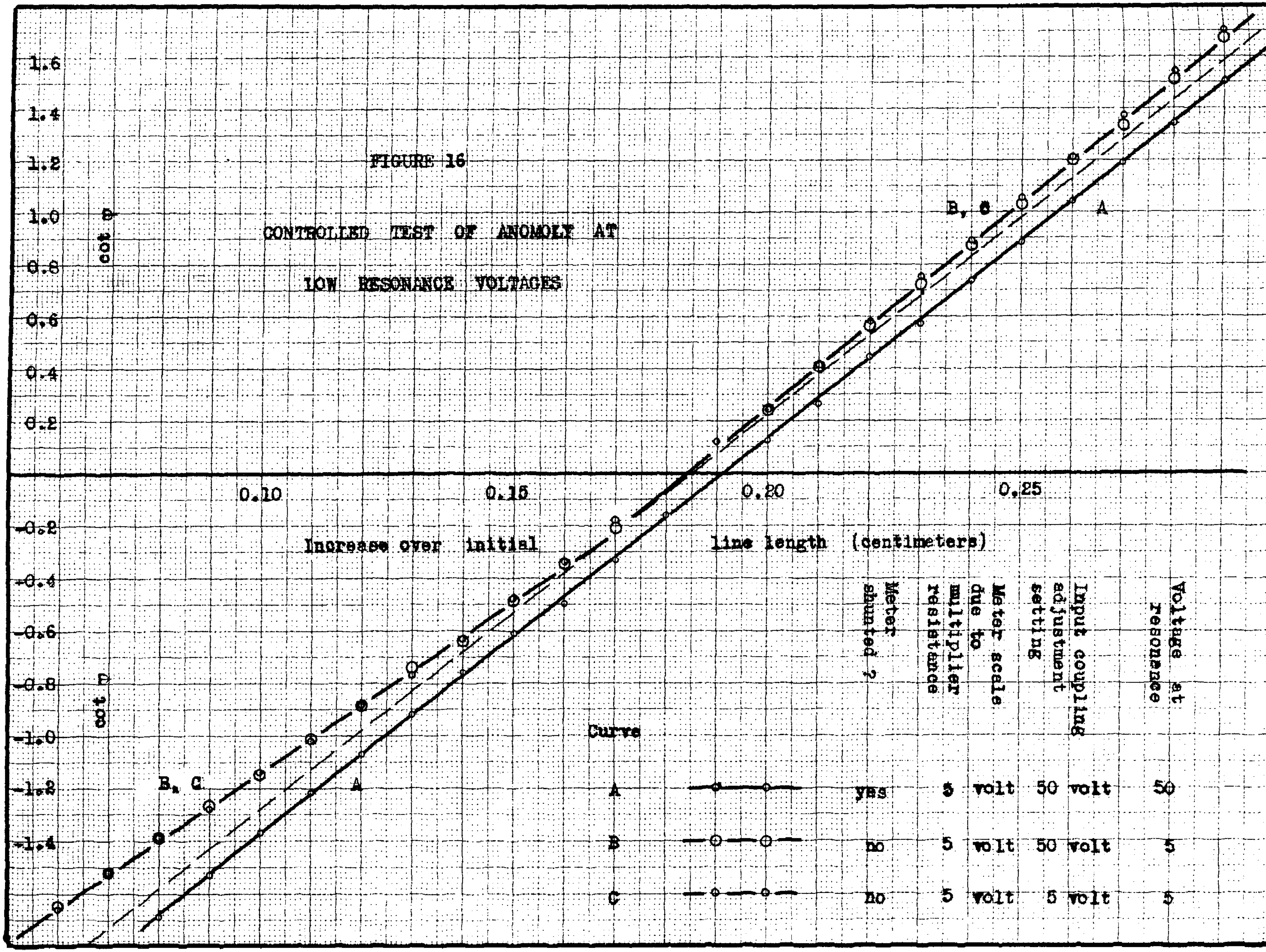
loss is higher than in Fig. 15. Curve A was taken with the meter shunted so that full scale deflection occurred with 50 volt peak voltage in the line instead of 5 volts. The couplings between oscillator and pickup coil and between feeding circuit and line were those normally used to obtain a 50 volt resonance in the line. Curve B was taken by unshunting the meter and decreasing the oscillator to pickup coil coupling till the voltage at resonance in the line was 5 volts. In Curve C, the oscillator to pickup coil coupling was restored to its original value and the feeding circuit to line coupling was reduced to lower the resonance voltage again to 5 volts.

It will be noticed that both curves representing 5 volts at resonance in the line are substantially coincident and show the same departure from the linear 50 volt run, and that this departure is greater on one side of resonance than on the other, namely for line lengths shorter than the resonant length. (The slight separation of the curves laterally was intentional, to avoid confusion). It is this dissymmetry of the error which makes it so difficult to explain, and which makes even the transit time explanation seem doubtful. With voltages at resonance of 25 volts or more the effect is no longer evident, and as a result all sample measurements were made with a voltage at resonance of between 40 and 50 volts peak in the line.

FIGURE 16

CONTROLLED TEST OF ANOMOLY AT

LOW RESONANCE VOLTAGES



5. OBSERVATION, RECORDING, AND CONVERSION OF DATA

5.1 Method of Taking Readings

At the beginning of each series of tests on a group of samples, a considerable warm up period was allowed. The first sample was placed in the line, the equipment turned on, and the line adjusted to resonance. An interval of about twenty minutes was normally required for thermal equilibrium to be established in the oscillator and the frequency to become stable. The line was kept in resonance during this period as a check on the frequency drift. When drift no longer became apparent, the feeding circuit was again adjusted to resonance. Another ten minute period was then allowed to elapse during which any further frequency drift was observed. If none was found, the resonance curve was taken.

Readings were normally taken from a half maximum value reading on one side of resonance, through resonance to a half maximum value reading on the other side, with a spacing such that a total of about twenty-five points was read. To eliminate effects due to backlash of the shorting ring, the motion employed was always in the same direction. The deflection of the thermocouple meter indicating the current in the feeding loop was observed at intervals during the run to make sure no change or reaction occurred. At the end of a run, the first few readings were rechecked to insure that no frequency drift, or other change, had taken place during the run.

The line was then reset at the resonant length, and after recording the thickness as given by the micrometer scale on the sample condenser tube, the sample was removed. The sample condenser cylinder was then replaced and screwed in until resonance was reestablished with no sample present, and the plate spacing required to do this was also read from the micrometer scale. As this resonance was approached it was usually necessary to reduce the feeding circuit to line coupling to keep the diode voltmeter on scale. After making the resonance run with no sample, the sample condenser plates were reset to the thickness of the sample (but with no sample present) and the line length required to reestablish resonance was recorded. With the recording of the sample dimensions and other characteristics, the run was completed.

During all tests the temperature of the room was held to as nearly a constant value as possible. This made it convenient to make critical runs (on low loss samples) at night when the temperature was fairly stable, with no rapid fluctuations and therefore could be more easily controlled.

In addition to the initial warm-up periods at the start of a series of tests, a ten or fifteen minute interval was allowed to elapse between successive runs, after having introduced the next sample and having made all necessary adjustments. This was done to allow the sample itself to come to thermal equilibrium with its surroundings.

5.2 Sample Data Sheet

On the following page is reproduced as a typical case, the data as recorded for the test of a sample of cast Lucite. In the next section, this data will be treated, as an example, by the methods actually used to determine the losses.

MATERIAL: Cast Lucite
AREA: Disc 3/4" in Diameter
d_s: 3.85 mm (Sample thickness)
d_o: 2.78 mm (Plate spacing for same capacity)
ℓ_o': 63.15 cm (Resonant length with $d = d_s$, no sample)

ℓ_1	e_s (with sample)	ℓ_1	e_o (no sample)
.8	21.1	.9	22.6
603.0	22.7	604.0	24.5
.2	24.6	.1	26.6
.4	26.6	.2	29.0
.6	28.9	.3	31.9
.8	31.4	.4	34.8
604.0	34.1	.5	38.2
.2	37.4	.6	41.6
.4	39.6	.7	45.0
.6	43.0	.8	47.7
.8	45.0	.9	48.9
605.0	46.2	605.0	48.6
.2	46.1	.1	47.0
.4	44.7	.2	44.0
.6	42.2	.3	40.1
.8	39.1	.4	37.3
606.0	36.5	.5	33.8
.2	33.5	.6	30.7
.4	30.7	.7	28.3
.6	28.5	.8	25.7
.8	26.0	.9	23.6
607.0	24.0	606.0	21.9
.2	22.3	.1	20.5

5.3 Conversion of Data - Example

5.31 Estimation of Resonance Reading

As a first step in the conversion of the observed data, it is necessary to estimate the diode voltmeter reading at exact resonance with and without the sample present, since ordinarily no observed points correspond exactly to this condition. This is best done by plotting on an enlarged scale the points near resonance. By drawing a smooth curve through these points the maximum can be accurately estimated. Thus for the case of cast Lucite one obtains:

Reading at resonance with sample = 46.3

Reading at resonance without sample = 49.0

Failure to estimate the reading at resonance with sufficient accuracy produces a detectable error in the plotted computed data whereby points which would otherwise lie on a straight line, lie instead on two rectangular half-hyperbolae as shown in Figure 17.

5.32 Computation of Equivalent Resistance of Sample

Using the resonance readings obtained by the method described, the quantities cot ϕ_s, and cot ϕ_o are obtained by means of the relations:

$$\cot \phi = \sqrt{\left(\frac{e_r}{e}\right)^2 - 1}$$

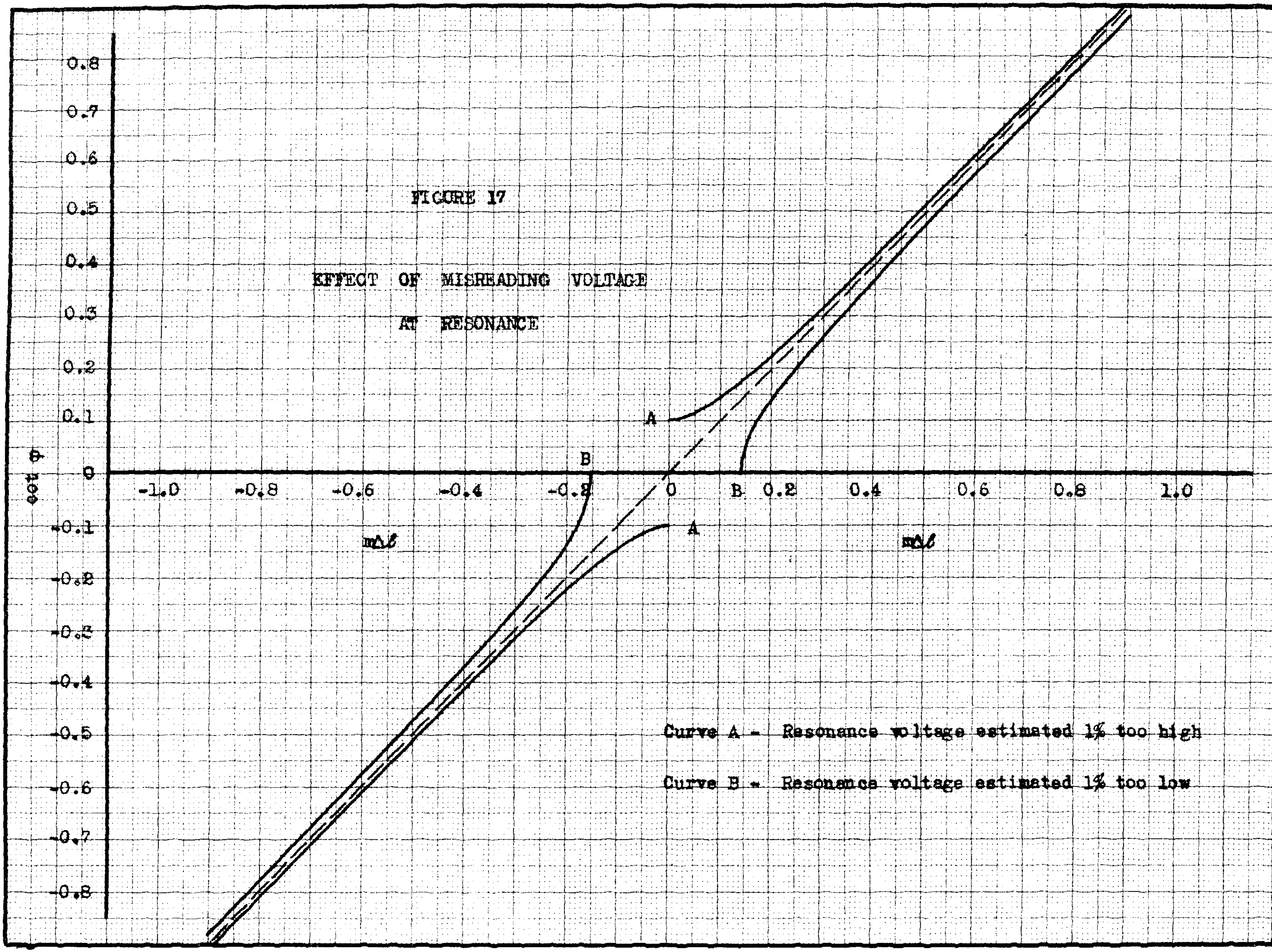


FIGURE 17
EFFECT OF MISREADING VOLTAGE
AT RESONANCE
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-1.0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1.0
A
B
mΔℓ
Curve A - Resonance voltage estimated 1% too high
Curve B - Resonance voltage estimated 1% too low

Thus for the cast Lucite, we obtain:

$$\cot \phi_s = \sqrt{\left(\frac{46.3}{e_s}\right)^2 - 1}$$

$$\cot \phi_o = \sqrt{\left(\frac{49.0}{e_o}\right)^2 - 1}$$

The results of these computations are tabulated below:

ℓ_1	$\cot \phi_s$	ℓ_1	$\cot \phi_o$
.8	-1.95	.9	-1.92
603.0	-1.78	604.0	-1.73
.2	-1.60	.1	-1.55
.4	-1.42	.2	-1.36
.6	-1.25	.3	-1.17
.8	-1.08	.4	-0.99
604.0	-0.92	.5	-0.80
.2	-0.73	.6	-0.62
.4	-0.60	.7	-0.43
.6	-0.40	.8	-0.23
.8	-0.24	.9	-0.06
605.0	- .05	605.0	0.12
.2	.10	.1	0.30
.4	0.26	.2	0.49
.6	0.45	.3	0.70
.8	0.63	.4	0.85
606.0	0.78	.5	1.05
.2	0.95	.6	1.24
.4	1.13	.7	1.41
.6	1.28	.8	1.62
.8	1.47	.9	1.82
607.0	1.65	606.0	2.00
.2	1.82	.1	2.17

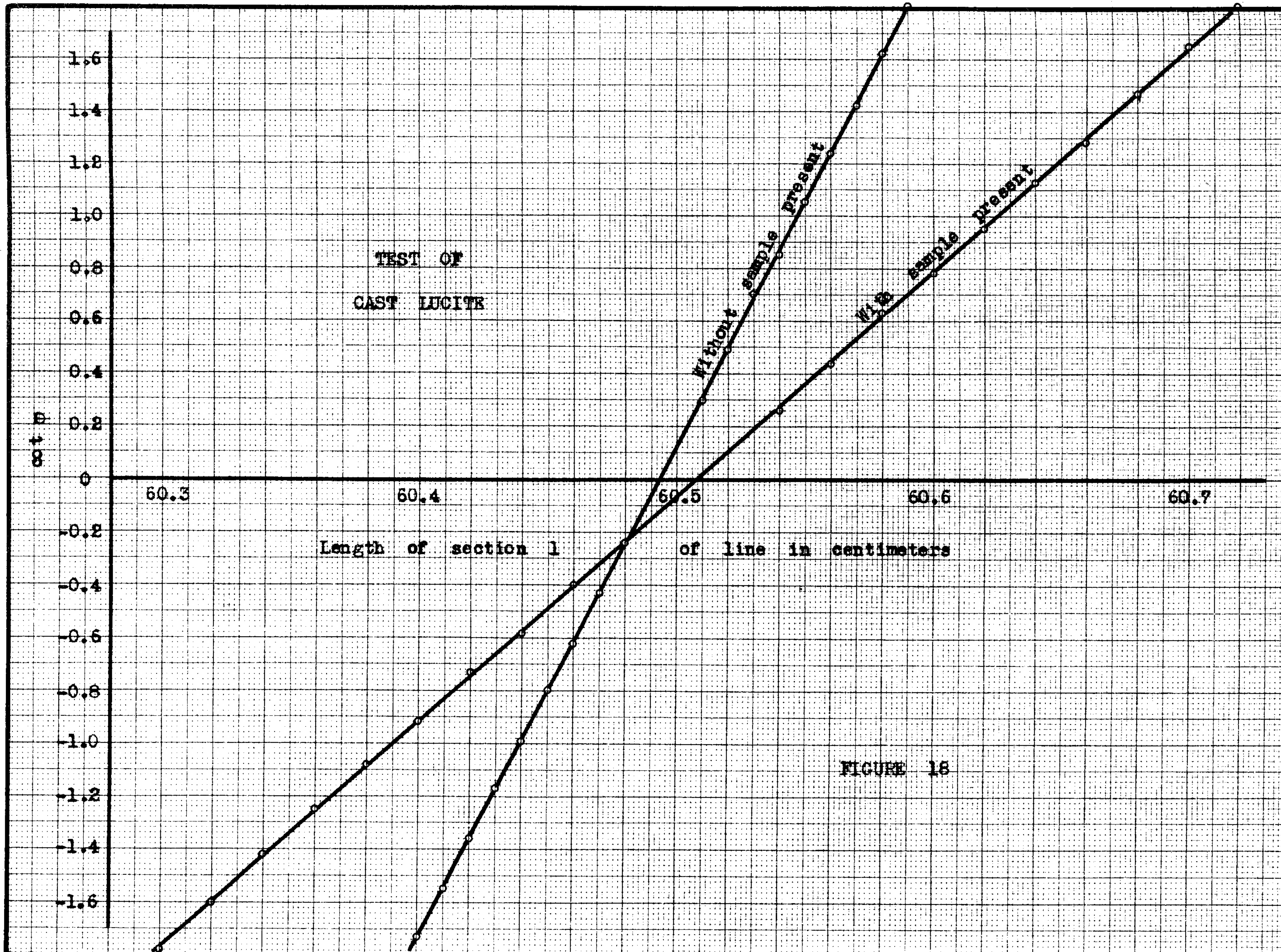

TEST OF
CAST LUCITE
Without sample present
With sample present
Length of section 1 of line in centimeters
60.3
60.4
60.5
60.6
60.7
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
-1.6

FIGURE 18

The above computed data is then plotted as shown in Figure 18, and the slopes of the "curves" obtained graphically. We have, by this means:

$$m_o \text{ (slope without sample)} = 18.55/\text{cm}$$

$$m_s \text{ (slope with sample)} = 8.53/\text{cm}$$

from which

$$\frac{m_o m_s}{m_o - m_s} = 15.8/\text{cm}$$

Now for the present line, at the frequency used,

$$\beta = \frac{\pi}{150} \text{ radians/cm} = 1.2 \text{ degrees/cm.}$$

Therefore $\beta \ell_o = 1.2 \times 60.5 = 72.6°$

This gives

$$\sin^2 \beta \ell_o = 0.910$$

and

$$B = \frac{\beta}{\sin^2 \beta \ell_o} = \frac{\pi}{150 \times 0.910} = 2.30 \times 10^{-2} \text{ rad./cm.}$$

R_s, the equivalent sample resistance is now determined from the relation

$$R_s = \frac{Z_o}{B} \quad \frac{m_o m_s}{m_o - m_s} \text{ ohms.}$$

to be

$$R_s = \frac{94 \text{ ohm} \times \text{cm}}{2.30 \times 10^{-2} \text{ rad}} \times 15.8/\text{cm}$$

$$R_s = 64{,}500 \text{ ohms}$$

5.33 Computation of Sample Capacity

(a) The recorded sample thickness, and plate spacing required to produce with no sample present the same total sample condenser capacity are, respectively,

$$d_s = 3.85 \text{ mm.}$$

and $$d_o = 2.80 \text{ mm.}$$

The sample diameter is 3/4", while the sample plates are 1-1/2" in diameter. Therefore the ratio of sample area to plate area $\frac{A_s}{A_o} = 1/4$, and from the relation

$$C \cong \left[\frac{1}{d_o} - \left(1 - \frac{A_s}{A_o}\right) \frac{1}{d_s} \right] \mu\mu\text{fds.}$$

$$C \cong \left[\frac{1}{0.280} - \left(1 - \frac{1}{4}\right) \frac{1}{0.385} \right]$$

$$\cong 1.65 \ \mu\mu\text{fds.}$$

(b) The line length at resonance with the sample present at the line center was $\ell_o = 60.51$ cm. With the sample plates at the same spacing but the sample absent, the line length required for resonance was $\ell_o' = 63.15$ cm.

From the relation

$$C_s = \left(\frac{10^{12}}{\omega Z_o} \; (\cot \beta \ell_o - \cot \beta \ell_o' \,) + \frac{1}{d_s} \; \left(\frac{A_s}{A_o}\right) \right) \mu\mu fds.$$

we get

$$C_s = \left(\frac{10^4}{2\pi \times 94} \; (0.3135 - 0.2515) + 0.65 \right) \mu\mu fds.$$

$$C_s = 1.70 \; \mu\mu fds.$$

The two methods (a) and (b) just illustrated for finding the sample capacity were always in roughly the agreement indicated here. The discrepancy was never more than a few per cent, and was always greatest and in the same direction for large thick samples of high dielectric constants, since for these the edge effects in method (a) were large. Since method (b) involves no edge effects nor other serious approximations the result by this method was always used.

5.34 Results for Cast Lucite

From the above calculations we have for cast Lucite:

$$\text{Dissipation Factor} = \text{D.F.} = \frac{1}{\omega CR}$$

$$\text{D.F.} = \frac{1}{2\pi \times 10^8 \times 1.70 \times 10^{-12} \times 64{,}500}$$

$$\text{D.F.} = 0.0145$$

and

$$\text{Resistivity} \quad \rho = \frac{A_s}{d_s} \; R = \frac{\left(\frac{3\pi(2.54)}{8}\right)^2}{0.385} \quad 64,500$$

$$\rho = 0.48 \text{ megohm - cm.}$$

The dielectric constant can also be calculated by the relation

$$\varepsilon = C_s \; \frac{A_o}{A_s} \; d_s = 1.70 \times 4 \times 0.385$$

$$\Sigma = 2.62$$

The values of the dielectric constant for the various samples tested and as calculated by this means have been tabulated along with the loss results.

6. TABULATION OF RESULTS

6.1 Dielectric Characteristics at 100 Megacycles, and 25°C.

Sample Material	Dissipation Factor	Max. Prob. Error (%)	Resistivity (megohm-cm.)	Max. Prob. Error (%)	Dielectric Constant
Amber (natural	0.00836	±6	0.824	±6	2.6
Bakelite: cast (clear)	0.1590	±5	0.019	±5	5.9
black laminated	0.0840	±5	0.0495	±5	4.3
black solid	0.0784	±5	0.0527	±5	4.4
orange laminated	0.0765	±5	0.061	±5	3.9
orange solid	0.0765	±5	0.064	±5	3.6
low loss (BM 262)	0.0083	±6	0.500	±6	4.4
Beeswax	0.0121	±6	0.638	±6	2.35
Catalan	0.1520	±5	0.0274	±5	4.5
Cellulose Acetate	0.0645	±5	0.0775	±5	3.6
Cellulose Nitrate	0.1250	±5	0.0285	±5	5.1
Ceresin Wax	0.00047	+40, -30	16.1	+40, -30	2.4
Glass (Pyrex)	0.00485	±6	0.823	±6	4.5

Sample Material	Dissipation Factor	Max. Prob. Error (%)	Resistivity (megohm-cm.)	Max. Prob. Error (%)	Dielectric Constant
Isolantite	0.00368	±6	1.02	±6	6.1
Koroseal	0.1078	±5	0.053	±5	3.2
Lucite: cast	0.0145	±6	0.480	±6	2.6
molded (pressed)	0.0145	±6	0.482	±6	2.6
Mica	0.00268	±8	1.55	±8	5.2
Micalex	0.00246	±6	1.29	±6	5.7
Paper: condenser, dry, waxy	0.00331	±6	0.281	±20	2.6
pyranol impregnated	0.0812	±5	0.0527	±20	4.3
Paraffin	0.00013	+100, -50	57.	+100, -50	2.45
Plexi-glass	0.0125	±5	0.551	±5	2.6
Polystyrene (XMS 10023)	0.000396	±23	~~1.94~~ 19.4	±23	~~2.4(6)~~ [illegible]
Quartz: Crystal	(0.00019)	?	(35.)	?	(6.5)
Clear, fused	0.000052	+50, -25	114.	+50, -25	3.0

Sample Material		Dissipation Factor	Max.Prob. Error (%)	Resistivity (megohm-cm.)	Max.Prob. Error (%)	Dielectric Constant
Redmenol		0.0842	±5	0.461	±5	4.7
Rubber (hard)		0.00905	±7	0.696	±7	2.9
Sulphur:	Yellow	0.00035	+50, -25	14.	+80, -40	3.4
	Amorphous	0.00049	±9	6.15	±15	6.0
Victron	Clear	0.00085	±15	8.65	±15	2.46
	Yellow	0.00087	±13	8.53	±13	2.45
Wood (ponderosa pine)						
	across grain	0.0937	±5	0.0835	±5	2.3
	along grain	0.124	±5	0.043	±5	3.4

6.2 Classification of Dielectrics as to Their Dissipation Factors at 100 Megacycles

(a) Very High Loss D.F. > 0.1)

Material	D.F.
Cast Bakelite	0.16
Catalan	0.15
Cellulose Nitrate	0.12
Koroseal	0.11
Ponderosa Pine (along grain)	0.125

(b) High Loss (0.1 > D.F. > 0.01)

Material	D.F.
Ordinary Bakelite	0.08
Beeswax	0.012
Cellulose Acetate	0.065
Lucite	0.0145
Pyranol Soaked Paper	0.080
Plexi-Glass	0.0125
Redmenol	0.084
Ponderosa Pine (across grain)	0.094

(c) Low Loss (0.01 > D.F. > 0.001)

Material	D.F.
Amber (natural)	0.0084
Low Loss Bakelite (BM262)	0.0083
Pyrex Glass	0.00485

Material	D.F.
Isolantite	0.0037
Mica	0.0027
Micalex	0.0025
Condenser Paper (dry)	0.0033
Rubber (hard)	0.0090

(d) Ultra Low Loss (0.001 > D.F.)

Material	D.F.
Ceresin Wax	0.00047
Paraffin	0.00013
Polystyrene (XMS10023)	0.00040
Quartz (fused, clear)	0.00005
Sulphur (yellow)	0.00035
(amorphous)	0.00049
Victron	0.00086

7. DISCUSSION OF RESULTS

7.1 Low Loss Materials

One of the primary objectives of any investigation such as this is the discovery of those substances most ideal according to some definite criterion. Low loss is of course the goal usually sought for in a dielectric for high frequency application. (There are cases for which this might not be true: cases in which it might be desirable to introduce a loss proportional to frequency, and thus, for example, evade the loss-phase laws of "minimum phase" networks). Examination of the results shows that among the samples tested, those having really low losses are few in number. Quartz in its usual role as an exceptional substance has the lowest loss of all though there appears at present little reason to suppose that the loss it has is a theoretical minimum. Paraffin, too, is seen to have very little loss: roughly twice that of quartz. Next, and in a group, come Ceresin wax, Polystyrene plastic, and sulphur, all having Dissipation Factors about 8 times that of quartz. Last in this group of substances arbitrarily designated "Ultra Low Loss", is Victron, an earlier plastic than Polystyrene, and with a Dissipation Factor roughly twice as great or about 8×10^{-4}.

All of the above substances have Dissipation Factors so low as normally to be negligible (with proper mechanical design of parts) even at frequencies of 100 megacycles per second. However the sad fact remains that with the exception of quartz, none of the substances in this group are capable of withstanding high temperatures, such as are involved in the construction and operation of a vacuum tube. It is not until we drop to Dissipation Factors ranging from 0.0025 to 0.005 that we begin to find substances, refractory, yet capable of being worked. Principal members of this group are Mica, and substances of high mica content, the high grade porcelains such as Isolantite, and the hard, high silica glasses such as Pyrex and Nonex.

In view of the extremely low loss in quartz, it would seem that the hard glasses should be a likely group of substances with which to experiment in an effort to develop new, refractory, low loss dielectrics. One such glass recently developed for other purposes is the new so-called "shrunk-glass" in which the softer constituents of an original cast glass piece are leached out by an etching bath and the remaining mass (nearly pure silica) is refired, whereupon it shrinks to close up the interstices left after etching and leave a final article of substantially pure quartz. This product should therefore have the low loss of quartz in addition to a workability approaching that of the ceramics. It was the author's hope to be able to obtain a sample of this "shrunk glass" for test, but at the time, none was available from the manufacturer.*

7.2 Anomalies and Interesting Features in the Results

The reader may have noticed that the crystal quartz showed a much higher loss and dielectric constant than the fused quartz sample. In the author's opinion this was due to the piezo-electric effect in the crystal quartz causing a reflected reactance and resistance to appear in the electrical circuit as a result of mechanical motion. True, the frequency used was far from the frequency for which the crystal was ground, but there are more modes of vibration in crystals than there are styles in women's hats and it is possible that one of these higher modes lay near 100 megacycles. This is not necessary, however, for even without a nearby resonance an energy loss of the amount required can be accounted for.

The anisotropic nature of wood was also apparent from the results for Ponderosa Pine (Normal Bridge Physics Laboratory Shop grade) which showed a higher loss when the

- -

*Corning Glass Co.

field acted along the grain than when it was applied across the grain. This anisotropism in wood has been mentioned by others in connection with conductivity measurements.[6]

Incidentally, the absence of any definite correlation between conductivity at low frequencies and loss at high frequencies is evidenced by the fact that while paraffin is both a good insulator and a low loss dielectric, both natural amber and Lucite which are better insulators have rather high losses.

While the measurement of dielectric properties is hardly a suitable or adequate method of chemical analysis, it is interesting to look at a few of the results in this light. Thus the final properties of Lucite are seen to be remarkably independent of its method of formation, while the two samples of Victron, though visibly very different were electrically identical. While the loss in Victron is over twice as great as the loss in Polystyrene, the dielectric constants as measured are seen to differ by less than 1%. This confirms other evidence (elasticity, brittleness, melting point, smell when hot etc.) of the molecular similarity of the two substances. In fact it seems probable that Victron is merely impure Polystyrene.

7.3 Comparison of Results

The results obtained admit of very little comparison, as there is little other data available. Also the large amount of variation between samples of substances of indefinite composition (as are porcelains, glasses etc.) make such data as is available hard to compare directly. Miller and Salzberg[7)] have published loss figures for a number of substances among them Polystyrene, and quartz. The figures they quote are

	D.F.	ε
Polystyrene	0.0006	2.5
Quartz	0.0004	3.5

The agreement with the present work is fair in the case of Polystyrene but very poor in the case of quartz, for which the loss quoted is 8 times as great as that found by the author. This disagreement is in excess of the maximum probable error by the present method for the quartz sample. It is possible that there might be this much variation in fused quartz as a result of included impurities. (The sample used by the author was exceptionally pure and optically perfect.) A discussion of the method used by Miller and Salzberg is given in Appendix VI and some possible sources of error in their method are mentioned.

Among the other substances for which data up to 10 megacycles are given elsewhere, the agreement obtained by extrapolation is as good as can be expected.

8. SUGGESTED MODIFICATIONS, IMPROVEMENTS, AND FUTURE RESEARCH USES.

It seldom happens that an investigator, having concluded some research project, is completely satisfied with the adequacy of his results, and with every detail of the method and equipment used. It is always possible, in retrospect, to find faults (secretly, or openly) with the work, as it was done, and to suggest improvements and further applications. This is not necessarily an unhealthy symptom. Far from it. It may in fact be argued that it is out of just such unrest that the state of any art advances. In this section, a comparison is first made of the theoretical capabilities of the method with actual capabilities of the apparatus used. Two ways of improving the sensitivity are suggested, and some suggestions made for the use of the revised apparatus.

8.1 Comparison of Theoretical and Actual Sensitivity

As has been pointed out, the sensitivity limit of the present method of loss measurement is fixed by the losses inherent in the line itself: were these halved, the sensitivity would be doubled. The theoretical lower limit for the losses in the line is the loss arising from the resistance of the line conductors themselves. The loss to be expected, were this the sole source, is calculated in

Appendix V, where it is shown that

$$\alpha \ell = 5 \times 10^{-4} .$$

The actual loss is also readily calculable, for example from Resonance Test #2, and was:

$$A \stackrel{\sim}{=} 12.6 \times 10^{-4}$$

Thus the line losses can account for only about 40% of the total loss and the theoretical sensitivity of the method with the line size used is roughly 2-1/2 times the realized sensitivity.

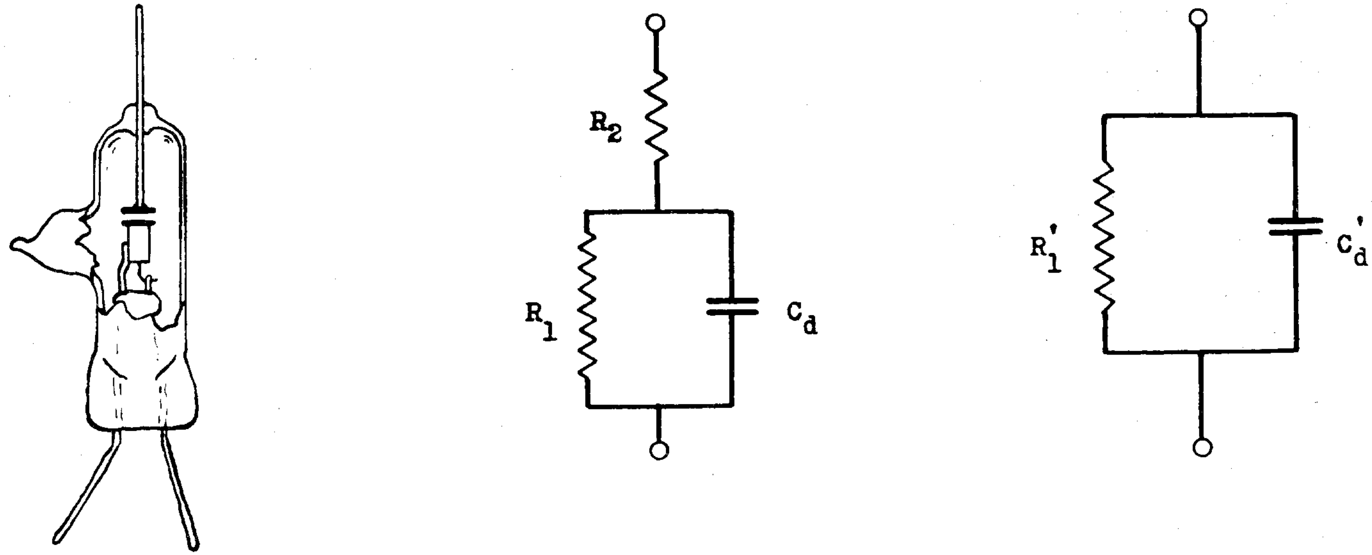
8.2 Localization of Other Losses

Perhaps 10%, but hardly more, of the total loss may have been due to increased resistance at the movable end connections. Accordingly, about half of the total loss with no sample present must be attributed to the diode detector and the Victron support bridge at the center of the line. Since, however, the Victron bridge was so constructed that the capacity it introduced was much less than 1 μμ fd, and since the Dissipation Factor of Victron is given by the measurements as 0.00086, the loss introduced by this bridge could not present an equivalent resistance across the line of less than 2×10^6 ohms, which corresponds to an "A" of 0.5×10^{-4} or roughly 4% of the total loss. Apparently, therefore, the diode detector is responsible for at least

45% of this total loss. The effective load placed across the line by the meter multiplier resistance is equivalent to a resistance one half as large or 5×10^6 ohms for the 50 volt scale. This and the presence of the by pass condensers and insulating sleeves inside the line (see Figure 8) and the associated filter chokes outside the line can perhaps claim another 5% of the total loss. Around 35% then, must be due to the tube itself. This is not surprising, although it is rather disheartening, for the tube used was the best available at the time.

8.3 Origin, Magnitude and Reduction of Loss in the Diode

Since the diode was operated as a peak voltmeter, with a load resistance very high compared with its plate resistance ($R_\ell : R_p > 1000:1$), and since the spacing of the tiny plate and cathode was only about 1 mm, the electronic losses (including transit time effects) must have been but a small portion of the tube's loss. The major portion is easily accounted for by dielectric and resistance losses in the glass bulb (on the inside surface of which was a considerable film of magnesium "getter"), and by resistance losses in the lead wires which had to carry the currents due to the stray capacity of bulb and elements. A likely equivalent circuit of the losses in the diode is therefore that given in Figure 19(b) which at any particular frequency



a) Section through diode

b) Equivalent circuit of loss in diode

c) Simplified equivalent circuit

FIGURE 19

DIODE CONSTRUCTION AND REPRESENTATION OF ITS LOSS can be converted to the equivalent circuit 19(c). It is to be remembered in computing these losses that the tube had applied to it at all times the total radio frequency voltage at the center of the line.

In the tube used, the capacity was about 2.5 μμ fds, of which approximately 1.5 μμ fds, was due to the glass envelope. Taking C = 1.5 μμ fds, and the Dissipation Factor of the glass to be that found for Pyrex, we get

$$R_1 \cong 215,000 \text{ ohms}$$

With the leads to the tube 0.02 cm in diameter and a total of 2 cm long,

$$R_2 \cong 0.1 \text{ ohm}$$

This proves negligible, and hence

$$R'_1 \cong 215,000 \text{ ohms}$$

which is equivalent to an "A" of 4.4×10^{-4}, or roughly 35% of the total loss, as expected. It is therefore possible with entirely reasonable values to account for all the "excess" loss present in the actual line as constructed.

Since there was no internal support insulation present in the diode used, and since the loss in the leads was negligible, it seems that practically the only way to reduce the tube's loss would be to redesign the glass envelope and construct it out of a lower loss glass. As to this redesign of the envelope, some benefit could be expected from a reduction in the wall thickness and in the lead in wire size as both of these changes would reduce the end to end bulb capacity. It is doubtful if the tube loss could be reduced by more than 30% by this means. A greater improvement could be obtained with a markedly lower-loss glass, and by the elimination of the getter film on the inside surface. By using all these means, it should be possible to reduce the tube loss till it be less than 10% of the total loss, in which case further improvement would be unwarranted.

There are of course other methods for measuring the resonance in the line. McRae[1)] used a thermocouple loop and galvanometer to read the current at the movable end of the line. It should be equally feasible to use a pickup loop and amplifier, followed by a detector to do this. However, while these schemes may have somewhat less inherent loss than the simple method used by the author, the added complexity, and instability, and inconvenience involved are potential sources of error which completely negate any advantages they may have.

Assuming success in reducing the diode loss, it would then be worthwhile to reduce some of the other losses by such means as silverplating the entire inside of the line, and then lacquering all current carrying surface to which no contact need be made. By bending every effort to loss reduction it should be possible to very nearly double the sensitivity of the method over its present value.

It should be mentioned that since the losses in the present type of transmission line at the frequencies used, or higher, are proportional to the square root of frequency (being controlled by skin effect) and since the loss in dielectrics is proportional to nearly the first power of frequency, the potentialities of the method are the greater the higher the frequency used.

8.4 Suggestions for Further Investigations

As well as for making continued measurements on many other deserving dielectrics, there are other applications for which the apparatus is equally well suited. By constructing an additional, and very simple lower sample condenser plate in the form of a shallow cup, it would be possible to make loss measurements equally easily on liquid samples such as oils.

Series of runs could also be made to determine the variation of the dielectric loss with temperature in various substances.

The measurement of the dielectric constants of other solids, liquids, and even gases is a possibility.

By bridging the line at the sample condenser location with the proper elements of a vacuum tube, it should be possible to determine the admittance looking into these elements and thus provide an experimental check on transit time theory[8].

These are a few of the uses which suggest themselves, uses for which the present apparatus could well be used now that it has been constructed.

9. CONCLUSION AND SUMMARY

A method, based upon an earlier one used by McRae[1], has been developed for the measurement of dielectric losses and constants at ultra high frequencies. A large number of samples have been successfully tested by the method at a frequency of one hundred megacycles per second, and the results of these measurements are given. The theory of the method is presented and the theoretical limitations considered. Means for attaining very nearly the maximum sensitivity set by these limitations are suggested.

10. ACKNOWLEDGMENT

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APPENDIX I

SENDING END-TO-MIDPOINT TRANSFER CONSTANT

Under the assumptions mentioned in Section 2.2, the resonant coaxial line used may be treated by the ordinary transmission line theory. We shall find it convenient, in the analysis, to break the line up into three sections each of which can be considered as a four terminal network. These sections are shown in the schematic diagram below:

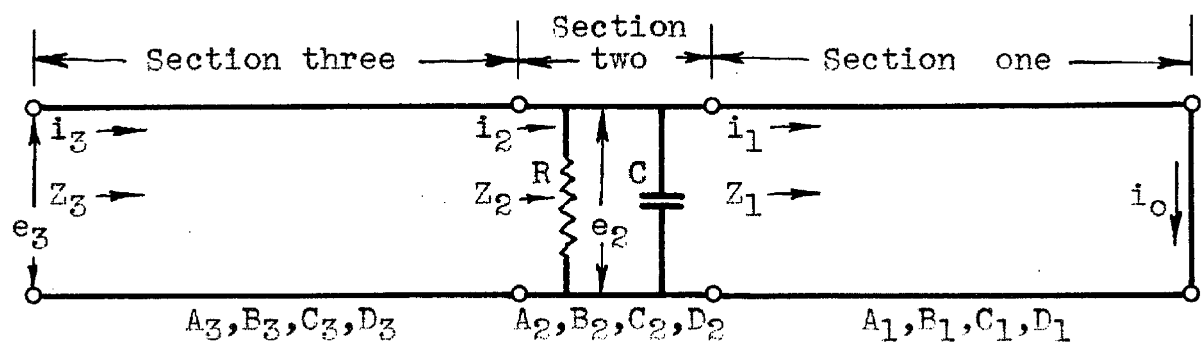

Fig. 20

Section 1 is that part of the line lying between the sample condenser and the movable short circuited end of the line. Section 2 is simply the shunt admittances of $\frac{1}{R}$ and $j\omega C$ representing the sample and holder and detector.

Section 3 is the quarter wavelength long section from the sample condenser to the short circuited input end. The voltage and current and impedance looking into each section as well as the constants thereof (with the exception of the undenoted R, and C) are denoted by a subscript which is that of the section in which they lie.

We desire to know the voltage at the center of the line, e_2, as a function of the line parameters ℓ_1, R, and C, with e_3, the input voltage, held constant.

By four-terminal network theory, we have:

$$\left.\begin{aligned} e_1 &= A_1 e_o + B_1 i_o \\ i_1 &= C_1 e_o + D_1 i_o \end{aligned}\right\} \qquad (1)$$

$$e_o = 0, \text{ so } Z_1 = \frac{B_1}{D_1} \qquad (1a)$$

$$\left.\begin{aligned} e_2 &= (A_1A_2+C_1B_2)\, e_o + (B_1A_2+D_1B_2) i_o \\ i_2 &= (A_1C_2+C_1D_2)\, e_o + (B_1C_2+D_1D_2) i_o \end{aligned}\right\} \qquad (2)$$

$$e_o = 0, \text{ so } Z_2 = \frac{B_1A_2+D_1B_2}{B_1C_2+D_1D_2} \qquad (2a)$$

$$\left.\begin{aligned} e_3 = {} & [A_3(A_1A_2+C_1B_2) + B_3(A_1C_2+C_1D_2)]e_o \\ & + [A_3(B_1A_2+D_1B_2) + B_3(B_1C_2+D_1D_2)]i_o \\ i_3 = {} & [C_3(A_1A_2+C_1B_2) + D_3(A_1C_2+C_1D_2)]e_o \\ & + [C_3(B_1A_2+D_1B_2) + D_3(B_1C_2+D_1D_2)]i_o \end{aligned}\right\} \qquad (3)$$

$$e_o = 0 \text{ so } Z_3 = \frac{A_3(B_1A_2+D_1B_2) + B_3(B_1C_1+D_1D_2)}{C_3(B_1A_2+D_1B_2) + D_3(B_1C_1+D_1D_2)} \qquad (3a)$$

Let us now define $K_T \equiv \frac{e_2}{e_3}$. Then we have from equations (2), (2a) and (3):

$$K_T = \frac{e_2}{e_3} = \frac{B_1A_2+D_1B_2}{A_3(B_1A_2+D_1B_2) + B_3(B_1C_2+D_1D_2)}$$

$$K_T = \frac{1}{1 + B_3\,\dfrac{B_1C_2+D_1D_2}{B_1A_2+D_1B_2}}$$

$$K_T = \frac{1}{1 + \dfrac{B_3}{Z_2}} \qquad (4)$$

Referring back to Figure 20, we see that:

$A_1 = \cosh \theta_1$	$A_2 = 1$	$A_3 = \cosh \theta_3$
$B_1 = Z_o \sinh \theta_1$	$B_2 = 0$	$B_3 = Z_o \sinh \theta_3$
$C_1 = \frac{1}{Z_o} \sinh \theta_1$	$C_2 = \frac{1}{R} + j\omega C$	$C_3 = \frac{1}{Z_o} \sinh \theta_3$
$D_1 = \cosh \theta_1$	$D_2 = 1$	$D_3 = \cosh \theta_3$

where Z_o = characteristic impedance of line.

θ = hyperbolic propagation angle.

Substituting the appropriate quantities in equation (2a), we get

$$Z_2 = \frac{Z_o \sinh \theta_1}{Z_o \sinh \theta_1 \left(\frac{1}{R} + j\omega C\right) + \cosh \theta_1}$$

$$\frac{1}{Z_2} = \frac{1}{R} + j\omega C + \frac{1}{Z_o \tanh \theta_1} \qquad (5)$$

Equation (5) is obvious directly since $Z_1 = \frac{B_1}{D_1} = Z_o \tanh \theta_1$.

Equation (5) now becomes:

$$K_T = \frac{1}{\cosh \theta_3 + \frac{Z_o}{R} \sinh \theta_3 + \frac{\sinh \theta_3}{\tanh \theta_1} + j\omega C\, Z_o \sinh \theta_3} \qquad (6)$$

Now

$$\left.\begin{aligned} \theta_1 &= \alpha\ell_1 + j\beta\ell_1 \\ \theta_2 &= \alpha\ell_2 + j\beta\ell_2 \end{aligned}\right) \qquad (7)$$

where α = attenuation constant of line

β = phase constant of line,

and so, by trigonometric reduction, we may write:

$$\frac{1}{\tanh \theta_1} = \coth \theta_1 = \frac{1 - j \coth \alpha\ell_1 \cot \beta\ell_1}{(\coth \alpha\ell_1 - j \cot \beta\ell_1)}$$

$$\coth \theta_1 = \frac{1 - j \coth \alpha\ell_1 \cot \beta\ell_1}{\coth \alpha\ell_1 (1 - j \tanh \alpha\ell_1 \cot \beta\ell_1)}$$

$$\coth \theta = \frac{\tanh \alpha\ell_1 [1 + \cot^2\beta\ell_1 - j(\coth \alpha\ell_1 - \tanh \alpha\ell_1) \cot \beta\ell_1]}{1 + \tanh^2\alpha\ell_1 \cot^2\beta\ell_1}$$

But since

$$\left.\begin{aligned} \alpha\ell_1 &\ll 10^{-2} \\ \frac{\pi}{6} < \beta\ell_1 &< \frac{\pi}{2} \end{aligned}\right\} \qquad (8)$$

we may safely neglect $\tanh^2\alpha\ell_1 \cot^2\beta\ell_1$ with respect to unity. We then have (remembering that $1 + \cot^2 x = \csc^2 x$ and that $1 - \tanh^2 y = \operatorname{sech}^2 y$) the relation

$$\coth \theta_1 = \frac{\tanh \alpha\ell_1}{\sin^2 \beta\ell_1} - j \frac{\cot \beta\ell_1}{\cosh^2 \alpha\ell_1}$$

Again because of relations (8), we may safely reduce this to

$$\coth \theta_1 = \frac{\alpha\ell_1}{\sin^2 \beta\ell_1} - j \cot \beta\ell_1 \qquad (9)$$

Also

$$\cosh \theta_3 = \cosh \alpha\ell_3 \cos(\tfrac{\pi}{2} + \delta) + j \sinh \alpha\ell_3 \sin(\tfrac{\pi}{2} + \delta)$$

$$\sinh \theta_3 = \sinh \alpha\ell_3 \cos(\tfrac{\pi}{2} + \delta) + j \cosh \alpha\ell_3 \sin(\tfrac{\pi}{2} + \delta) \quad (10)$$

where δ is a small angle which may arise as a result of the section of line, ℓ_3, not being an exact quarter wavelength long. Since

$$\left.\begin{aligned} \alpha\ell_3 &\ll 10^{-2} \\ \delta &\ll 1 \end{aligned}\right\}$$

equations (10) reduce to:

$$\left.\begin{aligned} \cosh \theta_3 &= j(\alpha\ell_3 + j \sin \delta) \\ \sinh \theta_3 &= j \end{aligned}\right\} \qquad (11)$$

Substituting the quantities given by equations (9) and (11) into our expression for K_T, we get:

$$jK_T = \frac{1}{\left[\alpha \ell_3 + \frac{Z_o}{R} + \frac{\alpha \ell_1}{\sin^2 \beta \ell_1} + j(\sin\delta + \omega C\, Z_o - \cot \beta \ell_1)\right]}$$

$$\cong \frac{1}{R_T + jQ_T} \qquad (12)$$

Because of the relative magnitudes of the quantities involved in the above expression can be still further simplified. Let us compute these quantities for the present line and at both extremes of the expected range of sample resistance.

For the line used

$$Z_o = 94 \text{ ohms}$$

$$\alpha = 3.3 \times 10^{-6}/\text{cm}$$

$$\beta = \frac{\pi}{150} \text{ rad/cm}$$

as shown in Appendix V. Since $\beta \ell_3 \cong \frac{\pi}{2}$, $\ell_3 \cong 75$ cm, and therefore

$$\alpha \ell_3 \cong 2.5 \times 10^{-4} .$$

Let us assume a 1" diameter sample, 1/8" thick, and for which $\varepsilon = 6$. In this case,

$$C_s \cong 8.5 \ \mu\mu\text{Fd.}$$

Allowing 2.5 μμFd for the tube capacity, and about 5 μμFd for stray capacity we find

$$C \cong 16 \text{ μμfds.}$$

This higher than average value is accurate enough for the present purpose, since variations in it will not affect the validity of our approximations, as we shall see. Neglecting sin δ for the moment, we see that in order to make jK_T real,

$$\cot \beta\ell_1 \cong \omega C Z_0 \cong 1.0$$

$$\beta\ell_1 \cong 45° = \frac{\pi}{4}$$

Therefore $\qquad \sin^2\beta\ell_1 = 0.5$

and $\qquad \ell_1 = 37.5 \text{ cm.}$

Depending on the sample R may be expected to lie anywhere between about 5000 ohms and 200,000 ohms (the upper limit being fixed by the voltmeter and support assembly). Hence we find by addition

$$\alpha\ell_3 = 2.5 \times 10^{-4}$$

$$5 \times 10^{-4} < \frac{Z_0}{R} \leqslant 200 \times 10^{-4}$$

that

$$\frac{\alpha\ell_1}{\sin^2\beta\ell_1} \cong 2.5 \times 10^{-4}$$

$$\overline{10^{-3} \lesssim R_T \lesssim 2 \times 10^{-2}}$$

Thus (since $jK_T = \frac{1}{R_T + jQ_T}$) anywhere from a 50 to a 1000 fold rise in voltage can be expected with $Q_T = 0$. "Resonance" will be indicated when jK_T is a maximum. Inasmuch as $R_T = f(\ell_1)$, the condition for resonance is not exactly that for which $Q_T = 0$; but, as we shall see, the difference is negligible.

Let ℓ_o be defined such that

$$\underline{\cot \beta\ell_o \equiv \omega C Z_o + \sin \delta} \tag{13}$$

Then in general, we may write $\ell_1 = \ell_o + \Delta\ell$ and therefore, by Taylor's series:

$$\cot \beta(\ell_o+\Delta\ell) = \cot \beta\ell_o - \frac{\beta\Delta\ell}{\sin^2\beta\ell_o} + \frac{\cot \beta\ell_o}{\sin^2\beta\ell_o}(\beta\Delta\ell)^2 + \cdots \tag{14}$$

This gives for jK_T

$$jK_T = \frac{1}{\left[\frac{\alpha(\ell_o+\Delta\ell)}{\sin^2\beta(\ell_o+\Delta\ell)} + \alpha\ell_3 + \frac{Z_o}{R}\right] - j\left[\frac{\beta\Delta\ell}{\sin^2\beta\ell_o} - \frac{\cot \beta\ell_o}{\sin^2\beta\ell_o}(\beta\Delta\ell)^2 + \cdots\right]}$$

Taking the lower extreme of 0.02 for R_T, we see that jK_T is reduced to the 70.71% value when

$$Q_T = 0.02 = \frac{\beta\Delta\ell}{\sin^2\beta\ell_o} - \frac{\cot \beta\ell_o}{\sin^2\beta\ell_o}(\beta\Delta\ell)^2 + \cdots$$

which, for the case assumed (C = 16 μμfds) gives

$$\beta\Delta\ell \cong 0.01$$

The omission of all terms in the expansion of a power of $(\beta\Delta\ell)$ greater than the first can introduce an error of 1% at most, and is therefore permissible. We then have

$$jK_T = \frac{1}{\left(\frac{\alpha(\ell_o+\Delta\ell)}{\sin^2\beta(\ell_o+\Delta\ell)} + \alpha\ell_3 + \frac{Z_o}{R}\right) - j\,\frac{\beta\Delta\ell}{\sin^2\beta\ell_o}} \tag{15}$$

Again, by Taylor's series:

$$\frac{\alpha(\ell_o+\Delta\ell)}{\sin^2\beta(\ell_o+\Delta\ell)} = \frac{\alpha\ell_o}{\sin^2\beta\ell_o} + \frac{1-2\,\beta\ell_o\cot\beta\ell_o}{\sin^2\beta\ell_o}(\alpha\Delta\ell) + \dots \tag{16}$$

$$\cong r_o + \delta r_o$$

If we let $\beta\Delta\ell = 0.02$ to get the maximum variation in R_T from one 70.71% point to the other, we get:

$$\delta r_o = \frac{1-2\left(\frac{\pi}{4}\right)}{0.5}(3.2 \times 10^{-6})(0.02) \cong -7.3 \times 10^{-8}$$

This is less than 0.001% of R_T. The variation is, as a matter of fact, even less for $\beta\ell_o > \frac{\pi}{4}$, and zero when $\beta\ell_o \cot\beta\ell_o = \frac{1}{2}$. As a result we can neglect all changes in R_T over the resonance range, as predicted, and write simply

$$K_T = \frac{1}{\frac{\beta\Delta\ell}{\sin^2\beta\ell_o} + j\left(\frac{\alpha\ell_o}{\sin^2\beta\ell_o} + \alpha\ell_3 + \frac{Z_o}{R}\right)} \tag{17}$$

In the actual arrangement

$$\frac{1}{R} = \frac{1}{R_s} + \frac{1}{R_o}$$

where

R_s = effective resistance of sample

R_o = effective resistance of all loss at line center with no sample

Therefore, if we let

$$A \equiv \frac{\alpha \ell_o}{\sin^2 \beta \ell_o} + \alpha \ell_3 + \frac{Z_o}{R_o} \qquad (18)$$

$$B \equiv \frac{\beta}{\sin^2 \beta \ell_o} \qquad (19)$$

We may write

$$K_T = \frac{1}{B\Delta\ell + j\left(A + \frac{Z_o}{R_s}\right)} \qquad (20)$$

The quantity "A" is seen to be a quantity which includes all the loss present in the line without a sample and with $\ell_1 = \ell_o$, and can be determined experimentally by taking such a run.

The method used to determine A and R_s from the experimental data is given in Appendix II.

APPENDIX II

CONVERSION OF DATA

In Appendix I we saw that

$$K_T = \frac{1}{B\Delta\ell + j(A + \frac{Z_o}{R_s})} \qquad (20)$$

Writing this in polar form, we have:

$$K_T = |K_T| \underline{|\phi} = \frac{1}{\sqrt{B^2\Delta\ell^2 + (A + \frac{Z_o}{R_s})^2}} \underline{|\phi} \qquad (21)$$

where $\phi = \cot^{-1} \dfrac{-B\Delta\ell}{A + \frac{Z_o}{R_s}}$

Thus if cot ϕ be plotted as a function of $\Delta\ell$, the result should be a straight line having a slope, m, given by

$$m = -\frac{\cot\phi}{\Delta\ell} = \frac{B}{A + \frac{Z_o}{R_s}} \qquad (22)$$

If $m_o \equiv$ slope without sample present

$m_s \equiv$ slope with sample present

we see that $$m_o = \frac{B}{A}$$

and $$m_s = \frac{B}{A + \frac{Z_o}{R_s}}$$

from which $$A = \frac{B}{m_o} \qquad (23)$$

and $$R_s = \frac{Z_o}{\frac{B}{m_s} - A} = \frac{Z_o}{B} \frac{m_o m_s}{m_o - m_s} \qquad (24)$$

in which $$B \equiv \frac{\beta}{\sin^2 \beta \ell_o}$$

In order to determine R_s by this method therefore, it is necessary to know the slopes m_o, and m_s, found by plotting cot ϕ as a function of $\Delta\ell$ (or ℓ_1).

If we let

$$K_{T_R} \equiv \text{value of } K_T \text{ at resonance,}$$

we see from equation (21) that

$$\sqrt{\left(\frac{K_{T_R}}{K_T}\right)^2 - 1} = \sqrt{\frac{B^2 \Delta\ell^2 - \left(A + \frac{Z_o}{R_s}\right)^2}{\left(A + \frac{Z_o}{R_s}\right)^2} - 1}$$

$$= \frac{B\Delta \ell}{A + \frac{Z_o}{R_s}} = \cot \phi$$

But

$$\sqrt{\left(\frac{K_{T_R}}{K_T}\right)^2 - 1} = \sqrt{\left(\frac{e_r}{e}\right)^2 - 1} \qquad (25)$$

where e_r = voltage read on diode detector at resonance

e = voltage read on diode detector at any point.

So that

$$\cot \phi = \sqrt{\left(\frac{e_r}{e}\right)^2 - 1} \qquad (26)$$

and in order to find m_o, or m_s, it is merely necessary to convert the appropriate data by the above equation, plot $\cot \phi$ as a function of $\Delta \ell$ (or ℓ_1) and measure the slope.

The sample resistance is then given by equation (24).

APPENDIX III

DEVELOPMENT OF SAMPLE CAPACITY FORMULAE

Method a)

Consider the parallel plate sample condenser between the plates of which is a slab of dielectric, small enough so that no portion extends to the edges of the plates. By the parallel plate capacity equation, then:

$$C \cong \left(\frac{A_o - A_s}{4\pi d_s} + C_s + C_o \right) \text{ cm} \tag{27}$$

where C = total capacity at line center

C_s = capacity of dielectric

C_o = excess stray capacity at line center

A_o = plate area in cm^2

A_s = dielectric area in cm^2

d_s = dielectric thickness - plate spacing in cm

By removing the dielectric, the plate spacing may be decreased till the capacity is again the same. Then:

$$C = \left(\frac{A_o}{4\pi d_o} + C_o \right) \text{ cm} \tag{28}$$

d_o = plate spacing for same capacity with no sample present.

Therefore from equations (27) and (28)

$$C_s \stackrel{\sim}{=} \left(\frac{A_o}{4\pi d_o} - \frac{A_o - A_s}{4\pi d_s} \right) \text{ cm} \tag{29}$$

or

$$C_s \stackrel{\sim}{=} \frac{A_o}{4\pi} \left[\frac{1}{d_o} - \left(1 - \frac{A_s}{A_o} \right) \frac{1}{d_s} \right] \text{cm} \tag{29a}$$

$$C_s \stackrel{\sim}{=} \frac{A_o}{(3.6)\pi} \left[\frac{1}{d_o} - \left(1 - \frac{A_s}{A_o} \right) \frac{1}{d_s} \right] \mu\mu\text{fds} \tag{29b}$$

For the apparatus used:

$$\frac{A}{(3.6)\pi} = \frac{\pi (0.75 \times 2.54)^2}{3.6\ \pi} = 1 \text{ cm } \mu\mu\text{fds}$$

and therefore

$$C_s \stackrel{\sim}{=} \left[\frac{1}{d_o} - \left(1 - \frac{A_s}{A_o} \right) \frac{1}{d_s} \right] \mu\mu\text{fds} \tag{30}$$

The method is approximate because of the presence of edge effects around the parallel plate condenser.

Method b)

From equation (13) (Appendix I) we have (with C in farads):

$$\cot \beta \ell_o = \omega C Z_o + \sin \delta \qquad (13)$$

If we let C' = capacity (in farads) of sample condenser without dielectric present but at plate spacing of d_s

ℓ_o' = resonant length (of ℓ_1) under these conditions.

Then $$\cot \beta \ell_o' = \omega Z_o (C' + C_o) + \sin \delta \qquad (31)$$

Subtracting from equation (13):

$$\cot \beta \ell_o - \cot \beta \ell_o' = \omega Z_o (C - C' - C_o)$$

But

$$C = \left(C_s + C_o + \frac{A_o' - A_s}{4\pi d_s} \frac{1}{9 \times 10^{11}}\right) \text{ farads}$$

$$C' = \frac{A_o'}{4\pi d_s} \frac{1}{9 \times 10^{11}} \text{ farads}$$

where

A_o' = <u>effective</u> area of sample condenser plates including edge effects.

Hence

$$C - C' - C_o = \left(C_s - \frac{A_s}{4\pi d_s} \frac{1}{9 \times 10^{11}}\right) \text{ farads}$$

Substituting, we get:

$$C_s = \left(\frac{1}{\omega Z_o} (\cot \beta \ell_o - \cot \beta \ell_o') + \frac{A_s}{4\pi d_s} \frac{1}{9 \times 10^{11}}\right) \text{ farads} \quad (32)$$

This expression is exact since there are no edge effects around the boundary of A_s, provided this boundary is well inside the boundary of A_o, as specified. For the apparatus used, equation (32) reduces to

$$C_s = \left(\frac{10^{12}}{\omega Z_o} (\cot \beta \ell_o - \cot \beta \ell_o') + \left(\frac{A_s}{A_o}\right) \frac{1}{d_s}\right) \mu\mu\text{fds} \quad (32a)$$

APPENDIX IV

BASIC SENSITIVITY OF THE METHOD

We have seen (Appendix II) that the equivalent resistance of the sample, R_s, is given by

$$R_s = \frac{Z_o}{B} \frac{m_o m_s}{m_o - m_s} \tag{24}$$

In which

Z_o = characteristic impedance of line

$B = \dfrac{\beta}{\sin^2 \beta \ell_o}$

β = phase constant of line

ℓ_o = length of section 1 of line at resonance

m_o = slope of plotted data without sample

m_s = slope of plotted data with sample

In practice it is only possible to determine m_o and m_s within a certain maximum probable error, and a corresponding error may thus be present in R_s. Quantitatively:

$$\delta R_s = \frac{\partial R_s}{\partial m_o} \delta m_o + \frac{\partial R_s}{\partial m_s} \delta m_s$$

$$\delta R_s = \frac{Z_o}{B} \left\{ \frac{(m_o - m_s) m_s - m_o m_s}{(m_o - m_s)^2} \delta m_o + \frac{(m_o - m_s) m_o + m_o m_s}{(m_o - m_s)^2} \delta m_s \right\}$$

which reduces to

$$\delta R_s = \frac{Z_o}{B}\,\frac{m_o^2\delta m_s - m_s^2\delta m_o}{(m_o - m_s)^2} \qquad (33)$$

Dividing through by R_s, we get

$$\frac{\delta R_s}{R_s} = \frac{m_o^2\delta m_s - m_s^2\delta m_o}{m_o m_s(m_o - m_s)}$$

$$\frac{\delta R_s}{R_s} = \frac{m_o\left(\frac{\delta m_s}{m_s}\right) - m_s\left(\frac{\delta m_o}{m_o}\right)}{(m_o - m_s)} \qquad (34)$$

This expression is largest when $\frac{\delta m_s}{m_s}$ and $\frac{\delta m_o}{m_o}$ are of opposite sign; in other words, if one slope is estimated too large, and the other too small. Thus

$$\left|\frac{\delta R_s}{R_s}\right|_{max} = \frac{m_o\left|\frac{\delta m_s}{m_s}\right|_{max} + m_s\left|\frac{\delta m_o}{m_o}\right|_{max}}{m_o - m_s} \qquad (35)$$

where $100\left|\frac{\delta m_s}{m_s}\right|_{max}$ and $100\left|\frac{\delta m_o}{m_o}\right|_{max}$ are the maximum probable percentage errors in estimating m_s and m_o. For given values of $\left|\frac{\delta m_o}{m_o}\right|$ and $\left|\frac{\delta m_s}{m_s}\right|$, equation (30) is the greater, the nearer m_s approaches m_o.

If we define:

$$\Delta m = m_o - m_s$$

$$\varepsilon = \left|\frac{\delta R_s}{R_s}\right|_{max}$$

we may rewrite equation (30) thus:

$$\varepsilon = \frac{m_o}{\Delta m}\left(\left|\frac{\delta m_s}{m_s}\right|_{max} + \left(1 + \frac{\Delta m}{m_o}\right)\left|\frac{\delta m_o}{m_o}\right|_{max}\right)$$

As $m_s \rightarrow m_o$, $\frac{\Delta m_o}{m_o} \rightarrow 0$, and we have

$$\varepsilon = \frac{m_o}{\Delta m}\left(\left|\frac{\delta m_s}{m_s}\right|_{max} + \left|\frac{\delta m_o}{m_o}\right|_{max}\right)$$

and accordingly

$$\Delta m = \frac{m_o}{\varepsilon}\left(\left|\frac{\delta m_s}{m_s}\right|_{max} + \left|\frac{\delta m_o}{m_o}\right|_{max}\right) \qquad (36)$$

But from equation (24)

$$R_s = \frac{Z_o}{B}\,\frac{m_o^2\left(1 - \frac{\Delta m}{m_o}\right)}{\Delta m}$$

and again as $m_s \rightarrow m_o$, we have

$$R_s \cong \frac{Z_o}{B}\,\frac{m_o^2}{\Delta m} \qquad (37)$$

Combining equations (31) and (32) we get

$$R_s = \frac{Z_o}{B} m_o \left(\frac{\varepsilon}{\left|\frac{\delta m_o}{m_o}\right|_{max} + \left|\frac{\delta m_s}{m_s}\right|_{max}} \right)$$

but since from Appendix II,

$$A = \frac{B}{m_o}$$

we may write

$$R_s = \frac{Z_o}{A} \left(\frac{\varepsilon}{\left|\frac{\delta m_o}{m_o}\right|_{max} + \left|\frac{\delta m_s}{m_s}\right|} \right) \qquad (38)$$

Now this expression has been developed for the case when the loss is low and therefore for $m_s \cong m_o$. In this case it is reasonable to assume that

$$\left|\frac{\delta m_o}{m_o}\right|_{max} \cong \left|\frac{\delta m_s}{m_s}\right|_{max}$$

and hence that:

$$R_s \cong \frac{Z_o}{A} \frac{\varepsilon}{2\left|\frac{\delta m_o}{m_o}\right|_{max}} \qquad (38a)$$

Thus we have a relation between the maximum probable error in reading the slopes m_o and m_s, the sample resistance and the error in the result.

APPENDIX V

CALCULATION OF LINE CONSTANTS

a) Characteristic Impedance.

Since in the line used, the resistance and leakage were negligibly small compared with the inductive reactance and capacitive susceptance, we have for the characteristic impedance of the line Z_o,

$$Z_o = \sqrt{\frac{\Lambda}{\Gamma}}$$

where

Λ = inductance per unit length

Γ = capacitance per unit length

now $\Lambda = 2\mu \log \frac{b}{a} = (2 \times 10^{-9})\mu \log \frac{b}{a} \ \frac{\text{henries}}{\text{cm}}$

$$\Gamma = \frac{\varepsilon}{2 \log \frac{b}{a}} = \frac{\varepsilon}{(18 \times 10^{11}) \log \frac{b}{a}} \ \frac{\text{farads}}{\text{cm}}$$

Hence

$$Z_o = 60\sqrt{\frac{\mu}{\varepsilon}} \log \frac{b}{a} \text{ ohms.}$$

For the present line, $\mu = 1$, $\varepsilon = 1$, $\frac{b}{a} = 4.8$ and we get

$$\underline{Z_o = 94 \text{ ohms}}$$

b) Phase constant of Line, β.

In a transmission line with negligible dissipation, the velocity of propagation is given by

$$v = \frac{c}{\sqrt{\mu \varepsilon}}$$

where

c = velocity of light

μ = permeability of medium

ε = dielectric constant of medium.

In our case therefore

$$v = c = \lambda f$$

where λ = wavelength

f = frequency

But

$$\beta = \frac{2\pi}{\lambda}, \text{ so}$$

$$\beta = \frac{2\pi f}{c} = \frac{\omega}{c}$$

Since

$$\omega = 2\pi \times 10^8 \text{ rad/sec}$$

$$c = 3 \times 10^{10} \text{ cm/sec}$$

we get

$$\beta = \frac{\pi}{150} \frac{\text{rad}}{\text{cm}} .$$

c) Attenuation constant of line, α.

The loop resistance of a copper coaxial line at high frequencies is very nearly[5]:

$$R_L = 41.6 \sqrt{f} \left(\frac{1}{a} + \frac{1}{b}\right) \times 10^{-9} \frac{\text{ohms}}{\text{cm}}$$

where a = outer radius of inner conductor in cm.

b = inner radius of outer conductor in cm.

and the attenuation constant is given by

$$\alpha = \frac{R_L}{2Z_o}$$

For the present line:

$$R_L = 41.6 \times 10^4 \times \frac{3.87}{2.59} \times 10^{-9}$$

$$R_L = 62.1 \times 10^{-5} \text{ ohms/cm.}$$

and thus

$$\alpha = \frac{62.1 \times 10^{-5}}{2 \times 94} = 3.3 \times 10^{-6}/\text{cm.}$$

For the whole line therefore

$$\alpha \ell \equiv \alpha (\ell_1 + \ell_3) = \alpha \times 150 \cong 5 \times 10^{-4}$$

APPENDIX VI

METHOD USED BY MILLER AND SALZBERG

A method of measuring admittances at ultra high frequencies, and one which also makes use of a tuned transmission line, has been described by Miller and Salzberg[7]. In their method a two wire transmission line considerably less than a quarter wavelength long was used; one end of the line being short circuited and the other loaded with the sample condenser and sample. Across the "open" (condenser) end was a vacuum tube voltmeter.

To measure a sample, a reading of the voltmeter was first made with the sample in place and the line tuned to resonance. The sample was then removed and the line was again tuned to resonance by the terminal capacity, but with a calibrated (?) resistor shunting the line at some point such that the same reading was again obtained on the vacuum tube voltmeter. From the position on the line of the resistor relative to the short-circuited end, and from the value of the resistance used, the equivalent resistance of the sample was computed by a simple relation.

This method, though convenient, has, in the author's opinion, several notable disadvantages. First: because of a fundamental sensitivity limitation imposed by the losses in the line itself (similar to that derived in Appendix IV) and the fact that these losses are higher on a two-wire line (due to radiation), the method is inherently less sensitive than that used by the author. Second: being unshielded, the apparatus used is more susceptible to errors due to stray capacities and couplings. Third: the measurements rest upon the validity of but two observations, and hence all the errors which may affect a single reading are not eliminated. Fourth: ~~Fifth~~, and perhaps most important, the current distribution in the line itself is not the same with and without the sample as a result of the stray capacity and loss present in the "standard" resistor placed across the line at an intermediate point. The effect of this change in current distribution is in such a direction as to alter the loss contributed by the line itself, between the sample and no-sample runs. This, and the other effects listed, could be expected to introduce an error into the results obtained by this method: an error large for low loss samples.

In addition the method is not absolute, as it depends upon the accuracy of the standard resistor with respect to which all measurements are made, and a standard resistance accurate at ultra high frequencies is difficult to obtain.

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