SOME TOPICS IN THEORETICAL HIGH-ENERGY PHYSICS

Thesis by

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ABSTRACT

This thesis discusses various aspects of theoretical high-energy physics. The first two sections describe methods for investigating QCD effects in e^+e^- annihilation to hadrons. The third section presents some predictions for various features of QCD jets. The fourth section shows that any fermions in the standard weak interaction model must have masses $\lesssim 100$ GeV. In the fifth section, the abundances of any new absolutely stable heavy particles which should have been produced in the early universe are estimated, and found to be inconsistent with observational limits. Finally, the sixth section describes the development of a baryon excess in the very early universe due to B, CP violating interactions.

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PREFACE

This thesis consists of a selection of short papers which summarize some of the research on theoretical high-energy physics that I have carried out during the last year and a half (June 1978 - October 1979). Many details have been omitted. In most cases, further details are described in published or soon to be published papers. These papers alone amount to some 800 typed pages, and their inclusion here would have rendered this thesis a somewhat lengthy document. Appropriate references necessary to locate them are given in the Introduction below. The chronological order in which the major parts of the works described in the papers below were performed was: 5, 1, 4, 2, 3, 6. Topics investigated during the last year and a half which have been entirely omitted below include:

Non-logarithmic terms and effective coupling

Weak effects in Σ^0 decay

Some cosmological effects of the Higgs mechanism.

INTRODUCTION

The first three papers in this thesis concern the extraction of measurable predictions from quantum chromodynamics (QCD). QCD is a gauge field theory which purports to describe the (color) interactions of quarks and gluons (for reviews on various aspects of QCD, see ref. [1]). At present, QCD is the only viable model for strong interactions. However, few definite quantitative consequences of QCD have yet been deduced. The main obstruction is that the fundamental quarks and gluons of QCD apparently cannot be permanently isolated as free particles, but are always confined within the observed hadrons by strong forces not amenable to treatment by perturbative methods (which are the only proven approach to investigating quantized field theories). Nevertheless, at distances much smaller than the inverse sizes of light hadrons (e.g. π), QCD interactions become sufficiently weak that precise calculations using perturbation theory and Feynman diagrams are possible. The simplest process to analyse in QCD is probably high-energy e^+e^- annihilation into hadrons, since it involves no quarks or gluons in the intial state. Measurable features of e^+e^- annihilation which are sensitive to the structure of events only at short distances (before QCD interactions become strong and hadrons form) may be calculated from QCD perturbation theory. The simplest observable is the total cross-section for hadronic e^+e^- annihilation. Since at accessible energies (distances) the effective QCD coupling constant is typically not particularly small ($\alpha_s^{eff} \sim 0.2$), it is prudent (and in fact formally necessary in order to uncover the scale of the characteristic logarithmic energy dependence implied by QCD) to evaluate not only $O(\alpha_s)$ but also $O(\alpha_s^2)$ terms in the perturbation series for the total cross-section. The necessary three-loop calculations are extremely complicated, and require the use of algebraic computer programs [2]; we have still not yet completed them [3]. In addition to the total cross-section, sufficiently coarse features of the angular distributions of energy in the final states of e^+e^- annihilation events should also be insensitive to hadron formation at large distances (. involving correspondingly small transverse momenta), and reliably estimated by perturbation theory. Papers 1 and 2 below discuss sets of observables which parametrize the energy distributions or 'shapes' of e^+e^- annihilation final states, and whose mean values (and higher moments) are typically sensitive to the structure of events only at short distances. At lowest order in QCD perturbation theory, e^+e^- annihilation proceeds through $e^+e^-\to\gamma^*\to q\bar{q}$; the final $q,\bar{q}$ 'fragment' into two jets of hadrons with small transverse momenta. At $O(\alpha_s)$, one of the outgoing quarks may emit a gluon (G) yielding $\gamma^*\to q\bar{q}G$: if the gluon has sufficiently large transverse momentum, then it will initiate a third jet of hadrons. The observables described in papers 1 and 2 allow direct quantitative tests of these QCD predictions, at least at high enough e^+e^- centre-of-mass energies ($\sqrt{s}$) that the smearing associated with the formation of hadrons is not overwhelmingly important. Simulations of the fragmentation of quarks and gluons into hadrons using a phenomenological model in papers 1 and 2 indicate that (if all particles in each event are measured) $\sqrt{s}$ >20, 30 GeV is required. In addition to theoretical advantages, the observables described in papers 1 and 2 have the practical advantage over other proposed observables that they require no minimization, and are thus less susceptible to biases. The observables have found some use in the analysis of experimental data from PETRA [4]. Beyond providing quantitative tests of QCD, the observables may also be used in a more phenomenological manner: for example, to identify the roughly spherical events expected from the production and decay of heavy quarks (or leptons) near threshold from the usual two-jet events. The observables are also useful in analysing processes other than e^+e^- annihilation: in ref. [5] they are applied to deep-inelastic lepton-hadron scattering. Further details of the work summarized in papers 1 and 2 may be found in the lengthy papers of refs. [6], [7] and [8], where other observables are also introduced.

At very short distances, the structure of e^+e^- annihilation events involves the emission of small numbers of gluons, typically with large transverse momenta, and may be analysed by direct explicit perturbative calculation. At very large distances, the quarks and gluons condense into hadrons in a presently incalculable but presumably universal manner. Nevertheless, there is a large region between these two extremes in which many gluons are emitted independently and with small transverse momenta.

The methods for analysing this region are similar to those used for the analogous case of electromagnetic shower development in matter, and are based on the leading logarithm approximation (in which only leading terms in (roughly) the logarithm of the distance divided by a fixed scale are retained). The third paper in this thesis makes several applications of the leading logarithm approximation to QCD jet development. Some details of derivations and results are given in ref. [7]. The consequences of the iterative picture for jet development described in paper 3 below are naturally investigated by use of Monte Carlo methods. Ref. [9] describes the construction and application of a Monte Carlo computer program which simulates the production of quarks and gluons in QCD jets, and embodies all presently known features of QCD final states. The final condensation of quarks and gluons in a jet into hadrons (which is irrelevant for sufficiently coarse measurements, such as those provided by the observables of papers 1 and 2) must be simulated by purely phenomenological means: ref. [10] discusses several suitable models. These models provide complete predictions for hadronic final states observed in e^+e^- annihilation at all energies.

The fourth and fifth papers in this thesis are concerned with an entirely different topic. They describe constraints on quarks and leptons more massive than those yet observed. Paper 4 shows that in the standard Weinberg-Salam $SU(2)_L XU(1)$ gauge model for weak interactions (reviewed in ref. [11]), no quarks or leptons may acquire masses in excess of about 100 GeV by the usual mechanism of spontaneous symmetry breakdown (with a single Higgs doublet). The observation of more massive quarks or leptons (which should be possible at the next generation of pp and $\bar{p}p$ colliding beam facilities) would provide the first definite evidence that the minimal $SU(2)_L XU(1)$ model for weak interactions is inadequate. (Predictions for heavy quark and lepton production cross-sections are given in ref. [12].) Paper 5 addresses the possibility of absolutely stable charged or strongly-interacting particles more massive than the proton. Such particles appear in several extensions of the minimal weak interaction model. It is shown in paper 5 that according to the standard hot big bang model for the early universe (reviewed in ref. [13]), a rather large number of such particles should have been produced (corresponding to present concentrations above about 10^{-10}/nucleon). The failure of terrestrial searches (sensitive to much smaller concentrations) to detect these particles (with masses below about 300 GeV) then implies either that they do not exist (thus placing severe constraints on weak interaction models) or that the standard cosmological model is grossly wrong. Ref. [14] estimates the concentrations of any stable leptons or hadrons which should be produced by interactions of high-energy cosmic rays with the earth's atmosphere: experimental limits exclude stable heavy hadrons (with masses below 100 GeV) produced even by this mechanism.

Very few of the comparatively small number of relevant observed large scale features of the present universe are satisfactorily explained even by the standard hot big bang model for the early universe. One important unexplained feature of the present universe is the local absence of large amounts of antimatter. If this is a global phenomenon, then in the early universe, an excess $\lesssim 10^{-8}$ of nucleons over antinucleons must have existed. It is an old idea that in models where baryon number and time reversal invariance are violated by interactions at very high energies (usually leading to proton decay with a very long lifetime), the baryon excess in the early universe should be calculable. Paper 6 describes the development of a baryon asymmetry in such models: any initial baryon number is probably destroyed at extremely high temperatures; a small excess is generated by non-equilibrium processes at lower temperatures. The details of this work are described in ref. [15]: several aspects of non-equilibrium thermodynamic systems with time reversal violating interactions discussed there perhaps have wider application than cosmology.

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Observables for the Analysis of Event Shapes in e^+e^- Annihilation and Other Processes

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We present a set of rotationally invariant observables which characterizes the "shapes" of events, and is calculable in quantum-chromodynamics perturbation theory for final states consisting of quarks and gluons (G). We include the effects of fragmentation to hadrons in comparing the shapes of events from the processes $e^+e^- \to q\bar{q}$, $e^+e^- \to q\bar{q}G$, and $e^+e^- \to$ heavy resonance $\to GGG$, and from heavy-quark and lepton production. We indicate how our analysis may be extended to deep-elastic lepton-hadron interactions and hadron-hadron collisions involving large transverse momenta.

Experiments[1] have shown that at high center-of-mass energies ($\sqrt{s}$) the final states in $e^+e^- \to$ hadrons usually consist predominantly of two jets of hadrons presumably resulting from the process $e^+e^- \to q\bar{q}$. Quantum chromodynamics (QCD) explains this basic two-jet structure,[2] but predicts that one of the outgoing quarks should sometimes emit a gluon (G), tending to lead to three-jet final states.

Previous attempts[3] to discriminate between two- and three-jet events concentrated on finding a "jet axis" by minimization, and then measuring the collimation of particles with respect to it. Instead, one may use observables which directly characterize the "shape" of each event. Since there is no natural axis defined in the final state of e^+e^- annihilation, it is convenient to consider rotationally invariant observables. A set of such observables is given by [$Y_l^m(\Omega)$ are the usual spherical harmonics and $P_l(\cos\varphi)$ the Legendre polynomials]

$$H_l \equiv \left(\frac{4\pi}{2l+1}\right)\sum_{m=-l}^{+l}\left|\sum_i Y_l^m(\Omega_i)\frac{|\vec{p}_i|}{\sqrt{s}}\right|^2$$

$$=\sum_{i,j}{}' \frac{|\vec{p}_i||\vec{p}_j|}{s}P_l(\cos\varphi_{ij}), \qquad (1)$$

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where the indices i and j run over the hadrons which are produced in the event, and φ_{ij} is the angle between particles i and j. When the first for the H_l is used, one must choose a particular set of axes to evaluate the angles (Ω_i) of their momenta, but the values of the H_l deduced will be independent of the choice. Energy-momentum conservation requires $H_1 = 0$ and $H_0 = 1$. In principle, all the other H_l carry independent information.[4] In practice, however, one need only consider the lower-order H_l; in this paper we concentrate on H_2 and H_3.

The information contained in the H_l may also be expressed by the "autocorrelation function"

$$F(\cos\beta) = 2\int \rho(\hat{\Omega})\rho(\hat{\Omega}\hat{R}(\alpha, \beta, \gamma))$$
$$= \sum_l (2l+1) H_l P_l(\cos\beta), \tag{2}$$

where $\rho(\hat{\Omega})$ is a continuous distribution of momentum and $\hat{\Omega}, \hat{R}$ are operators in the rotation group. For particle events, we define the two-detector energy correlation[5]

$$\tilde{F}_2(\sigma_1, \sigma_2) = \frac{16\pi^2}{|\sigma_1||\sigma_2|}\frac{E_1 E_2}{s}, \tag{3}$$

where E_i are energies incident on detectors covering the regions σ_i of total solid angle $|\sigma_i|$. We form the rotationally invariant observable F_2 by averaging $\tilde{F}_2$ over all possible positions for the detectors, while maintaining their relative orientation. In e^+e^- annihilation events, this may be achieved (apart from correlations with the beam axis and polarization) by averaging over events. In the limit $|\sigma_i| \to 0$, F_2 becomes a function solely of the angle β between the two point detectors, and is identical to $F(\cos\beta)$. F_2 may clearly be generalized to a correlation between n detectors (F_n). However, unlike the case of the H_l, there are infrared difficulties when the F_n are calculated in QCD perturbation theory.[6]

The ability of the H_l to distinguish between different processes is illustrated in Fig. 1. Final states of the process $e^+e^- \to q\bar{q}$ have $H_l = 1$ for even l and $H_l = 0$ for odd l. In contrast, the process $e^+e^- \to q\bar{q}G$ gives events with a wide distribution of H_l values, corresponding to a range of shapes. For example, the dependence of H_2 on the fractional energies $x_i (= 2E_i/\sqrt{s})$ of the final quarks and gluons in this case is given by

$$H_2 = 1 - 6(1-x_1)(1-x_2)(1-x_3)/x_1x_2x_3. \tag{4}$$

Each kinematic configuration, labeled by the x_i, leads to an event of a different shape, and each is characterized by a particular value of H_2.

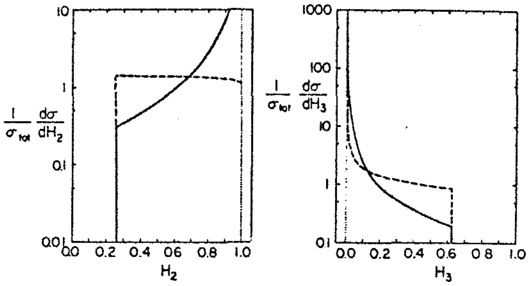

FIG. 1. The distributions in H_2 and H_3 for the processes $e^+e^- \to q\bar{q}$ (dotted lines), $e^+e^- \to q\bar{q}G$ (full lines), and $e^+e^- \to$ heavy resonance $\to GGG$ (dashed lines). The process $e^+e^- + q\bar{q}G$ alone yields an infinite total cross section, but when added to $e^+e^- \to q\bar{q}$ calculated through $O(g^2)$ the combination of processes [denoted by $e^+e^- \to q\bar{q}(G)$] gives a finite cross section. We have taken $\alpha_s = 0.25$ for the $e^+e^- \to q\bar{q}G$ distribution.

The H_l do not discriminate between final states differing by the inclusion of a vary low-energy particle or by the replacement of one particle by two collinear particles with the same total momentum. It is believed that these properties are sufficient to ensure that calculations involving the H_l are infrared finite in QCD perturbation theory.[3,6]

A convenient measure of the event shapes due to different processes is provided by the mean H_l. For the sum of the process $e^+e^- \to q\bar{q}G$ and $e^+e^- \to q\bar{q}$ calculated to lowest order in the QCD coupling constant $\alpha_s = g^2/4\pi$, we have

$$\langle H_2 \rangle = 1 + (2\alpha_s/3\pi)(33 - 4\pi^2) \simeq 1 - 1.4\alpha_s, \tag{5}$$

so that a center-of-mass energy $\sqrt{s} = 40$ GeV, $\langle H_2 \rangle \simeq 0.76$.

QCD suggests that heavy $Q\bar{Q}$ vector mesons (such as ψ, Υ) should decay to three gluons. Figure 1 shows that the H_2 and H_3 distributions due to this process are very different from those for $e^+e^- \to q\bar{q}G$. The flatter H_2 distribution for the GGG decay is reflected in a lower $\langle H_2 \rangle$:

$$\langle H_2 \rangle \simeq \frac{103\pi^2 - 1008}{16(\pi^2 - 9)} \simeq 0.62. \tag{6}$$

Our results above were obtained by making the idealization that final states consist of free quarks and gluons. In reality, one must consider the "fragmentation" of these quarks and gluons into hadrons, although at sufficiently high energy the values of the H_l should be the same whether they are calculated from Eq. (1) using the momenta of the actual hadrons in each event, or of their parent quarks and gluons. In order to estimate the shapes of realistic events at finite energy, one must go beyond the realms of present QCD theory and adopt an essentially phenomenological model for the generation of complete hadronic final states by the fragmentation of quarks and gluons. We use the model developed by Field and Feynman,[7] which agrees with available data.[8]

QCD predicts that, away from resonances, e^+e^- annihilation should be dominated by the processes $e^+e^- \to q\bar{q}$ and $e^+e^- \to q\bar{q}G$. The processes $e^+e^- \to q\bar{q}G$ can give rise to final states containing either two or three jets of hadrons. Two-jet events occur when some of the quarks and gluons have low energy or are nearly collinear[9] and they cannot be distinguished from $e^+e^- \to q\bar{q}$ events by measurements on the hadron final state. Only when $e^+e^- \to q\bar{q}G$ and $e^+e^- \to q\bar{q}$ [calculated through $O(g^2)$] are added is the jet-production cross section infrared finite. We denote this combination of processes by $e^+e^- \to q\bar{q}(G)$.

In Fig. 2, we present the H_2 distributions for realistic hadronic events resulting from $e^+e^- \to q\bar{q}$, $e^+e^- \to q\bar{q}(G)$, and $e^+e^- \to \zeta \to GGG$ (ζ is a heavy $Q\bar{Q}$ resonance). The modifications to the results in Fig. 1 due to the fragmentation of the quarks and

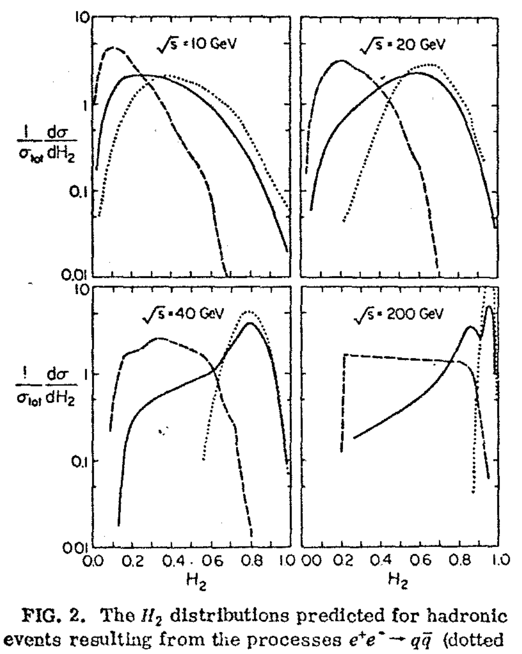

FIG. 2. The H_2 distributions predicted for hadronic events resulting from the processes $e^+e^- \to q\bar{q}$ (dotted lines), $e^+e^- \to q\bar{q}(G)$ (full lines), and $e^+e^- \to$ heavy resonance $\to GGG$ (dashed lines), at various center-of-mass energies $\sqrt{s}$.

gluons into hadrons are striking. (They also occur for the higher H_l and for other observables designed to identify three-jet events.[3]) Nevertheless, above $\sqrt{s} \simeq 10$ GeV, the H_2 distributions for the different types of events are clearly distinguished. By $\sqrt{s} \simeq 40$ GeV, the predictions are similar to those obtained in the idealization of free quarks and gluons (Fig. 1). H_2 and H_4 distributions are particularly effective at distinguishing $e^+e^- \to q\bar{q}(G)$ and $e^+e^- \to \zeta \to GGG$ events, while H_3 distributions are very sensitive to the presence of any pure $e^+e^- \to q\bar{q}$ component. The H_1 distributions for realistic events may be made more similar to the idealized ones of Fig. 1 by using only the higher-momentum hadrons in each event for the computation of the H_l.[10] Even the cut $|p_i| > 0.5$ GeV is sufficient to effect a great improvement. The H_l distributions[10] are little affected if only the charged particles in each event are detected. Our predictions are not particularly sensitive to the parameters of the jet development model (which may presumably in any case be determined from single-hadron momentum distributions), but it is still difficult to estimate the uncertainties in our results at energies where the fragmentation of the quarks and gluons has an important effect. Refinement of the jet model as further experimental data become available should allow more accurate predictions to be made.

The H_l are not specialized to the investigation of two- and three-jet events. They may also be used to identify events of other types. The pair production and weak decay of heavy mesons (containing a heavy quark Q and a light antiquark $\bar{q}$) and heavy leptons (L) should give events containing many hadron jets. For heavy leptons we assume the decay scheme $L \to \nu_L u\bar{d}$, while for heavy quarks (mesons) we consider the three possibilities $Q \to q'u\bar{d}$, $Q \to q'G$, and $Q\bar{q}_{\text{spect}} \to q'\bar{q}''$. Figure 3 shows our predictions for the H_2 distributions of heavy-quark and lepton production events. We take no account of hadron production by heavy quarks prior to their weak decays, so that our results for heavy-quark pair production should be valid only near threshold.

In addition to the H_l, one may consider the multipole moments[5]

$$B_l \equiv \sum_i (|\vec{p}_i|/\sqrt{s}) P_l(\cos\alpha_i), \tag{7}$$

where α_i are the angles made by the particles in the event with the beam axis. A $q\bar{q}$ final state with angular distribution $1 + \lambda\cos^2\alpha_i$ (the naive parton model predicts $\lambda = 1$) gives a broad distribution in B_2 with mean $2\lambda/5(\lambda+3)$, while the process $e^+e^- \to q\bar{q}(G)$ gives $\langle B_2\rangle \simeq 1/10 - 3\alpha_s/10\pi$, corresponding to $\lambda \simeq 1-4\alpha_s/\pi$, and $e^+e^- \to \zeta \to GGG$ gives $\langle B_2\rangle \simeq (72-7\pi^2)/80(\pi^2-9)$ or $\lambda \simeq (72-7\pi^2)/(13\pi^2-120) \simeq 0.35$. The H_l, being rotational invariants, are of course insensitive to correlations with the beam direction. They are, however, far superior in identifying the shape of events and distinguishing competing processes.

The H_l may also be used to analyze three-jet effects in deep-inelastic lepton-nucleon scattering. Making the idealization of free final quarks and gluons, and treating the nucleon fragments as a single particle, we find that in the virtual photon- (or W-) nucleon rest frame, two-jet events arising from $\gamma^* q \to q$ give $H_l \simeq 1$ for even l and $H_l \simeq 0$ for odd l, just as in e^+e^- annihilation. The three processes[11] $\gamma^* q \to q$, $\gamma^* q \to qG$, and $\gamma^* G \to q\bar{q}$ typically give a $\langle H_2\rangle$ which varies smoothly from $1-0.5\alpha_s$ at Bjorken x around 0.1 to $1-0.9\alpha_s$ at $x=0.8$. The distributions in the H_l are similar to those in e^+e^- annihilation. The effects of fragmentation to hadrons are governed by $s_{\gamma^* N} = Q^2(1/x-1)$.

For processes in which a natural plane (Π) is defined it is convenient to use the two-dimensional analogs of the H_l:

$$C_l \equiv \left|\sum_i \frac{|\vec{p}_i|_{\rm proj}}{\sqrt{s}} \exp il\varphi_i\right|^2, \tag{8}$$

where φ_i are the angles of the particles relative to an arbitrary axis in Π, and $|\vec{p}_i|_{\rm proj}$ are the magnitudes of their momenta projected onto Π. In deep-inelastic scattering, it is best to take the plane Π to be orthogonal to the γ^* (or W^*) direction. Then two-jet events give $C_l \simeq 0$, while three-jet ones can give nonzero values of C_{2l}.[12] Typically, in the free-quark approximation, $\langle C_{2l}\rangle$ is independent of l, and typically $\langle C_{2l}\rangle \simeq 0.06\alpha_s$ at $x=0.1$, rising to $0.15\alpha_s$ at $x=0.8$. In hadron-hadron collisions involving high transverse momenta, Π should be chosen as the plane perpendicular to the incoming hadrons. Once again, the distributions in C_l/C_0 distinguish two- and three-jet events. The obvious two-dimensional analog of F_2 [as defined in Eq. (2)] will also be useful.

A detailed discussion of the work summarized here is given in Ref. 6.

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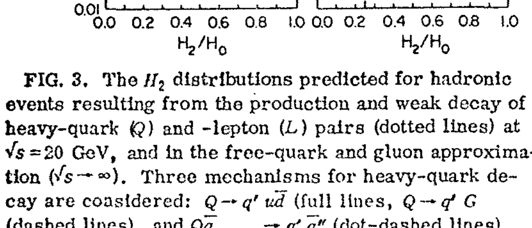

FIG. 3. The H_2 distributions predicted for hadronic events resulting from the production and weak decay of heavy-quark (Q) and -lepton (L) pairs (dotted lines) at $\sqrt{s}=20$ GeV, and in the free-quark and gluon approximation ($\sqrt{s}\to\infty$). Three mechanisms for heavy-quark decay are considered: $Q \to q' u\bar{d}$ (full lines, $Q \to q' G$ (dashed lines), and $Q\bar{q}_{\rm spect} \to q' \bar{q}''$ (dot-dashed lines). In the free-quark and gluon approximation the latter two processes give the same H_l distributions.

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[8]The model was adjusted so as to agree with observed single-hadron momentum distributions. Limited experimental tests of its predictions for the detailed structure of jets [W. G. Scott, in "Neutrinos—78," edited by Earle C. Fowler (Perdue Univ. Press, to be published)] have proved successful.

[9]The division between configurations of quarks and gluons which give two- and three-jet events is determined by the details of their fragmentation to hadrons. At present the division must be made almost arbitrarily, but our results are not sensitive to the choice (see Ref. 6).

[10]If incomplete final states are considered then only a fraction of the true energy of the event will be measured, so that it is convenient to use the effective H_l/H_0 rather than H_l for this case.

[11]All processes of $O(g^2)$, including those involving extra initial-state particles (e.g., $\gamma^*Gq \rightarrow q$), must be added in order to obtain an infrared-finite result. To $O(g^2)$, however, only the three-jet parts of $\gamma^*q \rightarrow qG$ and $\gamma^*G \rightarrow q\bar{q}$ contribute to $\langle H_{2l+1}\rangle$ and $\langle H_{2l}\rangle - 1$. $\gamma^*G \rightarrow q\bar{q}$ gives an insignificant contribution.

[12]One may also define two-dimensional analogs of the B_l. These provide an improved formulation of the tests of QCD proposed by H. Georgi and H. D. Politzer, Phys. Rev. Lett. 40, 3 (1978).

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TESTS FOR PLANAR EVENTS IN e^+e^- ANNIHILATION ☆

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We present a new class of observables which distinguish events containing two or three hadron jets from those containing a larger number. These observables, which essentially measure the coplanary of events, are calculable in QCD perturbation theory. Their use should allow the mechanism of Υ decay to be determined.

According to QCD, e^+e^- annihilation into hadrons at high center of mass energies ($\sqrt{s}$) proceeds dominantly through the process $e^+e^- \to q\bar{q}$, with some contribution from higher-order mechanisms such as $e^+e^- \to q\bar{q}G$. On vector meson resonances composed of heavy quark pairs (such as ψ and Υ, denoted generically ζ), QCD suggests that hadrons should be produced primarily through $e^+e^- \to \zeta \to GGG$, and should therefore form three jets. In this paper, we discuss tests for this mechanism, which distinguish it, for example, from those in which the hadrons are distributed isotropically rather than forming jets. In a previous paper [1], we considered the class of observables defined by (the P_l are the Legendre polynomials)

$$H_l = \sum_{i,j} \frac{|p_i||p_j|}{s} P_l(\hat{p}_i \cdot \hat{p}_j)\,, \tag{1}$$

where the sums run over all particles in an event, and the $\hat{p}_i$ are unit vectors along the momenta p_i. These observables provide a measure of the "shapes" of events in e^+e^- annihilation and allow some discrimination between isotropic and three-jet hadron production on resonance. For idealized two-jet events, $H_{2l} = 1$ and $H_{2l+1} = 0$, while for isotropic events $H_l = 0$ for $l \neq 0$. Three-jet events lead to intermediate values of the H_l. To make this more quantitative and include the effects of the fragmentation of quarks and gluons to hadrons, one must perform a detailed theoretical calculation [1]. Perhaps the most distinctive feature of three-jet events is the approximate coplanarity of the final state particles. Unfortunately, this property has no simple consequences for the H_l. However, if instead one considers observables of the form

$$\Pi_S = \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 S(\hat{p}_i, \hat{p}_j, \hat{p}_k)$$

$$\Psi_A = \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k) A(\hat{p}_i, \hat{p}_j, \hat{p}_k)\,, \tag{2}$$

where the functions S and A are respectively symmetric and antisymmetric polynomials in the scalar products of the unit vectors, then for coplanar events, the Π and Ψ vanish. These observables, therefore, provide a definitive test for coplanarity and hence should allow clean discrimination of two- and, particularly, three-jet final states from more complicated structures. The simplest example of the Π class of observables has $S = 1$ and will be denoted Π_1, while the simplest nontrivial member of the Ψ class (denoted by Ψ_1) has

$$\begin{aligned} A = [&(\hat{p}_i \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_k) \\ &+ (\hat{p}_k \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_i) - (\hat{p}_i \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_k) \\ &- (\hat{p}_k \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_j) - (\hat{p}_j \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_i)]\,. \end{aligned}$$

Note that while the Π are scalars, the Ψ are pseudoscalars, so that when averaged over events, $\langle \Psi \rangle = 0$. Of course, $\langle \Psi^2 \rangle$, for example, need not vanish.

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In [1] we argued that the moments of the H_l should be infrared stable when computed in QCD perturbation theory. This result should also hold for the Π and Ψ. In general, divergences in the mean values of observables are canceled if the observables take on the same value for all physically indistinguishable processes. One requirement is, therefore, that the addition of very soft particles should not affect the value of the observable. This is guaranteed for the Π and Ψ by the presence of a term proportional to the total momenta of the particles. The other condition for infrared stability is that the observables should be linear in the momenta of collinear particles. This is clearly satisfied by the Π and Ψ.

We showed in [1] that the H_l correspond to moments of two-detector energy correlation functions which are formed from the product of the energies incident on each of two detectors [2]. The Π and Ψ may be related to momenta of the analogous three-detector energy correlations ‡1. We sketch this relation below.

Let us define the multipole moments of an event by (the Y_l^m are the usual spherical harmonics)

$$A_l^m = \sum_i \frac{|p_i|}{\sqrt{s}} Y_l^m(\Omega_i) , \tag{3}$$

where the angles Ω_i are measured with respect to a set of axes chosen in the event. The H_l defined in eq. (1) may then be written as

$$H_l = \left(\frac{4\pi}{2l+1}\right) \sum_{m=-l}^{+l} |A_l^m|^2 , \tag{4}$$

which is clearly a rotational invariant and hence independent of the choice of axes used to measure the angles Ω_i. The three-detector energy correlation function may be decomposed in terms of natural generalizations of the H_l, given by

$$T_{l_1 l_2 l_3} = (4\pi)^{3/2} \sum_{m_1 m_2, m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \times A_{l_1}^{m_1} A_{l_2}^{m_2} A_{l_3}^{m_3} \tag{5}$$

where the 3-j symbol serves to combine the three spherical tensors into a rotational invariant ‡2. The H_l represent a special case of these observables:

$$T_{l_1 l_2 0} = (-1)^{l_1} \sqrt{2l_1+1}\, \sigma_{l_1 l_2} H_{l_1} . \tag{6}$$

For planar events, the three-detector energy correlation function clearly vanishes unless the three detectors lie in a plane. As we describe in detail elsewhere [2], this property of the three-detector energy correlation may be translated into the vanishing of certain linear combinations of the $T_{l_1 l_2 l_3}$ for planar events. These combinations fall into two classes corresponding to the Π and Ψ observables. Those involving only $T_{l_1 l_2 l_3}$ with $l_1 + l_2 + l_3$ even correspond to the Π and, for example

$$\Pi_1 = \frac{-2}{45} \{\sqrt{14}\, T_{222} + 3\sqrt{5}\, T_{220} - 5 T_{000}\} . \tag{7}$$

If $l_1 + l_2 + l_3$ is even, then $T_{l_1 l_2 l_3}$ is real, but if it is odd, then the $T_{l_1 l_2 l_3}$ are purely imaginary. However, for planar events, all the $T_{l_1 l_2 l_3}$ must be real ‡3 so that all $T_{l_1 l_2 l_3}$ with odd $l_1 + l_2 + l_3$ must vanish in that case. The Ψ may be written in terms of these $T_{l_1 l_2 l_3}$ and, for example,

$$\Psi_1 = \tfrac{8}{35} \operatorname{Im}[T_{234}] . \tag{8}$$

The formulae for the simpler Π and Ψ are given in table 1. Note that momentum conservation implies that $T_{l_1 l_2 l_3}$ vanishes if any of its indices $l_i = 1$. We have nevertheless retained such $T_{l_1 l_2 l_3}$ in table 1 so that our results may be applied to incomplete final stages where momentum is not conserved among the particles used to calculate the Π and Ψ.

In the approximation of free final quarks and gluons, events of the types $e^+e^- \to q\bar{q}(G)$ and $e^+e^- \to \zeta \to GGG$ will give zero for all the Π and Ψ. For an exactly isotropic event, however, all the $T_{l_1 l_2 l_3}$ vanish except for $T_{000} = 1$. In this case, therefore, $\Pi_1 = \frac{2}{9}$, $\Pi_2 = 0$, $\Pi_3 = 0$, $\Pi_4 = \frac{2}{15}$ and all $\Psi = 0$.

In order to simulate real hadronic events, we use the phenomenological model for quark and gluon frag-

‡1 Observables involving products of four or more momenta arising from energy correlations between four or more detectors do not appear to have any immediate application [5].

‡2 Note that the $T_{l_1 l_2 l_3}$ vanish for l_3 outside the range $|l_1 - l_2|$ to $|l_1 + l_2|$ (triangle inequality) or if the sum $l_1 + l_2 + l_3$ is odd and two of the l_i are equal (symmetry property of the 3-j symbols).

‡3 If the plane formed by the x and z axes is chosen to be in the plane of the event, then from (3) all the A_l^m are real so that the $T_{l_1 l_2 l_3}$ deduced from (5) will also be real.

Table 1
Examples of observables which vanish for coplanar events.

$$\Pi_1 \equiv \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 = -\tfrac{2}{45}[\sqrt{14}\, T_{222} + 3\sqrt{5}\, T_{220} - 5T_{000}]$$

$$\Pi_2 \equiv \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 [(\hat{p}_i \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_k) + (\hat{p}_k \cdot \hat{p}_i)]$$
$$= \tfrac{2}{525}[12\sqrt{21}\, T_{332} + 15\sqrt{7}\, T_{330} + 42\, T_{321} - 7\sqrt{6}\, T_{211} - 35\sqrt{3}\, T_{110}]$$

$$\Pi_3 \equiv \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 [(\hat{p}_i \cdot \hat{p}_j)(\hat{p}_j \cdot \hat{p}_k) + (\hat{p}_k \cdot \hat{p}_i)(\hat{p}_i \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_k)(\hat{p}_k \cdot \hat{p}_i)]$$
$$= \frac{-2}{3675}[12\sqrt{154}\, T_{433} + 84\sqrt{3}\, T_{431} - 6\sqrt{21}\, T_{332} - 21\sqrt{7}\, T_{330} - 126\, T_{321} - 49\sqrt{6}\, T_{211} + 49\sqrt{3}\, T_{110}]$$

$$\Pi_4 \equiv \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)^2 [(\hat{p}_i \cdot \hat{p}_j)^2 + (\hat{p}_j \cdot \hat{p}_k)^2 + (\hat{p}_k \cdot \hat{p}_i)^2]$$
$$= \frac{-2}{3675}[20\sqrt{77}\, T_{442} + 140\, T_{440} + 12\sqrt{70}\, T_{422} + 25\sqrt{14}\, T_{222} + 63\sqrt{5}\, T_{220} - 245\, T_{000}]$$

$$\Psi_1 \equiv \sum_{i,j,k} \frac{|p_i||p_j||p_k|}{(\sqrt{s})^3} (\hat{p}_i \times \hat{p}_j \cdot \hat{p}_k)[(\hat{p}_i \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_j) + (\hat{p}_j \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_k) + (\hat{p}_k \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_i)$$
$$- (\hat{p}_i \cdot \hat{p}_j)^2 (\hat{p}_j \cdot \hat{p}_k) - (\hat{p}_k \cdot \hat{p}_i)^2 (\hat{p}_i \cdot \hat{p}_j) - (\hat{p}_j \cdot \hat{p}_k)^2 (\hat{p}_k \cdot \hat{p}_i)] = \tfrac{8}{35}\,\mathrm{Im}[T_{234}]$$

mentation into hadrons developed by Field and Feynman [3]. To investigate the discrimination between planar and non-planar events provided by our observables, we shall compare events due to $e^+e^- \to \zeta \to GGG$ with ones which give the same single hadron momentum ($z = 2|p|/\sqrt{s}$) distribution but which arise from non-coplanar configurations of quarks and gluons. We chose two models for non-coplanar events. In the first (referred to as '6-jet'), we consider the production and decay of a pair of heavy quarks into three particles. This model was introduced in [1]. Although it gives rise to events which are non-planar and contain six hadron jets, it happens that with our quark and gluon fragmentation functions, they have roughly the same z distributions as $e^+e^- \to \zeta \to GGG$ events. For our second model (referred to as 'isotropic'), we generated $e^+e^- \to \zeta \to GGG$ events and then rotated the momentum of each of the particles randomly. This procedure gives roughly isotropic events but at the cost of some violation of momentum conservation.

In fig. 1, we show the distributions of simulated hadronic events in Π_1 at three center of mass energies while fig. 2 gives their distributions in H_2 [‡4]. In both cases, the free quark and gluon predictions are considerably modified by fragmentation to hadrons. This effect is particularly marked for the Π_1 distributions. Nevertheless, even at $\sqrt{s} = 10$ GeV (corresponding to the Υ region), the distributions allow clear discrimination between different mechanisms. Of course, at higher $\sqrt{s}$, the effects of fragmentation become less important, and the various processes are yet more

‡4 $e^+e^- \to q\bar{q}(G)$ denotes the sum of the processes $e^+e^- \to q\bar{q}G$ and $e^+e^- \to q\bar{q}$, calculated through $O(g^2)$. According to QCD, $e^+e^- \to q\bar{q}(G)$ should be the dominant process away from resonances. Details are given in [1].

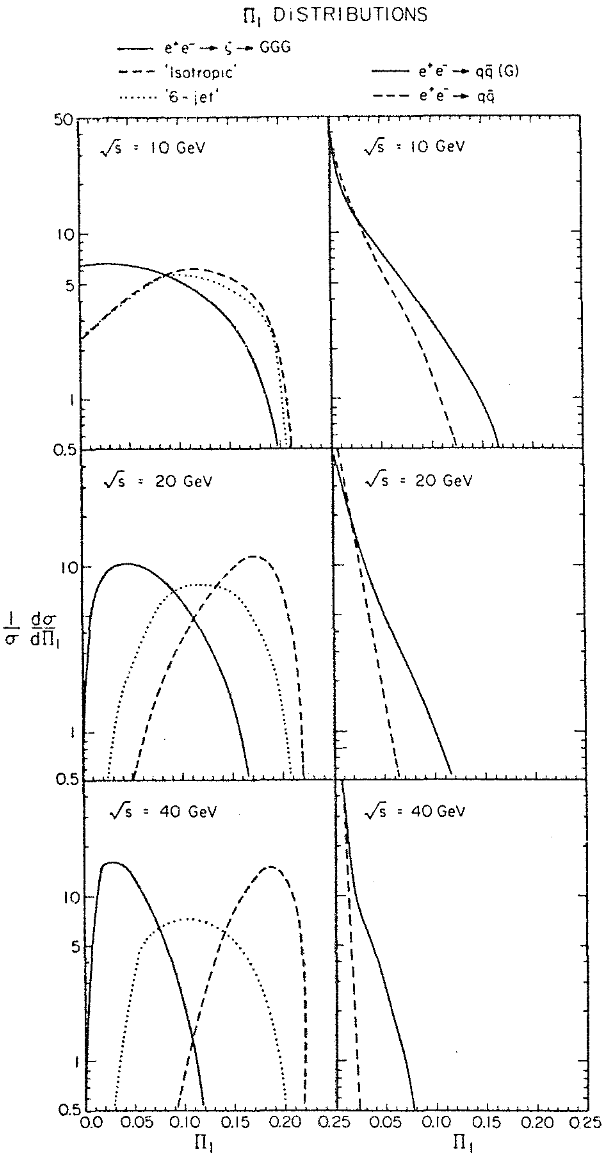

Fig. 1. The distributions $1/\sigma\ d\sigma/\Pi_1$ of simulated hadronic events in the coplanarity parameter Π_1 for various center of mass energies ($\sqrt{s}$). $e^+e^- \to \zeta \to GGG$, "isotropic" and "6-jet" are three illustrative mechanisms for heavy resonance (ζ) decay. According to QCD, $e^+e^- \to q\bar{q}(G)$ should be the dominant process of resonance [1]. In the free quark and gluon approximation, the processes $e^+e^- \to \zeta \to GGG$, $e^+e^- \to q\bar{q}$ and $e^+e^- \to q\bar{q}(G)$ should lead to $\Pi_1 = 0$. In the same approximation, the '6-jet' process leads to a roughly flat distribution in Π_1 over its kinematically allowed range ($0 \leqslant \Pi_1 \leqslant 2/9$). Completely isotropic events have $\Pi_1 = 2/9$. Note that in this and fig. 2, all curves are calculated by considering only hadrons with momenta above 0.5 GeV.

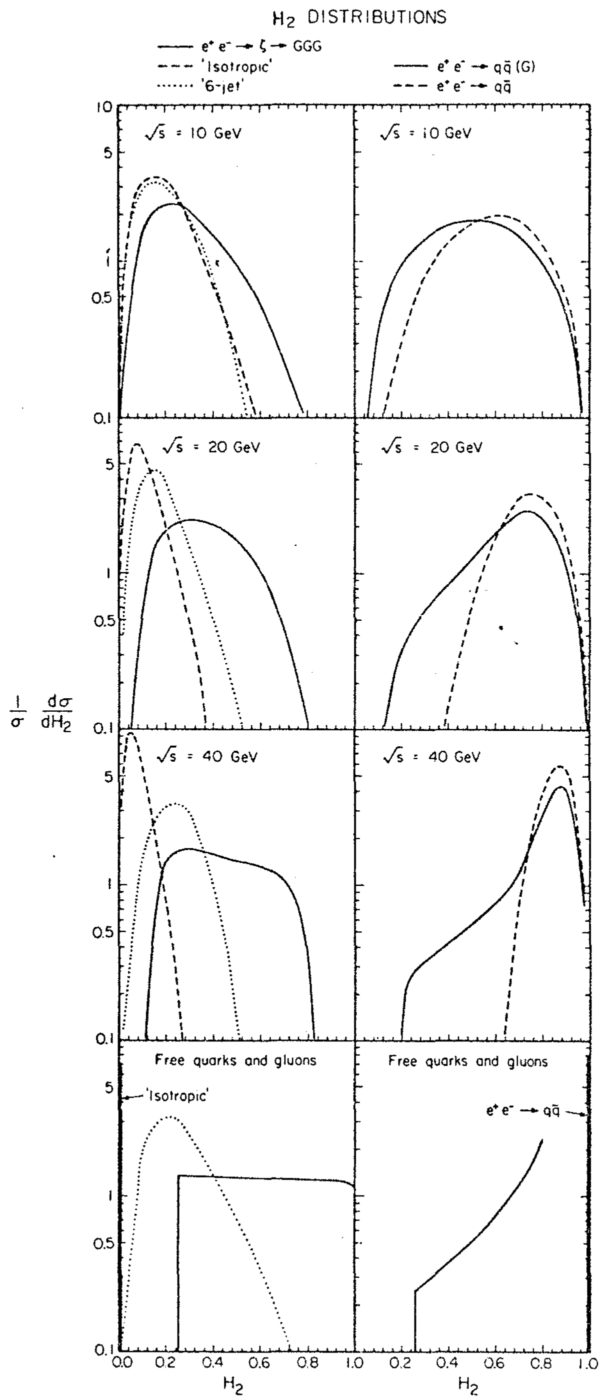

Fig. 2. The distributions $1/\sigma\ d\sigma/dH_2$ of simulated hadronic events in the shape parameter H_2, for the various center of mass energies ($\sqrt{s}$). The corresponding distributions in the free quark and gluon approximation are also given.

clearly separated. Note that the distributions in Π_1 are particularly suitable for distinguishing planar from non-planar processes and, for example, allow separation of $e^+e^- \to \zeta \to GGG$ events from isotropic or 6-jet ones. At $\sqrt{s} = 10$ GeV, isotropic and 6-jet events give indistinguishable Π_1 and H_2 distributions, but at higher $\sqrt{s}$ they differ. Figs. 1 and 2 show that it should be possible to determine whether Υ decay proceeds dominantly through $\Upsilon \to GGG$ by measuring the Π_1 and H_2 distributions of Υ production events. It should be pointed out, however, that if the decays are found to be more isotropic than would be expected for $\Upsilon \to GGG$, this does not represent a contradiction with present QCD theory since there is thus far no overwhelming evidence that low-order processes should dominate in Υ decay. Note that the results shown in figs. 1 and 2 depend on the quark and gluon fragmentation functions assumed. Our choices for these may be tested by measuring single hadron momentum distributions and if a significant difference were found, the calculations of the shape parameter distributions should be revised. In our discussion of ζ decays, we have always considered models which give the same z distributions. Thus the discrimination between different mechanisms illustrated in figs. 1 and 2 should not be affected by changes in the z distributions.

We find that the distribution of realistic hadronic events in the observables Ψ_1, Π_2 and Π_3 defined in table 1 does not differ significantly between the processes we consider. The distributions in Π_4 are qualitatively similar to those in Π_1 but distinguish slightly less between the various processes, and so we find that it is sufficient to measure Π_1 to test the coplanarity of events.

Our observables can also be used to analyse final states in which not all the particles are detected. For example, at $\sqrt{s} = 10$ GeV, the difference in $1/\sigma\, d\sigma/d\Pi_1$ between $e^+e^- \to \zeta \to GGG$ and isotropic events at $\Pi_1 = 0$ changes from the factor of about 3 shown in fig. 1 when all particles are measured to a factor of about 2 when only charged particles are detected.

Our previous work [1] showed that the H_l (and, in particular, H_2 and H_3) provide clear measures of the shapes of events. They are especially suited to discriminating two-jet events from events containing larger numbers of jets. Here we have introduced the observable Π_1 which tests for planar events and is, therefore, particularly suited to distinguishing two- or three-jet events from events with a more complicated structure.

We are grateful to R.D. Field and R.P. Feynman for the use of their jet development computer program and to the MATHLAB group of the MIT Laboratory for Computer Science for the use of MACSYMA.

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DoE RESEARCH AND
DEVELOPMENT REPORT

Jet Development in Leading Log QCD*

by

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ABSTRACT

A simple picture of jet development in QCD is described. Various applications are treated, including transverse spreading of jets, hadroproduced γ^* p_T distributions, lepton energy spectra from heavy quark decays, soft parton multiplicities and hadron cluster formation.

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According to QCD, high-energy e^+e^- annihilation into hadrons is initiated by the production from the decaying virtual photon of a quark and an antiquark, each with invariant masses up to the c.m. energy $\sqrt{s}$ in the original e^+e^- collision. The q and $\bar{q}$ then travel outwards radiating gluons which serve to spread their energy and color into a jet of finite angle. After a time $\gtrsim 1/\sqrt{s}$, the rate of gluon emissions presumably decreases roughly inversely with time, except for the logarithmic rise associated with the effective coupling constant, ($\alpha_s(t) \sim 1/\log(t/\Lambda^2)$, where $\sqrt{t}$ is the invariant mass of the radiating quark). Finally, when emissions have degraded the energies of the partons produced until their invariant masses fall below some critical $\sqrt{t_c}$ (probably a few times Λ), the system of quarks and gluons begins to condense into the observed hadrons.

The probability for a gluon to be emitted at times of $O(\frac{1}{\sqrt{s}})$ is small and may be estimated from the leading terms of a perturbation series in $\alpha_s(s)$. Any gluon produced at these early times will typically be at a large angle to the q, $\bar{q}$ directions (so that the jet it initiates is resolved) and will have an energy $\sim \sqrt{s}$: thus the wavelength of a gluon 'emitted from q' encompasses $\bar{q}$, so that interferences between the various amplitudes for gluon emissions are important. At times $\gg 1/\sqrt{s}$, the average total number of emitted gluons grows rapidly (see eq. 9) with time, and one must sum the effects of many gluons radiated at progressively smaller angles, but with energies $\sim \sqrt{s}$. Usually the wavelength of one radiated gluon does not reach the point at which the last was emitted, and hence at these times the sequence of gluon emissions in a jet may be treated independently from each other and from those in other jets. Below I shall mostly discuss the development of jets in this semiclassical regime, where the leading log approximation (LLA) may be used: some details of the results are contained in Refs. [1] and [2]. The ultimate transformation of the quarks and gluons in each jet into hadrons (which undoubtedly involves consideration of amplitudes, rather than probabilities) is quite beyond any perturbative methods, but, at least locally, depends only on the energy and quantum numbers of a jet, and not on the details of the process by which the jet was produced (except perhaps because of low-energy remnants from initial hadrons or nuclei). (The formation of a jet from an off-shell quark in many respects parallels the development of an electromagnetic shower from a high-energy electron in matter, for which the probabilistic LLA is accurate above a fixed critical energy below which ionization losses dominate.)

The times and distances quoted here are in the rest frame of the radiating quark. In the c.m. frame, they are dilated by $\gamma = E/E_0 \sim \sqrt{s/t}$. A parton off-shell an amount $\sqrt{t}$ should typically survive a time $\tau \sim 1/\sqrt{t}$ (this is clear on dimensional grounds or from the energy denominators $\Delta E \sim 1/\tau \sim E - |\vec{p}|$ in non-covariant perturbation theory). A system of partons apparently forms hadrons when the parton invariant masses $\sqrt{t} \sim \sqrt{t_c} \sim \Lambda$, corresponding to a distance $\sim \sqrt{s}/\Lambda^2$ in the c.m.s. (at this distance a string with $\kappa \sim \Lambda^2$ stretched between the q, $\bar{q}$ would have dissipated their original kinetic energy). Note that if confinement acted at a fixed time $\sim 1/\Lambda$ in the c.m.s., then $t_c \sim \Lambda\sqrt{s}$, and no scaling violations should occur in fragmentation functions (since $\log(s/\Lambda^2)/\log(t_c/\Lambda^2)$ is independent of s). (Such a mass would result from rescattering of a parton with $E \sim \sqrt{s}$ from a cloud of low energy partons with momenta $\sim \Lambda$: in e^+e^- annihilation, such a cloud forms only at $t_{cms} \gtrsim \sqrt{s}/\Lambda^2$, but in hadron reactions such spectators may seriously affect the structure of the final state.) The time of hadron formation may be investigated directly in collisions with nuclei: if $t_c \sim \Lambda^2$ then partons produced within a nucleus should form hadrons only far outside it, in a manner uninfluenced by its presence [F.1].

One approach in studying QCD jet development is to consider quantities which are insensitive to all but the short time region described by low-order perturbation theory. The simplest such observable is the total cross-section for e^+e^- annihilation to hadrons. QCD corrections modify the wavefunctions for the q, $\bar{q}$ even at the moment of production, and thereby correct the Born term. Attractive one-gluon exchange at short distances enhances the cross-section by a factor $1 + \alpha_s(s)/\pi$ [F.2], while the effects of the eventual confinement of the quarks (at short distances similar to the acquisition of an effective mass) are suppressed by an energy denominator to be $O(\Lambda^2/s)$. (Close to heavy $Q\bar{Q}$ production thresholds, the $Q,\bar{Q}$ have long wavelengths ($\sim 1/(m_Q v)$), and their wavefunctions are therefore sensitive to interactions at large times: such threshold regions must simply be smeared over.) In processes involving initial hadrons (e.g., $\gamma^*N \to X$), only scatterings which deflect initial partons outside the cylinders (of fixed transverse dimension $1/\sqrt{t_c^i} \sim 1/\Lambda$) formed by the incoming hadrons contribute to observable cross-sections. Just before a scattering involving momentum transfer Q, gluons will typically be emitted with differential cross-section $\sim dk_T/k_T$ up to $k_T \simeq Q$. The probability for gluon emission (which affects the cross-section by 'spreading' the initial parton) outside the initial cylinder $\sim \log(Q^2/t_c^i)$; because the size $1/\sqrt{t_c^i}$ of the initial hadron is fixed with Q^2, such terms give rise to 'scaling violations' which cause the cross-section to depend on Q^2/t_c^i. For a given initial hadron, the terms divergent as its size is taken to infinity are known to be universal and independent of the details of the parton scattering [4]; they are determined by processes which act at large times before the interaction.

One may obtain further information on the short distance structure of QCD processes from the angular distributions of hadronic energy in their final states. (A convenient set of shape parameters for this purpose is the $H_\ell = \sum_{i,j} E_i E_j/s\ P_\ell(\cos\phi_{ij})$ [5].) If in studying final states, hadrons with low energies are ignored and sets of hadrons separated by angles less than, say θ, are lumped together into 'jets', then the lumped energy distributions are typically sensitive to the structure of events only at times $\lesssim 1/(\theta\sqrt{s})$, since particles radiated later will usually not be 'resolved'. (In the $\langle H_\ell \rangle$, the behavior of the Legendre polynomials implies $\theta \sim 1/\ell$.) Nevertheless, it turns out that the residual effects of confinement at large distances are more important for shape parameters than for total cross-sections: they suffer $O(\Lambda/\sqrt{s})$ rather than $O(\Lambda^2/s)$ corrections [F.3]. As θ is decreased, measures of final state energy distributions become progressively more sensitive to nearly collinear emissions occurring with high probability, typically at times $\sim 1/(\theta\sqrt{s})$.

In diagrammatic calculations, the approximate independence of small transverse momentum gluon emissions from the q and $\bar{q}$ produced in e^+e^- annihilation (or the incoming and outgoing q in $\gamma^* q \to X$, etc.) is best revealed by using axial gauges $n.A = 0$ for the gluon propagator. In these gauges, interference terms are suppressed, and a probabilistic interpretation of single (ladder) diagrams is possible. The choice of n determines what fraction of the radiation appears to come from each of the quarks: if n is chosen symmetrically with respect to their momenta then they appear to radiate equally; if n is along one quark direction, then the gluons appear to come from the other quark, although some travel backwards with respect to its momentum. In a suitable gauge, the differential cross-section for emissions of k low transverse momentum gluons from an incoming or outgoing quark may be written in the simple product form [6,7]

$$\frac{d\sigma}{dz_1\ldots dz_k\, dt_1\ldots dt_k} \simeq [\frac{P_{qq}(z_1)}{2\pi}\frac{\alpha_s(t_1)}{\hat{t}_1}]\ldots$$

$$\ldots[\frac{P_{qq}(z_k)}{2\pi}\frac{\alpha_s(t_k)}{\hat{t}_k}], \quad (1)$$

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)_+$$

$$\int_0^1 (h(z))_+ f(z)dz = \int_0^1 h(z)(f(z)-f(1))dz,$$

where z_i is the relative longitudinal Sudakov variable (roughly energy fraction) of the $(i+1)^{th}$ quark with respect to the i^{th} quark ($z_i = (p^o_{i+1}+p^3_{i+1})/(p^o_i+p^3_i)$; 3 along $\vec{p}_i$) and t_i is the invariant mass of the i^{th} quark link ($\hat{t}_i = t_i/s$). The terms dropped from the leading log approximation (1) contain extra $\hat{t}_i$ factors; these may only be neglected if $\hat{t}_i << 1$ (although $t_i >> \Lambda^2$ for confinement effects to be ignored). Kinematics require that $t_i \leq t_{i-1}$, $0 \leq z_i \leq 1$. Many consequences of (1) follow simply from integrating over more restricted phase space volumes so as to select only jets obeying various criteria. In addition to radiation of real gluons, (1) includes virtual gluon corrections to quark lines or to vertices which contribute leading log terms at the points $z_i = 1$, $t_i = t_{i-1}$. If the external kinematic constraints imposed allow such diagrams to contribute (so that z_i integrals run right up to 1), then the $\int dz/(1-z)$ [F.4] divergences from the soft gluon emissions are canceled by the virtual diagrams. (The remaining infrared divergences, apparent at small t, arise from emission of hard gluons collinear to a massless quark and are cut off by the finite propagation time of the quark, implemented in perturbation theory by exchanges with other jets or by the effects of the cylinders of initial partons representing hadrons.) The contribution of a virtual correction to a quark link of mass $\sqrt{t}$ may be written (by introducing Sudakov variables into the internal loop integration) roughly as $\delta(1-z) \int_0^1 (1+z'^2)/(1-z')dz' \int_{t_{min}}^{t} dt'/t'$, where the internal t' integration is cut off by the same large distance effects as are the external t integrations, so that $t_{min} \simeq t_c$. For most applications, the virtual diagrams may then be included as in (1) simply by adding a divergent $-\delta(1-z)$ term to $P_{qq}(z)$ (hence the +); then the $\log(t/t_c)$ from internal loop integration will be reproduced by integration over the external t. This procedure will be sufficient so long as t_i is allowed to run up to t_{i-1} whenever z_i runs up to 1 (so that virtual diagrams contribute). (This will certainly be the case if the k_T, but not angles of emitted gluons are considered.)

The formula (1) accounts only for gluon emissions from the original quark: to describe radiation from the gluons produced, one must append similar product forms, with appropriate P_{qq} replaced by P_{GG}, P_{qG} or P_{Gq} [F.5] according to the type of emission. In many calculations, one is concerned with the behavior of only one or two partons, and in this case, one need essentially consider only the possible 'backbones' of the jet, which connect the initial parton to the partons considered (provide their structural support in the tree); further emissions from partons not in the backbone may be disregarded, since integrating their contributions to the cross-section over available phase space simply gives a factor one. To describe the production of the partons considered, one must sum over all possible backbones and integrate over the ordered t_i of the partons along them. The differential cross-section for a given backbone consisting of k partons i_1, i_2, i_3.... involves the product $P_{i_1 i_2}(z_1)P_{i_2 i_3}(z_2)$.... . When the required integrals of this are summed over k, they often form an exponential series, in which the exponent contains a matrix of (the z^n moments of) the $P_{ij}(z)$; ordered expansion of the matrix exponential accounts for all possible backbones with the correct combinatorial weights.

As a first application of eq. (1), I estimate the mean product of energies incident on two back-to-back detectors of angular size θ around an e^+e^- event. For the lowest-order process, $e^+e^- \rightarrow q\bar{q}$, the energy correlation $\$_B(\theta) \equiv 2\langle E_i E_{i'}\rangle/s$ (E_i is the energy incident per unit area on detector i; i' is antipodal to i) is 1 (for $\theta \neq 0$): if q enters one detector, $\bar{q}$ must be incident on the other. (For large ℓ, $\langle H_\ell\rangle \simeq [\langle E_i^2\rangle + (-1)^\ell \langle E_i E_{i'}\rangle]/(2s)$, where $\theta \simeq 1/\ell$; in dominantly two-jet processes $\langle E_i^2\rangle \ll \langle E_i E_{i'}\rangle$.) $\$_B(\theta)$ deviates from one when gluon emissions deflect energy outside angle $\simeq \theta$ cones around the $q,\bar{q}$ directions. To LLA, the energies of radiated gluons are negligible; their only effect is to deflect the original $q,\bar{q}$: $\$_B(\theta)$ thus becomes simply the total probability that the final $q,\bar{q}$ should have transverse momenta $k_T \leqslant \theta\sqrt{s}$. The i^{th} gluon emission imparts a $(k_T^2)_i = (1-z_i)(z_i t_i - t_{i+1}) \simeq (1-z_i)t_i$ to the quark. $\$_B(\theta)$ is the integral of the differential cross-section (1) (summed over all possible numbers of emissions) subject to the constraint $\Sigma(k_T^2)_i \leqslant \theta^2 s$; all radiated gluons must therefore be both soft ($(1-z_i)$ small) and nearly collinear to the quarks (t_i small). The necessary integrals are most conveniently calculated by subtracting from one those obtained by integrating outside the constraints $(1-z_i)t_i \leqslant \theta^2 s$. (In this way, one need only consider real emissions and is not concerned with delicate cancellations from virtual processes.) Consider first the emission of one gluon. To satisfy $k_T < \theta\sqrt{s}$, z_1 must be integrated from $\simeq 0$ only up to $\sim 1 - \theta^2 s/t_1$, rather than 1. The $1/(1-z_1)$ soft divergence in $P_{qq}(z_1)$ thus contributes a term $\sim \log(t_1/\theta^2 s)$ [F.6]. Integrating over t_1 from $\sim \theta^2 s$ to $\simeq s$ gives the final $O(\alpha_s)$ result $\$_B(\theta) \simeq 1 - \frac{8\alpha_s}{3\pi}\log^2\theta$. Notice that the variation of $\alpha_s(t_1 \sim 1/\log(t_1/\Lambda^2)$ over the range of the t_1 integration must be ignored to leading log accuracy compared to the $\log(t_1)$ result from the z_1 integral: its effects are formally of the same order as other subleading log corrections, which change the scale of the θ in the final result [F.7]. While the leading log terms are independent of the process by which the $q,\bar{q}$ were produced, the subleading logs are not universal. In the leading log approximation, the gluon emissions are all independent, except for the phase space restriction $t_i \le t_{i-1}$. Hence the contribution to $\not{s}_B(\theta)$ from k gluon emission $\simeq (-2\alpha_s/3\pi)^k \log^{2k}(\theta^2)/k!$: the crucial $1/k!$ arises from the nesting of the t_i integrations. Summing over k then gives [F.8]

$$\not{s}_B(\theta) \simeq \exp[-\frac{8\alpha_s}{3\pi}\log^2\theta]. \tag{2}$$

In contrast to the $O(\alpha_s)$ result, this form vanishes as $\theta \to 0$, reflecting the fact that the $q,\bar{q}$ will always be at least slightly deflected by radiation. However, at the small θ ($\lesssim \exp(-1/\alpha_s)$) where the leading log eq. (2) is damped, thus far uncalculated subleading log effects will probably dominate: when $\theta^2 \lesssim \Lambda^2/s$ (i.e., $\ell \gtrsim \sqrt{s}/\Lambda$ for $\langle H_\ell \rangle$), (2) must fail, since then the emissions no longer occur before hadronization. (Phenomenological simulations of hadron formation suggest that, in practice, perturbative results become inaccurate at much larger angles.) Note that if the variation of $\alpha_s(t)$ had been retained in the t_i integrals, (2) would become ($\alpha_s(t) = \beta_o/\log(t/\Lambda^2)$)

$$\begin{aligned}\not{s}_B(\theta) &\simeq \exp[-\frac{4\beta_o}{3\pi}(\log(1+\frac{\log(\theta^2)}{\log(s/\Lambda^2)})\log(\frac{\theta^2 s}{\Lambda^2}) - \log(\theta^2))] \\ &\simeq \exp[-\frac{8\alpha_s}{3\pi}\log^2\theta(1-\frac{\log\theta}{\log(s/\Lambda^2)}+\ldots)];\end{aligned} \tag{3}$$

the change cannot consistently be kept in the LLA.

Equation (2) gives approximately the probability that a produced or struck quark emits no gluons with total $k_T \geqslant \theta\sqrt{s}$ and, therefore, typically propagates without radiation for a time $\geqslant 1/\theta\sqrt{s}$. It is thus similar to the quark (Sudakov) form factor, which gives roughly the probability that no gluons are emitted before a time $\sim 1/p^2$, where $\sqrt{p^2}$ is a (regularizing) invariant mass assigned to final quarks and/or gluons. (In the Sudakov form factor, θ^2 is roughly replaced by $(p^2/s)^\lambda$; where λ depends on the precise method of regularization used [F.9].)

The results obtained above may be applied directly to estimate the transverse momentum (p_T) spectrum of virtual photons (γ^*) produced in hadron collisions. The leading log terms come from the process in which a q and a $\bar{q}$ from the initial hadrons suffer transverse deflections by the emission of gluons (but retain roughly their original energy) before annihilating to the γ^*. Then the p_T spectrum is obtained from the (derivative of the) deflection probability (2) as [9]

$$\frac{1}{\sigma_o}\frac{d\sigma}{dp_T^2} \simeq -\frac{4\alpha_s}{3\pi p_T^2}\log\left(\frac{p_T^2}{s}\right)\exp\left[-\frac{2\alpha_s}{3\pi}\log^2\left(\frac{p_T^2}{s}\right)\right] \qquad (4)$$

where σ_o is the cross-section without gluon emissions, and $\sqrt{s}$ is roughly the invariant mass ($\sqrt{Q^2}$) of the γ^* (which is formally indistinguishable from the incoming $q\bar{q}$ c.m. energy $\sqrt{s}$ in the LLA). However, as with eq. (2), this result is rarely adequate. At large p_T ($\sim \sqrt{s}$), the exact $O(\alpha_s)$ p_T spectrum (including subleading log terms not accounted for in (4)) should be sufficient; at small p_T higher-order terms could potentially be significant, but the leading logs of (4) are damped at small p_T and so may be overwhelmed by subleading log corrections (for $p_T^2 \leqslant \sqrt{s\Lambda^2}$). (Subleading logs from hard, but collinear (small t), emissions may be accounted for by keeping the full P(z) in the derivation of (4), rather than approximating $P_{qq}(z) \sim \log(1-z_{max})\delta(1-z)$; this yields a more complicated form in the exponent of (4), which is a convolution sampling $Q^2/S < 1$, and thus not simply a multiplicative correction to σ_o. Subleading logs from soft (but non-collinear) emissions plausibly exponentiate as in massive QED.) In practice, p_T must be measured with respect to the incoming hadrons rather than the $q,\bar{q}$, introducing a further spread in p_T^2 of order $t_c^i \sim \Lambda^2$.

A significant fraction of hadroproduced $^3S_1$ $Q\bar{Q}$ states (e.g., Υ; denoted here generically by ζ) probably arises from decays $\chi \to \zeta\gamma$ of even-spin χ produced by GG 'annihilation'. The resulting ζ p_T spectrum is given in LLA by replacing $4/3(= C_F)$ in eq. (4) with $3(= C_A)$ and is, therefore, broader than for γ^*, at least for $\Lambda^2 << p_T^2 << s$, m_ζ^2.

For deep-inelastic scattering, similar analysis shows that in the LLA, the distribution of final transverse momenta with respect to the γ^* direction (i.e., $\sum_i |p_T^i| = C_o$) should follow roughly the form (4) (in this approximation, only the q energy is significant). It is interesting to speculate on the differences between the p_T spectra in deep-inelastic scattering and the Drell-Yan process. While $p_T^2 > 0$ always, $s > 0$ for Drell-Yan but $s < 0$ for deep-inelastic scattering. Thus one might expect a subleading log difference between the integrated spectra by a large factor, perhaps $\sim \exp(2\pi\alpha_s/3)$.

In muon decay, the outgoing electron spectrum close to the endpoint $x = 2E_e/m_\mu \simeq 1 - O(m_e^2/m_\mu^2)$ is softened by emission of many low k_T photons. In the LLA, and taking $m_e = 0$, the methods used to derive (2) give the approximate formula

$$\frac{d\Gamma}{dx} \simeq \frac{d\Gamma_o}{dx} \exp(-\frac{\alpha}{2\pi} \log^2(1-x)), \qquad (5)$$

which is independent of the details of the decay; for μ decay, $d\Gamma_o/dx = 2x^2(3-2x)$. (If $m_e \neq 0$, then divergences from photons emitted nearly collinear to the e are regulated, leaving only those from soft photons and replacing $\log^2(1-x)$ by $2\log(m_e^2/m_\mu^2)\log(1-x)$: in this case, all subleading $\log(1-x)$ terms are known also to exponentiate.) Different $d\Gamma_o/dx$ cannot be distinguished in the LLA. With $d\Gamma_o/dx = 2x^2(3-2x)$ ($\mu \to eX$ (or $b \to \ell X$) spectrum), the $O(\alpha)$ term in the expansion of (5) implies a correction to the total decay rate of $(1-265/144\ \alpha/\pi) \simeq (1-1.84\ \alpha/\pi)$; with $d\Gamma_o/dx = 12x^2(1-x)$ ($\mu \to \nu_\mu X$ (or $C \to \ell X$) spectrum) $\Gamma/\Gamma_o \doteq (1-0.8\ \alpha/\pi)$ and for $d\Gamma_o/dx = 1$, $\Gamma/\Gamma_o \simeq (1-\alpha/\pi)$; the exact result for V-A μ decay is [10] $(1-(\pi^2-25/4)\alpha/(2\pi)) \simeq (1-1.81\ \alpha/\pi)$. One may guess the correction to Γ summed to all orders in α by integrating just the LLA (5), which yields (taking $d\Gamma_o/dx = 1$):

$$\Gamma \simeq \Gamma_o \frac{\pi}{\sqrt{2\alpha}} e^{\pi/2\alpha} \mathrm{erfc}\left(\sqrt{\frac{\pi}{2\alpha}}\right),$$

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-x^2} dx. \tag{6}$$

For $\alpha/\pi = 0.1$, this gives $0.87\ \Gamma_o$ compared to the $O(\alpha)$ result $0.8\ \Gamma_o$, while for $\alpha/\pi = 0.4$, it gives $0.79\ \Gamma_o$ compared to $0.6\ \Gamma_o$. These results for μ decay may also be applied to the lepton energy spectra and rates for semileptonic decays $Q \to q'\ell\nu$ of heavy quarks [11], after the substitution $\alpha \to 4\alpha_s/3$. In charm decays, $O(m_s/m_c)$ and $O(\Lambda/m_c)$ effects still dominate over $O(\alpha_s)$ ones, but for b decays QCD corrections should be relevant. Here (5) gives QCD corrections to the lepton spectrum from the weak decay of an on-shell massive quark. If, as in e^+e^- annihilation, the initial Q is produced off its mass shell, gluon emissions degrade its energy by a factor $\sim [\alpha_s(s)/\alpha_s(m_Q^2)]^{0.4}$ long before the weak decay occurs (m_Q acts as a cutoff for collinear hard gluon emissions).

A similar analysis gives the modification of the γ energy spectrum from $\zeta \to \gamma GG...$ due to radiation from the outgoing gluons as $\sim \exp(-3\alpha_s/(2\pi) \times \log^2(1-x))$. Integrating this over x (with $d\Gamma_o/dx$) suggests the rash guess $\Gamma \simeq (1-4.5 \frac{\alpha_s}{\pi})\Gamma_o$.

The momenta of partons produced in e^+e^- annihilation should lie roughly in a plane; deviations from coplanarity may be measured by $\Pi_1 \equiv \sum_{i,j,k} (\vec{p}_i x \vec{p}_j \cdot \vec{p}_k)/(\sqrt{s})^3(\hat{p}_i x \hat{p}_j \cdot \hat{p}_k)$ [5] ($\Pi_1 = 0$ for coplanar events and $\Pi_1 = 2/9$ for isotropic final states). The lowest-order contribution to $<\Pi_1>$ in e^+e^- annihilation is from $e^+e^- \to q\bar{q}GG$ (for which $\Pi_1^2 \sim (1-z_1)(1-z_2)(t_1t_2/s^2)^2$), and in the LLA this gives $1/\sigma\ d\sigma/d\Pi_1 \simeq 8/9(\alpha_s/\pi)^2|\log^3\Pi_1|/\Pi_1$ at small Π_1 ($e^+e^- \to q\bar{q}q'\bar{q}'$ gives only an $O(\log\Pi_1/\Pi_1)$ term). In ζ decays, $<\Pi_1>$ is larger; $\zeta \to GGGG$ (which is allowed, unlike the analogous positronium decay, as a direct consequence of the non-Abelian nature of the G couplings) gives $1/\sigma\ d\sigma/d\Pi_1 \simeq 3(\alpha_s/\pi)|\log\Pi_1|/\Pi_1$ ($\zeta \to GGq\bar{q}$ contributes $O(1/\Pi_1)$). In both cases, the integrated Π_1 distributions exponentiate when summed to all orders in α_s.

Now consider the energy correlation $\not{S}_F(\theta) = 2<E_i^2>/s$ which gives the mean square energy in a jet concentrated within a cone of angle θ. Whereas $\not{S}_B(\theta)$ contained $[\alpha_s\log^2\theta]^k$ terms, only $[\alpha_s\log\theta]^k$ appears in $\not{S}_F(\theta)$. (For large ℓ, $<H_\ell> \simeq [\not{S}_F(1/\ell) + (-1)^\ell\not{S}_B(1/\ell)]/2$; only for dominantly two-jet processes (e.g., $e^+e^- \to q\bar{q}...$) is $\not{S}_B$ significant: when the lowest order involves > 2 final partons (as in $\zeta \to GGG$), $\not{S}_F$ determines $<H_\ell>$). The deviations of $\not{S}_F(\theta)$ from one are dominated by radiations in which the emitted and recoiling parton make an angle > θ. To LLA, this angle is simply t_i/s, where $\sqrt{t_i}$ is the invariant mass of the radiating parton. Here the crucial difference between $\not{S}_F$ and $\not{S}_B$ becomes apparent: a given emission will not affect $\not{S}_F(\theta)$ so long as its products are collinear to within an angle $\simeq \theta$; however, in $\not{S}_B(\theta)$ they must rather have a relative transverse momentum $\lesssim \theta\sqrt{s}$ and thus be not only almost collinear (small t), but the radiated parton must also be soft (small 1-z). The greater restriction of phase space in the latter case forbids complete cancellation of soft gluon emission divergences and leads to double rather than single log terms. Note that because of the ordering of the t_i ($t_i >> t_{i+1}$), the dominant contributions to $\not{S}_F(\theta)$ come from the first few emissions; subsequent radiations must have much smaller angles and therefore will not spread jets sufficiently to affect $\not{S}_F(\theta)$. On the other hand, $k_T^2 \sim (1-z_i)t_i$ relevant for $\not{S}_B(\theta)$ are not ordered, and, in fact, $\not{S}_B(\theta)$ is typically dominated by a sequence of emissions imparting roughly equal k_T and is therefore considerably more influenced by incalculable large distance effects than $\not{S}_F(\theta)$.

To $O(\alpha_s)$, $e^+e^- \to q\bar{q}G$ spreads the $q,\bar{q}$ jets and modifies the $O(\alpha_s^o)$ result $\not{S}_F(\theta) = 1$ to $\not{S}_F(\theta) \simeq 1 - \int_{\theta^2 s}^{s} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int_o^1 z(1-z)P_{qG}(z)dz \simeq 1 + \frac{2\alpha_s}{\pi} \log\theta$ in the LLA [F.10]. (For a two-gluon-jet final state, this becomes $\not{S}_F(\theta) \simeq 1 + (\frac{42+F}{10}) \frac{\alpha_s}{\pi} \log\theta$.) In higher orders, $\not{S}_F(\theta)$ may be computed as the mean product of the absolute fractional energies ($<z_i^{abs} z_{i'}^{abs}> = <\prod_{j=1}^{i-1} z_j^2(1-z_i)>$) summed over all pairs of emitted partons with $t \geqslant \theta^2 s$. In calculating the contribution of the i^{th} emission, the z_j for $j < i$ obey $0 \leq z_j \leq 1$, and the virtual diagrams at $z_j = 1$ entirely cancel the soft divergences. (This is in contrast to the case of $\not{S}_B(\theta)$, where $(1-z_j) \leqslant \theta^2 s/t_j$, thus leaving uncanceled a $\log(t_j/\theta^2 s)$ term from the soft emission region.) The t_j integrals are, however, restricted according to $t_j \geqslant t_{j+1} \ldots\ldots \geqslant t_i \geqslant \theta^2 s$. In performing the t_j integrations, one must retain the variation of $\alpha_s(t_j) \sim 1/\log(t_j/\Lambda^2)$, leading to $[\log(\log(\theta^2 s/\Lambda^2)/\log(s/\Lambda^2))]^k/k!$ terms at each order. Then, summing over all possible jet backbones and numbers of emissions, one obtains the exponentiated form

$$\$_F(\theta) \simeq \frac{T(\theta)}{\pi}\left[\int_0^1 P(z)z(1-z)dz\right]\cdot\exp\left[-\int_0^1 P(z)z^2 dz\right.$$

$$\left.\log(T(\theta))(6/(33-2F))\right]\cdot I, \tag{7}$$

$$T(\theta) \simeq \frac{\alpha_s(s)}{\alpha_s(s\theta^2)} \simeq \frac{\log(s\theta^2/\Lambda^2)}{\log(s/\Lambda^2)}$$

where P is the matrix of kernels P_{ij}, and I is a vector in (q,G) space representing the initial parton. Hence, for quark and gluon jets [12,1]

$$\begin{aligned}(\$_F(\theta))_q &\simeq 1.2[T(\theta)]^{0.6} - 0.2[T(\theta)]^{1.4}\\ (\$_F(\theta))_G &\simeq 0.4[T(\theta)]^{0.6} + 0.6[T(\theta)]^{1.4}.\end{aligned} \tag{8}$$

Without knowledge of subleading log terms, one cannot determine the optimal argument of α_s (or T) to be used in applications of these formulae to jets produced in specific processes; plausible choices give rather different phenomenological estimates for spreading of jets. (From eq. (8), one may estimate $\langle H_\ell \rangle$ for $\zeta \to GGG\ldots$ at large ℓ. The lowest-order process has a differential cross-section barely distinguishable from three-body phase space and gives $\langle H_\ell \rangle \simeq 3/8$; higher order processes serve simply to multiply this by $\simeq (\$_F(1/\ell))_G$. Note that at high ℓ, the $\langle H_\ell \rangle$ for this 3-jet process $\sim \log\ell$, whereas for two jet processes such as $e^+e^- \to q\bar{q}\ldots.$, $\langle H_\ell \rangle \sim \log^2 \ell$.)

Most of the radiation in a jet consists of soft partons. One may estimate the multiplicity of partons with absolute fractional energies $E/\sqrt{s}$ above some small cutoff x_{min} by integrating the differential cross-section (1) with the restrictions $x_{min}/(\prod_{j=1}^{i-1} z_j) \leq z_i \leq 1$ and summing over all possible jet backbones. Consider first the emission of gluons in a gluon jet, so that the

z_i integrands are roughly $c_A/\pi\ 1/z_i$ (the multiplicity is dominated by soft gluons emitting soft gluons). The nested lower limits on the z_i integrals result in a triangular integration region (analogous to that for the t_i), and for k gluon emission gives $[(c_A/\pi)\log x_{min}]^k/k!$; the corresponding t integrals give a factor $[\log\log(s/\Lambda^2)]^k/k!$. The terms from k gluon emission therefore $\sim A^k/(k!)^2$. The sum over k may be performed by recalling the expansion of irregular Bessel functions: $I_n(2y) = \sum_{k=0}^{\infty} y^{2k+n}/(k!(k+n)!)$; their asymptotic expansion is $I_n(y) \sim e^y/\sqrt{2\pi y}$. To obtain a complete result, one must include the $O(1)$ as well as $O(1/z)$ parts of $P(z)$: such terms give no $\log(z)$ contributions and exponentiate to a power of α_s [F.11]. Summing over all possible emissions, one estimates that the number of gluons with fractional energies $\geqslant x_{min}$ in a gluon jet is (taking $F = 3$ and $t_c = \Lambda^2$)

$$\begin{aligned} \langle n_G\rangle_G &\simeq I_0(2\sqrt{A})[\alpha_s(s)]^{1.25} \\ A &\simeq \frac{c_A}{\pi}\log(\alpha_s(s))\log(x_{min}). \end{aligned} \tag{9}$$

In a quark jet, the probability for the first gluon emission is reduced by a factor $C_F/C_A = 4/9$, but the subsequent development remains the same, so that the number of gluons in (9) is just reduced by 9/4. Soft quarks emitted from gluons follow a dz rather than dz/z spectrum; most light quarks at small z thus arise from a series of gluon emissions followed by a single materialization $G \to q\bar{q}$, so that the last $2C_A\log(z)$ for gluon emissions is replaced by just F/2. Then the multiplicity of quarks with energies $\geqslant x_{min}$ in a gluon jet becomes

$$\langle n_q\rangle_G \simeq \frac{F}{2\pi}\log(1/\alpha_s(s))I_1(2\sqrt{A})/\sqrt{A}[\alpha_s(s)]^{1.25}. \tag{10}$$

The x distributions of soft partons in a jet may be found by differentiating (9) and (10) or directly by not integrating over the last emission in the construction of the series (or, alternatively, by inverting the behavior of z^{n-1} moments due to $0(\frac{1}{n-1})$ and 0(1) terms in the anomalous dimensions [7,12]).

Perturbation theory presumably ceases to be operative when the invariant masses of partons in a jet fall below $\sim \sqrt{t_c}$, but some properties of the parton system prepared may be relevant for subsequent condensation into hadrons. One of these features is the invariant mass distribution for pairs of final partons (each with $t \leqslant t_c$) in the jet [13]. Such partons may be taken as emitted from the backbone of the jet, which consists of a sequence of radiating partons with large t. The invariant mass of the k^{th} and $(k+1)^{th}$ real partons emitted is $M^2 \simeq (1-z_{k+1})t_k$. In computing the mean number of such pairs with $M^2 \geqslant M_o^2 >> t_c \sim \Lambda^2$, the corresponding limits on the z_i are $0 \leq z_i \leq 1$ $(i \leq k)$, $0 \leqslant z_{k+1} \leqslant 1 - M_o^2/t_i$, while the t_i satisfy $M_o^2 \leqslant t_i \leqslant t_{i-1}$ $(i \leq k)$, $t_c \leqslant t_{k+1} \leqslant t_k$. Performing the z_{k+1} integral introduces a crucial $\log(t_k/M_o^2)$. In all t_i integrals, the variation of this term overwhelms the running of $\alpha_s(t)$ in the LLA and prevents the appearance of log log(t) term. Instead, the final result $\sim \log^k(s/M_o^2)/k!$. Summing this over the position of the pair and dividing by the total number of pairs (i.e., $M_o^2 = t$), one obtains for the probability that a given pair has $M > M_o$ the power-law damped form $\sim (M^2/t_c)^{-\alpha_s\gamma}$, where γ depends on the types of partons in the pair and jet. (Note that in an asymptotically-free theory such as ϕ_6^3 with no soft divergences, the $\log(t_k/M_o^2)$ from the z_{k+1} integral is absent, and the spectrum $\sim [\log(M^2/t_c)]^{-p}$. Also note that the damped spectrum is independent of the color of the pair; for a sequence of n produced partons $M^2 \sim (1-z_{k+1}\cdots z_{k+n-1})t_k$.) If instead of considering a pair of 'final' partons each with $t \leqslant t_c$, one allows one parton in the pair to have arbitrary mass, then the pair mass spectrum is just $\$_B(M^2/s)$ and is only logarithmically damped.

External forces acting on a sufficiently small color singlet system of partons should cancel coherently, so that its later evolution is independent of the rest of the final state. The argument of the previous paragraph suggests that at a time $\sim 1/\sqrt{t_c}$, the invariant mass of a nearby pair of partons is peaked around $\sqrt{t_c} \sim 1$ GeV. It is therefore plausible that when such pairs constitute color singlet systems, they should condense directly into clusters of hadrons, probably isotropically in their rest frames. The relevant pairings are perhaps chosen according to the spatial separation of the final partons: A convenient and largely equivalent picture is that every parton trails a 'string' representing each spinor color index (hence two strings per gluon), and that it is the strings which eventually form hadrons. This picture implies that the ultimate fragmentation of gluon jets should be like pairs of quark jets and requires no further parameters. (Equations (9) and (10) support this when $N_c \to \infty$ so that $C_A/C_F \to 2$.) An alternative method would be to ignore the colors of partons and fragment each separately to hadrons when its t reaches $t_o \gg \Lambda^2$, using a phenomenological model fit at $s \simeq t_o$; predictions should be independent of t_o. The latter method is commonly applied to deduce the dependence of single hadron momentum spectra on s. For complete final states it is more difficult to implement: A Monte Carlo model based on the former method will be described in [14].

I am grateful to R. P. Feynman, R. D. Field, H. D. Politzer and especially G. C. Fox for discussions.

Footnotes

[F.1] In the deep inelastic scattering cross-section (σ), non-kinematical $O(\Lambda^2/Q^2)$ effects presumably arise from rescattering of the struck quark at large times. In a heavy nucleus, the effect of this rescattering $\sim A^{1/3}$. Thus, $O(\Lambda^2/Q^2)$ terms in σ should behave $\sim A^{4/3}$, while scaling (up to short-distance QCD corrections) terms should $\sim A^1$. This fact may allow extrapolations to obtain better estimates of the latter at small Q^2.

[F.2] The relevant scale for the variation of α_s is determined by the $O(\alpha_s^2)$ result [3] $1 + \alpha_s(s)/\pi + (2.0-0.1\ F)(\alpha_s/\pi)^2$, where α_s (or Λ) is defined to be extracted from measurements on another process using theoretical predictions calculated in the truncated minimal subtraction renormalization scheme, with Tr[1] = 4.

[F.3] This behavior (essentially kinematic in origin) is manifest when mass corrections are computed. For example, $\langle H_\ell \rangle$ or $\langle$thrust$\rangle$ typically contain $O(\sqrt{m^2/s})$ corrections, which are forbidden for σ by power-counting theorems for the corresponding Feynman diagrams.

[F.4] The appearance of these soft divergences is specific to vector field theories; they do not occur with scalar gluons.

[F.5] The kernels $P_{ij}(z)$ which represent the probability (in units of $\alpha_s/2\pi t$) that parton i will emit parton j carrying a fraction of its energy (strictly, longitudinal Sudakov variable) are given by [6,7]

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{1-z}\right)_+$$

$$P_{qG}(z) = C_F\left(\frac{1+(1-z)^2}{z}\right) = P_{qq}(1-z)$$

$$P_{Gq}(z) = \frac{F}{2}\,(z^2+(1-z)^2) \qquad \text{(summing over F flavors of light quarks)}$$

$$P_{GG}(z) = 2C_A\left(\frac{(1-z+z^2)^2}{z(1-z)}\right)_+ - \frac{F}{3}\,\delta(1-z)$$

where the color factors $C_A = N_C = 3$, $C_F = (N_C^2-1)/(2N_C) = 4/3$.

[F.6] The details of this derivation depend on the gauge used. $P_{qq}(z) \sim 1/(1-z)$ when n is approximately along the q ($\bar{q}$) direction, so that only radiation from the $\bar{q}$ (q) gives leading logs; otherwise $P_{qq} \sim 1/(1-z+t/s)$ ($\sim 1/x_G$) but both q and $\bar{q}$ radiate. The former approach is used here; in the latter, the t_1 integral becomes $\int_{\theta^2 s}^{s} \frac{dt_1}{t_1} \log(\theta^2 s/t_1 + t_1/s) \simeq \frac{1}{2}\int_{\theta^2 s}^{s} \frac{dt_1}{t_1} \log(\theta^2 s/t_1)$, thus compensating for the different number of contributing diagrams. (z is defined as the relative Sudakov variable; other choices differ by $O(t/s)$, but give different phase space boundaries.)

[F.7] Defining the differential energy correlation $F_2^{pt}(\chi) = \sum_{\text{partons}} 2E_iE_j/s\;\delta(\cos\phi_{ij}-\chi)$ (so that $H_\ell = \int_{-1}^{1} F_2^{pt}(\chi)P_\ell(\chi)/2\,d\chi$), the integral (which coincides with the previous definition of $\not{S}_B(-\cos^{-1}(\eta))$ to leading log order)

$$\int_\eta^1 F_2^{pt}(\chi)d\chi \simeq 1 - \frac{2\alpha_s}{3\pi}\left[\log^2\left(\frac{1+\eta}{2}\right) + 3\log\left(\frac{1+\eta}{2}\right) + 4.7 + \ldots\right].$$

[F.8] Again, details of derivation depend on gauge. The exponential form has been verified explicitly to $O(\alpha_s^2)$ in [8] (the more complicated terms found in [9] using the second gauge in [F.6] appear to be absent). For a final state of two gluon jets (e.g., from a ^1S_o, ^3P_o or $^3P_2$ $Q\bar{Q}$ state), the exponent here is multiplied by $C_A/C_F = 9/4$.

[F.9] In analogy with [F.8], off shell quarks give $P_{qq}(z) \sim 1/(1-z+p^2/s)$, but off shell gluons leave $P_{qq}(z) \sim 1/(1-z)$. The coefficient of $\log^2(p^2/s)$ in the Sudakov form factor for off-shell q is thus 1/2 that for off-shell G.

[F.10] In this case, the quantity defined in [F.5] becomes $\int_\eta^1 F_2^{pt}(\chi)d\chi \simeq 1 + \frac{\alpha_s}{\pi}\,[\log(1-\eta) - 0.40 + \ldots]$.

[F.11] To see this, first sum over the number of $O(1/z)$ kernels with a fixed set of $O(1)$ kernels. Note that the effects of the $O(1)$ kernels are of the same order as those of subleading log terms in the cross-section and therefore can only be considered indicative of such corrections.

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BOUNDS ON PARTICLE MASSES IN THE WEINBERG–SALAM MODEL ☆

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Various conditions necessary for the self-consistency of the Weinberg–Salam model are used to place constraints on fermion and Higgs Boson masses. We find that spontaneous symmetry breakdown cannot generate fermion masses in excess of about 100 GeV.

In the Weinberg–Salam $SU(2)_L \times U(1)$ model for weak interactions, the masses of all the gauge bosons, quarks and leptons are taken to arise from the Higgs mechanism. At the tree approximation, the couplings of the Higgs scalar field ϕ to itself determine the effective potential $V(\phi)$, which in turn determines the symmetry of the "vacuum". In this approximation $V(\phi)$ is independent of the couplings (which determine the masses attained after spontaneous symmetry breakdown) of fermions and gauge bosons to ϕ. If, however, one-loop corrections to $V(\phi)$ are included, then the gauge bosons and fermions will influence $V(\phi)$. The requirement that this influence should not serve to prevent the possibility of spontaneous symmetry breakdown places several constraints on the couplings in the theory, and hence on the ratios of masses of various particles. Linde and Weinberg [1] have derived a lower bound on the mass of the Higgs particle H by demanding that the energy density of the "vacuum" after spontaneous symmetry breakdown should not exceed its value when $\phi = 0$. In this note, we apply the more complete requirement that the conventional "vacuum" in which $\langle\phi\rangle \neq 0$ corresponds to the absolute, rather than only a local, minimum of $V(\phi)$, at least in the domain where $V(\phi)$ may be obtained reliably from perturbation theory. If all fermion and gauge boson masses are generated from the vacuum expectation value of a single ϕ field, then this constraint allows one to place an upper bound on the fermion masses. The exact form of the bound involves m_W, m_H and other parameters, but typically the mass m_f of the heaviest fermion must satisfy $m_f \lesssim 300$ GeV. While this range is not immediately accessible to experimental investigation, the very existence of such a bound, coming solely from considerations of self-consistency, places constraints on models for weak interactions. Our bound is equivalent to an upper limit on the dimensionless fermion–Higgs Yukawa coupling, f, and it ensures that f is perturbatively small; $m_f \lesssim 300$ GeV corresponds to $f^2/4\pi \lesssim 0.1$.

In a theory with more than one coupling constant, one-loop graphs can dominate over tree graphs, while perturbation theory remains reliable because all couplings are small. For example, with a gauge coupling g and ϕ^4 self-coupling λ (both small), but such that λ is of order g^4, a gauge boson loop can compete with $O(\lambda)$ tree graphs, while yet higher-order corrections remain unimportant. However, even if the couplings are small, the perturbation expansion breaks down when logarithms of field strengths become large [‡1]. In the following discussion, we shall simply require that the theory be consistent over the range of ϕ that can be explored perturbatively.

The complete formula for $V(\phi)$ in the one-loop approximation is [2]

$$V(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda\phi^4 + A\phi^4 \log(\phi^2/M^2)\,, \qquad (1a)$$

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[2] Feynman Fellow.

[‡1] Renormalization group improvement would be helpful only if the theory were asymptotically free.

where

$$A = \frac{1}{1024\pi^2}\left[\sum_{\text{gauge bosons}} 3g_i^4 - \sum_{\text{fermions}} f_i^4\right], \tag{1b}$$

and the g_i (f_i) are the couplings of the gauge bosons (fermions) to the Higgs particles. Note that, because of Fermi statistics, the fermion contribution to A is negative. The parameter M in eq. (1a) is a renormalization mass. In the Weinberg–Salam $SU(2)_L \times U(1)$ model, A is given by

$$A = \frac{1}{1024\pi^2}\left[3[2g^4 + (g^2 + g'^2)^2] - 4g^4 \sum_{\text{fermions}} \left(\frac{m_f}{m_W}\right)^4\right], e = g\sin\theta_W = g'\cos\theta_W \tag{2}$$

In our numerical estimates, we use $\sin^2\theta_W = 0.23$, so that $m_W \approx 77$ GeV. We have dropped the $O(\lambda^2)$ contributions of Higgs scalar loops to $V(\phi)$, since, as discussed below, these must be negligible if perturbation theory is to be valid ‡2.

For spontaneous symmetry breakdown to occur it is necessary that $V(\phi)$ should have a non-trivial local minimum at $\phi = \phi_0$ such that

$$\phi_0 \neq 0, \quad \delta V/\delta\phi|_{\phi=\phi_0} = 0,$$

$$\delta^2 V/\delta\phi^2|_{\phi=\phi_0} = m_H^2 \geqslant 0 . \tag{3}$$

To investigate the consistency of a theory based on the "vacuum" $\phi = \phi_0$, we shall assume such a theory and then find under what circumstances inconsistencies appear. In that case, the parameters μ^2 and M^2 appearing in the effective potential $V(\phi)$ may be eliminated in favor of ϕ_0 and m_H. It is convenient to introduce

$$S \equiv \phi/\phi_0 , \quad \Xi \equiv 4A\phi_0^2/m_H^2 , \tag{4a}$$

in terms of which

$$V(\phi) \equiv \tfrac{1}{8}m_H^2\phi_0^2\,\widetilde{V}(\phi) = \tfrac{1}{8}m_H^2\phi_0^2 S^2 \times [2\Xi S^2\log(S^2) - 3\Xi S^2 + 4\Xi + S^2 - 2] . \tag{4b}$$

The requirement [1] $V(\phi_0) < V(0)$ necessary to allow spontaneous symmetry breakdown becomes

‡2 For quark loops, higher-order QCD corrections are governed by an effective coupling evaluated on the scale of ϕ_0 (see eq. (3)), and may therefore safely be ignored.

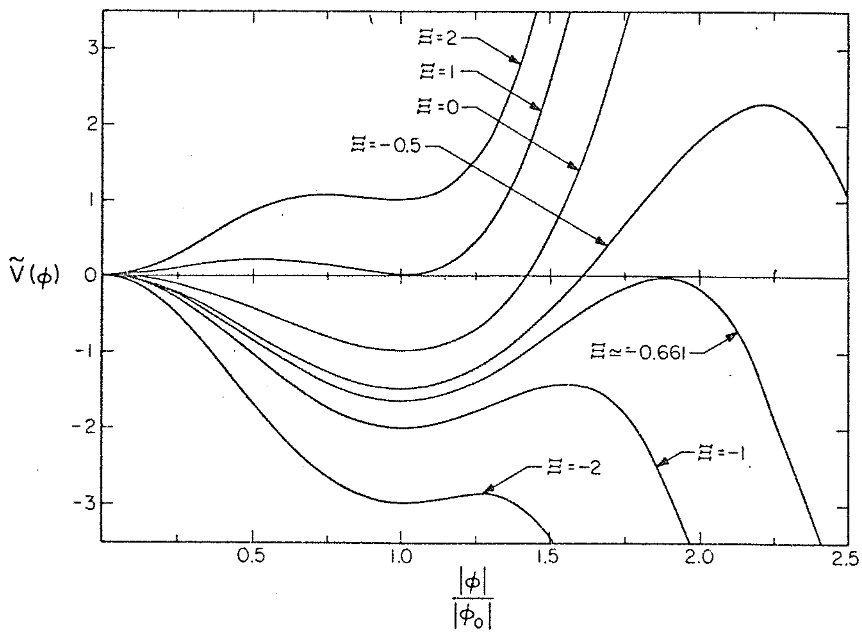

Fig. 1. The reduced effective potential $\widetilde{V}(\phi) = (8/m_H^2\phi_0^2)\,V(\phi)$ as a function of ϕ/ϕ_0 for various choices of the combination of couplings Ξ defined in eq. (4). For usual spontaneous symmetry breakdown to occur, $\phi = \phi_0$ must correspond to an absolute minimum of $V(\phi)$, at least within the range of ϕ accessible to perturbation theory.

$$V(\phi_0) = \tfrac{1}{8} m_{\mathrm{H}}^2 \phi_0^2 (\Xi - 1) < 0 \,, \tag{5}$$

so that $\Xi < 1$.

In fig. 1 we plot $\widetilde{V}(\phi)$ as a function of ϕ/ϕ_0 for various values of Ξ. As the Yukawa couplings f_i increase, Ξ decreases, as does $V(\phi_0)$. For negative Ξ, a new phenomenon occurs: $V(\phi)$ eventually turns over and goes to $-\infty$ as $\phi \to \infty$. However, since our expression for $V(\phi)$ is obtained from perturbation theory, we have no estimate of it for values of ϕ so large that $A \log(\phi^2/\phi_0^2) \gtrsim 1$. We therefore do not consider its behavior as $\phi \to \infty$, but rather, require that $V(\phi) > V(\phi_0)$ for all values of $\phi \neq \phi_0$ within the range over which $V(\phi)$ is reliably calculated. If this is not satisfied, then the theory is inevitably inconsistent. Fig. 2 shows the values ϕ_1 of ϕ for which $V(\phi_1)$ becomes less than $V(\phi_0)$, as a function of Ξ. (We also show the values of ϕ corresponding to the second local maximum of $V(\phi)$.) For large values of $\log(\phi_1/\phi_0)$, one finds

$$\Xi \approx -[4 \log(\phi_1/\phi_0)]^{-1} \,. \tag{6}$$

If the theory is to allow a stable "vacuum" in perturbation theory then ϕ_1 must lie outside the range of validity of perturbative approximations. In practice, our

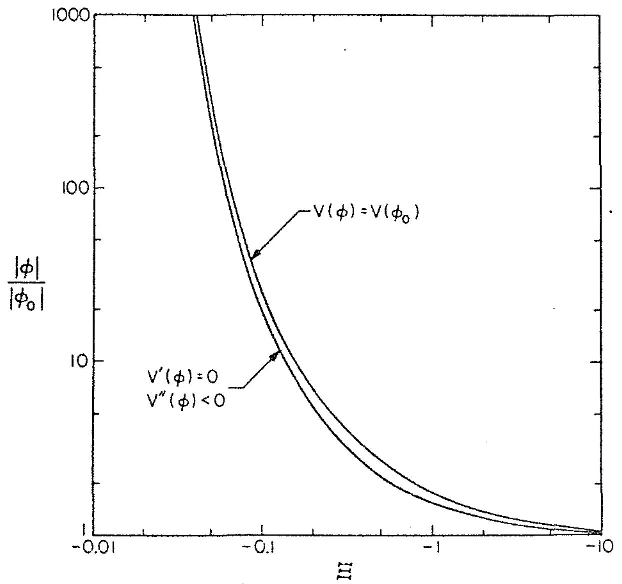

Fig. 2. The values of ϕ at which $V(\phi)$ drops below $V(\phi_0)$ and at which the second local maximum of $V(\phi)$ occurs, as a function of Ξ. These values of ϕ must be so large that our perturbative methods fail if the "vacuum" $\phi = \phi_0$ is to be stable.

final results are rather insensitive to the precise value of Ξ which is deemed unacceptable. Combining the Linde–Weinberg condition [1] with our requirements on $V(\phi)$ one obtains

$$1 > \Xi > -|\Xi_{\min}| \,, \tag{7}$$

where $|\Xi_{\min}|$ is presumably much less than 1 and perhaps as small as 0.005.

For any particular set of fields and couplings, one can translate these bounds on Ξ into bounds on ratios of particle masses. Consider first the case of the Weinberg–Salam $\mathrm{SU(2)_L \times U(1)}$ model with its one complex $\mathrm{SU(2)_L}$ doublet of Higgs fields and with a single heavy fermion. In this case (g and g' are defined in eq. (2))

$$\Xi = (m_{\mathrm{W}}^2/64\pi^2 m_{\mathrm{H}}^2 g^2)\ \{3[2g^4 + (g^2 + g'^2)^2] \\ -4(g m_{\mathrm{f}}/\ m_{\mathrm{W}})^4\} \,. \tag{8}$$

The first inequality in eq. (7) then becomes

$$m_{\mathrm{H}} \gtrsim (m_{\mathrm{W}}/8\pi g)\ \{3[2g^4 + (g^2 + g'^2)^2]. \\ -4(g m_{\mathrm{f}}/\ m_{\mathrm{W}})^4\}^{1/2} \\ \approx \{24[1.8 - 0.6\ \ (m_{\mathrm{f}}/m_{\mathrm{W}})^4]\}^{1/2}\,\mathrm{GeV} \,, \tag{9}$$

which reduces to the Linde–Weinberg bound [1] $m_{\mathrm{H}} \gtrsim 6$ GeV when $m_{\mathrm{f}} \to 0$. Note that this bound places no constraint on m_{H} when $m_{\mathrm{f}} \gtrsim 1{\cdot}3\, m_{\mathrm{W}} \approx 100$ GeV. A constraint is, however, provided by the second inequality in eq. (7), which yields

$$m_{\mathrm{f}} \lesssim (m_{\mathrm{W}}/\sqrt{\pi} g)\ \{3[2g^4 + (g^2 + g'^2)^2] \\ -(8\pi g m_{\mathrm{H}}/m_{\mathrm{W}})^2\, \Xi_{\min}\}^{1/4} \\ \approx 86\ \{1.8 - 250\, \Xi_{\min}\, m_{\mathrm{H}}^2/m_{\mathrm{W}}^2\}^{1/4} \,. \tag{10}$$

The regions in m_{f} and m_{H} allowed by the bounds (9) and (10) are illustrated in fig. 3 for various choices of $\Xi_{\min}$. If there are many fermions, then the m_{f} in eqs. (9) and (10) is obviously replaced by $(\Sigma_i\, m_{fi}^4)^{1/4}$; for quarks each color is counted separately.

Our bound on m_{f} does not come from the requirement that the Yukawa couplings of the Higgs bosons to the fermions should not be large; in fact, so long as $(m_{\mathrm{H}}/m_{\mathrm{f}})^{1/2}$ is not enormous it is much more stringent. However, for a perturbative investigation of the theory to be at all meaningful, it is necessary that higher and

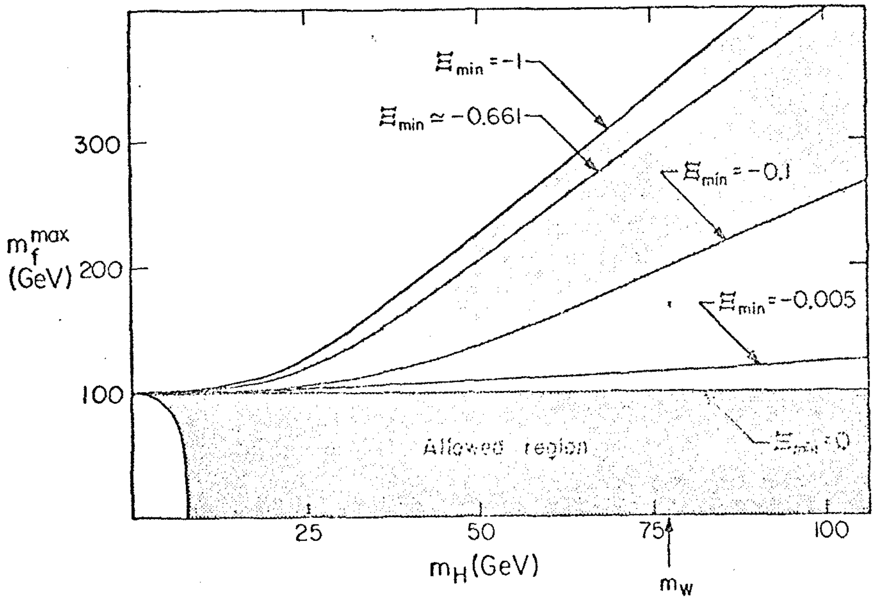

Fig. 3. The domains in the mass of the Higgs particle and of the heaviest fermion for which the Weinberg–Salam model is consistent. The value of Ξ_{min} depends on the region of validity of perturbation theory; Ξ_{min} is probably very small. The forbidden region in the lower left-hand corner represents the Linde–Weinberg bound.

higher orders in the perturbation series should give systematically smaller contributions. Experiments have shown that g and g' satisfy this condition, and our bounds on m_f ensure that it will hold for the f_i. The quartic self-couplings λ of the Higgs bosons must also obey the condition, so that [‡3]

$$\lambda/4\pi^2 = g^2 m_H^2 (1 + \tfrac{16}{3}\,\Xi + O(\Xi^2))/16\pi^2 m^2 \ll 1 ,$$

or

$$m_H \ll 4\pi m_W/g \approx 1000 \text{ GeV} . \qquad (11)$$

All predictions of the theory are obtained by perturbative methods, and, if the bound (11) were not satisfied then no predictions could be made [‡4].

We have given bounds on the Higgs particle mass (eqs. (9), (10) and (11)) which result from demanding consistency of the theory. However, by making the specific assumption that the term $\mu^2\phi^2$ in $V(\phi)$ vanishes [‡5], one may obtain a definite prediction for m_H [2]:

$$m_H \approx (m_W/4\sqrt{2}\pi)\{3[2g^4 + (g^2 + g'^2)^2] - 4\sum (g m_f/\ m_W)^4\}^{1/2} . \qquad (12)$$

If the fermion term can be ignored, then this gives $m_H \approx 9$ GeV – close to the range of present experiments.

In this paper, we have concentrated on the simplest workable model for weak interactions, since there is so far no compelling experimental evidence for a more complicated structure. In more complicated models our

‡3 For the purposes of computing higher-order corrections to the effective λ, we have defined $\lambda \equiv \frac{1}{3}\delta^4 V/\delta\phi^4|_{\phi=\phi_0}$.

‡4 Similar conclusions have been reached by demanding that the high-energy interactions of Higgs particles in the Born approximation should not violate unitarity [3].

‡5 If dimensional regularization is used, then the ϕ^2 counterterms generated at each order in the perturbation series must be proportional to the bare μ^2, since the renormalization mass (which allows the coupling constant to attain dimensions away from $d = 4$) can enter only in logarithms. Hence the vanishing of the renormalized μ^2 in $V(\phi)$ which was suggested in ref. [2] may be preserved naturally to all orders, despite the fact that no symmetry requires it. It would naively be guaranteed by scale invariance, but this is violated by renormalization. Nevertheless, the violations in perturbation theory are logarithmic and do not provide a scale for the mass.

bounds may be strengthened, weakened or may even disappear entirely. For example, if one introduces an extra Higgs field which couples only to certain fermions, then our bounds (7) cannot be used, because they involve the vacuum expectation value of the new Higgs field, which would only be determined directly from the mass of a gauge boson coupled to it.

To conclude, we have investigated the Weinberg–Salam $SU(2)_L \times U(1)$ model for weak interactions, and find that unless ratios of particle masses obey certain bounds, no meaningful predictions based on the model may be obtained by perturbative methods.

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Notes added:

It has been suggested (P.H. Frampton, Phys. Rev. Lett. 37 (1976) 1378; A.D. Linde, Phys. Lett. 70B (1977) 306) that the universe may have survived (without tunnelling) since it began in a metastable state, corresponding to a local, rather than a global minimum of $V(\phi)$. However, if the universe was once hot, then this situation could not occur, since regions of "true vacuum" (ϕ at global minimum of $V(\phi)$) as well as of "false vacuum" should be formed as the universe cooled, and these would quickly overwhelm the "false" regions. Thus such considerations of cosmology, in any case suspect because of the unphysical cosmological constant they imply, should not affect our bounds.

Bounds on masses and couplings, numerically similar to ours were derived by L. Maiani, G. Parisi and R. Petronzio (Nucl. Phys. B136 (1978) 115) using the constraint (unrelated to ours) that no coupling constants deduced from the renormalization group equation should diverge at energies below the Planck mass. We thank G. Parisi for bringing this work to our attention.

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ABUNDANCES OF NEW STABLE PARTICLES PRODUCED IN THE EARLY UNIVERSE ☆

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The standard model of the early universe is used to estimate the present abundances of possible absolutely-stable hadrons or charged leptons more massive than the proton. It is found that experimental limits on their present abundances indicate that no such particles exist with masses below about 16 GeV/c^2. Forthcoming experiments could increase this limit to masses up to around 300 GeV/c^2.

The standard model of the early universe has recently been used to place constraints on the masses and lifetimes of possible nearly-stable heavy neutrino-like particles predicted by various gauge models of weak interactions [1]. Several models of this kind imply the existence of absolutely-stable charged and/or strongly-interacting particles more massive than the proton (e.g. [2]). In this note, I show that rather large numbers of such particles would have been produced in the early universe, so that experimental limits on their terrestrial abundances may place stringent bounds on their masses.

Any new stable charged particles with masses below about 4 GeV/c^2 should already have been seen in e^+e^- interactions. The next generation of e^+e^- accelerators (PETRA, PEP) could extend this limit to masses up to 20 GeV/c^2. Attempts to produce pairs of new stable hadrons in 400 GeV proton interactions have probed up to masses ≈ 10 GeV/c^2 [2,3], but the production cross-sections for heavy hadrons near threshold are not known with sufficient accuracy for definite conclusions to be drawn [4].

The number density (n) of any species of stable particles spread uniformly throughout a homogeneous universe should obey the rate equation [1,5]

$$\frac{dn}{dt} = \frac{-3(dR/dt)}{R} n - \langle\sigma\beta c\rangle(n^2 - n_{eq}^2), \qquad (1)$$

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where R is the expansion scale factor for the universe and $\langle\sigma\beta c\rangle$ is the product of the low-energy annihilation cross-section and relative velocity for the particles, averaged over their energy distribution at time t. n_{eq} is their number density in thermal equilibrium. The first term in eq. (1) accounts for the dilution in n due to the expansion of the universe, while the second term arises from the annihilation and production of particles in interactions. Let

$$f = \frac{n}{T^3}, \qquad x = \frac{kT}{mc^2},$$

$$f_{eq} = \frac{n_{eq}}{T^3} = \frac{(2s+1)}{2\pi^2}\left(\frac{k}{\hbar c}\right)^3 \int_0^\infty \frac{u^2\,du}{\exp\sqrt{(u^2+x^{-2})}\pm 1}, \qquad (2)$$

where T is the equilibrium temperature, and in f_{eq} the upper (lower) sign is for fermions (bosons). Then, ignoring the curvature of the universe, which has no effect at the times we consider, eq. (1) becomes ‡1

$$\frac{df}{dx} = Z\,[f^2(x) - f_{eq}^2(x)], \qquad (3)$$

$$k^3 Z = \left(\frac{45}{8\pi^3 G}\right)^{1/2} \frac{m\langle\sigma\beta\rangle}{\sqrt{N_{eff}(T)}}(c^{11}\hbar^3)^{1/2}$$

$$\approx 4\times 10^{-29}\,\frac{\langle\sigma\beta\rangle[\mathrm{GeV}^{-2}]\,m[\mathrm{GeV}/c^2]}{\sqrt{N_{eff}(T)}}\,\mathrm{GeV}^3\,\mathrm{m}^3.$$

‡1 $\sigma[\mathrm{cm}^2] \approx 4\times 10^{-28}\,\sigma[\mathrm{GeV}^{-2}]$.

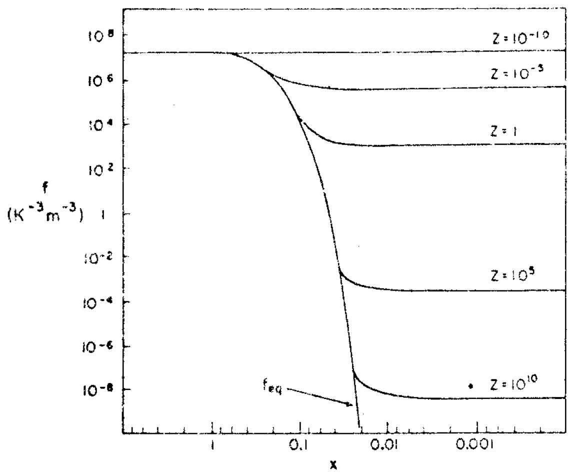

Fig. 1. Solutions to the differential eq. (3) for various values of the parameter Z (in units of $m^3\ K^3$). The curves give the number densities (divided by T^3) of particle species with a variety of low-energy annihilation cross-sections as a function of the average temperature of the universe ($x = kT/mc^2$). The equilibrium number density, $n_{eq} = f_{eq}T^3$, is also given.

If the temperature of the universe was arbitrarily high at early times [‡2] (and the cross-sections for particle interactions do not decrease too rapidly at very high energies), then all particle species should then have been in thermal equilibrium, so that the boundary condition in eq. (1) was $n(t = 0) = n_{eq}$ or $f(x = \infty) = f_{eq}$. The solutions of eq. (3) for various values of Z subject to this boundary condition are shown in fig. 1. As the universe cooled, the equilibrium number density of particle species fell dramatically around $x \approx 0.1$. The more strongly-interacting (higher Z) the particles were, the longer they will have remained in thermal equilibrium, and thus the lower their final number density will have been.

The parameter $N_{eff}(T)$ appearing in eq. (3) is the effective number of particle species in thermal equilibrium at temperature T. It determines the energy density and hence the expansion rate of the universe. Ultrarelativistic fermion (boson) spin states contribute 7/16 (1/2) to N_{eff}. (The observed spectrum of particles suggests that for $kT \lesssim 0.1$ GeV, $N_{eff} \approx 4.5$; for $0.1 \lesssim kT \lesssim 0.5$ GeV, $N_{eff} \approx 6$; for $0.5 \lesssim kT \lesssim 2$ GeV, $N_{eff} \approx 35$ (according to QCD quarks and gluons should contribute to N_{eff} as if they were free for $kT \gtrsim 0.5$ GeV); and for $2 \lesssim kT \lesssim 5$ GeV, $N_{eff} \approx 42$.)

The present number density of a particle species is given approximately by $n_p \approx f(0)T_p^3$, where T_p is the temperature which the microwave background radiation would now have if it had frozen out of thermal equilibrium at the same time and temperature ($T_f = mcx_f/k \approx mc^2/[k \log_e(10^{17}(m\langle\sigma\beta\rangle)\ [\mathrm{GeV}^{-1}/c^2]))]$) as the particle species under consideration. The difference between T_p and the present temperature of the actual microwave background radiation arises from the heating of the universe by the annihilation of other species. Specific entropy conservation gives $T_p \approx T_\gamma/(N_{eff}(T_f))^{1/3}$.

Eq. (3) may be solved approximately by assuming $f = f_{eq}$ for $T > T_f$, and neglecting f_{eq} compared to f for $T < T_f$. This gives

$$n_p \approx \frac{8 \times 10^{-8}}{\sqrt{N_{eff}(T_f)}\langle\sigma\beta\rangle\ [\mathrm{GeV}^{-2}]\ m\ [\mathrm{GeV}/c^2]}\ \mathrm{m}^{-3}\ , \quad (4)$$

which is the correct solution to eq. (3) within about a factor of 20 for the cases considered below.

To obtain estimates of n_p for particular types of particles, one must estimate $\langle\sigma\beta\rangle$. Charged stable heavy leptons ($L^\pm$) with $m_\mu \leqslant m_L \leqslant m_{Z^0}$ should annihilate primarily into two photons, and through a virtual photon to hadrons and lighter leptons, giving

$$\lim_{\beta\to 0}(\sigma\beta)_{L^+L^-} \approx \left(\frac{\pi\alpha^2}{m_L^2} + \frac{2\pi\alpha^2}{m_L^2}\,\frac{\sigma_{tot}(e^+e^-)(s = 4m_L^2)}{\sigma(e^+e^- \to \mu^+\mu^-)(s = 4m_L^2)}\right)\hbar^2\ . \quad (5)$$

This cross-section, together with the form for N_{eff} discussed above, may now be used to solve eq. (3) and to obtain an estimate for the present abundances of any charged stable heavy leptons. (The exact results are well-approximated by eq. (4).) One finds that for $4 \leqslant m_L \leqslant 10\ \mathrm{GeV}/c^2$, $n_p(L^\pm) \sim 10^{-5}\ \mathrm{m}^{-3}$, corresponding to an abundance of about one new stable charged heavy lepton in 10^5 nucleons. For $m_L \gtrsim 10\ \mathrm{GeV}/c^2$, the estimated present $L^\pm$ number density rises roughly linearly with m_L, except for slight decreases due to increases in the L^+L^- annihilation cross-section associated with the opening of new channels. The abundances of any $L^\pm$ produced in the early universe should therefore be rather

[‡2] Models predicting a maximum temperature for hadronic matter are disfavored by recent experimental results indicating the presence of pointlike weakly-interacting constituents within hadrons at short distances.

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large, and hence easily amenable to experimental investigation.

To estimate the present abundances of any stable heavy hadrons (H) (containing heavy quarks Q), one must assume a form for the low-energy $H\bar{H}$ annihilation cross-section. An upper bound on $\langle\sigma\beta\rangle_{H\bar{H}}$ is probably provided by the low-energy limit of $\sigma\beta$ for protons [6] $\approx 300\ \mathrm{GeV}^{-2}$. If $m_H \gtrsim 3\ \mathrm{GeV}/c^2$, then the universe at the freezing temperature for the H should have consisted of almost free quarks and gluons, so that a better estimate of $H\bar{H}$ annihilation may be given by the rate for electromagnetic annihilation (5) and for $Q\bar{Q} \to GG$ (obtained from the first term of eq. (5) by replacing α by α_s). The first estimate for $\langle\sigma\beta\rangle_{H\bar{H}}$ leads to $n_p(\mathrm{H}) \sim 10^{-11}\ \mathrm{m}^{-3}$ for $m_H = 5\ \mathrm{GeV}/c^2$, decreasing (roughly as $1/m_H$) to $\sim 10^{-12}\ \mathrm{m}^{-3}$ for $m_H = 100\ \mathrm{GeV}/c^2$. The second estimate for $\langle\sigma\beta\rangle_{H\bar{H}}$ suggests $n_p(\mathrm{H}) \sim 10^{-8}\ \mathrm{m}^{-3}$ for $m_H = 5\ \mathrm{GeV}/c^2$, increasing roughly as m_H, and perhaps reaching $\sim 10^{-4}\ \mathrm{m}^{-3}$ for $m_H = 100\ \mathrm{GeV}/c^2$. Since it seems most unlikely that the $H\bar{H}$ annihilation cross-section is smaller than its value according to the first estimate, any stable heavy hadrons (with masses below about $100\ \mathrm{GeV}/c^2$) should exist in concentrations above one in about 10^{12} nucleons.

These estimates for heavy hadron abundances may be applied to protons. They give a result $\simeq 10^{10}$ too small. The discrepancy is due to the assumption of homogeneity made in eq. (1); in fact, there must either be a net excess of baryons over antibaryons in the universe, or protons and antiprotons must have become spatially separated (presumably at $kT \gtrsim 50$ MeV) thereby preventing their annihilation [7]. Similar phenomena may have occurred for other stable particles. (An indication that they were not important comes from the result that the present chemical potential (μ) for all species of neutrinos is below 5×10^{-4} eV ‡3, while for $\bar{\nu}_e, \mu \leqslant 5 \times 10^{-6}$ eV [8].) Inhomogeneity can serve only to increase n_p, so that our estimates should be considered in fact as lower bounds on n_p.

The observed average mass density in the present universe is around $2 \times 10^{-26}\ \mathrm{kg\ m}^{-3}$. The requirement that yet unobserved new stable particles produced in the early universe should not contribute a larger mass density than is observed yields (from eq. (4)) $\sqrt{N_{\mathrm{eff}}\langle\sigma\beta\rangle} \gtrsim 7 \times 10^{-9}\ \mathrm{GeV}^{-2}$, which is irrelevant for all species of particles except those undergoing only weak interactions [1].

After their production in the early universe, stable heavy particles will presumably have followed the gravitational clumping of ordinary matter. Their number densities should not, however, usually have become sufficiently high for much annihilation to occur. Any L^- produced should have been combined into tightly-bound pL^- systems, while L^+ should occur in $\bar{\mathrm{p}}\mathrm{L}^+$ or, in the absence of many $\bar{\mathrm{p}}$, $\mathrm{L}^+\mathrm{e}^-$ composites. The fact that the lightest strange and charmed baryons do not undergo strong decay indicates that the lightest baryon carrying a new absolutely-conserved quantum number should not be able to decay into a meson carrying the same quantum number and should therefore be stable. These new stable baryons and mesons should be bound into ordinary nuclei. Any $\mathrm{L}^\pm$ and H produced in the early universe should therefore occur in terrestrial material.

Another source of heavy stable particles is pair production by the interaction of cosmic ray particles with the earth's atmosphere. Assuming that all L^+ will eventually get into water, this gives [4] $n_p(\mathrm{L}^+) \approx 10^{-22} [m_L(\mathrm{GeV}/c^2)]^5$/nucleon ‡4. The cosmic-ray-induced heavy hadron abundance should be about $2 \times 10^{-18} [m_H[\mathrm{GeV}/c^2]]^{-6}$/nucleon. These abundances are insignificant compared to those expected from heavy particle production in the early universe.

There have been a number of searches for heavy integer-charged stable particles, mostly in sea water. The best published experiment [9] found no such particles in 3×10^{18} nucleons, for almost all masses between 6 and $16\ \mathrm{GeV}/c^2$. When combined with the abundances expected form the early universe, this result suggests that no stable integer-charged particles exist with $1 \lesssim m \lesssim 16\ \mathrm{GeV}/c^2$. The most sensitive search yet made is presently being performed [10] using a mass spectrometer to scan the equivalent of 10^8 kg of sea water. This experiment should detect concentrations down to one new particle in $\sim 10^{20}$ nucleons, for $3 \lesssim m \lesssim 300\ \mathrm{GeV}/c^2$. Modern nuclear physics accelerator techniques, if applied to the same sample, should allow the sensitivity of 10^{-29} new particles per nucleon to be reached

‡3 This result comes from the requirement that the neutrinos should not so alter the expansion rate of the early universe as to affect the amount of ^{4}He produced [8].

‡4 If, however, stable $\mathrm{L}^\pm$ can come from the weak decays of hadrons, then their abundances should be comparable to those of their parent hadrons had those hadrons been stable.

[11]. Even if no heavy stable particles were produced in the early universe, a null result in this experiment would show that their abundance was in many cases below that expected just from their production in cosmic ray interactions. The conclusions that no such particles exist (with masses less than several hundred GeV/c^2) would then surely be inescapable, placing an important constraint on present and future models in particle physics.

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The Development of Baryon Asymmetry in the Early Universe

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ABSTRACT

The development of an excess of baryons over antibaryons due to CP and baryon number violating reactions during the very early stages of the big bang is calculated in simple models using the Boltzmann equation.

There are observational and theoretical indications that the local preponderance of baryons over antibaryons extends throughout the universe (at least since the time when the temperature $T \sim 100$ MeV) with an average ratio of baryon to photon densities [1] $n_B/n_\gamma \equiv Y_B \simeq 10^{-9}$. If baryon number (B) were absolutely conserved in all processes, this small baryon excess must have been present since the beginning of the universe. However, many grand unified gauge models [2] require superheavy particles (typically with masses $m_X \sim 10^{15}$ GeV $\equiv$ 1 IIeV) which mediate baryon- and lepton-number (L) violating interactions. Any direct evidence for these must presumably come from an observation of proton decay. In the standard hot big bang model [1], the temperature T (of light particle species) in the early universe fell with time t according to (taking units such that $\hbar = c = k = 1$) $T \sim \sqrt{m_P/2t}$ where $m_P = (45/8\pi^3)^{1/2}\, m_{\mathcal{P}}/\sqrt{\xi(T)} \simeq 5 \times 10^3/\sqrt{\xi}$ MeV, and $m_{\mathcal{P}} = G^{-1/2} \simeq 10^{19}$GeV is the Planck mass, while ξ gives the effective number of particle species in equilibrium ($\xi = \frac{1}{2}(\frac{7}{16})$ for each ultrarelativistic boson (nondegenerate fermion) spin state). At temperatures $T \gtrsim m_X$, B-violating interactions should have been important, and they should probably have destroyed or at least much diminished any initial baryon excess. (This occurs even when, for example, B-L is absolutely conserved, since then an initial baryon excess would presumably be accompanied by a lepton excess, so as to maintain the accurate charge neutrality of the universe.) It is interesting (and in some models necessary) to postulate that B-violating interactions in the very early universe could give rise to a calculable baryon excess even from an initially symmetrical state. For this to be possible, the rates for reactions producing baryons and antibaryons must differ, and hence the interactions responsible must violate CP invariance. We describe here a simple but general method for calculating B generation in any specific model. We clarify and extend previous estimates [3].

Let $M(i \to j)$ be the amplitude for transitions from the state i to j, and let $\bar{i}$ be the CP conjugate of i (particles $\to$ antiparticles and spins reversed). Then CPT invariance demands $M(i \to j) = M(\bar{j} \to \bar{i})$, while CP invariance would require $M(i \to j) = M(\bar{i} \to \bar{j}) = M(j \to i)$. Unitarity [4] (transitions to and from i must occur with total probability 1) demands $\sum_j |M(i \to j)|^2 = \sum_j |M(j \to i)|^2$; combining this with the constraint of CPT invariance yields (the sum over j includes all states and their antistates)

$$\sum_j |M(i \to j)|^2 = \sum_j |M(\bar{i} \to j)|^2 = \sum_j |M(j \to \bar{i})|^2 = \sum_j |M(j \to i)|^2 \text{ . } \quad (1)$$

In thermal equilibrium (and in the absence of chemical potentials representing nonzero conserved quantum numbers) all states j of a system with a given energy are equally populated. Then the last equality in (1) shows that transitions from these states (interactions) must produce i and $\bar{i}$ in equal numbers; thus no excess of particles over antiparticles may develop in a system in thermal equilibrium, even if CP is violated. In addition, the first equality in (1) shows that the total cross-sections for destroying particles and antiparticles must be equal. Since in thermal equilibrium no excess of i over $\bar{i}$ may develop, this implies that any initial excess must be destroyed.

The phase space distribution $f_i(p)$ (number per unit cell $d^3\vec{p}\, d^3x$ [5]) for a species i develops with time (on average) according to a Boltzmann transport equation. A closed system with no external influences obeys Boltzmann's H theorem (which holds regardless of T (i.e., CP) invariance [6]), so that from any initial state the $f_i(p)$ evolve (on average) to their equilibrium forms for which $f_{\bar{i}}(p) = f_i(p)$, and no baryon excess may survive.

However, in an expanding universe, extra terms must be added to the Boltzmann equations, and if some participating particles are massive [7], a baryon excess may be generated; the relaxation time necessary to destroy the excess often increases faster than the age of the universe [8].

Equation (1) requires that the total rates for processes with particle and antiparticle initial states be equal. CP violation allows the rates for specific conjugate reactions to differ; unitarity nevertheless requires ($T = i(1-S)$, $SS^\dagger = S^\dagger S = 1$) [9]:

$$|M(i\to j)|^2 - |M(\bar{i}\to\bar{j})|^2 = |T_{ij}|^2 - |T_{ji}|^2$$

$$= 2\,\mathrm{Im}[(\sum_n TT^\dagger)_{ij}\, T^*_{ji}] - |(\sum_n TT^\dagger)_{ij}|^2 . \qquad (2)$$

Hence the fractional difference between conjugate rates must be at least $O(\alpha)$ where α is some coupling constant [10]. Moreover, the loop diagrams giving CP violation must allow physical intermediate states n. (These loop corrections must usually also be B-violating to give a difference in rates when summed over all final states $\overset{(-)}{j}$ with a given (−)B [11].)

Let $\overset{(-)}{b}$ be an "(anti)baryon" with $B = \underset{(-)}{+} \frac{1}{2}$. For simplicity we assume here that all particles (including photons) obey Maxwell-Boltzmann statistics and have only one spin state. In our first (very simple) model, we consider CP, B violating 2↔2 reactions involving $\overset{(-)}{b}$ and a heavy neutral particle ϕ; we take their rates to be (this parametrization ensures unitarity and CPT invariance)

$$|M(bb\to\bar{b}\bar{b})|^2 = (1+\zeta)|M_o|^2/2$$

$$|M(bb\to\phi\phi)|^2 = |M(\phi\phi\to\bar{b}\bar{b})|^2 = (1-\zeta)|M_o|^2/2$$

$$|M(\bar{b}\bar{b}\to bb)|^2 = (1+\bar{\zeta})\ |M_o|^2/2$$

$$|M(\bar{b}\bar{b}\to\phi\phi)|^2 = |M(\phi\phi\to bb)|^2 = (1-\bar{\zeta})\ |M_o|^2/2 \tag{3}$$

where $\zeta-\bar{\zeta} = O(\alpha)$ measures the magnitude of CP violation. The number of a species i per unit volume $n_i \equiv \int d^3\vec{p}/(2\pi)^3\ f_i(\vec{p})$ decreases with time even without collisions in an expanding universe according to (R is the Robertson-Walker scale factor; dots denote time derivatives)

$$\frac{dn_i}{dt} = N_i\ \frac{d(1/V)}{dt} = -\frac{3\dot{R}}{R}\,n_i = \frac{3\dot{T}}{T}\,n_i = -\frac{3T^2}{M_P}\,n_i\,. \tag{4}$$

The n_i are also changed by collisions; the (average) time development of the ϕ and baryon number ($n_B' \equiv n_b - n_{\bar{b}}$) densities is given by the Boltzmann equations ($Y_i \equiv n_i/n_\gamma$ where γ is a massless particle; $|M_o|^2 = O(\alpha^2)$)

$$\begin{aligned} n_\gamma\dot{Y}_\phi &= \dot{n}_\phi + \frac{3\dot{R}}{R}\,n_\phi = 2\Lambda^{34}_{12}\Big[f_b(p_1)\ f_b(p_2)|M(bb\to\phi\phi)|^2 \\ &+ f_{\bar{b}}(p_1)\ f_{\bar{b}}(p_2)|M(\bar{b}\bar{b}\to\phi\phi)|^2 - f_\phi(p_1)\ f_\phi(p_2)(|M(\phi\phi\to bb)|^2 \\ &+ |M(\phi\phi\to\bar{b}\bar{b})|^2)\Big] \end{aligned} \tag{5a}$$

$$\begin{aligned} n_\gamma\dot{Y}_B &= \Lambda^{34}_{12}\Big[-f_b(p_1)\ f_b(p_2)(2|M(bb\to\bar{b}\bar{b})|^2 + |M(bb\to\phi\phi)|^2) \\ &+ f_{\bar{b}}(p_1)\ f_{\bar{b}}(p_2)(2|M(\bar{b}\bar{b}\to bb)|^2 + |M(\bar{b}\bar{b}\to\phi\phi)|^2) \\ &+ f_\phi(p_1)\ f_\phi(p_2)(|M(\phi\phi\to bb)|^2 - |M(\phi\phi\to\bar{b}\bar{b})|^2)\Big]\quad , \end{aligned} \tag{5b}$$

where the operator Λ represents suitable integration over initial and final state momenta. We assume that the $\overset{(-)}{b}$ undergo baryon-conserving collisions with a frequency much higher than the $O(\alpha^3)$ rate on which n_B changes (as is presumably the case in realistic models). They are therefore always in kinetic equilibrium with the rest of the universe, and hence Maxwell-Boltzmann distributed in phase space:

$$f_{\overset{(-)}{b}}(p) \simeq \exp[-(E \overset{-}{(+)} \mu)/T]$$

$$Y_B \equiv (n_b - n_{\bar{b}})/n_\gamma \simeq 2 \sinh(\mu/T) \quad . \tag{6}$$

μ is a baryon number chemical potential, which is changed only by B-violating processes, and would vanish if chemical equilibrium prevailed. Assuming $Y_B \ll 1$, one may use momentum conservation in (5) to write $f_{\overset{(-)}{b}}(p_1)\, f_{\overset{(-)}{b}}(p_2) \simeq \exp[-(E_3+E_4)/T](1 \overset{+}{(-)} Y_B) \simeq f_\phi^{eq}(p_3) f_\phi^{eq}(p_4)(1 \overset{+}{(-)} Y_B)$, where $f_\phi^{eq}(p) = \exp(-E/T)$ is the equilibrium distribution of ϕ at temperature T: The equilibrium ϕ number density $n_\phi^{eq} = T^3/(2\pi^2)(m_\phi/T)^2\, K_2(m_\phi/T)$, where K_2 is a modified Bessel function [12] (as $m_\phi \to 0$, $n_\phi^{eq} \to T^3/\pi^2$; as $T \to 0$, $n_\phi^{eq} \to (m_\phi T/2\pi)^{3/2} \exp(-m_\phi/T)$). Then substituting the parametrization (3) and performing phase space integrations, (5) becomes

$$\dot{Y}_\phi \simeq n_\gamma \langle \sigma_o v \rangle \{2[1 - (\tfrac{\zeta+\bar{\zeta}}{2})][(Y_\phi^{eq})^2 - Y_\phi^2] - (\zeta - \bar{\zeta})(Y_\phi^{eq})^2\, Y_B\} \quad , \tag{7a}$$

$$\dot{Y}_B \simeq n_\gamma \langle \sigma_o v \rangle \{(\tfrac{\zeta-\bar{\zeta}}{2})[Y_\phi^2 - (Y_\phi^{eq})^2] - [3 + (\tfrac{\zeta+\bar{\zeta}}{2})](Y_\phi^{eq})^2\, Y_B\} \; , \tag{7b}$$

where $\langle \sigma_o v \rangle$ is the cross-section corresponding to $|M_o|^2$ averaged over a flux of incoming particles in equilibrium energy distributions. Equation (7b) exhibits *the necessity of deviation from equilibrium for* $\dot{B}$ *generation, and the destruction of* Y_B in equilibrium.

We now turn to a slightly more realistic but more complicated model in which massive particles $\overset{(-)}{X}$ decay to $\overset{(-)}{b}$ with rates ($\gamma_X = O(\alpha)$)

$$
\begin{aligned}
|M(X \to bb)|^2 &= |M(\bar{b}\bar{b} \to \bar{X})|^2 = (1+\eta)\gamma_X/2 \\
|M(X \to \bar{b}\bar{b})|^2 &= |M(bb \to \bar{X})|^2 = (1-\eta)\gamma_X/2 \\
|M(\bar{X} \to bb)|^2 &= |M(\bar{b}\bar{b} \to X)|^2 = (1-\bar{\eta})\gamma_X/2 \\
|M(\bar{X} \to \bar{b}\bar{b})|^2 &= |M(bb \to X)|^2 = (1+\bar{\eta})\gamma_X/2 \quad .
\end{aligned}
\tag{8}
$$

Note that if $\overset{(-)}{X}$ decays preferentially produce b, then CPT invariance implies that $\bar{b}$ are preferentially destroyed in inverse processes; thus $\overset{(-)}{X}$ decays and inverse decays (DID) alone would generate a net B even if all particles were in thermal equilibrium, in contravention of the theorem (1) [13]. However, the CP violation parameter $(\eta-\bar{\eta})$ is $O(\alpha)$, and hence changes in n_B from DID are of the same order as $2\to 2$ scattering processes, such as $bb \to \bar{b}\bar{b}$. It will turn out that s-channel exchange of nearly on-shell $\overset{(-)}{X}$ in $\overset{(-)}{b}\,\overset{(-)}{b} \leftrightarrow bb$ cancels the DID contribution to $\dot{Y}_B$ so as to recover $\dot{Y}_B = 0$ in thermal equilibrium. In direct analogy with eqs. (5) and (7), and using the assumption (6), the equation for the evolution of the $\overset{(-)}{X}$ number density $n_{\overset{(-)}{X}} \equiv Y_{\overset{(-)}{X}} n_\gamma$ becomes

$$
\dot{Y}_X = -\langle\Gamma_X\rangle\{(Y_X - Y_X^{eq}) - \bar{\eta}\, Y_B\, Y_X^{eq}\} \quad ; \tag{9}
$$

the corresponding equation for $\dot{Y}_{\bar{X}}$ is obtained by charge conjugation ($Y_X \leftrightarrow Y_{\bar{X}}$, $Y_B \to -Y_B$, $\eta \leftrightarrow \bar{\eta}$). The $\langle\Gamma_X\rangle$ in (9) is the total $\overset{(-)}{X}$ decay width multiplied by the time dilation factor m_X/E_X and averaged over the equilibrium X energy distribution [14]. The baryon concentration evolves according to

$$\dot{Y}_B = \langle\Gamma_X\rangle\{\eta\, Y_X - \bar{\eta}\, Y_{\bar{X}} + (\eta - \bar{\eta})\, Y_X^{eq} - 2Y_B\, Y_X^{eq}\}$$

$$- \frac{2}{n_\gamma} \Lambda_{12}^{34}\, \{f_X^{eq}(p_1 + p_2)(|\mathcal{M}'(bb \to \bar{b}\bar{b})|^2$$

$$- |\mathcal{M}'(\bar{b}\bar{b} \to bb)|^2\}$$

$$- 2Y_B \Lambda_{12}^{34}\, \{f_X^{eq}(p_1 + p_2)(|\mathcal{M}'(bb \to \bar{b}\bar{b})|^2 + |\mathcal{M}'(\bar{b}\bar{b} \to bb)|^2)\} \tag{10}$$

where the first term is from DID (and does not separately vanish when $Y_{\overset{(-)}{X}} = Y_X^{eq}$), while the second two terms arise from $2 \to 2$ scatterings. The DID term accounts for sequential inverse decay and decay processes involving real $\overset{(-)}{X}$: *these are therefore subtracted from the true* $2 \to 2$ *scattering terms* by writing $|\mathcal{M}'(i \to j)|^2 = |\mathcal{M}(i \to j)|^2 - |\mathcal{M}_{RIX}(i \to j)|^2$, where $\mathcal{M}_{RIX}(i \to j)$ is the amplitude for $i \to j$ due to on-shell s channel X exchange. In the narrow X width approximation, $|\mathcal{M}_{RIX}(i \to j)|^2 \sim |\mathcal{M}(i \to \overset{(-)}{X})|^2\, |\mathcal{M}(\overset{(-)}{X} \to j)|^2/\Gamma_X$; the presence of the Γ_X denominator renders it $O(\alpha)$. According to the theorem (1), the CP violating difference of total rates $|\mathcal{M}(bb \to \bar{b}\bar{b})|^2 - |\mathcal{M}(\bar{b}\bar{b} \to bb)|^2 = O(\alpha^3)$. Hence $|\mathcal{M}'(bb \to \bar{b}\bar{b})|^2 - |\mathcal{M}'(\bar{b}\bar{b} \to bb)|^2 = |\mathcal{M}_{RIX}(\bar{b}\bar{b} \to bb)|^2 - |\mathcal{M}_{RIX}(bb \to \bar{b}\bar{b})|^2 + O(\alpha^3) = O(\alpha^2)$, and the second term in eq. (10) becomes $-2\langle\Gamma_X\rangle(\eta - \bar{\eta})Y_X^{eq}$, thereby elegantly cancelling the first term in thermal equilibrium. Finally, therefore,

$$\dot{Y}_B = \langle\Gamma_X\rangle\,\{(\eta - \bar{\eta})[(Y_X + Y_{\bar{X}})/2 - Y_X^{eq}]$$

$$+ (\eta + \bar{\eta})(Y_X - Y_{\bar{X}})/2\}$$

$$- 2Y_B\{\langle\Gamma_X\rangle Y_X^{eq} + n_\gamma\langle v\sigma'(bb \to \bar{b}\bar{b}) + v\sigma'(\bar{b}\bar{b} \to bb)\rangle\} \quad . \tag{11}$$

The differential equations (10) and (11) must now be solved with the initial condition $Y_X(t{=}0) = Y_X^{eq}(0)$, and possibly an initial baryon density

Y_B. Figure 1 shows the solutions with guesses for parameters based on the SU(5) model [2] ($m_X = 10^{15}$ GeV and 10^{14} GeV; $\alpha \simeq 1/40$ (vector decays), or 10^{-3} (scalar decays)). If all $\overset{(-)}{X}$ initially in thermal equilibrium decayed with no back reactions, the Y_B generated would be simply $\eta-\bar{\eta}$. For small α or large m_X/m_P this upper limit is approached. (At small $x \equiv m_X/T$, series solution of equations (10) and (11) gives $(Y_X + Y_{\bar{X}})/2 \simeq 1 - ax^5/20$; $(Y_X - Y_{\bar{X}})/2 \simeq (\eta-\bar{\eta})a^2x^8/160$; $Y_B \simeq (\eta-\bar{\eta})ax^5/20$, where $a = m_P\Gamma_X/m_X^2$.) For $T \ll m_X$, baryon number is destroyed by $2 \to 2$ reactions with $\sigma \sim \alpha^2T^2/m_X^4$ roughly like $Y_B(T) \sim \exp[\alpha^2 m_P T^3/m_X^4]$ [15], so that $Y_B \to$ constant as $T \to 0$, but if m_X is small, the final Y_B is much diminished from its value at higher T. The Y_B generated is always roughly linearly proportional to $\eta-\bar{\eta}$, but is a sensitive function of m_X/m_P and α; for realistic values of these parameters, a numerical solution is probably essential.

According to equation (11), any baryon excess existing at the Planck time $t_P = 1/m_P$ should be diminished by inverse decays at $T \gg m_X$ so that $Y_B(t)/Y_B(t_P) \sim \exp[-\alpha m_X m_P/T^2]$; any initial Y_B should be reduced by a factor $\sim \exp[-m_P/m_X]$ before CP violating processes can generate Y_B at $T \lesssim m_X$. B-violating $2 \to 2$ scatterings at temperatures $m_P > T > m_X$ should reduce an initial Y_B by a factor $\sim\exp[-m_P \int_{m_P}^{m_X} \langle v\sigma\rangle\, dT]$. One might expect that $\langle v\sigma\rangle \sim \alpha^2/m_X^2$ at high energies due to t-channel vector X exchange; however, the effective $\langle v\sigma\rangle$ presumably relevant for the Boltzmann equation is rather $\langle v\sigma_{eff}\rangle \sim \alpha^2/\lambda_D^2$ where the Debye screening length $\lambda_D \sim [\sqrt{32\alpha}\, T]^{-1}$. In this approximation $2 \to 2$ and higher multiplicity collisions are probably no more effective at destroying an initial Y_B than are inverse decays.

We conclude therefore that B-violating reactions in the very early universe might well destroy any initial baryon number existing around the Planck time $(1/m_P)$, requiring subsequent B and CP-violating interactions to generate the observed baryon asymmetry. The methods described here [16] allow a calculation of the resulting baryon excess in any specific model; the simple examples considered suggest that the observed Y_B should place stringent constraints on parameters of the model.

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4. V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, Relativistic Quantum Theory (Pergamon, Oxford, 1971), p. 239. In the present context the importance of unitarity was emphasized by Toussaint, et al., ref. 3.

5. We assume homogeneity and isotropy, so that $f(\vec{p},x) = f(\vec{p}) = f(p)$.

6. See, e.g., A. Aharony in Modern Developments in Thermodynamics (Wiley, New York, 1973), pp. 95–114.

7. It is not necessary that these participate directly in B violating reactions.

8. In the simple models discussed below, this phenomenon occurs if the universe is homogeneous and always cools faster than $T \sim m_P/(tm_P)^{1/5}$; in practice the quark excess will always be contained in baryons where their probability for collisions remains constant rather than falling as in a homogeneous expanding universe.

9. This constraint applies only if no initial or final particles may mix with their antiparticles (as in the K^o system). CP-violating mixing requires a difference $M(i \to j) - M(\bar{i} \to \bar{j}) \neq 0$ in amplitudes rather than rates.

10. Regardless of perturbation theory, CP violation is asymptotically suppressed by powers of log(s), where $\sqrt{s}$ is the invariant mass of the initial state.

11. D. V. Nanopoulos and S. Weinberg, "Mechanisms for Cosmological Baryon Production," Harvard University Preprint 79/4023.

12. I. S. Gradestyn and I. M. Ryzhik, Table of Integrals Series and Products (Academic Press, New York, 1965), p. 951.

13. This rather relevant point has also been noticed by A. D. Dolgov and Ya. B. Zeldovich (ref. 3), but was apparently neglected elsewhere.

14. Strictly, m_X/E_X should be averaged separately for the various terms of (9); if X is in kinetic equilibrium, however, the averages are equal. Note that we have implicitly assumed all produced and decaying $\overset{(-)}{X}$ to be exactly on their mass shells. However, particularly at high T, the mean $\overset{(-)}{X}$ collision time << $1/\Gamma_X$, so that the $\overset{(-)}{X}$ resonance is collision broadened, and produced or decaying $\overset{(-)}{X}$ may be far off shell. The m_X/E_X factor for inverse decays essentially arises from the fact that the incoming particles must subtend a sufficiently small angle to have invariant mass m_X; if produced $\overset{(-)}{X}$ are far off shell, the m_X/E_X in DID should disappear.

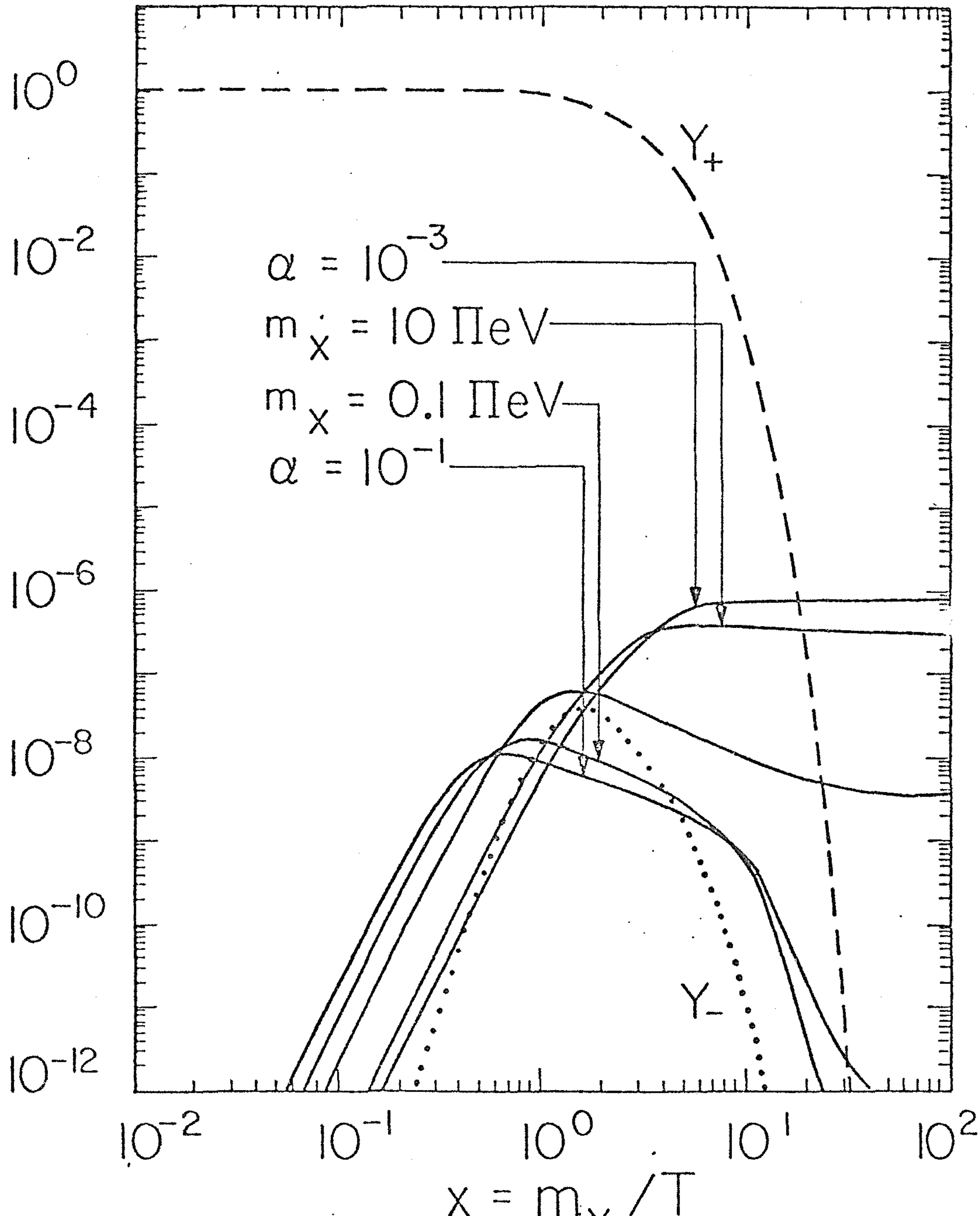
15. If $T \sim m_P/(tm_P)^{1/5}$ as in footnote 8, then $Y_B \sim \exp(-1/T)$; the universe expands sufficiently slowly for Y_B to relax to zero.

16. Many details are given in E. W. Kolb, and S. Wolfram, "Baryon Number Generation in the Early Universe," Caltech report OAP 547.

17. Results depend only on m_X through the dimensionless combination m_X/m_P; here we take $\xi = 100$ in the definition of m_P. Note that inhomogeneities in the early universe may be manifest in different expansion rates and hence different effective ξ for different regions. The final Y_B produced could vary considerably between the regions.

Figure Caption

The development of baryon number density (solid curves) as a function of inverse temperature in the model of eq. (11) for various choices of parameters (unless otherwise indicated, $\alpha = 1/40$ and $m_X = 1$ IIeV $\equiv 10^{15}$ GeV [17]). The dashed and dotted curves give $(Y_X + Y_{\bar{X}})/2$ and $(Y_X - Y_{\bar{X}})/2$, respectively. In all cases we have taken the CP violation parameter, $\eta - \bar{\eta} = 10^{-6}$.


10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
Y_+
$\alpha = 10^{-3}$
$m_X = 10$ TeV
$m_X = 0.1$ TeV
$\alpha = 10^{-1}$
Y_-
10^{-2}
10^{-1}
10^0
10^1
10^2
$x = m_X / T$