Pre-Auction Investment and Equivalence of Auctions

Thesis by

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Abstract

In this thesis we investigate some extensions of game theoretic auction models and models of R&D by allowing the participants' cost of producing an indivisible object to be determined by their R&D decisions prior to the auctioning of a fixed price production contract. We establish that when the production cost distributions are endogenously determined as a result of private investment expenditures which are only privately observable, first and second price auctions are equivalent : both give rise to the same level of total investment, same reserve price, same expected price to the buyer and same expected level of profits for the sellers, at the *symmetric* Nash equilibria. This is an extension of the equivalence results known in the context of standard independent private value auction models with risk neutral bidders. We also show using a discrete cost model that, when investment is observable, the requirement of subgame perfection eliminates the symmetric investment equilibrium from the set of equilibria in pure strategies, and the only pure strategy equilibria are asymmetric. The buyer's optimal response to this asymmetry in the investment equilibria is to reduce her reserve price so that equilibrium total investment level is lower when the buyer knows that the sellers know one another's investment levels. We also consider ex ante incentives to collude under first and second price auctions and find that equilibrium patterns of collusion differ significantly. Finally, we report some experimental results.

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Chapter 1

INTRODUCTION AND OVERVIEW

In this thesis we investigate some extensions of game theoretic auction models and models of R&D by allowing the participants' cost of producing an indivisible object to be determined endogenously by their R&D (or search) decisions *prior* to the auctioning of a *fixed price* production contract.

Procurement contracts and procurement auctions have been an area of extensive research activity in recent years ([1], [2], [3], [4], [12], [13], [19], [28], [29], [30], [31], [58], [64], [67], [68], [71]). The theory of auctions is an important part of the analytical tools used in most of the theoretical studies that address the economic problems in the procurement area.

In a standard auction model, a monopsonist who wants to buy an indivisible object faces a number of potential sellers. The case of a monopolist facing a number of potential buyers is essentially identical modulo some changes in the signs, and interpretations, of some of the variables in the models. Although most of the results on auctions we refer to in this study are stated for the monopoly case in the literature, in what follows we will be translating those results to the monopsony context. (See McAfee and McMillan [43] and Engelbrecht-Wiggans [17] for a survey of the literature on auctions.)

The cost conditions of each seller is known only to himself [1]. The buyer and the other sellers are uncertain about what the production cost of any particular seller might be. This uncertainty is modeled by specifying a probability distribution over the set of cost conditions a seller might possibly have. In this formulation, each auction procedure is viewed as a game of incomplete information (Harsanyi [26]), and standard tools from the theory of games are used to study equilibrium outcomes.

In all these auction models, the distribution of a seller's production cost is taken as the starting point in the analysis. Situations in which the distribution of a seller's production cost is determined as a result of his investment decisions prior to the auction remain to be studied[2].

Considered from the viewpoint of another body of research in economics, namely the game theoretic models of research and development (e.g., Dasgupta and Stiglitz [11]; Lee and Wilde [33]; Loury [36]; Sah and Stiglitz [63]; Reinganum [54],[55],[56]; Flaherty [21]; Fudenberg and Tirole [22]; Mariotti [38]), such situations constitute important special cases that may shed more light on some of the results on the relation between market structure and the social optimality of equilibrium R&D behavior in the R&D literature. In a typical R&D model, the return to R&D activity is a patent. The details of the workings of the market are represented in a general return function. In studying the importance of market structure for R&D, the focus of these models has been on one side of the market : the number of contestants in the race and the form of ensuing competition among the successful innovators. The structure of the demand side of the market is usually taken to be strate-

[1] Throughout this thesis the masculine pronouns will be used when referring to a seller, and the feminine when referring to the buyer.

[2] Cases where the production costs are determined by investment decisions *after* the auction can be analyzed within the standard auction framework as in Rob [60].

gically neutral by assuming a fixed demand function. In this sense, models of a situation in which returns to the R&D activity are determined in a subsequent auction held by a monopsonist can also be seen as a specialization of the R&D models.

An extension of the existing auction models to accommodate the pre-auction investment feature is important for several reasons. First of all, pre-auction investment adds a new dimension to the standard auction problem. This immediately implies a set of new questions regarding 1) the analysis of equilibrium behavior under a given auction institution, and comparison of alternative auction institutions, and 2) the characterization of the optimal auction institution for a given environment. The focus of this thesis is on the implications of pre-auction R&D for behavior under given auction institutions.

In the theory of auctions, two polar cases of the correlation in the joint distribution of production costs have been extensively studied: private costs and common costs. In this thesis, we consider auctions with *independent private costs.* Comparison of alternative auction procedures constitutes one of the two main bodies of research in the area. In particular, two most common types of auctions, English and sealed bid first price auctions, have been compared in terms of the equilibrium outcomes that arise under various environments. In the present framework, a natural question would be the comparison of the equilibrium levels of R&D the two auction institutions induce. Moreover, this new feature could affect the equilibrium outcomes along the dimensions already incorporated into the standard auction framework, e.g., equilibrium price distribution. This extension is also useful to gain new insight on, and study the "robustness" of, some results obtained in the standard framework such as the *revenue equivalence* .

1.1 Summary of the Thesis

The thesis contains five chapters. The common theme of all five chapters is the equivalence of first and second price auctions. In Chapters 2 and 3 we study first and second price auction with pre-auction investment. Chapter 4 is a study of collusion in first and second price auctions. Chapter 4 is related to Chapter 3 in that it studies the implications of results on bidding equilibrium obtained in Chapter 3. Finally, Chapter 5 reports some experimental results on the equivalence of the two auctions and on the model developed in Chapter 3.

Summary of Chapter 2

We consider a situation where a fixed number of sellers engage in private research and development activities to determine the production cost of an item that may be sold at a subsequent auction.

The specification of the relation between the R&D expenditure and the distribution of production cost makes an important difference with respect to the structure of the auctions that follow R&D. We consider two general specifications of the search technology. In both specifications, R&D is modeled following the paradigm of independent sampling from a known distribution: 1) non-sequential, and 2) sequential search.

We establish that when the production cost distributions are endogenously determined as a result of private investment expenditures, first and second price auctions are equivalent : Both give rise to the same level of total investment, same reserve price, same expected price to the buyer and same expected level of profits for the sellers, at the *symmetric* Nash equilibria. This is an extension of the equivalence results known in the context of standard independent private value auction models with risk neutral bidders.

We also show that total investment is independent of the number of sellers for any given reserve price. However, when we include the buyer's strategic decisions, equilibrium total investment is always less than the socially optimum level and increases with the number of sellers.

Final result from Chapter 2 is that if pre-auction search is sequential, the buyer can extract all surplus by setting a reserve price equal to the expected cost conditional on the optimal search policy.

Throughout Chapter 2 we assume that a seller's investment level is observed only by himself. Although all the results we obtained for second price auctions remain valid when sellers can observe one another's investment levels before the auction, bidding behavior of a seller will depend on the distribution of his rivals' production costs, and hence, on the rivals' investment levels. Therefore, the results we obtained for first price auctions depend on the investment unobservability assumption.

Summary of Chapter 3

In Chapter 3 we study the effect of observability of investment levels by the sellers at the auction stage on bidding and pre-auction investment behavior using a model where the production cost can take only two values. We show that when investment is observable at the bidding stage, all pure strategy equilibria are asymmetric in the investment stage under the first price auction. We study the implications of this asymmetry for the equivalence of the two auctions in several dimensions.

We obtain a number of results from the model regarding the equivalence of first and second price auctions. We know that observability of investment does not affect behavior in the second price auction, since bidding the true production cost is a dominant strategy regardless of how the production costs of the rival sellers are distributed. In the first price auction, however, the set of equilibrium outcomes changes when investment is observable: The requirement of subgame perfection eliminates the symmetric investment equilibrium from the set of equilibria in pure strategies, and the only pure strategy equilibria are asymmetric. Total investment level, however, is not affected by this asymmetry for a given reserve price. The buyer's optimal response to this asymmetry in the investment equilibria is to reduce her reserve price so that equilibrium total investment level is lower when the buyer knows that the sellers know one another's investment levels.

We also consider the effect of investment observability on the maximum number of sellers when the number of sellers is endogenous. Auctions with entry and with fixed entry scale are studied by McAfee and McMillan [44]. Samuelson [65] studied bidding with entry cost. In the context of common value auctions, Lee's [32] study of information acquisition can be interpreted as a model with an endogenous entry scale. We show that with a fixed entry cost, first price auctions attract fewer sellers than second price auctions when the number of sellers and the scale of entry are endogenous.

Summary of Chapter 4

In Chapter 4 we study the *ex ante* stability of collusive agreements under the first and second price auctions. Relative conduciveness of the auction procedures to collusive practices is an important aspect of the problem of comparing the two auction types, and this problem has been studied by several authors (Robinson [61]; Graham and Marshall [23]; McAfee and McMillan [45]; Mailath and Zemsky [37]; Feinstein et al. [20]; Zemsky [72]).

The focus of most of these studies has been on information revelation problems faced by a collusive ring in dealing with the private information of its members, and on the buyer's reaction to the existence of collusion among the sellers. The incentive problems within the collusive ring point to the difficulty of collusion under the first price auction (Robinson [61]) relative to the second price auction. The focus of our study is on the *ex ante* incentives to form collusive rings in the absence of enforcement problems.

In a first price auction, the existence of collusive rings affects bidding behavior. In particular, the bidding behavior would be different for a seller if he faces n rivals from the case where he faces one rival with "size " n. This effect on the bidding behavior feeds back into the initial incentives to form rings even in the absence of further incentive problems within a ring once it is formed. One such possible impediment to formation of rings under the first price auction is a positive *externality* of a ring on non-members in the sense that sellers outside the ring may prefer to face one large rival bidder rather than many small rivals.

We require a collusive ring structure to be immune to individual deviations in the sense that no member of any ring would have any *ex ante* incentive to leave the ring he is in, in order to join another ring or to act alone. This is a rather weak requirement of "stability" and yet, as we find, it restricts the possible ring structures considerably.

We show that in second price auctions, the grand collusive ring of all sellers is the only individually stable collusive ring structure. For first price auctions, we find that the set of all sellers need not be stable and that all individually stable ring structures have the same form : a collusive ring that contains at least three sellers (two sellers, if there are no more than two sellers) and a "competitive fringe" made up of the remaining sellers each of whom acts independently. The number of sellers involved in a collusive agreement is a decreasing function of the probability of high production cost. This number is independent of the collusive ring structure that actually forms. If the total number of firms exceeds three, then there are at least three firms that form a ring independent of the number of sellers.

We discuss the distribution of profits under an individually stable collusive ring structure, and find that *every* seller benefits from the existence of a ring, and that sellers outside the ring benefit more than the sellers inside. We also discuss the implications of this profit structure for collusion.

Summary of Chapter 5

Chapter 5 reports some experimental results on the model developed in Chapter 3. Laboratory experimental methods have been used extensively to study the common forms of oral and sealed bid auctions ([6],[7],[53]). Our experimental focus is on the pre-auction investment decisions, and on the implications of investment observability for investment behavior under first and second price auctions.

The aggregate predictions based on the model are largely borne out by the observations. At the level of market aggregates, the two auctions and the two observability treatments are indistinguishable at the pure strategy equilibria. Further tests on the observed distributions of investment under the two information treatments and the two auctions reveal that the model is not entirely accurate in explaining the observations. The discrepancies seem to be related to the the fact that the experimental setting is, in fact, a repeated game situation.

Chapter 2

PRE-AUCTION R&D BEHAVIOR IN INDEPENDENT PRIVATE VALUE AUCTIONS

2.1 Introduction

We consider a situation where a fixed number of sellers engage in private research and development activities to determine the production cost of an item that may be sold at a subsequent auction. An analytical framework to study the such a situation should cover several important dimensions. We will focus on the polar cases by representing each dimension by a dichotomous variable. The first dimension we consider is the *type of auction.* In this dimension we focus on two types of sealed bid auctions: first price and second price. The second dimension is related to the *conditions of information* regarding the production costs. In the theory of auctions two polar cases of the correlation in the joint distribution of production costs have been studied extensively: private costs, and common costs. In this study we will consider auctions with *independent private costs.* In the common costs framework, pre-auction investment in information acquisition has been studied by Lee

[32] and Matthews [42]. Dasgupta [12] studies the problem of pre-auction investment in a similar framework. He considers only first price auctions with a specific production technology, which is different from the one considered in this study. Most of his conclusions also hold in our case. Tan [67] investigates the optimal procurement contracts in a similar setting for general R&D technologies, which include the specification we consider as a special case.

The third dimension we consider is the specification of the *search technology,* i.e., the relation between the R&D expenditure and the distribution of production cost. We consider two general specifications of the search technology. In both specifications, R&D is modeled following the paradigm of independent *sampling* from a known distribution [1] .

1. *Non-sequential search* : the firm cannot observe the outcome of its previous investment before deciding on additional units of investment. In other words, the R&D outlay is committed once and for all.

2. *Sequential search* : the firm can observe the outcome of its previous investment before doing additional R&D.

We take the search technology to be part of the description of the economic environment. This amounts to an implicit assumption about the conditions of search so that these two polar cases arise as the optimal search strategies, given the respective conditions of search. For a discussion of these conditions, see Morgan and Manning [51]. The non-sequential search technology gives rise to auctions in which the range of possible production costs is the same regardless of the investment profile. Under sequential search, investment strategies affect the *support* of the distribution of production cost

[1]In what follows the terms "R&D," "investment" and "search" will be used interchangeably. "Cost" will be used to refer to *production cost* . For *cost of investment* we will be using "R&D cost," "investment cost" and "search cost."

at the auction stage [2]. This distinction brings out the necessity of studying two kinds of *asymmetric auctions* (or auctions with asymmetric beliefs) (Maskin and Riley [41]) with potentially different results.

Throughout this chapter, we assume that both the production cost and the level of investment of a seller is private information. When the production cost distributions of the sellers are determined endogenously, whether or not the sellers can observe one another's investment levels before submitting their bids in the auction can make a crucial difference for behavior in first price auctions even if the buyer can not observe the sellers' investment levels. A property of second price auctions, namely, that a seller's bidding strategy is independent of what he knows about other sellers, makes investment observability inconsequential. Therefore, all the results stated below for second price auctions remain valid when investment is observable.

In the next section, we present the model for the non-sequential search technology. In Section 2.2 we characterize the socially efficient investment level as a benchmark to be used in later sections. Section 2.3 contains the characterization of equilibrium under first and second price auctions. In Section 2.4 we discuss the equivalence of the two auction procedures. In Section 2.5 we study the equilibrium behavior of the buyer. Finally, in Section 2.6 we present some results for the case where search is sequential.

2.2 The Model

This chapter addresses the questions posed above in the context of an *independent private costs* auction model. We consider a situation where a fixed

[2]Pre-auction investment in Dasgupta [12] affects the support of the cost distribution. Although this roughly corresponds to the case with sequential search, the distribution of production costs is not derived explicitly from an underlying search technology.

set, N, of *risk neutral* participants, where

$$N = \{1, 2, \ldots, n\} \tag{2.1}$$

engage in private research and development activities to determine the production cost of an indivisible object that may be sold at a subsequent auction.

R&D takes the form of search from a set of possible techniques with a known distribution. Each technique is identified with a production cost, c. Cost of producing the item is determined as a random function of the research and development expenditure undertaken by the participant. If the firm does not do any R&D, its unit cost is some fixed number $\bar{c}$. By engaging in R&D the firm may potentially reduce its production cost down to $\underline{c}$. However, the outcome of R&D is uncertain. Each unit of R&D gives rise to a *random* production cost figure between $\underline{c}$ and $\bar{c}$ according to some fixed distribution $F(\cdot)$. Formally, each unit of R&D gives rise to a random production cost C^i according to a fixed probability law $F(\cdot)$ with a continuous positive density $F'(\cdot)$, where

$$C^i \in [\underline{c}, \bar{c}], \tag{2.2}$$

and

$$F(c^i) := \text{ Prob } \{C^i \leq c^i\}, \;\; F(\underline{c}) = 0, \;\; F(\bar{c}) = 1, \;\; F'(\underline{c}) > 0. \tag{2.3}$$

If a firm invests x in R&D, its production cost is a random variable C, where

$$C := \min \{C^1, C^2, \ldots, C^x\}. \tag{2.4}$$

The cumulative distribution function (cdf) for C is

$$G(c, x) = \text{ Prob } \{C \leq c\} = 1 - [1 - F(c)]^x, \quad c \in [\underline{c}, \bar{c}]. \tag{2.5}$$

The higher the R&D expenditure the more likely it is for the firm to have a low production cost. However, there are *diminishing returns* to R&D in the sense that each additional unit of R&D gives, on average, lower reduction in expected production cost than previous units. For each unit of R&D the firm has to pay s.

The buyer has an alternative source of supply with unit production cost $\tilde{c}$, which we assume to be available to all sellers. The buyer's valuation of the object is v. We assume that production is worthwhile at the current unit production cost, i.e.,

$$v \geq \tilde{c}, \tag{2.6}$$

and that the buyer's search cost, s_b, is "prohibitively high."

The *environment* $\Lambda := \{N, \tilde{c}, \underline{c}, \bar{c}, F, s, s_b, v\}$ is *common knowledge* .

2.3 Social Efficiency

We first consider the solution to the search problem from the point of view of a buyer who values the object at v and faces the same search technology and search cost as the sellers and whose current production cost is $\tilde{c}$. An equivalent interpretation is the problem faced by a social planner who maximizes the social surplus, assigning equal weights to the suppliers' and to the buyer's profits. It is clear that *ex ante* social efficiency in this setting requires that production take place at the lowest available production cost for every level of investment in R&D.

Expected social surplus for a given R&D level x is

$$S(x; \tilde{c}, s) = v - \{\tilde{c}[1 - F(\tilde{c})]^x + \int_{\underline{c}}^{\tilde{c}} tx[1 - F(t)]^{x-1} F'(t) dt\} - sx. \tag{2.7}$$

The second term in (2.7) is the expected production cost, given the investment level x, and the third term is the cost of search. Integration by parts of the second term in brackets in (2.7) gives

$$\int_{\underline{c}}^{\tilde{c}} tx[1-F(t)]^{x-1}F'(t)dt = \underline{c} + \int_{\underline{c}}^{\tilde{c}} [1-F(t)]^x dt - \tilde{c}[1-F(\tilde{c})]^x. \qquad (2.8)$$

Using (2.8) we have an equivalent expression for the social surplus:

$$S(x;\tilde{c},s) = v - \{\underline{c} + \int_{\underline{c}}^{\tilde{c}} [1-F(t)]^x dt\} - sx. \qquad (2.9)$$

The socially optimum level of investment,

$$\hat{x}(\tilde{c},s) := \arg\max_x S(x;\tilde{c},s)\,, \qquad (2.10)$$

is given by the first order condition [3]

$$\frac{\partial S(\hat{x};\tilde{c},s)}{\partial x} = -\int_{\underline{c}}^{\tilde{c}} [1-F(t)]^{\hat{x}} \ln[1-F(t)]dt - s \;\leq\; 0. \qquad (2.11)$$

If the inequality in (2.11) is strict, we have $\hat{x} = 0$. To guarantee that a positive amount of investment is worthwhile, we assume that search cost s is small enough; that is,

$$-\int_{\underline{c}}^{\tilde{c}} \ln[1-F(t)]dt - s > 0. \qquad (2.12)$$

For such s we have a *unique* interior solution since the second order condition is satisfied with strict inequality:

$$\frac{\partial^2 S(\hat{x};\tilde{c},s)}{\partial x^2} = -\int_{\underline{c}}^{\tilde{c}} [1-F(t)]^{\hat{x}}\{\ln[1-F(t)]\}^2 dt < 0. \qquad (2.13)$$

At the optimum, marginal cost of search, s, is equated to the marginal expected return to search in terms of reduced production cost. Using the implicit function theorem we have

$$\frac{\partial \hat{x}(\tilde{c},s)}{\partial \tilde{c}} \geq 0, \qquad (2.14)$$

[3] Although the proper interpretation of the specification given in (2.1) - (2.4) requires having $x \in \mathrm{N}^+$, we treat x as a real number and approximate first and second differences by derivatives.

and

$$\frac{\partial \hat{x}(\tilde{c}, s)}{\partial s} \leq 0. \tag{2.15}$$

The maximal social surplus for given $(\tilde{c}, s)$ is

$$\hat{S}(\tilde{c}, s) := S(\hat{x}(\tilde{c}, s); \tilde{c}, s). \tag{2.16}$$

By the envelope theorem $\hat{S}(\tilde{c}, s)$ is decreasing in both of its arguments.

The assumption that the buyer's search cost is "prohibitively high" is formalized by the following condition :

$$-\int_{\underline{c}}^{\tilde{c}} \ln[1 - F(t)]dt - s_b < 0, \tag{2.17}$$

which implies that if the search cost is s_b, the buyer would choose not to invest in R&D. Note that even if the buyer's search cost were lower than the critical level implied by the inequality (2.17), it would be socially inefficient for the buyer to engage in R&D as long as s_b exceeds s.

In the next section we will show that the socially optimal level of R&D, $\hat{x}$, can be realized using a first price or a second price sealed bid auction with a reserve price $\tilde{c}$.

For later reference we will also calculate the social surplus generated when the buyer faces a single seller and commits to a reserve price $p \leq \tilde{c}$ above which she does not purchase the item. If the outcome of R &D is a production cost above the reserve price p, then the seller will choose not to produce, and the production cost will be $\tilde{c}$. The level of investment undertaken by a single seller facing a reserve price p is determined as the solution to the problem

$$\max_{x} \; p - \int_{\underline{c}}^{p} xt[1 - F(t)]^{x-1}F'(t)dt - sx. \tag{2.18}$$

If the reserve price p is such that

$$-\int_{\underline{c}}^{p} \ln[1 - F(t)]dt - s > 0, \tag{2.19}$$

then there is a unique positive solution x^* to the problem (2.18), which satisfies

$$-\int_{\underline{c}}^{p} [1-F(t)]^{x^*} \ln[1-F(t)]dt - s \;=\; 0. \tag{2.20}$$

Social surplus would then be

$$\begin{aligned} S(p;\tilde{c},s) &= S(x^*(p,s);\tilde{c},s) \\ &= v - \{\tilde{c}[1-F(p)]^{x^*} + \int_{\underline{c}}^{p} tx^*[1-F(t)]^{x^*-1}F'(t)dt\} - sx^* \\ &= v - \{\underline{c} + (\tilde{c}-p)[1-F(p)]^{x^*} + \int_{\underline{c}}^{p} [1-F(t)]^{x^*}dt\} - sx^*. \end{aligned} \tag{2.21}$$

It is easy to verify that $S(p;\tilde{c},s)$ is increasing in p for $p < \tilde{c}$.

2.4 Pre-Auction R&D

We assumed that "in-house" search by the buyer is prohibitively costly. Therefore, the buyer has to rely on search by the sellers for a possible reduction in the price she pays for the object. One trivial solution to the buyer's problem is to post a take-it-or-leave-it price equal to the expected minimum cost plus the cost of investment at the socially efficient investment level $\hat{x}(s)$ to one of the sellers before any investment takes place, or equivalently, to hold an auction before any sellers do any R&D. Since search cost is common to all sellers, such an auction would enable the buyer to extract all the expected profit of the sellers ([35], [59]). We rule out this trivial solution by assuming that the sellers cannot commit to perform and that such contracts are not enforceable. Without enforcibility, a seller will breach the contract whenever the outcome of his R&D is a production cost above the contract price, although he undertakes the optimum amount of investment under such contracts.

We will first characterize the investment and bidding behavior in a second price and a first price sealed bid auction with an arbitrary reserve price p when the investment levels are only privately observable. *Ex ante* all the potential sellers are symmetrically situated.

The timing of events is as given in Figure 2.1. The first case we consider is where the buyer can commit to a *uniform* reserve price [4] before the sellers decide on their investment levels. We will first characterize the equilibrium seller behavior in the *subgame* following an announced reserve price p under the two auction procedures. The buyer's payoff from a given reserve price will be determined by the equilibrium behavior of the sellers for that reserve price.

Under both auction procedures, after the R&D decisions and before submitting bids, each seller learns the result of his R&D, i.e., what his production cost will be if he wins the procurement auction.

When investment levels are only privately observable, either auction with a given reserve price p is a "one-stage" simultaneous move game. The strategy set, M_i, of a player $i \in N$ is a pair $(x_i, B_i(\cdot))$ of an investment level, $x_i \in \mathbf{R}^+$, and a bidding function , $B_i(c_i)$, stating the bid that i would submit for every realization c_i of its production cost. For all $i \in N$

$$M_i := \mathbf{R}^+ \times \mathcal{B}, \tag{2.22}$$

where

$$\mathcal{B} := \{B : [\underline{c}, \bar{c}] \to \mathbf{R}\}. \tag{2.23}$$

[4]We assume that the buyer quotes a single reserve price that applies to all sellers. Although this assumption can be justified under the present scenario where the buyer can commit and investment levels are unobservable, optimal behavior by the buyer when she can observe investment levels would involve *non-uniform* reserve prices; i.e., reserve prices faced by each seller would depend on investment distribution. Furthermore, the buyer's optimal strategy would involve "discrimination" between sellers with different investment levels (see McAfee and McMillan [48].)

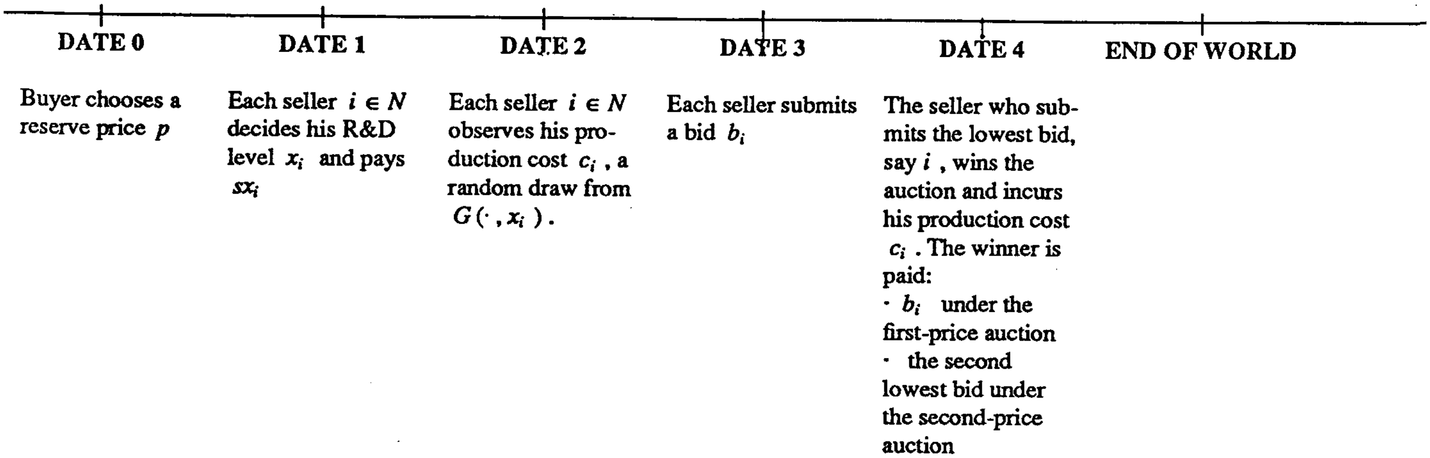

Figure 2.1 : Timing of Events

Let

$$M_{-i} = \prod_{j \neq i} M_j, \tag{2.24}$$

and

$$M = \prod_{i \in N} M_i. \tag{2.25}$$

Let

$$m_i = (x_i, B_i(\cdot)), \tag{2.26}$$

$$m_{-i} = (x_{-i}, B_{-i}(\cdot)), \tag{2.27}$$

and

$$\boldsymbol{m} = (\boldsymbol{x}\ , \boldsymbol{B}\ (\cdot)) = ((x_1, x_2, \ldots, x_n), (B_1(\cdot), B_2(\cdot), \ldots, B_n(\cdot)) \tag{2.28}$$

denote generic elements of M_i , M_{-i} and M , respectively.

The buyer's strategy set is a set of reserve prices which we identify with the set of non-negative real numbers $\mathbf{R}^+$. Her payoff, if she chooses a reserve price p, is then

$$\Pi_b(p, \boldsymbol{m}) = v - \text{ Expected price given } p \text{ and } \boldsymbol{m}\ . \tag{2.29}$$

We will denote the game in which the buyer can commit to a reserve price to be effective in a subsequent second price auction, before the sellers do R&D at a cost s by $\Gamma_b^{sp}(s)$. The corresponding game where the auction procedure is first price is denoted by $\Gamma_b^{fp}(s)$. Characterization of the sellers' Nash equilibrium behavior for a given reserve price p is common to the solution of equilibria under both games.

Both auctions as games have the same extensive form. They differ only in determination of the payoffs for any given strategy combination $\boldsymbol{m} \in M$. Under both auctions the seller who submits the lowest bid wins the auction, if the bid is lower than the reserve price p, and produces the object incurring his production cost. The auction winner is paid his bid for producing the object under the first price auction. Under the second price auction the payment to the winner is the second lowest bid submitted or the reserve price p, whichever is lower.

Let

$$\{\Pi_i^{fp}(\boldsymbol{m}; p, s) \ : \ i \in N\} \in \mathbf{R}^n, \tag{2.30}$$

and

$$\{\Pi_i^{sp}(\boldsymbol{m}; p, s) \ : \ i \in N\} \in \mathbf{R}^{\mathbf{n}} \tag{2.31}$$

be the payoffs under the first price and second price auctions, respectively. Finally, let

$$NE^{fp}(p,s) = \{\boldsymbol{m} \in M : \forall i \in N, m_i \in \arg \max_{\hat{m}_i \in M_i} \Pi_i^{fp}(\hat{m}_i, m_{-i}; p, s)\}, \tag{2.32}$$

and

$$NE^{sp}(p,s) = \{\boldsymbol{m} \in M : \forall i \in N, m_i \in \arg \max_{\hat{m}_i \in M_i} \Pi_i^{sp}(\hat{m}_i, m_{-i}; p, s)\} \tag{2.33}$$

be the set of *Nash equilibria* under the two auctions for given reserve price p.

Given a search cost s, the *Subgame Perfect Nash Equilibria* of the game $\Gamma_b^{fp}(s)$, $SPE_b^{fp}(s)$, is the set of pairs $(p^*, m^*(p^*, s))$ such that

(i) $m^*(p^*, s) \in NE^{fp}(p^*, s)$,

(ii) $p^* \in \arg\max_{p \in \mathbf{R}} \Pi_b^{fp}(p, m^*(p, s))$.

For the second price auction $SPE_b^{sp}(s)$ is defined similarly.

2.4.1 Second Price Auction

In characterizing the Nash equilibria under the second price auction we will make use of the following fact attributed to Vickrey [69]: In the second price auction "truth-telling," always submitting a bid equal to one's production cost, is a dominant strategy for every participant independent of the distribution of the rivals' costs and/or bids. In the present framework, this observation together with the fact that R&D costs are sunk at the bidding stage imply that regardless of the investment and bidding behavior of the rivals *and* regardless of the amount invested prior to the auction, a bidder $i \in N$ cannot do better by bidding any amount different from his production cost. Define

$$B^{sp}(c) = c \quad \text{for all} \quad c \in [\underline{c}, \bar{c}], \tag{2.34}$$

and let

$$\boldsymbol{B}^{sp}(\boldsymbol{c}) = (B^{sp}(c_1), \ldots, B^{sp}(c_n)), \tag{2.35}$$

where $\boldsymbol{c} = (c_1, \ldots, c_n)$.

Lemma 2.1 *For all $\boldsymbol{m} = (\boldsymbol{x}\ , \boldsymbol{B}(\cdot)) \in M$ and for all $i \in N$*

$$\Pi_i^{sp}(\boldsymbol{x}, B^{sp}(c), B_{-i}) \geq \Pi_i^{sp}(\boldsymbol{x}, \boldsymbol{B}\ (\cdot)). \tag{2.36}$$

Proof: For any realization c of his cost, seller i only lowers his probability of winning and cannot affect the price he gets in case he wins if he bids higher than c. If he bids lower than c, he incurs a loss when he wins, and he wins with positive probability. Noting that x_i is sunk at the bidding stage completes the proof. □

Proposition 2.1 *When investment is unobservable, the set of all Nash equilibria under the second price auction, for given* (p, s), *is*

$$NE^{sp}(p,s) = X^{**}(p,s) \times \boldsymbol{B}^{sp}, \tag{2.37}$$

where

$$X^{**}(p,s) := \{x \in \mathbf{R}^n \ : \ \sum_{i \in N} x_i = X^{sp}(p,s)\}, \tag{2.38}$$

and $X^{sp}(p,s)$ *solves*

$$\int_{\underline{c}}^{p} [1 - F(t)]^{X^{sp}(p,s)} \ln[1 - F(t)] dt = -s. \tag{2.39}$$

If investment is observable, the set of subgame perfect equilibria is the same as the set of Nash equilibria with observable investment.

Proof: Lemma 2.1 characterizes the bidding component of the equilibrium strategies. From Lemma 2.1, whether or not an investment profile $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ is observable before sellers submit their bids, bidding behavior is not affected since $B_i^{sp}(c) = c$ is a dominant strategy for all $i \in N$. Hence, we can replace the bidding part of the extensive form with the payoffs from the equilibrium bidding strategy $B^{sp}(\cdot)$ to get a reduced extensive form. Let sellers $j \neq i$ use the bidding strategy $B_{-i}^{sp} = \{B_j^{sp}(\cdot) \ : \ j \neq i\}$, and investment strategy $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n)$. Given the bidding and investment strategies of $j \neq i$, i wins the auction with a production cost c_i only if

$$z := \min_{j \neq i} \{c_j\} \ \geq \ c_i. \tag{2.40}$$

Using (2.5),

$$\text{Prob } \{z > c_i\} = [1 - F(c_i)]^{X_{-i}}, \tag{2.41}$$

where

$$X_{-i} := \sum_{j \neq i} x_j. \tag{2.42}$$

Hence, given the bidding strategies $B^{sp}(\cdot)$, i's payoff depends on x_{-i} only through X_{-i}. Seller i wins the auction if and only if one of the two mutually exclusive events, $c_i < p < z$ and $c_i < z < p$, occur since we can ignore ties for they occur with zero probability. The price i gets is p in the former case and z in the latter. Therefore, the expected profit of seller i, given the bidding strategies, is

$$\Pi_i^{sp}(x_i, X_{-i}; p, s) = [1 - F(p)]^{X_{-i}}\{p - \int_{\underline{c}}^{p} [1 - F(t)]^{x_i} dt\} +$$
$$\{\int_{\underline{c}}^{p}\{\int_{\underline{c}}^{z}(z-t)x_i[1-F(t)]^{x_i-1}F'(t)dt\}X_{-i}[1-F(z)]^{X_{-i}-1}F'(z)dz\}$$
$$-sx_i. \tag{2.43}$$

Integrating the second term in (2.43) by parts, and after cancellations, we get

$$\Pi_i^{sp}(x_i, X_{-i}; p, s) \;=\; \int_{\underline{c}}^{p} [1 - F(t)]^{X_{-i}} dt - \{\underline{c} + \int_{\underline{c}}^{p} [1 - F(t)]^{x_i + X_{-i}} dt\} - sx_i. \tag{2.44}$$

The first term in (2.44) is the expected price i receives and the second term is i's expected production cost conditional on his winning the auction. Thus, the sum of the first two terms represents the expected return to investment for seller i. Given X_{-i} and (p, s), seller i's expected profit is maximized at x_i^{**} which solves

$$\frac{\partial \Pi_i^{sp}(x_i^{**}, X_{-i}; p, s)}{\partial x_i} = -\int_{\underline{c}}^{p} [1 - F(t)]^{x_i^{**} + X_{-i}} \ln[1 - F(t)] dt - s = 0. \tag{2.45}$$

Consideration of sellers $j \neq i$ gives exactly the same first order condition as in (2.45), which proves (2.39) in Proposition 2.2. To rule out equilibria that involve randomization in the investment decisions, let sellers $j \neq i$ use mixed investment strategies $\{G_j(\cdot) \; : \; j \neq i\}$, where $G_j(y) =$ Prob $\{x_j \leq y\}$, and $y \in \mathbf{R}$. Let $G_{-i}(\cdot)$ be the cumulative distribution function of the sum $X_{-i} = \sum_{j \neq i} x_j$, when sellers $j \neq i$ use the given mixed investment strategies. For all $t \in [\underline{c}, \bar{c}]$, let

$$E_{G_{-i}}(t) = E[1 - F(t)]^{X_{-i}} \tag{2.46}$$

be the expected value of $[1 - F(t)]^{X_{-i}}$. Then, if we take the expected value of seller i's profit given in (2.44) with respect to the distribution $G_{-i}(\cdot)$, we can write seller i's expected profit as

$$E\Pi_i^{sp}(x_i; p, s) = \int_{\underline{c}}^{p} E_{G_{-i}}(t)dt - \underline{c} - \int_{\underline{c}}^{p} E_{G_{-i}}(t)[1 - F(t)]^{x_i}\, dt \; - \; sx_i. \tag{2.47}$$

If seller i uses a mixed strategy, say $G_i(\cdot)$, to determine his investment level, it should be the case that all investment levels in the support of $G_i(\cdot)$ give seller i the same expected profit. But then, $G_i(\cdot)$ can not have more than one point in its support, because $E\Pi_i^{sp}(x_i; p, s)$ given in (2.47) has a *unique* maximizer given by the solution x_i to

$$\frac{\partial E\Pi_i^{sp}(x_i; p, s)}{\partial x_i} = -\int_{\underline{c}}^{p} E_{G_{-i}}(t)\, [1 - F(t)]^{x_i} \ln[1 - F(t)]\, dt - s = 0. \tag{2.48}$$

First order condition (2.48) is necessary and sufficient for a maximum since the second order condition is satisfied with strict inequality:

$$\frac{\partial^2 E\Pi_i^{sp}(x_i; p, s)}{\partial {x_i}^2} == -\int_{\underline{c}}^{p} E_{G_{-i}}(t)\, [1 - F(t)]^{x_i} \{\ln[1 - F(t)]\}^2\, dt < 0 \tag{2.49}$$

for all x_i. Since this is true for all $i \in N$, an equilibrium cannot involve mixed investment strategies. □

From Proposition 2.1 we obtain the following properties of the Nash Equilibria under the second price auction as corollaries, which can be easily proved: Under a second price auction with reserve price p and search cost s, there is a plethora of investment equilibria. In particular, for each k,

($k = 1, 2, \ldots, n$), there are equilibria in which there are k *active* sellers with strictly positive investment levels, i.e., in which $n - k$ sellers do not invest . Consequently, there is a multiplicity of equilibrium price distributions for a given reserve price p. In particular, there is an equilibrium price "distribution" corresponding to the investment equilibrium with a single active seller, which is degenerate at the reserve price p.

The following proposition is easily proved as a corollary to Proposition 2.1.

Proposition 2.2 *Equilibrium* total *investment under a sealed bid second price auction with a reserve price p and search cost s, $X^{sp}(p, s)$, is*

1. *unique,*
2. *increasing in p, and decreasing in s,*
3. *independent of the number of sellers,*
4. *socially optimal if the reserve price p is set at the production cost using the currently available technology, $\tilde{c}$.*

For $p = \tilde{c}$, any seller i's objective function given in (2.44) has exactly the same form as the social planner's objective function given in (2.9). That is, given the level of total investment by the other sellers, each seller minimizes the expected production cost by "picking up the slack" between the socially optimal investment level and the total investment by his rivals. When the reserve price is $\tilde{c}$, the value of the "marginal" investment unit is the same regardless of the number of sellers, and equal to the social planner's marginal valuation. Social optimality of total investment level independent of the number of sellers is reminiscent of the result obtained by Sah and

Stiglitz [63] in a patent race model where the gains to firms are determined in a Bertrand competition. In contrast to previous studies of the relation between the market structure and R&D, the Sah and Stiglitz model implies that the total number of projects undertaken is unaffected by the number of firms. When the reserve price is set at the currently available production cost, the production decision is also *ex post* optimal, since the seller with the lowest cost will be awarded the contract.

2.4.2 First Price Auction

We will characterize the *symmetric* Nash equilibrium for the first price auction when investment decisions of the participants are only privately observable . Characterization of all the equilibria under the first price auction in the present framework is a difficult task, for we cannot rule out multiple bidding equilibria. The same problem did not arise in studying the second price auction because of the property of the second price auctions stated in Lemma 2.1 above.

Proposition 2.3 *The unique symmetric pure strategy Nash equilibrium under the first price auction with a reserve price p and search cost s is*

$$\{x^{fp}(p,s), B^{fp}(\cdot)\}, \tag{2.50}$$

where

$$x^{fp}(p,s) = (1/n)X^{fp}(p,s), \tag{2.51}$$

and $X^{fp}(p,s)$ *solves*

$$-\int_{\underline{c}}^{p} [1-F(t)]^{X^{fp}(p,s)} \ln[1-F(t)]dt = s, \tag{2.52}$$

and $B^{fp}(c)$ is given by:

$$B^{fp}(c) = c + \frac{\int_c^p [1 - F(t)]^{(n-1)x^{fp}} dt}{[1 - F(c)]^{(n-1)x^{fp}}}. \tag{2.53}$$

The proof proceeds by showing that $\{x^{fp}(p, s), B^{fp}(\cdot)\}$ is a best response to itself. Letting all $j \neq i$ use the strategy prescribed by (2.50) to (2.53) above, we can show that i's best response is using the same strategy: By the fact that x_i is sunk and not observable by the others at the bidding stage, we know that x_i cannot affect either i's or others' bidding strategies. Then it is easy to establish that i's best response is using the strategy given by (2.53) at the bidding stage for any production cost c he might have. Finally, using the expected profit from the bidding stage as the payoff in the investment game, we obtain the same expected payoff as a function of the player's investment strategies as in (2.44).

Proof of Proposition 2.3 : Let all $j \neq i$ use the strategy $(x, \tilde{B}(\cdot))$. Let $\beta = (n-1)x$ and $\psi = \tilde{B}^{-1}$. For any production cost c that he might have, seller i wins the auction with a bid $b \leq p$ if and only if

$$b \leq \tilde{B}(c_j) \quad \text{for all } j \neq i; \tag{2.54}$$

that is,

$$c_j \geq \psi(b) \quad \text{for all } j \neq i. \tag{2.55}$$

Therefore, i wins the auction with a bid $b \leq p$ with probability $[1 - F(\psi(b))]^{\beta}$, and his expected profit is

$$\pi(b, c) = (b - c)[1 - F(\psi(b))]^{\beta}. \tag{2.56}$$

The first order condition is

$$\frac{\partial \pi}{\partial b} = [1 - F(\psi(b))]^{\beta} - (b-c)\, \beta\, [1 - F(\psi(b))]^{\beta - 1}\, F'(\psi(b))\, \psi'(b) = 0. \tag{2.57}$$

Multiplying by $[1-F(\psi(b))]$ and rearranging, we obtain

$$\frac{\partial \pi}{\partial b} = [1-F(\psi(b))]^{\beta+1} - \beta\, \pi\, (b,c)\; F'(\psi(b))\; \psi'(b) = 0. \tag{2.58}$$

Evaluating $\pi(b,c)$ at the optimum $B(c)$,

$$\pi(c) = \frac{[1-F(\psi(B(c)))]^{\beta+1}}{\beta\, F'(\psi(B(c)))\; \psi'(B(c))}. \tag{2.59}$$

Since no bid above p is accepted, we have

$$\pi(c) = 0 \quad \text{for all} \quad c \geq p, \tag{2.60}$$

and thus,

$$B(p) = p. \tag{2.61}$$

The expected return to an investment of x_i for seller i is then

$$\Pi_i^{fp}(x_i, \beta; p, s) = \int_{\underline{c}}^{p} \pi(t)\; x_i\; [1-F(t)]^{x_i - 1} F'(t) dt - s x_i. \tag{2.62}$$

The first order condition for i's best investment response x_i is

$$\frac{\partial \Pi_i^{fp}(x_i, \beta; p, s)}{\partial x_i} = \int_{\underline{c}}^{p} \pi(t)\; [1-F(t)]^{x_i-1} F'(t)\{1 + x_i \ln{[1-F(t)]}\} dt - s = 0. \tag{2.63}$$

Imposing the symmetry condition

$$B(c) = \tilde{B}(c), \tag{2.64}$$

we have

$$\psi(B(c)) = c \quad \text{for all} \quad c, \tag{2.65}$$

and

$$\psi'(B(c)) = 1/B'(c). \tag{2.66}$$

Making use of (2.65) and (2.66) in (2.57), we obtain an ordinary differential equation for $B(c)$:

$$B'(c)\; [1-F(c)]^{\beta} - [B(c) - c]\; \beta\; [1-F(c)]^{\beta - 1} F'(c) = 0. \tag{2.67}$$

It can be easily verified that the solution to (2.67) is

$$B(c) = c + \frac{\int_c^p [1 - F(t)]^\beta dt}{[1 - F(c)]^\beta}, \tag{2.68}$$

which is the expression for $B^{fp}(c)$ in Proposition 2.3. Substituting $B(c)$ from (2.68) in the expression for $\pi(c)$, (2.62) reduces to

$$\Pi_i^{fp}(x_i, \beta; p, s) = \int_{\underline{c}}^{p} \{\int_t^p [1 - F(y)]^\beta dy\} \; x_i \, [1 - F(t)]^{x_i - 1} F'(t) \, dt - s x_i. \tag{2.69}$$

Integration by parts and some cancellation gives the expression in (2.44) for the expected returns to investment. Substituting (2.68) in (2.63) and using the symmetry condition $\beta = (n-1)x_i$, (2.63) reduces to (2.52). To prove that the symmetric equilibrium we found is the only symmetric pure strategy equilibrium, we note that for an arbitrary symmetric investment n-tuple $(x, x, \ldots, x)$, the symmetric bidding equilibrium given by (2.68) is unique. Given the uniqueness of symmetric bidding equilibrium, uniqueness of symmetric pure strategy investment equilibrium follows from the fact that (2.69) has a unique maximizer, which is strictly decreasing in β. □

We have not ruled out the existence of a symmetric equilibrium that involves randomization in investment decisions. We conjecture that such an equilibrium does not exist.

The results about the equilibrium total investment under the second price auction stated in Proposition 2.2 above hold verbatim at the *symmetric* Nash equilibrium under the first price auction.

Proposition 2.4 Total *investment in the symmetric pure strategy equilibrium under a sealed bid first price auction with a reserve price p and search cost s, $X^{fp}(p, s)$, is*

1. *unique,*

2. increasing in p, and decreasing in s,

3. independent of the number of sellers,

4. socially optimal if the reserve price p is set at the production cost using the currently available technology, $\tilde{c}$.

2.4.3 An Example

We give the explicit solution for the case where $F(\cdot)$ is the uniform distribution on $[0, 1]$ and the current production cost is 1. That is,

$$\underline{c} = 0, \quad \bar{c} = 1, \quad \tilde{c} = p = 1, \quad v \geq 1, \tag{2.70}$$

and

$$F(c) = c, \quad c \in [0, 1]. \tag{2.71}$$

And we take $s \in [0, .5]$ so that at least one unit of investment is undertaken. For the reserve price $p = 1$, the total investment at the symmetric equilibrium is given by

$$X(s) = (1/s)^{1/2} - 1, \tag{2.72}$$

so that each seller invests

$$x(s) = (1/n)[(1/s)^{1/2} - 1], \tag{2.73}$$

and uses the bidding strategy

$$B(c; s) = c \tag{2.74}$$

under the second price auction, and

$$B(c; s) = \frac{1 + (n-1)x(s)c}{1 + (n-1)x(s)} \tag{2.75}$$

under the first price auction.

Under both auctions, the expected production cost of the winning seller is

$$c_w(s) = \frac{1}{1 + X(s)}, \tag{2.76}$$

and the expected price paid by the buyer is

$$c_b(s) = \frac{1 + 2X(s) - x(s)}{[1 + X(s)][1 + (n-1)x(s)]}. \tag{2.77}$$

2.5 Equivalence of First and Second Price Auctions

The "Revenue" Equivalence Theorem (Myerson [52], Riley and Samuelson [57]) states that with risk neutral sellers and independent identically distributed private costs, the expected price to the buyer under any of the usual auction procedures (sealed bid or open, first price or second price) is the same. Moreover, these auctions combined with an appropriate reserve price minimize the expected price of the object to the buyer for a general class of production cost distributions.

In the independent private costs framework with risk neutrality and with exogenously given production cost distributions, first and second price auctions are equivalent in terms of the equilibrium expected payoffs to the sellers and to the buyer. In a second price auction, bidding one's true production cost is a dominant strategy for every seller so that the expected profit of a typical seller is the expected value of the difference between the lowest and the second lowest production costs. In a first price auction, the symmetric equilibrium bidding strategies are such that a seller's bid as a function of his production cost is the expected value of the second lowest production cost conditional on his having the lowest production cost among all sellers.

Because of this property of the symmetric bidding equilibrium in a first price auction the expected profit of a typical seller is equal to the expected profit of a typical seller in a second price auction. Since the expected total surplus is fixed, the buyer's expected profit is also the same under the two auctions.

A number of authors studied the implications for this equivalence result of weakening the assumptions of risk neutrality (Maskin and Riley [41]; Riley and Samuelson [57]; Holt [28]), and independence (Milgrom and Weber [50]) and symmetry of cost distributions across sellers (Maskin and Riley [41]). This body of research established that with risk averse sellers, first price auctions generate a lower expected price than the second price auctions. The ranking of the two auctions is reversed when the costs are correlated, i.e., when the independence of the cost distributions does not hold. With risk neutral bidders and independent identically distributed costs the equivalence has also been shown to hold in multi-object auctions in which each seller can supply only one unit (Harris and Raviv [25]), and a seller can supply more than one unit (Engelbrecht-Wiggans [18]).

In recent work on asymmetric auctions Maskin and Riley [41] conclude that in environments with risk neutral sellers and independently distributed costs, no general ranking of the two procedures is available when the cost distributions are not identical across sellers; that is, the assumption of *symmetry* is also crucial for the equivalence result to hold. However, we argue that one should make a distinction between *symmetry / asymmetry of cost distributions* (or of *beliefs*) and *symmetry / asymmetry of equilibrium behavior* in this context. Although this distinction is not made explicit in their study, Maskin and Riley [41] consider only *asymmetry of beliefs* . If there exist situations where the cost distributions are asymmetric and yet there exist symmetric equilibria at which the two auctions are equivalent, then one can argue that it is not so much the symmetry of beliefs as the *symmetry of behavior* that matters for the equivalence result. One can give examples of auctions where the bidding behavior is symmetric despite the *ex ante* asymmetry of the participants and where the revenue equivalence holds at the symmetric equilibria, but *only* at symmetric equilibria.

In the present framework such asymmetries arise endogenously despite the fact that all sellers are *ex ante* symmetric. In the present setting, there are a multiplicity of asymmetric equilibria despite the *ex ante* symmetry of the participants, and the same result obtains: The two auctions are equivalent at the symmetric equilibria but not at asymmetric equilibria.

We combine the results of the previous two sections as they relate to the comparison of the two auctions in the following proposition:

Proposition 2.5 *At the symmetric pure strategy equilibria of both first price and second price auctions with reserve price p and search cost s,*

1. *The equilibrium investment is the same under both auctions,*

2. *The equilibrium expected price is the same under both auctions.*

When multiple equilibria exist under both auctions, the proper comparison of the two auctions should be based on the sets of equilibrium outcomes in terms of payoffs to the participants; that is, for equivalence, the set of equilibrium outcomes in a first price auction should be the same as the set of equilibrium outcomes in a second price auction . Although we have not characterized the set of all equilibria for a first price auction, we can still show that the sets of equilibrium outcomes under the two auctions, although they intersect, do not coincide. This is proved in the next proposition.

Proposition 2.6 *With a fixed number of potential sellers, $n > 1$, there exists a Nash equilibrium outcome under a second price auction that cannot arise in any Nash equilibrium under a first price auction.*

Proof: Let $\mathbf{o}$ be the $(n-1)$-tuple of zeros, and consider the reserve price $\tilde{c}$ together with the strategy combination

$$\boldsymbol{m}^* = ((X^{sp}(\tilde{c}, s), \mathbf{o}), (\boldsymbol{B}^{sp})), \tag{2.78}$$

where all sellers except seller one invest zero and seller one invests $X^{sp}(\tilde{c}, s)$, and all sellers use the "truth-telling" strategy in the bidding stage. Clearly, m^* is a Nash equilibrium under the second price auction, and the resulting expected price is $\tilde{c}$. To show that $\tilde{c}$ cannot be the equilibrium expected price at any Nash equilibrium of the first price auction, first note that if the number of sellers investing positive amounts is at least two, the equilibrium price distribution cannot be degenerate at $\tilde{c}$ under the first price auction. Therefore, the only possible Nash equilibria with the desired price outcome must have the same investment component as m^*; i.e., all sellers except one seller, say seller j, invest zero. Given this investment profile, the only bidding equilibrium is every seller's bidding $\tilde{c}$ at the bidding stage. But then, the best response of every seller other than j to this strategy combination would be to invest $X^{sp}(\tilde{c}, s)$ and to bid infinitesimally less than $\tilde{c}$. Hence there cannot be a Nash equilibrium under the first price auction at which only one seller invests a strictly positive amount, and $\tilde{c}$ cannot be the expected equilibrium price under the first price auction.□

The two auctions are equivalent at equilibria other than the one considered in Proposition 2.5. Interestingly, all of these equilibria are also *symmetric,* or *pseudosymmetric,* in the sense that all *active* sellers invest the same amount and use the same bidding strategy in these equilibria. For each $k = 2, ..., n$ there is an equilibrium at which k sellers each invest $X^{sp}(p, s)/k$ and $n - k$ sellers invest zero under both auction procedures. The corresponding expected prices under the two auctions are trivially equivalent, and each active seller gets the same expected profit in equilibrium.

2.6 The Buyer's Optimal Reserve Price

The issue of commitment by the buyer in the present setting has two aspects. First the problem of commitment arises even in the absence of pre-auction R&D in a standard auction setting. To minimize her expected payment, the buyer has to commit to a reserve price. However, she would have incentives to change that reserve price, once the bids are submitted. With pre-auction R&D the same problem arises before and after the investment decisions of the sellers. Although once the investment costs of the sellers are sunk, the buyer always has an incentive to lower the reserve price she had announced just before the bids are submitted, credibility of such commitments helps solve "part" of the problem of "information rents" in the bidding stage because the higher the investment by each seller, the lower the cost of each seller and of his competitors in the auction. However, whatever the new reserve price chosen by the buyer in the interim, it suffers the same problem in terms of credibility as in a standard auction. In the sequel, we will assume that the commitment problem arises only in the R&D stage; i.e., the buyer cannot change her reserve price after the bids are submitted. The trade-off faced by the buyer is that a reserve price lower than the currently available production cost would lower the expected price she pays for any given investment distribution, but a lower reserve price may be associated with lower investment by the sellers.

Multiplicity of equilibria for any given reserve price p necessitates some equilibrium selection criterion before one can solve the buyer's problem. In order for the buyer to be able to compare alternative reserve prices, she has to have some beliefs about which of the multiplicity of equilibria would be played by the sellers following any reserve price announcement. An increase in the reserve price from a given level would create incentives for further investment by the sellers, but this additional investment may be distributed among the sellers in a number of ways. For instance, suppose at a reserve price $\tilde{p}$ a symmetric equilibrium is played by the sellers, where each seller invests $x(\tilde{p}, s)$, with total investment equal to $nx(\tilde{p}, s)$. A given change Δ in $\tilde{p}$ may increase or decrease the buyer's payoff, depending on which equilibrium is played. Although the total investment will increase if Δ is positive, the expected price paid by the buyer may increase or decrease depending on how the total investments in the two cases are distributed among the sellers. To take one extreme case, the new equilibrium may involve only one active seller, in which case the buyer would not increase the reserve price over $\tilde{p}$. On the other hand, if the equilibrium at the new reserve price were to be symmetric, the buyer might increase his reserve price from $\tilde{p}$.

The discussion above suggests that a continuum of reserve prices can be supported at a *subgame perfect equilibrium* of the two-stage game $\Gamma_b^{sp}(s)$ where the buyer commits to a reserve price in the first stage, and the sellers invest and submit bids in the second stage.

We start by considering the Nash equilibrium in the game where the buyer can not commit to a reserve price. Since sellers' investment levels are not observable, lack of commitment by the buyer to a reserve price makes the game between the buyer and the sellers a simultaneous move game. The buyer's reserve price and the sellers' investment and bidding decisions are determined simultaneously. The buyer chooses a reserve price, which minimizes the expected price she pays for the item, given her expectations about the investment and bidding strategies used by the sellers. Each seller chooses an investment and bidding strategy which maximizes his expected profit, given his expectations about the reserve price and the bidding and investment decisions of the rival sellers. In a Nash equilibrium, each player's expectation about the strategies chosen by other players coincides with the strategies actually chosen by the other players.

Since, by assumption, $\tilde{c}$ is available to every player, we take, without loss of generality, $\tilde{c} = \bar{c}$ and $\underline{c} = 0$. A trivial Nash equilibrium in the simultaneous move game is the one in which the buyer chooses a reserve price p below $p_*(s)$ and all sellers invest zero, where $p_*(s)$ is given by

$$- \int_0^{p_*} \ln\left[1 - F(t)\right] dt - s = 0. \tag{2.79}$$

We will rule out this trivial equilibrium by appealing to the fact that, given our assumption about the search cost s, every player would be at least as well off in an equilibrium that involves a positive investment level.

We will restrict attention to "symmetric" equilibria in which all sellers use the same strategy. If the buyer expects that each seller invests x and that each seller uses the dominant bidding strategy in the second price auction or the symmetric equilibrium bidding strategy corresponding to x in the first price auction, then her expected profit as a function of her reserve price p and the symmetric investment strategies x of the sellers would be the difference between the social surplus generated by the strategy combination (p, x) and the expected total profits of the n sellers for the same strategy combination.

The expected social surplus, given (p, x), is

$$S(p, x; n, s) = v - \{(\bar{c} - p)[1 - F(p)]^{nx} + \int_0^p [1 - F(t)]^{nx}\, dt\} - s\, nx. \quad (2.80)$$

The expected profit of a typical seller is

$$\Pi(p, x; s, n) = \int_0^p [1 - F(t)]^{(n-1)x}\, dt \; - \; \int_0^p [1 - F(t)]^{nx}\, dt \; - \; sx. \qquad (2.81)$$

Since the buyer's expected profit is the difference between the social surplus and the total expected profits of the sellers, for a given reserve price p, the buyer's expected profit as obtained from (2.80) and (2.81) is

$$\Pi_b(p, x; s, n) = v \; - \; \{(\bar{c} - p)\,[1 - F(p)]^{nx} \; + \\ n \int_0^p [1 - F(t)]^{(n-1)x}\, dt \; - \; (n-1) \int_0^p [1 - F(t)]^{nx}\, dt\}. \qquad (2.82)$$

The reserve price that maximizes $\Pi_b(p, x; s, n)$ also minimizes the buyer's expected cost:

$$C_b(p, x; s, n) = (\bar{c} - p)\,[1 - F(p)]^{nx} \; + \\ n \int_0^p [1 - F(t)]^{(n-1)x}\, dt - (n-1) \int_0^p [1 - F(t)]^{nx}\, dt. \qquad (2.83)$$

The first order condition for the buyer's minimization problem is

$$1 - [1 - F(p)]^x - (\bar{c} - p)\, x\, [1 - F(p)]^{x-1}\, F'(p) = 0, \qquad (2.84)$$

which is independent of the number of sellers. We can rewrite (2.84) in the form

$$(\bar{c} - p) \; = \; \frac{1 \; - \; [1 - F(p)]^x}{x\,[1 - F(p)]^{x-1}\, F'(p)}. \qquad (2.85)$$

To guarantee an interior solution to the buyer's minimization problem, we assume that the following condition is satisfied by $F(\cdot)$:

$$\varphi(x,p) = 2[1-F(p)]+(\bar{c}-p)\,\{(x-1)\,F'(p)-\frac{1-F(p)}{F'(p)}\,F''(p)\,\} \;>0, \quad (2.86)$$

for all $p \in (0,\bar{c})$ and all $x \geq 0$. For $x = 1$, this assumption is the same as the well-known hazard rate condition assumed in the optimal auction literature (McAfee and McMillan [47], [48]; Riley and Samuelson [57]; Guesnerie and Laffont [24]; Myerson [52]), when the production cost distribution is exogenous. Since the second order condition (2.86) is increasing in x for all p, assuming that (2.86) holds for $x = 0$ guarantees that it also holds for all x. Thus, we assume

Assumption A :

$$\varphi(0,p) = 2[1-F(p)] - (\bar{c}-p)\,\{F'(p) + \frac{1-F(p)}{F'(p)}\,F''(p)\,\} \;>0. \qquad (2.87)$$

It is easy to show that (2.87) is satisfied if $F(\cdot)$ is the uniform distribution.

The first order condition (2.84) implicitly defines the buyer's *reaction function* $p(x)$ for every level of symmetric investment strategies by the sellers. Differentiating the first order condition (2.84) with respect to x and rearranging, we obtain

$$\frac{d\,p(x)}{d\,x} = \frac{-F'(p)\,\{x\,\ln[1-F(p)]^{-1} - [1-(1-F(p))^{x}]\}}{x^{2}[1-F(p)]^{x-2}\,\varphi(x,p)}. \qquad (2.88)$$

Proposition 2.7 *Under Assumption A, the reaction function of the buyer, $p(x)$, is strictly decreasing in x :*

$$\frac{d\,p(x)}{d\,x} < 0, \qquad (2.89)$$

for all $x > 0$.

Proof: By Assumption A, the denominator in (2.88) is positive. The numerator is negative, since

$$\ln [1 - F(p)]^{-x} > 1 - [1 - F(p)]^{x} \tag{2.90}$$

for all $p < \bar{c}$ and all $x > 0$. □

Taking the limit as x approaches zero in the first order condition (2.84) we see that the buyer's optimal reserve price approaches p^*, where p^* satisfies

$$\bar{c} = p^* + \frac{[1 - F(p^*)] \ln \{1/[1 - F(p^*)]\}}{F'(p^*)}. \tag{2.91}$$

We obtained (2.91) by using L'Hopital's rule in (2.85). For $x = 0$ (2.91) has two roots, $\bar{c}$ and another root $p^* < \bar{c}$. However, if we require continuity of $p(x)$ with respect to x, the only solution is strictly less than $\bar{c}$.

For x to be a symmetric investment equilibrium for the n sellers, given the reserve price p, it should be the case that

$$- \int_0^p [1 - F(t)]^{nx} \ln [1 - F(t)]\, dt = s. \tag{2.92}$$

We write $x(n,p)$ for the symmetric investment equilibrium that occurs when there are n sellers and the reserve price is p. If a seller expects the buyer to quote the reserve price p and the rival sellers to invest $x(n,p)$, his best investment strategy is to invest $x(n,p)$. It is easily verified that

$$\frac{\partial x(n,p)}{\partial p} > 0, \tag{2.93}$$

and

$$\frac{\partial x(n,p)}{\partial n} < 0, \tag{2.94}$$

for all $p > p_*$.

Combining the facts about the reaction functions of the buyer and the sellers, we obtain

Proposition 2.8 *Under Assumption A, there exists a Nash equilibrium (p^o, x^o) in which the sellers use symmetric investment strategies. Furthermore, (p^o, x^o) is the unique equilibrium with this property.*

Proof: The best response functions given by (2.84) and (2.92) are continuous. Since $p(0) = p^* > p_*$, $x(n, p^*) > 0$, and $p(\cdot)$ and $x(n, \cdot)$ are strictly decreasing and strictly increasing, respectively, the best response functions have a unique intersection p^o, x^o. □

Since $p(0) = p^*$, and since $p(x)$ strictly decreasing the equilibrium reserve price p^o is strictly less than $\bar{c}$, therefore, equilibrium total investment nx^o is strictly less than the socially optimal level $\hat{x}$. As the number of sellers increases, both the equilibrium reserve price and the equilibrium total investment increase towards their respective socially efficient levels.

For the uniform uncertainty case, i.e., $F(c) = c$, with $\bar{c} = 1$, the first order condition for the buyer given by Equation (2.84) reduces to

$$p(x) = 1 - (\frac{1}{1+x})^{\frac{1}{x}}, \tag{2.95}$$

which approaches $1 - e^{-1} < 1$ as x approaches 0, and (2.92) reduces to

$$\frac{(1-p)^{1+nx}}{1+nx} \ln(\frac{1}{1-p}) - \frac{1-(1-p)^{1+nx}}{(1+nx)^2} + s = 0. \tag{2.96}$$

Substituting (2.95) in (2.96), we obtain an implicit expression that must be satisfied by equilibrium investment level x^o:

$$\frac{1}{1+nx^o} \{\frac{1-(1+x^o)^{-\frac{1+nx^o}{x^o}}}{1+nx^o} - (1+x^o)^{-\frac{1+nx^o}{x^o}} \ln(1+x^o)^{\frac{1}{x^o}}\} = s. \tag{2.97}$$

If the buyer can commit to a reserve price before the sellers undertake any investment, both the reserve price and the total investment levels are higher than the corresponding levels without commitment. That is, the equilibrium reserve price in the symmetric subgame perfect equilibrium will be higher than p^o, since the buyer's expected cost is decreasing in $x(n,p)$, and $x(n,p)$ is increasing in p. Since the total investment level is increasing in p, it also follows that the total investment level in the symmetric subgame perfect equilibrium of the game with the buyer's commitment will be closer to the socially efficient level.

One way to obtain the efficient reserve price $p = \bar{c}$ as a subgame perfect equilibrium is to exploit the multiplicity of Nash equilibria in the investment subgame under the second price auction.

Proposition 2.9 *Any $p \in (p_*, \bar{c}]$ can be supported in a subgame perfect equilibrium of the game $\Gamma_b^{sp}(s)$, where p_* is defined by*

$$-\int_{\underline{c}}^{p_*} \ln\left[1 - F(t)\right] dt - s = 0. \tag{2.98}$$

Proof: First, any reserve price under c_* gives rise to zero total investment at equilibrium and hence to an expected price of $\bar{c}$. Any reserve price strictly greater than c_* induces positive investment. With positive investment, even if it were undertaken by one seller only, the price paid by the buyer is $\bar{c}$ only when the minimum production cost resulting from investment exceeds c_* which occurs with probability less than one. Therefore, with non-zero probability, she pays a price less than or equal to c_*. She can, therefore, lower the expected price by increasing the reserve price slightly over c_*, which means that she cannot set a reserve price equal to c_* at an equilibrium. Secondly, any reserve price $\tilde{p}$ above $\bar{c}$ is equivalent to a reserve price equal to $\bar{c}$, since the equilibrium price is no greater than $\bar{c}$ under both reserve prices $\tilde{p}$ and $\bar{c}$. To see that $\bar{c}$ can be a subgame perfect equilibrium reserve price, let each seller invest $X^{sp}(p,s)/n$ in the subgame following the buyer's choice of p as the reserve price, for all p. Since the buyer's expected payment is decreasing in the level of investment per seller at symmetric equilibrium, and investment per seller is increasing in p for $p \leq \bar{c}$, her expected payment is minimized at $\bar{c}$. It remains to show that any reserve price in the interval $(c_*, \bar{c})$ can be an equilibrium reserve price. Take any $p \in (c_*, \bar{c})$ and let the sellers use the symmetric equilibrium strategy $X^{sp}(p', s)/n$ in each of the subgames following the reserve price choices $p' \in (c_*, p]$. For higher reserve prices p'', let all sellers except one invest zero. Since the buyer's expected payment is decreasing in the reserve price up to p and decreasing in the reserve price beyond p for this choice of equilibrium seller behavior, p is the buyer's best reserve price.□

In only one of these equilibria, the investment and production outcomes are efficient, namely, the subgame perfect equilibrium where the buyer sets the reserve price at the current production cost and the sellers use the symmetric investment strategy. In any equilibrium with a reserve price strictly less than the current production cost $\bar{c}$, not only the total investment is less than the socially optimal level but also production decision is inefficient in the sense that with positive probability, production is undertaken at the current cost $\bar{c}$ despite the availability of a seller with a lower production cost.

2.7 Sequential Search

In this section we consider the case where search technology takes the form of sequential sampling from a given distribution $F(\cdot)$. We assume that a decision maker can observe the outcome of his/her previous investment before he/she decides to commit more funds to further search, so that his/her investment decision may depend on the outcome of previous investments.

To obtain the non-sequential and sequential search strategies as the only optimal procedures, one can think of a situation where investment involves a *gestation lag*, which can be shortened only by incurring additional cost. In such a situation, a non-sequential search plan would be optimal if a gestation lag of 1 period,i.e., the time between the date investment decisions are made and the date the auction is held, is costless, but any shorter gestation lag is costly enough. The optimal search procedure would be sequential if, for instance, gestation lag could be *halved* after each observation costlessly so that one could make as many observations as one liked before the auction date.

If seller i takes k samples at a constant per unit cost s, the production cost is the minimum of the sample observations $(c^1, c^2, \ldots, c^k)$ as in the non-sequential search case:

$$c_k = \min\{c^1, c^2, \ldots, c^k\}. \tag{2.99}$$

To consider first the socially efficient search procedure, we note that the social surplus after k samples are taken is

$$S \equiv (v - c_k) - ks. \tag{2.100}$$

A well-known result in search theory (DeGroot [15], McCall [49], Morgan and Manning [51], Rob [60]) is that the optimal search procedure in this situation is a *stopping rule* : Continue sampling until the expected gain from one more observation drops below the search cost. The optimal stopping rule is characterized by a production cost level c^* such that the decision maker is indifferent between making one more observation and stopping if his current production cost is c^*. c^* is given by

$$\int_0^{c^*} F(t)\,dt = s, \tag{2.101}$$

and the expected surplus generated by following the optimal stopping rule is

$$S^* = v - c^*. \tag{2.102}$$

The optimal stopping rule for a single seller facing a reserve price $p \geq c^*$ is obtained, using the same reasoning, to be equal to c^*. After taking k samples the seller's profit is

$$\pi = (p - c_k) - ks, \tag{2.103}$$

where c_m is the minimum of the sample of k observations. As long as the reserve price p is at least as high as c^*, then the seller will continue sampling until he observes a production cost c^* or lower, and the seller's expected profit from following the optimal search procedure is

$$\pi^* = p - c^*. \tag{2.104}$$

Since search cost s is common knowledge, the buyer's optimal reserve price would be c^*, and the seller would make zero expected profits. Note that if a seller accepts a fixed price production contract with a price c^*, which is *ex ante* individually rational for a risk neutral seller with a zero reservation profit, then the contract will not be breached even without any enforcement, because optimal search behavior by the seller would realize a production cost at or below the contract price. This contrasts with the non-sequential search case where the production cost realization is above the reserve price with positive probability.

Therefore, as long as the seller's search cost s is common knowledge, the buyer can extract the entire social surplus by announcing a reserve price c^*. No matter how large the buyer's search cost s_b is, the expected price she pays for the object is the same as it would be if she did the search and production herself.

If there are two or more sellers, nothing essential changes since, in equilibrium, the buyer announces a take-it-or-leave-it price c^* to any seller, and one seller undertakes investment until he obtains a production cost c^* or lower, and the other sellers stay out.

Under second price auction this result does not change even with a reserve price p strictly above c^*; that is, for any reserve price $p \geq c^*$, the only investment equilibria are such that one seller uses the optimal stopping rule c^*, and the rest of the sellers stay out. This follows from the fact that bidding the true production cost is a dominant strategy under the second price auction, and that the production cost of a seller following the optimal stopping rule is at most c^*.

Cremer and McLean ([9], [10]) show that when the costs are correlated, it is possible for the buyer to extract all the surplus despite private information of the sellers. When search is sequential, we find that the buyer can extract all the surplus by simply choosing a reserve price.

2.8 Conclusions

We have established that when the production cost distributions are endogenously determined as a result of private investment expenditures, first and second price auctions are equivalent : Both give rise to the same level of total investment, same reserve price, same expected price to the buyer and same expected level of profits for the sellers, at the *symmetric* Nash equilibria. This is an extension of the equivalence results known in the context of standard independent private value auction models with risk neutral bidders.

We have also shown that total investment is independent of the number of sellers for any given reserve price. However, when we include the buyer's strategic decisions, the equilibrium total investment is always less than the socially optimum level and increases with the number of sellers.

Throughout this chapter we assumed that a seller's investment level is observed only by himself. Although all the results we obtained for second price auctions remain valid when sellers can observe one another's investment levels before the auction, bidding behavior of a seller will depend on the distribution of his rivals' production costs, and hence, on the rivals' investment levels. Therefore, the results we obtained for first price auctions depend on the investment unobservability assumption.

Although the Nash equilibrium we characterized for the unobservable investment case remains a Nash equilibrium when investment is observable, the proper equilibrium concept to be used in this case is subgame perfect Nash equilibrium, and it is not guaranteed that the symmetric Nash equilibrium we obtained will be subgame perfect when bidding strategies are allowed to depend on the rival's investment levels.

In general, one would expect that a seller's bidding strategy as a function of his production cost and the rival's investment levels would be such that he would bid more aggressively against a rival whose production cost distribution is more favorable. A seller's investment not only affects his production cost but also affects his probability of winning for any given production cost he might have. The latter effect is non-existent in second price auction simply because bidding strategies are independent of information conditions. It is also trivially non-existent if investment levels are only privately observable because bidding strategies can not be made contingent on the rivals' investment levels. A satisfactory study of this issue would require characterization of equilibrium bidding strategies for arbitrary distribution of investment levels. This is still an open problem for general continuous production cost distributions. A plausible conjecture is that if a symmetric equilibrium does exist in the observable investment case, it will involve less investment by every seller than the unobservable investment case. In the next chapter we will study the implications of investment observability for behavior in first price auctions in a setting where production cost can take only a discrete set of values.

Chapter 3

EQUIVALENCE OF AUCTIONS WITH OBSERVABLE PRE-AUCTION R&D

3.1 Introduction

In this chapter we study the effect of observability of investment levels by the sellers at the auction stage on bidding and pre-auction investment behavior using a model where the production cost can take only two values. We show that when investment is observable at the bidding stage, all pure strategy equilibria are asymmetric in the investment stage under the first price auction. We study the implications of this asymmetry for the equivalence of the two auctions in several dimensions. In this setting the second price auction dominates the first price auction.

In order to evaluate how observability affects investment behavior we need to know the equilibrium payoffs from the auction stage for arbitrary distribution of investment levels among the sellers. When the support of production cost distributions is a continuum, we are unable to characterize the bidding equilibrium in the first price auction because of asymmetries in these cost distributions that may arise when investment levels differ among sellers.

Few results are available on equilibrium bidding strategies when the distribution of production costs differ among sellers. Maskin and Riley [41] obtain the equilibrium bid functions for some special distributions, but the form of asymmetry is different from what we consider here. Namely, the distributions have different supports in the cases studied by Maskin and Riley, whereas in our framework the support of production cost distributions is the same for all sellers; i.e., investment does not affect the support of production cost distribution. Tirole [68] studies the implications of investment observability for the investment decisions for the single seller case.

We obtain a number of results from the model regarding the equivalence of first and second price auctions. We know that observability of investment does not affect behavior in the second price auction, since bidding the true production cost is a dominant strategy regardless of how the production costs of the rival sellers are distributed. In the first price auction, however, the set of equilibrium outcomes change when investment is observable: The requirement of subgame perfection eliminates the symmetric investment equilibrium from the set of equilibria in pure strategies, and all pure strategy equilibria are asymmetric. Total investment level, however, is not affected by this asymmetry for a given reserve price. The buyer's optimal response to this asymmetry in the investment equilibria is to reduce her reserve price so that the equilibrium total investment level is lower when the buyer knows that the sellers know one another's investment levels.

We also consider the effect of investment observability on the maximum number of sellers when the number of sellers is endogenous. Auctions with entry and with fixed entry scale are studied by McAfee and McMillan [44]. Samuelson [65] studied bidding with entry cost. In the context of common value auctions, Lee's [32] study of information acquisition can be interpreted as a model with endogenous entry scale. We show that with a fixed entry cost first price auctions attract fewer sellers than second price auctions when the number of sellers and the scale of entry are endogenous.

In the next section we present the model and characterize the socially optimal investment level in this framework as a benchmark. In Section 3.3 we obtain the equilibrium strategies under unobservable investment in first and second price auctions.

In Section 3.4, equilibrium outcomes under first price auction with observable investment are characterized. In Section 3.5, we find the optimum reserve price of the buyer and compare the two auctions. Finally, in Section 3.6, we use the results from the previous sections to compare the two auctions when the number of sellers is endogenous.

3.2 The Model

We study a model in which n risk neutral sellers can invest to affect the distributions from which their production costs are drawn. For every seller i initial production cost is $\bar{c}$. If seller i invests x_i in cost-reduction type research and development, his production cost, c_i, remains at $\bar{c}$ with probability $r(x_i)$, and it is reduced to $\underline{c}$ with probability $1 - r(x_i)$, where $r(x)$ is a decreasing function. The investment outcome for seller i is a random variable independent and is independent of other sellers' outcomes. The expected production cost is decreasing in investment level. For each unit of investment, a seller pays a constant price s. Each seller observes the outcome of his investment privately, before submitting his bid in the auction. To continue the sampling analogy, we let r_i be determined as the probability of $min\{c_1 \dots c_x\} = \bar{c}$ when x independent draws are made from a given distribution on $\{\underline{c}, \bar{c}\}$. That is, for some exogenously given $r \in (0,1)$, we let

$$r_i = r(x_i) = r^{x_i} \tag{3.1}$$

If seller i chooses to invest x_i, he incurs a certain cost of investment sx_i.

We continue to assume that the buyer can observe neither the sellers' investment levels nor the production costs, and the strategic choice of the seller is restricted to choosing a reserve price above which no purchase is made. We first characterize the equilibrium outcomes for an arbitrary reserve price p, and then find the optimal reserve price for the buyer. The buyer's value of item is $v > \bar{c}$. The parameters of the model $(v, r, \bar{c}, \underline{c}, s)$ are common knowledge.

3.2.1 Social Efficiency

As a benchmark to be used in later sections, we first find the socially optimum investment level. Social surplus as a function of the investment level is $v - [\bar{c}r(x) + \underline{c}(1 - r(\underline{x}))] - sx$. For $r(x) = r^x$, the social planner problem is

$$\max_x \quad [v - \{\underline{c} + (\bar{c} - \underline{c})r^x\} - sx]. \tag{3.2}$$

First order conditions are necessary and sufficient since the maximand is strictly concave in x. The socially optimum level of investment solves

$$-(\bar{c} - \underline{c})r^x \ln r - s = 0. \tag{3.3}$$

That is,

$$\hat{x} = \frac{1}{\ln r} \ln(\frac{s}{(\bar{c} - \underline{c}) \ln r^{-1}}), \tag{3.4}$$

and $r^{\hat{x}} = s/[(\bar{c} - \underline{c}) \ln r^{-1}]$ is the corresponding socially optimal failure probability.

We will also use the optimal investment level for a *single* seller facing a reserve price p in later sections. The equilibrium investment level for a single seller facing a reserve price p is the solution $x^*(p)$ to the expected profit maximization problem

$$\max_x \quad (1 - r^x)(p - \underline{c}) - sx. \tag{3.5}$$

To guarantee that a positive investment level is worthwhile in the absence of any rivalry, we assume

Assumption 3.1

$$s < (p - \underline{c}) \ln r^{-1}. \tag{3.6}$$

The solution $x^*(p)$ satisfies

$$r^{x(p)} = \frac{s}{(p - \underline{c}) \ln r^{-1}}. \tag{3.7}$$

That is,

$$x^*(p) = \frac{1}{\ln r} \ln[\frac{s}{(p - \underline{c}) \ln r^{-1}}]. \tag{3.8}$$

3.3 Equilibrium With Unobservable Investment

3.3.1 Second Price Auction

Bidding truthfully is a dominant strategy equilibrium in the second price auction. Given these bidding strategies, seller i's profit is zero unless he is the *only* seller whose production cost is $\underline{c}$, in which case his profit is $(p - \underline{c})$.

Therefore, for a given investment profile $\mathbf{x} = (x_1, \ldots, x_n)$ for the n sellers, seller i's expected profit is

$$\Pi_i(x_1, \ldots, x_n) = (1 - r^{x_i})r^{x_{-i}}(p - \underline{c}) - sx_i \quad i = 1, \ldots, n, \tag{3.9}$$

where

$$x_{-i} = \sum_{j \neq i} x_i. \tag{3.10}$$

In equilibrium $(x_1, \ldots, x_n)$ satisfies

$$r^{x_i + x_{-i}}(\ln r^{-1})(p - \underline{c}) = s. \tag{3.11}$$

Therefore, any distribution of investment levels among the n sellers which satisfy

$$r^{\sum x_i} = \frac{s}{(p - \underline{c}) \ln r^{-1}} \tag{3.12}$$

is an equilibrium, and the equilibrium total investment is independent of the number of sellers. At the symmetric equilibrium each seller invests

$$x = \frac{1}{n} \frac{1}{\ln r} \ln\left(\frac{s}{(p - \underline{c}) \ln r^{-1}}\right). \tag{3.13}$$

3.3.2 First Price Auction

When production costs can take only a discrete set of values, there does not exist an equilibrium in pure strategies in the first price auction. In equilibrium the sellers with the low production cost should use mixed bidding strategies, and sellers with the high production cost $\bar{c}$ should bid $\bar{c}$.

Since investment levels are unobservable, the bidding strategies cannot be made contingent on the investment levels. The sellers simultaneously choose investment levels and mixed bidding strategies for each cost level they might have as a result of their investment. Since sellers with the high production cost $\bar{c}$ always bid $\bar{c}$, each seller's strategic choice is reduced to $(x_i, F_i(\cdot))$, where x_i is seller i's investment, and $F_i(\cdot)$ is the cumulative distribution function the seller i uses to determine his bid when his production cost is $\underline{c}$; i.e., $F_i(b) = \text{Prob }\{b_i \leq b\}$.

We will characterize the symmetric equilibrium in which each seller invests the same amount and uses the same mixed bidding strategy.

Let sellers $j = 2, \ldots, n$ use the strategy $(x, F(\cdot))$. Seller 1's expected profit in the auction is then given by

$$\Pi_1(b, \underline{c}, x) = (b - \underline{c})[1 - (1 - r^x)F(b)]^{n-1}, \tag{3.14}$$

if his production cost is $\underline{c}$, and his expected profit is zero if his production cost is $\bar{c}$. Any bid used by seller one with positive probability should satisfy

$$\frac{\partial \Pi_1}{\partial b} = [1-(1-r^x)F(b)]^{n-1}-(b-\underline{c})(n-1)[1-(1-r^x)F(b)]^{n-2}(1-r^x)F'(b) = 0. \tag{3.15}$$

That is,

$$(b - \underline{c}) = \frac{1 - (1 - r^x)F(b)}{(n-1)(1 - r^x)F'(b)}, \tag{3.16}$$

and expected profit in the auction for any such bid for seller 1 is

$$(b - \underline{c})[1 - (1 - r^x)F(b)]^{n-1} = \frac{[1 - (1 - r^x)F(b)]^n}{(n-1)(1 - r^x)F'(b)}. \tag{3.17}$$

Therefore, if seller 1 invests x_1, his expected profit is

$$(1 - r^{x_1})(b - \underline{c})[1 - (1 - r^x)F(b)]^{n-1} - sx_1. \tag{3.18}$$

Maximization with respect to x_1 gives

$$r^{x_1}(b - \underline{c})[1 - (1 - r^x)F(b)]^{n-1} = \frac{s}{\ln r^{-1}}. \tag{3.19}$$

Imposing symmetry, $x_1 = x$ and using $F(p) = 1$,

$$r^{x+(n-1)x} = \frac{s}{(p - \underline{c}) \ln r^{-1}}. \tag{3.20}$$

That is,

$$r^x = [\frac{s}{(p-\underline{c})\ln r^{-1}}]^{\frac{1}{n}}, \tag{3.21}$$

$$x = \frac{1}{n}\frac{1}{\ln r^{-1}}\ln(\frac{s}{(p-\underline{c})\ln r^{-1}}). \tag{3.22}$$

Using $F(\underline{b}) = 0$, we obtain

$$(\underline{b}-\underline{c}) = \frac{s}{r^x \ln r^{-1}} = (p-\underline{c})r^{(n-1)x}, \tag{3.23}$$

$$\underline{b} = \underline{c} + (p-\underline{c})r^{(n-1)x}. \tag{3.24}$$

Finally, solving for $F(\cdot)$ we get

$$F(b) = \frac{1}{1-r^x}\{1-(\frac{b-\underline{c}}{\underline{b}-\underline{c}})^{\frac{1}{n-1}}\} \quad b \in [\underline{b}, p]. \tag{3.25}$$

We have proved

Proposition 3.1 *Symmetric equilibrium under the first price auction with unobservable investment is, for all* $i = 1, \ldots, n$

$$x_i = x = \frac{1}{n}\frac{1}{\ln r}\ln[\frac{s}{(p-\underline{c})\ln r^{-1}}], \tag{3.26}$$

$$F_i(b) = F(b) = \frac{1}{1-r^x}\{1-(\frac{b-\underline{c}}{b-c})^{\frac{1}{n-1}}\} \quad b \in [\underline{b}, p], \tag{3.27}$$

where $\underline{b} = \underline{c} + (p-\underline{c})r^{(n-1)x}$.

Note that total investment level is the same under both auctions and is independent of the number of sellers for any given reserve price p. If reserve price were set at $p = \bar{c}$, both auctions would result in the socially optimal investment level. Note that, the only possible source of social inefficiency in the present setup with two possible production costs is due to inefficiency in the investment level. That is, for any given investment level, the production decision is always efficient in the sense that production takes place at the lowest production cost for any reserve price.

3.4 Equilibrium With Observable Investment for First Price Auction

In this section we characterize the equilibria for the first price auction under investment observability. Since investment observability does not affect bidding behavior in second price auction the set of Nash equilibria under unobservable investment found in the previous section for second price auction is also the set of subgame perfect equilibria under observable investment.

3.4.1 Equilibrium with two sellers

We first consider the simplest case where there are only two sellers. The sellers' production costs can take only two values, $\bar{c}$ and $\underline{c}$, where $\underline{c} < \bar{c}$. Seller i's production cost c_i takes the value $\bar{c}$ with probability $r_i = r(x_i)$ and the value $\underline{c}$ with probability $1-r_i = 1-r(x_i)$, where x_i is seller i's investment in cost-reduction R&D. c_1 and c_2 are statistically independent.

Bidding Equilibrium

We start by characterizing the equilibrium payoffs in the bidding subgame for arbitrary r_i, $i = 1, 2$. We assume that the buyer announces a reserve price $p \in [\underline{c}, \bar{c}]$.

First, it is clear that a seller with production cost $\bar{c}$ always bids $\bar{c}$, and a seller with production cost $\underline{c}$ must randomize. Without loss of generality, assume that $r_1 < r_2$ and $\underline{c} < p < \bar{c}$.[1] Let F_i be the cumulative distribution of seller i's bid and S_i be the support of F_i, $i = 1, 2$.

The following Lemma can be established by modifying the arguments in

[1]If $p = \bar{c}$, a tie-braking rule that picks seller 1 as the winner in case of ties would give the same equilibrium. With $p < \bar{c}$, ties occur with zero probability in equilibrium.

Maskin & Riley [41].

Lemma 3.1

(i) $S_1 = S_2$.

(ii) F_1 *cannot have an atom except at* p.

(iii) At most one of F_i, $i = 1, 2$, *can have an atom at* p.

(iv) S_i *is an interval for* $i = 1, 2$.

Proof: Let $\underline{b}_i$ and $\bar{b}_i$ be the lowest and highest bids made by seller i if his production cost is $\underline{c}$. First, we establish that $F_i(b)$ is continuous over $[\underline{b}_i, \bar{b}_i)$. Suppose otherwise that seller 1 bids b_o with finite probability. Then there is some interval $[b_o + \epsilon, b_o]$ over which seller 2 will not bid because he can increase his winning probability by a finite amount with an infinitesimal decrease in his bid. But then seller 1 can increase his bid to $b_o + \epsilon$ without affecting his probability of winning, which contradicts the assumption that b_o is an optimal bid for seller 1. The same argument applies for seller 2. Therefore, the distribution $F_i(\cdot)$ cannot have any atom on $[\underline{b}_i, \bar{b}_i)$. Unless $\bar{b}_i$ equals the reserve price p, the same argument holds at $\bar{b}_i$. It cannot be optimal for *both* sellers to bid p with finite probability, since then either seller can increase his probability of winning by a finite amount with an infinitesimal reduction in his bid. Suppose S_1 contains bids b^0 and b^1 but no bids in (b^o, b^{oo}), where $b^o < b^{oo}$. Then S_2 cannot contain any bid in $[\, b^o \,,\, (b^0 + b^{oo}) \,/\, 2\,]$, since if this is the case, he can always increase his bid without changing his probability of winning. But then, seller 1 strictly prefers to bid $(b^o + b^{oo})/2$ rather than b^o. Therefore, S_i is an interval for both i; that is, $F_i(\cdot)$ is strictly increasing on S_i. Next, suppose $\bar{b}_1 < \bar{b}_2$. Then seller 2 cannot bid in the interval $(\bar{b}_1 < \bar{b}_2)$. The same argument applies for seller 1 if $\bar{b}_1 > \bar{b}_2$. Therefore, we have $\bar{b}_1 = \bar{b}_2 = \bar{b}$. For the same reason, $\underline{b}_1 = \underline{b}_2 = \underline{b}$. Finally, if $\bar{b} < p$, since there cannot be any atom except at p, seller 1 wins the auction at $\bar{b}$ if the production cost of seller 2 is $\bar{c}$. But then seller 1 strictly prefers to bid p.□

Using Lemma 3.1, equilibrium mixed strategies F_1 and F_2 should satisfy

$$\Pi_1(b,\underline{c}) = (b-\underline{c})[r_2 + (1-r_2)(1-F_2(b))] = \pi_1 \quad \text{for all } b \in S_1, \tag{3.28}$$

and

$$\Pi_2(b,\underline{c}) = (b-\underline{c})[r_1 + (1-r_1)(1-F_1(b))] = \pi_2 \quad \text{for all } b \in S_2, \tag{3.29}$$

where π_1 and π_2 are constants. Since $S_1 = S_2 = S$, we have

$$\pi_1 = \pi_2 = (\underline{b} - \underline{c}), \tag{3.30}$$

where $\underline{b} = inf\{b \mid b \in S\}$.

Using (3.28) and (3.29) we obtain the equilibrium bidding strategies as follows:

Proposition 3.2 *For $r_1 < r_2$, equilibrium mixed strategies for sellers with the low production cost $\underline{c}$ in first price auction with observable investment are given by*

$$F_1(b) = \begin{cases} \frac{b-\underline{b}}{(1-r_1)(b-\underline{c})} & \underline{b} \leq b < p \\ 1 & b = p \end{cases}, \tag{3.31}$$

$$F_2(b) = \frac{b-\underline{b}}{(1-r_2)(b-\underline{c})} \quad \underline{b} \leq b \leq p, \tag{3.32}$$

where $\underline{b} = r_2 p + (1-r_2)\underline{c}$.

For $r_1 < r_2$, seller 1 bids p with probability $[r_2 - r_1]/[1 - r_1]$ and randomizes according to F_1 with remaining probability. Similarly, for $r_1 > r_2$, seller 2's cumulative distribution function $F_2(.)$ has an atom at the reserve price p with mass $[r_1 - r_2]/[1 - r_2]$. Figure 3.1 illustrates the equilibrium mixed strategies for two sellers for $r_1 < r_2$.

The lowest bid in the support of equilibrium bidding strategies, $\underline{b}$, is a weighted average of the reserve price p and the low production cost $\underline{c}$, and the weight on p is the maximum of the two failure probabilities r_1 and r_2. Let $r_m = \max\{r_1, r_2\}$. The lowest bid is, then, given by

$$\underline{b} = r_m p + (1 - r_m)\underline{c}. \tag{3.33}$$

The equilibrium mixed strategy distribution for the seller with the higher probability of a high production cost stochastically dominates the bid distribution of the seller who is less likely to have a high production cost. That is, the more optimistic bidder bids less aggressively conditional on having the low production cost. Therefore, conditional on both sellers' having the low production cost, the seller who is *ex ante* more likely to have the low production cost wins *less often* with a *higher* price. However, the *ex ante* bid distribution of the seller with a more favorable cost distribution is stochastically dominated by the bid distribution of the seller with a less favorable cost distribution. Expected profit of a seller in the auction subgame is given, for $i = 1, 2$, by

$$\Pi_i(r_1, r_2) = (1 - r_i)(\underline{b} - \underline{c}) = (p - \underline{c})(1 - r_i)\max\{r_1, r_2\}. \tag{3.34}$$

Investment Equilibrium in Pure Strategies

Since we know the subgame payoffs for every configuration of production cost distributions, we can reduce the two-stage game to a one-stage one by using the equilibrium profits in the corresponding auction game for any choice of

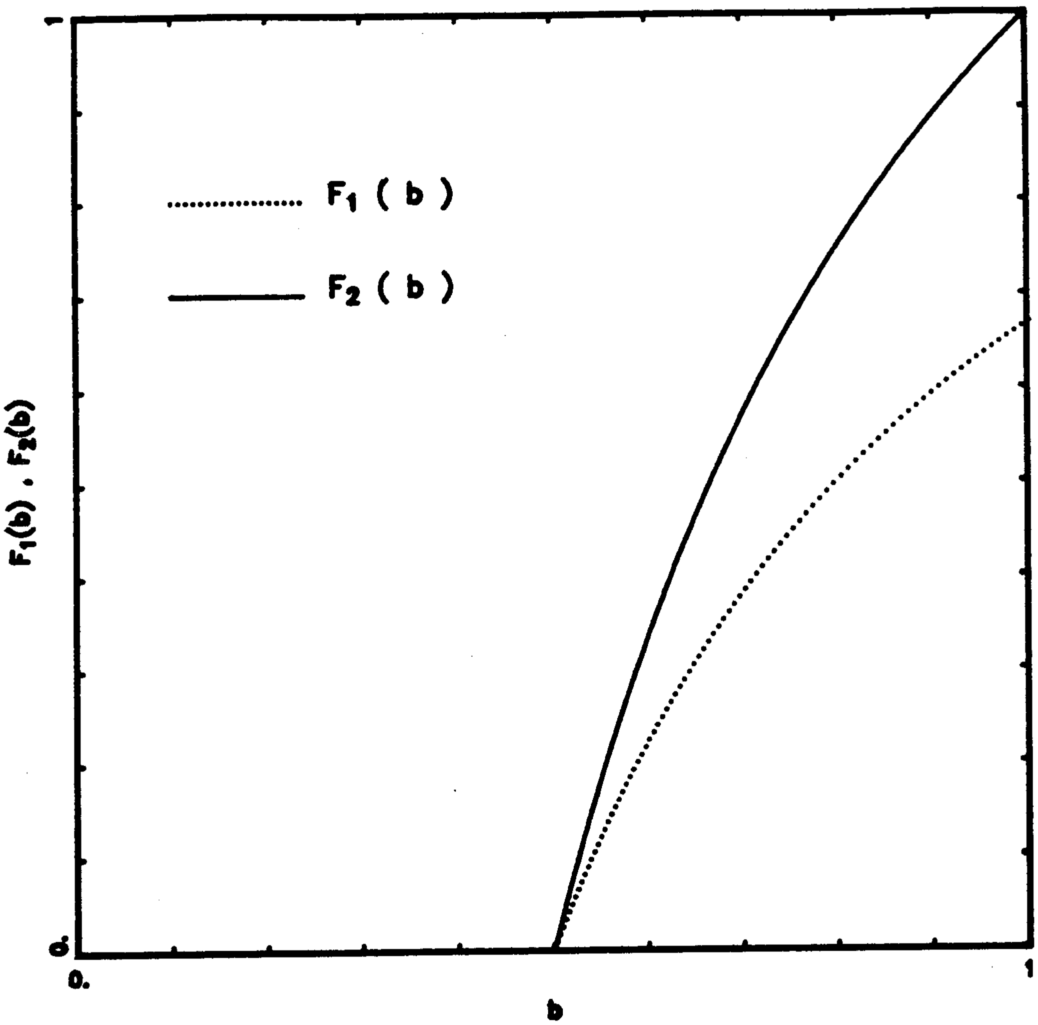

Figure 3.1 : Bidding Equilibrium in Mixed Strategies for Two Sellers.

($r_1 = .25$, $r_2 = .50$, $p = 1$, , $\underline{c} = 0$)

r_1 and r_2 by the two sellers.

The payoffs as a function of investment pairs (x_1, x_2) are given by

$$\Pi_i(x_1, x_2) = (p - \underline{c})(1 - r^{x_i}) \max\{r^{x_1}, r^{x_2}\} - sx_i. \tag{3.35}$$

That is,

$$\Pi_i(x_1, x_2) = \begin{cases} (1 - r^{x_i})r^{x_i}(p - \underline{c}) - sx_i & \text{if} \quad x_i \leq x_{-i}, \\ (1 - r^{x_i})r^{x_{-i}}(p - \underline{c}) - sx_i & \text{if} \quad x_i \geq x_{-i}\,. \end{cases} \tag{3.36}$$

for $i = 1, 2$. Figures 3.2 and 3.3 illustrate the payoff function for player 1 for two different levels of x_2.

Dividing the payoffs by $(p - \underline{c})$, we obtain an equivalent game where search cost is

$$t = s/(p - \underline{c}) \tag{3.37}$$

and the maximum potential profit from the auction is 1. We continue to use the same notation for the transformed payoffs:

$$\Pi_i(x_1, x_2) \quad = \quad (1 - r^{x_i})r^{x_{3-i}} - tx_i \quad \text{if} \quad x_i \geq x_{3-i}\,, \tag{3.38}$$

$$\Pi_i(x_1, x_2) \quad = \quad (1 - r^{x_i})r^{x_i} - tx_i \quad \text{if} \quad x_i \leq x_{3-i}\,. \tag{3.39}$$

Let x^* be the solution to

$$\max_x \, (1 - r^x) - tx. \tag{3.40}$$

That is, x^* is the optimum investment level for a seller whose rivals invest zero. Straightforward calculation gives

$$x^* = \ln[t/\ln r^{-1}]/\ln r, \tag{3.41}$$

$$r^* = r^{x^*} = t/\ln r^{-1}. \tag{3.42}$$

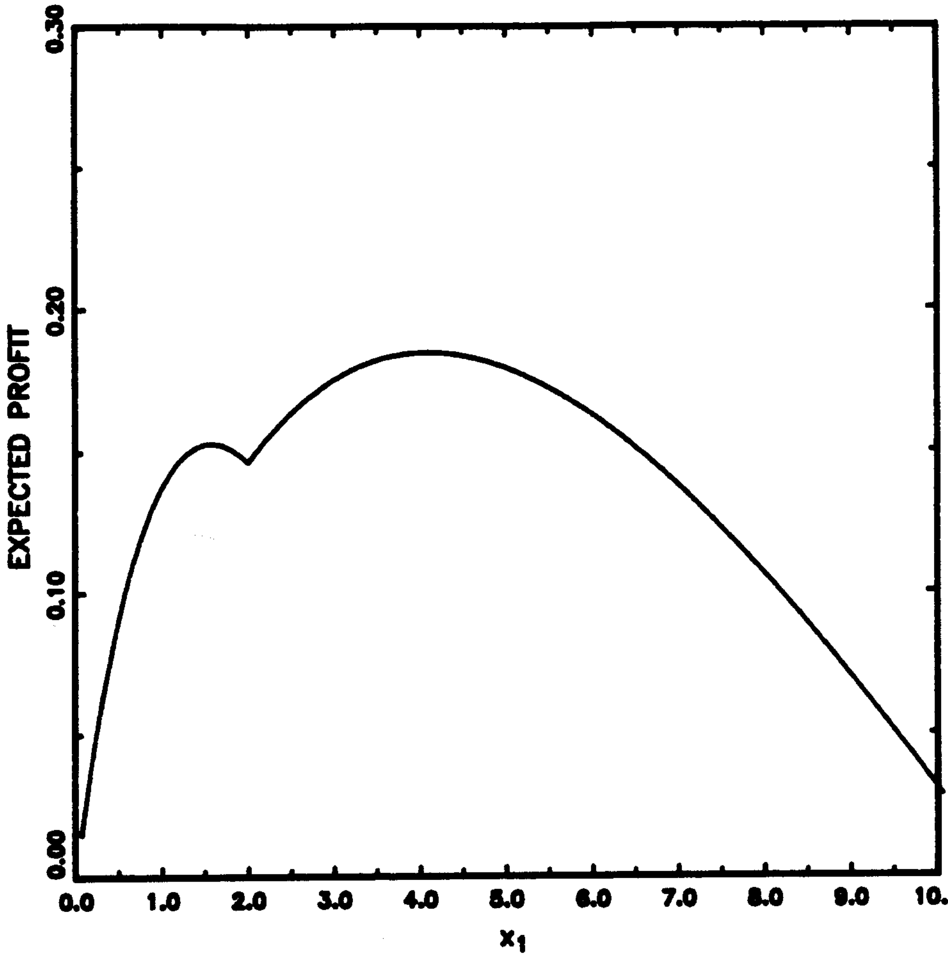

Figure 3.2 : Expected Profit of Seller 1 for $x_2 = 2$.

($r = .75$, $s = .05$, $p = 1$, $\underline{c} = 0$)

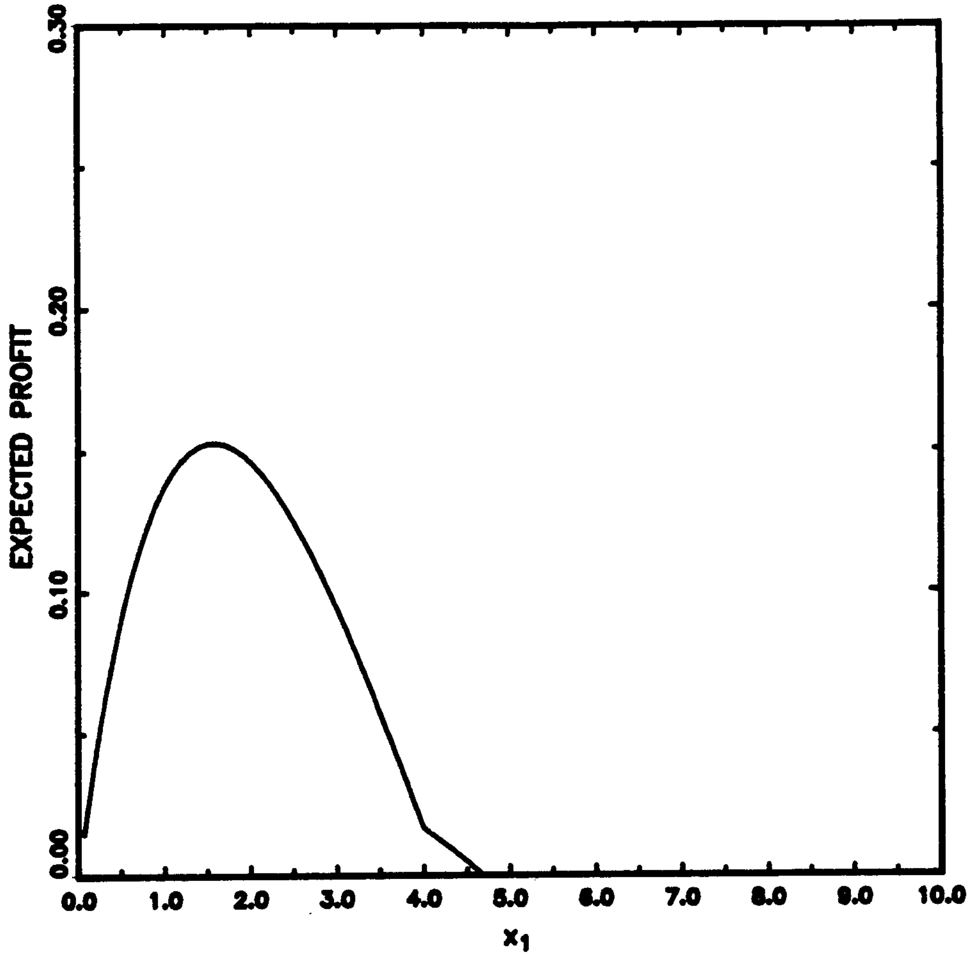

Figure 3.3 : Expected Profit of Seller 1 for $x_2 = 4$.

($r = .75$, $s = .05$, $p = 1$, $\underline{c} = 0$)

r^* is the corresponding failure probability with a single seller.

We first establish that there does not exist a symmetric pure strategy equilibrium for the payoff functions given in (3.38) and (3.39).

Proposition 3.3 *For* $r < 1$ *there does not exist a symmetric pure strategy investment equilibrium in a first price auction with observable investment.*

Proof: Suppose otherwise that (x, x) is a pure strategy equilibrium. We show that at any $(x, x) \neq (0, 0)$, either

(i) $\frac{d}{dx_i}[(1 - r^{x_i})r^{x_i} - tx_i] < 0$, so that it would pay i to reduce x_i, or

(ii) $\frac{d}{dx_i}[(1 - r^{x_i})r^{x} - tx_i] > 0$, so that it would pay i to increase x_i.

Suppose that neither (i) nor (ii) is true: Then

(i') $r^{x_i} \ln r(1 - 2r^{x_i}) - t \geq 0$ and

(ii") $-r^{2x} \ln r - t \leq 0$.

From (ii"), it follows that $-t \leq (\ln r)r^{2x}$. Thus, (i') and (ii") imply

$$r^x \ln r(1 - 2r^x) + (\ln r)r^{2x} \geq 0, \tag{3.43}$$

which is impossible unless $r = 1$. □

Note that the particular form of the relation between investment and probability of the low cost is not crucial for this result. Any decreasing function $r(x)$, $r'(x) < 0$ relating investment level and the probability of low cost would give the same results; and the proof would be the same as above. To obtain the best response correspondence for player 1, we maximize (3.38) and (3.39) separately.

$$\tilde{x}_1(x_2) = arg \max_x (1 - r^x)r^{x_2} - tx \tag{3.44}$$

$$\underline{x} = arg\max_{x}(1 - r^{x})r^{x} - tx \tag{3.45}$$

Carrying out the maximization in (3.44) and (3.45) we get

$$\tilde{x}_1(x_2) = x^* - x_2, \tag{3.46}$$

and $\underline{x}$ solves

$$r^{\underline{x}}(2r^{\underline{x}} - 1) = r^{x^*}. \tag{3.47}$$

The Equation (3.47) is a quadratic in $r^{\underline{x}}$ with root

$$r^{\underline{x}} = \frac{1}{4}(1 + \sqrt{1 + 8r^{x^*}}). \tag{3.48}$$

Note that $\frac{1}{2} < r^{\underline{x}} < 1$. We obtain the solution $\underline{x}$ as

$$\underline{x} = \ln[\frac{1}{4}(1 + \sqrt{1 + 8r^{x^*}})]/\ln r. \tag{3.49}$$

If we establish that $\tilde{x}(\underline{x}) > \bar{x}$, we have an equilibrium. This result is obtained from (3.47), noting that

$$\frac{r^{\bar{x}}}{r^{\underline{x}}} = (2 - \frac{1}{r^{\underline{x}}}) < 1\,, \tag{3.50}$$

since $\frac{1}{2} < r^{\underline{x}} < 1$.

To illustrate the best response functions define $\check{x}$ implicitly and uniquely by

$$(1 - r^{\underline{x}})r^{\underline{x}} - t\underline{x} = (1 - r^{x^*-\check{x}})r^{\check{x}} - t(x^* - \check{x}), \tag{3.51}$$

or equivalently,

$$r^{\check{x}} + t\check{x} = (1 - r^{\underline{x}})r^{\underline{x}} - t\underline{x} + r^{x^*} + tx^*. \tag{3.52}$$

For $x_2 > \check{x}$, the left-hand side of (3.52) is greater than the right-hand side and vice versa. If $x_2 = \check{x}$, seller 1 is indifferent between investing $\underline{x}$ and investing $x^* - \check{x}$. If x_2 exceeds $\check{x}$, the best response of seller 1 is to invest

$\underline{x}$,and if x_2 is strictly less than $\tilde{x}$ seller 1 invests $x^* - x_2$.Therefore, we obtain the best response function of player 1 to be

$$BR_1(x_2) = \begin{cases} \tilde{x}(x_2) & \text{if} \quad (1 - r^{\underline{x}})r^{\underline{x}} - t\underline{x} < (1 - r^{\tilde{x}(x_2)})r^{x_2} - t\tilde{x}(x_2) \\ \underline{x} & \text{otherwise} \end{cases} \tag{3.53}$$

Player 2's reaction function has the same form because of symmetry:

$$BR_2(x_1) = \begin{cases} \tilde{x}(x_1) & \text{if} \quad (1 - r^{\underline{x}})r^{\underline{x}} - t\underline{x} < (1 - r^{\tilde{x}(x_1)})r^{x_1} - t\tilde{x}(x_1) \\ \underline{x} & \text{otherwise} \end{cases} \tag{3.54}$$

See Figure 3.4 for the best response functions for the two players. Note that any investment level below $\underline{x}$ is dominated by $\underline{x}$ for both players. Given this, any investment level above $x^* - \underline{x}$ is dominated by $x^* - \underline{x}$ for both players. Let

$$\bar{x} = x^* - \underline{x}. \tag{3.55}$$

Note that $\bar{x} > \underline{x}$. We have proved

Proposition 3.4 *With two sellers, the only pure strategy investment equilibria in first price auction with observable investment are the asymmetric equilibria:*

$$\{(\underline{x}, \bar{x}) \ , \ (\bar{x}, \underline{x})\}. \tag{3.56}$$

Investment Equilibrium in Mixed Strategies

Each investment profile $\mathbf{x}$ determines a subgame. The equilibrium payoffs in each subgame are completely determined by $\underline{b}(\mathbf{x})$, the lowest bid in the support of mixed bidding strategies in the auction subgame. For risk-neutral sellers, equilibrium payoffs conditional on having $c_i = \underline{c}$ in the subgame are

$$\Pi_i(x_1, x_2) = (p - \underline{c}) \max\{r(x_i), r(x_j)\}(1 - r(x_i)). \tag{3.57}$$

That is,

$$\Pi_i(x_1, x_2) = \begin{cases} (p - \underline{c})r^{x_i}(1 - r^{x_i}) & \text{if} \quad x_i < x_j, \\ (p - \underline{c})r^{x_j}(1 - r^{x_i}) & \text{if} \quad x_i \geq x_j. \end{cases} \tag{3.58}$$

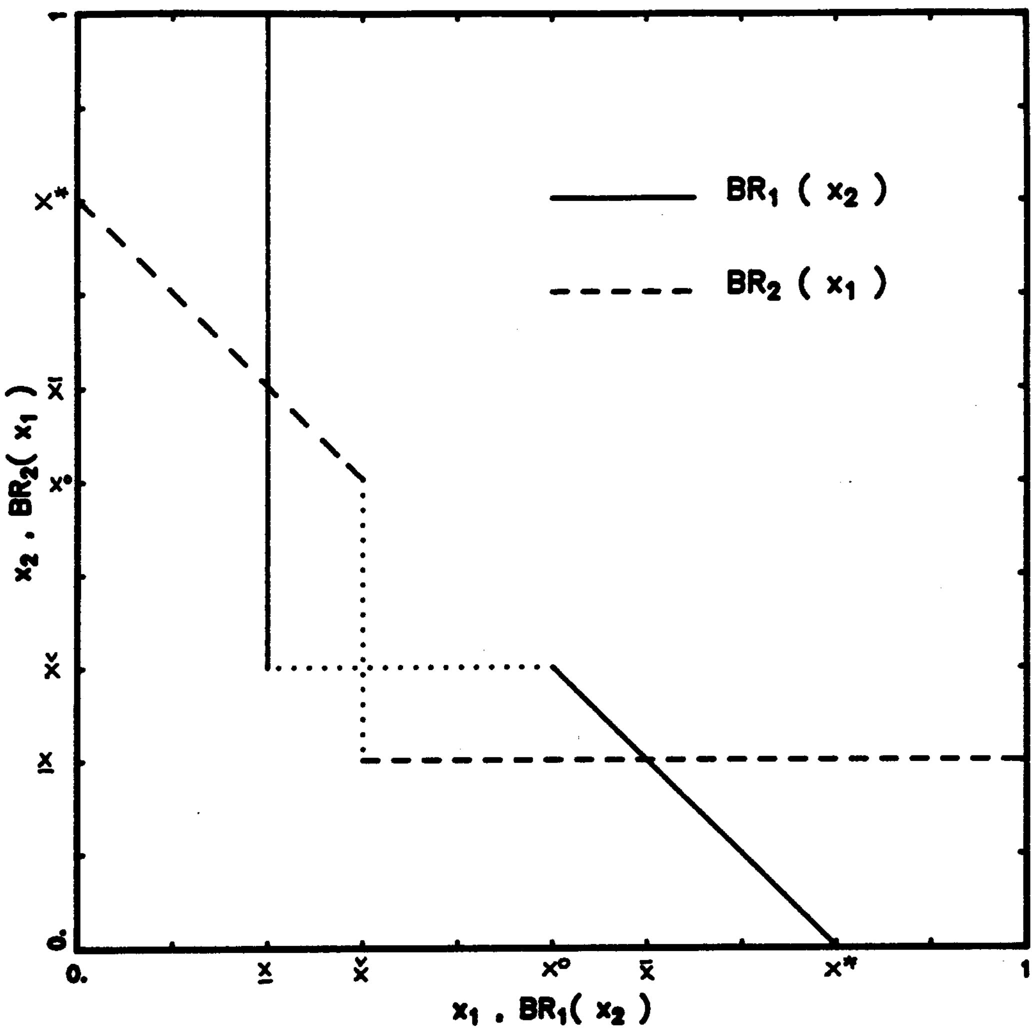

Figure 3.4 : Best Response Functions BR_1 and BR_2

We want to find a symmetric mixed strategy equilibrium in investment decisions.

Let seller 2 use the mixed strategy $G(x)$ on $x \in [x_o, x^o]$ for some $x_o, x^o \in \mathbf{R}$, where $G(x) = Prob\ \{x_2 < x\}$ is a continuous, piecewise differentiable distribution function.

We know that $x_o \geq \underline{x}$ and $x^o \leq \bar{x}$, since dominated strategies cannot be used in a mixed strategy equilibrium. Thus, dominance solvability reduces the support of any mixed strategy equilibrium to a subset of $[\underline{x}, \bar{x}]$. We will see that the support is a strict subset of $[\underline{x}, \bar{x}]$. In particular, $\bar{x}$ is never used in the mixed strategy equilibrium.

The expected payoff to seller 1 is

$$E\Pi_1(x_1) = (p-\underline{c})(1-r^{x_1})\{r^{x_1}[1-G(x_1)]+\int_{x_o}^{x_1} r^{\omega}G'(\omega)\,d\omega\}-sx_1 = K \quad (3.59)$$

for all $x_1 \in$ support $G(\cdot)$, where K is a constant. That is, $E\Pi_1(x_1)$ is independent of x_1 on $[x_o, x^o]$. If we substitute $G(x_o) = 0$ in (3.59) to get

$$K = (1 - r^{x_o})r^{x_o} - tx_o, \quad (3.60)$$

we obtain $x_o = \underline{x}$. Otherwise, always investing $\underline{x}$ dominates x_o, since $\underline{x}$ maximizes the right-hand side of (3.60).

Since $E\Pi_1(x_1)$ is independent of x_1 on $[x_o, x^o]$,

$$\begin{aligned}
\frac{\partial E(x_1)}{\partial x_1} &= (1 - r^{x_1})\{r^{x_1} \ln r[1 - G(x_1)] - r^{x_1}G'(x_1) + r^{x_1}G'(x_1)\} \\
&\quad -r^{x_1} \ln r\{r^{x_1}[1 - G(x_1)] + \int_{x_o}^{x_1} r^{\omega}G'(\omega)d\omega\} - t \quad (3.61) \\
&= (1 - r^{x_1})r^{x_1} \ln r[1 - G(x_1)] - r^{x_1}(\ln r)r^{x_1}[1 - G(x_1)] \\
&\quad -r^{x_1} \ln r \int_{x_o}^{x_1} r^{\omega}G'(\omega)d\omega - t \quad (3.62) \\
&= r^{x_1} \ln r\{(1 - r^{x_1})[1 - G(x_1)] - r^{x_1}[1 - G(x_1)] \\
&\quad - \int_{x_o}^{x_1} r^{\omega}G'(\omega)d\omega\} - t \quad (3.63)
\end{aligned}$$

$$= \quad r^{x_1}\ln r^{-1}\{\int_{x_o}^{x_1} r^{\omega}G'(\omega)d\omega - [1-G(x_1)][1-2r^{x_1}]\} - t = 0, \tag{3.64}$$

from which it follows that

$$\int_{x_o}^{x_1} r^{\omega}G'(\omega)d\omega = \frac{t}{r^{x_1}\ln r^{-1}} + [1-G(x_1)](1-2r^{x_1}). \tag{3.65}$$

Substituting (3.60) and (3.65) in (3.59),

$$(1-r^{x_1})\{r^{x_1}(1-G(x_1)+(1-2r^{x_1})(1-G(x_1))+\frac{t}{r^{x_1}\ln r^{-1}}\}-tx_1 = (1-r^{x_o})r^{x_o}-tx_o \tag{3.66}$$

for all $x_1 \in [x_o, x^o]$. Imposing symmetry, and using $x_o = \underline{x}$ and the fact that $t/[\ln r^{-1}] = r^{x^*}$, we get

$$G(x) = 1-\frac{1}{(1-r^x)^2}\{r^{\underline{x}}(1-r^{\underline{x}})+s(x-\underline{x})-(1-r^x)r^{x^*-x}\} \quad x \in [\underline{x}, x^o]. \tag{3.67}$$

Using $x_1 = x^o$ in (3.65), we obtain the expected value of r^x as

$$E(r^x) = \int_{\underline{x}}^{x^o} r^{\omega}G'(\omega)d\omega = r^{x^*-x^o}. \tag{3.68}$$

To obtain x^o we use $G(x^o) = 1$ which implies that x^o solves

$$r^{\underline{x}}(1-r^{\underline{x}}) - t\underline{x} = (1-r^{x^o})r^{x^*-x^o} - tx^o\,; \tag{3.69}$$

i.e., $x^o = x^* - \check{x}$, where $\check{x}$ is defined in (3.51)-(3.52). The solution to (3.69) is unique since at $x^o = \underline{x}$ the left-hand side is greater than the right-hand side, and the right-hand side is strictly increasing in x^o on $(\underline{x}, x^*)$. The fact that the right-hand side is strictly greater than the left-hand side at $x^o = \bar{x}$ $(= x^* - \underline{x})$ implies

$$x_o < \bar{x}. \tag{3.70}$$

We evaluate (3.65) at x_0 and x^o, and use the fact that r^{ω} is decreasing ω to obtain the following bounds:

$$x^* > x^o > x^*/2, \tag{3.71}$$

and

$$x^o > E(x) > x^* - x^o. \tag{3.72}$$

The symmetric mixed strategy equilibrium for two sellers is illustrated in Figure 3.5.

Note that symmetric mixed strategy equilibrium is Pareto-dominated by any of the two asymmetric pure strategy equilibria; that is, an asymmetric pure strategy equilibrium gives strictly higher expected profit to one seller than the symmetric equilibrium, while giving the other seller the same expected profit as in the symmetric equilibrium. We will study only the pure strategy equilibria in the investment game in the following sections.

3.4.2 Equilibrium with n sellers

Bidding Equilibrium

We start with characterizing the bidding equilibrium in the auction stage for arbitrary distribution of investment levels. We continue to use the same notation as in the previous section. For $i = 1, \ldots n$, let $r_i = Prob\{c_i = \bar{c}\}$, $c_i \in \{\underline{c}, \bar{c}\}$. The reserve price is p. Without loss of generality we assume

$$r_1 \leq r_2 \leq r_3 \leq \ldots \leq r_n. \tag{3.73}$$

To characterize the mixed strategy equilibrium we cannot use Lemma 3.1 of the previous section. Although Lemma 3.1 guaranteed that the equilibrium characterized for the two-seller case is essentially unique, for general n we need to extend Lemma 3.1 . First of all, part (ii) of Lemma 3.1 about *equality* of the supports of the mixed strategies is no longer necessary for general n. The other parts of the Lemma reduce the search for mixed strategy equilibrium to continuous cumulative distribution functions defined over

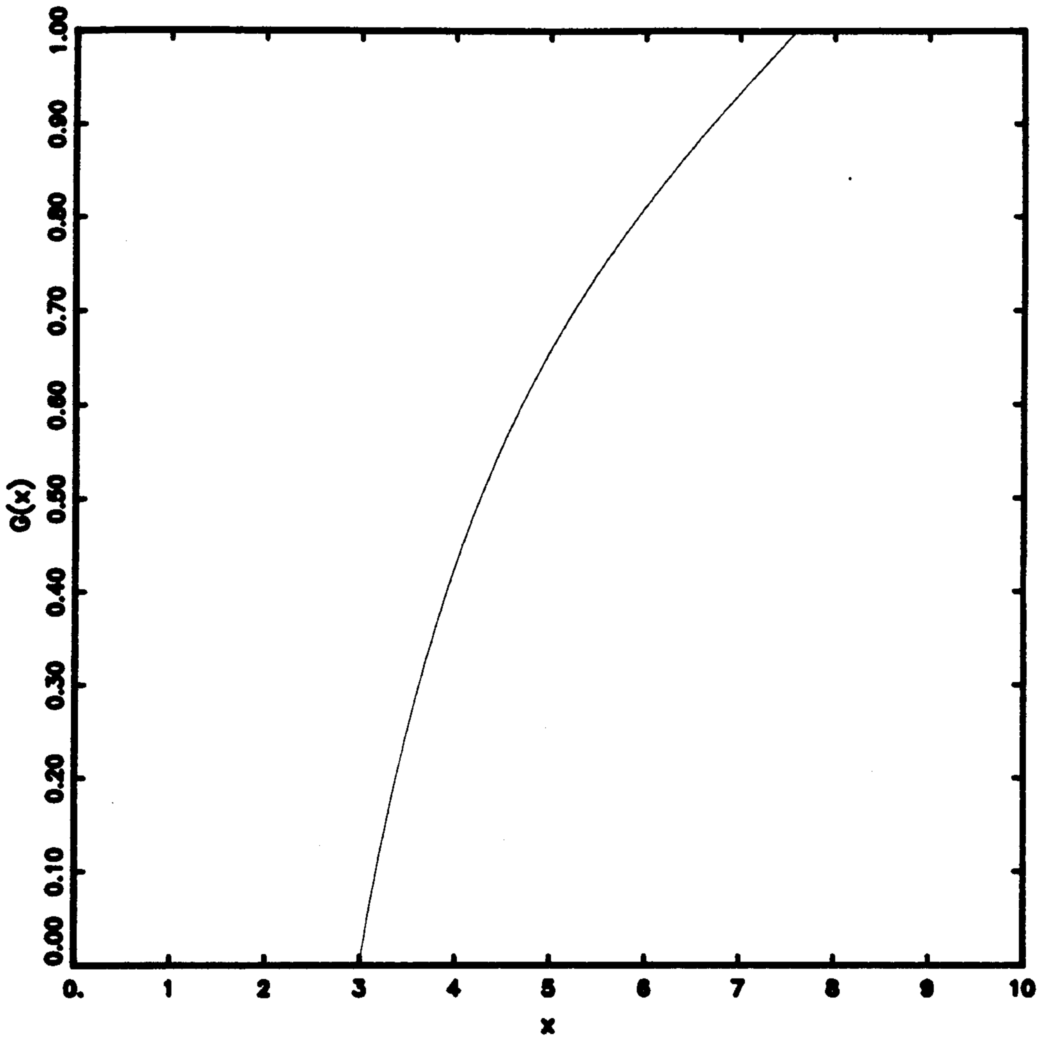

Figure 3.5 : Symmetric Investment Equilibrium in Mixed Strategies.

$(r = .85, \ s = .023, \ p = 1, \ \underline{c} = 0)$

intervals. We characterize the necessary conditions for equilibrium for an arbitrary number of sellers in series of Lemmas and find that the mixed strategy equilibrium with n sellers has features similar to the two-seller case.

In equilibrium, sellers with $c_i = \bar{c}$ bid $\bar{c}$. If $F_i(\cdot)$ is the equilibrium mixed strategy used by seller i when his production cost $c_i = \underline{c}$, from Lemma 3.1, the support of $F_i(\cdot)$ should be an interval. Let $S_i =$ support $F_i(\cdot)$ and

$$\underline{b}_i = \inf \{b : b \in S_i\} \tag{3.74}$$

and

$$\bar{b}_i = \ sup \{b : b \in S_i\}. \tag{3.75}$$

If $\{F_i(b) : i = 1, \dots n\}$ is a mixed strategy equilibrium then we must have for all $i \in \{1, \dots, n\}$:

$$(b - \underline{c}) \prod_{j \neq i} [1 - (1 - r_j)F_j(b)] = \pi_i \quad \text{for all } b \in S_i\,, \tag{3.76}$$

where π_i is a constant.

We know that at an equilibrium,

$$\pi_i \geq (p - \underline{c}) \prod_{k \neq i} r_k \tag{3.77}$$

for all $i \in N$, since seller i can get the amount in the right-hand side of (3.77) by always bidding p, and any bid above p is strictly dominated by a bid of p or less.

It follows directly from (3.76) that, for all $i, j \in N$, if $\underline{b}_i = \underline{b}_j$ then $\pi_i = \pi_j$.

Lemma 3.2 *For all $i, j \in N$, if $\underline{b}_i \geq \underline{b}_j$ then $\pi_i \geq \pi_j$.*

Proof: Since neither $F_i(\cdot)$ nor $F_j(\cdot)$ can have an atom at $\underline{b}_j$ we have $F_i(\underline{b}_j) = F_j(\underline{b}_j) = 0$. Therefore,

$$\pi_j = (\underline{b}_j - \underline{c}) \prod_{k \neq i,j} [1 - (1 - r_k)F_k(\underline{b}_j)]. \tag{3.78}$$

For seller i, since $\underline{b}_j \notin S_i$, we have

$$\pi_i \geq (\underline{b}_j - \underline{c}) \prod_{k \neq i,j} [1 - (1 - r_k) F_k(\underline{b}_j)]. \tag{3.79}$$

Therefore, $\pi_i \geq \pi_j$ if $\underline{b}_i \geq \underline{b}_j$. □

Lemma 3.3 *For all $j \in N$, $\underline{b}_j = \underline{b}$ for some $\underline{b} \in [\underline{c}, p]$.*

Proof: Suppose otherwise. Let $\underline{b}_i = \max \{\underline{b}_j : j \in N\}$. Take any $j \in N$ with $\underline{b}_j < \underline{b}_i$. From Lemma 3.2,

$$\pi_i \geq \pi_j \text{ for all } j \in N. \tag{3.80}$$

From (3.76),

$$\pi_i = (\underline{b}_m - \underline{c})[1 - (1 - r_j) F_j(\underline{b}_m)](\underline{b}_j - \underline{c}) \prod_{k \neq i,j} [1 - (1 - r_k) F_k(\underline{b}_j)]. \tag{3.81}$$

Now, if j always bids $\underline{b}_i$, his expected profit would be

$$(\underline{b}_i - \underline{c})(\underline{b}_j - \underline{c}) \prod_{k \neq i,j} [1 - (1 - r_k) F_k(\underline{b}_j)], \tag{3.82}$$

since $F_i(\underline{b}_i) = 0$. Comparing (3.82) with (3.81) we have

$$\pi_i < \pi_j, \tag{3.83}$$

which contradicts (3.80) . □

Lemma 3.4 *If $\underline{b}_i = \underline{b}_j$ and $r_i < r_j$, then F_i stochastically dominates F_j; i.e., for all $b \in S_i \cap S_j$*

$$F_i(b) < F_j(b). \tag{3.84}$$

Proof: Since $\underline{b}_i = \underline{b}_j$ we have $\pi_i = \pi_j$. Thus, substituting for π_i and π_j from (3.76), it must be the case that for all $b \in S_i \cap S_j$,

$$(b - \underline{c})[1 - (1 - r_i) F_i(b)] \{(\underline{b}_j - \underline{c}) \prod_{k \neq i,j} [1 - (1 - r_k) F_k(\underline{b}_j)]\} = \tag{3.85}$$

$$(b-\underline{c})[1-(1-r_j)F_j(b)]\{(\underline{b}_j-\underline{c})\prod_{k\neq i,j}[1-(1-r_k)F_k(\underline{b}_j)]\}. \qquad (3.86)$$

Cancellations in (3.86) give

$$\frac{F_i(b)}{F_j(b)}=\frac{1-r_j}{1-r_i}<1, \qquad (3.87)$$

since $r_i<r_j$. □

In view of Lemma 3.3 and Lemma 3.4, for $r_1\leq r_2\leq\cdots\leq r_n$,

$$F_1(b)\leq F_2(b)\leq\cdots\leq F_n(b), \qquad (3.88)$$

and for each $i\in N$, $S_i=[\underline{b},\overline{b}_i]$ where

$$\overline{b}_n\leq\overline{b}_{n-1}\leq\cdots\leq\overline{b}_2=\overline{b}_1=p. \qquad (3.89)$$

Lemma 3.5 *If $r_1<r_2$, then $F_1(\cdot)$ has an atom at p.*

Proof: Otherwise, by Lemma 3.4, there would be an interval $[\overline{b}_2,p]$ where only seller 1 bids with positive probability. Since, then, $\overline{b}_2$ and p have the same probability of winning, bidding p would be strictly preferred by seller 1. □

Since there can be at most one seller who bids p with strictly positive probability, and p is in the support of at least two bidding distributions, we have, for seller 1,

$$\pi_1=(p-\underline{c})\prod_{k=2}^{n}r_k\,. \qquad (3.90)$$

Lemma 3.6 $\underline{b}=\underline{c}+(p-\underline{c})\prod_{k=2}^{n}r_k$.

Proof: Immediate from (3.90) and the fact that $\pi_1=(\underline{b}-\underline{c})$. □

From Lemmas 3.1 through 3.6 we can restrict our search to n cumulative distribution functions with nested supports. For $i=2,3,\ldots,n$, define

$$\phi_i=1-\{[(p-\underline{c})(\prod_{k=2}^{i}r_k)]\}^{\frac{1}{i-1}}. \qquad (3.91)$$

The following set of functions constitute a mixed strategy Nash equilibrium.

Proposition 3.5 *In a first price auction with observable investment, equilibrium bidding strategies, for $r_1 \leq r_2 \leq \ldots \leq r_n$, are such that*

1. *if $c_i = \bar{c}$, then $b_i = \bar{c}$,*

2. *if $c_i = \underline{c}$, then seller i randomizes his bid according to $F(\cdot)$,*

where

$$F_1(b) = \begin{cases} \phi_n/(1-r_1) & b \in [\underline{b}, \bar{b}_n] \\ \phi_{n-1}/(1-r_1) & b \in [\bar{b}_n, \bar{b}_{n-1}] \\ \phi_{n-2}/(1-r_1) & b \in [\bar{b}_{n-1}, \bar{b}_{n-2}] \\ & \vdots \\ \phi_3/(1-r_1) & b \in [\bar{b}_4, \bar{b}_3] \\ \phi_2/(1-r_1) & b \in [\bar{b}_3, p] \end{cases}$$

$$F_2(b) = \begin{cases} \phi_n/(1-r_2) & b \in [\underline{b}, \bar{b}_n] \\ \phi_{n-1}/(1-r_2) & b \in [\bar{b}_n, \bar{b}_{n-1}] \\ \phi_{n-2}/(1-r_2) & b \in [\bar{b}_{n-1}, \bar{b}_{n-2}] \\ & \vdots \\ \phi_3/(1-r_2) & b \in [\bar{b}_4, \bar{b}_3] \\ \phi_2/(1-r_2) & b \in [\bar{b}_3, p] \end{cases}$$

$$F_3(b) = \begin{cases} \phi_n/(1-r_3) & b \in [\underline{b}, \bar{b}_n] \\ \phi_{n-1}/(1-r_3) & b \in [\bar{b}_n, \bar{b}_{n-1}] \\ \phi_{n-2}/(1-r_3) & b \in [\bar{b}_{n-1}, \bar{b}_{n-2}] \\ & \vdots \\ \phi_3/(1-r_3) & b \in [\bar{b}_4, \bar{b}_3] \end{cases}$$

$$\vdots$$

$$F_{n-1}(b) = \begin{cases} \phi_n/(1-r_{n-1}) & b \in [\underline{b}, \bar{b}_n] \\ \phi_{n-1}/(1-r_{n-1}) & b \in [\bar{b}_n, \bar{b}_{n-1}] \end{cases}$$

$$F_n(b) = \begin{cases} \phi_n/(1-r_n) & b \in [\underline{b}, \bar{b}_n] \end{cases} \tag{3.92}$$

where

$$\underline{b} = \underline{c} + (p - \underline{c}) \prod_{k=2}^{n} r_k, \tag{3.93}$$

and $\overline{b}_j$, for $j = 3, \ldots, n$, are determined by the conditions $F_j(\overline{b}_j) = 1$ to be

$$\overline{b}_j = \underline{c} + (p - \underline{c})(\prod_{k=2}^{j-1} r_k)/(r_j)^{j-2}. \tag{3.94}$$

Proof: The proof is obtained by solving the equations in (3.76) recursively starting from the rightmost interval $[\overline{b}_3, p]$, where there are only two sellers bidding with positive probability. The procedure is lengthy but simple algebra. It is easier to verify that the given functions satisfy (3.76). □

As in the two-sellers case, at this equilibrium each seller has the same expected utility conditional on having the low production cost . Also, the seller with the lowest probability of low production cost submits a bid equal to the reserve price with a positive probability. Each seller's expected profit, conditional on having the low production cost, is completely determined by the common lowest bid in the supports of equilibrium mixed strategies:

$$\underline{b} = \underline{c} + (p - \underline{c}) \max_j \{\prod_{i \neq j} r_i\}. \tag{3.95}$$

The expected profit of each seller with the low production cost is equal to the common lowest bid in the supports of mixed strategy distributions minus the production cost:

$$\pi(r_1, r_2, \ldots, r_n) = (p - \underline{c}) \max_j \{\prod_{i \neq j} r_i\}. \tag{3.96}$$

Figure 3.6 illustrates the equilibrium mixed strategies for three sellers for $r_1 < r_2 < r_3$.

Investment Equilibrium

For any given n-tuple of investment strategies $\mathbf{x} = (x_1, \ldots, x_n)$, if we let $r_i = r^{x_i}$, $i = 1, 2, \ldots, n$, equilibrium bidding strategies in the auction subgame

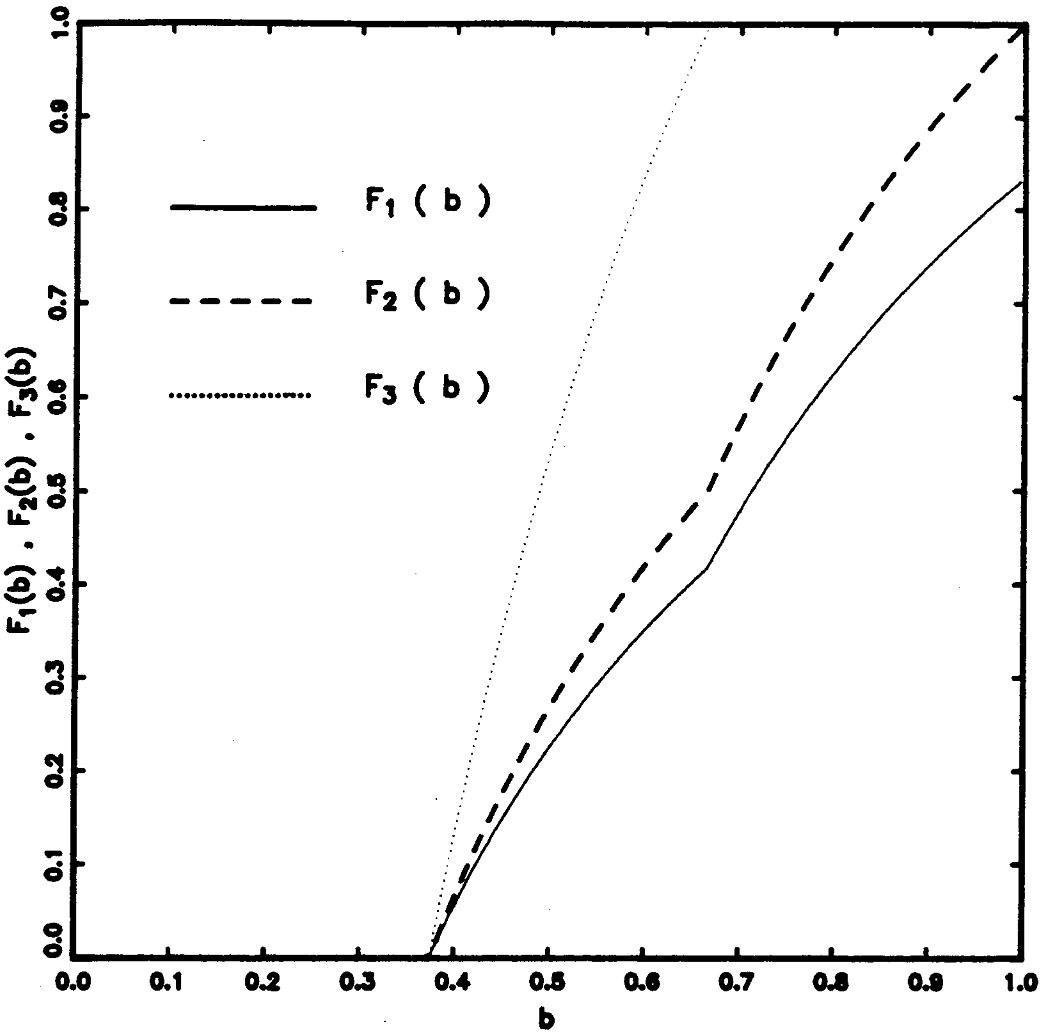

Figure 3.6 : Bidding Equilibrium in Mixed Strategies for Three Sellers.

($r_1 = .40$, $r_2 = .50$, $r_3 = .75$, $p = 1$, $\underline{c} = 0$)

give rise to the following payoff function for the investment game

$$\Pi_i(\mathbf{x}) = (1 - r^{x_i})\pi(\mathbf{x}) - sx_i \quad i = 1, 2, \ldots, n, \tag{3.97}$$

where

$$\begin{aligned} \pi(x_1, x_2, \ldots, x_n) &= \pi(r^{x_1}, r^{x_2}, \ldots, r^{x_n}) \\ &= (p - \underline{c}) \max_j \{\prod_{i \neq j} r^{x_i}\} \\ &= (p - \underline{c}) r^{x_T - x_m} \end{aligned} \tag{3.98}$$

and

$$x_T = \sum_{i=1}^{n} x_i, \tag{3.99}$$

$$x_m = \max\{x_1, x_2, \ldots, x_n\}, \tag{3.100}$$

$$m = \{j \; : \; x_j = x_m \text{ and } j < i \text{ for all } i \text{ such that } x_i = x_m\}. \tag{3.101}$$

We will need some additional notation to proceed. Let

$$x_{-m} = x_T - x_m, \tag{3.102}$$

$$x_{-i} = x_T - x_i, \tag{3.103}$$

$$x_{-im} = x_{-i} - x_m, \tag{3.104}$$

$$x_{mi} = \max\{x_j \; : \; j \neq i\}. \tag{3.105}$$

Again, as in the two-seller case, the payoff function of seller i has a kink at x_{mi}.

Dividing the payoff function in (3.97) by $(p - \underline{c})$, we write an equivalent payoff function for the investment game, using the above notation

$$\Pi_i(\mathbf{x}) = \begin{cases} (1 - r^{x_i}) r^{x_i + x_{-im}} - tx_i & \text{if } x_i < x_{mi}, \\ (1 - r^{x_i}) r^{x_{-i}} - tx_i & \text{if } x_i \geq x_{mi}. \end{cases} \tag{3.106}$$

where $t = s/(p - \underline{c})$.

Equilibrium strategies are given by the solution to the first-order conditions. For each k in $\{1, 2, \ldots, n\}$, we have an equilibrium satisfying

$$r^{x_k + x_{-k}} \ln r^{-1} = t, \tag{3.107}$$

$$(2r^{x_i} - 1) r^{x_i + x_{-ik}} \ln r^{-1} = t \quad i \neq k. \tag{3.108}$$

Using $r^{x^*} = t / \ln r^{-1}$, where x^* is the optimal investment level in the single-seller case, and the fact that

$$x_i + x_{-ik} = x_{-k}, \tag{3.109}$$

we rewrite the first-order conditions in the form

$$r^{x_k} = r^{x^* - x_{-k}}, \tag{3.110}$$

$$(2r^{x_i} - 1) = r^{x^* - x_{-k}} \quad i \neq k. \tag{3.111}$$

The $(n-1)$ equations in (3.111) have a unique symmetric solution $\underline{x}$, which satisfies

$$r^{(n-1)\underline{x}}(2r^{\underline{x}} - 1) = r^{x^*} \tag{3.112}$$

which is an n^{th} order polynomial in $r^{\underline{x}}$. Let $z = r^{x}$. If $\underline{z}$ is the unique positive real root of

$$2z^n - z^{n-1} = r^{x^*}, \tag{3.113}$$

we obtain $\underline{x}$ via

$$\underline{x} = (\ln \underline{z}) / \ln r\,, \tag{3.114}$$

and x_k is obtained from (3.111) to be

$$x_k = x^* - (n-1)\underline{x} =: \bar{x}. \tag{3.115}$$

Establishing that $\bar{x}$ is indeed greater than $\underline{x}$ proves that we have the equilibrium solution.

Note that the solution $\underline{x}(n, r, t)$ to the equations in (3.111) depends on (r, t) only through $r^* = t/\ln r^{-1}$. Therefore, we can write the solution, with some abuse of notation, as $\underline{x}(n, r^*)$.

Define

$$\mathbf{x}^k = (x_1^k, x_2^k, \ldots, x_n^k), \tag{3.116}$$

where

$$x_i^k = \begin{cases} \bar{x}(n, r^*) & \text{if } i = k \\ \underline{x}(n, r^*) & \text{otherwise .} \end{cases} \tag{3.117}$$

Proposition 3.6 *When investment levels are observable by sellers in the first price auction subgame, the set of pure strategy Nash equilibria in the investment game is*

$$\{\mathbf{x}^k : \ k = 1, 2, \ldots, n\}. \tag{3.118}$$

That is, there are n pure strategy equilibria, and all pure strategy equilibria are asymmetric. Furthermore, the total investment level $(n-1)\underline{x}(n, r^*) + \bar{x}(n, r^*) = x^*$ is independent of n.

Proof: Non-existence of a symmetric pure strategy equilibrium is established as in the two-seller case: If $(x, x, \ldots, x)$ is a symmetric pure strategy equilibrium, it should be the case that

$$r^{nx} \ln r^{-1} - t \leq 0 \tag{3.119}$$

and

$$(2r^x - 1) r^{(n-1)x} \ln r^{-1} - t \geq 0. \tag{3.120}$$

Substituting $(2r^x - 1) r^{(n-1)x} \ln r^{-1}$ for t in the first inequality, we get a contradiction: $r^{(n-1)x} - r^{nx} \leq 0$.

For the asymmetric pure strategy equilibria, all we need to show is that $\bar{x}(n, r^*) > \underline{x}(n, r^*)$. To avoid notational clutter, we will suppress the arguments of the functions $\bar{x}(\cdot,\cdot)$ and $\underline{x}(\cdot,\cdot)$, in the following. We will show that $r^{\bar{x}} < r^{\underline{x}}$, or $(r^{\bar{x}}/r^{\underline{x}}) < 1$. Using the definition of $\bar{x}$ via the first order condition, we get

$$\frac{r^{\bar{x}}}{r^{\underline{x}}} = \frac{r^{x^*}}{(r^{\underline{x}})^n}, \tag{3.121}$$

and from the fact that $r^{\underline{x}}$ solves $2(r^{\underline{x}})^n - (r^{\underline{x}})^{n-1} = r^{x^*}$, we know

$$\frac{1}{2} < r^{\underline{x}} < 1 \quad \text{for all } n \tag{3.122}$$

and

$$\frac{r^{\bar{x}}}{r^{\underline{x}}} = (2 - \frac{1}{r^{\underline{x}}}) < 1, \tag{3.123}$$

as required. Second order conditions are easily seen to be satisfied with strict inequality. Finally, $\bar{x} + (n-1)\underline{x} = x^*$ follows from the first order condition for k. □

The following lemma will be useful in proving some interesting facts about the structure of asymmetric pure strategy equilibria.

Lemma 3.7 *If $\underline{z}$ is the solution to*

$$\psi(z) := z^{n-1}(2z-1) = \omega, \tag{3.124}$$

for $\omega \in (0,1)$, then

$$\max\{\frac{1}{2}, \omega^{\frac{1}{n+1}}\} < \underline{z} < \frac{1}{2}(1 + \omega^{2/(n+1)}), \tag{3.125}$$

for all $\omega \in (0,1)$ and for all $n \geq 2$.

Proof: $\psi(z)$ is strictly increasing in z for $z > (n-1)/(2n)$, in particular, for $z > 1/2$. Thus, to prove the lemma, all we need to show is that $\psi(z)$

evaluated at $z = \frac{1}{2}(1 + \omega^{2/(n+1)})$ exceeds ω, and $\psi(z)$ evaluated at $z = \omega^{\frac{1}{n+1}}$ is less than ω. It is easy to show that both inequalities are true if and only if

$$2\omega^{\frac{1}{n+1}} - (\omega^{\frac{1}{n+1}})^2 < 1\,, \tag{3.126}$$

which is always true for $\omega^{\frac{1}{n+1}} \in (0,1)$. □

Proposition 3.7 *For all $n \geq 2$, and all $r^* \in (0,1)$,*

$$\underline{x}(n, r^*) < \frac{x^*}{n+1}, \tag{3.127}$$

and

$$\underline{x}(n, r^*) < \frac{\bar{x}(n, r^*)}{2}. \tag{3.128}$$

Proof: The inequality in (3.127) is obtained by taking $\omega = r^{x^*}$ in Lemma 3.7 and noting that $\underline{x}(n, r^*) = \ln(\underline{z})/lnr$. Inequality (3.128) is equivalent to inequality (3.127), since $\bar{x}(n, r^*) = x^* - (n-1)\underline{x}(n, r^*)$. □

To study the behavior of asymmetry as the number of sellers becomes large we totally differentiate the identity in (3.113) to obtain

$$\frac{d\, r^{\underline{x}}}{d\, n} = \frac{r^{\underline{x}}(1 - 2r^{\underline{x}}) \ln r}{2nr^{\underline{x}} - n + 1} > 0\,, \tag{3.129}$$

since both the numerator and the denominator are positive in view of the fact that $1 > r^{\underline{x}} > (1/2)$. Therefore,

$$\frac{d\, \underline{x}(n, r^*)}{d\, n} = \frac{(1 - 2r^{\underline{x}})}{1 + n(2r^{\underline{x}} - 1)} < 0. \tag{3.130}$$

Since (3.123) is an identity ,we differentiate the right-hand side with respect to n to obtain

$$\frac{d\,(2 - r^{-\underline{x}})}{d\, n} = r^{-\underline{x}}(\ln r)\frac{d\, \underline{x}(n, r^*)}{d\, n} > 0. \tag{3.131}$$

Therefore, the ratio $r^{\bar{x}}/r^{\underline{x}}$ is increasing in n. It follows that

$$\frac{d\,(n-1)\underline{x}(n,r^*)}{dn} > 0\,. \tag{3.132}$$

Increasing the number of sellers in a first price auction with observable investment means not only competition among more sellers, but also competition among sellers who are more alike. In Figure 3.7 we illustrate the high and low equilibrium investment levels, $\bar{x}(n,r^*)$ and $\underline{x}(n,r^*)$ for a fixed r^*.

For another comparative statics result, we differentiate the identity (3.113) with respect to r^* and obtain, as expected,

$$\frac{d\,\underline{x}(n,r^*)}{d\,r^*} = \frac{1}{r^{(n-1)\underline{x}}[1+n(2r^{\underline{x}}-1)]\ln r} < 0. \tag{3.133}$$

Finally, as r^* increases, the difference between $\bar{x}(n,r^*)$ and $\underline{x}(n,r^*)$ becomes smaller, i.e.,

$$\frac{d\,(2-r^{-\underline{x}})}{d\,r^*} = r^{-\underline{x}}(\ln r)\frac{d\,\underline{x}(n,r^*)}{d\,r^*} > 0. \tag{3.134}$$

Since $r^* = t/\ln r^{-1}$, and $t = s/(p-\underline{c})$, r^* is increasing in r and s, decreasing in p. Therefore, the asymmetry in equilibrium investment levels is greater; the lower the probability of high production cost, r, the lower the search cost, s, or the higher the reserve price, p.

A striking feature of the pure strategy equilibria is the form of asymmetry: There is a "competitive fringe" of "small" firms which are identical, and a "dominant" firm which is at least *twice* as "big" as a firm in the competitive fringe. The structure of these equilibria is very different from the asymmetries that occur under the second price auction. First, under the first price auction, *all* potential sellers are *active* in the sense that their investment levels are all positive, whereas under the second price auction, some seller(s) can " flood" the investment game so that other sellers optimally stay out. Second, there

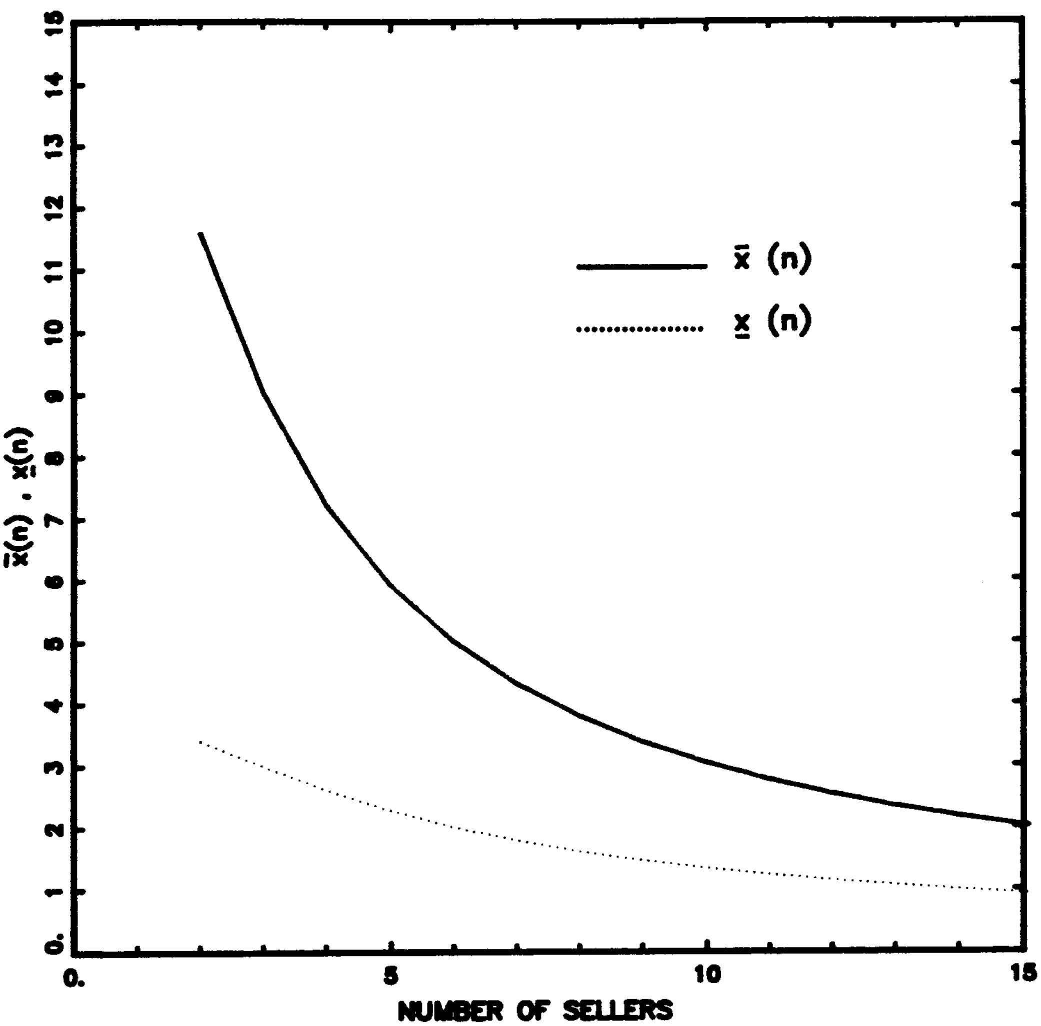

Figure 3.7 : High and Low Investment Levels in Equilibrium.

($r = .85$, $s = .0143$, $p = 1$, $\underline{c} = 0$)

is a single dominant firm under the first price auction, whereas the total investment can be distributed arbitrarily among the sellers under the second price auction.

This phenomenon of "being identical, behaving differently" (Mariotti [38]) is also observed in dynamic games of technology adoption (Reinganum [55], Mariotti [38]) where, without any *ex ante* heterogeneity among the firms, firms choose different adoption dates in an equilibrium, giving rise to a diffusive pattern. However, in those models, there is also a symmetric pure strategy equilibrium with simultaneous adoption, whereas we have only asymmetric pure strategy equilibria in our model. Furthermore, the asymmetry we obtain arises in a *one-shot* game.

The fact that all pure strategy equilibria are asymmetric is surprising in view of the *ex ante* symmetry among the sellers. This result is different from the non-existence of symmetric pure strategy equilibria observed in entry games (Dixit and Shapiro [16]), since we allow continuous strategy sets as opposed to the binary strategy sets in the entry games.

Finally, in most models of research and development, the investment outcomes become common knowledge at the market competition stage, and the return to successful innovation is given exogenously as a function of market structure. Thus, investment behavior affects the market competition only through the production cost realizations, which become common knowledge. When the production cost realization is private knowledge for each firm at the market competition stage, whether or not the levels of investment by each firm are observable makes a crucial difference, as we have seen, despite the fact that investment expenditures are sunk at the time of market competition.

3.5 The Buyer's Optimal Reserve Price

We have fixed the reservation price at an arbitrary level up to this point. In this section we will study the implications of the equilibria in the game among the sellers on the buyer's optimal reserve price choice. To simplify notation, we will normalize parameters so that $\bar{c} = 1$, $\underline{c} = 0$.

We will assume that investment levels are not observable by the buyer and that the buyer is restricted to set a uniform reserve price despite possible asymmetry among the sellers in the game following any reserve price announcement.

When the cost distribution is exogenously fixed, the buyer's reserve price is zero. Therefore any positive reserve price in the case where the probability of failure is endogenous can be interpreted as being due to moral hazard. The buyer's expected profit as a function of the reserve price she chooses is the difference between the social surplus generated and the expected profit of the sellers.

When there is only one seller facing a reserve price p, the equilibrium response of the seller gives $r(x^*(p)) =: r(p) = \hat{r}/p$ for $p > \hat{r}$, where $\hat{r} = s/lnr^{-1}$ is the optimal failure probability when the reserve price is set to 1, i.e., the socially optimal failure probability, and $x^*(p)$ solves $\max_x (1 - r^x)p - sx$.

The buyer's expected profit maximization problem is

$$\max_p \{v - [r(p) + sx(p)]\} - \{p(1 - r(p)) - sx(p)\}, \tag{3.135}$$

where the first term is social surplus and the second term is the seller's expected profit. This is equivalent to the expected price minimization problem.

$$\min_p [p + (1 - p)r(p)]. \tag{3.136}$$

The unique solution to this problem satisfies

$$p = \sqrt{\hat{r}}\,, \tag{3.137}$$

giving the expected price paid by the buyer as

$$c_b = \sqrt{\hat{r}}(2 - \sqrt{\hat{r}}). \tag{3.138}$$

Note that the optimal reserve price depends on s and r only through $s/\ln r^{-1} = \hat{r}$.

When investment is unobservable and sellers use symmetric pure strategies in the investment game following every reserve price announcement, the buyer's optimal reserve price would be the same under the two auctions, since the two auctions are equivalent at every reserve price. We obtain the optimal reserve price for the buyer in the following proposition for the unobservable investment case.

Proposition 3.8 *When investment is unobservable and the sellers use symmetric pure strategies in the investment game, the buyer's optimal reserve price p is*

$$p(n, \hat{r}) = (\hat{r})^{\frac{1}{n+1}}. \tag{3.139}$$

Proof: The buyer's expected profit is the difference between the expected total surplus and the expected total profit of the sellers. At the symmetric equilibrium in the investment game, total investment x^* is such that $r^{x^*} = \hat{r}/p = r(p)$ independent of n. Expected profit of a seller who invests x^*/n in equilibrium is

$$p(1 - r^{x^*/n})r^{\frac{n-1}{n}x^*} - sx^*/n, \tag{3.140}$$

so that total profit of the sellers is

$$np\{[r(p)]^{\frac{n-1}{n}} - r(p)\} - sx^*. \tag{3.141}$$

The buyer's optimization problem is, then,

$$\max_{p} \; v - \{r(p) + sx^*\} - \{np\{[r(p)]^{\frac{n-1}{n}} - r(p)\} - sx^*\}, \tag{3.142}$$

which is equivalent to

$$\min_{p} \; r(p) + np\{[r(p)]^{\frac{n-1}{n}} - r(p)\}. \tag{3.143}$$

Using $r(p) = \hat{r}/p$ and carrying out the minimization give $p(n, \hat{r}) = (\hat{r})^{\frac{1}{n+1}}$. □

The optimal reserve price is increasing, and the buyer's expected cost is decreasing in n when investment is unobservable and the sellers use symmetric investment strategies.

When investment levels are observable by the sellers at the bidding stage, the only pure strategy equilibria are asymmetric under the first price auction for all reserve prices. However, total investment in all these equilibria is the same and equal to the total investment in the unobservable investment case. Thus, total surplus for any given reserve price is the same under both auctions. Moreover, since all the asymmetric equilibria differ only in the names of the sellers the total profit of the sellers is the same across all n asymmetric equilibria in the observable investment case. Therefore, if the sellers use pure equilibrium strategies in the investment game the buyer's expected cost minimization problem is independent of whichever of the n pure strategy equilibria is played by the sellers. In any pure strategy equilibrium, $(n-1)$ sellers each get

$$(1 - r^{\underline{x}})r^{(n-1)\underline{x}} - s\underline{x}, \tag{3.144}$$

and one seller gets

$$(1 - r^{x^* - (n-1)\underline{x}})r^{(n-1)\underline{x}} - s(x^* - (n-1)\underline{x}), \tag{3.145}$$

where $\underline{x}(n, \hat{r}, p)$ solves

$$r^{(n-1)\underline{x}}(2r^{\underline{x}} - 1) = r^{x^*}. \tag{3.146}$$

Total profit of n sellers in any equilibrium is obtained from (3.144) and (3.145) to be

$$p(n+1)(r^{n\underline{x}} - \hat{r}/p). \tag{3.147}$$

Therefore, the buyer's expected cost minimization problem is

$$\min_p \frac{\hat{r}}{p} + (n+1)p[r^{n\underline{x}} - \hat{r}/p]. \tag{3.148}$$

Before proceeding to solve for the optimal reserve price for the first price auction with observable pre-action investment, we compare the expected price paid by the buyer under the first price auction and the expected price paid by the buyer under the second price auction when buyers use the same asymmetric investment strategies as in the first price auction, given a reserve price p. For given reserve price p, the expected price paid by the buyer under the first price auction ,$C_b^{fp}(p)$, is given by

$$C_b^{fp}(p) = (\hat{r}/p) + (n+1)pr^{n\underline{x}} - (n+1)\hat{r}. \tag{3.149}$$

If the sellers use the same asymmetric investment strategies under the second price auction, the total expected profits of the sellers would be

$$p[r^{(n-1)\underline{x}} + (n-1)r^{x^*-\underline{x}} - n\hat{r}/p] - sx^*. \tag{3.150}$$

Therefore, since the total surplus remains the same, the expected price paid by the buyer would be

$$C_b^{sp}(p) = (\hat{r}/p) + p[r^{(n-1)\underline{x}} - (n-1)r^{x^*-\underline{x}}] - n\hat{r}. \tag{3.151}$$

The difference between the two expected prices is

$$C_b^{fp}(p) - C_b^{sp}(p) = p(n-1)[r^{n\underline{x}} - r^{x^*-\underline{x}}], \tag{3.152}$$

where we used the fact, from (3.146), that

$$r^{(n-1)\underline{x}} = 2r^{n\underline{x}} - r^{x^*}. \tag{3.153}$$

We thus have

Proposition 3.9 *Given a reserve price p, a second price auction generates lower expected cost for the buyer than a first price auction when sellers use the same asymmetric investment strategies under both auctions, when investment is observable.*

Proof: In (3.152), $[r^{n\underline{x}} - r^{x^*-\underline{x}}] > 0$ if and only if

$$\underline{x}(n, \hat{r}, p) < x^*(\hat{r}, p)/(n+1), \tag{3.154}$$

or equivalently,

$$\underline{x}(n, \hat{r}, p) < \bar{x}(n, \hat{r}, p)/2. \tag{3.155}$$

Both inequalities are valid by Proposition 3.7 . □

To return to the optimal reserve price for the seller in a first price auction with observable investment, the first order conditions are

$$\frac{-\hat{r}}{p^2} + (n+1)r^{n\underline{x}} + n(n+1)pr^{n\underline{x}} \ln r \frac{d\,\underline{x}}{d\,p} = 0. \tag{3.156}$$

From (3.146),

$$\frac{d\,\underline{x}}{d\,p} = \frac{\hat{r}}{p^2(\ln r^{-1})r^{(n-1)\underline{x}}[1 + n(2r^{\underline{x}} - 1)]}. \tag{3.157}$$

After cancellations we get the following implicit characterization of the optimal reserve price under a first price auction:

Proposition 3.10 *Under a first price auction with observable investment, the buyer's optimal reserve price satisfies*

$$p^2(n+1)r^{n\underline{x}} - \hat{r}\{1 + pn(n+1)r^{\underline{x}}/[1 + n(2r^{\underline{x}} - 1)]\} = 0 \tag{3.158}$$

Proof: Substitute (3.157) in (3.156). □

Because of non-linearities we are unable to sign $\frac{dp}{dn}$. We limit ourselves to the two seller case for comparisons, since we have an explicit expression for $\underline{x}$ in this case.

For two sellers, the buyer's problem becomes

$$\min_{p}\{r(p) + p(2r^{\underline{x}} - r^{2\underline{x}} - r(p))\} \tag{3.159}$$

where $\underline{x} = \underline{x}(p)$ is the lower investment level is in the asymmetric pure strategy equilibrium defined above. Using the fact that $\underline{x}(p)$ solves

$$r^{\underline{x}}(2r^{\underline{x}} - 1) = r^{x^*}, \tag{3.160}$$

the problem becomes

$$\min_{p}[r(p) + 3p(r^{\underline{x}} - r(p))]\,. \tag{3.161}$$

Substituting the values for $r(p) = \hat{r}/p$, and

$$\underline{x}(p) = (1/4)(1 + \sqrt{1 + 8\hat{r}/p}) \tag{3.162}$$

and differentiating with respect to p, we get the first order conditions for the optimum reserve price to be

$$\frac{5}{8} - \frac{\hat{r}}{p^2} + \frac{3\hat{r}}{2p\sqrt{1 + \frac{8\hat{r}}{p}}} + \frac{5}{8}\sqrt{1 + \frac{8\hat{r}}{p}} = 0\,. \tag{3.163}$$

For comparisons, since we cannot get an explicit form for p, we evaluate the first order condition at the optimal reserve price for the symmetric two-seller case, $p = (\hat{r})^{\frac{1}{3}}$. We obtain

$$\frac{5}{8} - \hat{r}^{\frac{1}{3}} + \frac{3\hat{r}^{\frac{2}{3}}}{\sqrt{1 + 8\hat{r}^{\frac{2}{3}}}} + \frac{5}{8}\sqrt{1 + 8\hat{r}^{\frac{2}{3}}} > 0 \quad \text{for all } \hat{r}\,. \tag{3.164}$$

The inequality is obtained by evaluating the second term at $\hat{r} = 1$ and the last two terms at $\hat{r} = 0$.

Therefore, the optimal reserve price when the sellers can observe one another's investment levels is lower than the optimal reserve price when investment is not observable for the two seller case.

We evaluate the first order condition at the optimal reserve price for the one-seller case $p = \sqrt{\hat{r}}$. We obtain

$$\frac{3}{2}\sqrt{\hat{r}}\frac{1}{\sqrt{1+8\hat{r}^{\frac{1}{2}}}} + \frac{5}{8}\sqrt{1+8\hat{r}^{\frac{1}{2}}} - \frac{3}{8} > 0 \quad \text{for all } \hat{r}\,, \tag{3.165}$$

where we evaluated the first and second terms at $\hat{r} = 0$. Letting the subscripts denote the number of sellers and the superscripts denote the auction type, we have

Proposition 3.11

$$p_1^{fp} = p_1^{sp} < p_2^{fp} < p_2^{sp}. \tag{3.166}$$

As in the case with exogenously fixed cost distributions, the buyer responds to asymmetries by lowering her reserve price. Despite the fact that a lower reserve price will imply lower total investment and hence lower total surplus, the buyer increases her share of the total surplus by setting a lower reserve price. This is because at the asymmetric equilibria total profits of the sellers are higher than the level of total profits at the symmetric equilibrium with the same total investment.

Since total investment levels are the same under the two auction procedures for each reserve price p and total investment level is monotone increasing in p we have

Proposition 3.12 *For two sellers, first price auctions give rise to a lower total investment level than second price auctions in the subgame perfect equilibriaof the game with observable investment.*

3.6 Entry With Endogenous Scale

The structure of equilibrium also has an interesting implication for comparison of the two auction types with respect to entry incentives. Although we have so far assumed that the number of sellers is fixed exogenously, the problem of entry with endogenous "scale" can be studied within this framework without any change in the model.

Starting from a situation with n "established" firms, the "entrant" decides whether or not to enter and the scale of entry. Under the second price auction the established firms do not have any reason to change their scale decisions (investment *plans*) if entry occurs; hence, the entrant, knowing that his optimal scale choice would be zero, would stay out. Under the first price auction there is always room and incentive for new entry. The entrant knows that the established firms will adjust their investment plans in case of a new entrant. Without a fixed entry cost, entry would occur indefinitely, which contrasts with the no-entry outcome under the second price auction, even with zero fixed cost of entry. Even when the established firms have *committed* to their investment levels, the equilibrium bidding reactions make it worthwhile for the entrant to enter with a small positive investment level. We next show that when n established firms have committed to their equilibrium investment levels $(\underline{x}, \underline{x}, \ldots, \bar{x})$, a potential entrant's optimum entry scale is positive *and* small; i.e., *entry occurs and occurs at the fringe.*

Proposition 3.13 *Suppose n established firms have committed to invest-*

ment levels $x_i = \underline{x}$ *for* $i = 1, \ldots, n-1$ *and* $x_n = \bar{x}$*, where* $\underline{x}$ *and* $\bar{x}$ *are the equilibrium investment levels for* n *sellers in first a price auction. Then a potential entrant's optimal entry scale strategy* x_e *is positive and satisfies*

$$r^{x_e}(2r^{x_e} - 1) = r^{x^* - (n-1)\underline{x}}. \tag{3.167}$$

Proof: If the potential entrant enters with scale x_e, the equilibrium payoffs in the auction, conditional on having a low production cost, would be completely determined by the common lowest bid in the support of mixed bidding strategies:

$$\underline{b}(x_e, \underline{x}, \underline{x}, \ldots, \underline{x}, \bar{x}) = \underline{c} + (p - \underline{c})r^{x_e + x - m}. \tag{3.168}$$

Therefore, the entrant's expected profit is

$$\Pi_e(x_e, \underline{x}, \bar{x}, n) = (1 - r^{x_e})r^{x_e + x - m} - tx_e \quad \text{if } x_e < \bar{x} \tag{3.169}$$

and

$$\Pi_e(x_e, \underline{x}, \bar{x}, n) = (1 - r^{x_e})r^{x_T} - tx_e \quad \text{if } x_e \geq \bar{x}. \tag{3.170}$$

Since $(\underline{x}, \bar{x})$ is the equilibrium for the n-seller case, $(n-1)\underline{x} + \bar{x} = x^*$.

First, x_e cannot be greater than or equal to $\bar{x}$. Suppose $x_e \geq \bar{x}$; then it must satisfy

$$r^{x_e} r^{x^*} \ln r^{-1} = t; \tag{3.171}$$

i.e., $r^{x_e} = 1$, since $r^{x^*} = t / \ln r^{-1}$, a contradiction. Next, we show that $0 < x_e < \bar{x}$ solves

$$\max_{x_e}(1 - r^{x_e})r^{x_e + x - m} - tx_e. \tag{3.172}$$

The first order condition is

$$r^{x_e}(2r^{x_e} - 1) = r^{x^* - (n-1)\underline{x}}, \tag{3.173}$$

since $\bar{x} = x^* - (n-1)\underline{x}$. Let ω_e be the positive solution to the quadratic equation

$$2\omega^2 - \omega - r^{\bar{x}} = 0\,. \tag{3.174}$$

We obtain $x_e = \ln \omega_e / \ln r$.□

Note that the entrant's best response is similar to the determination of the low investment level in the two-seller case with r^{x^*} replaced by $r^{\bar{x}}$.

The foregoing discussion seems to suggest that with observable investment, first price auctions would attract more sellers than second price auctions. This interpretation of the above proposition is not correct, since we need to compare the *equilibrium* number of sellers and *equilibrium* scales under the two auctions. Certainly, the configuration of investment levels given above is not an equilibrium.

To compare equilibrium entry levels under the two auctions, we assume a fixed entry cost K, which must be incurred if the scale of entry is strictly positive. We continue to assume *ex ante* identical potential sellers and identify the scale of entry with the level of investment in cost reduction activity.

We know from previous work (McAfee and McMillan [44]) on entry in auctions with a fixed entry scale that with risk neutral sellers the two auctions attract the same number of sellers in the private values frame work. When sellers choose the scale of entry together with their entry decision, this equivalence may no longer hold. As we shall show, the equivalence of the two auctions with respect to the equilibrium number of sellers and equilibrium entry scales depends crucially on observability of entry scales at the bidding stage.

The multiplicity of investment equilibria under the second price auction with an exogenously given number of sellers carries over to the endogenous entry case: Any number of sellers and any configuration of entry scales such that 1) the seller with the lower scale makes just enough profit to cover the fixed cost entry K, and 2) the total investment of the entrants equal the scale that would be chosen by a single seller facing the same reserve price, would be an equilibrium in the entry game with an endogenous scale. The maximum equilibrium number of sellers would be given by the zero profit condition for each seller at the symmetric equilibrium; i.e., each seller chooses the scale $\frac{x^*}{n}$ and the expected profit each seller just covers K.

To focus on the equilibrium number of sellers, we will rule out the case where K is so large that at most one seller can expect to make enough profits when he chooses the optimal scale. We will assume that K is small enough so that the equilibrium number of sellers at the symmetric scale equilibrium under the second price auction is greater than two.

We will compare the minimum and maximum equilibrium number of sellers under the two auctions. It is easily established that the minimum equilibrium number of sellers under a second price auction is one, no matter how small K is. One seller paying K, choosing the maximum scale for the given reserve price and bidding truthfully is an equilibrium. Under the first price auction the minimum number of sellers is at least two for small K; if there is only one seller, he would bid the reserve price at the bidding stage, and the best response of any other potential seller would be to enter.

The maximum number of sellers in equilibrium is the same under the two auctions when the investment levels are *not* observable. At the symmetric investment equilibrium, each seller's expected profit is the same under both auctions, and the zero profit condition gives the same equilibrium number of sellers.

When invesment is observable nothing is affected under the second price auction. However, observability of investment changes the bidding behavior under the first price auction, as we have seen. The maximum number of sellers in this case would be given by the zero profit condition at the margin. Since the expected profit of a firm with low investment is lower than the expected profit of a firm at the symmetric investment equilibrium, the maximum number of sellers under the first price auction with observable investment is lower than the maximum number of sellers under the second price auction.

To formalize the above argument, let n_1 and n_2 denote the maximum equilibrium number of bidders under the first price auction and the second price auction, respectively. As before, x^* is the optimum level of investment for the single seller case for a given reserve price. Under the second price auction, n_2 satisfies

$$\Pi_2(n_2) = (1 - r^{u_2})r^{v_2} - tu_2 = K\,, \tag{3.175}$$

where

$$u_2 = \frac{x^*}{n_2}, \tag{3.176}$$

$$v_2 = \frac{n_2 - 1}{n_2}x^*. \tag{3.177}$$

Under the first price auction n_1 is given by

$$\Pi_1(n_1) = (1 - r^{u_1})r^{v_1} - tu_1 = K\,, \tag{3.178}$$

where $u_1 = \underline{x}(n_1)$, $v_1 = (n_1 - 1)\underline{x}(n_1)$ and $\underline{x}(n_1)$ satisfies

$$r^{(n_1-1)\underline{x}}(2r^{\underline{x}} - 1) = r^{x^*}. \tag{3.179}$$

Proposition 3.14 *The maximum equilibrium number of sellers is less under the first price auction than under the second price auction.*

Proof: Both $\Pi_2(n)$ and $\Pi_1(n)$ are decreasing in n and $\Pi_2(n)$ is strictly greater than $\Pi_1(n)$ evaluated at $n_1 = n_2 = n$ for any n. Therefore, $\Pi_2(n_2)$ is strictly greater than $\Pi_1(n_1)$, if $n_1 > n_2$. Hence, for the equality $\Pi_2(n_2) = \Pi_1(n_1)$, n_1 should be less than n_2. □

Chapter 4

EX ANTE STABILITY OF COLLUSION IN FIRST AND SECOND PRICE AUCTIONS

4.1 Introduction

In this chapter we study the *ex ante* stability of collusive agreements under the first and second price auctions. Relative conduciveness of the auction procedures to collusive practices is an important aspect of the problem of comparing the two auction types, and this problem has been studied by several authors (Robinson [61]; Graham and Marshall [23]; McAfee and McMillan [45]; Mailath and Zemsky [37]; Zemsky [72]).

The focus of most of these studies has been on information revelation problems faced by a collusive ring in dealing with the private information of its members, and on the buyer's reaction to the existence of collusion among the sellers. The incentive problems within the collusive ring point to the difficulty of collusion under the first price auction (Robinson [61]) relative to the second price auction. In this study, we focus on the *ex ante* incentives to form collusive rings in the absence of enforcement problems.

We assume that collusive agreements are reached prior to the realization of random private production costs and that *interim* incentive problems within a ring are already solved. That is, once a ring is formed and the ring members observe their individual production costs, possible incentives on the part of ring members to "cheat" on the ring are costlessly eliminated by the ring. We are, in effect, assuming that the realizations of private production costs of ring members are publicly observable within the ring, and the ring can costlessly prevent its members from submitting bids privately. These interim incentive problems faced by the ring have been the focus of previous studies on collusion in auctions.

The reason for our special assumptions is to focus on possible effects of the existence of collusive rings on bidding behavior. In particular, the bidding behavior would be different for a seller if he faced n rivals from the case where he faced one rival with "size " n. This effect on the bidding behavior might feed back into the initial incentives to form rings even in the absence of further incentive problems within a ring, once it is formed. One such possible impediment to formation of rings under the first price auction is a positive *externality* of a ring on non-members in the sense that sellers outside the ring may prefer to face one large rival bidder rather than many small rivals. This effect is similar to the one Stigler [66] noted in relation to merger agreements:

> the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. (...) Hence the promoter of a merger is likely to receive much encouragement from each firm – almost every encouragement, in fact, except participation.

The same effect is observed by Lien [34] in bribery games. Lien [34] solved the equilibrium bidding strategies in *all-pay* first price auction with three buyers when two buyers form a ring, and observed that the outsider's profit is higher than that of every ring member. Characterization of equilibrium in first price auctions with general continuous distribution of private values in the presence of asymmetries that result from the formation of rings has not been successfully solved. Thus, we use a simpler model in which we can solve for equilibrium bidding strategies, and obtain some interesting results on collusive ring formation under the two auction procedures. Whether the results would continue to hold under more general assumptions on the distribution of possible production costs remains an open question.

In the next section we develop the model and notation to study the problem. The notation and the notion of individual stability follows d'Aspremont and Gabszewicz [14]. We require a collusive ring structure to be immune to individual deviations in the sense that no member of any ring will have any *ex ante* incentive to leave the ring he is in, in order to join another ring or to act alone. This is a rather weak requirement of "stability" and yet, as we shall see, it restricts the possible ring structures considerably.

In Section 4.3, we show that under the second price auction, the grand collusive ring of all sellers is the only individually stable collusive ring structure. In Section 4.4, we study the stability of collusive ring structures under the first price auction. We find that the set of all sellers need not be stable and that all individually stable ring structures have the same form : a collusive ring of at least three sellers (two sellers, if there are no more than two sellers) and a "competitive fringe" made up of the remaining sellers each of whom acts independently. The number of sellers involved in a collusive agreement is a decreasing function of the probability of high production cost. This number is independent of the collusive ring structure that actually forms. For $n \geq 3$, there are at least three firms that form a ring independent of the number of sellers.

In Section 4.5 we discuss the distribution of profits under an individually stable collusive ring structure, and find that *every* seller benefits from the existence of a ring, and that sellers outside the ring benefit more than the sellers inside. We also discuss the implications of this profit structure for collusion. The final section contains some remarks on the restrictions of the model and on the possibility of extensions.

4.2 A Model of Collusion in Auctions

We assume a set of identical risk-neutral sellers:

$$N = \{1, 2, \ldots, n\} \tag{4.1}$$

Seller i's private production cost, c_i, can take only two values $\overline{c}$ and $\underline{c}$, where $\underline{c} < \overline{c}$, with probability r and $1 - r$, respectively, independently of the other sellers' production costs. Without loss of generality we normalize production costs so that $\overline{c} = 1$ and $\underline{c} = 0$, and, therefore, the expected production cost of a typical seller is r. Each seller submits a bid to supply an item in a single unit auction after he observes his production cost realization privately. A reserve price $0 < p < 1$ is given exogenously. If the production costs were to take more than two values, we would obtain exactly the same results by redefining $r = \text{Prob}\{c_i > p\}$, as long as the reserve price is less than the second lowest production cost in the support of production cost distribution.

We allow sellers to form *collusive rings* before the auction. By a *collusive ring*, or simply *ring*, we mean a group of sellers who agree to act in unison in the auction. A ring is thus identified with a subset S of the set of sellers N. By a *collusive ring structure*, we mean a partition of N into subsets S^k where each S^k is a ring. We denote a collusive ring structure with k rings by

$$C = \{S^1, S^2, \ldots, S^k\} \tag{4.2}$$

and let $\mathcal{C}$ denote the set of all partitions of N, i.e., the set of all possible ring structures. For any set $S = \{i, j, \ldots, l\}$, we will denote the finest partition of S by $< S >$:

$$< S >= \{\{i\}, \{j\}, \ldots, \{l\}\}, \tag{4.3}$$

and $[S]$ will denote the coarsest partition of S:

$$[S] = \{S\}. \tag{4.4}$$

With this notation, $< N >$ is the trivial ring structure with no collusion among the sellers, and $[N]$ is the ring structure where all sellers form one ring. We will also use the following notation: For $S \subseteq N$,

$$C^o(S) = \{S, < N - S >\} \tag{4.5}$$

$$C_o(S) = \{S, [N - S]\} \tag{4.6}$$

For a subset S of N, $C_o(S)$ is the ring structure where the complement of S, $N - S := \{i \in N : i \notin S\}$, form a single ring and $C^o(S)$ is the ring structure where the members of $N - S$ behave individually. The following definitions have similar interpretations: for disjoint subsets S, T, ..., U of N define

$$C^o(S, T, \ldots, U) = \{S, T, \ldots, U, < N - T - S - \ldots - U >\} \tag{4.7}$$

$$C_o(S, T, \ldots, U) = \{S, T, \ldots, U, [N - T - S - \ldots - U]\}. \tag{4.8}$$

$C(S)$ is the set of all partitions of N where members of S form one ring. $C(S,T)$ and $C(S,T,U)$, etc. are defined similarly. Finally, for $i \in N$ and $C \in \mathcal{C}$,

$$S(i,C) = \{S^m \in C : i \in S^m\}, \tag{4.9}$$

and $Z_i(C)$ is the set of all partitions $\tilde{C} \in \mathcal{C}$ such that for all $\tilde{S} \in \tilde{C}$, one of the following is true:

1. $\tilde{S} = S$ for some $S \in C$.

2. $\tilde{S} = S - \{i\}$ for some $S \in C$.

3. $\tilde{S} = S \cup \{i\}$ for some $S \in C$.

4. $\tilde{S} = \{i\}$.

$S(i,C)$ is the ring seller i belongs to in the ring structure C, and $Z_i(C)$ is the set of all ring structures obtained from C; when seller i leaves the ring he belongs to under C, and joins another ring or stays alone.

Given a ring structure $C = \{S^1, S^2, \ldots, S^k\}$, the payoffs to sellers are determined in a subsequent auction. At the auction stage the parameters of the model (N, r, p, C) are common knowledge. Thus, a ring structure $\{S^1, S^2, \ldots, S^k\}$ gives rise to an auction with k sellers, and the *ex ante* distribution of ring S^i's production cost is $(r^{\#S^i}, 1 - r^{\#S^i})$, i.e., ring S^i's production cost is $\bar{c}$ with probability $r^{\#S^i}$ and $\underline{c}$ with remaining probability, where $\#S^i$ is the size of the ring S^i, since each ring member's production cost is independently distributed. Therefore, the expected production cost for the ring is decreasing in the size of the ring. The ring's profit from the auction is shared equally among the ring members.

Let $\Pi(S, C)$ be the expected profit of a ring $S \in C$, given the ring structure C, and $\Pi(S, C)/\#S$ is the expected profit of a typical member S, given the ring structure C. Denote

$$\Pi_i(C) = \Pi(S(i, C), C)/\#S(i, C). \tag{4.10}$$

Definition 1 *A ring structure* $C = \{S^1, \ldots, S^k\}$ *is* **individually stable** *if for all* $i \in N$ *and for all* $\tilde{C} \in Z_i(C)$

$$\Pi_i(C) \geq \Pi_i(\tilde{C}). \tag{4.11}$$

Verbally, an individually stable ring structure is one where no seller can expect to increase his expected profit by leaving the ring he is in and joining another ring or staying alone.

Note that the definition of individual stability assumes that a seller j can join any existing ring S regardless of how he affects the profit of existing members of S. It is not *a priori* inconceivable that the existing members of S would reject seller j as would happen if an additional seller in the ring *diluted* the expected profit of existing members. If the existing members of a given ring are endowed with a *veto power* to reject potential new members, this should be reflected in the definition of individual stability, by restricting the possible moves of a seller j to a subset of $Z_j(C)$ in the definition. One way to justify this implicit assumption is to appeal to stylized facts about rings in practice. Graham and Marshall [23] report the following as Fact 4 in their study of collusion in second price auctions:

> Rings have open membership policies in the sense that bidders who are expected to be competitive at the main auction are invited to join.

Under second price auction this turns out to be an equilibrium outcome as Graham and Marshall note. We will see that this issue is solved *endogenously* in the present setup for *both* auction types; in other words, we obtain the mentioned stylized fact as a result rather than maintaining it as an assumption. That is, even if the existing members of a ring S had veto power to deny membership to a prospective member $j \notin S$, this veto power would *not* be exercised in situations where j *did* want to join S: Whenever $j \notin S$ has any incentive to join S so does every existing member of S to accept j. In the present setting, the binding constraint for a ring structure to be individually stable is not preventing members of one ring S from joining another ring T, but is keeping members of S from leaving S and acting alone, because it is always the case that when $j \in S$ has incentives to leave S, he does better by not joining any other ring $T \neq S$.

Another assumption implicit in the definition of individual stability is that when a member j of a ring S leaves S, the remaining members $S - \{j\}$ continue as one ring, and any other ring $T \neq S$ is not affected except when j joins T. One alternative would be to assume that defection of any member of a ring S would result in the ring's collapse. We do not pursue this alternative specification here because such behavior on the part of the remaining members will not be *credible* in the sense that they will remain as a ring if it is profitable to do so within the new ring structure that forms after j's departure.

Finally, note also that this definition of individual stability, as d'Aspremont and Gabszewicz [14] remark, can be seen as a Nash equilibrium in a non-cooperative game where the choice by every player of a strategy from an abstract strategy set results in a ring structure C with the corresponding payoff $\Pi(S(i,C), C)$ for each player $i \in N$.

We will denote the class of individually stable collusive ring structures by $\mathcal{C}_{IS}$:

$$\mathcal{C}_{IS} = \{C \in \mathcal{C} \ : \ C \text{ is individually stable}\}. \tag{4.12}$$

In order to study individually stable ring structures under first and second price auctions, we need to find $\{\Pi(S,C) : S \in C\}$ for all ring structures $C \in \mathcal{C}$ under the two auctions. We use superscripts to denote the auction type so that $\Pi^{sp}(S,C)$ is the expected profit of ring S under the second price auction when ring structure C forms, and $\Pi^{fp}(S,C)$ is the corresponding expected profit under the first price auction. $\mathcal{C}_{IS}^{fp}$ and $\mathcal{C}_{IS}^{sp}$ will be used to denote the class of individually stable collusive ring structures under first price and second price auctions, respectively.

4.3 Individually Stable Ring Structures Under Second Price Auction

It is easily established, using the fact that truthful bidding is a dominant strategy under the second price auction, that for all $C \in \mathcal{C}$ and for all $S \in C$

$$\Pi^{sp}(S,C) = p(1 - r^{\#S})r^{n-\#S}. \tag{4.13}$$

Thus, the expected profit of ring S depends only on its size and the total number of bidders, and is independent of how the complement of S is structured, i.e., independent of the amount of cooperation among sellers outside S. This is because the ring S wins the auction with a positive price if and only if every seller outside S has the high production cost *and* the ring S has the low production cost. These events occur independently with probabilities

$r^{n-\#S}$ and $(1 - r^{\#S})$, respectively. That is, for any subset S of N,

$$\Pi^{sp}(S, C(S)) = p(1 - r^{\#S})r^{n-\#S}. \tag{4.14}$$

In particular,

$$\Pi^{sp}(S, C_o(S)) = \Pi^{sp}(S, C^o(S)). \tag{4.15}$$

The expected profit of ring S does not change, regardless of how the sellers in $(N - S)$ form rings among themselves. It does not make any difference for a ring S if it faces $\#(N-S)$ individual rivals or one rival ring with membership size $\#(N - S)$.

We will see in the next section that how the non-members organize will matter for a given ring S under the first price auction.

We will make use of the following expansion of $(1 - r^m)$ for integer m in finding the individually stable collusive ring structures:

$$(1 - r^m) = (1 - r)(1 + r + r^2 + \ldots + r^{m-1}). \tag{4.16}$$

Given the expected profit function for a given ring structure under the second price auction, it is easily proved that coalition of the whole is the only individually stable ring structure.

Proposition 4.1 *$\{N\}$ is the only individually stable collusive ring structure under the second price auction independent of n, r and p.*

Proof: We will first show that $\{N\}$ is individually stable; i.e., for all $i \in N$,

$$\Pi_i^{sp}(\{N\}) \geq \Pi_i^{sp}(\{\{N - i\}, \{i\}, \}). \tag{4.17}$$

Substituting from (4.13) we get

$$p(1 - r^n)/n \geq p(1 - r)r^{n-1}. \tag{4.18}$$

Using (4.16), this is equivalent to

$$1 + r + r^2 + \ldots + r^{n-1} > nr^{n-1}, \tag{4.19}$$

which is always true for all n and all $r \in (0,1)$. Next, suppose $C = \{S^1, S^2, \ldots, S^k\}$ is individually stable. Let $S^m \in C$ be the ring with the largest size. We will show that any seller that belongs to a ring other than S^m would strictly prefer to join S^m. In order to establish this, we need to note that

$$\varphi(m) := (1 - r^m)(r^{n-m})/m \tag{4.20}$$

is increasing in m. Differentiating $\varphi(m)$, we get

$$\varphi'(m) = \frac{r^n(1 - r^{-m} - mr^{-m}\ln r)}{m^2}. \tag{4.21}$$

Therefore,

$$sign\ \varphi'(m) = sign\ (1 - r^{-m} - mr^{-m}\ln r). \tag{4.22}$$

Using the series

$$\ln r^m = (r^m - 1) - \frac{1}{2}(r^m - 1)^2 + \frac{1}{3}(r^m - 1)^3 - \ldots, \tag{4.23}$$

the right-hand side of (4.22) is easily seen to be strictly positive; therefore, $\varphi(m)$ is strictly increasing in m, for $m < n$. □

4.4 Individually Stable Ring Structures Under First Price Auction

For any given ring structure, the only bidding equilibrium is in mixed strategies under the first price auction, and the equilibrium depends on what collusive rings have formed.

We make use of the characterization of equilibrium bidding strategies under the first price auction from Chapter 4 to find the expected profits for a ring. Some additional notation will be needed to state the results. For a collusive ring structure $C \in \mathcal{C}$, let $m(C)$ denote the size of the largest ring in C.

$$m(C) = max\{\#S : S \in C\} \tag{4.24}$$

Proposition 4.2 *For any ring structure $C \in \mathcal{C}$, the expected profit of a ring $S \in C$ is given by*

$$\Pi^{fp}(S, C) = p(1 - r^{\#S})r^{n-m(C)}. \tag{4.25}$$

Proof: The equilibrium expected profit for every ring having the low production cost is $pr^{n-m(C)}$ in the mixed strategy equilibrium (see Chapter 4). Since the probability of having the low production cost is $(1 - r^{\#S})$ for the ring S, (4.25) is the expected profit of the ring S for the given ring structure C.□

The expected profit of a ring in a collusive ring structure C, then, depends only on its size and the size of the largest ring in the ring structure under the first price auction.

We first show that the coalition of the whole is no longer individually stable for sufficiently large n. Crampton and Palfrey [8] find a similar result in their study of cartel enforcement under uncertainty.

The largest n such that the grand coalition is individually stable depends on r. In the collusive ring structure $\{N\}$, each seller's expected profit is the same under the first price auction as in the second price auction, since

$$\Pi^{fp}(N, \{N\}) = p(1 - r^n). \tag{4.26}$$

Seller i's expected profit when he leaves N to form the collusive ring structure $\{\{N - i\}, \{i\}\}$ is, from (4.25),

$$\Pi_i^{fp}(\{\{N - i\}, \{i\}\}) = p(1 - r)r. \tag{4.27}$$

Proposition 4.3 *For any $r \in (0,1)$, there exists $\bar{n}$ such that if $\#N > \bar{n}$, then $\{N\}$ is not individually stable. The critical $\bar{n}$, which depends on r, is given by*

$$1 - r^n = n(1-r)r. \tag{4.28}$$

Proof: It is clear that if the LHS of (4.28) is less than the RHS for any n, N with $\#N = n$ is not individually stable. For $n \leq 3$, the LHS is greater than the RHS for all r. The LHS is monotone increasing in n and is always less than 1. The RHS is monotone increasing and is equal to 1 when $n = 1/[r(1-r)]$.Therefore, there exists $\bar{n}$

$$3 \leq \bar{n} \leq 1/[r(1-r)], \tag{4.29}$$

such that the two sides are equal, and for $n \geq \bar{n}$ the RHS exceeds the LHS, and hence $\{N\}$ with $\#N = n$ is not individually stable. In particular, if

$$\#N > 1/[r(1-r)], \tag{4.30}$$

then $\{N\}$ is not individually stable. □

When the production cost is distributed uniformly on $\{0,1\}$, i.e., $r = 1/2$, we have "three is company, and four is a crowd."

Another interesting question that arises in this context is the relation between the individual stability of the grand coalition N, for an exogenously given N, and the distribution of production cost, r. For given N we ask which distributions r of production cost make $\{N\}$ individually stable. We know from (4.26) and (4.27) that such r should satisfy, for $n = \#N$,

$$1 - n(1-r)r - r^n \ \geq 0. \tag{4.31}$$

Denote

$$R(n) = \{r \in [0,1) \ : \ 1 - n(1-r)r - r^n \ \geq 0\}. \tag{4.32}$$

For later use we define

$$r_n = \sup \{r \,:\, r \in R(n)\}. \tag{4.33}$$

Note that r_n is a decreasing sequence for $n \geq 3$.

It is easy to check that, given r, $\{N\}$ is individually stable only if $\{T\}$ is individually stable for $\#T \geq \#N$. Therefore, for $n > t$,

$$R(n) \subseteq R(t). \tag{4.34}$$

We also know that $R(2) = R(3) = [0,1)$; i.e., the collusive ring containing all sellers is individually stable for all r if the number of sellers does not exceed 3. (4.31) has a unique root in $[0,1)$ (see Figure 4.1), which is decreasing in n.

Therefore, as the probability of high production cost rises, it becomes more and more difficult to sustain a collusive ring involving all sellers for large n. In other words, the larger the number of sellers, the lower the expected production cost should be to sustain collusion among all sellers. We illustrate this relation between the total number of sellers and the distribution of production cost in Table 4.1.

Next, we show that any individually stable collusive ring structure can contain *at most* one non-trivial ring under the first price auction.

Proposition 4.4 *Under first price auction, the only individually stable collusive ring structures are of the form* $\{S, < N - S >\}$ *for some* $S \subseteq N$*; i.e.,*

$$\mathcal{C}_{IS}^{fp} \subseteq \{C^o(S) \,:\, S \subseteq N\}. \tag{4.35}$$

Proof: Suppose otherwise that a stable ring structure $C = \{S^1, S^2, \ldots, S^k\}$ exists such that $\#S^i \geq 2$ for more than one $i \in \{1, 2, \ldots, k\}$. Let $S^m \in C$

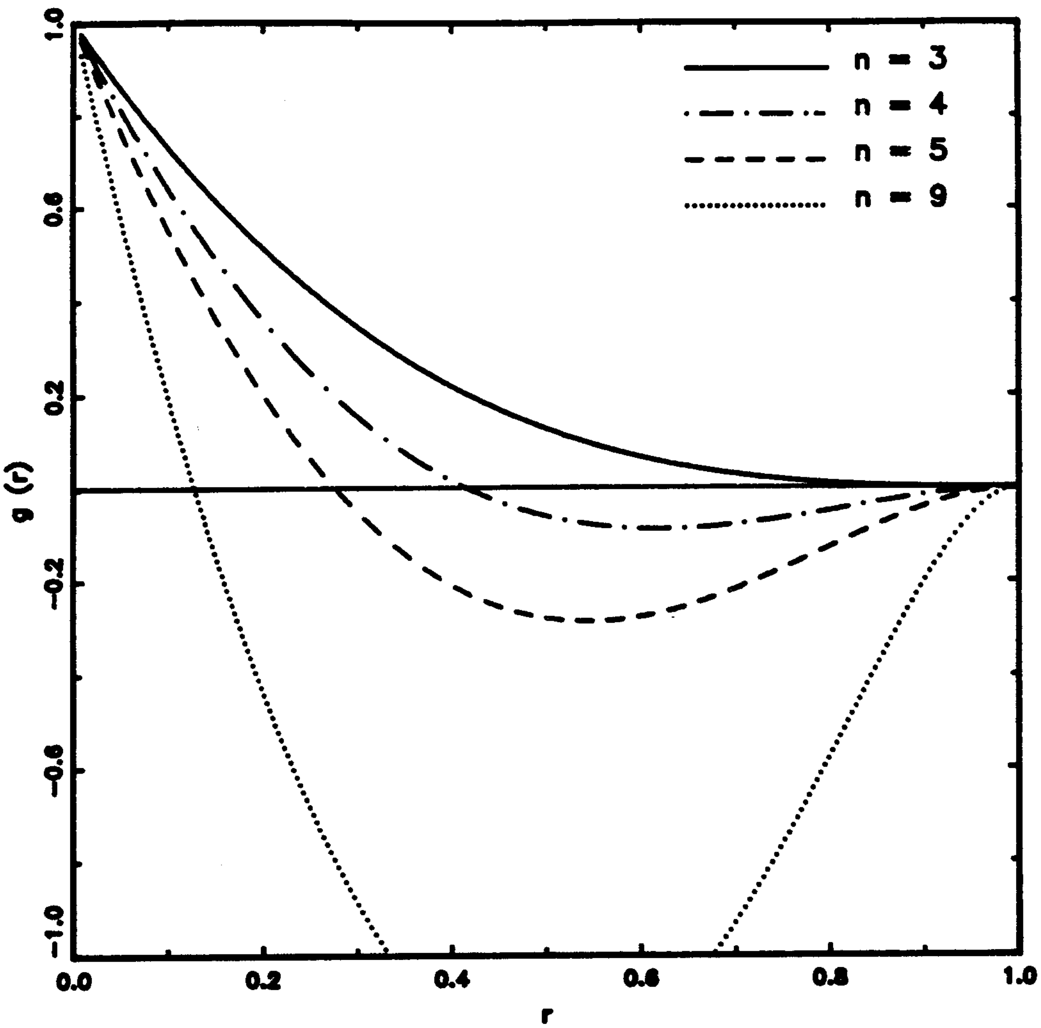

Figure 4.1 : The function $g(r) = 1 - n(1-r)r - r^n$.

n	$R(n)$
2	[0,1.0000)
3	[0,1.0000)
4	[0,0.4142)
5	[0,0.2756)
6	[0,0.2122)
7	[0,0.1726)
8	[0,0.1464)
9	[0,0.1273)
⋮	⋮
29	[0,0.0036)

Table 4.1: The range of expected production costs for which the grand coalition is individually stable.

be the ring with the largest number of members. Take any $S^i \neq S^m$. We will show that if S^i has more than one member, any $j \in S^i$ can increase his expected profit by leaving S^i and acting alone. That is, the ring structure

$$\tilde{C} = \{S^1, \ldots, S^i - \{j\}, \{j\}, \ldots, S^k\} \tag{4.36}$$

gives j higher expected profit than the ring structure C. For $m = \#S^m$, the expected profit of $j \in S^i$, from Proposition 4.2 above, is

$$\Pi_j(C) = p(1 - r^{\#S^i})r^{n-m}/\#S^i \tag{4.37}$$

under the ring structure C, and

$$\Pi_j(\tilde{C}) = p(1-r)r^{n-m} \tag{4.38}$$

under the ring structure $\tilde{C}$. If C is individually stable, (4.37) should be at least as big as (4.38). Since this would imply

$$1 - r^{\#S^i} \geq \#S^i(1-r), \tag{4.39}$$

showing that (4.39) cannot hold for any $r \in (0,1)$ would complete the proof. Using (4.16), (4.39) is true if and only if

$$1 + r + \ldots + r^{\#S^i - 1} \geq \#S^i, \tag{4.40}$$

which is impossible since $r \in (0,1)$. □

An interesting implication of Proposition 4.3 is related to the possibility of collusion when there are exogenous constraints, e.g., communication constraints or geographic distance, on the type of rings that can form, or when the sellers are *heterogeneous* in the sense that some seller is much more likely to have a low production cost than the other sellers. In such cases, presence of an exogenously given dominant firm in the auction precludes collusion among the rest of the sellers. Formally, let the probability of high production cost for one seller $m \in N$ be $r_m < r$; for the remaining sellers $j \in (N - \{m\})$, let $r_j = r$ for some $r \in (0,1)$. Suppose a ring member's share of the ring's profit is proportional to its size. Then, if $r_m < r^{\bar{n}}$, where $\bar{n}$ is defined in Proposition 4.2 above, the only individually stable collusive ring structure under the first price auction is $< N >$, i.e., no collusion exists !. This is reminiscent of the empirical finding by Clabault and Burton [5], that concentration and collusion do not mix well. Hay and Kelly [27], in their empirical study of price fixing conspiracies, report that contrary Clabault and Burton's conclusion, concentration is an important determinant of the ability of the firms to collude. This apparent contradiction can be resolved by noting that the relevance of concentration for collusion cannot be assessed by using *four-firm* concentration ratios. It is also easy to give examples showing that in an industry with heterogeneous sellers, a collusive ring composed of bigger sellers will be more likely than a collusive ring that brings together big and small sellers or only small sellers. That is, ring members will be more like one another than like non-members. In fact, Hay and Kelly's findings are consistent with this prediction.

Proposition 4.3 significantly restricts the set of possible ring structures that are individually stable. We need to check only the ring structures of the form $C^o(S)$ for some $S \subseteq N$. For C^o to be stable, it should be the case that a member of S gets at least as much expected profit as he would get by leaving S, *and* no seller outside S should have any incentive to join S. Therefore, S should satisfy

$$\frac{p(1-r^{\#S})r^{n-\#S}}{\#S} \geq p(1-r)r^{n-(\#S-1)}, \tag{4.41}$$

and

$$p(1-r)r^{n-\#S} \geq \frac{p(1-r^{\#S+1})r^{n-(\#S+1)}}{\#S+1}. \tag{4.42}$$

The left-hand side of (4.41) is what a member of S gets and the right-hand side is the expected profit of a member of S when he leaves S and acts alone. The left-hand side of (4.42) is the expected profit of a seller outside S, and the right-hand side is what this seller would get if he were to join S. After cancellations, we get

$$1-r^{\#S} \geq \#S(1-r)r \tag{4.43}$$

and

$$1-r^{\#S+1} \leq (\#S+1)(1-r)r. \tag{4.44}$$

Therefore, we have the following result:

Proposition 4.5 *$C^o(S)$ is individually stable if and only if*

1. *$\{S\}$ is individually stable, and*

2. *$\{S \cup \{j\}\}$ is not individually stable,*

where $j \in N - S$.

Proof: It is clear that the conditions for individual stability of $C^o(S)$ given by the inequalities (4.43) and (4.44) are exactly the same as the condition, given in Proposition 4.2, for the individual stability of the grand coalition of sellers when the total number of sellers is $\#S$ and $\#S+1$, respectively. □

Combining these results, we characterize all individually stable collusive ring structures under the first price auction in the following proposition.

Proposition 4.6 *The set of all individually stable collusive ring structures under the first price auction is*

$$\mathcal{C}_{IS}^{fp}(r,n) = \{C^o(S) \; : \; \#S = k \text{ and } r \in [r_{k+1}, r_k]\}. \tag{4.45}$$

Proof: From Proposition 4.4, all individually stable ring structures are of the form $C^o(S)$ for some $S \subseteq N$. And, from Proposition 4.5, if $r \in [r_{k+1}, r_k]$, the only individually stable ring structures of the form $C^o(S)$ are the ones with $\#S = k$. □

It follows that for $r \in (r_{k+1}, r_k)$, there are $\binom{n}{k}$ individually stable ring structures under the first price auction. For $n \geq 3$, there are at least three firms that form a ring independent of the number of sellers. All individually stable ring structures have the same form: a collusive ring of at least three sellers and a "competitive fringe" made up of the remaining sellers each of whom acts independently. The number of sellers involved in a collusive agreement is a decreasing function of the probability of high production cost. This number is independent of the collusive ring structure that actually forms; therefore, the following number is well defined:

$$m(r,n) = \{m(C) \; : \; C \in \mathcal{C}_{IS}^{fp}(r,n)\}. \tag{4.46}$$

Figure 4.2 illustrates the number of sellers involved in a collusive agreement, $m(C)$, when the total number of sellers is 10.

An interesting implication of the foregoing analysis, to the extent that production cost can take only two values, is that in a sample of independent auctions with uniformly distributed r, and $n \geq 3$, in more than 60 percent of the cases there will be a collusive ring of three sellers, and in more than 70 percent of the cases, there will be a collusive ring of three or four sellers.

4.5 Profit Comparisons and Individual Stability

If C *is* individually stable, we have the following result which is almost a restatement of Stigler's remark quoted in the introduction: A member of the competitive fringe gets strictly higher expected profit than a member of the collusive ring.

Proposition 4.7 *For all* $r \in (0,1)$ *and* $C \in \mathcal{C}_{IS}^{fp}(r,n)$, *if* $C = C^o(S)$ *and* $m = \#S < \#N$, *then, for all* $j \in (N-S)$ *and all* $i \in S$,

$$\Pi_j^{fp}(C) > \Pi_i^{fp}(C). \tag{4.47}$$

Proof: From (4.25) and (4.16 6) the inequality in (4.47) is equivalent to

$$m > 1 + r + \ldots + r^{m-1}, \tag{4.48}$$

which is always true for $r \in (0,1)$. □

The phenomenon that each firm in a collusive ring receives less expected profits than an outsider is also observed by Lien [34] in a three-person *all-pay* first price auction with identical uniform cost distribution for each firm.

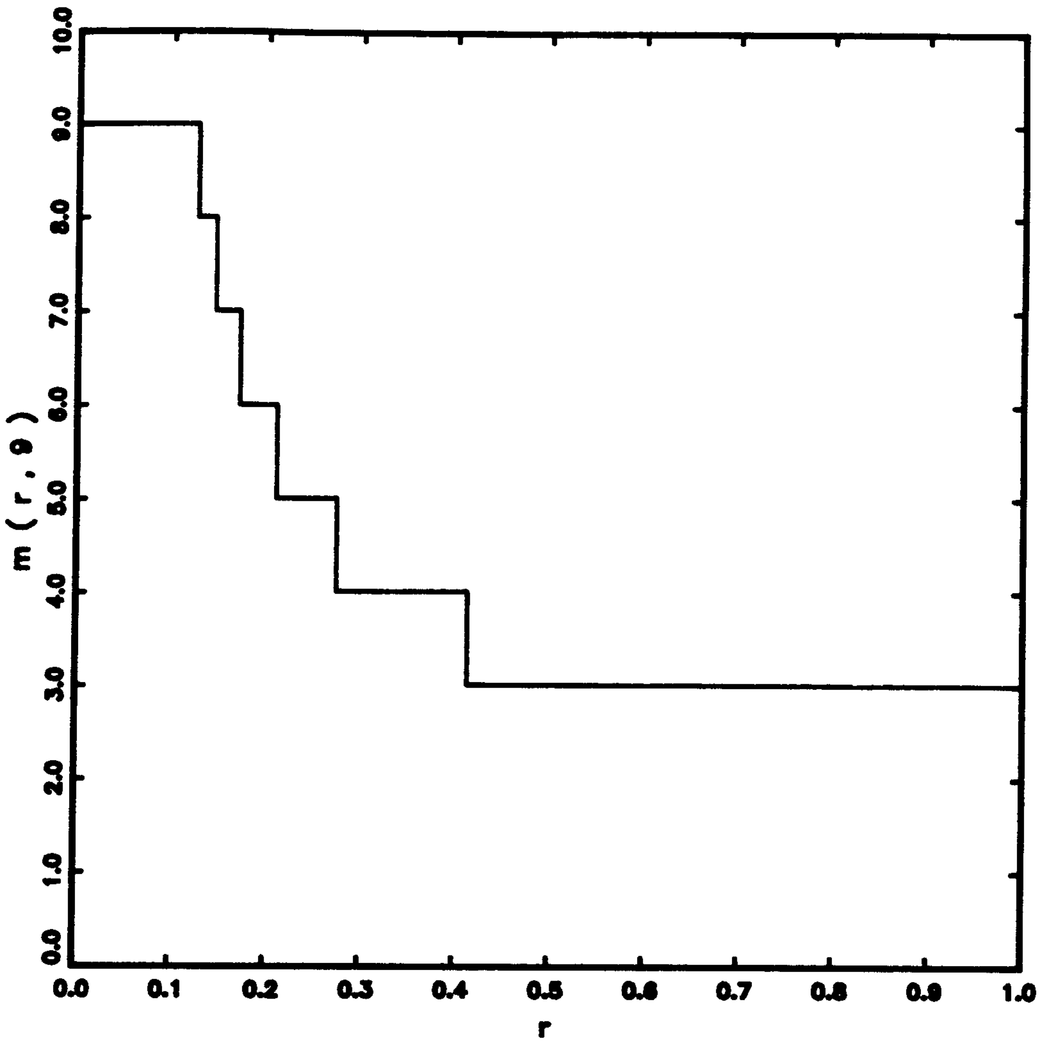

Figure 4.2 : Number of Sellers Involved in Collusion for $n = 9$.

In this auction the highest bidder obtains a project that pays a fixed predetermined sum and incurs a production cost, and every bidder pays the amount he submitted to the auctioneer. Lien solves for the equilibrium bidding strategies when two of the three firms form a ring, and observes that per capita expected profit of the ring is less than the expected profit of the outsider. Lien remarks that this result seems to indicate that "there is no strong incentive for any firm to propose a coalition," given the assumptions of the model, and conjectures that the payoff to each member of a collusive ring may not necessarily increase with the size of the ring when there are more than three bidders and more than two of them can collude. In our setup with discrete production costs, the conjecture turns out to be incorrect in the sense that the payoff to every seller is increasing in the size of the ring. It can also be shown that in the discrete cost analog of the case considered by Lien, Proposition 4.9 below continues to hold.

With the payoff functions given in (4.25), we have the following result:

Proposition 4.8 *For all* $r \in (0,1)$*, all collusive ring structures* $C \in \mathcal{C}$*, and all* $S \in C$*,* $\Pi^{fp}(S, C)$ *is increasing in* $m(C)$*.*

Proof: Immediate from (4.25) . □

If S^m is the largest ring in C, Proposition 4.8 states that the expected profit of *every* seller, members *and* non-members of S alike, is increasing in the number of sellers that belong to S^m. This phenomenon is the *positive* externality mentioned in the introduction. An immediate implication of this externality is that any firm would prefer to face one large rival rather than many small rivals with the same total "size." Note that we do not require C to be an individually stable ring structure for the above result.

As to the conclusion on the implication of the relative profit levels inside and outside the ring for incentives to form collusive rings, it rests on an implicit assumption about the *process* by which coalitions are formed; namely, the process starts from a situation with no cooperation and a seller proposes a coalition to another seller. In such a process, it is clear that any seller i who is approached by the proposer would *wish* that the proposer had approached some other seller j. However, given that, once he has been asked to join, his expected profit would be higher if he joined the ring than if he did not, seller i *would* join the ring, notwithstanding the fact that he would have liked it better if seller j were the one who received the offer. Stigler's quoted remark, and the conclusion of Proposition 4.8, can not be interpreted to mean that there would not be any merger or collusion.

Second, the proper comparison is not between expected profits of sellers inside and outside a collusive ring, but between what a seller gets by joining or leaving a ring. In Lien's study, a two-seller coalition is *exogenously* given, and the only comparison made is between expected per capita profit of the colluding sellers, sellers 1 and 2, and the expected profit of the third seller. It turns out that in Lien's model with three sellers, expected profits of *every* seller, including the third seller, would increase if all sellers formed one ring: Despite the fact that seller 3 gets higher expected profits than both seller 1 and seller 2 if sellers 1 and 2 form a ring and seller 3 stays out, he has every incentive to join sellers 1 and 2. This fact stands in sharp contrast with the conclusion on incentives to form coalitions based on relative profits inside and outside a ring.

The results in Propositions 4.7 and 4.8 are also similar to the ones obtained by d'Aspremont and Gabszewicz [14] in a collusive price-leadership model with complete information. They consider a model of an industry for a homogeneous product with linear market demand and n sellers each of whom has a linear marginal cost function. The class of collusive ring structures they consider is exogenously restricted to structures of the form $C^o(S)$ for $S \subseteq N$, and they show that in this class, the only individually stable collusive structures are the ones with a cartel size of three firms. Our results are stronger in some respects and weaker in others. First, we allow arbitrary ring structures. Second, we do not assume any asymmetry between a collusive ring and the competitive fringe, as all moves are simultaneous in the pricing game in our case. Finally, we introduce uncertainty, if in a limited way. Although our assumption of constant marginal cost is, in general, more restrictive than the assumption of increasing marginal cost considered by d'Aspremont and Gabszewicz, in this framework, increasing marginal cost assumption would make collusion among a larger number of firms easier compared to the constant marginal cost case because of the possibility of profitable reallocation of production among the cartel members. This possibility is non-existent in the constant marginal cost case. The only substantive restriction in our model compared to the model studied by d'Aspremont and Gabszewicz is our assumption of fixed quantity. We conjecture that this assumption can be relaxed without significant effects on the qualitative results.

4.6 Conclusions

We have shown that first price auctions are less conducive to formation of collusive rings than second price auctions, even in the absence of incentive compatibility problems within a ring in dealing with the private information of ring members. That is, the size of a collusive ring under a first price auction is restricted by the expected profit calculations of potential colluders even *before* each seller observes his private information. Other potential problems from which we abstracted in this study, e.g., illegality of price fixing, the buyer's strategic reactions and private information of the ring members, would tend to weaken even more the stability of collusive agreements under the first price auction. In the absence of these additional impediments to collusion, we have shown that individually stable ring structures under the first price auction have a special form : a ring, whose size depends on the expected production cost, together with a competitive fringe of remaining sellers, whereas a collusive ring of all sellers is the only individually stable structure under the second price auction.

Questions remain as to the robustness of the qualitative results with respect to the form of production cost distribution. The case of general continuous cost distributions present great difficulties in solving the equilibrium bidding strategies under the first price auction. Extensions to cost distributions with more than two possible costs seem to be relatively easier, and the case we considered can be interpreted as a situation where production costs can take more than two values, and the buyer's reserve price is less than or equal to the second lowest of all possible production costs. We conjecture that the qualitative results extend to arbitrary discrete cost distributions.

Chapter 5

EXPERIMENTS ON PRE-AUCTION INVESTMENT

5.1 Introduction

In this chapter we report some experimental results on the model developed in Chapter 3. Laboratory experimental methods have been used extensively to study the common forms of oral and sealed bid auctions ([6], [7], [53]). Our experimental focus is on the pre-auction investment decisions, and on the implications of investment observability for investment behavior under first and second price auctions. We used two treatment variables:

1. Type of Auction: First Price and Second Price.

2. Investment Observability: Observable and Unobservable.

The aggregate predictions based on the model are largely borne out by the observations. At the level of market aggregates, the two auctions and the two observability treatments are indistinguishable at the pure strategy equilibria. Further tests on the observed distributions of investment under the two information treatments and the two auctions reveal that the model is not entirely accurate in explaining the observations. The discrepancies seem to be related to the the fact that the experimental setting is, in fact, a repeated game situation.

5.2 Experiments

We conducted six experiments. Experiments were conducted using special software at the Caltech Economics and Political Science Laboratory computer network. In each experiment twelve subjects participated. All subjects were undergraduate students at California Institute of Technology. Average hourly earnings of a subject were about ten dollars.Each experiment took about two hours to complete.

Twelve subjects participated in each experiment. A subject's earnings from the experiment were determined by the decisions he made in a market. The type of currency used in the market was francs. All earnings were in terms of francs, which were accumulated throughout the experiment. At the end of the experiment, accumulated earnings in francs were converted to dollars at privately known exchange rate for each subject, and each subject was paid in dollars.

An experiment consisted of a sequence of sessions. In each session, several parallel markets were in operation with a given number of sellers in each market. The number of subjects was the same in every market. In all experiments each subject was randomly assigned to a market at the beginning of the experiment. One unit of an object was to be bought at auction in every market from the seller who submitted the lowest bid. The payment to the auction winner depended on the auction type. In the first price auction, the payment received by the winner for the object was his bid. In the second price auction, the auction winner received an amount equal to the second lowest bid. The instruction sheet for the second price auction is given in the appendix.

The winner of the auction had to incur a production cost that could take only two values, $\bar{c} = 2000$ or $\underline{c} = 1000$. The production cost of a seller was determined as a random function of his investment decision prior to the auction. If a seller did not make any investment, his production cost was $\bar{c}$. By paying s francs, a seller could draw a random production cost which was equal to $\underline{c}$ with probability $1 - r = 0.15$. This was operationalized by making a draw from a bingo cage that contained a given number of balls $100r\%$ of which were marked with $\bar{c}$ and $100(1-r)\%$ with $\underline{c}$. A seller could make as many independent draws with replacement as he liked from the bingo cage by paying s francs per draw. His production cost, if he made any draws, was the lowest number on the balls drawn from the cage. At the beginning of the experiment each subject was given 4000 francs which they could use to pay for the draws. The bingo cage analogy was used to explain the relation between the investment decisions and the distribution from which the production cost would be drawn. In the experiments, the actual random numbers were obtained by using a random number generator rather than bingo cages.

At the beginning of a session a seller was asked to report the number of draws he wanted to make for the given bingo cage. Then, his production cost was determined as a random function of his draws. Each seller observed his production cost privately. Under the observable investment treatment, every seller in a market could observe the investment decisions of his rivals before he was asked to submit his bid. Under the unobservable investment

Exp.	# Subjects	# Markets	r	s	$p - \underline{c}$	Franks/$
1	12	4	0.85	10	700	0.0015
2	12	4	0.85	10	700	0.0015
3	12	4	0.85	10	700	0.0015
4	12	4	0.85	0	700	0.0015
5	12	6	0.85	16	700	0.0015
6	12	6	0.85	16	700	0.0015

Table 5.1: Parameters used in the Experiments.

treatment, investment decisions were private knowledge. After the bids were submitted, the winner and the price were announced in every market. A seller's profit for the session was determined as the difference between the price and the sum of production and investment costs, if he won the auction. Otherwise, the seller would lose an amount equal to the cost of draws he made for the session.

The parameters used in the experiment are given in Table 5.1. In all experiments, the production cost could take only two values, $\bar{c} = 2000$ or $\underline{c} = 1000$. A reserve price $p = 1700$ above which no purchase was to be made was announced at the beginning of the experiment. In Table 5.1 r is the probability of high production cost, and s is the search cost.

Table 5.2 exhibits the sequence of auction and investment observability treatments in the experiments. In the table FU(50) stands for fifty sessions of first price auction with unobservable investment, FO(30) for thirty sessions of first price auction with observable investment. SU($\cdot$) and SO($\cdot$) are interpreted similarly.

In experiments 1 and 2, we conducted eighty sessions of four parallel first price auctions with three sellers in each. In the first fifty sessions investment decisions were only privately observable. In the last thirty sessions,

Exp.	# Periods	# Markets	# Sellers	Sequence
1	80	4	3	FU(50)/FO(30)
2	80	4	3	FU(50)/FO(30)
3	90	2	3	SU(50)/FU(40)
3	90	2	3	SO(50)/FO(40)
4	50	4	3	F(50)
5	90	3	2	FU(50)/SU(40)
5	90	3	2	FO(50)/SO(40)
6	90	3	2	FU(50)/SU(40)
6	90	3	2	FO(50)/SO(40)

Table 5.2: Experimental Design.

investment decisions of sellers in each market were publicly announced to the sellers in that market before sellers submitted their bids.

In experiment 3, we conducted 50 sessions of four parallel second price auctions followed by forty sessions of first price auctions. Sellers in market 3 and market 4 could observe one another's investment levels before submitting bids, and in markets 1 and 2 investment decisions were unobservable by others. Therefore, we conducted one hundred second price auctions and eighty first price auctions under each investment observability treatment.

Experiment 4 was conducted immediately following experiment 3 with the same subjects who participated in experiment 3. In experiment 4, we conducted fifty sessions of four parallel first price auctions with exogenously given cost distributions. The cost distributions were exogenously given to be the ones corresponding to the asymmetric pure strategy investment equilibrium; i.e., production cost distribution of one seller was the same as the production cost distribution of the seller with the high investment level in the investment game, and two sellers had a production cost distribution corresponding to the low investment level. With the parameters used in experiments 1, 2 and 3, the only pure strategy equilibria in first price auction with investment observability were such that one seller invested 9 units and two sellers invested 3 units each. In experiment 4, we exogenously assigned cost distributions to three sellers in each market so that the probability of having the high production cost was r^9 for one seller and r^3 for the remaining two sellers.

In experiments 5 and 6, every session consisted of six parallel markets with two sellers in each market. In three markets, markets 1,2 and 3, the sellers could not observe one another's investment levels before bidding,and in markets 4, 5 and 6, investment was publicly observable at the bidding stage. In both experiments, fifty sessions of first price auction were followed by forty sessions of second price auction. Experiment 5 was the only experiment in the series in which the sellers within the same investment observability treatment were matched randomly in every session. In all other experiments, the sellers were assigned randomly to markets at the beginning of the experiment, and the assignments were fixed throughout the rest of the experiment.

5.2.1 Predictions

The choice of the parameters used in the experiments was mainly motivated by several considerations. The first consideration was the need to obtain equilibrium predictions with integer values. Secondly, we wanted to use parameters consistent with a reasonable difference between the high and the low investment levels in equilibrium under the first price auction with observable investment.

In this section we present the equilibrium predictions for the given the experimental parameters.

First Price Auction

In first price auctions with unobservable investment, we have predictions only on the symmetric equilibrium. With the given parameters, the symmetric equilibrium investment level is 5 units for each of the three sellers in experiments 1, 2, and 3, and 6 units for each seller in experiments 5 and 6. In first price auctions with observable investment, the set of pure strategy subgame perfect equilibria is uniquely determined by two numbers; namely, a high investment level, $\bar{x}$, and a low investment level, $\underline{x}$. With the given parameters, the high investment level is 9, and the low investment level is 3 in experiments 1, 2, 3, 5 and 6. In experiments 5 and 6, investment levels below 3 units are dominated. Given that neither seller's investment level is below 3 units, investment levels above 9 units can also be eliminited. As a result of this two-stage iterative elimination of dominated strategies, the set of undominated strategies is the set of investment pairs (x_1, x_2), where $3 \leq x_i \leq 9$ for $i = 1, 2$.

In experiment 4, one of the three sellers, seller 1, was exogenously assigned a production cost distribution such that the probability of the high production cost was r^9. The remaining two sellers, sellers 2 and 3, were assigned the production cost distribution such that the probability of the high production cost was r^3. For these parameters, the equilibrium bidding strategy for seller 1, when his production cost is $\underline{c}$, is to randomize according to

$$F^*(b) = F(b; 9, 3, 3) := \{1 - r^3\sqrt{(p - \underline{c})}\} \,/\, (1 - r^9)\,. \qquad (5.1)$$

The corresponding equilibrium bidding strategy for sellers 2 and 3 is

$$F_*(b) = F(b; 3, 3, 9) := \{1 - r^3\sqrt{(p - \underline{c})}\} \,/\, (1 - r^3)\,. \qquad (5.2)$$

Second Price Auction

In second price auctions with unobservable investment, the set of Nash equilibria is determined by a unique number which represents the total equilibrium investment level. In experiments 1, 2 and 3, any triple of investment levels that add up to 15 is a Nash equilibrium. In experiments 5 and 6, any pair of investment levels that add up to 12 is a Nash equilibrium. Under the observable investment treatment, the set of subgame perfect equilibria is the same as the same as the set of Nash equilibria under unobservable investment treatment.

Collusion

Under both auctions and both observability treatments, if the sellers in a market collude by correlating their investment strategies, then there will be a single active seller in every period who undertakes the unique total equilibrium investment level. Thus, with perfect collusion, one seller invests 15 units in experiments 1,2 and 3, and 12 units in experiments 5 and 6, and the remaining sellers stay out of the market.

5.3 Experimental Results

Figures 5.1.A through 5.1.F exhibit the results from experiment 1. Figure 5.1.A gives the per seller investment level averaged over four markets in every period. Figure 5.1.B is the time series of per seller investment level by market. Figures 5.1.C through 5.1.F show the individual investment levels in markets 1 through 4, respectively. Figures 5.2.A through 5.2.F contain the same information for experiment 2. Figures 5.3.A through 5.3.F present the corresponding time series for experiment 3. The only difference is that in Figure 5.3.A, there are two time series representing the average the investment levels in markets with observable and unobservable investment, respectively. The dotted lines in the graphs for individual investment decisions show the symmetric pure strategy equilibrium for the first price auction with unobservable investment and for the second price auction. The two parallel dotted lines in the first price auction panel under observable investment show the asymmetric pure strategy equilibria. Figures 5.5.A through 5.5.I and Figures 5.6.A through 5.6.I exhibit the corresponding time series for experiments 5 and 6, respectively.

Figure 5.4 exhibits the observed and predicted cumulative frequencies of bids conditional on low production cost in experiment 4.

Given the experimental parameters the model predicts a unique aggregate investment, which is fifteen units in experiments 1 through 3 and twelve units in experiments 5 and 6. If we treat each auction as an independent observation unit, then the average over all auctions of the aggregate investment must be close to the values predicted by the model. Table 5.3 reveals that the observed total investment figures do not differ significantly from the predicted ones at conventional significance levels.

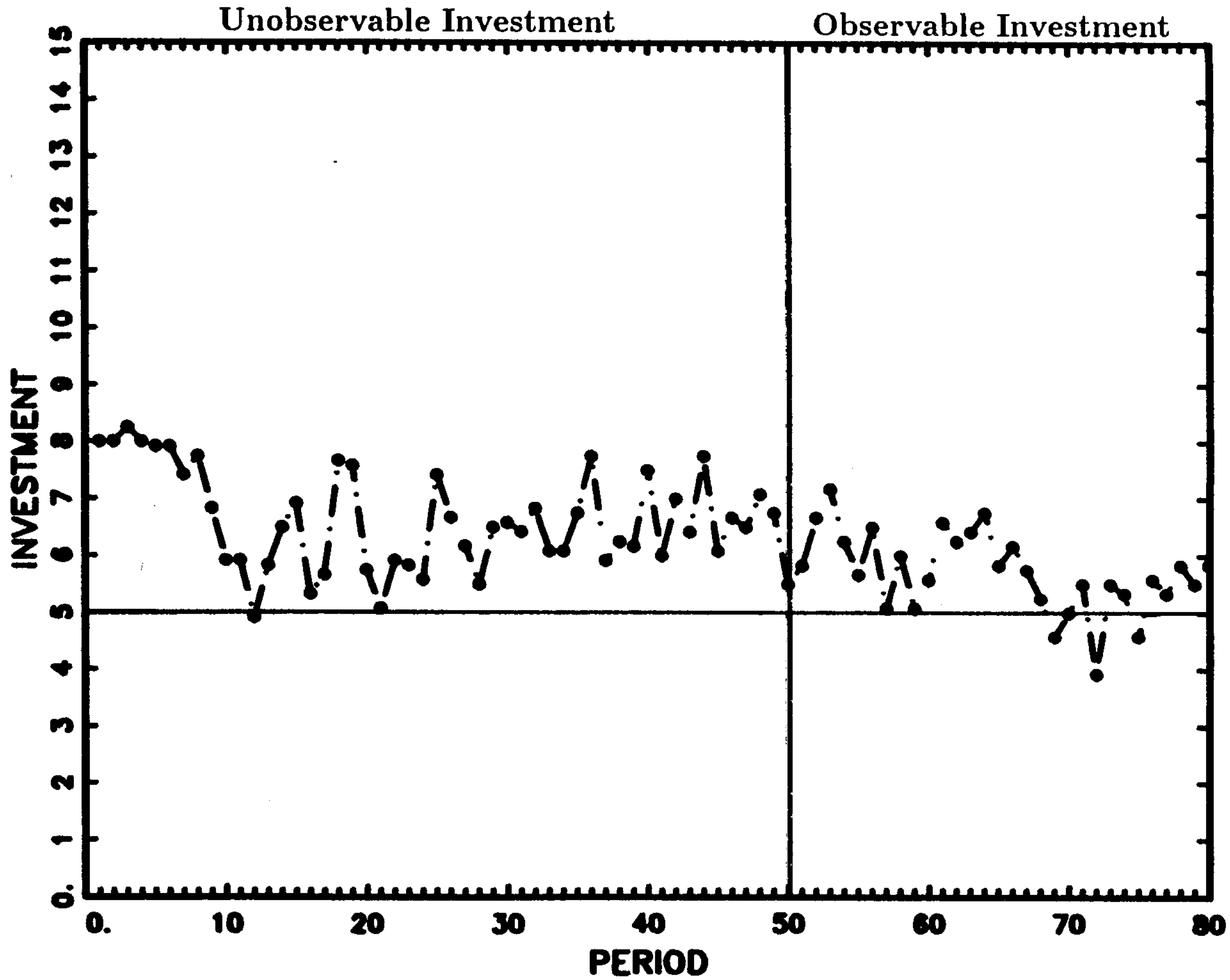

Figure 5.1.A : Experiment 1 - First Price Auction

Average Investment Level

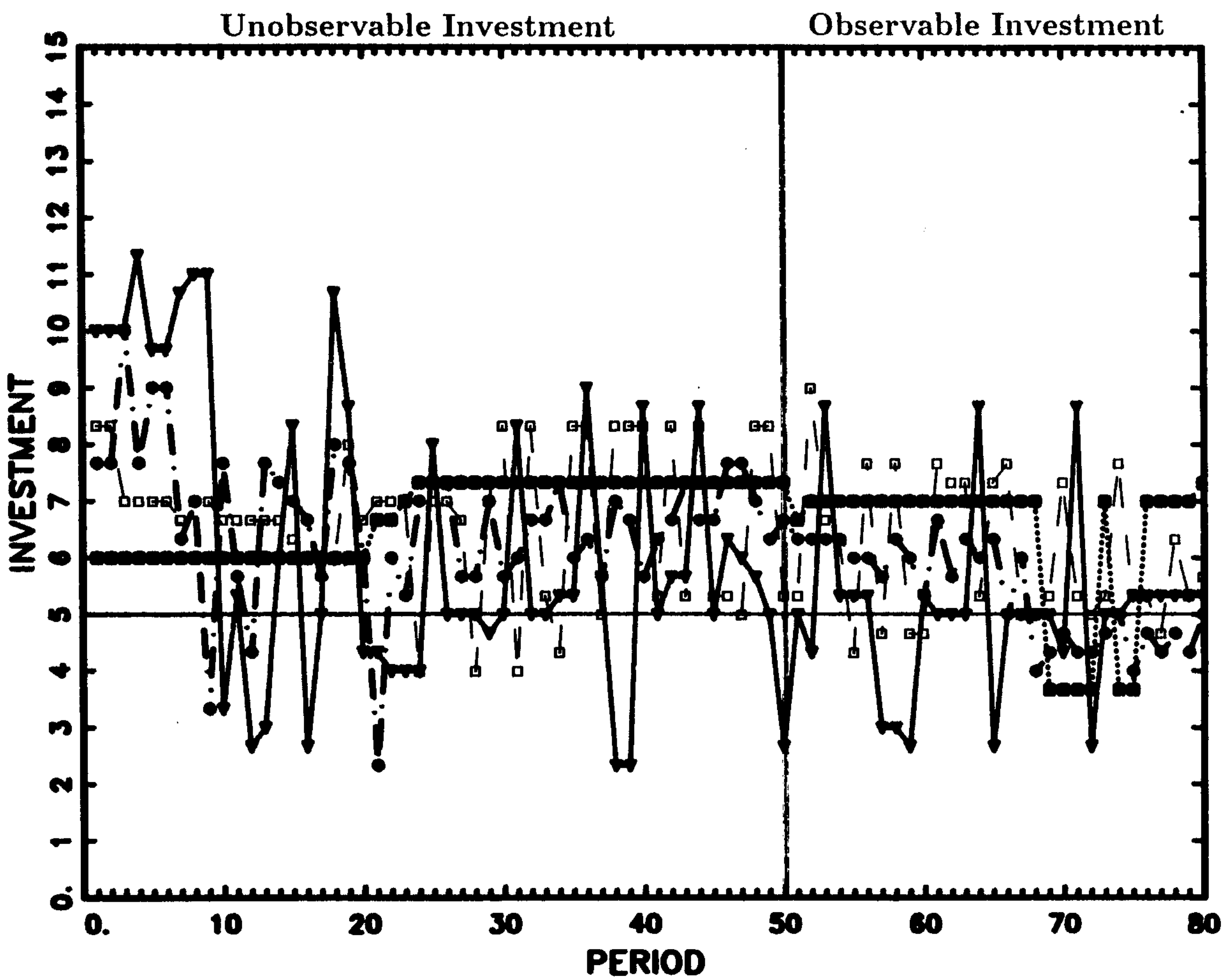

Figure 5.1.B : Experiment 1 - First Price Auction

Average Investment Level by Market

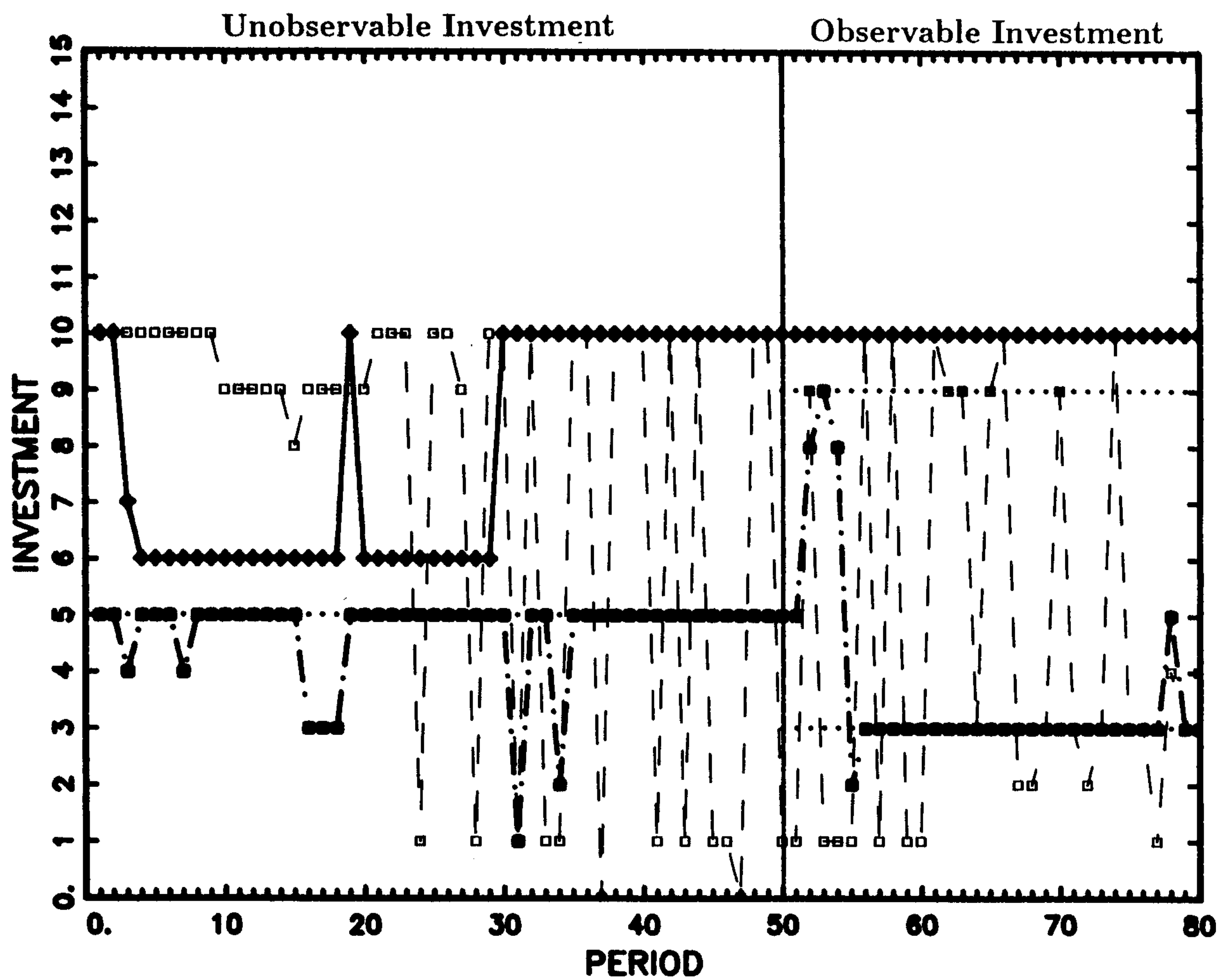

Figure 5.1.C : Experiment 1 - First Price Auction

Individual Investment Decisions in Market 1

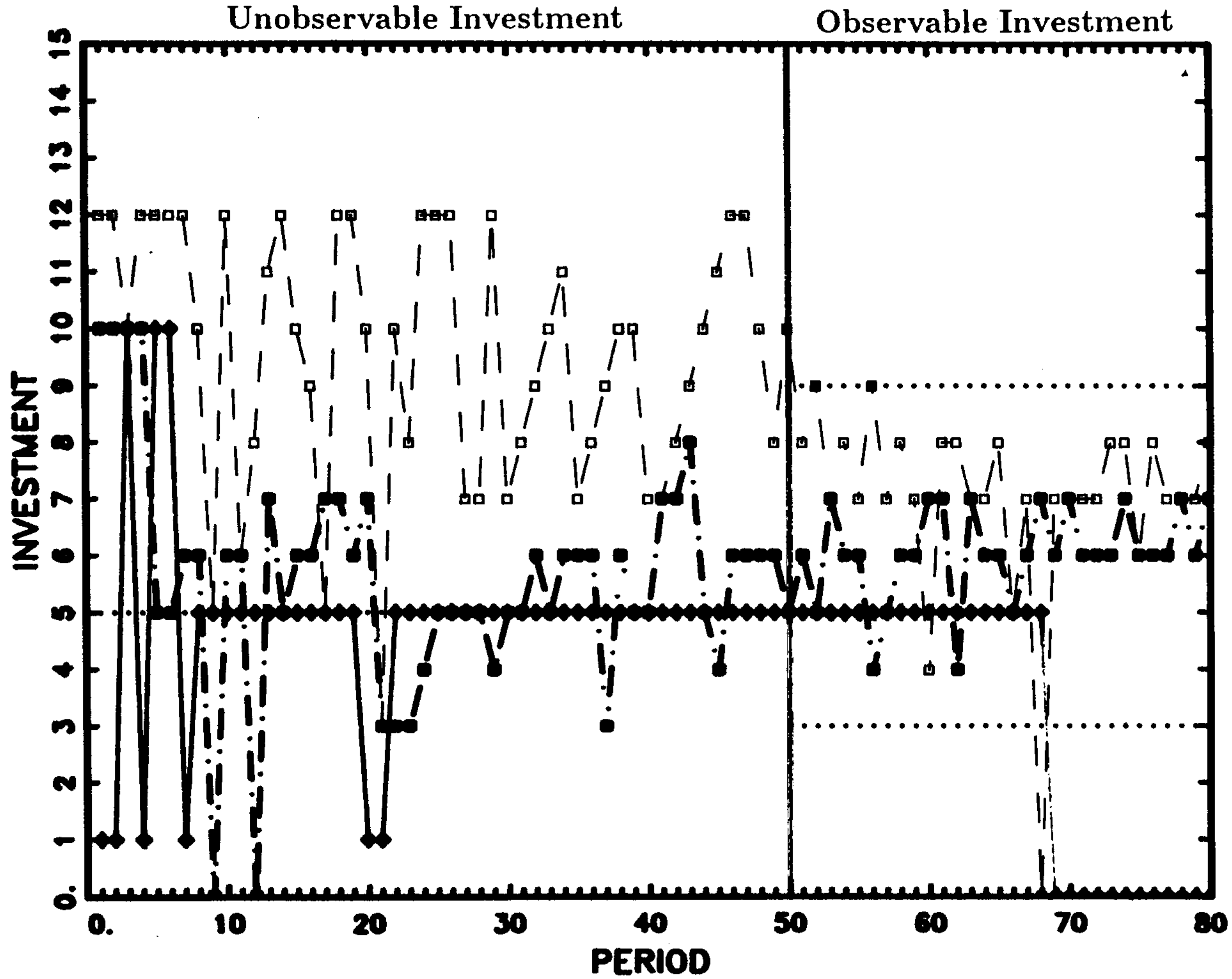

Figure 5.1.D : Experiment 1 - First Price Auction

Individual Investment Decisions in Market 2

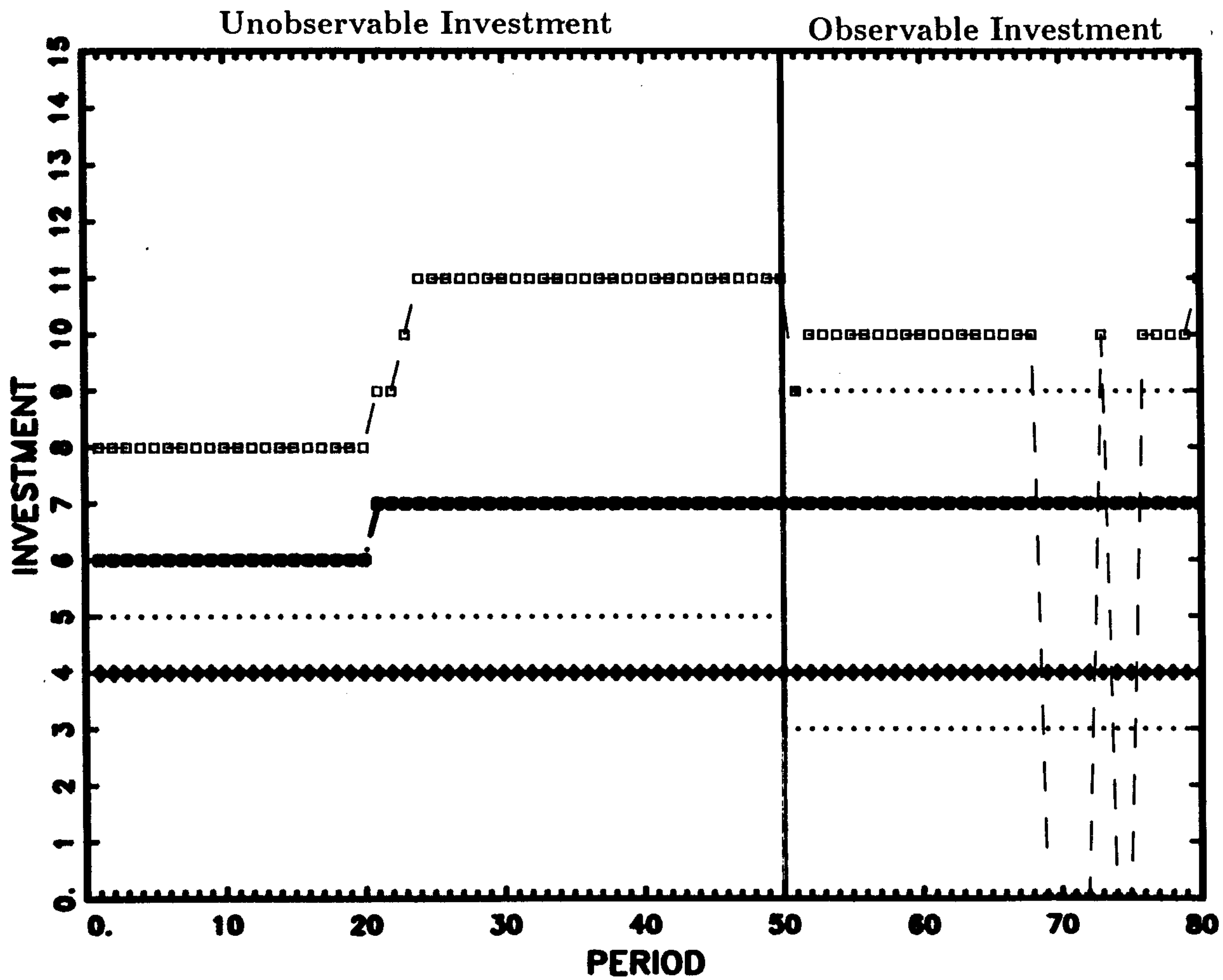

Figure 5.1.E : Experiment 1 - First Price Auction

Individual Investment Decisions in Market 3

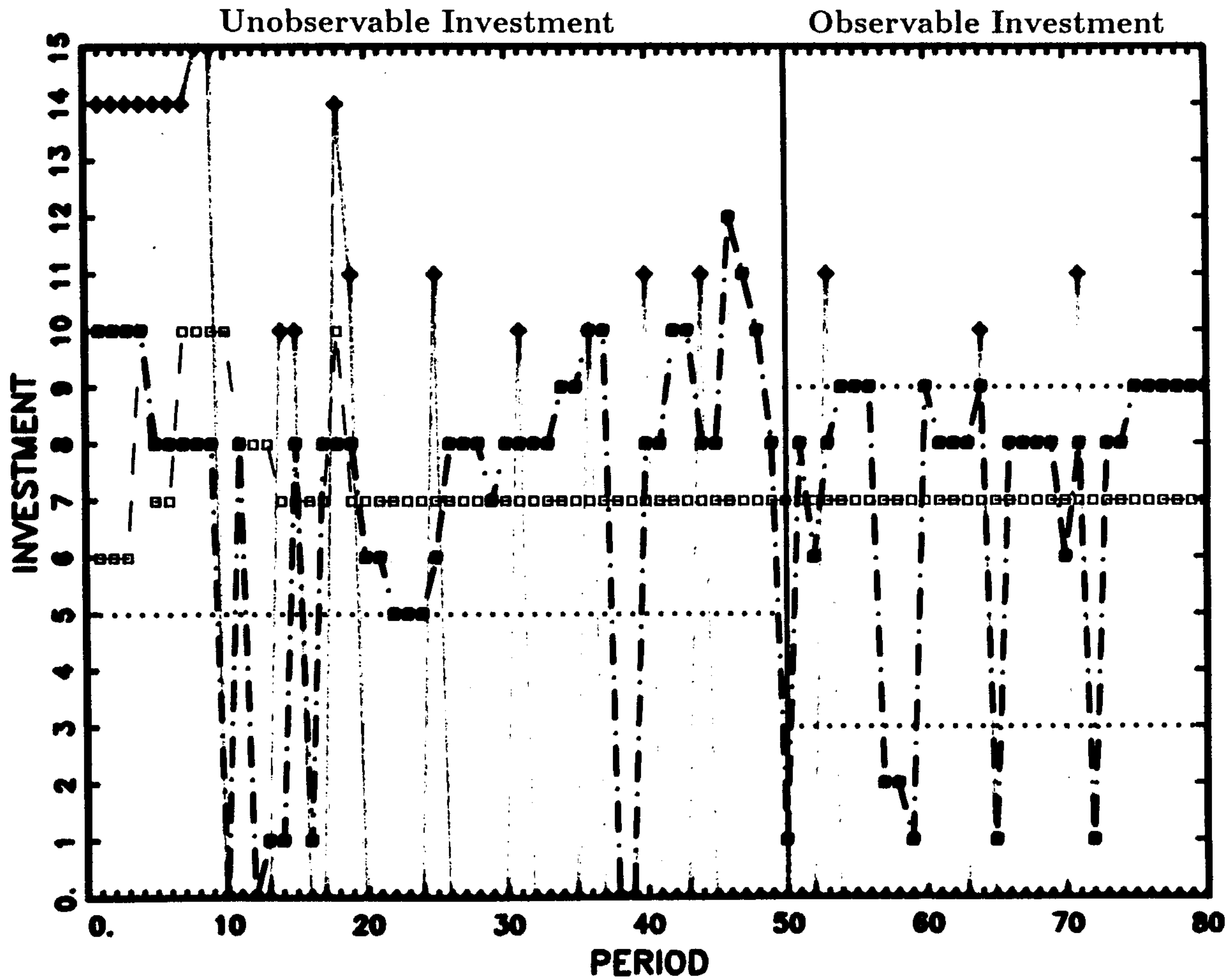

Figure 5.1.F : Experiment 1 - First Price Auction

Individual Investment Decisions in Market 4

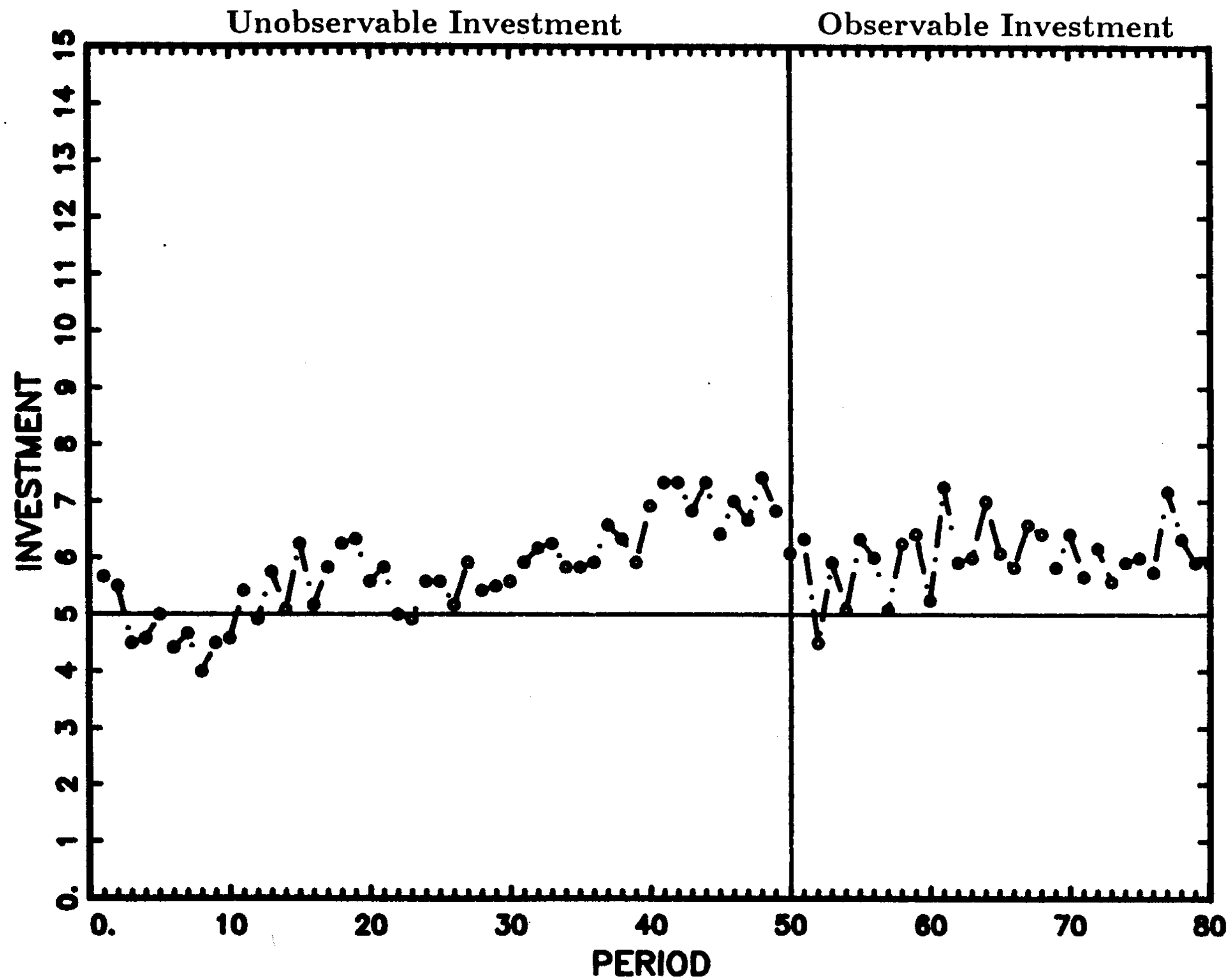

Figure 5.2.A : Experiment 2 - First Price Auction

Average Investment Level

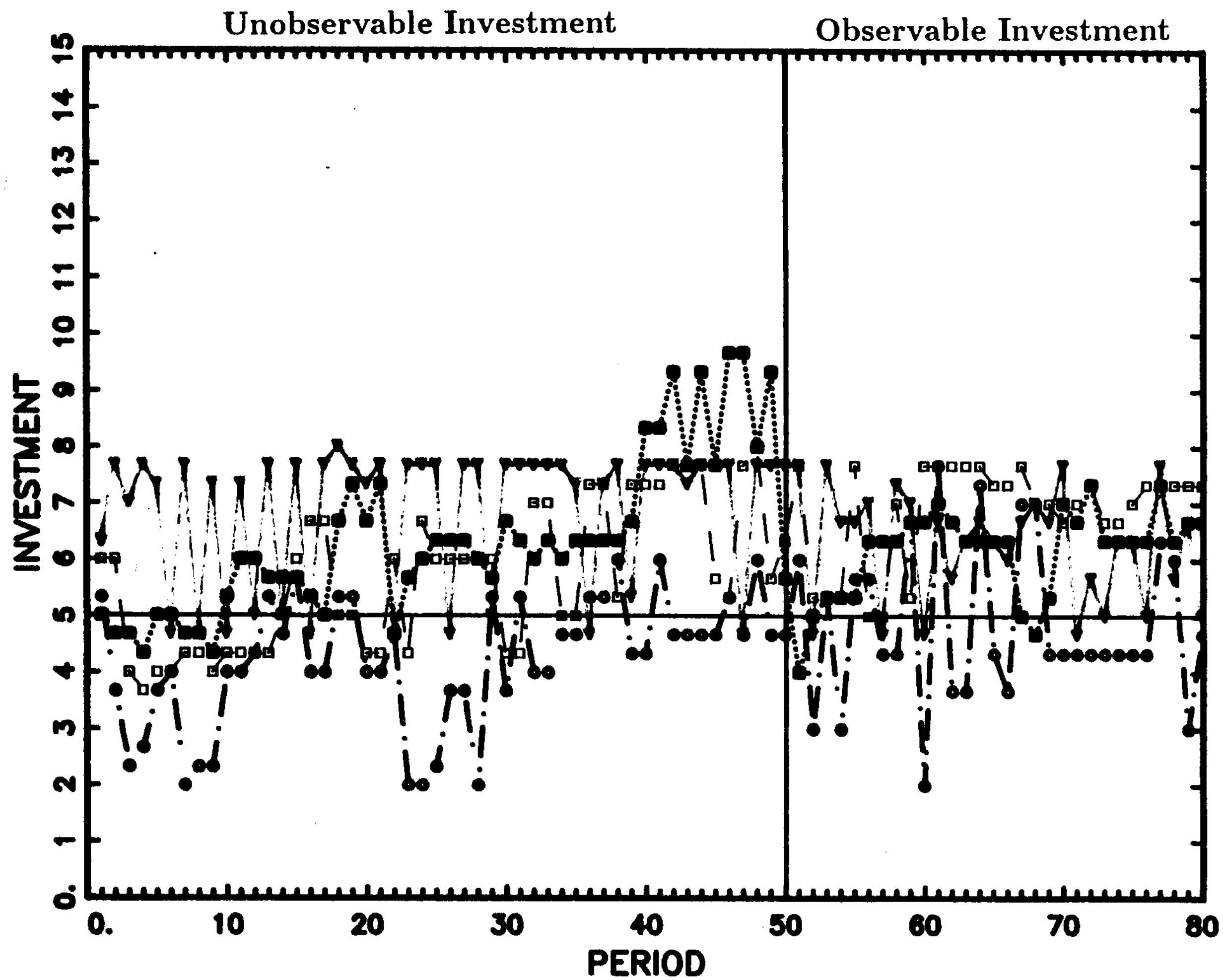

Figure 5.2.B : Experiment 2 - First Price Auction

Average Investment Level by Market

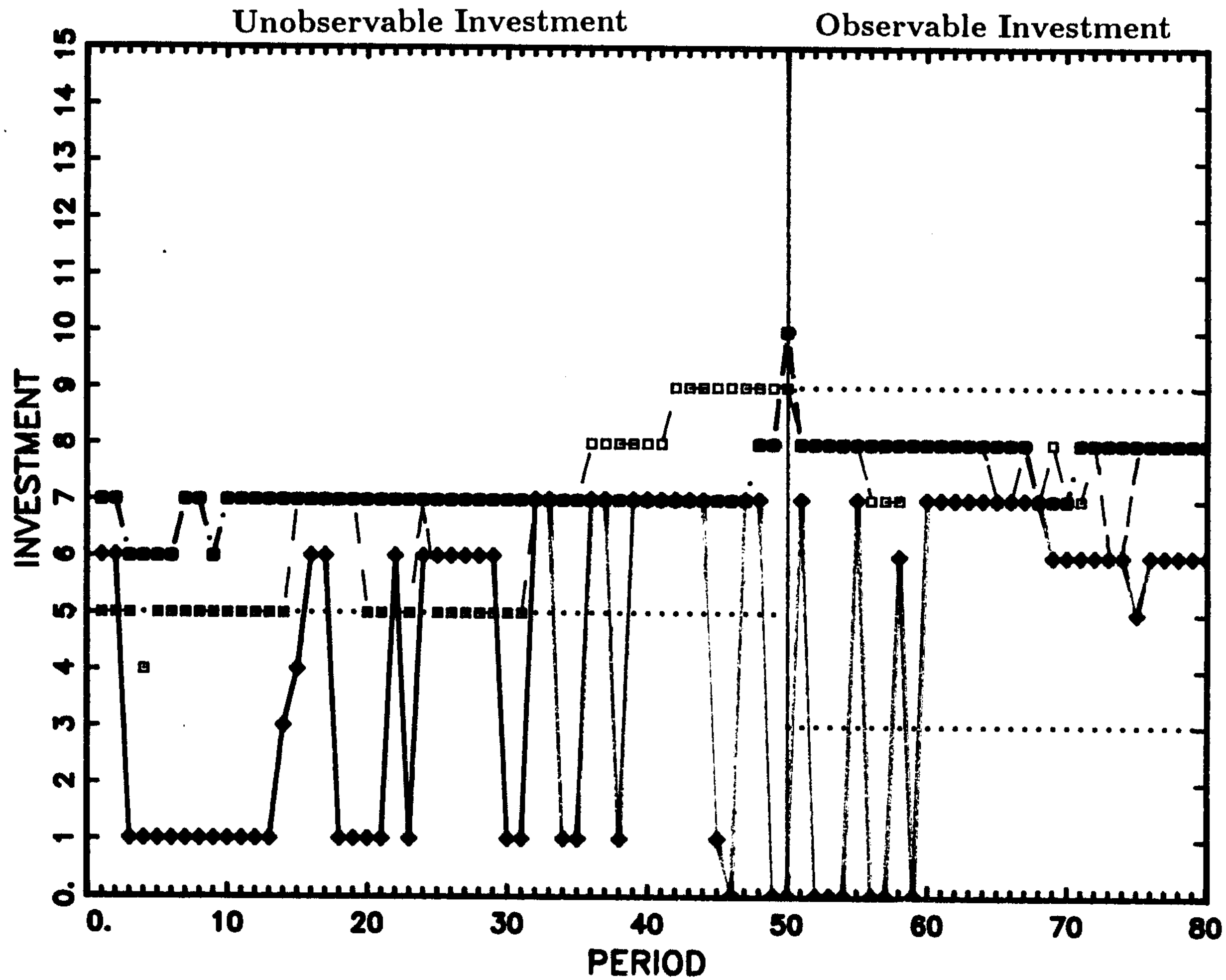

Figure 5.2.C : Experiment 2 - First Price Auction

Individual Investment Decisions in Market 1

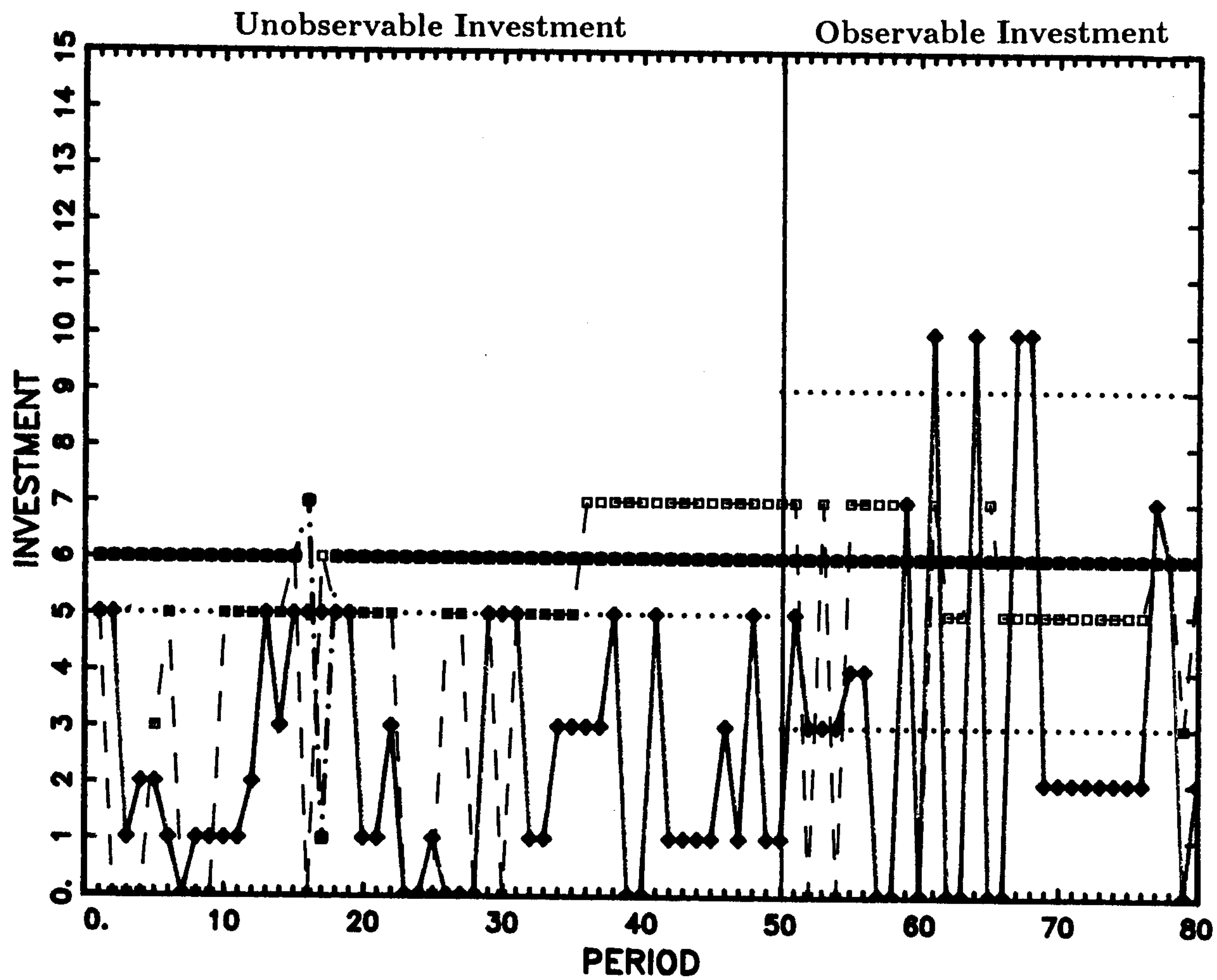

Figure 5.2.D : Experiment 2 - First Price Auction

Individual Investment Decisions in Market 2

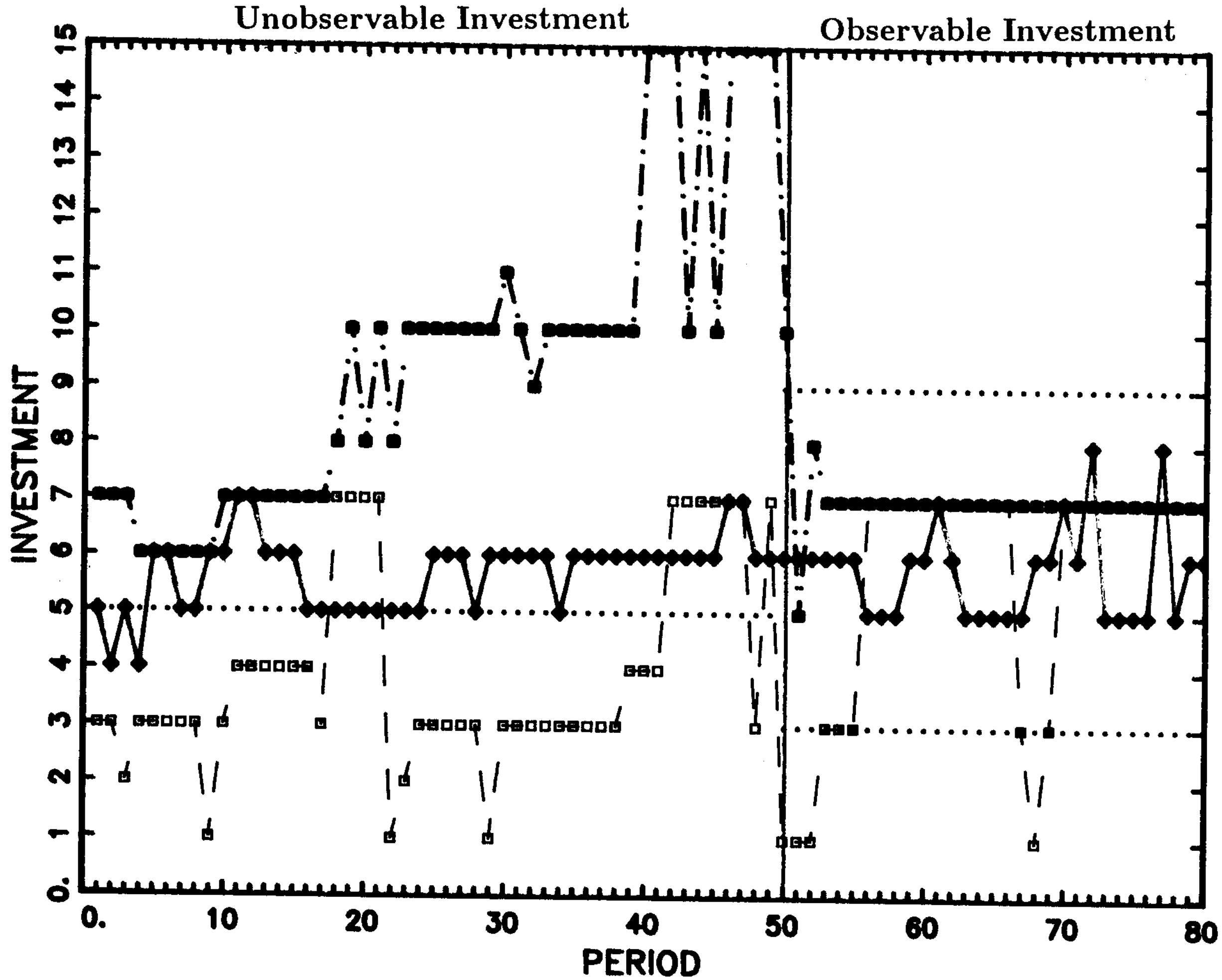

Figure 5.2.E : Experiment 2 - First Price Auction

Individual Investment Decisions in Market 3

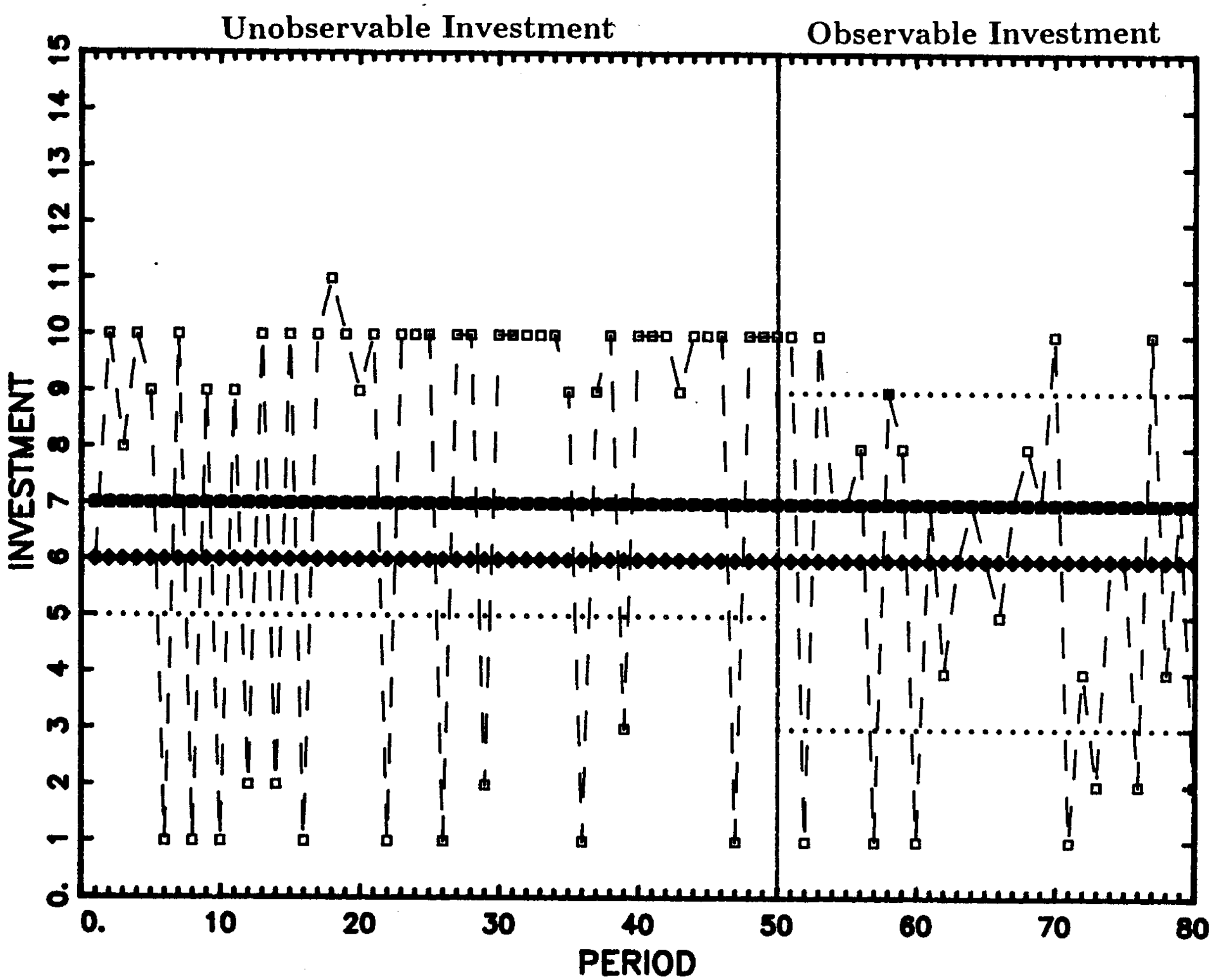

Figure 5.2.F : Experiment 2 - First Price Auction

Individual Investment Decisions in Market 4

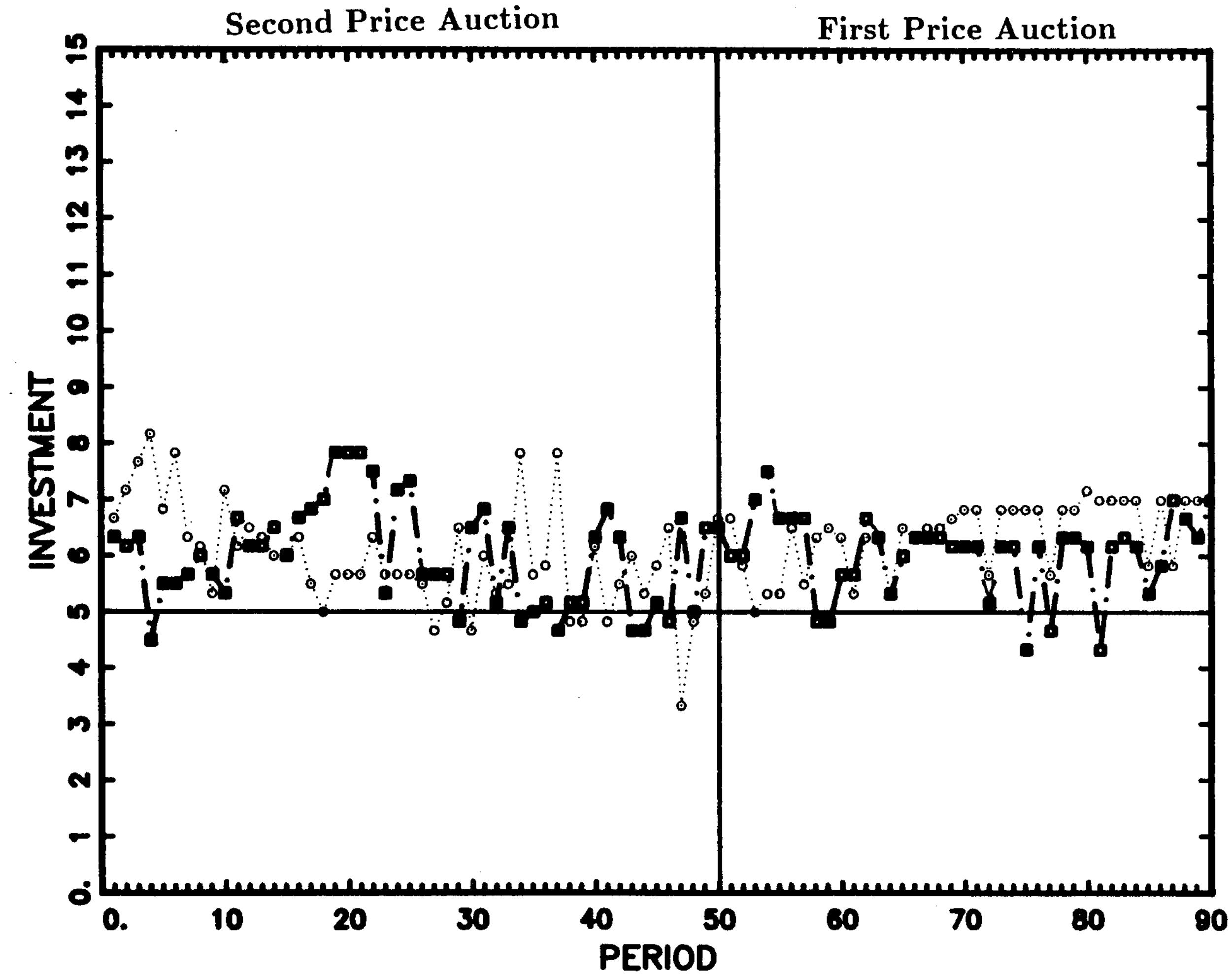

Figure 5.3.A : Experiment 3 - Second and First Price Auctions

Average Investment Level

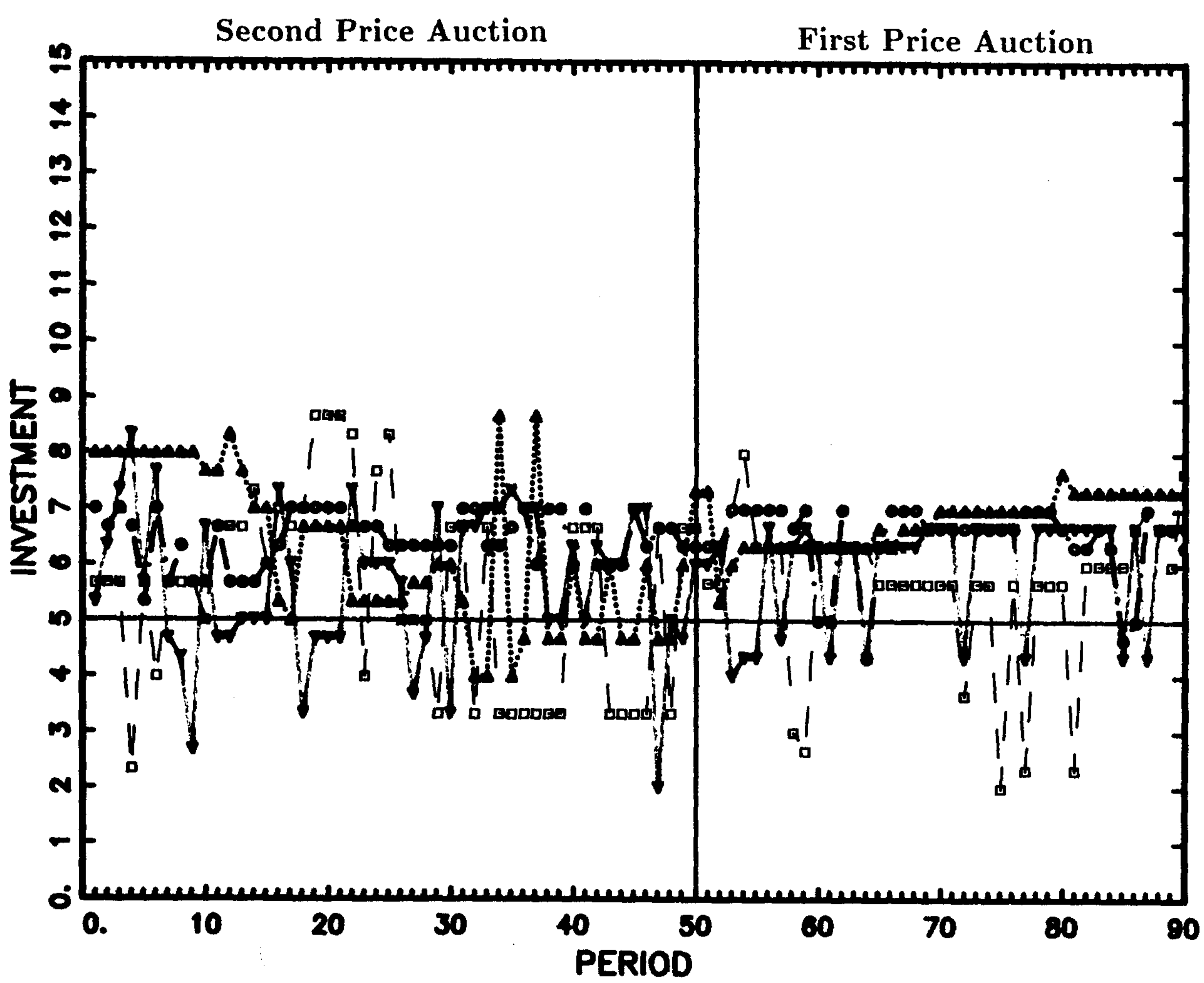

Figure 5.3.B : Experiment 3 - Second and First Price Auctions

Average Investment Level by Market

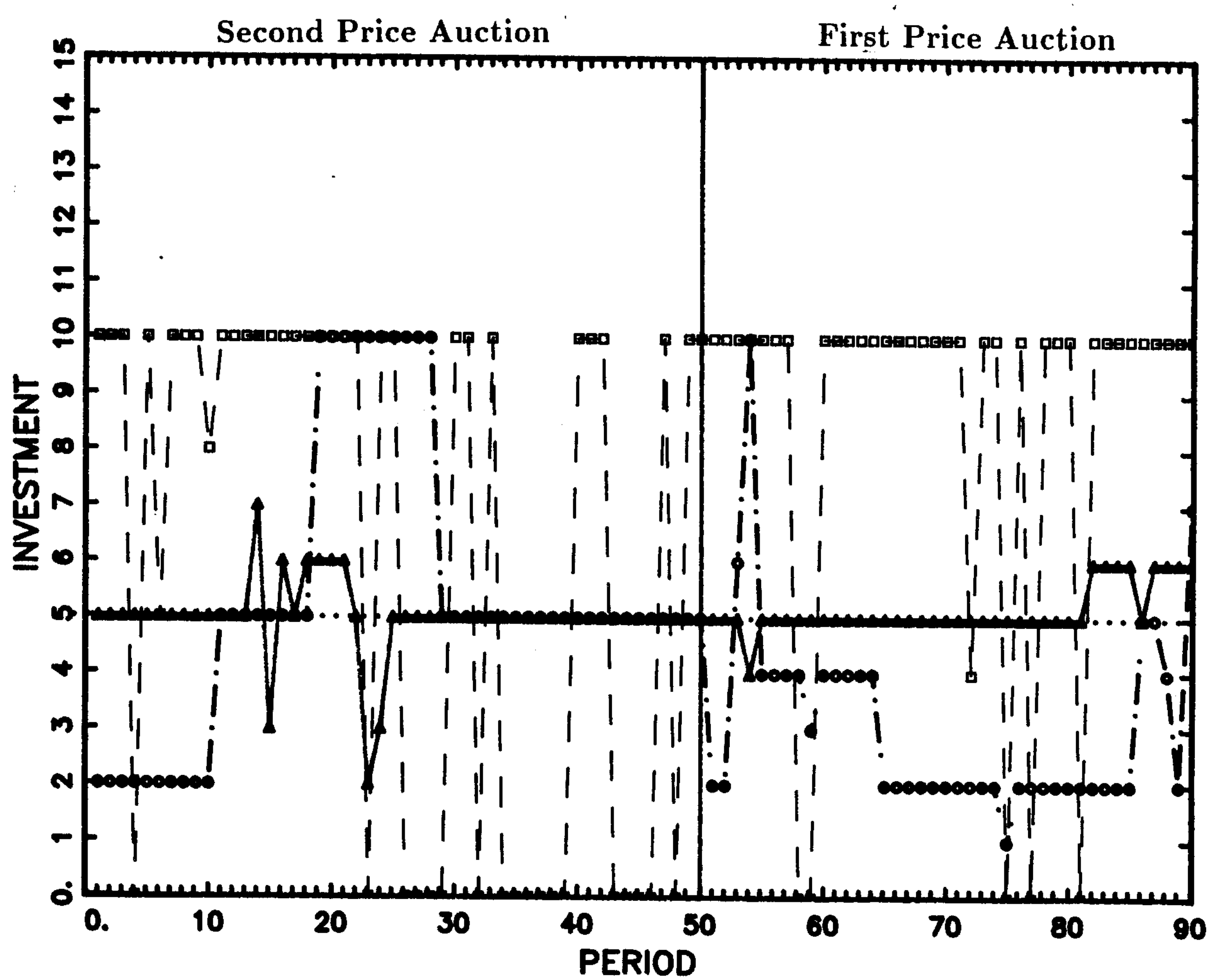

Figure 5.3.C : Experiment 3 - Second and First Price Auctions

Individual Investment Decisions in Market 1

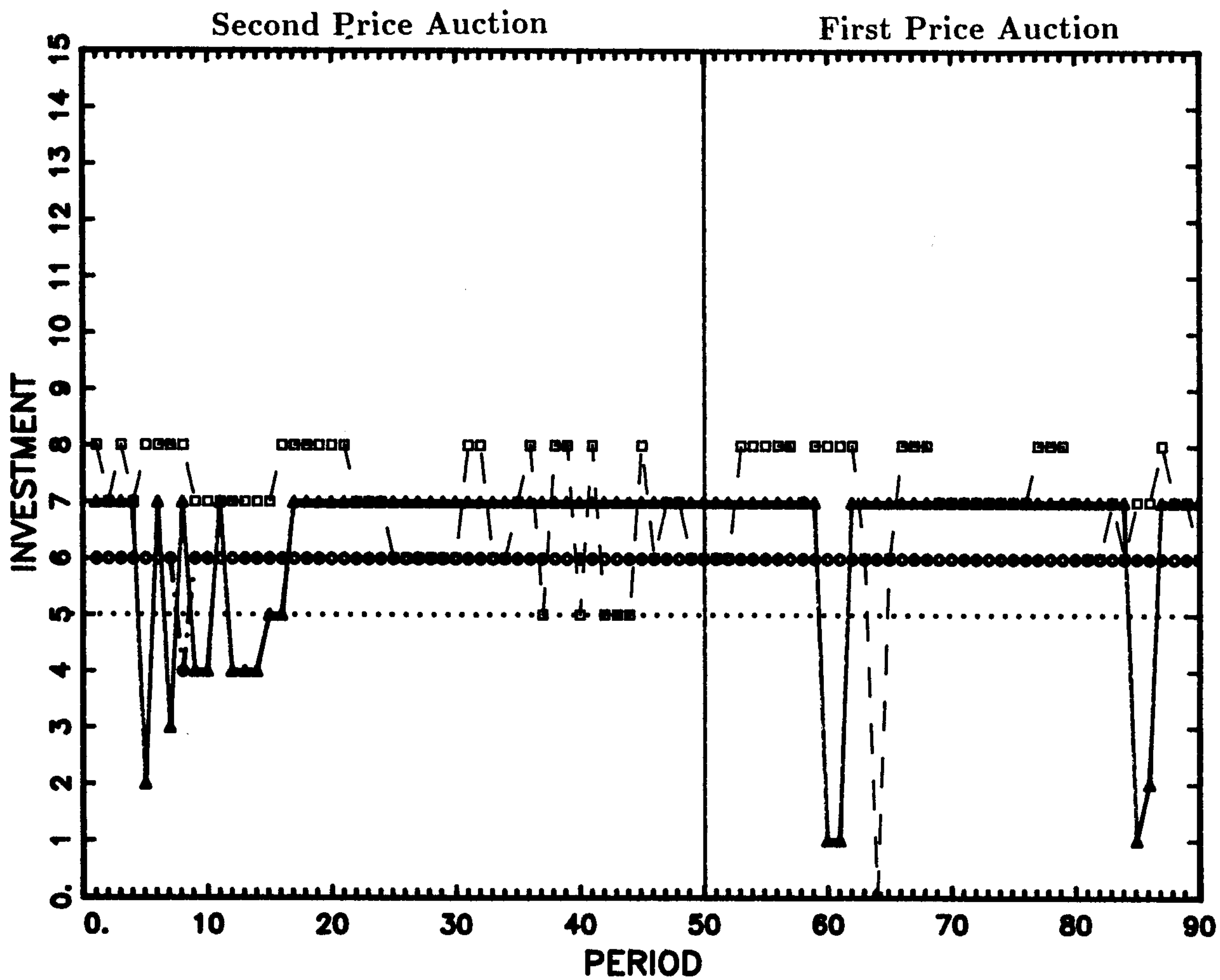

Figure 5.3.D : Experiment 3 - Second and First Price Auctions

Individual Investment Decisions in Market 2

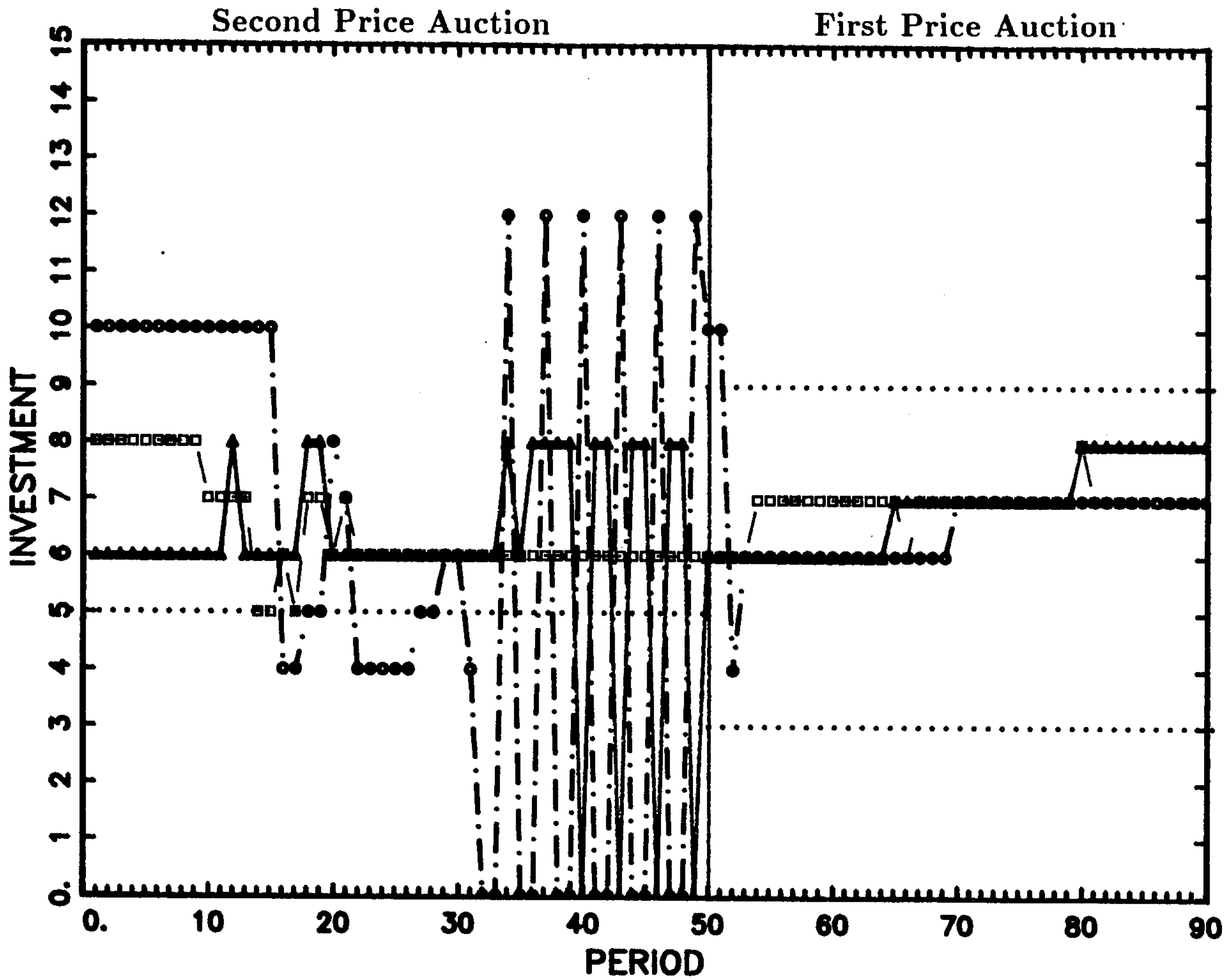

Figure 5.3.E : Experiment 3 - Second and First Price Auctions

Individual Investment Decisions in Market 3

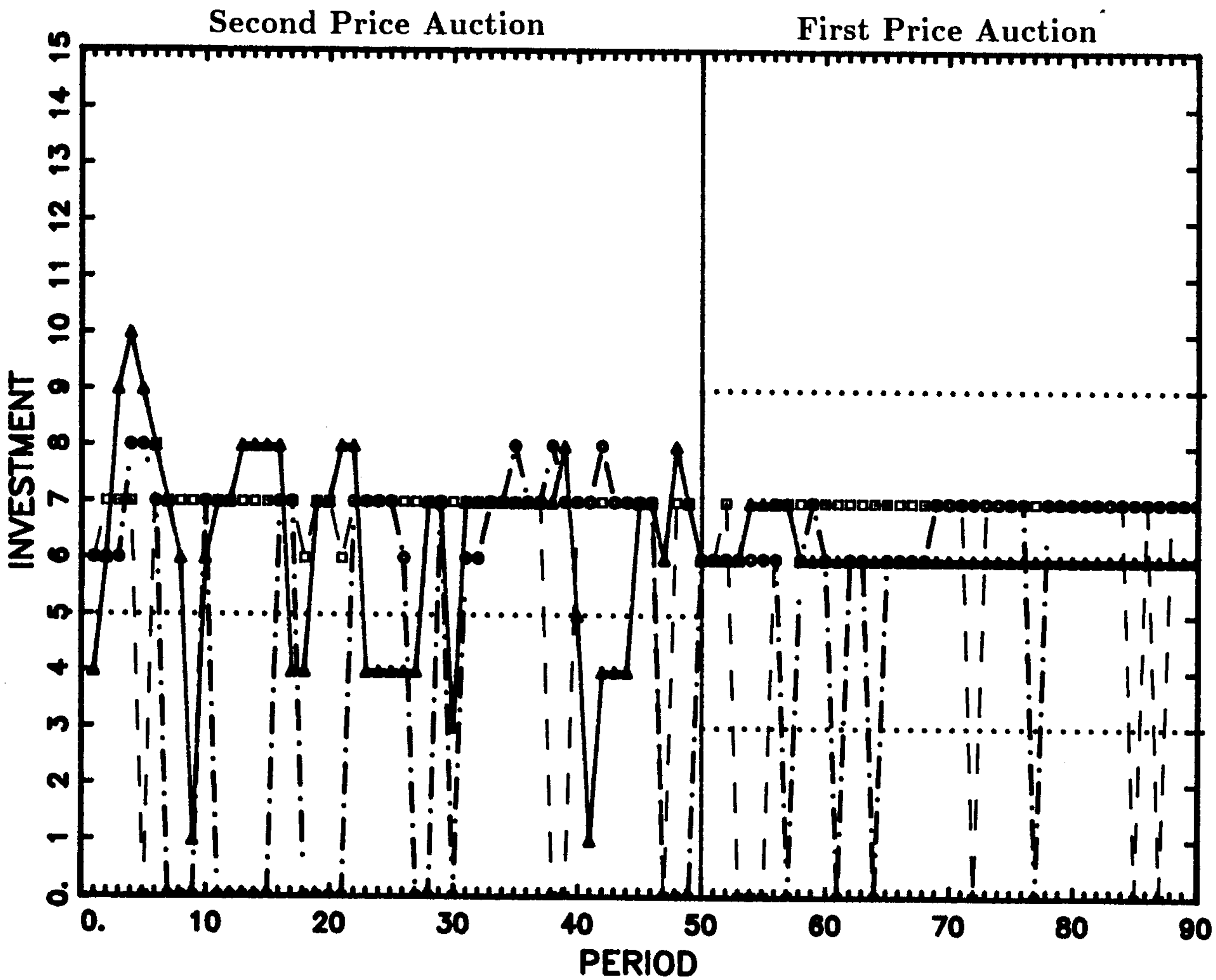

Figure 5.3.F : Experiment 3 - Second and First Price Auctions

Individual Investment Decisions in Market 4

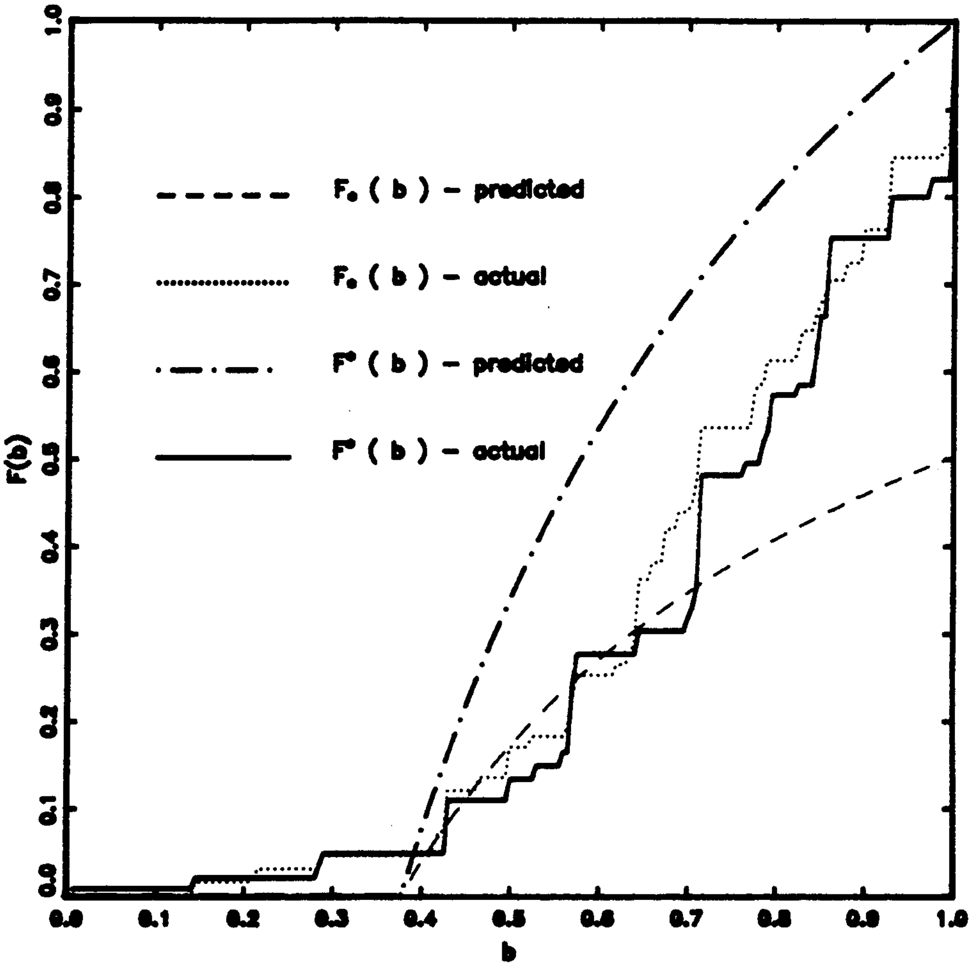

Figure 5.4.A : Experiment 4 - Predicted and Actual Bidding Strategies

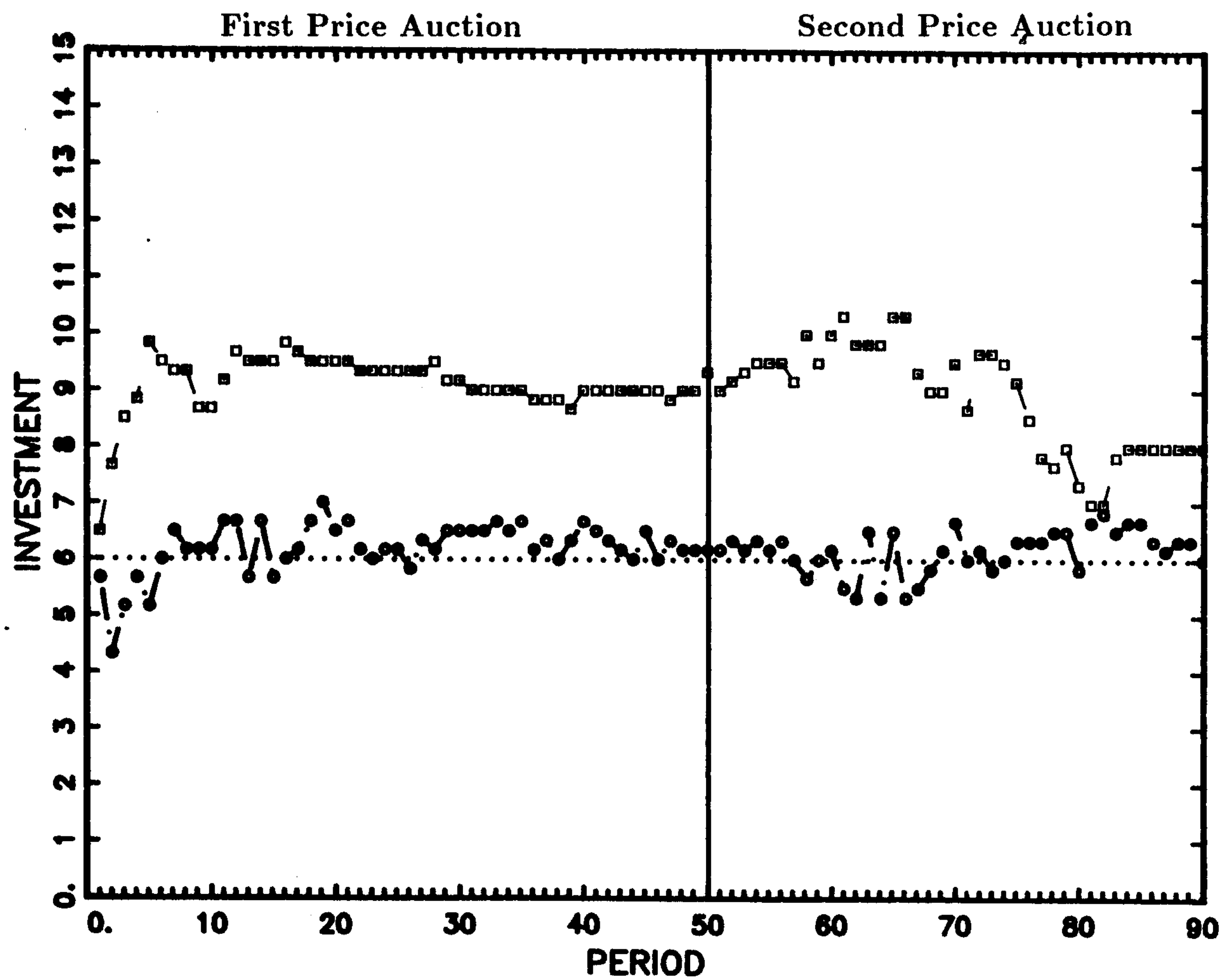

Figure 5.5.A : Experiment 5 - First and Second Price Auctions

Average Investment Level

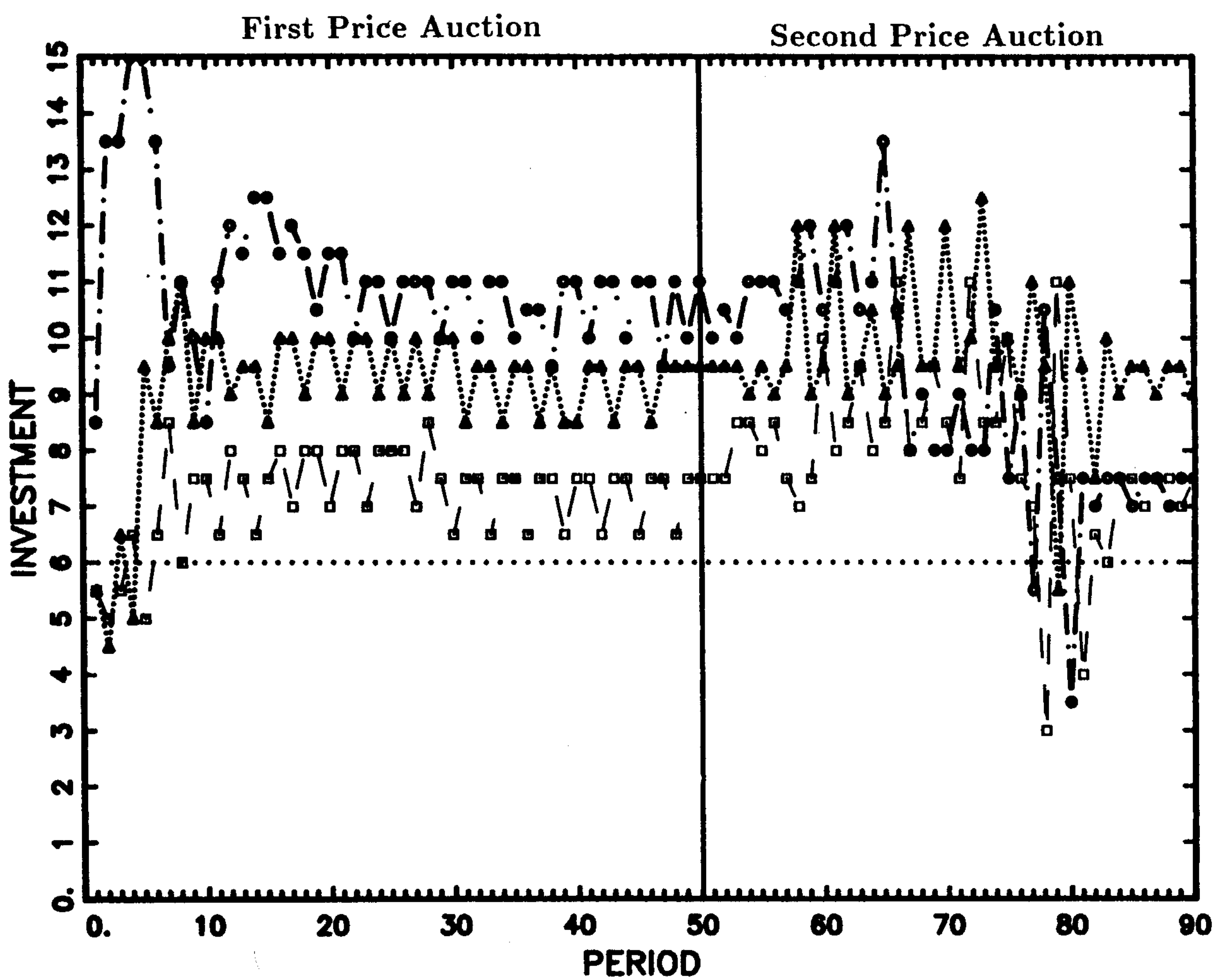

Figure 5.5.B : Experiment 5 - First and Second Price Auctions Average Investment Level in Markets with Unobservable Investment

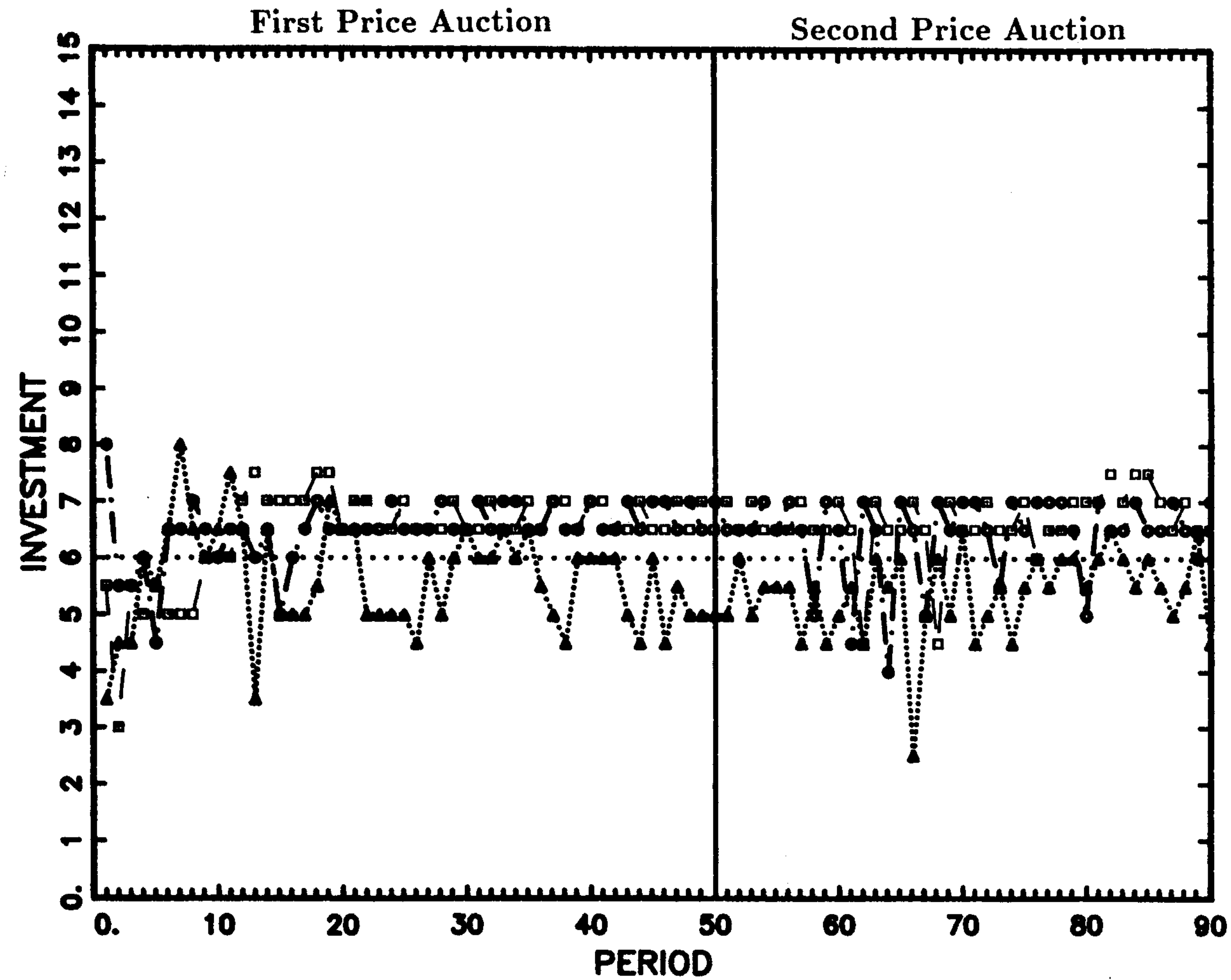

Figure 5.5.C : Experiment 5 - First and Second Price Auctions Average Investment Level in Markets with Observable Investment

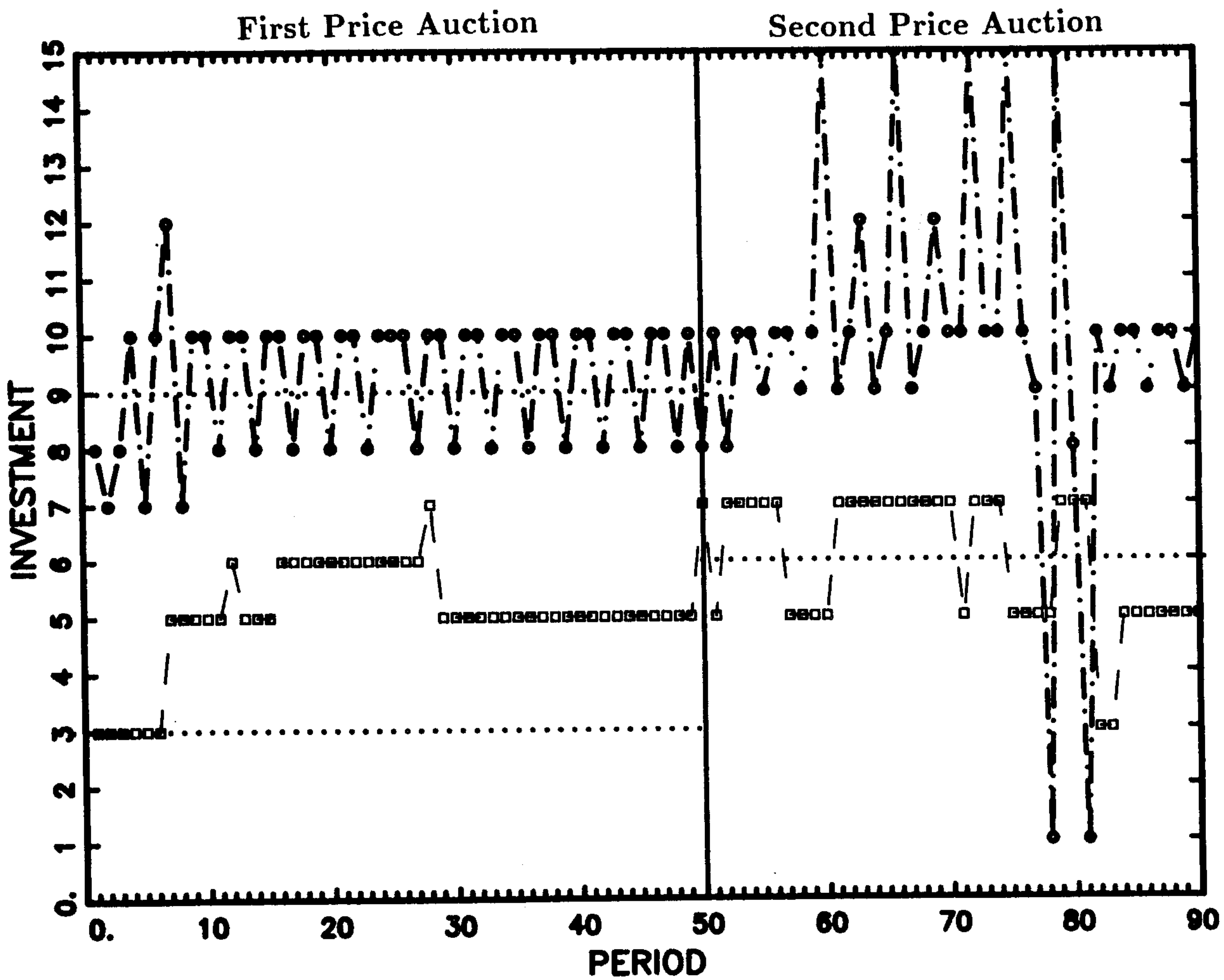

Figure 5.5.D : Experiment 5 - First and Second Price Auctions
Individual Investment Decisions in Market 1

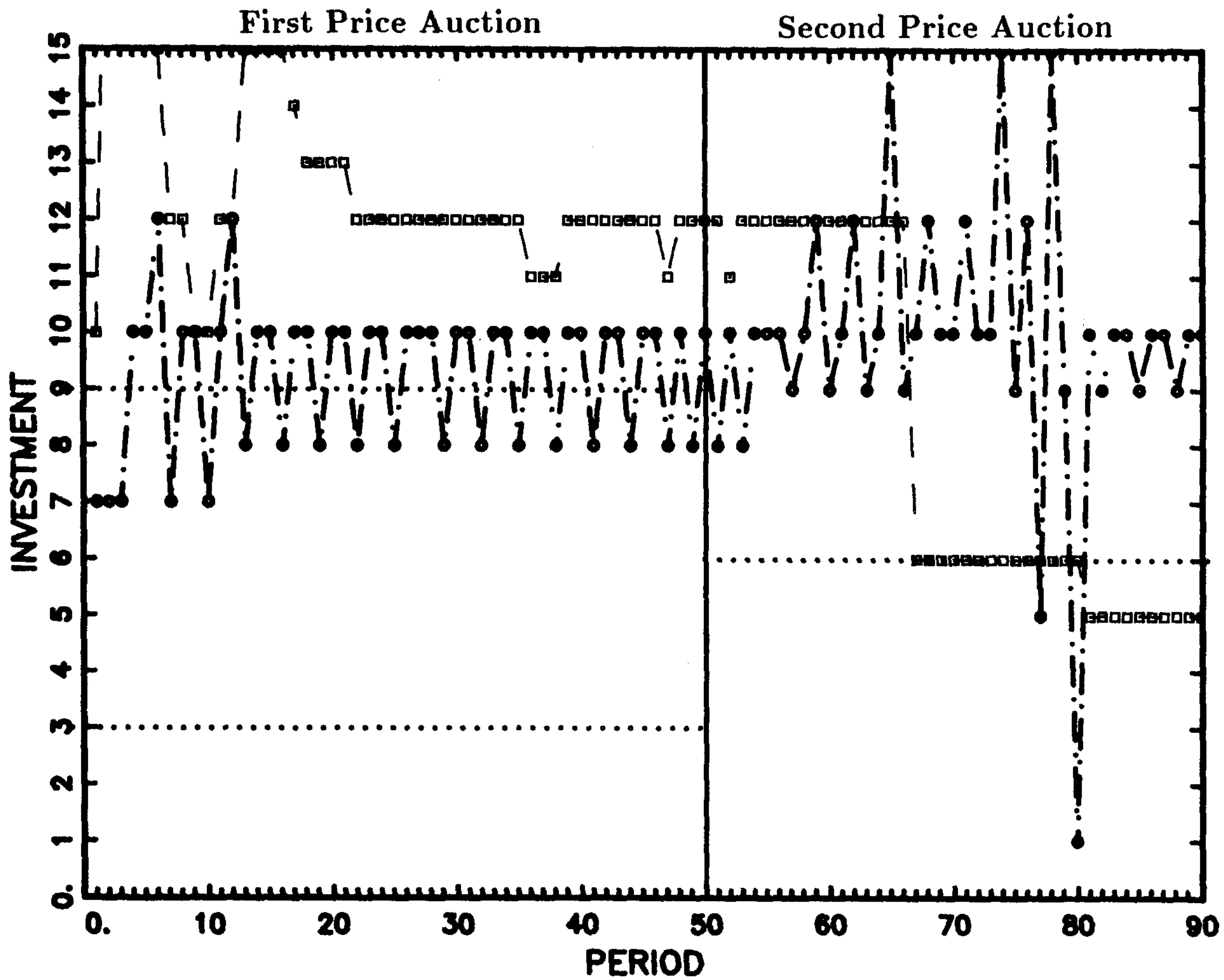

Figure 5.5.E : Experiment 5 - First and Second Price Auctions
Individual Investment Decisions in Market 2

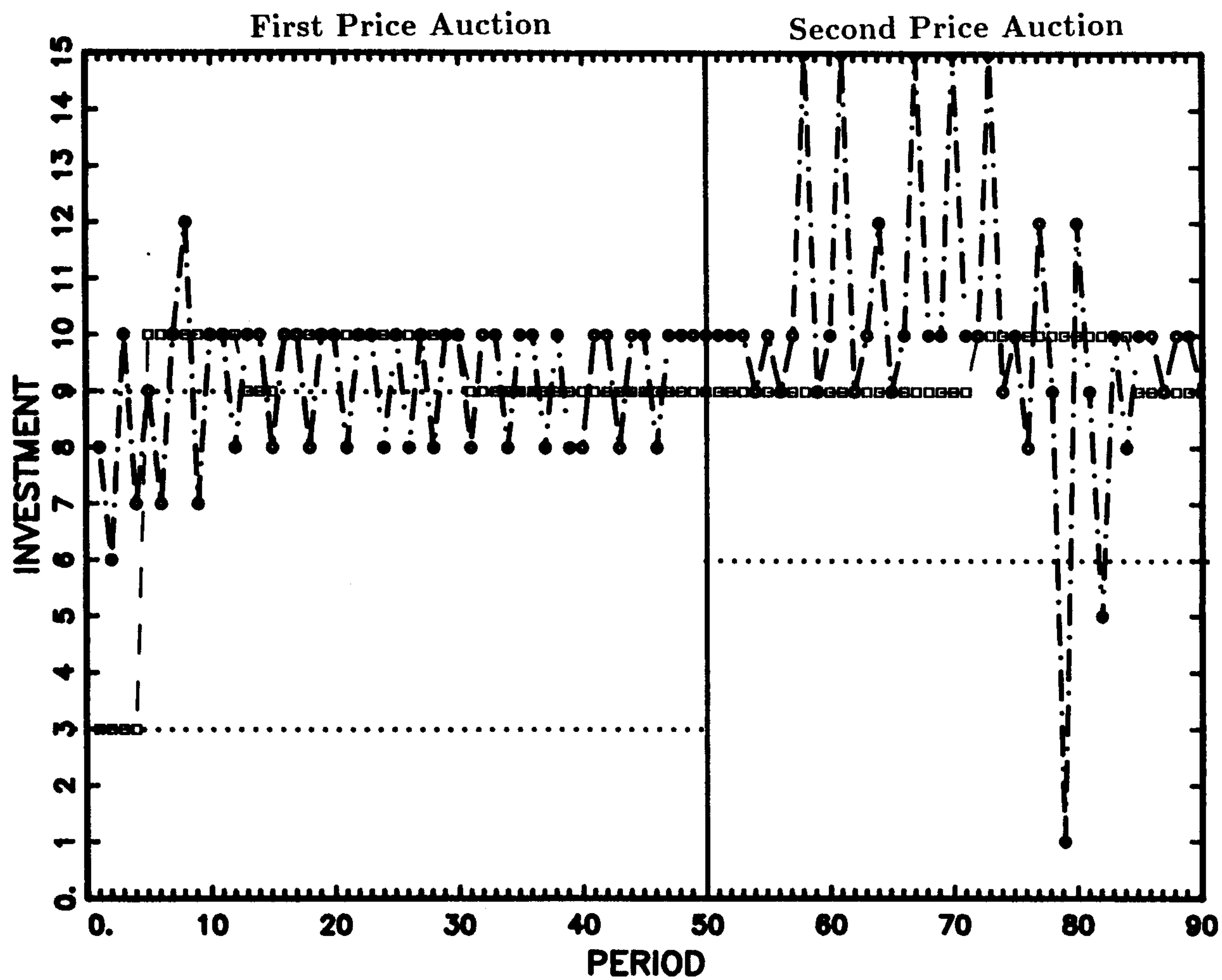

Figure 5.5.F : Experiment 5 - First and Second Price Auctions
Individual Investment Decisions in Market 3

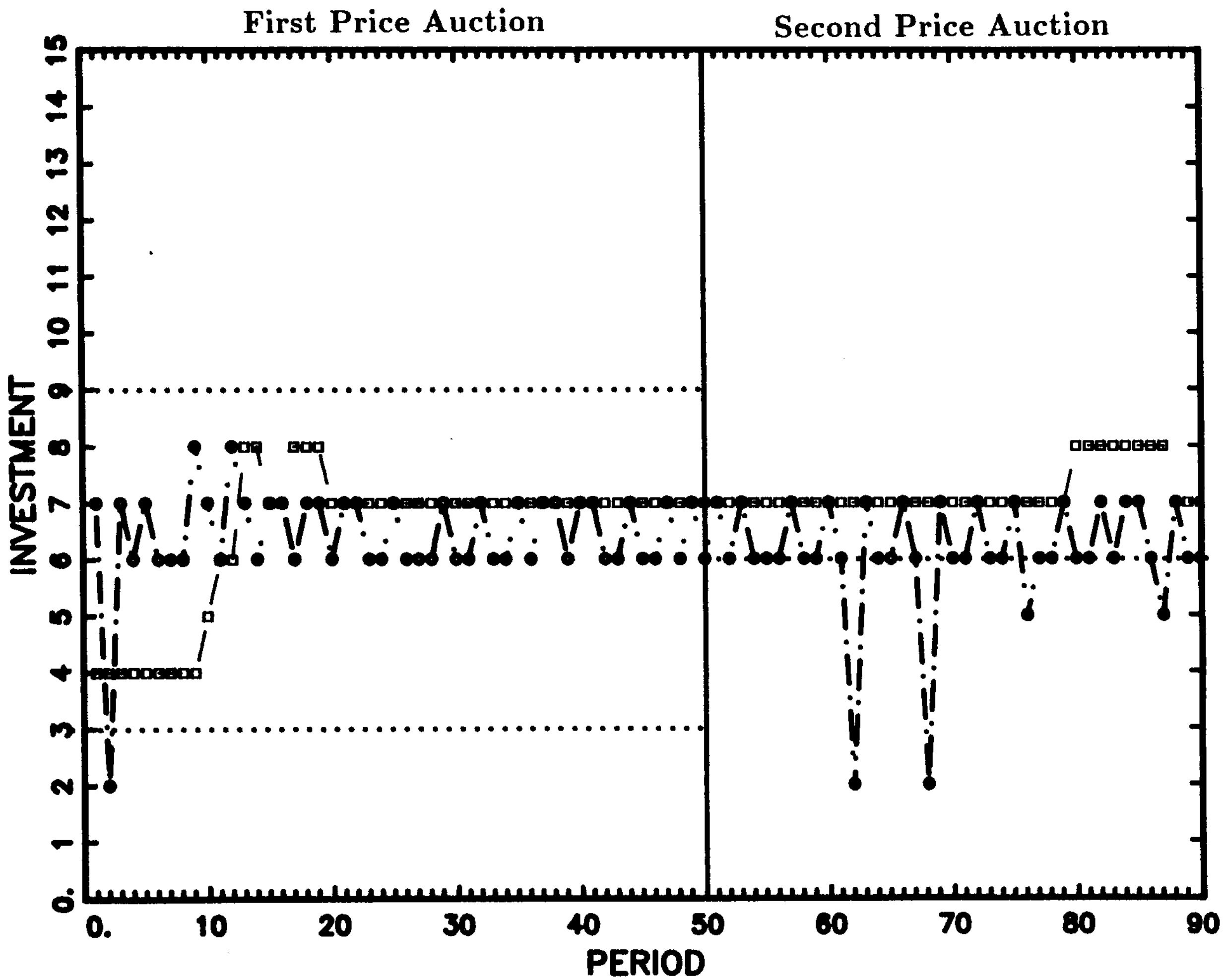

Figure 5.5.G : Experiment 5 - First and Second Price Auctions
Individual Investment Decisions in Market 4

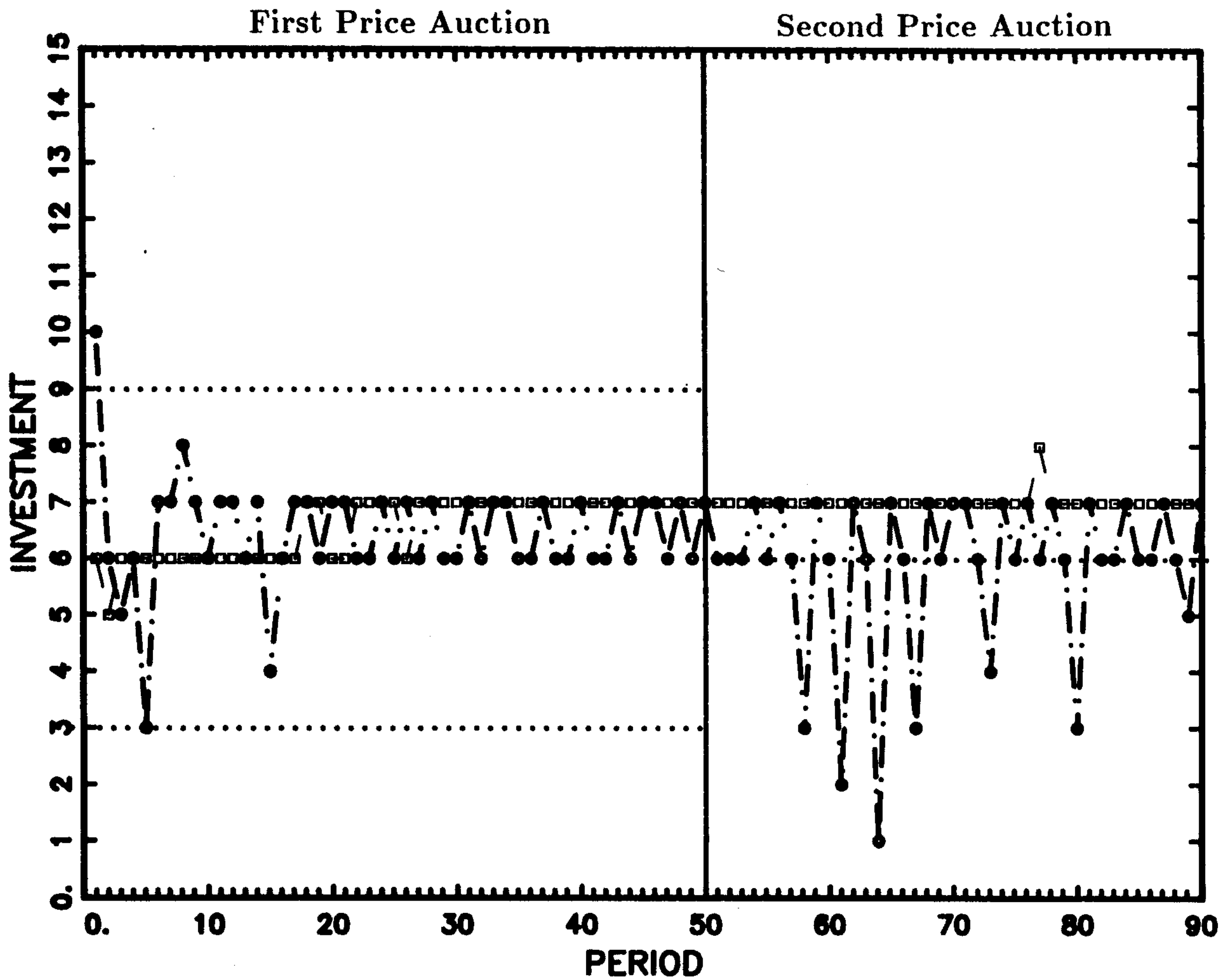

Figure 5.5.H : Experiment 5 - First and Second Price Auctions
Individual Investment Decisions in Market 5

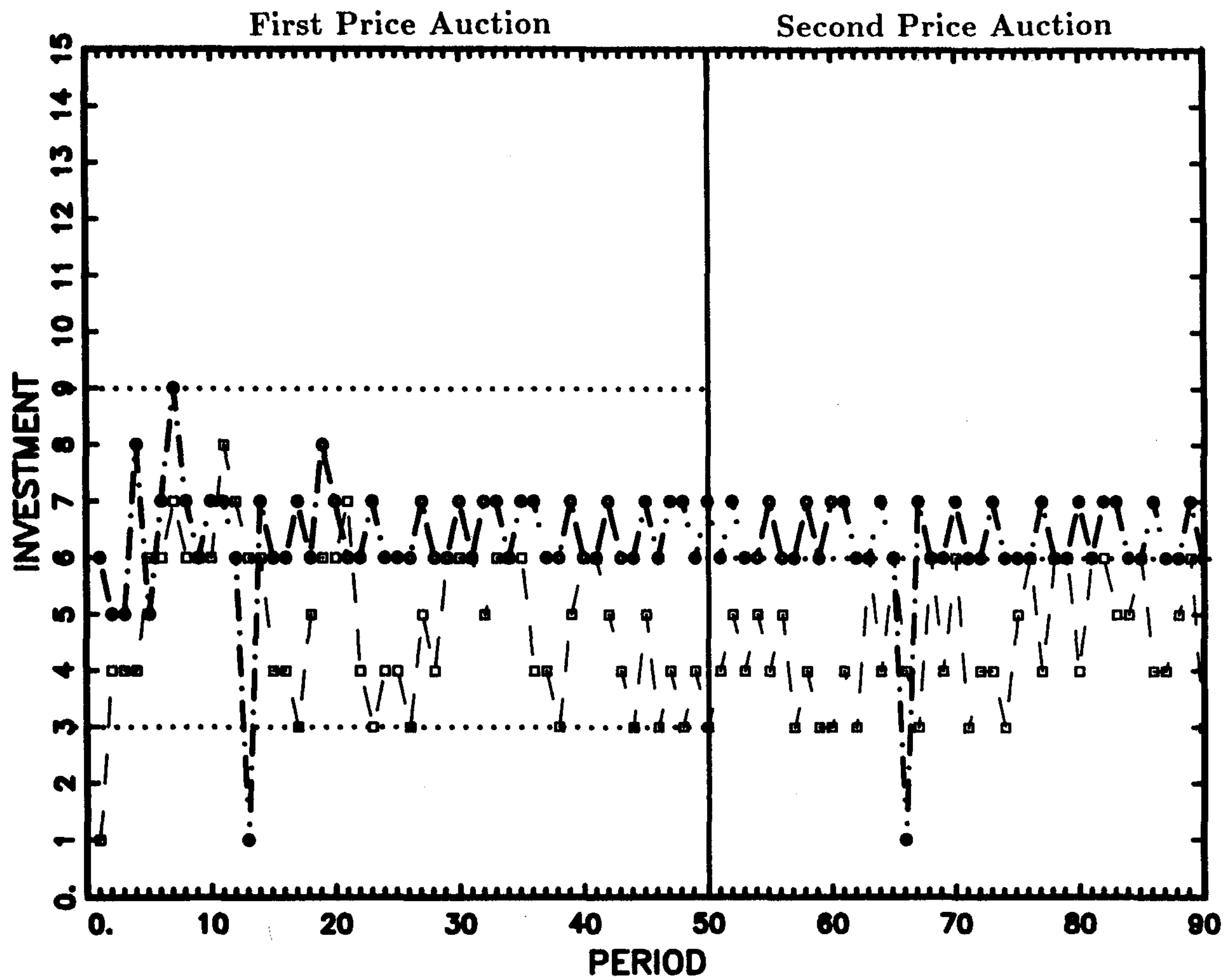

Figure 5.5.I : Experiment 5 - First and Second Price Auctions
Individual Investment Decisions in Market 6

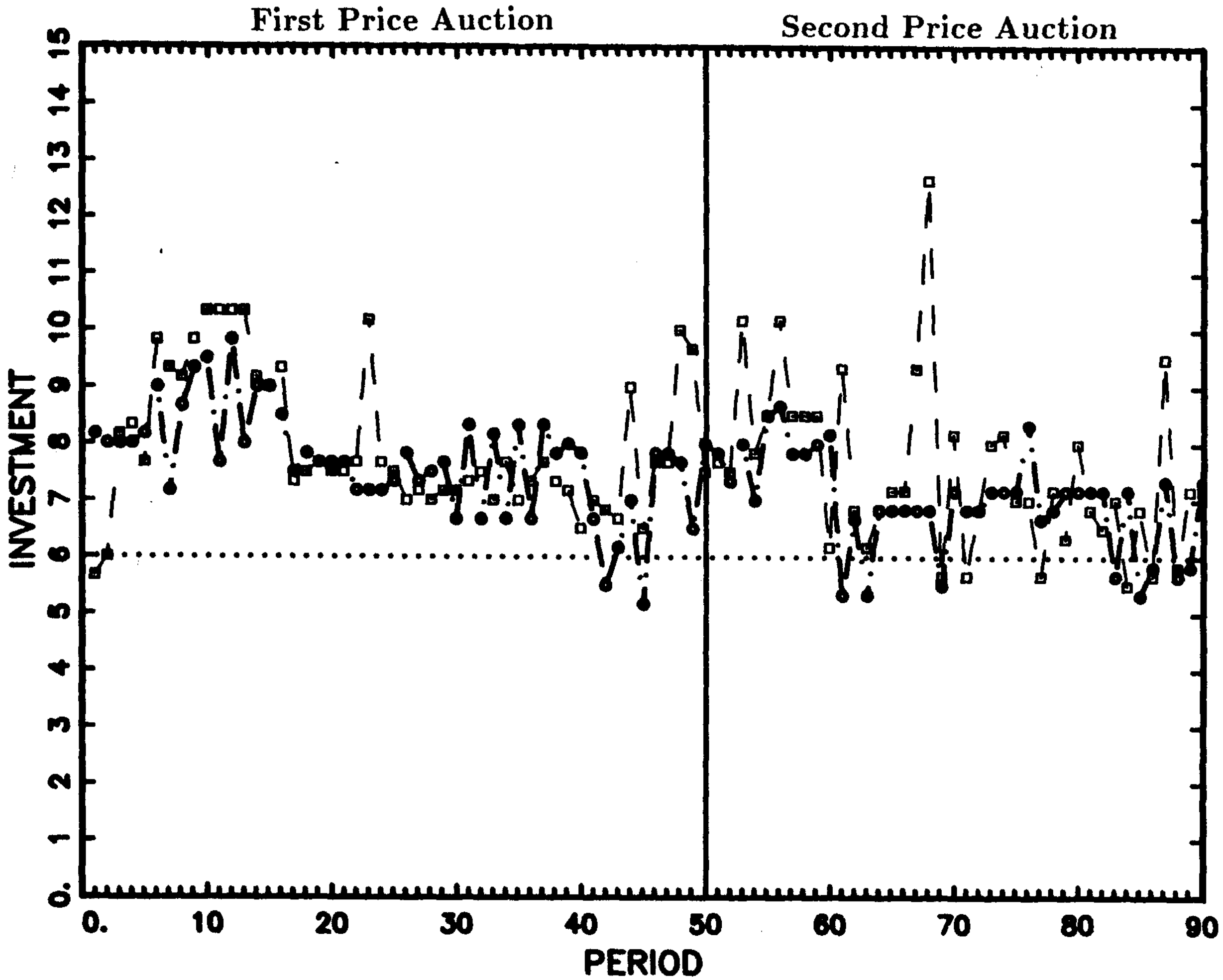

Figure 5.6.A : Experiment 6 - First and Second Price Auctions
Average Investment Level

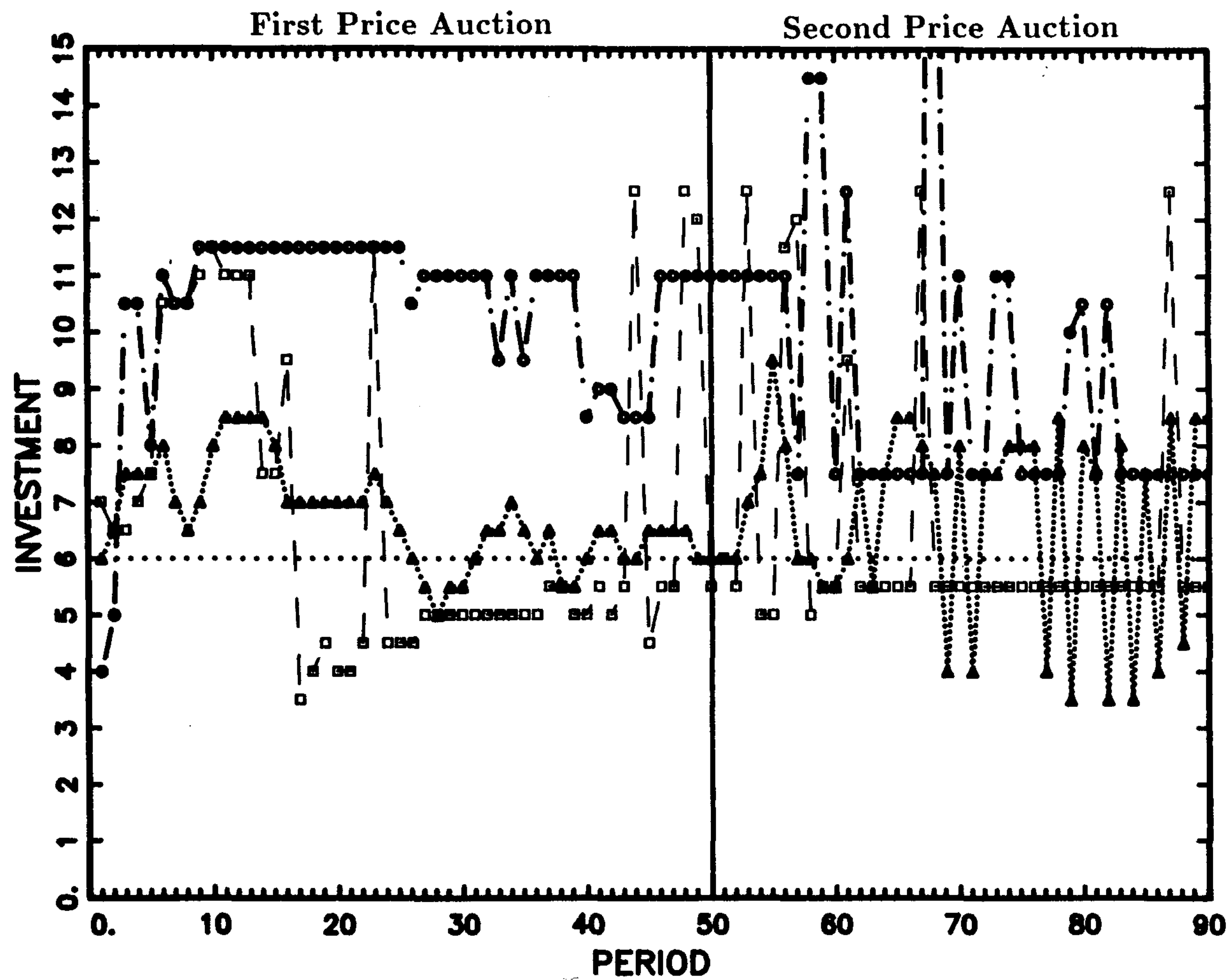

Figure 5.6.B : Experiment 6 - First and Second Price Auctions
Average Investment Level in Markets with Unobservable Investment

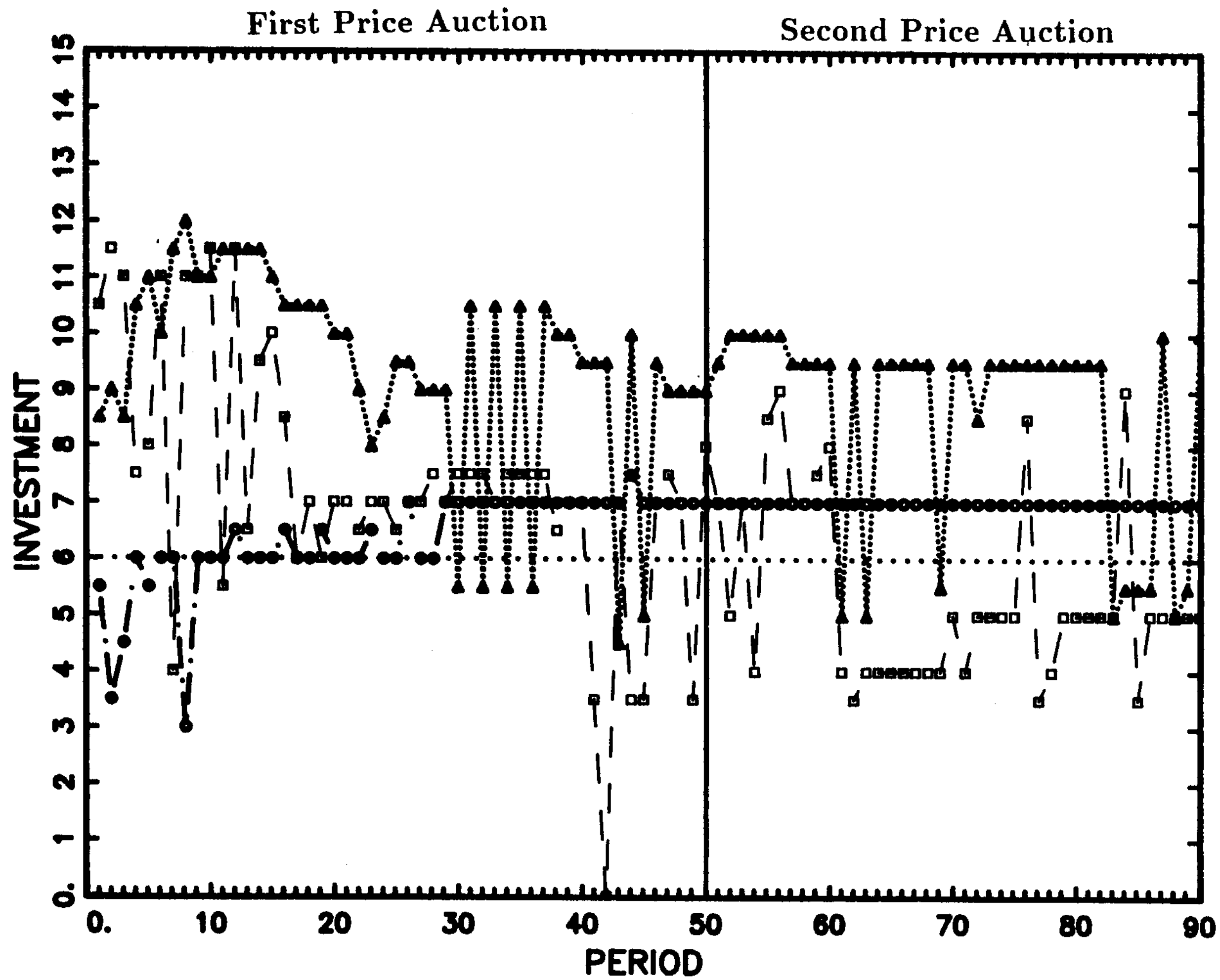

Figure 5.6.C : Experiment 6 - First and Second Price Auctions Average Investment Level in Markets with Observable Investment

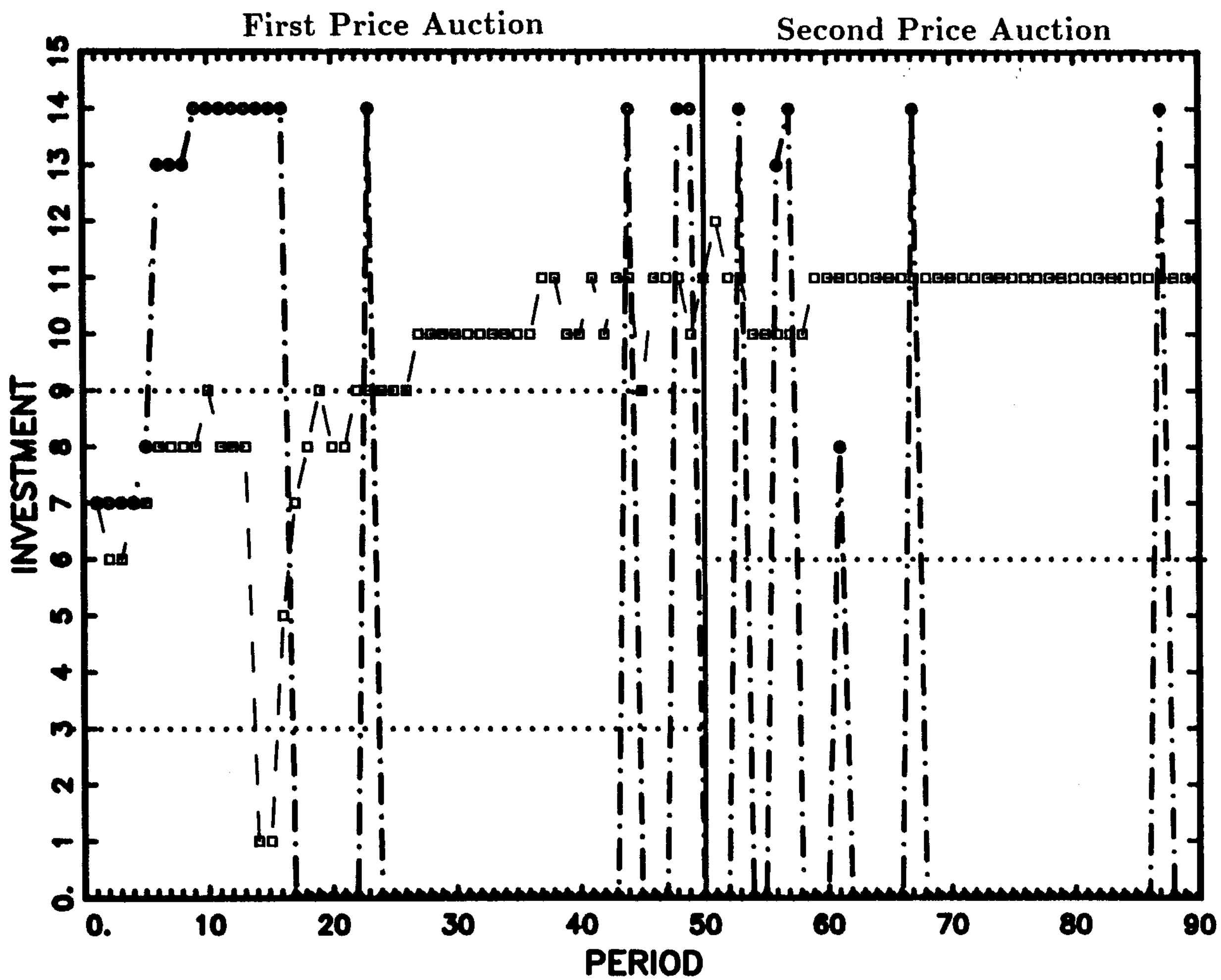

Figure 5.6.D : Experiment 6 - First and Second Price Auctions
Individual Investment Decisions in Market 1

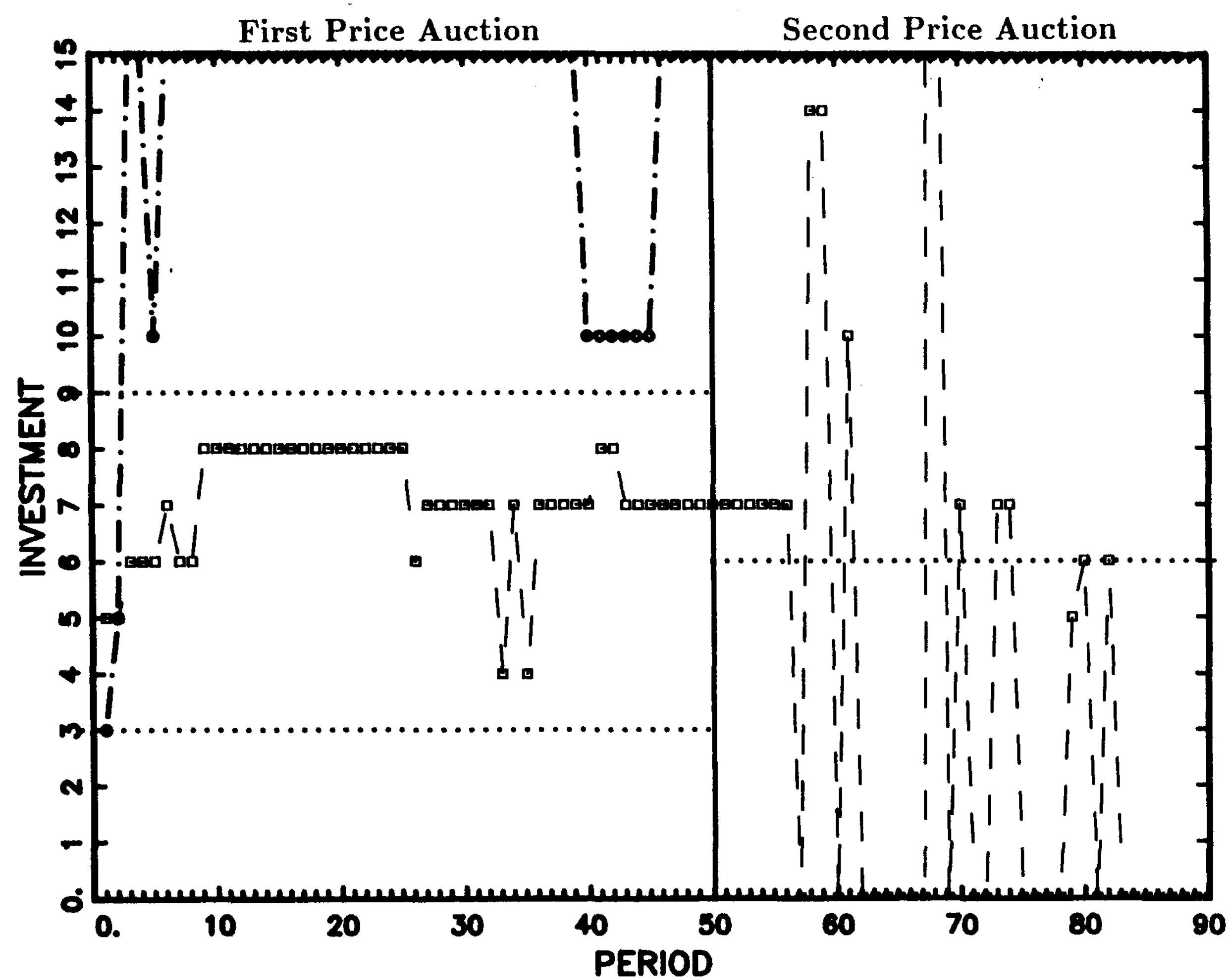

Figure 5.6.E : Experiment 6 - First and Second Price Auctions
Individual Investment Decisions in Market 2

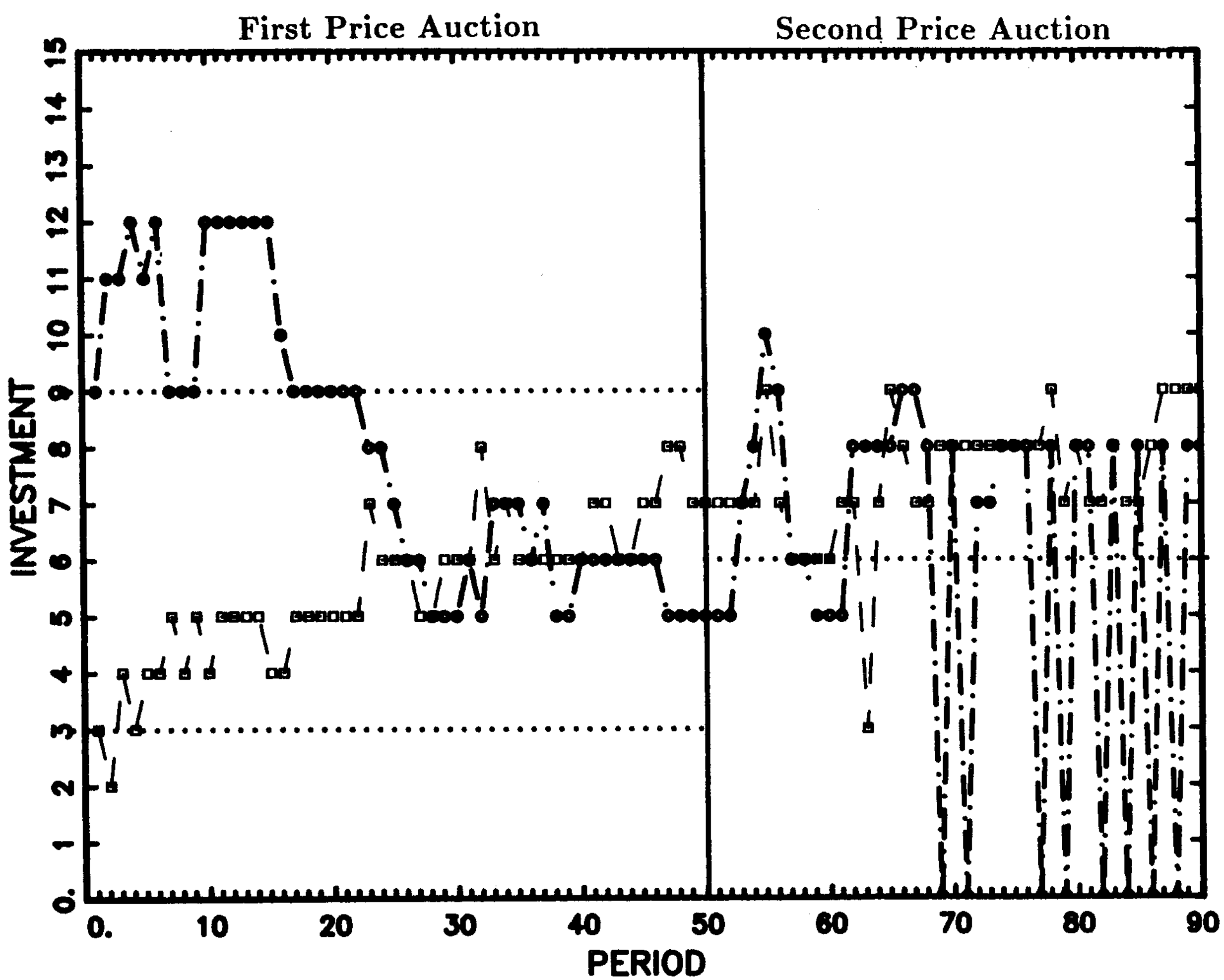

Figure 5.6.F : Experiment 6 - First and Second Price Auctions
Individual Investment Decisions in Market 3

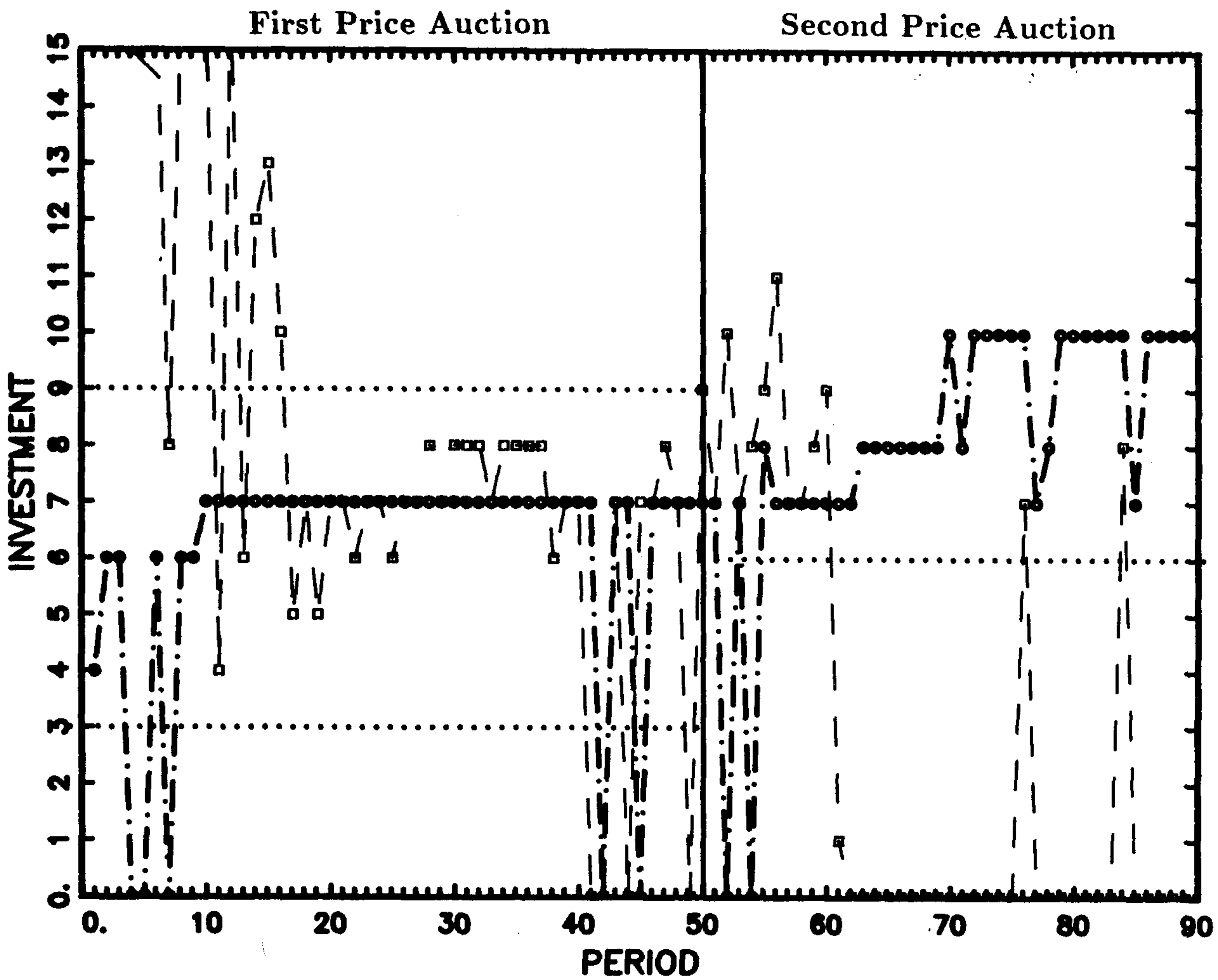

Figure 5.6.G : Experiment 6 - First and Second Price Auctions
Individual Investment Decisions in Market 4

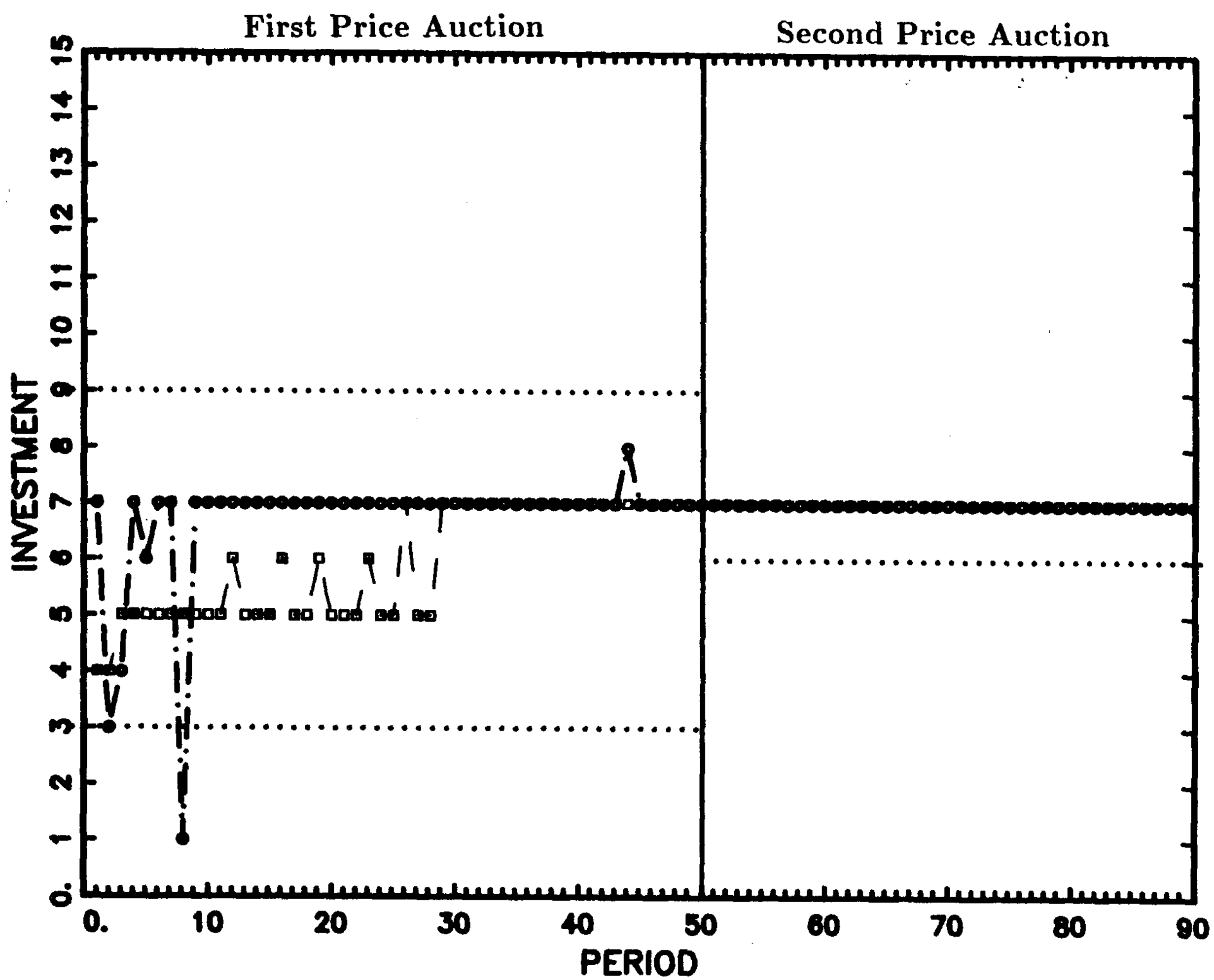

Figure 5.6.H : Experiment 6 - First and Second Price Auctions
Individual Investment Decisions in Market 5

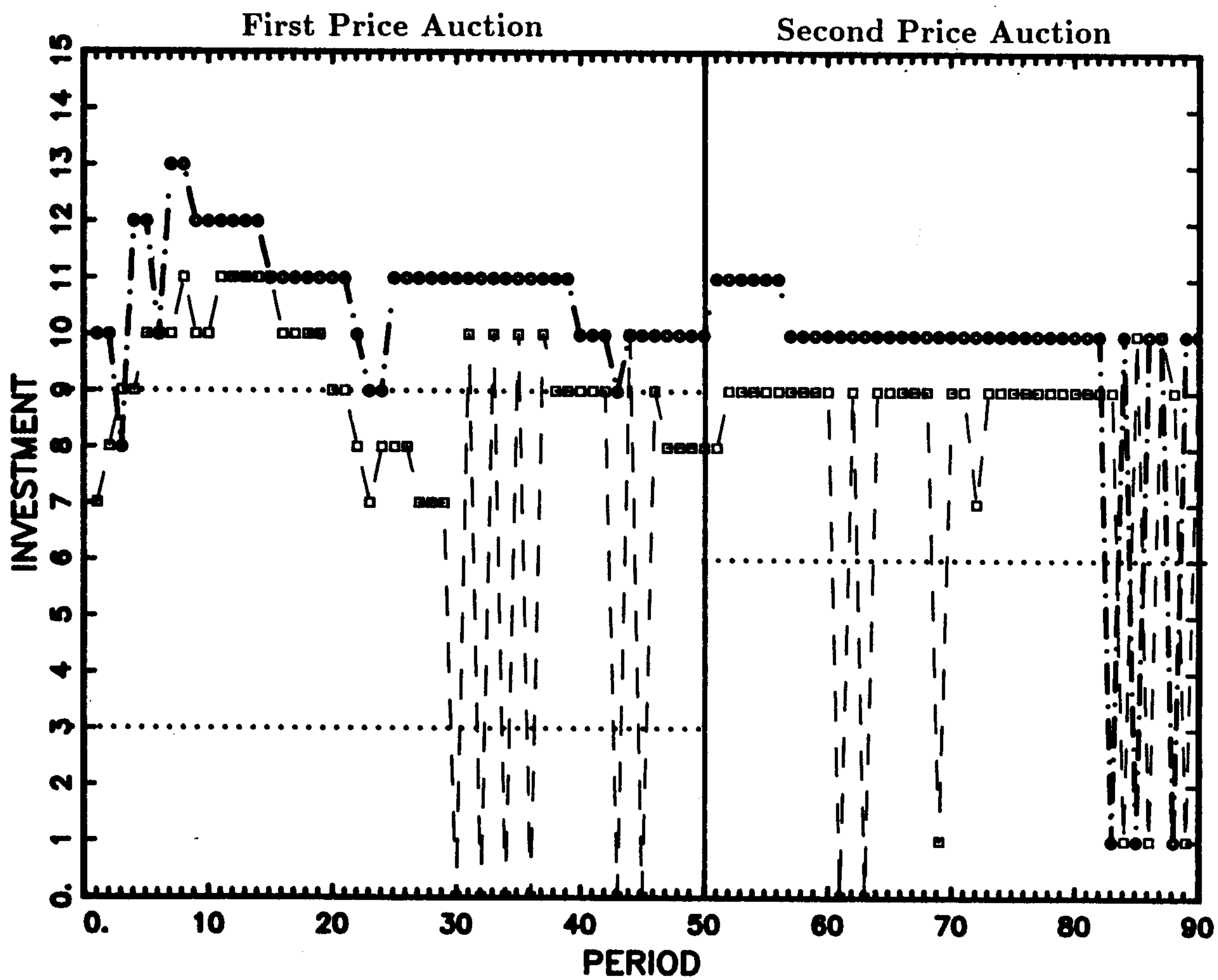

Figure 5.6.I : Experiment 6 - First and Second Price Auctions
Individual Investment Decisions in Market 6

Experiment	Prediction	Mean	Std. Dev.	# Obs.	t
1. FP / U	15	19.850	4.928	200	0.894
1. FP / O	15	17.133	4.085	120	0.522
2. FP / U	15	17.365	4.853	200	0.487
2. FP / O	15	18.092	3.839	120	0.805
3. FP / U	15	18.150	3.519	80	0.895
3. FP / O	15	19.200	2.636	80	1.593
3. SP / U	15	18.010	4.098	100	0.734
3. SP / O	15	17.880	4.233	100	0.680
5. FP / U	12	18.193	3.942	150	1.571
5. FP / O	12	12.393	1.760	150	0.223
5. SP / U	12	17.791	3.569	120	1.623
5. SP / O	12	12.317	1.782	120	0.178
6. FP / U	12	15.993	5.159	150	0.774
6. FP / O	12	15.413	4.364	150	0.782
6. SP / U	12	15.033	5.685	120	0.534
6. SP / O	12	13.958	3.888	120	0.504

Table 5.3: Experimental Results : Aggregate Investment.

Although the aggregate predictions based on the model are largely borne out by the observations, we need to use some test based on individual decisions because, at the level of market aggregates, the two auctions and the two observability treatments are indistinguishable at the pure strategy equilibria. Further tests on the observed distributions of investment under the two information treatments and the two auctions reveal that the model is not entirely accurate in explaining the observations.

In Table 5.4 we list the set of hypotheses tested using Kolmogoroff-Smirnoff statistics of the observed frequencies. In the table $G^{ab}(z)$ stands for the distribution of the variate z under treatment ab, where a is the auction type, and b is investment observability treatment. X is the aggregate investment in a market, and x is individual investment.

Hypotheses H^1 and H^5 state that investment observability does not affect the distribution of aggregate or individual investment levels under the second price auction. Hypotheses H^2 and H^6 claim that the same is true for the first price auction. Hypothesis H^3 states that with respect to the aggregate investment distributions they induce, first and second price auctions are equivalent if investment is unobservable. Hypothesis H^4 makes the same statement when investment is observable. Hypotheses H^7 and H^8 claim the corresponding equivalences with respect to individual investment levels.

With the parameters used in experiments 1, 2 and 3, the model predicts that the only pure strategy equilibria in the first price auction with investment observability are such that one seller invests 9 units and two sellers invest 3 units each. In experiment 4 we exogenously assigned cost distributions to three sellers in each market so that the probability of having the high production cost is r^9 for one seller and r^3 for the remaining two sellers. $F_*(\cdot)$ in Table 5.4 is the observed cumulative frequency of bids conditional on low production cost by two sellers whose cost distributions are given exogenously to be the same as those of sellers with low investment, and $F^*(\cdot)$ is the observed cumulative frequency of bids by sellers whose production costs are distributed more favorably. Finally, $F(\cdot, 9, 3, 3)$ is the equilibrium mixed bidding strategy used by the seller whose production cost is determined by making 9 draws from the given bingo cage with $r = 0.85$ and whose rivals make 3 draws each. Similarly, $F(\cdot, 3, 3, 9)$ is the equilibrium mixed bidding strategy used by a seller who makes 3 draws from the cost distribution r against rivals who make 3 and 9 draws. Hypotheses H^9 and H^{10} claim that the observed bids are generated by the equilibrium mixed bidding strategies.

Hypothesis	Explanation
H^1	$G^{su}(X) = G^{so}(X)$
H^2	$G^{fu}(X) = G^{fo}(X)$
H^3	$G^{su}(X) = G^{fu}(X)$
H^4	$G^{so}(X) = G^{fo}(X)$
H^5	$G^{su}(x) = G^{so}(x)$
H^6	$G^{fu}(x) = G^{fo}(x)$
H^7	$G^{su}(x) = G^{fu}(x)$
H^8	$G^{so}(x) = G^{fo}(x)$
H^9	$F_*(b) = F(b; 3, 3, 9)$
H^{10}	$F^*(b) = F(b; 9, 3, 3)$
H^{11}	$F_*(b) = F^*(b)$

Table 5.4: Hypotheses .

Finally, hypothesis H^{11} states that the the distribution of bids conditional on a low production cost is the same for both types of sellers.

The Kolmogoroff-Smirnoff statistic is used to compare the empirical distributions of the related variables. Table 5.5 contains the Kolmogoroff-Smirnoff statistics corresponding to the hypotheses in Table 5.4 for experiments 1 through 6. The empty entries in the table indicate that the corresponding hypothesis is irrelevant for the given experiment. Table 5.6 is taken from Sachs [62] . It contains the bounds for the Kolmogoroff-Smirnoff statistic corresponding to several significance levels. If the observed value of the Kolmogoroff-Smirnoff statistics exceeds the tabulated bound for a given significance level, we reject the null hypothesis that the two sets of observations are generated by the same underlying distribution at the corresponding significance level.

The statistics reported in Table 5.5 for the set of hypotheses H^1 through H^8 reveal a mixed picture for the related equivalence hypotheses. Table

Hypothesis	Exp. 1	Exp. 2	Exp. 3	Exp.4	Exp. 5	Exp. 6
H^1	–	–	1.202	–	2.453	2.776
H^2	1.025	0.924	1.186	–	2.483	1.327
H^3	–	–	0.867	–	0.721	1.796
H^4	–	–	1.550	–	0.463	1.306
H^5	–	–	2.898	–	4.291	3.195
H^6	1.575	1.908	3.560	–	4.817	3.470
H^7	–	–	1.453	–	1.722	2.021
H^8	–	–	2.906	–	0.548	1.385
H^9	–	–	–	3.395	–	–
H^{10}	–	–	–	4.075	–	–
H^{11}	–	–	–	0.819	–	–

Table 5.5: Experimental Results : Kolmogoroff-Smirnoff Statistics .

Significance Level	.20	.15	.10	.05	.01	.001
Bound for KS	1.073	1.138	1.224	1.358	1.628	1.949

Table 5.6: Bounds for Kolmogoroff-Smirnoff Statistic .

Hypothesis	Exp. 1	Exp. 2	Exp. 3	Exp.4	Exp. 5	Exp. 6
H^1	–	–		–	*	*
H^2				–	*	
H^3	–	–		–		*
H^4	–	–	*	–		
H^5	–	–	*	–	*	*
H^6	*	*	*	–	*	*
H^7	–	–	*	–	*	*
H^8	–	–	–	*		–
H^9	–	–	–	*	–	–
H^{10}	–	–	–	*	–	–
H^{11}	–	–	–		–	–

Table 5.7: Experimental Results : Hypotheses rejected at the 5% significance level.

5.7 gives a list of the hypotheses that cannot be rejected at the 5% significance level. In all experiments, the statistics indicate that the hypotheses H^5,H^6 and H^7 can be rejected at the 5% significance level. In the first three experiments, the pattern of rejected hypotheses is largely in line with the expectations based on the model. On the basis of the model, we expect that the distributions of aggregate investment levels under the two auctions and observability treatments will be the same. This expectation is contradicted only in the case of hypothesis H^4, indicating that the two auctions give rise to different aggregate investment distributions under investment observability. In the last two experiments, the evidence for the respective aggregate hypotheses is reversed.

With respect to the distribution of individual investment levels, despite our having multiple investment equilibria, we do not expect investment observability to affect the distribution of individual investment levels under second price auctions under the maintained hypothesis that the observations are independent samples from a given distribution. This expectation is contradicted by the experimental observations. This may be an indication of the existence of repeated game factors. The fact that the distribution of individual investment levels differs significantly with respect to auction type under both observability treatments can be interpreted as a vague support for the equilibrium predictions of the model. The general pattern of rejected hypotheses in the experiments with two sellers seems to contradict the expectations based on the model.

The test statistics for hypotheses H^9 and H^{10} indicate that the observed bid distributions differ significantly from the equilibrium predictions of the model even when the sellers are exogenously forced to play the equilibrium strategies in the investment game. The result on hypothesis H^{11} comes as a real surprise: Despite the fact that the distributions from which the production costs are drawn are very different, we cannot reject the hypothesis that bidding behavior is symmetric.

In Figures 5.8 and 5.9 we present the observations on individual investment levels in experiments 5 and 6. The vertical axis in the figures measures the maximum of the investment levels by two sellers in a market, and the horizantal axis measures the minimum. The solid line joining the point (0,12) and the point (6,6) gives the set of pure strategy equilibria under the second price auction. The point (6,6) is the unique symmetric pure strategy equilibrium under the first price auction with unobservable investment. The pure strategy equilibrium under the first price auction with observable investment is given by the point (3,9). Under the first price auction with observable investment, investment pairs inside the triangle given by the dotted lines and the forty-five degree line represent the undominated strategy pairs. Finally, if the two sellers colluded by coordinating their investment strategies perfectly, then we would observe investment pairs at point (0,12). Figures 5.7.A through 5.7.C present the equilibrium predictions.

Figure 5.8.A and Figure 5.8.B exhibit the distribution of investment pairs under the first price auction and the second price auction, respectively, in experiment 5. Figures 5.9.A and 5.9.B exhibit the same information from experiment 6. In all figures, the panels on the left show the observed investment pairs under unobservable investment treatment, and the panels on the right show the observed investment pairs when investment is observable. In all figures, the modal observation is shown by black shading. The numbers in the cells represent the number of observations.

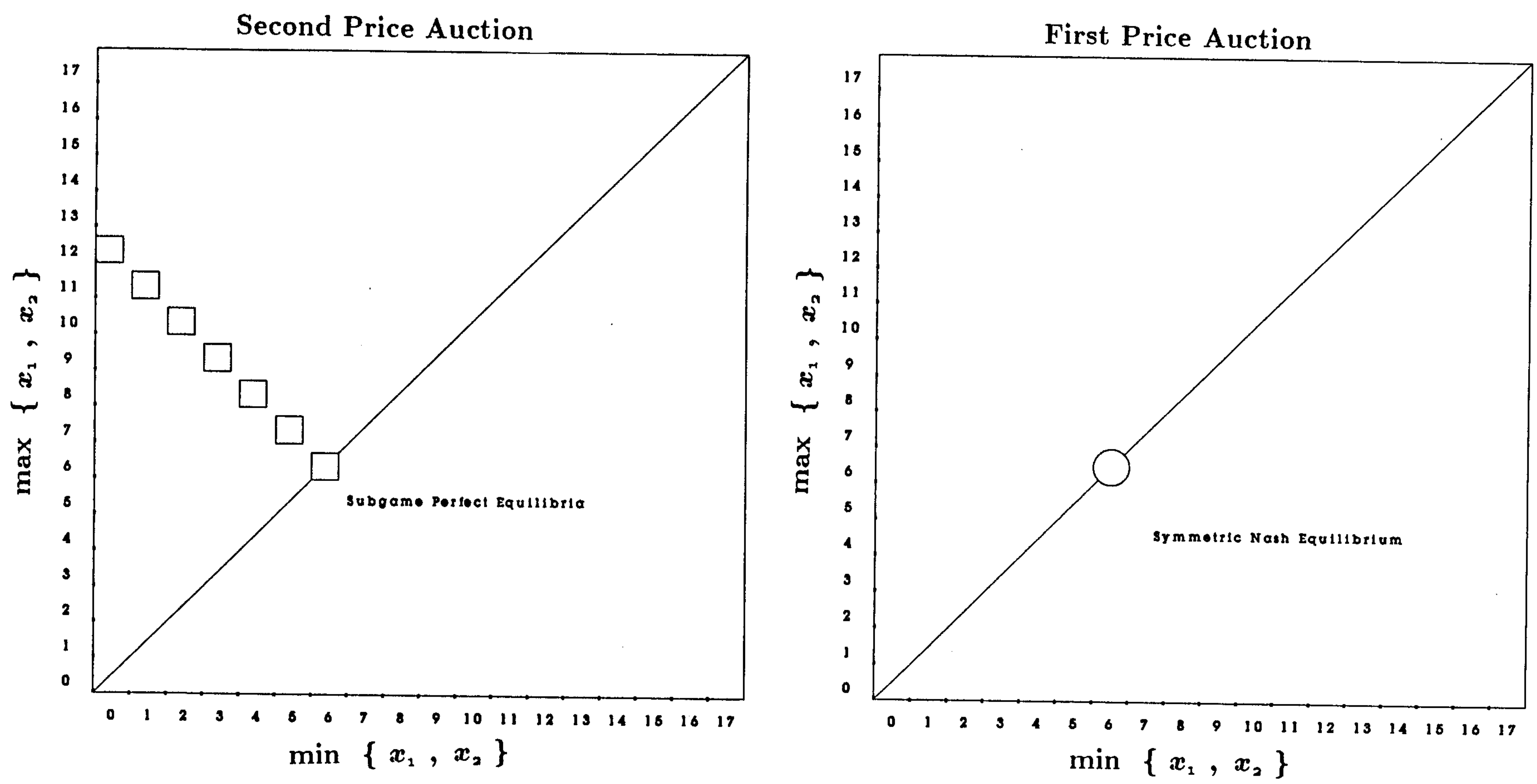

Figure 5.7.A : Equilibrium Predictions for Experiments 5 and 6. (Unobservable Investment)

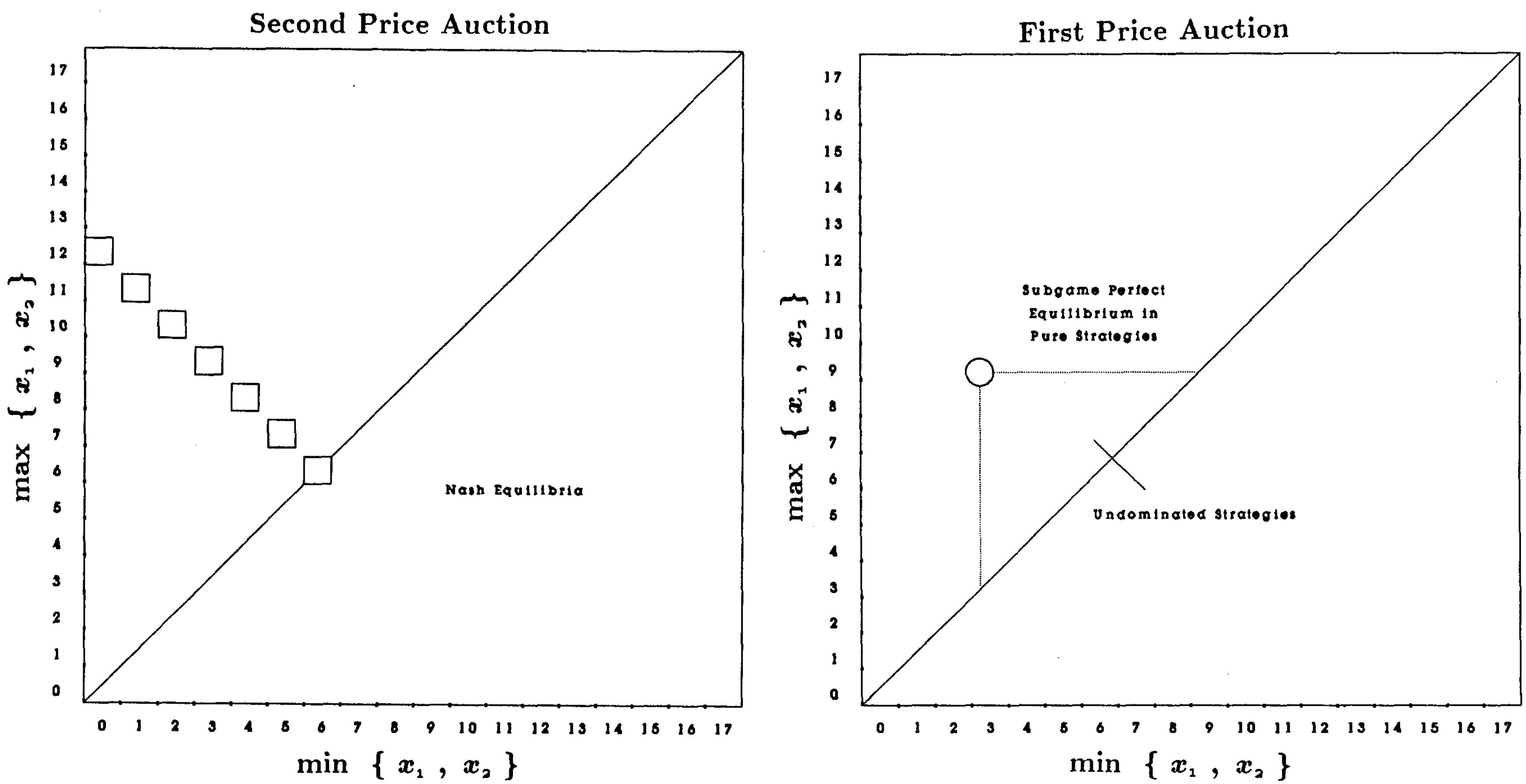

Figure 5.7.B : Equilibrium Predictions for Experiments 5 and 6. (Observable Investment)

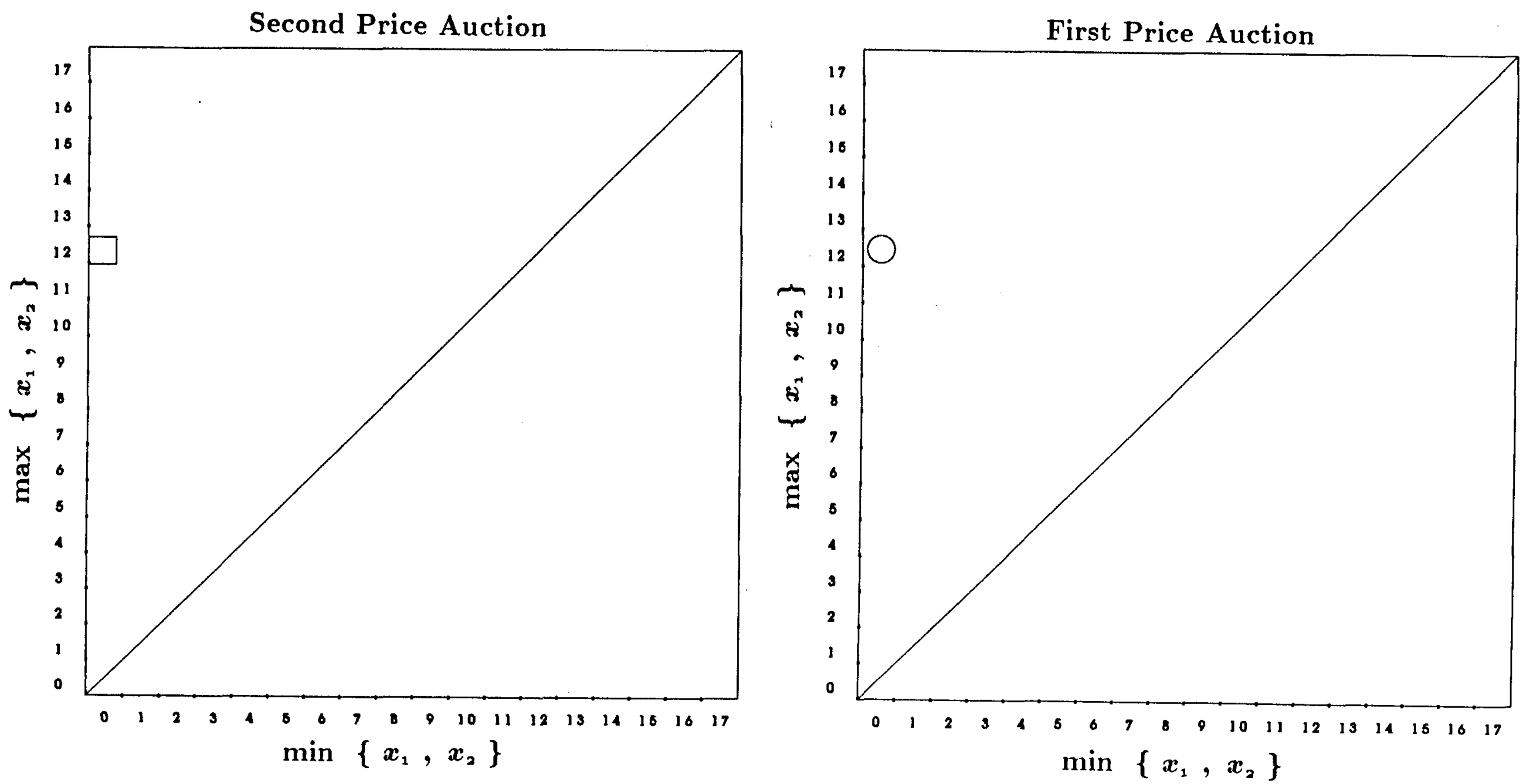

Figure 5.7.C : Equilibrium Predictions for Experiments 5 and 6. (Perfect Collusion)

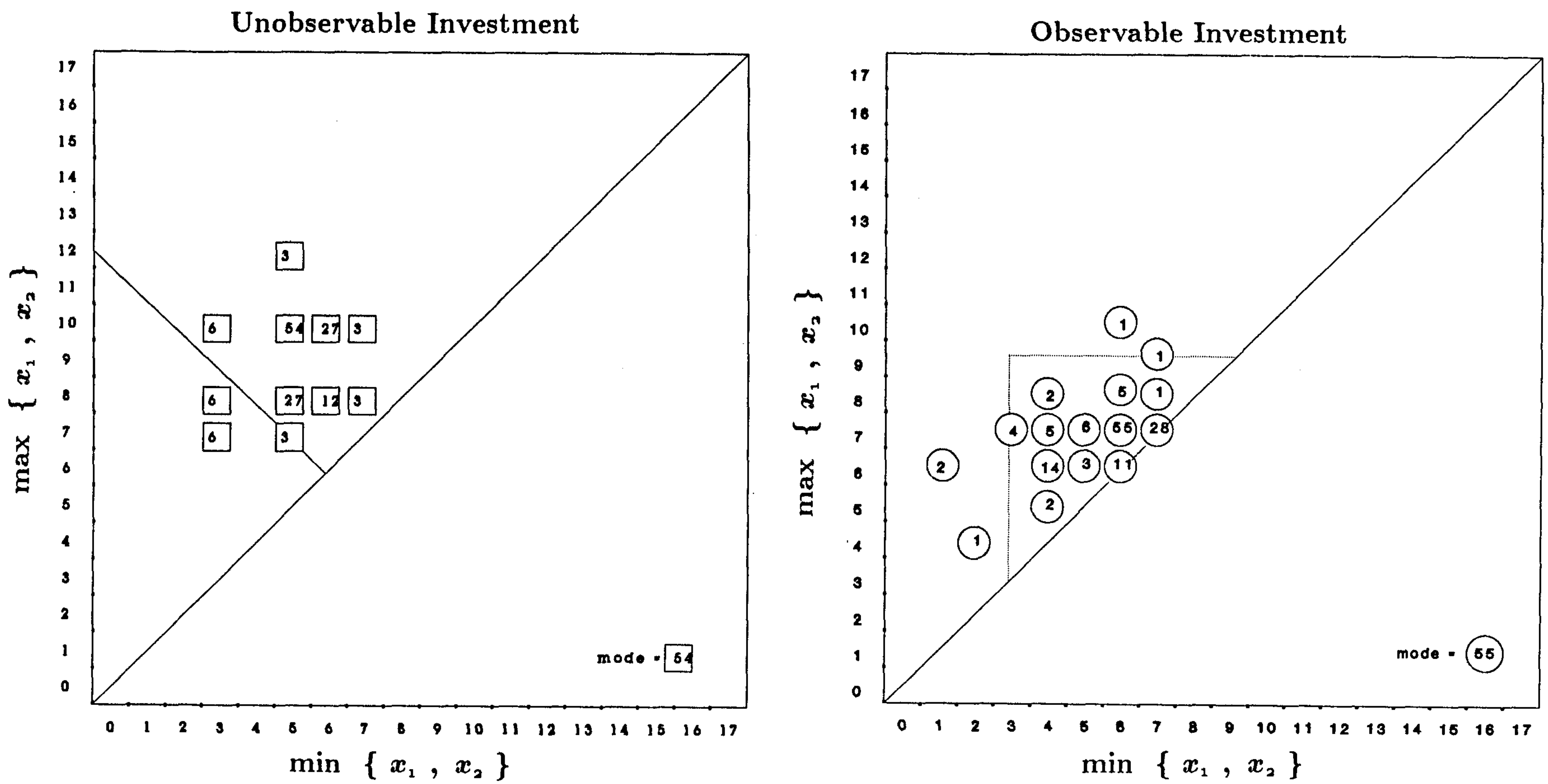

Figure 5.8.A : Experiment 5 - First Price Auction.

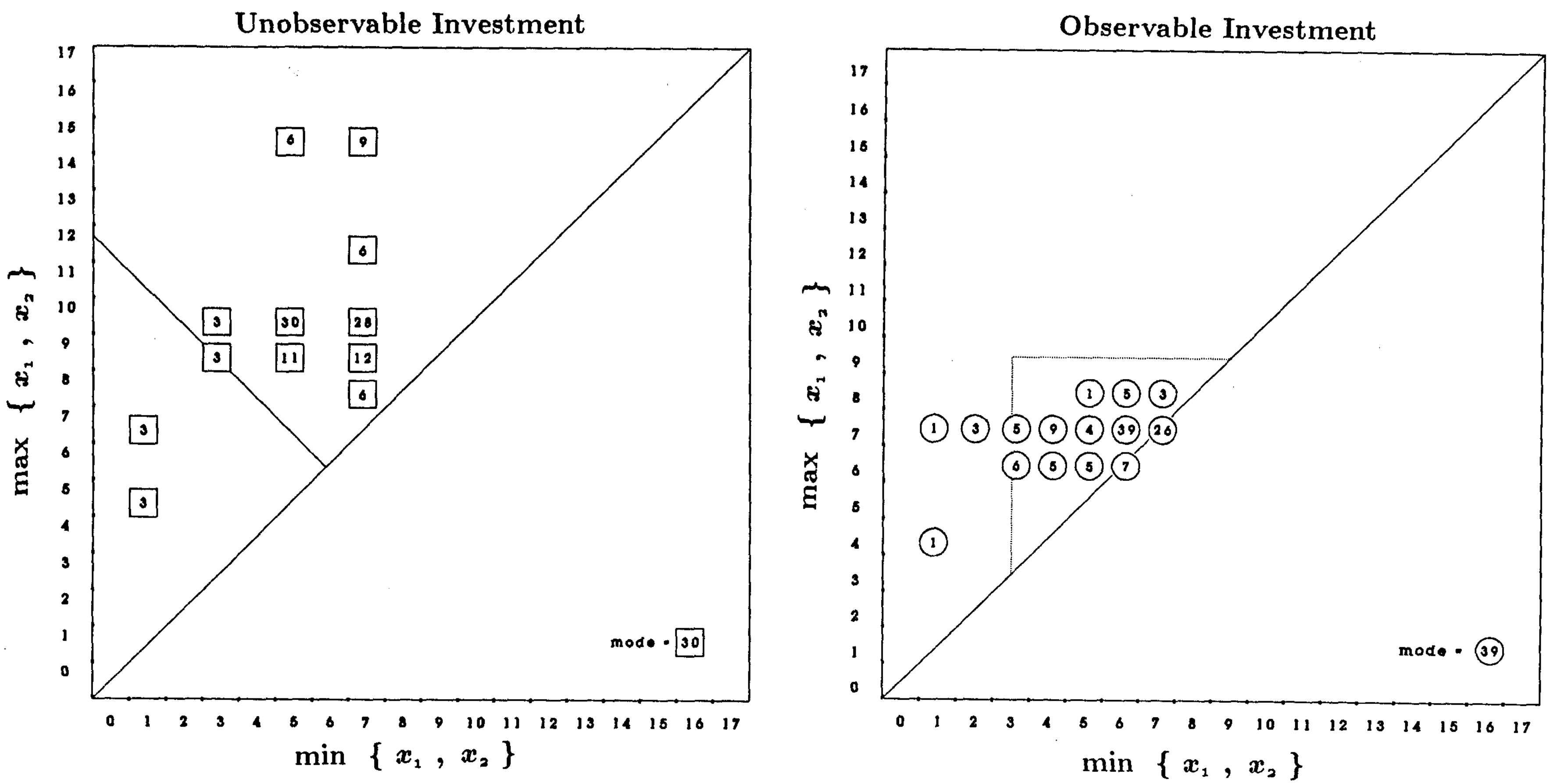

Figure 5.8.B : Experiment 5 - Second Price Auction.

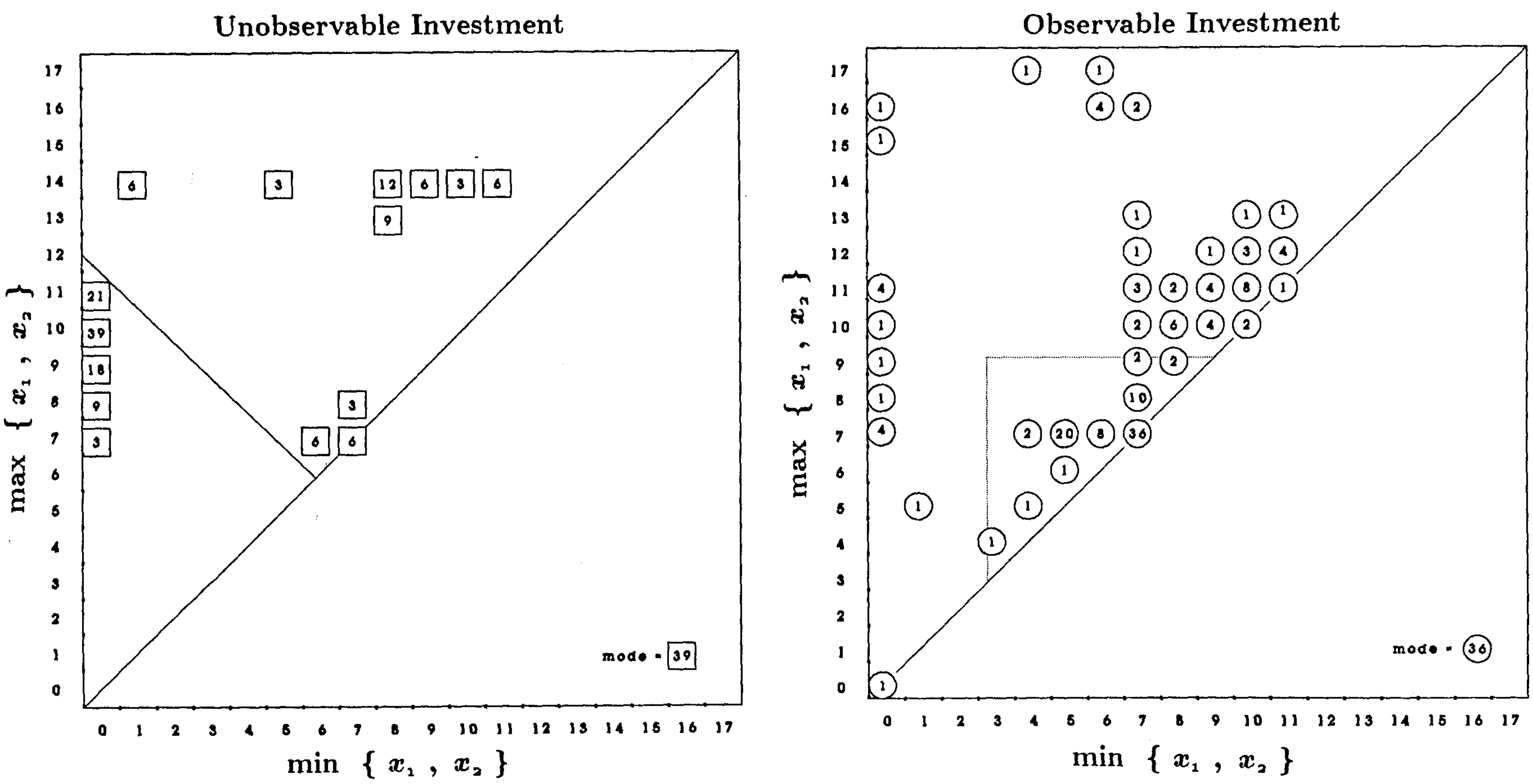

Figure 5.9.A : Experiment 6 - First Price Auction.

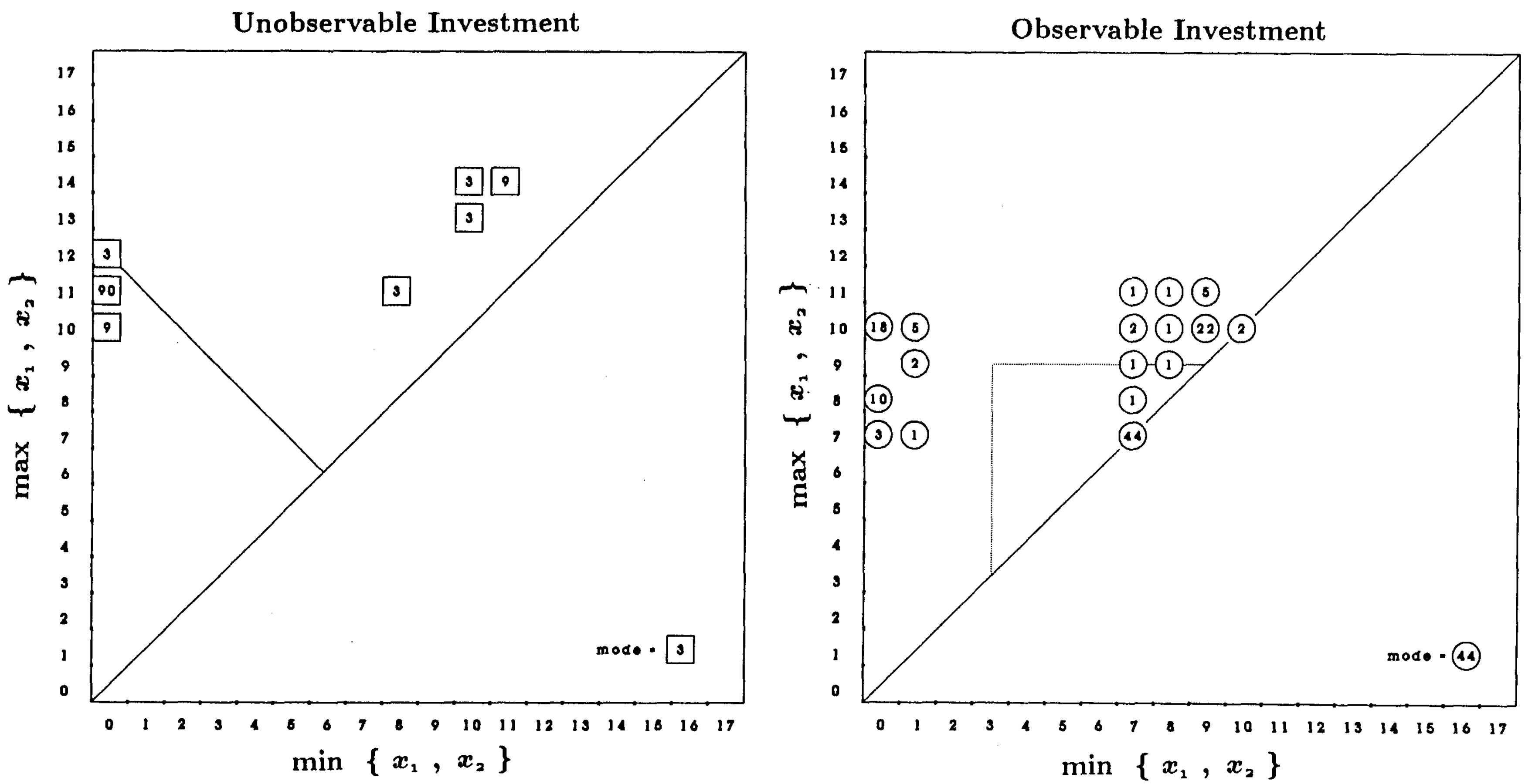

Figure 5.9.B : Experiment 6 - Second Price Auction.

In Experiment 5, the distribution of observed investment pairs under first and second price auctions are remarkably close to each other. The modes of both distributions are the same under both observable and unobservable investment. With observable investment, the number of observations that lie outside the triangle enclosed by the dotted lines is 4 out of 150 under the first price auction and 5 out of 120 under the second price auction. Therefore, almost all observations are concentrated within the same region under both auctions with observable investment. When investment is observable, 107 out of 150 observations are concentrated within one unit of the symmetric equilibrium (6,6) under the first price auction, and under the second price auction 81 out of 120 observations are within the same region. With unobservable investment, investment pairs are more scattered in both auctions.

Experiments 5 and 6 were conducted under the same parametric conditions except that in experiment 5 the sellers were matched randomly in every session, whereas in experiment 6 random matching was done at the beginning of the experiment. Therefore, although a seller did not know the identity of his opponent, he knew that he was competing with the same seller in every period in experiment 6. This variation allows us to study the relative occurrence of collusion attempts under the two auctions with observable and unobservable investment.

In Figures 5.9.A and 5.9.B, we present the distribution of investment pairs under first and second price auctions, respectively, for experiment 6. The modal observation under investment observability is the same as in experiment 5 under both auctions. With unobservable investment, the modal observation is (0,10) under the first price auction and (0,12) under the second price auction. A striking feature of the observations reported in Figures 5.9.A and 5.9.B is the frequency of observations where one seller invests zero. Note that in experiment 5, we did not observe any investment pairs with one component equal to zero under either auction and either observability treatment. This contrasts rather sharply with the distribution of investment pairs in experiment 6 : The number of investment pairs with a zero component is 81 out of 150 in the first price auction and 102 out of 120 in the second price auction when investment is unobservable. With observable investment, the corresponding numbers are 14 out of 150 in the first price auction, and 31 out of 120 in the second price auction. Although not all observations with a zero component indicate collusion, the number of occurrences of observations with a zero component in experiment 6 is quite large to be attributed to pure chance. This is especially true for the first price auction since a seller investing zero is never an equilibrium in the one-shot game.

5.4 Conclusions

The aggregate predictions based on the model were largely borne out by the observations in the first three experiments with three sellers. In experiments with two sellers, we obtained mixed results. At the level of market aggregates, the two auctions and the two observability treatments are indistinguishable at the pure strategy equilibria. Further tests on the observed distributions of investment under the two information treatments and the two auctions reveal that the model is not entirely accurate in explaining the observations. The discrepancies seem to be related to the the fact that the experimental setting is, in fact, a repeated game situation.

An aspect of the information conditions used in the experiments together with repeated game problems makes it difficult to relate any observed difference between the observable and unobservable investment treatments to the subgame perfection requirement in the one-shot, two-stage investment-bidding game. This is because, in the model, observability matters only through its effect on the subsequent bidding stage, not through its effect on subsequent repetitions of the game. In this sense, more experiments in which all sellers can observe the investment decisions *after* the bids are submitted and the winner is announced would be useful. This would, in a sense, equalize the effect of repeated game issues across different treatments.

Appendix to Chapter 5

INSTRUCTIONS

General Instructions

This is an experiment in the economics of market decision making. Various research foundations have provided funds for this research. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money which will be paid to you in cash.

In this experiment your earnings are determined by the the decisions you make in a market in a sequence of sessions.

The type of currency used in this market is francs. All earnings will be in terms of francs. Each franc is worth [$] to you. Do not reveal this information to anyone. At the end of the experiment your francs will be converted to dollars at this rate, and you will be paid in dollars. Notice that the more francs you earn, the more dollars you earn.

Specific Instructions

Your earnings are determined by the decisions you make in a market. You make your decisions in this market in a sequence of sessions.

At the beginning of the experiment, the participants will be matched randomly in four groups, with three participants in each group[1].

One unit of an object will be bought using an auction in each group. The auction procedure is the sealed bid first price auction. In this procedure, each participant submits a bid stating the price at which he/she is willing

[1]In experiments 5 and 6, the instructions read "six groups with two participants."

to sell the object. The bidder who submits the lowest bid wins the auction; that is, he/she gets to sell the object and receives his/her bid as payment for the object.

The cost to you of producing one unit of the object is determined as follows:

A bingo cage contains balls numbered from to . In a random draw from this cage, each ball is equally likely to be drawn.

You decide the number of draws you would like to make from this bingo cage. Your cost of producing the object is the lowest number on the balls you have drawn. The balls are drawn **with replacement**; that is, after each draw, the number on the ball is recorded and the ball is put back in the cage before the next draw.

You may draw as many balls as you like by paying [Francs] per draw. If you decide to make **k** draws, you pay [$\times$ **k**] Francs. Your cost of producing the object is the lowest number among the **k** balls. That is, this is how much you have to pay to produce the object if you win the auction. You do not incur this production cost if you do not win the auction. This cost is your private information, nobody else knows the result of your draws.

If you do not make any draws, your production cost is [Francs].

At the beginning of the experiment, you will be given [Francs] as Cash On Hand which you may use to pay for the draws you make.

After you observe the price per draw and the range of production costs, you will be asked to report how many draws you would like to make. Then you will privately observe your production cost, and you will be asked to submit your bid. In each group, the subject who submits the lowest bid will win the auction. The winner in each group will receive an amount equal to his/her bid as the price for the object.

No bid above [Francs] will be accepted. In case of a tie, the winner will be determined randomly.

Your profit is determined as follows :

if you win the auction:

your profit = your bid - your production cost - price per draw × number of draws you make

if you do not win the auction:

your profit = - price per draw × number of draws you make

After every session, the price and the winner in your group will be announced publicly.

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