SPECTRA OF POPULATION II CEPHEID VARIABLE STARS.

Thesis by

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It has been a pleasure to have received so much encouragement from so many individuals during the course of this research. Dr. Baade originally suggested this topic for a thesis. I have had valuable conversations and correspondence with Drs. Abt, Arp, Kraft, Munch, Osterbrock, and Whitney. Dr. Greenstein, as thesis advisor, has given me encouragement and advice throughout the entire project.

ABSTRACT

The physical properties of the population II cepheids have been studied using the following material: photoelectric observations of Arp, low dispersion spectra of Joy, high dispersion spectra taken by Sanford and Abt, and numerous moderate dispersion spectra obtained by the author.

Radial velocity curves for ten cepheids are shown. It is shown that the velocity curves for all population II cepheids with periods greater than 15 days are probably discontinuous. The difference between the RV Tauri star and the star whose velocity and light curves repeat well is shown to be due to a delay in the new outrush of gas during alternate cycles of the RV Tauri star. Displacement curves are derived for eight cepheids that probably have discontinuous velocity curves.

The photoelectric observations are interpreted to show that the surface temperature of M5 No. 42 varies from 7200°K to 4800°K and of W Virginis from 6200°K to below 4800°K. The electron pressure for these two stars varies from 100 dynes/cm^2 to 0.1 dynes/cm^2 and 10 dynes/cm^2 to less than 0.1 dynes/cm^2 respectively. The changes in radii obtained by applying Steffan's Law to the changes in temperature and luminosity agree reasonably well with integration of the velocity curve. Values of the surface gravity obtained from the colors at maximum radius and from the deceleration of the velocity curve yield masses in the vicinity of one to three solar masses. The use of the period density relation is consistent with masses between 1.2 and 2.0 solar masses.

The spectra of the population II cepheids are generally of type A5 to F0 at earliest and F5 to G0 at latest. No correlation of spectral type with period can be found. A correlation between spectral type and light curve seems to be present.

Consideration of the emission lines and of certain absorption lines that are effected by dilution leads to the conclusion that the proper model for W Virginis must contain a shock wave moving out through the atmosphere of the star.

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ABSTRACT

I. INTRODUCTION

A - Historical Background

In his classic paper on the Radial Velocities of Cepheid Variable Stars(1) Dr. Joy noted certain remarkable properties of the star W Virginis. His description is as follows: "Bright hydrogen lines are present on 11 plates taken during the increase in the star's light from phase .594 to maximum....The large proper motion and high galactic latitude of this star are notable." Joy followed this up by studying Barnard's variable (No. 154) in Messier 3 and noted the similarity between it and W Vir(2).

When Baade introduced the concept of the two population types(3), it became apparent that the cepheids could be divided into the two populations. The classical cepheids of population I are found in the spiral arms of the Galaxy while stars like W Vir and M3 No. 154 populate the galactic halo and the globular clusters. In this respect they are similar to the RR Lyrae stars.

The early work of Mrs. H. B. S. Hogg on the light curves of variables in globular clusters was beset with large scatter but nevertheless the difference between the classical and population II cepheids can be noted. She was the first to note the alternating minima of M2 No. 11 and M56 No. 6 and thus relate the W Vir stars to the RV Tauri stars(4,5).

In an important paper, H. C. Arp studied 19 variables in globular clusters by means of accurate two color photographic photometry(6). He supplemented this by three color photoelectric observations of W Vir, M5 Nos. 42 and 84, and M10 Nos. 2 and 3(7). Arp noted that the light curves of most of the cluster variables are very similar to W Vir. Several stars showed alternating deep and shallow minima. The variables M3 No. 154 and

M5 No. 42 have light curves that differ markedly from phase, the minimum is broad and the rise to maximum is extremely rapid. These stars have a bluer color and higher absolute magnitude by about .5 mag. than stars of the W Vir type with the same period.

In the meantime Joy extended his spectroscopic study to 32 variables in globular clusters including several semi-regular variables(8). Stars with periods from one to a hundred days were studied. He noted that spectral types ranged from A to F in the stars of shortest period and became later with period until they ranged from G to M in the 65 to 100 day group. Emission lines were strongest in the W Vir and RV Tauri groups, generally weaker among the semi-regulars and absent in the stars of the 1-2 day group.

The presence of double lines in the spectrum of W Vir was discovered by Sanford(9). A detailed study of W Vir has been completed by Abt(10). He studied the changes in radius, temperature, relative abundances and opacity. He also discussed two possible pulsation models. For the details of this very careful study the reader is referred to Abt's paper.

B - Problems Related to Population II Cepheids

It seems best at this point to list some of the problems related to the population II cepheids. It has been the purpose of this work to shed as much light as possible on these problems within the limitations of time and equipment.

1 - What are the spectroscopic differences between the population I and II cepheids?

2 - What are the differences between the RV Tauri stars and the cepheids whose light curves repeat well from cycle to cycle?

One star of each type is found in M5. They have virtually the same period so certainly the period is not the deciding factor.

3 - What are the differences (if any) between cepheids on the different lines of Arp's period-luminosity diagram?

4 - Which population II cepheids have double lines and which do not?

5 - What information can be obtained about the radii of population II cepheids from integration of the velocity curves and interpretation of the colors?

6 - What are the masses of the population II cepheids?

7 - What can be learned by classification of the spectra on the MKK system and comparison of the spectra among the cepheids?

8 - Can a definite decision be made between the shockwave model and the interpenetrating shell model for W Vir?

9 - What evolutionary track in the color-magnitude diagram leads a star into the region of the population II cepheids?

II. CHOICE OF MATERIAL

In order to follow up the questions discussed above, it was decided to study intensively a few population II cepheids with interesting properties that are bright enough to observe at moderate dispersion. The two variables in M5 were the obvious starting point. Both have approximately the same period, 26 days, and are bright enough to observe at the 60" telescope. No. 42 has the anomolous light curve discussed above. It is, intrinsically, one of the brightest known cepheids in a globular cluster. No. 84 alternates at minimum and is therefore an RV Tauri star. To these was added TW Capricorni, a field star of period 28 days, having a light curve very similar to M5 No. 42 (11). Plates of W Vir and the classical cepheid of period 27 days, T Monocerotis, were obtained for comparison purposes. A few plates were taken of other variables in globular clusters for special purposes. In the clusters, variables of short period are too faint so a few plates of field stars of periods from two to ten days were taken. In addition Dr. Joy kindly loaned me his plates for reexamination.

The new Cassegrain spectrograph on the 60" telescope (12) was used for all plates obtained by the author. Grating C and the 4" and 8" cameras were used for almost all plates. In the blue region of the spectrum these give 80 and 40 A/mm respectively. When looking for double lines a few plates were taken with the 16" camera giving 20 A/mm. In addition, 3 plates were obtained at the 200" telescope, two by Dr. Munch and one by Dr. Greenstein. These have a dispersion of 18 A/mm having been taken with the 18" camera at the coude spectrograph. These plates were of great importance in the program and I am extremely grateful to Drs. Greenstein and Munch.

III. LIGHT CURVES AND PHASES

Arp's light curves of variables No. 42 and 84 are reproduced in Fig. 1 and 2. Certain properties of the light curves are particularly notable. The curve of No. 42 showed very little scatter both in 1952 and 1954. The difference between deep and shallow minima of No. 84 is .4 in m_{pg}. This was unchanged in 1954. No. 84, however, showed much more scatter about the mean light curve than No. 42. This may be responsible

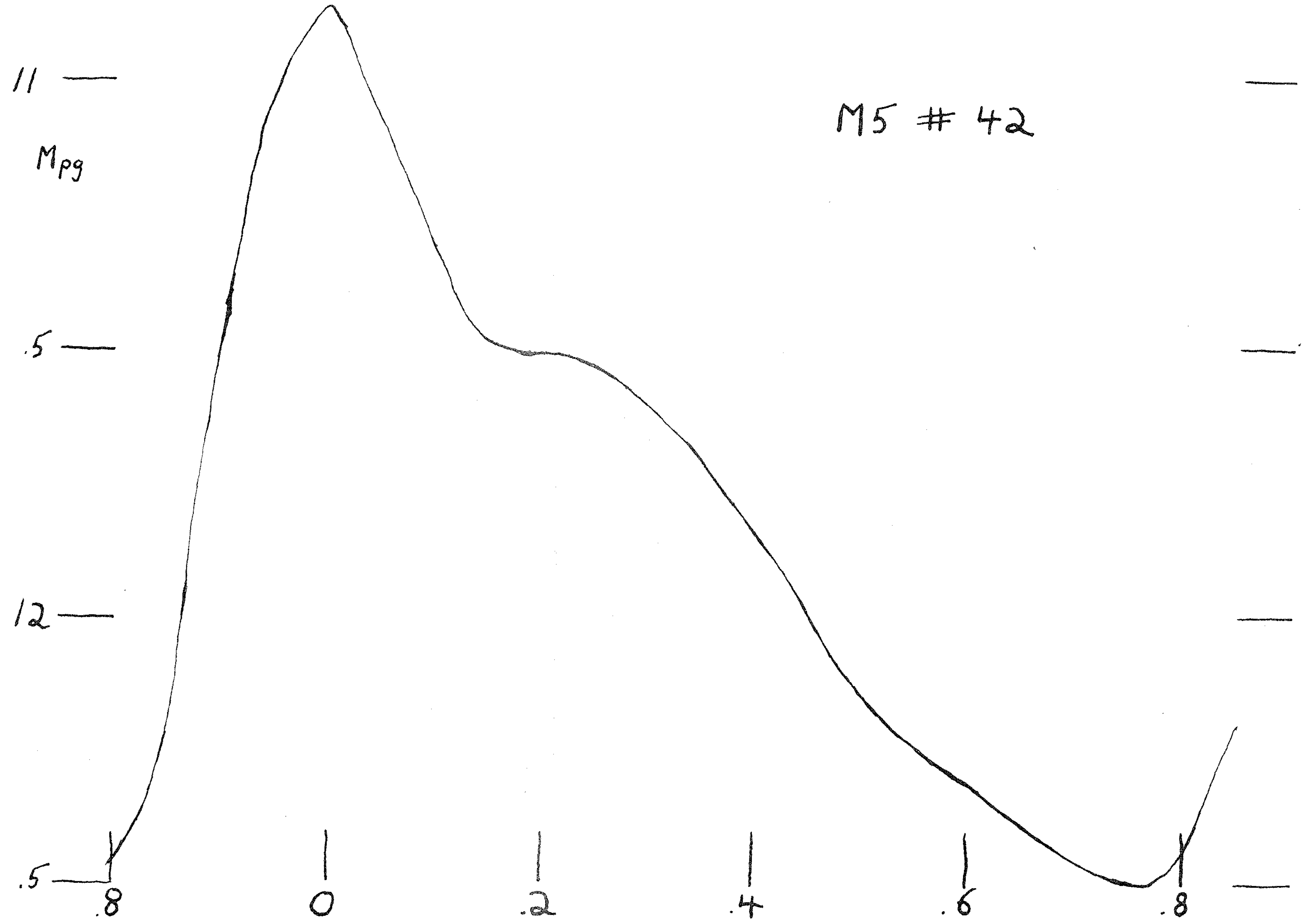


Fig. 1.

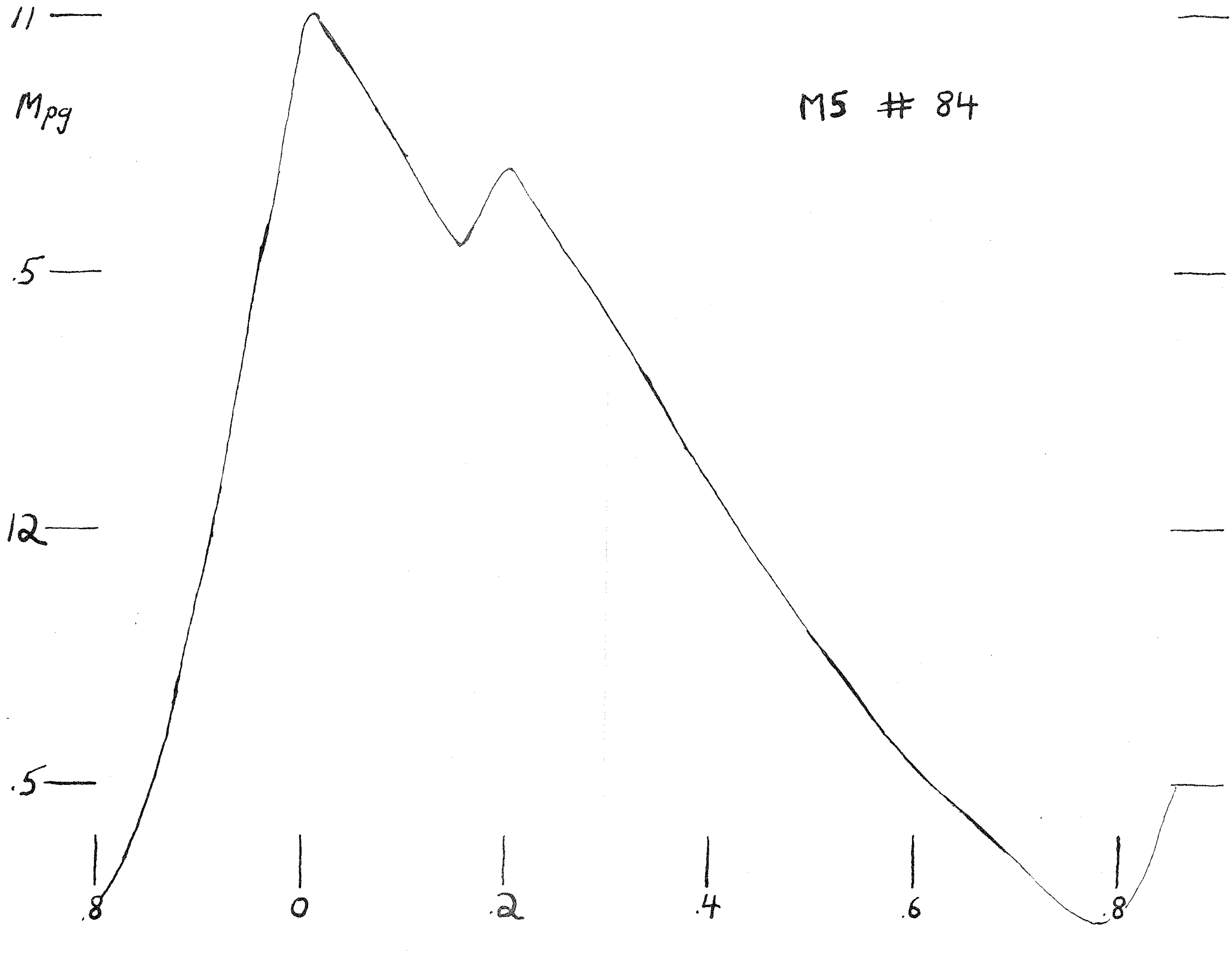


Fig. 2. Mean Light Curve

for the difficulties in establishing an ephemeris to be discussed below.

While making spectroscopic observations in 1956 plates of M5 were also obtained with a direct plateholder in the spectrograph especially constructed for this purpose. The combination of the long focal length and 4" x 5" plates results in only a small field being available on a plate. There are no stars bright enough in the immediate vicinity of M5 to obtain an accurate calibration curve at maximum light. Additional comparison stars in the small available field were established from four Newtonian plates taken by Arp in 1952. All comparison stars are identified on Fig. 4 and m_{pv} listed in Table 1. The lack of comparison stars in m_{pg} is so serious that no attempt was made to obtain light curves in this color. The plates were obtained through Kodak Wratten 12 and 2a filters on 103aD plates. This approximates m_{pv} very closely.

TABLE 1

Comparison Stars in M5

Star No.	m_{pv}	Probable error
1	11.98	.02
2	12.07	.04
3	12.32	.03
4	13.60	.04
5	13.73	.03
6	11.93	.06

Since the cycle was not completely covered, the curves need not be reproduced here. The objective was to determine the phase of the variables and whether or not any changes in the stars' behavior had occurred in the two years since they had been observed. This sort of change is most noticeable at minimum light.

No. 42 showed no change in behavior and the ephemeris of Arp(6) was found to hold. No. 84 continued to show alternation at minimum of the same magnitude as before. No interchange of deep and shallow minima has been observed for this star for four years. The ephemeris given by Arp(6) was epoch of maximum J.D. 2434097.9 + 26.4945 days. Uncertainty as to the number of periods since the previous observed maximum led to the other possibilities of the period being 26.531 days and 26.458 days. After his photoelectric observations in 1954 Arp changed these elements to J.D. 2434901.8 + 26.62 days. The 1956 observations do not fit this ephemeris. Combination of the 1952 and 1956 data yield J.D. 2435669.33 + 26.54 days for maximum light. This has been used to determine the phases of the spectroscopic observations. The discrepancy in Arp's 1954 observations can be explained in two ways. Most likely the star acted peculiarly as RV Tauri stars are prone to do, or possibly the proximity of this star to the cluster center makes it unsuitable for photoelectric observations due to the background cluster light.

The best light curve of TW Cap is published by S. Gaposchkin(11), and is shown in Fig. 3. In order to determine the phase in 1956 Mr. Thomas Cragg kindly made a series of visual observations with the Mt. Wilson 6" refractor. By observing the star on the rapidly rising branch of the light curve, Cragg obtained a good epoch of maximum, J.D. 2435636.6 ± 1

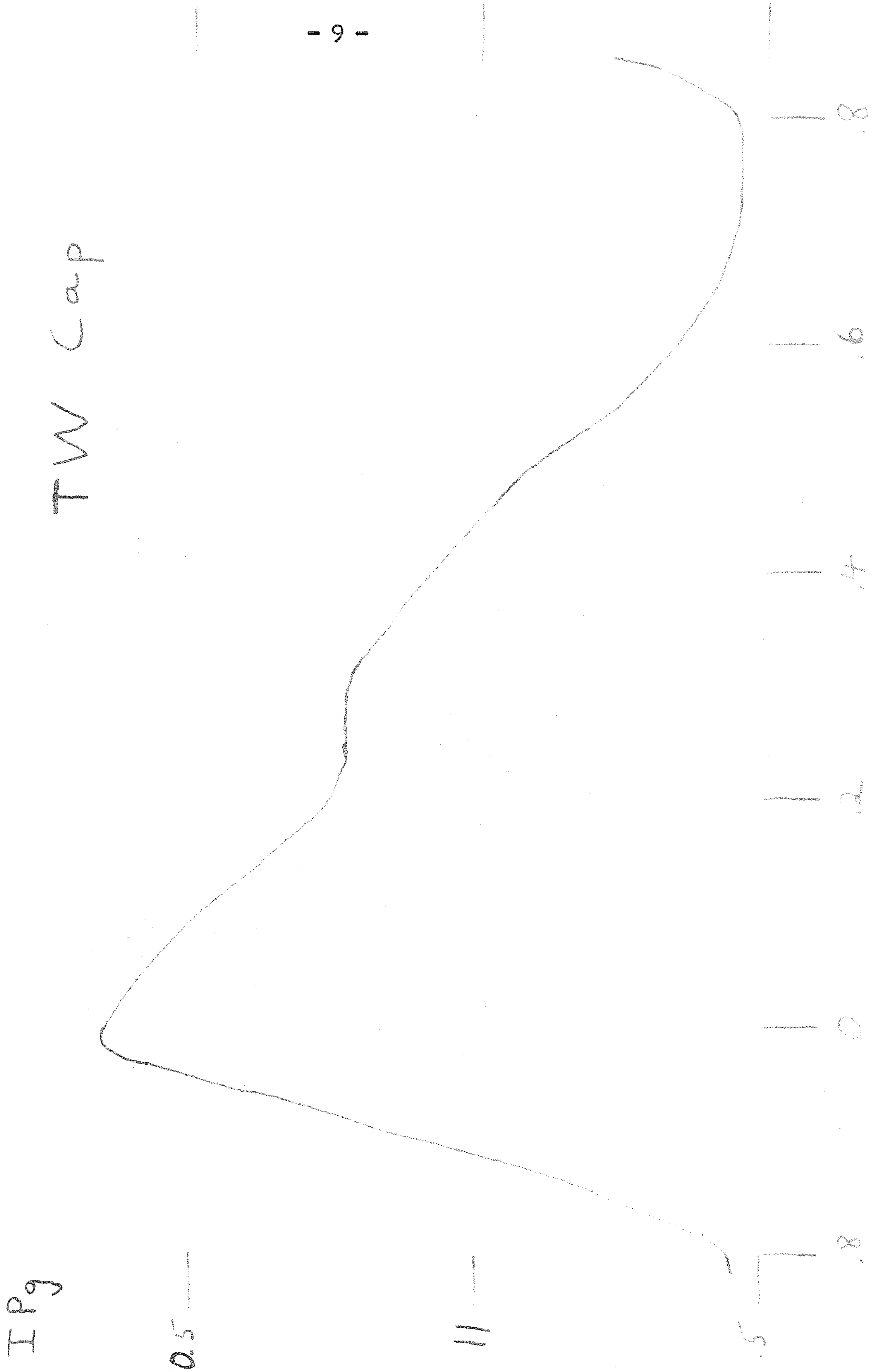


Fig. 3.

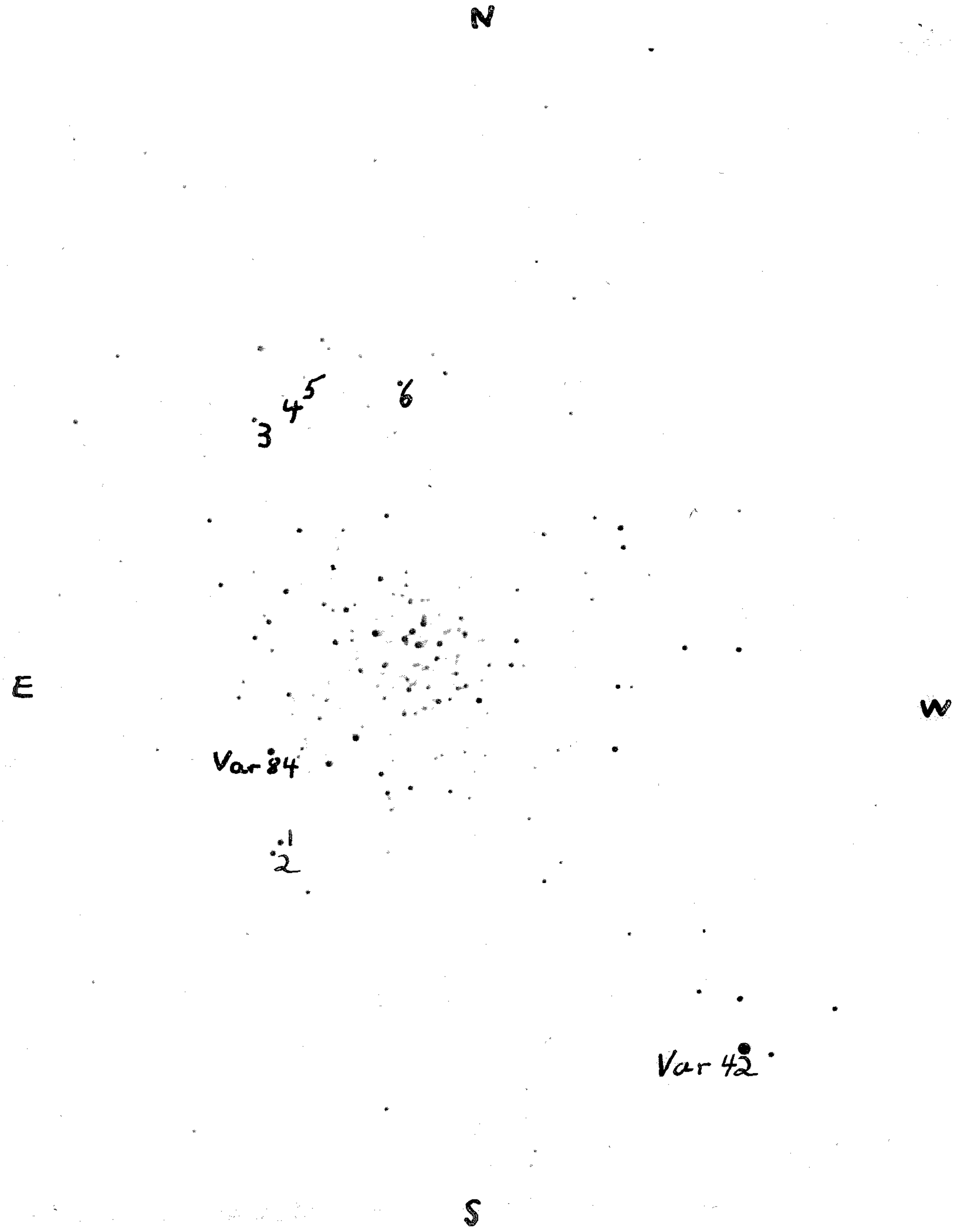


Fig. 4. Direct Photograph of M5.

Variables and Comparison Stars Are Identified.

which combines with Gaposchkin's maximum to yield a period of 28.556 days. According to Gaposchkin the period of TW Cap has been decreasing at the rate of about 1.7 days per 1000 years which is ten times the rate of decrease of W Vir. This star is presently being observed photoelectrically by both Jaschek and Gascoigne but their results are not available as yet.

Partial spectroscopic coverage was obtained for M3 No. 154, M10 No. 2, and M2 No. 11. The light curves of these stars are shown in Figs. 5-7. Phases are computed using Arp's(6) ephemerides checked by a few direct plates taken in 1957, except for M2 No. 11 for which the light curve was completely redetermined in 1953, 1954(13) and again in 1956(14). The 1953 light curve is of the best quality and is used here. Among the field stars, spectra were obtained of four. W Vir was taken for comparison studies. Arp's 1954(7) light curve is shown in Fig. 8. The other stars are AL Virginis, period 10.30 days; UY Camelopardalis, period 6.16 days; and UY Eridani, period 2.21 days. These light curves are shown in Figs. 9-11. They are taken from Mrs. C. Payne-Gaposchkin(15), Baker(16), and S. Gaposchkin(17). No light curves will be shown for stars discussed only briefly from material in the Mt. Wilson Observatory files.

IV. THE SPECTROSCOPIC OBSERVATIONS

All usable plates obtained in 1956 and 1957 are listed in Table 2.

The dispersions associated with the plate numbers are X_f, 80 A/mm; X_e, 40 A/mm, X_d, 20 A/mm; P_d, 18 A/mm.

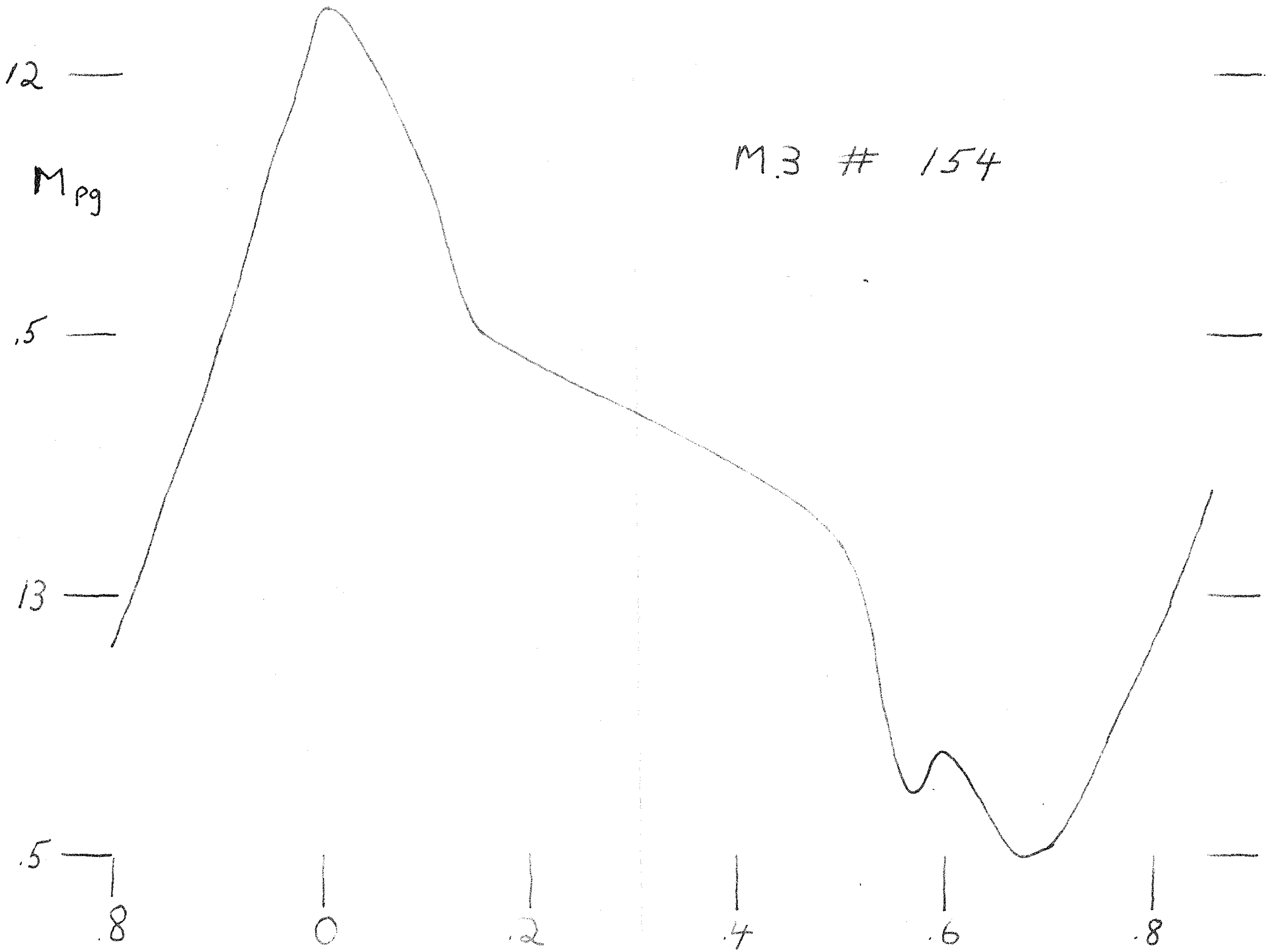


Fig. 5

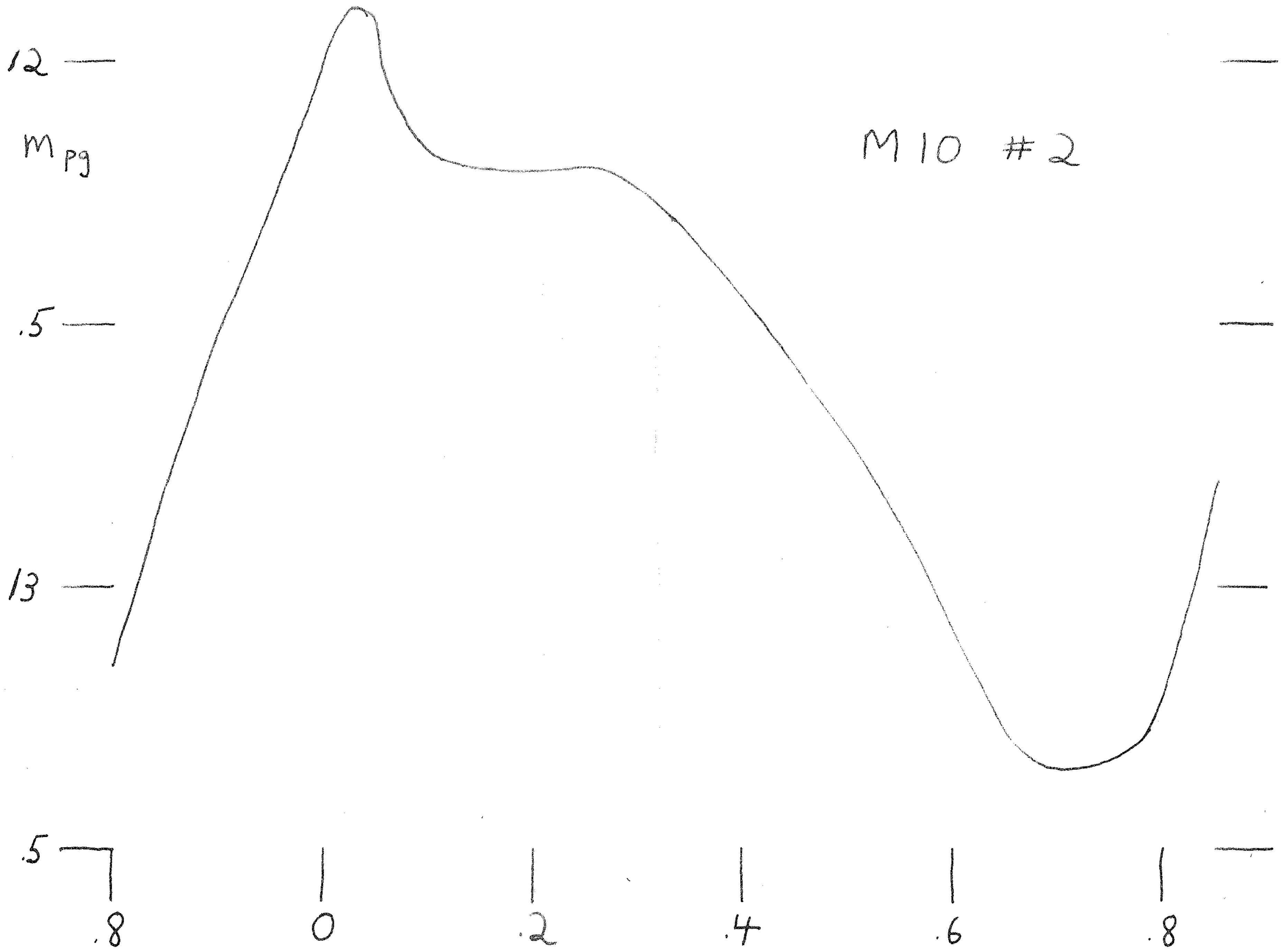


Fig. 6. Mean Light Curve

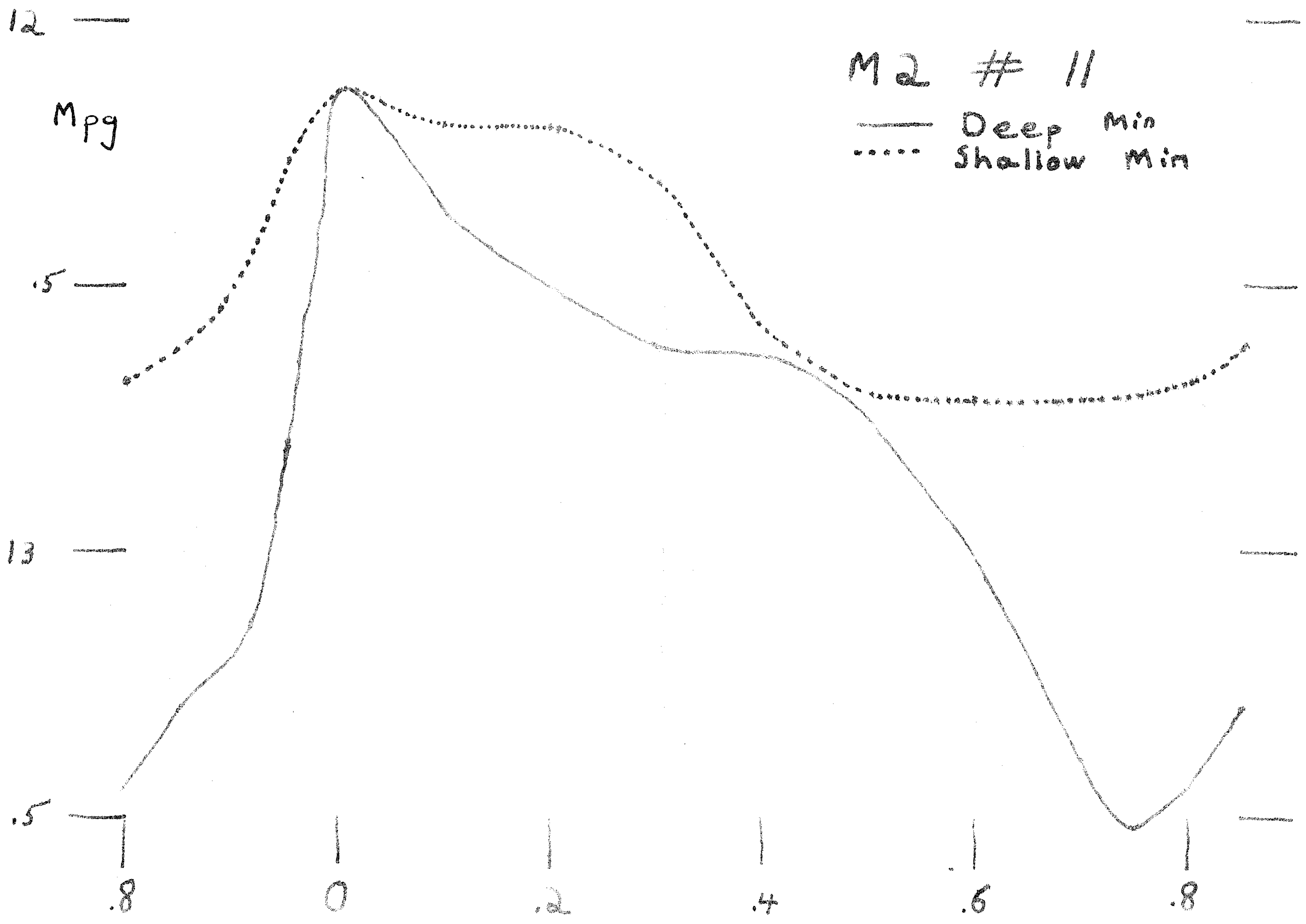


Fig. 7.

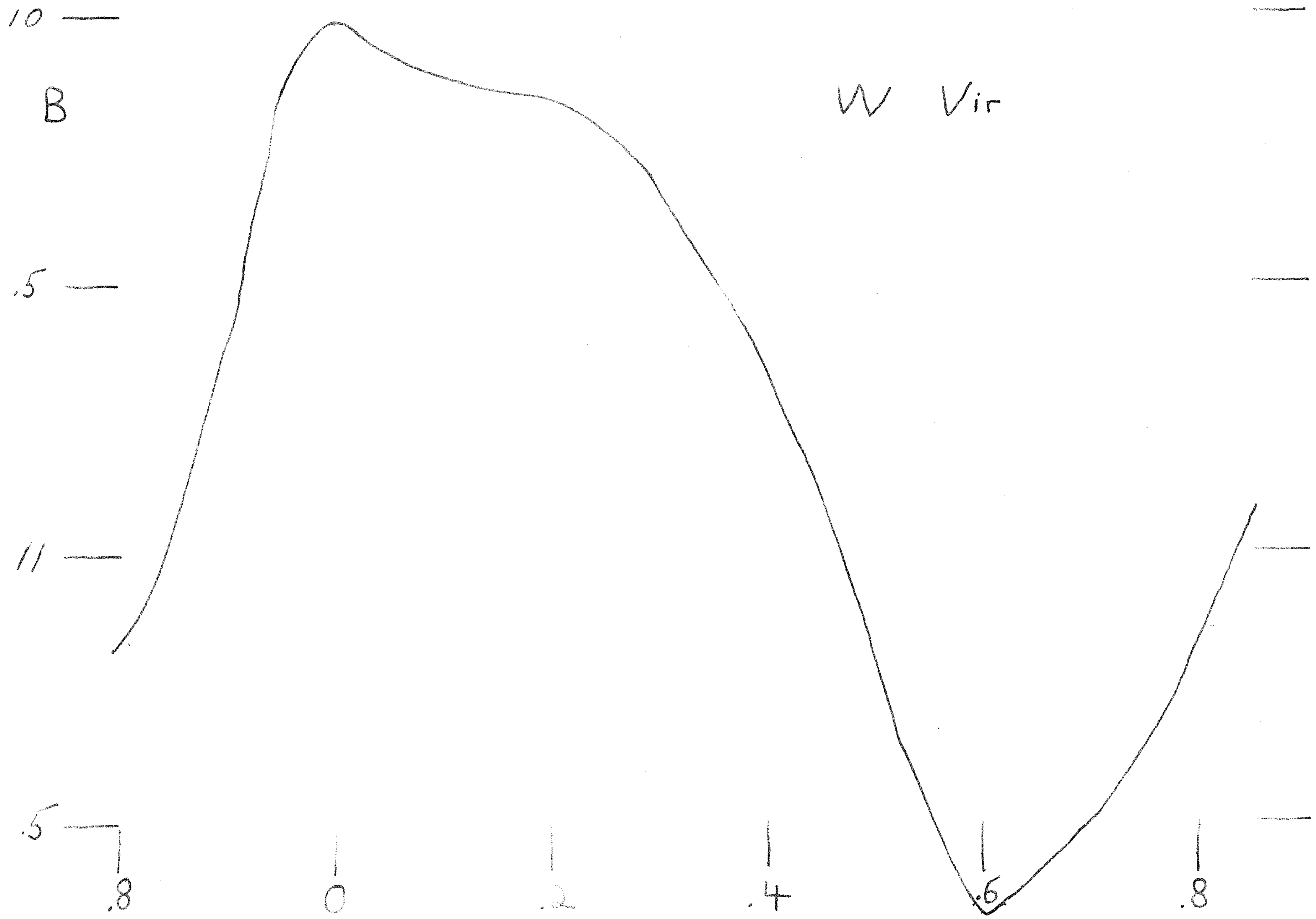


Fig. 8.

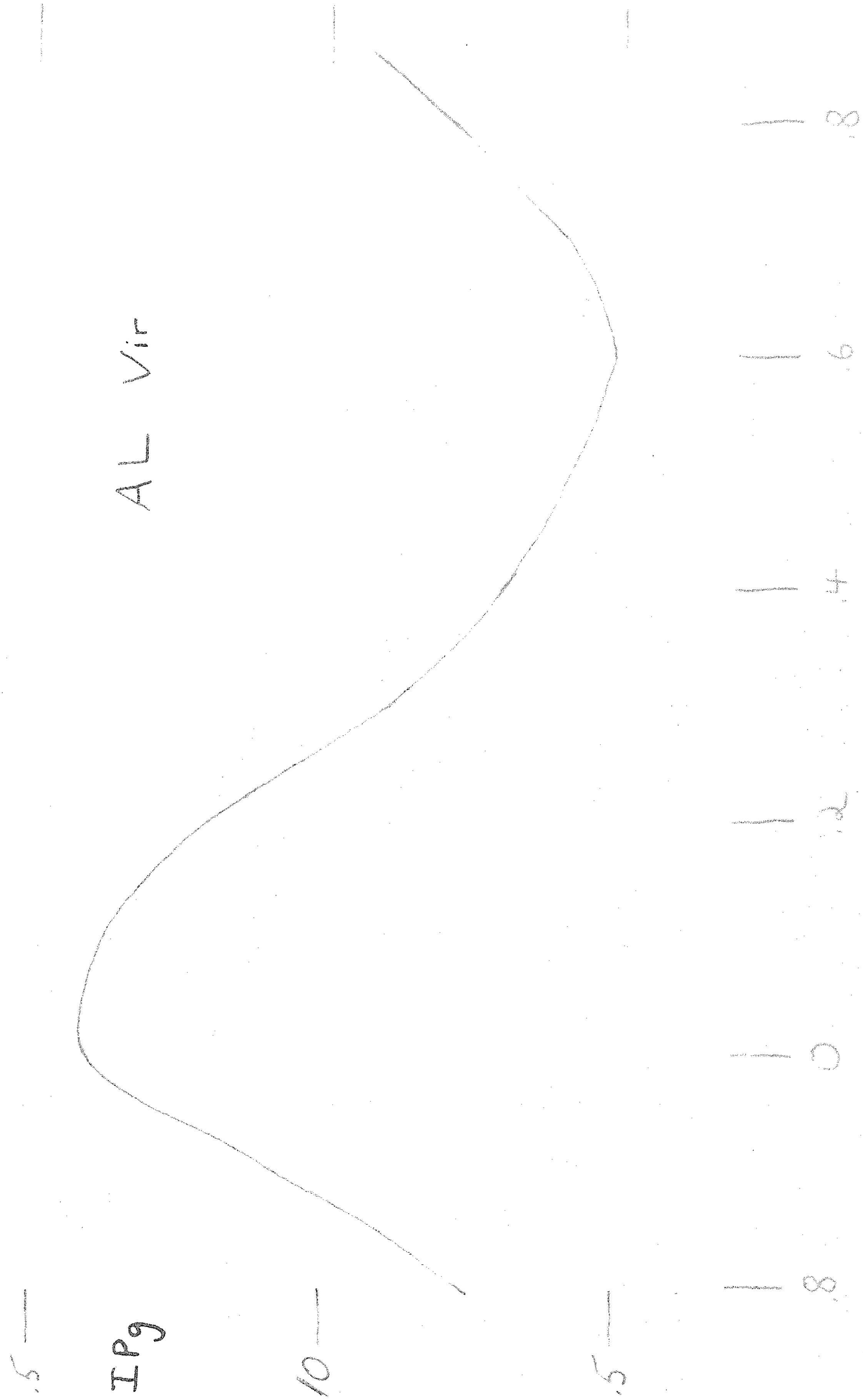


Fig. 9.

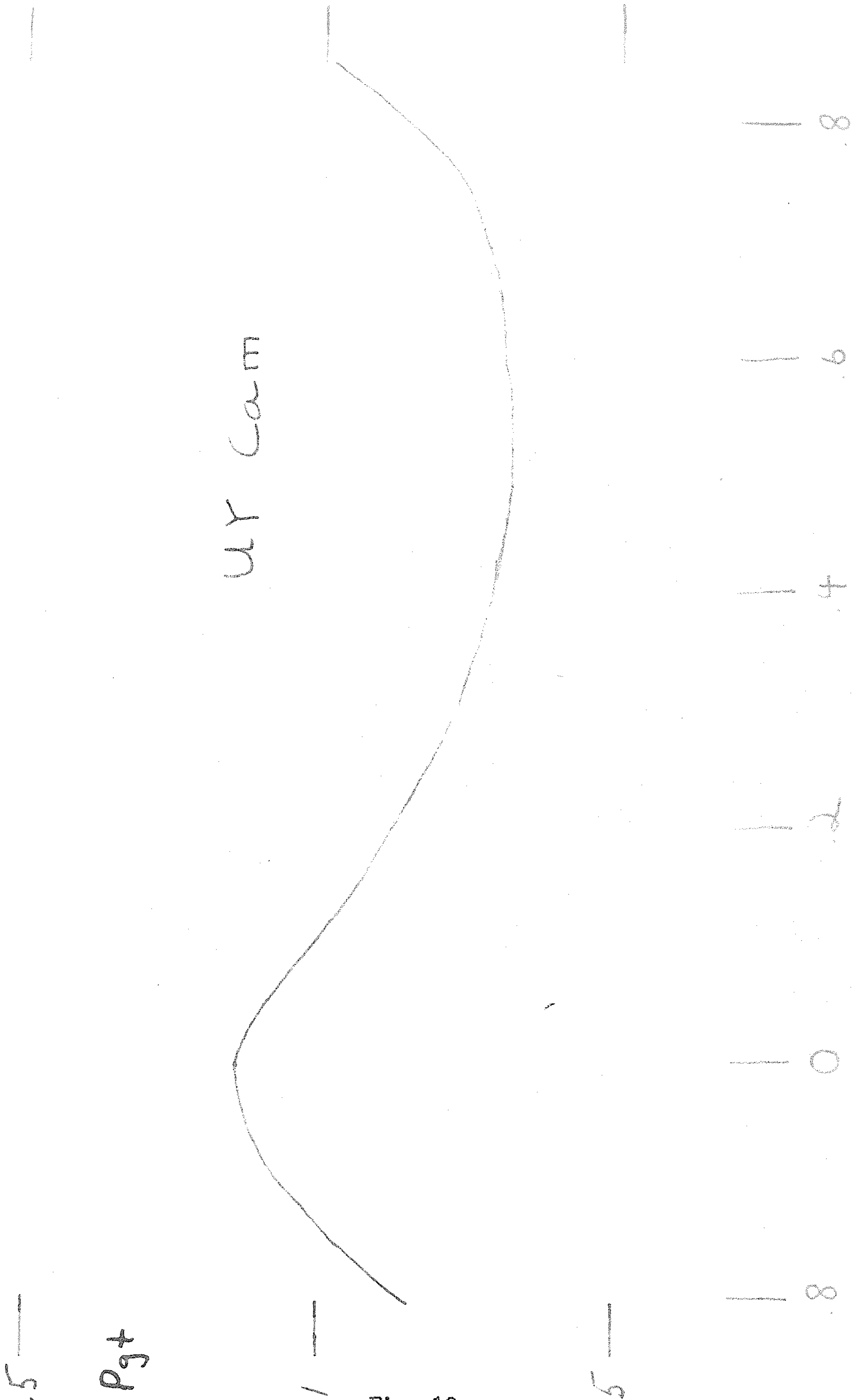


Fig. 10.

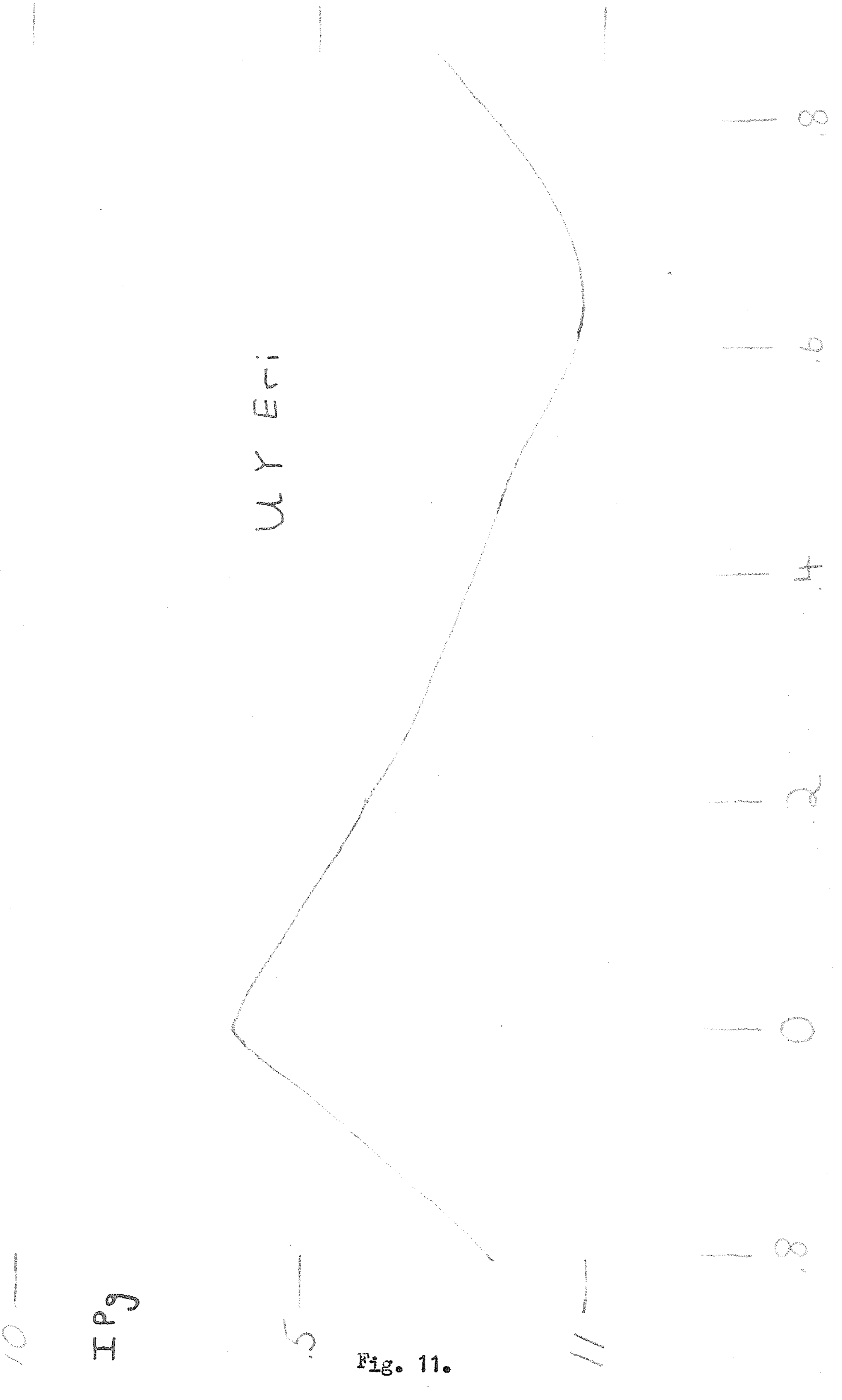


Fig. 11.

TABLE 2

Star	Plate No.	Mid exposure J.D. 2435 +	Cycle & Phase
M5 No. 42	X_f 1281	524.96	1.21
	X_3 1332	553.94	2.35
	X_f 1341	554.95	2.39
	X_d 1417	570.29	2.99
	X_e 1428	572.90	3.09
	X_f 1547	593.75	3.90
	X_e 1561	601.81	4.21
	X_e 1568	609.83	4.56
	X_f 1575	612.83	4.63
	X_f 1582	614.88	4.72
	P_d 2635	622.84	5.03
	X_e 1653	631.25	5.35
	X_f 1662	633.75	5.45
	X_f 1814	657.73	6.38
	X_f 2488	904.95	15.98
M5 No. 84	X_f 1251	511.98	1.06
	X_f 1260	512.98	1.09
	X_f 1277	522.96	1.47
	X_e 1423	571.90	3.32
	P_d 2587	592.80	4.11
	X_f 1548	593.88	4.15
	X_e 1556	596.83	4.26

TABLE 2 (continued)

	X_e 1559	597.75	4.30
	X_f 1564	608.79	4.71
	X_f 1578	613.83	4.90
	X_e 1585	615.85	4.98
	X_e 1651	630.79	5.54
	X_f 1656	632.75	5.62
	X_f 1665	634.79	5.69
	X_f 1676	636.79	5.77
	P_d 2700	651.73	6.34
	X_e 1789	651.75	6.34
	X_f 1796	653.73	6.41
	X_f 1811	656.75	6.52
	X_f 1817	658.73	6.60
	X_f 2495	905.90	15.91
	X_f 2516	909.91	16.04
	X_f 2558	932.90	16.92
	X_f 2568	933.83	16.96
TW Cap	X_f 1549	593.96	1.51
	X_f 1565	608.96	2.03
	X_f 1576	612.98	2.17
	X_e 1654	631.90	2.84
	X_f 1663	633.90	2.91
	X_e 1674	635.85	2.97

TABLE 2 (Continued)

	X_e 1803	654.83	3.61
	X_f 1818	658.92	3.78
	X_d 1823	665.08	4.00
	X_e 1827	671.85	4.23
	X_e 1841	673.85	4.30
	X_e 1936	735.71	6.47
	X_f 1945	736.73	6.50
	X_f 1994	756.66	7.20
	X_f 2012	761.63	7.38
	X_f 2607	953.98	14.10
	X_f 2614	954.98	14.14
	X_f 2638	980.92	15.06
	X_f 2643	983.97	15.16
	X_f 2656	993.92	15.51
	X_f 2667	995.92	15.58
	X_f 2675	997.90	15.65
T Mon	X_e 1129	501.81	1.48
	X_f 1148	502.79	1.52
	X_e 1190	507.79	1.70
	X_e 1235	509.79	1.78
	X_f 1250	511.85	1.86
	X_f 1276	523.77	2.30

TABLE 2 (continued)

	X_e 1331	553.73	3.40
	X_f 1346	554.73	3.44
	X_d 1416	569.67	5.00
	X_e 1421	571.65	5.07
	X_e 1426	572.63	5.10
	X_f 1545	593.65	5.88
	X_e 1551	594.65	5.92
	X_e 1555	596.63	5.99
	X_e 1558	597.65	6.03
	X_e 1560	601.65	6.18
W Vir	X_e 1422	571.71	1.04
	X_f 1546	593.68	2.31
	X_f 1581	614.72	3.53
	X_f 2475	890.94	19.63
	X_f 2494	905.77	20.42
	X_f 2502	908.83	20.60
	X_f 2514	909.81	20.66
	X_f 2636	980.75	24.76
M2 No. 11	X_f 1815	657.92	1.97
	X_f 1961	737.67	4.36
	X_e 2004	758.30	4.97
	X_f 2083	771.71	5.37

TABLE 2 (Continued)

M3 No. 154	X_f 2605	953.80	1.98
M10 No. 2	X_f 2655	993.77	1.99
AL Vir	X_f 2515	909.93	
	X_f 2559	932.96	
	X_f 2567	933.77	
	X_f 2609	954.78	
	X_f 2673	997.77	
UY Cam	X_f 2557	932.67	
	X_f 2566	933.71	
	X_f 2598	943.75	
	X_f 2604	953.67	
	X_f 2608	954.67	
	X_f 2635	980.69	
UY Eri	X_f 1959	736.97	
	X_f 1996	756.88	
	X_f 2005	758.92	

V. RADIAL VELOCITIES

All plates were measured for radial velocity in the usual way by comparison with an iron spectrum impressed at the spectrograph. The lines used depended upon the dispersion. Laboratory wavelengths were used and a few seemingly unblended lines were discarded because of large residuals. Tests of the four inch camera by Bonsack(18) show that the camera generally gives very good radial velocity results for standard stars except in a few scattered cases when a plate would show a discrepancy of up to fifty kilometers per second. This has been carefully discussed by Bonsack and myself and we agree that with the very wide slit of this spectrograph a bright standard star, which needs only one or two fast trails for adequate exposure, may easily be guided up one side of the slit and thus give an incorrect velocity. This is no problem for long exposures. Seven plates of velocity standards in different parts of the sky were taken with the 8" camera. Six of these were very close to the accepted velocity and one was off by 10 km/sec. No velocity tests of the 16" camera have been taken. The most important point here is that plates taken with four different cameras on two telescopes fall on smooth curves. The probable error of the mean velocity is about 4 km/sec at 80 A/mm and 2 km/sec at 40 A/mm.

A - The Velocity Curves Drawn Continuously

The drawing of velocity curves depends upon the presence or not of double lines near light maximum. These are not resolvable with a dispersion of less than 20 A/mm. Since most population II cepheids are so faint that high dispersion plates are an impossibility with present equipment. I will first discuss the available material from the point of view of continuous velocity curves even if double lines have been observed. This enables a fairly large number of stars to be discussed together. Since future work on fainter variables will be limited to low dispersion, comparisons can then be made with the continuous velocity curves shown here for the brighter stars.

The velocity curves of the absorption lines are shown in Figs. 12-21. The curves for M5 No. 42 and No. 84 and TW Cap are drawn entirely from spectra listed in Table 2. The curve for T Mon is from Sanford(19). The curves for M3 No. 154, M10 No. 2, W. Virginis, MZ Cygni, AL Virginis, and AP Hercules are from Joy (1,2,8).

The first four curves and that of W Vir are the best determined. M5 No. 42 shows a smooth curve which correlates with the regularity of its light curve. The falling branch of the velocity curve is very rapid corresponding to the rapid rise in the light curve. The velocity amplitude is 40 km/sec.

The RV Tauri star M5 No. 84 shows strikingly different behavior than No. 42 which is in the same cluster and has almost the same period. Odd and even cycles are drawn separately in Fig. 13. There is a difference of about .3 in the phase of the falling branch. There is no doubt that this is real. It was strongly suspected on the basis of the 1956 observations and confirmed by two plates specially taken on each of two successive cycles in March and April 1957. When the velocity declines late the secondary maximum in the light curve is strong. This must indicate that this is the real maximum of the star and that after a deep minimum the new wave or surge of gas is late in arriving. The ~~decline in~~ velocity amplitude is 60 km/sec, but it is no more than 50 in any one cycle.

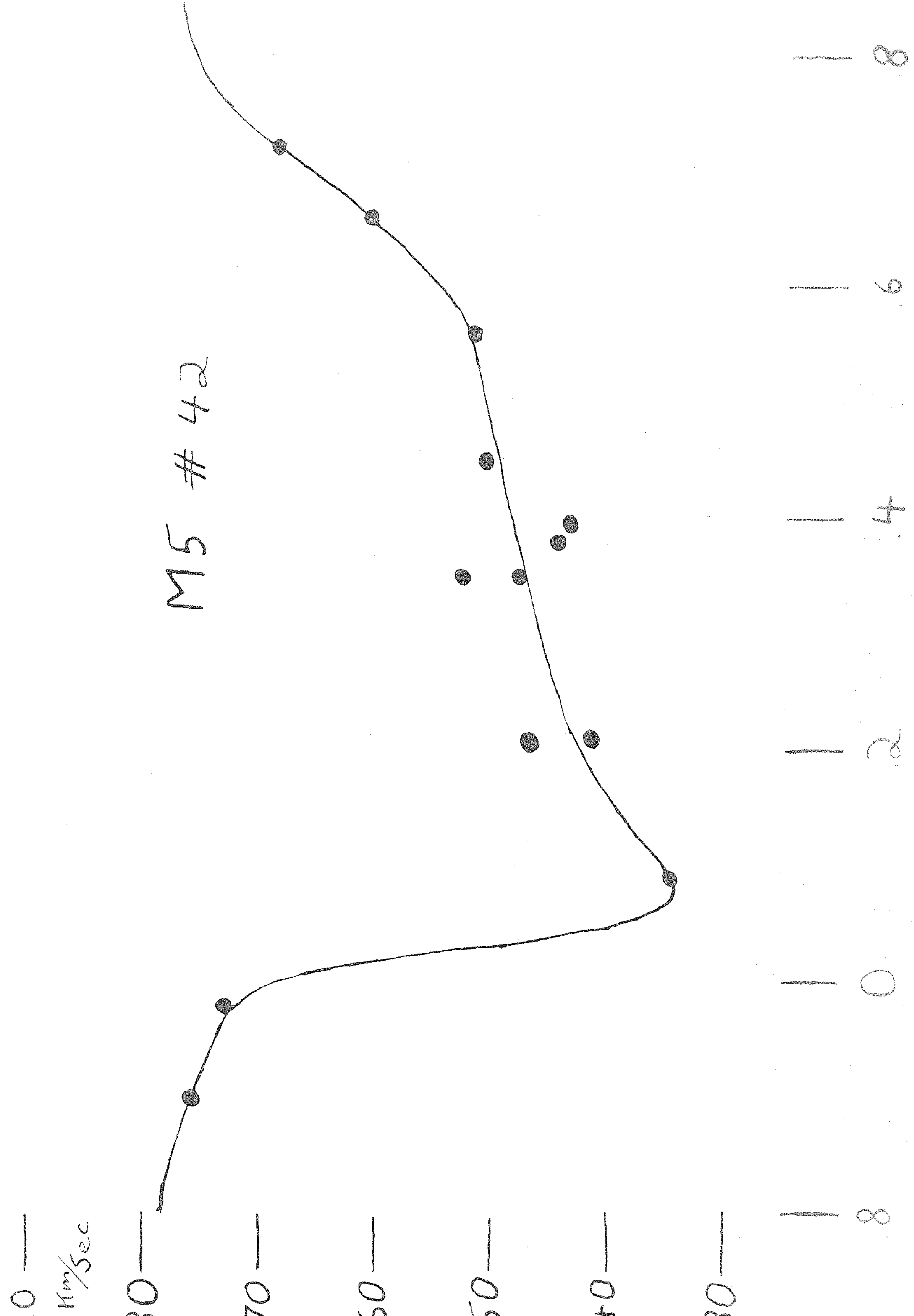


Fig. 12.

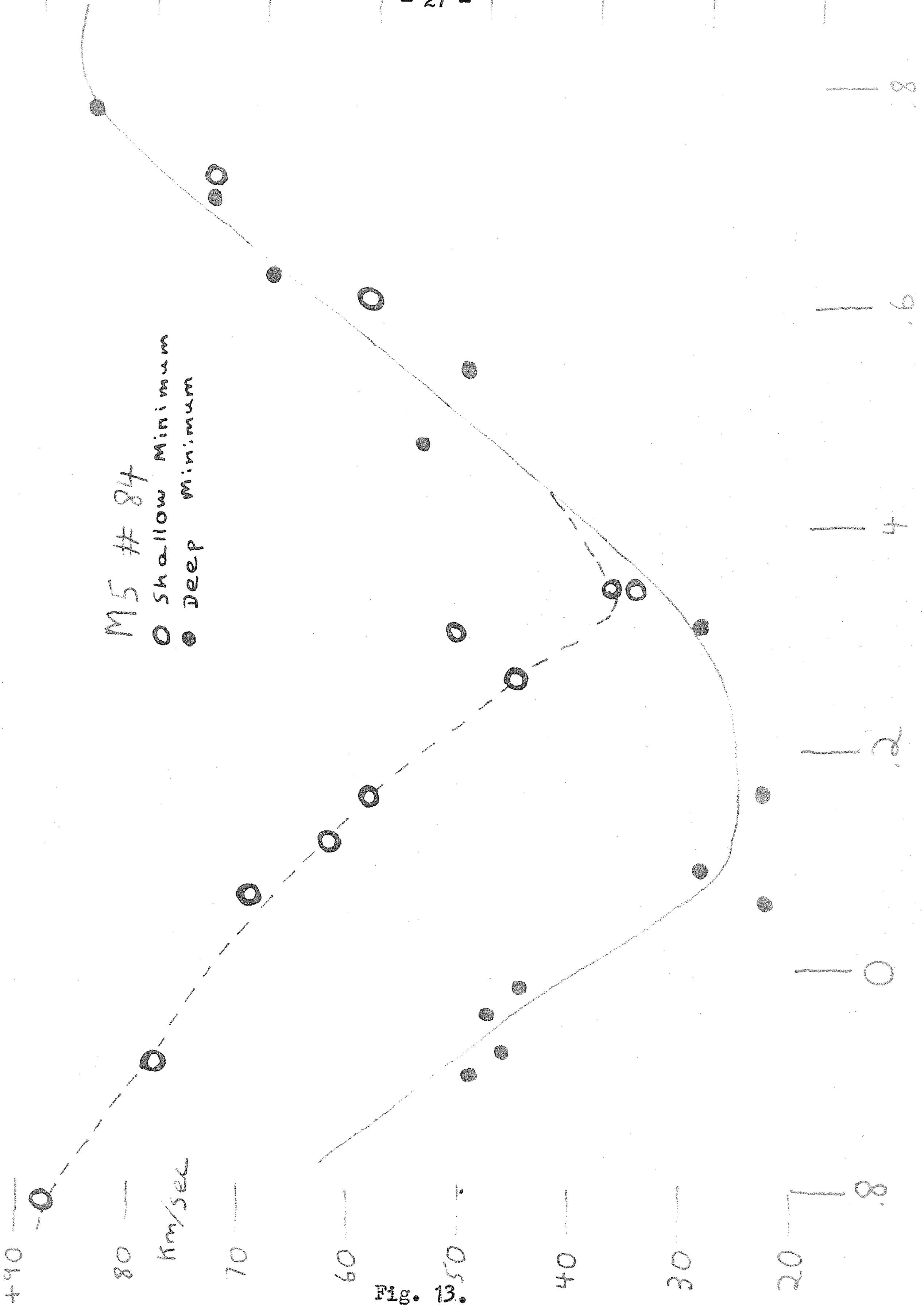

M5 # 84
O Shallow Minimum
● Deep Minimum
km/sec
+90
80
70
60
50
40
30
20
.8
.6
.4
.2
0
.8

Fig. 13.

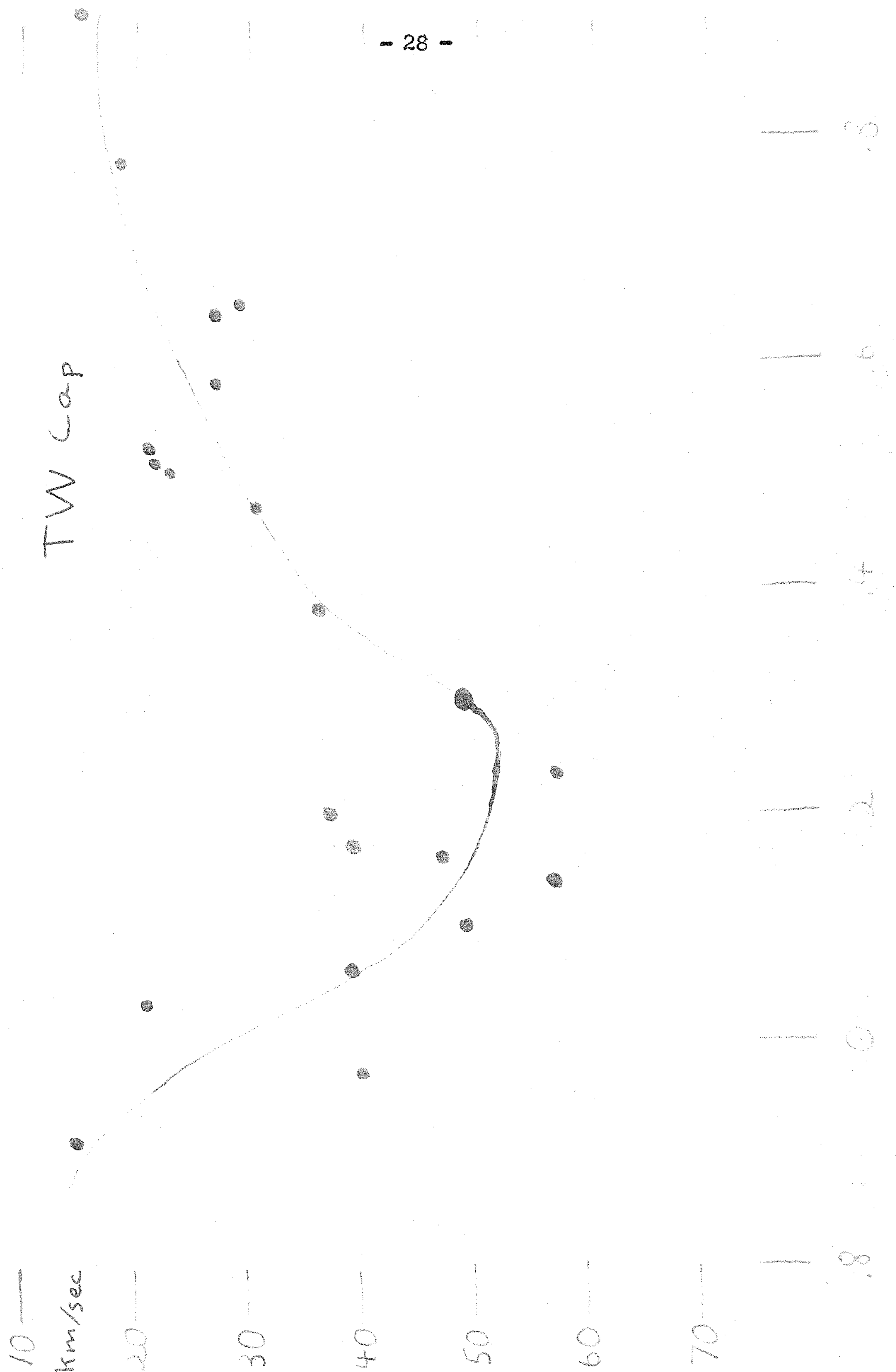


Fig. 14.

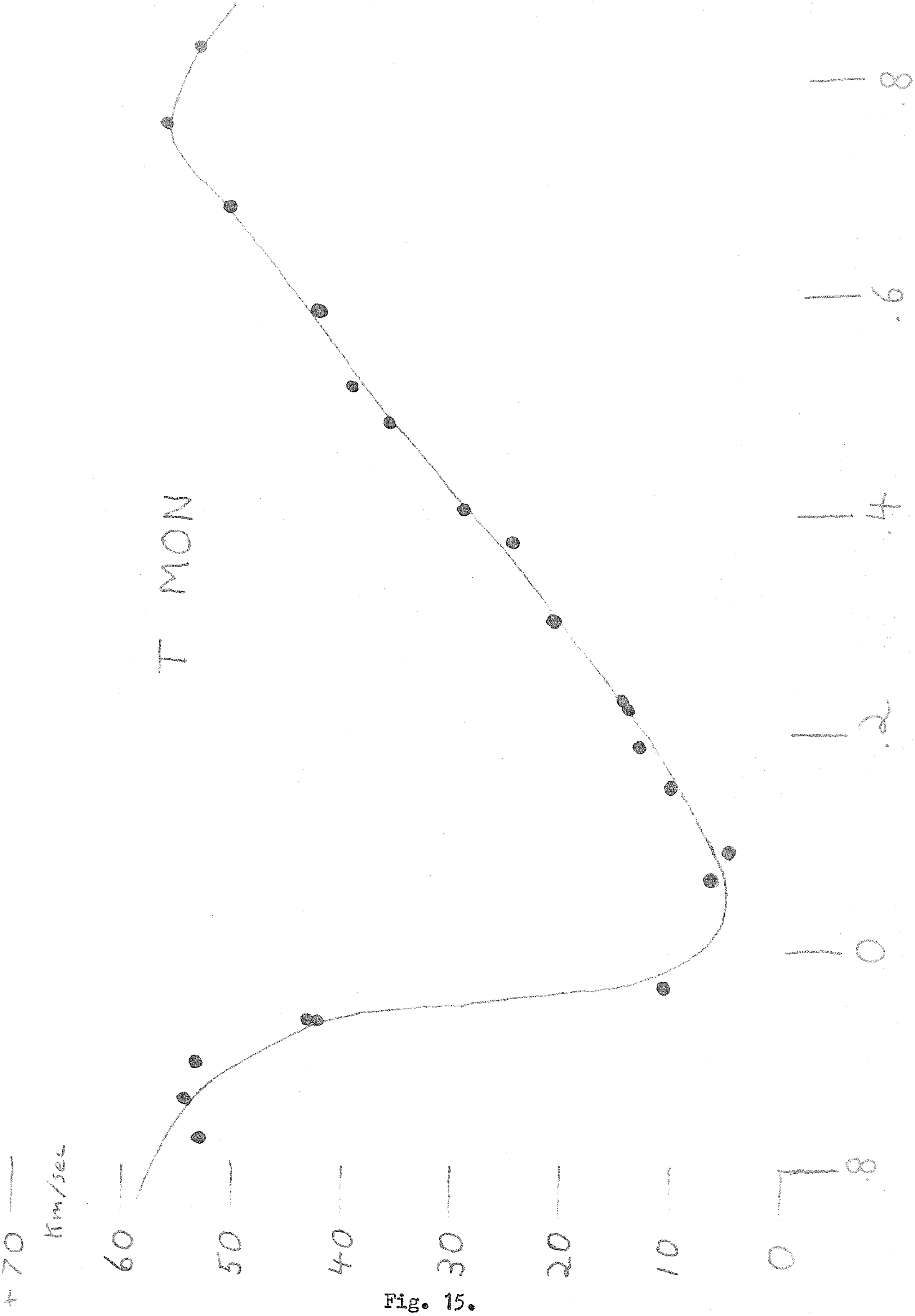


Fig. 15.

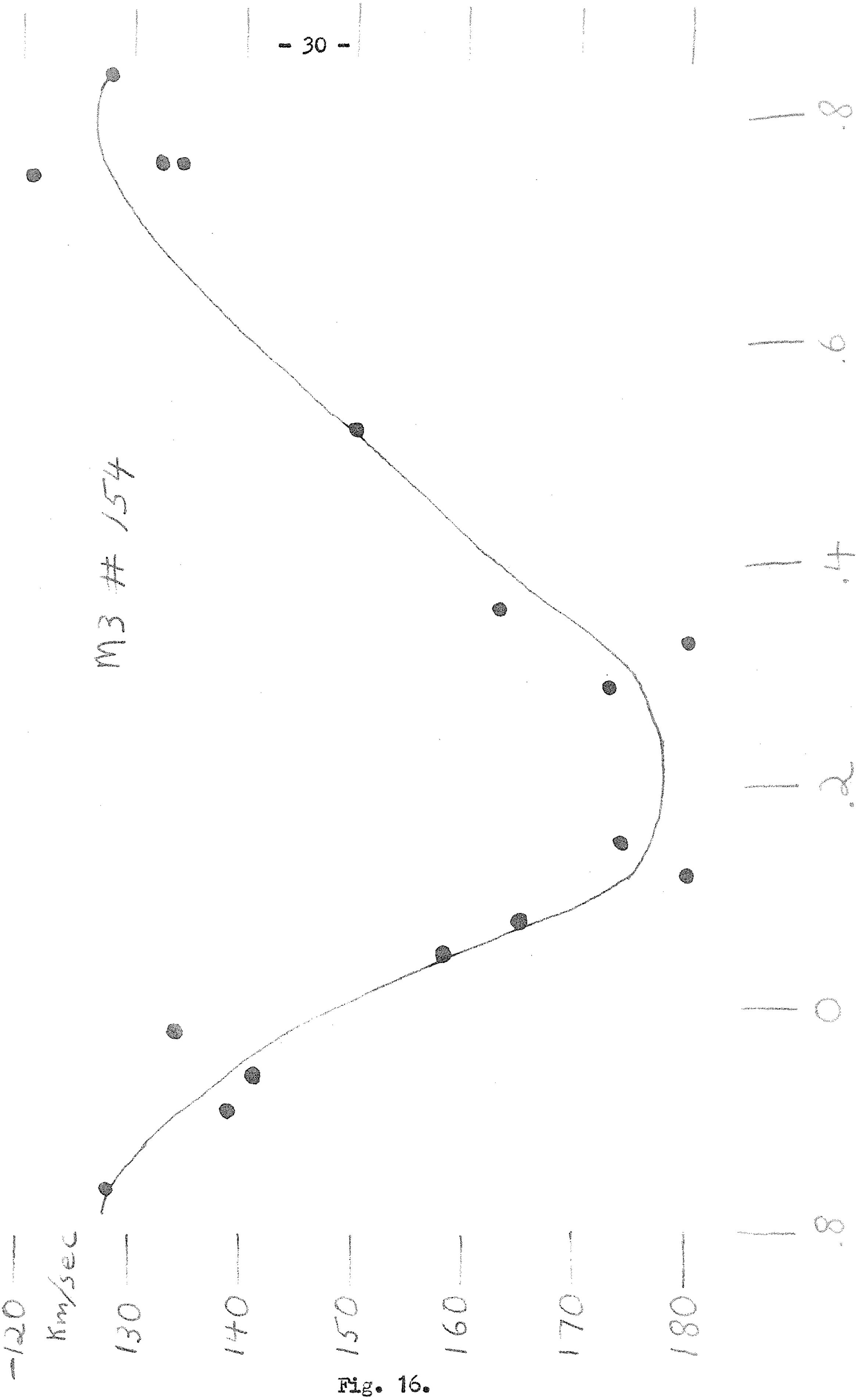


Fig. 16.

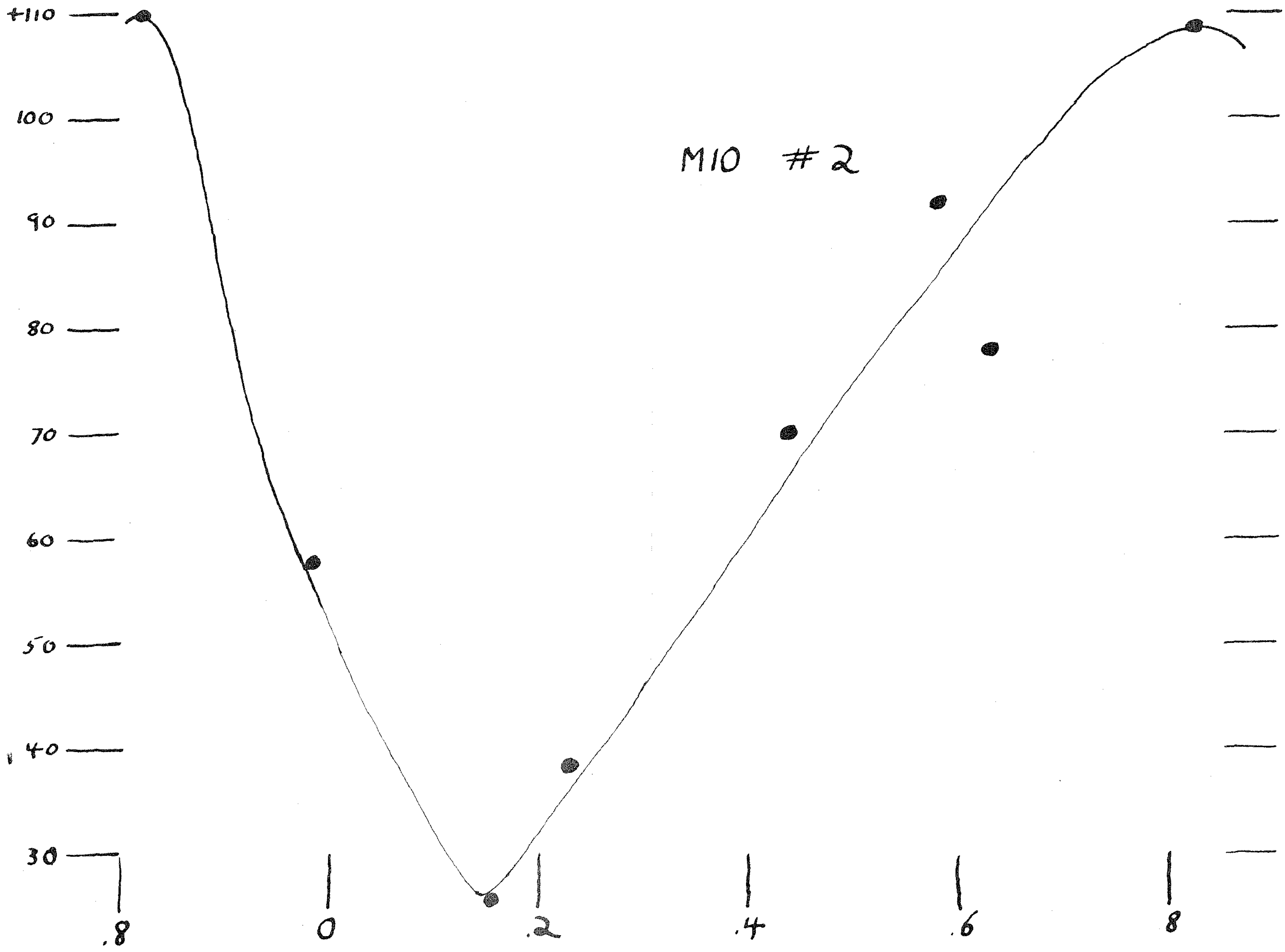
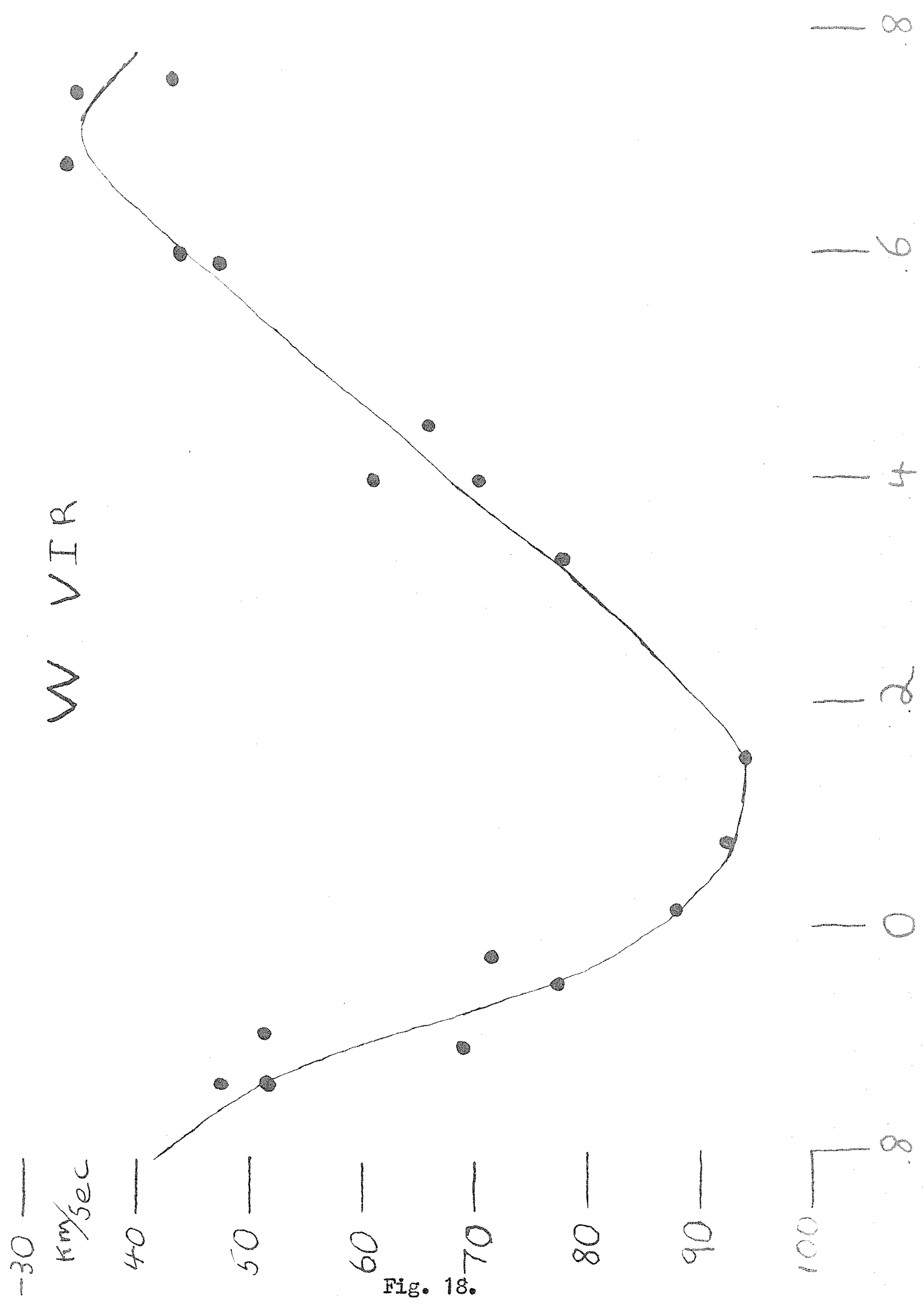


Fig. 17.



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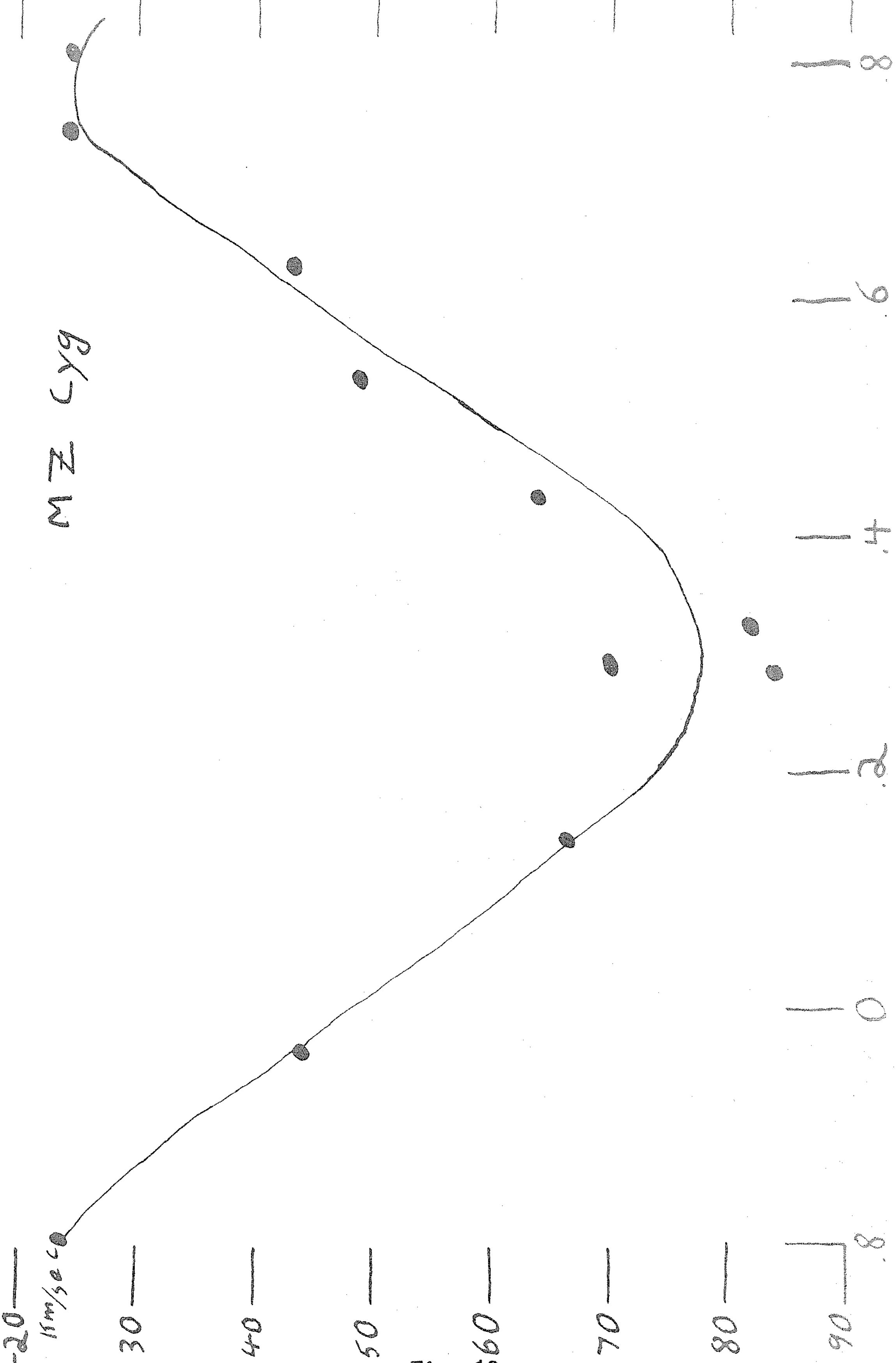

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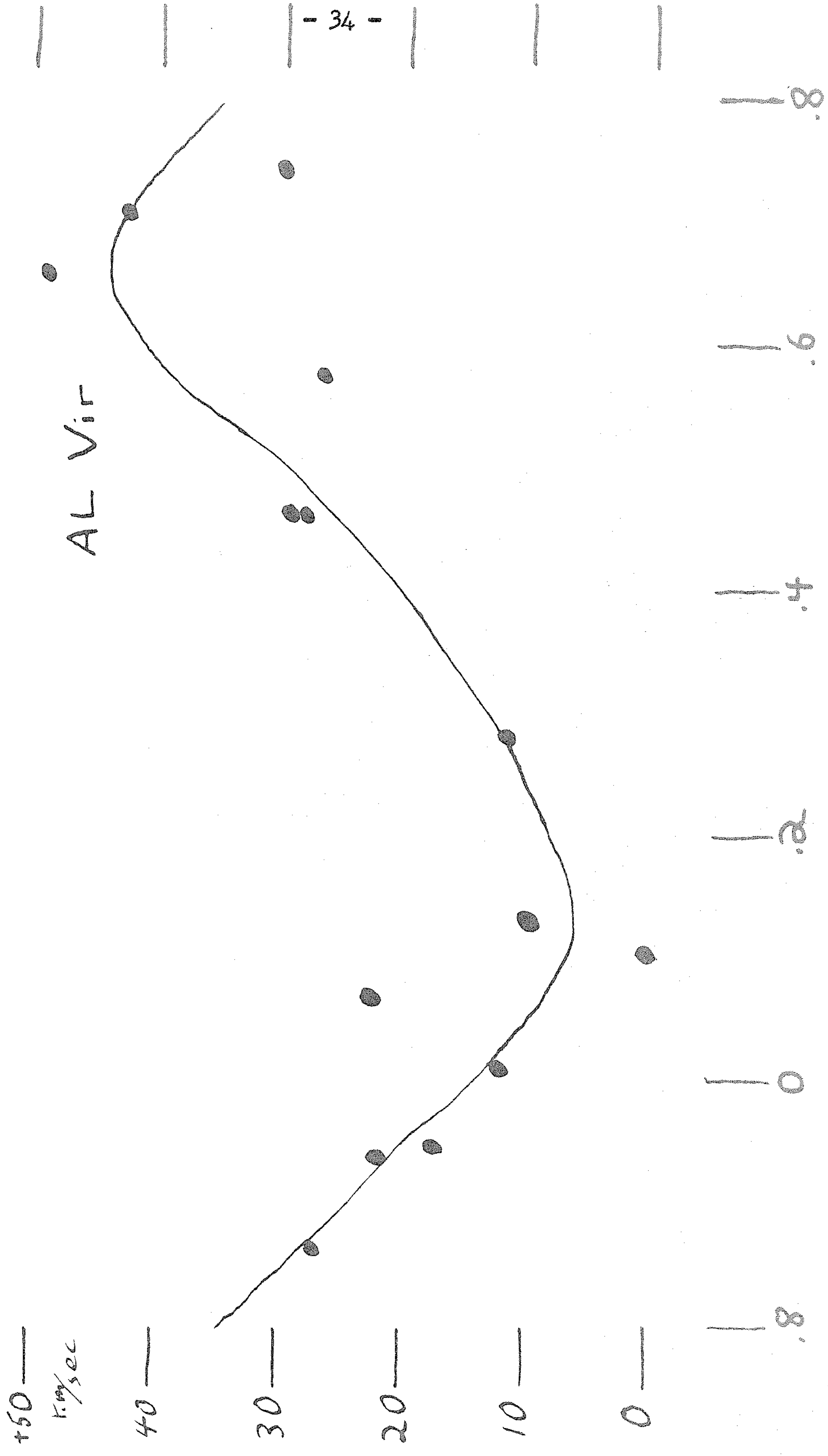

Fig. 20.

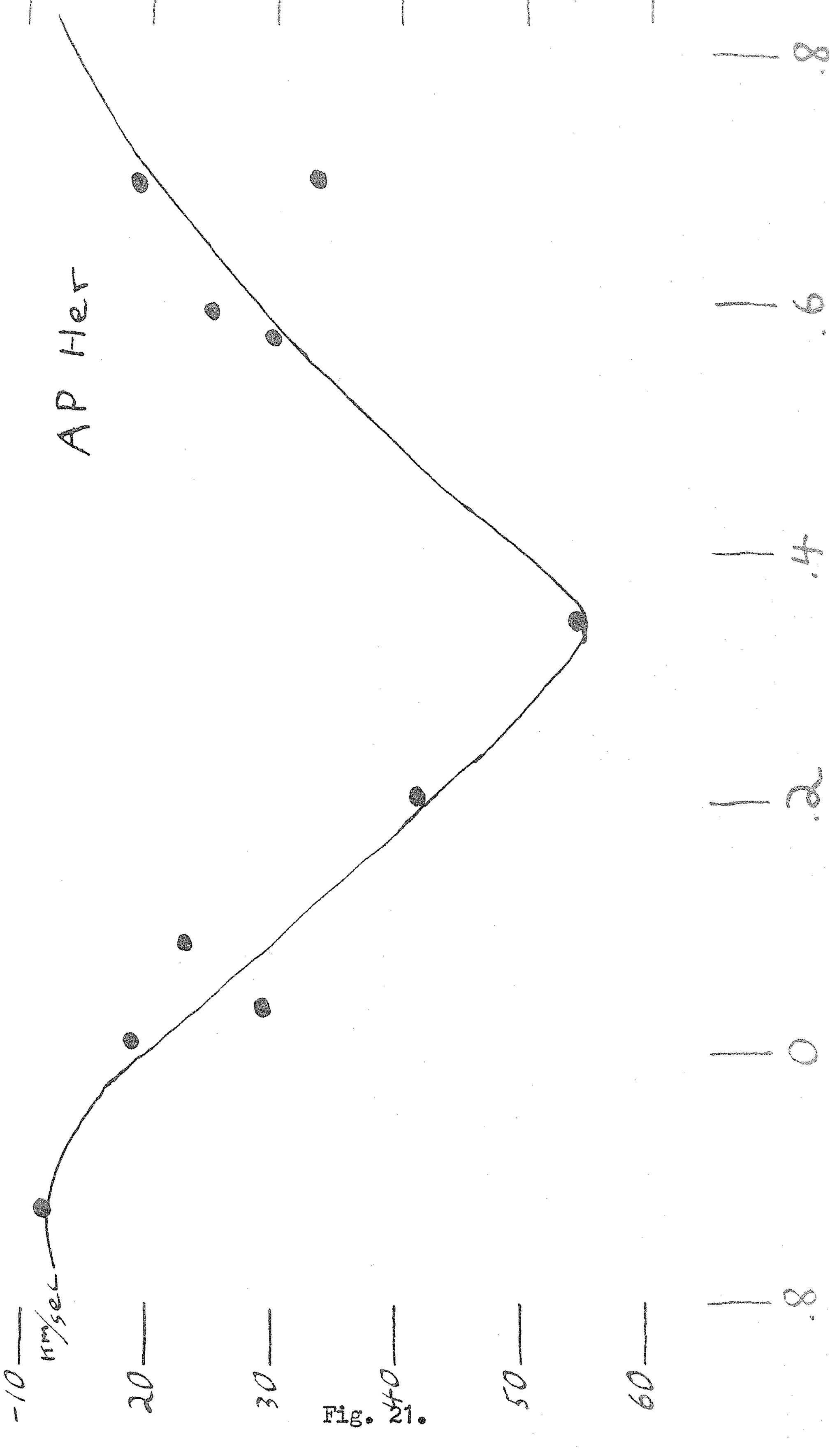

Fig. 21.

Although there are a large number of plates available of TW Cap the curve is poorly determined due probably to non-repetition from cycle to cycle. This non-repetition is not a regular alternation of odd and even cycles as it is for M5 No. 84. The velocity amplitude is about 40 km/sec.

The velocity curve of the classical cepheid T Mon is discussed in detail by Sanford(19). It is very closely the image of the light curve which is characteristic of classical cepheids. The fall is rapid corresponding to the rapid rise in the light curve. The velocity amplitude of the neutral iron lines is about 50 km/sec, while that of ionized strontium is close to 65 km/sec. The matter of the velocity differences for different elements and ions noted by Sanford will be discussed later.

M10 No. 2 has a very large amplitude, 80 km/sec, but this depends upon only one plate of Joy, E 1456, which showed a velocity of + 109 km/sec and is a plate of good quality considering the low dispersion, 110 A/mm at $H\gamma$. If this plate is spurious the amplitude of the velocity curve is still 70 km/sec which is quite large. Otherwise, the curve appears to be similar to the other stars.

Both AL Virginis and AP Herculis have velocity curves with an amplitude of about 40 km/sec and not too different from sinusoidal in shape but possibly with sharp corners. Their light curves are similar also.

B - Double Lines and Discontinuous Velocity Curves

It is at this point that the presence of double lines must be discussed. Their presence in W Vir is well known, and they may be considered as one of the important differences between population II and classical cepheids. The presence of double lines in the classical cepheid X Cygni has been reported by Kraft$^{(20)}$, but the appearance of the spectrum at the phase of doubling is completely different from that of W Vir.

There is no question as to the presence of double lines for M5 No. 42. They were first noted on X_d 1417, which required an exposure of 14 hours. This was done on two successive nights during which the total change in phase was .05 which is sufficiently short. Plate P_d 2635 obtained by Munch is of excellent quality and shows that nearly all strong unblended lines are double.

Two Palomar plates of No. 84 fail to show double lines. P_d 2587 was taken at the phase at which they were to have been expected. The strong lines are extremely broad but are definitely not double. This plate was taken at a late maximum when, according to Fig. 13, the amplitude of doubling is expected to be less than for an early maximum. An attempt to get a plate on the other cycle was frustrated by the weather. Now Abt$^{(21)}$ in his study of the RV Tauri star U Monocerotis found line doubling of only about 35 km/sec. This would be at the very limit of detectability at 18 A/mm especially if the lines had appreciable intrinsic width. Considering this and evidence to be presented in Section X-A, in the figures that follow, the velocity curve of M5 No. 84 will be shown double with as small an amplitude of doubling as appears reasonable.

Double lines appear on only one plate of TW Cap, X_d 1823, obtained in 9 hours on two nights on Mt. Wilson. The presence of double lines does not assist in drawing a mean curve through the badly scattered points.

Thus double lines are present on stars with either the W Vir type light curve or the M5 No. 42 type light curve. I will now assume that the other stars with a period greater than 15 days would also show double lines if it were possible to obtain plates of sufficiently high dispersion. To determine the duration of line doubling is extremely difficult observationally. While the presence of double lines can be discovered at maximum light, any determination of the time at which double lines first appear involves high dispersion work at the vicinity of minimum light when the star is at least a magnitude fainter. I will draw the curves under the assumption that double lines are present during the declining branch of the velocity curve as is known to be the case for W Vir.

There is at present no evidence for double lines in stars of period less than about 15 days. The velocity curve of AL Vir does not show a rapid decline typical of stars with double lines. This star is bright enough that a search for double lines is well within the realms of possibility, and such a search would be important even if it yielded negative results. The velocity differences between hydrogen and neutral iron in T Mon can be interpreted as indicating double lines are present but blended or somehow obscured. Kraft's discovery of double lines in X Cyg and certain spectrophotometric evidence to be presented later support this. I will therefore show T Mon with a discontinuous velocity curve also.

The discontinuous velocity curves are shown in Fig. 22-30.

VI. THE γ VELOCITY OF THE STARS

Joy(8) first noted that in the mean the cepheids in globular clusters had the same mean velocity as the cluster in which they were found ± 10 km/sec. This is of the greatest importance when integrating the discontinuous velocity curves. In order to determine the γ velocity of the stars in M5 all possible data for the velocity of the cluster were assembled.

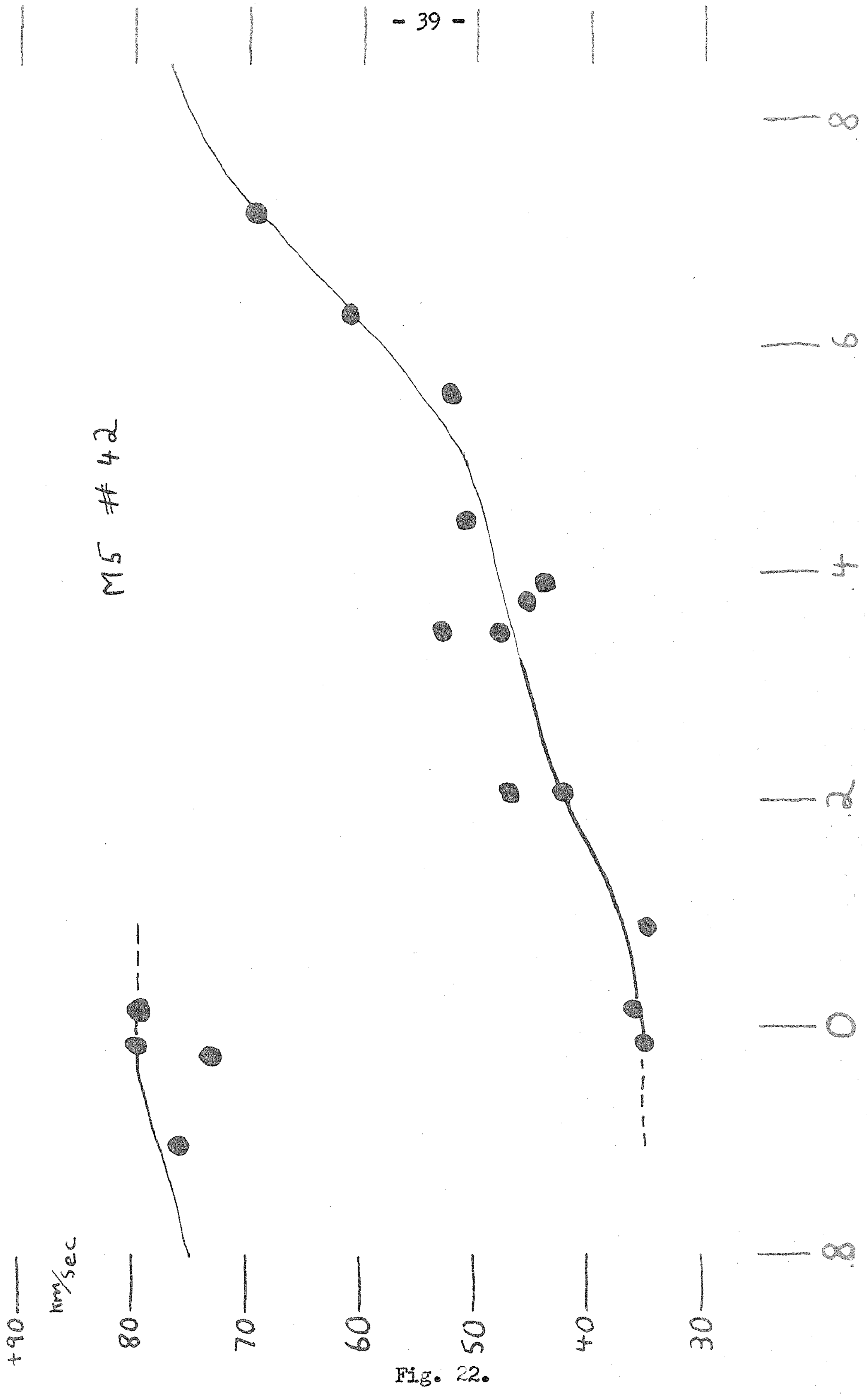

Fig. 22.

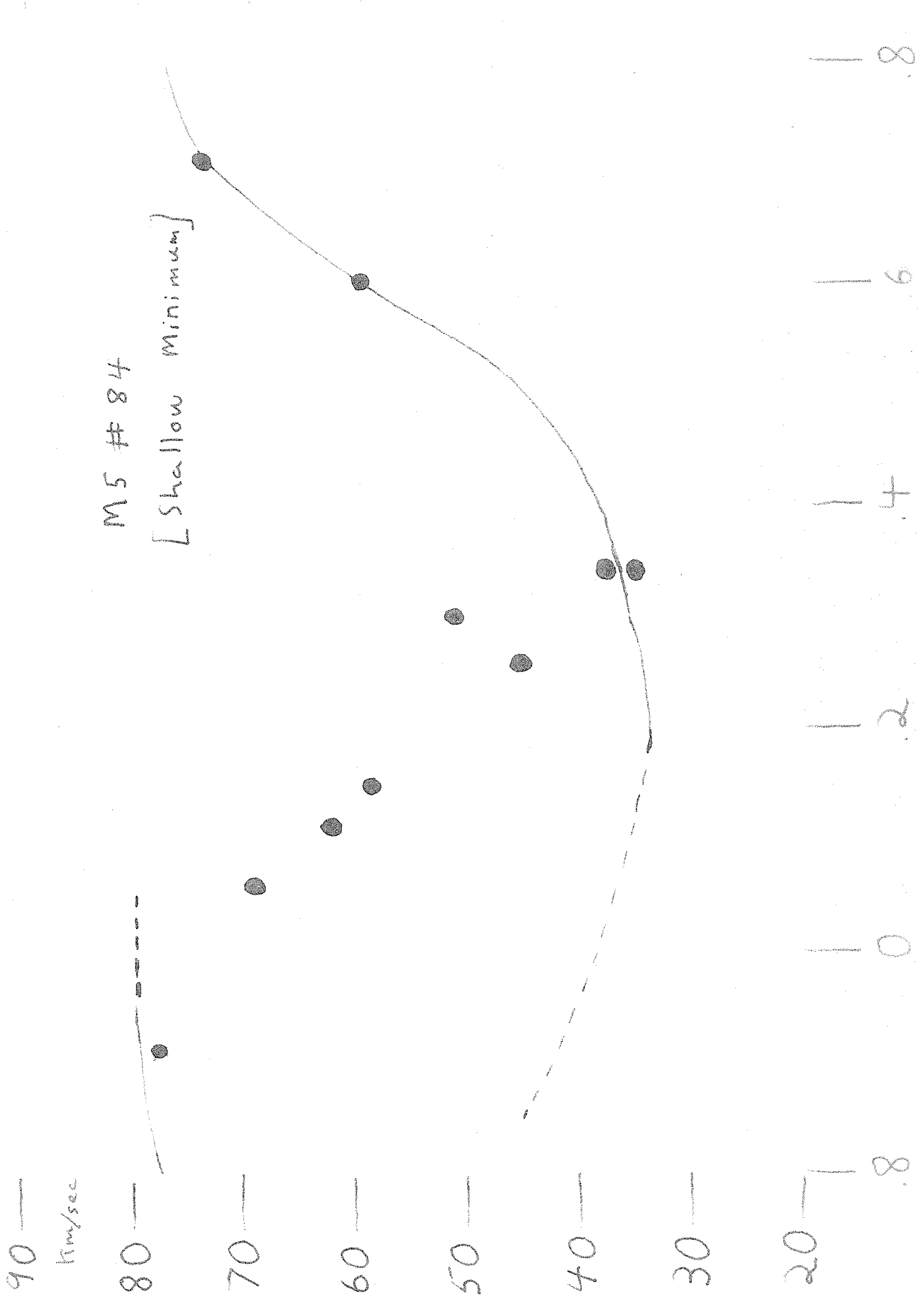

Fig. 23.

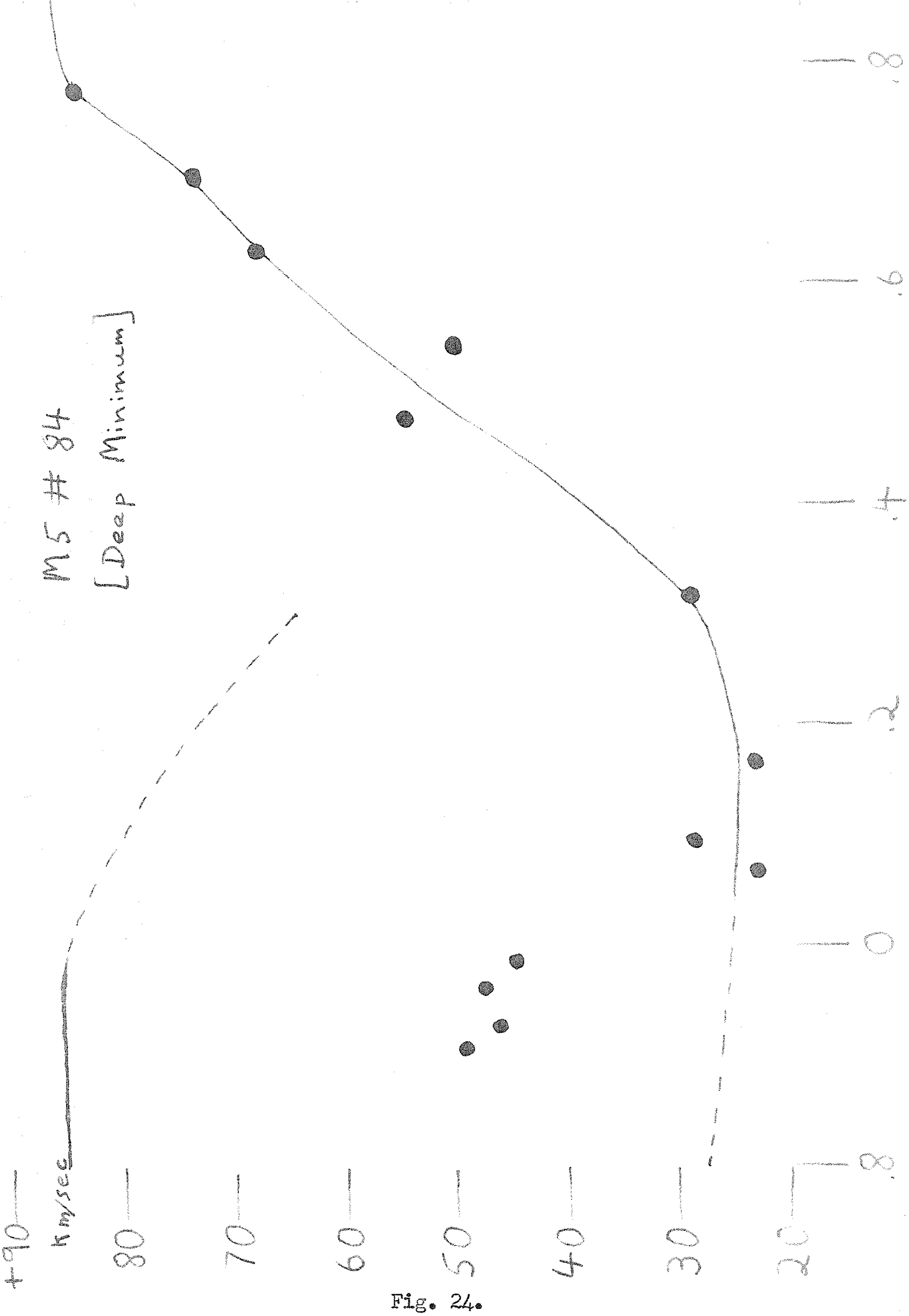

Fig. 24.

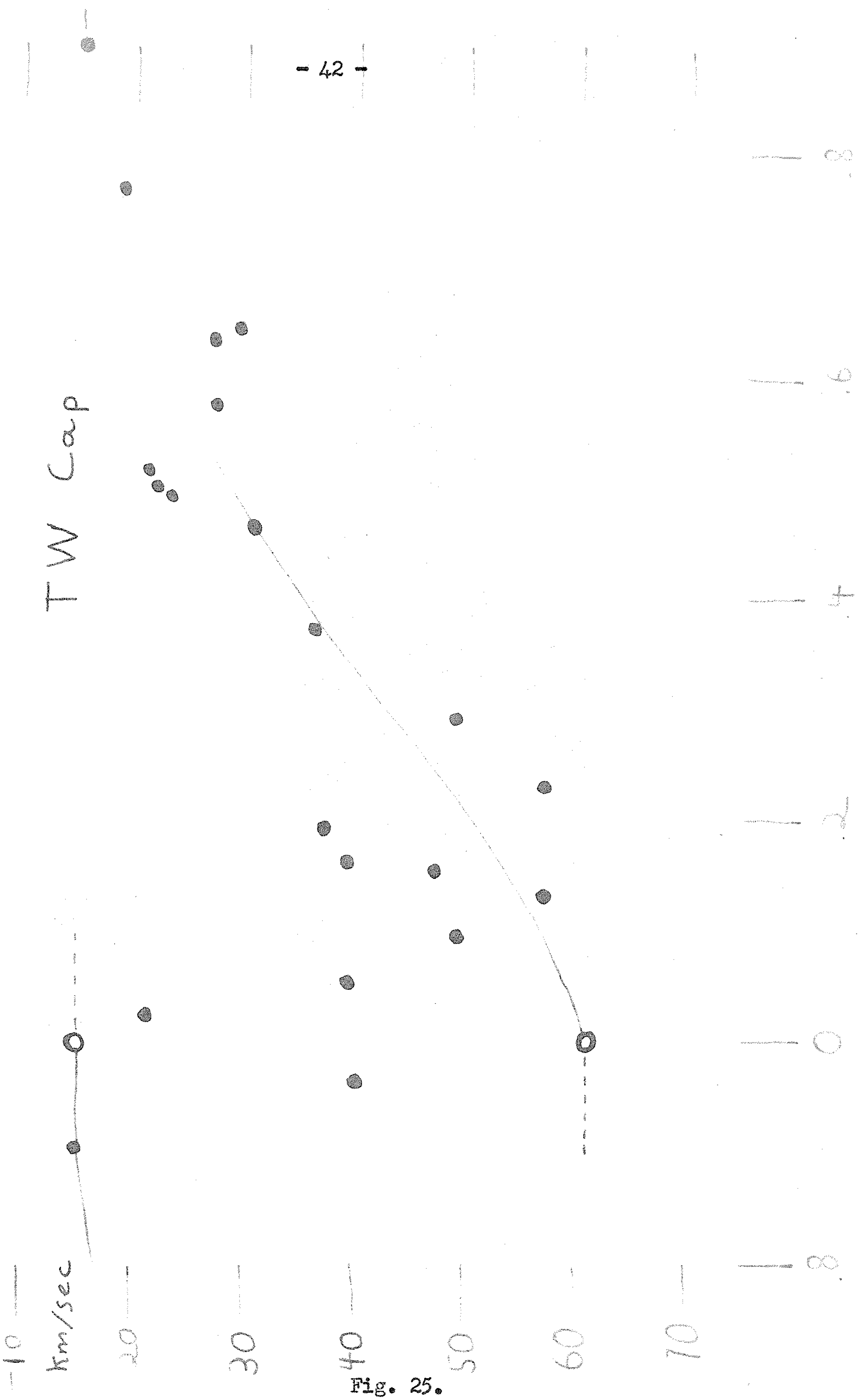

Fig. 25.

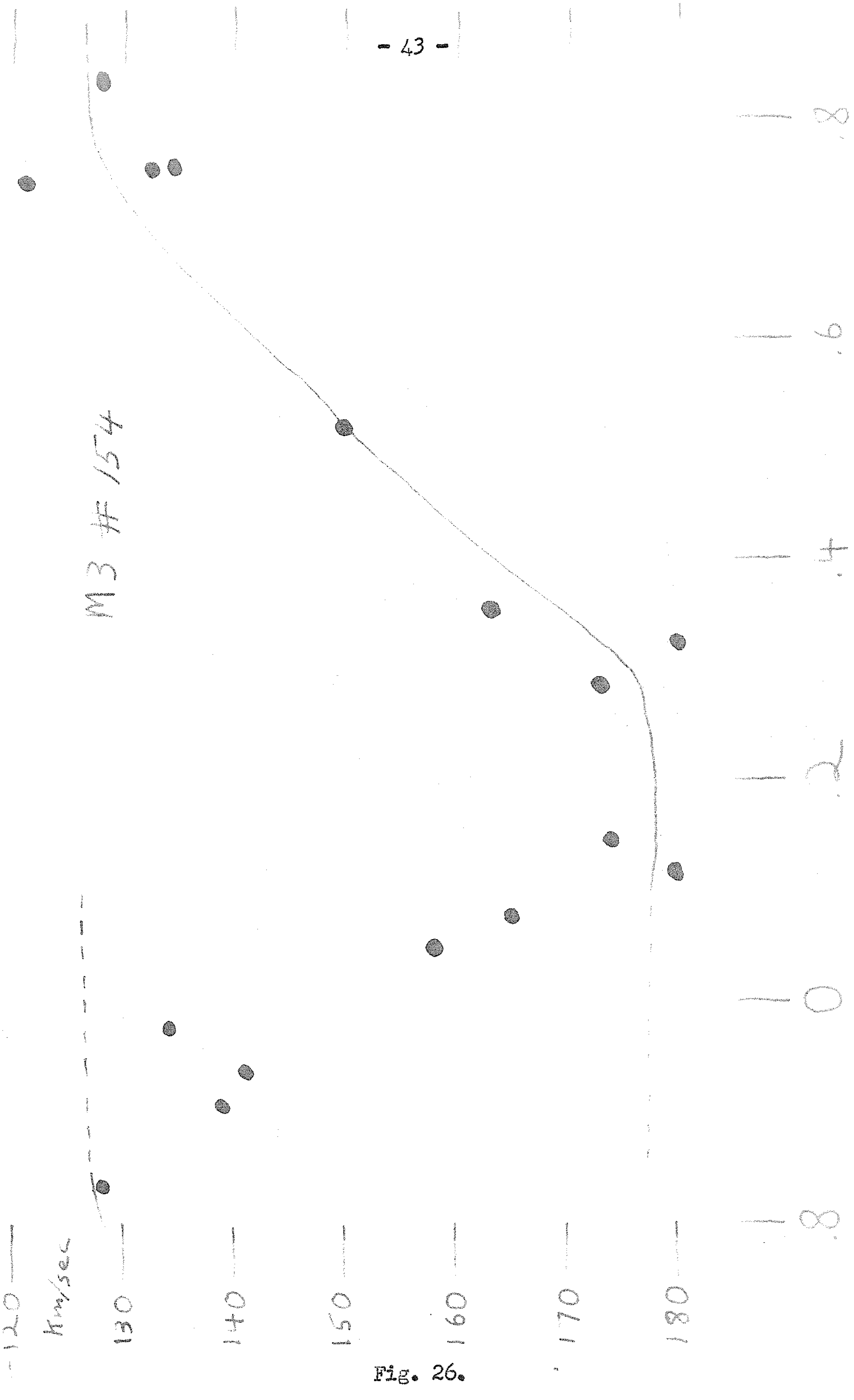

Fig. 26.

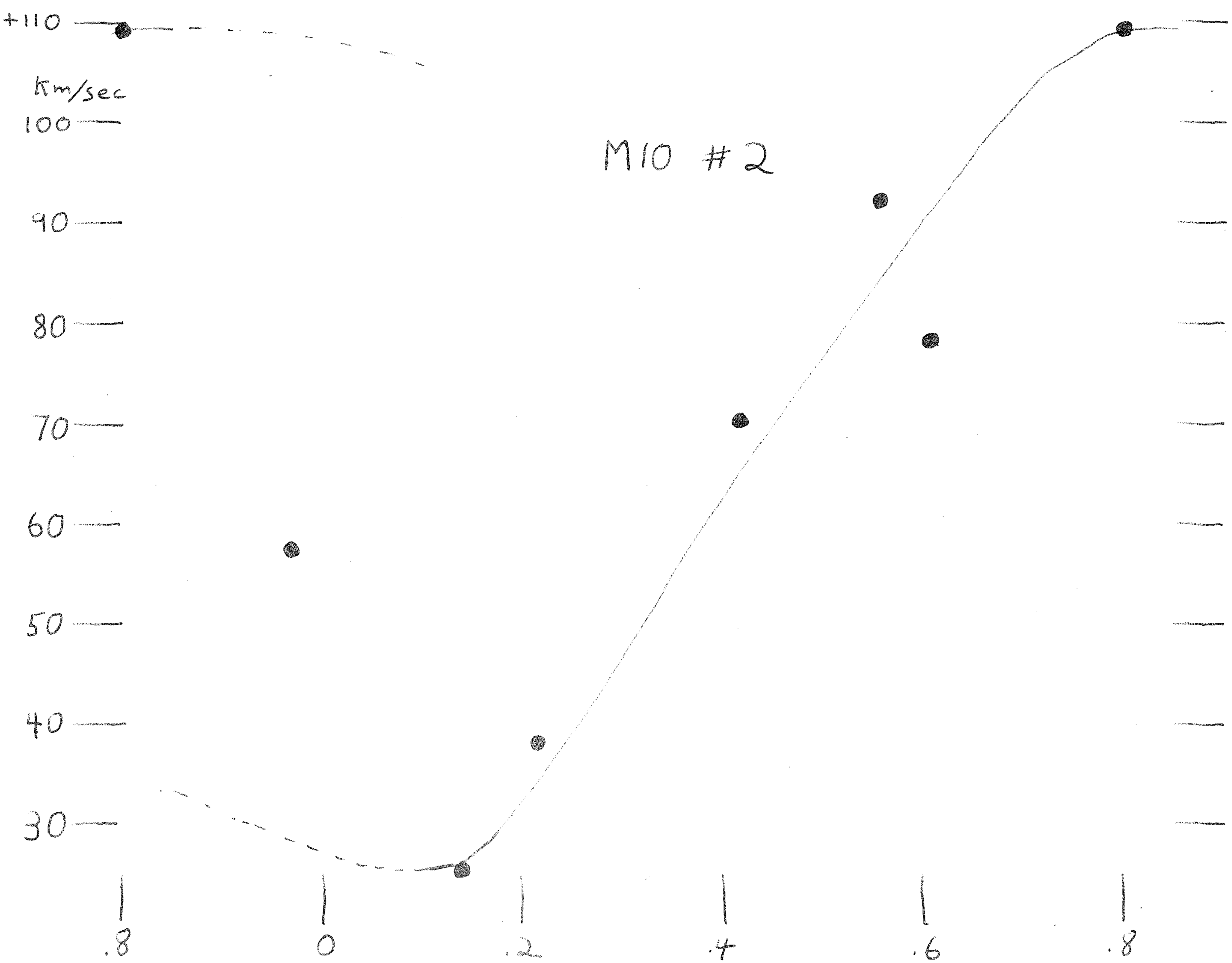

Fig. 27.

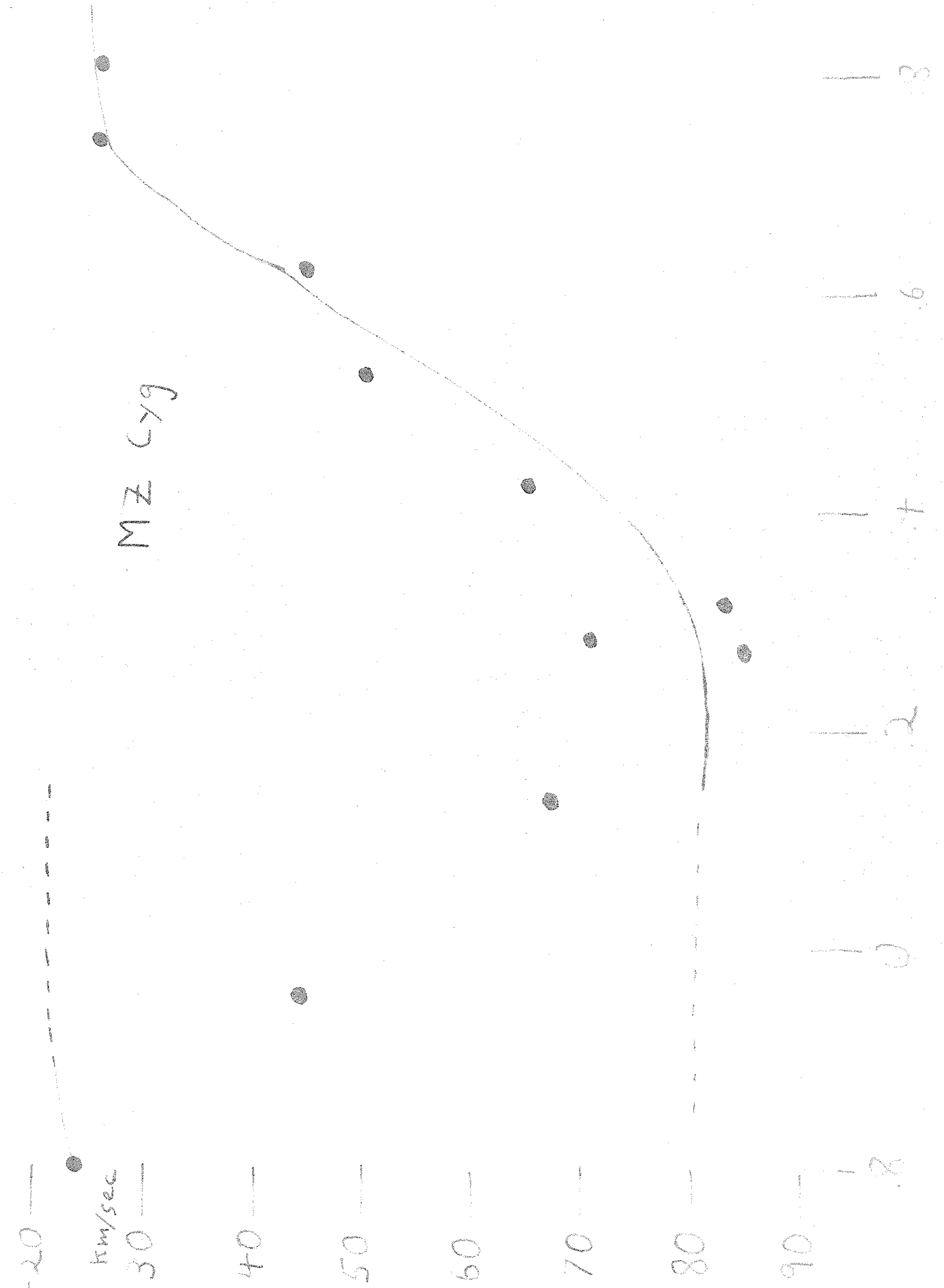

Fig. 28.

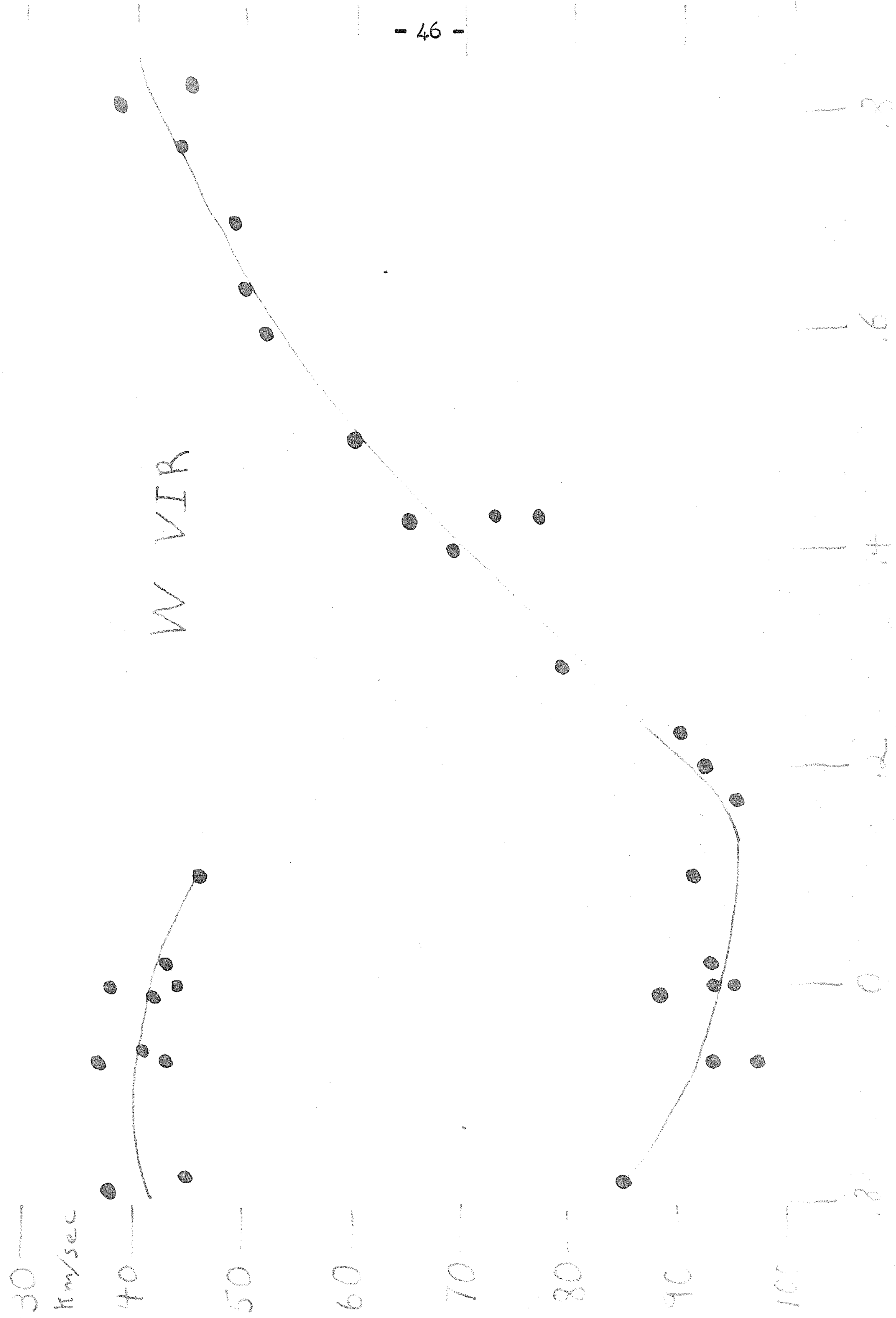

Fig. 29.

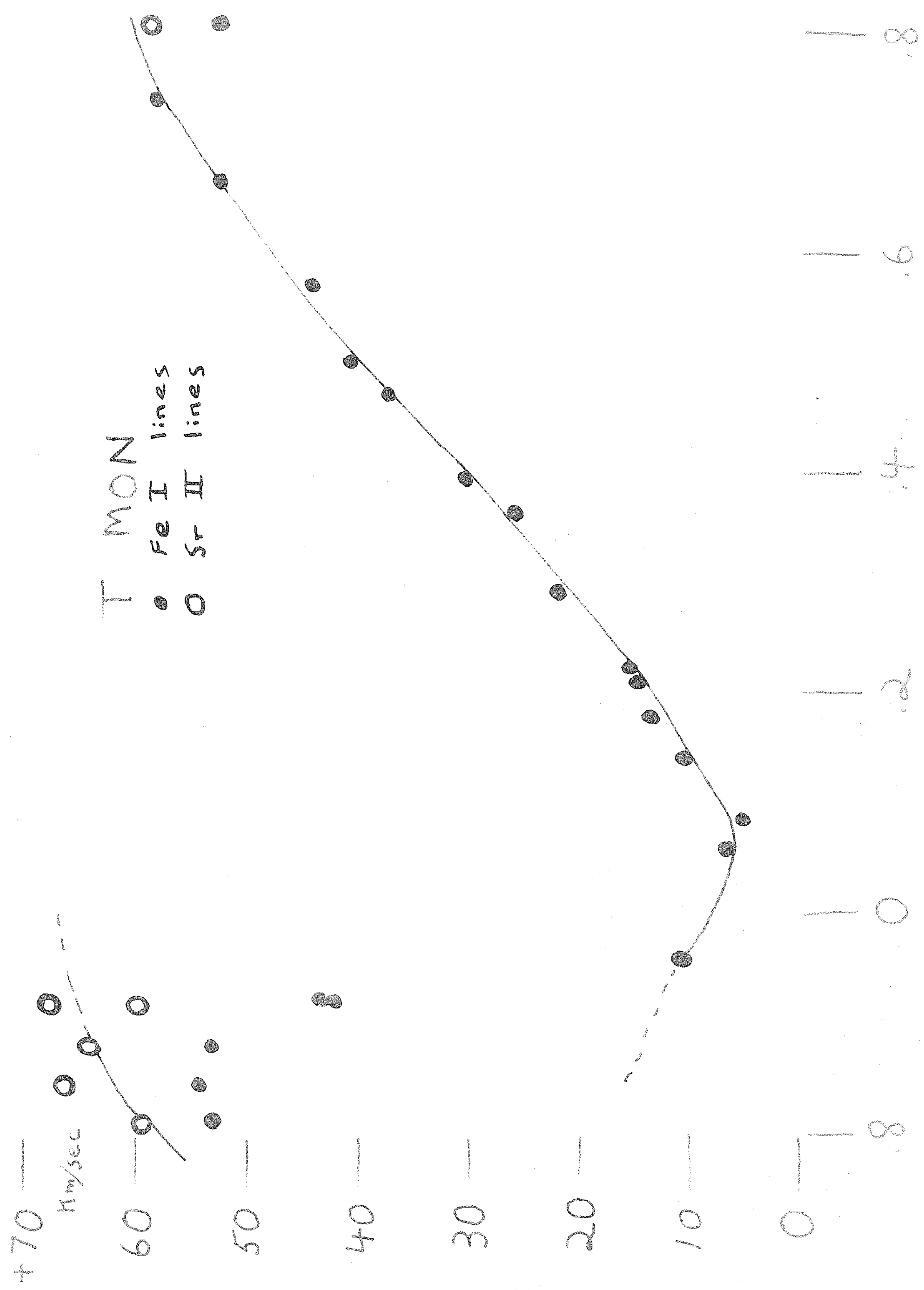

Fig. 30.

Four low dispersion and one higher dispersion plate of Mayall are available[(22)]. Three plates of individual red giants in the cluster have been taken, two by O. C. Wilson and one by A. J. Deutsch who have kindly loaned them to me for velocity measurements. In addition one plate of the integrated light of the cluster was taken at the 60" Cassegrain spectrograph using the 4" camera. These data are summarized in Table 3.

It is not obvious how to combine such diversified material. The mean velocity was computed using three weighting methods: 1) all of equal weight, 2) integrated spectra given double weight, 3) weights assigned inversely to the probable errors. The resulting means are respectively 54, 55, and 55 km/sec. This is quite gratifying. The median velocity of M5 No. 42 is + 57 km/sec. and of M5 No. 84 is + 54 km/sec. Considering that the expected deviation of individual stars in a globular cluster is roughly 6 km/sec[(23)], it seems very likely that we are observing no systematic inward or outward motion in population II cepheids greater than a few kilometers. It can be safely stated that $V_\gamma = 0 \pm 6$ km/sec; to pin this down more accurately would involve a very extensive program of observation.

VII. DISPLACEMENT CURVES

We now have the necessary information to construct displacement curves for the stars. This is done most easily by numerically integrating the velocity curve at intervals of .05 of the period. It seems best to assume symmetrical expansion and contraction as was concluded in the previous section.

TABLE 3

The Radial Velocity of M5

Observer	Plate No.	Object	Dispersion (A/mm)	Velocity (km/sec)	P.E. (km/sec)	Remarks
Mayall		M5 Integrated	426	+ 56	14	mean of 4 plates
Mayall		M5 Integrated	130	+ 61	?	
Deutsch	P_e 2014	Apr IV-27	38	+ 58.1	1.3	
Wilson	P_e 888	Barnard 172	38	+ 51.6	2.1	
Wilson	P_e 889	Barnard 168	38	+ 52.5	1.2	
Wallerstein	X_f 2503	M5 Integrated	80	+ 49.7	2.9	

There is some question as to the meaning of this sort of integration. If the absorption lines are formed in the same layer throughout the cycle of observation then it is meaningful to say that the integration of the velocity curve yields true displacements. If, however, we are looking at absorption lines that are formed at varying depth in the atmosphere throughout the cycle the situation becomes more dubious. If the velocity is the same throughout this depth then the integration still has full meaning. This has been considered by Whitney(24) who concludes that for δ Cephei and η Aquilae it is legitimate to integrate the radial velocity curves. For W Vir Abt finds from studying the lines that the continuous opacity varies by as much as a factor 200. This is a non-negligible factor but it is not obvious that the procedure must be abandoned.

The question of comparing radius results derived from the lines with results derived from the continuum [i.e. the colors] is a much more dubious proposition and will be discussed later.

Integrated radii for the velocity curves of Figs. 22-30 are shown in Figs. 31-39. A surface velocity of 24/17 of the radial velocity was used(25). However, if the lines are formed in a shell that is far above the photosphere it is wrong to introduce this factor and the changes of radius shown here must be reduced by the factor 24/17. In studying U Mon Abt(21) chose not to use this factor. Where the radii appear to cross the old cycle is drawn in with a dashed line. Whether or not there is an actual penetration of shells depends upon the chosen model. This will be discussed in Section X-D.

Table 4 lists the velocity amplitudes and radius changes of Figs. 22-39. Also included are two typical cycles for U Mon. These have been corrected by the factor 24/17 to make them comparable with the other stars. The data show no correlation with period. The two RV Tauri stars

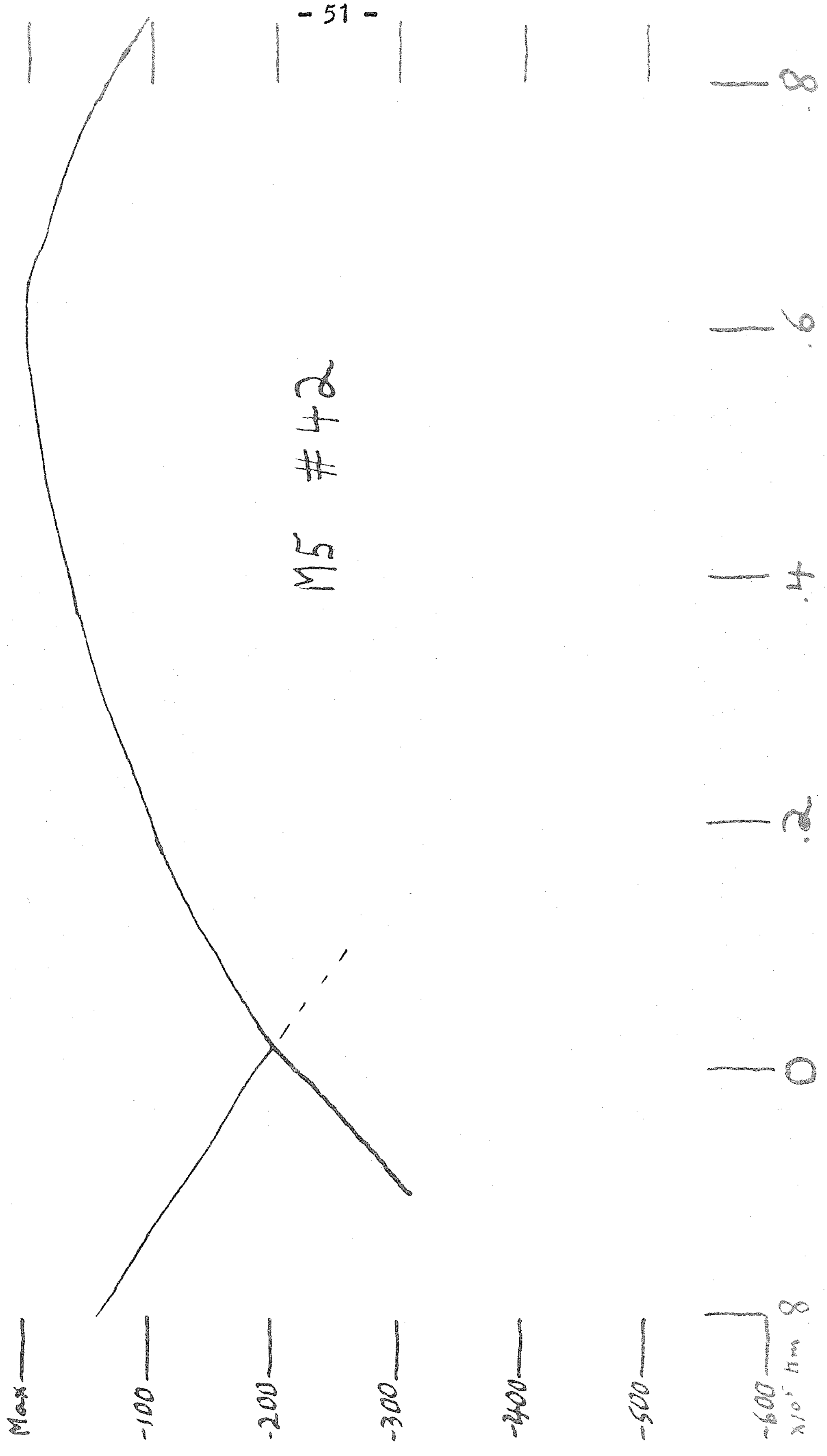

Fig. 31.

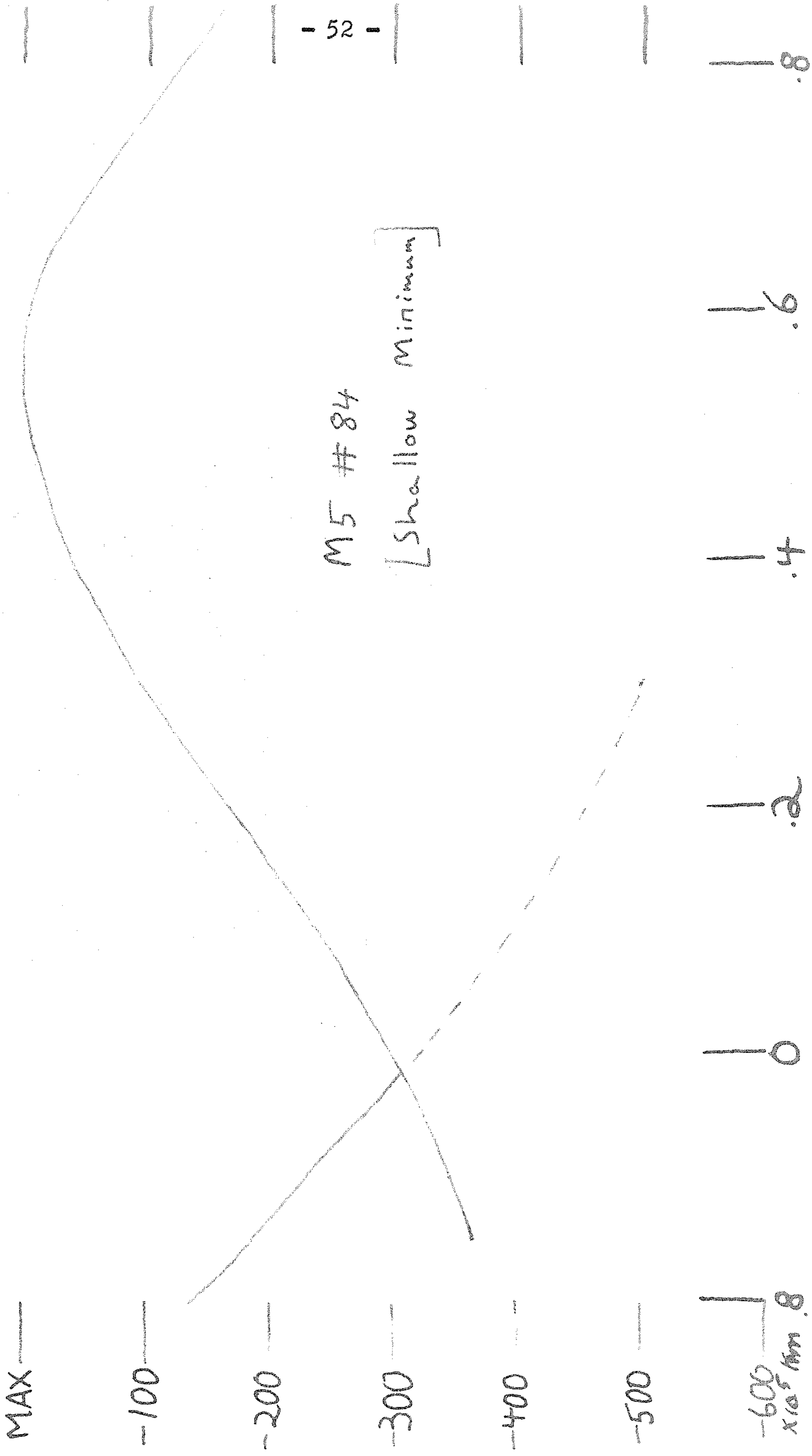
MAX
-100
-200
-300
-400
-500
-600
x10⁵ km
.8
.6
.4
.2
0
.8
M5 #84
[Shallow Minimum]

Fig. 32.

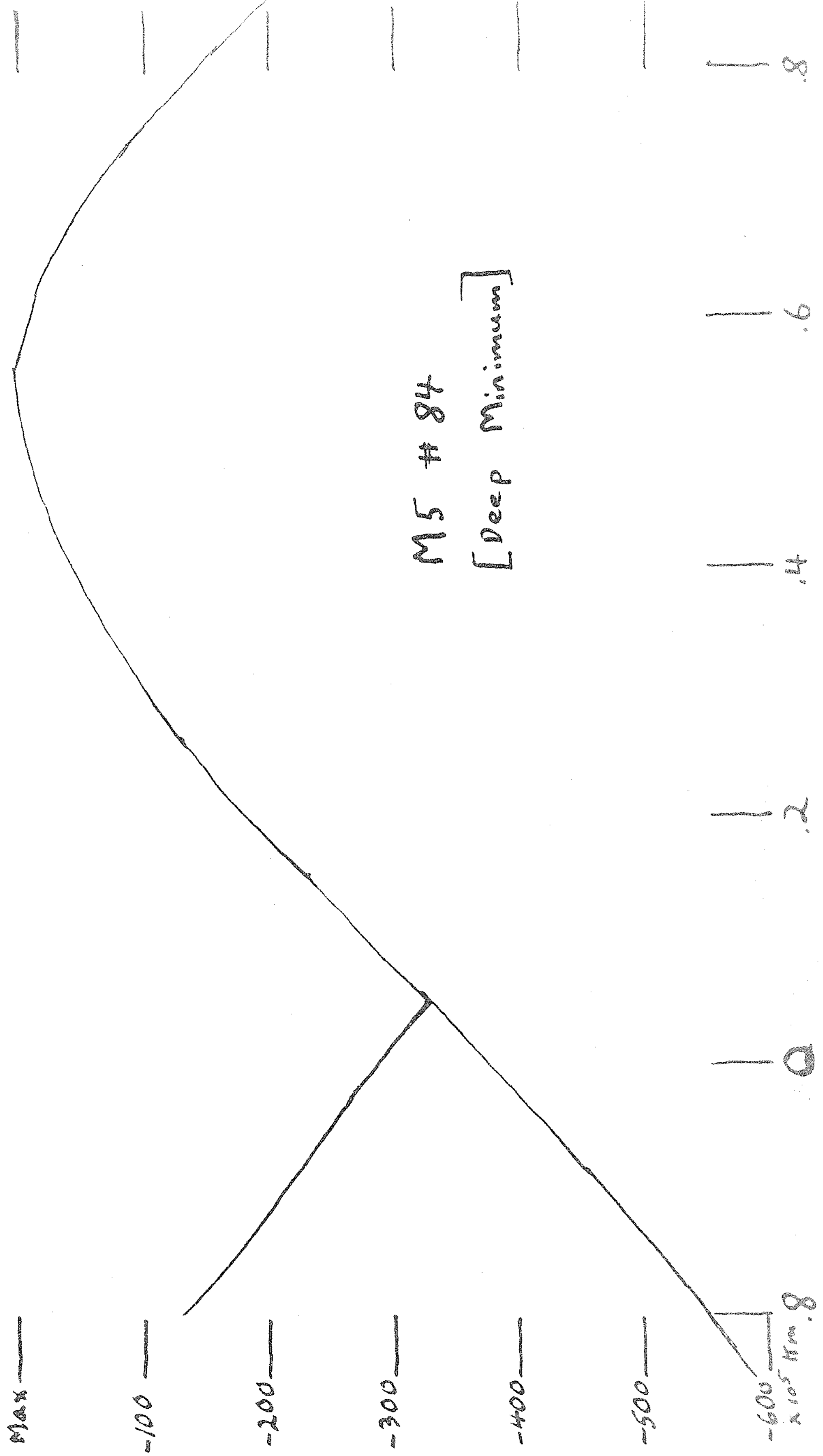

Fig. 33.

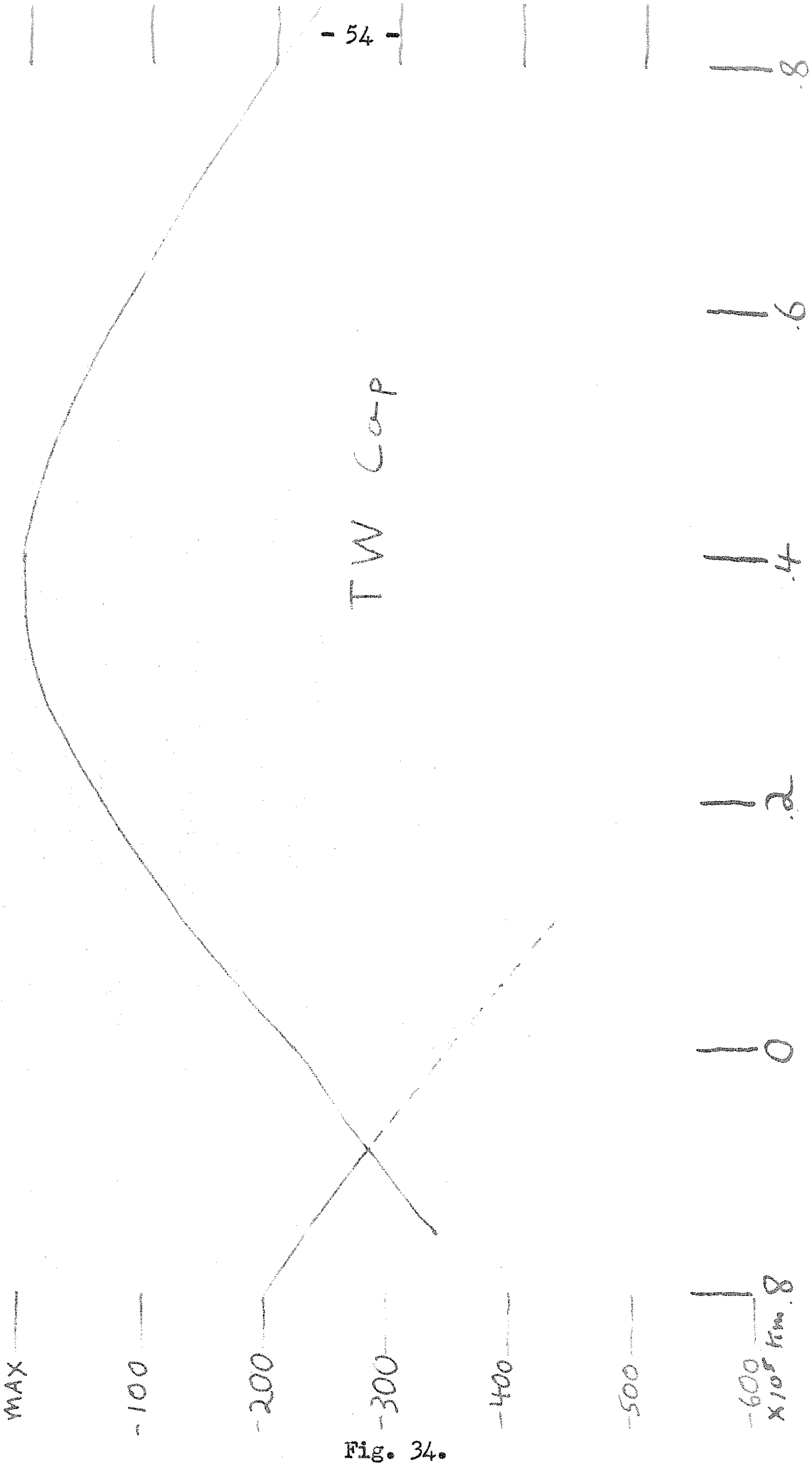

Fig. 34.

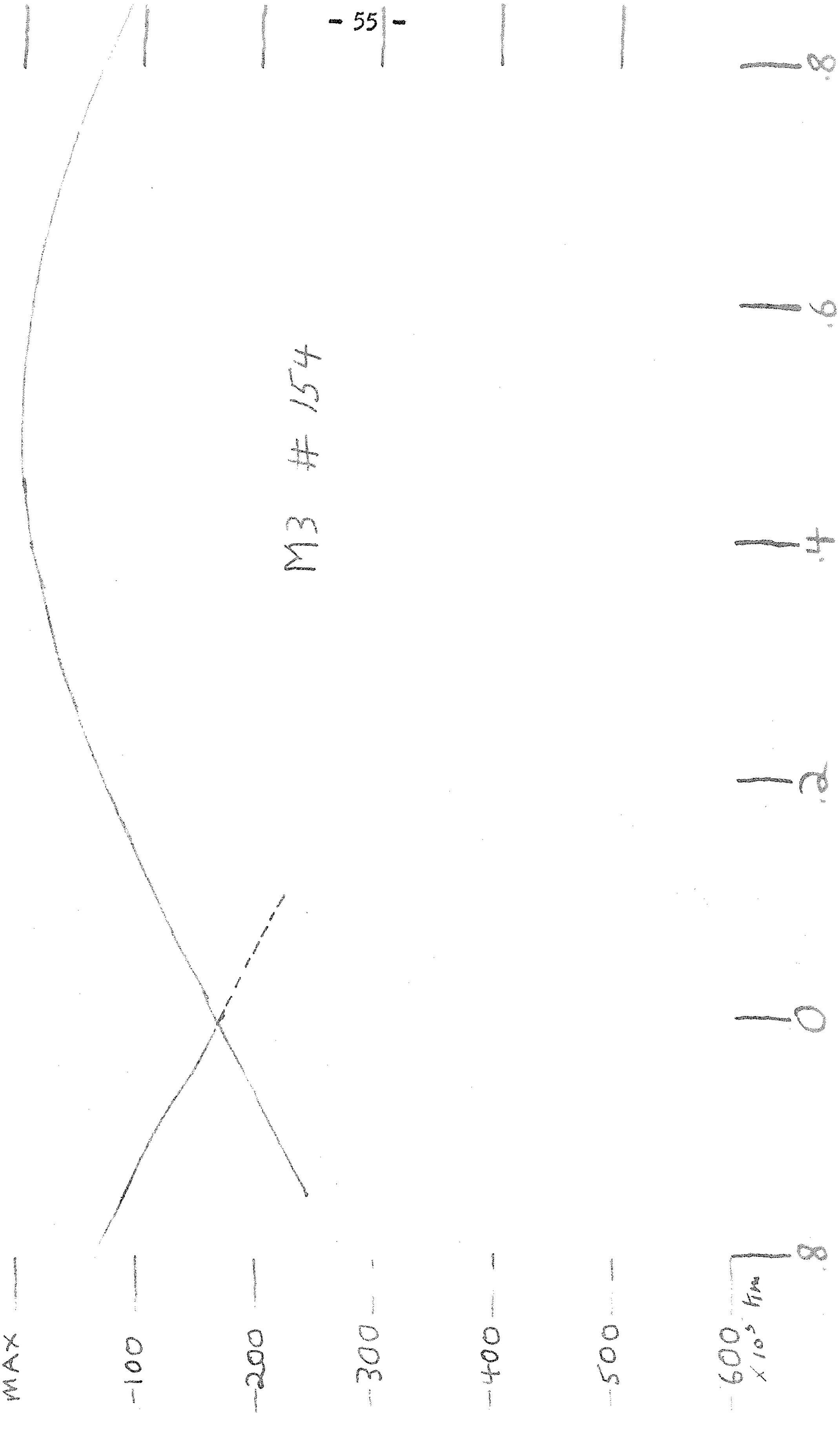

Fig. 35.

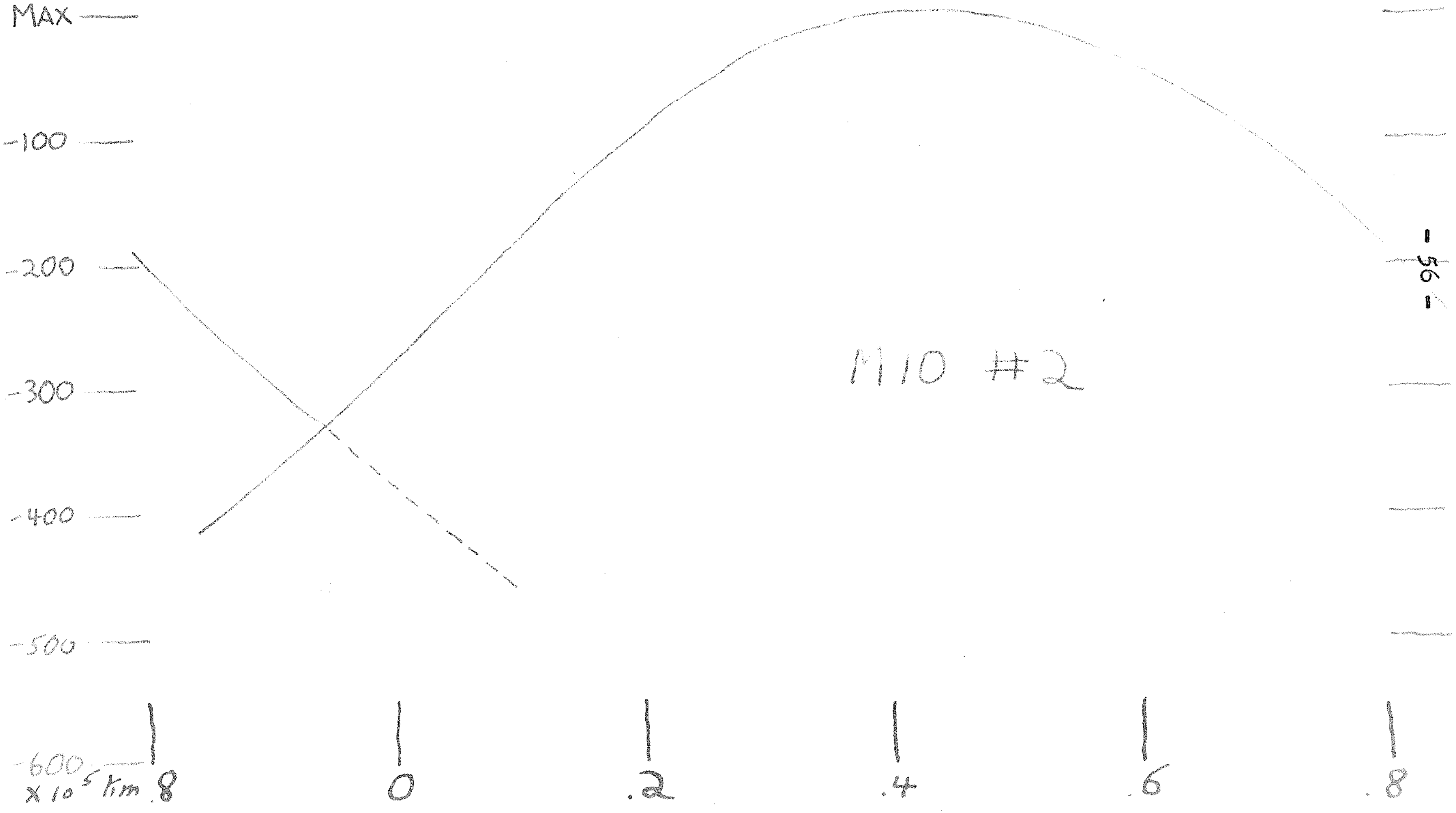

Fig. 36.

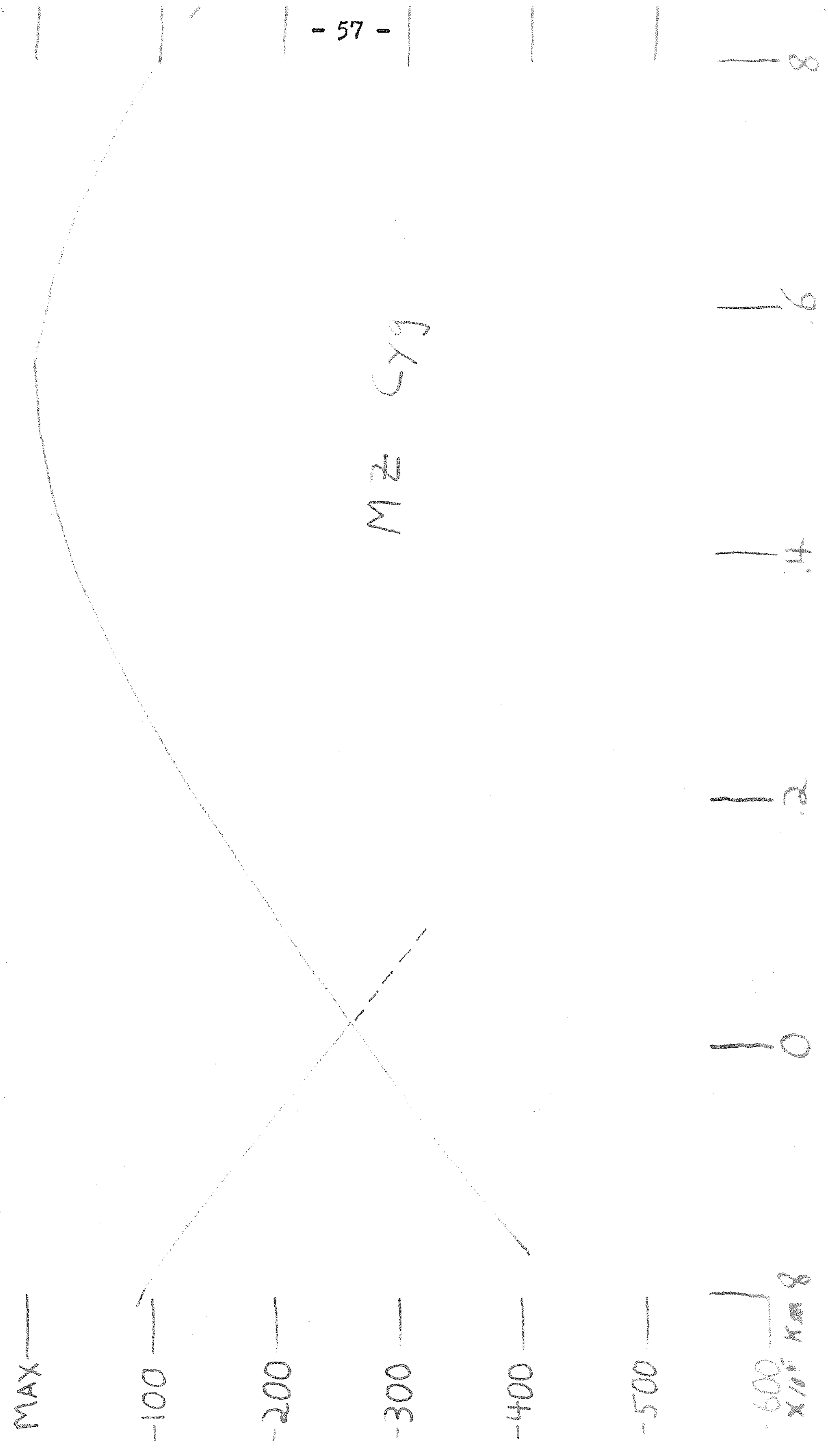

Fig. 37.

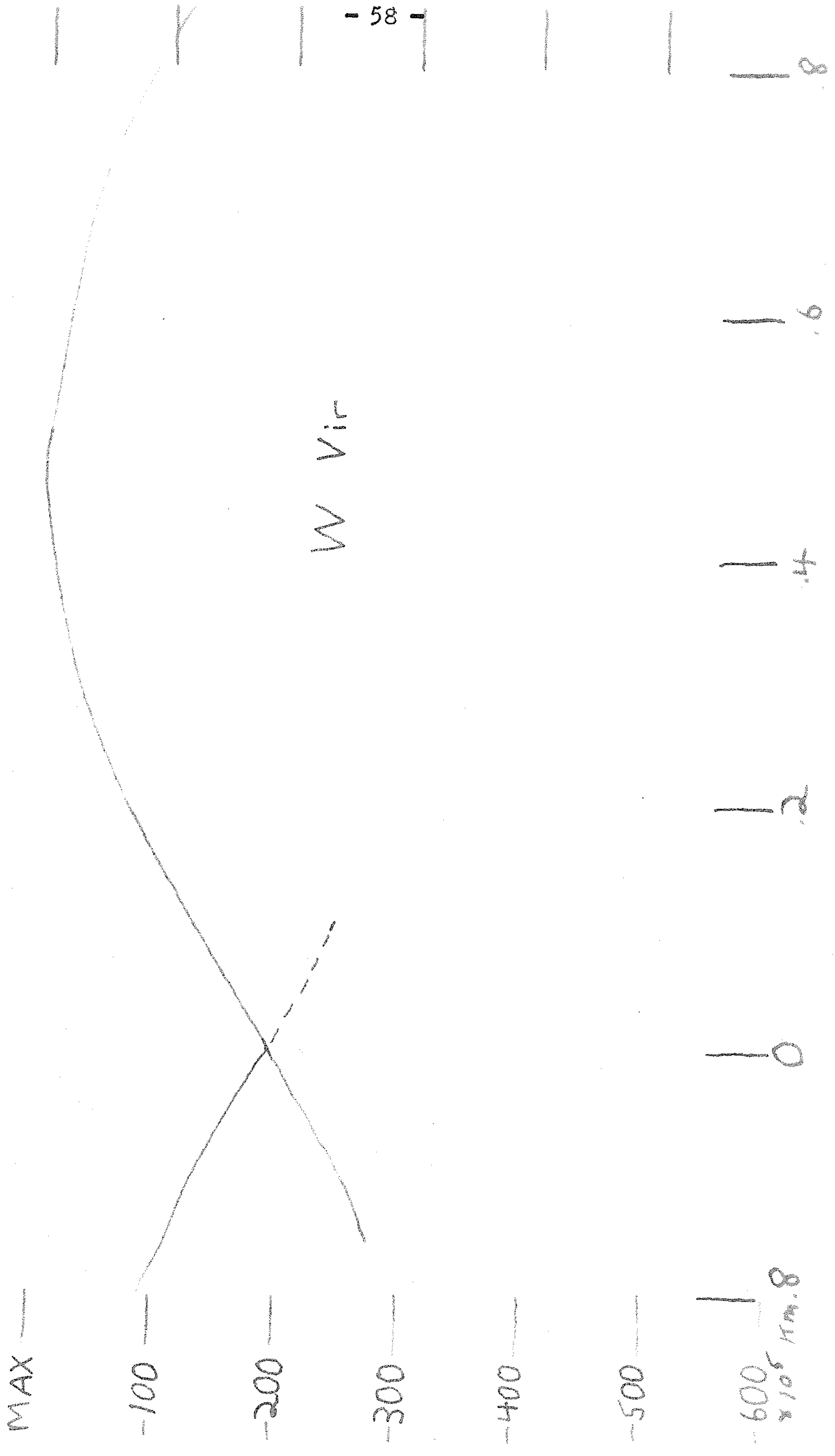

Fig. 38.

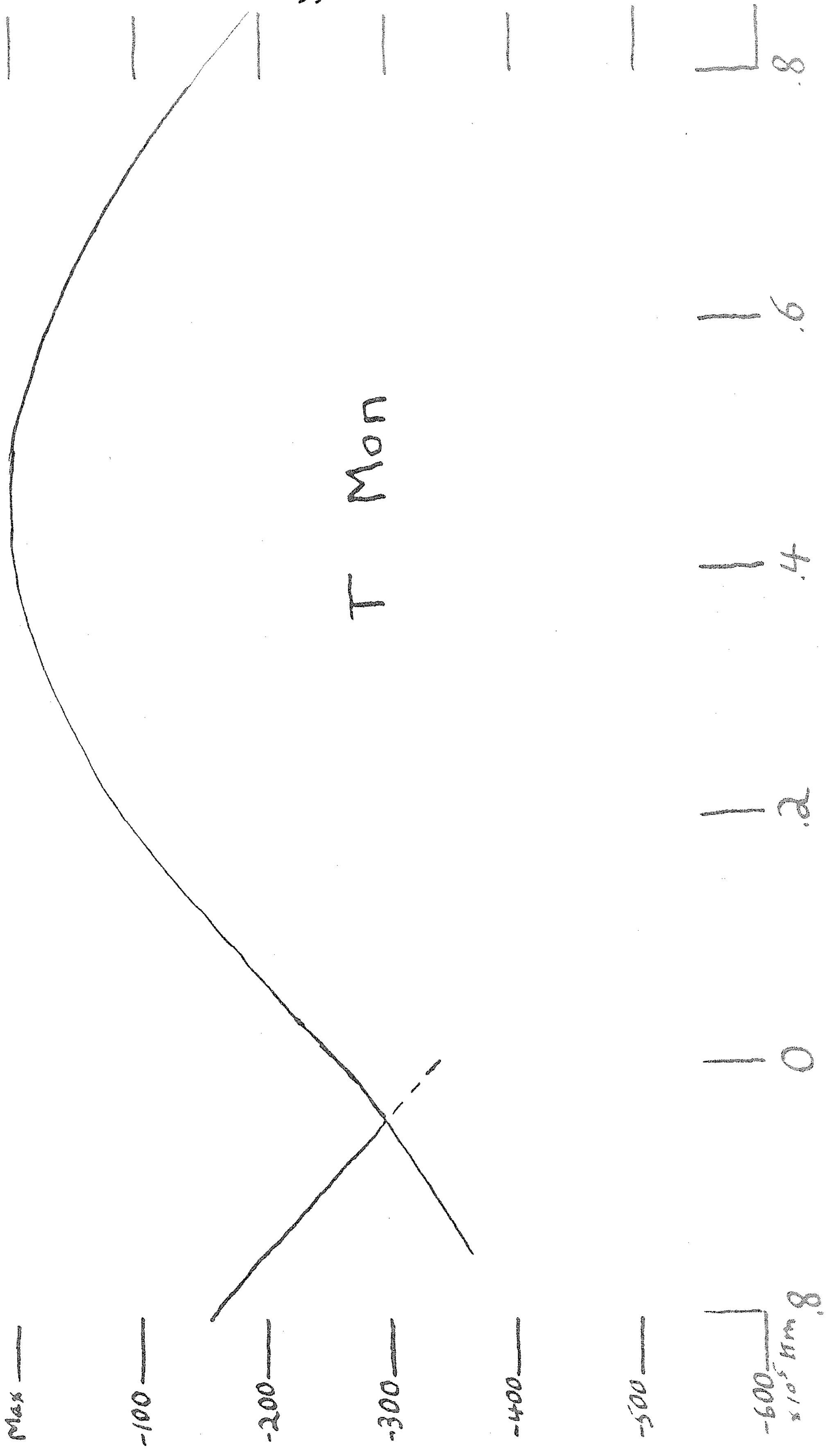

Fig. 39.

show a greater expansion associated with a deep minimum. The difference is greater for U Mon than M5 No. 84 as would be expected considering the much greater degree of alternation of light of U Mon. The classical cepheid T Mon cannot be distinguished from the population II cepheids on the basis of displacement if the velocity curve is drawn discontinuously.

TABLE 4

Velocity Amplitudes and Displacements of Cepheids

Star	Period (days)	Velocity Amplitude (km/sec)	Mean Displacement ($\times 10^6$ km)	Remarks
U Mon	46	30	17	shallow min.
U Mon	46	40	52	deep min.
TW Cap	29	46	37	
T Mon	27	59	35	
M5 No. 84	27	55	34	shallow min.
M5 No. 84	27	62	55	deep min.
M5 No. 42	26	45	28	
Mz Cyg	21	59	35	
M10 No. 2	19	85	43	
W Vir	17	54	26	
M3 No. 154	15	55	23	

VIII. PHOTOELECTRIC RESULTS

A - Effective Temperatures and Radii

Arp's photoelectric observations of W Vir and M5 Nos. 42 and 84, deserve further analysis than he gave them in the light of recent work on the interpretation of photoelectric colors(26). Using the work of Bonsack et al., it is possible to derive the effective temperature and electron pressure from the B-V and U-B of a star of type A or F. Figure 40 is the same as Fig. 2 of that paper with further points computed by Tifft and myself. Each point is labeled by $\log P_e$ above and θ_e below. Lines of constant $\log P_e$ and θ_e in the B-V, U-B plane are shown. For large values of θ_e line blanketing begins to take effect but this is not of great importance for spectral types earlier than F5 and it will be shown later that except for W Vir these stars do not get later than F5. Once it is possible to eliminate the effect of the electron pressure on the colors, Stefan's law can be applied to obtain absolute radii. This is essentially the absolute photometric method used by Whitney(24). From the effective temperature the bolometric correction is obtained from Kuiper(27). The absolute magnitude of the M5 variables is known, assuming the RR Lyrae stars are at $M_v = 0.0$; and the absolute magnitude of W Vir can be obtained by considering it to be the same as similar stars in globular clusters. The results are shown in Tables 5 and 6 for W Vir and M5 No. 42 respectively. The columns show the phase, reciprocal effective temperature, log of effective electron pressure, bolometric correction, and radius in units of the sun's radius. For W Vir the colors are too red between phases .50 and .70 for interpretation by this method. This analysis yields that W Vir

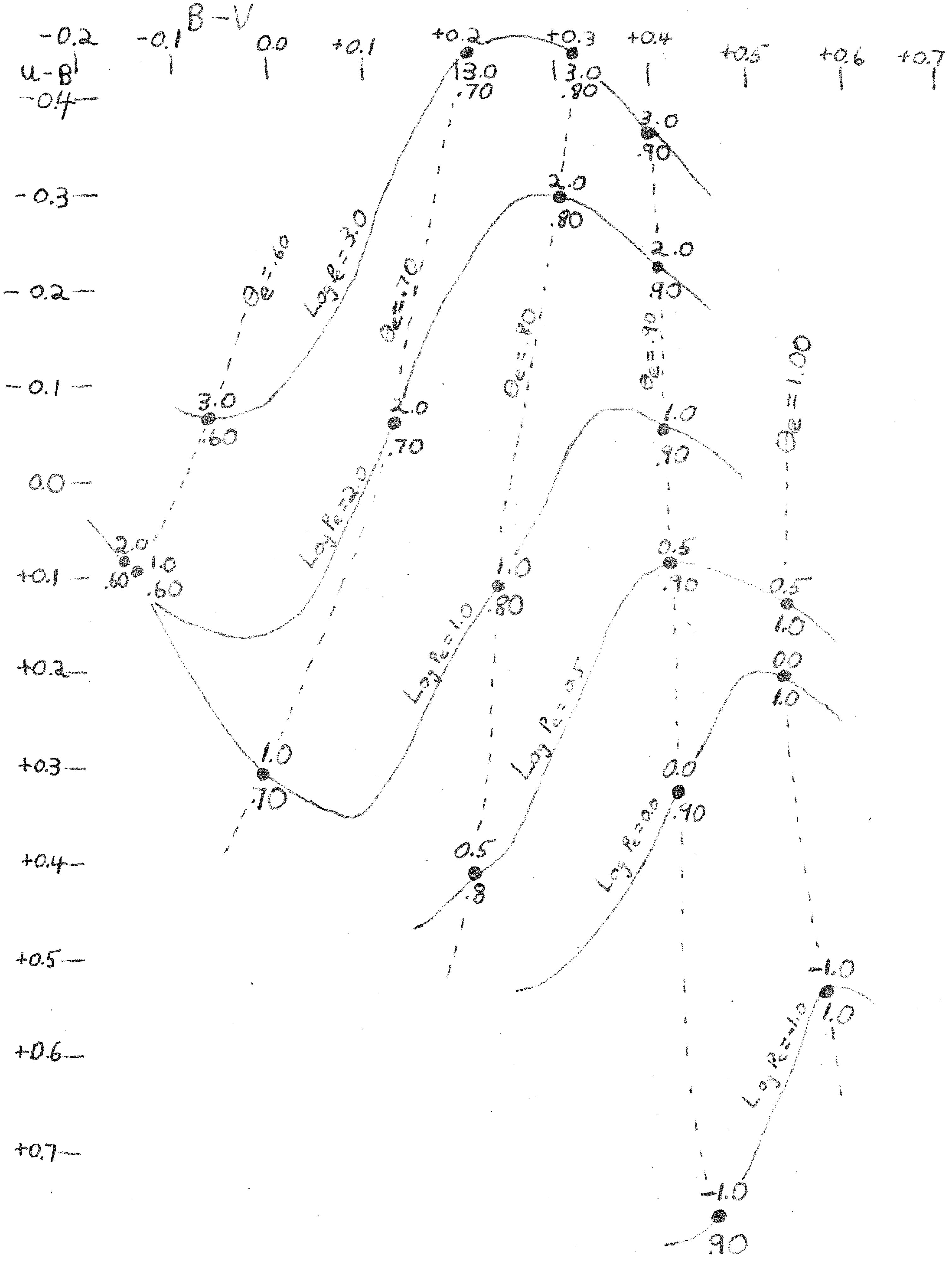

Fig. 40. Θ_e and log P_e as a Function of B - V and U - B.

varies from 30 to 55 solar radii. This is a displacement of 17.5×10^6 km. This is less than the integration yielded as is expected since photoelectrically we are dealing with a continuous change and cannot trace two layers or shells simultaneously. M5 No. 42 varies from 51 to 88 solar radii. This is an amplitude of 26×10^6 Km., which is essentially the same as 28×10^6 Km. obtained by the integration method. Arp's data for M5 No. 84 are incomplete but a little judicious extrapolation can give at least an indication of the radius at minimum radius and at maximum radius of a deep cycle. The result is a variation from 41 to 79 solar radii, i.e., an amplitude of 26.7×10^6 Km., which is the same amplitude as for No. 42.

TABLE 5

Interpretation of Photoelectric Colors of W Virginis.

Phase	Θ_e	log P_e	B.C.	R/R
.00	0.83	+ 0.8	0.05	36.3
.05	0.87	+ 0.5	.10	41.5
.10	0.90	+ 0.3	.15	45.8
.20	0.95	− 0.3	.24	52.3
.30	0.96	− 0.6	.26	54.6
.40	1.00	− 1.2	.33	54.6
.80	1.00	+ 0.1	.33	38.9
.85	0.92	+ 0.4	.16	31.9
.90	0.85	+ 0.7	.08	29.8
.95	0.82	+ 0.9	.04	31.8
.98	0.81	+ 0.9	.03	34.2

TABLE 6

Interpretation of Photoelectric Colors of M5 No. 42

Phase	θ_e	log P_e	B.C.	R/R
.00	.72	+ 1.7	.01	54.2
.10	.80	+ 1.0	.02	57.0
.20	.89	+ 0.2	.13	70.1
.30	.95	- 0.3	.24	80.9
.40	1.00	- 0.6	.32	87.0
.50	1.03	- 0.8	.42	88.0
.60	1.05	- 1.0	.47	81.6
.70	1.05	- 0.5	.42	75.5
.80	1.00	0.0	.32	65.8
.85	.92	+ 0.4	.20	57.3
.90	.86	+ 0.6	.09	57.0
.95	.73	+ 1.6	.01	51.7
.98	.70	+ 2.0	.05	51.0

B - Surface Gravity

The next important quantity that must be determined is the surface gravity. The simplest method is to consider the deceleration of the velocity curve. This assumes that other forces on the material are much smaller than gravity. These forces are probably hydrostatic or turbulent forces that tend to support the material, and will be a minimum at maximum radius which is certainly the best place for this method of determination. Thus, gravity determined by differentiating the velocity curve will be a lower limit. It is also important to note that this value of gravity refers to the region in which the lines are formed. The resulting gravities are: W Virginis, 9.5 dynes; M5 No. 42, 4.4 dynes; M5 No. 84, 6.1 dynes.

A completely different way of determining gravity is by using the colors. Using the values of Θ_e and $\log P_e$ derived above, one can enter Sandage's(28) Fig. 3. It seems best to use $\log A = 4.2$. This yields $g = 5$ dynes for M5 No. 42 at maximum radius and $g = 10$ dynes at maximum radius for W Vir. For M5 No. 84 the value of g comes out too high by a factor 10. This is probably due to extrapolation of already questionable color observations. The fit for the other two stars is remarkably good.

It might be expected that application of Sandage's calculations to these stars at minimum radius would show an increase in gravity corresponding to changes in radius noted above, i.e. an increase for W Virginis of a factor 3.4 and for M5 No. 42 by a factor 3.0. By using Sandage's calculations for W Vir the gravity at minimum radius is up by a factor 250 and for M5 No. 42 it is up by a factor 10,000, as compared with maximum radius! It is concluded that something is seriously wrong. The ultraviolet excess, which may well be caused by the collision between layers, can account for about a factor ten. Some possible explanations of the remaining discrepancy are:

1) The values of Θ_e and P_e from the simple models of Bonsack et al. do not apply or are zero pointed wrong for this type of star.
2) Sandage's calculations do not apply to this type of star.
3) Kuiper's bolometric corrections do not apply to this type of star.
4) At certain phases we are looking to much greater depth in the atmospheres of these stars than we had previously supposed. This would mean that near maximum light we are looking through an infalling shell like layer onto an expanding photosphere characterized by the observed colors. For M5 No. 42 at minimum radius these colors correspond

to a main sequence A_5 star with an ultra-violet excess of .1 mag. In my opinion this can be only partly the case. It is difficult to see just why the colors do not yield a smaller radius and higher temperature at this phase. It would be surprising if the work of Bonsack et al. and Sandage referred to above applies accurately to a pulsating atmosphere in which we look at light from two layers simultaneously.

C - The Masses

If we can consider the gravity obtained at maximum radius to be meaningful we can compute the masses of W Virginis and M5 No. 42. Using the gravity and radii listed above the mass of W Virginis is 1.1 solar masses and the mass of M5 No. 42 is 1.4 solar masses. Arp has derived the radius of W Virginis and obtains 90 solar radii at maximum but notes that this does not obey Stefan's Law. With this radius and a surface gravity of 10 dynes the mass comes out to be 2.95 solar masses. For M5 No. 84 using the value of gravity from the deceleration at maximum radius, 6.1 dynes, the mass is 1.8 solar masses. Since Arp's radius for W Virginis of 90 solar radii seems to be too large we conclude that these stars have masses in the interval of one to two solar masses.

D - The Period - Density Relation

It is instructive at this point to discuss the period density relation. This relation is at present the most successful link between theory and observation for the classical cepheids. The period density relation states $P\sqrt{\bar{\rho}} = Q$ where P is the period, $\bar{\rho}$ is the mean density and Q is a constant. Stated in terms of the quantities we have been discussing, the relation is $P\sqrt{\frac{M}{M_\odot}\left(\frac{R_\odot}{R}\right)^3} = Q$. Epstein$^{(29)}$ has shown that integration of the pulsation equation for red giant models yields $Q = .041$ days if M and R are in solar units. At the time of Epstein's work the empirical period density relation for classical cepheid yielded $Q = .09$ days. However, with the change in the zero point of the period ~~mass~~-luminosity law Whitney$^{(24)}$ has pointed out that the empirical pulsation constant for δ Cephei and η Aquilae now should be .043 and .048 respectively. Sandage$^{(28)}$ has shown that the pulsation constant for RR Lyrae and for the cluster type variables in M3 is .041 days under certain assumptions, namely that the masses are 1.2 solar masses, $M_v = 0.0$ and that RR Lyrae is reddened by .12 mag (C.I.).

For population II cepheids the use of the period density relation is hampered by the fact that we are sure of only one of four quantities in the equation. It seems best to proceed as follows. For masses between 1 and 10 solar masses we compute the necessary radii to satisfy the relation for each of the three stars for $Q = .041$ and .09. The results are shown in Table 7. We now look for reasonable combinations. We want $\bar{R}/R$ to be 42 for W Vir, 70 for M5 No. 42 and 60 for M5 No. 84. If we assume that the masses are all the same a possible solution is

$$Q = .041$$

$$M/M_\odot = 1.4$$

$R/R_\odot$ = 47.5 for W Virginis,
62.0 for M5 No. 42
and 63.0 for No. 84.

An interesting solution is $M/M_\odot = 2.0$ for all stars. This yields the proper radius for M5 No. 42, 69.5 R, a bit too large a radius for No. 84,

TABLE 7

$\bar{R}/R_\odot$ as a Function of Mass and Pulsation Constant, Q.

	W Vir		M5 No. 42		M5 No. 84	
$m/m_\odot$	Q = .041	Q = .09	Q = .041	Q = .09	Q = .041	Q = .09
1.0	42.3	25.0	55.0	33.0	56.0	33.5
1.2	45.0	26.5	58.5	34.5	59.5	35.0
1.4	47.5	28.0	62.0	36.5	63.0	37.0
1.6	49.5	29.0	64.5	38.0	64.5	38.5
1.8	51.5	30.0	67.0	39.5	68.0	40.0
2.0	53.5	31.5	69.5	41.0	70.5	42.0
2.5	57.5	34.0	75.0	44.0	76.5	45.0
3.0	61.0	36.0	79.0	46.5	80.5	47.5
3.5	64.5	38.0	84.0	49.5	85.5	50.5
4.0	67.0	39.5	87.0	51.0	88.5	52.0
4.5	70.0	41.0	91.0	53.5	92.5	54.5
5.0	73.0	43.0	95.0	56.0	97.0	57.0
6.0	77.0	45.5	100.0	59.0	102.0	60.0
7.0	81.0	47.5	105.0	62.0	107.0	63.2
8.0	85.0	50.0	111.0	65.5	113.0	67.0
9.0	88.0	52.0	114.0	67.0	116.0	69.0
10.0	91.0	53.5	118.0	69.5	120.0	71.0

and a radius that is considerably too large for W Vir. However, a pulsation constant of .057 yields the desired radius for W Vir. That W Vir might be described by a larger pulsation constant is not unreasonable. Epstein(29) points out that the existence of a deep hydrogen convection zone would tend to increase the pulsation constant. Now W Vir is redder and has an appreciably later spectrum than the M5 variables and thus might well have a deeper hydrogen convection zone. On the other hand, the pulsation constant could be kept the same for all stars if we permit the masses to differ. If we give W Virginis 1.0 solar masses and M5 No. 42, 2.0 solar masses and keep the pulsation constant at .041 the radii come out as desired. It is reasonable to say that these stars may be losing mass from their pulsating shells and thus all population II cepheids do not have the same mass. This would mean that W Vir is an "old" cepheid relative to M5 No. 42.

These radii and masses were computed on the assumption that the absolute magnitude of the RR Lyrae stars is 0.00. If this is not correct the effect is quite appreciable. For example, if $m_{pv} = +0.60$ for the RR Lyrae stars then the radii computed above must be reduced by a factor 1.33 and the masses reduced by a factor 1.8. This is about the maximum change in the absolute magnitude of the RR Lyrae stars that can still give reasonable results here.

Further speculation on the basis of the period density relation hardly seems warranted on the basis of the present data. It can be seen that good three or more color photometry on more cepheids is badly needed.

IX. SPECTRAL CLASSIFICATION AND COMPARISON*

A - Classification

One of the primary purposes of this program was the spectral classification and comparison of as many population II cepheids as possible. In order to get the best possible accuracy in classification and giving due consideration to the faintness of the stars, the spectra taken with the 60" were widened to between .20 and .30 mm. on the plate. A wide net of standard stars listed by Johnson and Morgan(30) were taken with the same equipment and comparisons were made in the usual way on a spectrocomparator.

The population II cepheids do not fit the standards nearly as well as the classical cepheids do. Thus, classifications are a compromise. Generally, the hydrogen lines have not been used due to their excessive weakness and tendency to go into emission at certain phases. This is very unfortunate and detracts considerably from the accuracy of the classification, since the hydrogen lines are particularly useful in the classification of F Stars.

A further difficulty in classification of the spectra of these stars is the variability of the opacity. Abt has pointed out that the continuous opacity in the atmosphere of W Vir varies by a factor 200. During the phases between maximum extension and minimum light Abt has shown that the excitation and effective temperature for W Vir are decreasing while the electron pressure and hence the opacity are increasing. It might be expected that the increasing electron pressure would make the spectrum appear similar to a standard star of lower luminosity, i.e. if the spectrum were F5 Ib at maximum extension it might be G0 III at minimum light.

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* It is requested in the following sections that the typographical error of subscripting the second letter of element symbols be excused rather than requiring that the remaining part of the thesis be retyped.

However, for example, for M5 No. 42 the ratio of Ti II and F_e II to F_e I does not change in the direction of a lower luminosity class but rather all the lines are weak at minimum light. This can lead to systematic errors in classification as a function of phase which are almost impossible to eliminate. That is, the changes in temperature of a star cannot be determined merely by comparison of the spectra with MKK standards when the star's opacity is highly variable.

The internal accuracy of classification is probably about two spectral subclasses. I doubt if any errors exist as large as five spectral subclasses. I have classified a large number of spectra of population II cepheids in the Mt. Wilson files. These have a dispersion of 75 and 110 A/mm at Hγ. A comparison of Joy's classification and mine shows that I classify systematically early by an average of three spectral subclasses. Comparing a group of giant and supergiant spectra classified by Adams et al.(31) with the same stars classified by Morgan and his collaborators, shows a mean difference of .15 spectral subclasses between the Mt. Wilson and MKK systems for this type of star. Thus the remaining .15 difference must be attributed to personal error and difference of lines used by Joy and myself.

Table 8 lists the most important data for each star. The list includes the classical cepheid T Mon, the cepheids in clusters, and finally the population II cepheids in the general field. Within each group the stars are listed according to period. For each star the earliest and latest spectrum observed is listed followed by a statement as to the presence of hydrogen emission. For ten stars the material available is sufficient that a more complete discussion is necessary than can be listed in a single "remarks" column. These stars will be discussed now.

TABLE 8

Summary of Spectral Types and Emission Line Intensities

Star	Period (Days)	Earliest Spectrum	Latest Spectrum	Hydrogen Emission	Remarks
T Mon	27	F_8	K_0	No	Typical classical cepheid; see discussion
M2 No. 11	34	F_0	G_0 or later	present	Spectrum might get later than G_0 during a deep min.
M5 No. 84	27	A_5	F_{4-6}	present	see discussion
M5 No. 42	26	A_6	F_5	present	see discussion
M10 No. 2	19	F_0	G_0	strong	
M3 No. 154	15	A_6	G_0	strong	
M13 No. 2	5	A_8	F_8	No	
M13 No. 1	1.5	A_2	F_0	No	
TW Cap	29	A_5	F_4	present	see discussion
MZ Cyg	21	F_5	G_5	?	see discussion
W Vir	17	F_0	G_0	very strong	see discussion
AP Her	10	F_0	F_5 or later	No	Spectral range may be greater
AL Vir	10	F_0	F_8	No	see discussion
UY Cam	62	A_3	A_6	No	period may be wrong, see discussion
CN Lyr	2.3	F_0	F_5 or later	No	spectral range, may be greater
UY Eri	2.2	A_5(?)	F_2(?)	No	see discussion
BL Her	1.3	A_6	F_3	?	

T Monocerotis: This star is a typical classical cepheid of Eggen's type C. The spectrum was found to vary from F8 Iab to K0Ib about as described by Code$^{(32)}$. The effects described by Struve$^{(33)}$ are easily recognizable on these plates. The hydrogen lines are as strong as F0 at maximum light and certain ionized lines are abnormally strong just before maximum. T Mon was chosen as a comparison star but its spectrum is so different from most of the stars discussed here that it proved to be almost useless in this respect.

M2 No. 11: Plates taken just before maximum on both cycles of this RV Tauri star showed an F0 spectrum. By phase .3 the spectrum is F5. Among Joy's better plates the latest is G0, but the star may get later at the bottom of a deep minimum.

M5 No. 84: Just before light maximum the spectrum is as early as A5 Ib-II. In a cycle with a deep minimum the spectrum declines rapidly to F4 but remains at about F4 including plates .32, .47, .62, .69 and .77. On the other cycle it becomes a little later, F6, at phase .26 and then gets earlier again; becoming F5 at .30, F3 at .60 and F0 at .71. The luminosity class seems a little lower than Ib, perhaps Ib-II, certainly not III.

Hydrogen is seen in emission from .80 to about .20. Many lines appear weak. Most notable is Mg II, λ 4481. Also very much weakened is λ4178 of F_e II and Y II. On plates of 40 A/mm these are resolved and it is the F_e II component that is weak. The effect can be seen at λ 4297 of F_e II, but not nearly as strongly. λ 4172 of F_e II and T_i II is also weakened. Again the plates at 40 A/mm indicate that it is the F_e II component that weakens. Of the other lines of F_e II multiplet 27, λ 4417 weakens but to a lesser extent; λ 4233 does not show the effect but may be supported by an F_e I line; λ 4352 lies between Hγ and the mercury emission line λ 4358 and is difficult to judge; λ 4303 and λ 4385 are badly blended. Also often weakened are λ 4260 and other lines of F_e I multiplet 152. This is probably an indication of the low iron excitation temperature. The M_n I blend λ 4030-4 shows considerable weakening. Neither S_r II nor S_c II can be seen to be particularly weak in this star.

M5 No. 42: The spectral type just before maximum light is A6 Ib-II and declines to F5 by phase .35 but gets no later. Hydrogen emission lasts from about .80 to about .07. Hydrogen absorption is generally weak for the spectral type. There seems to be a veiling of the lines starting at phase .60 and is especially noticeable at .72.

This may be due to the increasing opacity but the veiling has ended by phase .90 when the metalic lines are of type F0 Ib and are sharp and clear. At .90 the hydrogen lines are still veiled by incipient emission. Many lines are generally weakened. The weakness of M_g II, λ 4481 is most marked. Also particularly noticeable is S_c II, λ 4320. λ 4260 of F_e I is weak showing the low F_e I temperature found by Abt for W Virginis may apply to M5 No. 42 also. As in M5 No. 84, the M_n I blend, λ 4030-4 is weak. Since these lines are zero volt lines they could not be weakened by the low apparent excitation temperature but rather would be strengthened by it. It is likely that this is due to a real abundance difference. There is a slight, but not marked weakness of F_e II.

TW Capriconi: Just before maximum the spectrum is A5 Ib. It declines steadily to F4 Ib at .38 but becomes earlier by minimum when it is F_o with weak or veiled lines. The hydrogen lines are weak when in absorption, particularly just before emission begins and after it ends.

Hydrogen is in emission from about phase .75 to .00. At .03 the hydrogen lines are filled in but no emission line is present.

The absorption lines of this star are generally weaker than in the M5 variables. The spectrum resembles M5 No. 84 more than No. 42 or W Vir. This is especially the case so far as the weakness of F_e II is concerned. λ 4172 and λ 4178 are particularly weak throughout the cycle. λ4352 and λ 4233 also show this weakness particularly near maximum light. On a single plate taken in the green region of the spectrum at phase .15 the three F_e II lines of multiplet No. 42 are strikingly weak. λ 4260 of F_e I is generally weak while the star is of the type F. The blends at 4384 and 4488 containing F_e I and other contributors are also weak. λ4320 of S_c II is weak on several plates, but there is no marked weakness of either of the S_r II lines. The weakness of the F_e II lines would place this star in luminosity class III; however, the T_i II and S_r II lines appear strong enough to classify it as Ib.

MZ Cygni: Joy's plates show a spectral type from F5 Ib to G5 Ib. The G band of CH is strong when the star is of G type and the CN break at λ4215 is weak but noticeable. The spectrum is definitely later than that of W Vir. Hydrogen is weak before maximum, but no emission lines were detected.

W Virginis: The spectrum of W Vir has been described elsewhere(1),(9),(10), and it was not the purpose of this work to amplify Abt's work. Nevertheless it seems worthwhile briefly to describe the spectrum of W Vir as it appears at the dispersion used for the other stars so that comparisons can be made.

The spectrum varies from F0 Ib at light maximum to G0 Ib at light minimum. The appearance of lower luminosity in the spectrum of

M5 No. 42 and M5 No. 84 when compared with W Vir seems puzzling at first glance since M_{pv} is probably higher for these two stars. However, the luminosity classification is really only an indication of electron pressure and it can be seen in Tables 5 and 6 that W Vir does have a lower electron pressure than M5 No. 42, and would be expected to show a spectrum that appeared to be of higher luminosity on the MKK system.

Hydrogen emission lines are extremely strong and last from phase .70 to .10. At other phases hydrogen is weak for its spectral class and incipient emission may be present. Helium emission at λ 4472 was originally suspected on one plate by Joy. Helium emission at λ 5876 was found by Kraft on two of Sanford's plates after I had suggested that he look for it there. The spectrum does not appear too different from the standards although the lines are generally weak. M_g II, λ 4481 is generally quite weak. If the luminosity class Ib is assigned on the basis of the strength of F_e II and T_i II, then the S_r II lines λ 4215 and particularly λ 4077 are much too weak. This was noted by Abt(10) and interpreted as an abundance difference of a factor five. The weakness of S_c II which Abt found to be under abundant by a factor three is hardly noticeable. Neither CH nor CN is present.

AL Virginis: Plates taken in 1957 show a spectrum varying from F5 to F8 Ib-II; but the cycle was not completely covered. Joy's plates show a range from F0 to F8. The spectrum does not show any of the peculiarities that the other stars show and even the hydrogen lines are normal for the spectral type.

UY Cameleopardalis: The spectrum of this star seems to vary from A3 III to A6 III. M_g II, λ 4481 is a bit weak. This star has not been studied very carefully photometrically and I suspect that the period may be in error and that it may be an RR Lyrae type star. Hence, it will not be considered further.

UY Eridani: This star of period 2.21 days is very interesting and deserves further study than I was able to give it. The radial velocity is about + 160 km/sec and the galactic latitude is 50°; so it is certainly a population II cepheid. Three good plates at 80 A/mm are available. The luminosity class is II-III. On one plate the K line of C_a II is as late as F0 or later; the hydrogen lines are F5 or a bit later; and the metallic lines are about A5. On a second plate, the K line indicates A7; C_a I, λ 4226 indicates A5; and the hydrogen lines are F4. This same plate shows M_g II, λ 4481 extremely weak. The third plate is A6 according to the K line; F2 according to the hydrogen lines, and M_g II, λ 4481 is again very weak.

B - Illustrations of the Spectra

Figure 41 shows spectra with an original dispersion of 40 A/mm taken very close to maximum light. Some standards are also shown. Particular points to note are: The early spectral types of TW Cap and M5 No. 84. Spectral type close to F0 for W Vir and later than F5 for T Mon. Hydrogen emission for all variables except T Mon which has hydrogen lines that are too strong for its metallic line spectrum. The extreme weakness of M_g II, λ 4481 in the population II cepheids. Weakness of F_e II in TW Cap and M5 No. 84. Weakness of the M_n I blend λ 4030-4 in the variables. The strength of T_i II in the variables. Weakness of S_r II, λ 4077 in W Vir.

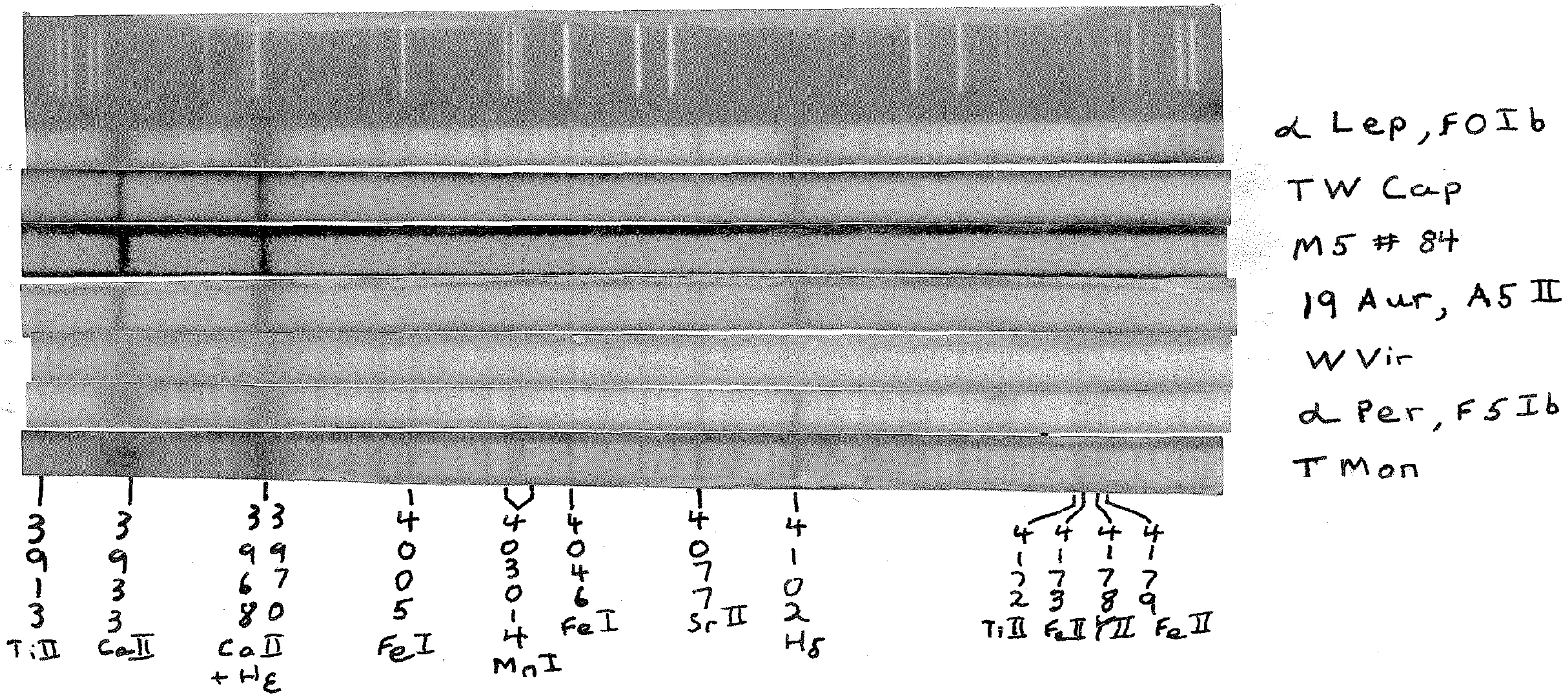

Fig. 41a: Spectra of Cepheids Near Light Maximum, Original Dispersion 40 A/mm.

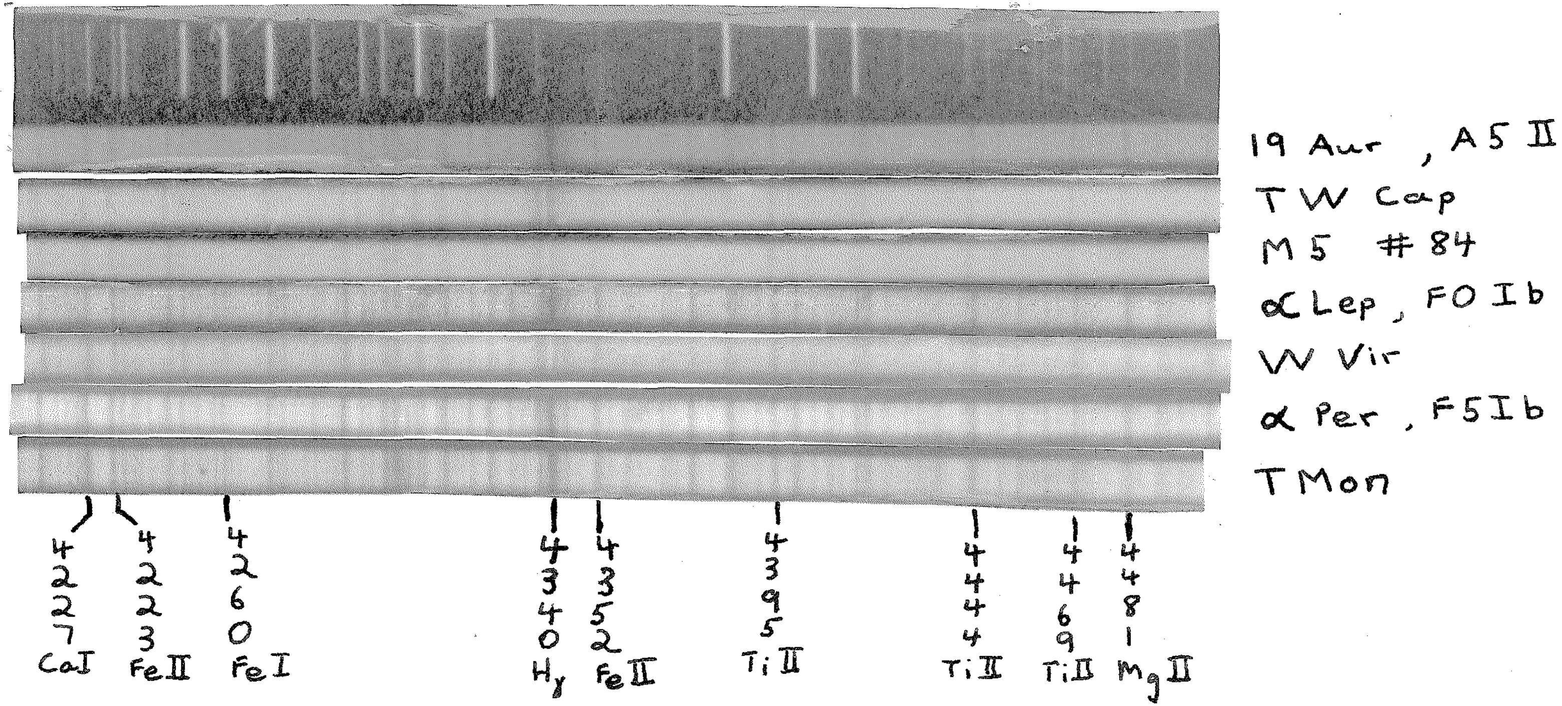

Fig. 41b: Spectra of Cepheids Near Light Maximum, Original Dispersion 40 A/mm.

Figure 42 shows spectra with an original dispersion of 80 A/mm taken very close to minimum light. Standards at F5 Ib and G2 Ib are also shown. Note: The spectral types are rather early except for W Vir which is about G0. The hydrogen lines are extremely weak and the metallic lines are weakened except again for W Vir. The G band is absent in W Vir and the weakness of S_r II, λ 4077 and λ 4215 is notable. M_g II, λ 4481 is virtually missing in some stars and weak in others. The mercury emission lines at λ 4358 and λ 4047 from the spectrum of the Los Angeles sky are strong for the fainter stars.

Figure 43 shows spectra taken on the rising branch of the light curve along with standards at F0 II and F5 Ib. Of particular note are the hydrogen lines. The emission of W Vir is much stronger than the other variables. For M5 No. 42 $H\gamma$ is almost filled in but shows no emission. The weakness of S_r II, λ 4077 in W Vir is very marked. F_e II is distinctly weak for M5 No. 84 and TW Cap, but not for M5 No. 42.

Figure 44 shows the high dispersion plates. Plates of both 20 A/mm and 18 A/mm have been enlarged to the same scale and shown together. There are many points of interest. M5 No. 42 is closer in spectral type to A7 than F2. The structure of $H\delta$ in M5 No. 42 is particularly interesting and will be discussed in the next chapter. The metallic lines in M5 No. 42 are double. The point of particular interest here is the ratio of intensities of the components. It can be seen that for S_r II, λ 4077 the long component is more intense while the nearby lines of F_e I, multiplet 43, have short components that are more intense. M5 No. 84 at ϕ = .11 has very broad lines but they do not appear double. M5 No. 84 at ϕ = .34 has broad lines also, particularly among the strong lines.

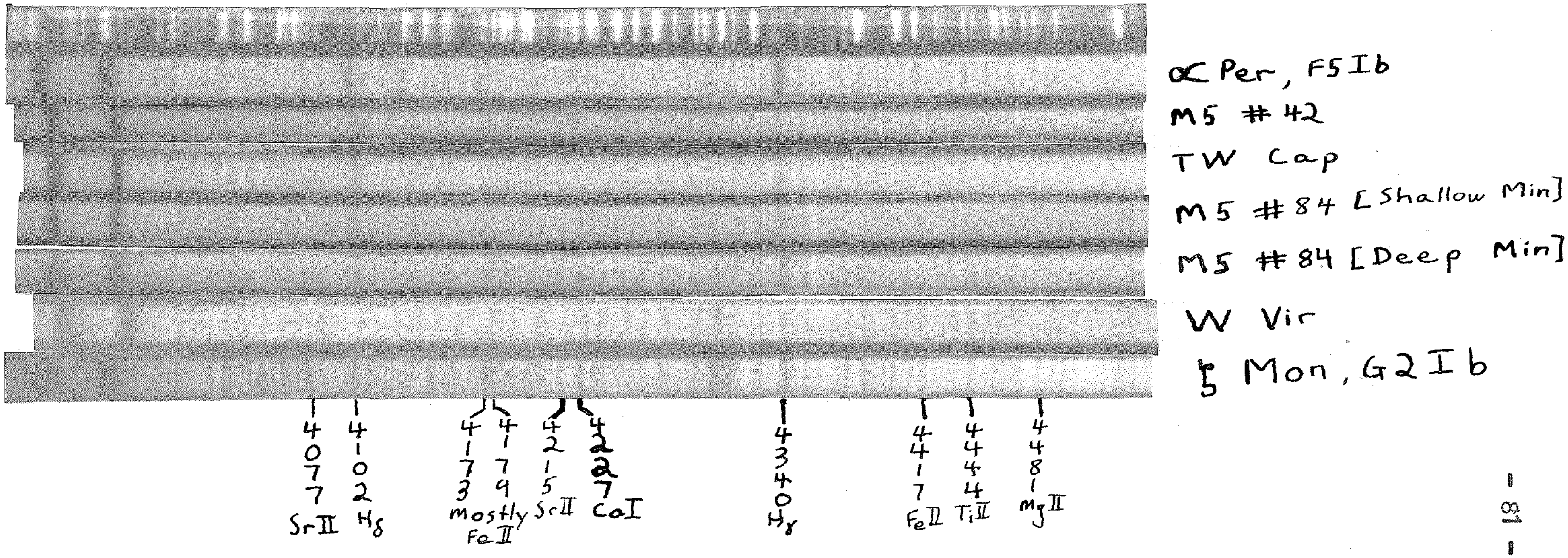

Fig. 42: Spectra of Cepheids Near Light Minimum, Original Dispersion 80 A/mm.

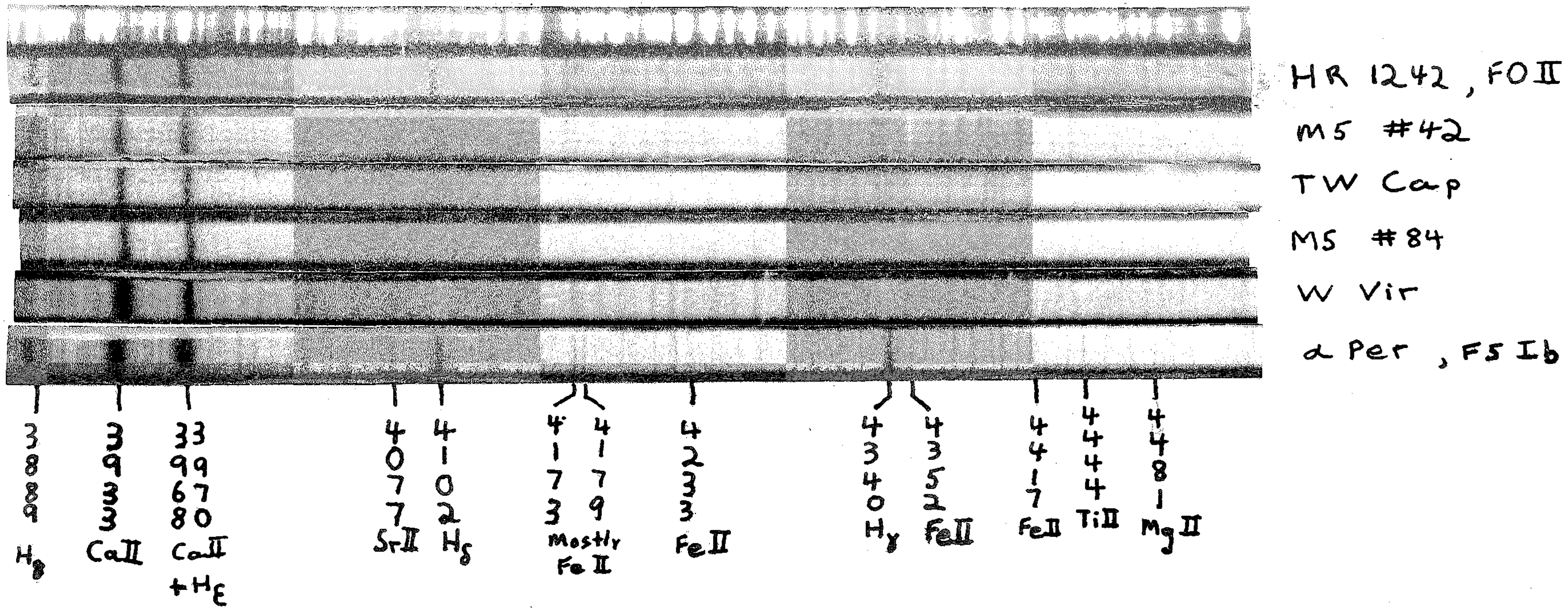

Fig. 43: Spectra of Cepheids During Increasing Light, Original Dispersion 80 A/mm.

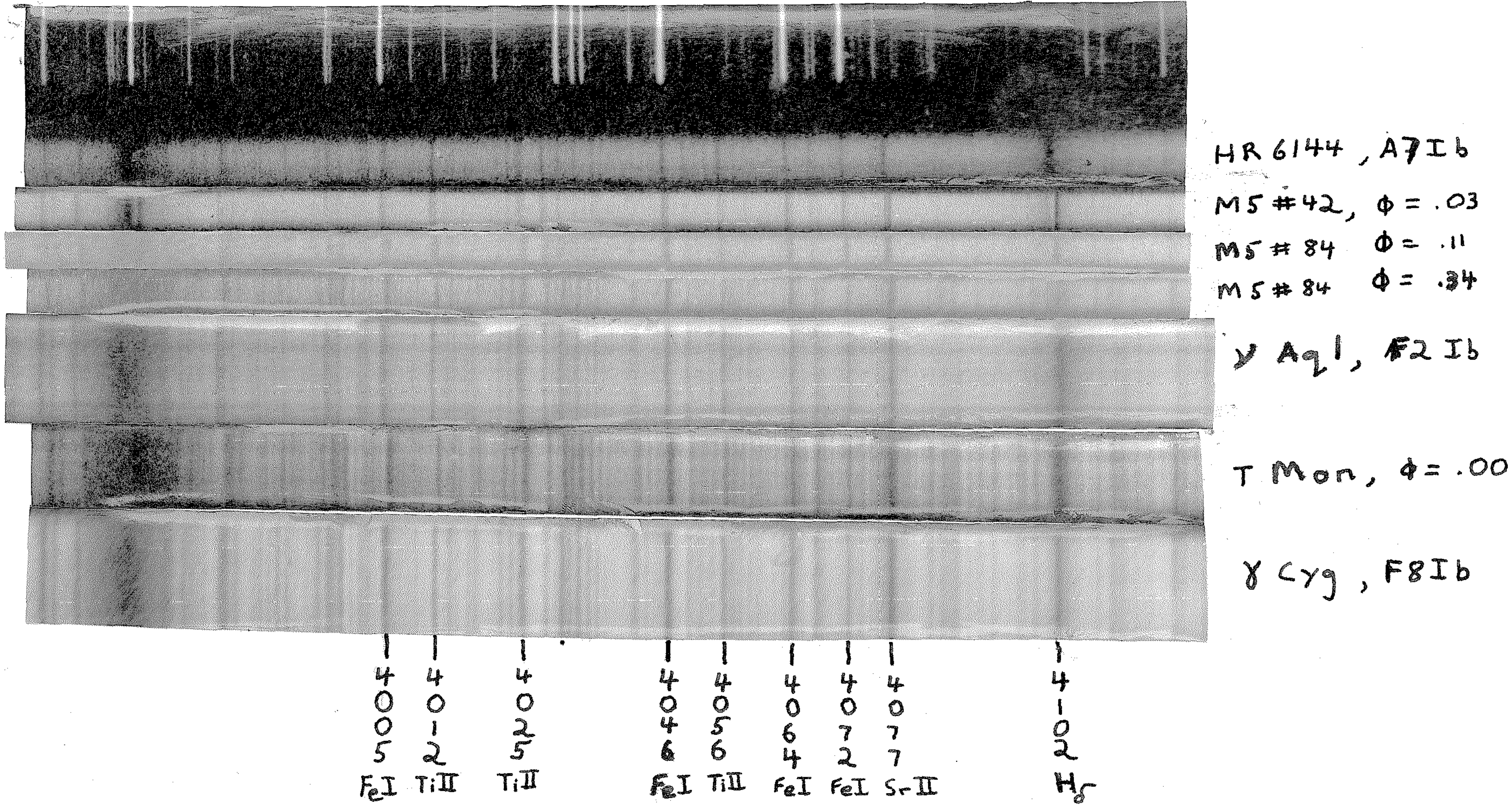

Fig. 44: High Dispersion Spectra of Cepheids.

Many weak lines appear that cannot be seen at $\phi = .11$. The spectrum of T Mon at maximum is very similar to γ Cygni. H_δ is much too strong. It can be seen to have a broader and deeper core than ν Aquilae but lacks the wings seen in ν Aquilae. S_r II, λ 4077 is also much too strong.

C - Discussion of the Spectra

These stars are more individualistic than the classical cepheids and therefore groupings and correlations will be more difficult to find. E.g., in W Vir S_r II is weak relative to F_e II, while in TW Cap the ratio of intensities is exactly opposite.

The difference in the hydrogen lines in the vicinity of light maximum between the classical and population II cepheids of period greater than about 15 days is so great that the two types of stars can be separated by using spectra of quite low dispersion.

No serious differences between the stars in clusters and the field stars can be seen. There seems to be little, if any, correlation of spectral type with period. Emission seems to have a marked maximum among stars of period 15-19 days. Emission is definitely weaker among the stars of period 20-30 days.

Among the stars of period greater than 15 days a marked correlation between spectral type and light curve is present. Stars with a light curve similar to W Vir, i.e., with a high shoulder just after maximum, are at spectral type F0 at earliest. On the other hand, stars with the shoulder occurring later reach type A5 or A6 just before maximum light. These can be called TW Cap stars to differentiate them from the W Vir type. It is interesting to note that both groups extend into the RV Tauri stars.

This separation on the basis of early spectral type was recognized before the spectra of M10 No. 2 and M3 No. 154 were taken. One plate of each of these stars was then taken at the proper phase, i.e., just before maximum light, and the two stars showed spectra as expected. The three TW Cap stars in the clusters lie on the uppermost of the three lines in Arp's(6) period-luminosity diagram.

There are several stars that are barely bright enough for spectra to be taken that should definitely prove this classification into two types. They are all in the Southern Hemisphere. On the basis of light curves published by Mrs. Gaposchkin(34) the following stars should belong to the W Vir group: UX Lupi, RX Libra, and RV Normae. On the other hand the light curve of V1077 Sagittarii indicates that it should be of the TW Cap type. The published light curves of ST Puppis and RR Microscopii(35) are indecisive and a photometric and spectroscopic study of these stars by Southern Hemisphere observers would be worthwhile. ST Pup is brighter than W Vir by almost a magnitude.

The stars AL Vir and AP Her may belong to the W Vir group but they show no emission in the hydrogen lines and probably form a small group of their own. The southern star, κ Pavonis, may be a member of this group.

As has been previously pointed out(8), the stars of the shortest period seem to be quite closely related to the RR Lyrae stars.

The high dispersion plates and the plates at 40 A/mm of best quality were measured completely in order to compile a list of elements present. The list follows that of α Carinae, F0 Ib, and α Perseii, F5 Ib, so closely that there is no need to reproduce it here(36,37). Elements reported to be only faintly present in α Car and α Per could not be detected because of the lower dispersion used here. Considering the three stars M5 Nos. 42 and 84 and TW Cap as a group the following element and ions seem weak relative to α Car and α Per: C_o I, L_a II, M_n II, N_i II, and V II.

X. SPECTROPHOTOMETRY

A - The Metallic Lines

Abt found an appreciable temperature difference between the two spectra when double lines were present both in W Vir and U Mon. The only plate available that is of sufficient quality for spectrophotometry and shows double lines is P_d 2635 of M5 No. 42. Rather than go through a full curve of growth analysis for a single plate the following semi-quantitative procedure was adopted. For lines that appeared double and relatively unblended the residual intensity at the center of each component of the line was measured. This was done for F_e I, F_e II, T_i II, and S_r II. The results are shown in Table 9. The columns are the wavelength of the line; the multiplet number; the excitation potential; the residual intensity at the center of the line of the violet component, V; the residual intensity at the center of the line of the red component, R; and V/R. There seems to be no change in V/R as one goes from low to high excitation potential in F_e I. It is concluded that the F_e I excitation temperature is approximately the same for the two layers. However, in going to F_e II, T_i II, and S_r II the ratio changes steadily toward the greater line intensity of the red component. This is shown for the mean value of V/R. For F_e I it is 0.80; for F_e II it is 0.94; for T_i II it is 1.04; and for S_r II it is 1.12. Figure 44 clearly illustrates this effect. It is apparent from this that there is a substantial difference in electron pressure between the two

TABLE 9

The Residual Central Intensities of Metallic Lines on P_d 2635 of M5 No. 42

Element or Ion		Multiplet No.	Excitation Potential	V	R	V/R
F_e I	3860	4	0.00 e.v.	.53	.55	.96
	3923	4	0.05	.62	.79	.79
	3896	4	0.11	.52	.67	.78
	3920	4	0.12	.64	.84	.76
	4045	43	1.48	.55	.65	.85
	4063	43	1.55	.57	.77	.74
	4005	43	1.55	.59	.68	.87
	4405	41	1.55	.61	.81	.75
	3956	278	2.72	.57	.78	.73
	4135	357	2.82	.65	.73	.89
F_e II	4233	27	2.57	.59	.51	1.16
	4352	27	2.69	.59	.70	.85
	4583	38	2.79	.61	.67	.91
	4556	37	2.82	.65	.77	.84
	4522	38	2.83	.69	.74	.93
	4508	38	2.84	.67	.72	.93
T_i II	4012	11	0.57	.67	.67	1.00
	4444	19	1.08	.63	.46	1.37
	4450	19	1.08	.70	.77	.91
	3913	34	1.11	.50	.42	1.19
	4501	31	1.11	.61	.50	1.22
	3900	34	1.13	.49	.46	1.08
	4468	31	1.13	.61	.50	1.22
	4464	40	1.16	.66	.77	.86
	4534	50	1.23	.58	.49	1.18
	4564	50	1.23	.60	.60	1.00
	4590	50	1.23	.71	.80	.89
	4572	82	1.56	.61	.50	1.22
	4054	87	1.88	.68	.80	.85

TABLE 9 (continued)

S_r II	4077	1	0.00	.50	.42	1.19
	4215	1	0.00	.49	.46	1.08

layers in that the new layer has the higher pressure but little difference in temperature. This is just the opposite of Abt's findings for W Vir as shown in his Table 6 and Fig. 9. Dilution effects may be of real importance here. For λ 4260 of F_e I, multiplet No. 152, the red component is extremely weak and unmeasureable. This multiplet arises from a non-metastable state and would be weakened by dilution. λ 4481 of M_g II shows no red component whatsoever showing that apparently the red components are more affected by dilution than the violet components.

When considering the radial velocity measurements by Sanfrod of T Monocerotis and SV Vulpeculae(19) careful account must be taken of his Figures 1 and 2 which show the velocity residuals relative to F_e I of F_e II, T_i II and S_r II. These residuals are just what would be expected if these stars actually had double lines whose ratios of central intensities are similar to the ratios in M5 No. 42 but are so badly broadened and blended that the individual components cannot be seen. The velocity difference between H and F_e I was the greatest but the hydrogen lines in M5 No. 42 cannot be treated in this way. It is these considerations that make it reasonable to draw a double line velocity curve for T Mon as was done in Fig. 30.

The question is immediately raised as to whether the velocity differences among elements and ions can be noted for the population II cepheids as well. Actually it must be noticeable; the only question is at what dispersion is the accuracy of measurement sufficient to detect it. A careful check has been made and it turns out that even the best plates at 80 A/mm do not show it, but good quality plates at 40 A/mm show the effect quite definitely for F_e I as compared to T_i II. Since there are only two S_r II lines available, random errors can hide the effect at 40 A/mm, but sufficient F_e I and T_i II lines are available that the mean velocities are good and the effect is marked. This can now be used as a method of detecting incipient doubling for stars that are too faint to observe with high enough dispersion to resolve the lines.

Both M5 No. 84 and TW Capricornus show the effect. Plate X_a 1585 of M5 No. 84 shows a difference of 15 km/sec between T_i II and F_e I. P_d 2587 shows a five km/sec difference. These were taken before deep and shallow minima respectively. Plate X_e 1674 of TW Cap shows a difference of 5 km/sec. It is this factor that decides unquestionably in favor of a double velocity curve for these stars.

B - The Helium Lines

Plates C_e 5092 and C_e 5647 show emission in the D_3 line of Helium I. According to Abt they are taken at phase .959 and .005 respectively. The radial velocity of the helium emission is close to that of the hydrogen emission. Several other plates show no emission in D_3. The best plate for photometry is C_e 5647 for which the emission line profiles were reconstructed as well as could be done. The total energy emitted in the H_α line is 3.9 times the total intensity of D_3. For plate 5092 the ratio appears to be similar, but the center of the H_α emission line is overexposed and accurate measurement is not possible. The Boltzman factors for $\theta = 0.7$ show that in thermodynamic equilibrium the upper level of the H_α would be populated by about 10^8 times more atoms than the upper level of D_3 if the hydrogen to helium ratio were unity. Clearly D_3 is too strong and thermodynamic equilibrium at $\theta = .7$ is not present.

If we consider the possibility that the hydrogen and helium lines are formed at a shock front where the temperature would be much higher, some reasonable numbers can be derived. Abt has stated that if a shock wave is present in W Vir during the phases of double lines and emission lines then the temperature at the shock front would be of the order of 100,000°K. By using the combined Boltzmann and Saha equations as given for example in Aller$^{(38)}$ p. 81, it turns out that the ratio of the fraction of helium atoms in the upper state of D_3 to the fraction of hydrogen atoms in the upper state of H_α is virtually independent of temperature for temperatures between about 20,000 and 100,000°K. This can be seen from Aller's equation 14,

$$\frac{N_1 P_e}{N_{o,r}} = \frac{(2\pi m)^{3/2} (KT)^{5/2}}{h^3} \left[\frac{2B_1(T)}{g_{o,r}} \right] \left[e^{-(I-\chi_r)/KT} \right] ,$$

since $I - \chi_r$ is very nearly 1.5 electron volts for both upper states. Thus for unit ratio of hydrogen to helium per atom, the ratio of atoms in the upper level of H_α to the number in the upper level of D_3 reduces to

$$\frac{N_{H_\alpha}}{N_{H_e D_3}} = \frac{B_1(T)_{H_e}}{B_1(T)_H} \frac{g_H}{g_{H_e}}$$

where $B_1(T)$ is the partition function of the ionized atom and g is the statistical weight of the upper state in question. For hydrogen $B_1(T) = 1$ and $g = 18$. For helium $g = 15$ and $B_1(T) = 2.0$ for 50,000°K. For

100,000°K, $B_1(T)$ is about 2.5. For temperatures above 100,000°K it becomes increasingly difficult to calculate $B_1(T)$, since the upper states of ionized helium have very appreciable populations under the assumption of thermodynamic equilibrium. If we use the value 2.2 for $B_1(T)$ the result will not be far off for temperatures below 100,000°K. The resulting value for $N_{H_\alpha} / N_{H_e D_3}$ is 2.64.

Now the photometry yielded a ratio of energy emitted in the two lines of $E_H/E_{H_e} = 3.9$. This is related to the number of atoms in the upper levels by the relation

$$\frac{E_{H_\alpha}}{E_{H_e D_3}} = 3.9 = \frac{N_{H_\alpha}}{N_{H_e D_3}} \; \frac{A_{H_\alpha}}{A_{H_e D_3}} \; \frac{\nu_{H_\alpha}}{\nu_{H_e D_3}}$$

where the A's are the Einstein transition probabilities. For H_α $A = .435 \times 10^8$ per sec, and for D_3 $A = .647 \times 10^8$ per sec. Putting in the numerical values yields that the observed intensity ratio can be accounted for by a ratio of hydrogen to helium of 2.9 hydrogen atoms for every helium atom. This is about two and one-half times as much helium as Mathis(39) found for the ratio of hydrogen to helium in the Orion Nebula. It cannot be said that this is significant until departures from thermodynamic equilibrium in W Vir are taken into account. Increasing the temperature above 100,000°K only results in a larger apparent abundance of helium since an appreciable number of atoms will lie in the excited states of ionized helium.

C - The Hydrogen Lines

W Vir shows the strongest hydrogen emission in any population II cepheid yet studied. Typical H_δ profiles are shown by Abt(10) Fig. 6.

A few H_δ profiles for the stars studied here are shown in Fig. 45. The general appearance of the profiles is very similar to W Vir at a similar phase except for the weakness of the emission. The maximum intensities of emission lines relative to the continuum are listed in Table 10.

Plate P_d 2635 of M5 No. 42 taken at phase .03 deserves careful study. Three entirely different layers of gas seem to be contributing to the contour. The very sharp absorption core has the same velocity as the red component of the metallic lines. The emission line is not strong but clearly has the velocity of the violet component of the metallic lines. The broad wings are shown more clearly in M5 No. 42 than they were in W Vir since they are not swamped by the strength of the emission lines. As in W Vir they have the same velocity as the emission line and the violet metallic lines. The broad contour does not have the flat bottom typical of rotation. It seems most likely that at this phase we are looking into a region that has a high enough electron pressure to produce stark broadened wings.

It is important to determine the physical condition that will produce this profile. The profile of H_δ of P_d 2635 is shown in greater detail in Fig. 46. Also shown are profiles of H_δ of α Persei and α Carinae(36,37). An attempt has been made to match the contour with theoretical contours of Verweij(40). The sharp absorption core fits Verweij's contours for $\theta = 0.9$. At this temperature the contour is very insensitive to gravity and any gravity from $\log g = 1$ to $\log g = 4$ is satisfactory. The broad wings fit Verweij's contours for $\theta = 0.8$. Again the result is not very sensitive to gravity and a value of $\log g$ between 2 and 4 is satisfactory. Now in section 8 the analysis of the colors led to $\theta = 0.75$ and $\log g = 4.2$ at this phase. On the other hand the log g

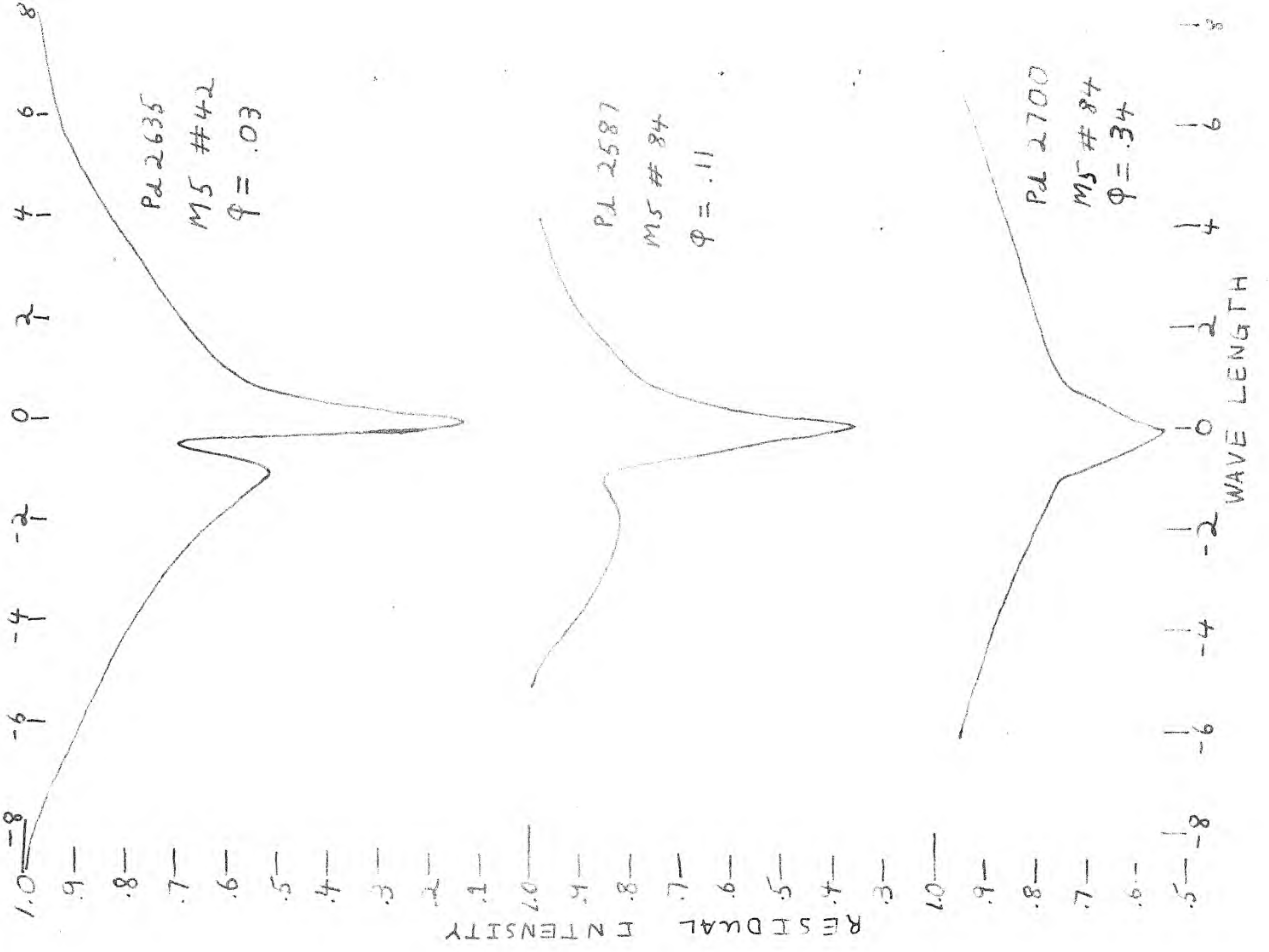

Fig. 45: Profiles of H_δ

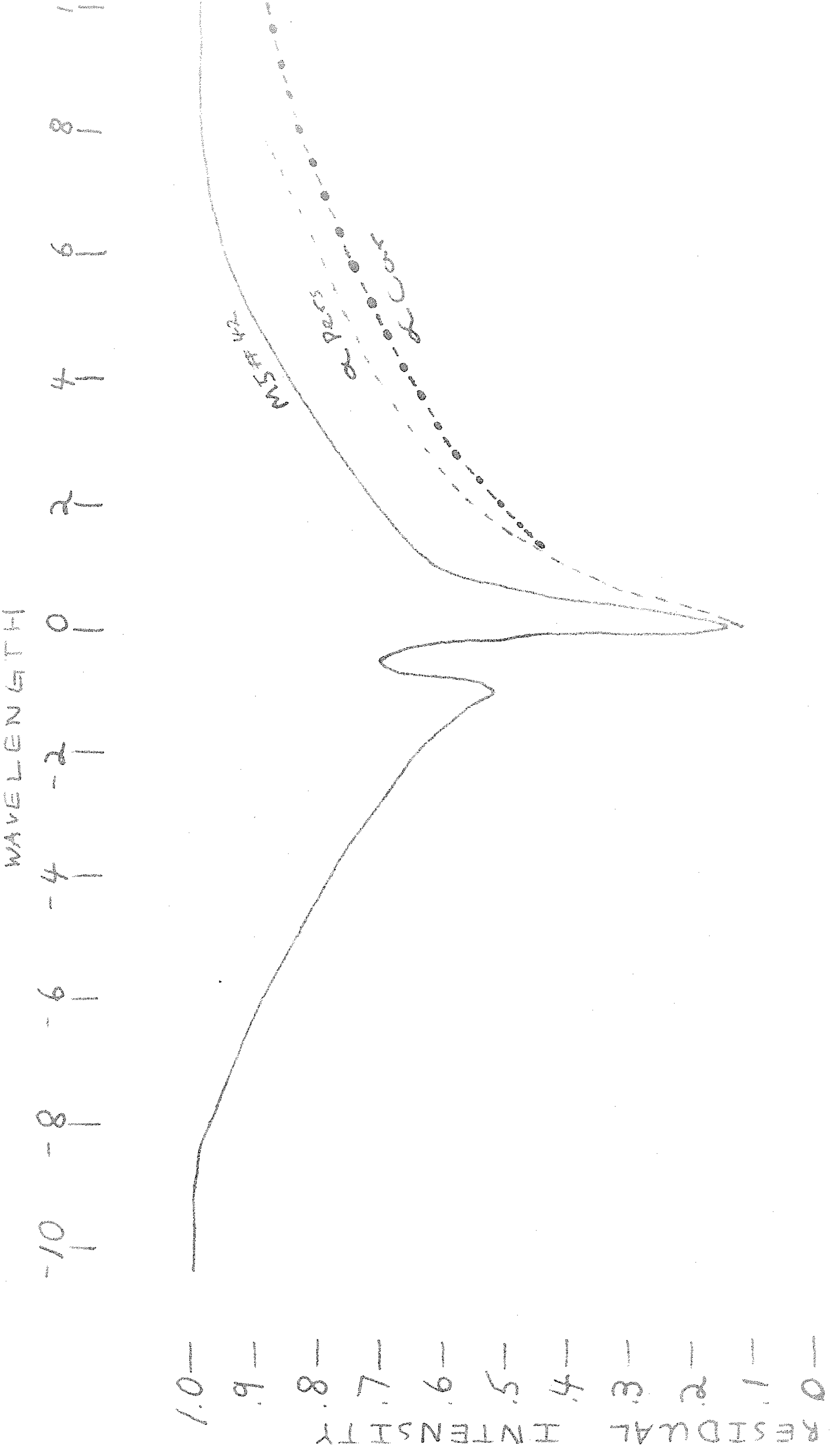

Fig. 46: Profile of Hδ of M5 No. 42 at Phase .03.

determined at maximum radius and taking into account the change in radius purely from the radial velocities is 1.3 at this phase. It seems likely that the broad contour is formed down in the same layers as the continuous spectrum and that the sharp core is formed in the pulsating shell that we observe spectroscopically. This is not definitely proved here due to the insensitivity of the hydrogen lines to gravity when the temperature is near $\theta = 0.8$. Another factor that has not been taken into account is the filling in of the broad contour by continuous emission from the overlying layers.

TABLE 10

Emission Line Maximum Intensities in Units of the Continuum

Star	Plate	Phase	H_β	H_γ	H_δ	H_ϵ	[illegible]
M5 No. 42	P_d 2635	.03	1.13	.85	.68	.44	
M5 No. 84	X_e 1585	.98	1.33	1.00	.85		
M5 No. 84	P_d 2587	.11	1.03	.97	.85		
TW Cap	X_e 1674	.97	1.00	.88			

D - A Model for W Virginis

In order to interpret the observations that have been presented not only here but also principally by Abt(10) and Arp(7) it is necessary to answer the following questions:

1. To what extent is the spectrum affected by dilution?
2. At what depth is the continuum and line spectrum being formed?
3. When double lines are present which layer is above and which is below?

The answer to each question is a function of phase. All three questions are very closely interrelated.

The weakness in these stars of λ4481 of M_g II and λ 4260 of F_e I makes one suspicious that the line spectrum is affected by dilution. It is unlikely that M_g is drastically under abundant since the lines of M_g I, multiplet 2, are normal in TW Cap. It is also unlikely that excess second ionization would affect M_g II and not other ions. It seems, therefore, that dilution must be responsible for the weakness of λ4481. If the dilution is caused by geometry it means that the atmosphere must be about one or two stellar radii from the photosphere. This means that the Schuster-Schwarzschild model must describe the line spectrum. However, Abt(21) has shown by the analysis of the F_e lines in U Mon that the Milne-Eddington model must be used; i.e. the line and continuum are formed in the same region. As he points out, his arguments for U Mon hold just as well for W Vir so geometrical dilution must be excluded. It seems therefore that the dilution must be radiative, i.e. there is a deficiency of radiation in the region necessary to excite the lower level of λ4481, namely 8.8 e.v. This is very reasonable when one considers the observations of the far ultra-violet solar spectrum(41). Definite breaks in the spectrum can be seen at about λ2400 and λ2100. These breaks very likely are due to the ionization of sodium (5.06 e.v.) and aluminum (5.76 e.v.). Now although spectra below λ2100 have not yet been taken, it will be expected that the radiation at 8.8 e.v. will be reduced by the ionization of silicon (8.1 e.v.), iron (7.9 e.v.) and magnesium itself (7.6 e.v.). Each of these three elements is cosmically at least ten times as abundant as sodium and aluminum(42) so the depression of a star's energy curve due to their ionization will surely be very great. Just why this should show up in the population II cepheids and not in the normal supergiants is not clear. Perhaps detailed model atmospheres will be able to show this.

It seems very likely that during the time when double lines appear the effect will be much more noticeable in the component formed higher in the atmosphere, that it is in the outer of the two "shells".

The red component of λ 4260 of F_e I is also weakened. This may well be due also to radiative dilution of a different sort. This line is excited by the zero volt line λ 5166.3, which lies only one angstrom to the violet of λ 5167.3 of M_g I. During phases of double lines the doubling is about 0.8Å. Thus, if the descending material is above the rising material the shift of 0.8Å will place the strong M_g I line very close to the line necessary to excite λ 4260. In the sun, according to the Utrecht Atlas, the residual intensity at 0.2A from the center of λ 5167.3 is 40 percent.

It is now possible to determine which layer is above and which is below during the phases when double lines are observed. It seems logical that if λ 4481 and λ 4260 are affected by dilution then the outer layer is going to show the weaker component no matter what the cause of the dilution is. The high dispersion plates of W Vir taken by Sanford and Abt have been examined with this in mind. It will be recalled that during phases of double lines the violet component of the metallic lines (outward moving material) first appear at phase .825 and these lines gradually strengthen as the red component (inward moving material) decreases in intensity. Now at phase .825 Abt(10, pp. 86-87) has shown that the outward moving material is below the inward moving material. λ 4481 becomes useful at phase .959 when on C_e 5092 it can be seen that the violet component is much stronger than the red component. However, for both the T_i II and F_e II lines the red component is stronger. This must be interpreted as meaning that the rising gas is still below the falling gas. This is again the situation on C_e 5647 at phase .005. On plate C_e 6207 at phase .040 the violet component of λ4481 is by far the stronger while the two components of the T_i II lines are of equal strength. After that the violet components of all lines are stronger. It is concluded that the rising gas is at all times below the descending gas. These same properties are noted on P_d 2635 of M5 No. 42.

This means that the shock wave model must be accepted. It also means that when the radii appear to be crossing in the curves drawn from integration of the velocity curves, actually the descending material that we are observing is the material at less and less depth in the atmosphere and always above the rising layer.

As mentioned above, Abt has shown that for U Mon and W Vir the Milne-Eddington model is necessary in order to interpret the line spectrum. This holds even in the vicinity of maximum extension of the stars when the continuous opacity is at a minimum. Therefore, the only phase at which we are able to see deeper than a region in which lines are being formed is when the hydrogen emission lines are present and when double lines are seen.

It is now possible to picture the sequence of events in W Vir. At maximum extension we are looking at an outer layer that is optically thick. Both the line and continuous radiation ~~is~~ are coming from this layer. This agrees with the fact that the surface gravity as determined by both the colors and the deceleration of the material is very similar. This layer now begins to descend. At phase .60 the emission lines of hydrogen appear with the velocity of the approaching material. Apparently the approaching material has been hit by the falling material at the shock front. It is likely that much of the energy of the collision at the shock front goes into the ionization of hydrogen and helium resulting in emission lines of recombination. This would also be responsible for the ultra-violet excess that appears at this phase. This is taking place at considerable optical depth with the result that the absorption lines of the new, rising, material are completely filled in and the emission lines are cut down very considerably. The strength of the observed emission lines depends upon two factors: the energy that must be dissipated at the shock front and the opacity of the overlying layers. The emission lines grow~~n~~ very rapidly in intensity until phase .80 - .85 and then gradually fade away. Thus, the energy of the collision at the shock front must be much greater at first when the shock is moving through the material of higher density than it is at a later time when the opacity of the overlying layer is less. At phase .825 we begin to see the metallic lines of the rising material. Now the infalling material never penetrates the outflowing material but rather is reversed at the shock front so that as the red components weaken with phase they are being formed in a steadily thinning layer, which is always above the outflowing material. By phase .98 the continuum is coming almost entirely from the outward flowing layer and the star appears to be at minimum radius. It is at about this point that the stark broadened wings of the hydrogen lines can first be seen. By phase .10 the rising material has reversed just about all of the infalling material and even in the lines the optical depth of the infalling material is very small. After this, any infalling material is completely transparent.

The rising material continues to expand until the gravity of the star stops it and the cycle is repeated.

This model is similar to the one discussed by Whitney(43). It is gratifying to note that his model at phase .10 is very close to the situation described here.

XI. EVOLUTIONARY CONSIDERATIONS

This discussion will admittedly be speculative and it will not be possible to give definite answers to the very interesting questions that will be raised; however, I believe it to be worthwhile to sum up the availalbe information that applies to the problem.

Figure 47 shows the combined color-magnitude diagram for seven globular clusters. This is from the work of Arp. The sequences that contain the vast majority of cluster stars are shown. Individual stars that fall off these sequences are not shown except for the cepheids in the clusters. The zero point of absolute magnitude for all clusters is determined by the position of the Variable Gap which is assumed to lie at $M_v = 0.0$.

It is important first to determine whether or not nonvariable stars are to be found in the same part of the color-magnitude diagram of the globular clusters as are the cepheids. This can be studied with the aid of Figs. 6-12 of Arp's paper on the "Color Magnitude Diagram for Seven Globular Clusters"(44). Five of those clusters contain one fairly bright star in the cepheid region. In addition, five stars appear just above the Variable Gap. All these stars are listed in Table 11. Spectra of three of the brighter stars have been obtained. M10, star II-72, is of spectral type G2V and has the radial velocity of + 29 km/sec. According to Mayall(22)

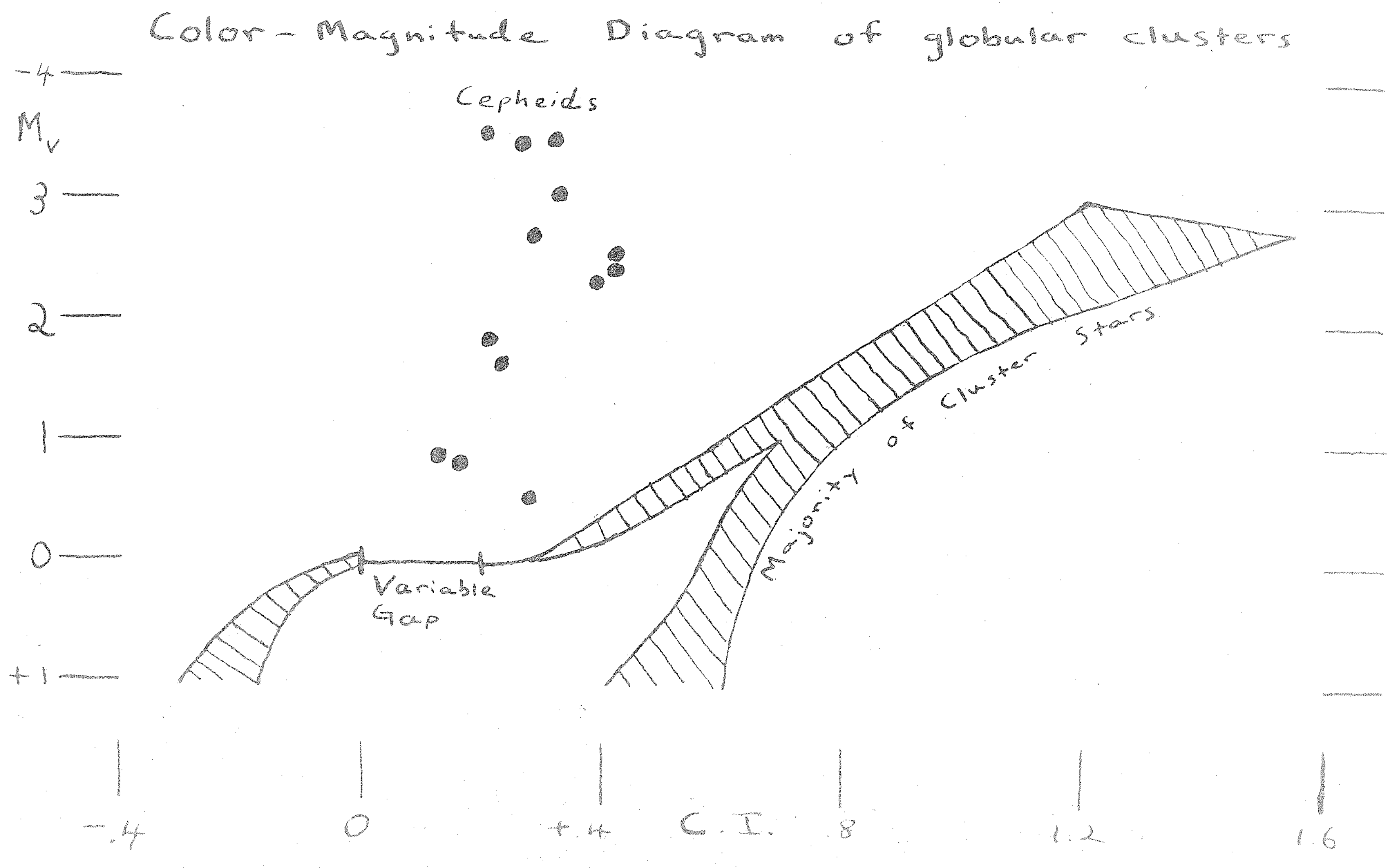
Color - Magnitude Diagram of globular clusters
-4
Mv
3
2
1
0
+1
Cepheids
Variable Gap
Majority of Cluster Stars
-.4
0
+.4
C. I.
8
1.2
1.6

Fig. 47.

the velocity of M10 is + 65 km/sec. It is therefore, not a member. M13, star IV-60, is of spectral type F8V and, by inspection, its radial velocity is not - 225 km/sec. It too is not a member. M15, star III-34, is also of spectral type F8V and also by inspection it can be seen that its velocity is not - 115 km/sec. The other stars which have M_{pv} brighter than - 1.00 (if they are members) are rather remote from the cluster centers. Although spectra of all the brighter stars have not been taken, it seems safe to conclude that they are probably not members. The five stars with M_{pv} between - .19 and - .59 are reasonably close to their cluster's center and are too faint to observe spectroscopically except possibly with the 200" telescope. In the absence of any contrary evidence, it is concluded that they may very well be cluster members. It seems that the region of the population II cepheids (at least above M_{pv} = - 1.00) is a region of instability and any globular cluster star entering this region will become a cepheid.

TABLE 11

Nonvariable Stars in Globular Clusters and With Colors and Magnitudes Similar To Cepheids in Globular Clusters.

Cluster	Star	M_{pv}	CI
M2	II-94	-1.34	+ .26
	III-73	-0.19	+ .07
	IV-34	-0.26	+ .08
M3	1402	-2.98	+ .48
M5	IV-46	-0.22	+ .26
M10	II-72	-2.97	+ .76
	IV-50	-0.59	+ .54
M13	IV-60	-2.13	+ .60
	I-43	-0.49	+ .30
M15	III-34	-3.43	+ .45

In considering the evolution problem we will consider two masses of cepheids: $M/M_{\odot} = 2.0$ and 1.2. If the value suggested for the mass is 2.0 $M_{\odot}$, the star surely was on the main sequence above the present turn off point. If it has not lost mass it was originally an A star. Also its age must be less than the age of the vast majority of cluster stars. An indication of an extension of the main sequence above the turn off point has been noted for M3(45) and in M13 there is a star of type B2 that is a cluster member and has the colors of a main sequence star(46). No models of globular cluster stars of $M/M_{\odot} = 2.0$ have been computed. It is therefore, impossible to say what sort of track such a star might follow and how it might get into the instability zone. The two most likely possibilities are either that the cepheids are on their way to the right in the color-magnitude diagram moving parallel to and above the usual track of the stars of mass 1.2 $M_{\odot}$ or they have already passed through a red giant phase and are on their way back. There is no clue at present as to which is the more likely.

If $M/M_{\odot} = 1.2$ we must determine where and why the cepheids deviate from the usual evolutionary track of globular cluster stars. One possibility is that they are a transitory stage of the RR Lyrae stars; i.e. an RR Lyrae star for a short time moves up into the cepheid region and then moves down again. This would explain the steady and rather rapid change in the periods of W Vir and TW Cap. A statistical test is of some interest here. There should be a correlation between the number of cepheids and the number of RR Lyrae stars in a cluster. This is not the case as is shown in Table 12 where the number of cepheids and number of RR Lyrae stars are listed for eight most carefully studied clusters. It is concluded that this evolutionary track is highly unlikely.

A second and more likely scheme of evolution if $M/M_{\odot} = 1.2$ is that the cepheids move to the left from the red giant branch along a track above the horizontal branch. There is no immediate argument against this, but we have no idea as to why a few stars should so deviate. Possibly the present work by Hoyle and his coworkers(47,48) will shed light on this.

TABLE 12

Number of Cepheids and RR Lyrae Stars in Globular Clusters.

Cluster	No. of Cepheids	No. of RR Lyrae Stars
M2	4	13
M3	1	171
M5	2	93
M10	2	0
M13	3	4
M15	1	60
M92	0	12
ω Cen	8	137

XII. SUMMARY

Since there has been no single theme to this work, it seems best to summarize the results at this time. I will present the best answers possible to the questions raised in section I-B.

1. The cepheids of population I have spectra from F5 to F8 at earliest to G0 to K0 at latest. The cepheids of population II have spectra from A5 to F0 at earliest to F5 to about G0 at latest. The emission lines of hydrogen that appear in population II cepheids of period greater than 15 days do not appear in the classical cepheids. Rather, the hydrogen lines of the classical cepheids are strong and deep near maximum light. This difference in the behavior of the hydrogen lines before and at maximum light serves to classify a star as a member of one or the other group. The population II cepheids with periods near 10 days do not show emission lines of hydrogen and the classification of these stars into the two populations is not as easy as for stars of longer period.
2. The main difference between M5 No. 42 and the RV Tauri star, M5 No. 84, seems to be in the timing of the new outrush of gas from below. A deep minimum occurs when the new outburst is delayed and a shallow minimum occurs when the new outburst arrives early. Whether this can be shown to apply to other RV Tauri stars is questionable.
3. The cepheids on the top line of Arp's period-luminosity diagram have earlier spectra than the stars of lower luminosity. Also, M5 No. 42 has a higher electron pressure than W Vir. This helps to show a real difference between population II cepheids of different luminosities but does not settle the question of whether or not the lines noted by Arp in the period-luminosity diagram are discreet.

4. Double lines have now been observed in W Vir, M5 No. 42, and TW Cap. It seems likely that they are present in the other stars of period greater than 15 days. Double lines have not been found in stars with a period of 10 days. AL Vir should be observed at 10 A/mm in order to search for double lines, and κ Pavonis should certainly be studied by southern observers in this connection. Among the stars of period just over one day BL Hercules is presently being studied by Abt using coude plates and his results are awaited with interest.

5. Studies of the radii from integration of the velocity curves and from interpretation of the colors yield generally consistent results. The determination of surface gravity from the colors yields much too high a gravity near minimum radius to make sense.

6. A discussion of the masses leads to values ranging from one to three solar masses. A value in the interval 1.2 to 2.0 solar masses seems most likely.

7. The spectral classification showed these stars range from A5 to F5 or F0 to about G0 depending in the type of light curve. No period-spectrum relation could be determined. The lines are generally weak for the spectral type. The strength of T_i II results in the luminosity class of Ib or II being assigned almost all the spectra. However, relative to T_i II, S_r II is weak in W Vir and F_e II is weak in M5 No. 84 and especially TW Cap. M_g II, λ 4481, is weak in almost all population II cepheids.

8. Considering the strength of M_g II, λ 4481 when double lines are present in W Vir, it was shown that the old, infalling, material is always above the new outflowing material and hence the interpentrating shell model

must be abandoned and the shock wave model accepted. This is consistent with the presence of emission lines of hydrogen and helium.

9. A discussion of possible evolutionary tracks in the color-magnitude diagram lead to no definite conclusions as to how a star reaches the instability zone of the population II cepheids.

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