

# Nonlinear Optics in Ultra-high-Q Whispering-Gallery Optical Microcavities

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To my parents,  
Hans and Karola,

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# List of Publications

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- [2] S. M. Spillane, T. J. Kippenberg, O. J. Painter, and K. J. Vahala. Ideality in a fiber-taper-coupled micro-resonator system for application to cavity Quantum Electrodynamics. *Physical Review Letters*, 91(4):art. no.–043902, 2003.
- [3] S. M. Spillane, T. J. Kippenberg, and K. J. Vahala. Ultralow-threshold Raman laser using a spherical dielectric microcavity. *Nature*, 415(6872):621–623, 2002.
- [4] B. K. Min, T. J. Kippenberg, and K. J. Vahala. Compact, fiber-compatible, cascaded Raman laser. *Optics Letters*, 28(17):1507–1509, 2003.
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- [7] T. J. Kippenberg, S. M. Spillane, D. K. Armani, and K. J. Vahala. Fabrication and coupling to planar high-Q silica disk microcavities. *Applied Physics Letters*, 83(4):797–799, 2003.
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# Abstract

Optical microcavities confine light at resonant frequencies for extended periods of time and fundamentally alter the interaction of light with matter. They are the basis of numerous applied and fundamental studies, such as cavity QED, photonics and sensing. Of all resonant geometries, surface tension-induced microcavities, such as silica micro-spheres, exhibit the highest Q-factor to date of nearly 9 billion. Despite these high Q-factor and the intense interest in these structures, the nonlinear optical properties of silica micro-spheres have remained nearly entirely unexplored. In this thesis the nonlinear optical phenomena which can occur in ultra-high-Q microcavities are investigated. To efficiently excite the whispering-gallery modes, tapered optical fibers are used and the coupling to ultra-high-Q modes studied. It is found, that microcavities with ultra-high enter a regime where scattering of light into the degenerate pair of clockwise and counter-clockwise mode is the dominant scattering process. In this regime the coupling properties are significantly altered, but the cavities still retain their ability to achieve significant cavity build-up fields. This allowed exceeding the threshold for all common nonlinearities encountered in silica. In particular, stimulated Raman scattering is observed in taper fiber coupled silica micro-spheres at threshold levels typically in the micro-Watt range, which usually is considered the regime of linear optics. Cascaded Raman lasing is also observed in these structures. The tapered optical fiber in these experiments functions to both pump WGMs as well as to extract the nonlinear Raman fields. In addition, the tapered-fiber coupling junction is highly ideal, making it possible to strongly over-couple ultra-high-Q cavities with negligible junction loss. This feature allows for the observation of very high internal differential photon conversion efficiencies approaching unity. Whereas

micro-spheres are both compact and efficient nonlinear oscillators, their fabrication properties lack the control and parallelism typical of micro-fabrication techniques. A synergistic approach of micro-fabrication and a laser assisted reflow process, allows to create toroidally silica microcavities on a chip. In this thesis it is demonstrated, that these cavities can exhibit ultra-high-Q whispering-gallery modes, allowing to achieve ultra-high-Q modes on a chip. This results is a nearly four-order of magnitude improvement with respect to other wafer-scale microcavities. In addition their azimuthal mode-spectrum is strongly reduced. Nonlinear oscillation in these cavities has also been studied, and stimulated Raman scattering observed, allowing to achieve the first Raman laser on a chip. The devices show improved performance compared to micro-spheres due to a strongly reduced azimuthal mode spectrum, which allowed to observe single mode emission. The enhanced geometric control of these cavities is also studied and found to profoundly alter the nonlinear optical processes the toroid microcavities. Reduction of toroidal cross section is observed to cause a transition from stimulated Raman to parametric oscillation regime. This allowed to observe Kerr nonlinearity induced parametric oscillation in a microcavity for the first time. The parametrically generated "twin beams" exhibit high conversion efficiency and show near unity signal-to-idler ratio.

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# Glossary of Acronyms

**WGM** Whispering-gallery mode

**XPM** Cross-phase modulation

**FWM** Four-wave mixing

**UHQ** Ultra-high Q

**SPM** Self-phase modulation

**SEM** Scanning electron microscope

**SRS** Stimulated Raman scattering

**SBS** Stimulated Brillouin scattering

**STIM** Surface-tension-induced microcavity

**LL** Light-in light-out

**FSR** Free-spectral-range

**OSA** Optical spectrum analyzer