Essays on Law and Economics

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To my parents.

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Abstract

This thesis studies three legal problems through the lens of economic theory.

In the first chapter, I study a model of group identification in which individuals' opinions as to the membership of a group are aggregated to form a list of group members. Potential aggregation rules are studied through the axiomatic approach. I introduce two axioms, *meet separability* and *join separability*, each of which requires the list of members generated by the aggregation rule to be independent of whether the question of membership in a group is separated into questions of membership in two other groups. I use these axioms to characterize a class of *one-vote* rules, in which one opinion determines whether an individual is considered to be a member of a group. I then show that the only anonymous one-vote rule is *self-identification*, in which each individual determines for himself whether he is a member of the group.

The second chapter introduces a *path-based* measure of convexity to be used in assessing the compactness of legislative districts. Our measure is the probability that a district will contain the shortest path between a randomly selected pair of its points. The measure is defined relative to exogenous political boundaries and population distributions.

In the third chapter, I introduce a new model of community standards relevant to the judicial determination of obscenity. In the model, standards are defined as subjective judgments restricted only by a simple reasonableness condition. A set of individual standards is then methodically aggregated to form the community standard. I define several axioms which reflect legal concerns expressed by the judiciary. The axioms require that the community standard (a) preserve unanimous agreements about the entire standard, (b) become more permissive when all individuals become more permissive, and not discriminate, ex ante, (c) between individuals and (d) between works. I then show that the only method which satisfies these properties is *unanimity rule*, in which a work is considered obscene if and only if all members of the community consider it to be obscene. I also consider several variants of the model and provide characterizations in these related models.

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Chapter 1

Introduction

1.1 Introduction

This thesis consists of three chapters which apply social science methods to legal problems. Two of these chapters use axiomatic models to study legal institutions, following the work of Spitzer (1979) who pioneered the use of axiomatic methods in legal analysis. One of these chapters is an axiomatic study of classification rules. These rules are relevant in several contexts including (a) the allocation of legal rights and (b) the collection of data for social science research purposes. The other axiomatic chapter presents a formal model of "community standards" used in obscenity law, and characterizes the "unanimity rule" as the only aggregation method to satisfy four basic axioms. The "unanimity rule" closely resembles the unanimity rule used in juries in much of the United States. The remaining chapter introduces a new measure of compactness of legislative districts. This measure is designed to be used both by researchers and practitioners in the area of voting rights law.

1.1.1 Group Identification

Classification problems abound in law, policy, social science, and philosophy. Federal agencies classify individuals for statistical purposes. Legal systems allocate rights and restrictions to groups, whether these be particular minority groups or the group of licensed drivers. Philosophers try to understand whether groups exist. Sociologists analyze social groups, anthropologists examine cultures, political scientists study collective action, and historians use groups to explain past human behavior. In economics there has been a significant increase in the use of group membership as a variable in formal modeling following the recent work of Akerlof and Kranton (2000).

Government policymakers and social science researchers share a common interest in finding objective standards by which individuals can be classified into groups. In some cases this is a simple problem. Two yardsticks are sufficient to determine whether an individual is in the group of people not shorter than six feet. In other cases, however, traditional tools of measurement are not practical. For example, the U.S. Government maintains that racial and ethnic groups are not defined by biology or genetics, but rather are a social phenomenon. In these cases, the potential data available to policymakers and researchers is in the form of subjective beliefs held by members of the society. The policymakers and researchers need a method to objectively classify individuals on the basis of these subjective beliefs.

The relevant model of group identification was first introduced by Kasher and Rubinstein (1997). In this model, individuals have opinions about the composition of groups. These opinions are then aggregated according to some rule to form a list of group members. Theirs and subsequent papers study aggregation rules through an axiomatic approach: various properties are proposed and rules satisfying these properties are characterized.

The most prominent rule studied in the literature is self-identification, in which individuals are classified into groups on the basis of their opinions about themselves. Self-identification is an important rule because it is used heavily by the government and by social science researchers. The literature contains two basic characterizations of self-identification.

Kasher and Rubinstein (1997) first characterized self-identification as the only rule satisfying symmetry, a weak independence condition, and the "liberal principle," which required that individuals can force certain outcomes.[1] Later, Samet and Schmeidler (2003) introduced a separate characterization of self-identification as the only rule satisfying monotonicity, non-degeneracy, a stronger independence condition, and another property labeled self-determination. Other rules studied in the literature include consent rules (Samet and Schmeidler, 2003) and recursive rules. (Dimitrov et al., 2007; Houy, 2006)

I introduce a new characterization of self-identification using a property I call *separability*, which requires that social rules preserve certain relationships between groups. To study this concept, I extend the Kasher-Rubinstein model by introducing multiple groups with predefined relationships.

For example, Federal statistical policy envisions many racial groups, including Asians, Whites, people who are members of either group ("Asian or White"), and people who are members of both ("Asian and White"). The set of groups also includes the non-members of each group (e.g., "Non-Whites" and "Non-Asians"). Opinions must preserve these relationships. A person who believes that his neighbor is "Asian and White" must also believe that his neighbor is Asian. In this extension, aggregation rules are allowed to work differently for different groups. A rule need not aggregate opinions about Asians in the same way that it aggregates opinions about Whites. However, a consequence of the axioms studied in the chapter is that a rule must be independent of the group under consideration.

To study separability in this context, I introduce two axioms. *Meet separability* requires social rules to yield the same list of "Asian and White" people regardless of which of two possible approaches

[1]This follows a refinement of the Kasher-Rubinstein characterization by Sung and Dimitrov (2005).

is used. The first approach is to aggregate opinions about “Asian and White” people. The second approach is to generate two lists by aggregating opinions about Asians and about Whites separately, and then to take the intersection of the two lists.

Join separability is similar, except that it is defined with respect to disjunction instead of conjunction. It requires social rules to yield the same list of “Asian or White” people regardless of which of two possible approaches is used. The first approach is to aggregate opinions about “Asian or White” people. The second approach is to generate two lists by aggregating opinions about Asians and about Whites separately, and then to take the union of the two lists.

In addition to the separability axioms, I require *non-degeneracy*. This axiom requires that the opinions be relevant in determining whether each person is a member of a group. Using these three axioms I characterize a family of rules called *one-vote* rules. These rules associate with each person a single opinion that determines whether that person is a member of the group.

An *anonymity* axiom requires that the qualification of individuals does not depend on their names. Anonymity is appealing because government policymakers cannot arbitrarily favor some people above others. Non-governmental researchers gathering data may not be bound by this normative requirement, but may lack an external scientific basis to draw distinctions between individuals. I show that self-identification is the only anonymous one-vote rule.

1.1.2 Measuring Bizarreness (joint with Chris Chambers)

Hundreds of years ago, legislators discovered that the ultimate composition of a legislature is not independent of the means through which district boundaries are drawn. Hoping to stave off unemployment, legislators learned to master the art of *gerrymandering*: carefully drawing district boundaries to increase their electoral chances and political power. Like certain forms of painting and ballet, this art became more and more noticeable by the odd shapes it produced.[2]

Past attempts on the part of political reformers to fight gerrymandering have led to the introduction of vague legal restrictions requiring districts to be “compact and contiguous.”[3] The vagueness of these legal terms has led to the introduction of several methods to measure district “compactness.”[4] However, none of these methods is widely accepted, in part because of problems identified by Young (1988) and Altman Altman (1998). We argue that these laws were introduced with the aim of eliminating bizarrely shaped districts. To this end we introduce a measure of “bizarreness.”

The primary problem with gerrymandering is that elections become less competitive when legis-

[2] In 1812 a district was said to resemble a salamander; one hundred eighty years later, another was likened to a “Rorschach ink blot test.” *Shaw v. Reno*, 509 U.S. at 633.

[3] Thirty-five states require congressional or legislative districting plans to be “compact”, forty-five require “contiguity”, and only Arkansas requires neither. (NCSL, 2000) There may also be federal constitutional implications. See *Shaw v. Reno*, 509 U.S. 630 (1993); *Bush v. Vera*, 517 U.S. 959 (1996).

[4] “Contiguity” is generally understood to require that it be possible to move between any two places within the district without leaving the district. See for example Black’s Law Dictionary which defines a “contiguous” as touching along a surface or a point. (Garner, 2004)

lators draw district lines to strengthen their reelection chances. The "bizarre" shapes which result are merely a side-effect of this process.[5] Reformers have focused on compactness because, while there is no consensus as to how district boundaries should be drawn, bizarre shapes are clearly identifiable as a symptom of gerrymandering.

Part of the difficulty of defining a measure of compactness is that there are many conflicting understandings of the concept. According to one view the compactness standard exists to eliminate elongated districts. In this sense a square is more compact than a rectangle, and a circle may be more compact than a square. According to another view compactness exists to eliminate bizarrely shaped districts.[6] According to this view a rectangle-shaped district would be better than a district shaped like a Rorschach blot.[7]

We follow the latter approach. While it may be preferable to avoid elongated districts, the classic sign of a heavily-gerrymandered district is bizarre shape.[8] To the extent that elongation is a concern, it should be studied with a separate measure. These are two separate issues, and there is no obvious way to weigh tradeoffs between bizarreness and elongation.

The basic principle of *convexity* requires a district to contain the shortest path between every pair of its' points. Circles, squares, and triangles are examples of convex shapes, while hooks, stars, and hourglasses are not. The most striking feature of bizarrely shaped districts is that they are extremely non-convex. We introduce a measure of convexity with which to assess the bizarreness of the district.

The *path-based* measure we introduce is the probability that a district will contain the shortest path between a randomly selected pair of its' points.[9] This measure will always return a number between zero and one, with one being perfectly convex. To understand how our measure works, consider a district containing two equally sized towns connected by a very narrow path, such as a road. Our method would assign this district a measure of approximately one-half. A district containing n towns connected by narrow paths would be assigned a measure of approximately $1/n$.[10]

Ideally, a measure of compactness should consider the distribution of the population in the district. Population can be incorporated by using the probability that a district will contain the

[5] However, the U.S. Supreme Court has held that "bizarre shape and noncompactness" of districts is not only evidence of unconstitutional manipulation of district boundaries but also "part of the constitutional problem." See *Shaw v. Reno*, 509 U.S. 630 (1993); *Bush v. Vera*, 517 U.S. 952, 959 (1996).

[6] Writing for the majority in *Bush v. Vera*, Justice O'Connor referred to "bizarre shape and noncompactness" in a manner which suggests that the two are synonymous, or at least very closely related. If so then a compact district is one without a bizarre shape, and a measure of compactness is a measure of bizarreness.

[7] The majority opinion in *Shaw v. Reno* noted that one district had been compared to a "Rorschach ink blot test" by a lower court and a "bug splattered on a windshield" in a major newspaper. 509 U.S. at 633.

[8] Note that the term gerrymander was coined in 1812 by a political cartoonist who sought to link then-Massachusetts Governor Elbridge Gerry to a salamander-shaped legislative district. Had the controversial district merely resembled a rectangle, the process of district manipulation would possibly be referred to as a *gerrytangle*.

[9] A version of this measure was independently discovered by Lehrer (2007).

[10] Alternatively one might use the reciprocal, where the measure represents the equivalent number of disparate communities strung together to form the district. The reciprocal will always be a number greater or equal to one, where one is perfectly convex. A district containing n towns connected by narrow paths would be assigned a measure of approximately n.

shortest path between a randomly selected pair of its' residents. In practice our information will be more limited — we will not know the exact location of every resident, but only the populations of individual census blocks. We can solve this problem by weighting points by population density.

One potential problem is that some districts may be oddly shaped simply because the states in which they are contained are non-convex. We solve this problem by measuring the probability that a district will contain the shortest path *in the state* between a randomly selected pair of its' points. Districts are not penalized for bizarre shapes which are a consequence of the shape of the state.

Our measure considers *whether* the shortest path in a district exceeds the shortest path in the state. Alternatively, one might wish to consider the *extent* to which the former exceeds the latter. We introduce a parametric family of measures which vary according to the degree that they "penalize" deviations from convexity. At one extreme is the measure we have described; at the other is the degenerate measure, which gives all districts a measure of one regardless of their shape.

The chapter also contains computations of our measure for fifteen districts in three states.

1.1.3 A Model of Community Standards

In 1957, the United States Supreme Court ruled that obscenity is not protected by the U.S. Constitution and that "contemporary community standards" are to be used in determining whether particular works are obscene.[11] The Supreme Court has never explained what "community standards" are or how, if at all, they are related to the standards of the individuals who comprise the community.

I introduce a new model in which community standards are formed by aggregating a set of individual standards. In the model, standards are defined as judgments — categorizations of possible works as either "obscene" or "not obscene." Every possible judgment is allowed provided it satisfies the following restriction: neither individuals nor the community may consider one-hundred percent of the works to be obscene. I define several basic normative properties of aggregation methods which reflect legal concerns expressed by the judiciary. I then show that the only method which satisfies these properties is *unanimity rule*, in which a work is considered obscene if and only if all members of the community consider it to be obscene.

Unanimity rule was described as the law of England Lord Patrick Devlin in his classic work, *The Enforcement of Morals* (Devlin, 1965). Lord Devlin argued that it was proper for governments to prohibit behavior felt to be immoral by the community. He suggested that, in some sense, unanimous agreement within a society is necessary to justify regulation of immorality: "the moral judgment of society must be something about which any twelve men or women drawn at random might after discussion be expected to be unanimous."

To ascertain the moral standards of the community, Lord Devlin's understanding of the Law of

[11] *Roth v. United States*, 354 U.S. 476 (1957), upheld in *Miller v. California* 413 U.S. 15 (1973).

England can be described in the following way. First, the community consists of all "right-minded" or "reasonable" persons within the society.[12] Next, an act is deemed immoral if and only if every reasonable person believes the act to be immoral. "Immorality then, for the purpose of the law, is what every right-minded person is presumed to consider to be immoral." (Devlin, 1965).[13]

The basic model can be described as follows. First, there is a community, which can be any group of individuals. The Supreme Court has required that the community be defined in geographic terms and contain all adults in that community, including the young, the old, the religious, the irreligious, the sensitive, and the insensitive.[14] Lord Devlin (1965) seems to have argued that the community consists only of reasonable persons. Others might propose to restrict the definition to clerics, to parents, or to some other community of interest. The model is general enough to include all of these as special cases.

Next, there is an infinite set of all possible works. We might loosely understand this as the set of possible artworks but it might also include literary works, scientific publications, and other forms of human expression. The space of works is modeled as a non-atomic measure space. The decision to use a non-atomic measure space rather than a discrete space is made to simplify the exposition. Parallel conclusions would be reached if the space of works were modeled as discrete and appropriate modifications were made to the axioms.

Individuals from the community have standards as to which works in the set are obscene. An individual standard is simply a division of the set into two groups: the obscene and the non-obscene (or permissible). Individual standards are assumed to be well-informed and made after deliberation and reflection. There is a single restriction on allowable standards: the set of works judged to be obscene must be of less than full measure. Reasonable individuals should all believe that some works, even those lacking serious literary, artistic, political, and scientific value, are non-obscene.[15] I do not require individuals to believe that some works must be obscene — there is no reason why individuals *must* be offended by anything.

These individual standards are then aggregated to form a community standard. The community standard is subject to the same restriction as the individual standards: the set of works judged to be obscene must be of less than full measure. I place no other restrictions on the class of allowable standards. Individual standards and community standards are assumed to be subjective.

The model introduced in this chapter is general and can be applied to problems other than the question of which works are legally obscene. I will describe three different types of legal standards

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[12] Whether an individual is "right-minded" or "reasonable" does not seem to be directly connected to the specific content of that individual's beliefs; otherwise Devlin's rule would be circular and ill-defined.

[13] Whether Devlin's rule is certainly practicable is a debatable proposition. He certainly felt that the rule would lead to convictions in 1958, but whether that should remain the case in the more tolerant environment of the twenty-first century is unclear. However, the mere possibility that some communities would find little to prohibit does not invalidate Devlin's rule. He argued that a community should be able to prohibit that which it found immoral, and not that every community must find some works to be immoral.

[14] See *Roth v. United States*, 354 U.S. 476 (1957) and *Pinkus v. United States*, 436 U.S. 293 (1978).

[15] Individuals who do not satisfy this restriction would be found to be unreasonable as a matter of law.

to which the model can be applied.

First, standards of offensiveness are used to determine whether speech, or other forms of expression, may be prohibited on the grounds that it is offensive. Obscenity doctrine provides the clearest example of a prohibition on offensive expression; other examples include the prohibitions on the broadcast of indecent and profane speech regulated by the Federal Communications Commission.

Second, standards of proof are used to determine whether defendants are guilty (or liable) in criminal (and civil) cases. Commonly used standards of proof include (a) the proof beyond a reasonable doubt standard, (b) the clear and convincing standard, and (c) the preponderance of the evidence standard. Here, instead of a set of works, we have a set of cases as in Kornhauser (1992a,b) and Lax (2007), and individuals choose the subset of cases that lead to conviction. The results of the chapter support the use of unanimity rule in determining which works are obscene.

Third, standards of behavior are used to evaluate behavior in civil and criminal trials. Examples of standards of behavior include the reasonable person standard studied by Rubinstein (1983), the business judgment rule, and fiduciary duties. To model this standard, we replace the set of works with a set of actions. Because the range of allowable behavior depends on the circumstances in which an actor finds herself, individuals have multiple standards, one for each set of circumstances. I show in the chapter that the main result does not change in the case of two (or more) standards. Even if we allow for interdependent aggregation, unanimity rule is the unique aggregation rule that satisfies the axioms.

Chapter 2

Group Identification

This chapter has been previously published in Miller (2008).

2.1 Introduction

In 1997, the White House decided that, for purposes of Federal data collection, you are African-American if you claim you are.[1] In addition, the White House decided that you can be both African-American and White. These policy changes were first implemented in the 2000 decennial census.

The policymakers charged with revising the policy were guided by several principles. Two of these principles stand out. First, they rejected the view that race can be objectively defined. Second, they desired that results be comparable across Federal agencies with different needs. They chose a standard set of racial categories and allowed agencies to use additional categories "provided they can be aggregated to" the standard ones. Other principles included minimizing cost, respecting individual dignity, and the understanding that "the standards are not intended to be used to establish eligibility for participation in any federal program."

Increases in the "number of persons born who are of mixed race or ethnicity" led to the relatively uncontroversial decision to allow individuals to be counted as members of multiple racial groups. The decision to use self-identification was more controversial. Some Federal agencies were concerned that changes in the method of data collection could make it difficult to study historical trends. However, other methods were also known to have problems in this regard and could lead to an individual being identified differently among data sets.

I argue that the decision to switch to self-identification was appropriate in light of the policymakers' concerns. To study this question I set forth a model of group identification in which individuals are classified into groups on the basis of opinions. I introduce several properties which reflect the

[1] Statistical Policy Directive No. 15, Race and Ethnic Standards for Federal Statistics and Administrative Reporting, 62 FR 58782, October 30, 1997.

policymakers' concerns and show that self-identification is the only data collection method that satisfies these properties.

I follow a model of group identification first introduced by Kasher and Rubinstein (1997). Individuals have opinions as to which members of society are members of a particular group. A *social rule* is a systematic method for aggregating opinions of agents. Opinions and outcomes are binary. Each individual is either believed (or determined) to be a member of the group, or not. Social rules are studied through an axiomatic approach: various properties are proposed and rules satisfying these properties are characterized.

I extend the Kasher-Rubinstein model by introducing multiple groups with predefined relationships. Federal statistical policy envisions many racial groups, including Asians, Whites, people who are members of either group ("Asian or White"), and people who are members of both ("Asian and White"). The set of groups also includes the non-members of each group (e.g. "Non-Whites" and "Non-Asians").

Opinions must preserve these relationships. A person who believes that his neighbor is "Asian and White" must also believe that his neighbor is Asian. A social rule takes the group label as an argument, so that it is allowed to work differently for different groups. It need not aggregate opinions about Asians in the same way that it aggregates opinions about Whites. However, a consequence of the axioms described below is that a rule must be independent of the group under consideration.

The focus of this chapter is to understand the implications of a property called *separability*, which requires that social rules preserve certain relationships between groups. I introduce two axioms which implement different aspects of this concept.

Meet separability requires social rules to yield the same list of "Asian and White" people regardless of which of two possible approaches is used. The first approach is to aggregate opinions about "Asian and White" people. The second approach is to generate two lists by aggregating opinions about Asians and about Whites separately, and then to take the intersection of the two lists.

Join separability is a similar, except that it is defined with respect to disjunction instead of conjunction. It requires social rules to yield the same list of "Asian or White" people regardless of which of two possible approaches is used. The first approach is to aggregate opinions about "Asian or White" people. The second approach is to generate two lists by aggregating opinions about Asians and about Whites separately, and then to take the union of the two lists.

In addition to the separability axioms, I require *non-degeneracy.* This axiom requires that the opinions be relevant in determining whether each person is a member of a group. There are cases when this axiom is not appropriate. To find the group of people whose height exceeds six feet, we do not need opinions. Two yardsticks are sufficient. In the absence of objective standards, opinions are needed to classify people into groups.

Using these three axioms I characterize a family of rules called *one-vote* rules. These rules associate with each person a single opinion that determines whether that person is *qualified* (determined to be a member of the group).

An *anonymity* axiom requires that the qualification of individuals does not depend on their names. Anonymity is appealing because government policymakers cannot arbitrarily favor some people above others. Non-governmental researchers gathering data may not be bound by this normative requirement, but may lack an external scientific basis to draw distinctions between individuals. I show that self-identification is the only anonymous one-vote rule.

The problem of group identification extends beyond the sphere of Federal agencies trying to classify individuals for statistical purposes. Legal systems allocate rights and restrictions to groups, whether these be particular minority groups or the group of licensed drivers. Philosophers try to understand whether groups exist. Sociologists analyze social groups, anthropologists examine cultures, political scientists study collective action,[2] and historians use groups to explain past human behavior. In economics there has been a significant increase in the use of group membership as a variable in formal modeling following the recent work of Akerlof and Kranton (2000).

2.1.1 Related Literature

The model of group identification was first introduced by Kasher and Rubinstein (1997). Their paper included a characterization of self-identification as the only rule satisfying symmetry, a weak independence condition, and the "liberal principle," which required that individuals can force certain outcomes.[3] Much of the notation in this chapter was introduced by Samet and Schmeidler (2003), who studied a family of consent rules characterized by anonymity, monotonicity, and a strong independence condition. They also characterize self-identification as the only rule satisfying monotonicity, non-degeneracy, the stronger independence condition, and another property labeled self-determination.

Nearly every paper which considers the Kasher-Rubinstein model studies social rules which satisfy an *independence* axiom.[4] The stronger version of this axiom, found in Kasher and Rubinstein (1997), Samet and Schmeidler (2003), Ju (2005), and Çengelci and Sanver (2006), requires that whether a particular individual is determined to be a member of a group is independent of the opinions regarding all of the other individuals. A weaker version of this axiom, found in Kasher and Rubinstein (1997), Sung and Dimitrov (2005), and Dimitrov et al. (2007),[5] allows an individual's status to be affected by opinions about other individuals if some other individual's status is also affected. This chapter departs completely from the requirement of independence.

[2] See Olson (1971).

[3] This follows a refinement of the Kasher-Rubinstein characterization by Sung and Dimitrov (2005).

[4] An exception is Houy (2006) which classifies individuals as members of a group if they are "indirectly designated by all the individuals in the society."

[5] Using the weaker independence condition, Dimitrov et al. (2007) characterize a recursive procedure for determining group membership.

The Kasher-Rubinstein framework is applicable in determining which individuals meet a particular standard, such as the set of students eligible for merit scholarships. A related but conceptually distinct problem involves ranking individuals according to a standard. For example, a school might wish to create a ranking of students. The latter problem has been studied axiomatically by Palacios-Huerta and Volij (2004) in the context of developing a cardinal ranking of scientific publications.

2.2 The Model

2.2.1 The Model and the Notation

I extend the model introduced by Kasher and Rubinstein (1997) and use the notation introduced by Samet and Schmeidler (2003). There is a set $N \equiv \{1, ..., n\}$ of individuals, $n \geq 3$.[6] There is a set of groups, $\mathscr{G}$, that forms a Boolean algebra under conjunction, disjunction, and negation. Each element $a \in \mathscr{G}$ describes membership in a group. For example, if $a, w \in \mathscr{G}$ are the groups "Asian" and "White," respectively, then $\mathscr{G}$ also contains the groups $a \wedge w$ ("Asian and White"), $a \vee w$ ("Asian or White") and $\bar{a}$ ("Non-Asian"). The collection $\mathscr{G}$ also contains a minimal group $\mathtt{0}$ ("no one") and a maximal group $\mathtt{1}$ ("everyone"). I denote by $\mathcal{G} \equiv \mathscr{G} \setminus \{\mathtt{0}, \mathtt{1}\}$ this set minus these minimal and maximal groups.

A categorical **view** about a group is an N-vector, the j-th component of which is 1 if individual j is viewed as a member, and 0 otherwise. The set of views is denoted by $\mathcal{V} \equiv \{0, 1\}^N$. A **profile** is a vector of views $P = (P_1, ..., P_n) \in \mathcal{V}^N$ where P_i represents individual i's view. I write P_{ij} to denote individual i's opinion about individual j. A **qualification problem** is a pair $(P, a) \in \mathcal{V}^N \times \mathcal{G}$. A **social rule** is a function $f : \mathcal{V}^N \times \mathcal{G} \to \mathcal{V}$ which associates each qualification problem with a **social opinion** $f(P, a) \equiv (f_1(P, a), ..., f_n(P, a))$.

As usual, for a set of the form $\{0, 1\}^I$, I write $x \geq y$ if $x_i \geq y_i$ for all $i \in I$, $x > y$ if $x \geq y$ and $x \neq y$, and $(\bar{x}_i) = (1 - x_i)$. I define the **meet** ($\wedge$) as the coordinatewise minimum, so that $(x \wedge y)_i = \min\{x_i, y_i\}$, and the **join** ($\vee$) as the coordinatewise maximum, so that $(x \vee y)_i = \max\{x_i, y_i\}$. Lastly, I denote $\mathbf{0}$ and $\mathbf{1}$ as the elements of the set composed entirely of zeros and ones, respectively.

For any two qualification problems (P, a) and (Q, b), I define $(P, a) \wedge (Q, b) \equiv (P \wedge Q, a \wedge b)$, $(P, a) \vee (Q, b) \equiv (P \vee Q, a \vee b)$, and $(\overline{P, a}) \equiv (\bar{P}, \bar{a})$. A direct implication of this definition is that the opinions with respect to groups $a \wedge \bar{a} = \mathtt{0}$ and $a \vee \bar{a} = \mathtt{1}$ are represented by $(P, a) \wedge (\overline{P, a}) = (\mathbf{0}, \mathtt{0})$ and $(P, a) \vee (\overline{P, a}) = (\mathbf{1}, \mathtt{1})$. Everyone believes that no one is a member of the group "no one," and that everyone is a member of the group "everyone." I define $f(\mathbf{0}, \mathtt{0}) \equiv \mathbf{0}$ and $f(\mathbf{1}, \mathtt{1}) \equiv \mathbf{1}$.

A set of qualification problems is **consistent** if, for any two qualification problems (P, a) and (Q, b) in the set, the following three properties hold.

[6]With the exception of Theorem 2.2.6, all results would hold if I allowed the case where n=2.

1. When a and b encompass the entire set (e.g., "Asian or White" and "Non-Asian"), then everyone should believe that everyone is a member of a or b.
   - If $a \vee b = \mathbf{1}$, then $(P, a) \vee (Q, b) = (\mathbf{1}, \mathbf{1})$.
2. When a and b are mutually exclusive (e.g., "Asian and White" and "Non-White"), then everyone should believe that no one is a member of both.
   - If $a \wedge b = \mathbf{0}$, then $(P, a) \wedge (Q, b) = (\mathbf{0}, \mathbf{0})$.
3. When a includes b (e.g., "White" includes "Asian and White"), then everyone should believe that members of b are also members of a.
   - If $a \vee b = a$, then $(P, a) \vee (Q, b) = (P, a)$.

I denote by $\mathscr{C}$ the set of consistent two-element sets of qualification problems.

2.2.2 The Axioms

Let a and w be the groups of Asians and Whites, respectively, and let (A, a) and (W, w) describe the opinions about these groups. Then $a \wedge w$ is the group of "Asian and White" people and $(A, a) \wedge (W, w) \equiv (A \wedge W, a \wedge w)$ describes the opinions about that group.

There are two ways to generate a list of "Asian and White" people. The *single ballot approach* will directly generate a list of "Asian and White" people: $f((A, a) \wedge (W, w))$. The *two ballot approach* will generate two lists; one of Asians, $f(A, a)$, and one of Whites, $f(W, w)$. One can then generate a list of "Asian and White" people by taking the names common to both lists. This is the meet of the two lists: $f(A, a) \wedge f(W, w)$. The first axiom, *meet separability*, requires that these lists be the same.

Meet separability: For every consistent set of qualification problems $\{(P, a), (Q, b)\} \in \mathscr{C}$, $f((P, a) \wedge (Q, b)) = f(P, a) \wedge f(Q, b)$.

Similarly, $a \vee w$ is the group of "Asian or White" people and $(A, a) \vee (W, w) \equiv (A \vee W, a \vee w)$ describes the opinions about that group.

There are two ways to generate a list of "Asian or White" people. The *single ballot approach* will directly generate a list of "Asian or White" people: $f((A, a) \vee (W, w))$. The *two ballot approach* will generate two lists; one of Asians, $f(A, a)$, and one of Whites, $f(W, w)$. One can then generate a list of "Asian or White" people by taking the names which appear on either list. This is the join of the two lists: $f(A, a) \vee f(W, w)$. The second axiom, *join separability*, requires that these lists be the same.

Join separability: For every consistent set of qualification problems $\{(P, a), (Q, b)\} \in \mathscr{C}$, $f((P, a) \vee (Q, b)) = f(P, a) \vee f(Q, b)$.

The third axiom is adapted from Samet and Schmeidler (2003). This axiom excludes constant rules — rules for which there exists an individual who is, or is not, a member of the group regardless of which names are on the ballots.

Non-degeneracy: For every individual j and every group $a \in \mathcal{G}$ there exist profiles P, P' such that $f_j(P, a) = 1$ and $f_j(P', a) = 0$.

Each of the separability axioms implies *monotonicity*, which requires that no names be removed from the list of qualified persons as additional names are added to the ballots.

Monotonicity: For every group $a \in \mathcal{G}$, $P \geq P'$ implies that $f(P, a) \geq f(P', a)$.

The proof of the following lemma is straightforward and is left for readers.

Lemma 2.2.1 *If a social rule f satisfies either of the meet separability or join separability axioms then it satisfies monotonicity.*

Social rules satisfy *group independence* if they use the same method to aggregate opinions about every group.

Group independence: For all groups $a, b \in \mathcal{G}$ and every profile P, $f(P, a) = f(P, b)$.

None of these axioms directly requires group independence. One might use one method to aggregate opinions about Asians, a different method to aggregate opinions about Whites, and a third method to aggregate opinions about "Asians and Whites". However, rules which satisfy non-degeneracy and either separability axiom necessarily satisfy group independence, as I show in the following proposition.[7] Every rule discussed in the chapter satisfies this property. To simplify the notation I will sometimes drop the group label and write "$f(P)$" in place of "$f(P, a)$ for every $a \in \mathcal{G}$".

Proposition 2.2.2 *If a social rule f satisfies the non-degeneracy axiom and either of the meet separability or join separability axioms then it satisfies group independence.*

Proof.

Non-degeneracy and monotonicity directly imply that $f(\mathbf{1}, a) = \mathbf{1}$ for all $a \in \mathcal{G}$.

Let the pair $\{(P, a), (\mathbf{1}, b)\} \in \mathscr{C}$ such that $(*)$ $a \neq a \wedge b \neq b$. By meet separability, $f(P, a) \wedge f(\mathbf{1}, b) = f(P \wedge \mathbf{1}, a \wedge b)$ and thus $f(P, a) = f(P, a \wedge b)$. From $(*)$ it follows that the pair $((\mathbf{1}, a), (P, b))$ is consistent, and thus $f(P, b) = f(P, a \wedge b)$. Therefore $f(P, a) = f(P, b)$. For every $a, b \in \mathcal{G}$ such that $(*)$ does not hold there exists an element $c \in \mathcal{G}$ such that $a \neq a \wedge c \neq c \neq c \wedge b \neq b$ and thus $f(P, a) = f(P, b)$ for all $a, b \in \mathcal{G}$.

The second half of the proof is the dual of the first and can be proved by replacing "meet" $(\wedge)$ with "join" $(\vee)$ and 1 with 0. ■

[7] To understand why non-degeneracy is necessary, consider any rule where $f(P, a) = f(P, a \wedge b)$ and $f(P, b) = \mathbf{1}$. This rule satisfies meet separability but neither non-degeneracy nor group independence.

2.2.3 The Main Characterizations

I now define three families of rules which are characterized by combinations of axioms from the preceding subsection. In each family rules associate with each individual a non-empty set of relevant opinions which uniquely determine whether the individual is qualified. The families differ by the degree of cohesiveness that the relevant opinions must demonstrate for the individual to be qualified, as well as by the size of the relevant set. The relevant opinions are neither required to be about nor otherwise related to the individual with whom the set is associated.

2.2.3.1 Agreement Rules

The first such family of rules are **agreement** rules. An individual is qualified as a member of a group if every opinion in the relevant set is in favor of qualification. These rules can equivalently be defined by associating with each individual a minimal profile P in which $P_{ij} = 1$ if and only if i's opinion about j is relevant.[8] This family of rules is characterized by meet separability and non-degeneracy.

Agreement rules: For every individual j there exists a profile $P^{j-} > \mathbf{0}$ such that, for all groups $a \in \mathcal{G}$, $f_j(P, a) = 1$ if and only if $P \geq P^{j-}$.

These rules are characterized in the following theorem:

Theorem 2.2.3 *A social rule f satisfies the meet separability and non-degeneracy axioms if and only if it is an agreement rule. Moreover, both axioms are independent.*

Proof.

By Proposition 2.2.2 rules satisfying meet separability and non-degeneracy are group independent. Let $P, Q \in \mathcal{V}^N$ and $j \in N$. Define $\mathcal{P}_j \equiv \{P \in \mathcal{V}^N : f_j(P) = 1\}$. We know that $\mathcal{P}_j \neq \varnothing$ by the non-degeneracy axiom.

Define $P^{j-} \equiv \wedge_{P \in \mathcal{P}_j} P$. For all profiles $P', P'' \in \mathcal{P}_j$, $f_j(P') = f_j(P'') = 1$. By the meet separability axiom, $f_j(P' \wedge P'') = 1$. It follows by an induction argument that $f_j(\wedge_{P \in \mathcal{P}_j} P) = f_j(P^{j-}) = 1$. Therefore, $P^{j-} \in \mathcal{P}_j$.

Clearly, for all profiles $P \in \mathcal{P}_j$, $P \geq \wedge_{P \in \mathcal{P}_j} P = P^{j-}$. Furthermore, $P^{j-} \neq \mathbf{0}$, otherwise $f_j(P) = 1$ for all profiles P, which would violate the non-degeneracy axiom.

Lastly, I show that for all profiles P such that $P \geq P^{j-}$, $P \in \mathcal{P}_j$. Let $P \geq P^{j-}$. By monotonicity, $f_j(P) \geq f_j(P^{j-})$. Because $f_j(P^{j-}) = 1$ it follows that $P \in \mathcal{P}_j$. Hence $P \in \mathcal{P}_j$ if and only if $P \geq P^{j-}$. It follows that, for all $a \in \mathcal{G}$, $f_j(P, a) = 1$ if and only if $P \geq P^{j-}$.

The independence of the axioms is proved in the appendix. ■

[8] P is the minimal profile which leads to qualification. Note that the minimal profile cannot be $\mathbf{0}$; otherwise the set of relevant opinions would be empty.

2.2.3.2 Nomination Rules

The second family of rules are **nomination** rules. An individual is qualified as a member of a group if any opinion in the relevant set is in favor of qualification. These rules can equivalently be defined by associating with each individual a maximal profile P in which $P_{ij} = 0$ if and only if i's opinion about j is relevant.[9] This family of rules is characterized by join separability and non-degeneracy.

Nomination rules: For every individual j there exists a profile $P^{j+} < \mathbf{1}$ such that, for all groups $a \in \mathcal{G}$, $f_j(P, a) = 0$ if and only if $P \leq P^{j+}$.

These rules are characterized in the following theorem:

Theorem 2.2.4 *A social rule f satisfies the join separability and non-degeneracy axioms if and only if it is a nomination rule. Moreover, both axioms are independent.*

Proof. This is the dual of Theorem 2.2.3 and can be proved by exchanging 0 and 1 and replacing "meet" ($\wedge$) with "join" ($\vee$), $\geq$ with $\leq$, and P^{j-} with P^{j+}.

The independence of the axioms is proved in the appendix. ■

2.2.3.3 One-Vote Rules

The third family of rules are **one-vote** rules, for which the relevant set associated with each individual consists of a single opinion. The individual is qualified if that opinion is in favor of qualification. One-vote rules are characterized by meet separability, join separability, and non-degeneracy.

One-vote rules: For every individual j there exists (i, k) in $N \times N$ such that, for all groups $a \in \mathcal{G}$, $f_j(P, a) = P_{ik}$.

From this follows the main result:

Theorem 2.2.5 *A social rule f satisfies the meet separability, join separability, and non-degeneracy axioms if and only if it is a one-vote rule. Moreover, all three axioms are independent.*

Proof. That the one-vote rules satisfy the three axioms is trivial. I show that any social rule that satisfies the three axioms is necessarily a one-vote rule. Let $j \in N$. Because f satisfies meet separability and non-degeneracy it must be an agreement rule (by Theorem 2.2.3). Therefore, there must exist a profile $P^{j-} > \mathbf{0}$ such that $f_j(P) = 1$ if and only if $P \geq P^{j-}$. This implies that there exists (i, k) in $N \times N$ such that $P_{ik}^{j-} = 1$. It follows that if $P_{ik} = 0$ then $f_j(P) = 0$ and therefore $P_{ik} \geq f_j(P)$.

Because f satisfies join separability and non-degeneracy it must be an agreement rule (by Theorem 2.2.4). Therefore, there must exist a profile $P^{j+} < \mathbf{1}$ such that $f_j(P) = 0$ if and only if

[9] P is the maximal profile leading to disqualification. Note that the maximal profile cannot be $\mathbf{1}$; otherwise the set of relevant opinions would be empty.

$P \leq P^{j+}$. Let $P^* \in \mathcal{V}^N$ such that all elements are 1 except that $P^*_{ik} = 0$. We know that if $P^*_{ik} = 0$ then $f_j(P^*) = 0$ and therefore $P^* \leq P^{j+}$. Because $P^{j+} < \mathbf{1}$ it follows that $P^* = P^{j+}$ and thus $P^{j+}_{ik} = 0$. This implies that if $P_{ik} = 1$ then $f_j(P) = 1$ and therefore $f_j(P) \geq P_{ik}$. It follows that $f_j(P, a) = P_{ik}$ for every group $a \in \mathcal{G}$.

The independence of the axioms is proved in the appendix. ■

2.2.4 Other Results

2.2.4.1 Self-identification

Kasher and Rubinstein (1997) introduced **self-identification**, in which each person decides whether to qualify herself.[10]

Self-identification: For every $j \in N$ and for every $a \in \mathcal{G}$, $f_j(P, a) = P_{jj}$.

The principle of equality of persons restricts an aggregation rule from making arbitrary distinctions among members of the society.[11] Samet and Schmeidler (2003) applied this principle through an **anonymity** condition which requires that the list of the qualified individuals does not depend on their names.[12] Names are switched through a permutation π of N. Thus, for a given permutation π, i is the new name of the individual formerly known as $\pi(i)$. For a given profile $P \in \mathcal{V}^N$, I let πP be the profile in which the names are switched. Then $(\pi P)_{ij} = P_{\pi(i)\pi(j)}$. I denote $\pi f(P, a) \equiv (f_{\pi(1)}(P, a), f_{\pi(2)}(P, a), ..., f_{\pi(n)}(P, a))$. The fourth axiom, anonymity, requires that if individual i is qualified in profile πP, then individual $\pi(i)$ is qualified in profile P.

Anonymity: For every permutation π of N and every group $a \in \mathcal{G}$, $f(\pi P, a) = \pi f(P, a)$.

Self-identification is the only one-vote rule which satisfies the anonymity axiom.[13]

Theorem 2.2.6 *Self-identification is the only rule that satisfies the meet separability, join separability, non-degeneracy, and anonymity axioms. Moreover, all four axioms are independent.*

Proof. That self-identification satisfies the four axioms is trivial. I show that any rule that satisfies the four axioms must necessarily be self-identification. Let $j \in N$. Let f satisfy the meet separability,

[10] Self-identification is referred to as "the liberal rule" by Samet and Schmeidler and as the "strong liberal collective identity function" by Kasher and Rubinstein.

[11] In axiomatic economic theory this principle dates back at least as far as May (1951). This principle can be motivated either from a normative belief in equality such as that found in the writings of Locke and Jefferson, or from a positive concern that a researcher lacks a scientific basis upon which to draw such distinctions. Both motivations are relevant to government bureaucrats forbidden from arbitrarily preferring some people over others.

[12] Samet and Schmeidler call this condition symmetry. I have changed the name to minimize confusion with a different axiom of the same name introduced by Kasher and Rubinstein.

[13] A different approach to the principle of equality can be found in the *symmetry* condition of Kasher and Rubinstein (1997), which requires that if any two individuals are symmetric with respect to their views about others and others' views toward them, then either both or neither are qualified. Self-identification is not the only one-vote rule which satisfies the symmetry axiom; however, it is the only one-vote rule which satisfies symmetry and a stronger form of non-degeneracy. For more explanation and a proof see Miller (2006).

join separability, non-degeneracy, and anonymity axioms. By Theorem 2.2.5 f must be a one-vote rule, and therefore there must be a pair of individuals i and k such that $f_j(P,b) = P_{ik}$. Because the pair of individuals may differ for every individual j, I denote these individuals $i(j)$ and $k(j)$. Therefore, $f_j(P) = P_{i(j)k(j)}$. Let π be a permutation of N. Then, $f_j(\pi P) = (\pi P)_{i(j)k(j)} = P_{\pi(i(j))\pi(k(j))}$, and $f_{\pi(j)}(P) = P_{i(\pi(j))k(\pi(j))}$. By the anonymity axiom, it follows that $P_{\pi(i(j))\pi(k(j))} = P_{i(\pi(j))k(\pi(j))}$, which implies that $\pi(i(j)) = i(\pi(j))$ and $\pi(k(j)) = k(\pi(j))$, which hold if and only if $i(j) = k(j) = j$. Thus, for every individual $j \in N$ and every issue $a \in \mathcal{G}$, $f_j(P,a) = P_{jj}$.

The independence of the axioms is proved in the appendix. ■

2.2.4.2 Negation

Individuals in the model vote consistently: if (P,a) describes the opinions about group a, then $(\overline{P,a})$ describes the opinions about the group $\bar{a}$. Then $f(\overline{P,a})$ is the list of members of group $\bar{a}$ and the list of *non-members* of group a is given by $\overline{f(P,a)}$.

The fifth axiom, *negation*, requires that these two lists be the same.[14]

Negation: For all profiles P and all groups $a \in \mathcal{G}$, $\overline{f(P,a)} = f(\overline{P,a})$.

The negation axiom has another interpretation: it requires that social rules classify each person in the society as a member of a group or its complement but not both. The following proposition is necessary and sufficient to show that Theorems 2.2.5 and 2.2.6 remain true if one of the separability axioms is replaced by negation.

Proposition 2.2.7 *If a social rule f satisfies two of the meet separability, join separability, and negation axioms it satisfies the third.*

The proof is straightforward and is left to readers.

2.3 Conclusion

I have extended the Kasher-Rubinstein model of group identification to allow social rules to aggregate opinions about different groups in different manners. I have introduced a concept of group separability and have shown that any non-degenerate rule satisfying both of the separability axioms is necessarily a one-vote rule, in which for each individual there is exactly one opinion which determines whether that person is qualified. The only anonymous one-vote rule is self-identification,

[14]Negation is related to the *self-duality* axiom introduced by Samet and Schmeidler. Both axioms require the social rule to preserve complementation and both are motivated by the view that one should be able to learn who is a member of a group by asking about the non-members of the group. Because of changes in the model, however, the axioms have rather different implications. Unlike self-duality, negation does not require a social rule to treat membership and non-membership in the same manner, nor does it require non-degeneracy.

under which each person determines for herself whether she is qualified. How we interpret these results depends on our understanding of the primitive.

The primary motivation set forth is to understand rules used to generate group data for research. This describes both Federal policy regarding data collection and the creation of data sets by social scientists. Here researchers decide that group identification should be a function of beliefs, either because the group cannot be objectively defined, or because the researchers lack other means to determine who is a member of a group. In this case it is important that seemingly trivial decisions made (whether to use one survey or two to generate the relevant data) should not have an unknown effect on the results of the research. Consequently, the results of this chapter recommend the use of one-vote rules, especially when it is not clear how those later researchers will use the data. In cases where the researchers lack a basis for preferring some persons over others, self-identification should be used.

One-vote rules are also particularly nice because they require fewer opinions and therefore may be cheaper to generate. If the data set contains information on a small subset of S individuals out of a much larger society of N people, the one-vote rule requires the person creating the data set to seek out S opinions out of a total of N^2. Self-identification has other desirable properties. It is “liberal” in the sense that each person chooses whether she is a member of a group. Federal policymakers argued that it respects human dignity because it does “not tell an individual who he or she is, or specify how an individual should classify himself or herself.”

Another possible understanding of the social rules studied in this chapter is as a voting mechanism used to determine the composition of groups endowed with certain legal rights or obligations. The anonymity axiom seems desirable when allocating rights in a democratic society; consequently the separability axioms suggest the use of self-identification. However, it seems rather clear that individuals may have an incentive to distort their beliefs when group membership leads to a direct and tangible benefit or cost. Self-identification would not make much sense in this case. This tells us that sensible aggregation rules used to allocate legal rights will violate the separability axioms. An agenda setter may be able to influence the allocation of rights by dividing one question into two.

A third understanding is more philosophical. A popular view holds that a group is a social construct and only exists as a function of the beliefs about its composition. In this sense a social rule is part of the definition of a group. The separability axioms are very natural in this context because a given set of beliefs will always lead to a unique list of group members. The results of the chapter suggest that there are limits on the method through which the beliefs can be aggregated. There cannot be groups defined by majority opinions, while there can be groups defined by self-inclusion. An alternate view is that a group is a social construct but that it exists as a function of beliefs other than the binary views considered in the model. It is impossible to evaluate this claim without adding more structure to the model. The case where opinions take the form of a totally ordered set is discussed next.

2.3.1 Generalizations of the Characterization Theorems

A possible extension to the group identification model would be to weaken the assumption that opinions about qualification are binary by replacing the domain of possible opinions from $\{0, 1\}$ to an arbitrary totally ordered set $\mathcal{D}$ containing minimal and maximal elements 0 and 1. Examples previously examined in the literature include the "trichotomous domain" studied by Ju (2005) in which $\mathcal{D} = \{0, \frac{1}{2}, 1\}$ (here $\frac{1}{2}$ has the meaning "no opinion"), and the unit interval $[0, 1]$ studied by Ballester and Garcia-Lapresta (2005). In this case profiles are elements of $\mathcal{D}^{N \times N}$. Social rules are mappings $f : \mathcal{D}^{N \times N} \times \mathcal{G} \to \mathcal{V}$. All of the axioms have natural analogues for this more general case.

If $\mathcal{D}$ is finite, then Theorems 2.2.3 and 2.2.4 remain unchanged, as the only assumption about $\mathcal{D}$ used in the proof is that $\mathcal{D}$ is finite. If $\mathcal{D}$ is not finite then neither of these theorems hold, as proofs rely on a finite induction argument. Proposition 2.2.2, however, is still applicable. In neither case does Theorem 2.2.5 hold; however, the rules characterized by meet separability, join separability, and non-degeneracy are very similar. As with one-vote rules, each individual is associated with a single relevant opinion. The individual is qualified if that opinion exceeds a cutoff point and is not qualified if the opinion is below that cutoff point. The rule additionally specifies whether the individual is qualified if that opinion is exactly at the cutoff point. For a definition of these rules and a proof see appendix B.

2.3.2 Weakening the Axioms

A potential criticism of the result stems from the formulation of the axioms. One might object in that they govern relationships between irrelevant groups. For example, one might want a rule to aggregate opinions about British, Americans, and British-Americans consistently but not care about how the rule aggregates opinions about people who are either British or American. A weaker form of these axioms would apply only to pairs of groups in a subset of $\mathcal{G}$, where the subset is carefully chosen so that the axioms only place restrictions on relationships between the relevant groups.

With respect to the relevant groups, the results of Proposition 2.2.2 and Theorems 2.2.3, 2.2.4, and 2.2.5 would be entirely the same if the weakened forms of the axioms were used. The only difference would be with respect to the irrelevant groups — these axioms would not apply to them and therefore any non-degenerate rules would suffice.

2.3.3 Separating Voters from Issues

This chapter has focused on the question of group identification, in which the binary opinions of n persons on n issues are simultaneously aggregated.[15] Alternatively, one might consider a more general model involving the simultaneous aggregation of the binary opinions of n persons on m issues, where $n \neq m$. All of the results in sections 2.3 and 2.4.2 are applicable to the more general case of simultaneous aggregation of binary opinions on multiple issues.

The special case where $m = 1$ corresponds a problem in the literature known as "judgment aggregation". In this case the set $\mathcal{G}$ corresponds to a set of logical propositions. For example, $a \in \mathcal{G}$ and $b \in \mathcal{G}$ might represent two elements of a crime, while $a \wedge b \in \mathcal{G}$ might represent the crime itself. A potentially desirable property is that it should not matter whether the court aggregates judgments about to the elements (a and b) or about the crime ($a \wedge b$). As is clear from Theorem 2.2.3, only agreement rules satisfy this property.[16] The first formal impossibility result in judgment aggregation was proved by List and Pettit (2002).

The judgment aggregation problem can be extended to the case where $m > 1$. For example, we might consider the case of two or more criminal co-defendants. The question of whether one defendant committed the first element of the crime is potentially related to the question of whether the other defendant committed that same element. The property discussed above implies that we must use an agreement rule. In some cases these rules may be plausible. If two defendants are being tried for conspiracy, for example, it might make sense to require that the jurors unanimously convict both to convict either.[17]

2.4 Appendix

2.4.1 Independence of the Axioms

Claim 1 *The meet separability, join separability, non-degeneracy, and anonymity axioms are independent.*

Proof. I present four rules. Each violates one axiom while satisfying the remaining three. This is sufficient to prove the claim.

Rule 1: Consider the rule f in which, for every $j \in N$, $f_j(P) = 1$ if and only if $P_{ij} = 1$ for some $i \in N$. This is a nomination rule but not a one-vote rule and therefore satisfies join separability and non-degeneracy but not meet separability (by Theorems 2.2.4 & 2.2.5).

[15] Each of the n issues is the issue of whether a particular individual is a member of the group.

[16] When $m = 1$ agreement rules are also known as oligarchic rules.

[17] The idea here is that one cannot commit conspiracy without a co-conspirator. In general, American law has dealt with this problem by establishing rules which determine which opinions are to be aggregated. For example, the law might require that the court aggregate opinions about the crime and not about the elements. This is possible because all opinions are known. This solution is less plausible for the case of aggregating opinions about group membership because in that case it is not necessarily possible to know all opinions at the time they are aggregated.

Lastly, to show that it satisfies anonymity, let $j \in N$ and let π be a permutation of N. According to this rule, $f_j(P) = 1$ if and only if $P_{ij} = 1$ for some $i \in N$. Then $f_j(\pi P) = 1$ if and only if $(\pi P)_{ij} = P_{\pi(i)\pi(j)} = 1$ for some $i \in N$. Because this is true for any $i \in N$, $f_j(\pi P) = 1$ if and only if $P_{i\pi(j)} = 1$ for some $i \in N$. Furthermore, $\pi f_j(P) = f_{\pi(j)}(P) = 1$ if and only if $P_{i\pi(j)} = 1$ for some $i \in N$. Therefore, $\pi f_j(P) = f_j(\pi P)$ for all $j \in N$.

Rule 2: Consider the rule f in which, for every $j \in N$, $f_j(P) = 1$ if and only if $P_{ij} = 1$ for all $i \in N$. This is an agreement rule but not a one-vote rule and therefore satisfies meet separability and non-degeneracy but not join separability (by Theorems 2.2.3 & 2.2.5).

Lastly, to show that it satisfies anonymity, let $j \in N$ and let π be a permutation of N. According to this rule, $f_j(P, a) = 1$ if and only if $P_{ij} = 1$ for all $i \in N$. Then $f_j(\pi P) = 1$ if and only if $(\pi P)_{ij} = P_{\pi(i)\pi(j)} = 1$ for all $i \in N$. Because this must be true for all $i \in N$, $f_j(\pi P) = 1$ if and only if $P_{i\pi(j)} = 1$ for all $i \in N$. Furthermore, $\pi f_j(P) = f_{\pi(j)}(P) = 1$ if and only if $P_{i\pi(j)} = 1$ for all $i \in N$. Therefore, $\pi f_j(P) = f_j(\pi P)$ for all $j \in N$.

Rule 3: Let $a \in \mathcal{G}$ such that $a > b$ for no $b \in \mathcal{G}$. Consider the degenerate rule f in which $f(P, b) = \mathbf{1}$ if and only if $b \geq a$, and in which $f(P, b) = \mathbf{0}$, otherwise. This trivially satisfies the meet separability, join separability, and anonymity axioms, but violates non-degeneracy.

Rule 4: Consider the rule f in which, for every $j \in N$, $f_j(P) = 1$ if and only if $P_{1j} = 1$. This is a one-vote rule but is not self-identification and therefore satisfies the meet separability, join separability, and non-degeneracy axioms but violates anonymity. ■

2.4.2 The Generalized Model

Formally, let $\mathcal{D}$ be a totally ordered set with a minimal element 0 and a maximal element 1. Let an aggregation rule be a mapping $f : \mathcal{D}^{N \times N} \times \mathcal{G} \to \mathcal{V}$.

One-opinion rules: For every individual j there exists (i, k) in $N \times N$ and $(d, r) \in \mathcal{D} \times \{0, 1\} \setminus \{(0, 1), (1, 0)\}$, such that, for all groups $a \in \mathcal{G}$, $f_j(P, a) = 1$ if $P_{ik} > d$ and $f_j(P, a) = 0$ if $P_{ik} < d$ and $f_j(P, a) = r$ if $P_{ik} = d$.

Note that in the case $\mathcal{D} = \{0, 1\}$ a one-opinion rule is equivalent to a one-vote rule.[18]

Theorem 2.4.1 *A social rule f satisfies the meet separability, join separability, and non-degeneracy axioms if and only if it is a one-opinion rule.*

Proof. That one-opinion rules satisfy the axioms is trivial. I show that any rule that satisfies the three axioms is a one-opinion rule. Let f satisfy the three axioms. By Proposition 2.2.2 f must satisfy group independence.

Let $P, Q \in \mathcal{V}^N$ and $j \in N$.

[18] In this case non-degeneracy implies that $r = d$.

Define $\mathcal{P}_j \equiv \{P \in \mathcal{D}^{N\times N} : f_j(P) = 1\}$, $P^{j-} \equiv \wedge_{P\in\mathcal{P}_j} P$, and $P^{j+} \equiv \vee_{P\notin\mathcal{P}_j} P$.

Note that $f_j(P) = 1$ implies that $P \geq P^{j-}$ and that $f_j(P) = 0$ implies that $P \leq P^{j+}$. Therefore, for all $P \in \mathcal{D}^{N\times N}$, $P \geq P^{j-}$ and/or $P \leq P^{j+}$.

First, I establish that $P^{j-}_{ik} > 0$ for at most one pair $(i,k) \in N \times N$.

Suppose, contrariwise, that $\left|\{(i,k) \in N \times N : P^{j-}_{ik} > 0\}\right| > 1$. Without loss of generality, assume that $P^{j-}_{11} > 0$. Let $P^* \in \mathcal{V}^N$ such that all elements are 0 except that $P^*_{11} = 1$. Because $P^* \not\geq P^{j-}$ it follows that $f_j(P^*) = 0$. Let $P^\circ \in \mathcal{V}^N$ such that all elements are 1 except that $P^\circ_{11} = 0$. Because $P^\circ \not\geq P^{j-}$ it follows that $f_j(P^\circ) = 0$. Join separability implies that $f_j(P^* \vee P^\circ) = f_j(\mathbf{1}) = 0$. Meet separability implies that, for all $P \in \mathcal{D}^{N\times N}$, $f_j(P) = f_j(P \wedge \mathbf{1}) = f_j(P) \wedge f_j(\mathbf{1}) = 0$. The contradiction proves that $P^{j-}_{ik} > 0$ for at most one pair $(i,k) \in N \times N$. Similarly one can show that $P^{j+}_{ik} < 1$ for at most one pair $(i,k) \in N \times N$.

Second, I establish that $P^{j-}_{ik} > 0$ implies that $P^{j+}_{i'k'} = 1$ for all $(i',k') \neq (i,k)$.

Without loss of generality, assume that $(i,k) = (1,1)$. Suppose, contrariwise, that $P^{j-}_{11} > 0$ and that $P^{j+}_{12} < 1$. Let P° be as previously defined. We know that $P^\circ \not\geq P^{j-}$ and that $P^\circ \not\leq P^{j+}$. This contradiction proves that $P^{j-}_{ik} > 0$ implies that $P^{j+}_{i'k'} = 1$ for all $(i',k') \neq (i,k)$. Similarly one can show that $P^{j+}_{ik} < 1$ implies that $P^{j-}_{i'k'} = 0$ for all $(i',k') \neq (i,k)$.

Third, I establish that there exists (i,k) in $N \times N$ and $(d,r) \in \mathcal{D} \times \{0,1\} \setminus \{(0,1),(1,0)\}$, such that $f_j(P) = 1$ if $P_{ik} > d$ and $f_j(P) = 0$ if $P_{ik} < d$ and $f_j(P) = r$ if $P_{ik} = d$. The are four cases.

Case A: $P^{j-}_{ik} > 0$ for some $(i,k) \in N \times N$. First, it is clear that $P^{j-}_{ik} \geq P^{j+}_{ik}$; otherwise $P^{j+} >> P^{j-}$ which is a contradiction. Second, it is clear that there is no $x \in \mathcal{D}$ such that $P^{j-}_{ik} > x > P^{j+}_{ik}$. Otherwise, there is a profile P' such that $P'_{ik} = x$. But $P' \not\geq P^{j-}$ and $P' \not\leq P^{j+}$, and this is a contradiction. Then $P^{j-}_{ik} = d$ and $f_j(P^{j-}_{ik}) = r$. (Note that if $d = 1$ then $r = 1$ due to non-degeneracy.) Thus $f_j(P) = 1$ if $P_{ik} > d$, $f_j(P) = 0$ if $P_{ik} < d$, and $f_j(P) = r$ if $P_{ik} = d$.

Case B: $P^{j+}_{ik} < 1$ for some $(i,k) \in N \times N$. This is the dual of case A. Then $P^{j+}_{ik} = d$ and $f_j(P^{j+}_{ik}) = r$. (Note that if $d = 0$ then $r = 0$ due to non-degeneracy.) Thus $f_j(P) = 1$ if $P_{ik} > d$, $f_j(P) = 0$ if $P_{ik} < d$, and $f_j(P) = r$ if $P_{ik} = d$.

Case C: $P^{j-} = \mathbf{0}$. (This case is not possible if $\mathcal{D}$ is finite due to non-degeneracy.) There must be exactly one $(i,k) \in N \times N$ such that $P^{j+}_{ik} = 0$; otherwise $P^{j+} >> P^{j-}$, which would be a contradiction. Thus $d = r = 0$ and $f_j(P) = 1$ if $P_{ik} > d$ and $f_j(P) = r$ if $P_{ik} = d$.

Case D: $P^{j+} = \mathbf{1}$. This is the dual of case C. Thus, $d = r = 1$ and $f_j(P) = 0$ if $P_{ik} < d$ and $f_j(P) = r$ if $P_{ik} = d$. ■

Chapter 3

A Measure of Bizarreness

This chapter reproduces the paper "A Measure of Bizarreness," written jointly with Christopher P. Chambers. The authors split equally the ideas and the writing of the paper.

3.1 Introduction

The upcoming decennial census will result in a new legislative redistricting process to be completed in 2012. That year will also mark the two-hundredth anniversary of the *Gerrymander* — that monster of American politics — the bizarrely shaped legislative district drawn as a means to certain electoral ends.

An early diagnosis of this malady did not lead to an early cure. Already in the nineteenth century, reformers introduced anti-gerrymandering laws requiring districts to be "compact" and "contiguous",[1] but the disease spread unabated. District shapes have grown more odd over time as politicians have used modern technology to increase their control over elections. In 1812 a district was said to resemble a salamander; one hundred eighty years later, another was likened to a "Rorschach ink blot test."[2]

Redistricting reform has been hampered by a lack of agreement among experts as to what a good district plan should look like (Cipra, 2009). Some believe that legislatures should mirror the racial, ethnic, or political balance of the population. Others believe that it is more important that districts be competitive or, alternatively, stable. This lack of an ideal has made it difficult to design an algorithm which will yield a districting plan that all will accept.

Rather than make districts better by moving them closer to an ideal, we try to make districts "less worse" by moving them further from an identifiable problem. That problem is bizarre shape. We introduce a new method to measure the bizarreness of a legislative district. The method provides

[1] Thirty-five states require congressional or legislative districting plans to be "compact", forty-five require "contiguity", and only Arkansas requires neither (NCSL, 2000). There may also be federal constitutional implications. See *Shaw v. Reno*, 509 U.S. 630 (1993); *Bush v. Vera*, 517 U.S. 959 (1996).

[2] *Shaw v. Reno*, 509 U.S. at 633.

courts with an objective means to identify the more egregious gerrymanders which weaken the citizens' confidence in the electoral system.

As with so many other aspects of redistricting, there is little agreement as to reason for restricting bizarre shapes. Some argue that while the shape of legislative districts is not important in and of itself, compactness restrictions constrain the set of choices available to gerrymanderers and thereby limit their ability to control electoral outcomes. Others believe that bizarrely shaped districts cause direct harm in that the "pernicious" messages that they send to voters and their elected representatives.[3]

Laws restricting the shapes of legislative districts have been unsuccessful, in part because courts lack objective criteria to determine whether a particular shape is acceptable. Lawyers, political scientists, geographers, and economists have introduced multiple methods to measure district "compactness."[4] However, none of these methods is widely accepted, in part because of problems identified by Young (1988), Niemi et al. (1990), and Altman (1998).

Part of the difficulty of defining a measure of compactness is that there are many conflicting understandings of the concept. According to one view the compactness standard exists to eliminate elongated districts. In this sense a square is more compact than a rectangle, and a circle may be more compact than a square. According to another view compactness exists to eliminate oddly shaped districts.[5] According to this view a rectangle-shaped district would be better than a district shaped like a Rorschach blot.

We follow the latter approach. While it may be preferable to avoid elongated districts, the sign of a heavily-gerrymandered district is bizarre shape. To the extent that elongation is a concern, it should be studied with a separate measure.[6] These are two separate issues, and there is no natural way to weigh tradeoffs between bizarreness and elongation.

We note that, in some cases, bizarrely shaped districts may be justified by compliance with the Voting Rights Act of 1965.[7] It is not clear whether any of these bizarre shapes could have been avoided by districting plans which satisfy the constraints of the act.[8] Whether a bizarrely shaped district is necessary to satisfy civil rights law is a matter for the courts.[9] Our role is only to provide

[3] "Put differently, we believe that reapportionment is one area in which appearances do matter." *Shaw v. Reno*, 509 U.S. at 647. The direct harm that arises from the ugly shape of the legislative districts is generally referred to as an "expressive harm" (Pildes and Niemi, 1993).

[4] "Contiguity" is generally understood to require that it be possible to move between any two places within the district without leaving the district. See, for example, Black's Law Dictionary which defines a "contiguous" as touching along a surface or a point (Garner, 2004).

[5] Writing for the majority in *Bush v. Vera*, Justice O'Connor referred to "bizarre shape and noncompactness" in a manner which suggests that the two are synonymous, or at least very closely related. If so then a compact district is one without a bizarre shape, and a measure of compactness is a measure of bizarreness.

[6] Elongated districts are not always undesirable. See Figure 3.5.

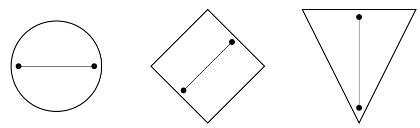
[7] See 42 U.S.C. 1973c.

[8] Individuals involved in the redistricting process often attempt to satisfy multiple objectives when creating redistricting plans. It may be the case that the bizarreness of these districts could be reduced by sacrificing other objectives (such as creating safe seats for particular legislators) without hurting the electoral power of minority groups. As a matter of law, it is not clear that the Voting Rights Act necessarily requires bizarre shapes in any case.

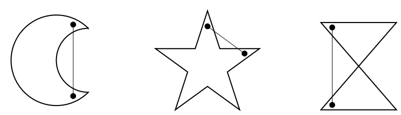
[9] The Supreme Court has held that, irrespective of the Voting Rights Act, "redistricting legislation that is so bizarre

a factual standard by which the court can determine whether districts are bizarrely shaped.

The basic principle of *convexity* requires a district to contain the shortest path between every pair of its points. Circles, squares, and triangles are examples of convex shapes, while hooks, stars, and hourglasses are not. (See Figure 3.1.) The most striking feature of bizarrely shaped districts is that they are extremely non-convex. (See Figure 3.2.) We introduce a measure of convexity with which to assess the bizarreness of the district.



(a) Convex Shapes



(b) Non-Convex Shapes

Figure 3.1: Convexity

The *path-based* measure we introduce is the probability that a district will contain the shortest path between a randomly selected pair of its points.[10] This measure will always return a number between zero and one, with one being perfectly convex. To understand how our measure works, consider a district containing two equally sized towns connected by a very narrow path, such as a road. (See Figure 3.3(a).) Our method would assign this district a measure of approximately one-half. A district containing n equally-sized towns connected by narrow paths would be assigned a measure of approximately $1/n$. [11] (See Figure 3.3(b).) If the n towns are not equally-sized, the measure is equivalent to the Herfindahl-Hirschman Index (Hirschman, 1964).[12]

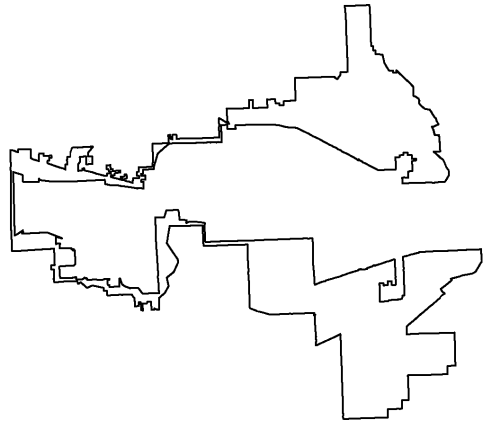
Ideally, a measure of compactness should consider the distribution of the population in the district. For example, consider the two arch-shaped districts depicted in Figure 3.4. The districts are of identical shape, thus the probability that each district will contain the shortest path between a randomly selected pair of its points is the same. However, the populations of these districts are distributed rather differently. The population of district A is concentrated near the bottom of the arch, while that of district B is concentrated near the top. The former district might represent two

on its face that it is 'unexplainable on grounds other than race"' is subject to a high level of judicial scrutiny. *Shaw v. Reno*, 509 U.S. at 643. See Pildes and Niemi (1993).

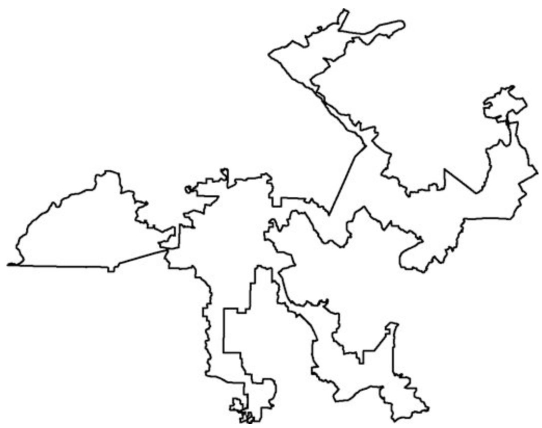
[10] A version of this measure was independently discovered by Lehrer (2007).

[11] Alternatively one might use the reciprocal, where the measure represents the equivalent number of disparate communities strung together to form the district. The reciprocal will always be a number greater or equal to one, where one is perfectly convex. A district containing n towns connected by narrow paths would be assigned a measure of approximately n.

[12] If x_i is the size of town i, then the measure of the district is $\sum_{i=1}^{n} x_i^2 \left[\sum_{j=1}^{n} x_j\right]^{-2}$.



(a) 4th District, Illinois



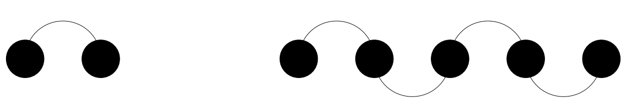
(b) 13th District, Georgia

Figure 3.2: Congressional Districts, 109th Congress

communities connected by a large forest, while the second district might represent one community with two forests attached.

Population can be incorporated by using the probability that a district will contain the shortest path between a randomly selected pair of its residents. In practice our information will be more limited — we will not know the exact location of every resident, but only the populations of individual census blocks. We can solve this problem by weighting points by population density. The population-weighted measure of district A is approximately one-half, while that of district B is nearly one.[13]

[13]Note that the population-weighted approach measures the compactness of the districts' populations, and not the compactness of their shapes. A district may have a perfect score even though it has oddly shaped boundaries in unpopulated regions. The ability to draw bizarre boundaries in unpopulated regions is of no help to potential



(a) Two Circular Towns (b) Five Circular Towns

Figure 3.3: Towns Connected with Narrow Paths.

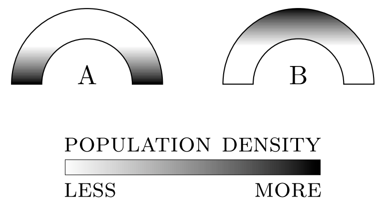


Figure 3.4: Same Shapes, Different Populations

One potential problem is that some districts may be oddly shaped simply because the states in which they are contained are non-convex. Consider, for example, Maryland's Sixth Congressional District (shown in Figure 3.5 in gray). Viewed in isolation, this district is very non-convex — the western portion of the district is almost entirely disconnected from the eastern part. However, the odd shape of the district is a result of the state's boundaries, which are fixed. We solve this problem by measuring the probability that a district will contain the shortest path *in the state* between a randomly selected pair of its points. The adjusted measure of Maryland's Sixth Congressional District would be close to one.

Figure 3.5: 6th District, Maryland, 109th Congress

Our measure considers *whether* the shortest path in a district exceeds the shortest path in the state. Alternatively, one might wish to consider the *extent* to which the former exceeds the latter. We introduce a parametric family of measures which vary according to the degree that they "penalize" deviations from convexity. At one extreme is the measure we have described; at the other is the degenerate measure, which gives all districts a measure of one regardless of their shape.

3.1.1 Related Literature

3.1.1.1 Individual District Compactness Measures

A variety of compactness measures have been introduced by lawyers, social scientists, and geographers. Here we highlight some of basic types of measures and discuss some of their weaknesses. A more complete guide may be found in surveys by Young (1988), Niemi et al. (1990), and Altman (1998).

gerrymanderers.

Most measures of compactness fall into two broad categories: (1) dispersion measures and (2) perimeter-based measures. Dispersion measures gauge the extent to which the district is scattered over a large area. The simplest dispersion measure is the length-to-width test, which compares the ratio of a district's length to its width. Ratios closer to one are considered more compact. This test has had some support in the literature, most notably Harris (1964).[14]

Another type of dispersion measure compares the area of the district to that of an ideal figure. This measure was introduced into the redistricting literature by Reock (1961), who proposed using the ratio of the area of the district to that of the smallest circumscribing circle. A third type of dispersion measure involves the relationship between the district and its center of gravity. Measures in this class were introduced by Boyce and Clark (1964) and Kaiser (1966). The area-comparision and center of gravity measures have been adjusted to take account of district population by Hofeller and Grofman (1990), and Weaver and Hess (1963), respectively.

Dispersion measures have been widely criticized, in part because they consider districts reasonably compact as long as they are concentrated in a well-shaped area (Young, 1988). We point out a different (although related) problem. Consider two disjoint communities strung together with a narrow path. *Disconnection-sensitivity* requires the measure to consider the combined region less compact than at least one of the original communities. None of the dispersion measures are disconnection-sensitive. An example is shown in Figure 3.6.[15]

Perimeter measures use the length of the district boundaries to assess compactness. The most common perimeter measure, associated with Schwartzberg (1966), involves comparing the perimeter of a district to its area.[16] Young (1988) objected to the Schwartzberg measure on the grounds that it is overly sensitive to small changes in the boundary of a district. Jagged edges caused by the arrangement of census blocks may lead to significant distortions. While a perfectly square district will receive a score of 0.785, a square shape superimposed upon a diagonal grid of city blocks will have a much longer perimeter and a lower score, as shown in Figure 3.7(a).[17] Figure 3.7 shows four shapes, arranged according to the Schwartzberg ordering from least to most compact.

Taylor (1973) introduced a measure of indentation which compared the number of reflexive (inward-bending) to non-reflexive (outward-bending) angles in the boundary of the district. Taylor's measure is similar to ours in that it is a measure of convexity. Figure 3.8 shows six districts and

[14]The length-to-width test seems to have originated in early court decisions construing compactness statutes. See *In re Timmerman*, 100 N.Y.S. 57 (N.Y. Sup. 1906).

[15]The length-width measure is the ratio of width to length of the circumscribing rectangle with minimum perimeter. See Niemi et al. (1990). All measures are transformed so that they range between zero and one, with one being most compact. The Boyce-Clark measure is $\sqrt{\frac{1}{1+bc}}$, where bc is the original Boyce-Clark measure (Boyce and Clark, 1964). The Schwartzberg measure used is the variant proposed by Polsby and Popper (1991) (originally introduced in a different context by Cox (1927)), or $(\frac{1}{sc})^2$, where sc is the measure used by Schwartzberg (1966).

[16]This idea was first introduced by Cox (1927) in the context of measuring roundness of sand grains. The idea first seems to have been mentioned in the context of district plans by Weaver and Hess (1963) who used it to justify their view that a circle is the most compact shape. Polsby and Popper (1991) have also supported the use of this measure.

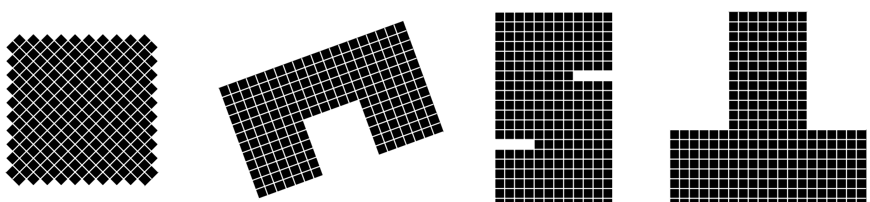
[17]The score of the resulting district will decrease as the city blocks become smaller, reaching 0.393 in the limit.

Figure 3.6: District **II** is Formed by Connecting District **I** to a Copy of Itself. Disconnection-Sensitivity Implies that **I** is More Compact.

I **II**

COMPACTNESS MEASURES

Dispersion Measures	*District:*	**I**	**II**
Length-Width		0.63	1.00
Area to Circumscribing Circle		0.32	0.44
Area to Convex Hull		0.57	0.70
Boyce-Clark		0.15	0.29
Other Measures			
Path-Based Measure		0.84	0.42
Schwartzberg		0.29	0.14
Taylor		0.40	0.20



(a) 0.432 (b) 0.448 (c) 0.456 (d) 0.503

Figure 3.7: Schwartzberg Measure

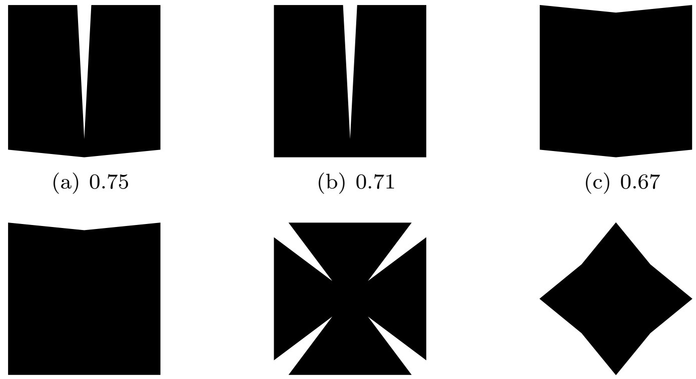
their Taylor measures, arranged from best to worst.

Lastly, Schneider (1975) introduced a measure of convexity using Minkowski addition. For more on the relationship between convex bodies and Minkowski addition, see Schneider (1993).

3.1.1.2 Districting-Plan Compactness Measures

In addition to these measures of individual legislative districts, several proposals have been introduced to measure entire districting plans. The "sum-of-the-perimeters" measure, found in the Colorado Constitution, is the "aggregate linear distance of all district boundaries."[18] Smaller numbers indicate greater compactness. An alternative method was introduced by Papayanopoulos (1973). His proposal can be described through a two-stage process. First, in each district, the sum total

[18]Colo. Const. Art. V, Section 47



(a) 0.75 (b) 0.71 (c) 0.67

(d) 0.60 (e) 0.33 (f) 0.00

Figure 3.8: Taylor's Measure

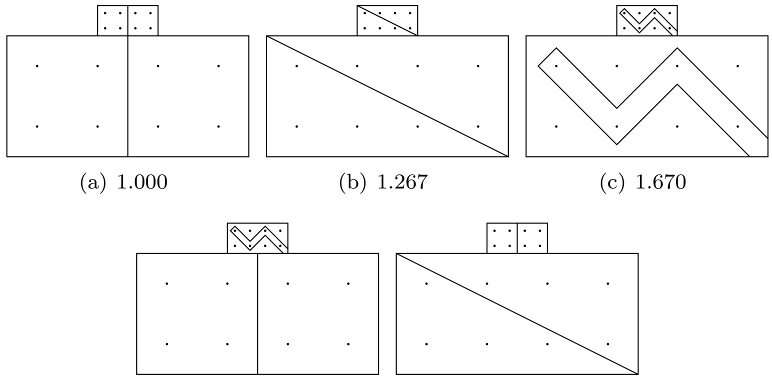
of the distances between each pair of residents is calculated. The measure for the plan is then the sum of these scores across the districts. Smaller numbers again indicate greater compactness. More recently, Fryer and Holden (2007) have proposed a related measure which uses quadratic distance and which is normalized so that an optimally compact districting plan has a score of one.

A potential problem, raised by Young (1988), is that these measures penalize deviations in sparsely populated rural areas much more severely than deviations in heavily populated urban areas. For example, Figure 3.9 shows five potential districting plans for a four-district state with sixteen equally sized population centers (represented by dots). The upper portion of the state represents an urban area with half of the population concentrated into one-seventeenth of the land. Papayanopoulos scores are given, although we note that the sum-of-the-perimeters and Fryer-Holden measures give identical ordinal rankings of these districting plans.

According to these measures, the ideal districting plan divides the state into four squares (Figure 3.9(a)). The plan with triangular districts is less compact (Figure 3.9(b)), and the plan with wave-shaped districts fares the worst (Figure 3.9(c)). However, the measure is more sensitive to deviations in areas with lower population density. The plan in Figure 3.9(d), which divides the rural area into perfect squares and the urban area into low-scoring wave-shape districts, is considered more compact than the plan in Figure 3.9(e), which divides the rural area into triangles and the urban area into perfect squares.

An alternative approach is to rank state-wide districting plans using the scores assigned to individual districts. Examples include the *utilitarian* criterion, which is the average of the districts' scores (see Papayanopoulos (1973)), and the *maxmin* criterion, which is simply the lowest of the scores awarded the districts under the plan. This approach allows for the ranking of both individual districts and entire districting plans as required by Young (1988).

The ideal criterion depends in large part on the individual district measure with which it is used. We advocate the use of the maxmin criterion with our path-based measure on the grounds that it will



(a) 1.000 (b) 1.267 (c) 1.670

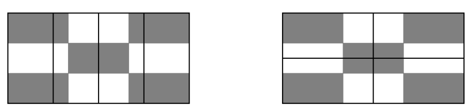
(d) 1.134 (e) 1.213

Figure 3.9: Urban Gerrymandering

restrict gerrymandering the most. The maxmin criterion is also consistent with the U.S. Supreme Court's focus on analyzing individual districts as opposed to entire districting plans.[19] However, if some districts must necessarily be non-compact (a common problem with the Schwartzberg measure) then the utilitarian criterion may be more appropriate.

3.1.1.3 Other literature

Vickrey (1961) showed that restrictions on the shape of legislative districts are not necessarily sufficient to prevent gerrymandering. In Vickrey's example there is a rectangular state in which support for the two parties (white and gray) are distributed as shown in Figure 3.10. With one district plan, the four legislative seats are divided equally; with the other district plan, the gray party takes all four seats. In both plans, the districts have the same size and shape.



(a) 2 gray, 2 white (b) All gray, no white

Figure 3.10: Vickrey's Example

Compactness measures have been touted both as a tool for courts to use in determining whether districting plans are legal and as a metric for researchers to use in studying the extent to which districts have been gerrymandered. Other methods exist to study the effect of gerrymandering – the most prominent of these is the seats-votes curve, which is used to estimate the extent to which the district plan favors a particular party as well as the responsiveness of the electoral system to changes in popular opinion. For more see Tufte (1973).

[19]This focus might stem from the Court's understanding of the right to vote as an individual right, and not a group or systemic right. This understanding may have influenced other measures used in the redistricting context, such as the 'total deviation' test. See Edelman (2006).

3.2 The Model and Proposed Family of Measures

3.2.1 The Model and Notation

Let $\mathcal{K}$ be the collection of compact sets in $\mathbb{R}^n$ whose interiors are path-connected (with the usual Euclidean topology) and which are the closure of their interiors. Elements of $\mathcal{K}$ are called **parcels**. For any set $Z \subseteq \mathbb{R}^n$ let $\mathcal{K}_Z \equiv \{K \in \mathcal{K} : K \subseteq Z\}$ denote the restriction of $\mathcal{K}$ to Z.

Consider a path-connected set $Z \subseteq \mathbb{R}^n$ and let $x, y \in Z$. Let $\mathcal{P}_Z(x, y)$ be the set of continuous paths $g : [0,1] \to Z$ for which $g(0) = x$, $g(1) = y$, and $g([0,1]) \subset Z$. For any path g in $\mathcal{P}_Z(x, y)$, we define the length $l(g)$ in the usual way.[20] We define the distance from x to y within Z as:

$$d(x, y; Z) \equiv \inf_{g \in \mathcal{P}_Z(x,y)} l(g).$$

We define $d(x, y; \mathbb{R}^n) \equiv d(x, y)$. This is the Euclidean metric.

Let $\mathcal{F}$ be the set of density functions $f : \mathbb{R}^n \to \mathbb{R}_+$ such that $\int_K f(x)dx$ is finite for all parcels $K \in \mathcal{K}$. Let $f_u \in \mathcal{F}$ refer to the uniform density.[21] For any density function $f \in \mathcal{F}$, let F be the associated probability measure so that $F(K) \equiv \int_K f(x)dx$ represents the population of parcel K.[22]

We measure compactness of districts relative to the borders of the state in which they are located. Given a particular state Z,[23] we allow the measure to consider two factors: (1) the boundaries of the legislative district, and (2) the population density.[24] Thus, a **measure of compactness** is a function $s_Z : \mathcal{K}_Z \times \mathcal{F} \to \mathbb{R}_+$.

3.2.2 The Basic Family of Compactness Measures

As a measure of compactness we propose to use the expected relative difficulty of traveling between two points within the district. Consider a legislative district K contained within a given state Z. The value $d(x, y; K)$ is the shortest distance between x and y which can be traveled while remaining in the parcel K. To this end, the shape of the parcel K makes it relatively more difficult to get from points x to y the lower the value of

$$\frac{d(x, y; Z)}{d(x, y; K)}. \tag{3.1}$$

Note that the maximal value that expression (3.1) may take is one, and its smallest (limiting) value is zero. Alternatively, any function $g(d(x, y; Z), d(x, y; K))$ which is scale-invariant, monotone

[20] That is, suppose $g : [0,1] \to Z$ is continuous. Let $k \in \mathbb{N}$. Let $(t_0, ..., t_k) \in \mathbb{R}^{k+1}$ satisfy for all $i \in \{0, ..., k-1\}$, $t_i < t_{i+1}$. Define $l_t(g) = \sum_{i=1}^{k} \|g(t_k) - g(t_{k-1})\|$. The length (formally, the **arc length**) of g is then defined as $l(g) = \sup_{k \in \mathbb{N}} \sup_{\{t \in [0,1]^k : t_i < t_{i+1}\}} l_t(g)$.

[21] We define $f_u(x) = 1$.

[22] Similarly, the uniform probability measure $F_u(K)$ represents the area of parcel K.

[23] The state Z is typically chosen from set $\mathcal{K}$ but is allowed to be chosen arbitrary; this allows the case where $Z = \mathbb{R}^n$ and the borders of the state do not matter.

[24] The latter factor can be ignored by assuming that the population has density f_u.

decreasing in $d(x,y;K)$, and monotone increasing in $d(x,y;Z)$ is interesting; expression (3.1) can be considered a canonical example. The numerator $d(x,y;Z)$ is a normalization which ensures that the measure is affected by neither the scale of the district nor the jagged borders of the state. We obtain a parameterized family of measures of compactness by considering any $p \geq 0$; so that $\left[\frac{d(x,y;Z)}{d(x,y;K)}\right]^p$ is our function under consideration, defining

$$\left[\frac{d\,(x,y;Z)}{d\,(x,y;K)}\right]^{\infty} = \begin{cases} 1, \text{ if } \frac{d(x,y;Z)}{d(x,y;K)} = 1 \\ 0, \text{ otherwise} \end{cases} .$$

Note that for $p = 0$, the measure is degenerate. This expression is a measure of the relative difficulty of travelling from points x to y. Our measure is the expected relative difficulty over all pairs of points, or:

$$s_Z^p\,(K,f) \equiv \int_K \int_K \left[\frac{d\,(x,y;Z)}{d\,(x,y;K)}\right]^p \frac{f(y)\;f(x)}{(F(K))^2}\;dy\;dx. \tag{3.2}$$

We note a few important cases. First, the special case of $p = +\infty$ corresponds to the measure described in the introduction, which considers whether the district contains the shortest path between pairs of its points.[25] Second, we can choose to measure either the compactness of the districts' shapes (by letting $f = f_u$) or the compactness of the districts' populations (by letting f describe the true population density). Third, if $Z = \mathbb{R}^n$, our measure describes the compactness of the legislative district without taking the state's boundaries into consideration.

3.2.3 Discrete Version

Our measure may be approximated by treating each census block as a discrete point. This may be useful if researchers lack sufficient computing power to integrate the expression described in (3.2).

Let $Z \in \mathbb{R}^n$ be a state as described in subsection 3.2.1 and let $K \in \mathcal{K}_Z$ be a district. Let $\mathscr{B} \equiv \mathbb{R}^n \times \mathbb{Z}_+$ be the set of possible census blocks, where each block $b_i = (x_i, p_i)$ is described by a point x_i and a non-negative integer p_i representing its center and population, respectively. Let $Z^* \in \mathscr{B}^m$ describe the census blocks in state Z and let $K^* \subset Z^*$ describe the census blocks in district K. The approximate measure is given by:

$$s_{Z^*}^p\,(K^*) \equiv \left[\sum_{b_i \in K^*} \sum_{b_j \in K^*} \left[\frac{d\,(x_i,x_j;Z)}{d\,(x_i,x_j;K)}\right]^p p_i\;p_j\right] \left[\sum_{b_i \in K^*} \sum_{b_j \in K^*} p_i\;p_j\right]^{-1} .$$

[25] Mathematically, there may be two shortest paths in a parcel connecting a pair of residents. The issue arises when one state is not simply connected. For example, two residents may live on opposite sides of a lake which is not included in the parcel. In this general case, our measure is the probability that at least one of the shortest paths is contained in the district for any randomly selected pair of residents.

3.3 Data

To illustrate our measure we have calculated scores for all districts in Connecticut, Maryland, and New Hampshire during the 109th Congress. (See Figures 3.11, 3.12, and 3.13.) Because of limitations in computing power we use the approximation described in Section 3.2.3.

Dark lines represent congressional district boundaries, while shading roughly follows population distributions. Table 3.1 contains scores for both our path-based measure as well as the Schwartzberg measure.[26] The small numerals in parentheses give the ordinal ranking of the district according to the respective measure. Thus, according to our measure, Connecticut's Fourth District is the most compact, with a nearly perfect score of 0.977, followed by Maryland's Sixth District (0.926). Maryland's Third District is the least compact with a score of 0.140, which makes it slightly less compact than seven equally sized communities connected with a narrow path. (See Figure 3.3). The Schwartzberg measure ranks Connecticut's Second District as most compact and Maryland's First District as least compact. For these fifteen districts, the ordinal rankings agree on fewer than seventy-five percent of the pairwise comparisons.

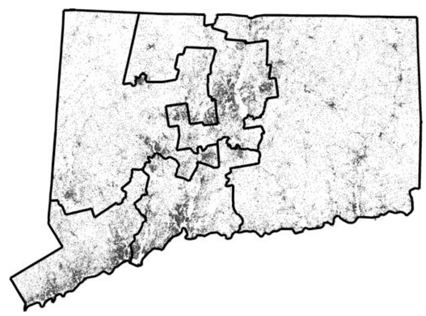
Table 3.1: Legislative District Scores

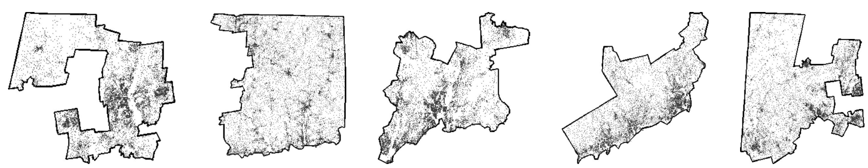
District		*Measure:* **Path-Based**	**Schwartzberg**
Connecticut:	1st	0.609 (8)	0.161 (9)
	2nd	0.860 (4)	0.412 (1)
	3rd	0.891 (3)	0.235 (4)
	4th	0.977 (1)	0.305 (3)
	5th	0.481 (12)	0.228 (5)
Maryland:	1st	0.549 (10)	0.016 (15)
	2nd	0.294 (14)	0.019 (14)
	3rd	0.140 (15)	0.029 (13)
	4th	0.366 (13)	0.083 (11)
	5th	0.517 (11)	0.066 (12)
	6th	0.926 (2)	0.119 (10)
	7th	0.732 (6)	0.174 (8)
	8th	0.657 (7)	0.204 (7)
New Hampshire:	1st	0.801 (5)	0.228 (6)
	2nd	0.561 (9)	0.370 (2)

The measures give strikingly different results with respect to Connecticut's Fifth District and Maryland's Sixth District. Both assign a high rank to one of the districts and a low rank to the other, but the order is reversed. The difference primarily stems from two factors: state boundaries and population.

[26] To calculate perimeters for the Schwartzberg measure we summed the lengths of the line segments that form the district boundary. In some cases, natural state boundaries (such as the Chesapeake Bay) added significantly to the total length. The Census data we used did not allow us to calculate district tri-junctions (as recommended by Schwartzberg (1966)), although it seems unlikely that this would have a substantial effect on the calculation in this case. We do not know whether practitioners use a different method to calculate these scores.

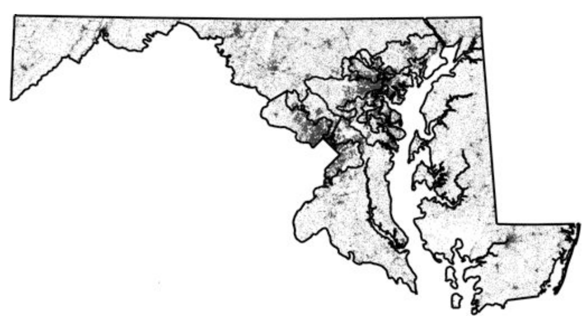
Figure 3.11: Connecticut



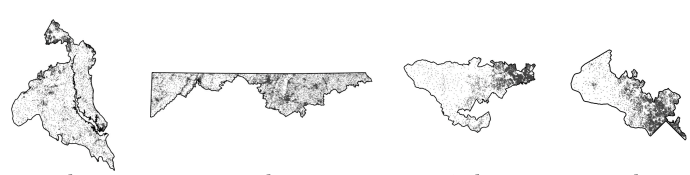


1st 2nd 3rd 4th 5th

Figure 3.12: Maryland



1st 2nd 3rd 4th



5th 6th 7th 8th

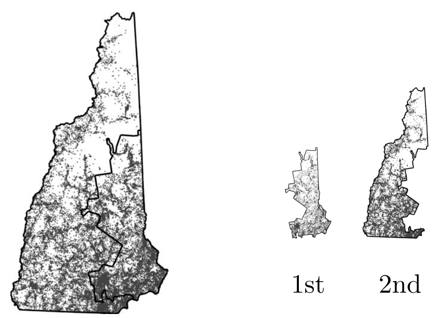
Maryland's Sixth District has a very low area-perimeter ratio owing to its location in the sparsely populated panhandle of western Maryland and to the ragged rivers which makes up its southern and eastern borders. Our path-based measure, however, takes the state boundaries into account and thus gives this district a high score.

Connecticut's Fifth District, however, has a much higher area-perimeter ratio: the generally square shape of the district compensates for the two appendages protruding from its eastern side. However, the appendages reach out to incorporate several urban areas into the district. (See for example, the southeastern portion of the northern appendage and the eastern part of the southern appendage.) Because the major population centers are relatively disconnected from each other, our path-based measure assigns this district a low score of 0.481, which is slightly less compact than two equally sized communities connected with a narrow path. (See Figure 3.3).

3.4 Conclusion

We have introduced a new measure of district compactness: the probability that the district contains the shortest path connecting a randomly selected pair of its points. The measure can be weighted for population and can take account of the exogenously determined boundaries of the state in which the district is located. It is an extreme point in a parametric family of measures which vary according to the degree that they "penalize" deviations from convexity.

Figure 3.13: New Hampshire



Chapter 4

A Model of Community Standards

4.1 Introduction

In 1957, the United States Supreme Court ruled that obscenity is "utterly without redeeming social importance" and is not protected by the U.S. Constitution. The court held that "contemporary community standards" are to be used in determining whether particular works are obscene.[1] The Supreme Court has never explained what "community standards" are or how, if at all, they are related to the standards of the individuals who comprise the community. Lower courts have provided only limited guidance describing the community standard as an "aggregation or average".

I introduce a new model in which community standards are formed by aggregating a set of individual standards. In the model, standards are defined as judgments — categorizations of possible works as either "obscene" or "not obscene." Every possible judgment is allowed provided it satisfies the following restriction: neither individuals nor the community may consider one-hundred percent of the works to be obscene. I define several basic normative properties of aggregation methods which reflect legal concerns expressed by the judiciary. I then show that the only method which satisfies these properties is *unanimity rule*, in which a work is considered obscene if and only if all members of the community consider it to be obscene.

4.1.1 The Problem of Community Standards

In communities that are perfectly homogeneous, where each individual's belief is identical, it should be simple to determine the community standard. However, as the Supreme Court has recognized, few communities are perfectly homogeneous. For this reason the Court has required the jury to consider the views of a diverse set of individuals, including the young and the old, the religious and the irreligious, the sensitive and the insensitive.[2] But when the community is heterogeneous, it is

[1] *Roth v. United States*, 354 U.S. 476 (1957). The Supreme Court retained the community standards test when it refined the definition of obscenity sixteen years later in *Miller v. California* 413 U.S. 15 (1973).

[2] See *Roth v. United States*, 354 U.S. 476 (1957) and *Pinkus v. United States*, 436 U.S. 293 (1978).

not obvious how the conflicting views of the citizenry should be combined.

Some commentators, including Sadurski (1987), have argued that the community standard is an average or median in a mathematical sense. But as another commentator has pointed out, "the notion of an average standard ... implies the existence of a spectrum of tolerance that can be ranked along a single dimension, from least tolerant to most tolerant. The problem with this approach is that a single dimension of tolerance does not exist." (Boyce, 2008).[3] No court nor commentator has yet identified an acceptable objective method to order judgments or levels of tolerance along a single dimension.

A different approach was taken by Lord Patrick Devlin in his classic work, *The Enforcement of Morals* (Devlin, 1965). Lord Devlin argued that it was proper for governments to prohibit behavior felt to be immoral by the community. He suggested that, in some sense, unanimous agreement within a society is necessary to justify regulation of immorality: "the moral judgment of society must be something about which any twelve men or women drawn at random might after discussion be expected to be unanimous." To ascertain the moral standards of the community, Lord Devlin's understanding of the Law of England can be described in the following way. First, the community consists of all "right-minded" or "reasonable" persons within the society.[4] Next, an act is deemed immoral if and only if every reasonable person believes the act to be immoral. "Immorality then, for the purpose of the law, is what every right-minded person is presumed to consider to be immoral." (Devlin, 1965).[5]

While the U.S. Supreme Court adopted the principle that certain acts (the distribution and sale of obscene material) can be criminalized on the grounds of offense to community morals, American courts have never adopted a specific rule to ascertain the moral standards of the community. Individual jurors are instructed to ascertain these standards on the basis of their experience and familiarity with the community, and are not instructed as to the method through which differing beliefs should be combined.[6] For over fifty years the Supreme Court has simply ignored this question, allowing the incarceration of defendants convicted under a vague and murky legal doctrine.

[3]Boyce (2008), however, assents to the principle that community standards "must in some sense be an aggregate of the standards of the individuals who comprise the community."

[4]Whether an individual is "right-minded" or "reasonable" does not seem to be directly connected to the specific content of that individual's beliefs; otherwise Devlin's rule would be circular and ill-defined.

[5]Whether Devlin's rule is certainly practicable is a debatable proposition. He certainly felt that the rule would lead to convictions in 1958, but whether that should remain the case in the more tolerant environment of the twenty-first century is unclear. However, the mere possibility that some communities would find little to prohibit does not invalidate Devlin's rule. He argued that a community should be able to prohibit that which it found immoral, and not that every community must find some works to be immoral.

[6]The views of the individual jurors themselves are combined through the unanimous jury rule which closely corresponds to Devlin's rule: an individual is convicted of an immoral act only when every juror considers the act to be immoral.

4.1.2 The Model

The basic model can be described as follows. First, there is a community, which can be any group of individuals. The Supreme Court has required that the community be defined in geographic terms and contain all adults in that community, including the young, the old, the religious, the irreligious, the sensitive, and the insensitive.[7] Lord Devlin (1965) seems to have argued that the community consists only of reasonable persons. Others might propose to restrict the definition to clerics, to parents, or to some other community of interest. The model is general enough to include all of these as special cases.

Next, there is an infinite set of all possible works. We might loosely understand this as the set of possible artworks but it might also include literary works, scientific publications, and other forms of human expression. The space of works is modeled as a non-atomic measure space. The decision to use a non-atomic measure space rather than a discrete space is made to simplify the exposition. Parallel conclusions would be reached if the space of works were modeled as discrete and appropriate modifications were made to the axioms.

Individuals from the community have standards as to which works in the set are obscene. An individual standard is simply a division of the set into two groups: the obscene and the non-obscene (or permissible). Individual standards are assumed to be well-informed and made after deliberation and reflection. There is a single restriction on allowable standards: the set of works judged to be obscene must be of less than full measure. Reasonable individuals should all believe that some works, even those lacking serious literary, artistic, political, and scientific value, are non-obscene.[8] I do not require individuals to believe that some works must be obscene — there is no reason why individuals *must* be offended by anything.

These individual standards are then aggregated to form a community standard. The community standard is subject to the same restriction as the individual standards: the set of works judged to be obscene must be of less than full measure. I place no other restrictions on the class of allowable standards. Individual standards and community standards are assumed to be subjective.

An aggregation rule is a systematic method of deriving the community standard from the individuals judgments. Aggregation rules are studied through the axiomatic approach: several normative properties are formalized as axioms and the unique rule satisfying these axioms is characterized.

I suggest two distinct approaches to understanding aggregation rules. First, the aggregation rule may be understood as an actual procedure used to determine whether a work is obscene. It specifies how the standards of the members of the community (or of a jury) are to be combined.

Second, an aggregation rule may be understood as a jury instruction. As mentioned above, the community standards are to be determined by the trier of fact as part of a mental exercise. The

[7]See *Roth v. United States*, 354 U.S. 476 (1957) and *Pinkus v. United States*, 436 U.S. 293 (1978).
[8]Individuals who do not satisfy this restriction would be found to be unreasonable as a matter of law.

aggregation rule instructs the trier of fact on how to aggregate these many envisioned individual standards into a single community standard. Legislators attempting to codify community standards into law might undertake a similar thought exercise.

4.1.3 The Main Result

I introduce four axioms. Each is, in some way, a desirable property for any objective aggregation rule.

The first axiom, *homogeneity*, requires that if there is a single standard shared by every member of the community, then that standard is also the community standard. In some sense, if this axiom is not satisfied, then the community standard must be derived from something other than the individual judgments.

The second axiom, *responsiveness*, requires the community standard to "respond" in the same direction (more permissive or less) as the community. If every individual standard becomes more permissive, then the community standard should become more permissive as well. Responsiveness prevents the perverse result in which a defendant is convicted *because* the individuals in the community became more tolerant.

The third axiom, *anonymity*, requires that the aggregation rule not discriminate between individuals. In general, the law requires equal treatment of individuals. More specific to this case, the Supreme Court has explicitly held that the views of all adult members of the community must be taken into account in determining the community standard.

The fourth axiom, *neutrality*, requires that the aggregation rule not discriminate, ex ante, between works. This axiom assumes that all judgments are subjective and is relevant when there is no method by which works can be objectively compared. No court nor commentator has yet identified a plausible method of comparison. The lack of an objective method is largely what makes even personal views on obscenity difficult to define through a rule. Supreme Court Justice Potter Stewart believed that only "hard-core pornography" could be prohibited as obscenity but he could not define even that term. He only knew it when he saw it.[9] A natural method to compare works would be to judge them by their parts; however, this method was expressly disallowed by the Supreme Court.[10]

Together, these four axioms characterize the unanimity rule, under which a work is deemed obscene when every individual considers it to be obscene.

4.1.4 Multiple Standards

The U.S. Supreme Court has held that contemporary community standards are to be used in evaluating two elements of obscenity: (a) whether the work appeals to the prurient interest, and (b)

[9] Concurring opinion in *Jacobellis v. Ohio*, 378 U.S. 184 (1964).
[10] *Roth v. United States*, 354 U.S. 476 (1957).

whether the work is patently offensive.[11] This implies that there are, at least, three types of judgments individuals can make: (1) which works appeal to the prurient interest, (2) which works are patently offensive, and (3) which works are obscene; that is, which both appeal to the prurient interest and are patently offensive.

The first two types of judgments are not logically related. As a matter of law, a work may appeal to the prurient interest but not be patently offensive; alternatively, a work may be patently offensive but not appeal to the prurient interest. Were one judgment to imply the other, there would be no need for both elements to appear in the test. Each of the first two types of judgments, however, is clearly related to the third. If a work both appeals to the prurient interest and is patently offensive, then it also appeals to the prurient interest.

If there is a single community standard for obscenity, as has been assumed in this chapter, then the judgments being aggregated are of the third type. We might label the resulting standard the prurient interest and patently offensive community standard. However, one could infer from the Supreme Court opinions that there are two community standards, (a) the prurient interest community standard and (b) the patently offensive community standard.

A model of two community standards would take the following form. Individuals would make two separate judgments about which works (1) appeal to the prurient interest and (2) are patently offensive. The judgments would then be aggregated to form (a) the prurient interest community standard and (b) the patently offensive community standard. These two community standards need not be aggregated independently — it is conceivable, for example, that the individual judgments about which works are patently offensive are somehow relevant in determining the prurient interest community standard.

The main result of this chapter does not change in the case of two (or more) standards. Even if we allow for interdependent aggregation, unanimity rule is the unique aggregation rule that satisfies the four axioms.

4.1.5 Other Standards

The model introduced in this chapter is general and can be applied to problems other than the question of which works are legally obscene. I will describe three different types of legal standards to which the model can be applied.

First, standards of offensiveness are used to determine whether speech, or other forms of expression, may be prohibited on the grounds that it is offensive. Obscenity doctrine provides the clearest

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[11] *Miller v. California*, 413 U.S. 15 (1973). The full test provided in Miller is: (a) whether the average person, applying contemporary community standards would find that the work, taken as a whole, appeals to the prurient interest; (b) whether the work depicts or describes, in a patently offensive way, sexual conduct specifically defined by the applicable state law; and (c) whether the work, taken as a whole, lacks serious literary, artistic, political, or scientific value. The third element is an "objective" standard and does not vary from community to community. The test provided in Miller remains the current law.

example of a prohibition on offensive expression; other examples include the prohibitions on the broadcast of indecent and profane speech regulated by the Federal Communications Commission.

Second, standards of proof are used to determine whether defendants are guilty (or liable) in criminal (and civil) cases. Commonly used standards of proof include (a) the proof beyond a reasonable doubt standard, (b) the clear and convincing standard, and (c) the preponderance of the evidence standard. Here, instead of a set of works, we have a set of cases as in Kornhauser (1992a,b) and Lax (2007), and individuals choose the subset of cases that lead to conviction. The results of the chapter support the use of unanimity rule in determining which works are obscene.

Third, standards of behavior are used to evaluate behavior in civil and criminal trials. Examples of standards of behavior include the reasonable person standard studied by Rubinstein (1983), the business judgment rule, and fiduciary duties. To model this standard, we replace the set of works with a set of actions, and individuals have multiple standards, one for each set of circumstances, describing which actions are unreasonable in that circumstance.

4.2 The Model

4.2.1 Notation and the Model

The **community** is a set $N \equiv \{1, ..., n\}$ of individuals. The space of **works** is denoted by (W, Σ, μ), where W is the set of works, Σ is the σ-algebra of subsets of works, and μ is a measure on (W, Σ). The space (W, Σ) is assumed to be isomorphic to $([0, 1], \mathscr{B})$, where $\mathscr{B}$ is the set of Borel subsets of $[0, 1]$. I assume that μ is countably additive, non-atomic, non-negative, and finite.[12] Let Φ be the set of all automorphisms of (W, Σ) that preserve the measure μ.

Let $\mathcal{J} \equiv \{J \in \Sigma : \mu(J) < \mu(W)\}$ be the set of **judgments**. The requirement that judgments must be of less than full measure is a reasonableness condition that reflects the idea that not all works can be obscene, or should be prohibited. Let $M \equiv \{1, ..., m\}$ denote the set of **issues**. For example, if there is only a single standard of obscenity then $m = 1$, while if there is both a standard of "appeal to the prurient interest" and "patently offensive" then $m = 2$. The set M can be finite or countably infinite. A **standard** is an M-vector of judgments, one for each issue. The set of standards is denoted $\mathcal{S} \equiv \mathcal{J}^M$. A **profile** is an N-vector of standards, $S = (S_1, ..., S_n) \in \mathcal{S}^N$, where S_i represent individual i's standard. I write S_{ij} to denote individual i's judgment about issue j. A rule $f : \mathcal{S}^N \to \mathcal{S}$ is a function mapping each profile into a **community standard**, denoted $f(S) = (f_1(S), ..., f_m(S))$.

For $K \in \mathbb{N}$ and for any two sets S and T of the form $\mathcal{J}^K$, I define $\sqcap$ as the coordinatewise intersection, so that $(S \sqcap T)_k \equiv S_k \cap T_k$, and I define $\sqcup$ as the coordinatewise union, so that

[12] The space of actions is taken from the model of non-atomic games studied in Aumann and Shapley (1974) and Dubey and Neyman (1984).

$(S \sqcup T)_k \equiv S_k \cup T_k$. Note that there exist $S, T \in \mathcal{J}^K$ such that $S \sqcup T \notin \mathcal{J}^K$. I define $S \sqsubseteq T$ to mean that $S_k \subseteq T_k$ for every $k \in K$. When $S \sqsubseteq T$ I write that S is as **permissive** as T, because every work that a particular person permits in profile T is permitted by that person in profile S.[13] I define $(\phi S)_k \equiv \phi(S_k)$.

4.2.2 Axioms

The first axiom, *homogeneity*, requires that if the community is perfectly homogeneous, so that every individual in the community has identical views about the entire standard, then this commonly held belief is the community standard. In some sense, if this axiom is not satisfied, then the community standard must be derived from something other than the individual judgments. This axiom excludes *degenerate* rules, under which the community standard is predetermined and does not change as a result of the opinions.[14]

Homogeneity: If $S_i = S_j$ for all $i, j \in N$, then $f(S) = S_1 = ... = S_n$.[15]

Suppose that the individual standards change and that every individual's new standard is as permissive as was that individual's old standard (so that $S_i \sqsubseteq S_i^*$ for all $i \in N$). The second axiom, *responsiveness*, requires the resulting community standard to be as permissive as the prior community standard (so that $f(S) \sqsubseteq f(S^*)$). In other words, the community standard must "respond" in the same direction (more permissive or less) as the individuals in the community. Responsiveness prevents the perverse result in which a defendant is convicted *because* the individuals in the community became more permissive. This axiom excludes *variable threshold* rules, under which the degree of consent required to deem a work obscene varies.

Responsiveness: If $S \sqsubseteq S^*$, then $f(S) \sqsubseteq f(S^*)$.

The principle of anonymity requires that each individual's view must be treated equally. Individuals' names are switched through a permutation π of N. For a given permutation, $\pi(i)$ is the new name of the individual formerly known as i. For a given profile S, $\pi S \equiv \left(S_{\pi(1)}, ..., S_{\pi(n)}\right)$ is the profile that results once names are switched. The third axiom, *anonymity*, requires that permutations of the individuals' names do not affect the community standard. This axiom excludes *dictatorships*, under which a pre-selected individual decides which works are obscene.

Anonymity: For every permutation π of N, $f(S) = f(\pi S)$

[13] Note that for $K = 1$, the symbols $\sqcap$ and $\cap$ are interchangeable, as are the symbols $\sqcup$ and $\cup$, and the symbols $\sqsubseteq$ and $\subseteq$.

[14] The examples provided in this section are not meant as an exhaustive list of all rules excluded by these axioms.

[15] A weaker axiom would require that, if $S_i = S_j$ for all $i, j \in N$, then $f(S) \sqsubseteq S_1 = ... = S_n$. All theorems in this section would remain true if we replaced homogeneity with this weaker axiom.

The principle of *neutrality* is similar. It requires that a rule not discriminate, ex ante, between works on the basis of their names. Differences between works in the community standard should come from the beliefs and not from the rule. Works' names switched through an automorphism $\phi \in \Phi$. For a given profile S, $f(\phi S)$ is the community standard derived from the profile that results when the names are switched; while $\phi f(S)$ is the community standard that results when the names are switched only after the aggregation. The neutrality axiom requires that these two community standards be the same. This axiom excludes rules that deem a particular work obscene regardless of the opinions.

Neutrality: For every automorphism $\phi \in \Phi$, $\phi\left(f(S)\right) = f(\phi S)$.

4.2.3 The Unanimity Rule

Under the "unanimity rule", a work is considered obscene if it is considered obscene by every individual. If there are multiple issues, then for each issue a work is prohibitable only when it is considered prohibitable by every individual.

Unanimity Rule: For every $S \in \mathcal{S}^N$, $f(S) = \sqcap_{i\in N} S_i$.

The main result of this chapter is that a rule satisfies all four axioms if and only if it is unanimity rule.

Theorem 4.2.1 *The unanimity rule is the only rule that satisfies homogeneity, responsiveness, anonymity, and neutrality. Moreover, all four axioms are independent.*

Proof.

That unanimity rule satisfies the four axioms is trivial. I will show that any rule which satisfies the four axioms must be unanimity rule. Let f satisfy the four axioms.

Step 1: I show that any work considered obscene by every individual must be considered obscene by the community, or that $\sqcap_{i\in N} S_i \sqsubseteq f(S)$ for all $S \in \mathcal{S}^N$.

Let $S \in \mathcal{S}^N$. Define S^* as the profile such that $S_j^* \equiv \sqcap_{i\in N} S_i$ for all $j \in N$. By homogeneity, $f(S^*) = \sqcap_{i\in N} S_i$. Because $S^* \sqsubseteq S$, responsiveness implies that $f(S^*) \sqsubseteq f(S)$. Thus $\sqcap_{i\in N} S_i \sqsubseteq f(S)$.

Step 2: I show that if there is a profile T such that (a) $T_{ik} \cup T_{jl} = W$ unless $i = j$ and $k = l$, and (b) $\mu\left(T_{ik}\right) = \mu\left(T_{jk}\right)$ for all $i, j \in N$ and $k \in M$, then $f(T) \sqsubseteq \sqcap_{i\in N} T_i$.

Let $T \in \mathcal{S}^N$ such that conditions (a) and (b) are met. Without loss of generality, let $w \notin T_{11}$. To prove that $f(T) \sqsubseteq \sqcap_{i\in N} T_i$, it is sufficient to show that $w \notin f_1(T)$.

Suppose, contrariwise, that $w \in f_1(T)$. Then, by neutrality, $W \setminus T_{11} \sqsubseteq f_1(T)$. By anonymity and neutrality, $W \setminus T_{i1} \sqsubseteq f_1(T)$ for all $i \in N$. Thus $\sqcup_{i\in N}\left(W \setminus T_{i1}\right) \sqsubseteq f_1(T)$. By step 1, $\sqcap_{i\in N} T_{i1} \sqsubseteq f_1(T)$, which implies that $f_1(T) = W$. But this is a contradiction, which proves that $w \notin f_1(T)$, and therefore that $f(T) \sqsubseteq \sqcap_{i\in N} T_i$.

Step 3: I show that any work not considered obscene by every individual must not be considered obscene by the community, or that $f(S) \sqsubseteq \sqcap_{i \in N} S_i$ for all $S \in \mathcal{S}^N$.

Let $S \in \mathcal{S}^N$. Without loss of generality, let $w \notin S_{11}$. To prove that $f(S) \sqsubseteq \sqcap_{i \in N} S_i$, it is sufficient to show that $w \notin f_1(S)$. Let T be a profile such that: (1) $T_{ik} \cup T_{jl} = W$ unless $i = j$ and $k = l$, (2) $\mu(T_{ik}) = \mu(T_{jk})$ for all $i, j \in N$ and $k \in M$, (3) $w \notin T_{11}$, and (4) $S \sqsubseteq T$. By step 2, $f(T) \sqsubseteq \sqcap_{i \in N} T_i$. Because $S \sqsubseteq T$, responsiveness implies that $f(S) \sqsubseteq f(T) \sqsubseteq \sqcap_{i \in N} T_i$. Because $w \notin T_{11}$ it follows that $w \notin f_1(S)$. This proves that $f(S) \sqsubseteq \sqcap_{i \in N} S_i$.

Step 4: Steps 1 and 3 directly imply that $f(S) = \sqcap_{i \in N} S_i$. The independence of the axioms is proved in the appendix. ■

4.2.4 Independence

Unanimity rule is clearly *independent* in the sense that the community standard's judgment about a particular work given a particular issue depends only on the individual judgments about that work given that issue. This *independence* property can be broken into two strong axioms, *work-independence* and *issue-independence*. A rule is *work-independent* if the determination as to whether a particular work is obscene depends only on the opinions about that particular work.

Work-Independence: If there exists $w \in W$ and $S, S' \in \mathcal{S}^N$ such that $w \in S_{ij}$ if and only if $w \in S'_{ij}$ for all $i \in N$ and $j \in M$, then $w \in f_j(S)$ if and only if $w \in f_j(S')$.

A rule is *issue-independent* if the collective judgment for each issue depends only on the opinions about that issue.

Issue-Independence: If there exists $j \in M$ and $S, S' \in \mathcal{S}^N$ such that $S_{ij} = S'_{ij}$ for all $i \in N$, then $f_j(S) = f_j(S')$.

It has long been known that when there is only a single issue ($m = 1$) and the set of works is finite, the unanimity rule is the unique rule satisfying homogeneity, responsiveness, anonymity, neutrality, and work-independence. (Monjardet, 1990; Nehring and Puppe, 2006). If there are multiple issues ($m > 1$), then it is clear that unanimity rule would be the unique rule satisfying these five axioms and issue-independence. In the infinite setting described in subsection 4.2.1, neither of these strong independence axioms must be assumed, but both are implied by the combination of homogeneity, responsiveness, anonymity, and neutrality.

Given this prior result, a natural question is whether either independence axiom is somehow implied by the model or some (non-full) subset of the axioms. The answer to this question is no — while all four axioms together are sufficient to imply work-independence and issue-independence, all four are also necessary to rule out non-independent rules.

Theorem 4.2.2 *The combination of the homogeneity, responsiveness, anonymity, and neutrality axioms is sufficient to imply work-independence and necessary to exclude rules that violate work-independence.*

Theorem 4.2.3 *Let $m \geq 2$. The combination of the homogeneity, responsiveness, anonymity, and neutrality axioms is sufficient to imply issue-independence and necessary to exclude rules that violate issue-independence.*

4.3 Other results

4.3.1 Finite Set of Works

In the previous section I assumed that the set of works is continuous and that each judgment must be of less than full measure. In this subsection I examine the implications of this assumption by allowing W to be finite and requiring only that there be at least one non-obscene work.

Consider the model specified in Section 4.2.1, with the following changes. Let $\mathcal{W}$ describe an infinite set of works, and let $W \subseteq \mathcal{W}$ be a collection of works. For each $W \subseteq \mathcal{W}$, let $\mathcal{J}_W \equiv 2^W \setminus W$ be the set of non-full subsets of W, and let $\mathcal{S}_W \equiv \mathcal{J}_W^N$ be the set of standards over W. For each $W \subseteq \mathcal{W}$, let $f^W : \mathcal{S}_W^N \rightarrow \mathcal{S}_W$ be a function mapping from an N-vector of standards into a single standard. Let Φ_W denote the set of permutations of W.

The axioms all have natural analogues in this setting, where f is replaced by f^W, $\mathcal{S}$ is replaced by $\mathcal{S}_W$, and Φ is replaced by Φ_W. The following characterization of the unanimity rule follows directly from Monjardet (1990) and Nehring and Puppe (2006).[16]

Theorem 4.3.1 *The unanimity rule is the only rule that satisfies homogeneity, anonymity, neutrality, work-independence, and issue-independence. Moreover, all five axioms are independent.*

Proof. That unanimity rule satisfies the five axioms is trivial. I will show that any rule which satisfies the five axioms must be unanimity rule. Let f^W satisfy the five axioms.

Issue-independence and work-independence imply that, for each issue $j \in M$ and each work $w \in W$, there exists a group of coalitions $G_{jw} \subseteq 2^N$ such that $w \in f_j^W(S)$ if and only if $\{i \in N : w \in S_{ij}\} \in G_{jw}$. Neutrality implies that there exists a single such group of coalitions G_j for each issue j such that $G_j = G_{jw}$ for all $w \in W$. Anonymity implies that there is a collection of quotas, $Q_j \subseteq \{0, ..., n\}$, such that $w \in f_j^W(S)$ if and only if $|\{i \in N : w \in S_{ij}\}| \in Q_j$. Homogeneity implies that $Q_j \neq \varnothing$.

[16]Both Monjardet (1990) and Nehring and Puppe (2006) used stronger axioms which additionally included responsiveness. However, as I show in the proof, responsiveness is implied by the other five axioms.

Let $j \in M$, let $x \in \{0, ..., n-1\}$, and let $S \in \mathcal{S}_W^N$ such that, for all $w \in W$, $|\{i \in N : w \in S_{ij}\}| = x$. Then $f_j^W(S) = W$ if $x \in Q_j$ and $f_j^W(S) = \varnothing$, otherwise. Clearly $f_j^W(S) \neq W$ and therefore $\{0, ..., n-1\} \not\subseteq Q_j$. Because $Q_j \neq \varnothing$ it follows that $Q_j = \{n\}$ and therefore $f^W(S) = \sqcap_{i \in N} S_i$.

The independence of the axioms is proved in the appendix. ■

Without the independence axioms, the four axioms of homogeneity, responsiveness, anonymity, and neutrality are not by themselves sufficient to characterize the unanimity rule. The other rules that satisfy these axioms have a special property — their outcomes differ from the unanimity rule outcome only when individuals consider a very small number of works to be non-obscene.

To formalize this concept, let $\mathcal{S}_{W^{mn}} = \{S \in \mathcal{S}_W : |W \setminus S_j| \geq m * n \text{ for all } j \in M\}$ be the set of standards in which each individual considers at least $m * n$ works to be acceptable for each issue, where m is the number of issues and where n is the number of individuals in the community. A rule has the *MN-Property* if, whenever each individual considers at least $m * n$ works to be acceptable for each issue, the outcome coincides with the unanimity rule outcome.

MN-Property: For each $S \in \mathcal{S}_{W^{mn}}^N$, $f^W(S) = \sqcap_{i \in N} S_i$.

The four axioms are sufficient to imply the MN-Property.

Lemma 4.3.2 *If an aggregation rule satisfies homogeneity, responsiveness, anonymity, and neutrality, then it satisfies the MN-Property.*

Proof. Let f^W satisfy the four axioms.

Step 1. I show that for any profile $S \in \mathcal{S}_{W^{mn}}^N$, $\sqcap_{i \in N} S_i \sqsubseteq f^W(S)$.

Let $S \in \mathcal{S}_{W^{mn}}^N$. Let $S' \equiv (\sqcap_{i \in N} S_i)^N$, the N-vector for which each element is $\sqcap_{i \in N} S_i$. Clearly, $S' \sqsubseteq S$. By homogeneity, $f^W(S') = \sqcap_{i \in N} S_i$. Responsiveness implies that $\sqcap_{i \in N} S_i \sqsubseteq f^W(S)$.

Step 2: I show that if there is a profile $T \in \mathcal{S}_W^N$ such that (a) $T_{ik} \cup T_{jl} = W$ unless $i = j$ and $k = l$, and (b) $|T_{ik}| = |T_{jk}|$ for all $i, j \in N$ and $k \in M$, then $f(T) \sqsubseteq \sqcap_{i \in N} T_i$.

Let $T \in \mathcal{S}_W^N$ such that conditions (a) and (b) are met. Without loss of generality, let $w \notin T_{11}$. To prove that $f(T) \sqsubseteq \sqcap_{i \in N} T_i$, it is sufficient to show that $w \notin f_1(T)$.

Suppose, contrariwise, that $w \in f_1(T)$. Then, by neutrality, $W \setminus T_{11} \sqsubseteq f_1(T)$. By anonymity and neutrality, $W \setminus T_{i1} \sqsubseteq f_1(T)$ for all $i \in N$. Thus $\sqcup_{i \in N} (W \setminus T_{i1}) \sqsubseteq f_1(T)$. By step 1, $\sqcap_{i \in N} T_{i1} \sqsubseteq f_1(T)$, which implies that $f_1(T) = W$. But this is a contradiction, which proves that $w \notin f_1(T)$, and therefore that $f(T) \sqsubseteq \sqcap_{i \in N} T_i$.

Step 3: I show that $f(S) \sqsubseteq \sqcap_{i \in N} S_i$ for all $S \in \mathcal{S}_{W^{mn}}^N$.

Let $S \in \mathcal{S}_{W^{mn}}^N$ and let $w \notin S_{11}$. To show that $f^W(S) \sqsubseteq \sqcap_{i \in N} S_i$. it is sufficient to show that $w \notin f_1^W(S)$. Let $S^* \in \mathcal{S}_W^N$ be a profile such that (a) $w \notin S_{11}^*$, (b) $S_{ik}^* \cup S_{jl}^* = W$ unless $i = j$ and $k = l$, (c) $|W \setminus S_{ij}^*| = 1$ for all $i \in N$ and $j \in M$, and (d) $S \sqsubseteq S^*$. Note that such a profile S^* is guaranteed to exist for all $S \in \mathcal{S}_{W^{mn}}^N$. By step 2, $f(S^*) \sqsubseteq \sqcap_{i \in N} S_i^*$. Because $S \sqsubseteq S^*$, responsivness implies that $f_1^W(S) \sqsubseteq f_1^W(S^*)$, and therefore $w \notin f_1^W(S)$.

Step 4: Steps 1 and 3 directly imply that $f(S) = \sqcap_{i \in N} S_i$ for all $S \in \mathcal{S}^N_{W^{mn}}$. ■

Lemma 4.3.2 explains why the axioms imply one result in the continuous model and another in the finite model. Any rule that satisfies the four axioms will coincide with unanimity rule when the set of non-obscene works is "large" relative to the number of individuals and issues — and not relative to the size of the entire set of works. In the continuous case, a set F with measure $\mu(F) = \frac{\mu(W)}{100}$ is, in some sense, the same relative size as a finite single-element set G out of a hundred-element set W. Both F and G are one percent of the whole. However, while G has one element, F has uncountably many elements, and thus only F is large relative to any integers n and m. Similarly, if W were countably infinite and the set of non-obscene works was also required to be countably infinite, the four axioms would imply unanimity rule.

For every work, it is reasonable to assume that there are similar works about which every individual would feel exactly the same way. Take a painting and add a small spot of blue paint; there is probably a place on the painting (or picture frame) where the spot would not affect any individual's judgment about the painting.

Formally, we can describe the set of similar issues in the following way. For each $W \subseteq \mathcal{W}$, let $W' \subseteq \mathcal{W}$ be a "similar" set of works, so that $|W| = |W'|$ and $W \cap W' = \varnothing$. For each $w \in W$ let $w' \in W'$ denote its counterpart. Let $\psi : \mathcal{S}_W \to \mathcal{S}_{W \cup W'}$ be the replication function such that $w \in S_{ij}$ if and only if $w, w' \in \psi(S)_{ij}$. For each $S \in \mathcal{S}^N_W$, let $\psi(S) = (\psi(S_1), ..., \psi(S_n))$. For a set $W \in \mathcal{W}$, let $\psi(W) = W \cup W'$.

A natural requirement is that the community standard preserve replications. For a given profile S, $f^{\psi(W)}(\psi(S))$ is the community standard derived from the replicated profile, and $\psi\left(f^W(S)\right)$ is the community standard derived from the profile and then replicated. The next axiom, *replication invariance*, requires that these two community standards be the same.

Replication Invariance: For each $W \subseteq \mathcal{W}$ and $S \in \mathcal{S}^N_W$, $\psi\left(f^W(S)\right) = f^{\psi(W)}(\psi(S))$.

Replication invariance, when combined with the other four axioms, is sufficient to characterize the unanimity rule without a direct assumption of independence.

Theorem 4.3.3 *An aggregation rule satisfies homogeneity, responsiveness, anonymity, neutrality, and replication invariance if and only if it is unanimity rule. Furthermore, the five axioms are independent.*

Proof. Let $S \in \mathcal{S}^N_W$ and let $z \equiv \min\{x \in \mathbb{N} : x \geq \log_2(m * n)\}$. For all $x > 1$, let $\psi^x(S) = \psi\left(\psi^{x-1}(S)\right)$. Repeated application of the replication invariance axiom implies that $\psi^z\left(f^W(S)\right) = f^{\psi^z(W)}(\psi^z(S))$. Because $\psi^z(S) \in \mathcal{S}^N_{\psi^z(W)^{mn}}$, it follows from Lemma 4.3.2 that $f^{\psi^z(W)}(\psi^z(S)) = \sqcap_{i \in N} \psi^z(S_i)$. Therefore, $f^W(S) = \sqcap_{i \in N} S_i$.

The independence of the axioms is proved in the appendix. ■

4.3.2 Ordered Works

The neutrality axiom implicitly assumes that there is no objective ordering on the set of works. No objective method to compare works (with respect to obscenity) has ever been developed by courts or by commentators. However, there are circumstances in which this assumption might appear to be too strong. In this subsection I consider the case where there is only a single issue, and the set of works is simply the real line. The non-obscene sets are taken to be open convex intervals of the real line, with the interpretation that if x and y are non-obscene, then $z \in [x, y]$ should also be non-obscene.

Consider the model specified in Section 4.2.1, with the following changes. Let the set of works $W = \mathbb{R}$ be the real line, and let $\mathcal{J}$ denote the set of convex open intervals in $\mathbb{R}$. Here elements of $\mathcal{J}$ correspond to judgments about which works are non-obscene or permissible. To simplify the model, let $m = 1$. Let Φ denote the set of strictly monotonic mappings $\phi : \mathbb{R} \to \mathbb{R}$. When $S \sqsubseteq T$ I write that T is as permissive as S. A rule f^* is the *least permissive* if, for every rule f and all profiles $S \in \mathcal{S}^N$, $f^*(S) \sqsubseteq f(S)$. Similarly, a rule f^* is the *most permissive* if, for every rule f and all profiles $S \in \mathcal{S}^N$, $f(S) \sqsubseteq f^*(S)$.

The *median-rule* is the rule in which the highest and lowest endpoints of the set of works considered non-obscene by the community standard are the median highest and median lowest in the community. (If n is even, then the median-rule uses the $\frac{n}{2}^{th}$ highest and lowest endpoints.)

Median-rule: For all $S \in \mathcal{S}^N$

$$f^{med}(S) = \left\{x \in \mathbb{R} : |\{i \in N : S_i \cap [x, \infty) \neq \varnothing\}|, |\{i \in N : S_i \cap (-\infty, x] \neq \varnothing\}| \geq \tfrac{n}{2}\right\}.$$

The *maximal-rule* is the rule in which the highest and lowest endpoints of the set of works considered non-obscene by the community standard are the highest and lowest in the community.

Maximal-rule: For all $S \in \mathcal{S}^N$

$$f^{max}(S) = \{x \in \mathbb{R} : |\{i \in N : S_i \cap [x, \infty) \neq \varnothing\}|, |\{i \in N : S_i \cap (-\infty, x] \neq \varnothing\}| \geq 1\}.$$

The median-rule and the maximal-rule are two of many rules that satisfies the four axioms in this setting. However, every other rule is more permissive than the median-rule, and is less permissive than the maximal rule.

Proposition 4.3.4 *The median-rule is the least permissive rule that satisfies homogeneity, responsiveness, anonymity, and neutrality.*

Proposition 4.3.5 *The maximal-rule is the most permissive rule that satisfies homogeneity and responsiveness. It also satisfies anonymity and neutrality.*

Proof.

Part A. I first show that the median-rule and the maximal-rule satisfy all four axioms. For $S \in \mathcal{S}^N$, and for $t \in \left\{1, \frac{n}{2}\right\}$, define
$f^t(S) = \{x \in \mathbb{R} : |\{i \in N : S_i \cap [x, \infty) \neq \varnothing\}|, |\{i \in N : S_i \cap (-\infty, x] \neq \varnothing\}| \geq t\}$.

To show that the f^t satisfies homogeneity, let $S' \in \mathcal{S}$ and let $S \equiv (S')^N$. If $w \notin S'$, convexity implies either that $S_i \cap [w, \infty) = \varnothing$ for all $i \in N$, or that $S_i \cap (-\infty, x] = \varnothing$ for all $i \in N$, which implies that $w \notin f^t(S)$. If $w \in S'$, convexity implies either that $S_i \cap [w, \infty) \neq \varnothing$ for all $i \in N$, or that $S_i \cap (-\infty, x] \neq \varnothing$ for all $i \in N$, which implies that $w \in f^t(S)$.

To show that the f^t satisfies responsiveness, let $S, T \in \mathcal{S}^N$ such that $S \sqsubseteq T$, and let $w \in f^t(S)$. I will show that $w \in f^t(T)$. That $w \in f^t(S)$ implies both that $|\{i \in N : S_i \cap [w, \infty) \neq \varnothing\}| \geq t$ and that $|\{i \in N : S_i \cap (-\infty, w] \neq \varnothing\}| \geq t$. Because $S_i \sqsubseteq T_i$ for all $i \in N$, $S_i \cap [w, \infty) \neq \varnothing$ implies that $T_i \cap [w, \infty) \neq \varnothing$, and $S_i \cap (-\infty, w] \neq \varnothing$ implies that $T_i \cap (-\infty, w] \neq \varnothing$. It follows that $w \in f^t(T)$.

To show that the f^t satisfies anonymity is trivial. To show that the f^t satisfies neutrality, let $S \in \mathcal{S}^N$ and $\phi \in \Phi$. It is sufficient to show that either condition (a) $S_i \sqcap [x, \infty) \neq \varnothing$ if and only if $\phi(S_i) \sqcap [\phi(x), \infty) \neq \varnothing$, and $S_i \sqcap (-\infty, x] \neq \varnothing$ if and only if $\phi(S_i) \sqcap (-\infty, \phi(x)] \neq \varnothing$, or condition (b) $S_i \sqcap [x, \infty) \neq \varnothing$ if and only if $\phi(S_i) \sqcap (-\infty, \phi(x)] \neq \varnothing$, and $S_i \sqcap (-\infty, x] \neq \varnothing$ if and only if $\phi(S_i) \sqcap [\phi(x), \infty) \neq \varnothing$, must be true for all $x \in \mathbb{R}$ and $i \in N$.

Let $i \in N$ and $x \in \mathbb{R}$. If $x \in S_i$ then trivially $\phi(x) \in \phi(S_i)$ and both conditions hold. Assume then that $x \notin S_i$. Then either condition (1) $S_i \sqcap [x, \infty) \neq \varnothing$ or condition (2) $S_i \sqcap (-\infty, x] \neq \varnothing$ must be true but not both.

First, assume that ϕ is monotonically increasing, or that $x < y$ implies that $\phi(x) < \phi(y)$. If (1), let $y \in S_i \sqcap [x, \infty)$. Then $x < y$ and therefore $\phi(x) < \phi(y)$. It follows that $\phi(y) \in S_i \sqcap [\phi(x), \infty)$, and that $\phi(y) \notin S_i \sqcap (-\infty, \phi(x)]$, and therefore condition (a) holds. If (2), let $z \in S_i \sqcap (-\infty, x]$. Then $z < x$ and therefore $\phi(z) < \phi(x)$. It follows that $\phi(z) \notin S_i \sqcap [\phi(x), \infty)$, and that $\phi(z) \in S_i \sqcap (-\infty, \phi(x)]$, and therefore condition (a) holds.

Alternately, assume that ϕ is monotonically decreasing, or that $x < y$ implies that $\phi(y) < \phi(x)$. If (1), let $y \in S_i \sqcap [x, \infty)$. Then $x < y$ and therefore $\phi(y) < \phi(x)$. It follows that $\phi(y) \notin S_i \sqcap [\phi(x), \infty)$, and that $\phi(y) \in S_i \sqcap (-\infty, \phi(x)]$, and therefore condition (b) holds. If (2), let $z \in S_i \sqcap (-\infty, x]$. Then $z < x$ and therefore $\phi(x) < \phi(z)$. It follows that $\phi(z) \in S_i \sqcap [\phi(x), \infty)$, and that $\phi(z) \notin S_i \sqcap (-\infty, \phi(x)]$, and therefore (b) holds. This concludes the proof of Part A.

Part B. I show that if a rule f satisfies homogeneity and responsiveness, then $f(S) \sqsubseteq f^{max}(S)$ for all profiles $S \in \mathcal{S}^N$.

For $X \subseteq \mathbb{R}$, let $conv\,(X) = \{y \in \mathbb{R} : \text{ there exists } x, z \in X \text{ such that } x \geq y \geq z\}$. Note that $f^{max}(S) = conv\,(\sqcup_{i \in N} S_i)$.

Let $S \in \mathcal{S}^N$, and let $S' \equiv (conv\,(\sqcup_{i \in N} S_i))^N$. By homogeneity, $f\,(S') = conv\,(\sqcup_{i \in N} S_i)$. Because $S \sqsubseteq S'$, responsiveness implies that $f(S) \sqsubseteq conv\,(\sqcup_{i \in N} S_i) = f^{max}(S)$. This completes the proof of Proposition 4.3.5.

Part C. I show that if a rule f satisfies homogeneity, responsiveness, anonymity, and neutrality, then $f^{med}(S) \sqsubseteq f(S)$ for all profiles $S \in \mathcal{S}^N$.

Let $\mathcal{S}^* \equiv \left\{S \in \mathcal{S}^N : \text{for all } i \neq j, S_i \sqcap S_j = \varnothing \text{ and } \inf(S_i) \neq \sup(S_j)\right\}$.

For all $S \in \mathcal{S}^*$, let $S_{(i)}$ be the i-th highest element of S, so that $i > j$ implies that $x > y$ for any $x \in S_{(i)}$ and $y \in S_{(j)}$. Let $z \equiv \min\left\{x \in \mathbb{N} : x \geq \frac{n}{2}\right\}$.

Step 1. I show that for all $S \in \mathcal{S}^*$ and $i \in N$, $S_i \sqcap f(S) \in \{S_i, \varnothing\}$.

Let $S \in \mathcal{S}^*$, $i \in N$, $x, y \in S_i$, and $\phi \in \Phi$ such that, for all $i \in N$, $\phi(\inf(S_i)) = \inf(S_i)$, $\phi(\sup(S_i)) = \sup(S_i)$, and where $y = \phi(x)$. Then $\phi(S) = S$. By neutrality, $x \in f(S)$ if and only if $y = \phi(x) \in f(\phi(S)) = f(S)$.

Step 2. I show that for all $S \in \mathcal{S}^*$ and $i \in N$, $S_{(i)} \sqsubseteq f(S)$ if and only if $S_{(n+1-i)} \sqsubseteq f(S)$.

Let $S \in \mathcal{S}^*$, $i \in N$, $x \in S_{(i)}$, $y \in S_{(n+1-i)}$, and $\phi \in \Phi$ such that, for all $i \in N$, $\phi\left(\inf\left(S_{(i)}\right)\right) = \sup\left(S_{(n+1-i)}\right)$, $\phi\left(\sup\left(S_{(i)}\right)\right) = \inf\left(S_{(n+1-i)}\right)$, and where $y = \phi(x)$. Let π be the permutation such that $\pi(j) = n+1-j$ for all $j \in N$. Then $\pi\phi(S) = S$. By anonymity and neutrality, $x \in f(S)$ if and only if $y = \phi(x) \in f(\pi\phi(S)) = f(S)$.

Step 3. I show that $S_{(z)} \sqsubseteq f(S)$ for all $S \in \mathcal{S}^*$.

Let $S \in \mathcal{S}^*$. Suppose, contrariwise, that f satisfies the four axioms but that $S_{(z)} \sqcap f(S) = \varnothing$. Because $f(S)$ is convex, this implies that $S_{(j)} \sqcap f(S) = \varnothing$ for either (a) $j \geq z$ or (b) $j \leq z$. It follows from step 2 that $S_{(j)} \sqcap f(S) = \varnothing$ for all $j \in N$. Part B implies that $f(S) \sqsubseteq conv\left(\sqcup_{i\in N} S_i\right) \setminus \left(\sqcup_{i\in N} S_i\right)$.

Let $v \in f(S)$, and without loss of generality, assume that $v \in \left(\sup\left(S_{(i+1)}\right), \inf\left(S_{(i)}\right)\right)$ for some $i < n$. Construct a profile S^v such that $S_{(j)} = S^v_{(j)}$ for all $j \neq i+1$ and $S^v_{(i+1)} \equiv \left(\inf\left(S_{(i+1)}\right), v + \delta\right)$ for some suitably small δ. Let $x \in S_{(i+1)}$.

Let $\phi \in \Phi$ such that, for all $j \in N$, $\phi(\inf(S_j)) = \inf\left(S^v_j\right)$, $\phi(\sup(S_j)) = \sup\left(S^v_j\right)$, and $v = \phi(x)$. Then $\phi(S) = S^v$. Because $S \sqsubseteq S^v$, responsiveness implies that $f(S) \sqsubseteq f(S^v)$ and therefore $v \in f(S^v)$. Because $x \notin f(S)$, neutrality implies that $v = \phi(x) \notin f(\phi(S)) = f(S^v)$. This contradiction show that $S_{(z)} \sqsubseteq f(S)$ for all $S \in \mathcal{S}^*$.

Step 4. I show that $f^{med}(S) \sqsubseteq f(S)$ for all $S \in \mathcal{S}^N$.

Let $S \in \mathcal{S}^N$, and let $x \in f^{med}(S)$. I will show that $x \in f(S)$.

For each individual $i \in N$, let $a_i \equiv \inf(S_i)$ and $b_i \equiv \sup(S_i)$. Note that $S_i = (a_i, b_i)$. If $b_i \neq b_j$ for all $i, j \in N$, construct a profile S^+ such that $S^+_i = (b_i - \varepsilon, b_i)$, with ε chosen suitably small such that $S^+ \in \mathcal{S}^*$, $S^+ \sqsubseteq S$, and such that $x < y$ for all $y \in S^+_{(z)}$. If there exists $i, j \in N$ such that $b_i = b_j$ then construct the profile S^+ so that $S^+_i = (b_i - \varepsilon, b_i)$, $S^+_j = (b_i - 3\varepsilon, b_i - 2\varepsilon)$, etc., again with ε chosen suitably small such that $S^+ \in \mathcal{S}^*$, $S^+ \sqsubseteq S$, and such that $x < y$ for all $y \in S^+_{(z)}$.

Similarly, if $a_i \neq a_j$ for all $i, j \in N$, construct a profile S^- such that $S^-_i = (a_i, a_i + \varepsilon)$, with ε chosen suitably small such that $S^- \in \mathcal{S}^*$, $S^- \sqsubseteq S$, and such that $x > y$ for all $y \in S^-_{(z)}$. If there exists $i, j \in N$ such that $a_i = a_j$ then construct the profile S^- so that $S^-_i = (a_i, a_i + \varepsilon)$, $S^-_j = (a_i + 2\varepsilon, a_i + 3\varepsilon)$, etc., again with ε chosen suitably small such that $S^- \in \mathcal{S}^*$, $S^- \sqsubseteq S$, and such that $x > y$ for all $y \in S^-_{(z)}$.

By step 3, $S^+_{(z)} \sqsubseteq f(S)$. By steps 2 and 3, $S^-_{(n+1-z)} \sqsubseteq f(S)$. By convexity, $conv\left(S^+_{(z)} \sqcup S^-_{(n+1-z)}\right) \sqsubseteq f(S)$. By construction, for all $w \in S^+_{(z)}$ and $y \in S^-_{(n+1-z)}$, $w > x > y$ which implies that $x \in conv\left(S^+_{(z)} \sqcup S^-_{(n+1-z)}\right) \sqsubseteq f(S)$. This completes the proof of Proposition 4.3.4. ■

4.3.3 Related issues

I have assumed that the issues in M are not logically related and do not imply one another. That would not be a reasonable assumption if, for example, we were to include three issues, "appeal to prurient interest," "patent offensiveness," and "obscenity." The last issue is the intersection of the previous two.

To describe this formally, consider the model specified in Section 4.2.1, with the following changes. Let $M \equiv \{a, b, a \wedge b\}$, with the interpretation a="appeals to the prurient interest", b="patently offensive", and $a \wedge b$ = "obscene". Let $\mathcal{S} \subseteq \mathcal{J}^M$ be the set of standards such that, for all $S_i \in \mathcal{S}$ and $S_{ia} \sqcap S_{ib} = S_{i(a \wedge b)}$.

If we add an additional assumption of issue-independence, this formal setup allows us to remove two unecessary axioms: responsiveness and neutrality. The combination of the issue-independence, homogeneity, and anonymity axioms is sufficient to characterize the unanimity rule. This theorem is related to the doctrinal paradox of Kornhauser and Sager (1986) which was first formalized by List and Pettit (2002).

Theorem 4.3.6 *An aggregation rule satisfies homogeneity, anonymity, and issue-independence if and only if it is unanimity rule. Furthermore, the three axioms are independent.*

Proof. That unanimity rule satisfies the three axioms is trivial. To prove the converse, let f satisfy the three axioms. I will show that f must be unanimity rule.

Issue-independence implies that there are functions $g_a, g_b, g_{a \wedge b} : \mathcal{J}^N \to \mathcal{J}$ such that, for all $S \in \mathcal{S}^N$, $f(S) = \left(g_a\left((S_{ia})_{i \in N}\right), g_b\left((S_{ib})_{i \in N}\right), g_{a \wedge b}\left(\left(S_{i(a \wedge b)}\right)_{i \in N}\right)\right)$ such that, for all $x, y \in \mathcal{J}^N$, $g_a(x) \sqcap g_b(y) = g_{a \wedge b}(x \sqcap y)$. Furthermore, $g_{a \wedge b}(x)$ must be responsive. To see why, assume that $x \sqsubseteq z$. Clearly, $g_a(x) \sqcap g_b(z) = g_{a \wedge b}(x) = g_a(z) \sqcap g_b(x)$. This implies that $g_{a \wedge b}(x) \sqsubseteq g_a(z) \sqcap g_b(z)$ and therefore $g_{a \wedge b}(x) \sqsubseteq g_{a \wedge b}(z)$.

Homogeneity implies that, for all $x \in \mathcal{J}^N$, $g_a(x) = g_b(x) = g_{a \wedge b}(x)$. To see why, suppose, contrariwise, that there is an $x \in \mathcal{J}^N$ such that $g_a(x) \neq g_b(x)$. We know that $g_a(x) \sqcap g_b(x) = g_{a \wedge b}(x)$, this implies that either $g_a(x) > g_{a \wedge b}(x)$ or $g_b(x) > g_{a \wedge b}(x)$ or both. Without loss of generality, assume that $g_a(x) > g_{a \wedge b}(x)$. For all $z \in \mathcal{J}^N$, $g_a(x) \sqcap g_b(z) = g_{a \wedge b}(x \sqcap z)$. Let $z \equiv (g_a(x))^N$, the N-vector for which every element is equal to $g_a(x)$. By homogeneity, $g_b(z) = g_a(x)$ which implies that $g_a(x) \sqcap g_a(x) = g_a(x) = g_{a \wedge b}(x \sqcap z)$. But because $g_{a \wedge b}(x) \geq g_{a \wedge b}(x \sqcap z)$, this violates the assumption that $g_a(x) > g_{a\wedge b}(x)$ and proves that, for all $x \in \mathcal{J}^N$, $g_a(x) = g_b(x)$. Therefore, $g_a(x) = g_{a\wedge b}(x)$. Let $g(x) \equiv g_a(x)$.

Let $x \in \mathcal{J}^N$, and let π be the permutation such that $\pi(n) = 1$ and, for all $i < n$, $\pi(i) = i + 1$. By anonymity, $g(x) = g(\pi x)$. It follows that $g(x) = g(x) \sqcap g(\pi x) = g(x \sqcap \pi x)$. By induction, this implies that $g(x) = g(x \sqcap \pi x \sqcap \pi\pi x \sqcap ...) = g(\sqcap_{i\in N} x_i, ..., \sqcap_{i\in N} x_i)$. From homogeneity it follows that $g(x) = \sqcap_{i\in N} x_i$ which implies that for all $S \in \mathcal{S}^N$, $f(S) = \left(\sqcap_{i\in N} S_{ia}, \sqcap_{i\in N} S_{ib}, \sqcap_{i\in N} S_{i(a\wedge b)}\right) = \sqcap_{i\in N} S_i$.

The independence of the axioms is proved in the appendix. ■

4.4 Conclusion

I have introduced a new model of community standards used in determining whether potentially obscene material is protected by the free speech and press guarantees of the United States Constitution. In the model, both individual and community standards are taken to be judgments — categorizations of possible works as either "obscene" or "not obscene." Every possible judgment is allowed provided it satisfies the following restriction: neither individuals nor the community may consider all works to be obscene. Community standards are derived systematically from the individual standards. Every possible method of deriving the community standards is considered. The methods are then evaluated according to normative axioms.

The axioms require that the community standard (a) preserve unanimous agreements about the entire standard, (b) become more permissive when all individuals become more permissive, and not discriminate, ex ante, (c) between individuals and (d) between works. Together, these four axioms characterize the unanimity rule, under which a work is deemed obscene when every individual considers it to be obscene. Every other conceivable method of deriving a community standard from individual standards must violate one or more of these axioms. Whether this result is positive or negative depends on the specific interpretation of the model.

If the jury is taken to be a perfectly representative sample of the society, then unanimity rule coincides with the unanimous jury rule, the dominant rule in criminal trials in the United States.[17] Similarly, if we assume that the community consists of all reasonable persons who live in a society, then the result support Lord Devlin's argument that community standards are connected to unanimity rule.

However, there are strong reasons for believing that unanimity rule is not always used in the United States. The primary reason is that there are still convictions for obscenity. American society

[17] In civil cases, the unanimous jury rule is used in Federal courts, in the District of Columbia, and in twenty-seven states out of fifty. In criminal cases, the unanimous jury rule is used everywhere but Puerto Rico. The correspondence is not perfect, however. The rule generally requires that a jury must unanimously agree to find for either the plaintiff or the defendant. When the jury is not unanimous the result is a mistrial, which is a victory for the defense except that the case can be retried.

has become much more diverse in the past half-century, even in places generally thought to be conservative bastions. Empirical research supports the claim that many of these convictions are for material considered non-obscene by a many individuals in the relevant communities. (Linz et al., 1991, 1995).

There is an additional problem which occurs if the accused is a member of the community. In most criminal prosecutions the defendant's incentives are generally not aligned with those of the tribunal. Lord Devlin dealt with this problem by allowing the court to infer what the defendant's honest belief would be if the defendant was reasonable and had thought about the act in question. If the defendant's actual views are relevant, then unanimity rule may be unworkable in the United States. The self-incrimination clause of the Fifth Amendment to the United States Constitution prevents the court from asking the defendant to reveal facts (including beliefs) that would lead to conviction.

If, despite this, we decide to press forward with the unanimity rule, and if the relevant community consists of all reasonable individuals within the relevant geographical region, then the unanimity rule could be implemented through a jury instruction. Jurors would be instructed to find a work obscene only if every reasonable person in the community would consider it obscene. However, for this rule to be meaningful, whether a person is deemed 'reasonable' must not depend on that persons judgment.

If unanimity rule is not used, however, then the law can take one of two paths. First, the law could rely upon a rule that violates one of the four axioms. The rule would not respect unanimous judgments of the society, or convict individuals because society becomes more permissive, or discriminate between individuals or works.

Second, the law could cut the connection between the judgments of individuals in the community and the applicable legal standard. There is nothing, per se, wrong with such an approach. It would, however, represent a total sea change in the approach of the Supreme Court.

4.5 Appendix

4.5.1 Proof of Theorem 4.2.1: Independence of the Axioms

Claim *The homogeneity, responsiveness, anonymity, and neutrality axioms are independent.*

Proof. I present four rules. Each violates one axiom while satisfying the remaining three. This is sufficient to prove the claim.

Rule 1: Consider the degenerate rule in which $f_j(S) \equiv \varnothing$ for all $j \in M$ and all $S \in \mathcal{S}^N$. This trivially satisfies the responsiveness, anonymity, and neutrality axioms but violates homogeneity.

Rule 2: Consider the rule in which $f(S) \equiv \sqcup_{i \in N} S_i$, if $\sqcup_{i \in N} S_i \in \mathcal{S}$, and $\sqcap_{i \in N} S_i$ otherwise. This trivially satisfies the homogeneity, anonymity, and neutrality axioms. To see why it violates responsiveness, let S be a profile such that (a) $\sqcup_{i\in N} S_i = W^M$ for all $i \in N$, (b) $\sqcap_{i\in N} S_i = \varnothing^M$ for all $i \in N$, and (c) $S_{1j} \neq \varnothing$ for all $j \in M$. Let S^* be a profile where $S_i^* = S_i \sqcap S_1$ for all $i \in N$. Clearly $S^* \sqsubseteq S$. Because $\sqcup_{i\in N} S_i \notin \mathcal{S}$, it follows that $f(S) = \varnothing^M$, while $f(S^*) = S_1$. Because $S_1 \not\sqsubseteq \varnothing^M$ the example shows that this rule violates responsiveness.

Rule 3: Consider the rule in which $f(S) \equiv S_1$ for all $S \in \mathcal{S}^N$. This trivially satisfies the homogeneity, responsiveness, and neutrality axioms but violates anonymity.

Rule 4: Let $w^* \in W$. Consider the rule in which, for all issues $j \in M$, $f_j(S) \equiv (\cap_{i\in N} S_{ij}) \cup \{w \in W : w \in \cup_{i\in N} S_{ij} \text{ and } w = w^*\}$. This trivially satisfies the homogeneity, responsiveness, and anonymity axioms but violates neutrality. ■

4.5.2 Proof of Theorem 4.2.2

Proof. Any rule that satisfies the four axioms is necessarily unanimity rule, which satisfies work-independence. To show that all four axioms are necessary to exclude rules which violate work-independence, I provide four rules. Each violates one of the four axioms in addition to work-independence.

Rule 1: Consider the degenerate rule in which, for all $j \in M$, $f_j(S) \equiv \sqcap_{i\in N} S_{ij}$ if $\mu(\cap_{i\in N} S_{ij}) > 0$, else $f_j(S) \equiv \varnothing$. This trivially satisfies the responsiveness, anonymity, and neutrality axioms but violates homogeneity and work-independence.

Rule 2: Consider the rule in which $f(S) \equiv \sqcup_{i\in N} S_i$, if $\sqcup_{i\in N} S_i \in \mathcal{S}$, and $\sqcap_{i\in N} S_i$ otherwise. This satisfies homogeneity, anonymity, and neutrality but violates responsiveness and work-independence..

Rule 3: Consider the rule in which $f_j(S) \equiv \sqcap_{\{k\in N:\mu(S_{1j}\cup S_{kj})<\mu(W)\}} S_k$ for all $S \in \mathcal{S}^N$. This satisfies homogeneity, responsiveness, and neutrality but violates anonymity and work-independence.

Rule 4: Let $w', w^* \in W$. Consider the rule in which, for all issues $j \in M$, $f_j(S) \equiv (\cap_{i\in N} S_{ij}) \cup \{w \in W : \{w, w'\} \sqsubseteq \cup_{i\in N} S_{ij} \text{ and } w = w^*\}$. This trivially satisfies the homogeneity, responsiveness, and anonymity axioms but violates neutrality and work-independence. ■

4.5.3 Proof of Theorem 4.2.3

Proof. Any rule that satisfies the four axioms is necessarily unanimity rule, which satisfies issue-independence. To show that all four axioms are necessary to exclude rules which violate work-independence, I provide four rules. Each violates one of the four axioms in addition to issue-independence.

Rule 1: Consider the degenerate rule in which, for all $j \in M$, $f_j(S) \equiv \sqcap_{i\in N} S_{i1}$. This trivially satisfies the responsiveness, anonymity, and neutrality axioms but violates homogeneity and issue-independence.

Rule 2: Consider the rule in which, for all $j \in M$, $f_j(S) \equiv \sqcap_{i\in N} S_{ij} \sqcap$

$\{w \in W: \text{ for all } j \in M, \{i \in N : w \in S_{ij}\} \in \{\varnothing, N\}\}$. This trivially satisfies the homogeneity, anonymity, and neutrality axioms but violates responsiveness and issue-independence.

Rule 3: Consider the rule in which $f_1(S) \equiv \sqcap_{i \in N} S_{i1}$ and, for $j > 1$, $w \in f_j(S)$ if and only if $w \in S_{1j}$ and $w \in S_{kj}$ for all $k \in \{i \in N : w \in S_{i1}$ if and only if $w \in S_{11}\}$. This trivially satisfies the homogeneity, responsiveness, and neutrality axioms but violates anonymity and issue-independence.

Rule 4: Consider the rule in which $f_1(S) \equiv (\cap_{i \in N} S_{ij}) \cup \{w \in W : w \in \cup_{i \in N} S_{ij}$ and $w = w^*\}$ and, for $j > 1$, $f_j(S) \equiv \sqcap_{i \in N} S_{ij}$. This trivially satisfies the homogeneity, responsiveness, and anonymity axioms but violates neutrality and work-independence. ■

4.5.4 Proof of Theorem 4.3.1: Independence of the Axioms

Claim *The homogeneity, anonymity, neutrality, work-independence, and issue-independence axioms are independent.*

Proof. I present five rules. Each violates one axiom while satisfying the remaining four. This is sufficient to prove the claim.

Rule 1: Consider the degenerate rule in which $f_j^W(S) \equiv \varnothing$ for all $j \in M$ and all $S \in \mathcal{S}^N$. This trivially satisfies the anonymity, neutrality, work-independence, and issue-independence axioms but violates homogeneity.

Rule 2: Consider the rule in which $f^W(S) \equiv S_1$ for all $S \in \mathcal{S}^N$. This trivially satisfies the homogeneity, neutrality, work-independence, and issue-independence axioms but violates anonymity.

Rule 3: Let $w^* \in W$. Consider the rule in which, for all issues $j \in M$, $f_j^W(S) \equiv (\cap_{i \in N} S_{ij}) \cup \{w \in W : w \in \cup_{i \in N} S_{ij}$ and $w = w^*\}$. This trivially satisfies the homogeneity, anonymity, work-independence, and issue-independence axioms but violates neutrality.

Rule 4: Consider the rule in which $f^W(S) \equiv \sqcup_{i \in N} S_i$, if $\sqcup_{i \in N} S_i \in \mathcal{S}$, and $\sqcap_{i \in N} S_i$ otherwise. This trivially satisfies the homogeneity, anonymity, neutrality, and issue-independence axioms, but violates work-independence.

Rule 5: Let $U \equiv \{w \in W : w \in S_{ij}$ whenever $w \in S_{kj}$ for all $i, k \in N$ and $j \in M\}$. Consider the rule in which $f_j^W(S) = U \sqcap_{i \in N} S_{ij}$. This rule clearly satisfies the homogeneity, anonymity, neutrality, and work-independence axioms but violates issue-independence. ■

4.5.5 Proof of Theorem 4.3.3: Independence of the Axioms

Claim *The homogeneity, responsiveness, anonymity, neutrality, and replication invariance axioms are independent.*

Proof. I present five rules. Each violates one axiom while satisfying the remaining four. This is sufficient to prove the claim.

Rule 1: Consider the degenerate rule in which $f_j^W(S) \equiv \varnothing$ for all $j \in M$ and all $S \in \mathcal{S}^N$. This trivially satisfies the responsiveness, anonymity, neutrality, and replication invariance axioms but violates homogeneity.

Rule 2: Consider the rule in which $f^W(S) \equiv \sqcup_{i \in N} S_i$, if $\sqcup_{i \in N} S_i \in \mathcal{S}$, and $\sqcap_{i \in N} S_i$ otherwise. This trivially satisfies the homogeneity, anonymity, neutrality, and replication invariance axioms but violates responsiveness.

Rule 3: Consider the rule in which $f^W(S) \equiv S_1$ for all $S \in \mathcal{S}^N$. This trivially satisfies the homogeneity, responsiveness, neutrality, and replication invariance axioms but violates anonymity.

Rule 4: Let $w^* \in W$, and let $g : \mathcal{W} \to \mathbb{R}$ be a function mapping each element of $\mathcal{W}$ to a unique element of the real line, such that (a) $g(w) \geq g(w^*)$ for all $w \in \cup_{i=1}^{\infty} \psi^k(w^*)$ and (b) $g(w^*) \geq g(w)$ for all $w \in \cup_{w \in W \setminus \{w^*\}} \cup_{i=1}^{\infty} \psi^k(w)$. Without loss of generality, assume that $\arg\max_{w \in W} g\left(W \setminus \sqcap_{i \in N} S_{ij}\right) \in S_{(1)j}$. Let $X_j \equiv \left\{x \in W : g(x) > \max_{w \in W}\left(W \setminus \sqcap_{i \neq (1)} S_{ij}\right)\right\}$, and let $V_j \equiv \{x \in W : g(x) \geq g(w^*)\}$. Consider the rule in which, for all issues $j \in M$, $f_j^W(S) \equiv \sqcap_{i \in N} S_{ij} \sqcup (X_j \sqcap V_j)$. This satisfies the homogeneity, responsiveness, anonymity, and replication invariance axioms but violates neutrality.

Rule 5: Let $P_j \equiv \{w \in W : |\{i \in N : w \in S_{ij}\}| \geq |\{i \in N : v \in S_{ij}\}| \textit{ for all } v \in W\}$. Consider the rule where $f_j^W(S) \equiv W \setminus P_j$ when $|W \setminus S_{ij}| = 1$ for all $i \in N$, and where $f_j^W(S) \equiv \sqcap_{i \in N}$ otherwise. This rule satisfies the homogeneity, responsiveness, anonymity, and neutrality axioms but fails replication invariance. ■

4.5.6 Proof of Theorem 4.3.6: Independence of the Axioms

Claim *The homogeneity, anonymity, and issue-independence axioms are independent.*

Proof. I present three rules. Each violates one axiom while satisfying the remaining two. This is sufficient to prove the claim.

Rule 1: Consider the degenerate rule in which $f_j(S) \equiv \varnothing$ for all $j \in M$ and all $S \in \mathcal{S}^N$. This satisfies anonymity and issue-independence but violates homogeneity.

Rule 2: Consider the rule in which $f(S) \equiv S_1$ for all $S \in \mathcal{S}^N$. This satisfies homogeneity and issue-independence but violates anonymity.

Rule 3: Let $w^* \in W$. Consider the rule in which, for issues $j \in \{a, b\}$, $f_j(S) \equiv (\cap_{i \in N} S_{ij}) \cup \{w \in W : w \in \cup_{i \in N} S_{ij} \text{ and } w = w^*\}$, and where $f_{a \wedge b}(S) \equiv f_a(S) \sqcap f_b(S)$. This satisfies homogeneity and anonymity but not issue-independence. ■

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