Chapter 6. Conclusion

We created simple stochastic models of spatially heterogeneous stress in three dimensions. By breaking up the stress tensor into three invariant quantities (principal stresses) and three orientation angles (a rotation amplitude, ω, about a rotation axis $[\theta,\phi]$), we were able to produce filtered heterogeneous 3D matrices of the full stress tensor with properties that are approximately unchanged upon rotation of coordinate system. We generated random principal stresses $(\sigma_1,\sigma_2,\sigma_3)$ using Gaussian white noise and random orientations $(\omega,[\theta,\phi])$ using random unit quaternions, then filtered each quantity in three dimensions. The spatial smoothing parameter we used in the filtering is α, which is the spectral falloff of any 1D cross section through our 3D grids. We find that the larger the value of α, the greater the spatial smoothing. For our 201x201x201 grids and spatial smoothing $\alpha \le 1.0$, any orientation bias due to filtering is small and can be eliminated by stacking grids with a different random rotation applied to the stress tensors within each stacked grid. Subtracting out the pressure, we then added our filtered heterogeneous deviatoric stress in 3D, $\boldsymbol{\sigma}'_H(\mathbf{x})$, to a spatially uniform background stress, $\boldsymbol{\sigma}'_B$. This introduces our second stress heterogeneity parameter, HR, which uses a ratio of I'_2 s (second invariants of deviatoric stress tensors), which are functions of the deviatoric principal stresses (σ'_1, σ'_2, and σ'_3), as a measure of the relative amplitude of the spatially heterogeneous stress, $\boldsymbol{\sigma}'_H(\mathbf{x})$, to the amplitude of the spatial mean, $\boldsymbol{\sigma}'_B$. Last, we add a stress-rate $\dot{\boldsymbol{\sigma}}'_T$, due to far-field plate loading, to bring points to failure via our Hencky-Mises plastic yield criterion.

We showed analytically that, in the presence of extremely heterogeneous stress and our plastic yield criterion, we would expect bias to which points fail as earthquakes for $HR >> 1$, a bias towards $\dot{\boldsymbol{\sigma}}'_T$. Assuming that only a small percentage of the possible failure points in the Earth actually fail, we found that if the spatial stress heterogeneity is large in comparison to the spatial mean, the most likely points to fail will have an average stress rotated toward $\dot{\boldsymbol{\sigma}}'_T$. Numerically testing this with our 3D filtered heterogeneous stress, we computed $\bar{\boldsymbol{\sigma}}'_{Failure}\left(\mathbf{x}_{i_{Failure}}\right)$ from the first 2,000 failures for a variety of simulations and show that $\bar{\boldsymbol{\sigma}}'_{Failure}\left(\mathbf{x}_{i_{Failure}}\right) \approx \boldsymbol{\sigma}'_B$ for $HR << 1$, $\bar{\boldsymbol{\sigma}}'_{Failure}\left(\mathbf{x}_{i_{Failure}}\right) \approx \dot{\boldsymbol{\sigma}}'_T$ for $HR >> 1$, and $\bar{\boldsymbol{\sigma}}'_{Failure}\left(\mathbf{x}_{i_{Failure}}\right)$ is rotated approximately halfway between $\boldsymbol{\sigma}'_B$ and $\dot{\boldsymbol{\sigma}}'_T$ for $HR \approx 2.0$.

Current stress studies using focal mechanism inversions [*Angelier*, 1975; 1984; *Carey and Brunier*, 1974; *Etchecopar, et al.*, 1981; *Gephart*, 1990; *Gephart and Forsyth*, 1984; *Mercier and Carey-Gailhardis*, 1989; *Michael*, 1984; 1987] assume that there is no bias toward $\dot{\boldsymbol{\sigma}}'_T$ in their measured focal mechanism orientations. It is assumed that the set of earthquakes used in the inversions are a good random sampler of the mean stress state in the real Earth; therefore, the tensor obtained from these inversions is equated with the spatial mean, $\boldsymbol{\sigma}'_B$. However, according to our studies, if there is significant heterogeneity, the interpretation of focal mechanism inversions is not that simple; one must take into account the bias toward $\dot{\boldsymbol{\sigma}}'_T$.

To determine whether or not this bias toward $\dot{\boldsymbol{\sigma}}'_T$ is important in the real Earth, we compared our synthetic focal mechanisms produced from spatially heterogeneous stress to real focal mechanism data and estimated our heterogeneous ratio, HR. The parameter α has little to no effect on the percent bias toward $\dot{\boldsymbol{\sigma}}'_T$ for $\alpha \le 1.0$. Based on our numerical simulations, if $HR \ge 1.0$, there will be a minimum 25–35% bias toward $\dot{\boldsymbol{\sigma}}'_T$ in the stress inversions. Our first step was to estimate the model noise that must be added to our synthetic focal mechanisms, i.e., how much noise there is in real focal mechanism calculations due to errors in determining the mechanisms. Our next step was to calculate the average focal mechanism difference (an average angular difference) as a function of distance for simulations with varying amounts of stress heterogeneity, HR, and compare our results to a figure from Hardebeck's recently submitted paper [in review, 2006]. Hardebeck determined these quantities for three regions, Southern California; East Bay, San Francisco; and the Loma Prieta region. We attempted to model the Southern California and East Bay, San Francisco. We also compared focal mechanisms from Hardebeck's focal mechanism catalogue [*Hardebeck and Shearer*, 2003] for Southern California to our synthetic simulations. Applying Michael's inversion program, "slick" [1984; 1987], to focal mechanisms within seven non-aftershock regions using A and B quality data and then to our synthetic focal mechanisms with model noise added, we compare misfit angle statistics. Using these two methods, our best estimate is $HR = 1.25$ for Southern California and East Bay, San Francisco in aftershock free areas. According to our simulations, this would generate an ≈ 40% bias toward the stress rate, $\dot{\boldsymbol{\sigma}}'_T$. This is a non-trivial bias; hence, we conclude that stress studies that use focal mechanism data sets and standard stress inversion tools [*Angelier*, 1975; 1984; *Carey and Brunier*, 1974; *Etchecopar, et al.*, 1981; *Gephart*, 1990; *Gephart and Forsyth*, 1984; *Mercier and Carey-Gailhardis*, 1989; *Michael*, 1984; 1987] to determine the stress state in the crust need to be reinterpreted. We illustrate how one might subtract out this bias toward $\dot{\boldsymbol{\sigma}}'_T$ in our new heterogeneous stress paradigm to produce a more accurate estimate of $\boldsymbol{\sigma}'_B$. This new method of interpreting stress studies is significantly more complicated than current methods, but also generates a new parameter, the heterogeneity ratio, HR.

We also attempted to parameterize the spatial smoothing, α, by comparing our numerical simulations to Hardebeck's [2006] plot of average focal mechanism difference as a function of distance. We estimated an $\alpha \approx 0.8$ for non-aftershock regions. This parameter is more difficult to constrain than HR; clearly, more work can be done to refine this estimate. The exact value of α does not affect the main conclusion of this thesis, that stress heterogeneity biases stress inversion results toward $\dot{\boldsymbol{\sigma}}'_T$, but α is very important for determining the strength of the crust as a function of lengthscale.

Caveats and Future Work

As mentioned in Chapter 1, the Introduction, in our attempt to create a simple, statistical model of spatial stress heterogeneity in the Earth's crust, assumptions have been made that could have affected our results. For example, we do not update the stress field after each event; therefore, our results are best compared to stress inversions of background seismicity in between large earthquakes. If a large earthquake occurs and we wish to model its effect on the surrounding crust, namely how it produces aftershocks, we would have to modify our initial equation in Chapter 1, to take into account any 3D stress pertubations. The term we would add is:

$\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E)$ is the stress perturbation from major events that occur at time T_E (e.g., Landers earthquake). While we assume that these large events make extremely complex variations in stress in the immediate vicinity of the rupture, the stress variations can be approximately modeled with simple source models at larger distances from the rupture.

This produces our new stress equation:

$$\boldsymbol{\sigma}'(\mathbf{x},t)=\boldsymbol{\sigma}'_B+\dot{\boldsymbol{\sigma}}'_T t+\boldsymbol{\sigma}'_H(\mathbf{x})+\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E). \tag{0.1}$$

A future direction of research would be to use equation (0.1) to study aftershocks from a moderate to large earthquake and simulate the apparent stress rotations. The first step would be to model the pre-event $\boldsymbol{\sigma}'_B$ and $\dot{\boldsymbol{\sigma}}'_T t$ along with a spatially heterogeneous stress, $\boldsymbol{\sigma}'_H(\mathbf{x})$, with appropriate spectral properties, to produce the synthetic pre-event background seismicity. The next step would be to add a source model of Landers, Northridge, Loma Prieta, or another earthquake, calculate the static stress change within the surround medium, $\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E)$, ask which points exceed the failure threshold as a result of $\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E)$, and count these points as aftershocks. The last step would be to reapply the stress rate, $\dot{\boldsymbol{\sigma}}'_T t$, on this updated system to produce synthetic focal mechanisms that would represent the seismicity after the aftershock sequence has died off.

The point of this modeling would be to see if the pre-event seismicity, aftershock sequence, and post-aftershock seismicity have similar or different stress inversion orientations. We predict that the 3D static stress perturbation, $\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E)$, will cause a rotation of the average failure mechanism, directly after the mainshock; therefore, stress inversions of aftershock sequences will produce a tensor rotated relative to any inversions of premainshock seismicity. Indeed, such rotations have been seen for Landers [*Hardebeck and Hauksson*, 2001], Northridge [*Zhao, et al.*, 1997], and other earthquakes. We also predict that after the aftershock sequence has died off, if the orientation of $\dot{\boldsymbol{\sigma}}'_T t$ due to plate tectonics has remained constant, then the average focal mechanism orientation and stress inversion results will rotate back to the premainshock orientation. Our predictions are based on what we learned from Chapter 4, that whatever is perturbing the system in time, be it the stress buildup due to far-field plate loading, $\dot{\boldsymbol{\sigma}}'_T t$, or the transient processes initiated by the mainshock, $\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E)$, that affect the aftershock sequence, are what primarily determine the orientations of earthquake failures if stress is spatially heterogeneous in the crust. Therefore, in our paradigm, prior to the mainshock, $\dot{\boldsymbol{\sigma}}'_T t$, is the most important perturbation to the system, during the aftershock sequence processes related to $\boldsymbol{\sigma}_E(\mathbf{x})H(t-T_E)$, is the most important perturbation to the system (why the average failure orientation would rotate), and after the aftershock sequence has ceased, $\dot{\boldsymbol{\sigma}}'_T t$, is the most important perturbation to the system (why the average failure orientation would rotate back to the premainshock orientation).

This is a significantly different interpretation of "apparent" stress rotations. Currently, if there is a significant rotation of stress inversion results after a mainshock, it is assumed that the mainshock produced a nearly complete stress drop and that the magnitude of the background stress, $\boldsymbol{\sigma}'_B$, is approximately equal zero. In our interpretation, $\boldsymbol{\sigma}'_B$, no longer has to approximately equal zero, it can have a significant, non-zero magnitude. Instead, it is the interaction between the spatially heterogeneous stress and the perturbations to the system that "appears" to rotate the stress tensor, when in fact it is just changing the bias as a function of time; i.e., which spatially heterogeneous stress tensors have preference for failure changes depending upon the current perturbation to the system.

So far, we have tested this hypothesis with initial work on the reported stress rotation after the Northridge earthquake [*Zhao, et al.*, 1997] with our heterogeneous stress models. Immediately after the Northridge earthquake, Zhao and Kanamori reported an approximately 17° rotation of the P axis and found that within the months following the earthquake, it rotated back to the pre-Northridge orientation (Figure 6.1). Our initial models appear to replicate this reported rotation in average focal mechanism orientations. In our heterogeneous stress models, the average focal mechanism orientations are biased toward whatever is perturbing the system in time; therefore, any rotations in our system for large HR are a function of the perturbation, not the background stress + perturbation. We confirmed that our numerical models with heterogeneous stress are capable of generating significant rotations in average focal mechanism orientations even with non-zero background stress, $\boldsymbol{\sigma}'_B$. They also appear capable of generating the rotation back to the premainshock orientation. In essence, our heterogeneous stress models produce focal mechanism orientations biased from any aftershock time-dependent processes immediately after the mainshock. Then as aftershock processes die off and the stress rate from long-term tectonic processes become more important, the average focal mechanism orientations are predicted to rotate back. The time scale of this process is predicted to depend on the amount of slip in the mainshock compared to the long-term strain rate.

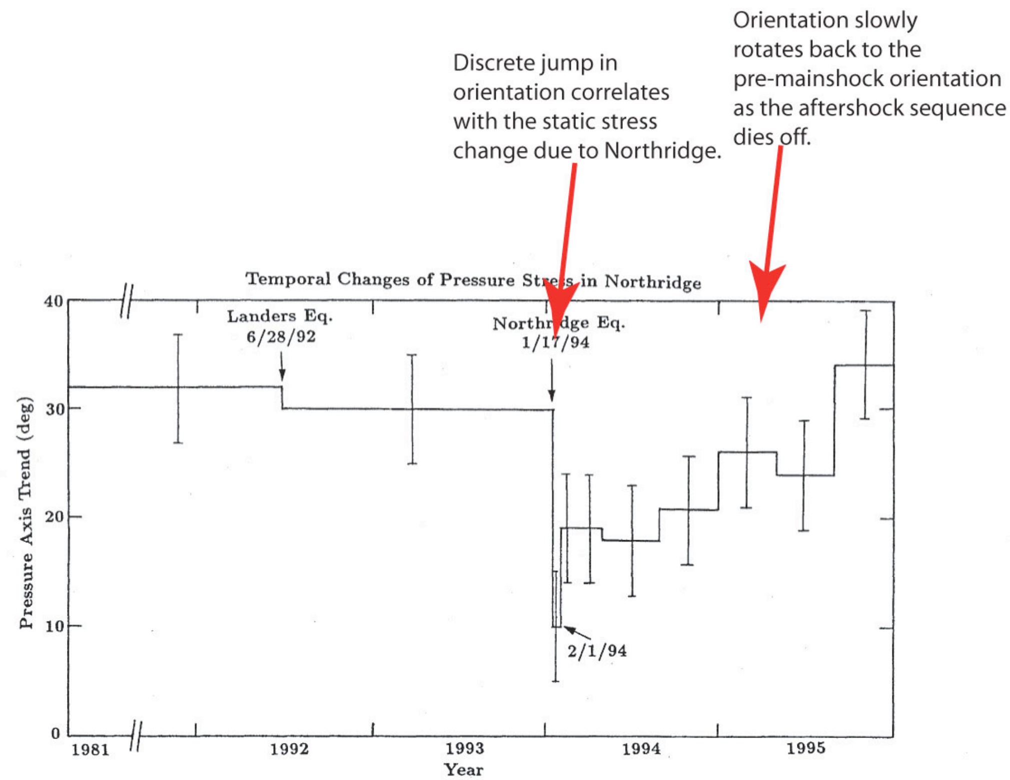

Figure 3. Azimuth of the principal pressure axis versus time in the Northridge area. The vertical bars denote the uncertainty of the azimuth estimates.

Figure 6.1. *Figure modified from Zhao et al. [1997] shows the rotation of the pressure axis as a function of time. There is a discrete jump in orientation of about 17° at the time of the Northridge earthquake then a slow rotation back over the course of two years.*

An additional route for new research would be to combine our spatially heterogeneous stress aftershock model with rate and state friction to study aftershock patterns and decays. In essence, instead of letting all the points that exceed the failure threshold after mainshock fail simultaneously, we would apply the rate and state friction law. Those heterogeneous points that exceed the failure threshold the most would fail first, and those that exceed it by a small amount would fail last. It would be a natural, physical way to produce the time delay for some points and explain why some fail quickly while others take much longer. We would compare our statistics to those of real aftershock sequences. In turn, this comparison with real data could provide additional constraints for our two statistical parameters, HR and α.

Another comparison/test of our spatially heterogeneous stress aftershock model would be to see if we can reproduce spatial/depth variations in the aftershock orientations. Kerkela and Stock [1996; personal communication, 2006] in their borehole breakout studies of the San Fernando Valley found variability in the orientation of maximum compressive stress as a function of depth that may be compatible with our aftershock models; however, it is yet to be tested.

One other limitation of our method that could lead into future research and refinements is that we do not allow failure on non-optimal slip planes. We do not allow spatial variability in the static or dynamic coefficient of frictions, μ. It is possible that some of the heterogeneity seen in the data is due to non-optimally orientated fault planes and variable strength faults, which would lower our estimate of stress heterogeneity. Given the borehole breakout data presented in Chapter 1 that strongly indicate heterogeneity of stress orientations, we are fairly sure there is some short wavelength spatially heterogeneous stress in tectonically active regions, but of course the question is how much. A future area for research would be to try to simply model this without using dynamic simulations. To derive statistics of fault orientations, sizes, sources on the faults (i.e., heterogeneous slip) and evolve it through time, allowing for fresh fractures as well as failure on pre-existing planes. It would involve many more assumptions that could complicate the problem and possibly add in hidden biases due to the statistics of fault/source generation, but if it could be done, it would provide a good comparison to our current method. The truest way to model the pre-existing failure and generate the heterogeneous stress field would be to employ dynamics fault failures for all faults throughout all time, but this is far beyond our current numerical capacity.

There are many other possible directions for future work, such as developing new ways of generating our 3D spatially heterogeneous stress field, using Weibull statistics or other distributions; adding in finite fault ruptures; or updating the stress field after each event. This thesis is meant to open the door for studying the effect of 3D stress heterogeneity on focal mechanisms and seismicity patterns in the real Earth and show that heterogeneity must be taken into account when interpreting stress inversions for the crust.

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