Appendix A. Equations for Translating Between Stress Matrices, Fault Parameters, and P-T Axes

Coordinate Systems and Rotations

We use the same right-handed coordinate system as Andy Michael's program, *slick [Michael, 1984; 1987]*, which is East, North, and Up.

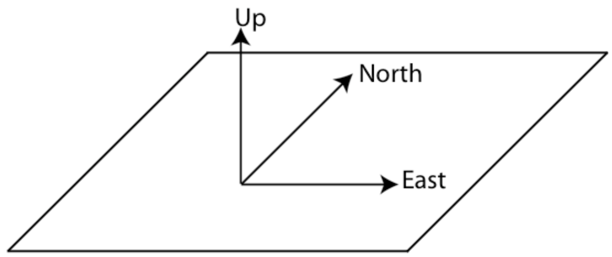


Figure A.1. *Right-handed coordinate system used in generating code that is compatible with Andy Michael's stress inversion programs. All vectors generated will have a format* $\mathbf{v} = [\hat{E}, \hat{N}, \hat{U}p]$.

Therefore, our stress matrices will have the following Cauchy stress tensor format:

$$\sigma_{ij} = \begin{matrix} \sigma_{EE} & \sigma_{EN} & \sigma_{EU} \\ \sigma_{NE} & \sigma_{NN} & \sigma_{NU} \\ \sigma_{UE} & \sigma_{UN} & \sigma_{UU} \end{matrix} . \tag{A.1}$$

For any component of σ_{ij}, j indicates the direction of the force applied, and i describes the normal of the plane on which the force is acting. Following physics sign convention for σ_{ij}, where tension is positive and pressure is negative, if the force vector is acting in the positive direction and the normal is also in the positive direction, then the component $\sigma_{ij} > 0$. Conversely, if the force vector is acting in the negative direction and the normal is in the positive direction, $\sigma_{ij} < 0$. For example, $\sigma_{EN} > 0$ describes one of two scenarios: 1) A force acting in the $\hat{N}$ direction on a plane with a normal in the $\hat{E}$ direction. 2) A force acting in the $-\hat{N}$ direction on a plane with a normal in the $-\hat{E}$ direction. The diagonal components of σ_{ij} describe the normal tractions (forces normal to the plane on which they are acting), and the off-diagonal components describe the shear tractions (forces tangent to the plane on which they are acting). Figure A.2 graphically shows all the components of the stress tensor. Figures A.3 and A.4 show 2D examples in more detail.

As noted in Figure A.2, our Cauchy stress matrix must be symmetric resulting in:

$$\sigma_{ij} = \begin{matrix} \sigma_{EE} & \sigma_{EN} & \sigma_{EU} \\ \sigma_{EN} & \sigma_{NN} & \sigma_{NU} \\ \sigma_{EU} & \sigma_{NU} & \sigma_{UU} \end{matrix} . \tag{A.2}$$

Figure A.3 shows in detail our convention for normal stresses and Figure A.4 shows in detail our convention for shear stresses.

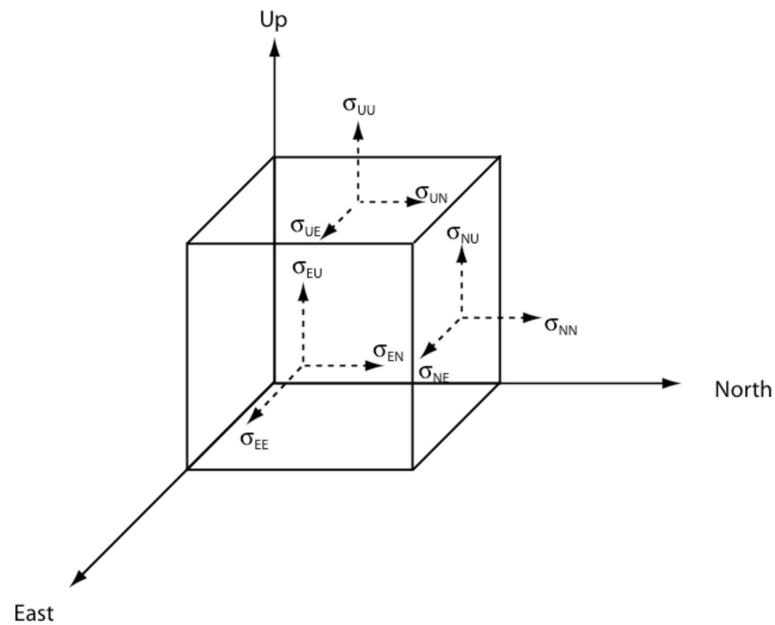


Figure A.2. *The stress vectors are shown for three of the six exterior faces on a box. Note that in our convention, the diagonal elements of our stress tensor (* σ_{EE} *,* σ_{NN} *, and* σ_{UU} *) for tension are* >0 *and for compression are* <0*. In this figure all the elements of* σ_{ij} *are positive. For example,* σ_{EE}*,* σ_{NN}*,* σ_{UU} *are all pointing in the same direction as their respective normal vectors, resulting in tension. The off-diagonal elements,* σ_{EN}*,* σ_{NU}*,* σ_{UE}*, have either a traction in the positive direction and a positive normal or have a traction in the negative direction and a negative normal for the given coordinate system. Since we are interested in systems where there is no net rotation, the matrix must be symmetric, i.e.,* $\sigma_{NE} = \sigma_{EN}$*,* $\sigma_{UE} = \sigma_{EU}$*, and* $\sigma_{UN} = \sigma_{NU}$ *[illustration adapted from Housner and Vreeland, 1965].*

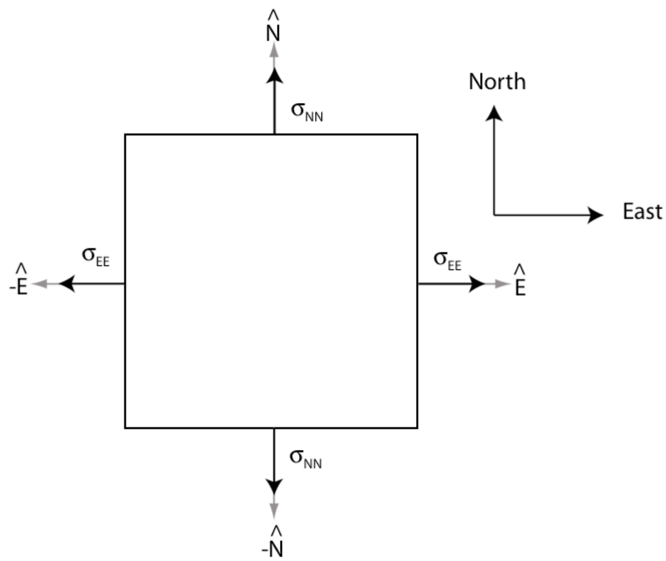


Figure A.3. *In this 2D example, σ_{EE} and σ_{NN} are both positive, i.e., the traction vectors are always pointing in the same direction as the normal vectors, resulting in $E-W$ tension and $N-S$ tension.*

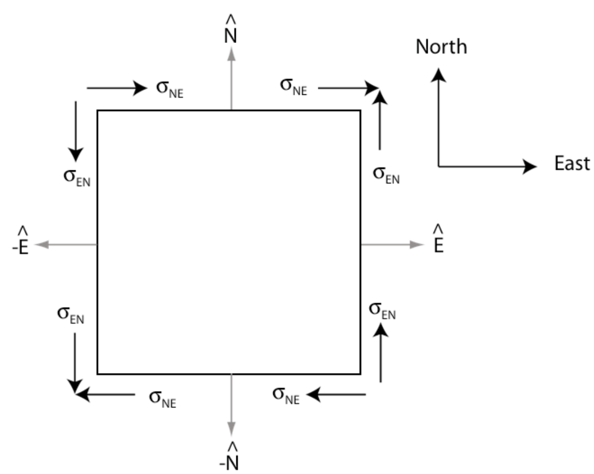


Figure A.4. *Again, σ_{EN} and σ_{NE} are positive using our convention that a traction aligned in a positive direction on a plane with a positive normal, or a traction aligned in a negative direction on a plane with a negative normal, produce a positive component in our stress tensor. Note, due to rotational symmetry, $\sigma_{EN} = \sigma_{NE}$.*

In Figure A.4, if σ_{EN} is the only stress being applied, then we could write our Cauchy stress tensor as follows,

$$\sigma_{ij}^{(\mathbf{a})} = A \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.3}$$

where A is the scalar amplitude of σ_{EN}.

If we wish to rotate the stress tensor, $\sigma_{ij}^{(\mathbf{a})}$, then we can apply any combination of the following rotation matrices,

$$\mathbf{R}(\psi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{pmatrix} \quad \text{for a counter-clockwise rotation } \psi \text{ of the stress tensor about the } \hat{E} \text{ axis} \tag{A.4}$$

$$\mathbf{R}(\delta) = \begin{pmatrix} \cos\delta & 0 & \sin\delta \\ 0 & 1 & 0 \\ -\sin\delta & 0 & \cos\delta \end{pmatrix} \quad \text{for a counter-clockwise rotation } \delta \text{ of the stress tensor about the } \hat{N} \text{ axis} \tag{A.5}$$

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{for a counter-clockwise rotation } \theta \text{ of the stress tensor about the } \hat{Up} \text{ axis.} \tag{A.6}$$

In Figure A.5, we apply the third rotation matrix, $\mathbf{R}(\theta)$, to rotate $\sigma_{ij}^{(\mathbf{a})}$ about the positive $\hat{\mathrm{U}}\mathrm{p}$ axis by $\theta = 45°$ to produce a new Cauchy stress tensor, $\sigma_{kl}^{(\mathbf{b})}$. We can write out the rotation as the following set of steps:

$$
\begin{aligned}
\sigma_{kl}^{(\mathbf{b})} &= R_{ki}\sigma_{ij}^{(\mathbf{a})}R_{lj} = \mathbf{R}(\theta)\boldsymbol{\sigma}^{(\mathbf{a})}\left(\mathbf{R}(\theta)\right)^{T} \\
&= A\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= A\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\sin\theta & \cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= A\begin{pmatrix} -\cos\theta\sin\theta - \sin\theta\cos\theta & \cos\theta\cos\theta - \sin\theta\sin\theta & 0 \\ -\sin\theta\sin\theta + \cos\theta\cos\theta & \sin\theta\cos\theta + \cos\theta\sin\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= A\begin{pmatrix} -\sin 2\theta & \cos 2\theta & 0 \\ \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 0 \end{pmatrix},
\end{aligned}
\tag{A.7}
$$

and for $\theta = 45°$, we find that our rotated stress is simply

$$
\sigma_{kl}^{(\mathbf{b})} = A\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{A.8}
$$

If we rotate the coordinate systems instead of the stress tensors themselves, the rotation matrices are the transpose of those used for rotating the stress tensors.

$$
\mathbf{R}^{T}(\psi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{pmatrix} \quad \text{for a counter-clockwise rotation } \psi \text{ of the coordinates about the } \hat{E} \text{ axis} \tag{A.9}
$$

$$
\mathbf{R}^{T}(\delta) = \begin{pmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{pmatrix} \quad \text{for a counter-clockwise rotation } \delta \text{ of the coordinates about the } \hat{N} \text{ axis} \tag{A.10}
$$

$$
\mathbf{R}^{T}(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{for a counter-clockwise rotation } \theta \text{ of the coordinates about the } \hat{Up} \text{ axis.} \tag{A.11}
$$

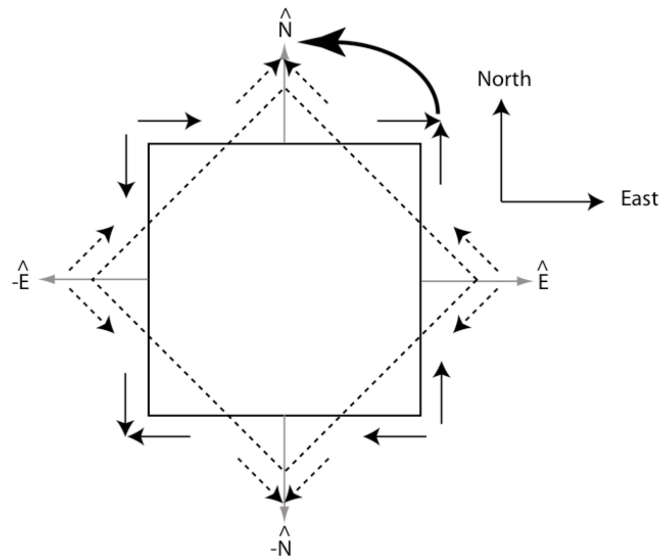


Figure A.5. *The dotted lines indicate the unrotated stress tensor and the solid lines are for the rotated stress tensor. Rotation of the stress field counter-clockwise by 45°. Note that it results in $E-W$ compression and $N-S$ tension. This agrees with our tensor, $\sigma_{kl}^{(\mathbf{b})}$, where $\sigma_{EE}^{(\mathbf{b})}<0$, i.e., compression in the $E-W$ direction, and $\sigma_{NN}^{(\mathbf{b})}>0$, i.e., tension in the $N-S$ direction.*

Therefore, if we start with

$$\sigma_{ij}^{(\mathbf{a})} = A\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.12}$$

and rotate the coordinate system by 45° about the $\hat{U}p$ axis, our stress tensor in the new coordinate system is:

$$\begin{aligned}
\sigma_{kl}^{(\mathbf{c})} &= R_{ik}\sigma_{ij}^{(\mathbf{a})}R_{jl} = \left(\mathbf{R}(\theta)\right)^T \boldsymbol{\sigma}^{(\mathbf{a})}\mathbf{R}(\theta) \\
&= A\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= A\begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \sin\theta & \cos\theta & 0 \\ \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= A\begin{pmatrix} \cos\theta\sin\theta + \sin\theta\cos\theta & \cos\theta\cos\theta - \sin\theta\sin\theta & 0 \\ -\sin\theta\sin\theta + \cos\theta\cos\theta & -\sin\theta\cos\theta - \cos\theta\sin\theta & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= A\begin{pmatrix} \sin 2\theta & \cos 2\theta & 0 \\ \cos 2\theta & -\sin 2\theta & 0 \\ 0 & 0 & 0 \end{pmatrix},
\end{aligned} \tag{A.13}$$

and for $\theta = 45°$, we find that our stress tensor in the rotated coordinate system is

$$\sigma_{kl}^{(\mathbf{c})} = A\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{A.14}$$

Figure A.6 graphically shows this rotation of the coordinate system.

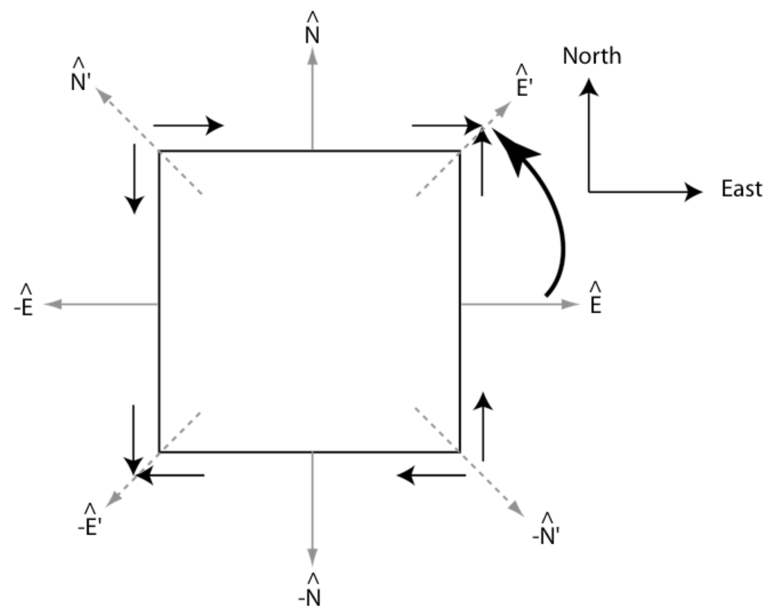


Figure A.6. *Rotation of the coordinate system counter-clockwise by 45°. The solid normal vectors represent the unrotated coordinates and the dashed normal vectors represent the rotated coordinates. Note that the rotation results in compression in the primed, or new $N-S$ direction, and tension in the new $E-W$ direction.*

Any symmetric stress matrix, $\boldsymbol{\sigma}$, can be represented in terms of a diagonal matrix $\boldsymbol{\sigma}'$, which contains the eigenvalues of $\boldsymbol{\sigma}$, and a rotation matrix $\mathbf{V}$, which contains eigenvectors of $\boldsymbol{\sigma}$, where $\boldsymbol{\sigma} = \mathbf{V}\boldsymbol{\sigma}'\mathbf{V}^T$. Once we know $\mathbf{V}$, which rotates our coordinate system to the primed coordinate system, we can rotate our stress tensor $\boldsymbol{\sigma}$ or anything else into this coordinate system. For example, $\boldsymbol{\sigma}' = \mathbf{V}^T\boldsymbol{\sigma}\mathbf{V}$. This is called the principal coordinate system, and the diagonal values of $\boldsymbol{\sigma}'$ are the principal stresses, where

$$\boldsymbol{\sigma}' = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}$$

and the principal stresses, σ_1, σ_2, and σ_3 are ordered from most to least compressive, i.e., from smallest (most negative) to largest (most positive) given that in our convention compression yields negative values and tension yields positive values. $\mathbf{V} = V_{ij}$ is the eigenvector matrix associated with the eigenvalues where V_{i1} is the eigenvector for σ_1, V_{i2} is the eigenvector for σ_2, and V_{i3} is the eigenvector for σ_3. σ_1 is the maximum compressive stress, σ_3 is the minimum compressive stress, and σ_2 is the intermediate compressive stress. Therefore, for a deviatoric matrix where the trace has been subtracted, one finds that $\sigma_1 < 0$ (compression), $\sigma_3 > 0$ (tension), and $\sigma_2 = -(\sigma_1 + \sigma_3)$. σ_1, σ_2, and σ_3, are the eigenvalues of $\boldsymbol{\sigma}$, and $\vec{x}_1 = \mathbf{V}\hat{E} = V_{i1}$, $\vec{x}_2 = \mathbf{V}\hat{N} = V_{i2}$, and $\vec{x}_3 = \mathbf{V}\hat{U}p = V_{i3}$ are the eigenvectors in the new principal coordinate system.

Translating a Stress Matrix into Strike, Dip, and Rake Earthquake Fault Parameters

Ultimately, we wish to ask, given a particular stress state described by our stress tensors, what is the failure orientation? What synthetic fault parameters (strike, dip, and rake) are produced when the material fails? These questions can only be answered once a fracture criterion, that determines the timing, locations, and possibly the orientations of the failures, has been chosen. The two fracture criteria we applied in this project were the Hencky-Mises plastic yield condition [*Housner and Vreeland*, 1965] and the Coulomb-Mohr criterion. Appendix B compares these two fracture criteria in detail; however, for this section we need only compare how they affect the orientation of failure relative to the principal eigenvectors of the stress tensor.

In the Hencky-Mises plastic yield condition, failure always occurs on a plane at $45°$ between the σ_1 and σ_3 axes. There are two possible failure planes, and they are perpendicular to one another. In this particular case, the two possible failure planes match the two possible planes on a focal sphere. The optimally oriented planes of our plastic yield criterion are also equivalent to the failure planes in the Coulomb-Mohr criterion with coefficient of friction $\mu = 0$. If the Coulomb-Mohr failure criterion is used instead, then the failure planes will occur at $\pm\theta$ relative to the σ_1 axis, where $\theta \leq 45°$. The formula for determining θ depends on the coefficient of friction, μ, where, $\theta = \frac{\pi}{4} - \frac{\tan^{-1}(\mu)}{2}$. If $\mu > 0$, then $\theta < 45°$ and the two failure planes are no longer perpendicular to one another; thus the two possible failure planes will be associated with different focal mechanisms. Therefore, the focal mechanism will depend on which failure plane one chooses.

Once one knows θ relative to the σ_1 axis based on the fracture criterion, then it is fairly simple to calculate all four possible slip and normal vectors by rotating the σ_1 and σ_3 axes (Figures A.7 and A.8). The slip and normal vectors can then be converted to strikes, dips, and rakes. Two possible triplets of strikes, dips, and rakes are associated with each failure plane depending on which side of the failure plane one considers fixed. Typically, we choose the strike, dip, and rake with dip ≤ 90. This results in one triplet of strike, dip, and rake for each failure plane; given that we have two failure planes, we now have two triplets of strike, dip, and rake to randomly choose between when we create our synthetic focal mechanism catalog. However, we find that it is helpful to have all four sets of slip and normal vectors when attempting to determine the minimum angle between two different focal mechanisms. It might be possible to reduce the problem to one set of slip and normal vectors per failure plane if we specify that the slip and normal vectors bound a compressional quadrant. However, for the numerical calculations in this thesis, we use all four possible sets of slip and normal vectors per failure plane when calculating the minimum angular difference between pairs of focal mechanisms.

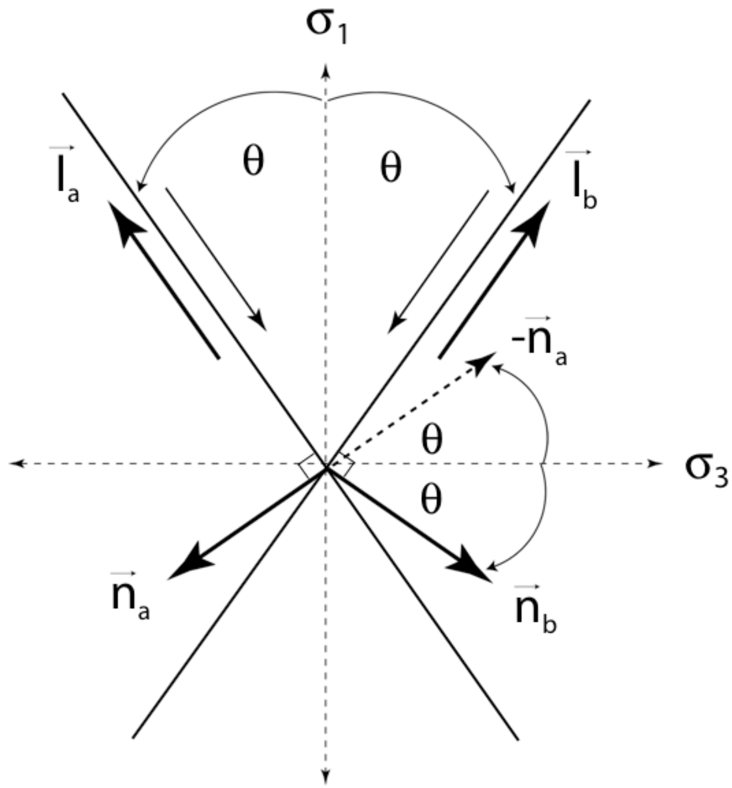


Figure A.7. *Slip and normal vectors for the two possible failure planes can be generated by rotating the* σ_1 *and* σ_3 *eigenvectors about the* σ_2 *axis. For example, our first slip vector,* $\vec{l}_a$*, is the* σ_1 *eigenvector rotated counter-clockwise through an angle* θ*.* $\vec{l}_b$*, the slip vector for the alternate failure plane, is the* σ_1 *eigenvector rotated clockwise through an angle* θ*.* $-\vec{n}_a$*, the negative of the normal vector associated with* $\vec{l}_a$*, is the* σ_3 *eigenvector rotated counter-clockwise through an angle* θ*. Last,* $\vec{n}_b$*, the normal associated with the slip vector* $\vec{l}_b$*, is the* σ_3 *eigenvector rotated clockwise through an angle* θ*.*

Figures A.7 and A.8 graphically show the rotation of eigenvectors in the principal coordinate system to produce our slip and normal vectors that will be converted into strikes, dips, and rakes. The procedure to determine the four possible slip and normal vectors from an arbitrary symmetric stress tensor, $\boldsymbol{\sigma}$, is as follows:

1) Calculate the eigenvalues and eigenvectors of $\boldsymbol{\sigma}$, where $\boldsymbol{\sigma} = \mathbf{V}\boldsymbol{\sigma}'\mathbf{V}^T$ and $\mathbf{V}$ is the eigenvector matrix.
2) Rotate the coordinate system of the σ_1 and σ_3 eigenvectors into the principal coordinate system, using the transpose of the eigenvector matrix, $\mathbf{V}^T$.
3) In the principal coordinate system, rotate the σ_1 and σ_3 eigenvectors about the σ_2 axis as shown in Figures A.7 and A.8 to produce the slip and normal vectors for the failure planes.
4) Rotate the coordinate system of the slip and normal vectors back into the unprimed E, W, and Up coordinate system using the eigenvector matrix, $\mathbf{V}$.

So the equations might look like $\vec{l}_a = \mathbf{VR}(\theta)\mathbf{V}^T\vec{x}_1$, $\vec{l}_b = \mathbf{VR}^T(\theta)\mathbf{V}^T\vec{x}_1$,

$\vec{n}_a = -\mathbf{VR}(\theta)\mathbf{V}^T\vec{x}_3$, $\vec{n}_b = \mathbf{VR}^T(\theta)\mathbf{V}^T\vec{x}_3$, and $\vec{l}_c = -\vec{l}_a$, $\vec{l}_d = -\vec{l}_b$, $\vec{n}_c = -\vec{n}_a$, $\vec{n}_d = -\vec{n}_b$,

where

$$\mathbf{R}(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \tag{A.15}$$

and $\vec{x}_1$, $\vec{x}_2$, $\vec{x}_3$ are the eigenvectors for the σ_1, σ_2, and σ_3 axes.

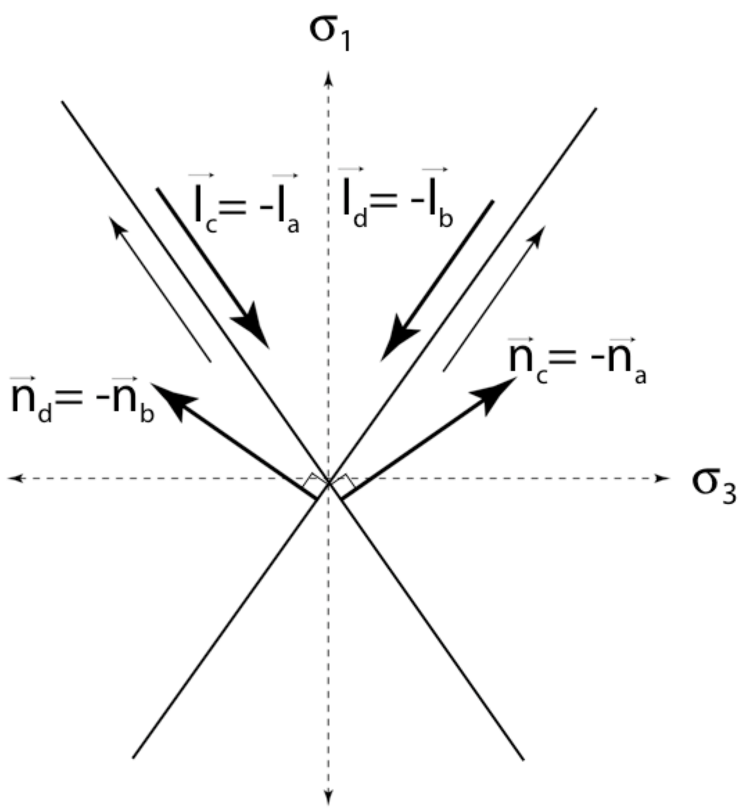


Figure A.8. *The last two sets of slip and normal vectors are simply the slip and normal vectors on the other side of the fault planes. In general, the slip vector on side two equals the negative of the slip vector on side one. The normal vector on side two equals the negative of the normal vector on side one. In a sense, these are the alternate slip and normal vectors for the two possible failure planes.*

Once we have a slip vector, normal vector pair, we can begin determining the strike, dip, and rake. Figures A.9 and A.10 illustrate how the strike and dip of a plane can be calculated from a given normal vector. The equation for the strike of the plane is

$$\theta = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = -\tan^{-1}\left(\frac{n_N}{n_E}\right), \tag{A.16}$$

and the equation for the dip of the plane is,

$$\delta = \tan^{-1}\left(\frac{\sin\delta}{\cos\delta}\right) = \tan^{-1}\left(\frac{\sqrt{n_E^2 + n_N^2}}{n_U}\right). \tag{A.17}$$

Note that if the normal vector points down, one must first switch the sign of the normal and slick vectors, $\vec{n} = -\vec{n}$ and $\vec{l} = -\vec{l}$, if $n_U < 0$ before calculating the strike and the dip of the plane.

Last, Figure A.11 graphically shows how one calculates the rake of the rupture given the strike angle and slip vector. We calculate what is the strike vector, $\vec{h}$, then find the angle between $\vec{h}$ and $\vec{l}$, which is rake, λ.

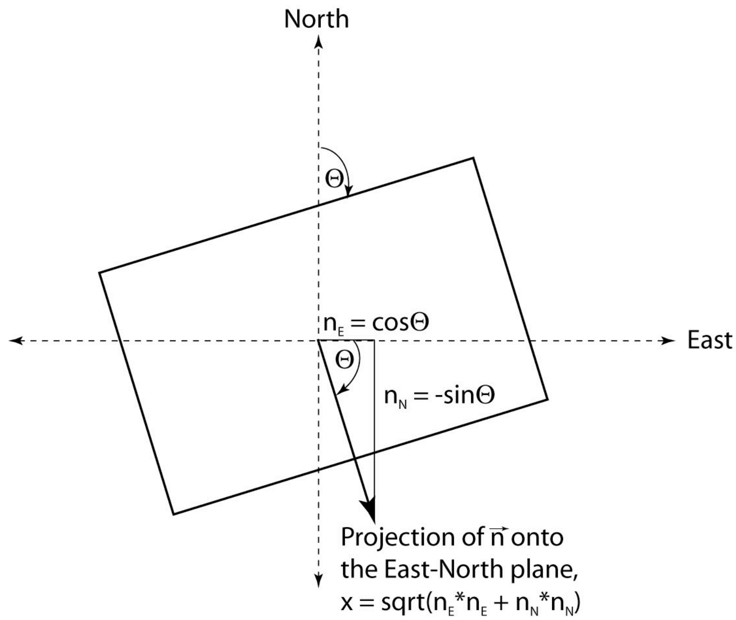


Figure A.9. *How to calculate the strike of a plane given the normal vector*.

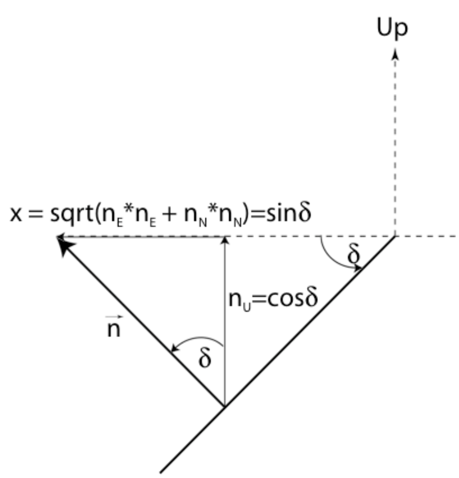
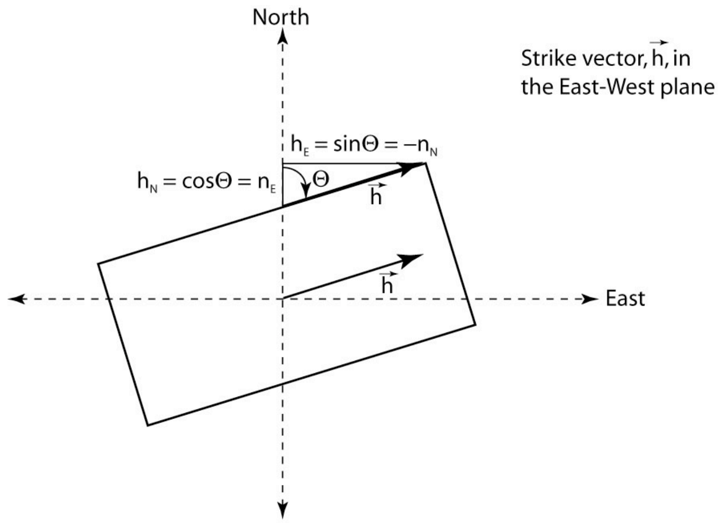


Figure A.10. *How to calculate the dip of a plane given the normal vector*.



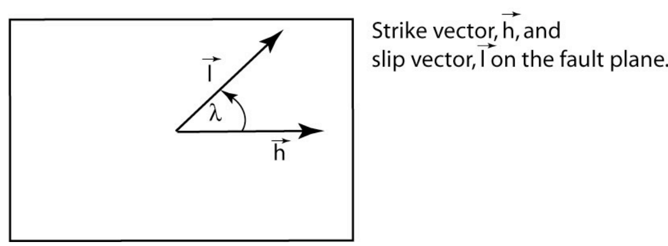


Figure A.11. *How to calculate the rake of a rupture. First, determine the strike vector,* $\vec{h}$ *, which by definition always has zero for the* $\hat{U}p$ *component. Then use the definition of a dot product of two vectors to derive the rake angle,* λ *.*

The formula for the strike vector will be

$$\vec{h} = \begin{pmatrix} \sin\Theta \\ \cos\Theta \\ 0 \end{pmatrix} = \begin{pmatrix} -n_N \\ n_E \\ 0 \end{pmatrix}. \tag{A.18}$$

Using the definition of the dot product, we can then determine the angle λ ,

$$\vec{h} \cdot \vec{l} = \| \vec{h} \| \ \| \vec{l} \| \cos\lambda$$
$$\lambda = \cos^{-1}\left(\frac{\vec{h} \cdot \vec{l}}{\| \vec{h} \| \ \| \vec{l} \|} \right). \qquad \text{(A.19)}$$

These formulas work for either set of conjugate planes.

Now if one starts with the strike, dip, and rake of a failure, one knows the coefficient of friction, and one wishes to determine the stress tensor, the procedure is the inverse of what has just been done. One calculates the slip and normal vectors of the plane, then rotates these by $\pm\theta$ in the principal coordinate frame to produce the eigenvectors. We will not go through the derivation, but one can look to Jarosch and Aboodi [1970] for how to calculate the slip and normal vectors from strike, dip, and rake. For our particular coordinate system, the equations are,

$$\vec{l} = \begin{pmatrix} l_E \\ l_N \\ l_U \end{pmatrix} = \begin{pmatrix} \sin(\Theta)\cos(\lambda) - \cos(\Theta)\cos(\delta)\sin(\lambda) \\ \cos(\Theta)\cos(\lambda) + \sin(\Theta)\cos(\delta)\sin(\lambda) \\ \sin(\delta)\sin(\lambda) \end{pmatrix}$$
$$\vec{n} = \begin{pmatrix} n_E \\ n_N \\ n_U \end{pmatrix} = \begin{pmatrix} \cos(\Theta)\sin(\delta) \\ -\sin(\Theta)\sin(\delta) \\ \cos(\delta) \end{pmatrix}. \qquad \text{(A.20)}$$

After we have rotated, $\vec{l}$ and $\vec{n}$ into our eigenvectors, $\hat{x}_1$ and $\hat{x}_3$, we can reconstruct the stress tensor exactly if we also know the eigenvalues. If not, then there is an ambiguity as to the magnitude the stress tensor. For example, if we have the stress tensor $\boldsymbol{\sigma}$ with its associated eigenvector matrix $\mathbf{V}$ and eigenvalue matrix $\boldsymbol{\sigma}'$, we can reconstruct $\boldsymbol{\sigma}$ exactly.

$$\boldsymbol{\sigma} = \mathbf{V}\boldsymbol{\sigma}'\mathbf{V}^T \qquad \text{(A.21)}$$

See the following example.

$$\boldsymbol{\sigma} = 0.618\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0.5257 & -0.8507 \\ 0 & -0.8507 & -0.5257 \\ 1.000 & 0 & 0 \end{pmatrix} \tag{A.22}$$

$$\boldsymbol{\sigma}' = \begin{pmatrix} -0.618 & 0 & 0 \\ 0 & -0.382 & 0 \\ 0 & 0 & 1.000 \end{pmatrix}$$

In this case, indeed, $\boldsymbol{\sigma} = \mathbf{V}\boldsymbol{\sigma}'\mathbf{V}^T$. However, what if one does not know the eigenvalues (principal stresses) and one has to guess their values? For example, assume one might choose $\boldsymbol{\sigma}'_{Guess}$ to be

$$\boldsymbol{\sigma}'_{Guess} = \begin{pmatrix} -1.0000 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.0000 \end{pmatrix}. \tag{A.23}$$

In this case our best guess for the stress matrix is, $\boldsymbol{\sigma}_{Guess} = \mathbf{V}\boldsymbol{\sigma}'_{Guess}\mathbf{V}^T$, where

$$\boldsymbol{\sigma}_{Guess} = 0.618\begin{pmatrix} 1.1708 & 0.7236 & 0 \\ 0.7236 & 0.4472 & 0 \\ 0 & 0 & -1.6180 \end{pmatrix}. \tag{A.24}$$

One can see that $\boldsymbol{\sigma} \neq \boldsymbol{\sigma}_{Guess}$. They may be close but not quite equal to one another. They do, however, produce the same strike, dip, and rake since the same eigenvector matrix, $\mathbf{V}$, is used for both $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_{Guess}$.

Translating a Stress Matrix into P and T axes

Viewing $\vec{P}$ (Pressure) and $\vec{T}$ (Tension) axes on an equal area plot is an excellent way to visualize earthquake focal mechanism orientations for a large number of earthquakes. The definition of the $\vec{P}$ and $\vec{T}$ vectors is

$$\begin{aligned} \vec{P} &= \frac{1}{\sqrt{2}}\left(\vec{n} - \vec{l}\right) \\ \vec{T} &= \frac{1}{\sqrt{2}}\left(\vec{n} + \vec{l}\right) \\ \vec{B} &= \vec{n} \times \vec{l} \end{aligned} \tag{A.25}$$

where $\vec{n}$ is the normal vector to a shear dislocation plane, $\vec{l}$ is the slip vector, and $\vec{B}$ is the vector normal to $\vec{n}$ and $\vec{l}$. $\vec{P}$ and $\vec{T}$ vectors are rotated $\pm 45°$ relative to $\vec{n}$ and $\vec{l}$. In the case of optimally oriented planes, i.e., $\mu = 0$, $\vec{P}$ corresponds to the σ_1 (most compressive) eigenvector of the stress matrix, $\vec{T}$ corresponds to the σ_3 (least compressive) eigenvector of the stress matrix, and $\vec{B}$ corresponds to the intermediate, σ_2 eigenvector of the stress matrix. Except for Appendix C, which explicitly discusses the Coulomb failure criterion, all our results assume optimally oriented planes with the plastic yield criterion; hence, this correspondence between the $\vec{P}$ and $\vec{T}$ vectors and our stress matrix eigenvectors for optimally oriented planes is especially useful.

For example, in Figure A.12 we have the following stress matrix being represented,

$$\sigma_{ij} = A\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{A.26}$$

where there is tension (blue) in the $E-W$ direction and compression (red) in the $N-S$ direction. Since $\mu=0$, the two possible failure planes are at $\theta=\pm 45°$ from the σ_1 axes. We could take any one of the possible four sets of normal vectors and slip vectors to reproduce the same $\vec{P}$ and $\vec{T}$ axes.

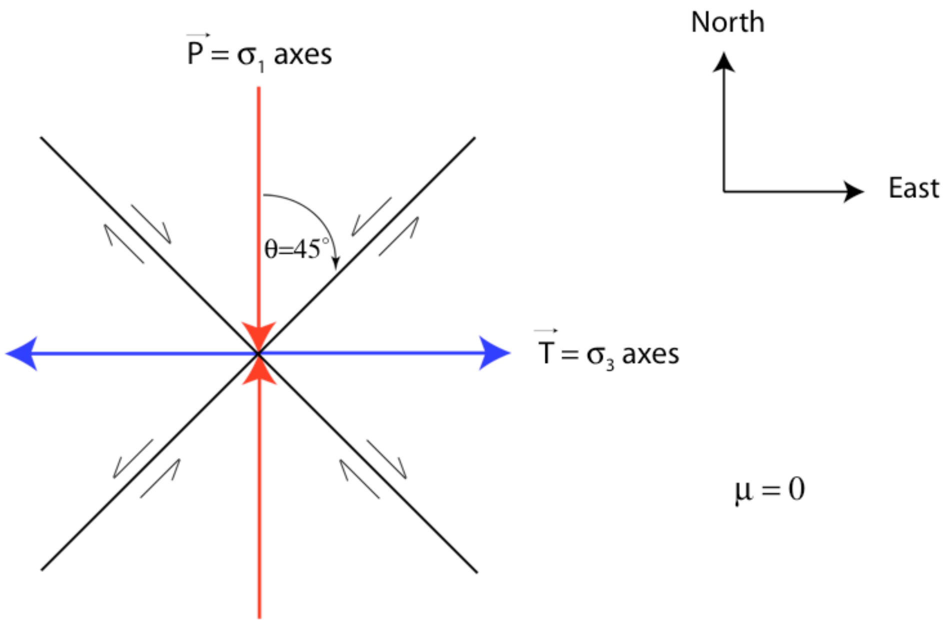


Figure A.12. *$N-S$ compression (red) and $E-W$ tension (blue). In this case of optimally oriented planes, $\mu=0$, the stress matrix eigenvectors align with the $\vec{P}$ and $\vec{T}$ axes. The two possible failure planes are $45°$ from the $\vec{P}$ and $\vec{T}$ axes.*

Figure A.14 is an equal area plot of $\vec{P}$ and $\vec{T}$ axes for 1,000 synthetic earthquakes where each red asterisk represents a $\vec{P}$ axes for a single event and each blue circle represents a $\vec{T}$ axes for a single event. The average $\vec{P}$ and $\vec{T}$ orientation is approximately the same as Figure A.12. The distance from center represents the dip or plunge, δ, of the $\vec{P}$ or $\vec{T}$ vectors, where a plunge of 90° corresponds to the center and a plunge of 0° would plot at the circumference of the circle. The azimuth, or angular distance from the top of the circle, represents the azimuth from N, θ, for the $\vec{P}$ or $\vec{T}$ vectors. Figure A.13, a cartoon of an equal area plot, visually shows these relations.

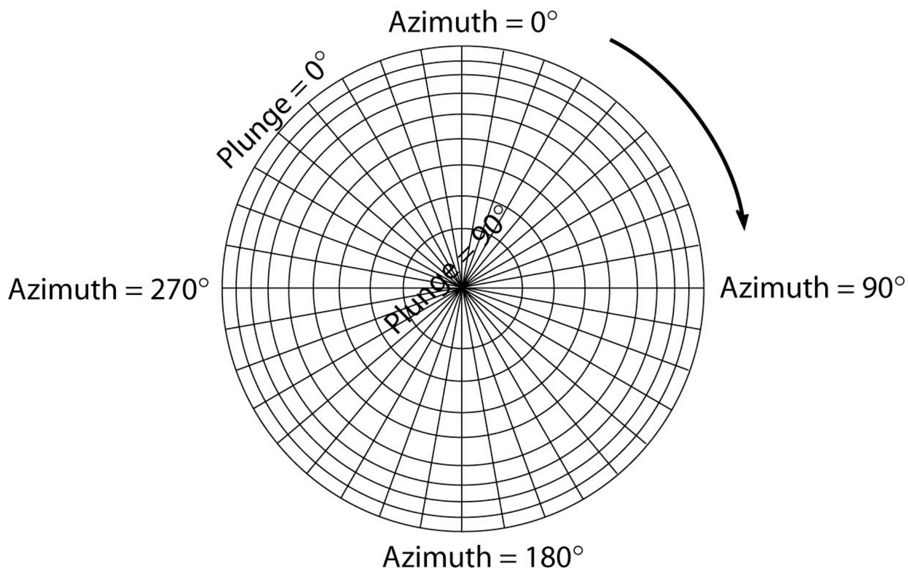


Figure A.13. *A cartoon of a typical equal area plot for P-T azimuths and plunges. The longitude,* θ*, is the azimuth of the circle, and plunge,* δ*, is plotted as a function of radial distance where,* $\delta = 90°$ *at the center and* $\delta = 0°$*, at the circumference. Note the radial lines are not necessarily to scale.*

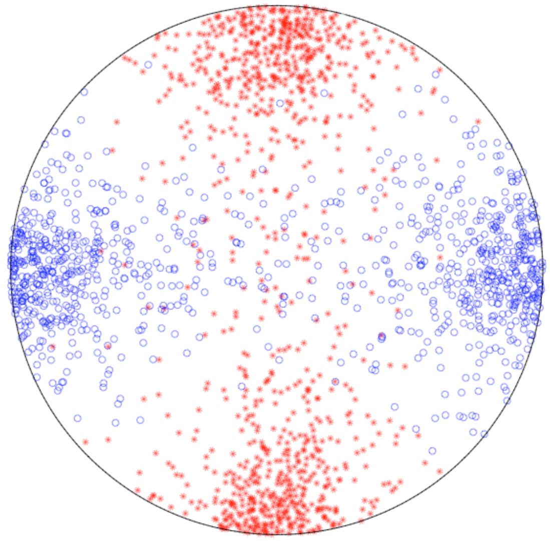


Figure A.14. *Equal area plot of* $\vec{P}$ *and* $\vec{T}$ *vectors for 1,000 synthetic earthquakes with an average compression axis in the* $N-S$ *direction and an average tension axis in the* $E-W$ *direction. The red asterisks are the* $\vec{P}$ *vectors and the blue circles are the* $\vec{T}$ *vectors. The azimuth,* θ*, of the vectors is represented by the angular distance from the top of the circle in the clockwise direction. The dip or plunge of the vectors,* δ*, is represented by the radial distance where the center of the circle is a* $\delta = 90°$*, and the circumference is a* $\delta = 0°$.

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