A STUDY OF THE EFFECT OF BOUNDARY LAYER CONTROL ON AN AXIAL FLOW COMPRESSOR STAGE

Thesis by

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SUMMARY

The problem of increasing the pressure ratio per stage of an axial flow compressor is studied in part by considering the effect of boundary layer control by area suction on the three dimensional turbulent boundary layer flow on the casing in one stage of a compressor.

The effect of this boundary layer control, on the prevention of separation is investigated. Crossflow and crosswise variation of cross-flow in the boundary layer are factors contributing to the tendency toward separation and the effect of boundary layer control on this phenomenon is determined. This analysis is limited to the boundary layer on the casing and does not take into consideration the possible separation taking place at the blades.

The boundary layer equations are used to obtain the momentum integral equations with suction applied at the surface. The method of small perturbations is applied to the equations in order to simplify them so that a solution may be reached with relatively little effort. Expressions are assumed far the boundary layer profiles, flow path outside the boundary layer, and shear stresses. These are in a form which closely resembles the actual conditions.

In order to show what effect boundary layer control exerts, a simplified single stage compressor is studied. The results show that crossflow can be reduced by applying suction to the boundary layer with a resultant decreased tendency toward separation of the flow, but this is reached only after a certain minimum suction velocity is attained, If the suction velocity is less than this minimum, in spite of the decrease in boundary layer thickness the crossflow will be increased over that of a solid wall and likewise the tendency toward separation will increase.

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SYMBOLS

C	Suction velocity in the y direction.
C_1	Constant defining the suction velocity.
C_2	Empirical constant defining shear stress.
C_α	Constant used in assumed definition of $\frac{W'}{U^\circ}$ giving stagger angle.
C_t	Constant used in assumed definition of $\frac{W'}{U^\circ}$ giving the turning angle in a blade row.
H	Form factor of boundary layer.
I_1	Constant determined by boundary layer velocity profile.
I_2	Constant determined by boundary layer velocity profile.
ℓ	Length of one stage of compressor.
m	Exponent used for R_θ in connection with $\frac{\tau_{ox}}{\rho U^{\circ 2}} = \frac{C_2}{(R_\theta)^m}$
p	Static pressure.
R_e	Reynolds number based on axial distance
R_θ	Reynolds number based on momentum thickness
s	Spacing between blades measured in the z direction.
u, v, w	Velocities inside the boundary layer in the x, y, z directions respectively.
U, W	Velocities outside the boundary layer in the x and z directions respectively.
x, y, z	Coordinate axes.
δ^*	Displacement thickness in the boundary layer.

SYMBOLS (cont'd)

ε	Dimensionless scale factor used in the assumed velocity profile of the tangential flow.
η	Ratio of vertical position to boundary layer thickness.
θ	Momentum thickness in the boundary layer.
Θ	Generalized momentum thickness in the boundary layer.
κ	Dimensionless scale factor used in the assumed velocity profile of the axial flow.
ρ	Density.
τ	Shear force.
ωr	Rotational velocity of compressor rotor blades.

SUPERSCRIPTS

o	Zero order terms of the small perturbation procedure.
$'$	First order terms of the small perturbation procedure.

SUBSCRIPTS

o	The value of the term at the wall, or where $\eta = 0$
i	Initial values of the term, taken where suction is first applied.
x, z	The term referred to the particular coordinate axis defined by the subscript.
xz, zx	The term referred to the axis defined by the first symbol as modified by the flow in the direction of the second symbol.

I - INTRODUCTION

In the design of an axial flow compressor it would be desirable to obtain a higher pressure per stage while still maintaining the high efficiencies presently attained. The application of axial flow compressors to turbojet engines in aircraft has dictated this desirability since a higher pressure ratio per stage would require fewer stages to reach the same overall pressure rise without affecting the performance of the engine. This reduced number of stages would decrease the weight of the compressor and thereby increase the overall performance of the aircraft.

The present day designs are limited, however, by the real fluid effects to a relatively small pressure rise (20-25%) per stage. Experience has shown that any increase over this amount causes a sharp decrease in efficiency below the approximately 90% currently attainable. Consequently the issue at hand is the increase in pressure ratio per stage without an accompanying decrease in efficiency.

The real fluid effects which are of importance in compressors are the separation of the boundary layer from the blades and casing walls and also the general lowering of the momentum due to crosswise transport of boundary layer air.

The separation of boundary layer in compressors is caused not only by the usual unfavorable pressure gradients in the direction of the flow, but also by the crosswise variation of crossflow in the boundary layer. This crossflow is created by the curvature of the passage through which the flow travels. An indication of conditions believed to exist is shown in Fig. 1. As the flow travels through the curved passage, a crosswise pressure gradient is induced. The fluid in the boundary layer on the casing is moving at a lower velocity than that outside the boundary layer and since the pressure in the layer is the same as outside the layer, the particle paths in the boundary layer have greater curvature than those outside the layer. The result is a crosswise flow of air in the boundary layer which under conditions existing in compressors periodically increases and decreases the momentum of the fluid moving in the direction of the flow outside the boundary layer. The result of these changes is an increased tendency toward local boundary layer separation over that caused by an unfavorable pressure gradient in the direction of motion. This occurs at certain points usually located on the casing walls and at the exit from the rotor blading.

The local separation of boundary layer is not the only undesirable effect of crossflow. In addition, this crosswise flow, which takes place at right angles to the flow outside the boundary layer, sets up a circulatory motion or "secondary flow", that results in the transfer of boundary layer air into the center of the blade passage and replenishment of the region close to the walls with high momentum air. Consequently there is a tendency to equalize the momentum state between the main flow and the low momentum fluid, and an additional increased frictional loss which the high momentum air experiences when brought near to the walls. The general influence of this flow is to lower the momentum of the main body of flow. (See Ref. 1.)

From the above discussion it is apparent that the chief source of undesirable real fluid effects in compressors is the existence of crossflow in the boundary layer. By eliminating the crossflow on the casing, one may expect not only to reduce the likelihood of local boundary layer separation, but also to prevent the decrease in over-all momentum of the flow. Consequently, due to such elimination it should be possible to increase the angle through which the flow is turned, and thus to increase the pressure rise per stage without the usual accompanying loss in efficiency.

One promising method of boundary layer control in two dimensional flow is the application of suction. Results have indicated that by use of suction, the separation of two dimensional boundary layer may be retarded. (See Ref. 2.) The effect of suction on three dimensional boundary layer flow, however, has not been investigated and it is not clear at once whether suction will similarly prevent separation in the presence of crossflow. Furthermore it is not known if there are any additional requirements on this suction and if so, what they are.

The present study is concerned with the problem of increasing the amount of turning and thereby the pressure rise in a blade passage by applying suction to the boundary layer at the casing. The effect of this boundary layer control on three dimensional boundary layer flow will be investigated. Once the requirements for suction which minimizes the effect of crossflow in the boundary layer for any given turning are determined, the estimates of gain from such control in terms of increased pressure ratio can easily be made.

Various means of boundary layer control have been proposed for airfoils. (See Ref. 3.) One such method is by a slot cut in the surface with the boundary layer drawn off through this slot. The most efficient position of this slot for the prevention of separation was determined to be at the point of separation. Since this changes under different conditions, this type of control appears not sufficiently flexible for practical application. Another method is to apply suction through a porous material over an entire area of the surface. This introduces the question, when this type boundary layer control is applied to a compressor, of where the suction should be applied to be most efficient. The structural properties of the porous materials themselves become of great importance in a compressor application. The surface roughness is another consideration since this would appear to be greater than for a solid wall, with presumably a greater shearing force at the wall.

At the present time there are materials available which can be used in this type of application and their properties are adequate from this viewpoint. Electroplated mesh has a very smooth surface and can be made to any desired degree of porosity. Porous bronze is not quite so smooth, but has the advantage of greater strength. Through the use of powder metallurgy techniques it should be possible to obtain almost any desired combination of strength and porosity. For example, a change in materials as well as sintering temperatures or pressures can be used in obtaining the desired results. Ref. 4 shows the results of test runs on boundary layer control using various materials.

If in a particular application the desired strength for a compressor casing could not be found inherently in one of the porous metals, a double shell could be constructed. The outer shell could be made the load-carrying member and the inner shell the porous member. The area in between could serve as air removal duct. In this same manner of using two-shell construction, the distribution of suction could be varied in any manner desired by using a porous surface of uniform porosity and then backing this suction surface in the area where less flow is desired by some such material as steel wool.

Several important points must be considered when discussing the most efficient method of applying suction to a compressor boundary layer. First, it is highly desirable to maintain the high efficiency of the compressor and therefore it is necessary to remove the minimum amount of air consistent with the realization of the desired results. Second, the tendency toward separation is not the same in each stage of a multistage compressor but increases in the latter stages due to increase in boundary layer thickness. In considering this first point, it seems logical to assume that one way of removing a minimum of air is to distribute the suction in such a way that it is proportioned to the thickness of the boundary layer. Thus where the momentum loss is greatest, the greatest amount of air will be removed.

As the boundary layer grows thinner under the influence of the suction applied, the suction velocity itself will be reduced, thus reducing the amount of air removed. The second point raises the question of just where would be the best place to apply this optimum suction distribution. If it were applied over the entire casing, more air would probably be removed than if it were applied to only the later stages and the results might only be wasted on the early stages. Since this analysis is limited to a single stage, it is not anticipated that any definite conclusions can be drawn to satisfy this question.

II - DERIVATION OF GENERAL EQUATIONS

<u>Prandtl's Simplification</u>: Prandtl's simplification of the equations of motion results in the equations for boundary layer flow. The method of simplification is given in Ref. 5, and a brief resume will be given for the convenience of the reader.

Considering steady turbulent flow with no impressed forces, the equations of motion may be written:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)+\frac{\partial p}{\partial x}=\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \tau_{xz}}{\partial z}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)+\frac{\partial p}{\partial y}=\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)+\frac{\partial p}{\partial z}=\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zy}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Inasmuch as the shearing flow has an extension much smaller in the y direction than either the x or z directions, the velocity v becomes of the order of the extension in y direction and may be neglected when compared to the velocities u and w. Assuming that the accelerations are of the same order of magnitude as the shear forces, the rate of change of shear in the x and z directions become negligible when compared to that in the y direction. Furthermore, the small transport of momentum in the vertical direction reduces the momentum equation in the y direction to $\frac{\partial p}{\partial y} \doteq 0$. Consequently the change of pressure through the boundary layer in the y direction may be considered equal to zero and the pressure considered as determined by the flow outside the boundary layer. Thus the equations for the three dimensional turbulent boundary layer flow can be written:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_x}{\partial y} \qquad (1.a)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{\rho}\frac{\partial \tau_z}{\partial y} \qquad (1.b)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (1.c)$$

Care must be taken that the application of these simplified equations is restricted to a region whose extension in the y direction is very much less than the extension in the x or z directions.

These equations are concerned with the flow over a plane surface, but the results obtained can be extended to a circular cylinder without change. (See Ref. 6.) Fig. 2 shows the coordinate system to which equations (1.a), (1.b), and (1.c) will be referred for further analysis.

The restriction placed on these equations in the process of simplification still applies, so that the length in the x direction and the radius of curvature in the z direction must be very much greater than the thickness of the boundary layer in the y direction. Furthermore, the radius of curvature must be a constant for the flat plate equations to be used directly with the cylindrical coordinates, so that this analysis will be restricted to the boundary layer on the casing of a compressor. In addition, steady incompressible flow conditions are assumed to exist.

Since these equations are to be considered when suction is applied to the boundary layer over an area of the wall, the vertical velocity at the wall is not zero as normally considered. It has an actual value which will be negative in algebraic sign following the convention of positive velocities in the positive directions of their respective axes. Consequently the boundary conditions are:

at $\quad y = 0: \qquad u = w = 0, \quad v = c < 0$

at $\quad y = h: \qquad u = U, \quad w = W$

where h is a constant greater than the boundary layer thickness. Since the flow under consideration is turbulent, all the terms used in these equations are mean values with respect to the time.

<u>Momentum Integral Equations:</u> The pressure is considered as determined

by the flow conditions outside the boundary layer and by writing equations (1.a) and (1.b) in terms of the velocities outside the boundary layer the change of pressure in the x and z directions can be determined. The shear stress outside of the boundary layer is considered equal to zero. Thus:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = U\frac{\partial U}{\partial x} + W\frac{\partial U}{\partial z}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = U\frac{\partial W}{\partial x} + W\frac{\partial W}{\partial z}$$

substituting these terms into equations (1.a) and (1.b) and integrating with respect to y over the constant height h (following the method of reference 7) leads to the momentum integral equations.

$$\frac{\tau_{0x}}{\rho U^2} = \frac{1}{U}\frac{\partial U}{\partial x}\theta_x\left(H_x + 2\right) + \frac{\partial \theta_x}{\partial x} - \frac{C}{U} \tag{2.a}$$

$$+\frac{W}{U}\left[\frac{\partial \theta_{xz}}{\partial z} + \theta_{xz}\left(\frac{1}{W}\frac{\partial W}{\partial z} + \frac{1}{U}\frac{\partial U}{\partial z}\left\{\frac{\delta^*_z}{\theta_{xz}} + 1\right\}\right)\right]$$

$$\frac{\tau_{0z}}{\rho U^2} = \frac{W}{U}\frac{\partial W}{\partial z}\theta_z\left(H_z + 2\right) + \frac{\partial \theta_z}{\partial z} - \frac{CW}{U^2} \tag{2.b}$$

$$+\frac{W}{U}\left[\frac{\partial \theta_{zx}}{\partial x} + \theta_{zx}\left(\frac{1}{U}\frac{\partial U}{\partial x} + \frac{1}{W}\frac{\partial W}{\partial x}\left\{\frac{\delta^*_x}{\theta_{zx}} + 1\right\}\right)\right]$$

Various displacement and momentum thicknesses are defined as follows:

$$\delta_x^* = \int_0^h \left(1 - \frac{u}{U}\right) dy \qquad \delta_z^* = \int_0^h \left(1 - \frac{w}{W}\right) dy$$

$$\theta_x = \int_0^h \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad \theta_z = \int_0^h \frac{w}{W}\left(1 - \frac{w}{W}\right) dy$$

$$\theta_{zx} = \int_0^h \frac{u}{U}\left(1 - \frac{w}{W}\right) dy \qquad \theta_{xz} = \int_0^h \frac{w}{W}\left(1 - \frac{u}{U}\right) dy$$

$$H_x = \frac{\delta_x^*}{\theta_x} \qquad H_z = \frac{\delta_z^*}{\theta_z}$$

Equations (2.a) and (2.b) show that for the three dimensional boundary layer, the momentum losses are made up of the two dimensional effects plus additional terms caused by the three dimensional interaction of momentum transport in the boundary layer. Setting the suction velocity C equal to zero reduces the equations to those determined in Ref. 2. To see the effect which the suction velocity has on the momentum loss, equations (2.a) and (2.b) will be written with the assumption that the velocities outside of the boundary layer are constant.

$$\frac{t_{ox}}{\rho U^2} + \frac{c}{U} = \frac{\partial \theta_x}{\partial x} + \frac{W}{U} \frac{\partial \theta_{xz}}{\partial z}$$

$$\frac{t_{oz}}{\rho U^2} + \frac{cW}{U^2} = \frac{\partial \theta_z}{\partial z} + \frac{W}{U} \frac{\partial \theta_{zx}}{\partial x}$$

The right hand sides of these equations indicate the total change in the momentum loss in the boundary layer. Assuming that the shear stresses at the wall for a porous material are the same as for a solid boundary and remembering that C has a negative value, it can be seen that the total change in the momentum loss in the boundary layer must become smaller as the left hand side of the equations becomes smaller. This occurs as the suction velocity becomes a larger negative quantity. Therefore a boundary layer to which suction is applied over an area of the wall will have a smaller momentum loss than one which is bounded by a solid wall, and the greater the suction velocity the smaller the momentum loss will be.

III - SIMPLIFIED EQUATIONS

Method of Small Perturbations: Equations (2.a) and (2.b) are non-linear differential equations and with their interconnecting terms it would be very difficult to find a solution in their present form. They can be simplified by the method of small perturbations. In this method the flow is considered as having a mean value upon which small variations must be very much less than the mean value. If the three dimensional effects are considered as these small variations, the amount of turning of the flow accomplished in a blade passage must then be a small quantity. The small turning angle is one restriction which is put on the following analysis, but under real conditions it appears that the three dimensional effects actually are small when compared to the mean flow. Therefore the following assumptions will be made:

$$u = u^{\circ}(x, y) + u'(x, y, z)$$

$$v = v^{\circ}(x, y) + v'(x, y, z)$$

$$w = w'(x, y, z)$$

$$\pi = \pi^{\circ}(x) + \pi'(x, y, z)$$

$$\varpi = \varpi'(x, y, z)$$

where: $\frac{u'}{u^\circ}, \frac{v'}{v^\circ}, \frac{w'}{u^\circ}, \frac{\mathcal{U}'}{\mathcal{U}^\circ}, \frac{\varpi'}{\mathcal{U}^\circ} << 1.0$

Assumed Flow Conditions Outside the Boundary Layer: In addition to the small perturbation procedure, an assumption can be made which will further simplify the equations, still permitting retention of a sufficiently significant model. Thus consider:

$$\mathcal{U}^\circ = \text{constant}$$
$$\mathcal{U}' = 0$$
$$\varpi' = \varpi'(x, z)$$

These assumptions preclude any acceleration in the axial direction outside the boundary layer (a condition which is imposed by continuity on cascades operating in incompressible flow) but consider the tangential flow undergoing acceleration. The tangential velocity is small compared to the axial velocity, but is determined by both the axial and tangential positions.

Substituting these expressions for the velocities and neglecting second order terms, equations (2.a) and (2.b) may be written in the following simplified manner:

$$\frac{\tau^\circ_{ox}}{\rho\,\mathcal{U}^{\circ 2}} = \frac{\partial \theta^\circ_x}{\partial x} - \frac{c}{\mathcal{U}^\circ} \qquad (3.a)$$

$$\frac{\tau'_{ox}}{\rho\,\mathcal{U}^{\circ 2}} = \frac{\partial \theta'_x}{\partial x} + \frac{\partial}{\partial z}\left[\frac{\varpi'\theta^\circ_{xz}}{\mathcal{U}^\circ}\right] \qquad (3.b)$$

$$\frac{\tau'_{oz}}{\rho\,\mathcal{U}^{\circ 2}} = \frac{\delta^{*\circ}_x}{\mathcal{U}^\circ}\,\frac{\partial \varpi'}{\partial x} + \frac{\partial}{\partial x}\left[\frac{\varpi'\theta^\circ_{zx}}{\mathcal{U}^\circ}\right] - \frac{c\,\varpi'}{\mathcal{U}^{\circ 2}} \qquad (3.c)$$

Appropriate expressions for the displacement and momentum thicknesses are:

$$\Theta_x^{\circ} = \int_0^h \frac{u^{\circ}}{U^{\circ}} \left(1 - \frac{u^{\circ}}{U^{\circ}}\right) dy$$

$$\Theta_x^{\prime} = \int_0^h \frac{u^{\circ}}{U^{\circ}} \left(2 \frac{u^{\circ} U^{\prime}}{U^{\circ 2}} - \frac{u^{\prime}}{u^{\circ 2}} - \frac{u^{\prime}}{U^{\circ}}\right) dy$$

$$\delta_x^{*\circ} = \int_0^h \left(1 - \frac{u^{\circ}}{U^{\circ}}\right) dy$$

$$\Theta_{xz}^{\circ} = \int_0^h \frac{w^{\prime}}{W^{\prime}} \left(1 - \frac{u^{\circ}}{U^{\circ}}\right) dy$$

$$\Theta_{zx}^{\circ} = \int_0^h \frac{u^{\circ}}{U^{\circ}} \left(1 - \frac{w^{\prime}}{W^{\prime}}\right) dy$$

Characteristic Scale Factors: Reasonable profile for the boundary layer velocity distribution must now be assumed before equations (3.a), (3.b) and (3.c) can be evaluated further. The zero order distribution will be taken as the familiar 1/7th power law for turbulent boundary layers:

$$\frac{u^{\circ}}{U^{\circ}} = F_1 \quad \text{where} \quad F = \left(\frac{y}{\delta}\right)^{1/7} = \eta^{1/7}$$

The first order term of the axial flow component is the variation due to accelerations caused by the component of velocity at right angles.

This profile is assumed to be a fractional part of the zero order distribution and its form will be similar to that indicated by measurements carried out in an actual compressor. (See Ref. 8) Therefore:

$$\frac{u'}{U^\circ} = KFG, \quad \text{where} \quad G = \left(1 - \frac{y}{\delta}\right)^2 = (1-\eta)^2$$

The profiles of F and GF are shown in Fig. 3. K is a dimensionless scale factor and is a function of X and Z.

The velocity distribution in the z direction will be considered as having the same form as that in the X direction and can be written as:

$$\frac{w'}{W'} = F\left[1 + \varepsilon G\right]$$

where $\varepsilon = \varepsilon(x,z)$ is another dimensionless scale factor.

The functions F and G are considered as zero order terms and since $\frac{u'}{U^\circ}$ is a first order term and very much less than unity, K must be a term of the first order and very much less than unity. The term $\frac{w'}{W'}$ is of zero order and therefore ε must be of zero order.

Flow conditions in the boundary layer are completely described when the value of the mean flow, and the small variations which occur about this mean flow are determined. Therefore by evaluating the momentum thickness, θ_x°, K, and ε the flow in the boundary layer can be completely described since the momentum thickness is a measure of the mean flow while K and ε determine the small variations. It is assumed that the profile shape does not change with suction.

Consider, as an example, the velocity profile in the axial direction just at the point where separation is about to occur. At this point, $u^o + u' = 0$ as $\eta \to 0$, so that at the point of separation $u^o = -u'$ at the wall. From the definitions of the velocity profiles,

$$u^o = \bar{u}^o F$$

$$u' = \bar{u}^o K F G$$

so that $\frac{u'}{u^o} = KG$ and as $\eta \to 0$, $\frac{u'}{u^o} = K$.

At the point of separation $\frac{u'}{u^o} = -1$ and $K = -1$, so that to prevent separation of the axial flow K must be greater than -1.

Considering α as the angle between the axial velocity component and the actual flow direction in the boundary layer, the following relation may be observed:

$$\tan \alpha = \frac{w'}{u^o + u'} = \frac{\bar{w}' F [1 + \mathcal{E} G]}{\bar{u}^o F [1 + KG]}$$

Since K is a first order term and very much less than unity, the following approximation may be made:

$$\frac{w'}{u^o + u'} = \frac{\bar{w}'}{\bar{u}^o} \left[1 + \mathcal{E} G \right]$$

and as $\eta \to 0$

$$\left. \frac{w'}{u^o + u'} \right]_{\eta \to 0} = \frac{\bar{w}'}{\bar{u}^o} (1 + \mathcal{E})$$

The shear stresses have the same directions as the velocities at the wall so that the following relation may also be defined:

$$\tan \alpha = \frac{\tau'_{oz}}{\tau^{o}_{ox} + \tau'_{ox}}$$

Being a first order term, τ'_{ox} may be considered negligible with respect to τ^{o}_{ox} so that the following approximation may be made:

$$\tan \alpha = \frac{\tau'_{oz}}{\tau^{o}_{ox}} = \frac{\varpi'}{\varpi^{o}}(1+\varepsilon)$$

From the above relations involving ε, it can be seen that if $\varepsilon = 0$ the flow at the wall has the same deviation from the axial direction as the flow outside the boundary layer, since then

$$\left.\frac{w'}{u^{o}+u'}\right]_{\eta \to 0} = \frac{\varpi'}{\pi^{o}}$$

But if $\varepsilon \neq 0$ the flow at the wall has a different deviation from the axial direction than the flow outside the boundary layer. Therefore the quantity ε may be considered a measure of the crossflow in the boundary layer. Determining values of ε for varying amounts of suction velocity would show in what manner, if any, the boundary layer suction would change the crossflow.

<u>Solution of the Momentum-Integral Equations</u>: In order to solve the equations describing the flow in the boundary layer, some expression

must be introduced for the shear stress at the wall. Falkner in Ref. 4 proposed the form which will be used in this analysis. This form has the advantage of being very convenient for purposes of calculation, and also agrees quite well with experimental data. The empirical expression may be written:

$$\frac{\tau^{o}_{ox}}{\rho U^{o2}} = \frac{C_2}{(R_\theta)^m}$$

C_2 is a constant determined for fully turbulent flow; $C_2 = 0.006534$ and $m = 1/6$.

It has been shown already that the tangential shear stress at the wall may be written in terms of the shear stress due to the mean flow:

$$\tau'_{oz} = \tau^{o}_{ox} \frac{w'}{U^o}(1+\varepsilon)$$

The Falkner expression for shear, although used here with zero pressure gradient, is also used when such pressure gradient exists. It is known that such an application gives good results as far as momentum loss is concerned. Consequently, the effect of changes in the axial velocity on the shear in the x direction is neglected and τ'_{ox} is assumed zero.

Having assumed the velocity profiles and expressions for the shear stresses in the boundary layer, it is now possible to proceed with the solution of equations (3.a), (3.b), and (3.c). The terms θ'_x, θ^{o}_{xz}, etc. may now all be written in terms of θ^{o}_x and the scale factors K and ε.

$$\delta_x^{*\circ} = \Theta_x^\circ H$$

$$\Theta_{zx}^\circ = \Theta_x^\circ (1 - \varepsilon I_1)$$

$$\Theta_x' = -\Theta_x^\circ K I_1$$

$$\Theta_{xz}^\circ = \Theta_x^\circ (1 + \varepsilon I_2)$$

where

$$H = \frac{\int_0^1 (1-F)\,d\eta}{\int_0^1 (1-F)F\,d\eta} = 1.2857$$

$$I_1 = \frac{\int_0^1 F^2 G\,d\eta}{\int_0^1 (1-F)F\,d\eta} = 2.1304$$

$$I_2 = \frac{\int_0^1 (1-F)FG\,d\eta}{\int_0^1 (1-F)F\,d\eta} = 0.5425$$

The generalized momentum thickness Θ is also introduced to eliminate the Reynolds number from the equations.

$$\Theta = \Theta_x^\circ (R_\theta)^n$$

where $$R_\theta = \frac{u^\circ \Theta_x^\circ}{\nu}$$

In view of the previous discussion of the optimum distribution of the suction velocity, that velocity will be prescribed as directly proportional to the momentum loss of the unperturbed flow. Thus, the greatest suction will be applied to the thickest boundary layer and a lesser amount of air will be drawn off where the momentum loss of the unperturbed flow is low. Therefore

$$\frac{c}{\mathcal{U}^\circ} = C_1 \frac{\theta^\circ_x}{x} = C_1 \frac{\Theta}{x(R_\theta)^n}$$

where C is the parameter determining the values of suction velocity desired. Equations (3.a), (3.b) and (3.c) can thus be written containing three unknown Θ, K, and $\mathcal{E}$.

$$\frac{\partial \Theta}{\partial x} - \left[C_1 (1+n) \frac{\partial \log x}{\partial x}\right] \Theta = C_2 \qquad (4.a)$$

$$\frac{\partial K}{\partial x} + \left(\frac{1}{1+m}\right)\left(\frac{\partial \log \Theta}{\partial x}\right) K = \frac{1}{I_1 \mathcal{U}^\circ}\left[\frac{\partial \varpi'}{\partial z} + I_2 \frac{\partial}{\partial z}\left(\mathcal{E}\varpi'\right)\right] \qquad (4.b)$$

$$\frac{\partial \mathcal{E}}{\partial x} + \left\{\left[\frac{I_1 + 1}{I_1(1+m)}\right] \frac{\partial \log \Theta}{\partial x} + \frac{\partial \log \varpi'}{\partial x} - \frac{C_1}{I_1} \frac{\partial \log x}{\partial x}\right\} \mathcal{E} = \frac{H+1}{I_1} \frac{\partial \log \varpi'}{\partial x} \qquad (4.c)$$

These equations can be solved for the three unknown by first determining Θ from (4.a), proceeding to solve (4.c) for $\mathcal{E}$ with the value of Θ previously determined, and then substituting into (4.b) finally solving for K. The solutions of the equations are:

$$\Theta = \frac{C_2}{1 - C_1(1+n)} X + \left(\frac{X}{X_i}\right)^{C_1(1+n)}\left[\Theta_i - \frac{C_2}{1 - C_1(1+n)} X_i\right] \qquad (5.a)$$

$$\mathcal{E} = \frac{\Theta^{-\left(\frac{I_1+1}{I_1(1+\mu)}\right)} X^{\frac{C_1}{I_1}}}{\varpi'} \left[\left(\frac{H+1}{I_1}\right) \int \frac{\partial \varpi'}{\partial x} \Theta^{\frac{I_1+1}{I_1(1+\mu)}} X^{-\frac{C_1}{I_1}} dx + C_3 \right] \tag{5.b}$$

$$K = \frac{\Theta^{-\frac{1}{1+\mu}}}{I_1 \pi^\circ} \left[\int \left[\frac{\partial \varpi'}{\partial z} + I_2 \frac{\partial}{\partial z} (\mathcal{E} \varpi') \right] \Theta^{\frac{1}{1+\mu}} dx + C_4 \right] \tag{5.c}$$

IV - APPLICATION TO A SPECIFIC MODEL

Model of a Compressor Stage: To determine the effect of suction applied to the boundary layer, a simplified model of a compressor stage will be utilized to determine the solutions of equations (5.a), (5.b) and (5.c). Various values for C_1 will then be assumed showing how the boundary layer changes with variation in suction velocity and whether the crossflow is affected. A sketch of this model is shown in Fig. 4. The flow through the approach passage is considered purely axial in direction. No suction is applied prior to the rotor blade row and the boundary layer grows unhindered in this region. Boundary layer suction is commenced at the leading edge of the rotor and continued throughout the length of the stage. This boundary layer suction is distributed in the assumed manner $\frac{C}{a^\circ} = C_1 \frac{\Theta_x^\circ}{x}$. This means that the suction must be varied continuously over the length of the stage conceivably by varied porosity.

It is assumed that the flow through the stages of a compressor is periodic in that the turning accomplished in a rotor passage is exactly reversed in a stator passage. The mean path of fluid particles follows a spiral path through a compressor at a constant deviation from the axial direction. There is a variation about this mean path which varies periodically in both axial and tangential directions. Furthermore, the flow will be changed in different designs by utilizing blades of different camber and thus changing the amount of turning accomplished.

With these ideas in mind, it is felt that an expression approximately describing the flow through a compressor can be written as:

$$\frac{w'}{U^o} = \left[1 - \cos\left(2\pi \frac{z}{s}\right)\right] C_t \sin^2\left(\pi \frac{x}{l}\right) + C_\alpha$$

(See Fig. 5.) The constant s is the distance between corresponding points on adjacent blades measured in the z direction. This will not be a constant value for an actual compressor stage due to the radial divergence of the blades, but for simplicity of calculation it will be considered constant. The length of the compressor stage is given by l . The term C_α is a measure of the inclination of the mean flow path from the axial direction and because $\frac{w'}{U^o}$ is assumed a small quantity, the value of this term must be small. The trigonometric terms take into account the periodic nature of the flow.

The constant C_t is a measure of the amplitude of the variation from the mean path and has a direct relation with the turning angle of the flow through a blade passage. The turning angle is denoted by ϕ , and $\phi = \phi_2 - \phi_1$ where ϕ_1 is the angle between the absolute flow direction and the axial direction at the inlet to the blade row, and ϕ_2 is the same angle at the outlet. Fig. 6 illustrates this definition, and

$$\tan \phi_1 = \frac{w'}{U^o} \text{ at } x = x_i$$

$$\tan \phi_2 = \frac{w'}{U^o} \text{ at } x = x_i + \frac{l}{2}$$

$\frac{\varpi'}{U^\circ}$ is a first order term and the approximation $\tan\phi_1 = \phi_1$ can be used. Considering the length of one passage only, the term $\sin^2(\pi\frac{x}{\ell})$ can be written as $\sin^2(\pi\frac{x_i'-x}{\ell})$ so that:

$$\phi_1 = \frac{\varpi'}{U^\circ}\Big]_{xi} = C\alpha$$

$$\phi_2 = \frac{\varpi'}{U^\circ}\Big]_{xi+\ell/2} = C_t\left[1-\cos\left(2\pi\frac{z}{s}\right)\right] + C\alpha$$

Therefore $\phi = \phi_2 - \phi_1 = C_t\left[1-\cos\left(2\pi\frac{z}{s}\right)\right]$

Now solving for the average value of the turning for one blade passage where $\frac{z}{s}$ goes from 0 to 1;

$$\phi_{ave} = \frac{C_t}{S}\int_0^s\left[1-\cos\left(2\pi\frac{z}{s}\right)\right]dz$$

Gives $\phi_{ave} = C_t\,.$

Given various amounts of average turning accomplished in a blade passage, the value of C_t is immediately given. Values of turning angles up to approximately 20° are used in compressor design and while the approximations made in this analysis are not strictly valid for these high values, it is hoped that the indicated trends will be correct.

Furthermore there is a direct relation between the turning angle and the pressure rise occurring in a blade passage. This can be determined in the following manner:

$$\Delta P_t = \rho U^\circ (\omega r)\left[\tan \phi_2 - \tan \phi_1\right]$$

or

$$\frac{\Delta P_t}{\rho U^\circ (\omega r)} = C_t \left[1 - \cos\left(2\pi \frac{x}{s}\right)\right]$$

The average value for this expression is determined in the same manner as for the average turning angle and actually equals the same value.

$$\left[\frac{\Delta P_t}{\rho U^\circ (\omega r)}\right]_{ave} = C_t = \phi_{ave}$$

Therefore any increase in the average turning angle will be reflected in an increase of the pressure rise across the passage.

Data reported in Ref. 8 obtained from experimental runs was considered in order to assume realistic initial values for the boundary layer dimensions. The following values are assumed for this application:

$\Theta_i = 0.0618$ in. $\quad \ell = 5$ in. $\quad X_i = 20$ in.

$Re = 52.5 \times 10^4 \quad S = 3.36$ in.

Upon substituting the expression for the flow outside the boundary layer and the assumed initial conditions into equations (5.a), (5.b) and (5.c) the following equations are obtained. These are only applicable to the initial stage of the chosen compressor model since they do not contain the terms which would be obtained from the constants of integration C_3 and C_4.

$$\Theta = \frac{C_2}{1-C_1(1+n)} X + \left(\frac{X}{X_i}\right)^{C_1(1+n)} \left[\Theta_i - \frac{C_2}{1-C_1(1+n)} X_i\right] \qquad (6.a)$$

$$\frac{\varpi'}{U^\circ} \frac{\varepsilon}{\left[1-\cos\left(2\pi\frac{z}{s}\right)\right]} = \left[\Theta^{-\frac{I_1+1}{I_1(1+n)}} X^{\frac{C_1}{I_1}}\right]\left[\frac{H+1}{I_1}\right]\left[\frac{2\pi}{\ell} C_t\right]$$

$$\int \sin\left(\pi\frac{X}{\ell}\right)\cos\left(\pi\frac{X}{\ell}\right)\Theta^{\frac{I_1+1}{I_1(1+n)}} X^{-\frac{C_1}{I_1}}\, dX \qquad (6.b)$$

$$\frac{SK}{\sin\left(2\pi\frac{z}{s}\right)} = \frac{\Theta^{-\frac{1}{1+n}}\, 2\pi}{I_1}\int\left\{C_t \sin^2\left(\pi\frac{X}{\ell}\right) + \beta\xi\right\}\Theta^{\frac{1}{1+n}} dX \qquad (6.c)$$

where ξ in (6.c) is equal to $\frac{\varpi'}{U^\circ}\frac{\varepsilon}{\left[1-\cos\left(2\pi\frac{z}{s}\right)\right]}$.

Values of C_1 equal to 0, -20, -50, -100 and -150 were used to give various amounts of suction velocity. The corresponding average suction velocities over the total stage length are given in Table I. Values of C_t corresponding to average turning angles of 5°, 10°, 15°, and 20° were used to show the effect of suction velocity on the boundary layer parameters. The solutions of equations (6.b) and (6.c) were obtained numerically using Simpson's rule for which the total length was divided into ten equal intervals of one-half inch. Since at the stage inlet the flow is purely axial with no first order variations of velocity, the constants of integration C_3 and C_4 in equations (5.b) and (5.c) connected to the initial conditions are both equal to zero and do not appear in (6.b) and (6.c).

V - DISCUSSION OF RESULTS

The effect of variation in suction velocity and turning angle on the boundary layer parameters is shown in Figures 7, 8 and 9. The values of these parameters are shown over the axial length of the compressor stage. The smallest value of X corresponds to the inlet to the rotor at which point suction of the boundary layer commences. The greatest value of X is the outlet of the stator. The dimensions of X are in inches and both the rotor and stator are 2.5 inches long.

For the distribution of suction assumed in this analysis, only small average suction velocities would be required to drastically reduce the momentum thickness of the unperturbed flow in the boundary flow, as shown in Fig. 7. This momentum thickness and its growth over the length of a compressor stage for a solid boundary are shown as a reference for comparison with the momentum thickness experienced with boundary layer suction applied. For the solid wall, the momentum thickness increases continuously throughout the stage. As suction velocity is applied and is increased, the loss decreases and tends to become a constant value over a certain portion of the stage. This can be seen in Fig. 7 for $C_1 = -150$, where the momentum loss is practically a constant value over 80% of the stage.

The deviation of the direction of the flow at the wall from the direction of the main body of flow outside the boundary layer is indicated in Fig. 8. Again, the results obtained for a solid boundary are plotted for reference. This reference curve shows a greater deviation at the outlet of the rotor than at any other point in the stage. For small values of suction velocity an increase in this deviation takes place; for $\left.\frac{c}{U^{\circ}}\right]_{ave} = 0.624\%$ the deviation is almost double the maximum deviation experienced on the solid wall. This maximum deviation still occurs at the outlet of the rotor. As the suction velocity is increased above this value, the deviation begins to decrease until at $\left.\frac{c}{U^{\circ}}\right]_{ave} = 0.703\%$ it is again at approximately the value experienced on the solid wall. A shift of the point of maximum deviation has taken place, however, and it is now almost at the midpoint of the rotor. Thus the change in direction of flow at the wall is following the change in direction of flow outside the boundary layer more closely. Further increase in the suction velocity reduces the deviation and an improved flow condition results. For the highest value of suction velocity computed, the deviation has been considerably reduced and is periodic in nature over the stage length. In the second stage, the initial values of the characteristic factors would thus be lower than for a solid wall and the beneficial results would carry over to the second stage. The deviation differs across the blade passage with the variation in $\frac{z}{s}$, and for $\frac{z}{s} = \frac{1}{2}$, the deviation will have its greatest value. The increased deviation of the boundary layer flow for low suction velocities is believed to be the result of an increased gradient in the crosswise direction due to the decreased thickness of the boundary layer. The suction appears to have some minimum value that must be reached before the effects of this gradient increase can be overcome.

The effect of turning angle and suction velocity on the tendency toward separation of the axial velocity component can be determined from Fig. 9, for any amount of spacing between blades. The value of for this analysis is known and the value at which axial velocity separation will occur at some particular point is shown on the curves. Since $S = 3.36$, the value of SK which would lead to separation somewhere in the passage must be greater than 3.36. The point at which axial separation will occur first is where $\frac{z}{s} = 3/4$. Correlating Fig. 8 and Fig. 9 shows that the tendency for separation of the axial velocity is greatest for the greatest crossflow. It can be seen from Fig. 9 that the stator is the area in which separation has the greatest tendency to occur. This figure refers specifically only to the axial velocity component. For true separation of the flow to take place, the total velocity at the wall must be **zero**. This means that the variation about the mean velocity in both the axial and tangential directions must be equal to and in the reverse direction of these mean velocities. Thus, to determine the point of true separation, Fig. 8 and Fig. 9 must be compared to find that common point in the passage where $\frac{\varpi'}{\varpi^{\circ}}\varepsilon = -1$ and $SK = -3.36$. It must be remembered, however, that the approximate method of computation used in this analysis does not permit accurate evaluation at separation and beyond, but the results are shown to indicate the trends.

The results indicate that for this first stage true separation will not take place up to turning angles of 20°. For the solid wall with no suction applied, separation of the axial component just begins to take place at 20° turning angle, approximately half way through the stator. At this same value of turning angle, the very low values of suction velocity show axial flow separation over the entire stator. The higher suction velocities do not show any tendency for axial flow separation to take place. A greater amount of turning could be accomplished before these higher suction velocities would allow axial flow separation to take place. Thus, there is an indication that crossflow can be reduced using reasonable values of suction velocity. A value of $\left.\frac{c}{U^\circ}\right]_{ave}$ greater than 0.7% should produce beneficial results. Any value less than this, however, would be detrimental.

The results seem to indicate that the effect of boundary layer control would be more pronounced if suction were started prior to the rotor inlet. The momentum loss could thus be reduced to a very low and practically constant value throughout the stage. Crossflow could thus be kept to a minimum and therefore the flow should be even further improved. In a multistage compressor, this would indicate that suction should be applied prior to the first stage which undergoes separation effects.

Concluding Remarks: It has been shown that boundary layer control can produce improved flow conditions in an axial flow compressor stage by reducing the crossflow in the boundary layer and thereby reducing the tendency toward separation at the casing. Therefore, it should be possible to increase the pressure rise per stage of an axial flow compressor by utilizing a greater turning angle and applying boundary layer control on the casing. It remains to analyze whether the advantages gained from a reduced number of stages with suction required to achieve a given pressure ratio outweigh the loss of partially compressed air arising from suction and the necessary weight of suction equipment.

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TABLE I

C_1	$\left.\frac{c}{u_0}\right]_{ave}$
0	0
-20	0.624%
-50	0.703%
-100	0.837%
-150	0.874%

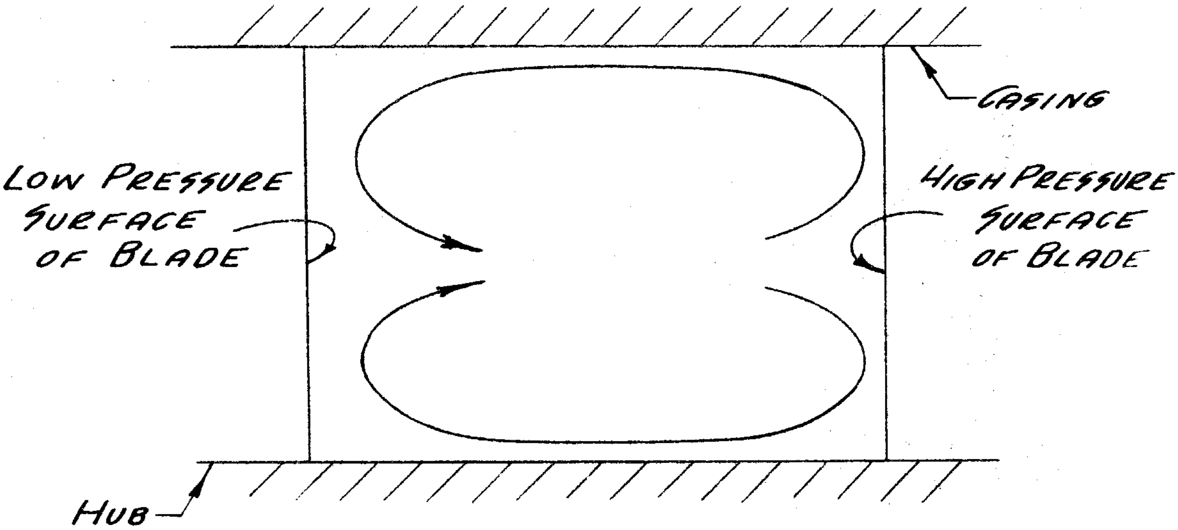

FIG. 1

CROSSFLOW IN A BLADE PASSAGE

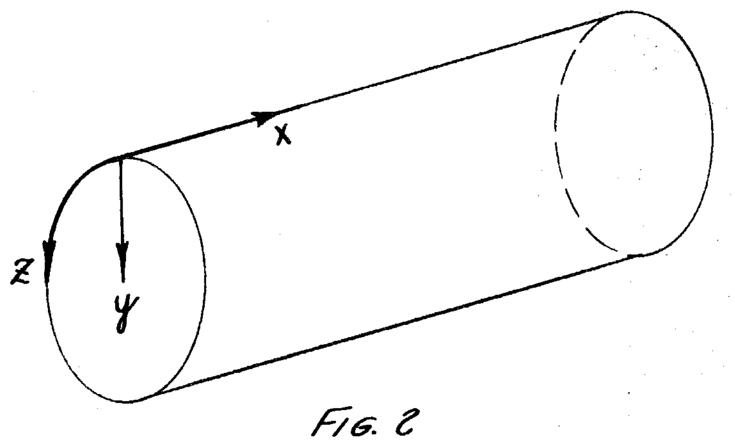

FIG. 2

COORDINATE AXES USED IN DERIVATION OF EQUATIONS

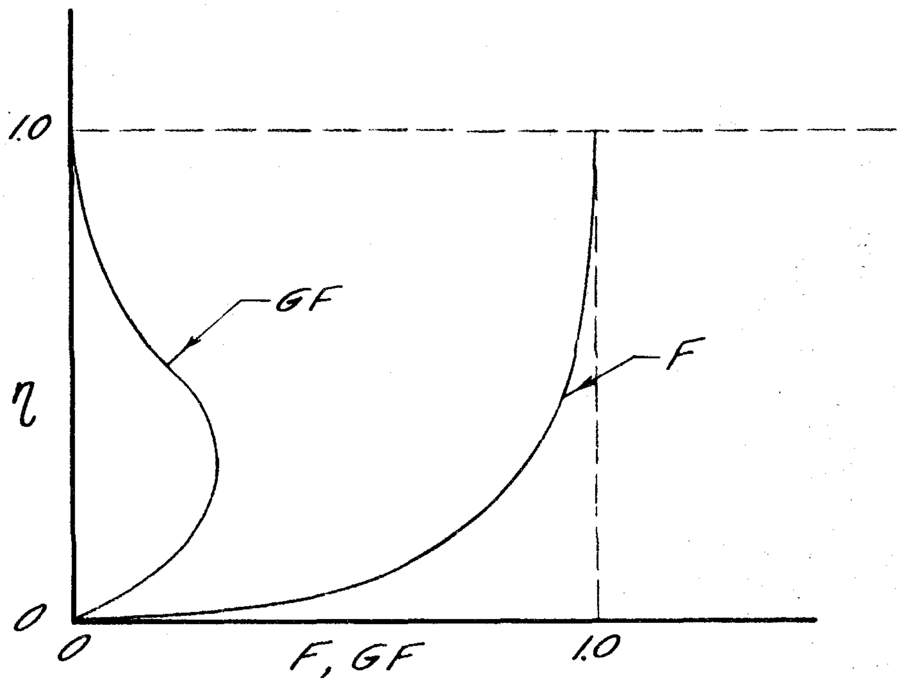

FIG. 3

BOUNDARY LAYER VELOCITY PROFILE FUNCTIONS F AND GF

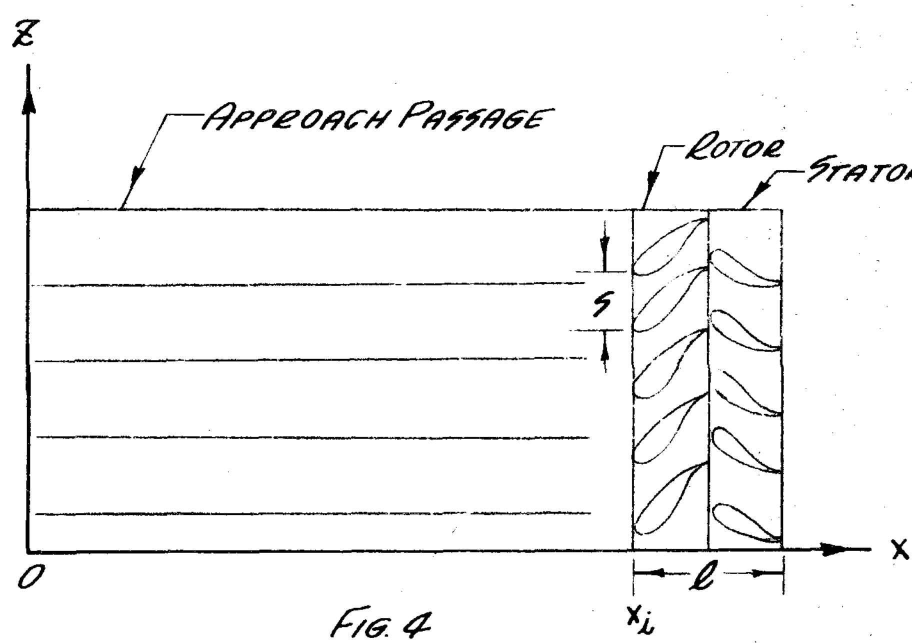

FIG. 4

ASSUMED MODEL FOR WHICH COMPUTATIONS WERE CARRIED OUT

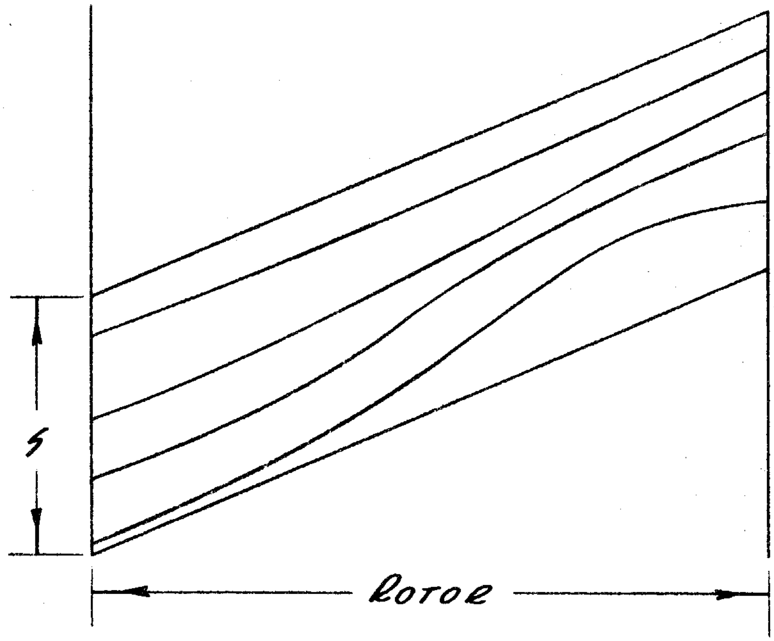

Fig. 5

Flow Path of $\frac{\varpi'}{u_o}$ (Outside the Boundary Layer)

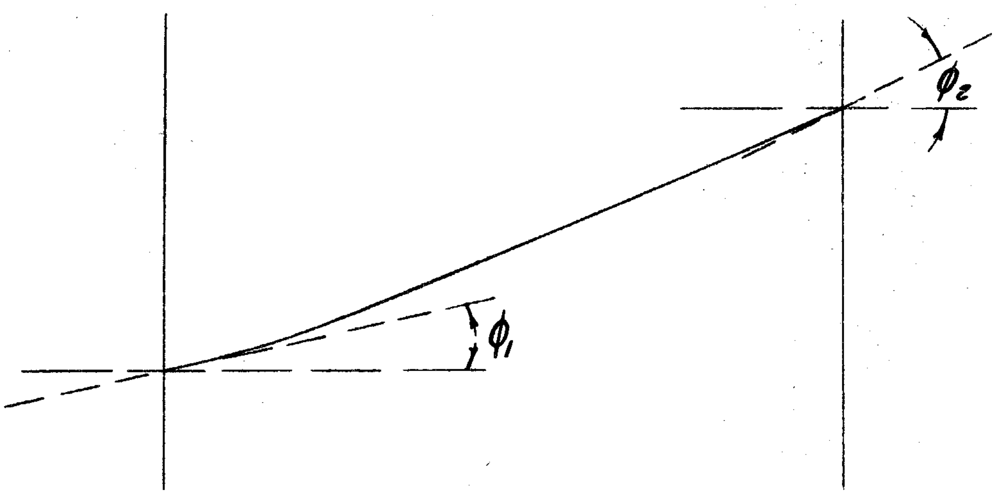

Fig. 6

Inlet and Outlet Flow Angles

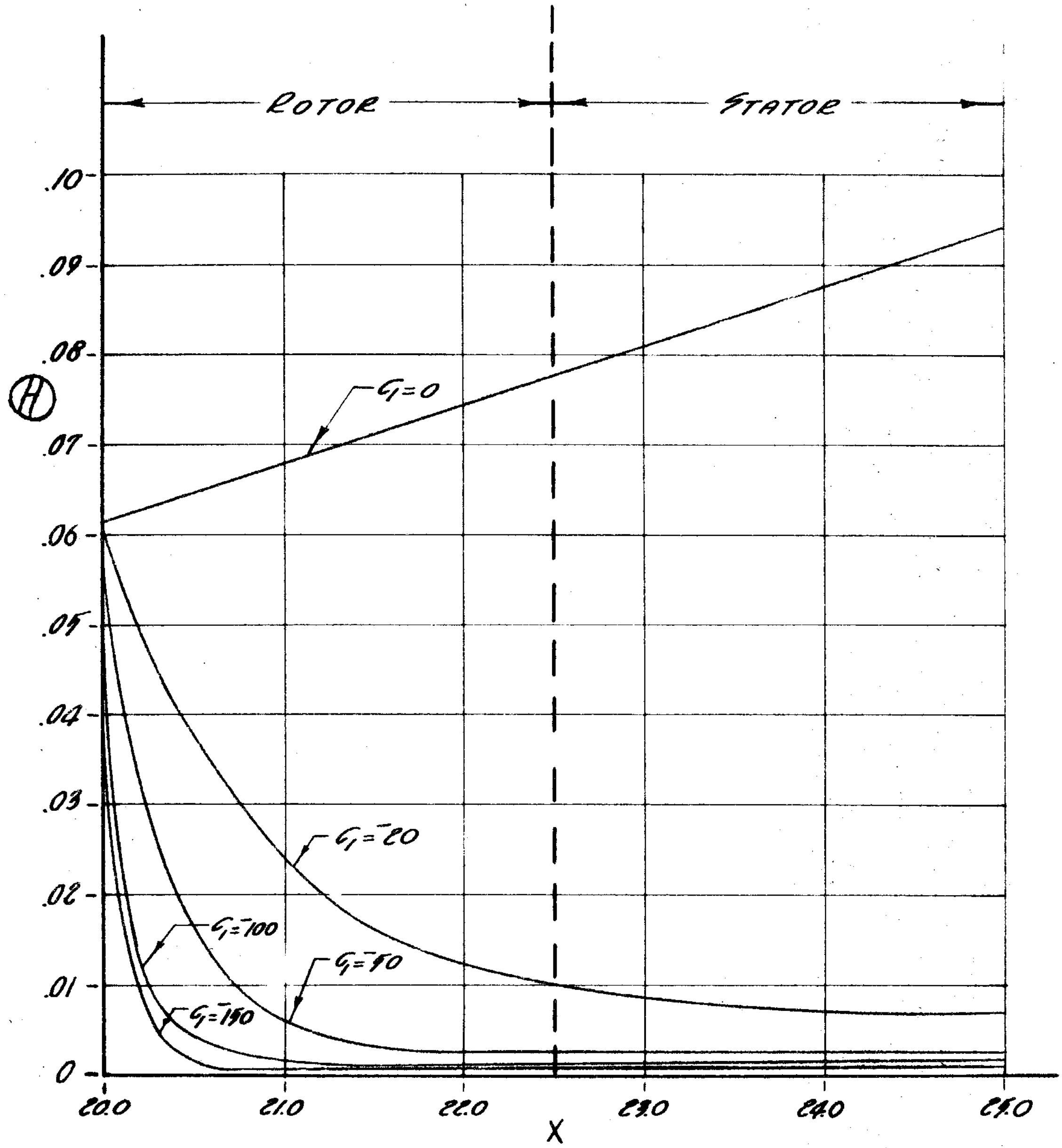

FIG. 7

GENERALIZED MOMENTUM LOSS THICKNESS

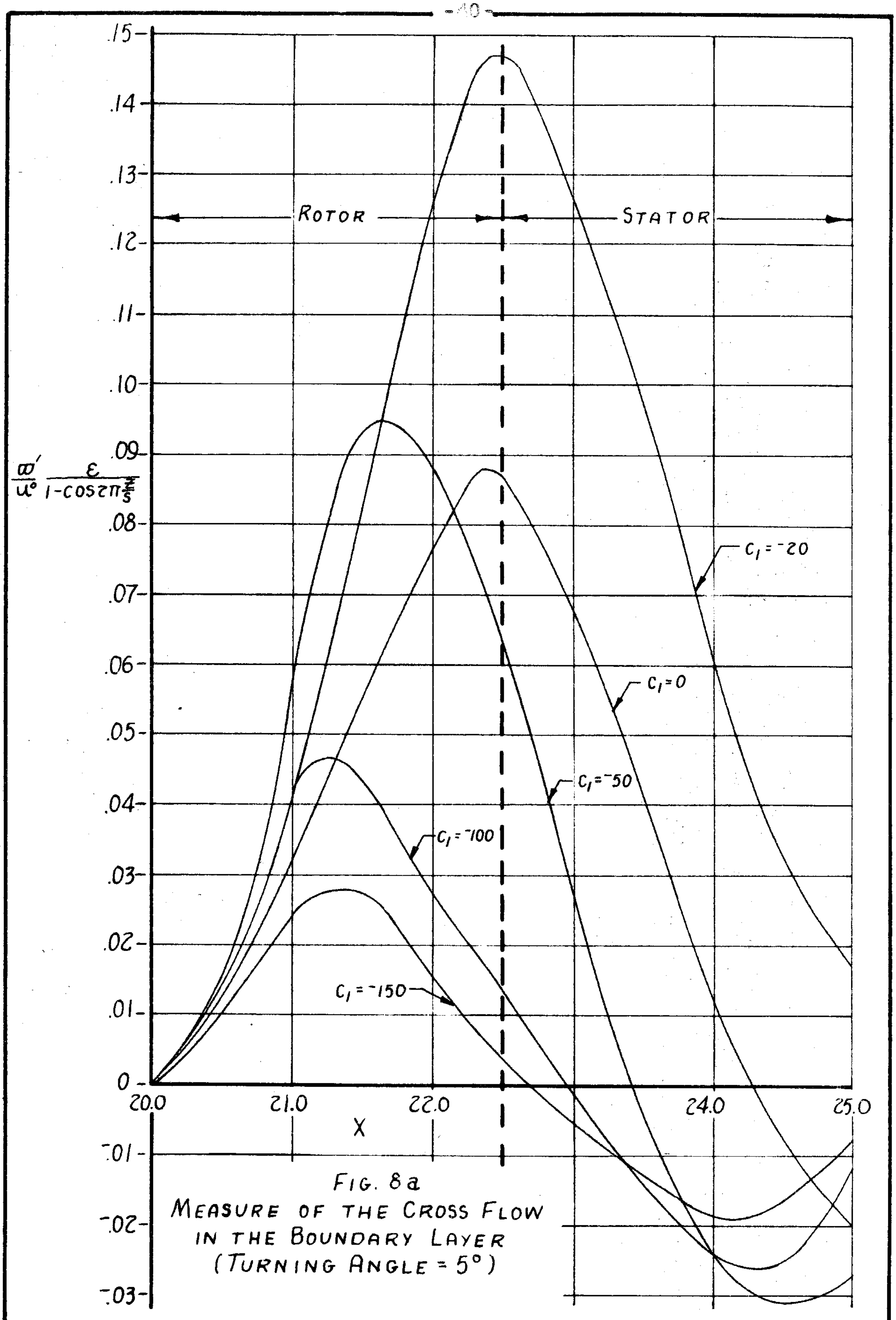

FIG. 8a
MEASURE OF THE CROSS FLOW IN THE BOUNDARY LAYER
(TURNING ANGLE = 5°)

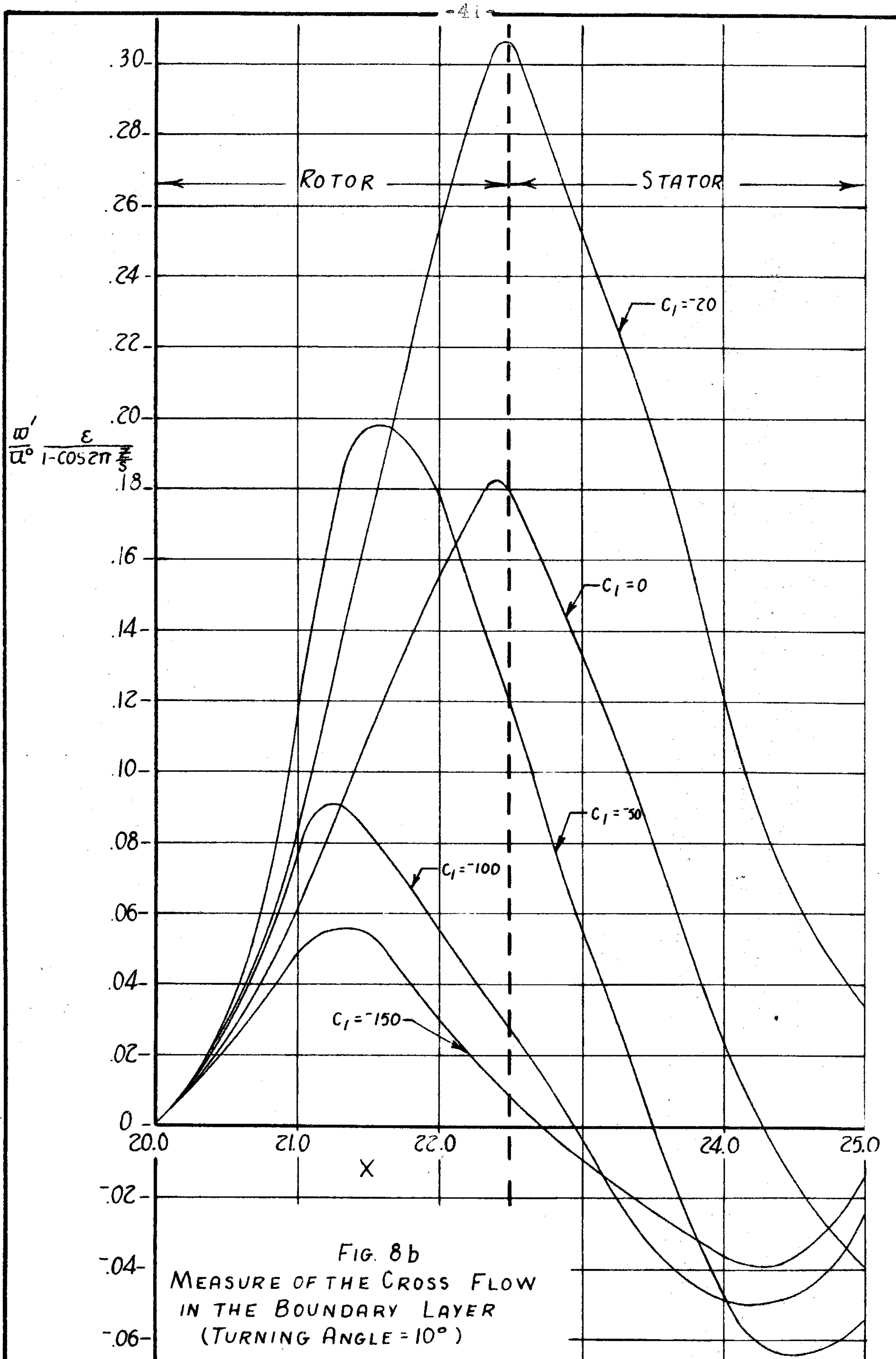

FIG. 8b
MEASURE OF THE CROSS FLOW IN THE BOUNDARY LAYER
(TURNING ANGLE = 10°)

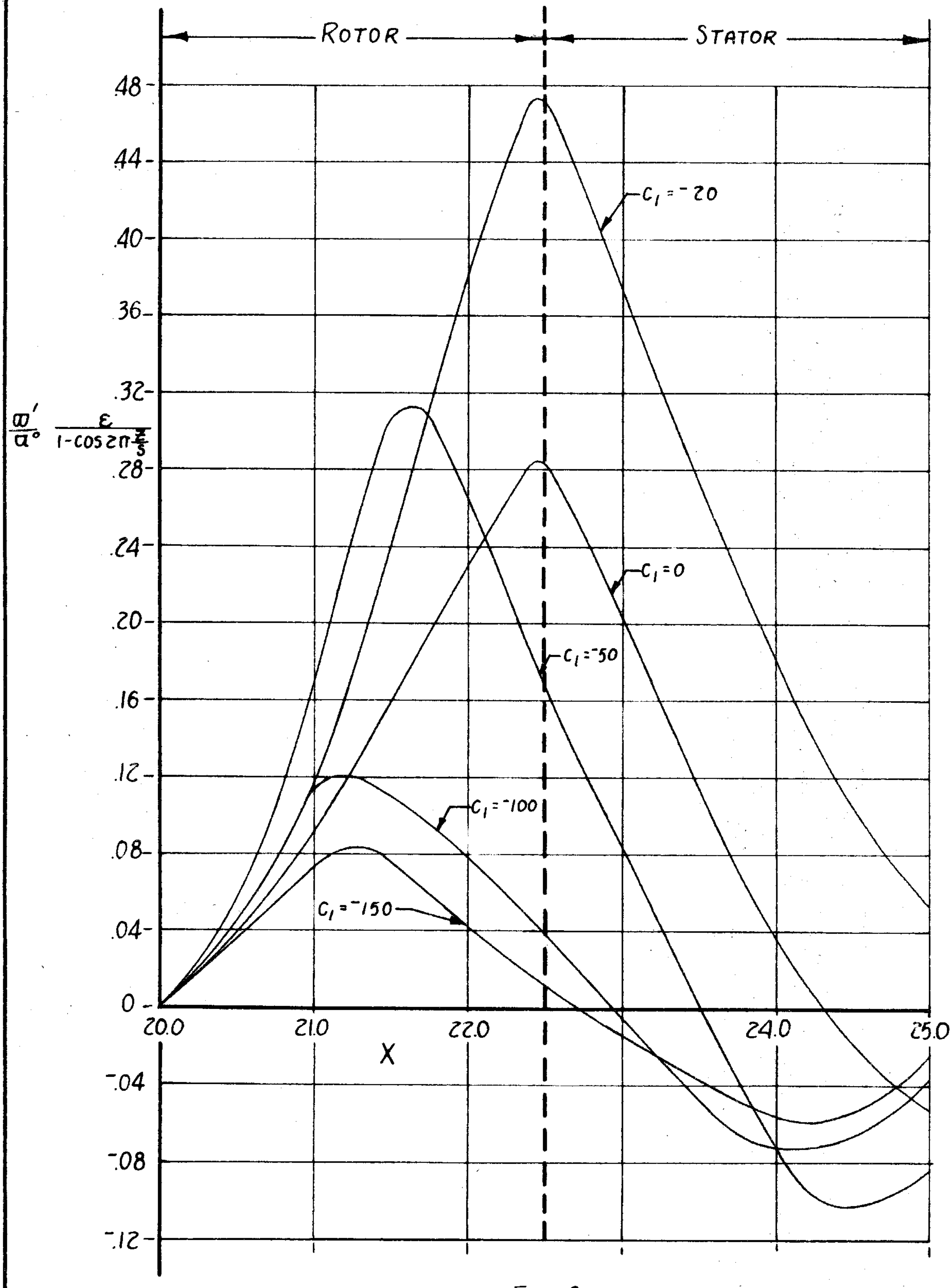

Fig. 8c
Measure of the Cross Flow in the Boundary Layer
(Turning Angle = 15°)

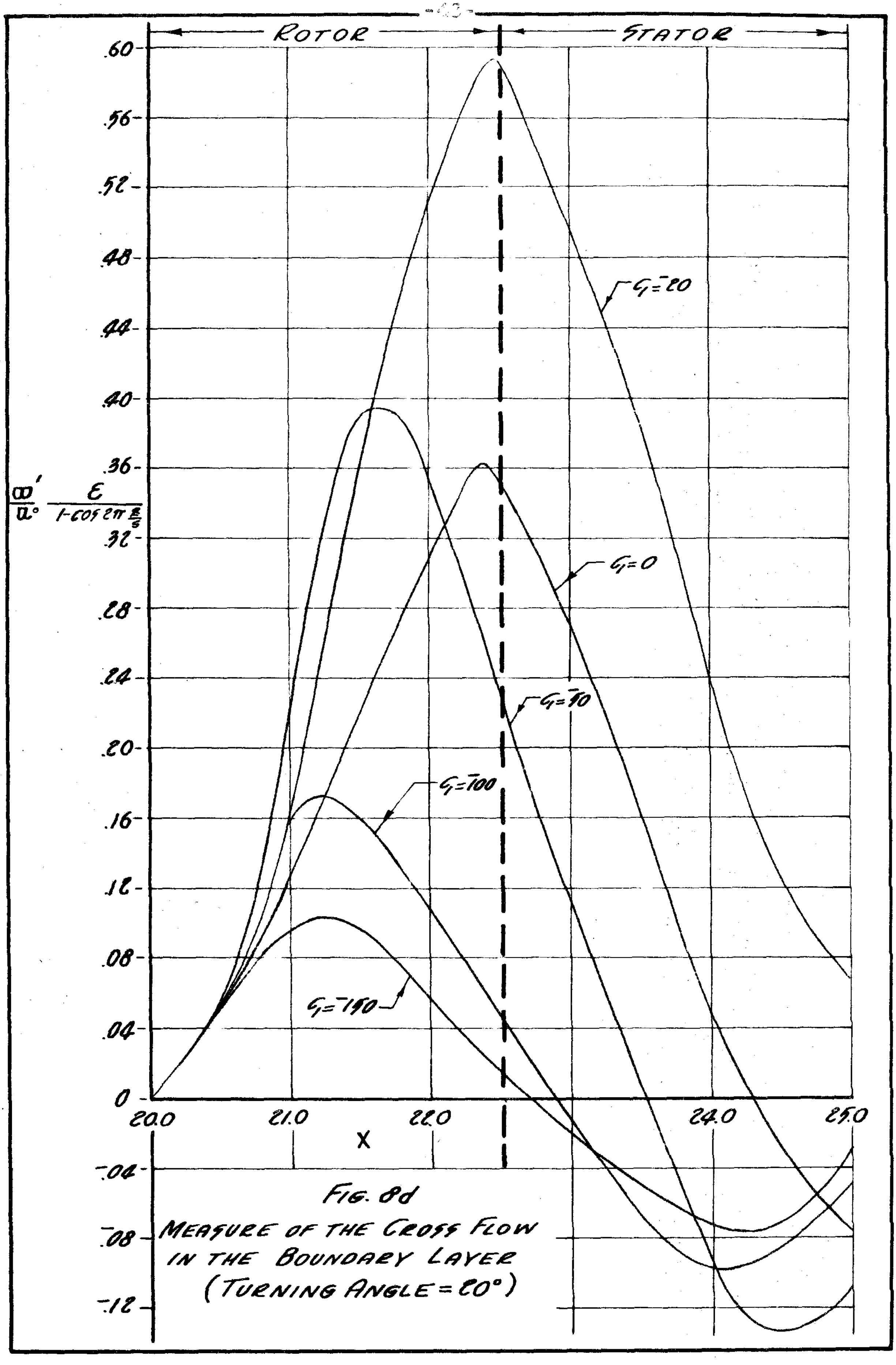

FIG. 8d
MEASURE OF THE CROSS FLOW IN THE BOUNDARY LAYER
(TURNING ANGLE = 20°)

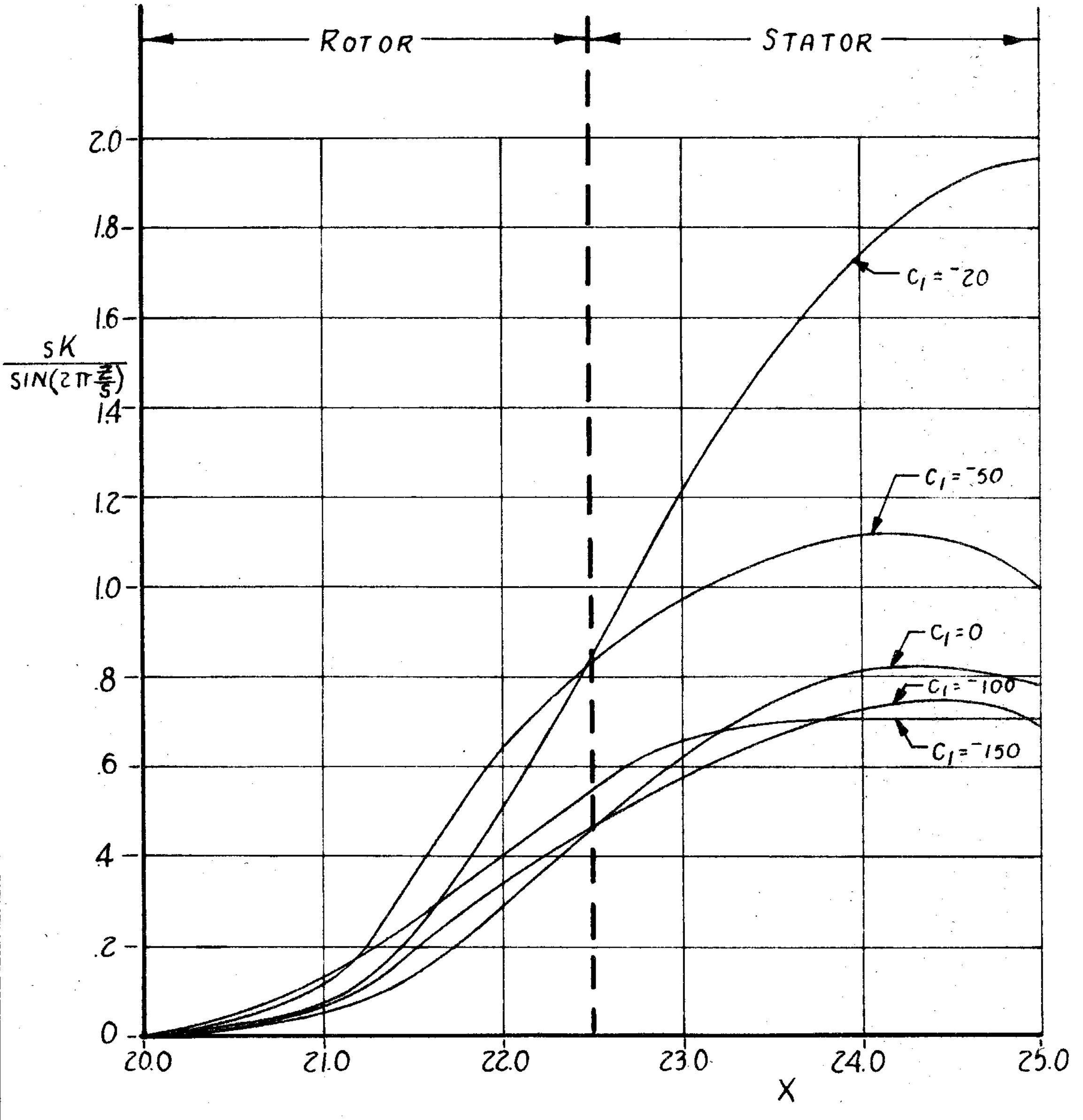

FIG. 9a
INDICATION OF AXIAL VELOCITY ACCELERATION AND SEPARATION
(TURNING ANGLE = 5°)

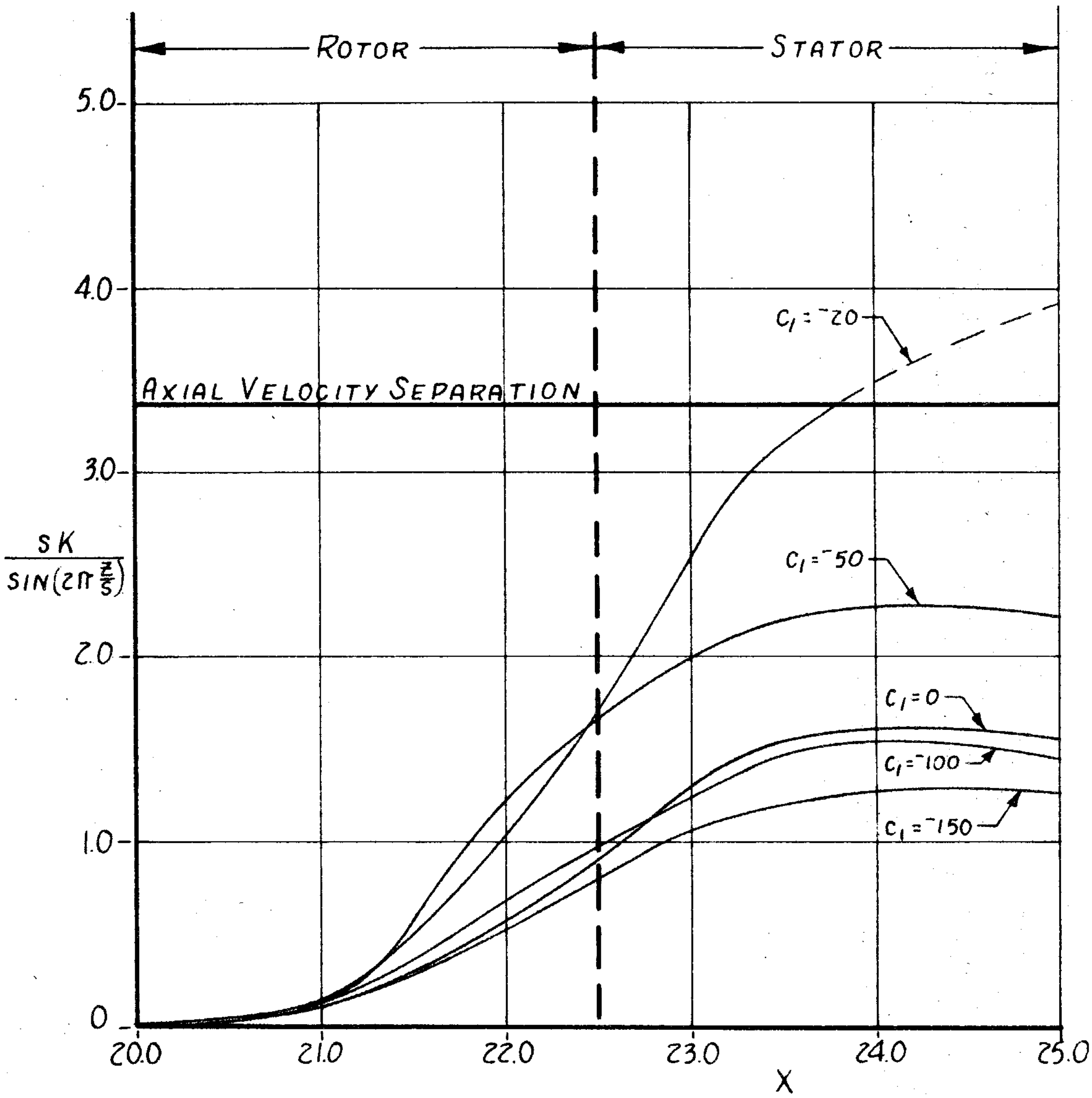

FIG. 9b
INDICATION OF AXIAL VELOCITY ACCELERATION AND SEPARATION
(TURNING ANGLE = 10°)

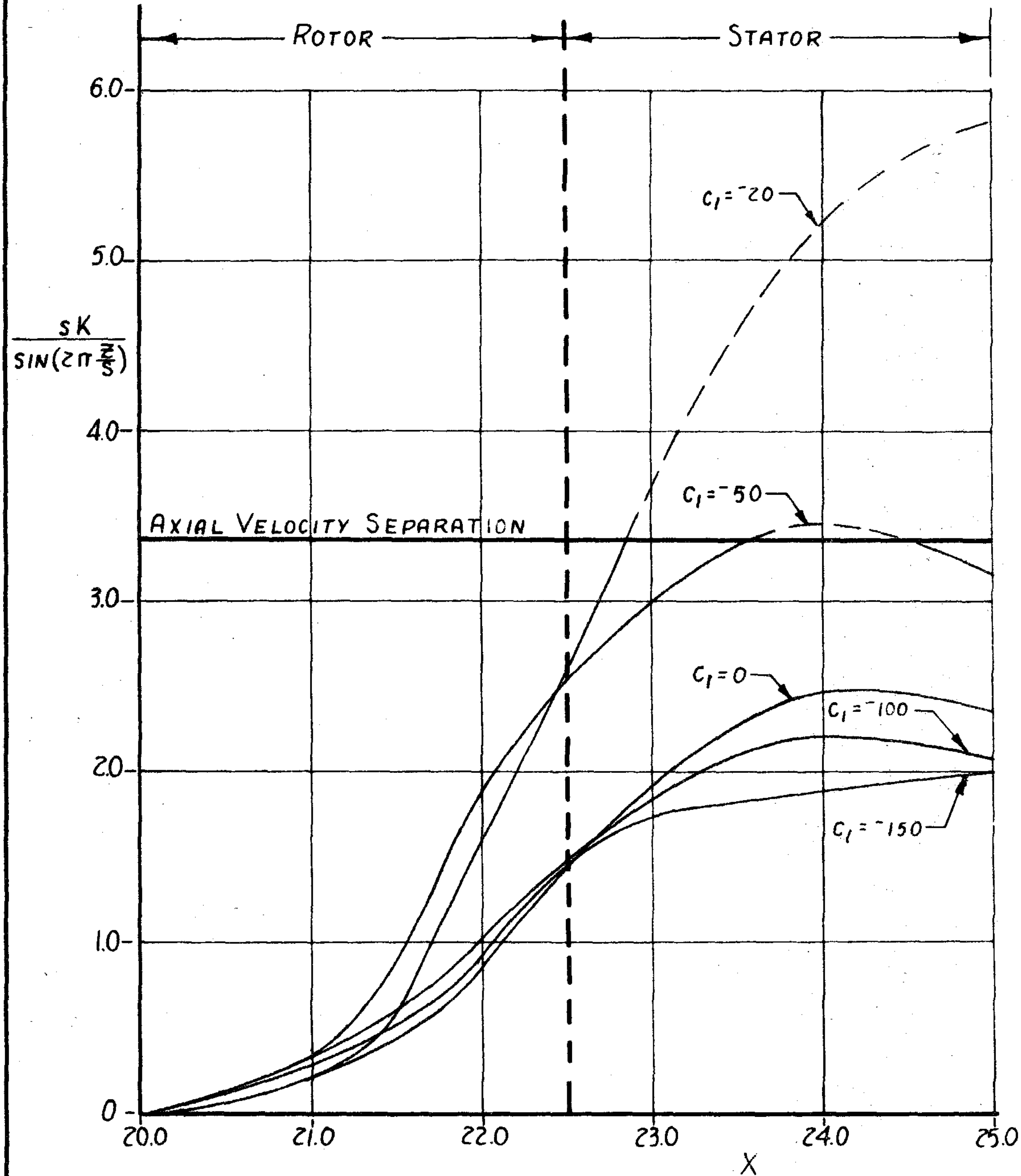

FIG. 9c
INDICATION OF AXIAL VELOCITY
ACCELERATION AND SEPARATION
(TURNING ANGLE = 15°)

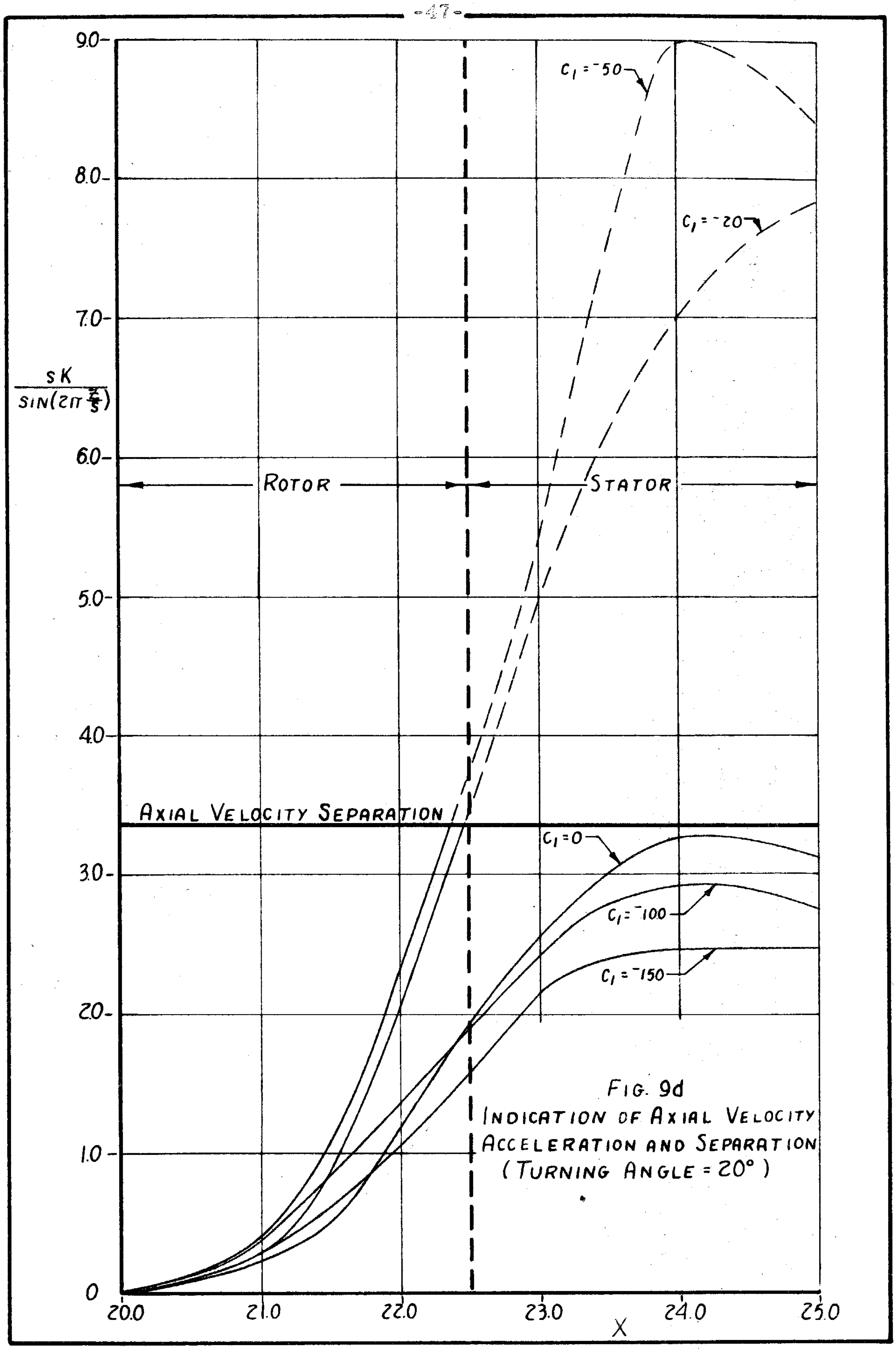

Fig. 9d
Indication of Axial Velocity Acceleration and Separation
(Turning Angle = 20°)