DYNAMIC STUDIES OF THE DISPLACEMENTS OF BUILDING FRAMES DUE TO VIBRATIONS

Thesis by

James H. Jennison

In Partial Fulfillment of the Requirements for the Degree of Master of Science

California Institute of Technology

Pasadena, California.

1936

Preface

Inasmuch as there are a number of separate studies included in this paper, it has been difficult to organize the results into a well unified report. However, the material contained herein can be divided into three main divisions; first, the descriptions of two theoretical analyses, and their application to the Bank of America Building, San Jose, California, described in Chapters I to VI; second, the tests made using a shaking table to produce transient vibrations, described in Chapters VII to X; and third, the tests of simple bents subjected to established simple harmonic ground motions, described in Chapter XI. All of the work must be considered to be of theoretical importance only, in that many simplifications are made, and the application of the results to design of earthquake - resistant structures is a rather remote possibility. However, it is felt that the results are interesting in the light of vibrational theory, and that the work has some importance in illustrating the nature and effects of vibrations similar to the earthquakes to which structures are often subjected.

Sincere appreciation is hereby expressed to Prof. R. R. Martel for his interest and many valuable suggestions. Grateful acknowledgement is also made to

Mr. Paul Kartzke, who assisted in the early stages of the work, and to Mr. Le Van Griffis who gave valuable aid in carrying out a number of the tests.

CONTENTS

Part I

Part II

Part III

PART I

Chapter I

The Westergaard Analysis

This analysis of the motion of tall buildings under the influence of an earthquake shock is presented in an article by Prof. H. M. Westergaard, in the Engineering News-Record of November 30, 1933, under the title, Earthquake-Shock Transmission in Tall Buildings.

The ideal building considered is one which has uniform weight and stiffness throughout the height of the building. Moreover, the deformation considered is that due to shear only. Thus the building may be considered as a vertical beam in which deformation in shear only is considered. Since the actual building deforms primarily in shear (that is, with horizontal displacement of one floor relative to another, producing flexure in the columns), and only to a very small extent in bending, producing lengthening and shortening of the columns, the assumption that the building deforms only in shear is justifiable. It is further assumed that the building and its loading are sufficiently symmetrical to produce little or no tendency to rotate.

Prof. Westergaard points out that an approximate analysis derived on the assumption that deformation is due entirely to shear does not apply in case the shock is so sudden that stresses will rise and fall during the short time that it takes for a wave of deformation to travel through one or two stories. However, the more significant earthquakes develop less rapidly.

The dynamics of shear deformation is simple, and the equations of motion have the same form as those of a water hammer, or of longitudinal sound waves in a rod. The horizontal motion of the ground may be resolved into two components parallel to the two sets of vertical frames. The shock then travels as a shear wave from the ground to the top of the building, is then reflected downward, travels to the ground, and is again reflected upward.

The mathematical presentation of the theory is as follows:

The following notation is used:

t = time.

x = vertical distance, positive upward.

y = horizontal deflection, function of x and t, positive toward the right.

s = horizontal shear at point x.

k = total stiffness of the columns.

w = weight of the building per unit of height.

g = acceleration of gravity.

v = velocity of shear wave.

h = height of the building.

T = fundamental period of the building.

The relation between the shear s, stiffness k, and slope $\frac{\delta y}{\delta x}$ may be written as

$$s = k \frac{\delta y}{\delta x}$$

Using the dynamic relation that force equals mass times acceleration applied to a differential of height, we obtain the equation

$$\frac{\delta s}{\delta x} = \frac{w}{g} \frac{\delta^2 y}{\delta t^2}$$

Combining these two equations,

$$k \frac{\delta^2 y}{\delta x^2} = \frac{w}{g} \frac{\delta^2 y}{\delta t^2}$$

A solution of this differential equation is

$$y = f\left(t \pm \frac{x}{v}\right)$$

where $$v = \sqrt{\frac{k\,g}{w}}$$

The physical meaning of this equation is that the magnitude of the deflection remains the same, but it has been moved along the time axis by an amount $\frac{x}{v}$ which is the time it takes for the wave to travel from the bottom of the building to the point x.

When the wave reaches the top of the building, a new wave is reflected downward. The correct boundary condition at the top is that the shear is zero. Now from the relation

$$s = k \frac{\delta y}{\delta x}$$

the shear at any point x is

$$s_1 = -\frac{k}{v} f'\left(t - \frac{x}{v}\right)$$

Equating the total shear at the top to zero, it is found that the deflection at the top is doubled , and the new descending wave is of the form

$$y_2 = f\left(t + \frac{x - 2h}{v} \right)$$

When this wave reaches the ground another wave is reflected, and the boundary condition in this case is that the new deflection wave cancels the deflection due to the downward wave, or $y_3 = -y_2$. Therefore,

$$y_3 = - f\left(t - \frac{x + 2h}{v}\right)$$

If the ground is actually in motion at the time of this reflection, the actual ground motion must be added to the reflected wave.

It may be of interest to note that the shears depend upon the derivitive f', or the velocity of the ground, as Prof. Westergaard has pointed out.

The value of v may be obtained by the use of the equation

$$v = \sqrt{\frac{kg}{w}}$$

or by using the relation

$$v = \frac{4h}{T}$$

in case the period of the building is known.

The analysis can be applied to buildings which have offsets by simply considering a certain percentage of the wave to be reflected at the offset, and the remainder to travel to the top where it will be reflected. Damping can also be considered by multiplying the amplitude of the wave by a damping factor at each reflection. Thus, the motion would eventually die out due to damping and incomplete reflections at the ground until the building returned to rest.

A graphical application of the analysis results in curves representing displacements of the particular floor considered, plotted against time.

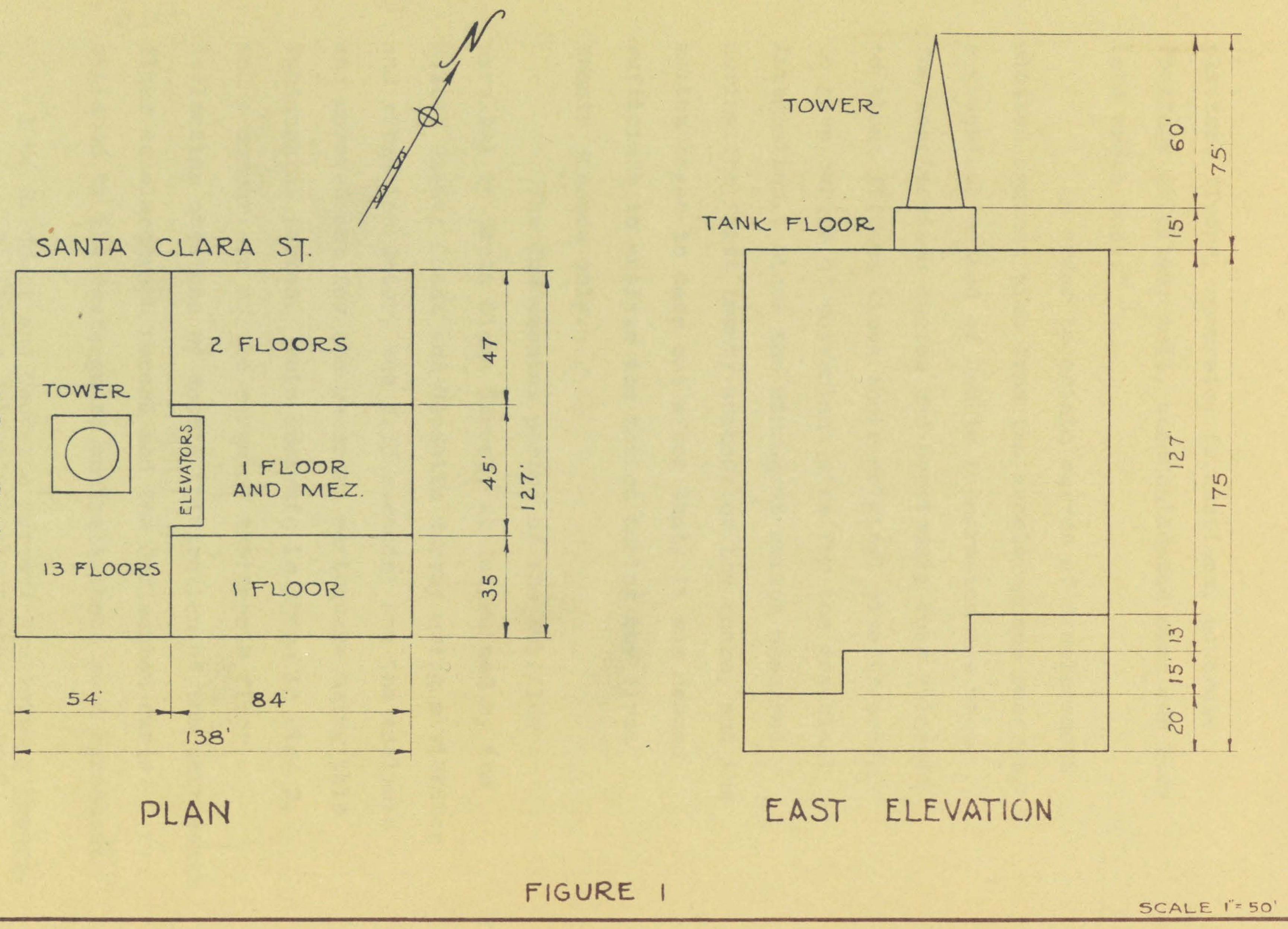
Chapter II

Application of the Westergaard Analysis to the Bank of America Building, San Jose, California.

It was thought that the application of this analysis to an actual building under the influence of an actual earthquake would be of particular interest.

Accelerograph records obtained by the United States Coast and Geodetic Survey in the Bank of America Building at San Jose, California, during the western Nevada earthquake of June 25, 1933, were available. These records gave a plot of vertical, longitudinal, and transverse components of acceleration at the thirteenth floor of the building and at the basement. The most violent motion was in a longitudinal direction (parallel to Santa Clara Street), and only this component was analyzed.

Before proceeding to a description of the analysis, it may be well to describe the building briefly. The structure has thirteen stories and a tower. An inspection of the drawings of Figure 1, which give the main dimensions of the building, will show that the building was not as symmetrical as might have been desired for a simple application of the Westergaard theory. In addition to the offset, a one story building abuts on the east. "The frame is steel, fireproofed with concrete; walls, concrete with face brick; partitions, lath and



SANTA CLARA ST.
FIRST ST.
TOWER
13 FLOORS
ELEVATORS
2 FLOORS
1 FLOOR AND MEZ.
1 FLOOR
47
45'
35
127'
54'
84'
138'
PLAN
TOWER
TANK FLOOR
60'
75'
15'
127'
175
13'
15'
20'
EAST ELEVATION
SCALE 1"= 50'

FIGURE 1

plaster; floors, concrete; foundations, concrete footings on piles; soil, unconsolidated sand and clay (old creek bed)."[1]

In order to obtain curves of displacement plotted against time from the accelerograph records, a graphical method of double integration was used. The acceleration curves had previously been enlarged to about fifteen times their original size in order to give curves of convenient size for the graphical integration. Since the strongest motion occurred during the first twenty seconds of the quake, and the motion began to damp out after that, it was deemed sufficient to analyze the motion during the first twenty seconds only.

The fundamental period of the building parallel to Santa Clara Street, as determined by the United States Coast and Geodetic Survey using a vibrator and vibration meter, was 1.33 seconds; and the analysis was carried out for the recorded earthquake using this fundamental period. This analysis is shown in Fig. 2, and a comparison of the curve of thirteenth floor deflection obtained by double integration of the thirteenth floor accelerograph record and the deflection curve obtianed by the Westergaard analysis shows good agreement

1 U. S. Coast and Geodetic Survey: Preliminary Report, California Seismological Program, Forced Vibration Tests.

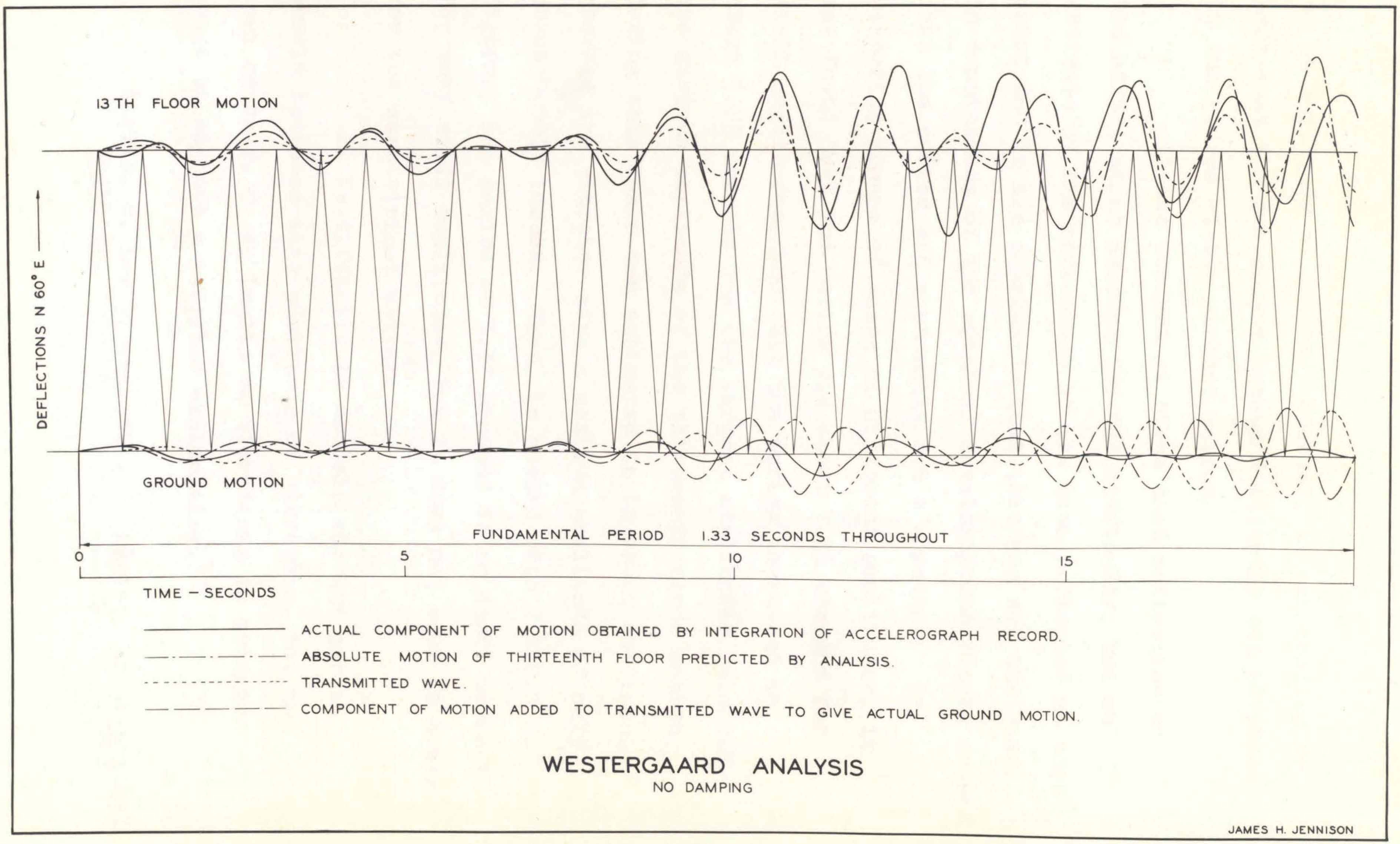

13 TH FLOOR MOTION
DEFLECTIONS N 60° E
GROUND MOTION
FUNDAMENTAL PERIOD 1.33 SECONDS THROUGHOUT
0
5
10
15
TIME - SECONDS
ACTUAL COMPONENT OF MOTION OBTAINED BY INTEGRATION OF ACCELEROGRAPH RECORD.
ABSOLUTE MOTION OF THIRTEENTH FLOOR PREDICTED BY ANALYSIS.
TRANSMITTED WAVE.
COMPONENT OF MOTION ADDED TO TRANSMITTED WAVE TO GIVE ACTUAL GROUND MOTION.
WESTERGAARD ANALYSIS
NO DAMPING
JAMES H. JENNISON

FIGURE 2

as to amplitude of motion. However, it can readily be seen that the two curves become distinctly out of phase in the course of the violent motion.

It was thought at first that reflection at the offset might account for the difficulty, but an estimate of the percentage of the wave reflected at that point led to the conclusion that this was not the case. An examination of the original accelerograph record showed that the period was apparently not a constant. By measuring groups of waves of different amplitudes, it was found that the period was about 1.35 seconds for small amplitudes, and that the period increased to about 1.65 seconds for the largest amplitudes recorded. The maximum amplitude of the thirteenth floor motion during this quake was estimated to be about one inch, whereas the vibrator gave a maximum amplitude of only about 0.0017 inches. Thus, in accord with U.S.C.G.S. figures, the period of 1.33 seconds is no doubt correct for very small amplitudes, but it does not seem to apply for the more violent motion.

It is difficult to explain why the period should increase with amplitude of vibration, but the work of Prof. C. E. Inglis on vibrations in bridges seems to suggest a possible explanation.[1]

1 Inglis, C. E.; Vibrations in Bridges; The Structural Engineer, July, 1935, Vol. XIII No. 7, p.294.

Inglis found that short span railway bridges exhibited a change in period of vibration when the spring friction of the locomotive springs was overcome, thus producing a pronounced benefit in the elimination of prolonged resonance.

In the case of a tall building, it seems possible that there may be a great number of very minute cracks, especially where brick facing is used, as in the case of the Bank of America Building. For mild vibrations, the friction between these surfaces would be sufficient to prevent relative motion, and the entire building would vibrate as though constructed of an entirely continuous material. However, as vibration becomes more violent, the maximum force that can be developed by friction along the cracks or joints is exceeded, and a slight amount of slip may occur along a great number of extremely small cracks. Slip of some of the joints in the steel frame might also possibly occur. Since the forces involved would vary with different cracks, the change of stiffness would be gradual; but, as more and more slip occurs, the building would become more limber; or, in other words, the period would be lengthened. As the vibration subsides, the forces involved gradually reach the point where friction can prevent slip along the cracks, and the period decreases again.

This explanation is merely suggested as a possible reason for the apparent change in period. It should not be construed to be a dogmatic opinion, but merely an explanation of an observed effect which seems to be reasonable. Moreover, it should be pointed out that the movements along the cracks, as well as the cracks themselves, are thought of as being of extremely small magnitude.

The analysis was carried out again, using the period of 1.33 seconds for small amplitudes, and changing the period to 1.65 seconds when the amplitude of vibration became large. This brought the motion derived from the analysis nicely into phase with the motion recorded during the actual quake, but the amplitudes of the strong motion were much too great, as can be seen from an examination of Fig. 3. Comparison of Fig. 2 and Fig. 3 gives an idea of the importance of resonance.

In these first two analyses, there was no damping considered. However, a study of the original accelerograph record showed that considerable damping prevailed in the actual structure. The damping apparently reduced the amplitude about 10% during each complete cycle of vibration, and consequently a third analysis was made in which a damping factor was employed. Each time that the wave was reflected, its amplitude was reduced by 2.5%. The periods used were 1.33 seconds for small vibrations, and 1.65 seconds for large vibrations. As in the previous analysis, shown in Fig. 3, this was

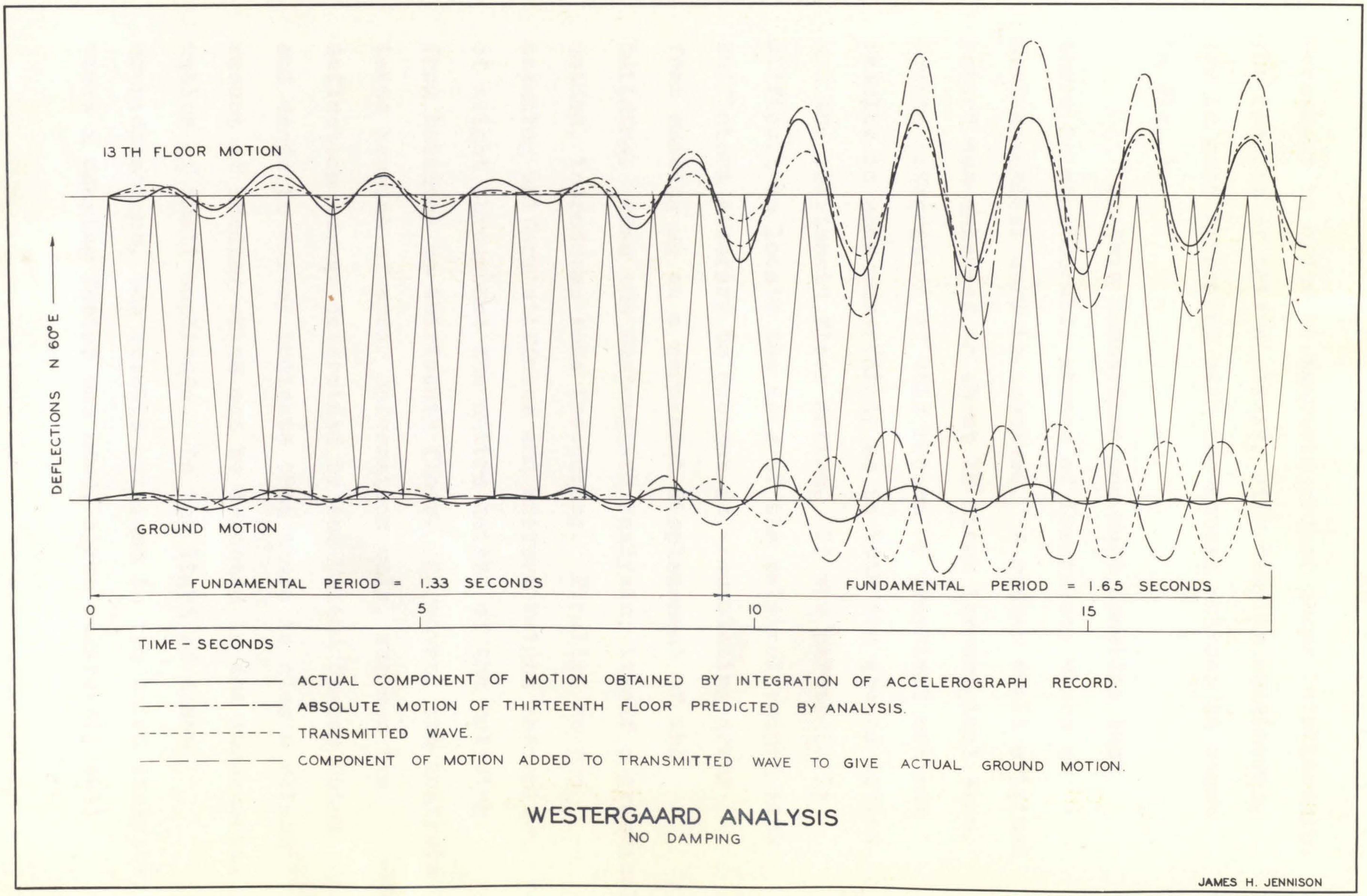

13 TH FLOOR MOTION
DEFLECTIONS N 60°E
GROUND MOTION
FUNDAMENTAL PERIOD = 1.33 SECONDS
FUNDAMENTAL PERIOD = 1.65 SECONDS
0
5
10
15
TIME - SECONDS
ACTUAL COMPONENT OF MOTION OBTAINED BY INTEGRATION OF ACCELEROGRAPH RECORD.
ABSOLUTE MOTION OF THIRTEENTH FLOOR PREDICTED BY ANALYSIS.
TRANSMITTED WAVE.
COMPONENT OF MOTION ADDED TO TRANSMITTED WAVE TO GIVE ACTUAL GROUND MOTION.
WESTERGAARD ANALYSIS
NO DAMPING
JAMES H. JENNISON

FIGURE 3

necessary in order to maintain proper phase relationship. The results of this analysis, with damping considered, are in good agreement with the actual motion, as shown in Fig. 4.

In this connection one must consider that there is considerable chance of inaccuracy where so much graphical work is involved. A rather small original record was enlarged to about 15 times its original size. Double integration of this record by graphical methods results in some uncertainty as to both the ground motion and the thirteenth floor motion. It was particularly difficult to locate the axis of the original record with sufficient accuracy to prevent an accumulating error from showing up as a permanent displacement of the building. Then the Westergaard Analysis, itself a graphical method, introduces some inaccuracy. Finally, we are assuming uniform stiffness and uniform weight per unit of height throughout the entire height of the building from basement to thirteenth floor. Moreover, the analysis takes account of shear deformation only, whereas the deflection curves determined by the United States Coast and Geodetic Survey indicate that there is also a slight amount of bending which can be detected in the fundamental motion of small amplitude. In the light of these considerations, the results obtained in the third analysis, where a damping factor was used, agree remarkably well

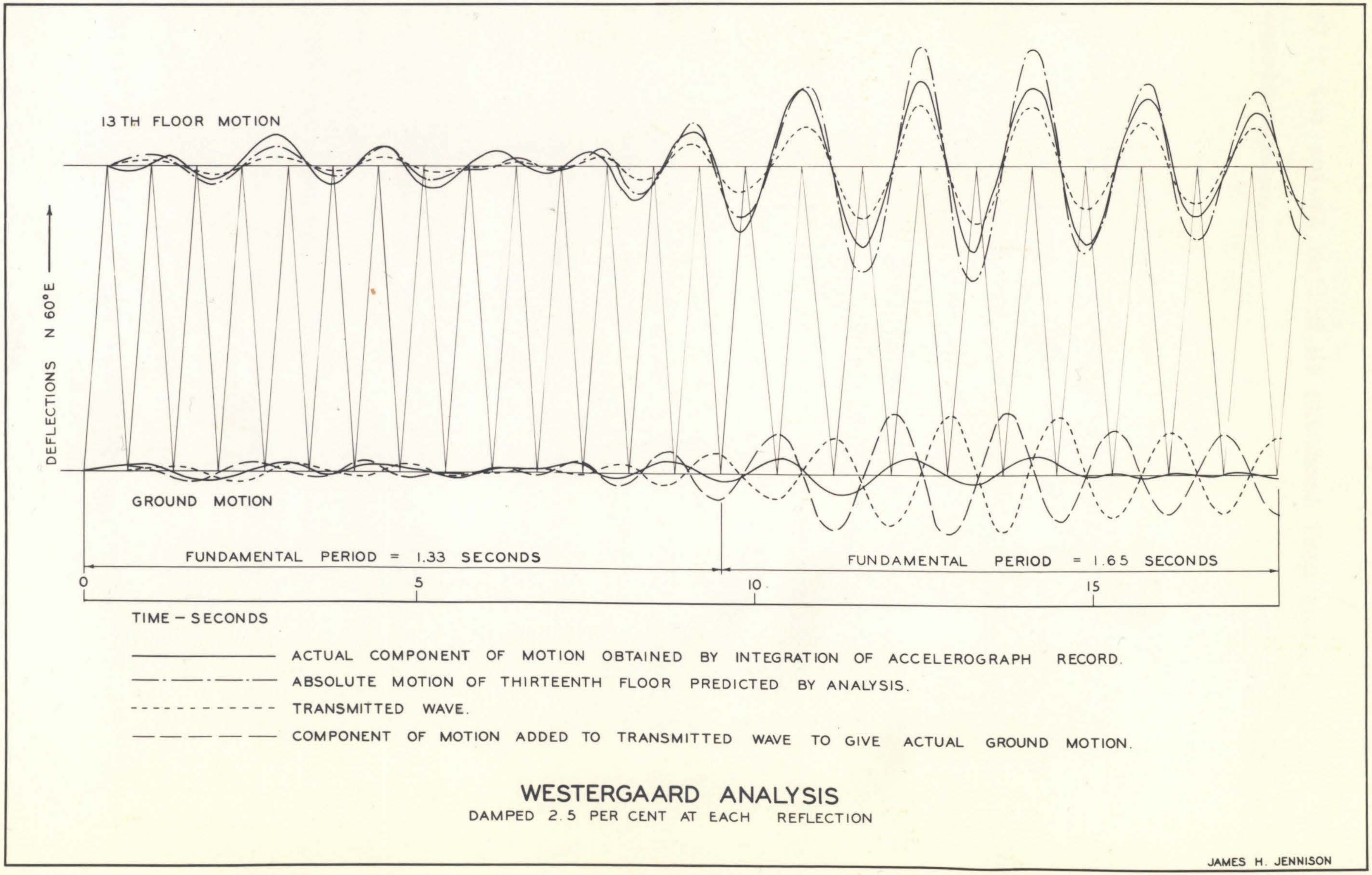

13 TH FLOOR MOTION
DEFLECTIONS N 60°E
GROUND MOTION
FUNDAMENTAL PERIOD = 1.33 SECONDS
FUNDAMENTAL PERIOD = 1.65 SECONDS
0
5
10
15
TIME – SECONDS
ACTUAL COMPONENT OF MOTION OBTAINED BY INTEGRATION OF ACCELEROGRAPH RECORD.
ABSOLUTE MOTION OF THIRTEENTH FLOOR PREDICTED BY ANALYSIS.
TRANSMITTED WAVE.
COMPONENT OF MOTION ADDED TO TRANSMITTED WAVE TO GIVE ACTUAL GROUND MOTION.
WESTERGAARD ANALYSIS
DAMPED 2.5 PER CENT AT EACH REFLECTION
JAMES H. JENNISON

FIGURE 4

with the actual motion as obtained from the accelerograph.

Chapter III

The Biot Analysis

The theory developed by Dr. M. Biot while at the California Institute of Technology was first presented at the National Applied Mechanics Meeting in June, 1932, and it has since been presented in two papers by M. Biot, namely (1) Theory of Elastic Systems Vibrating Under Transient Impulse With An Application to Earthquake-Proof Buildings, and (2) Theory of Vibration of Buildings During Earthquake. Unfortunately, the mathematics of the derivation is somewhat complicated, and the physical significance of the terms in the final equations is obscure. Consequently, the full derivation will not be presented here, but only the results and the methods used in their application.

The Biot Analysis is based on the fact that the motion of a building during an earthquake has the character of a transient oscillation. Applying the principles used by Heaviside in the analysis of transient electric currents, Dr. Biot, considering shearing forces only, makes an analysis of the vibrations in a building. In the case of forced vibrations, an oscillating system tends to approach a steady state of vibration with the same period as the applied force; and, in the case of resonance, the maximum amplitude is limited only by the internal friction. However, when the system is under the influence of transient vibrations, the steady state harmonic oscillation does not have time to develop; and it is the response of systems to such transient impulses that Dr. Biot considers. The analysis is based on the fact that any vibration in an elastic, undamped system may be considered as a superposition of harmonics. The analysis disregards damping.

The theory, as developed by Dr. Biot, considers four separate types of vibration separately, and then combines the results in a general theorem. The four types of motion are as follows:

1. Free oscillations.
2. Forced harmonic oscillations.
3. Sudden constant accelerations.
4. Oscillations due to arbitrary horizontal accelerations.

The general theorem derived has the form

$$U_k(x) = K_k F(\nu_k)$$

where $U_k(x)$ = deflection relative to the ground of a floor at a distance x from the roof.

K_k = constant depending on the properties of the building.

$F(\nu_k)$ = function involving constants and the earthquake motion.

This equation can be written with appropriate subscripts for the fundamental and the various harmonics.

For an analysis considering vibrations in the fundamental period, the above equation becomes

$$U_o(x) = K_o\ F(\nu_o)$$

The constant K_o can be found from the relation

$$K_o = 2\pi\nu_o\ j_o\ t^2\ B_o\ \cos\ (\lambda_o\ \xi)$$

The terms in the above equation can be evaluated as follows;

T_o = fundamental period of the building, in seconds per oscillation, as found by experiment.

ν_o = the frequency of the building in the fundamental mode of vibration, in oscillations per second. $= \frac{1}{T_o}$

$j_o = \frac{1}{T_o{}^2}$

λ_o = the value obtained from the relation

$\lambda_o \tan \lambda_o = \alpha$

where $\alpha = R\ n$

where n = number of stories above the first.

R = ratio of the rigidity of the first floor to that of the others.

The value of R may be found experimentally by deflecting the floors by applying a lateral force.

Then

$$R = \frac{d_n}{d_1}$$

where d_n = deflection of any floor above the second relative to the floor beneath it.

d_1 = deflection of second floor relative to ground floor with the same lateral force applied.

If the columns have very constant cross sections, the value of R may be found from the equation

$$R = \frac{L_n^3}{L_1^3}$$

where L_n = height of each story above the first.

L_1 = height of first story.

t_o is found from the following equation.

$$t_o = \frac{T_o \lambda_o}{2\pi}$$

B_o is found from the equation

$$B_o = \frac{2\alpha\beta_o}{\lambda_o^4}$$

where

$$\beta_o = \frac{\cos\lambda_o}{1 + \frac{\alpha\cos\lambda_o}{\lambda_o^2}}$$

$$\xi = \frac{x}{h}$$

where x = distance from the top of the building to the point at which the deflection is desired.

and h = height of the building above the first story.

These dimensions and the deflections of the building are illustrated in Figure 5.

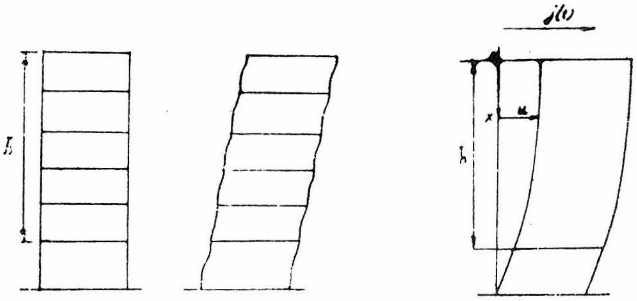


Figure 5.

The function $F(\nu_0)$ can be evaluated from the equation

$$F(\nu_0) = (2\pi \nu_0 T_0)^2 \sqrt{\left[\int_0^T G(t) \sin 2\pi\nu_0 t \, dt\right]^2 + \left[\int_0^T G(t) \cos 2\pi\nu_0 t \, dt\right]^2}$$

t is the independent variable, time; and G(t) is a function of time expressing the ground motion in terms of t.

The only remaining difficulty lies in evaluating the integrals under the radical sign. For any given ground motion, this integration can be carried out graphically as follows: The sine and cosine functions, when integrated, become cosine and sine functions respectively. These curves are plotted, and the curve of the ground motion is plotted at right angles to them, giving due consideration to the signs of the areas. The sign convention used is that the areas are positive when the ordinate of the ground motion and the slope of the sine or cosine curve have like signs. The positive areas are plotted to the left of the sine or cosine curve. The areas are negative when the ordinate of the ground motion and the slope of the sine or cosine curve have unlike signs, and the negative areas are plotted to the right of the sine or cosine curve. The cumulative area, with due regard to sign, may then be measured with a planimeter. At frequent intervals, the cumulative algebraic sum of the areas is noted, and the deflection is calculated at these points.

The result is an envelope within which the motion of the building, considering only vibrations in the fundamental, should remain. If it is desired to consider the harmonics, the subscripts 1, 2, 3, etc., for the first, second, and third harmonics, respectively, can be used with the above formulas to calculate new constants. The graphical integration can be carried out for the harmonics in a manner similar to the integration for the fundamental. The deflections obtained are then added to the corresponding deflection in the fundamental mode.

The assumptions underlying the Biot analysis should be kept clearly in mind. They are as follows:

1. The building must be of fairly regular shape.
2. The most important deformation must be due to horizontal shear.
3. The shearing rigidity and mass of each story above the first must be constant.
4. Damping is neglected.

Chapter IV

Application of the Biot Analysis to the Bank of America Building, San Jose, California.

The records used for this analysis were the same as those described in Chapter II, in connection with the Westergaard analysis. The analysis of the fundamental motion was first carried out considering the fundamental period to have a constant value of 1.33 seconds. The calculations shown on the following page are based upon a fundamental period of 1.33 seconds. The graphical integration is shown in Figure 6, and the results of the integration and the values calculated for the envelope are shown in the table on page 25. The envelope obtained is plotted at the top of Fig. 6 as a dotted line; and an examination shows that the envelope contracts due to the motion getting out of phase, which occurs at the same time as this difficulty was observed in the Westergaard analysis. To eliminate this, the period was changed to 1.65 seconds when the motion became violent, and the remainder of the analysis was repeated. It was found upon investigation that a change in fundamental period alone did not affect the constant in the final equation

$$U_o = 0.582 \sqrt{A_1^2 + A_2^2}$$

The results of the graphical integration, and the new

CALCULATION OF CONSTANTS
— FUNDAMENTAL —

$T_o = 1.33$ SEC. (U.S.C.G.S. FORCED VIBRATION STUDY)

$\nu_o = \frac{1}{T_o} = \frac{1}{1.33} = 0.752$ OSCILLATIONS PER SEC.

$j_o = \frac{1}{T_o^2} = \frac{1}{1.33^2} = 0.565$

$n = 12$

$R = 2$ (ESTIMATED FROM U.S.C.G.S. DEFLECTION CURVE)

$\alpha = Rn = 24$

$\lambda_o \tan \lambda_o = \alpha = 24$

$\lambda_o = 1.508$ RADIANS

$t_o = \frac{T_o \lambda_o}{2\pi} = \frac{1.33 \times 1.508}{2\pi} = 0.319$

$$\beta_o = \frac{\cos \lambda_o}{1 + \frac{\alpha \cos \lambda_o}{\lambda_o^2}} = \frac{0.0625}{1 + \frac{24 \times 0.0625}{1.508^2}} = 0.0375$$

$$B_o = \frac{2\alpha \beta_o}{\lambda_o^4} = \frac{2 \times 24 \times 0.0375}{1.508^4} = 0.349$$

$K_o = 2\pi \nu_o j_o t_o^2 B_o \cos(\lambda_o \xi)$

FOR 13TH FLOOR, $\xi = \frac{x}{h} = \frac{1}{12}$

$\cos(\lambda_o \xi) = 0.993$

$K_o = 2\pi \times 0.752 \times 0.565 \times 0.319^2 \times 0.349 \times 0.993 = 0.0938$

$$F(\nu_o) = (2\pi \nu_o T_o)^2 \sqrt{\left(\int_0^T G(t) \sin 2\pi \nu_o t \, dt\right)^2 + \left(\int_0^T G(t) \cos 2\pi \nu_o t \, dt\right)^2}$$

$F(\nu_o) = 39.4 \sqrt{(\text{AREA \#1})^2 + (\text{AREA \#2})^2}$

$$U_o = \frac{K_o F(\nu_o)}{E\, y\, 2\pi \nu_o P} = \frac{0.938 \times 39.4 \sqrt{A_1^2 + A_2^2}}{1 \times 1.5 \times 2\pi \times 0.752 \times \frac{1}{1.1136}} = 0.582\sqrt{A_1^2 + A_2^2}$$

(1) E = ENLARGEMENT FACTOR.

(1.5) y = AMPLITUDE OF SINE AND COSINE CURVES.

P = PLANIMETER FACTOR TO REDUCE AREA TO SQ. IN.

$2\pi \nu_o = 4.725$

Fundamental Mode of Vibration

Areas and Envelope Values

	Period = 1.33 sec.		
Point	A_1	A_2	U_o
0	0	0	0
1	-0.15	-0.10	0.105
2	+0.06	-0.30	0.178
3	+0.58	-0.22	0.347
4	+0.38	+0.01	0.222
5	-0.06	+0.03	0.039
6	-0.19	+0.06	0.116
7	-0.53	+0.06	0.310
8	-0.43	-0.05	0.252
9	-0.07	-0.10	0.071
10	+0.01	-0.06	0.035
11	-0.24	-0.37	0.257
12	-0.18	-0.72	0.432
13	-0.30	-1.07	0.646
14	-0.05	-1.34	0.831
15	+0.25	-1.50	0.885
16	+0.94	-1.39	0.977
17	+1.68	-0.81	1.09
18	+1.08	-0.18	0.638
19	+0.27	-0.19	0.192
20	-0.03	-0.55	0.320
21	-0.09	-1.06	0.62
22	+0.73	-1.46	0.95
23	+0.88	-1.46	0.991
24	+0.65	-1.52	0.961
25	+0.90	-1.55	1.04
26	+0.79	-1.41	0.94
27	+1.07	-1.33	0.99
28	+1.34	-1.18	1.04
29	+1.06	-1.06	0.872
30	+1.02	-1.11	0.881
31	+1.25	-1.01	0.934

	Period = 1.65 sec.		
Point	A_1	A_2	U_o
15	+0.17	-1.68	0.981
a	+0.61	-2.15	1.30
b	+1.66	-2.33	1.67
c	+2.32	-2.15	1.84
d	+2.73	-1.80	1.90
e	+3.32	-1.35	2.09
f	+3.18	-0.90	1.92
g	+3.07	-0.98	1.87
h	+3.17	-0.93	1.92
i	+3.17	-0.90	1.92
j	+3.25	-1.11	2.00
k	+3.57	-1.27	2.20
l	+3.55	-1.17	2.17
m	+3.43	-0.95	2.07

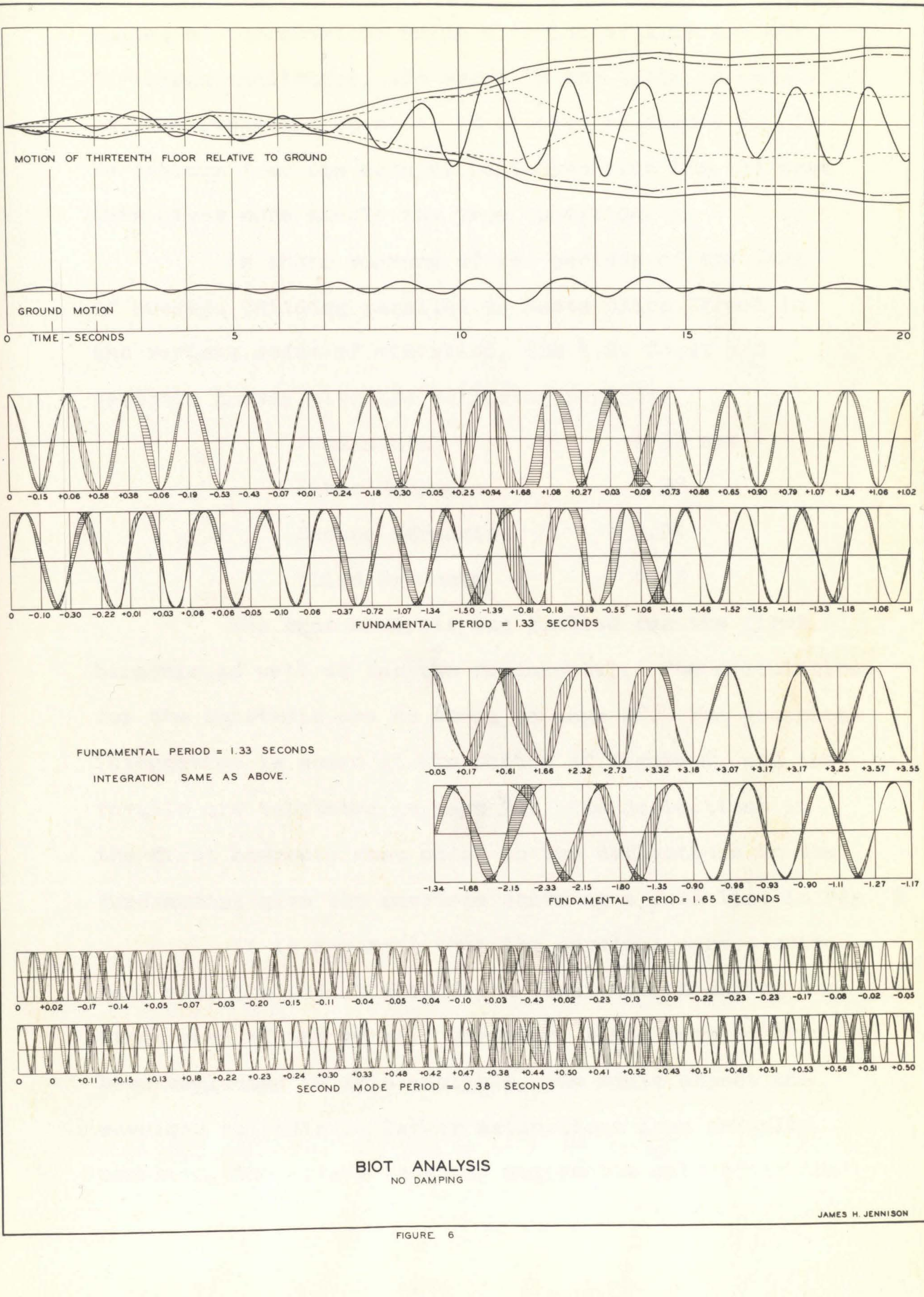

MOTION OF THIRTEENTH FLOOR RELATIVE TO GROUND
GROUND MOTION
0 TIME - SECONDS 5 10 15 20
0 -0.15 +0.06 +0.58 +0.38 -0.06 -0.19 -0.53 -0.43 -0.07 +0.01 -0.24 -0.18 -0.30 -0.05 +0.25 +0.94 +1.68 +1.08 +0.27 -0.03 -0.09 +0.73 +0.88 +0.65 +0.90 +0.79 +1.07 +1.34 +1.06 +1.02
0 -0.10 -0.30 -0.22 +0.01 +0.03 +0.06 +0.06 -0.05 -0.10 -0.06 -0.37 -0.72 -1.07 -1.34 -1.50 -1.39 -0.81 -0.18 -0.19 -0.55 -1.06 -1.46 -1.46 -1.52 -1.55 -1.41 -1.33 -1.18 -1.06 -1.11
FUNDAMENTAL PERIOD = 1.33 SECONDS
FUNDAMENTAL PERIOD = 1.33 SECONDS
INTEGRATION SAME AS ABOVE.
-0.05 +0.17 +0.61 +1.66 +2.32 +2.73 +3.32 +3.18 +3.07 +3.17 +3.17 +3.25 +3.57 +3.55
-1.34 -1.68 -2.15 -2.33 -2.15 -1.80 -1.35 -0.90 -0.98 -0.93 -0.90 -1.11 -1.27 -1.17
FUNDAMENTAL PERIOD = 1.65 SECONDS
0 +0.02 -0.17 -0.14 +0.05 -0.07 -0.03 -0.20 -0.15 -0.11 -0.04 -0.05 -0.04 -0.10 +0.03 -0.43 +0.02 -0.23 -0.13 -0.09 -0.22 -0.23 -0.23 -0.17 -0.08 -0.02 -0.05
0 0 +0.11 +0.15 +0.13 +0.18 +0.22 +0.23 +0.24 +0.30 +0.33 +0.48 +0.42 +0.47 +0.38 +0.44 +0.50 +0.41 +0.52 +0.43 +0.51 +0.48 +0.51 +0.53 +0.56 +0.51 +0.50
SECOND MODE PERIOD = 0.38 SECONDS
BIOT ANALYSIS
NO DAMPING
JAMES H. JENNISON

FIGURE 6

values of the envelope using a period of 1.65 seconds for large amplitudes, are shown in the table on page 25, and the resulting envelope is plotted in Figure 6. It is obvious from the results (dot-dash line Fig. 6) that this gives more nearly the true condition.

In their summary of the periods of the Bank of America Building parallel to Santa Clara Street in the various modes of vibration, the U.S. Coast and Geodetic Survey give the following periods.

Fundamental	1.33 sec.
First Harmonic	0.38
Second Harmonic	0.24
Third Harmonic	0.18

The Biot analysis was applied for the first harmonic as well as for the fundamental. The calculations for the constants are as shown on page 28. The graphical integration is shown at the bottom of Figure 6, and the results are tabulated on page 29. The deflections in the first harmonic when added to the deflections in the fundamental give the envelope shown as a full line in Fig. 6.

It is apparent that the envelope agrees quite well with the actual thirteenth floor motion during the first part of the quake. Moreover, since damping has been neglected in this analysis, one would expect the envelope to indicate larger deflections than actually occurred, the effect becoming noticeable only after the

CALCULATION OF CONSTANTS
— FIRST HARMONIC —

$T_1 = 0.38$ SEC.

$\gamma_1 = \frac{1}{T_1} = \frac{1}{0.38} = 2.63$

$j_1 = \frac{1}{T_1^2} = 6.92$

$n = 12$

$R = 2$

$\alpha = 24$

$\lambda_1 \quad \tan \lambda_1 = 24$

$\lambda_1 = 259°20' = 4.53$ RAD.

$$t_1 = \frac{T_1 \lambda_1}{2\pi} = \frac{0.38 \times 4.53}{2\pi} = 0.274$$

$$\beta_1 = \frac{\cos \lambda_1}{1 + \frac{\alpha \cos \lambda_1}{\lambda_1^2}} = \frac{0.185}{1 + \frac{24 \times 0.185}{4.53^2}} = 0.152$$

$$B_1 = \frac{2\alpha\beta_1}{\lambda_1^4} = \frac{2 \times 24 \times 0.152}{4.53^4} = 0.0173$$

$$\cos(\lambda_1 \mathcal{E}_1) = \cos \frac{259°20'}{12} = 0.93$$

$$K_1 = 2\pi \gamma_1 j_1 t_1^2 B_1 \cos(\lambda_1 \mathcal{E}_1)$$

$$K_1 = 2\pi \times 2.63 \times 6.92 \times 0.274^2 \times 0.0173 \times 0.93 = 0.138$$

$$F(\gamma_1) = 39.4 \sqrt{A_1^2 + A_2^2}$$

$$U_1 = \frac{K_1 \; F(\gamma_1) \; P}{E \; y \; 2\pi\gamma_1} = 0.138 \times 39.4 \times 1.114 \times 4 \sqrt{A_1^2 + A_2^2}$$

E = ENLARGEMENT FACTOR.

y = AMPLITUDE OF SINE AND COSINE CURVES.

P = PLANIMETER FACTOR TO REDUCE AREA TO SQ. IN.

$$U_1 = 0.49 \sqrt{A_1^2 + A_2^2}$$

lapse of a considerable time interval. Thus, the results give exactly the condition which one might expect.

First Harmonic

Areas and Envelope Values

A_1	A_2	U_1
0	0	0
+0.02	0	0.0098
-0.17	+0.11	0.0982
-0.14	+0.15	0.101
+0.05	+0.13	0.0682
-0.07	+0.18	0.0945
-0.03	+0.22	0.109
-0.20	+0.23	0.149
-0.15	+0.24	0.139
-0.11	+0.30	0.157
-0.04	+0.33	0.163
-0.05	+0.46	0.227
-0.04	+0.42	0.206
-0.10	+0.47	0.235
+0.03	+0.38	0.187
-0.43	+0.44	0.302
+0.02	+0.50	0.245
-0.23	+0.41	0.230
-0.13	+0.52	0.263
-0.09	+0.43	0.215
-0.22	+0.51	0.272
-0.23	+0.48	0.260
-0.23	+0.51	0.286
-0.17	+0.53	0.272
-0.08	+0.56	0.277
-0.02	+0.51	0.250
-0.05	+0.50	0.246
-0.05	+0.51	0.251

Chapter V

Comparison of the Biot and Westergaard Analyses and Conclusions Regarding Them

The results of the application of these two methods of analysis to an actual building in an actual earthquake show that, for a case where the ground motion has a period of the same order of magnitude as the fundamental period of the building, the results of both analyses check the recorded motion of the building. In fact, for practical purposes, the results are equally good. In both cases, it is necessary to know the fundamental period of the building to a degree of accuracy which might be rather difficult to obtain in a case where the actual behavior of the building was unknown. The fact that the fundamental assumptions regarding the symmetry and homogeneity of the building are only approximately correct did not seem to cause much trouble in either analysis.

The Westergaard analysis has several distinct advantages. First of all, the principle underlying it is simple and the physical significance of each step is apparent as the work progresses. Moreover, the result, being a curve of motion rather than an envelope, gives a more comprehensible representation of the behavior of the building. Then too, the graphical work can be carried out much more rapidly than the graphical integration in the Biot analysis can be performed; and no special instrument is needed for the Westergaard analysis, whereas a planimeter is required for the Biot analysis.

Damping can be easily considered in the Westergaard analysis, whereas the Biot analysis neglects damping. However, since the envelope can be reduced by a certain percentage at the plotted points and the cumulative effects carried on from point to point, the effect of damping on the Biot analysis can be quickly approximated.

One of the most encouraging things brought out by comparison of the two analyses is the fact that the two analyses produce approximately the same results when the same conditions and assumptions are imposed; so that one is confident that both theories are about equally valid.

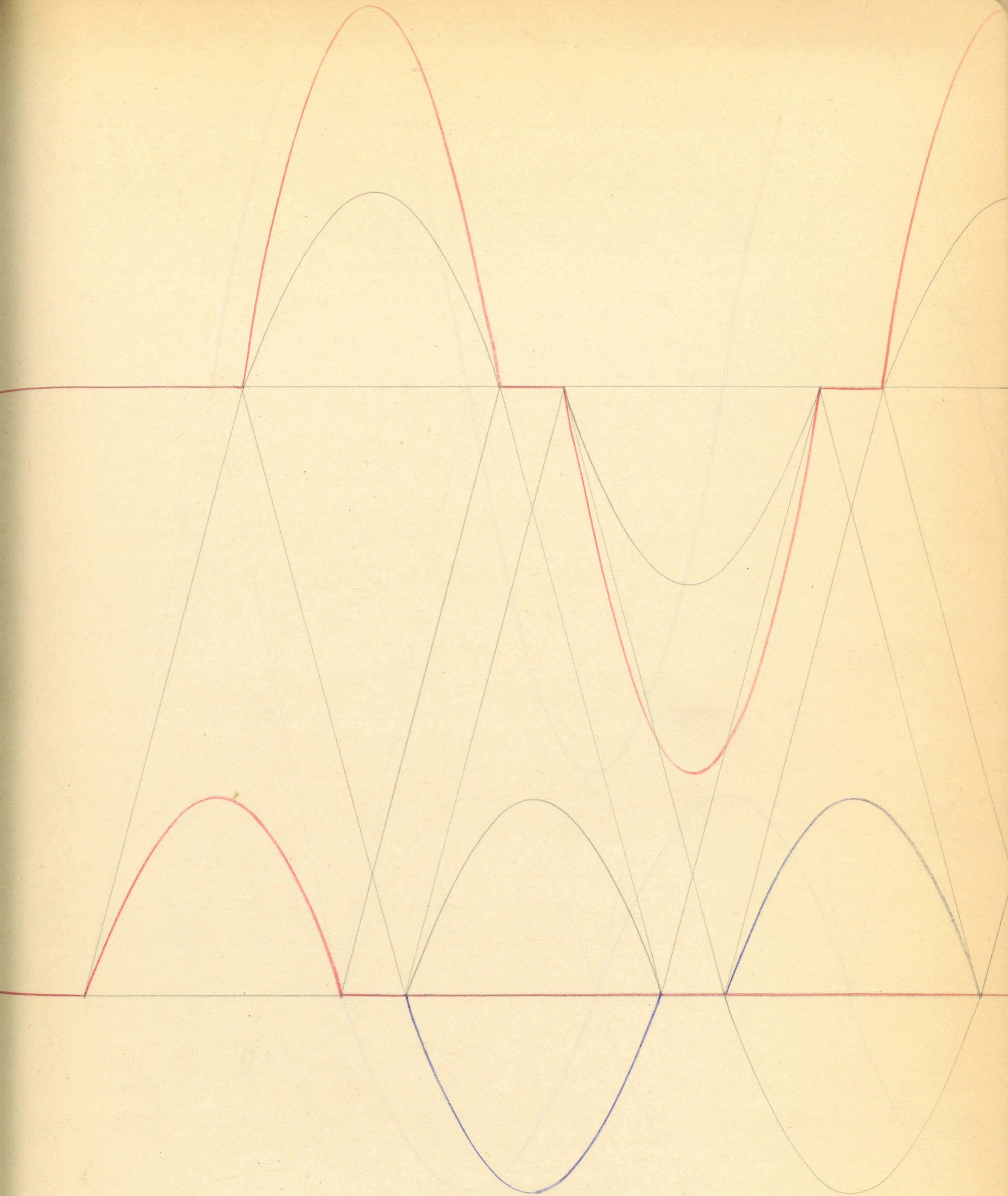
Chapter VI

Application of the Westergaard Analysis to Hypothetical Buildings

Analyses of theoretical buildings of various periods were made with the ground motion assumed, first, as one hump of a normal sine wave, and, second, as a half cycle of an off-set sine wave. The analyses are shown in Figures 7 to 19; and Figure 20 shows the curves of maximum relative displacement plotted against the period ratio. In observing that for a period ratio of one half or greater the maximum amplitude of relative displacement is constant at 3, one should bear in mind that all of these analyses are for a ground motion consisting of a single hump.

The results indicate that the gradual start and finish of the hump of the off-set wave give a considerably less violent shock when the period of the building is small compared to that of the ground motion. For both types of ground motion, however, the maximum amplitude of motion was obtained for the cases where the fundamental period of the building was one half, or more, of the period of the ground motion. Moreover, as the period of the building was made a smaller fraction of the period of the ground motion, the maximum displacement relative to the ground diminishes rapidly, as is shown in Figure 20; but the decrease was somewhat more rapid in the case of the off-set sine wave than in the case of the normal sine curve.

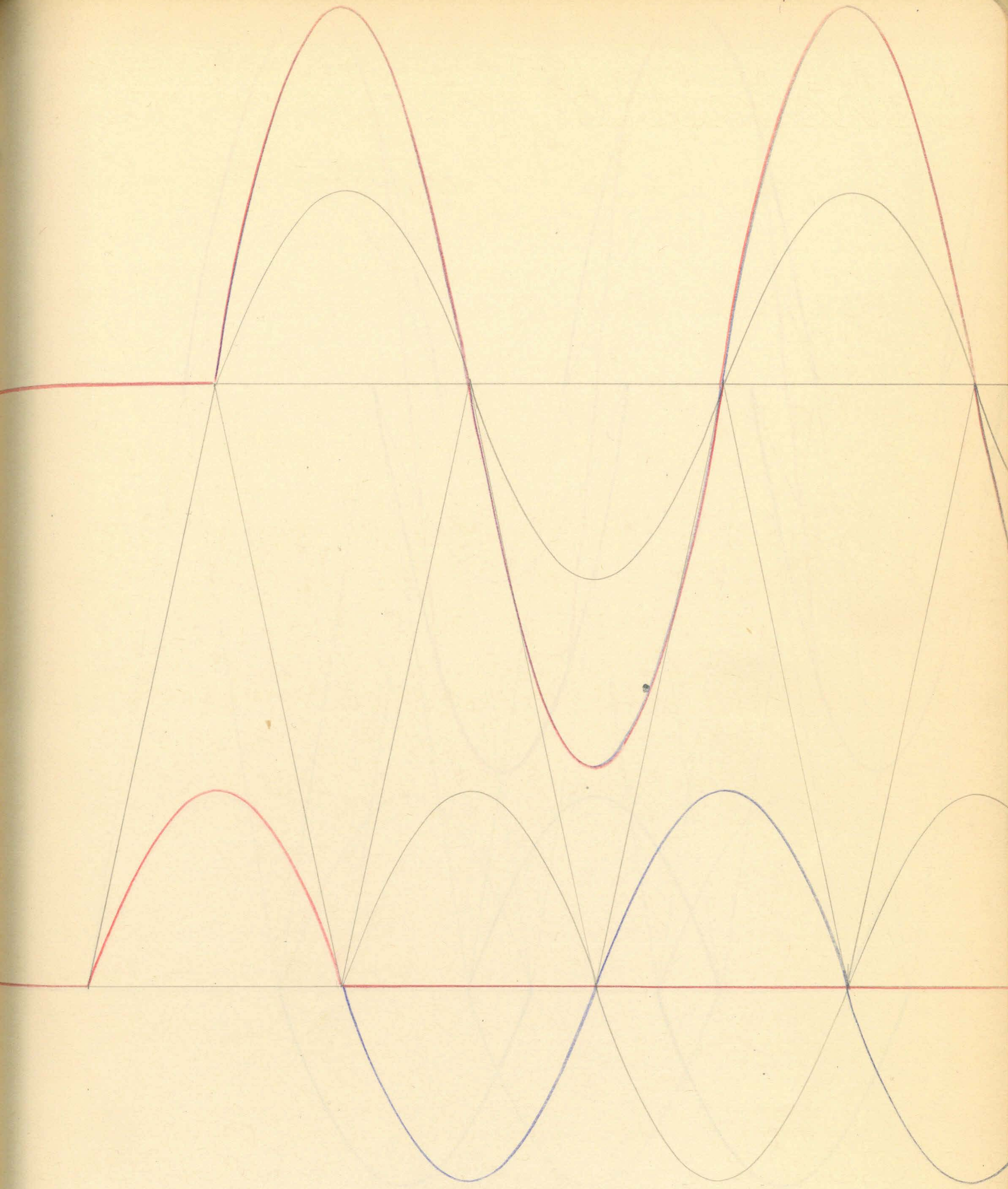
The smoother curves shown in Figures 14 to 19 as compared to Figures 7 to 13 would also indicate much lower accelerations resulting from the off-set sine wave than would be caused by the normal sine curve. For period ratios of one half or greater, the maximum deflections are the same for both cases, and so the difference in the magnitude of accelerations becomes the outstanding effect. Thus, both from the standpoint of deflections, and from the standpoint of accelerations, the gradual curve of the off-set sine curve is beneficial.



Transmitted Wave.
Motion Due to Reflection.
Total Motion.

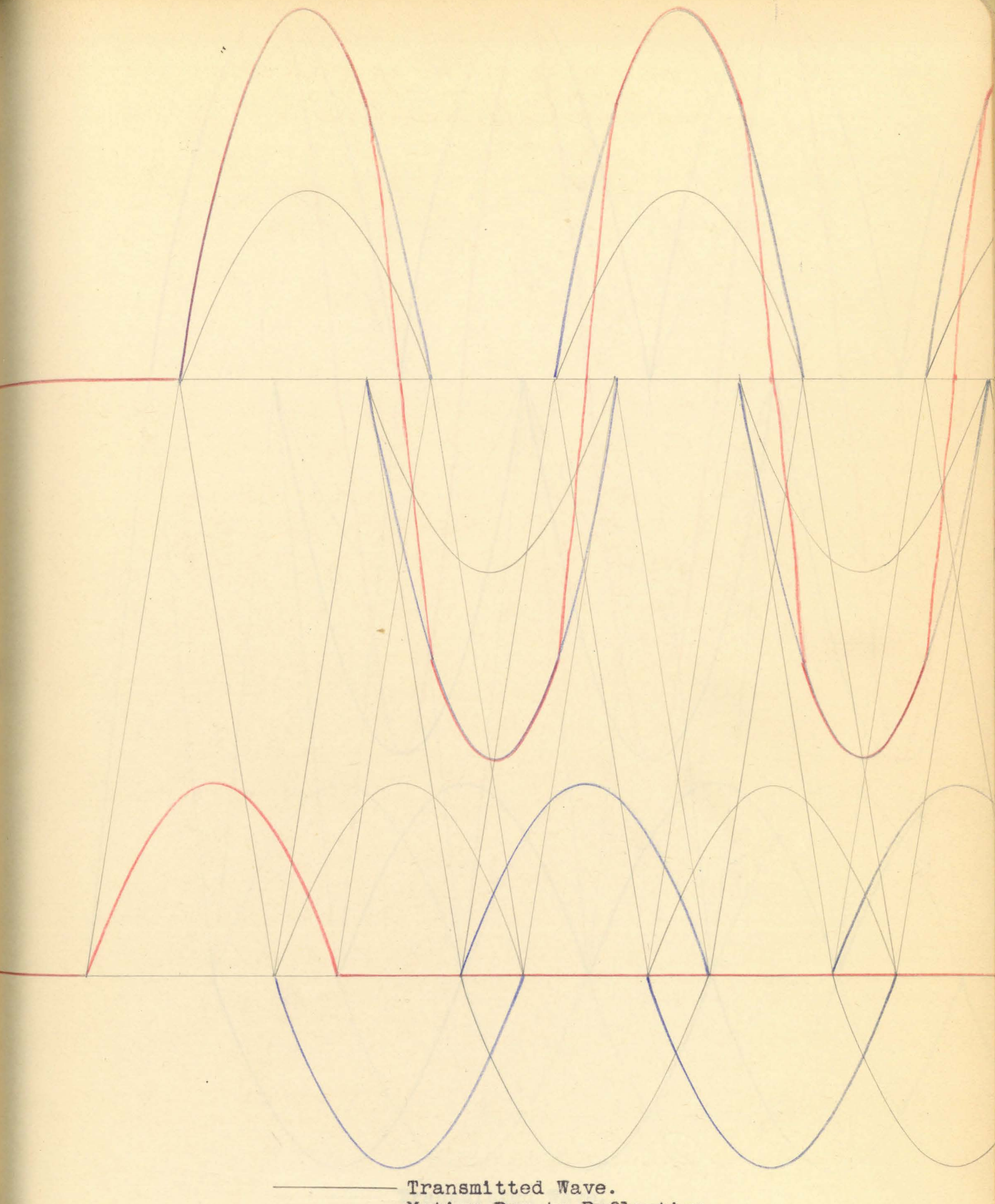
PERIOD OF BUILDING 1¼ TIMES PERIOD OF GROUND MOTION

Fig. 7



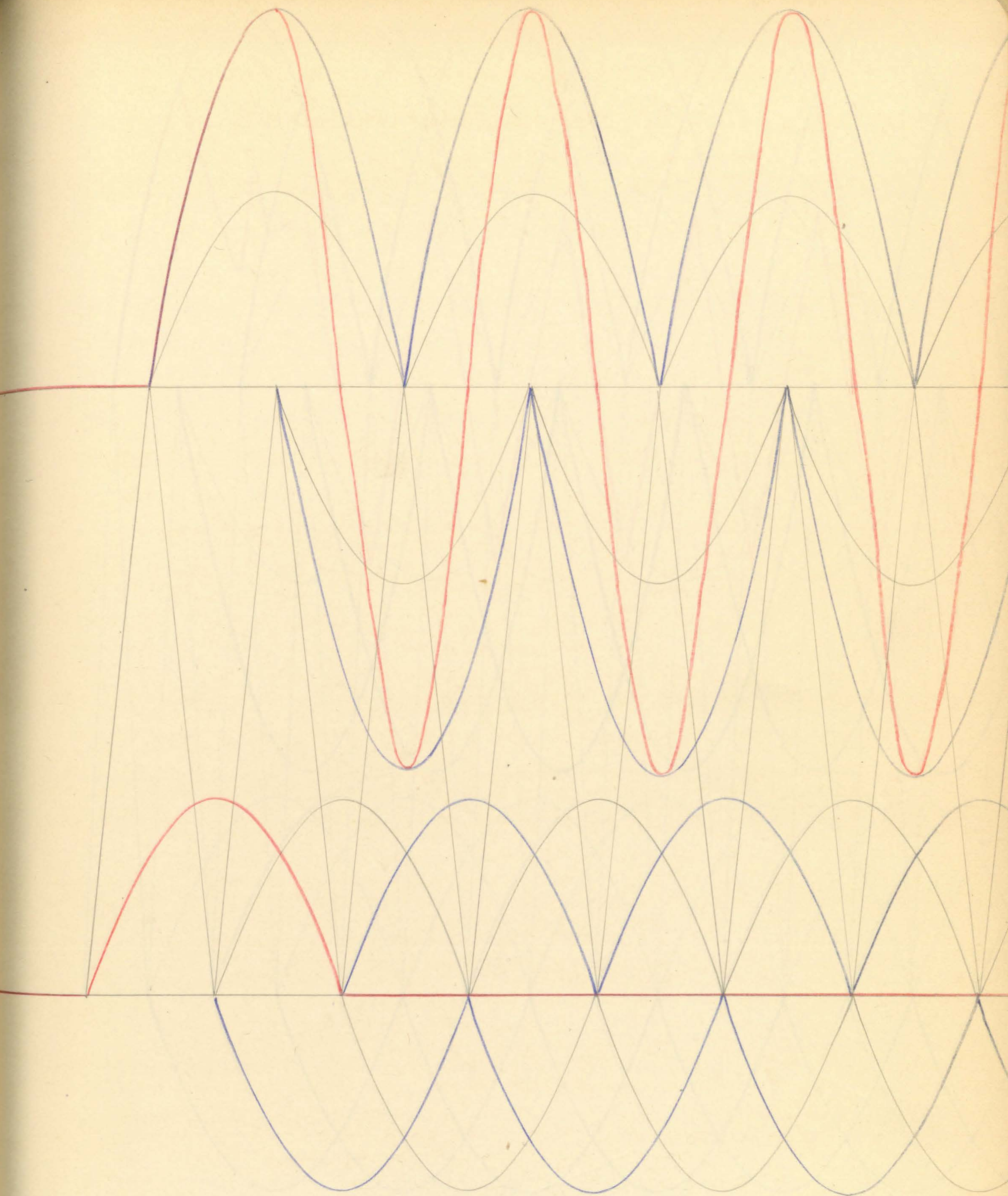
PERIOD OF BUILDING EQUAL TO PERIOD OF GROUND MOTION

Fig. 8



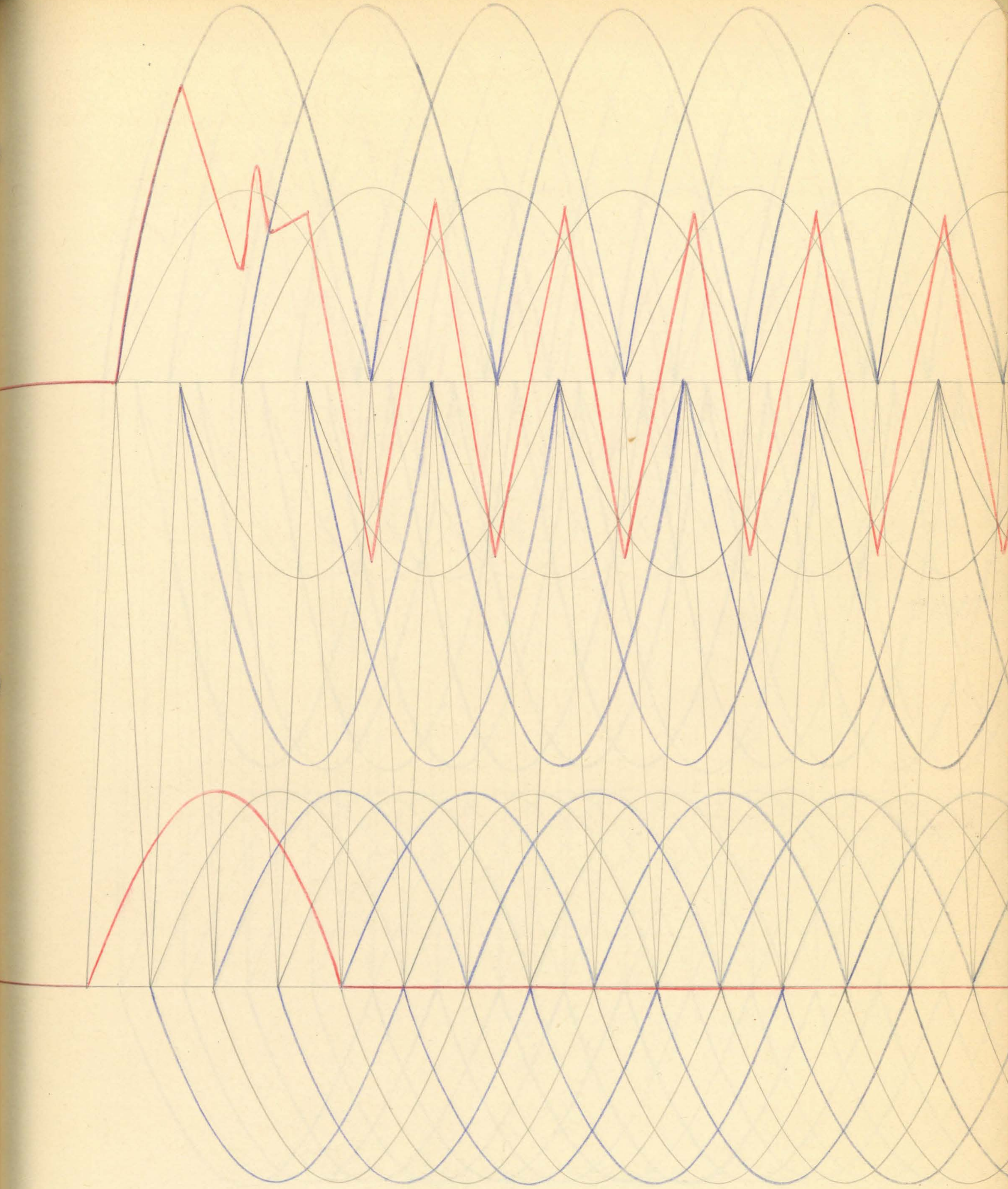
PERIOD OF BUILDING THREE-FOURTHS PERIOD OF GROUND MOTION

Fig. 9



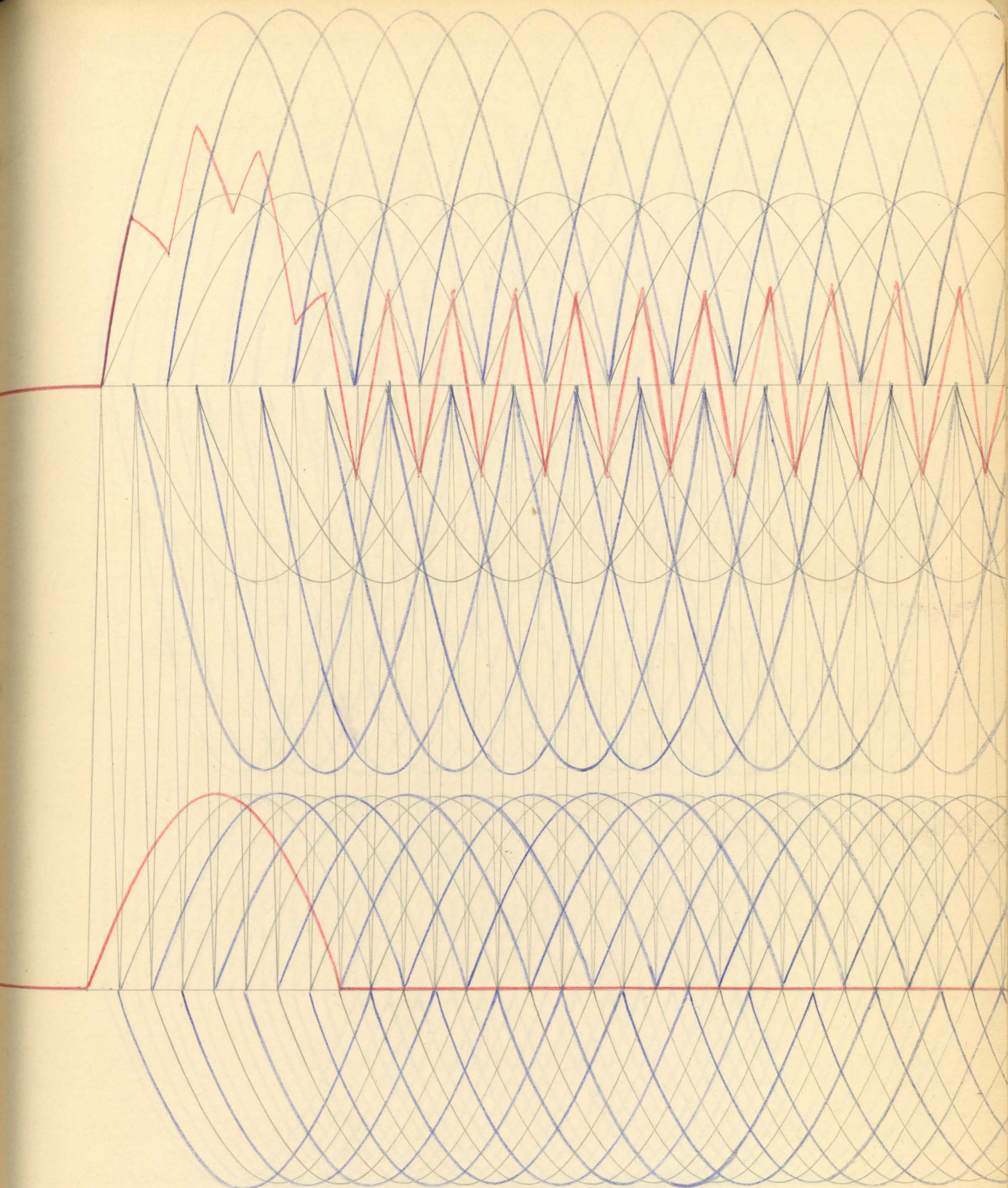
Transmitted Wave.
Motion Due to Reflection.
Total Motion.

PERIOD OF BUILDING ONE-HALF PERIOD OF GROUND MOTION
Fig 10



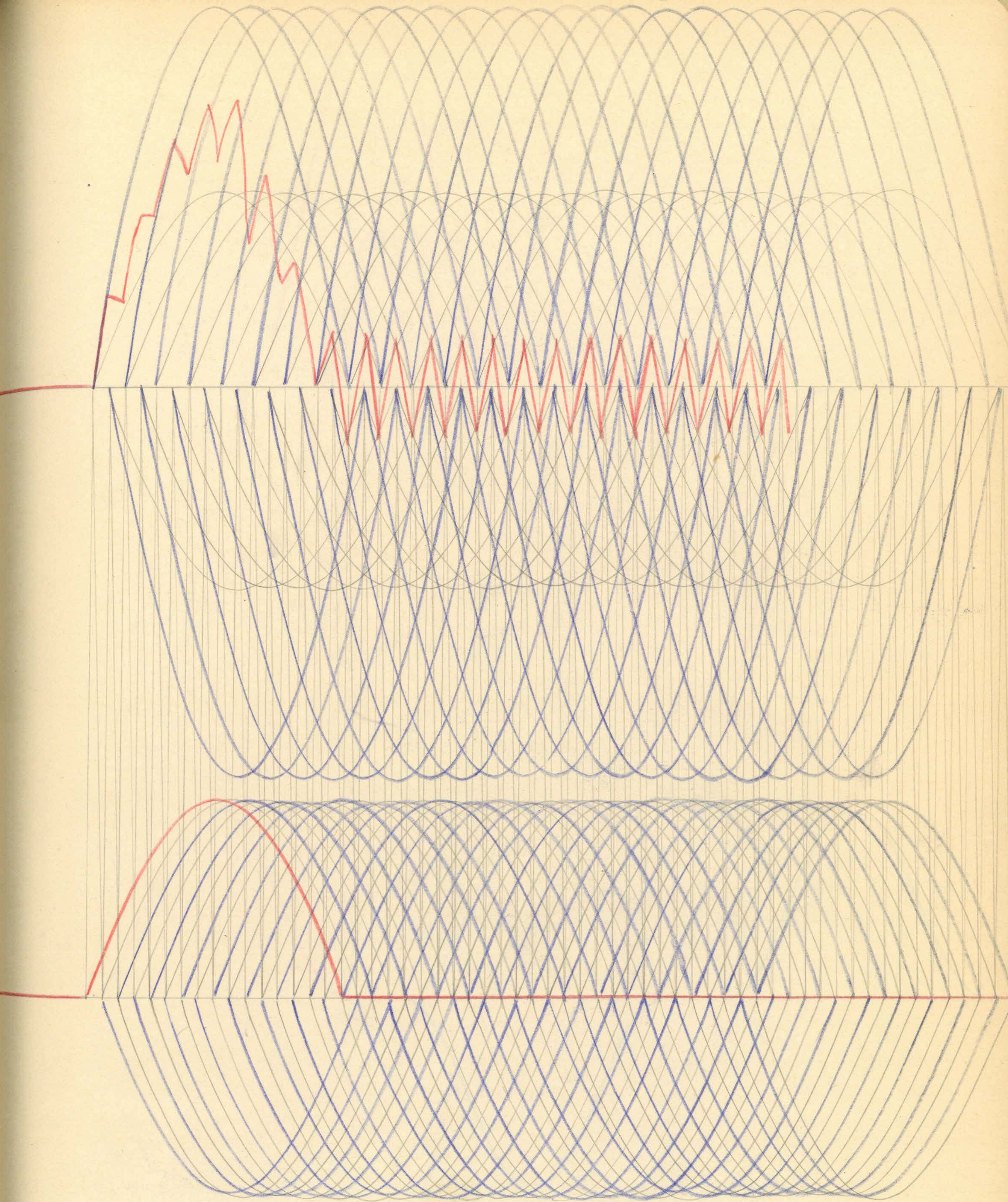
PERIOD OF BUILDING ONE-FOURTH PERIOD OF GROUND MOTION

Fig. 11



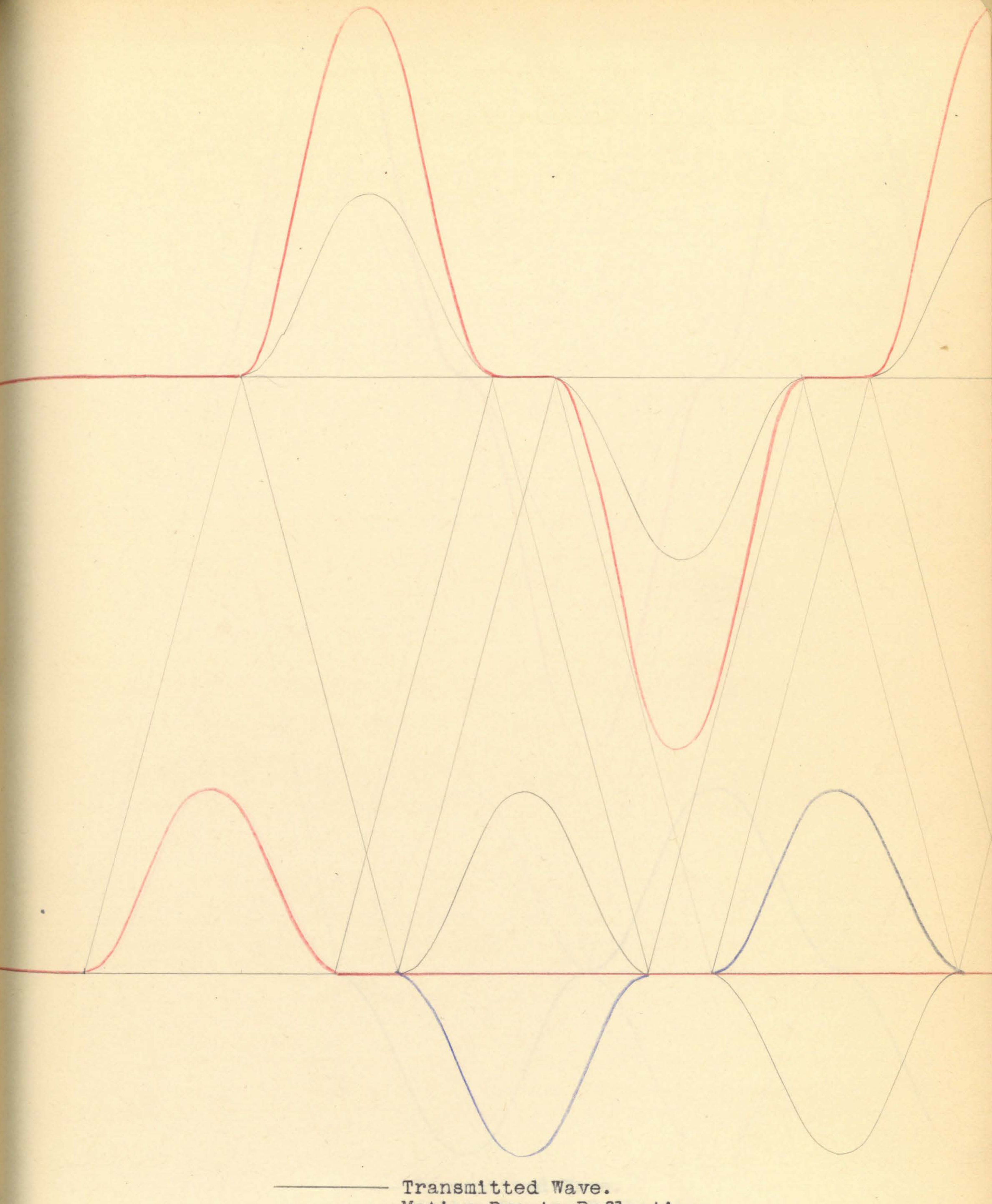
Transmitted Wave.
Motion Due to Reflection.
Total Motion.

PERIOD OF BUILDING ONE-EIGHTH PERIOD OF GROUND
Fig. 12



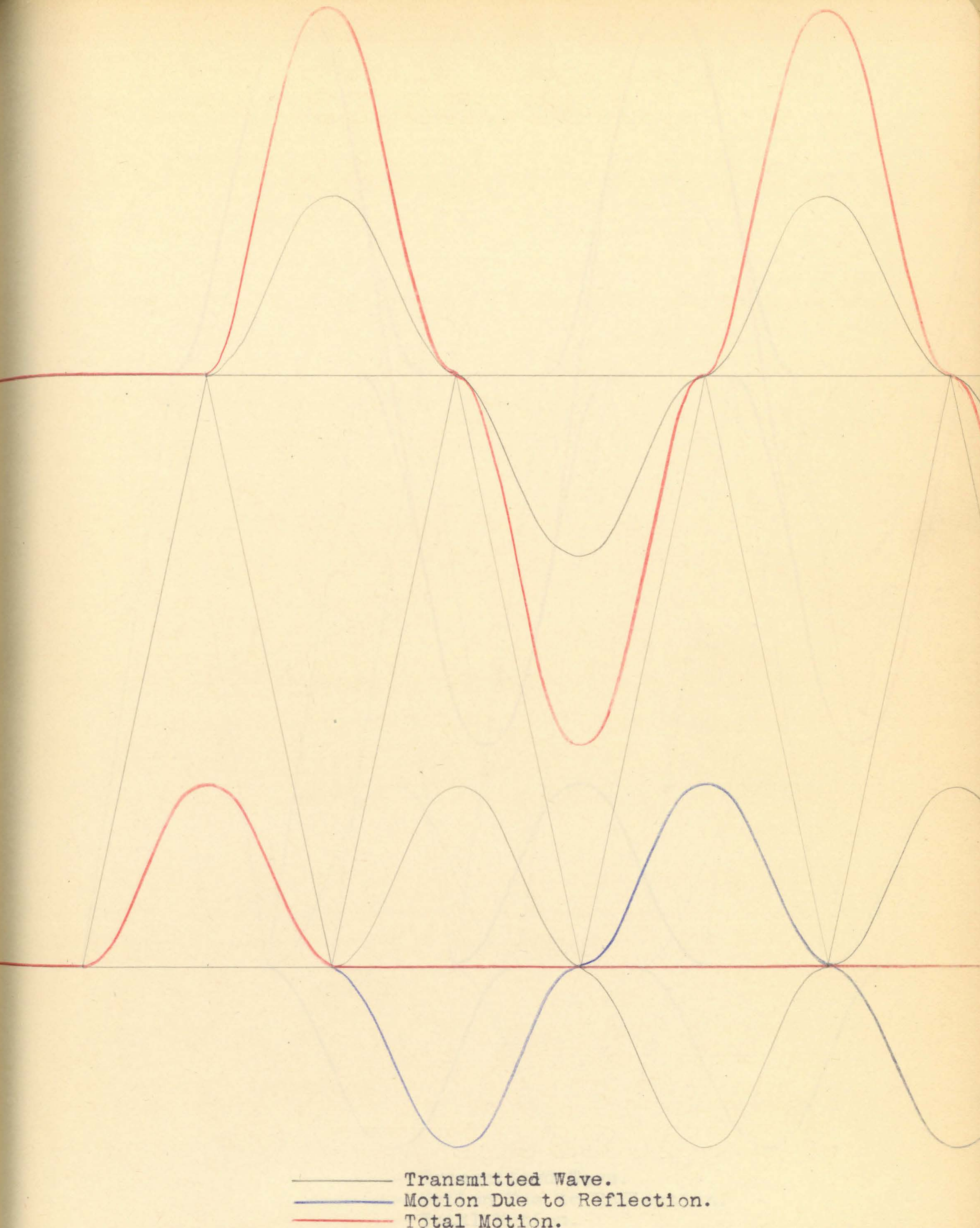
PERIOD OF BUILDING ONE-SIXTEENTH PERIOD OF GROUND MOTION.

Fig. 13



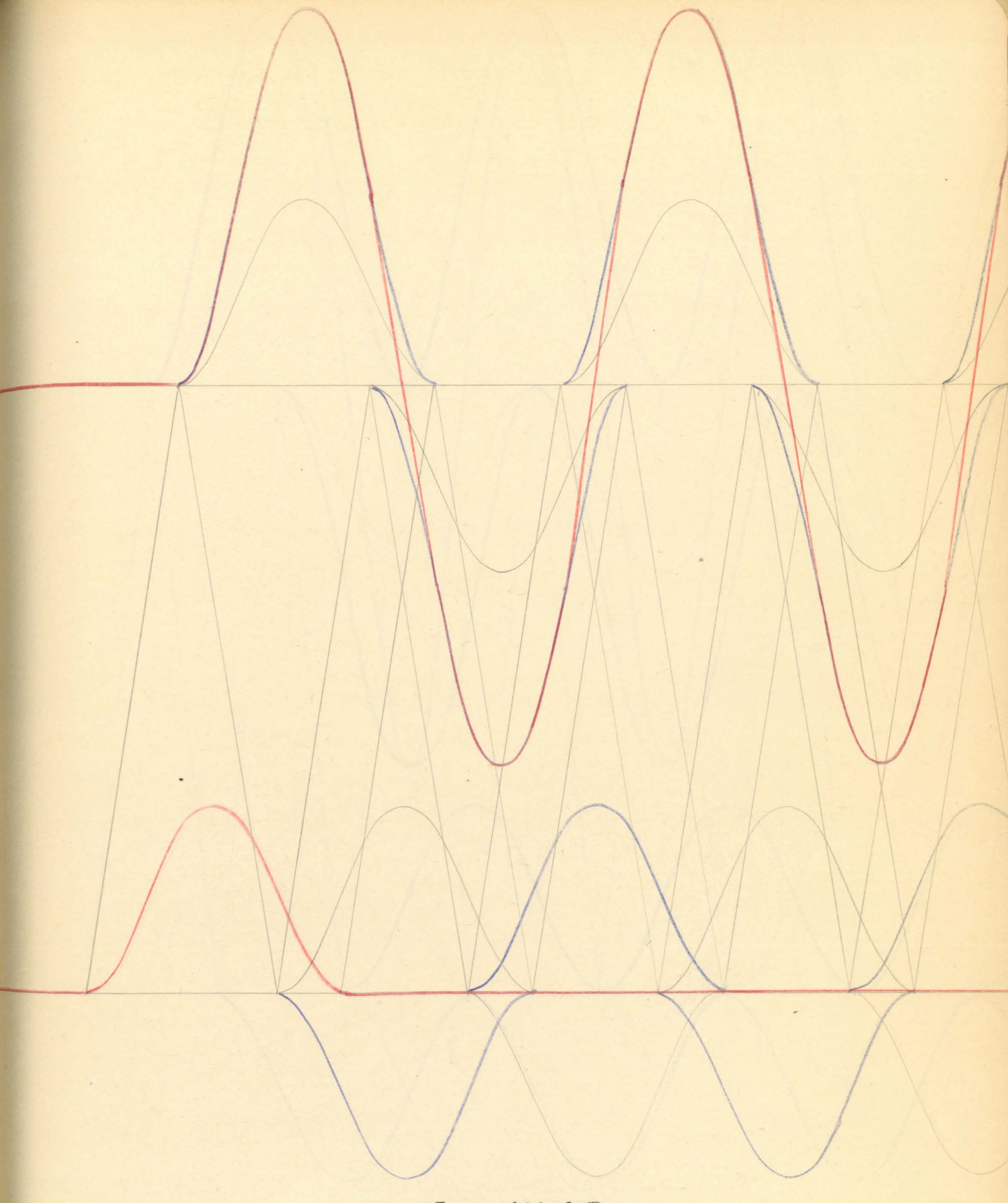
PERIOD OF BUILDING $1\frac{1}{4}$ TIMES PERIOD OF GROUND MOTION

Fig. 14



PERIOD OF BUILDING EQUAL TO PERIOD OF GROUND MOTION

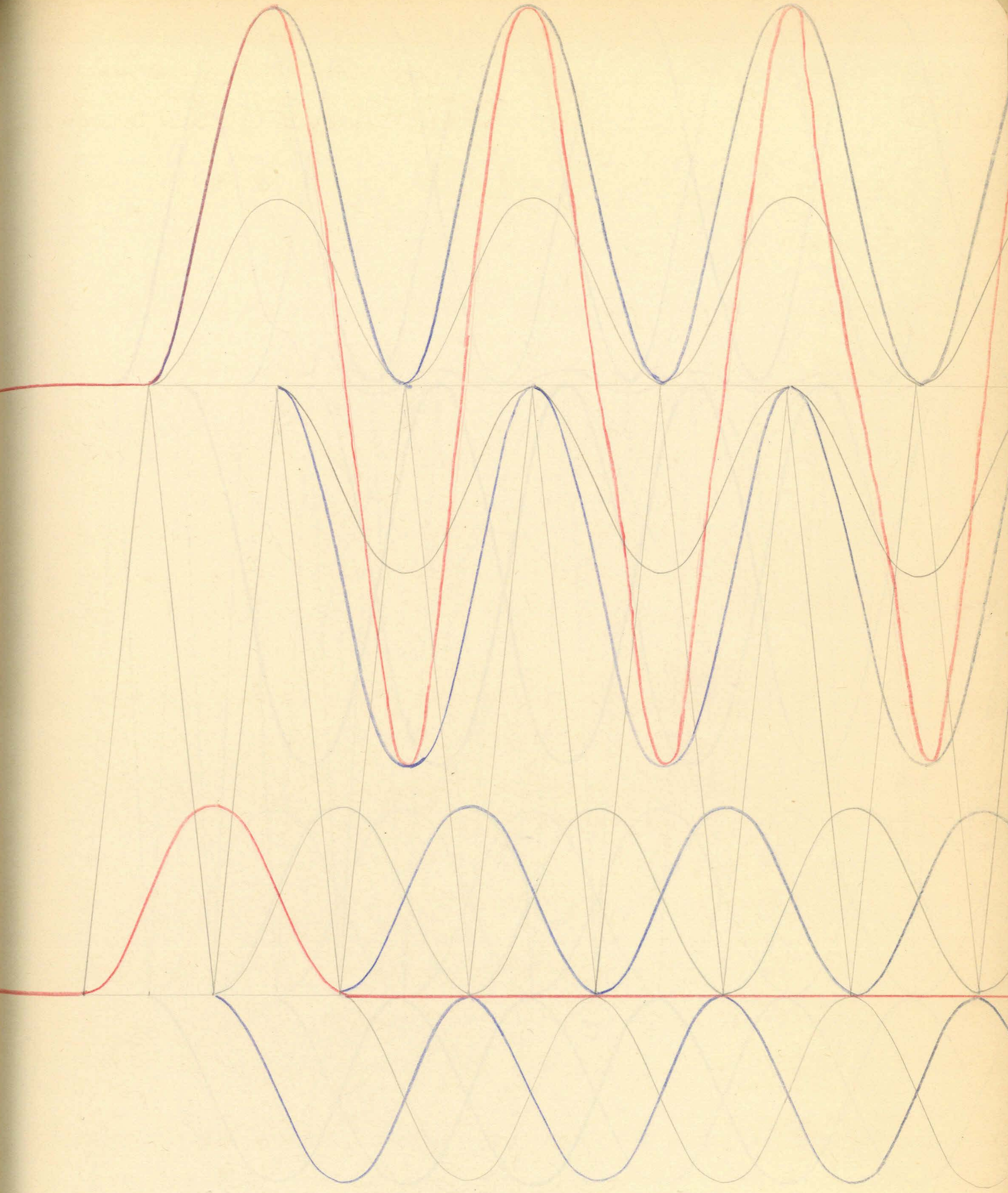
Fig. 15



Transmitted Wave.
Motion Due to Reflection.
Total Motion.

PERIOD OF BUILDING THREE-FOURTHS PERIOD OF GROUND MOTION

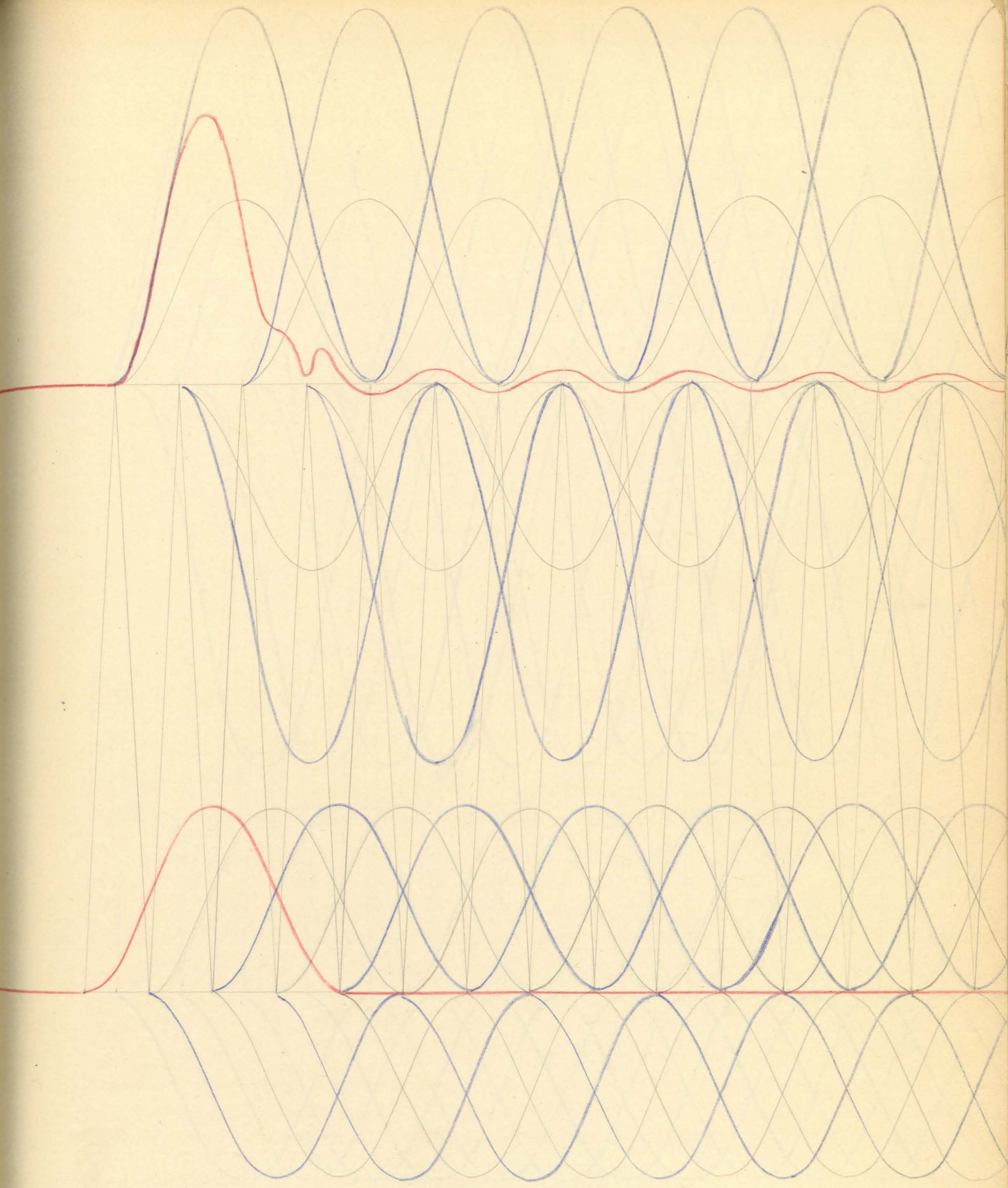
Fig. 16



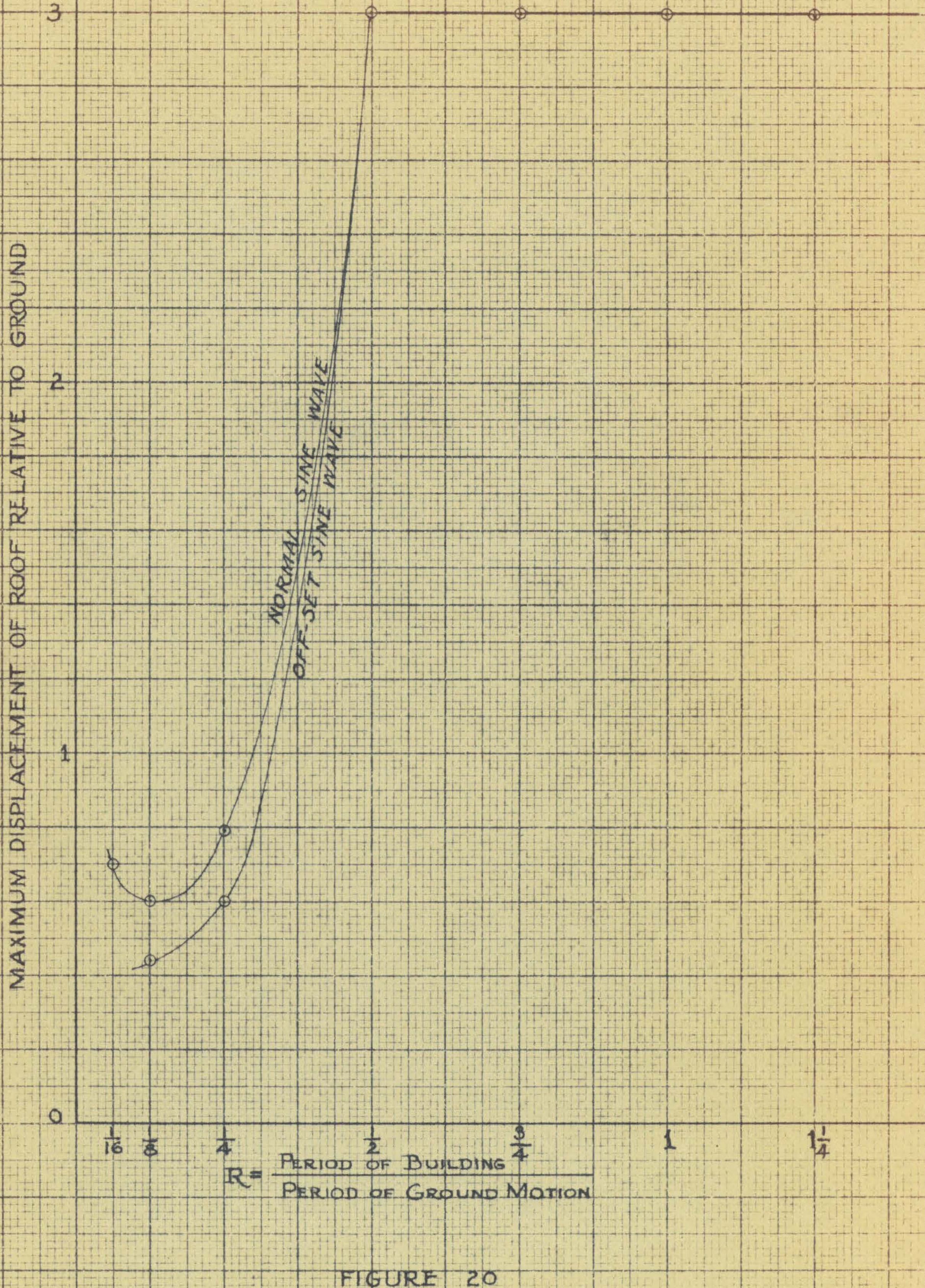
Transmitted Wave.
Motion Due to Reflection.
Total Motion.

PERIOD OF BUILDING ONE-HALF PERIOD OF GROUND MOTION

Fig. 17



PERIOD OF BUILDING ONE-FOURTH PERIOD OF GROUND MOTION
Fig. 18



MAXIMUM DISPLACEMENT OF ROOF RELATIVE TO GROUND
0
1
2
3
NORMAL SINE WAVE
OFF-SET SINE WAVE
1/16
1/8
1/4
1/2
3/4
1
1 1/4
R = Period of Building / Period of Ground Motion

FIGURE 20

PART II

Chapter VII

Description of Shaking Table Used in Analysis of Transient Vibrations

The shaking table used for the studies of transient vibrations is shown in Figures 21, 22, and 23. The complete apparatus is shown in Figure 21,

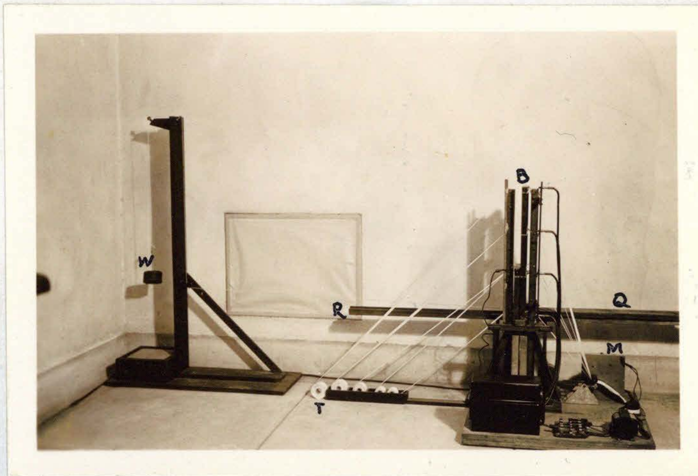


Figure 21

while Figures 22 and 23 show the main portion of the apparatus in closer detail. The bent (B) is mounted upon a base between rigid columns which support tapes (T) upon which the record of motion is secured. The motion of the base, or ground, is produced by releasing the quake stick (Q) which is pulled along the track (R) by the weights (W) and a system of cord and pulleys. The edge of the quake stick has an irregular outline, and a roller attached to the base presses against this irregular surface, causing an oscillating motion of the base as the quake stick is drawn past the roller. Somewhat different ground motions can be obtained by using different weights to pull the quake stick, since there is considerable friction in the system, thus causing increased weight to give greater velocity.

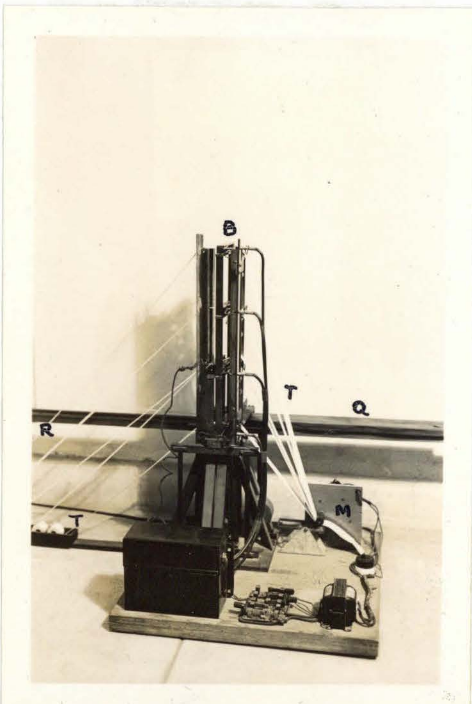


Figure 22

Figure 23

The record is obtained on the tapes (T) by means of a spark gap attached to each floor of the bent. The tapes are drawn through the spark gap at constant speed by the synchronous motor (M), the speed being such that one inch on the tape equals

0.358 seconds when the motor is operated on 50 cycle alternating current. The spark punches minute holes in the tapes, which trace the curve of motion for each floor.

The portion of the tape containing the record is placed between cover glasses and inserted in a slide machine adapted to give the desired enlargement. The curve is thus projected upon a sheet of paper as a series of light dots,and the record is traced with a pencil. Different degrees of enlargement can be obtained for various purposes, but,for most of the work, an enlargement of two times was found convenient.

Chapter VIII

Comparison of the Behavior of One Story and Three Story Bents of the Same Fundamental Period

A one story bent was constructed which had the same fundamental period as a three story bent, the one story bent being of the same height as one floor of the three story. Both were mounted on the same base, side by side, as shown in Fig. 24, and records

Figure 24

were obtained for the motion of the top of the one story bent and the three floors and roof of the three story bent, for a variety of ground motions. Both bents had a period of 0.323 seconds.

The results of the tests are given in Figures 25 to 34, the red line indicating the motion of the one story bent, and the blue line the motion of the three story bent. For the purpose of comparison, the roof motion of the one story bent was traced with the roof curve of the three story and also with the second floor motion.

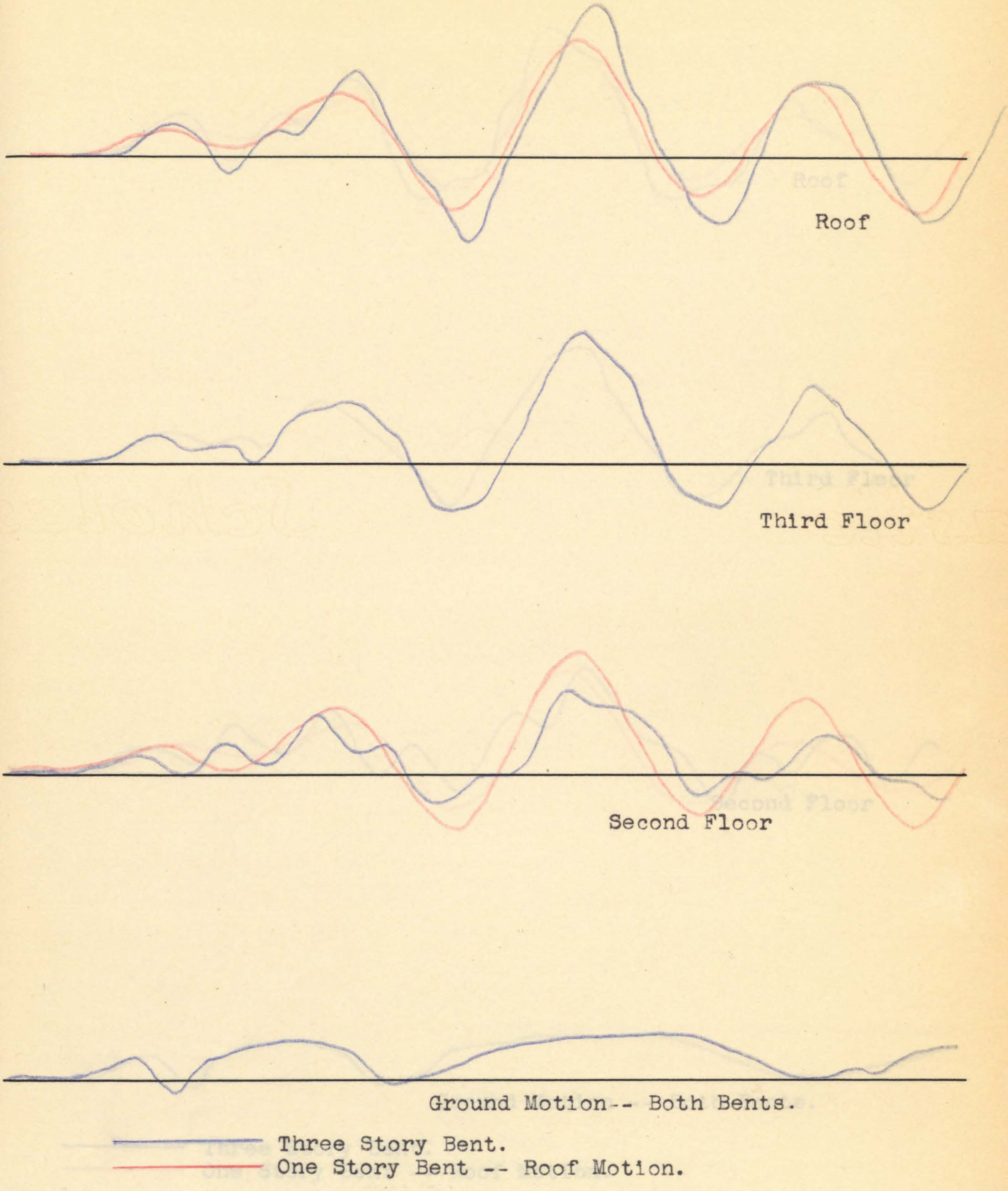
A study of these records reveals that the motion of the one story bent gives an average curve for the vibration of the three story; in other words, the motion of the one story bent is of the same form as the roof of the three story bent would have if harmonics did not influence the motion. This is brought out particularly by Figures 25 and 26, which show the motion with a long, gentle ground vibration giving resonance with the fundamental, and Figures 29 to 31, which show the effect of resonance with the harmonic on the three story bent. It should also be noted that, for a ground motion such as that of Figure 28, there was a much more violent motion of the three story bent than occurred in the case of the one story building where no harmonics were possible.

An approximate estimate of the maximum velocities and accelerations was made, and the results are tabulated on the following page. It can readily be seen that both the accelerations and the velocities of the floors of the three story bent were much greater than the corresponding values for the one story bent.

	Maximum Acceleration Ft/sec²	Rec. No.	Max. Sustained Velocity Ft/sec.²	Rec. No.
One Story Bent	11	155	0.58	155
Three Story Bent	57	155	3.1	149
Ground Motion	205	155	3.3	155

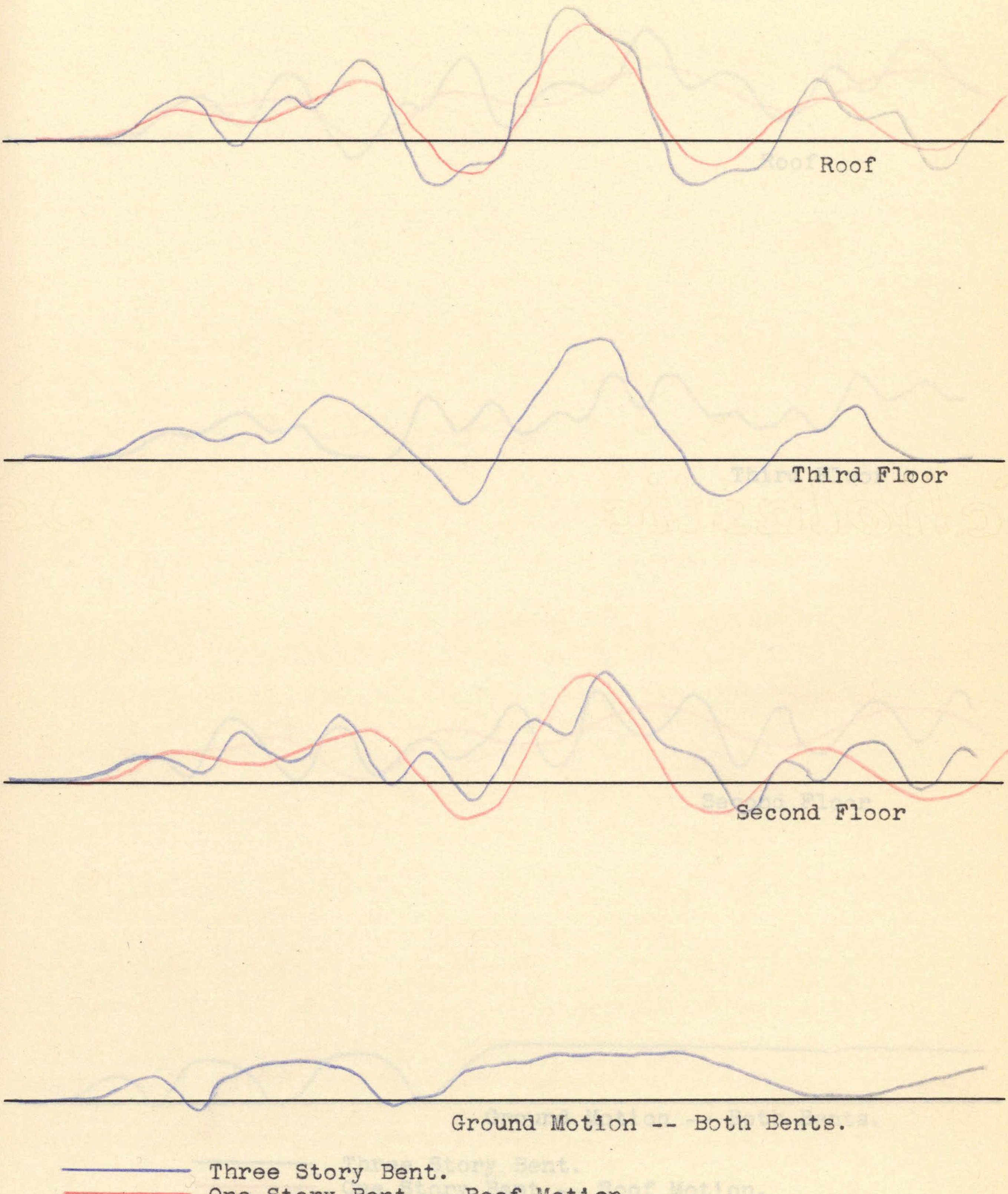
The Westergaard analysis was applied to Record No. 149 of Fig. 30, and the analysis and its results are given in Figure 35. A quake which caused the three story building to vibrate in the harmonics was purposely selected, and it can readily be seen that the motion predicted by the analysis checks the motion of the three story building fairly well. The overlapping of the deflection waves takes care of the harmonics. However, there are small oscillations in the analysis curve that did not appear in the actual motion due to the absence of harmonics of higher orders that the second harmonic. Moreover, the record for the one story bent, with no harmonics, is much different from the curve predicted by the Westergaard analysis. In this connection, it is well to bear in mind that the Westergaard analysis assumes uniform distribution of the weight of the building throughout the height.

This assumption is violated to quite a degree by the three story bent, with the mass concentrated at three points; and, of course, the one story bent, with the mass concentrated at the top, is entirely outside of the range to which the Westergaard analysis is applicable. The results seem to indicate that three stories is about the lowest number of stories to which the Westergaard analysis can safely be applied.



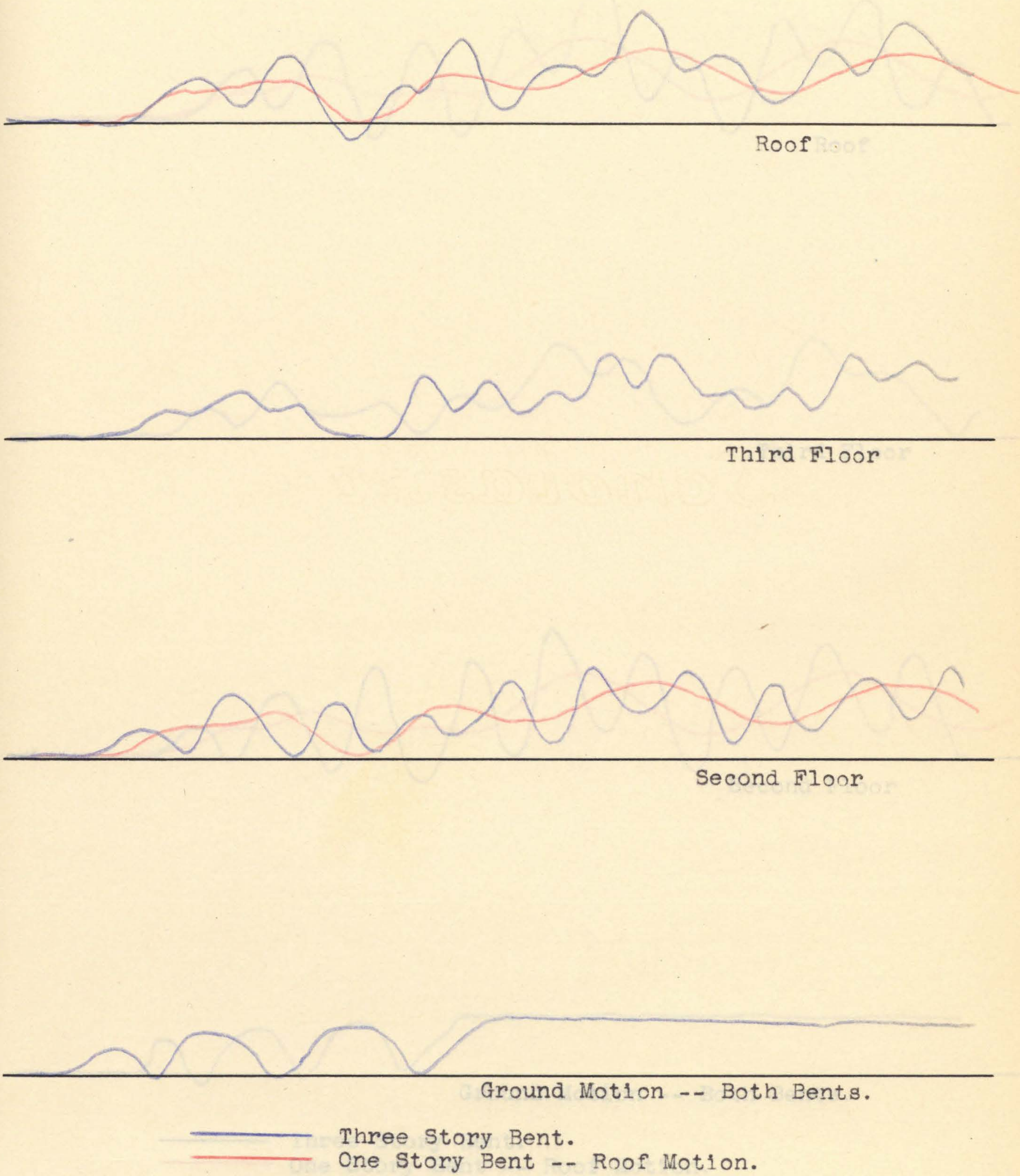
Ground motion caused by pulling quake stick through slowly by hand.
Record No. 157 6-18-35
Enlarged 2X

Fig. 25



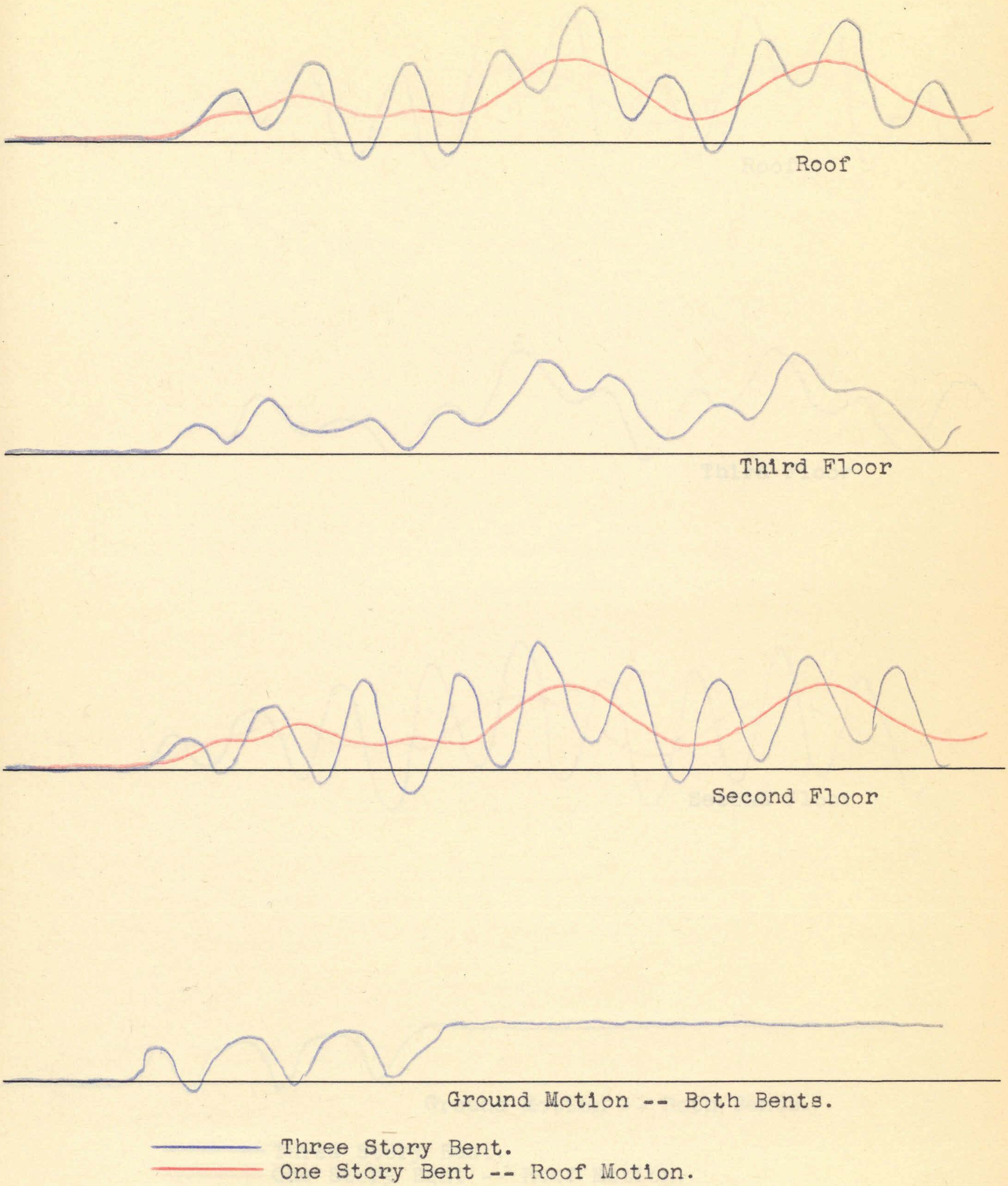
Ground motion caused by pulling quake stick through slowly by hand.
Record No. 158 6-18-35.
Enlarged 2X

Fig. 26



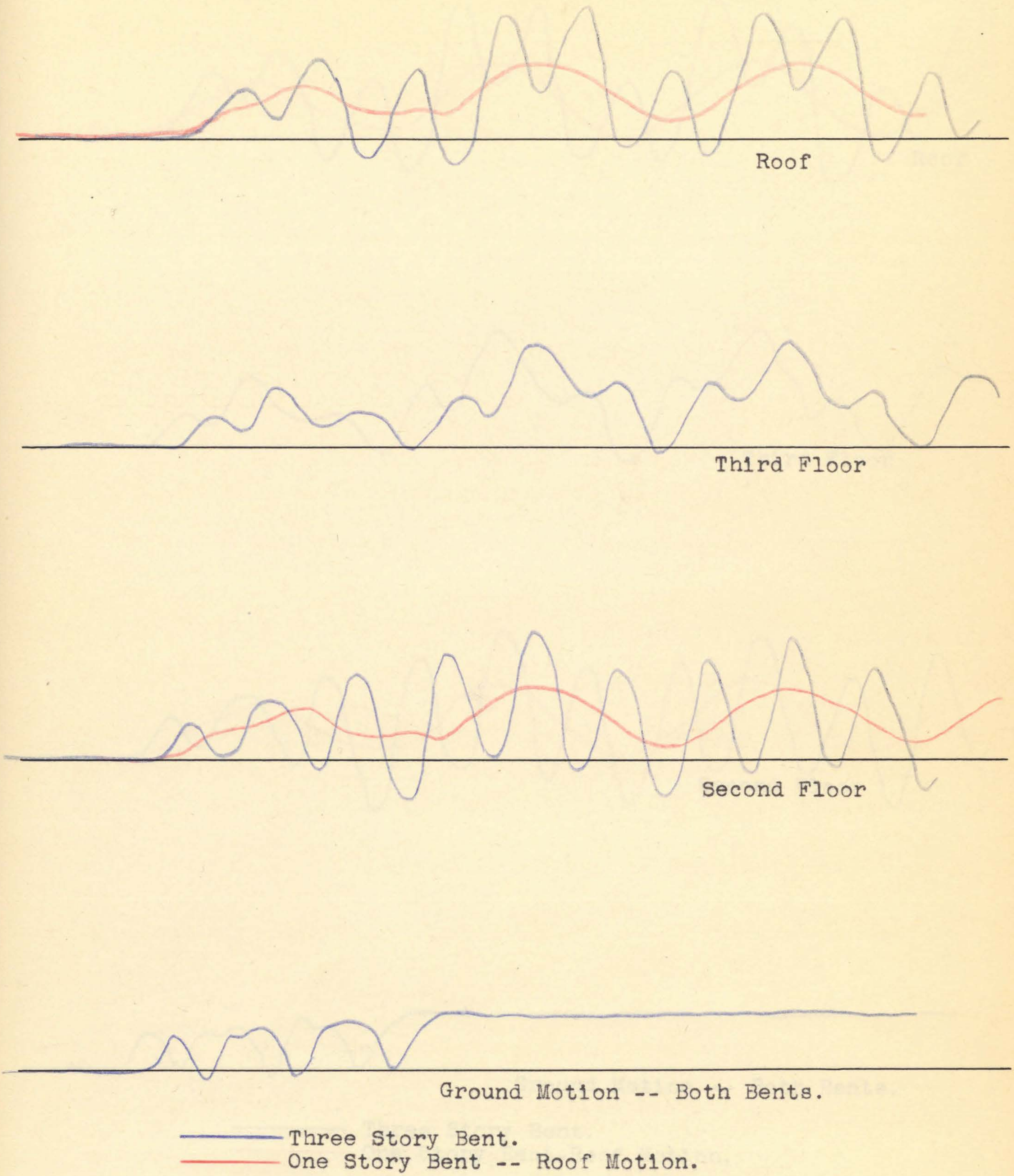
Quake Stick No. 1 Pulled by Two Pounds Weight.
Record No. 143 6-15-35
Enlarged 2X

Fig. 27



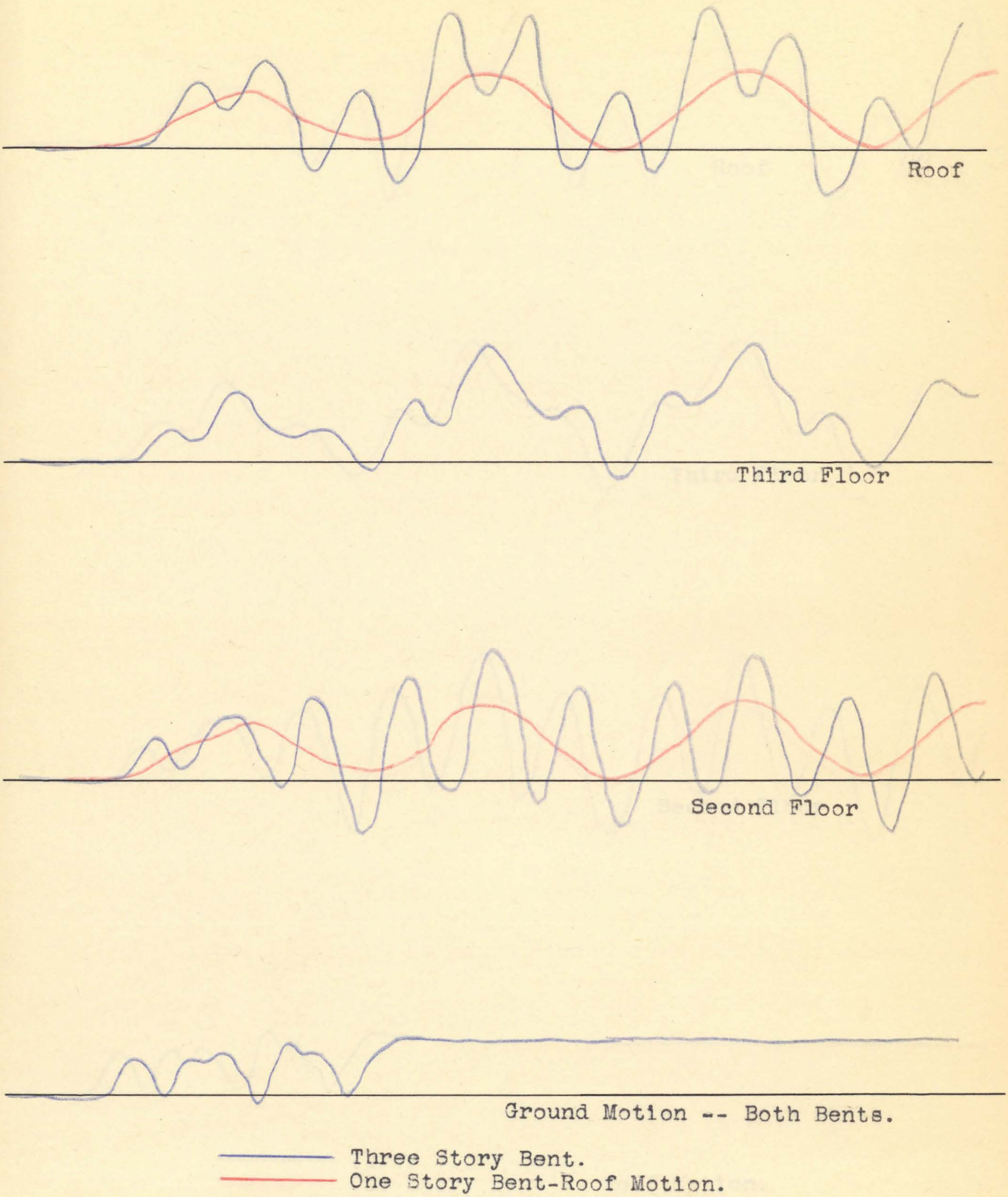
Quake Stick No. 1 Pulled by Three Pounds Weight.
Record No. 145 6-15-35
Enlarged 2X

Fig. 28



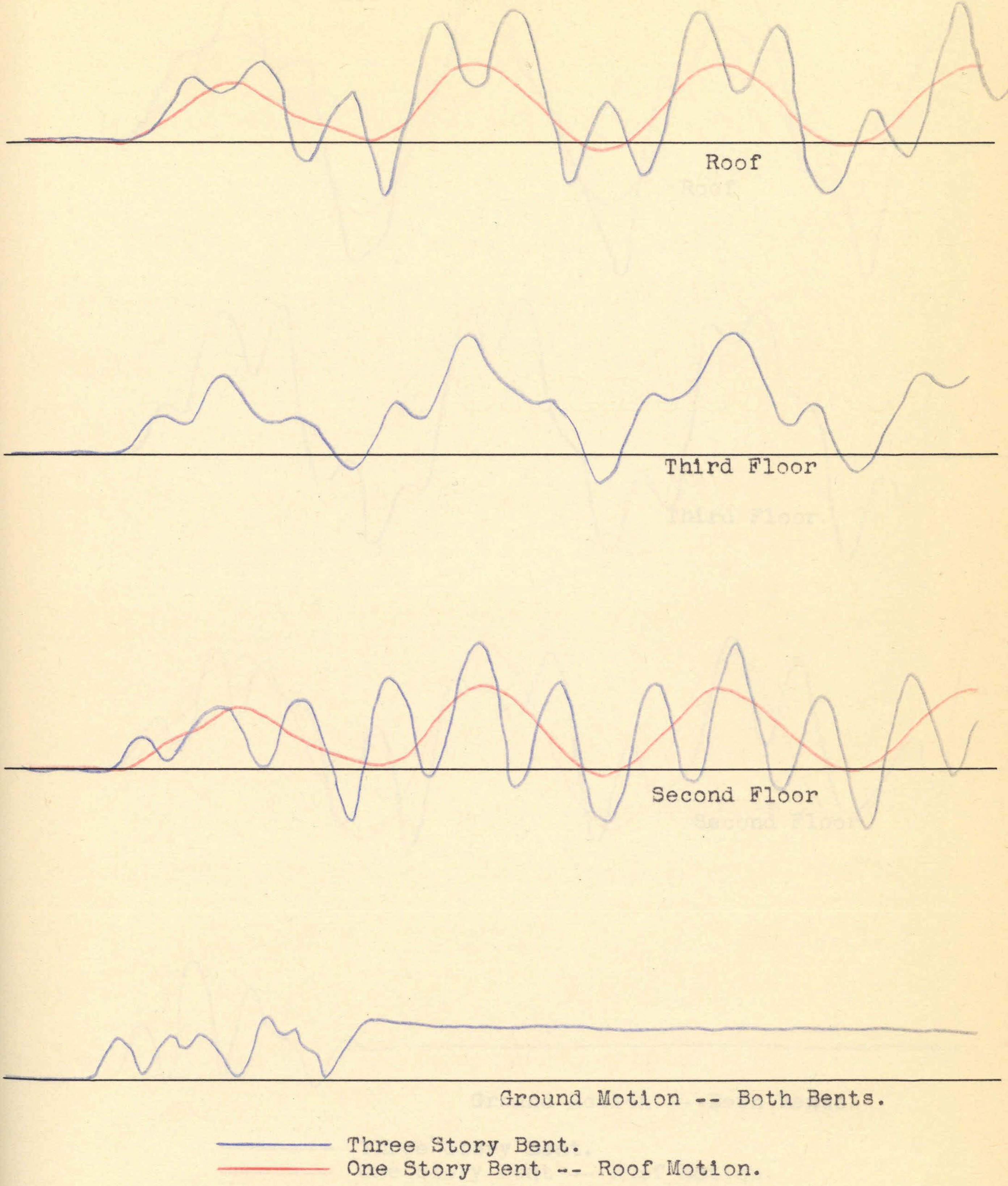
Quake Stick No. 1 Pulled by Four Pounds Weight.
Record No. 147 6-15-35
Enlarged 2X

Fig. 29



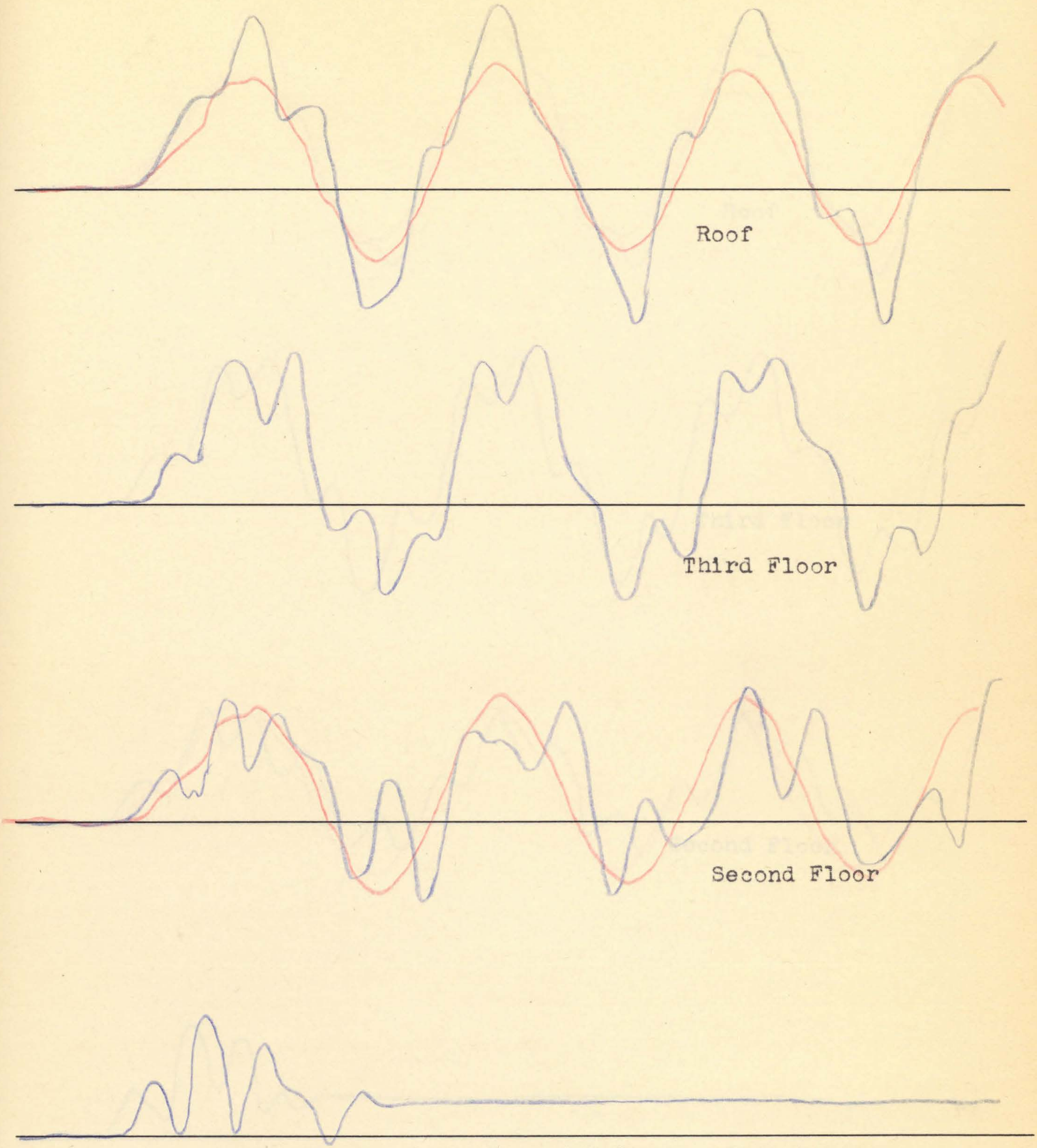
Quake Stick No. 1 Pulled by Five Pounds Weight.
Record No. 149 6-15-35.
Enlarged 2X

Fig. 30



Quake Stick No. 1 Pulled by Six Pounds Weight.
Record No. 151 6-15-35.
Enlarged 2X

Fig. 31

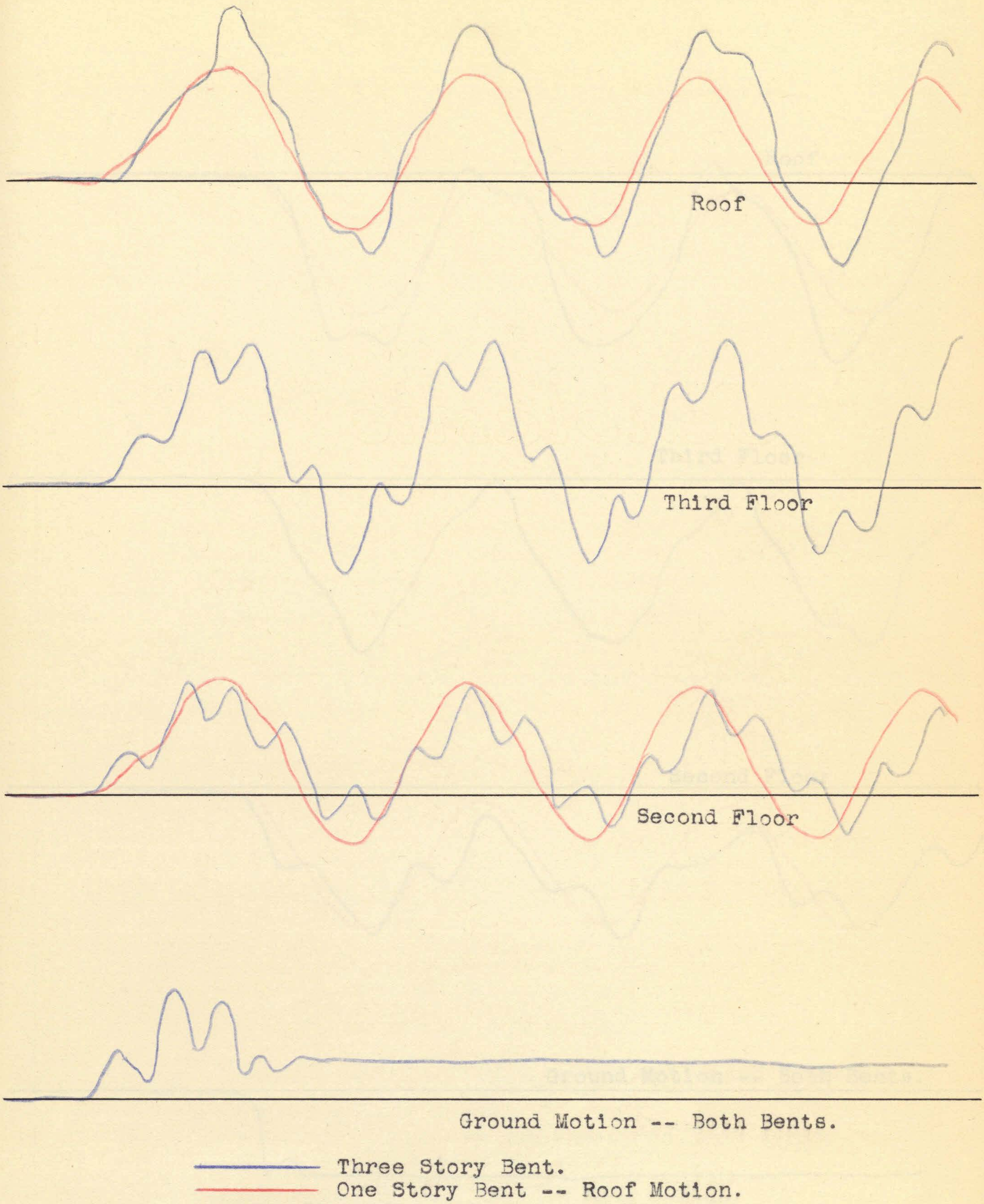


Ground Motion -- Both Bents.

——— Three Story Bent.
——— One Story Bent -- Roof Motion.

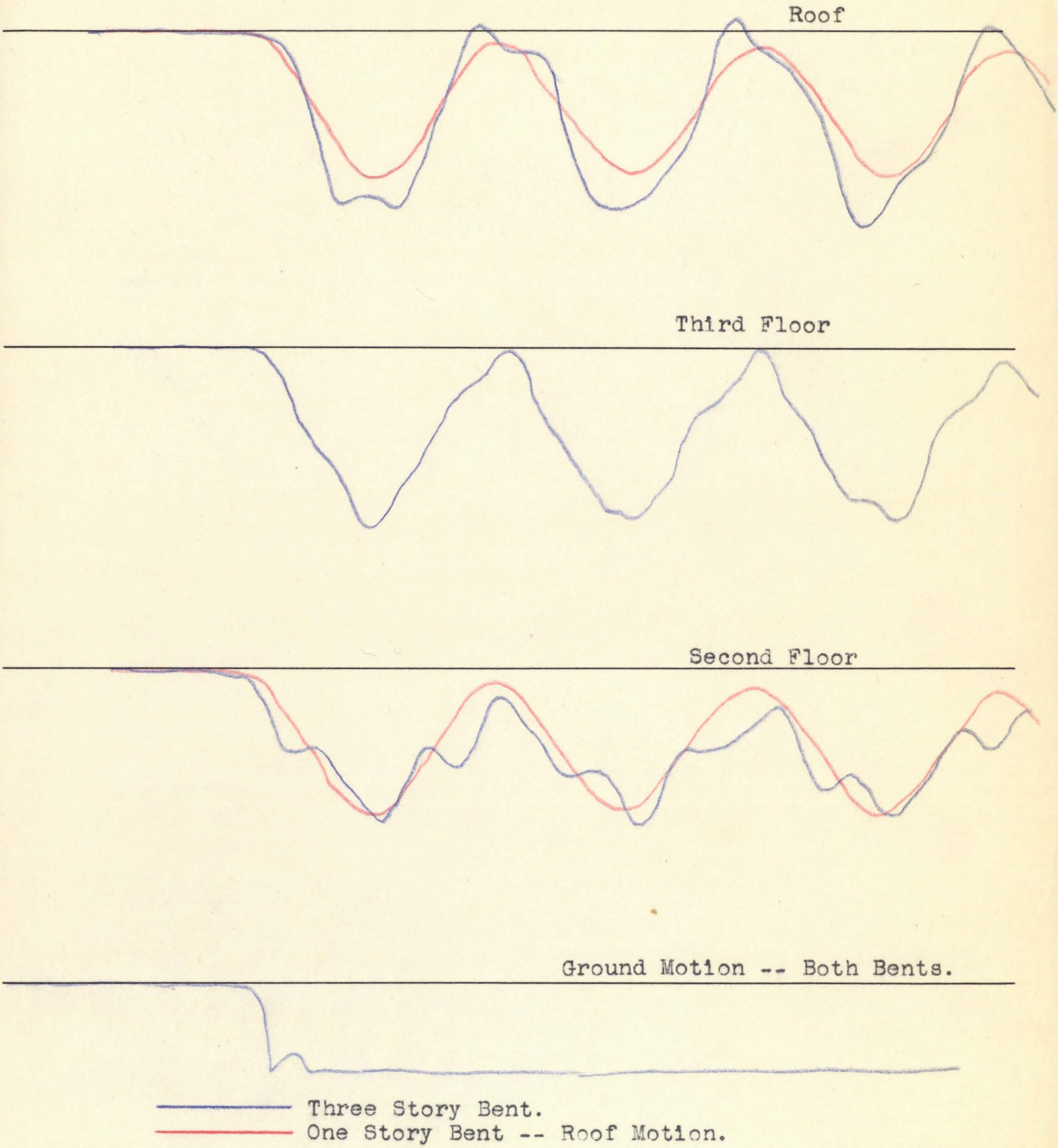
Quake Stick No. 1 Pulled Through Rapidly By Hand.
Record No. 155 6-18-35.
Enlarged 2X

Fig. 32



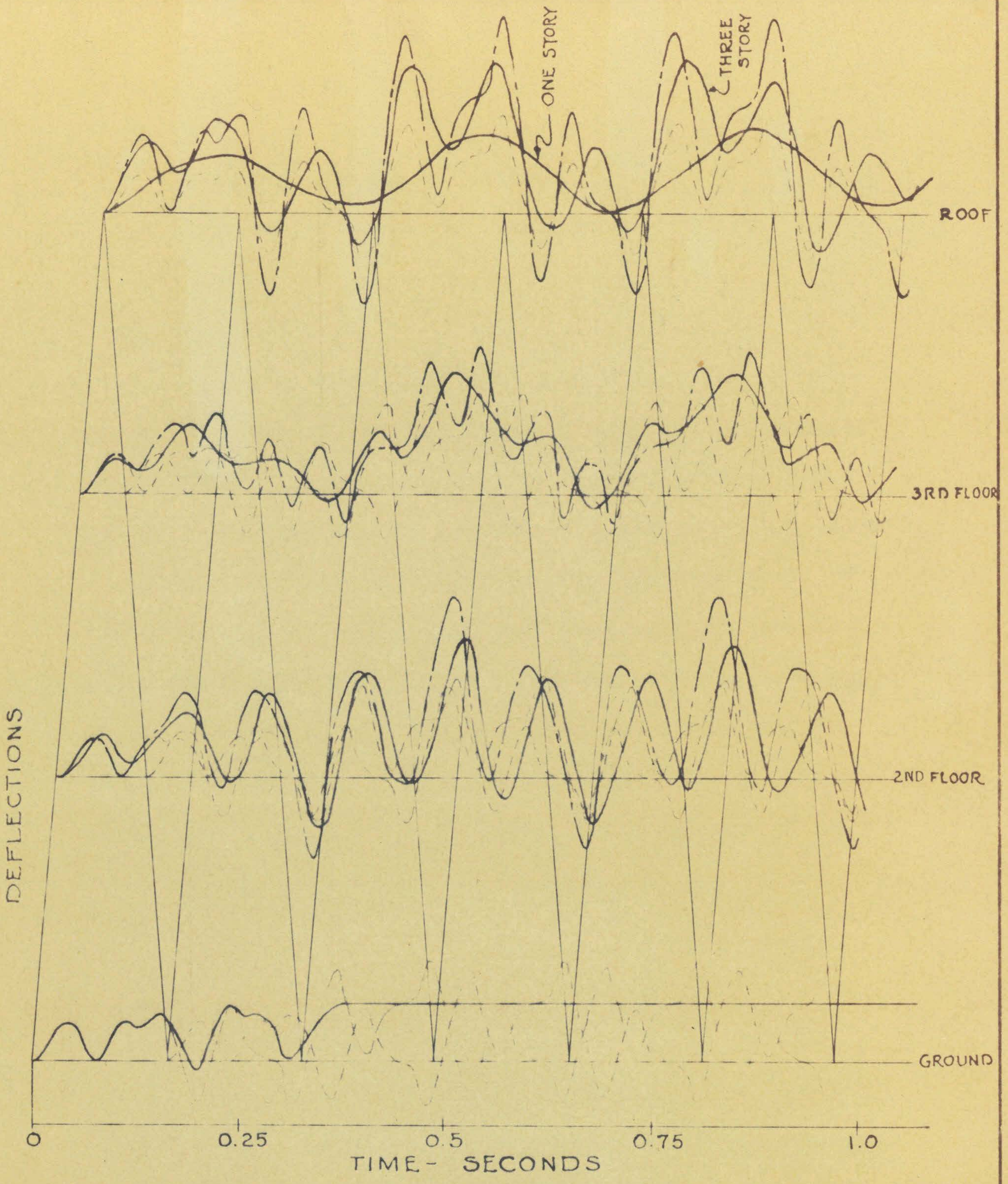
Quake Stick No. 1 Pulled Through Very Rapidly By Hand.
Record No. 156 6-18-35
Enlarged 2X

Fig. 33



Ground Motion Caused by Displacement and Sudden Release of Base.
Record No. 160 6-18-35.
Enlarged 2X

Fig. 34



ACTUAL MOTION FROM SHAKING TABLE RECORD.
TOTAL DEFLECTION PREDICTED BY ANALYSIS.
TRANSMITTED WAVE.
MOTION ADDED TO TRANSMITTED WAVE TO GIVE ACTUAL GROUND MOTION.

FUNDAMENTAL PERIOD OF BOTH ONE AND THREE STORY BENTS = 0.323 SEC.

WESTERGAARD ANALYSIS

FIGURE 35

Chapter IX

Bumping Between Buildings of Different Fundamental Periods

Description of Tests:

Since the buildings in business districts are built against one another, in many cases, without an intervening space, it is interesting to contemplate the effect that bumping between the buildings may have on their motions. With this in mind, a series of tests were made on the shaking table using a one story and a three story building. The two bents were mounted on the same base, end to end, as shown in Figure 36, so that they would bump together when vibration occurred. Tests were made for two different stiffnesses of the

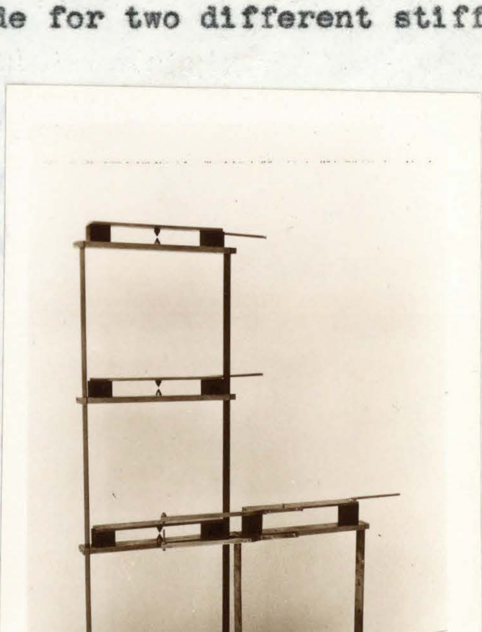


Figure 36

one story building, and in each case the motion was studied for the following four conditions:

1. No bumping between buildings.
2. Buildings touching when at rest.
3. Buildings separated 0.1" when at rest.
4. Two buildings tied together so that they are forced to vibrate together.

Series 1, Figures 37 to 41, shows the results of the tests for a fundamental period of 0.321 seconds for the three story building and 0.275 seconds for the one story building. Series 2, Figures 42 to 46, shows a similar series of curves for a fundamental period of 0.324 seconds for the three story building, and a period of 0.258 seconds for the one story building.

Conclusions:

The primary effect of bumping seems to be a change of the average period of each building, bringing the effective periods closer together. Consequently, there can be no generalization as to whether bumping will reduce or increase the amplitude of vibration. The result depends entirely upon the degree of resonance with the quake under consideration.

In discussing the curves given in Figures 37 to 46, the effects upon the three story building will first be considered, and then the effects upon the one story building will be discussed.

In the tests of Series 1, Figures 37 to 41, the three story bent dominated the motion. As might be expected, Fig. 37 shows that, for vibration approximately in the fundamental of the three story bent, the one story building had little effect on the three story. However, Figure 38 shows that tying the two buildings together gives resonance to harmonic vibration for the quake of that test; so that tying the buildings would be worse than either bumping or free vibration. Fig. 39 also shows detrimental results from tying the buildings, but in this case the curve for bumping under the condition with the buildings touching when at rest shows much smaller amplitudes of vibration for the three story building, than were prevalent when the building was free to vibrate without bumping. Fig. 41 shows a slight benefit from tying, over free vibration, but not as small amplitudes as in the cases where bumping took place.

Series two with stiffer columns in the one story bent, where the vibration of the three story bent is due primarily to the fundamental (Fig. 42), gives a more pronounced effect from bumping than was observed in Series 1; however, the maximum amplitudes are not greatly effected. The other curves of Series 2 are somewhat similar to the corresponding curves of Series 1, and lead to the same conclusion; namely, that the best condition (free, bumping, or tied) depends upon the ground motion and no general statement can be made as to which is best.

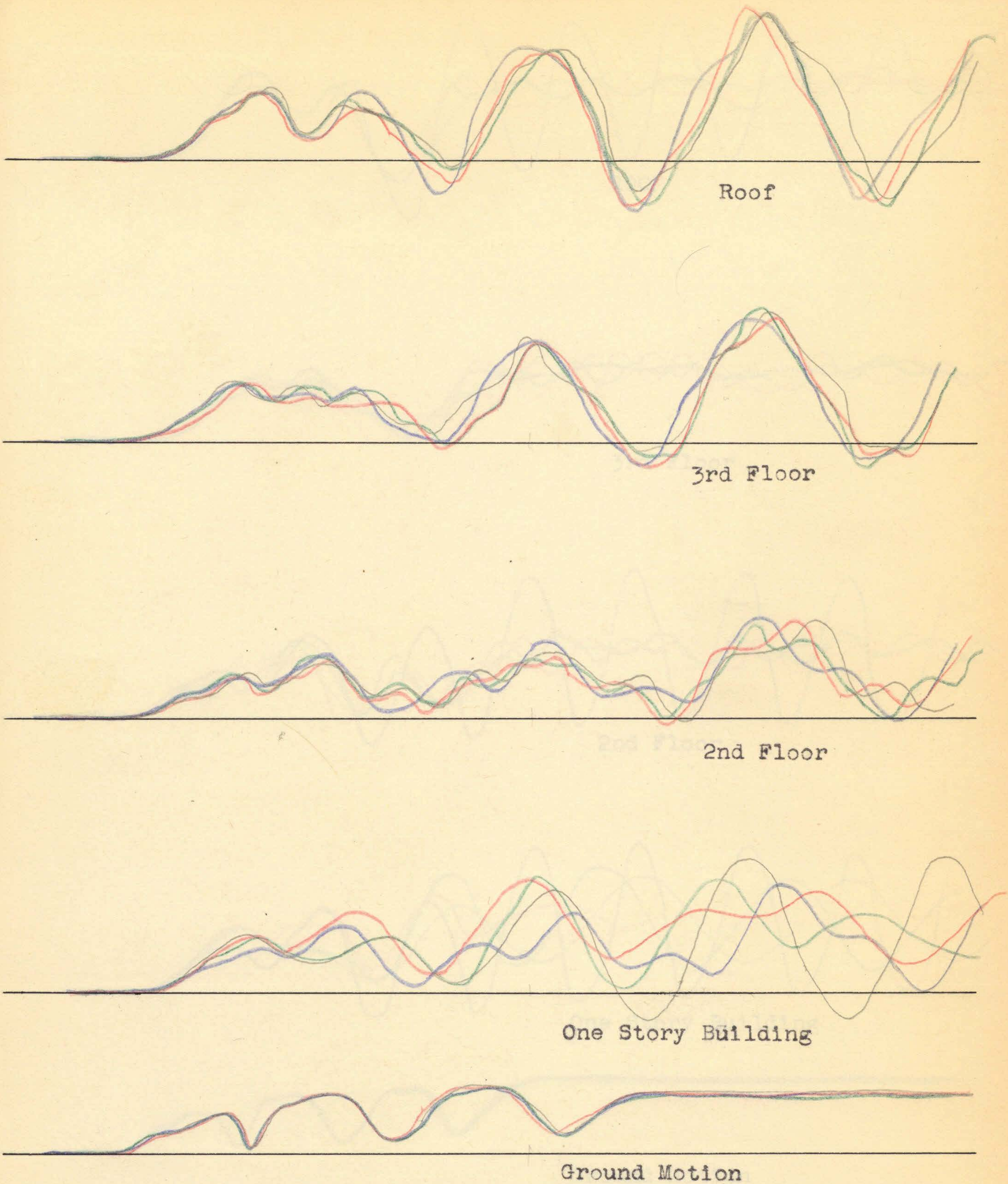
Now, let us consider the motion of the one story bent. The same conclusion seems to apply to it. Figures 37 and 42 give the most violent vibration for the condition with no bumping. Figures 38 to 41 and 43 to 46 give the largest amplitudes for the condition where the buildings are tied together. Figures 38 and 43 show less amplitude of vibration when bumping occurs than when the building vibrates freely, while Figures 39, 40, 41, 44, 45, and 46 give, for cases when the buildings bump, amplitudes which are either greater than, or are of about the same magnitude as, the amplitudes of vibration when no bumping takes place. Therefore, benefit or detriment depends upon the quake; in most cases, however, the building that would have extremely violent vibration if oscillating alone has its amplitude of vibration somewhat reduced at the expense of the other building, which suffers an increase in amplitude.

In the case of an actual building, the battering action against an adjacent structure may have harmful results. The rapid vibrations due to bumping are especially noticeable in the curves for the three story building Figures 39, 40, 41, and 44. These shocks due to bumping are probably more serious than the appearance of the curves would indicate.

SERIES 1

Fundamental Periods

One Story Building	0.275 seconds
Three Story Building	0.321 seconds
Combined Period	0.312 seconds

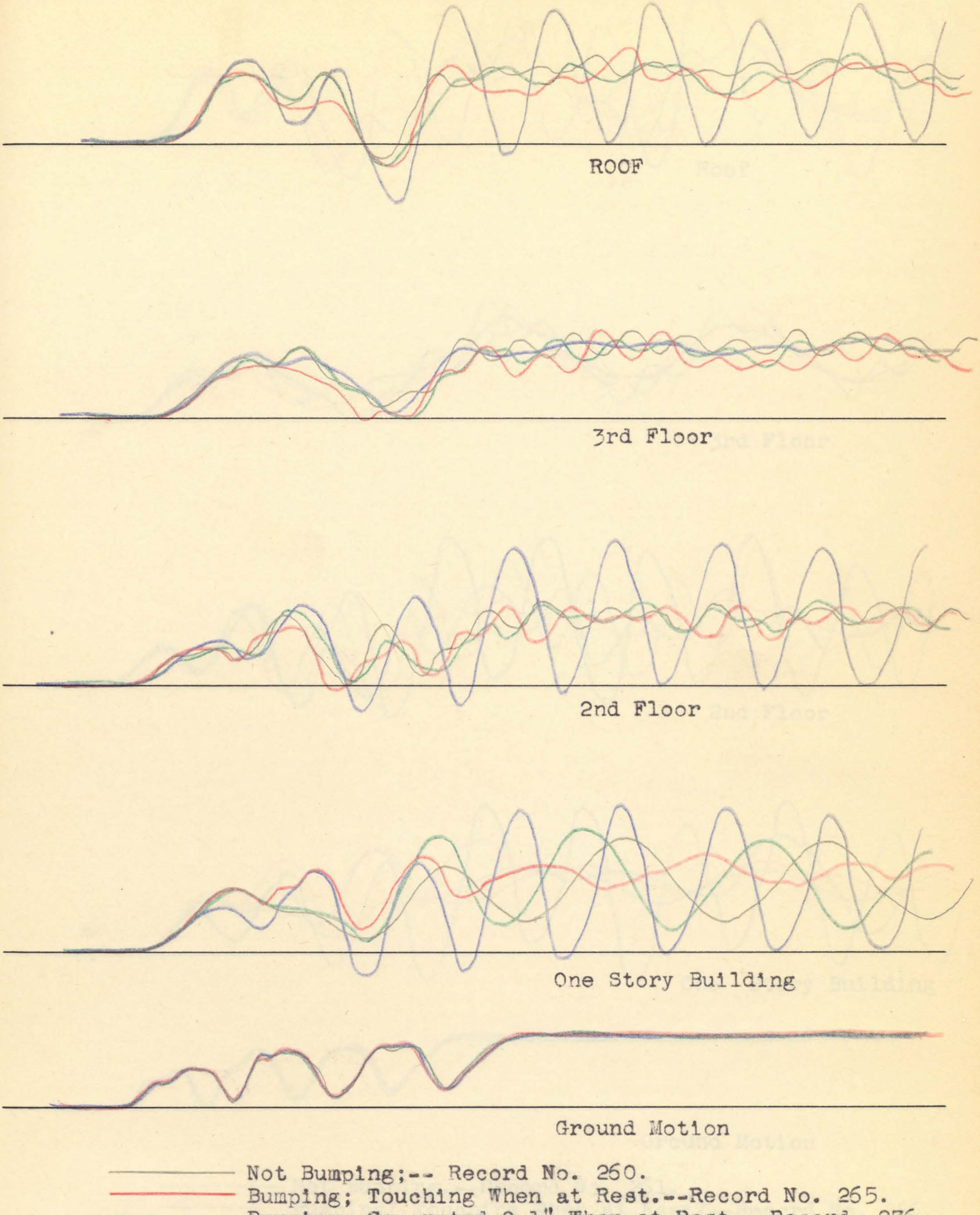


Not Bumping.--Record NO. 259.
Bumping; Touching when at rest.--Record No. 264.
Bumping; Separated 0.1" when at rest.--Record 275.
Two Buildings tied together.-- Record No. 270.

Enlarged 2X. 8-29-35

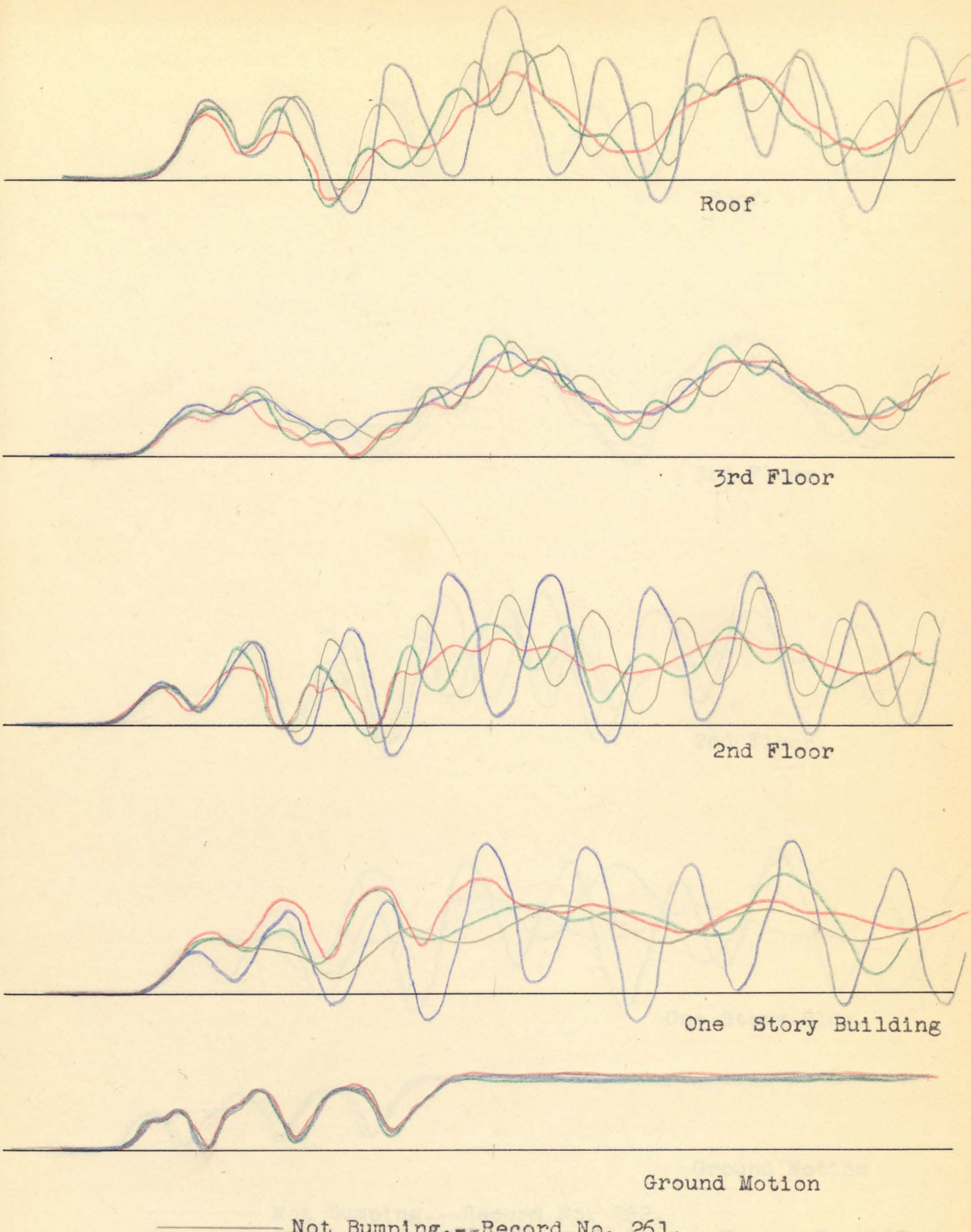
Quake Stick No. 1 Pulled by Two Pounds Weight

Fig. 37



Quake Stick No. 1 Pulled by Three Pounds Weight

Fig. 38

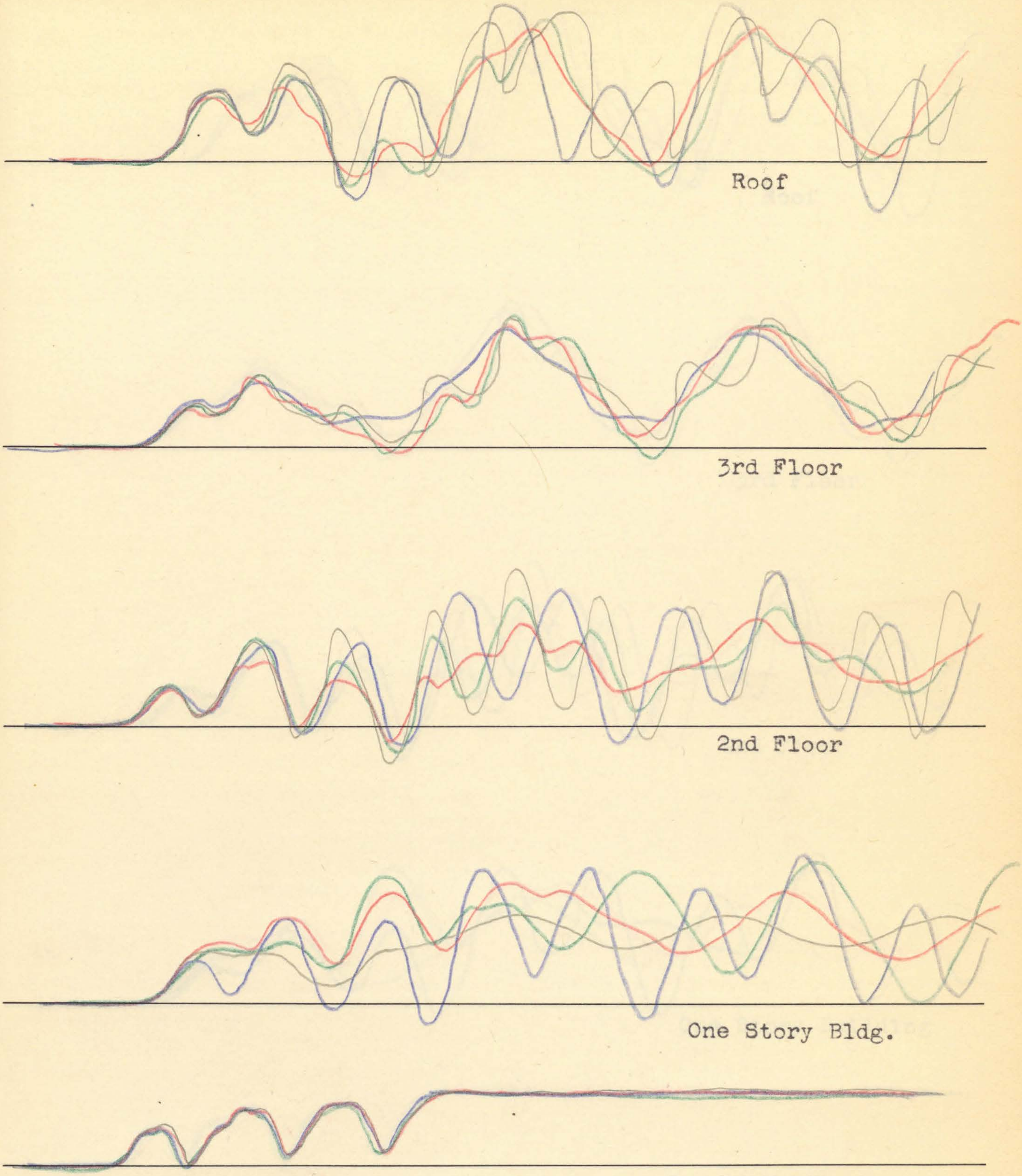


Not Bumping.--Record No. 261.
Bumping; Touching When at Rest/--Record No. 266.
Bumping; Separated 0.1" When at Rest.--Record 277.
Two Buildings Tied Together.--Record No. 272.

Enlarged 2X. 8-29-35

Quake Stick No. 1 Pulled by Four Pounds Weight

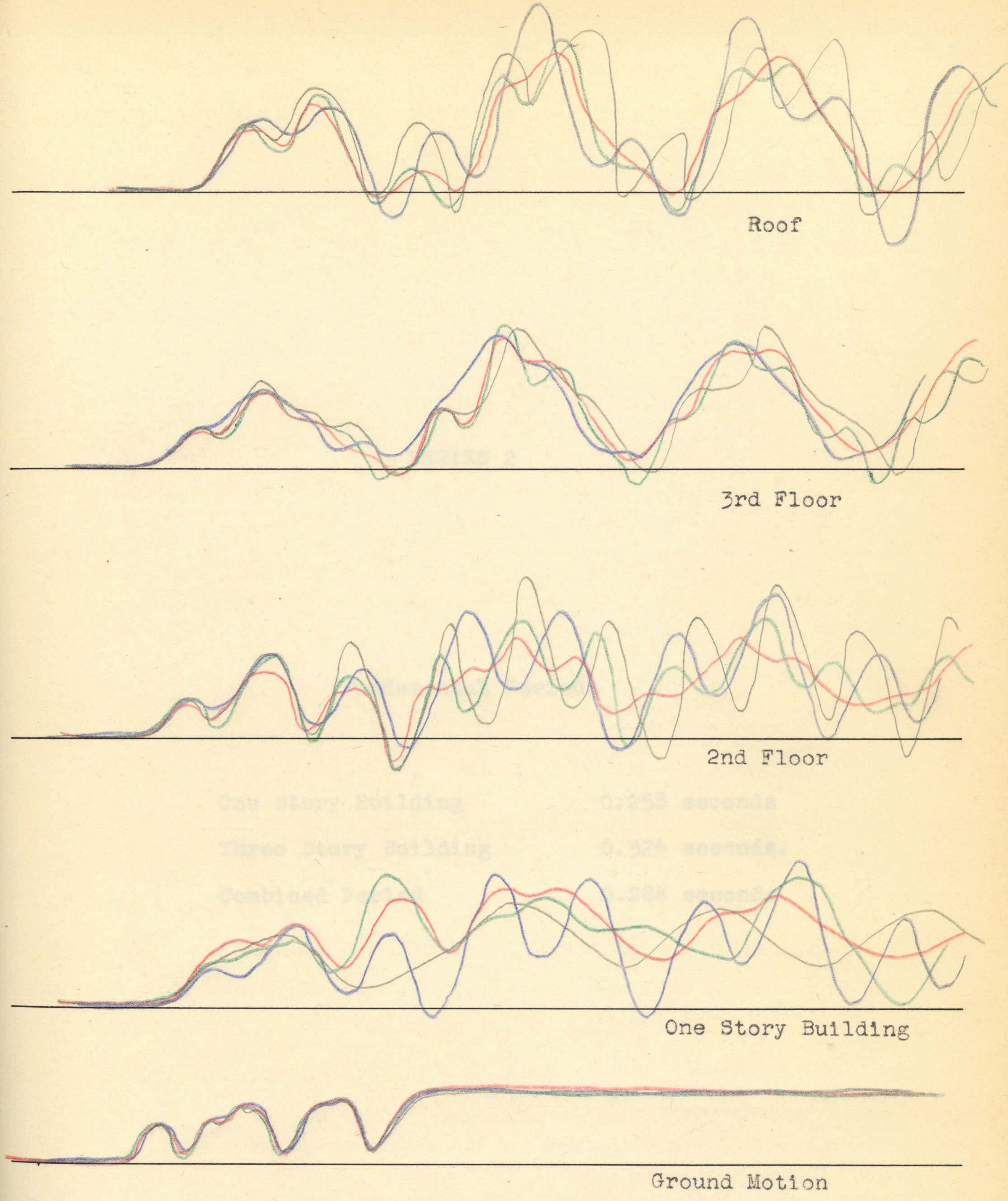
Fig. 39



Not Bumping.--Record No. 262.
Bumping: Touching When at Rest.--Record No. 267.
Bumping; Separated 0.1" When at Rest.--Record 278.
Two Buildings Tied Together.--Record No. 273.
Enlarged 2X. 8-29-35.

Quake Stick No. 1 Pulled by Five Pounds Weight

Fig. 40



Not Bumping.--Record No. 263.
Bumping; Touching When at Rest.--Record No. 268.
Bumping; Separated 0.1" When at Rest.--Record 279.
Two Buildings Tied Together,--Record No. 274.
Enlarged 2X. 8-29-35.

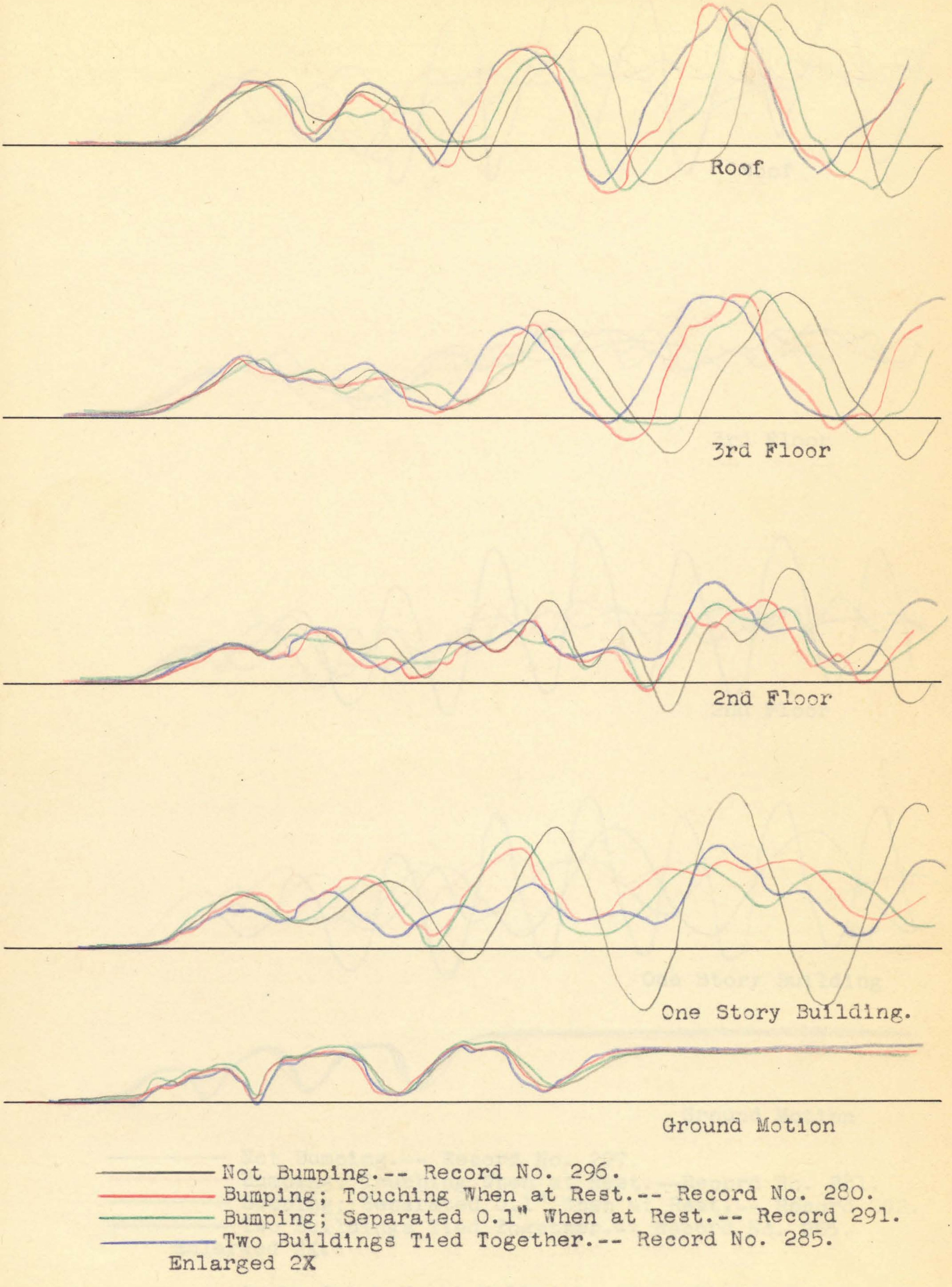
Quake Stick No. 1 Pulled by Six Pounds Weight

Fig. 41

SERIES 2

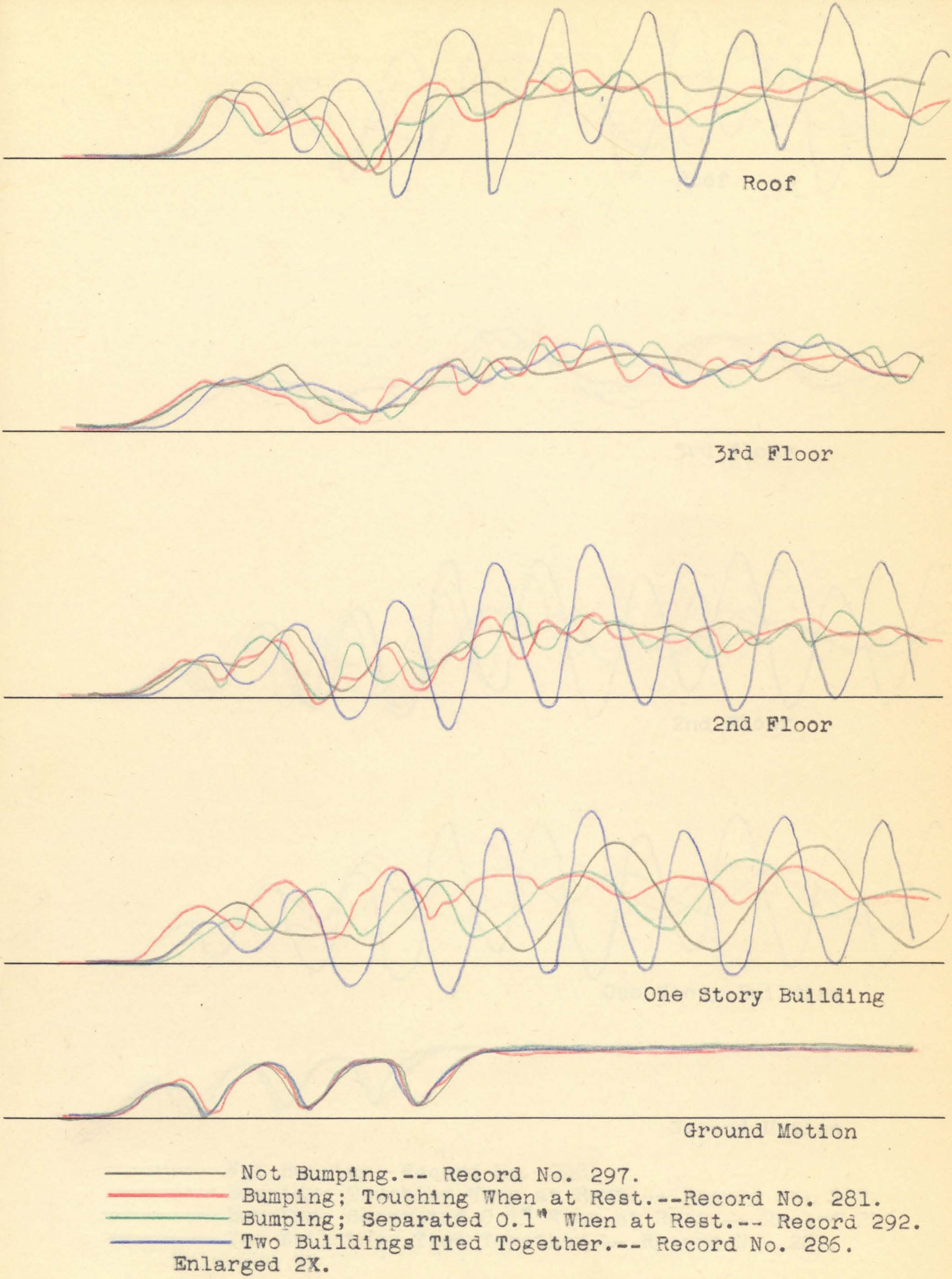
Fundamental Periods

One Story Building	0.258 seconds
Three Story Building	0.324 seconds.
Combined Period	0.284 seconds



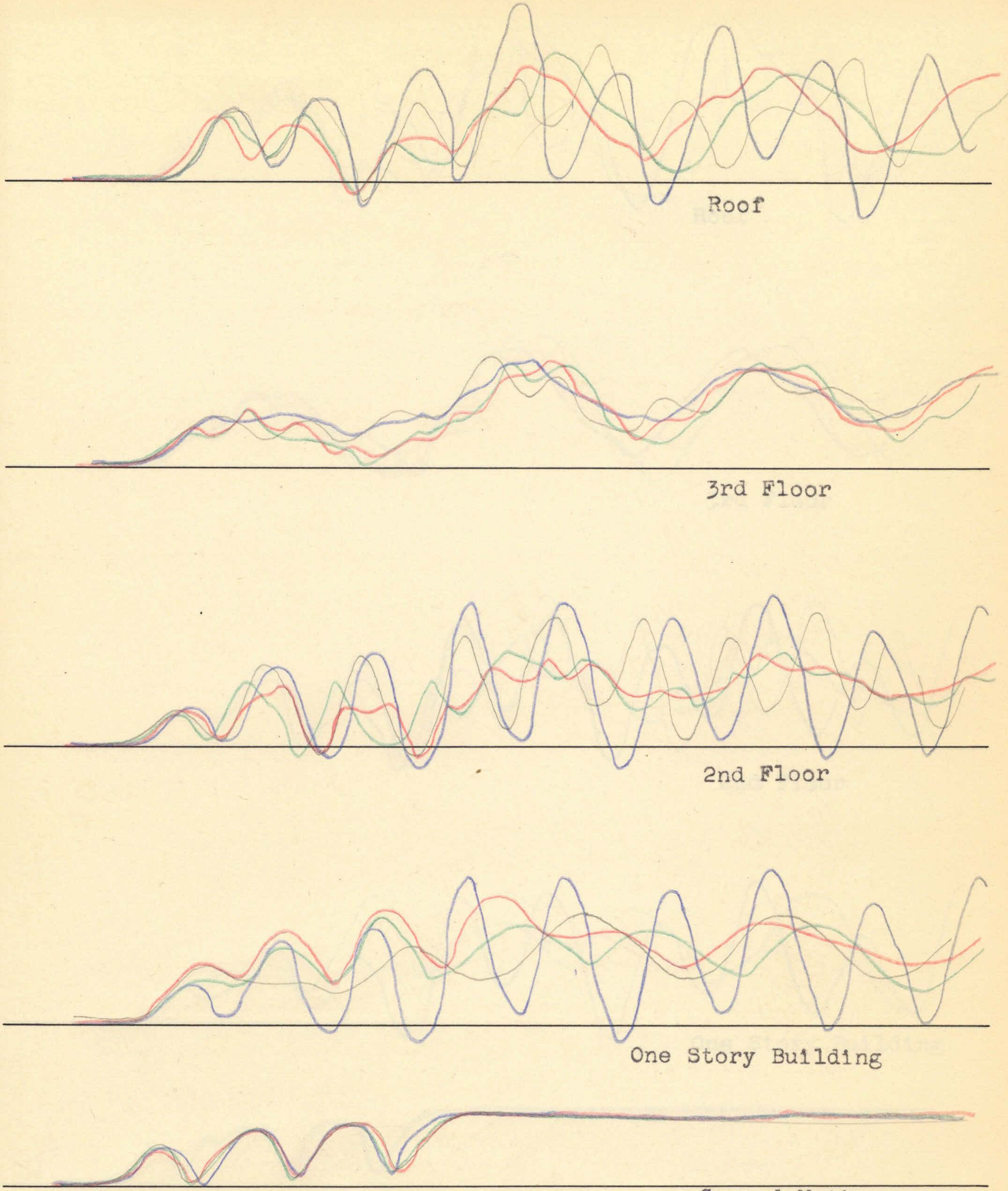
Quake Stick No. 1 Pulled by Two Pounds Weight.

Fig. 42



Quake Stick No. 1 Pulled by Three Pounds Weight.

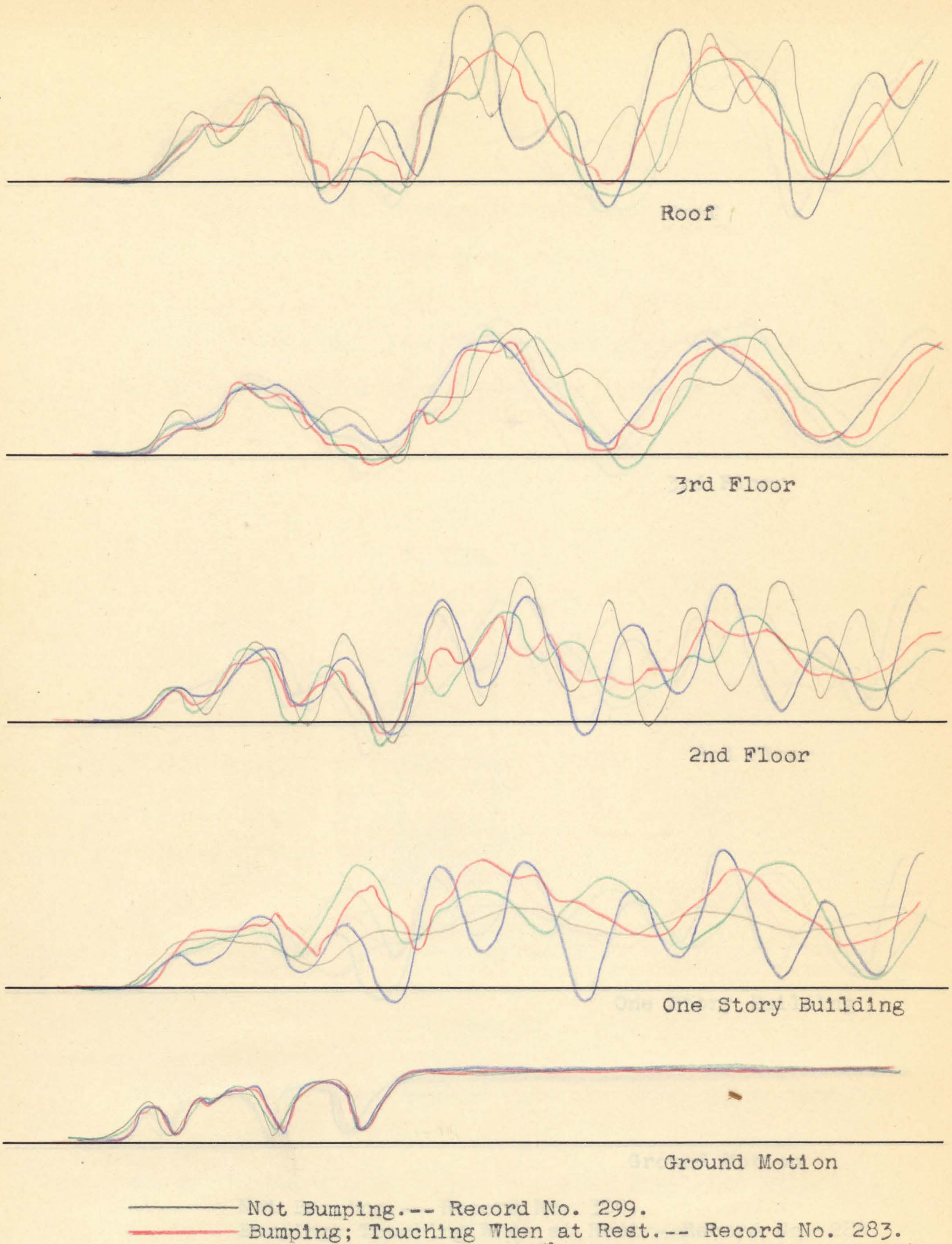
Fig. 43



Not Bumping; Record No. 298.
Bumping; Touching When at Rest.-- Record No. 282.
Bumping; Separated 0.1" When at Rest.-- Record 293.
Two Buildings Tied Together.-- Record No. 287.
Enlarged 2X

Quake Stick No. 1 Pulled by Four Pounds Weight.

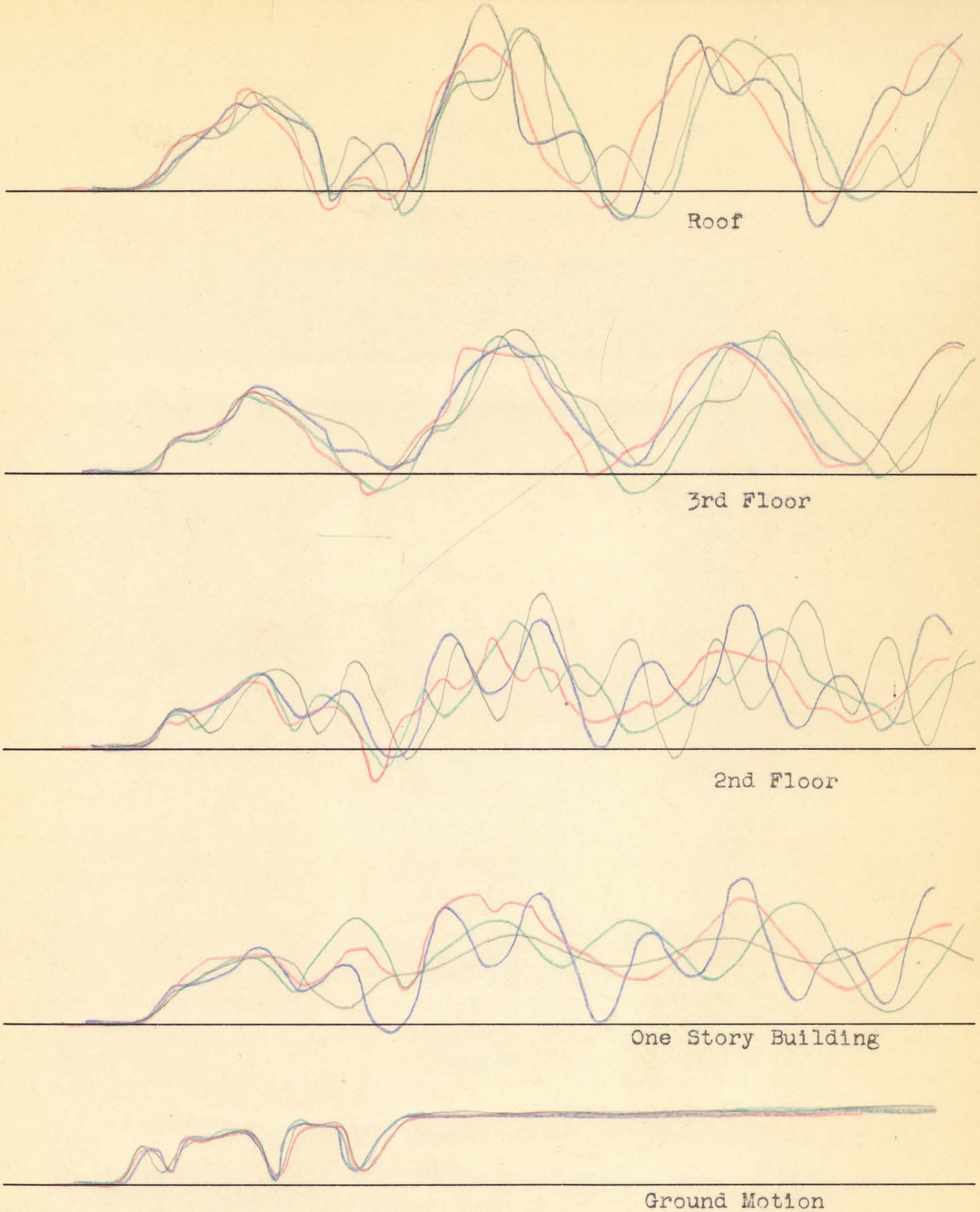
Fig. 44



Not Bumping.-- Record No. 299.
Bumping; Touching When at Rest.-- Record No. 283.
Bumping; Separated 0.1" When at Rest.-- Record 294.
Two Buildings Tied Together.--Record No. 288.
Enlarged 2X

Quake Stick No. 1 Pulled by Five Pounds Weight.

Fig. 45



Not Bumping.-- Record No. 300.
Bumping; Touching When at Rest.--Record No. 284.
Bumping; Separated 0.1" When at Rest.-- Record 295.
Two Buildings Tied Together.-- Record No. 289.
Enlarged 2X

Quake Stick No.1 Pulled by Six Pounds Weight.

Fig. 46

Chapter X

Effects of Tension Diagonals on One and Three Story Bents

Diagonal bracing composed of rubber strips as shown in Figure 47 was used for a series of tests.

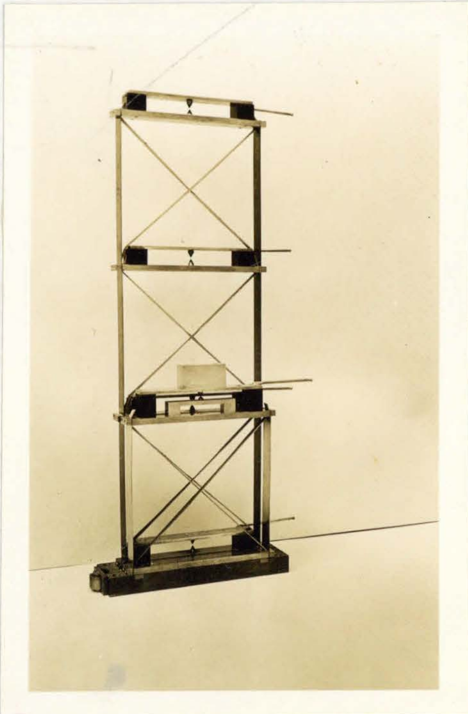


Figure 47

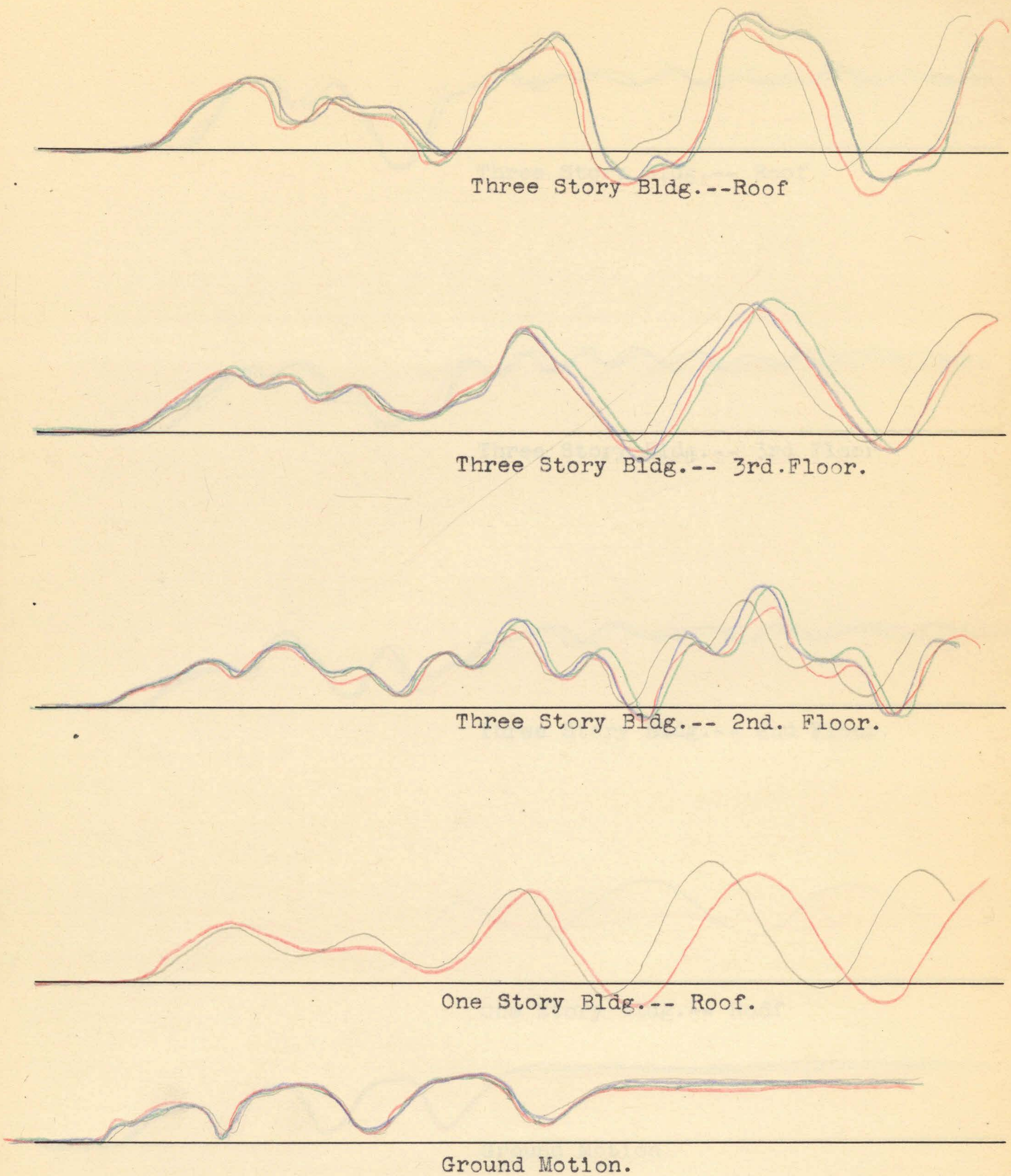
Diagonals were than removed from the lower floor, and the behavior of the buildihg studied with diagonals in the upper two floors only. The condition with diagonals in the upper ~~two~~ floor only was also tried. Two sets of diagonal springs were also used; first, long, light springs, and second, short stiff springs were attached as diagonal bracing.

Various initial tensions were used. The rubber strips and long springs were given about 1.07 oz. initial tension for the first tests, and 3.68 oz. and 10.5 oz., respectively, for a succeeding series of trials. The short springs were used first with no initial tension, and then with initial tension.

Figures 48 to 67 show the results of these tests. As can readily be seen, there was little effect other than a changed period resulting in a slight phase difference and a slightly different response to resonance. Although there was fairly good damping obtained using rubber strips as diagonals, the short time of the transient vibration was insufficient to realize any benefit from the damping. It may be observed that, in order to use tension diagonals effectively as bracing, there should be a large increase in tension, with small deformation, rather than a very gradual increase, as was the case for the rubber diagonals and also for the springs used.

Figures 68 to 72 show maximum amplitude of free vibration plotted against time. In other words, the curves represent the envelope of vibration as the sinusoidal oscillation, resulting from releasing the bent from a displaced position, is damped out. Figs. 68, 69, and 70 show that the rubber diagonals were capable of absorbing a considerable amount of energy, although a longer period of time than that required for a "quake" on the shaking table was required to produce noticeable effects. Figures 71 and 72 show that short stiff springs, especially when loose, have some damping effect, but that the long limber springs produced very little damping.

The study, taken as a whole, illustrates the fact that, where transient vibrations are involved, resonance controls the vibration, and damping characteristics have little influence.

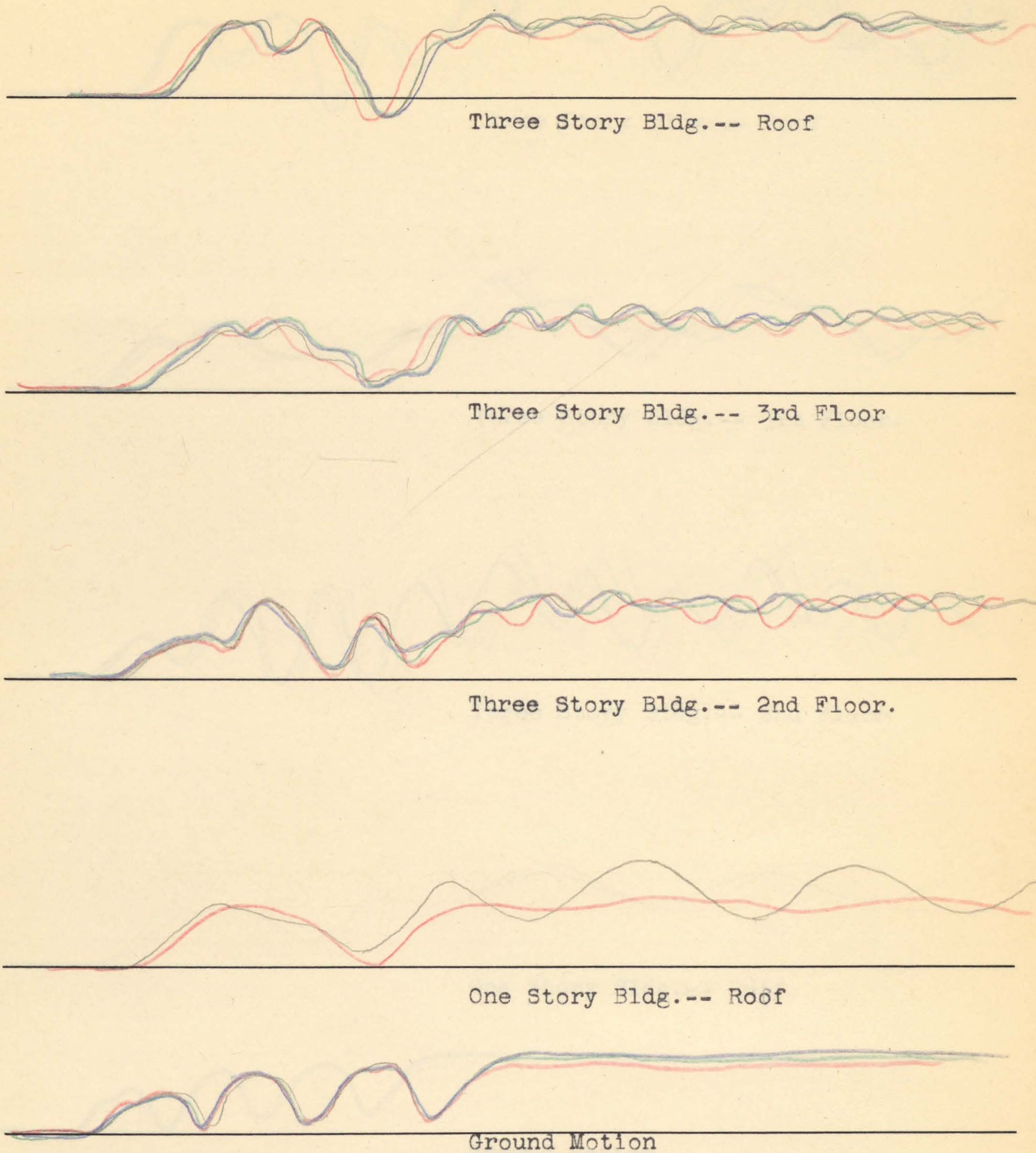


Rubber Diagonals at All Floors; 1.07 oz. Tension.-- Rec. 184B.
Rubber Diagonals at 2nd and 3rd Floors; 1.07 oz. Tension:189B.
Rubber Diagonals at 3rd Floor; 1.07 oz. Tension.-- Rec. 196B.
No Diagonals.-- Record No. 201A.

Enlarged 2X 7-5-35

Quake Stick No. 1 Pulled by 2 Pounds Weight.

Fig. 48

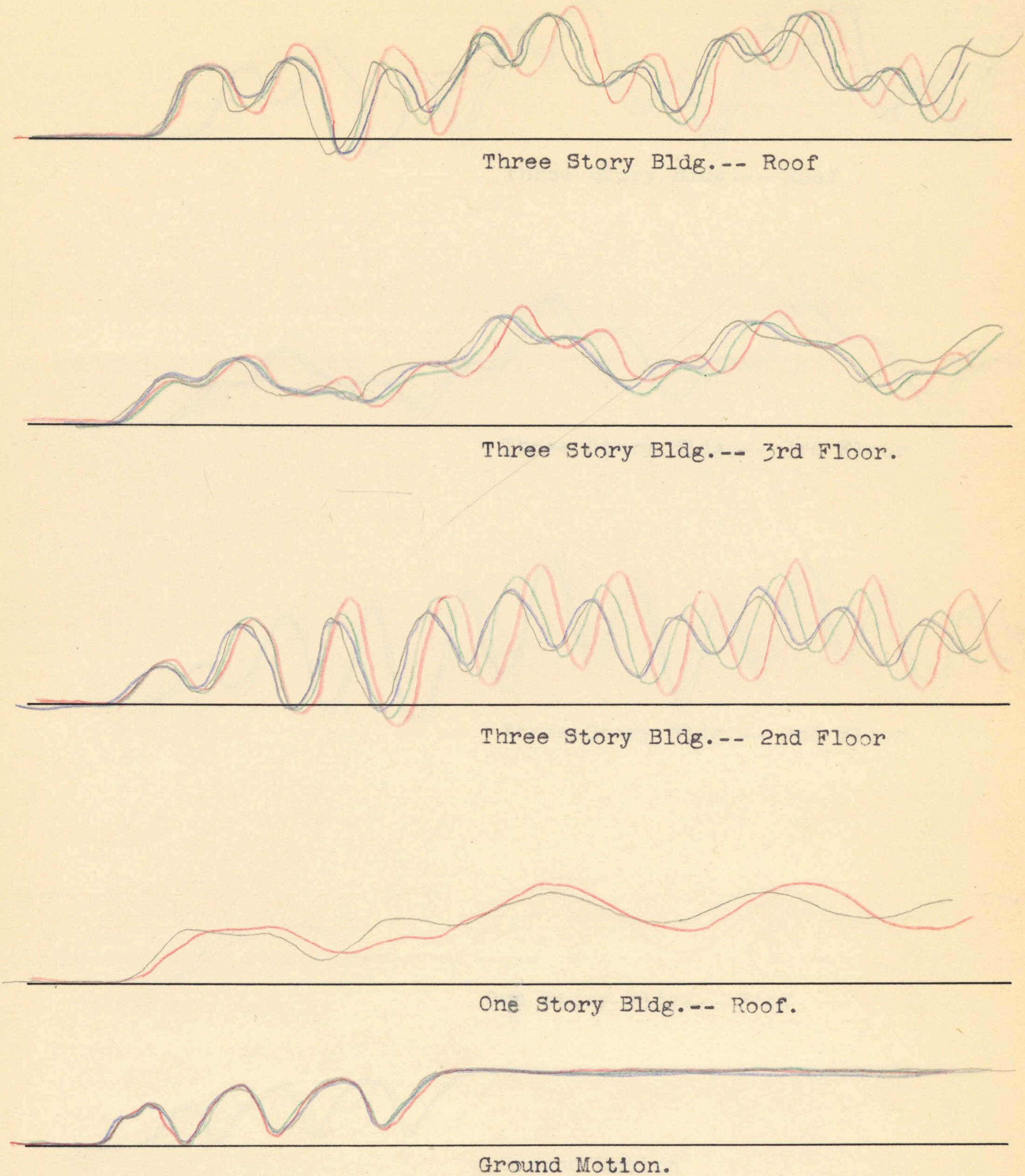


Rubber Diagonals at All Floors; 1.07 oz. Tension.--Rec. 185A.
Rubber Diagonals at 2nd & 3rd Floors; 1.07 oz. Tension.-190B.
Rubber Diagonals at 3rd Floor; 1.07 oz. Tension.-- Rec. 197B.
No Diagonals.-- Record No. 202B.

Enlarged 2X 7-5-35

Quake Stick No. 1 Pulled by 3 Pounds Weight.

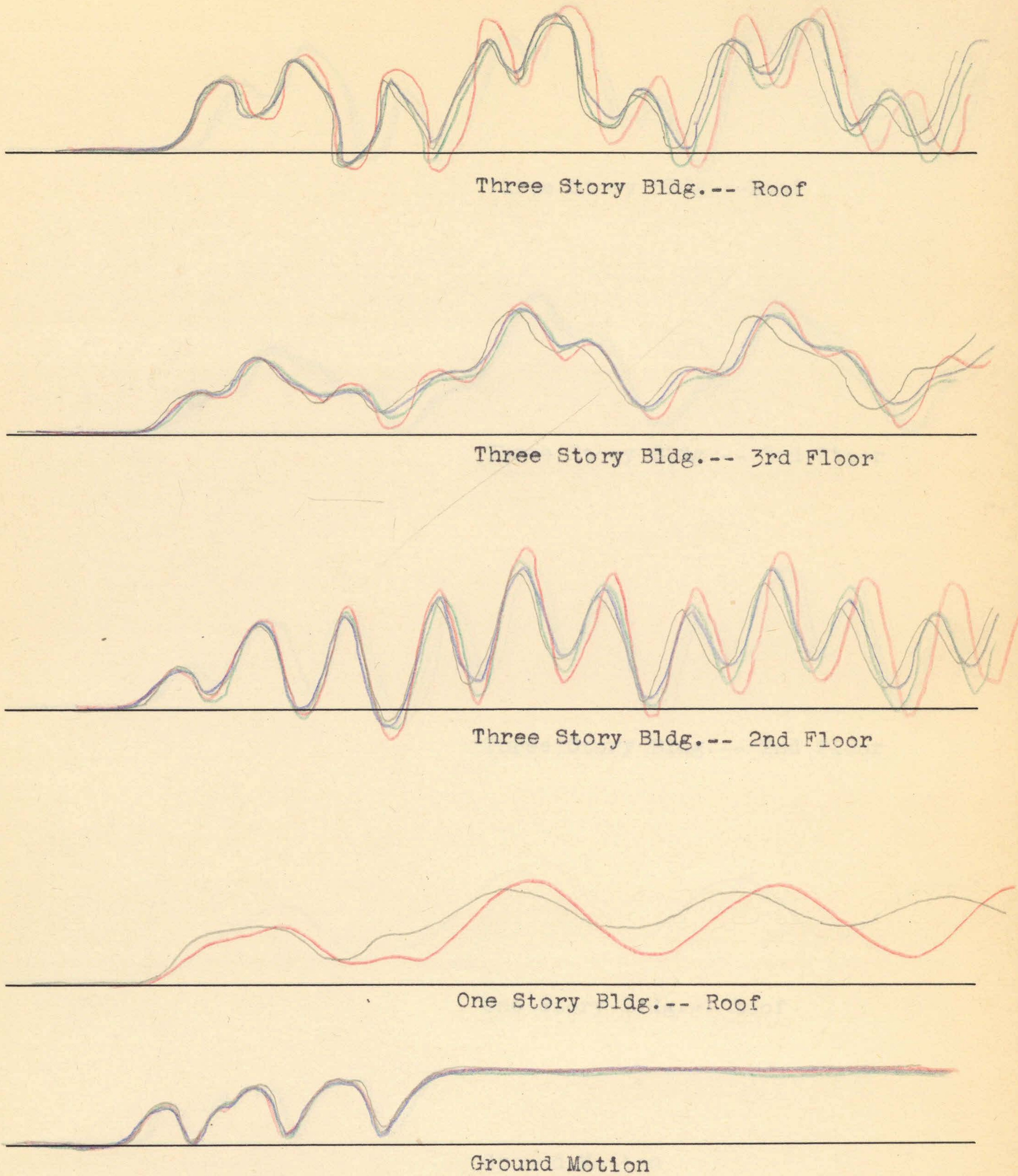
Fig. 49



Rubber Diagonals at All Floors; 1.07 oz. Tension.-- Rec.186A.
Rubber Diagonals at 2nd & 3rd Floors; 1.07 oz. Tension.-191A.
Rubber Diagonals at 3rd Floor; 1.07 oz. Tension.-- Rec. 198A.
No Diagonals.-- Record No. 203A.
Enlarged 2X 7-5-35

Quake Stick No. 1 Pulled by 4 Pounds Weight.

Fig. 50

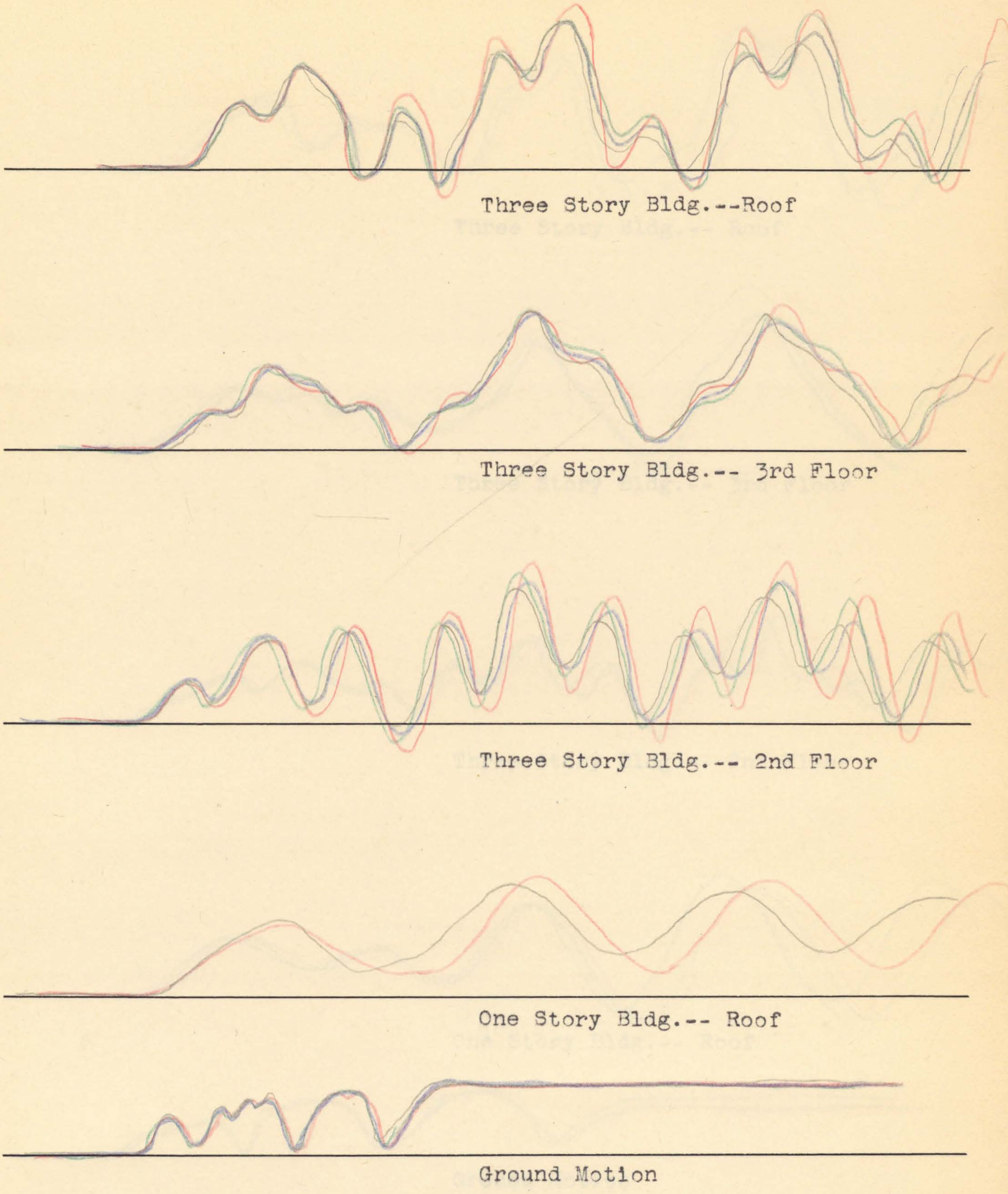


Rubber Diagonals at All Floors; 1.07 oz. Tension.-- Rec. 187A.
Rubber Diagonals at 2nd & 3rd Floors; 1.07 oz. Tension.-192A.
Rubber Diagonals at 3rd Floor; 1.07 oz. Tension.-- Rec. 199A.
No Diagonals.-- Record No. 204A.
Enlarged 2X

7-5-35

Quake Stick No. 1 Pulled by 5 Pounds Weight.

Fig. 51



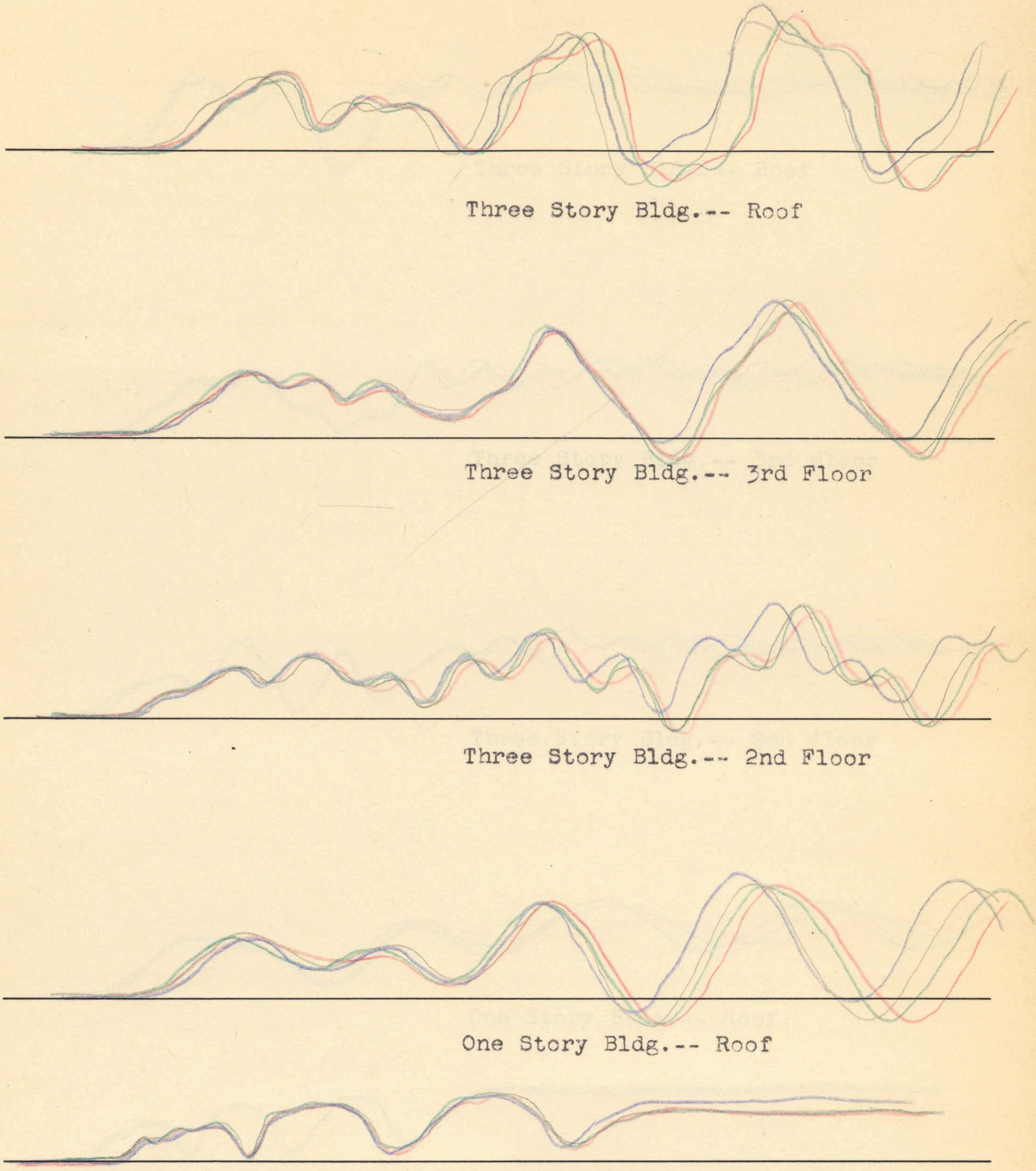
——— Rubber Diagonals at All Floors; 1.07 oz. Tension.-- Rec. 188A.
——— Rubber Diagonals at 2nd & 3rd Floors; 1.07 oz. Tension.-193A.
——— Rubber Diagonals at 3rd Floor; 1.07 oz. Tension.-- Rec. 200B.
——— No Diagonals.-- Record No. 205A.

Enlarged 2X

7-5-35

Quake Stick No. 1 Pulled by 6 Pounds Weight.

Fig. 52

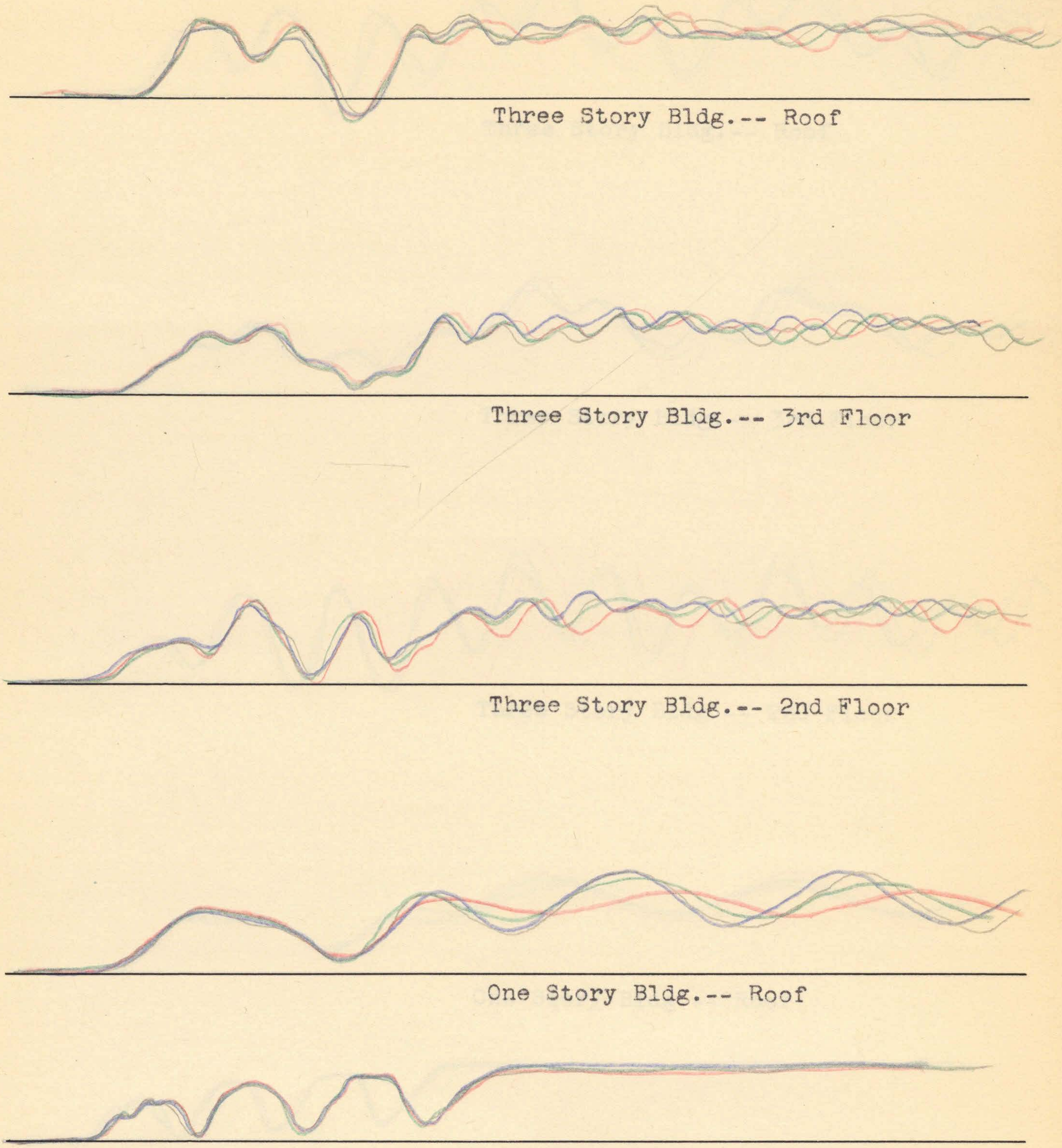


Rubber Diagonals; 1.07 oz. Tension.-- Record 184B.
Rubber Diagonals; 3.68 oz. Tension.-- Record 208A.
Rubber Diagonals; 10.5 oz. Tension.-- Record 217A.
No Diagonals.-- Record No. 201A.

Enlarged 2X 7-8-35

Quake Stick No. 1 Pulled by 2 Pounds Weight.

Fig. 53

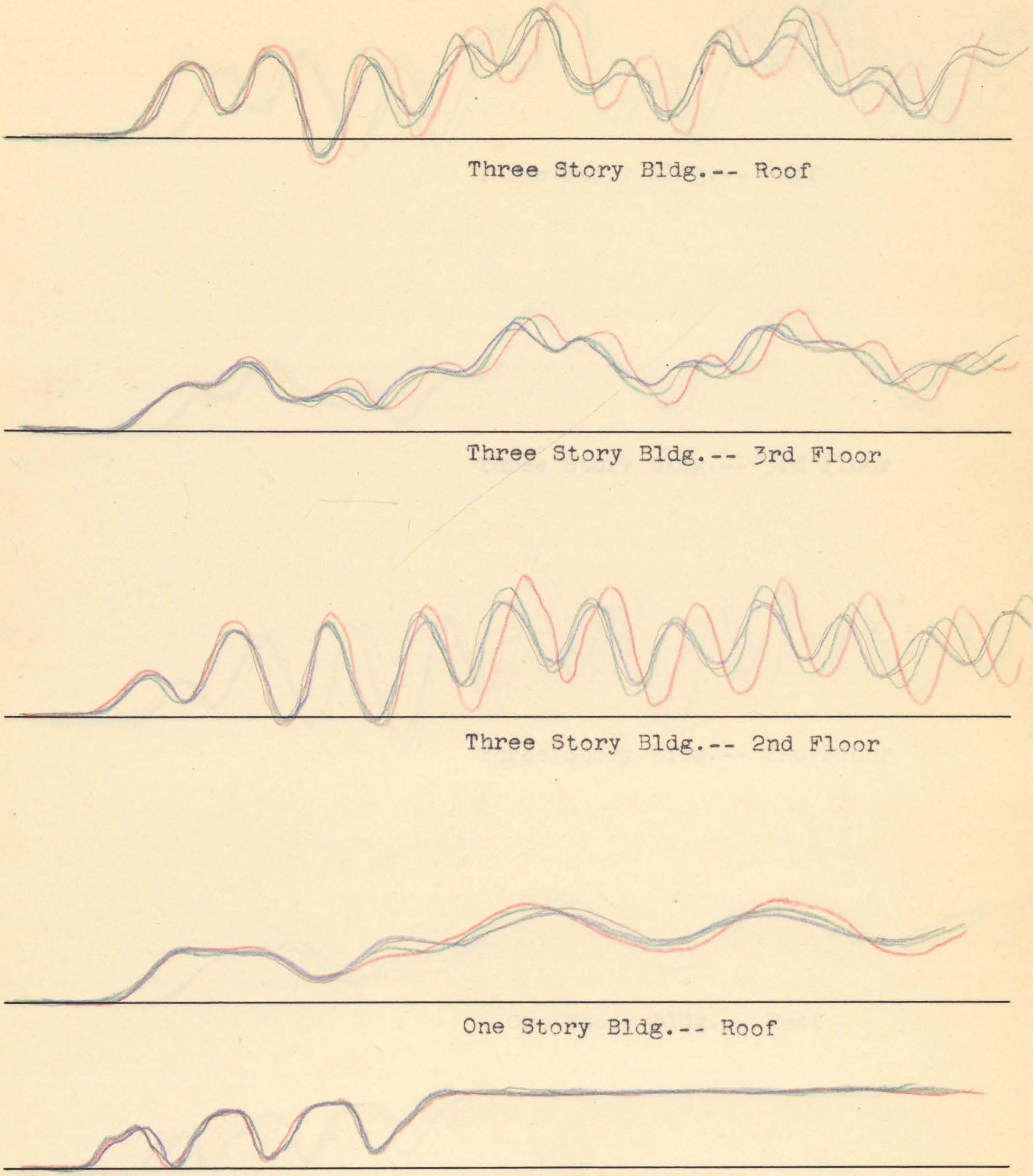


Rubber Diagonals; 1.07 oz. Tension.-- Record 185B.
Rubber Diagonals; 3.68 oz. Tension.-- Record 209A.
Rubber Diagonals; 10.5 oz. Tension.-- Record 218A.
No Diagonals.--Record No. 202A.

Enlarged 2X 7-8-35

Quake Stick No. 1 Pulled by 3 Pounds Weight.

Fig. 54

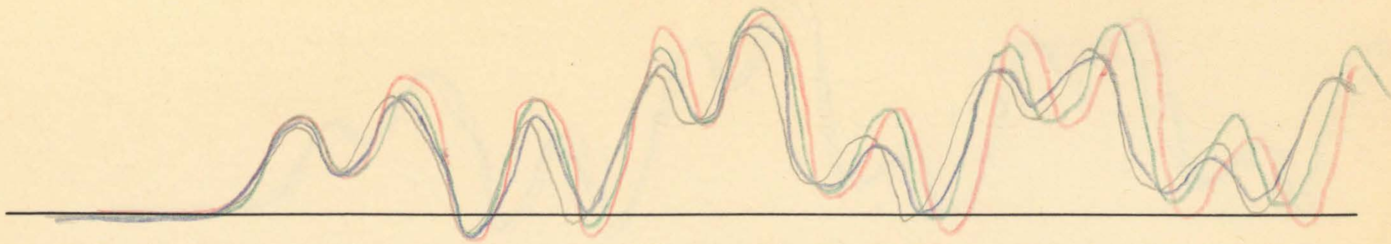


Rubber Diagonals; 1.07 oz. Tension.-- Record 186A.
Rubber Diagonals; 3.68 oz. Tension.-- Record 210A.
Rubber Diagonals; 10.5 oz. Tension.-- Record 219A.
No Diagonals.-- Record No. 203A.
Enlarged 2X

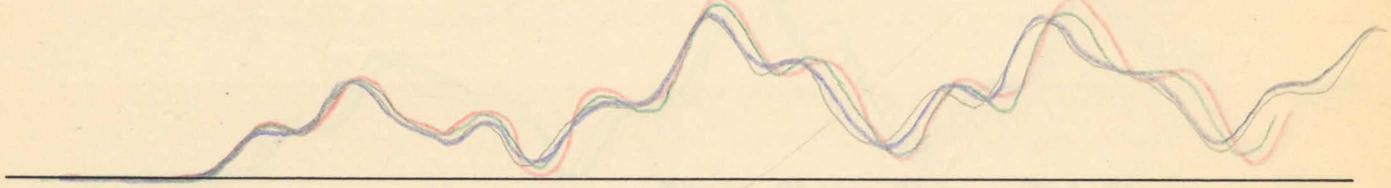
7-8-35

Quake Stick No. 1 Pulled by 4 Pounds Weight.

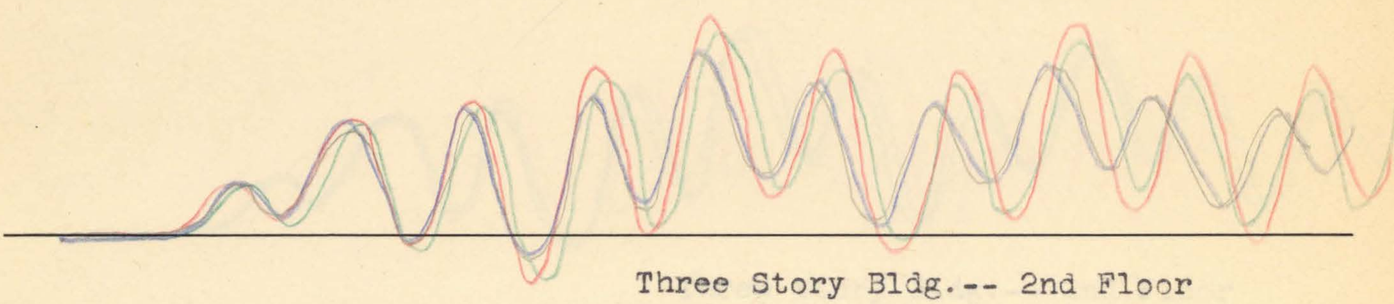
Fig. 55



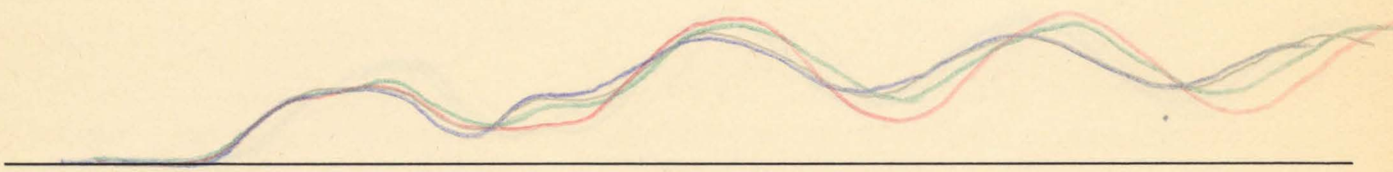
Three Story Bldg.-- Roof.



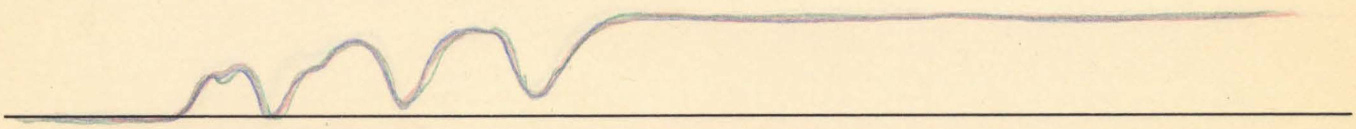
Three Story Bldg.-- 3rd Floor



Three Story Bldg.-- 2nd Floor



One Story Bldg.-- Roof



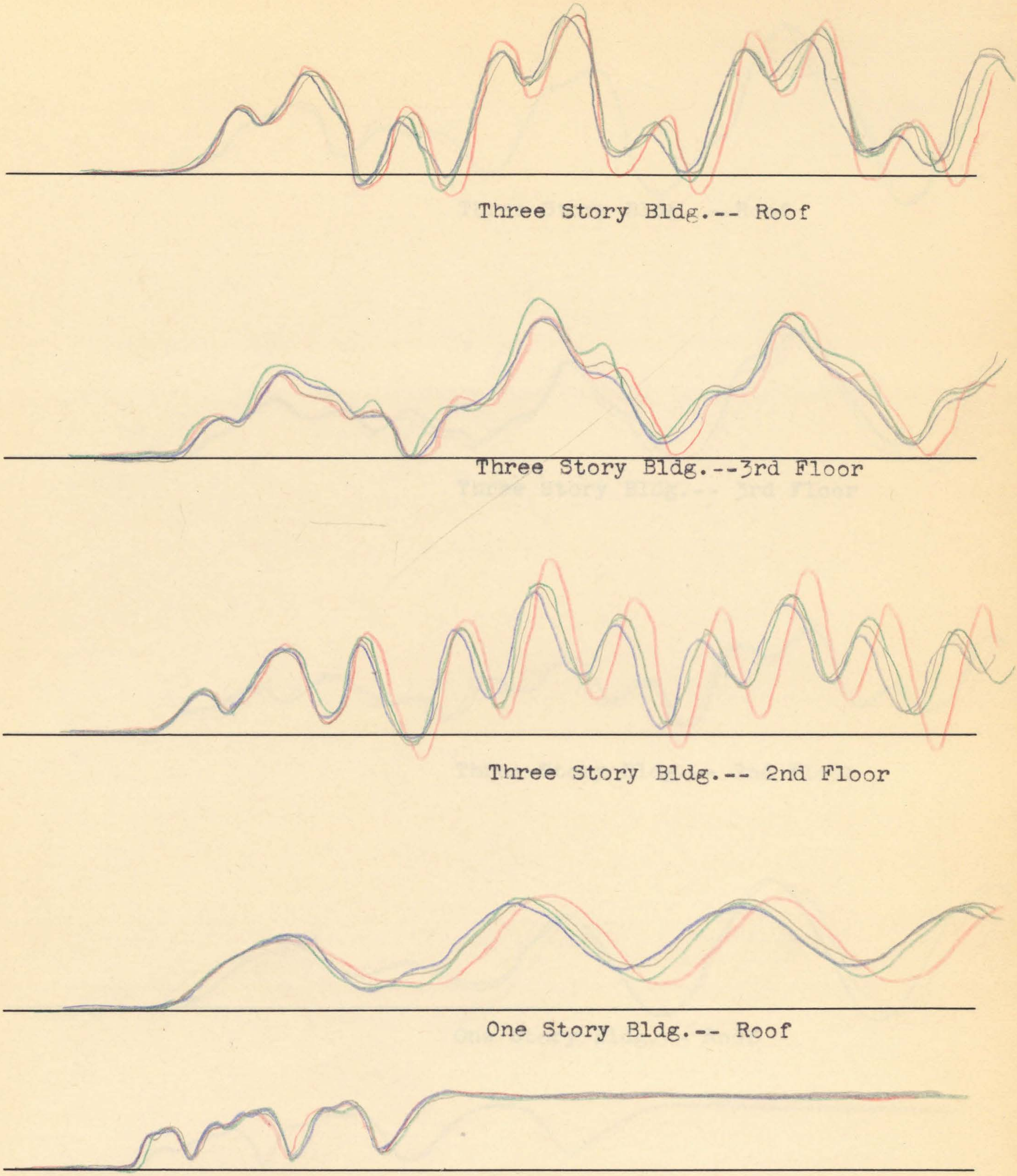
Ground Motion

——— Rubber Diagonals; 1.07 oz. Tension.-- Record 187A.
——— Rubber Diagonals; 3.68 oz. Tension.-- Record 211 A.
——— Rubber Diagonals; 10.5 oz. Tension.-- Record 220A.
——— No Diagonals.-- Record No. 204A.

Enlarged 2X 7-8-35

Quake Stick No. 1 Pulled by 5 Pounds Weight.

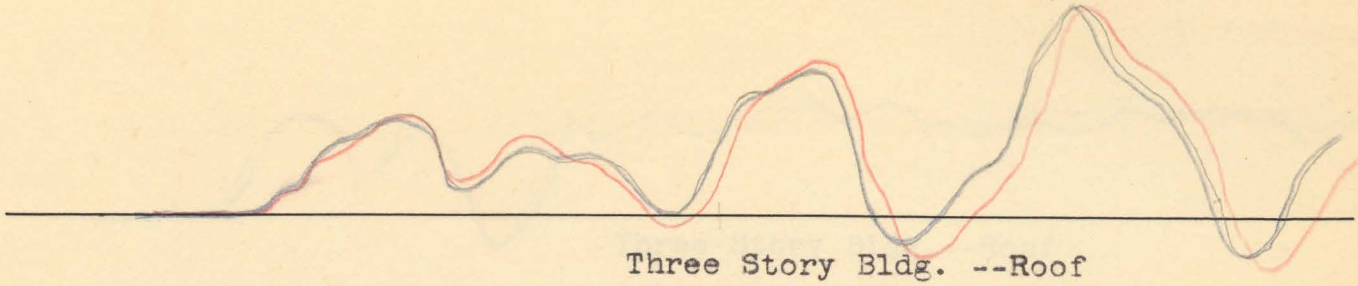
Fig. 56



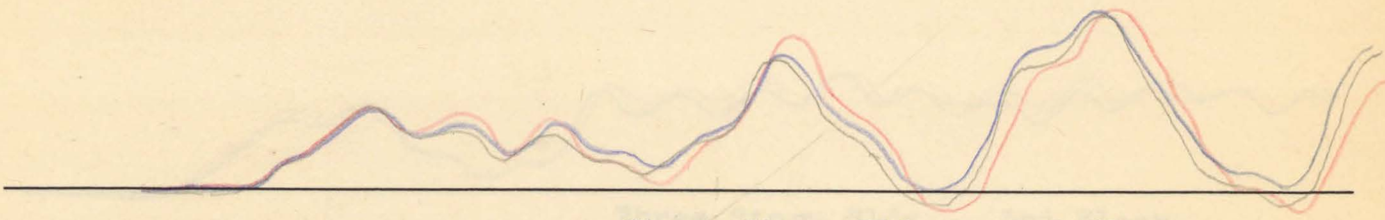
Rubber Diagonals; 1.07 oz. Tension-- Record 188B.
Rubber Diagonals; 3.68 oz. Tension-- Record 212A.
Rubber Diagonals; 10.5 oz. Tension-- Record 221A.
No Diagonals-- Record No. 205B.
Enlarged 2X. 7-8-35

Quake Stick No. 1 Pulled by 6 Pounds Weight.

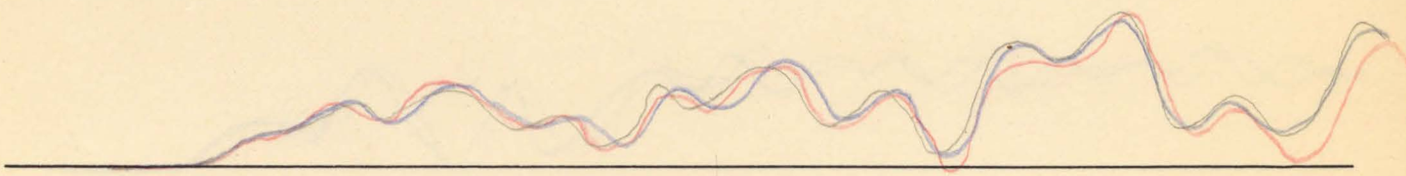
Fig. 57



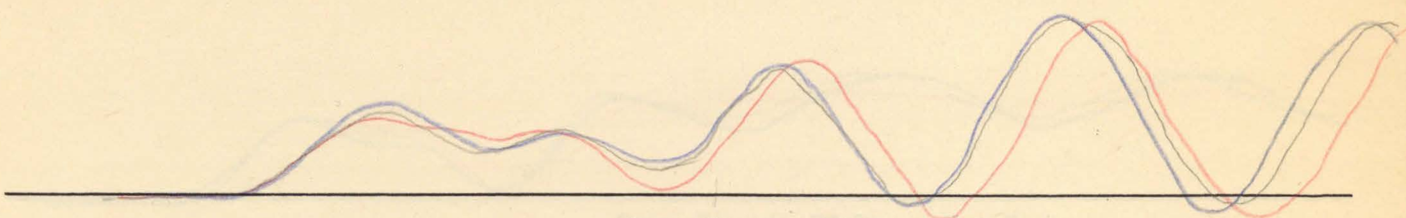
Three Story Bldg. --Roof



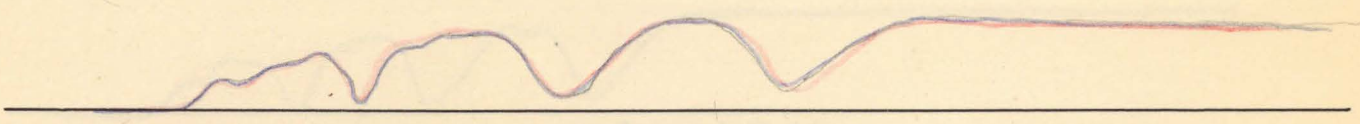
Three Story Bldg.-- 3rd Floor



Three Story Bldg.-- 2nd Floor



One Story Bldg.-- Roof

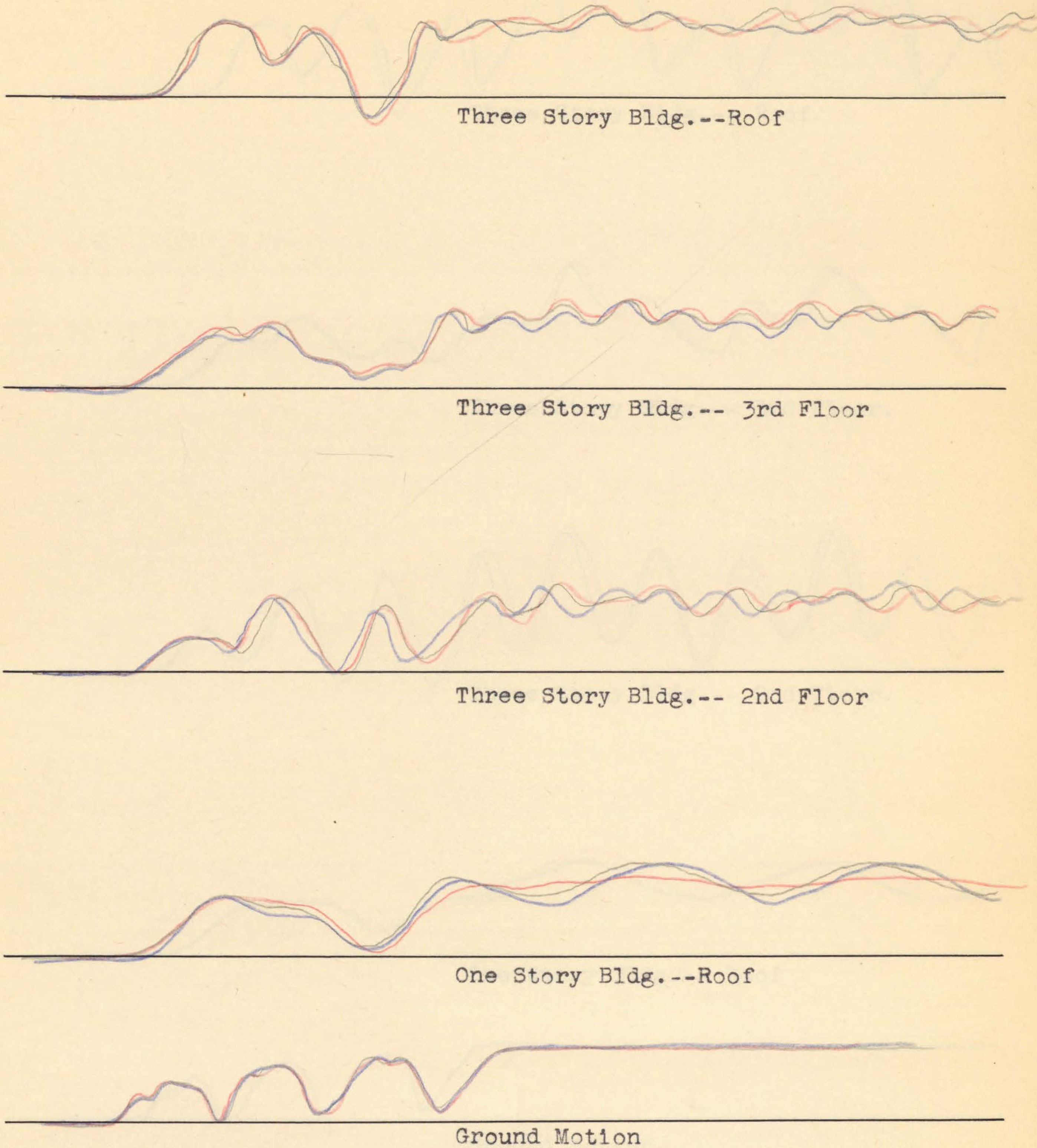


Ground Motion

—— Long Springs; 1.07 oz Tension.--Record No. 224A.
—— Long Springs; 3.68 oz. Tension.-- Record No. 229A.
—— No Diagonals; Record No. 236A.
Enlarged 2X. 7-10-35

Quake Stick No. 1 Pulled by 2 Pounds Weight.

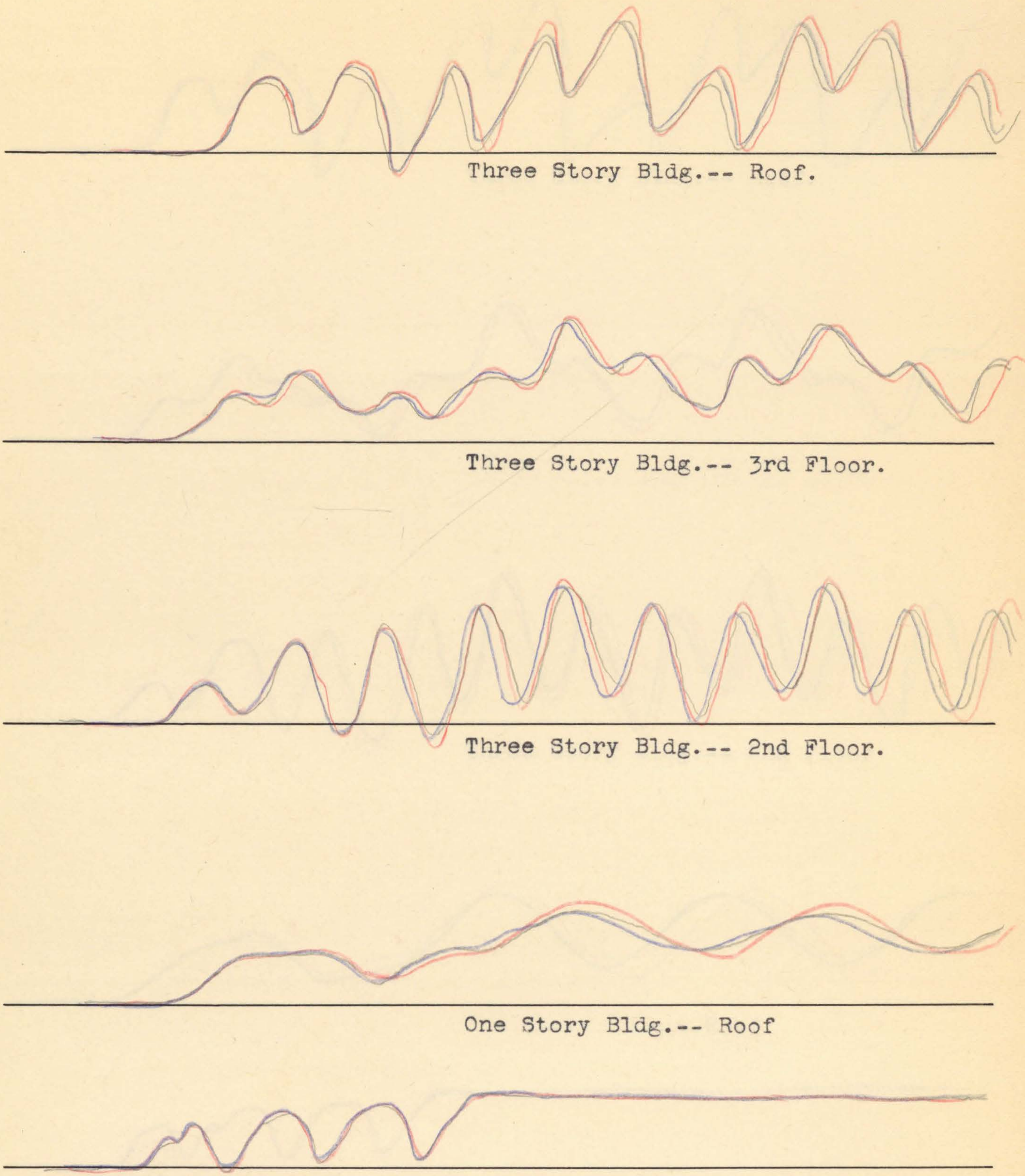
Fig. 58



——Long Springs; 1.07 oz. Tension.-- Record No. 225A.
——Long Springs; 3.68 oz. Tension.-- Record No. 230B.
——No Diagonals; Record No. 237A.
Enlarged 2X. 7-10-35

Quake Stick No. 1 Pulled by 3 Pounds Weight.

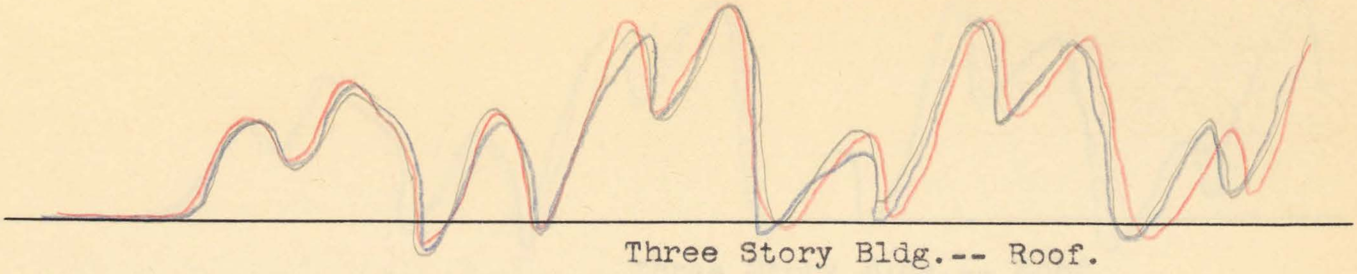
Fig. 59



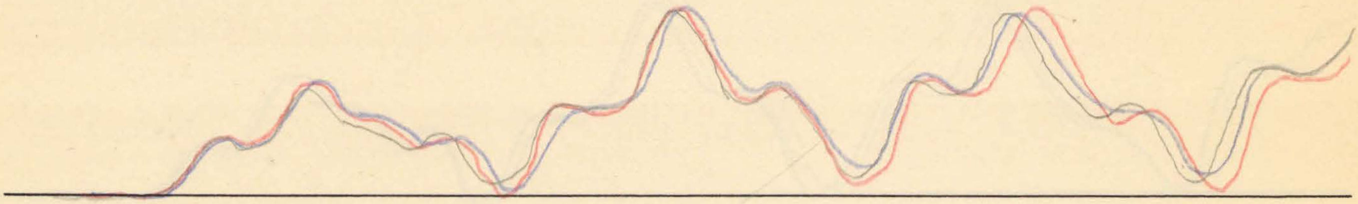
Long Springs; 1.07 oz. Tension.-- Record No. 226.
Long Springs; 3.68 oz. Tension.-- Record No. 231.
No Diagonals; Record No. 238.
Enlarged 2X. 7-10-35.

Quake Stick No.1 Pulled by 4 Pounds Weight.

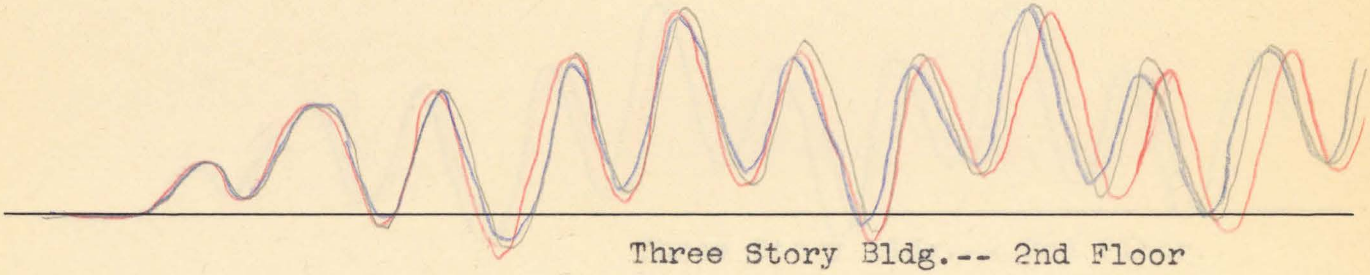
Fig. 60



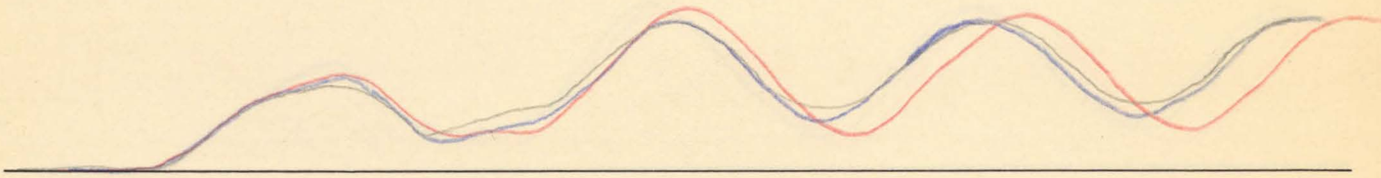
Three Story Bldg.-- Roof.



Three Story Bldg.-- 3rd Floor.



Three Story Bldg.-- 2nd Floor



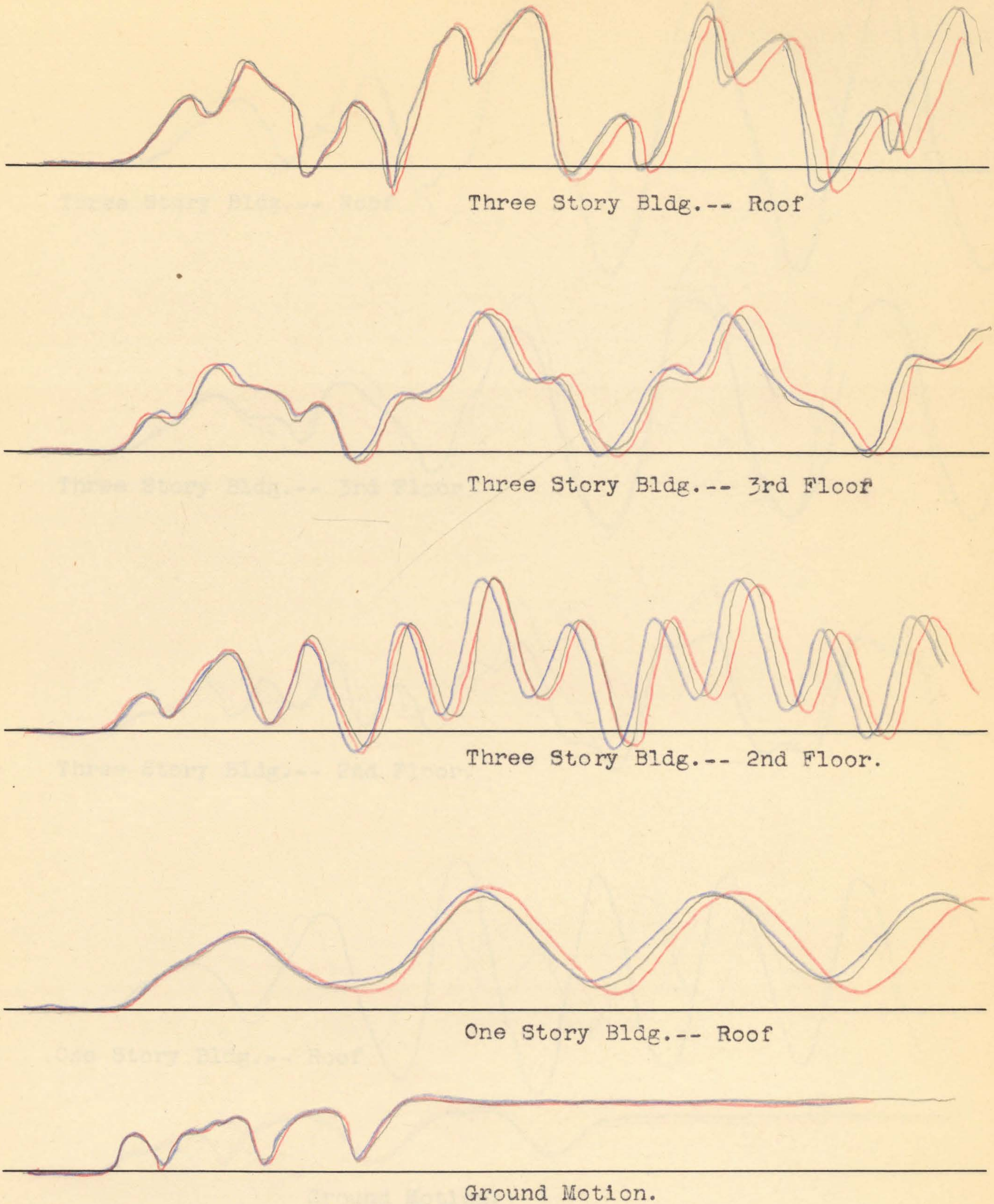
One Story Bldg.--Roof.

Ground Motion.

——Long Springs; 1.07 oz. Tension.-- Record No. 227.
——Long Springs; 3.68 oz. Tension.-- Record No. 232.
——No Diagonals; Record No. 239.
Enlarged 2X. 7-10-35.

Quake Stick No. 1 Pulled by 5 Pounds Weight.

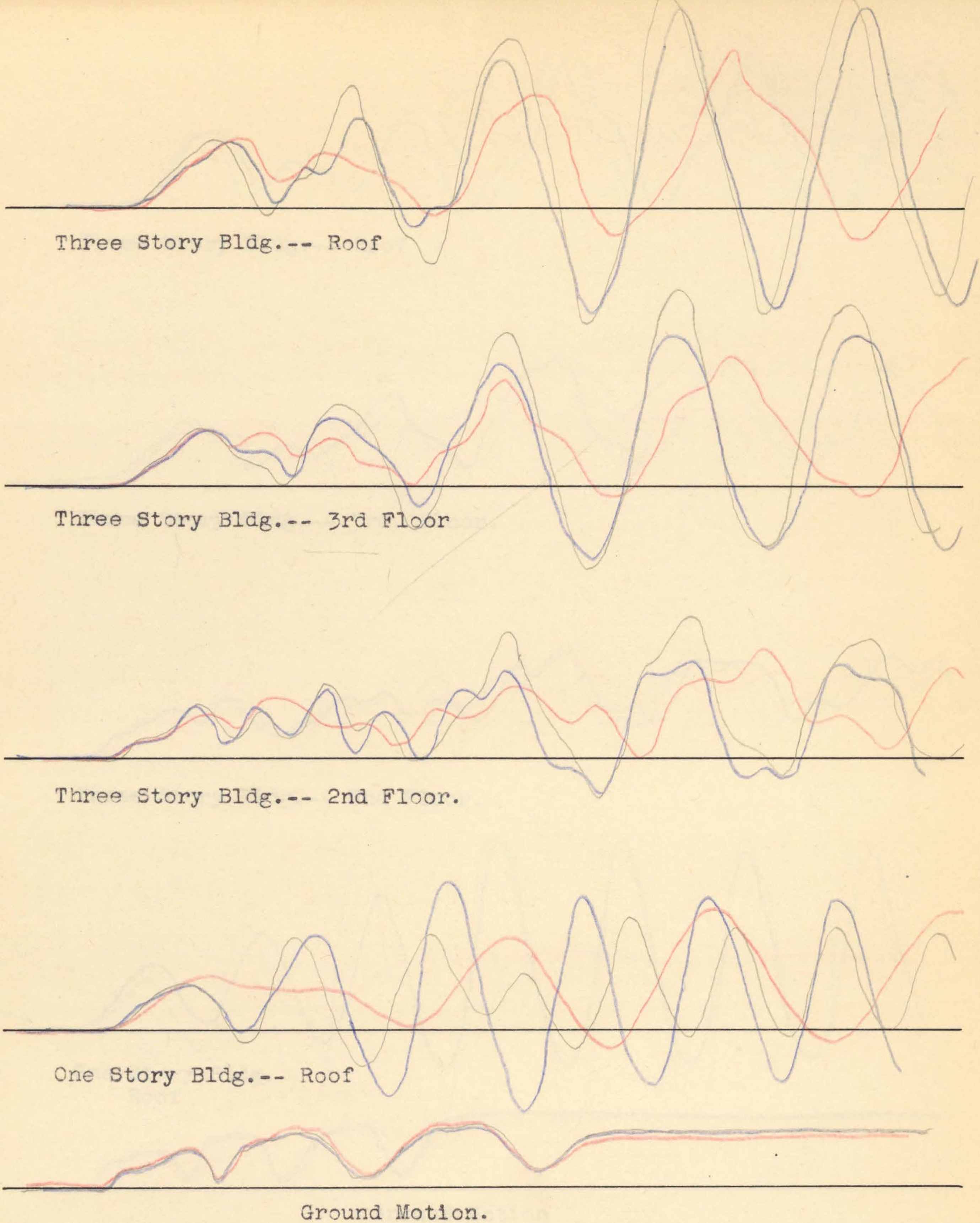
Fig. 61



Long Springs; 1.07 oz. Tension.-- Record No. 228.
Long Springs; 3.68 oz. Tension.-- Record No. 233.
No Diagonals; Record No. 240.
Enlarged 2X. 7-10-35.

Quake Stick No. 1 Pulled by 6 Pounds Weight.

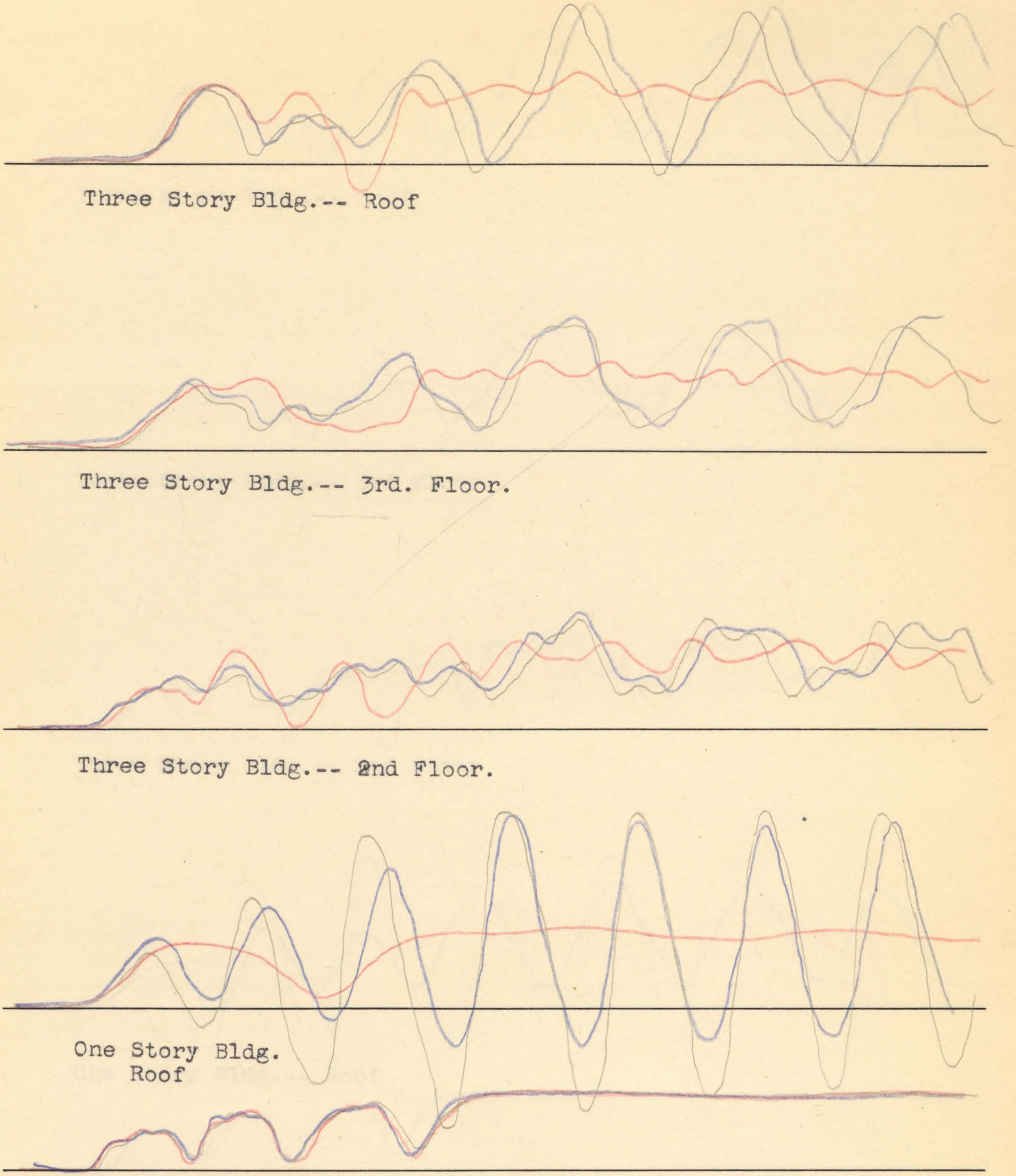
Fig. 62



——Short Stiff Springs; Loose.-- Record No. 243A.
——Short Stiff Springs; Stretched.-- Record No. 250B.
——No Diagonals.-- Record No. 236A.
Enlarged 2X. 7-16-35

Quake Stick No. 1 Pulled by 2 Pounds Weight.

Fig. 63

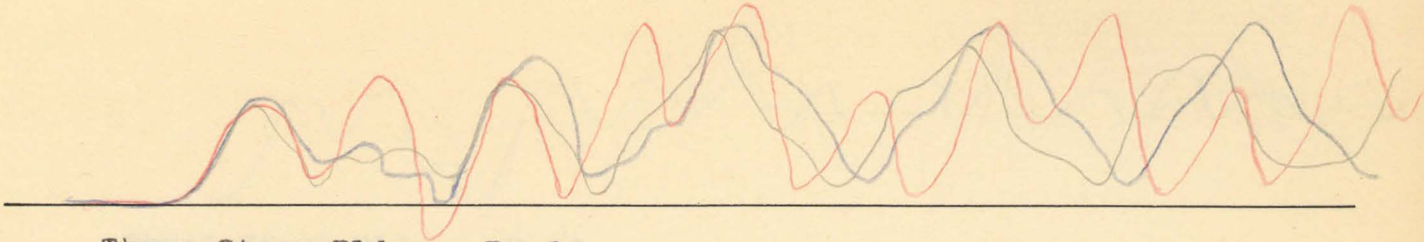


Short Stiff Springs; Loose.-- Record No. 244.
Short Stiff Springs; Stretched.-- Record No. 251.
No Diagonals.-- Record No. 237.
Enlarged 2X.

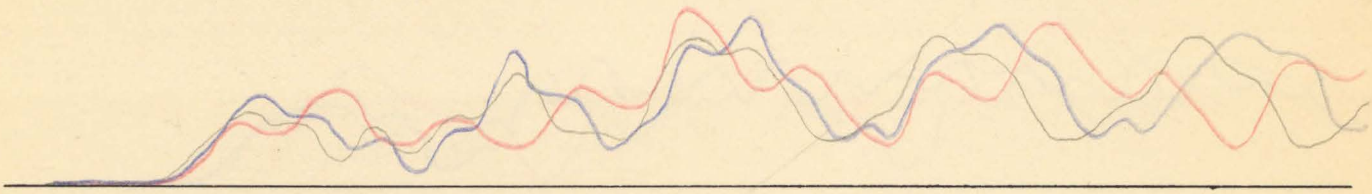
7-16-35.

Quake Stick No. 1 Pulled by 3 Pounds Weight.

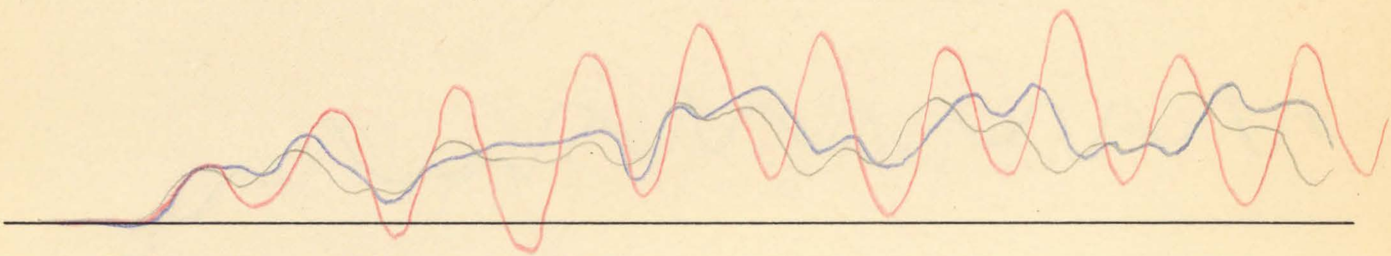
Fig. 64



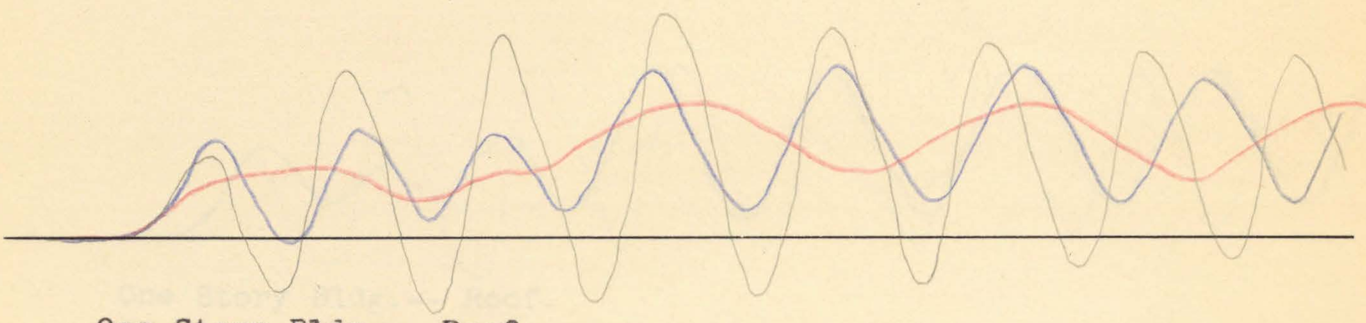
Three Story Bldg.-- Roof



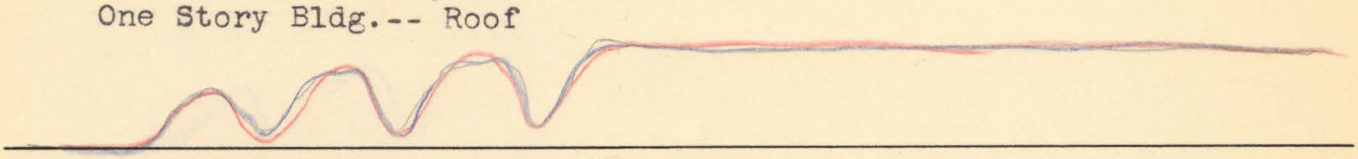
Three Story Bldg.-- 3rd Floor.



Three Story Bldg.-- 2nd Floor



One Story Bldg.-- Roof

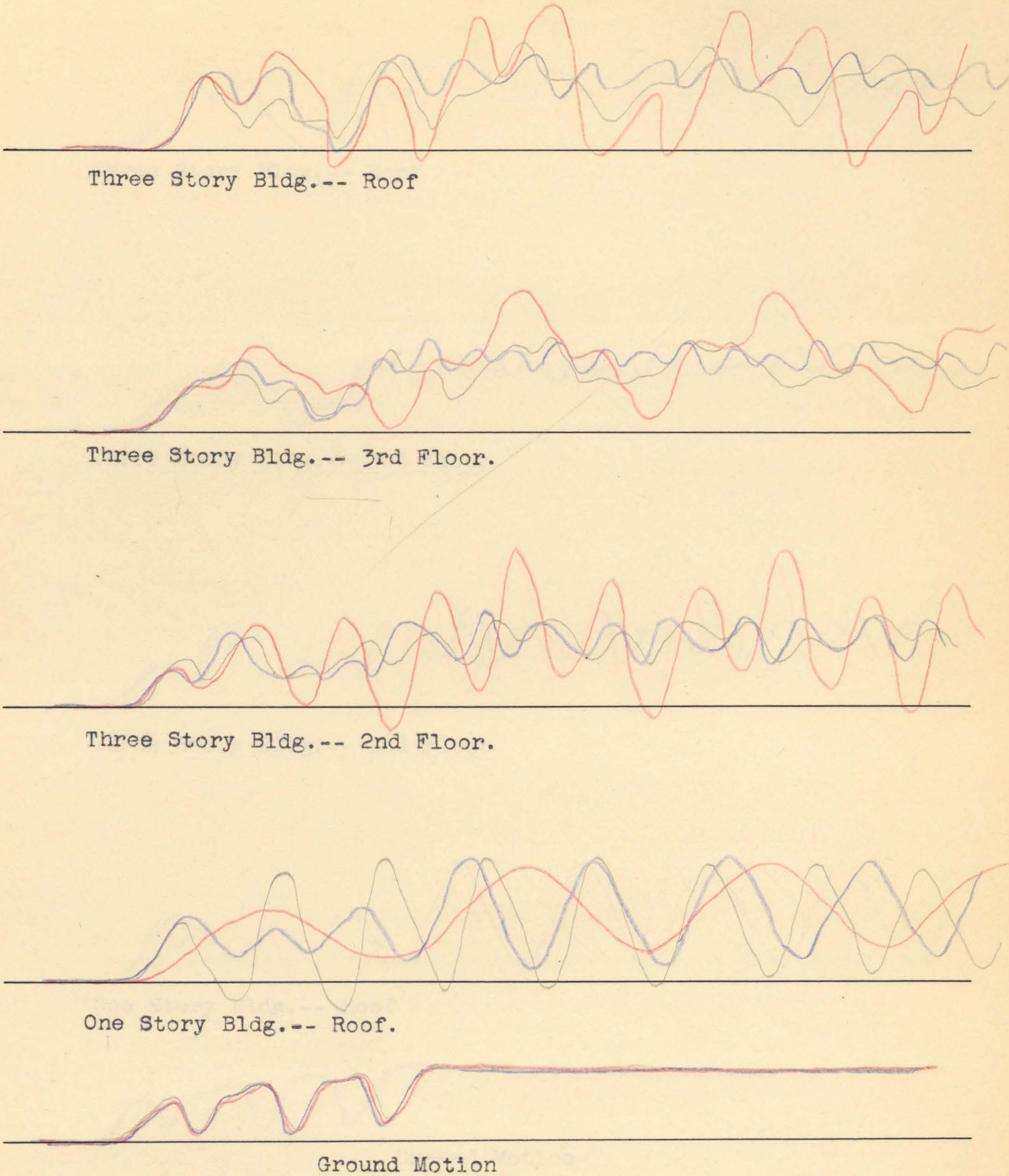


Ground Motion.

——— Short Stiff Springs; Loose.-- Record No. 245.
——— Short Stiff Springs; Stretched.-- Record No. 252.
——— No Diagonals.-- Record No. 238.
Enlarged 2X. 7-16-35.

Quake Stick No. 1 Pulled by 4 Pounds Weight.

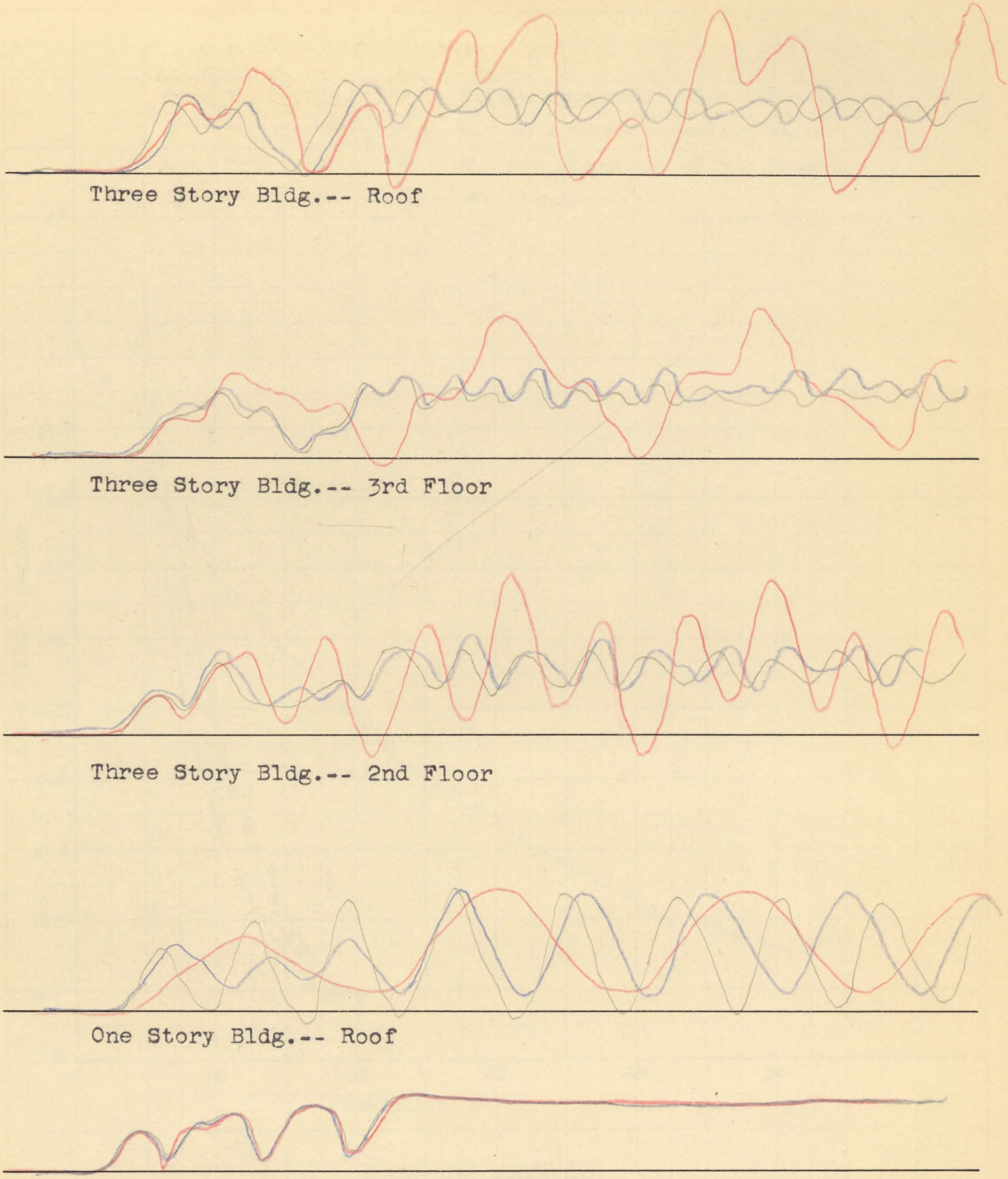
Fig. 65



Short Stiff Springs; Loose.-- Record No. 246.
Short Stiff Springs; Stretched.-- Record No. 253.
No Diagonals.-- Record No. 239.
Enlarged 2X. 7-16-35.

Quake Stick No. 1 Pulled by 5 Pounds Weight.

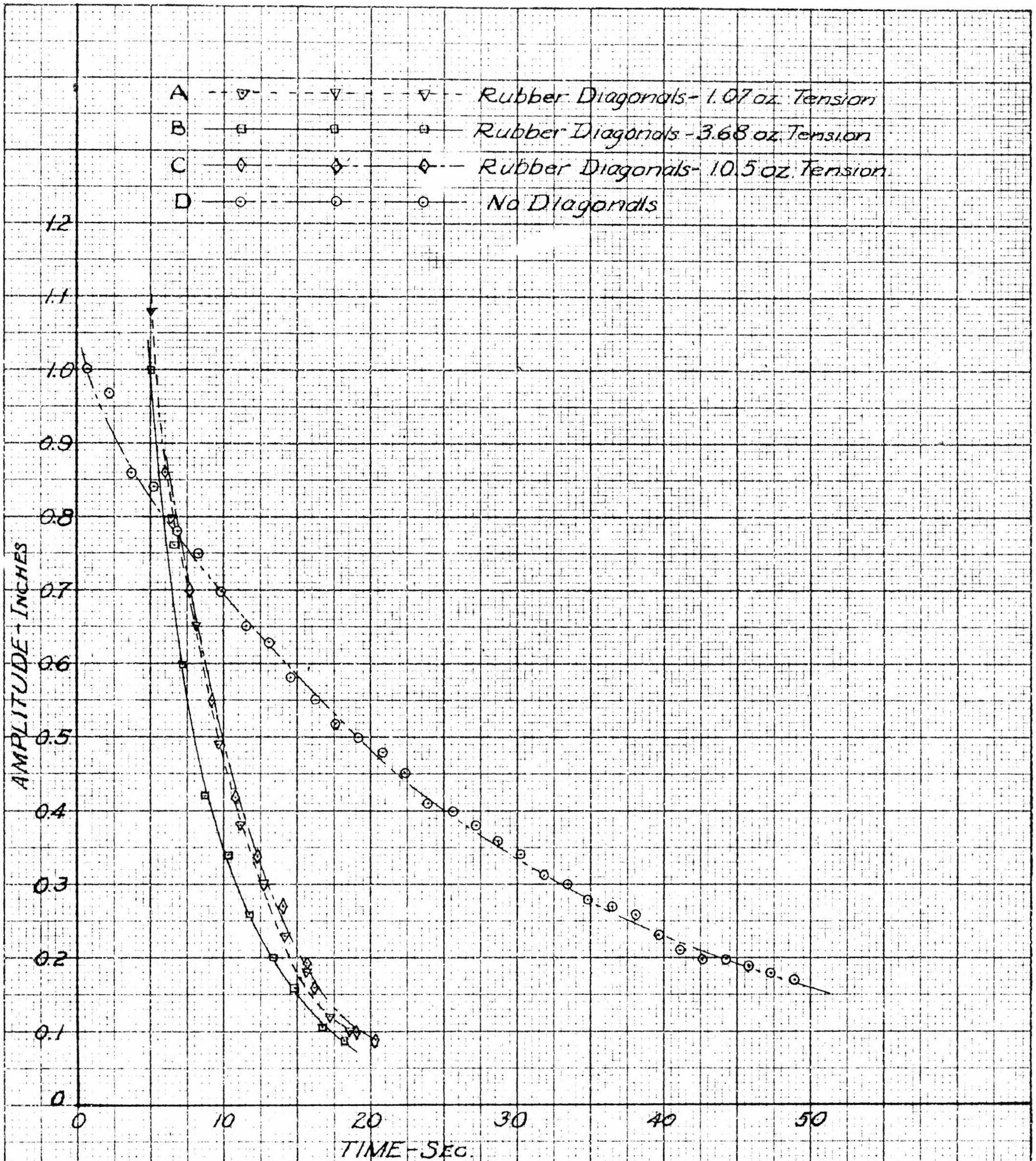
Fig. 66



——Short Stiff Springs; Loose.-- Record No. 247.
——Short Stiff Springs; Stretched.-- Record No. 254B.
——No Diagonals.-- Record No. 240.
Enlarged 2X. 7-16-35.

Quake Stick No. 1 Pulled by 6 Pounds Weight.

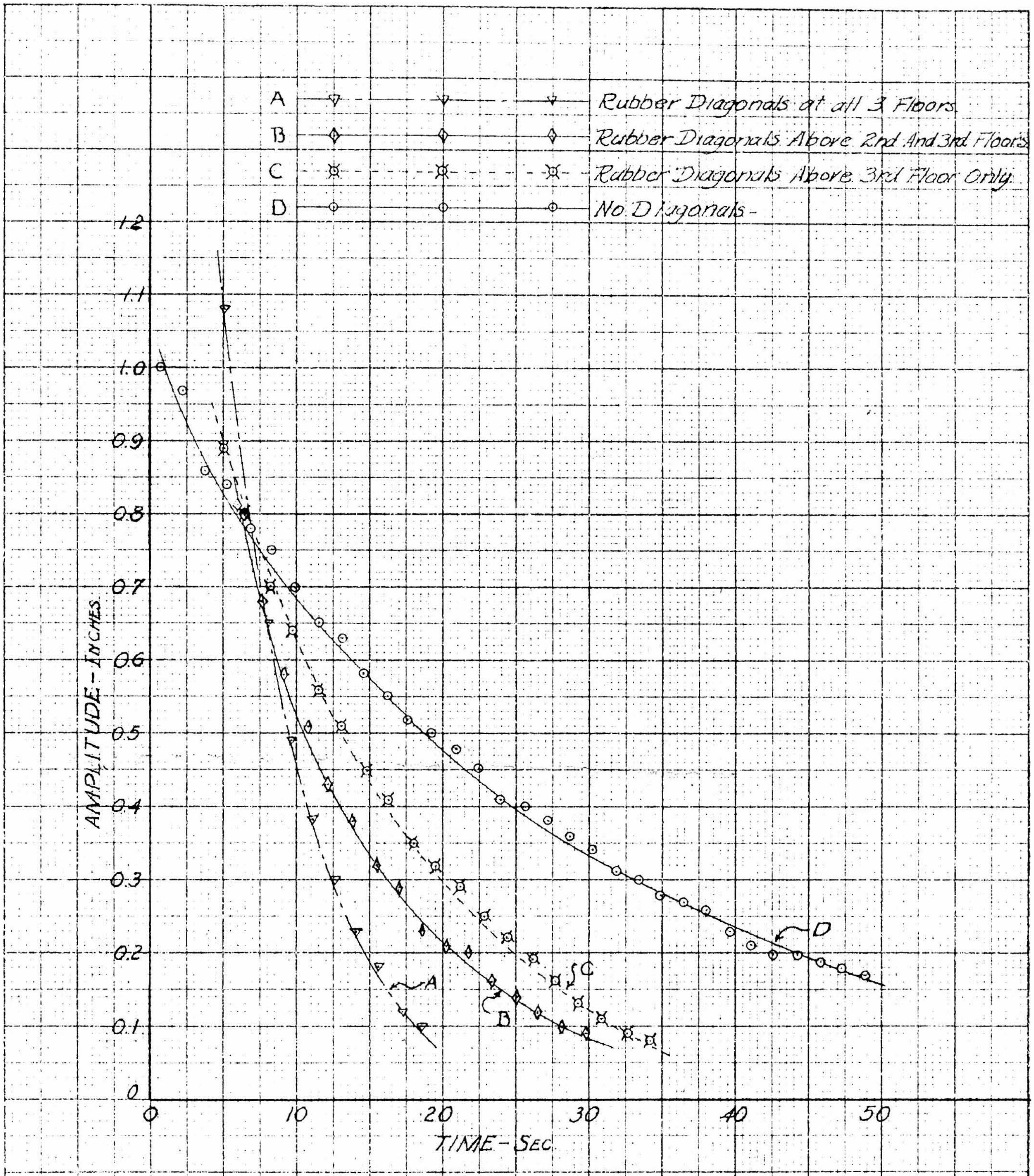
Fig. 67



DAMPING CURVES

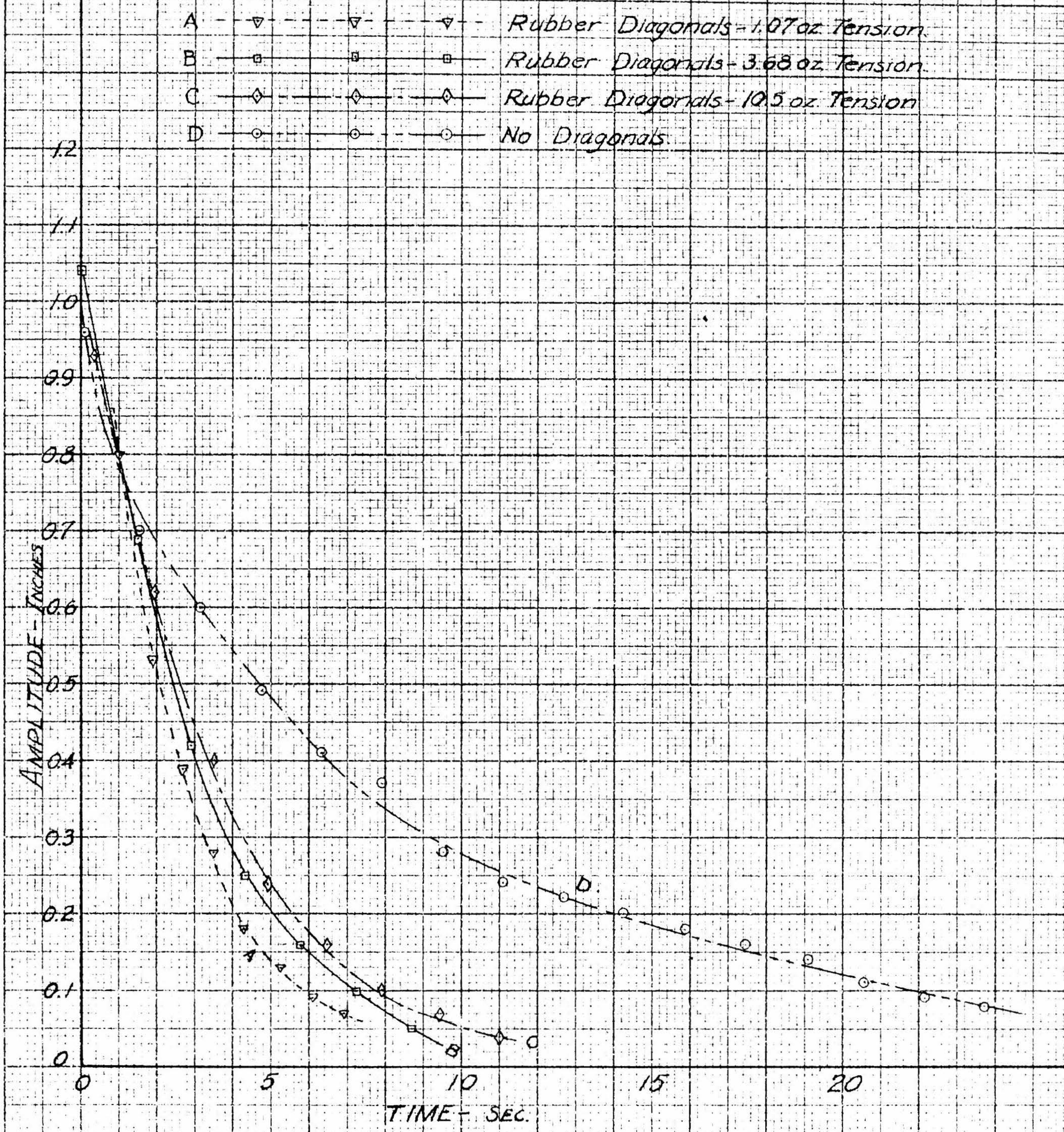
THREE STORY BUILDING

FIGURE 68



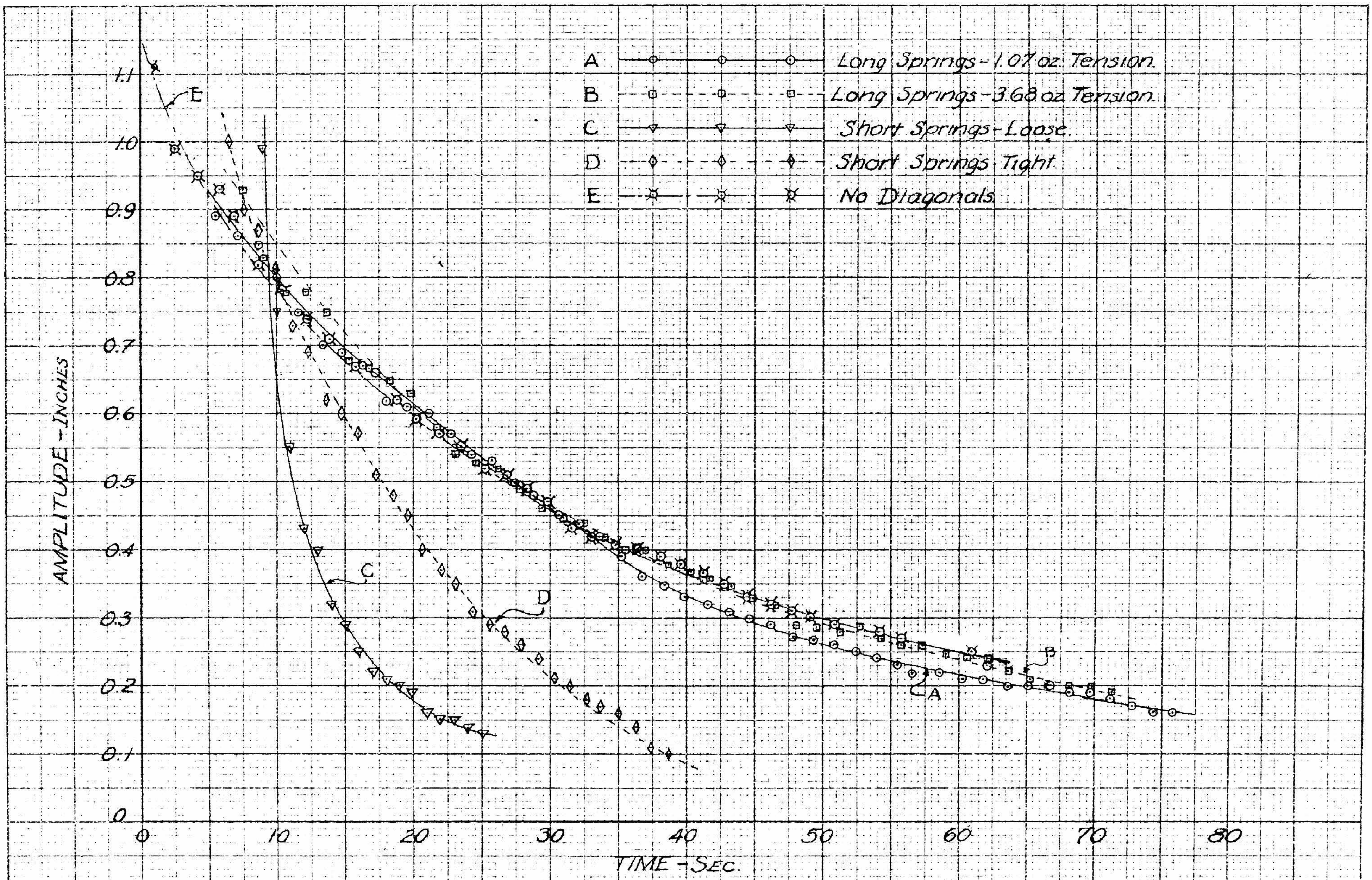
DAMPING CURVES
THREE STORY BUILDING

FIGURE 69



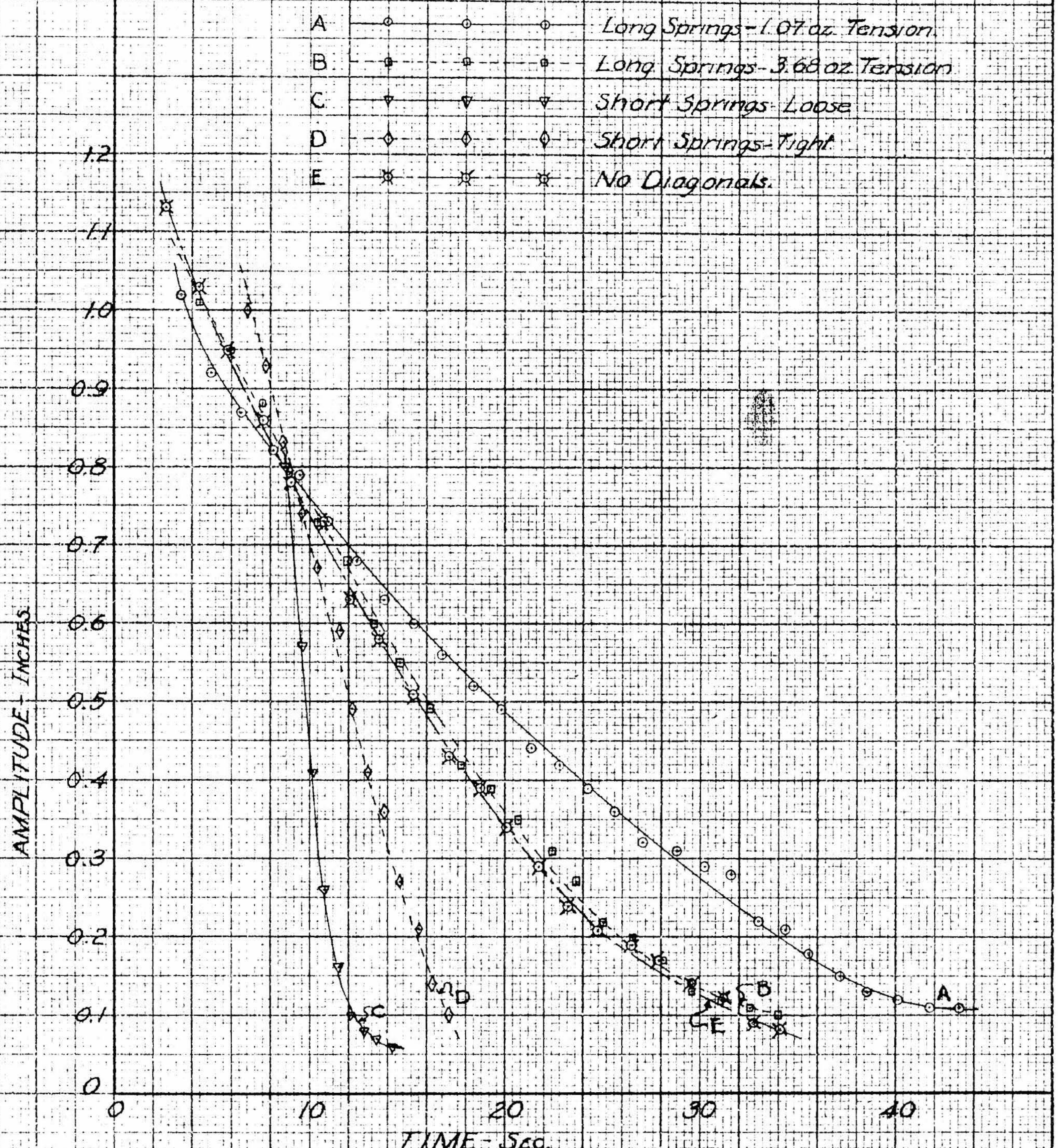
DAMPING CURVES
ONE STORY BUILDING

FIGURE 70



DAMPING CURVES
THREE STORY BUILDING

FIGURE 71



DAMPING CURVES
ONE STORY BUILDING

FIGURE 72

PART III

Chapter XI

Behavior of Simple Bents Subjected to Established Simple Harmonic Ground Motion

Description of Apparatus:

The apparatus used in the study of bents under the influence of established simple harmonic ground motion is shown in Figure 73. The base upon which the bent was mounted was caused to oscillate with simple harmonic motion by means of a motor-driven crank and connecting rod mechanism. The speed of the motor was adjusted by means of a rheostat, and the speed of the crank was measured by a tachometer. The speed was held constant at a given value until the motion of the bent achieved a steady state condition, and then the amplitude of the top floor motion was measured by means of a micrometer screw.

Two types of bents were used. Both were two story frames with steel columns and rigid girders, of the dimensions shown in Figure 74-A. However, one bent had columns consisting of flat steel strips, as shown in the sectional view Figure 74-B, while the other had columns consisting of sections of flexible steel tape with the convex side outward, as shown in Figure 74-C.

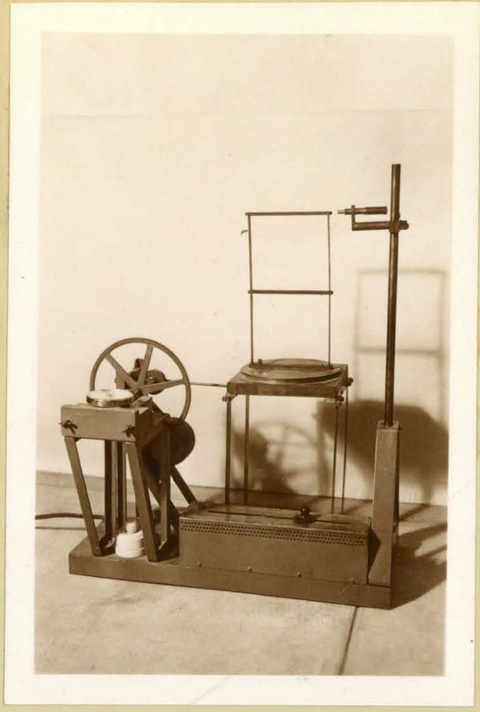


FIGURE 73

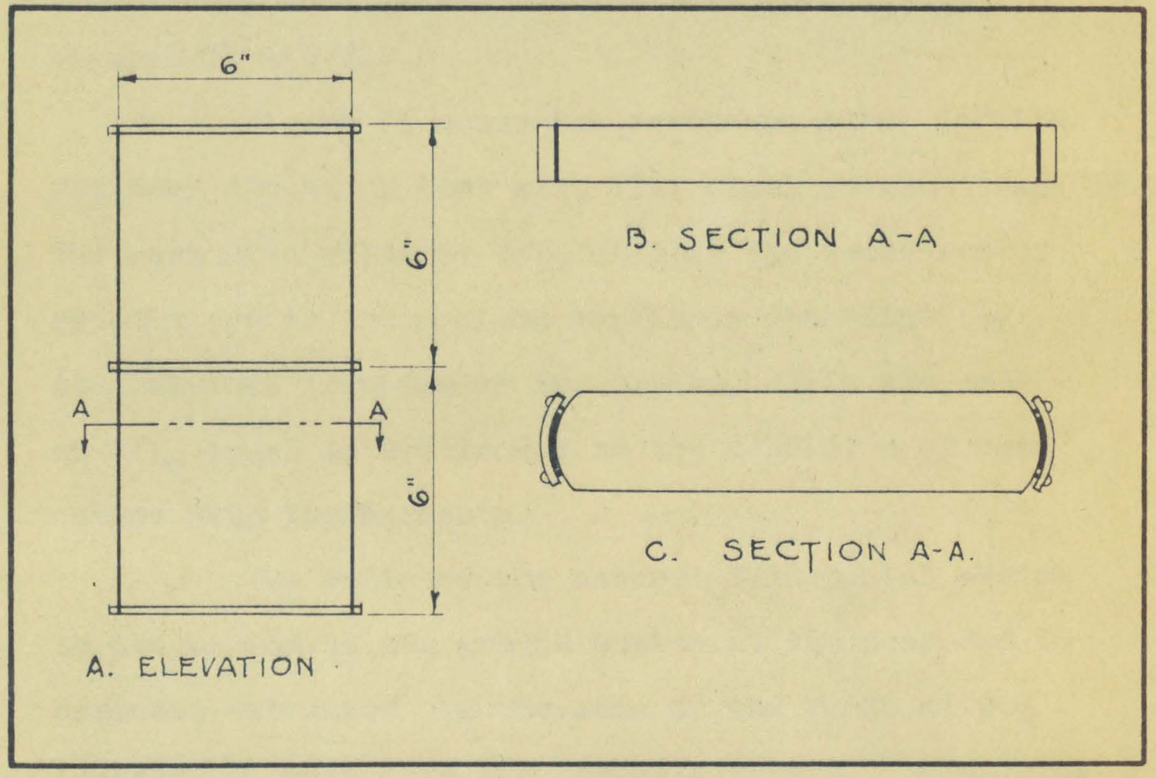


FIGURE 74

The object was to study the effects of buckling in the columns with curved cross section, as compared to the behavior of the ordinary bent with flat columns.

Results and Conclusions:

Simultaneous observations of the amplitude of vibration of the top of the bent and of the speed of rotation of the crank were taken. From this data, the ratios of amplitude of the top of the bent to amplitude of ground motion ($\frac{A}{A_o}$), and natural period to period of ground motion, ($\frac{T}{T_o}$), were calculated. The results were plotted to give the curves shown in Figures 75 and 76.

Figure 75 shows the resonance curve for the ordinary two story bent with flat steel columns (Fig. 74-B). The peak at a value of T/T_o of 1 is the condition of resonance, with the maximum amplitude determined by the internal friction of the system, while the peak at T/T_o equal to 2.6 is due to the condition of resonance with the harmonic.

The ratio of the natural fundamental period to the period of the ground motion at the peak due to harmonic vibration is the same as the ratio of the fundamental period to the harmonic period. This value

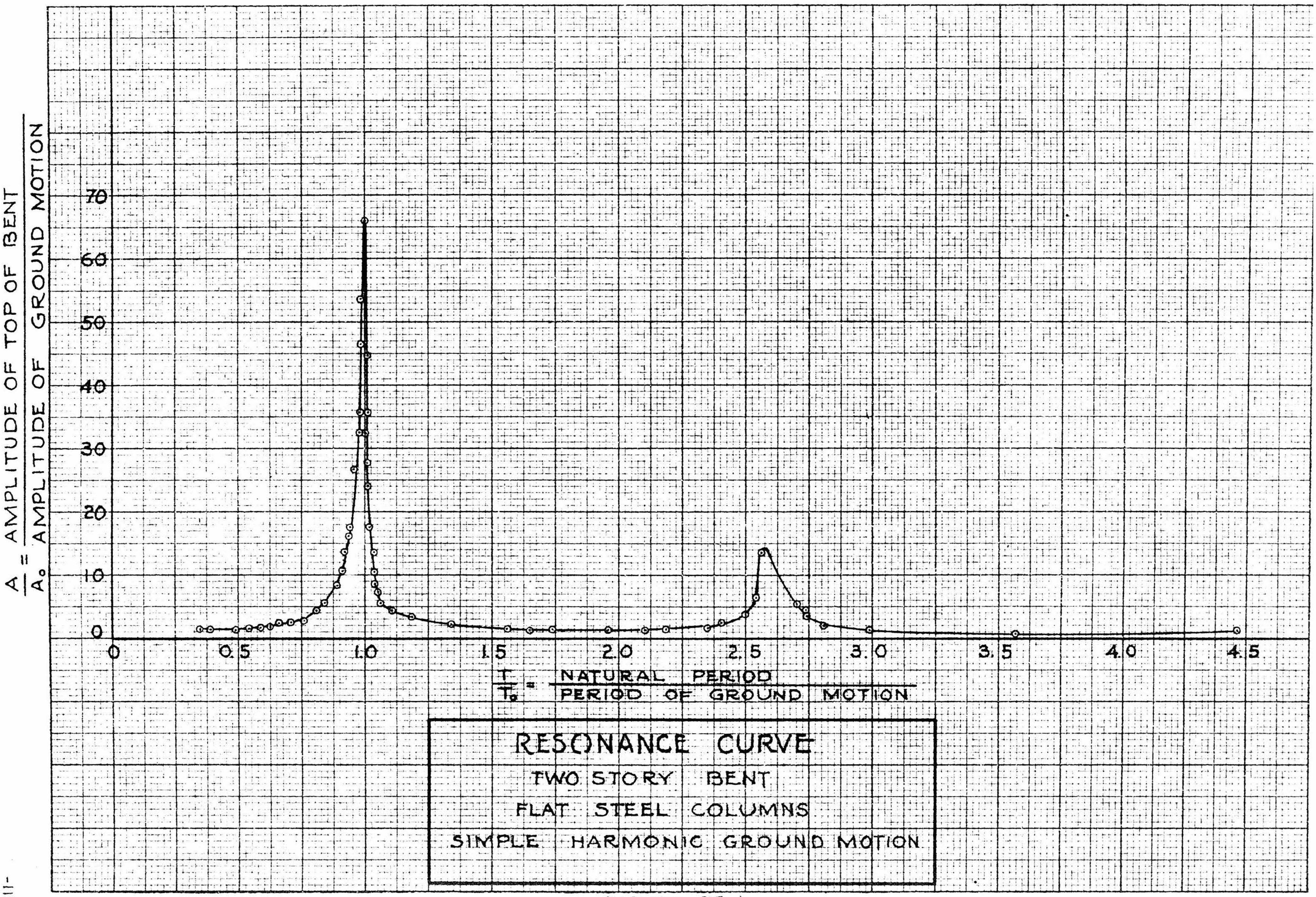

A/Ao = AMPLITUDE OF TOP OF BENT / AMPLITUDE OF GROUND MOTION
0
10
20
30
40
50
60
70
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
T/To = NATURAL PERIOD / PERIOD OF GROUND MOTION
RESONANCE CURVE
TWO STORY BENT
FLAT STEEL COLUMNS
SIMPLE HARMONIC GROUND MOTION

FIGURE 75

can be calculated using a theorètical equation, for bents with identical stories and floors, presented by Merit P. White in his thesis.[1]

$$\omega = \left[2 \sin \frac{\pi}{2}\left(\frac{2n - 1}{2L + 1}\right)\right]\sqrt{\frac{K}{m}}$$

where ω = angular frequency.

L = number of stories.

n = order of vibration. (1 for fundamental, 2 for first harmonic, 3 for second harmonic, etc.)

K = stiffness of pair of columns.

Then $$T/T_o = \frac{\omega_2}{\omega_1} = \frac{\sin \frac{\pi}{2}\left(\frac{2\times 2 - 1}{2\times 2 + 1}\right)}{\sin \frac{\pi}{2}\left(\frac{2 - 1}{2\times 2 + 1}\right)}$$

$$T/T_o = \frac{\sin 54^\circ}{\sin 18^\circ} = 2.61$$

It can be readily seen that the experimental peak occurs at the proper ratio of T/T_o as predicted by this theoretical calculation.

Figure 76 shows three curves of the same type for the bent with curved steel tape columns (Figure 74-C). With the amplitude of the ground motion equal to 0.0125 inches, the ordinary type of resonance curve is obtained. The accelerations were sufficiently small that practically no buckling

1 White, Merit P., <u>Analytical Studies of the Dynamic Response of Certain Structures to Assumed Ground Movements</u>. Thesis for Degree of Doctor of Philosophy California Institute of Tech. 1935.

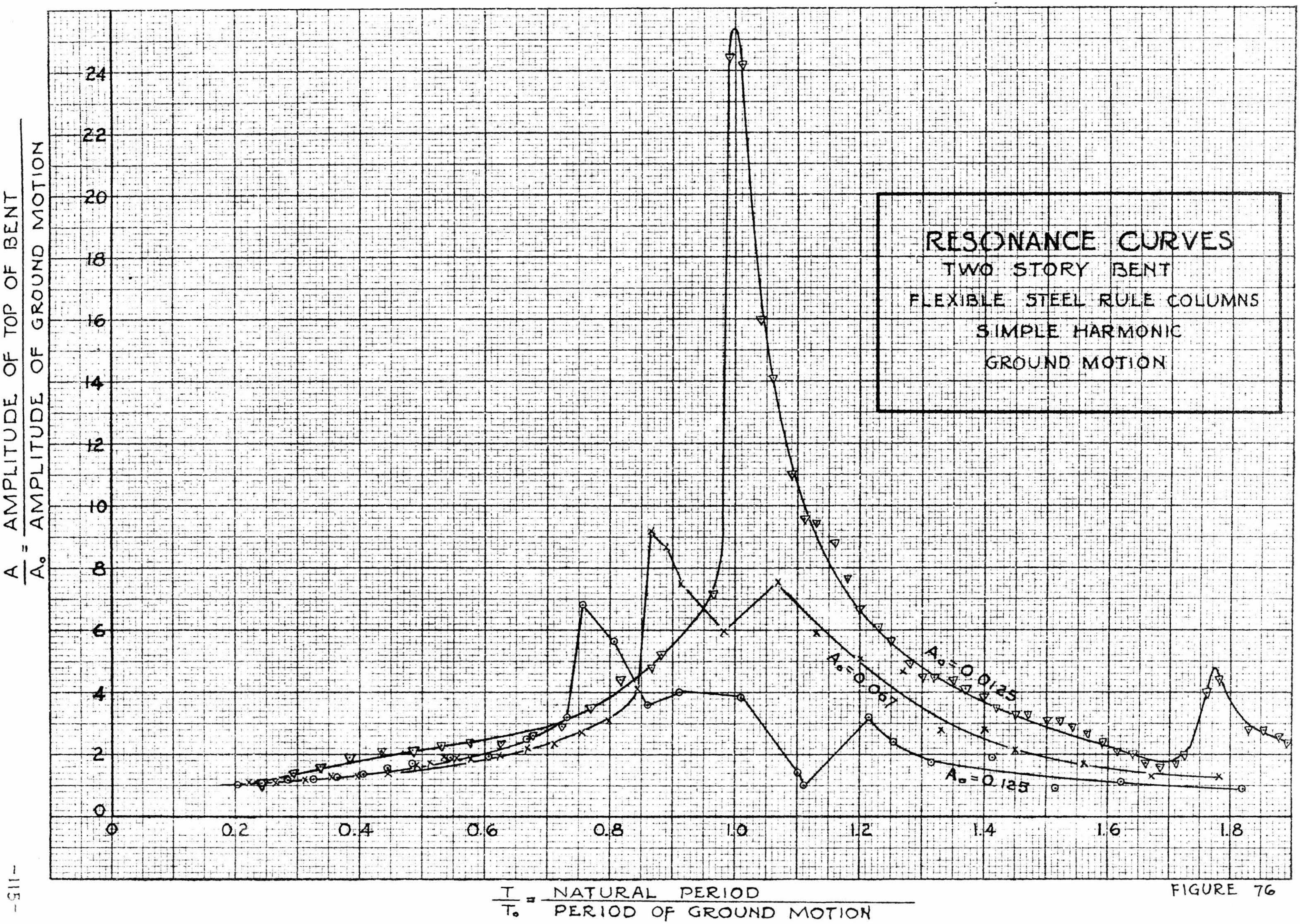

RESONANCE CURVES
TWO STORY BENT
FLEXIBLE STEEL RULE COLUMNS
SIMPLE HARMONIC
GROUND MOTION
$A_o = 0.0125$
$A_o = 0.067$
$A_o = 0.125$
$\frac{A}{A_o} = \frac{\text{AMPLITUDE OF TOP OF BENT}}{\text{AMPLITUDE OF GROUND MOTION}}$
$\frac{T}{T_o} = \frac{\text{NATURAL PERIOD}}{\text{PERIOD OF GROUND MOTION}}$

FIGURE 76

occurred, and it can be seen that a curve of a shape similar to Figure 75 for the flat columns was obtained. The peak at T/T_o of one is due to resonance with the fundamental, whereas the peak at T/T_o equal to 1.77 is caused by the harmonic. The reason for the lower ratio of T/T_o than for the bent with flat columns , for resonance with the harmonic, is that there was a fluttering type of buckling at the high speeds necessary for a ratio of T/T_o of 1.77, and a consequent reduction in stiffness.

With eccentricity of the crank adjusted to give an amplitude of ground motion (A_o) of 0.067 inches, the ground motion was sufficiently violent that buckling of the columns took place, to a greater or less degree, for all ratios of T/T_o greater than 0.85. When buckling takes place, the columns are suddenly reduced in stiffness to a marked degree, and the period of the bent is made much longer. This is the reason for the peak being displaced to the left ($T/T_o = 0.86$). Various degrees of buckling and resonance combine to give the jagged curves shown, since the columns were snapping and buckling intermittently. It should be mentioned that the moment required to cause buckling of a convex column in one direction is much less than the moment required to buckle the column in the other direction. Sufficient buckling occurred at the high crank speeds to destroy the effect of the harmonic entirely.

The curve with the amplitude of the ground motion equal to 0.125 inches shows the effects of the violent buckling that took place after T/T_o exceeded 0.73. The jagged nature of the curve again is an indication of the various degrees of buckling.

A comparison of the three curves of Figure 76 shows that the buckling was very effective in reducing the amplitude of vibration relative to the intensity of the ground vibration. The curves indicate that the relative amplitude (A/A_o) decreases as the amplitude of the ground motion (A_o) increases.

Another effect that can be observed is the shift of the peaks toward the left, due to decreasing the stiffness when buckling occurs. The greater the extent of buckling, the greater will be the decrease in the average stiffness, with a consequent movement of the peak of the curve toward the left.

In order to apply the system of buckling members to an actual structure, for the purpose of reducing amplitude of vibration in earthquakes, it would, of course, be necessary to support the structure by other members that would not be seriously effected by the motion of the structure. The buckling members would not be used to support the load at all, but merely to give the sudden changes in stiffness required to prevent excessive vibration due to resonance.