I

FLUX OF LOW ENERGY PROTONS AT SEA LEVEL

II

OTHER PHENOMENA INVOLVING PARTICLES OF GREATER THAN MINIMUM IONIZATION

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In conclusion, the writer wishes to express his gratitude to Professor Robert B. Leighton for first arousing his interest in cosmic rays, and to Professor Carl D. Anderson for his many helpful discussions, his sincere interest, and above all his very friendly approach to all problems.

ABSTRACT

A proportional counter has been installed in a six-inch cloud chamber with a magnetic field of 6400 gauss. By use of the proportional counter as a selection device, pictures of more than 200 low energy protons have been obtained in a time of somewhat over a year's duration. From these the absolute intensity of the low-energy proton component of cosmic rays has been determined at sea level. A spectrum of fifty electrons resulting from mu-meson decays has been obtained as well as other data on mu-mesons. Photographs of typical low-energy protons, mu-meson decays, and other interesting events are presented. In addition several interesting pictures are discussed in detail.

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I: INTRODUCTION

After a successful summer at high altitude in 1947, it was decided to reinstall the six-inch cloud chamber in the aft pressurized cabin of a B-29 for the summer of 1948. It was hoped that by use of a more refined selection device sufficient meson decays could be secured to be able to obtain a spectrum. In addition, it was felt that enough other significant pictures would be obtained to make the reinstallation worth-while.

The work of the summer of 1948 resulted in no meson decays. The number of flights in 1947 had been somewhat in excess of two per week, while in 1948 there was less than one flight per week and several of these were of short duration. Perhaps, it would be of value to point out that in the summer of 1948 while the chamber was installed in the plane the chamber was reset and waiting to take a picture less than 0.5 % of the time, while in the operation on the ground it has been found that the figure is between 65 and 70 %. In addition to this, one must also remember that, even though the counting rate goes up by a large factor at altitude, percentagewise the number of pictures taken at altitude of the type which one is attempting to select is normally much smaller. This is because the accidental coincidence rate rises sharply in spite of all precautionary measures. Of course this must be qualified somewhat because the counting rates for some rare events go up from an almost negligible number at sea level to a more appreciable number at altitude. Penetrating showers afford a very good example of this, since they increase by a factor of about 500 between sea level and 35,000 ft.

After the return of the chamber to Pasadena it was decided to install a proportional counter in the chamber for exploratory purposes because as far as was known this had not been previously done. The results of this installation form the subject of two theses. One by M. L. Merritt[1] which deals with the behavior of the proportional counter itself, and this work which deals with the results obtained by its use in the course of some fifteen months of as continuous operation as could be maintained.

II: THE APPARATUS

Since the six-inch cloud chamber in its present form has been amply described by R. V. Adams[2] and R. R. Rau[3] in their theses, it is felt that a brief mention of significant changes is all that is needed. The major change from the standpoint of operation has been the use of a proportional counter as a selection device for expansion; however, the details of this form the subject of the thesis of M. L. Merritt[1] and our interest in it will be limited to the use of his results from time to time.

In order to obtain a greater useful volume in the chamber it has been deepened somewhat. This was done by cutting some of the iron off the back pole face as well as by allowing the chamber to extend a little further into the camera slot. The total depth is now 6 cm and the usable (illuminated) depth is 4.5 cm of which 2.5 cm are occupied by the counter. The removal of some of the pole face has reduced the field to a figure of 6400 gauss at the normal operating current of 200 amps. The field decreases by about 10 % as one goes to the edge of the chamber. In order to take account of this a map has been made of the field strength in the chamber. By use of this map the average values of the field along various paths can be obtained. These can be used for computation of the true momentum of a particle traversing the chamber. For any tracks which pass through the center of the counter an average field value of 6230 gauss is accurate to better than 2 % provided that a correction is made for the front to back position of the track in the chamber. A correction for the front to back position of the track is necessary because there is a difference in magnification of 7 % from front to back of the counter as well as a difference of field of about the same amount.

The light system has been modified from that described previously. A General Electric FT-127 flash tube is still used, but it is now mounted on the outside of the coils; and two spherical lenses with a cylindrical reflector behind the light are used to get a parallel beam in the chamber. Recently a cylindical lense has been tried with a pair of reflectors at top and bottom as well as a right angle reflector behind the light. This tends to raise the light intensity about a stop with a slight decrease in uniformity of lighting. The film used is linagraph panchromatic which from observation has a speed about a stop faster than linagraph orthochromatic for the light from the flash tube. The faster speed of the panchromatic far more than offsets the poorer contrast which it possesses. It has been found that one of the most important considerations is the film age. In some cases film has so deteriorated that by the time of its expiration date its speed is down a stop.

The chamber uses the same copper tubing as a conductor of the current and of the cooling water. This fact causes some embarrassing problems because the high mineral content of the water allows a quite vigorous electrolytic action to take place at the tube ends due to voltage differences which are 120 volts at some points. To counteract this, short sections of copper tubing which are in front of the main tubes and electrically connected with them have been installed. In this way the auxiliary tubes which are expendable are eaten away and can be replaced without damage to the original coils. The deposits formed by the electrolysis rapidly clog the pipes, and experience has shown that the most effective way of cleaning the pipes is to circulate water with a hydrochloric acid concentration of about 2 % through the pipes at a temperature of 60 - 70° C. The tubes are then flushed out and a 2 % ammonia solution used as wash. Going through this procedure once every 4 to 6 weeks has proved successful in keeping the pipes clean.

It was originally hoped that meson mass measurements could be made by use of the counter as an absorber; hence it was desirable to reduce errors introduced by multiple scattering in the gas to a minimum. Since the scattering radius of curvature varies inversely as the Z of the gas used, an attempt was made to use helium rather than argon. A compromise had to be effected because tracks in helium become quite diffuse before the drops have grown enough to be photographed, while at the same time the ratio of track intensity to background is much poorer because of the decidedly smaller number of ions produced along the track. A mixture of 1/4 argon and 3/4 helium at 2 atmospheres pressure was found to be usable, although not as satisfactory as helium from the scattering standpoint. For a 50 Mev/c meson with a track length of half the chamber the probability of a 500 Mev/c multiple scattering is 0.006 % in the mixture as compared with 5 % in argon.[4] Another way of viewing this is to say that there is a 5 % probability of a 1000 Mev/c multiple scattering in the mixture.

The mixture has the disadvantage that the photographs do not have as much contrast as those with pure argon and the track width is half again as wide and hence not as easy to measure with the comparator. The mixture has the advantage that it seems to give much better results with the stationary atmosphere technique than argon; but most important is the fact that in the mixture tracks do not become continuous until they ionize in excess of ten times minimum; thus allowing good estimates of ionization up to this point. In argon tracks of three to four times minimum ionization are continuous, and from there on up estimates of ionization are quite difficult.

The choice of an absorber for the chamber was dictated by the fact that, if possible, it was desired to observe meson decay electrons. To reduce radiation to a minimum an absorber of low atomic number was needed; hence aluminum was used. Since it was originally also hoped that meson mass measurements could be made from the change in momentum of the mesons upon passage through the counter, there was a limit to the number of gm/cm^2 which could be used. It would be nice to get as large a change in curvature as possible as well as the maximum number of observable mesons, but this can make the radius of curvature in the upper half of the chamber so high that the accuracy gained from a large change of curvature can be lost in the inaccuracy arising from the inability to accurately measure the curvature. It was decided that in view of all considerations a density of about 1 gm/cm^2 would be best. Actually the size aluminum available resulted in an absorber thickness of 0.89 gm/cm^2.

A cloud chamber which one wishes to use for high accuracy momentum determinations must be free of distortion. It had been found in the past that the six-inch chamber was indeed good in this respect; however, upon installation of a counter in the chamber, the problem became somewhat worse for two reasons. First, the track length available for measurement was about 40 % of that in the whole chamber and second, the counter caused a larger distortion due both to the setting up of additional convection currents before the expansion and the new geometry which is afforded to the expansion by the counter. It has been found that by use of a heater of about one tenth of a watt on top of the chamber a stable atmosphere with a temperature differential of 0.6° C from bottom to top can be set up in the chamber. In this way one can eliminate distortions due to convection currents which are in the chamber before expansion and at the same time maintain a temperature difference which is not of a large enough magnitude to adversely affect the supersaturation near the top of the chamber. In order to overcome the difficulties due to the poor geometry, one must try to take the picture as soon after the expansion as possible. For this reason the chamber uses absolute ethyl alcohol whose drops grow much faster from our own observations (1.3 times as fast according to Hazen[5]) than those of the 35 % water, 65 % ethyl alcohol mixture which is preferred by many because of its lower expansion ratio. Our experience has been that there is much more to gain in getting rid of distortion by taking the pictures faster than is lost by the greater turbulence of an expansion which is larger by some 30 - 40 %.

III: RESULTS

The sea level flux of protons with ranges between 2.5 and 15.0 gm/cm^2 of air.

Ever since the first Wilson cloud chambers were built for the purpose of studying cosmic rays, one finds mention of occasional particles which were identified as protons. Few indeed were seen by early observers, but Anderson(6) and Brode(7) in 1936 were each able to report an appreciable number. The most that early observers were able to say was that they were very sparse and constituted a minute fraction of the total cosmic ray intensity. In 1940 Rochester and Bound(8) reported experiments at sea level which gave a rough indication of the intensity of the proton flux. At the same time by placing 20 cm of lead over the chamber they were able to reach the conclusion that because of the large reduction in proton flux (actually because they observed only eight protons with out lead and one with lead their statistics were rather poor) the lead either strongly absorbed the proton producing component, or lead was not a good medium for production of protons; and in any event they concluded that these protons which they observed could not be primary protons since there is no reason why their numbers should be radically different at depths of 90 and 110 cm of lead equivalent. Leprince-Ringuet(9) and co-workers in 1945 measured the intensity at 1000 meters and with a somewhat larger number of protons arrived at a figure of proton flux which was comparable to Rochester and Bound's. Adams[(10)] in measurements at 30,000 feet obtained figures from which one can estimate the flux at sea level by extrapolation on the assumption that the absorption coefficient is 125 gm/cm^2 and that the shape of the spectrum does not change at lower altitudes. Since the absorption coefficient's value is only tentatively estimated, and the extrapolation is over several absorption lengths, the value so obtained for the flux at sea level must be regarded as a first approximation. However, according to Rossi[(11)] the three experiments above form the best present-day (1948) determinations of the proton flux of cosmic rays at sea level.

During the operation of the six-inch chamber with a proportional counter in it somewhat over 35,000 pictures have been taken in a sensitive time of approximately 5,000 hours. For a variety of reasons, such as insufficient light, spurious flashing of the flashtube, faulty operation of the proportional counter, etc.; all periods of operation which for any reason could be suspected as not being indicative of normal operation have been eliminated to leave some 30,000 pictures in about 4,000 hours for selection.

The chamber was fired upon the passage of a particle through two coincidence counters above the chamber and then its subsequent passage through the proportional counter in the chamber with an ionization in excess of a minimum amount determined by the electronic circuits. At all times during which this data was collected the electronic circuits were set so that the cut-off point corresponded to an ionization of 2 times minimum. A proton of 500 Mev/c has an ionization of 3 times minimum; thus it is assumed that the probability of not counting a proton of 500 Mev/c is negligible. It is possible to do this, because in contrast to the slow tail-off of ionization on the high ionization side of most probable ionization the tail-off on the low side is very abrupt[1].

The lower limit of 185 Mev/c below which no protons should be seen is set by the thickness of the upper wall of the counter. Pictures should be taken with a very high degree of certainty of all protons of momenta between 185 and 500 Mev/c which pass through the coincidence counters above the chamber and then go through the proportional counter in the chamber. Protons of this range of momenta are easily identifiable because of the fact that their ionization is significantly different from that of any other known particles.

For the calculation of the proton flux only those protons were used which when viewed in stereo went through the coincidence counters as well as the proportional counter. This criterion removed seventeen protons which were associated with showers but could not have gone through the top counters, five random protons, and eight protons which were associated with stars occurring in the material (2.1 gm/cm^2) between the top coincidence counter and the chamber. In all the pictures only one particle was seen which was identified as a proton going up, and among the more than 200 tracks identified as protons there were no other cases that were even doubtful. It should be borne in mind that the counter arrangement would cut off at 285 Mev/c for protons going up. For protons going down it is found that half the protons have momenta in excess of 285 Mev/c; so one is probably justified in saying that for the momentum range 185 - 500 Mev/c the flux of protons going up is certainly not much greater than 2 % of those going down.

In 3207 hours of operation with no lead above the chamber 118 protons were observed which satisfied the above requirements. This is to be compared with 26 protons obtained in 847 hours under the same conditions with 10 cm of lead above the chamber. It is found that there are 0.036 ± 0.003 protons per hour without the lead and 0.031 ± 0.006 per hour with the lead. Without the lead 68 % of the protons have momenta less than 350 Mev/c compared with 65 % with lead. The results give a raw data figure of 2.96×10^{-6} protons cm^{-2} sec^{-1} sterad^{-1} with lead and 2.47×10^{-6} protons cm^{-2} sec^{-1} sterad^{-1} with 10 cm of lead above the chamber. It is to be noted that the counting rates are not significantly different statistically and certainly give no hint of the radical transition effect observed under 20 cm of lead by Rochester and Bound at sea level and apparently confirmed by Powell[12] in experiments at 14,120 ft. with the use of lead plates in a cloud chamber.

No large fraction of the observed protons can be attributed to the slowing down of high energy protons (greater than 150 Mev); for, if this were the case, in view of the spectrum (Fig. 3) which is sharply peaked at the cut-off point at the low end*, one would expect a diminution by a factor of greater than ten in the number of protons observed under the lead. The isotropic angular distribution (Fig. 5) also supports the above conclusion, since high energy protons are observed in emulsion work to have a distribution which is peaked in the vertical direction[14].

Since the primary purpose of the present experiment was to obtain the absolute value of the proton flux at sea level, the data obtained without lead must be corrected for the average value of 320 gm/cm^2 of cement in the building above the apparatus. Cement predominantly consists of elements of low Z; in fact the average Z for

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*The present spectrum is in substantial agreement with that obtained by Widhalm[13] in 1940 by use of photographic plates. His spectrum of 372 protons obtained at 3,465 meters showed a decided peak at 14.5 Mev.

cement is less than twice that for air; hence it is assumed that in absence of evidence to the contrary there is no large transition effect in going from air to cement. (By exposing plates at varying atmospheric depths Lattimore[15] obtained an absorption coefficient of 190 gm/cm^2 for air and by burying the plates in a glacier at 3600 meters he obtained an absorption coefficient of 196 gm/cm^2 for ice.) Thus the data is corrected by use of an absorption coefficient of 190 gm/cm^2 for the cement above the chamber and the fact that Pasadena is not at sea level. When this is done and without considering the 2.1 gm/cm^2 between the counters and the chamber, one obtains a value of 1.3×10^{-5} protons cm^{-2} sec^{-1} sterad^{-1} at sea level in the momentum range from 185 Mev/c to 500 Mev/c.

From the uncorrected (Fig. 4)* range curve it is found that the integral range spectrum has the form $N = N_0 e^{-\frac{R}{6.3}}$ (R in gm/cm^2). From this the conclusion can be drawn that 28 % of the protons which are found below a 2.1 gm/cm^2 absorber at sea level are produced in it. To correct the data for the particles which are actually incident on the counters, an estimate must be made of what fraction of the protons which are observed in the chamber were produced in the 2.1 gm/cm^2 between the counters and the chamber; and indeed produced such that the counters are set off. If pictures were obtained of

*It turns out that the uncorrected and the corrected range curves are the same.

all of these protons, the figure would be 28 %; however, observations show that approximately half the stars (since the mean range for the protons is so short we assume they are predominantly produced in stars) are produced by neutral particles[14]. If one also notes that the radiation which produces stars is thought to be isotropically distributed at least within fifteen degrees of the vertical[14], then it can be concluded that only a relatively small fraction of the ionizing rays which produce stars would cross through the counters. Then too, one must remember that 8 protons from obvious stars in the 2.1 gm/cm^2 have been discarded. If all the facts above are considered, one can conclude that the number of false counts certainly must be less than 5 %. As additional proof of this fact it was found that in the course of operation there were observed only three protons which were formed in the top of the counter (0.45 gm/cm^2), and one of these was in a star which, if occurring in the glass above the chamber, would have not been counted. If one again assumes that half of the protons are produced by neutrals and considers the solid angles involved, one can set a value of 3 to 5 as the maximum number of protons which are observed which were formed in the 2.1 gm/cm^2. Since other considerations introduce much larger errors, the error introduced by the 2.1 gm/cm^2 has been neglected.

Thus it is assumed that our distribution must merely be shifted 2.1 gm/cm^2 to get the correct one. The effect of this shift is to quite radically alter the shape of the momentum spectrum (Fig. 1 and 2) as well as to shift the points strongly towards the higher end especially in the lower range. The energy spectrum (Fig. 3) is less severely affected and the points merely shift slightly towards the high end. The range spectrum is linearly shifted by 2.1 gm/cm^2. Because of this, it is felt that giving the flux in terms of the range interval is probably the most satisfactory. When this is done, a value of 1.3×10^{-5} protons cm^{-2} sec^{-1} sterad^{-1} in the range interval of 2.5 to 15.0 gm/cm^2 of air is the result. This gives a value of 1.0×10^{-6} protons gm^{-1} sec^{-1} sterad^{-1} as the average differential value of the proton flux in the range interval between 2.5 and 15.0 gm/cm^2. This value is somewhat higher than the values given by Rossi (Page 18); but it must be remembered that his values are for protons of greater ranges than those of the present experiment.

The conclusion is that at least 95 % of the protons which are observed are produced with ranges less than 25 gm/cm^2. Since there is no large transition effect, one must also conclude that either the production of protons per gm/cm^2 in lead is not very different from that in air and at the same time the proton producing radiation exhibits no tendency towards preferential absorption by other means in lead, or that these two effects have a characteristic behavior in lead such that the combination of the two tends to give no large transition effect.

From emulsion work Lattimore[15] comes to the conclusion that the great majority of the low energy protons do not come from the characteristic protons of energies of less than 15 Mev observed in stars. He concludes that the average production energy of the protons is about 32 Mev (his plates are not sensitive above about 90 Mev), and that what is observed is these protons after they have been slowed down. Because of the fact that the cut-off due to the top wall thickness of the counter is at 18 Mev, one can only say that the present results are consistent with this.

TENTATIVE ESTIMATES OF THE NUMBER OF PROTONS

IN VARIOUS ENERGY RANGES*

Range Interval in gm/cm^2	Unit	Experimental Information	Adopted Sea Level Value
No. of Particles $6 < R < 100$	cm^{-2} sec^{-1} $sterad^{-1}$	10^{-2} at 9000 m (Adams)	3×10^{-5}
No. of Particles $100 < R < 1000$	cm^{-2} sec^{-1} $sterad^{-1}$	7×10^{-4} at 9000 m (Adams)	2×10^{-5}
Differential Range Spectrum at $R = 20$	gm^{-1} sec^{-1} $sterad^{-1}$	5×10^{-7} at sea level (Rochester)	5×10^{-7}
Differential Range Spectrum at $R = 100$	gm^{-1} sec^{-1} $sterad^{-1}$	4×10^{-7} at 1000 m (Leprince-Ringuet)	1.7×10^{-7}
No. of Particles $2.5 < R < 15.0$	cm^{-2} sec^{-1} $sterad^{-1}$	3.0×10^{-6} at Pasadena (Present Data)	1.3×10^{-5}

*The four first estimates were taken from Rossi[11]. He used an absorption coefficient of 125 gm/cm^2 to obtain the sea level values. In the present work an absorption coefficient of 190 gm/cm^2 has been used.

Figure 1

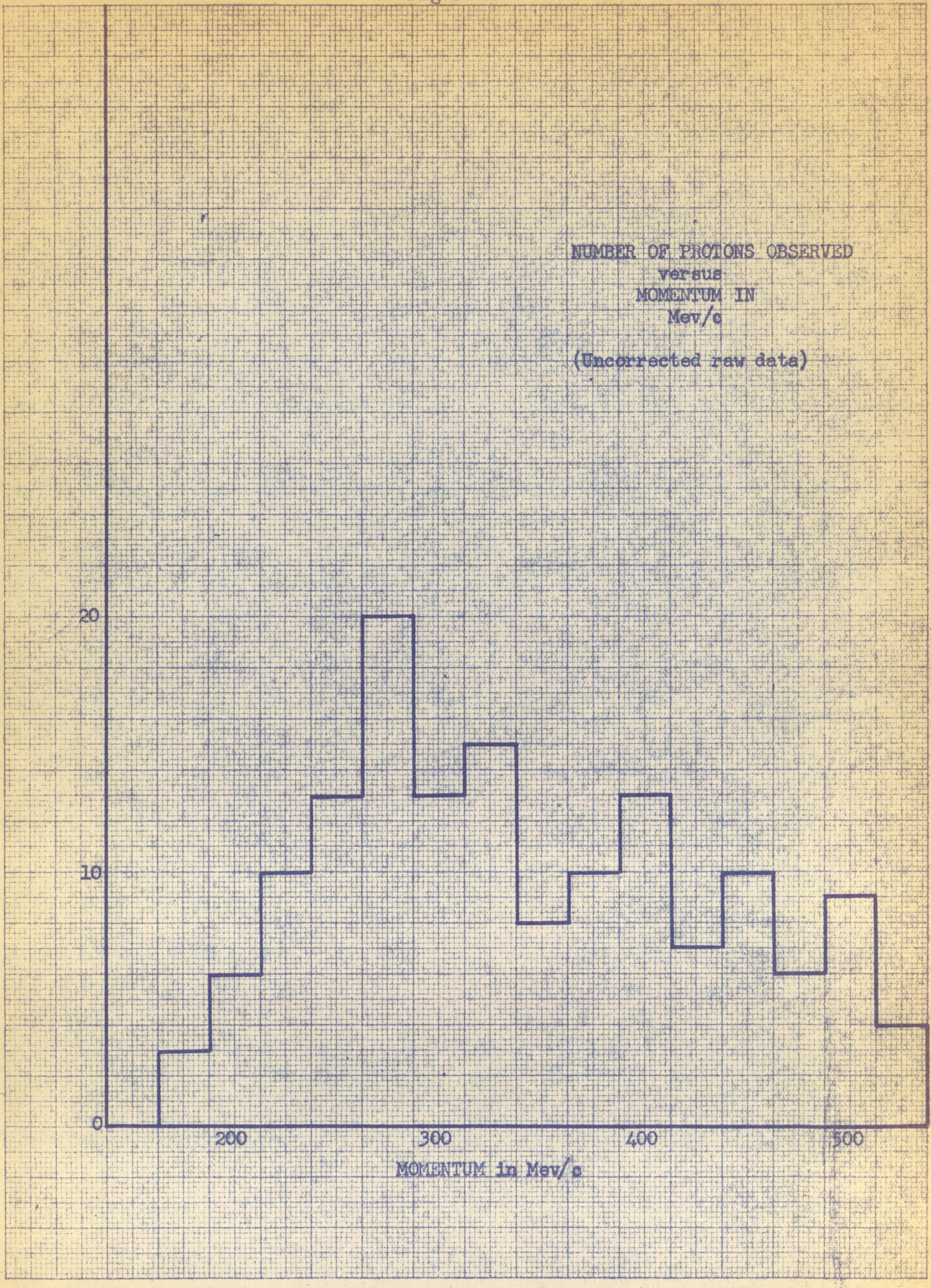

NUMBER OF PROTONS OBSERVED
versus
MOMENTUM IN
Mev/c
(Uncorrected raw data)
20
10
0
200
300
400
500
MOMENTUM in Mev/c

Figure 2

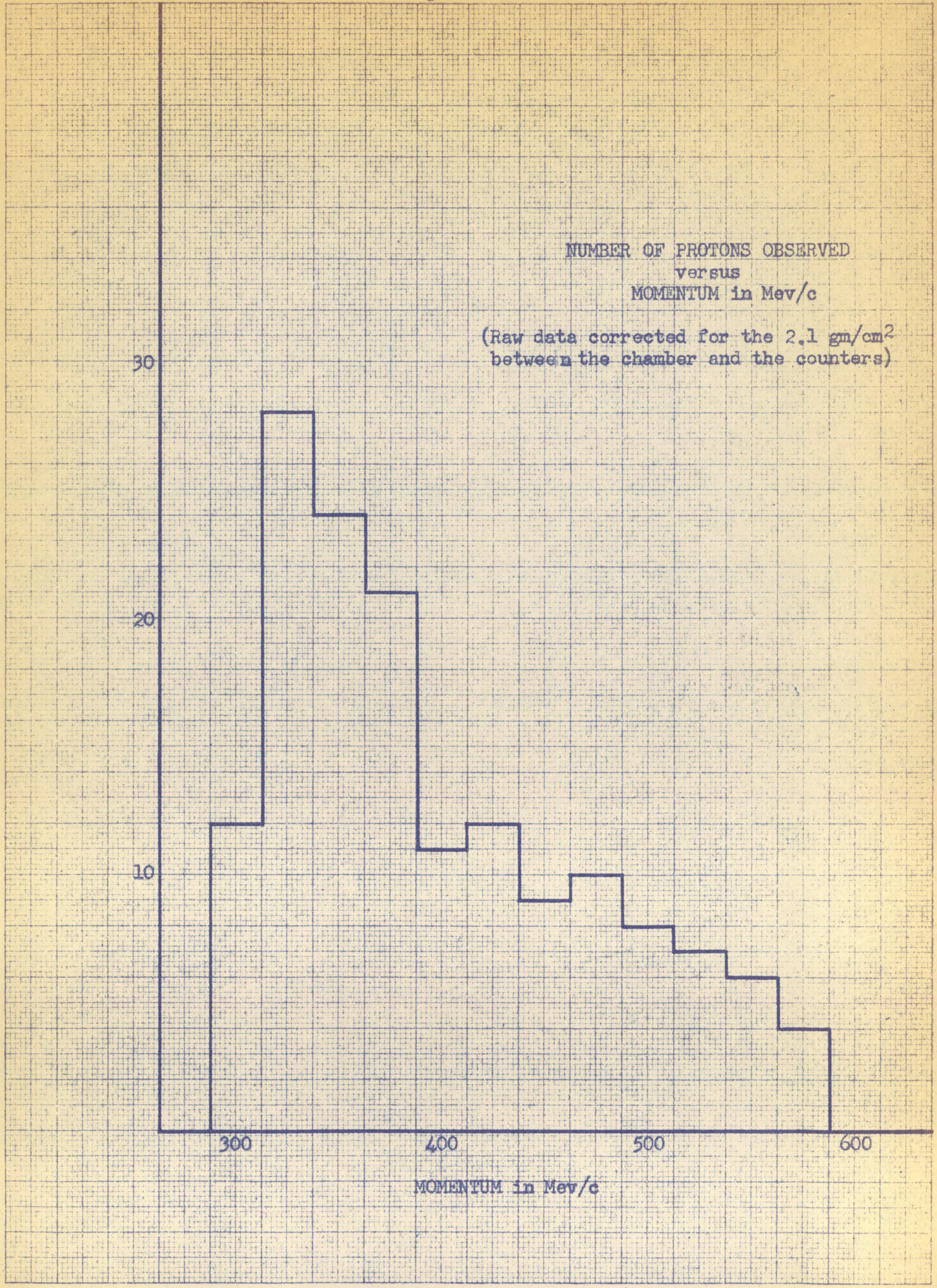

NUMBER OF PROTONS OBSERVED
versus
MOMENTUM in Mev/c
(Raw data corrected for the 2.1 gm/cm2
between the chamber and the counters)
30
20
10
300
400
500
600
MOMENTUM in Mev/c
KEUFFEL & ESSER CO., N. Y. NO. 359-14

Figure 3

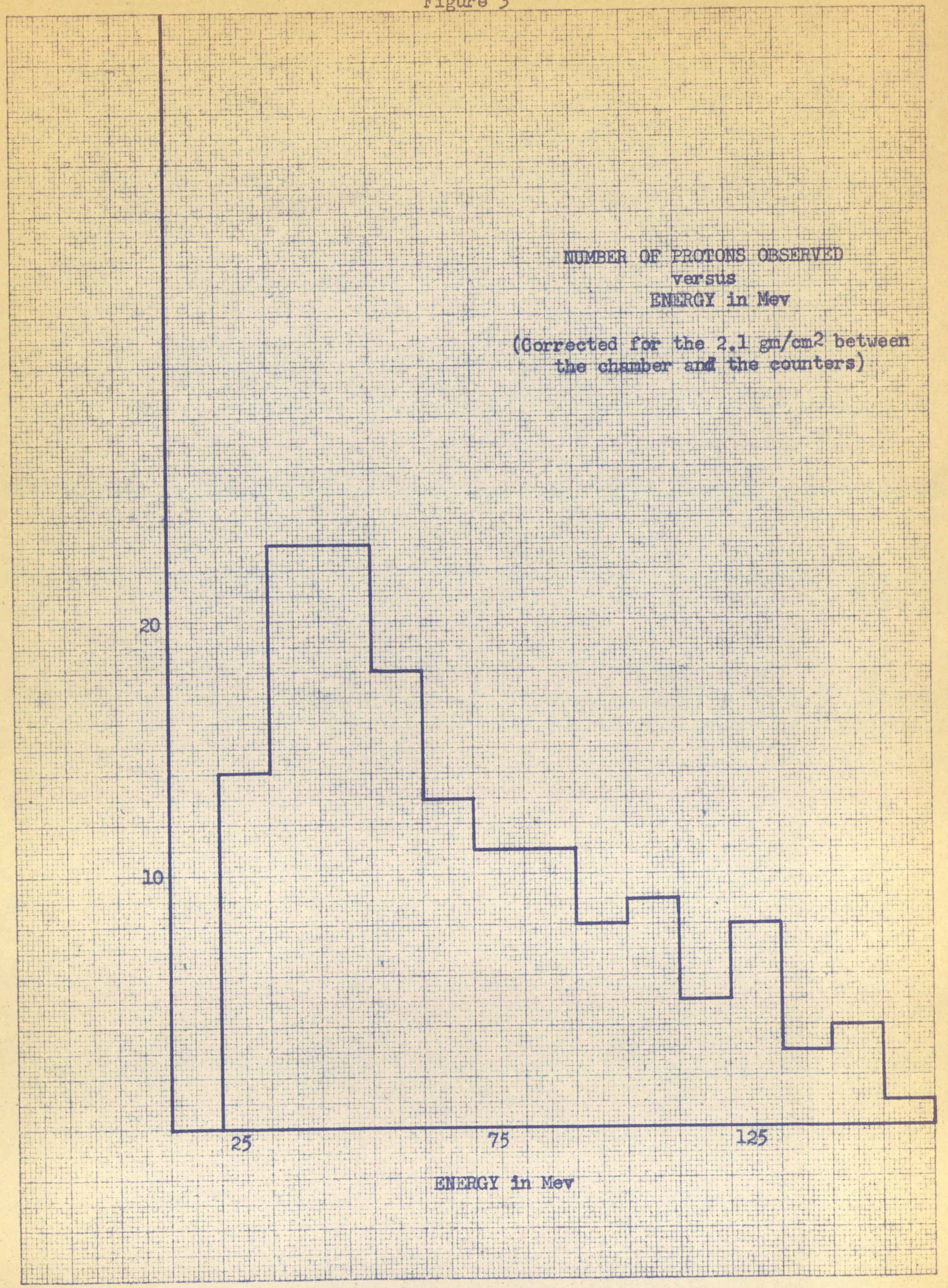

NUMBER OF PROTONS OBSERVED
versus
ENERGY in Mev
(Corrected for the 2.1 gm/cm2 between
the chamber and the counters)
20
10
25
75
125
ENERGY in Mev
KEUFFEL & ESSER CO. N. Y. NO. 359-14
Millimeters. 5 mm. lines accented, cm. lines heavy.
MADE IN U.S.A.

Figure 4

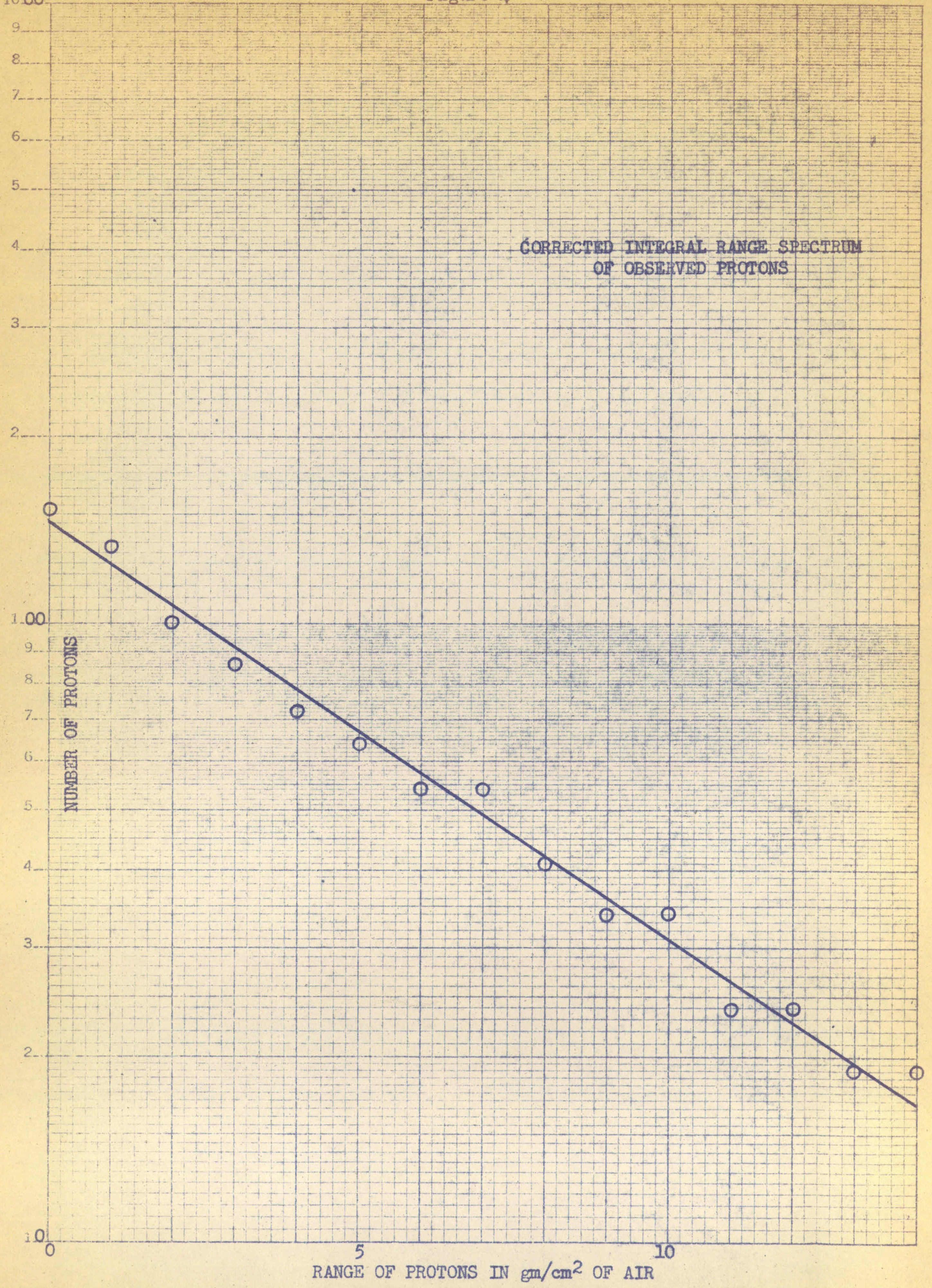
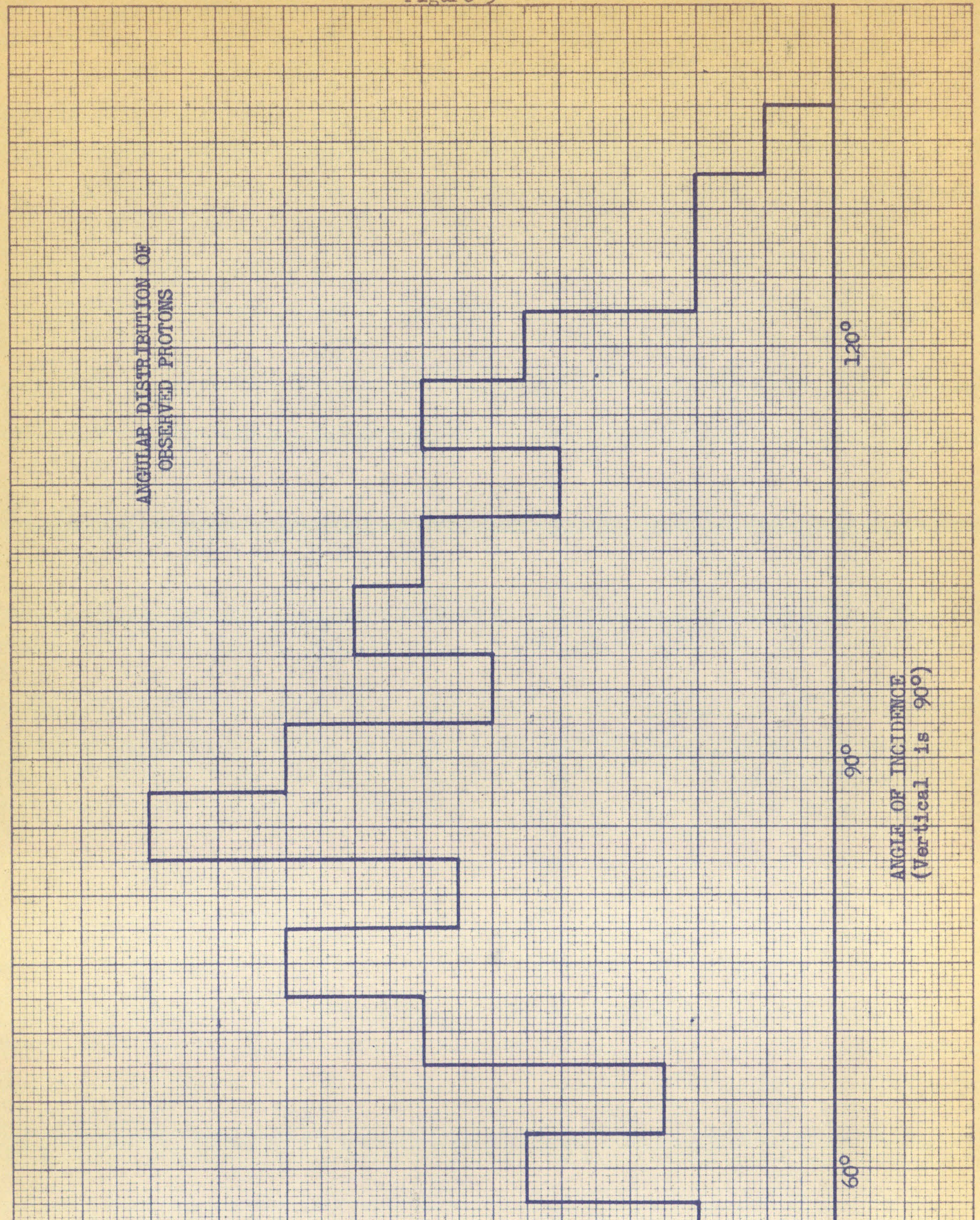

CORRECTED INTEGRAL RANGE SPECTRUM
OF OBSERVED PROTONS
NUMBER OF PROTONS
RANGE OF PROTONS IN gm/cm2 OF AIR
10.00
1.00
10
0
5
10

Figure 5


ANGULAR DISTRIBUTION OF
OBSERVED PHOTONS
ANGLE OF INCIDENCE
(Vertical is 90°)
60°
90°
120°
0
10
20

Mu-Meson Decay Spectrum

Due to the extremely small size of the chamber and the small amount of material in it one needs a very selective device in order to obtain any quantity of decays. Upon installation of the proportional counter verification was obtained that this mode of operation would be favorable to the selection of mu-meson decays. A year's operation has resulted in seventy mu-meson decay photographs of which fifty are usable.

In obtaining the decay spectrum the requirements have been set such that an electron track of one inch length is required. This is to rule out the possibility of not using high energy tracks because their energy is not measurable, while at the same time using a low energy track of the same length. No tracks have been used when the correction to the electron energy was greater than 4 Mev for the material which it traversed before appearing in the chamber. The correction for the energy lost before appearance in the chamber was in addition to the corrections previously applied for position in the chamber. By use of the stereo views the tracks were corrected where necessary for motion perpendicular to the plane of the chamber.

A decay spectrum obtained by use of a proportional counter has the desirable feature of not having a cut-off at either end of the spectrum. The only expected sources of error in a spectrum obtained in this way will come from chamber geometry. Since the mesons decay in an absorber, there is a possibility of not recording very low energy decays; however, the fact that no decays are considered which require a correction of greater than 4 Mev reduces this significantly. The effect of chamber geometry has been eliminated through the requirement of a track length in excess of an inch. The above considerations would seem to make it reasonable to assume that systematic errors have been eliminated. The error in all decay energy measurements is taken to be $1\frac{1}{2}$ Mev or 5 % of the energy whichever is the larger.

The spectrum obtained (Fig. 6) shows a lack of decays in the range 21 to 30 Mev; however, due to the rather small number of cases observed in any interval, one has difficulty saying that this gap is statistically significant. Other decay spectrums, such as those of Leighton[16] et al and Davies[17] et al, would lead one to believe in a more uniform distribution of the energies of the decay electrons. Possibly it is noteworthy that when Leighton et al obtained their decay spectrum they observed much the same phenomenon in their first forty-odd cases. The present spectrum has an average decay energy of 32 Mev as compared with the value of 34 Mev in the previous work. It is to be noted that there is one decay at 62 Mev. This would seem to show that decay electrons must be emitted with energies very near to the maximum possible. Since this is a positive meson, it could pick up some energy in leaving the nucleus in whose vicinity it may have decayed. This and the inherent error in measurement could reduce the value to 56 Mev which is not in bad agreement with the maximum value of 54.5 Mev obtained by use of 212 m_e as the mass of the mu-meson.

There is no good *a priori* reason for not accepting this spectrum since it has almost the same weight statistically as the previous spectrum; and, if it had been observed prior to the other, it would have certainly been given a certain credence; however, it is presented only because it is felt that some note should be made of its existence. It is not offered in the attempt to prove anything about the true shape of the spectrum other than that it is probably highly desirable to obtain a decay spectrum of some one thousand cases before saying that one theory or another is proved or disproved.

From the photographs one is able to obtain some other data on mesons, which while not of primary importance, should possibly be noted in passing.

In a period of 3207 hours, 46 cases were observed of mesons which stopped in the bottom of the counter. Since the ionization of these would be expected to be 3 to 4 times minimum on passing through the counter gas, the counting efficiency should be very high for them (the same arguments as were previously used for the 3 times minimum protons can be used); hence an estimate of the absolute intensity of mesons in the momentum range 37 to 45 Mev/c can be made by use of this data. A figure of 2.5×10^{-6} gm^{-1} sec^{-1} $sterad^{-1}$ is obtained for the differential range intensity at sea level with 320 gm/cm^2 of absorber (cement) over the apparatus. This is smaller by a factor of two than the value given by Rossi[11].

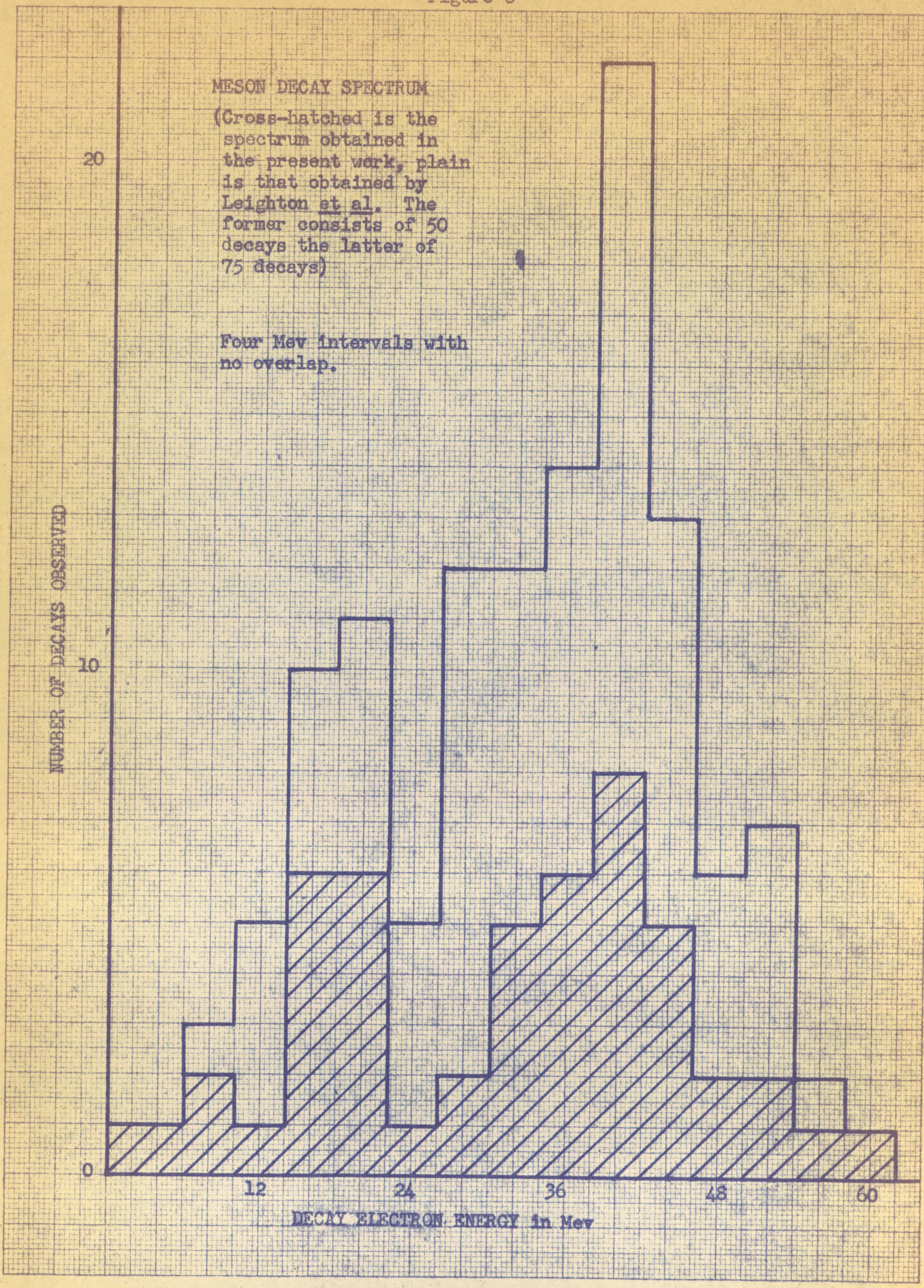
For the total operating time 57 mesons stop in the bottom wall of the counter, 34 of these are positive and 23 negative. Of the positives 21 are observed to decay and of the negatives 6. The proportion of positive decays observed is what would be expected from geometrical considerations. If one takes the positive decays as a measure of the number of decays which one fails to observe due to geometry, then the fraction of negative mesons which decay is 0.42. This is in agreement with the value of 0.40 which is given by Ticho[18] as the fraction of the negative mesons which decay in aluminum. This would seem to show the invalidity of Chang's[19] *a priori* hypothesis that those mesons which show a decay in very thin foils in a chamber are positive and those which do not are negative. This assumption was given some plausibility at the time because he observed about equal numbers of mesons which did and did not show decays upon coming to rest in an aluminum foil. The point is brought up because Bradner[20] in his recent monograph on artificially produced mesons says: "However, it has been shown by Chang that charged particles are rarely if ever emitted when negative mu-mesons are stopped in nuclei from Al to Pb", and at the same time gives reference to Ticho's work.

Since mesons of 70 Mev/c are ionizing in excess of twice minimum, it was felt that they could be distinguished from electrons at momenta less than this. By this means one can separate out mesons between 37 Mev/c and 70 Mev/c (37 Mev/c being the lower cut-off of the counter). When this is done, it is found that there are 147 positive mesons and 128 negative mesons, which confirms previous results[18] which indicate a slight positive excess for low energy mesons. In the above, due to the setting of the firing point of the counters, a certain number of the mesons in this momentum interval have been missed, especially near the upper limit. Nevertheless it will give a lower limit to the absolute flux. This lower limit is 1.8×10^{-6} gm^{-1} sec^{-1} $sterad^{-1}$ which is consistent with the absolute value of 2.5×10^{-6} gm^{-1} sec^{-1} $sterad^{-1}$ obtained above. However, this is but a third of the value which Rossi's[11] curve gives, and it is hard to believe that the counter has been so biased as to count only one third of the mesons in this range. The data is more in agreement with that of Wilson[21] whose values are considerably lower at the low end than those of Rossi's curve.

DECAY ELECTRONS

Energy in Mev	Decay Substance	Sign of Meson	Energy in Mev	Decay Substance	Sign of Meson
2	Glass	P	35	Aluminum	P
6	Glass	N	35	Aluminum	P
9	Glass	P	37	Aluminum	P
10	Aluminum	N	37	Aluminum	N
14	Copper	P	38	Aluminum	P
15	Glass	P	38	Aluminum	P
15	Glass	N	39	Aluminum	P
16	Glass	P	39	Aluminum	N
17	Aluminum	N	39	Aluminum	N
17	Aluminum	P	42	Aluminum	P
18	Glass	P	42	Aluminum	P
20	Glass	N	42	Aluminum	N
21	Aluminum	N	42	Glass	P
21	Aluminum	P	42	Glass	P
21	Glass	P	45	Aluminum	N
21	Glass	N	45	Aluminum	P
22	Glass	P	45	Glass	P
24	Glass	P	46	Aluminum	P
27	Aluminum	P	46	Glass	N
30	Glass	N	47	Aluminum	P
31	Aluminum	P	50	Glass	P
31	Glass	P	52	Aluminum	N
32	Aluminum	P	54	Glass	P
32	Glass	P	56	Aluminum	P
33	Glass	N	62	Glass	P

Figure 6


MESON DECAY SPECTRUM
(Cross-hatched is the spectrum obtained in the present work, plain is that obtained by Leighton et al. The former consists of 50 decays the latter of 75 decays)
Four Mev intervals with no overlap.
NUMBER OF DECAYS OBSERVED
20
10
0
12
24
36
48
60
DECAY ELECTRON ENERGY in Mev

Interesting Pictures

Illustrations 5, 6, 7, 9, 11, and 12 were obtained with the chamber filled with argon at two atmospheres. All the other illustrations were obtained with a mixture of 1/4 argon and 3/4 helium at a total pressure of two atmospheres. In order to aid the reader in following the discussion of the pictures, several universal curves relating momentum, energy, range, and ionization have been added at the end. Since the rest mass in Mev of a mu-meson is quite close to 100, and that of the proton is reasonably close to 1000; one can use these graphs directly for reasonably close approximations by merely getting the decimal point in the correct place.

Illust. 1 shows a typical proton of 225 Mev/c passing through the counter.

Illust. 2 is a proton of 215 Mev/c which stops in the counter.

Illust. 3 is the only proton found in this set of pictures which was going up. Note the small electron shower which is present and also seems to be going up.

Illust. 4 depicts the one star which occurred in the gas. Four heavily ionizing and two minimum ionizing particles appear.

Illust. 5 presents a positive mu-meson which goes through the counter and then decays in the bottom of the chamber. The electron, which is faintly visible, goes almost straight back.

Illust. 6, 7, 8, 9, and 10 show typical mu-meson decays in an absorber.

Illust. 11 depicts the production of two pairs in the counter with no other tracks visible. The energies from left to right are 75, 150, 60, and 200 Mev.

Illust. 12. This negative track shows a momentum of 39 ± 5 Mev/c and an ionization of less than two times minimum above the counter, below the counter it has a momentum of 40 ± 5 Mev/c and an ionization of three times minimum. It shows an apparent scattering through an angle of 37°. Since the track length is so short after scattering, all that can be said is that its momentum is in excess of 30 Mev/c. The track seems to show about the same ionization before and after the bend, although the ionization may decrease somewhat after the bend.

If one considers ionization and momentum as well as range, this cannot be a pi-meson above the counter. If it is a mu-meson; ionization, momentum, and range are not consistent by a considerable factor above the counter. Below the counter and before the scattering it is perfectly consistent with a mu-meson. By stretching things considerably one could say that this is a mu-meson which comes in, goes through the counter, and then suffers a large angle scattering in the bottom of the chamber.

A preferable explanation is to say that the particle is going up. Then one can say that the particle in the top is the decay electron of the mu-meson in the chamber bottom (if going up the mu-meson should have decayed in the upper counter wall). Then the only question remaining is what causes the change in direction of the particle in the lower half of the chamber. There are two possibilities, a scattering of a mu-meson or the decay of a pi-meson. Since a pi-meson of some 65 to 70 Mev/c and ionization of about three times minimum could decay into a mu-meson at this angle, the interpretation of the picture as a pi-meson, mu-meson, electron decay is probably preferable.

Illust. 13. A positive particle of about two times minimum ionization enters the chamber, and then passes through the counter. Below the counter it is ionizing about three times minimum and has a momentum of 53$\pm$ 3 Mev/c. Just above the bottom wall of the chamber a particle of minimum ionization is seen to go off at a slight angle with the original track.

This picture can be interpreted as the decay in flight of a 53 Mev/c mu-meson. The ionization both above and below the counter is consistent with a mu-meson of this momentum. The change of momentum upon passage through the counter is consistent with a mu-meson. When viewed in stereo, the angle at which the tracks diverge is in excess of 60°. It can be shown by a straight forward relativistic calculation that the maximum angle at which a pi-mu-meson decay can take place in the laboratory system is given by $\Theta = \sin^{-1} \frac{39.6}{P}$, where P is the momentum of the pi-meson in the laboratory system. This leads to a value of 48° as the maximum angle possible if this is a pi-mu-meson decay. With this possibility ruled out a mu-decay in flight seems to be the most reasonable explanation.

Illust. 14. Track A (upper left) shows a momentum of 95 ± 20 Mev/c, an ionization of about minimum, and a positive charge if going down. Track B (upper right) has a momemtum of 115 ± 10 Mev/c, an ionization of 3 to 4 times minimum, and is negative if going down. Track C (lower left) is essentially straight, but of such ionization as to leave no doubt that it is a proton if it is a known particle. B is the only particle which could have tripped the coincidence counters since A is inclined at a considerable angle to the plane of the chamber.

If one takes the momentum and ionization of B at face value, there results a mass of about 450 m_e. A is probably an electron, although there is some small possibility that it is a mu-meson. Since B cannot be a mu-meson or a proton, it must, (if a known particle) be a pi-meson; however, since its range would be far greater than the top wall thickness of the counter, one would have to say that it was scattered along the wall if one wishes to call this the star at the end of a negative pi-meson. None of the protons observed in the chamber at greater than minimum ionization showed any nuclear interaction; so it is probably reasonable to rule out a proton coming up and causing the event. The conclusion that is probably most reasonable is that an unseen particle causes a nuclear disintegration in the top wall of the counter.

Illust. 15. This positive track has a momentum of 100 ± 20 Mev/c and an ionization of 2 to 3 times minimum above the counter. Its momentum is 85 ± 10 Mev/c and its ionization 4 to 5 times minimum below the counter. The particle shows what appears to be a scattering about 1 cm above the counter. If a scattering, it would not seriously affect the momentum because it is of such a small angle; however it could be interpreted as a decay. The change of momentum and ionization of this particle during passage through the counter are both consistent with a mass of 450 m_e. The momentum and ionization in both the top and bottom are also consistent with this mass. It is very hard to call this a mu-meson; thus one is forced to conclude that it is a pi-meson even though it would seem to be necessary to stretch the observed facts to the limit. Perhaps it is significant that on the surface this appears to be a particle of about the same mass as track B in Illust. 14.

Illust. 16. Three tracks appear in this picture. Track A which is in the upper left has a momentum of 360 ± 80 Mev/c, shows an ionization of between 5 and 10 times minimum for a singly charged particle, and is negatively charged if going down. Track B, which is in the lower left has a momentum of 50 ± 4 Mev/c, an ionization of about one-third that of A (or from $1\frac{1}{2}$ to 3 times minimum), and is positively charged if going down. There is also a third very faint track in the right side of the chamber. This third track is negative if going down and seems to pass through the coincidence counter above the chamber; thus it could be the particle which caused the picture to be taken.

On an ionization--momentum basis A is almost certainly a proton (if only known particles are considered). B seems to be very consistent in ionization and momentum with a mu-meson, although there is a possibility that it is a pi-meson. This seems unlikely because a pi-meson of this momentum is ionizing in excess of 5 times minimum. If B is going up, then this could be taken to represent the capture of a negative mu-meson by an aluminum nucleus. There would seem to be very strong experimental evidence against this, and theoretical objections as well. If one wishes to disregard the apparent lack of consistency of ionization and momentum for a pi-meson; then this can be readily interpreted as the capture of a negative pi-meson with the subsequent release of a proton upwards.

Another possibility is that an unseen particle produces a nuclear disintegration with the ejection of a proton upwards and a pi-meson downwards. The ejection of a mu-meson as a primary particle in a star is not in accord with experimental evidence; thus the pi-mu-meson decay takes place in the counter before B appears in the chamber. This would require that the pi-meson decay in less than

0.004 of a lifetime. This seems improbable, but what does probability mean for a single event? If one wishes to consider the possibility of negative protons, then one has to postulate some mechanism in which their annihilation can result in the production of a low energy mu-meson.

The hypothesis that this is a star produced by an unseen particle would seem to be the most reasonable.

Illust. 17. In this photograph a positive particle, if going down, of 50 Mev/c and three times minimum ionization, which is very consistent with a mu-meson, comes in and apparently stops in the counter as would be expected from its momentum. A negative electron, if going up, of 12 Mev/c comes from a point which when viewed in stereo is within 2 mm of the point where the positive meson would have been expected to decay. This negative electron is time coincident. Another short stub is present (it can be seen where the meson enters the counter). In stereo it appears to be of greater than minimum ionization, but it is so foreshortened that this cannot be certain. Due to its shortness and the fact that it is distorted at the point of entry into the front glass, nothing can be said about its direction.

It should be pointed out that the mu-meson does not have sufficient range, if going up, to trip the coincidence counter so as to take the picture. This picture can be best interpreted as a mu-meson which goes into the counter and decays. The decay electron produces a knock-on at a point very close to the decay in the bottom wall of the counter. The knock-on is the observed negative electron, while the positive electron from the decay is seen as the stub.

Illustration 1

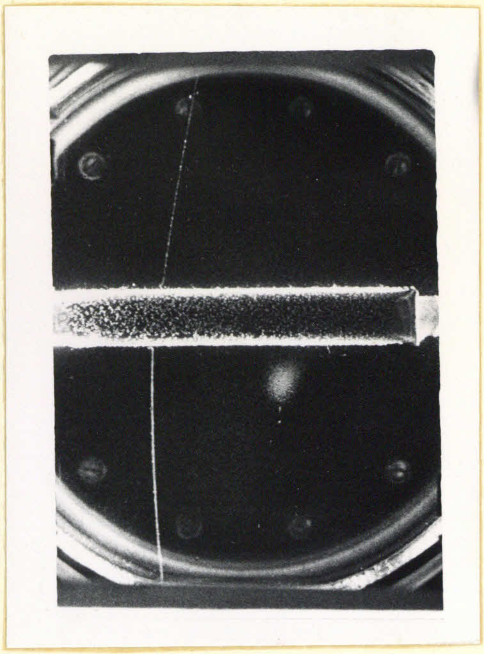
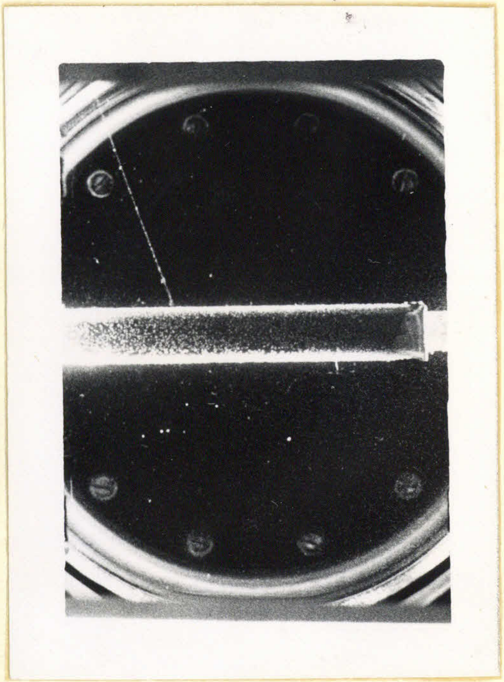


Illustration 2



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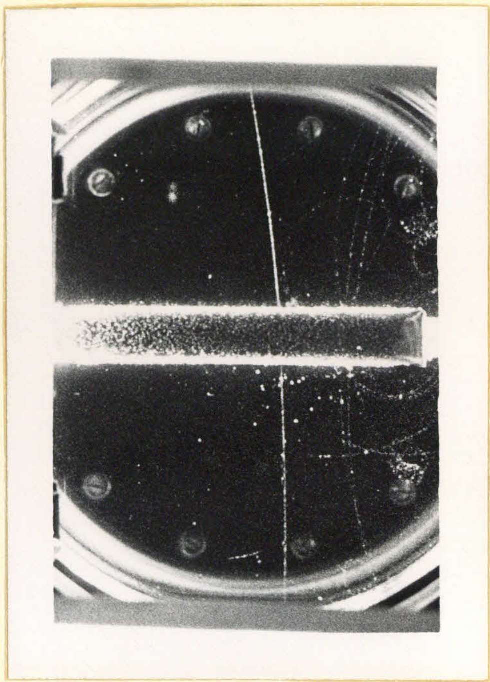

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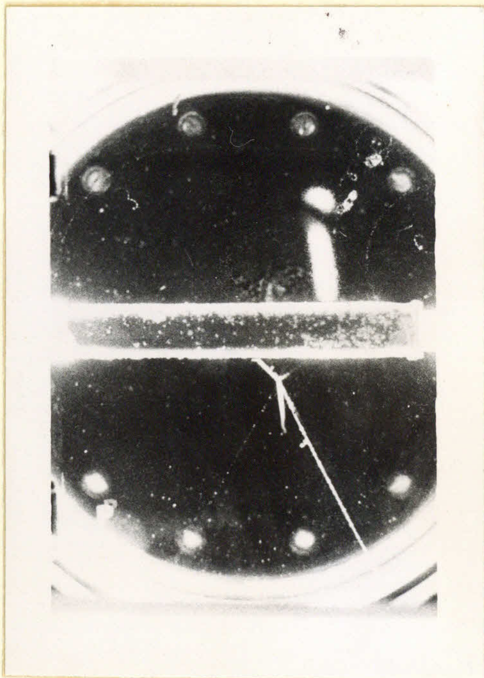

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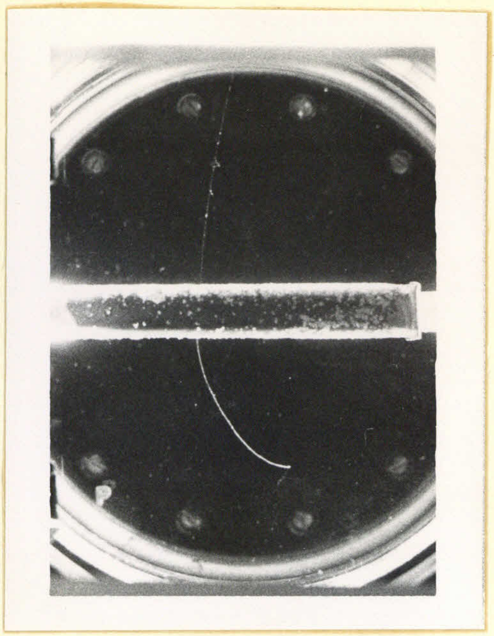


Illustration 6

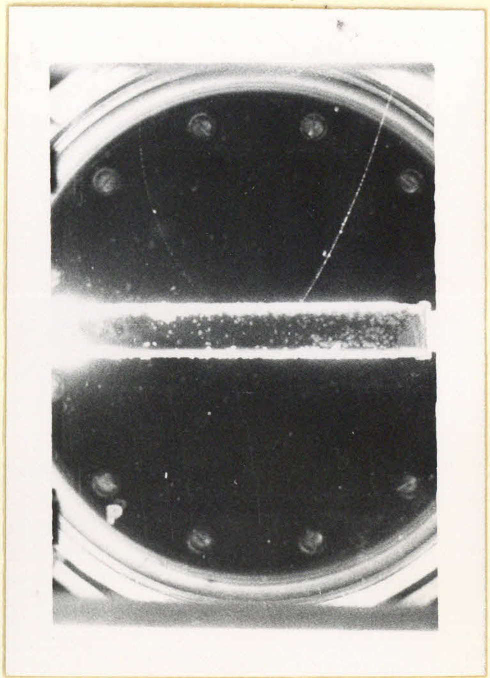


Illustration 7

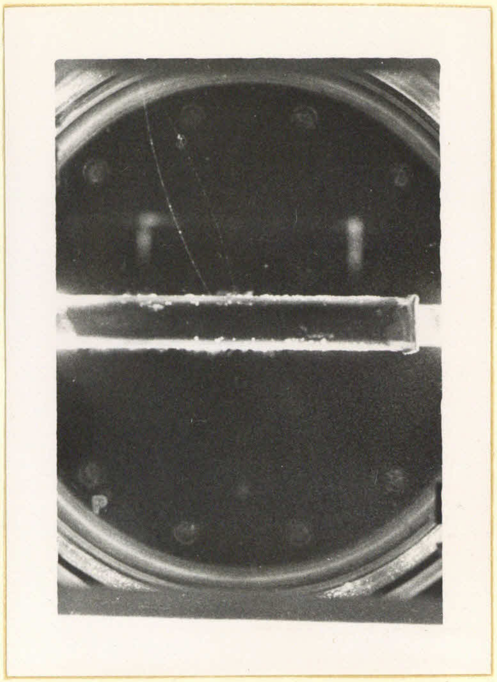
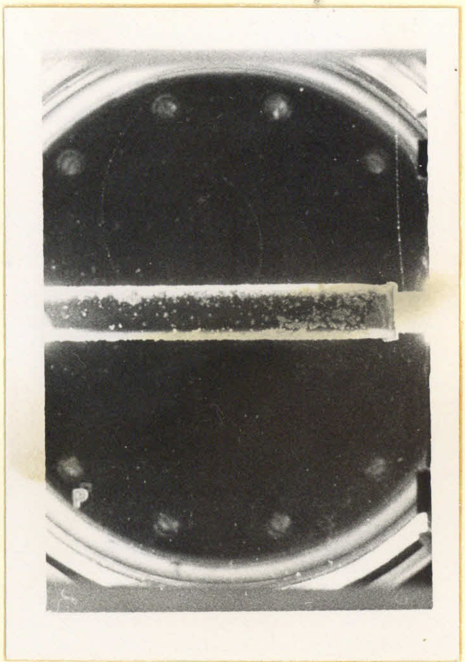


Illustration 8



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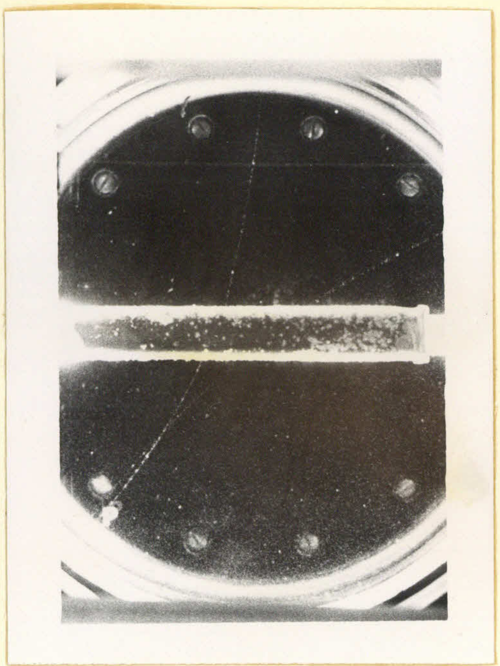

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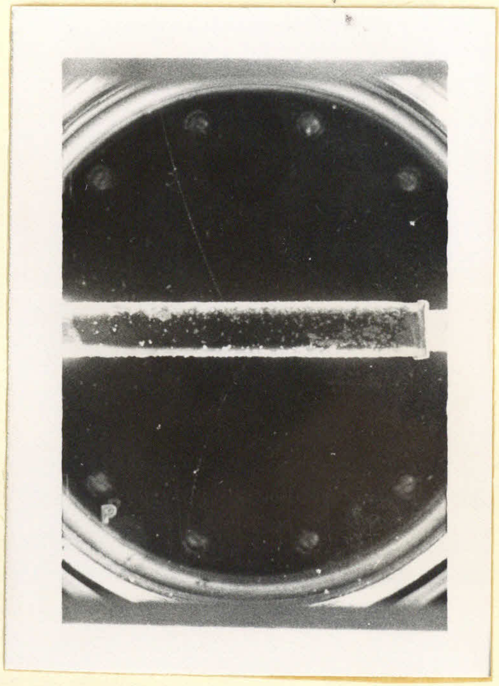
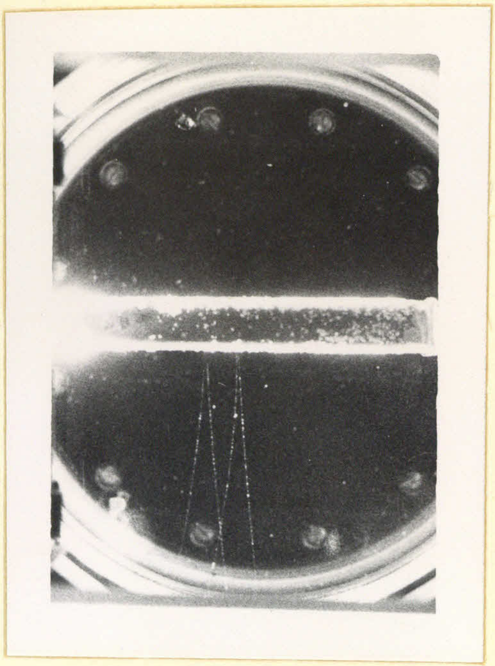


Illustration 11



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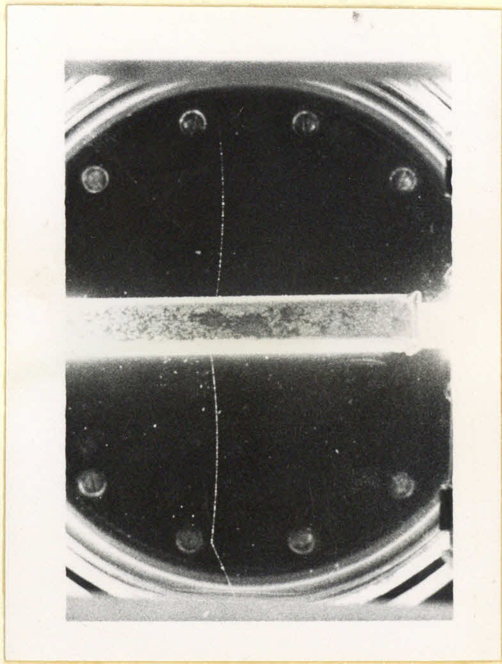


Illustration 13

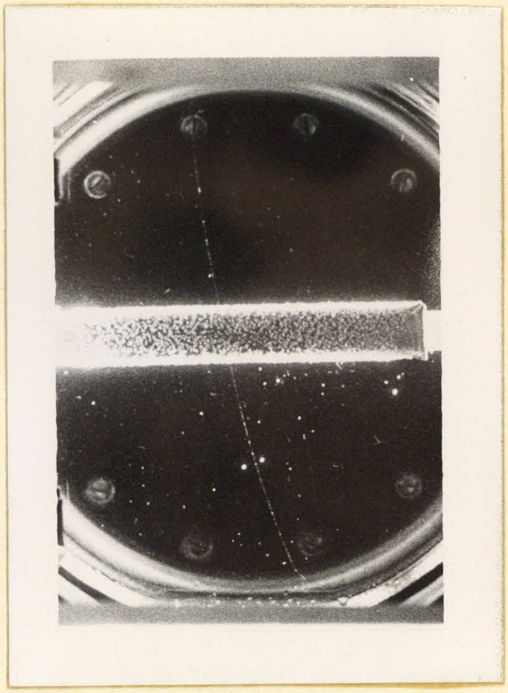


Illustration 14

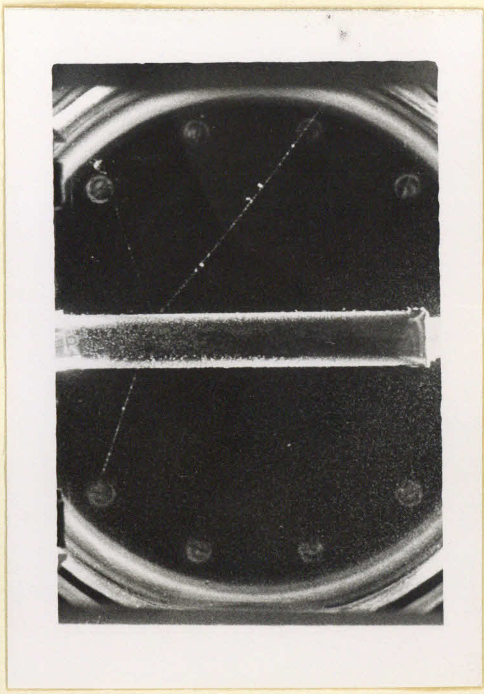


Illustration 15

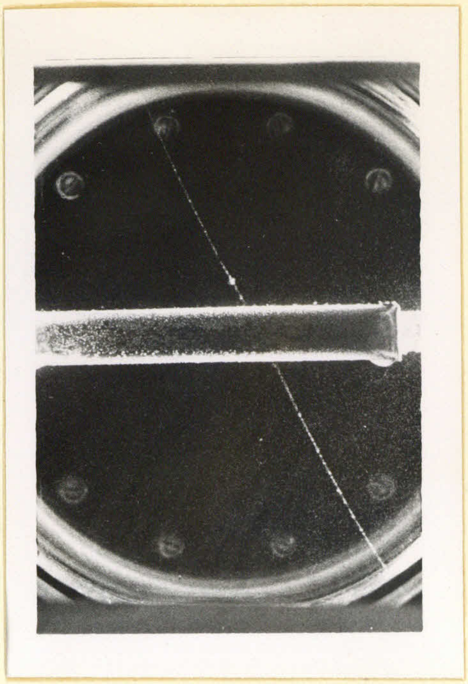


Illustration 16

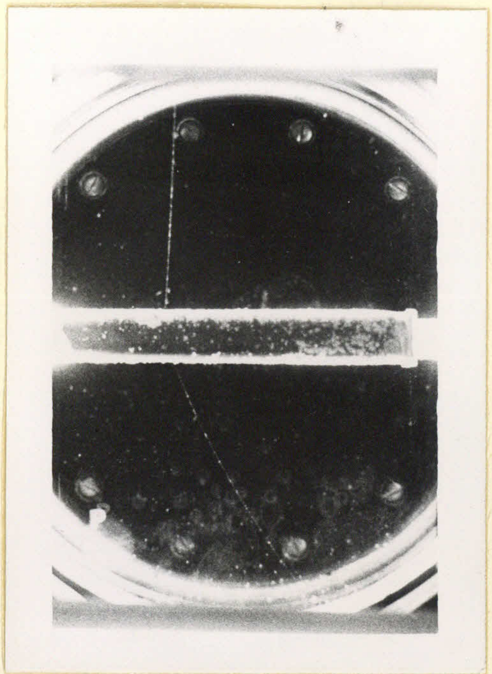


Illustration 17

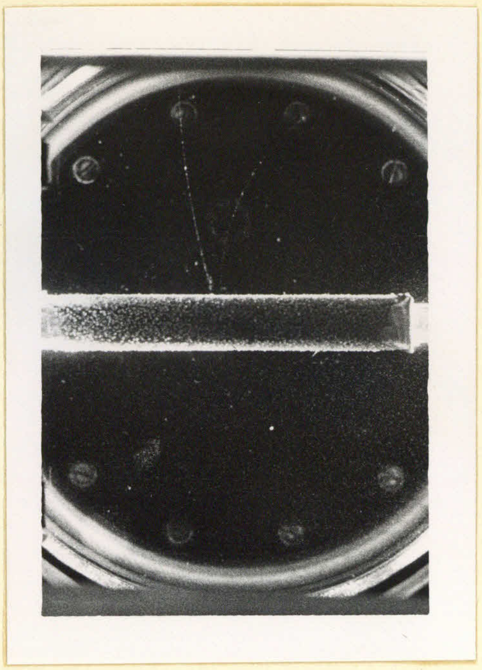


Figure 7

IONIZATION versus MOMENTUM in AIR for singly charged particles

Momentum in $M_o c$ units
Ionization is the ratio to minimum value of 2.236×10^{-3} Mev/cm (NTP)

Derived from proton ionization tables per J. H. Smith, Phys Rev 71 32 (1947) with $M_p = 9.376 \times 10^8$ ev

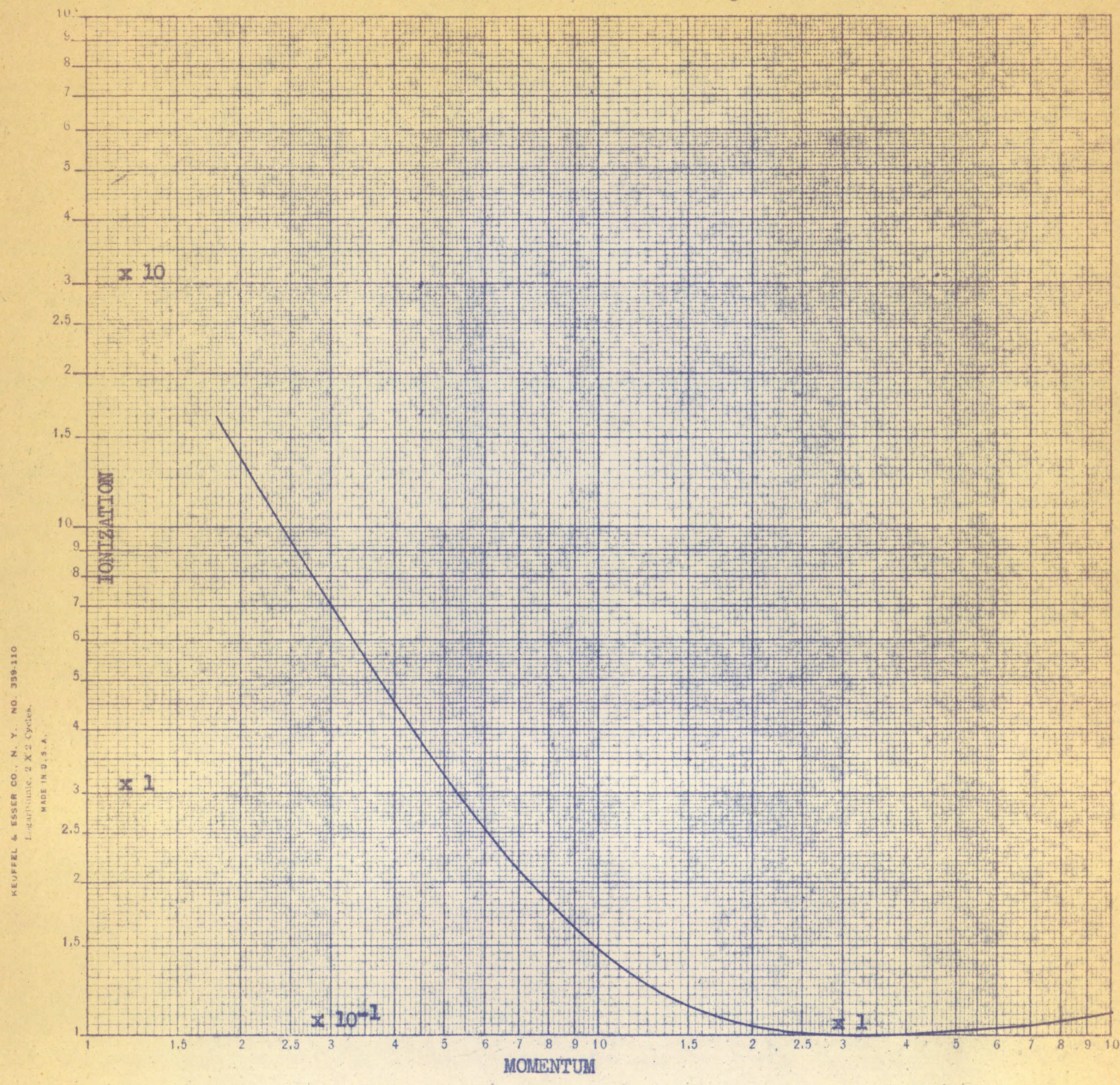


Figure 8

ENERGY VERSUS MOMENTUM IN NATURAL UNITS

Energy in M_oc^2 units
Momentum in M_oc units

Velocity in c units and ionization
times minimum in air indicated on graph

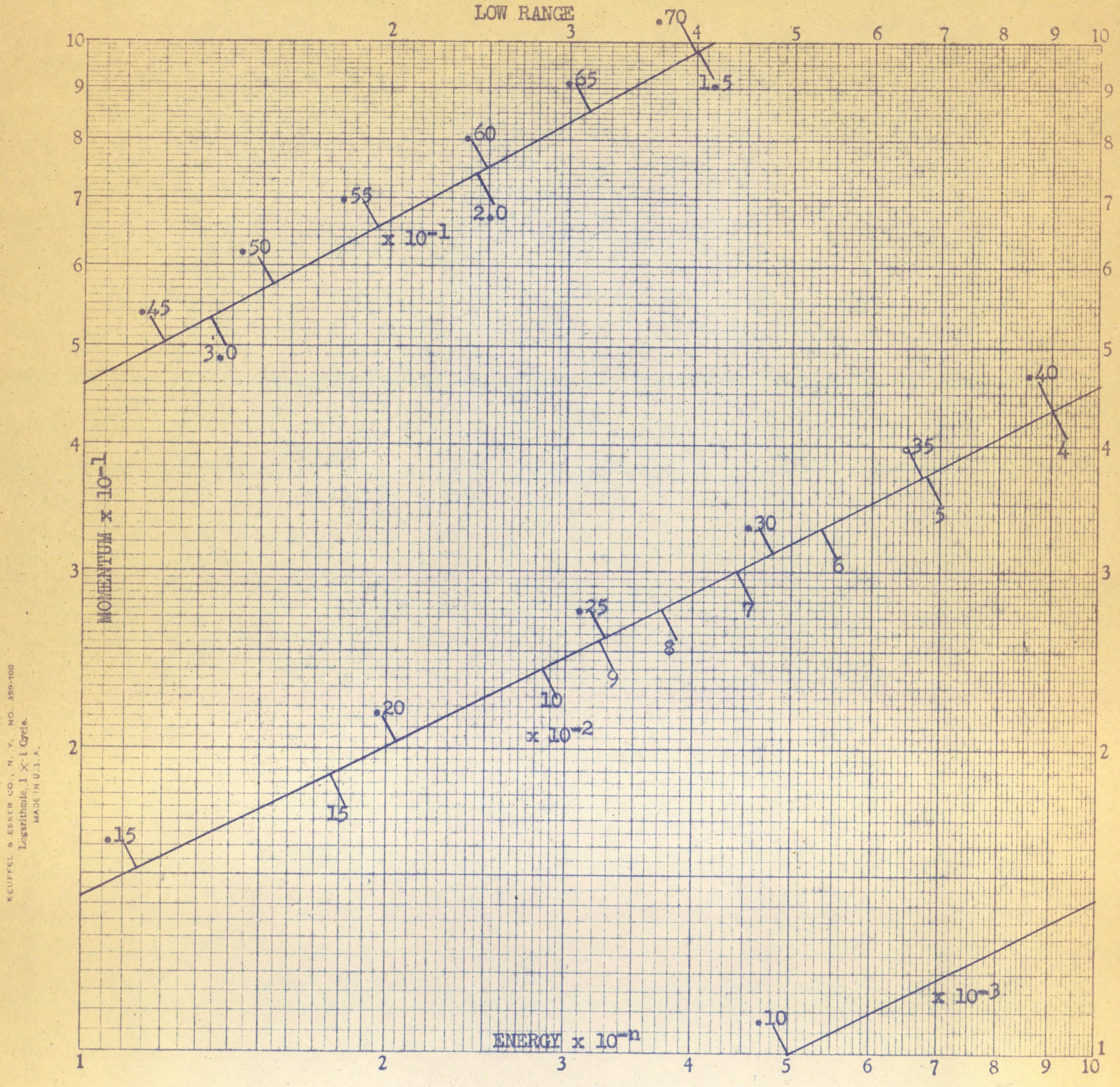


Figure 9

ENERGY VERSUS MOMENTUM IN NATURAL UNITS

Energy in M_oc^2 units
Momentum in M_oc units

Velocity in c units and ionization times minimum in air indicated on graph

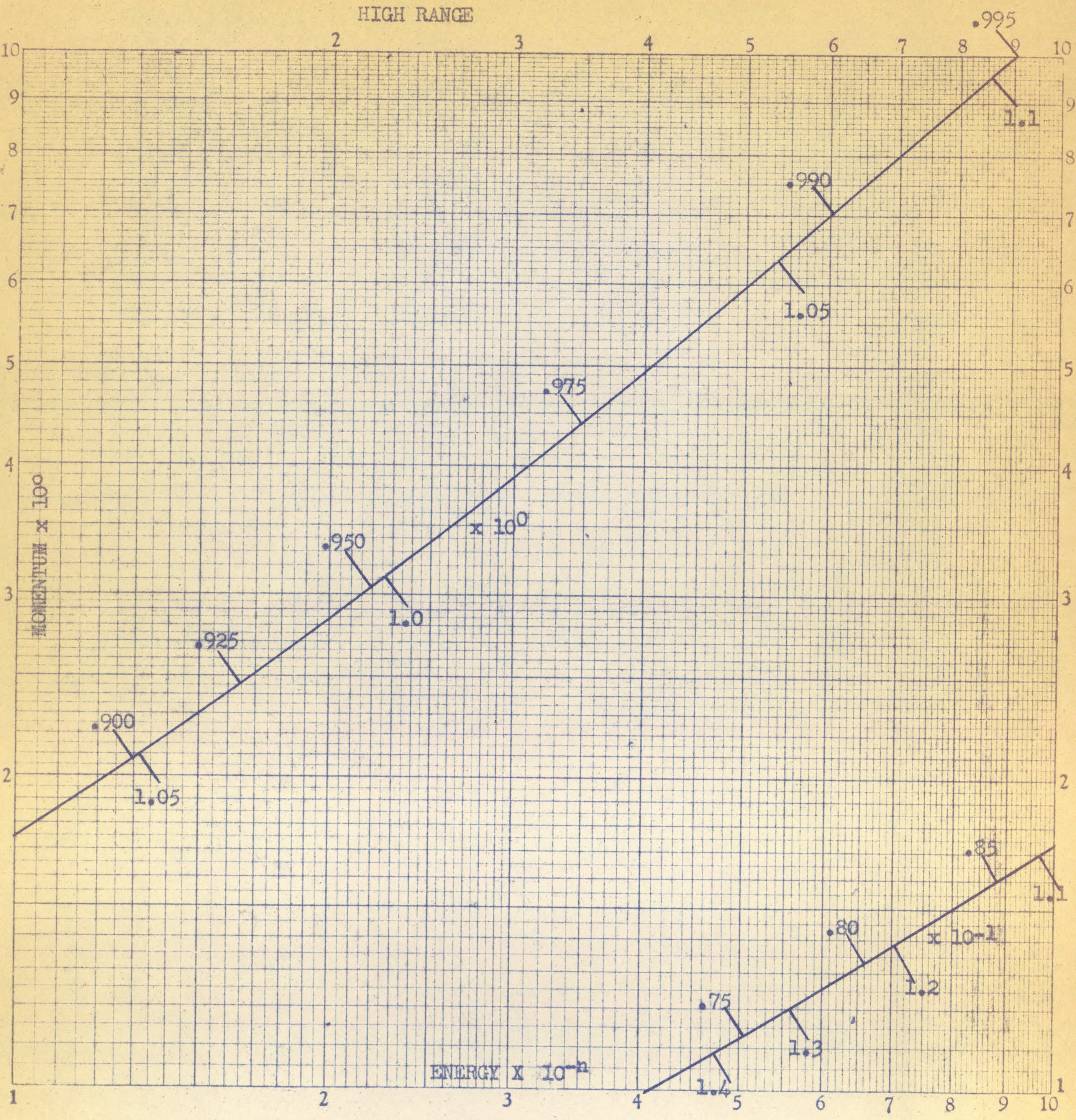


Figure 10
RANGE versus MOMENTUM
for Singly Charged Particles in
ALUMINUM

Range in gm-cm^{-2}/(M_oc^2 in Mev)
Momentum in M_oc units

Derived from proton range tables per J. H. Smith, Phys Rev 71 32 (1947) M_p = 938.17Mev

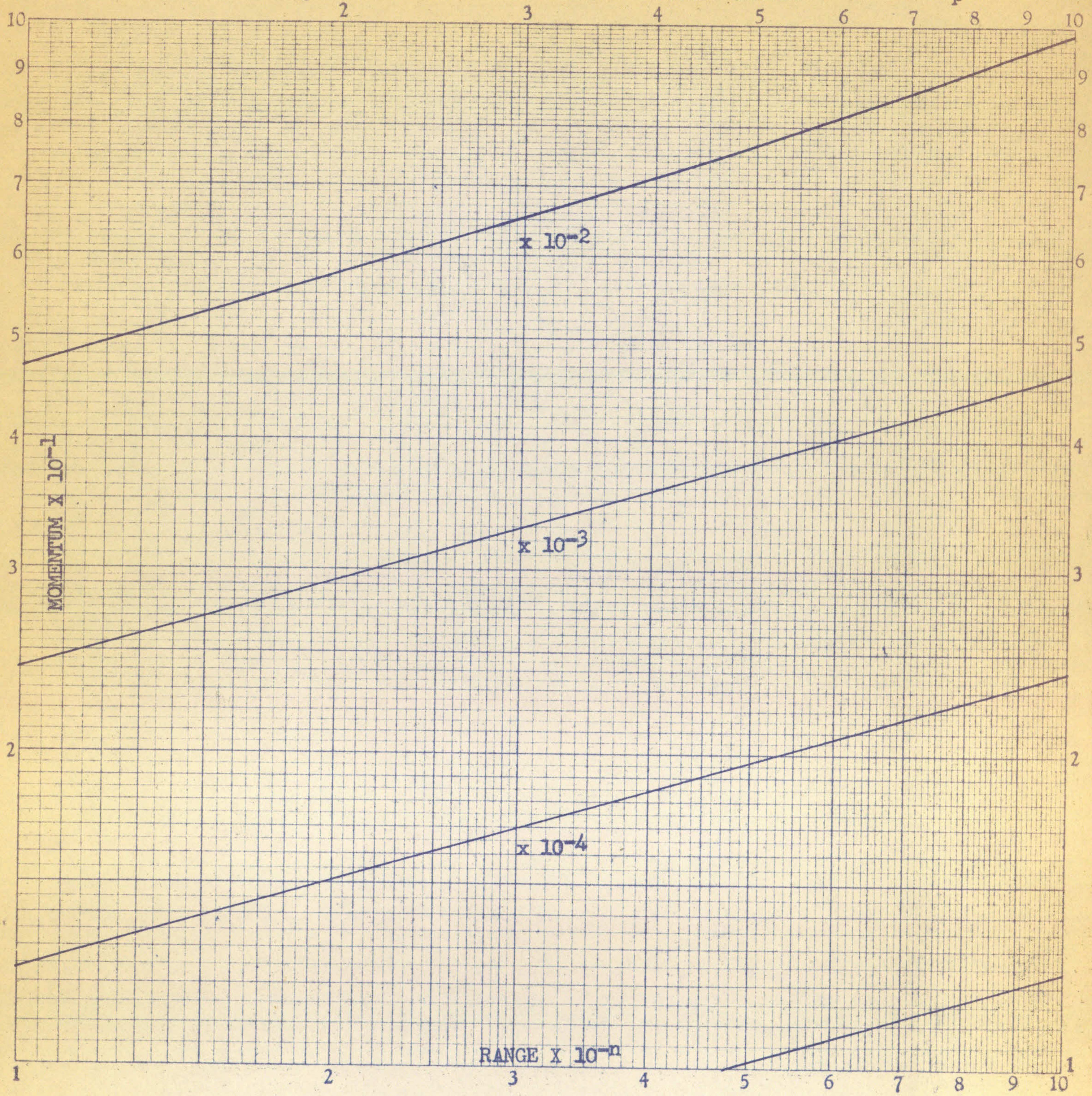
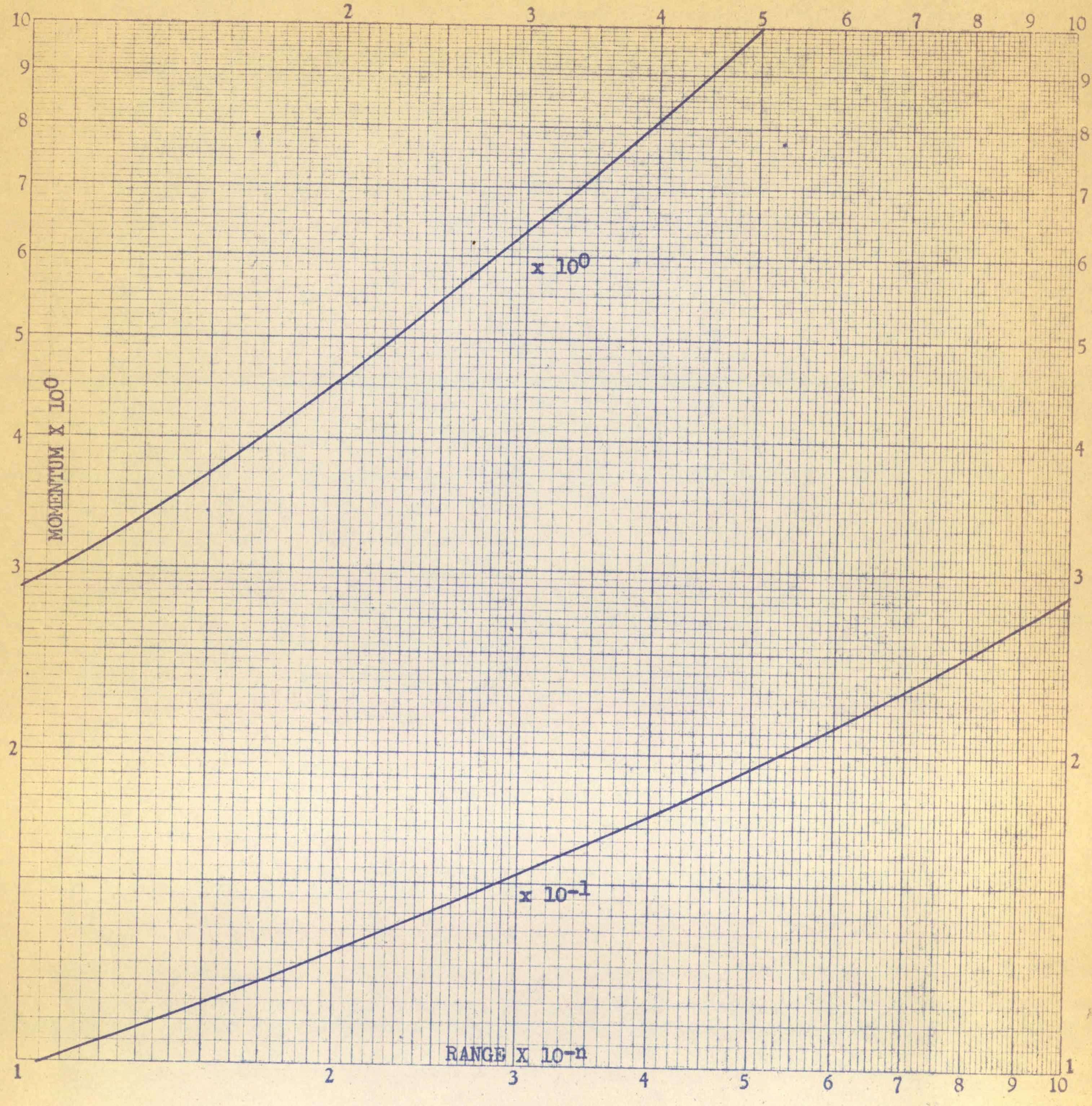


Figure 11
RANGE versus MOMENTUM
for Singly Charged Particles in
ALUMINUM

Range in gm-cm^{-2}/(M_oc^2 in Mev)
Momentum in M_oc units

Derived from proton range tables per J. H. Smith, Phys Rev 71 32 (1947) M_p = 938.17 Mev



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APPENDIX

LEAST SQUARE FIT OF COMPARATOR MEASUREMENTS OF COSMIC RAY TRACKS TO PARABOLAS

SECTION 1

Derivation

In the use of a cloud chamber with a magnetic field one is always faced with the necessity of devising a method of measuring the curvature of the tracks due to the field. This has been done with varying degrees of subtlety by the people who have been concerned with it. Previously for the six-inch chamber the coordinates of points along the track have been measured with a comparator[(2)]. These were plotted on graph paper with one scale exaggerated and then compared with a series of standard parabolas. This method is somewhat objectionable because in the last analysis it is found that the curvatures and limits of error which are determined are quite dependent on the judgment of the person performing the operation.

The use of a least square fit has the decided advantage that it eliminates all human judgment except that of the original comparator measurements. By using the comparator with an eyepiece which has a double cross hair spaced slightly in excess of the normal track width this human judgment has essentially been reduced to a point where it introduces errors of a much smaller size than other inherent errors in the cloud chamber determination of magnetic rigidity.

A circle which passes through the origin and has its center on the x -axis is described by the equation

$$x^2 - 2Rx + y^2 = 0$$

If the radius of curvature R is large, so that for all points measured the values of x are small compared with R, this circle will be closely approximated by the parabola

$$x = \frac{y^2}{2R}$$

The original circle must now be fitted with that parabola of the above form which gives the smallest least square error when compared with the circle. It will be convenient for the purpose of calculation to take the points y_n at even intervals when making the comparator measurements. This distance will be designated by Δ.

Since it is convenient to have the center of curvature of the track being measured as near as possible to the x -axis, an attempt should be made to line the track up along the y -axis when making the comparator measurements. (A correction will subsequently be derived for the case where the center of curvature of the track is off the x -axis).

An attempt is made to fit a parabola of the form

$$x_n = a + bn + cn^2 + \varepsilon_n$$

to the track in such a manner that the quantity

$$E = \sum \varepsilon_n^2 = \sum (x_n - a + bn - cn^2)^2$$

is a minimum.

This requires that

$$-\frac{\partial E}{\partial a} = 0 = 2\sum(x_n - a - bn - cn^2)$$

$$-\frac{\partial E}{\partial b} = 0 = \sum n(x_n - a - bn - cn^2)$$

$$-\frac{\partial E}{\partial c} = 0 = \sum n^2(x_n - a - bn - cn^2)$$

Upon rewriting the equations become

$$\sum x_n = (\sum n^0)a + (\sum n^1)b + (\sum n^2)c$$

$$\sum n x_n = (\sum n^1)a + (\sum n^2)b + (\sum n^3)c$$

$$\sum n^2 x_n = (\sum n^2)a + (\sum n^3)b + (\sum n^4)c$$

Introduction of the abbreviated forms

$$N_r = \sum n^r \qquad S_r = \sum n^r x_n$$

leads to

$$N_0 a + N_1 b + N_2 c = S_0$$

$$N_1 a + N_2 b + N_3 c = S_1$$

$$N_2 a + N_3 b + N_4 c = S_2$$

Taking an odd number of points of measurement and allowing the values of n assigned to these to range over a symmetric interval (all this can be readily done and at no loss of generality), and remembering that

$$N_1 = N_3 = 0$$

one obtains

$$a = S_0 M_4 - S_2 M_2$$

$$b = \frac{S_1}{N_2}$$

$$c = M_0 S_2 - M_2 S_0$$

where the notation

$$M_r = \frac{N_r}{N_0 N_4 - N_2^2}$$

has been adopted. (Values of M_r are tabulated in Table II).

The value of c so obtained permits the calculation of R, where R is the radius of curvature of the track on the emulsion, from the relationship

$$R = \frac{\Delta^2}{2c}$$

where it is assumed that effects due to b are negligible. This R will have to be multiplied by the appropriate factor to convert it to curvature in the chamber.

In general the above calculation of R is of more than enough precision. However, for tracks of small radii of curvature (15 cm or less) and for tracks which cannot be successfully centered, it is useful to evaluate a correction term.

A circle can be described by

$$(x-h)^2 + (y-k)^2 = R^2$$

or

$$x = h - \sqrt{R^2 - (y-k)^2}$$

$$= h - \sqrt{R^2 - k^2}\sqrt{1 - \frac{y^2 - 2ky}{R^2 - k^2}}$$

Since y and k are $<< R$, this can be expanded as

$$x = h - \sqrt{R^2 - k^2}\left[1 - \frac{y^2 - 2ky}{2(R^2 - k^2)} - \frac{1}{8}\left(\frac{y^2 - 2ky}{R^2 - k^2}\right)^2 \cdots\right]$$

$$= \left(h - \sqrt{R^2 - k^2}\right) - \frac{k}{\sqrt{R^2 - k^2}}\, y + \left[\frac{1}{2\sqrt{R^2 - k^2}} + \frac{k^2}{2\sqrt{(R^2 - k^2)^3}}\right] y^2$$

when this is compared with

$$x_n = a + bn + cn^2$$

$$= a + \frac{b}{\Delta}\, y + \frac{c}{\Delta^2}\, y^2$$

it is seen by equating coefficients that

$$\frac{c}{\Delta^2} = \frac{1}{2\sqrt{R^2-k^2}}\left[1-\frac{k^2}{R^2-k^2}\right] \qquad \frac{b}{\Delta} = -\frac{k}{\sqrt{R^2-k^2}}$$

$$\frac{c}{\Delta^2} = -\frac{1}{2}\frac{b}{\Delta k}\left[1+\frac{b^2}{\Delta^2}\right] \qquad k = -\frac{b\Delta}{c}\left[1+\frac{b^2}{\Delta^2}\right]$$

$$R^2 = k^2\left[1+\frac{\Delta^2}{b^2}\right]$$

$$R = \frac{b\Delta}{2c}\left[\frac{\Delta^2}{b^2}+3\right]^{\frac{1}{2}}$$

$$R = \frac{\Delta^2}{2c} + \frac{3}{4}\frac{b^2}{c}$$

This gives a correction to the first calculation. In practice Δ is always taken equal to 0.05 cm, and b's in the range of 0.0001 to 0.001 are obtained for tracks of moderate curvature. In rare cases the correction amounts to 1 %; so in general it is not made.

For small circles:

$$x = h - \sqrt{R^2-y^2}$$

where the origin is taken to be on the x- axis

$$x = h - R + \frac{y^2}{2R} + \frac{y^4}{8R^3} + \frac{y^6}{16R^5}$$

but $h = R$ and $y_n = n\Delta$

therefore $$x_n = \frac{\Delta^2}{2R}n^2 + \frac{\Delta^4}{8R^3}n^4 + \frac{\Delta^6}{16R^5}n^6$$

The quantities

$$S_0 = \sum x_n = \frac{\Delta^2}{2R} N_2 + \frac{\Delta^4}{8R^3} N_4$$

$$S_1 = \sum n x_n = 0$$

$$S_2 = \sum n^2 x_n = \frac{\Delta^2}{2R} N_4 + \frac{\Delta^4}{8R^3} N_6$$

are now formed

but

$$c = \frac{N_0 S_2 - N_2 S_0}{N_0 N_4 - N_2^2} = \frac{\Delta^2}{2R} + \frac{\Delta^4}{8R^3} f(n)$$

where

$$f(n) = \frac{N_0 N_6 - N_2 N_4}{N_0 N_4 - N_2^2}$$

is tabulated in Table III.

An appreciation of the accuracy of this method is gained if one calculates the coordinates of a perfect circle of 5 cm radius and then uses the standard conditions encountered in practice where $\Delta =$ 0.05 cm and $n =$ 17. Upon fitting a parabola to the circle by this method the value of the curvature obtained is $R =$ 5.000 . This corresponds to a track in the chamber with a momentum of 65 Mev/c.

SECTION 2

Errors in Least Square Fits

To assign a probable error use is made of

$$E = \sum \epsilon_n^2 = \sum (x_n - a - bn - cn^2)^2$$

which can be developed in the form

$$E = \sum x_n^2 - (aS_0 + bS_1 + cS_2)$$

If $E=0$, it would be possible to make a perfect fit of a parabola to the track. In general this will not be true, but for the moment let it be supposed that a, b, + c can be determined so that this is true. One now wishes to find the effect if these are changed by δa, δb, + δc.

$$E' = \sum (x_n - [(a+\delta a) + (b+\delta b)n + (c+\delta c)n^2])^2$$

$$E' = N_0(\delta a)^2 + N_2(\delta b)^2 + N_4(\delta c)^2 + 2N_2\,\delta a\,\delta c$$

It is required, for a given δc, that E' be a minimum for δa and δb. Therefore

$$\frac{dE'}{d\delta a} = 2N_0\,\delta a + 2N_2\,\delta c = 0$$

$$\frac{dE'}{d\delta b} = 2N_2\,\delta b = 0$$

Hence

$$\delta a = -\frac{N_2}{N_0}\delta c$$

$$\delta b = 0$$

Taking the second derivative shows these to be a minimum.

As a result

$$E' = \frac{N_0N_4 - N_2^2}{N_0}(\delta c)^2 = \frac{(\delta c)^2}{M_0}$$

or

$$\delta c = \sqrt{M_0 E'} = \sqrt{M_0 N_0}\, E_{rms}$$

where the quantity E_{rms}, which represents the root mean square deviation of the fit, has been introduced. This is independent of N_0, the number of points used in obtaining the fit.

Setting $$\sigma = \frac{1}{R} = \frac{2c}{\Delta^2}$$

and since $$L = (N_0 - 1)\Delta = \text{ track length.}$$

One obtains

$$\delta\sigma = \frac{2\delta c}{\Delta^2} = \frac{2(N_0-1)^2}{L^2}\delta c$$

To evaluate δc it is noted that in the derivation above it has in effect been assumed that the total E is produced by a direct change of curvature. Since the calculated value of curvature has so adjusted the parabola that the track has equal rms scatter on each side of the parabola, this will certainly be an outside limit. Hence

$$\delta c = k\sqrt{M_0 N_0}\, E_{rms} \qquad k < 1$$

then

$$\delta\sigma = 2k\,\frac{(N_0-1)^2}{L^2}\sqrt{M_0 N_0}\, E_{rms}$$

From the table it is seen that in the range $N_0 = 13 - 21$ in which one is interested the quantity

$$\sqrt{M_0 N_0}\,(N_0-1)^2$$

changes rather slowly; therefore a mean value of 12 can be used for the purposes of calculation.

Upon assigning a reasonable value to k the expression will be one which indicates the probable error in curvature due to measurement alone. To obtain a value for k one can argue heuristically that the value $k=1$ is the outside limit, and since the parabola has been adjusted to give equal rms scattering on each side of the track a value of 1/2 would seem to be reasonable. If one checks the value of 1/2 with the error limits set by the manual parabola fitting method, it is found that it is quite reasonable. If the somewhat more refined method of having a given track measured more than once by several people is used, the value 1/2 again seems to give a consistent result.

With the adoption of the value $k = 1/2$, the final error limit estimate is obtained.

$$\delta\sigma = \frac{12}{L^2} E_{rms}$$

The limits of E_{rms} for which this is valid are somewhat ambiguous depending on the individual cases; however a lower limit of 3×10^{-4} is set by the process of the comparator measurement itself. This is so because it is found to be difficult to repeat single readings with a rms spread of less than this figure. When

E_{rms} for a curve is lower than this, 3×10^{-4} is used as the value of error. Experience shows that tracks with $E_{rms} > 10^{-3}$ can result from distortions or scatterings of the track. Such tracks should be projected particularly to see whether any obvious distortions or scatterings are present. Indeed it has been a matter of policy to carefully project and examine all tracks which are considered interesting enough to measure with the comparator.

SECTION 3

Calculating Procedure

(This can be most readily done with a calculating machine)

(1) Calculate

$$S_0 = \sum X_n$$
$$S_1 = \sum n X_n$$
$$S_2 = \sum n^2 X_n$$

(2) From (1) and Table II calculate

$$a = M_4 S_0 - M_2 S_2$$
$$b = S_1 / N_2$$
$$c = M_0 S_2 - M_2 S_0$$

(3) Calculate

$$\sum X_n^2$$

(4) From (3) and (1 and 2) calculate

$$E_{min} = \sum X_n^2 - a S_0 - b S_1 - c S_2$$

(5) From (4) obtain

$$E_{rms} = \sqrt{\frac{E_{min}}{N_0}}$$

(6) From (2) obtain

$$\sigma = \frac{2c}{\Delta^2} = \frac{1}{R}$$

(7) From (5) obtain

$$\delta\sigma = \frac{12}{L^2} E_{rms}$$

(8) Calculate

$$\sigma_h = \sigma + \delta\sigma \qquad \sigma_\ell = \sigma - \delta\sigma$$

(9) From (8) obtain

$$R = \frac{1}{\sigma} \qquad R_+ = \frac{1}{\sigma_\ell} \qquad R_- = \frac{1}{\sigma_h}$$

where R is the radius of curvature of the track in the emulsion.

TABLE I

n	n^2	n^4	n^6	N_0	N_2	N_4	N_6	$N_0N_6 - N_2N_4$
1	1	1	1	3	2	2	2	2
2	4	16	64	5	10	34	130	310
3	9	81	729	7	28	196	1,588	5,628
4	16	256	4,096	9	60	708	9,780	45,540
5	25	625	15,625	11	110	1,958	41,030	235,950
6	36	1,296	46,656	13	182	4,550	134,342	918,346
7	49	2,401	117,649	15	280	9,352	369,640	2,926,040
8	64	4,096	262,144	17	408	17,544	893,928	8,038,824
9	81	6,561	531,441	19	570	30,666	1,956,810	19,699,770
10	100	10,000	1,000,000	21	770	50,666	3,956,810	44,080,190
11	121	14,641	1,771,561	23	1,012	79,948	7,499,932	91,591,060
12	144	20,736	2,985,984	25	1,300	121,420	13,471,900	178,951,500
13	169	28,561	4,826,809	27	1,638	178,542	23,125,518	331,937,190
14	196	38,416	7,529,536	29	2,030	255,374	38,964,590	611,563,890
15	225	50,625	11,390,625	31	2,480	356,624	60,965,840	1,005,513,520
16	256	65,536	16,777,216	33	2,992	487,696	94,520,272	1,659,982,544

TABLE II

n	N_0	M_0	M_2	M_4
1	1	1.5000000	1.0000000	1.00000000
2	5	7.1428571×10^{-2}	1.4285714×10^{-1}	0.48571428
3	7	1.1904761×10^{-2}	4.7619047×10^{-2}	0.33333333
4	9	3.2467532×10^{-3}	2.1645022×10^{-2}	0.25541125
5	11	1.1655011×10^{-3}	1.1655011×10^{-2}	0.20745920
6	13	4.9950049×10^{-4}	6.9930069×10^{-3}	0.17482517
7	15	2.4240465×10^{-4}	4.5248868×10^{-3}	0.15113122
8	17	1.2899896×10^{-4}	3.0959752×10^{-3}	0.13312693
9	19	7.3713696×10^{-5}	2.2114108×10^{-3}	0.11897390
10	21	4.4577847×10^{-5}	1.6345210×10^{-3}	0.10755148
11	23	2.8232636×10^{-5}	1.2422360×10^{-3}	0.09813662
12	25	1.8580453×10^{-5}	9.6618357×10^{-4}	0.09024154
13	27	1.2631047×10^{-5}	7.6628352×10^{-4}	0.08352490
14	29	8.8281512×10^{-6}	6.1797058×10^{-4}	0.07774070
15	31	6.3201537×10^{-6}	5.0561229×10^{-4}	0.07270705
16	33	4.6206165×10^{-6}	4.1893590×10^{-4}	0.06828655

TABLE III

N_0	$M_0N_0(N_0-1)^2$	f(N)
3	8.49	1.000
5	9.56	4.429
7	10.39	9.571
9	10.94	16.429
11	11.32	25.000
13	11.60	35.286
15	11.82	47.286
17	11.99	61.000
19	12.13	76.429
21	12.24	93.571
23	12.33	112.429
25	12.41	133.000
27	12.48	155.286
29	12.54	186.172
31	12.60	205.000
33	12.66	232.429

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