A STUDY OF THE TWO-MESON HYPOTHESIS

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ABSTRACT

An attempt was made to obtain some indication theoretically as to the tensor character of the heavy and the light mesons observed in cosmic rays. For this purpose the two-meson hypothesis of Bethe and Marshak was accepted. The heavy meson which interacts with nucleons was assumed to be either a scalar, a vector, or a pseudoscalar field in agreement with nuclear force theories. The light meson was assumed to be either a scalar, a pseudoscalar, or a spin $\frac{1}{2}$ field in agreement with the evidence on meson burst production. The associated secondary neutral particle was assumed to have the same tensor character as the light meson. Conservation of spin and statistics limited consideration to seven mixed fields. For each of these fields the decaytime of the heavy meson into the light meson and the lifetime of the light meson for nuclear capture was computed. Using the most recent values of the masses of the heavy and light mesons, the decaytime of the heavy meson is τ(scalar$\rightarrow$spin $\frac{1}{2}$)$\sim 1.22\times10^{-8}$sec., τ(pseudoscalar$\rightarrow$spin $\frac{1}{2}$)$\sim 1.94\times10^{-9}$sec., τ(vector$\rightarrow$spin $\frac{1}{2}$)$\sim 1.88\times10^{-8}$sec., τ(vector$\rightarrow$pseudoscalar or scalar)$\sim 6.31\times10^{-7}$sec., τ(scalar$\rightarrow$pseudoscalar or scalar)$\sim 1.97\times10^{-8}$sec.. These values are to be compared with the experimental value of 10^{-8}sec.. In view of the zero mass of the neutral secondary particle which makes it possible to identify it with a neutrino and the ease of nuclear production of the heavy meson, the indication is that the heavy and light mesons are scalar and spin $\frac{1}{2}$ fields, respectively.

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I INTRODUCTION

Recently two very striking difficulties with the theory of the meson were brought to light. The first was found by Conversi, Pancini, and Piccioni[1] and verified more quantitatively by other experimenters.[2] This work consisted of an investigation of the decay of mesons stopped by various materials. It was found that negative mesons which stopped in heavy material such as iron did not spontaneously decay indicating that these mesons were captured by the nuclei of the stopping material. On the other hand, negative mesons which stopped in light materials such as carbon seemed to decay as frequently as not. This result was in sharp contrast to the predicted results that mesons are captured by nuclei with a lifetime of the order 10^{-18} seconds[3] and in further contradiction with the ease of production of mesons in cosmic rays.[4] With regard to this result Fermi, Teller, and Weisskopf[5] investigated the time for a slow negative meson to be stopped in matter and to be captured into the K-orbit of the atoms of the material. The time computed was about 10^{-13} seconds, which time is far too short to account for any part of the decaying negative mesons.

The second difficulty arose from the observations of Lattes, Muirhead, Occhialini, and Powell[6] on cosmic-ray mesons. These observations were made with the aid of specially prepared photographic emulsions which were exposed to the cosmic rays. A study was made on the behavior of mesons which stopped in the emulsion. It was observed that stopped mesons could be divided into essentially two categories, those which produced secondary phenomena and those which did not. The latter mesons could be interpreted as principally positive mesons which after stopping were prevented by the Coulomb field of the nuclei from interacting with them(7) and hence these mesons spontaneously decayed into light particle products which were not detectable by the photographic emulsion. On the other hand the secondary phenomena of the stopped mesons were of two kinds. The most numerous such process was the stopping of the meson being accompanied by the emission of heavy particles and stars. This indicated that these were negative mesons which were captured by the nuclei of the emulsion and thereby caused nuclear disintegrations. However, the second such process consisted of the stopping of the meson being accompanied by the emission of another meson. It was suggested that the appearance of this secondary meson could be attributed to the nuclear capture of a negative meson with the subsequent emission of a positive meson from the nucleus. The change of the atomic number of the nucleus by two would then account for the energy of the secondary meson. An investigation into this possibility was made by Frank(8) with the negative result that no nucleus present in the emulsion would release sufficient energy when its atomic number changed by two in order to account for the energy of the secondary meson.

Marshak and Bethe(9) and independently Lattes, Muirhead, Occhialini, and Powell proposed that the phenomena observed in the experiments on meson capture by nuclei and on mesons which stop in photographic emulsions could be explained if one assumed the existence of two kinds of mesons. The basic concepts of this two-meson hypothesis are: (1) one meson is heavier than the other; (2) the heavy meson can spontaneously decay into the light meson with the emission of a light or uncharged particle; (3) the heavy meson interacts strongly with nuclei; and (4) the light meson is to be identified with the sea level cosmic-ray meson. Properties (3) and (4) offer an explanation of the anomalous nuclear capture of mesons found by Conversi. On the other hand, properties (1) and (2) give the interpretation of the process observed by Lattes, Muirhead, Occhialini, and Powell as being simply the spontaneous decay of the heavy meson into the light one. The above assumptions have been substantiated by further results obtained from the artificial production of mesons with the Berkeley cyclotron.(10)

We shall assume here that the two-meson hypothesis is a true picture of the experimental findings and then investigate more quantitatively the nature of the two kinds of mesons. This will be done by comparing the experimental lifetimes for the nuclear capture of a light meson and for the spontaneous decay of the heavy meson with these same quantities calculated on the basis of specific models for the meson fields.

II DETAILED NATURE OF THE TWO-MESON FIELD

The two-meson field has associated with it two processes unique to the assumed structure of the field. The first is the decay of the π-meson* into a μ-meson with the simultaneous emission of a third particle (or perhaps more than just one particle). The second is the capture of the μ-meson by a nucleon. The first process immediately raises the question as to the character of the third particle. From the fact that it was undetected in the photographic emulsions indicates that it must be a charged particle with mass considerably less than the mass of the meson, a photon, or a neutral particle. The possibility that it is a light charged particle may be discredited for the following reasons. To conserve charge in the decay process we would have to assume that either one of the particles involved in the decay had two electron charges associated with it or there was actually more than one additional particle accompanying the decay. The former possibility demands the introduction of a particle as yet unobserved; the latter alternative is inconsistent with the experimental results obtained in the photographic emulsions which indicated a unique energy for the created μ-meson. The assumption of a decay photon is contradicted by the absence of photons in the capture of the μ-meson, as will be discussed later. Then the only alternative

* It has become customary to call the heavy meson a π-meson and the light meson a μ-meson. This terminology will be used here.

is that the decay process is accompanied by a neutral particle-thus

$$\pi^{\pm} \rightarrow \mu^{\pm} + \mu^{0}$$

Measurements by Lattes, Bishop, and Gardner(11) show the mass of the π-meson to be about 283 times the electron mass. Fretter and Brode,(12) on the other hand, find a mass of 212 times the electron mass for the μ-meson. Using conservation of energy and momentum, one finds for the mass of the neutral particle

$$\frac{m_n}{m_e} = \left\{ \left[\left(\frac{m_h}{m_e} - \frac{m_l}{m_e} \right) - \frac{E_l}{m_e c^2} \right]^2 + \left(\frac{m_l}{m_e} \right)^2 - \left[\frac{m_l}{m_e} + \frac{E_l}{m_e c^2} \right]^2 \right\}^{1/2}$$

where m_n is the mass of neutral particle,

m_h is the mass of π-meson,

m_l is the mass of μ-meson,

m_e is the mass of electron,

E_l is the kinetic energy of μ-meson created by the decay.

From the range measurements in the photographic emulsions it was found that $E_l \approx 3.8$ Mev.(13) Thus one has $m_n \leq 29\ m_e$. With regard to the lifetime of the π-meson for decay into a μ-meson one has the result of the work on the artificially produced mesons at Berkeley(14) which indicates a time slightly greater than 10^{-8} seconds.

Considering the capture of the μ-meson by a nucleus, we have the results of Fermi, Teller, and Weisskopf which indicate that this process most probably takes place when the μ-meson is in the K-orbit of the capturing nucleus. Since according to the structure of the two-meson field the

μ-meson is not directly coupled to nucleons, one can only account for the capture by means of a intermediate-state process. The process of capture is thus assumed to occur by means of one of possibly three reactions:

$$\text{(i) } \mu\text{-meson} \rightarrow \left\{\begin{array}{l} \pi\text{-meson (virtual)} + \text{Neutral particle} \\ \pi\text{-meson (virtual)} + \text{Proton} \end{array}\right\} \rightarrow \text{Neutron}$$

$$\text{(ii) Proton} \rightarrow \left\{\begin{array}{l} \pi\text{-meson (virtual)} + \text{Neutron} \\ \pi\text{-meson (virtual)} + \mu\text{-meson} \end{array}\right\} \rightarrow \text{Neutral particle}$$

$$\text{(iii) } \mu\text{-meson} + \text{Proton} \rightarrow \text{Neutral meson} + \text{Neutron}^{*}$$

The choice of these methods of capture lies in the fact that they are immediate consequences of the assumed coupling scheme of the two-meson field. Any other capture processes would have to be considered in addition to the above rather than as alternatives (provided, of course, the Bethe-Marshak hypotheses are preserved). Thus the above capture scheme represents the simplest possible choice consistent with our basic assumptions. This process of capture is moreover consistent with the experimental results which indicate that the nuclear capture of the μ-meson is predominantly unaccompanied by the emission of heavy particles or stars or any other observable process.[15] Since, in the capture process, the nucleus would receive the entire rest energy of the μ-meson, it would be consistent with the absence of stars only if we assume that a secondary particle is emitted in the μ-meson capture. Investigations on the nature of this particle seem conclusively to exclude the possibility that it is a photon.[16]

*The direct transition may be present as a result of the coupling of the fields but will be second order in the coupling constants as are (i) and (ii) in analogy to β-decay theory.

Thus simplicity of form and minimization of the number of hypotheses favors the above capture process. Several investigators have measured the lifetime for nuclear capture of the μ-meson.[2] This was done indirectly by measuring the spontaneous decay of the negative μ-meson and comparing it with the positive μ-meson. The lifetime of nuclear capture is found to depend strongly on the atomic number of the nucleus being about equal to the probability of free decay for $Z \approx 13$.

III FORMULATION OF THE MIXED FIELD

Theoretical considerations give five possibilities for a meson field, two spin 1 fields (vector and pseudovector), two spin 0 fields (scalar and pseudoscalar), and one spin $\frac{1}{2}$ field (Dirac). The choice of the field to describe the π-meson is made under the assumption that the π-meson is to account for nuclear forces. Since the only reasonably successful nuclear force fields arise from a scalar meson field, a pseudoscalar meson field, a vector meson field,

TABLE 1. Mixed Fields

π-Meson	μ-Meson	Neutral Particle
Vector	Spin $\frac{1}{2}$	Spin $\frac{1}{2}$
Scalar	Spin $\frac{1}{2}$	Spin $\frac{1}{2}$
Pseudoscalar	Spin $\frac{1}{2}$	Spin $\frac{1}{2}$
Vector	Scalar	Scalar Vector Pseudovector Pseudoscalar
Scalar	Scalar	Scalar Vector Pseudovector
Pseudoscalar	Scalar	Pseudoscalar Vector Pseudovector
Vector	Pseudoscalar	Scalar Vector Pseudovector Pseudoscalar
Scalar	Pseudoscalar	Vector Pseudovector Pseudoscalar
Pseudoscalar	Pseudoscalar	Vector Scalar

or a mixture of these three, we shall assume that the π-meson is one of these. For the field to represent the μ-meson we shall assume either a spin $\frac{1}{2}$ or a spin 0 field as these seem to agree best with the calculations of Christy and Kusaka[17] on bursts produced by mesons. The field to represent the neutral particle is limited by conservation of spin and statistics in the decay and capture processes and by the requirement of a relativistically invariant interaction between the three particles which we shall assume does not depend on second or higher derivatives of the field potentials. Table 1. gives the possibilities for the mixed field. We shall here make the arbitrary assumption that the μ-meson and the neutral particle are the same kind of field. Using the notation introduced by Kemmer,[18] we have for the Lagrangian densities of the scalar, vector, and pseudoscalar fields:

$$L_{scalar} = -\hbar c\kappa\left(\varphi^*\varphi + \chi^*_\alpha\chi^\alpha\right) \tag{1}$$

$$L_{vector} = -\hbar c\kappa\left(\varphi^*_\alpha\varphi^\alpha + \tfrac{1}{2}\chi^*_{\alpha\beta}\chi^{\alpha\beta}\right) \tag{2}$$

$$L_{pseudoscalar} = -\hbar c\kappa\left(\tfrac{1}{6}\varphi^*_{\alpha\beta\gamma}\varphi^{\alpha\beta\gamma} + \tfrac{1}{24}\chi^*_{\alpha\beta\gamma\delta}\chi^{\alpha\beta\gamma\delta}\right) \tag{3}$$

and of the Dirac field:

$$L_{Dirac} = -\hbar c\left(\psi^+\left[\gamma^\mu\partial_\mu + \kappa\right]\psi\right) \tag{4}$$

The mixing of these fields is in general not unique, that is, there may be several interaction terms possible in a given field. The prescription used to fix on the interactions is that they shall involve to a minimum extent velocity dependent forces. As an example, in the scalar-scalar* field we have the four possible invariant interactions:

(a) $\varphi^*(\pi)$ $\varphi(\mu)$ φ(neutral)

(b) $\varphi^*(\pi)$ $\chi_\alpha(\mu)$ χ^α(neutral)

(c) $\chi_\alpha^*(\pi)$ $\chi^\alpha(\mu)$ φ(neutral)

(d) $\chi_\alpha^*(\pi)$ $\varphi(\mu)$ χ^α(neutral)

We decide in favor of (a) only as it contains no velocity dependence as is present as a result of the χ factors in (b), (c), and (d).

With the above restrictions, the choice of interaction Lagrangian densities is taken to be:

(a) Vector-spin $\frac{1}{2}$

$$L_1 = -\hbar c K_v\left(g_1 v_\alpha^{(\mu)} \varphi_v^{\alpha *} + c.c.\right) - \hbar c K_v\left(f_1 \tfrac{1}{2} u_{\alpha\beta}^{(\mu)} \chi_v^{\alpha\beta *} + c.c.\right)$$

(b) Scalar-spin $\frac{1}{2}$

$$L_2 = -\hbar c K_s\left(g_2 w^{(\mu)} \varphi_s^{*} + c.c.\right) - \hbar c K_s\left(f_2 v_\alpha^{(\mu)} \chi_s^{\alpha *} + c.c.\right)$$

(c) Pseudoscalar-spin $\frac{1}{2}$

$$L_3 = -\hbar c K_p\left(g_3 \tfrac{1}{6} t_{\alpha\beta\gamma}^{\mu} \varphi_p^{\alpha\beta\gamma *} + c.c.\right) - \hbar c K_p\left(f_3 \tfrac{1}{24} s_{\alpha\beta\gamma\delta}^{(\mu)} \chi_p^{\alpha\beta\gamma\delta *} + c.c.\right)$$

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*The designation of the mixed field is made by specifying first the character of the heavy particle and then that of the light and neutral particles.

(d) Vector-Scalar

$$L_4 = -\hbar c \kappa_n \left(g_4 \varphi_c \chi_{n\alpha} \varphi_v^{\alpha *} + \text{c.c.} \right) - \hbar c \kappa_c \left(f_4 \chi_{c\alpha} \varphi_n \varphi_v^{\alpha *} + \text{c.c.} \right)$$

(e) Scalar-Scalar[19]

$$L_5 = -\hbar c \kappa_s \left(g_5 \varphi_s^* \varphi_c \varphi_n + \text{c.c.} \right)$$

(f) Vector-Pseudoscalar

$$L_6 = -\hbar c \kappa_n \left(g_6 \tfrac{1}{6} \varphi_c^{\alpha\beta\gamma} \chi_{n\alpha\beta\gamma\delta} \varphi_v^{\delta *} + \text{c.c.} \right) - \hbar c \kappa_c \left(f_6 \tfrac{1}{6} \varphi_n^{\alpha\beta\gamma} \chi_{c\alpha\beta\gamma\delta} \varphi_v^{\delta *} + \text{c.c.} \right)$$

(g) Scalar-Pseudoscalar

$$L_7 = -\hbar c \kappa_s \left(g_7 \tfrac{1}{24} \chi_{c\alpha\beta\gamma\delta} \chi_n^{\alpha\beta\gamma\delta} \varphi_s^* + \text{c.c.} \right) \tag{5}$$

where $\kappa = \frac{mc}{\hbar}$; the subscripts s, v, p stand for scalar, vector, and pseudoscalar, respectively, and refer to the character of the π-meson. The subscripts c and n are used to distinguish between the charged μ-meson and neutral particle. The quantities $w^{(\mu)}$, $v_\alpha^{(\mu)}$, $u_{\alpha\beta}^{(\mu)}$, $t_{\alpha\beta\gamma}^{(\mu)}$, and $s_{\alpha\beta\gamma\delta}^{(\mu)}$ are the covariant Dirac interactions between the spin $\frac{1}{2}$ μ-meson and the neutral spin $\frac{1}{2}$ particle. It has been observed by Nelson[20] and by Dyson[21] that the interaction g_3 is equivalent to f_3. Also it may be shown that the interaction g_2 is the same as f_2. Moreover, an integration by parts shows that the interactions g_4 and g_6 are equivalent to f_4 and f_6, respectively. Only in the vector-spin $\frac{1}{2}$ field are the f_1- and g_1- interactions independent. Hence, we shall assume for the following that $f_2 = g_3 = f_4 = g_6 = 0$.

Finally, the interaction Lagrangian densities of the π-meson with the nucleons may be written as:

$$L'_{\text{scalar}} = -\hbar c\kappa_s\left(r_s w^{(N)}\varphi_s^{*} + \text{c.c.}\right) - \hbar c\kappa_s\left(s_s v_\alpha^{(N)}\chi_s^{\alpha *} + \text{c.c.}\right) \qquad (6)$$

$$L'_{\text{pseudoscalar}} = -\hbar c\kappa_p\left(r_p \tfrac{1}{24} S^{(N)}_{\alpha\beta\gamma\delta}\chi^{\alpha\beta\gamma\delta *} + \text{c.c.}\right) - \hbar c\kappa_p\left(s_p \tfrac{1}{6} t^{(N)}_{\alpha\beta\gamma}\varphi^{\alpha\beta\gamma *} + \text{c.c.}\right) \qquad (7)$$

$$L'_{\text{vector}} = -\hbar c\kappa_v\left(r_v v_\alpha^{(N)}\varphi_v^{\alpha *} + \text{c.c.}\right) - \hbar c\kappa_v\left(s_v \tfrac{1}{2} u^{(N)}_{\alpha\beta}\chi_v^{\alpha\beta *} + \text{c.c.}\right) \qquad (8)$$

The Hamiltonian functions for the mixed fields are derived from the Lagrangians of equations (1) to (8) by the usual method. In only vector-scalar and vector-pseudoscalar cases is any difference from the standard canonical procedure found. This arises in the elimination of φ_0 from the vector Hamiltonian. One uses for this purpose the equation of motion of φ_0, namely,

$$\frac{\partial L}{\partial \varphi_0} - \frac{\partial}{\partial x^\alpha}\frac{\partial L}{\partial\left(\frac{\partial \varphi_0}{\partial x^\alpha}\right)} = 0$$

This equation, however, besides containing a term in φ_0 alone contains terms in the product of φ_0 with the potentials of the μ-meson and neutral particle fields. Fortunately these latter terms are second order in the coupling constants of the π-meson with the μ-mesons and hence may be neglected. Further, it follows immediately from the Hamiltonian functions that the two fields containing the pseudoscalar light particles are identical with the corresponding two fields containing the scalar light particles. Thus the interaction Hamiltonian densities for the mixed fields are, in vector notation:

(a) $H_1 = \hbar c\kappa_v\Big\{\Phi_N^{*}\frac{\tau_x + i\tau_y}{2}\Big[r_v\bar{\alpha}\cdot\psi_v^{*} - s_v\big(\bar{\gamma}\cdot\pi_v - \tfrac{1}{\kappa_v}\beta\bar{\sigma}\cdot\text{curl}\,\psi_v^{*}\big)$

$$+ g_v \bar{\gamma}\cdot\Psi_v^* + r_v(\bar{\alpha}\cdot\Pi_v + {}^1/_{K_v}\,\mathrm{div}\,\Psi_v^*)\big]\Psi_N + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_v\Big\{\tilde{\psi}_m C\big[g_1\bar{\alpha}\cdot\psi_v^* - f_1(\bar{\gamma}\cdot\pi_v - {}^1/_{K_v}\beta\bar{\sigma}\cdot\mathrm{curl}\,\psi_v^*)$$

$$+ f_1\bar{\gamma}\cdot\Psi_v^* + g_1(\bar{\alpha}\cdot\Pi_v + {}^1/_{K_v}\,\mathrm{div}\,\Psi_v^*)\big]\psi_c + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_v\Big\{f_1^* g_v(\tilde{\psi}_c C\bar{\gamma}\psi_m)\cdot(\Phi_N^*\tfrac{\tau_x+i\tau_y}{2}\bar{\gamma}\Psi_N) + g_1^* r_v(\tilde{\psi}_c C\psi_m)(\Phi_N^*\tfrac{\tau_x+i\tau_y}{2}\Psi_N) + \mathrm{c.c.}\Big\}$$

(b) $$H_2 = \hbar c K_s\Big\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\big[r_s\beta\psi_s^*\big]\Psi_N + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_s\Big\{\tilde{\psi}_m C\big[g_2\beta\psi_s^*\big]\psi_c + \mathrm{c.c.}\Big\}$$

(c) $$H_3 = \hbar c K_p\Big\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\big[r_p i\beta\gamma^5\psi_p^*\big]\Psi_N + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_p\Big\{\tilde{\psi}_m C\big[f_3 i\beta\gamma^5\psi_p^*\big]\psi_c + \mathrm{c.c.}\Big\}$$

(d) $$H_4 = H_6 = \hbar c K_v\Big\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\big[r_v\bar{\alpha}\cdot\psi_v^* - g_v(\bar{\gamma}\cdot\pi_v - {}^1/_{K_v}\beta\bar{\sigma}\cdot\mathrm{curl}\,\psi_v^*)$$

$$+ g_v\bar{\gamma}\cdot\Psi_v^* + r_v(\bar{\alpha}\cdot\Pi_v + {}^1/_{K_v}\,\mathrm{div}\,\Psi_v^*)\big]\Psi_N + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_m\Big\{g_4\big[-{}^1/_{K_v}\psi_c\pi_m^*\,\mathrm{div}\,\Psi_v^* + {}^1/_{K_m}(\Psi_v^* + \Pi_v)\cdot\mathrm{grad}\,\psi_m\psi_c\big] + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_m\Big\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\big[-r_v g_4^*\psi_c^*\pi_m\big]\Psi_N + \mathrm{c.c.}\Big\}$$

(e) $$H_5 = H_7 = \hbar c K_s\Big\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\big[r_v\beta\psi_s^*\big]\Psi_N + \mathrm{c.c.}\Big\}$$

$$+\hbar c K_s\Big\{g_5\psi_s^*\psi_c\psi_m + \mathrm{c.c.}\Big\} \qquad (9)$$

In these expressions the numerical subscript on H corresponds to the mixed fields of equation (5), τ_x and τ_y are the isotopic spins, and Φ_N and Ψ_N are the nucleon wave functions. Delta function interactions between nucleons and between mesons are not included. The tilde over the Dirac wave functions indicates the charge conjugate solution.[22] The Majorana-Racah[23] representation of the neutral spin $\frac{1}{2}$ particle is used here with the representation in which $C = -i\alpha_2\beta$.

IV LIFETIME OF THE π-MESON

According to the usual quantum mechanical perturbation formulae we have that the probability per second for the decay of the π-meson is

$$w_{A\to F} = \frac{2\pi}{\hbar} \sum_{\text{SPIN}} |H_{AF}|^2 \rho(E_0) V \tag{10}$$

where H_{AF} is the Hamiltonian matrix element connecting the initial state A with the final state F, $\sum_{\text{SPIN}} |H_{AF}|^2$ means the summation of $|H_{AF}|^2$ over the spins of the final particles, and $\rho(E_0)$ is the density of final states per unit energy and volume evaluated at the energy E_0 of the final state. V is the volume of the hohlraum. By integrating $w_{A\to F}$ over all possible relative directions of the μ-meson and neutral particle, we obtain the lifetime of the π-meson, namely:

$$\frac{1}{\tau} = \int_{\Omega_c} w_{A\to F} \tag{11}$$

The density of states $\rho(E_0)$ is the same for all the mixed fields and is given by

$$\rho(E_0) = \frac{1}{(2\pi\hbar)^3} p_c^2 \frac{dp_c}{dE_0} d\Omega_c$$

A straightforward calculation gives

$$\rho(E_0) = \frac{1}{(2\pi\hbar c)^3} \frac{1}{8E_0^3} \left\{\left[E_0^2 + (m_\pi^2 - m_c^2)c^4\right]^2 - 4E_0^2 m_\pi^2 c^4\right\}^{1/2} \left[E_0^4 - (m_\pi^2 - m_c^2)^2 c^8\right] \tag{12}$$

And if we take proper coordinates for the π-meson, E_0 becomes the rest energy of the π-meson.

We now substitute the plane wave expansions for the particle wave functions into the Hamiltonian matrix elements for this transition, simplify and use proper coordinates for the π-meson. Substituting the result of this calculation together with equation (12) into equation (11), we then find the following values for the proper lifetime of the π-meson (Appendices 1 to 7):

$$(a)\ \frac{1}{\tau_{01}} = \left\{ |K_v^{3/2} g_1|^2 \frac{2m_v^2 + (m_\mu + m_c)^2}{3m_v^2} + 2K_v^3\, \mathrm{Rl}\, f_1 g_1^* \frac{m_\mu + m_c}{m_v} + |K_v^{3/2} f_1|^2 \frac{m_v^2 + 2(m_\mu + m_c)^2}{3m_v^2} \right\}$$

$$\frac{1}{16\pi} \frac{m_v c^2}{\hbar} \left[1 - \left(\frac{m_\mu}{m_v} + \frac{m_c}{m_v}\right)^2\right]^{1/2} \left[1 - \left(\frac{m_\mu}{m_v} - \frac{m_c}{m_v}\right)^2\right]^{3/2}$$

$$(b)\ \frac{1}{\tau_{02}} = \frac{|K_s^{3/2} g_2|^2}{16\pi} \frac{m_s c^2}{\hbar} \left[1 - \left(\frac{m_\mu}{m_s} + \frac{m_c}{m_s}\right)^2\right]^{3/2} \left[1 - \left(\frac{m_\mu}{m_s} - \frac{m_c}{m_s}\right)^2\right]^{1/2}$$

$$(c)\ \frac{1}{\tau_{03}} = \frac{|K_p^{3/2} f_3|^2}{16\pi} \frac{m_p c^2}{\hbar} \left[1 - \left(\frac{m_\mu}{m_p} + \frac{m_c}{m_p}\right)^2\right]^{1/2} \left[1 - \left(\frac{m_\mu}{m_p} - \frac{m_c}{m_p}\right)^2\right]^{3/2}$$

$$(d)\ \frac{1}{\tau_{04}} = \frac{1}{\tau_{06}} = \frac{|K_v^{3/2} g_4|^2}{384\pi} \frac{m_v c^2}{\hbar} \frac{m_\mu}{m_v} \frac{m_c}{m_v} \left[1 - \left(\frac{m_\mu}{m_v} + \frac{m_c}{m_v}\right)^2\right]^{3/2} \left[1 - \left(\frac{m_\mu}{m_v} - \frac{m_c}{m_v}\right)^2\right]^{3/2}$$

$$(e)\ \frac{1}{\tau_{05}} = \frac{1}{\tau_{07}} = \frac{|K_s^{3/2} g_5|^2}{32\pi} \frac{m_s c^2}{\hbar} \frac{m_\mu}{m_s} \frac{m_c}{m_s} \left[1 - \left(\frac{m_\mu}{m_s} + \frac{m_c}{m_s}\right)^2\right]^{1/2} \left[1 - \left(\frac{m_\mu}{m_s} - \frac{m_c}{m_s}\right)^2\right]^{1/2} \qquad (13)$$

The differences between the exponents on the brackets in $1/\tau_{04}$ and $1/\tau_{05}$ arises from the velocity dependent coupling in the vector case, and this latter fact combined with the angular correlations from the spin of the vector meson accounts for the different numerical coefficient. The small differences in the first three cases can be accounted for by the differences in the couplings.

V LIFETIME FOR NUCLEAR CAPTURE OF THE μ-MESON

The detailed mechanism for capture of the μ-meson has already been explained in Part II. Quantitatively we can express the lifetime of this process by

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \int_{\Omega} \sum_{\text{spin}} |H_{AF}|^2 \rho(E_0) V \tag{14}$$

For the density of states $\rho(E_0)$ we have in all cases

$$\rho(E_0) = \frac{1}{(2\pi\hbar c)^3} \hbar c k_n E_n \tag{15}$$

where E_n is the energy of the neutral particle, hk_n is its momentum.

In all cases except those involving the vector π-meson, the Hamiltonian contains only matrix elements for the capture process by means of an intermediate state; in the case of the vector field one finds also elements for the direct transition. Thus one has two possible intermediate states for the capture process and in addition for the vector fields one has a direct transition. The evaluation of the matrix elements is straightforward except that certain simplifying assumptions have been made. The first of these is that the energy imparted to the nucleus is small compared to the rest energy of the π-meson. A simple calculation based on the energy imparted to a free nucleon by a μ-meson with momentum equal to its mean momentum in the K-orbit of the atom indicates that this is justifiable. Further support for this lies in the absence of stars accompanying the capture process. The second assumption is that plane wave functions may be used for the π-meson in the virtual state. The justification for this lies in the fact that momenta of the virtual π-meson up to its rest mass times c contribute about equally to the process. Since the bound states of the meson are only a small fraction of these momenta, we can neglect them without significant error. Finally, the third assumption was that the distance of the μ-meson in the K-orbit of the capturing nucleus from the center of the nucleus was large compared to the radius of the nucleus. For light elements, $Z<10$, this is a very good approximation, the radius of the K-orbit for such atoms being about ten times the nuclear radius.

The result of the computation of H_{AF} combined with equations (14) and (15) give the following lifetimes (Appendices 1 to 7):

$$(a)\ \frac{1}{\tau_1}=\frac{1}{4\pi}\frac{\hbar c k_m}{\hbar}\frac{E_m}{\hbar c K_v}\left[\frac{K_v^2}{k_m^2+K_v^2}\right]^2\frac{|\psi_c(0)|^2}{K_v^3}\Bigg\{\Bigg[|K_v^{3/2}r_v|^2\,\frac{E_m-\hbar c k_m}{E_m}\left|\sum\int dV\,\Phi_N^*\,\frac{\tau_x+i\tau_y}{2}\,\Psi_N e^{-i\bar{k}_m\cdot\bar{x}}\right|^2$$

$$+|K_v^{3/2}s_v|^2\,\frac{E_m+\hbar c k_m}{E_m}\,\frac{2}{3}\left(\frac{k_m}{K_v}\right)^2\left|\sum\int dV\,\Phi_N^*\,\frac{\tau_x+i\tau_y}{2}\,\Psi_N e^{-i\bar{k}_m\cdot\bar{x}}\right|^2\Bigg]|K_v^{3/2}g_1|^2$$

$$+\Bigg[|K_v^{3/2}r_v|^2\,\frac{E_m+\hbar c k_m}{E}\left|\sum\int dV\,\Phi_N^*\,\frac{\tau_x+i\tau_y}{2}\,\Psi_N e^{-i\bar{k}_m\cdot\bar{x}}\right|^2$$

$$+|K_v^{3/2}s_v|^2\,\frac{E_m-\hbar c k_m}{E_m}\,\frac{2}{9}\left(\frac{k_m}{K_v}\right)^2\left|\sum\int dV\,\Phi_N^*\,\frac{\tau_x+i\tau_y}{2}\,\Psi_N e^{-i\bar{k}_m\cdot\bar{x}}\right|^2\Bigg]\left(\frac{k_m}{K_v}\right)^2|K_v^{3/2}f_1|^2\Bigg\}$$

$$(b)\ \frac{1}{\tau_2} = \frac{|K_s^{3/2} g_2|^2 |K_s^{3/2} r_s|^2}{4\pi} \frac{\hbar c k_n}{\hbar} \frac{E_n}{\hbar c K_s} \left[\frac{E_n - \hbar c K_n}{E_n}\right] \left[\frac{K_s^2}{k_n^2 + K_s^2}\right]^2 \frac{|\psi_c(0)|^2}{K_s^3} \left|\sum \int dv\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \Psi_N e^{-i\bar{k}_n\cdot\bar{x}}\right|^2$$

$$(c)\ \frac{1}{\tau_3} = \frac{|K_p^{3/2} f_3|^2 |K_p^{3/2} r_p|^2}{48\pi} \frac{\hbar c k_n}{\hbar} \frac{E_n}{\hbar c K_p} \left(\frac{k_n}{K_N}\right)^2 \left[\frac{E_n + \hbar c K_n}{E_n}\right] \left[\frac{K_p^2}{k_n^2 + K_p^2}\right]^2 \frac{|\psi_c(0)|^2}{K_p^3} \left|\sum \int dv\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \sigma \Psi_N e^{-i\bar{k}_n\cdot\bar{x}}\right|^2$$

$$(d)\ \frac{1}{\tau_4} = \frac{1}{\tau_6} = \frac{|K_v^{3/2} g_4|^2 |K_v^{3/2} r_v|^2}{4\pi} \frac{\hbar c k_n}{\hbar} \left(\frac{E_n}{\hbar c K_v}\right)^2 \frac{K_n}{K_v} \left[\frac{K_v^2}{k_n^2 + K_v^2}\right]^2 \frac{|\psi_c(0)|^2}{K_v^3} \left|\sum \int dv\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \Psi_N e^{-i\bar{k}_n\cdot\bar{x}}\right|^2$$

$$(e)\ \frac{1}{\tau_5} = \frac{1}{\tau_7} = \frac{|K_s^{3/2} g_5|^2 |K_s^{3/2} r_s|^2}{4\pi} \frac{\hbar c k_n}{\hbar} \frac{K_n}{K_s} \left[\frac{K_s^2}{k_n^2 + K_s^2}\right]^2 \frac{|\psi_c(0)|^2}{K_s^3} \left|\sum \int dv\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \Psi_N e^{-i\bar{k}_n\cdot\bar{x}}\right|^2 \qquad (16)$$

In these expressions $\psi_c(0)$ is the wave function of the μ-meson evaluated at the center of the nucleus. The summation in the nuclear matrix elements is to be taken over all protons in the nucleus. In $1/\tau_3$, $K_N = \frac{m_N c}{\hbar}$ where m_N is the mass of the nucleon. We note that all these expressions are quite similar. The complexity in $1/\tau_1$ results from the two independent nuclear interactions and the two independent meson couplings; the factors of k_n^2 arise from the velocity dependence of two of these interactions. $1/\tau_4 = 1/\tau_6$ has the distinctive factor E_n^2 which also arises from the velocity dependence of the interaction. $1/\tau_3$ has an additional factor k_n^2 which is due to the fact that, as in the theory of β-disintegration, the pseudoscalar nuclear matrix element is small.

VI NUMERICAL RESULTS

For the determination of the lifetime of the π-meson use was made of the experimental results that the mass of μ-meson is 212 electron masses and the lifetime for nuclear capture of the μ-meson in material with Z=10 is 3.3×10^{-6} seconds. It is further necessary to determine the excitation of the nucleus after capture of the μ-meson. However, it is clear from equation (16) that the probability of capture is not strongly affected by the amount of this excitation. A reasonable estimate based principally on the absence of stars accompanying the capture of the μ-meson is about 10 Mev. Moreover, even if this estimate is off by say a factor of two as seems to be the maximum possible error, one will see that no serious errors are thereby introduced. For the wave function of the μ-meson in the K-orbit of the capturing atom we use the ordinary hydrogenic Schrödinger function.

The principal problem associated with the evaluations of the lifetimes is the determination of the nuclear matrix elements. The exact values of these elements can be shown to be less than Z. The precise value however is rather difficult to obtain and for our purposes we shall find it adequate to take the maximum value of Z. In so doing we probably are not making a greater error, at least for light nuclei, than our other approximations warrant. We observe that as a result of this assumption and the assumed form ψ_μ for the μ-meson, the capture probability of the μ-meson depends directly on the fourth power of Z as first remarked by

Wheeler.[24]

The nuclear-meson coupling constants r and s correspond to the usual notation as follows:

$$\frac{|K^{3/2} r|^2}{4\pi} \rightarrow \frac{|g|^2}{\hbar c} \qquad \frac{|K^{3/2} s|^2}{4\pi} \rightarrow \frac{|f|^2}{\hbar c}$$

Hence, from the meson theory of nuclear forces we have:

$$|K_v^{3/2} r_v|^2 = \frac{4\pi}{5} \qquad |K_v^{3/2} s_v|^2 = \frac{4\pi}{5}$$

$$|K_s^{3/2} r_s|^2 = \frac{4\pi}{10}$$

$$|K_p^{3/2} r_p|^2 = \frac{4\pi}{10}\left(\frac{2K_N}{K_p}\right)^2$$

With the aid of the above numbers, the proper lifetime of the π-meson as a function of its mass was determined and plotted in Figure 1. In the calculation of τ_{01} the value of the ratio of f_1/g_1 which makes τ_{01} a maximum was chosen; this occurs for $f_1/g_1 = 0$, that is, $f_1 = 0$.

Before we can make any definitive assertions concerning these results, we must obtain some notion of the uncertainties in these numbers. There are three principal sources of uncertainty. The first is the evaluation of the nuclear matrix elements. Estimates made by Wheeler[25] and Christy[26] indicate that these elements for $Z \sim 10$ are probably greater than $Z/3$. The excitation energy of the nucleus is a second source of error. As remarked previously, it seems very unlikely that this can be off by more than a factor of two.

The third significant uncertainty is the nuclear coupling constants. These could easily be incorrect to a factor of two and possibly even more. The combination of these errors make it quite possible for the above computed values for τ_0 to be too small by a factor of about three or too large by a factor of about ten.

From the results of the Berkeley experiments on the artificially produced mesons, one can say that the lifetime of the π-meson is slightly greater than 10^{-8} seconds. Thus it appears unlikely that the vector-scalar or pseudoscalar or the pseudoscalar-spin $\frac{1}{2}$ fields suitably represent the π- and μ-mesons. Further from the ease of nuclear meson production a vector π-meson seems doubtful, leaving as the most probable fields the scalar-scalar or pseudoscalar or the scalar-spin $\frac{1}{2}$ fields.

Subsequent to the above calculations more precise information on the masses of the π-and μ-mesons and the value of the nuclear coupling constant has become available. For the sake of completeness, the effect of these recent experimental results will be included. The new data give for the mass of the π-meson 283 electron masses, for the mass of the μ-meson 215 electron masses, for the energy of the μ-meson in the decay of the π-meson 4.05 Mev, and for the nuclear coupling constants

$$|K_\sigma^{3/2}\Omega_\sigma|^2 = |K_\sigma^{3/2}S_\sigma|^2 = |K_s^{3/2}\Omega_s|^2 = \left(\frac{K_P}{2K_N}\right)^2 |K_p^{3/2}\Omega_p|^2 = \frac{4\pi}{3}$$

On the basis of these numbers it is found that:

(a) $\tau_{01} = 1.88\times10^{-8}$ seconds

(b) $\tau_{02} = 1.22\times10^{-8}$ seconds

(c) $\tau_{03} = 1.94\times10^{-9}$ seconds

(d) $\tau_{04} = \tau_{06} = 6.31\times10^{-7}$ seconds

(e) $\tau_{05} = \tau_{07} = 1.97\times10^{-8}$ seconds

where the more recent estimate of $\frac{2}{3}$ for the nuclear matrix element has been used. One implication of the new data is that the mass of the neutral secondary particle is zero; and as a result it seems reasonable, in order to avoid introducing a new kind of particle, to identify the neutral particle with a neutrino with the result that the μ-meson is necessarily spin $\frac{1}{2}$.(27)(28) Combining this new information with the above conclusion, it seems that the π- and μ-mesons are most probably scalar and spin $\frac{1}{2}$ fields, respectively, though the pseudoscalar π-meson is not impossible as a result of the uncertainties in the theory of the nuclear forces.

VII THEORETICAL POSSIBILITY OF A PHOTON "NEUTRAL PARTICLE"

A calculation similar to the above has been performed on a vector-scalar-photon field to determine whether a photon decay particle is theoretically consistent with the present data on the lifetime of the π-meson. The coupling between the fields was specified by the Lagrangian density:

$$L = -\hbar c \kappa_v \left[f \varphi_v^{\alpha *} \left(\partial_\alpha - \frac{ie}{\hbar c} A_\alpha \right) \varphi_s + c.c. \right]$$

Assuming the mass of the π-meson to be 283 electron masses, the mass of the μ-meson to be 215 electron masses, and the nuclear matrix element equal to $Z/3$, the lifetime is (Appendix 8) $\tau_0 = 4.59 \times 10^{-7}$ seconds. This is far too long compared with the experimental value. Thus one can disregard the possibility of decay photons in agreement with experiment.

VIII DISCUSSION

The evidence available at present seems to leave little doubt that one has to deal with a two-meson field. There does not yet appear to be sufficiently accurate experimental data to fix with certainty the character of the two-meson field though the indication is that the π-meson is most probably a scalar field and the μ-meson and neutral particle are spin $\frac{1}{2}$ fields.

APPENDIX 1 VECTOR-SPIN $\frac{1}{2}$ FIELD

Formulation of the Field

The Lagrangian density of the field is

$$L = L_{VECTOR} + L_{CDIRAC} + L_{MDIRAC} + L_1 + L'_{VECTOR} \tag{17}$$

where these terms are defined in equations (2), (4), (5), and (8). The canonical momenta are by definition

$$p(\varphi_i) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_i}{\partial t}\right)} = \hbar \zeta^*_{0i} = \hbar\left(\chi^*_{0i} + \xi^*_v \mu^{(N)*}_{0i} + f^*_1 \mu^{(M)*}_{0i}\right)$$

$$p(\psi_{c\alpha}) = \frac{\partial L}{\partial\left(\frac{\partial \psi_{c\alpha}}{\partial t}\right)} = i\hbar\psi^*_{c\alpha}$$

$$p(\psi_{m\alpha}) = \frac{\partial L}{\partial\left(\frac{\partial \psi_{m\alpha}}{\partial t}\right)} = i\hbar\psi^*_{m\alpha} \tag{18}$$

where i=1, 2, 3 and α = 1, 2, 3, 4. The momentum $p(\varphi_0)$ is zero and this causes difficulty in the quantization of the field. The procedure in this case is to obtain the Hamiltonian and then eliminate φ_0 from it, thus effectively leaving three coordinate functions and three momenta for the vector field. One uses for this the equation of motion for φ_0 which is

$$\frac{\partial L}{\partial \varphi_0^*} - \sum_\alpha \frac{\partial}{\partial x^\alpha}\frac{\partial L}{\partial\left(\frac{\partial \varphi_0^*}{\partial x^\alpha}\right)} = 0$$

or

$$K_v \eta_0 = -\frac{\partial \zeta_{0i}}{\partial x^i} = K_v\left(\varphi_0 + r_v v_0^{(N)} + g_1 v_0^{(M)}\right) \tag{19}$$

Now by definition the Hamiltonian density is

$$H_1 = \sum_i p(\varphi_i)\frac{\partial \varphi_i}{\partial t} + \sum_\alpha p(\psi_{c\alpha})\frac{\partial \psi_{c\alpha}}{\partial t} + \sum_\alpha p(\psi_{m\alpha})\frac{\partial \psi_{m\alpha}}{\partial t} - L \tag{20}$$

Thus substituting equations (17), (18), and (19) into equation (20) we find

$$\begin{aligned}
H_1 = {} & \hbar c\kappa_0\left[\varphi_i^*\varphi_i + \eta_0^*\eta_0 + \zeta_{0i}^*\zeta_{0i} + \tfrac{1}{2}\chi_{ik}^*\chi_{ik}\right] + H_{c\,DIRAC} + H_{m\,DIRAC} \\
& + \hbar c\kappa_0\left[r_0\left(v_i^{(N)}\varphi_i^* - v_0^{(N)}\eta_0^*\right) - s_0\left(\mu_{0i}^{(N)}\zeta_{0i}^* - \tfrac{1}{2}\mu_{ik}^{(N)}\chi_{ik}^*\right) + c.c.\right] \\
& + \hbar c\kappa_0\left[g_1\left(v_i^{(M)}\varphi_i^* - v_0^{(M)}\eta_0^*\right) - f_1\left(\mu_{0i}^{(M)}\zeta_{0i}^* - \tfrac{1}{2}\mu_{ik}^{(M)}\chi_{ik}^*\right) + c.c.\right] \\
& + \hbar c\kappa_0\left[|s_0|^2\mu_{0i}^{(N)*}\mu_{0i}^{(N)} + |r_0|^2|v_0^{(N)}|^2\right] \\
& + \hbar c\kappa_0\left[|f_1|^2|\mu_{0i}^{(M)}|^2 + |g_1|^2|v_0^{(M)}|^2\right] \\
& + \hbar c\kappa_0\left[f_1^* s_0\mu_{0i}^{(M)*}\mu_{0i}^{(N)} + g_1^* r_0 v_0^{(M)*}v_0^{(N)} + c.c.\right]
\end{aligned} \tag{21}$$

Changing to vector notation we have finally

$$\begin{aligned}
H_1 = {} & \hbar c\kappa_0\left[|\pi|^2 + |\psi|^2 + \tfrac{1}{\kappa_0^2}|\mathrm{curl}\,\psi|^2 + |\Pi|^2 + |\Psi|^2 + \tfrac{1}{\kappa_0^2}|\mathrm{div}\,\Psi|^2\right] + H_{c\,DIRAC} + H_{m\,DIRAC} \\
& + \hbar c\kappa_0\Big\{\Psi_N^*\frac{\tau_x + i\tau_y}{2}\Big[r_0\bar{\alpha}\cdot\psi^* - s_0\left(\bar{\gamma}\cdot\pi - \tfrac{1}{\kappa_0}\beta\bar{\sigma}\cdot\mathrm{curl}\,\psi^*\right) \\
& \qquad + s_0\bar{\gamma}\cdot\Psi^* + r_0\left(\bar{\alpha}\cdot\Pi + \tfrac{1}{\kappa_0}\mathrm{div}\,\Psi^*\right)\Big]\Psi_N + c.c.\Big\} \\
& + \hbar c\kappa_0\Big\{\tilde{\Psi}_m C\Big[g_1\bar{\alpha}\cdot\psi^* - f_1\left(\bar{\gamma}\cdot\pi - \tfrac{1}{\kappa_0}\beta\bar{\sigma}\cdot\mathrm{curl}\,\psi^*\right)
\end{aligned}$$

$$+ f_1 \bar{\gamma}\cdot\Psi^* + g_1(\bar{\alpha}\cdot\Pi + 1/\kappa_\nu \operatorname{div}\Psi^*)\Big]\psi_c + \text{c.c.}\Big\}$$

$$+\hbar c\kappa_\nu\Big[|s_\nu|^2\Big|\Phi_N^* \frac{\tau_x+i\tau_y}{2}\bar{\gamma}\,\Psi_N\Big|^2 + |r_\nu|^2\Big|\Phi_N^* \frac{\tau_x+i\tau_y}{2}\Psi_N\Big|^2\Big]$$

$$+\hbar c\kappa_\nu\Big[|f_1|^2|\tilde{\psi}_m C\bar{\gamma}\psi_c|^2 + |g_1|^2|\tilde{\psi}_m C\psi_c|^2\Big]$$

$$+\hbar c\kappa_\nu\Big[f_1^* s_\nu(\tilde{\psi}_c C\bar{\gamma}\psi_m)\cdot\Big(\Phi_N^*\frac{\tau_x+i\tau_y}{2}\bar{\gamma}\Psi_N\Big) + g_1^* r_\nu(\tilde{\psi}_c C\psi_m)\Big(\Phi_N^*\frac{\tau_x+i\tau_y}{2}\Psi_N\Big) + \text{c.c.}\Big] \tag{22}$$

Lifetime of the Vector Meson

The decay of the vector π-meson takes place by the reaction

$$\pi^- \rightarrow \mu^- + \mu^0$$

The lifetime of this process is

$$\frac{1}{\tau_{01}} = \frac{2\pi}{\hbar}\int_{\Omega_c}\sum_{\text{spins}}|H_{AF}|^2\frac{P_c^2}{(2\pi\hbar)^3}\frac{dP_c}{dE_F}\,d\Omega_c\,V \tag{23}$$

where the naught on τ indicates that proper coordinates for the vector π-meson are to be used for the calculation. The integration is taken over all relative directions of the light spin $\frac{1}{2}$ mesons and the sum is over the spins of the spin $\frac{1}{2}$ mesons.

The element of H_1 giving rise to the decay process is the coefficient in H_1 of

$$b_{\nu k_\nu s}\, b^*_{c k_c s}\, a^*_{m k_m r}$$

wherein plane waves are assumed for the wave functions with $\bar{k}_\nu = 0$ and thus $\bar{k}_c + \bar{k}_n = 0$. This element is

$$H_{AF} = \frac{\hbar c K_\nu}{2V^{1/2}} \tilde{P}_{m k_m n} \bar{C} \left(i g_1 \bar{\alpha} \cdot \bar{J}_{\nu k_\nu \epsilon} - f_1 \bar{\gamma} \cdot \bar{J}_{\nu k_\nu \epsilon} \right) \tilde{P}_{c k_c s}$$

By Casimir's spur technique we have

$$\sum_{\text{SPIN}} |H_{AF}|^2 = -\frac{(\hbar c K_\nu)^2}{4V} \operatorname{Spur} \left[\left(i g_1^* \bar{\alpha} \cdot \bar{J}_{\nu k_\nu \epsilon} + f_1^* \bar{\gamma} \cdot \bar{J}_{\nu k_\nu \epsilon} \right) \left(\frac{\hbar c (\bar{\alpha} \cdot \bar{k}_m + \beta K_m) + E_{m k_m}}{2 E_{m k_m}} \right) \left(i g_1 \bar{\alpha} \cdot \bar{J}_{\nu k_\nu \epsilon} - f_1 \bar{\gamma} \cdot \bar{J}_{\nu k_\nu \epsilon} \right) \left(\frac{\hbar c (\bar{\alpha} \cdot \bar{k}_c - \beta K_c) + E_{c k_c}}{2 E_{c k_c}} \right) \right]$$

A calculation of this spur gives

$$\begin{aligned} \sum_{\text{SPIN}} |H_{AF}|^2 = \frac{(\hbar c K_\nu)^2}{4V} \frac{1}{E_{m k_m} E_{c k_c}} \Big\{ & |g_1|^2 \left[\hbar^2 c^2 \left(k_c^2 - 2 (\bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_c)^2 \right) + \hbar^2 c^2 K_m K_c + E_{m k_m} E_{c k_c} \right] \\ & + |f_1|^2 \left[-\hbar^2 c^2 \left(k_c^2 - 2 (\bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_c)^2 \right) + \hbar^2 c^2 K_m K_c + E_{m k_m} E_{c k_c} \right] \\ & + 2 \mathrm{Rl}\, g_1^* f_1 \left[K_m E_{c k_c} + K_c E_{m k_m} \right] \hbar c \Big\} \end{aligned}$$

Then it follows that

$$\begin{aligned} \int_{\Omega_c} \sum_{\text{SPIN}} |H_{AF}|^2 d\Omega_c = \frac{(\hbar c K_\nu)^2}{V} \frac{\pi}{E_{m k_m} E_{c k_c}} \Big\{ & |g_1|^2 \left[\hbar^2 c^2 \left(k_c^2/3 + K_m K_c \right) + E_{m k_m} E_{c k_c} \right] \\ & + |f_1|^2 \left[\hbar^2 c^2 \left(-k_c^2/3 + K_m K_c \right) + E_{m k_m} E_{c k_c} \right] \\ & + 2 \hbar c \, \mathrm{Rl}\, g_1^* f_1 \left[K_m E_{c k_c} + K_c E_{m k_m} \right] \Big\} \end{aligned} \tag{24}$$

Using

$$\bar{k}_c + \bar{k}_m = 0$$

and

$$E_{ck_c} + E_{mk_m} = E_F = m_v c^2$$

we find after some algebra that

$$\hbar^2 c^2\left(\frac{k_c^2}{3} + K_m K_c\right) + E_{mk_m} E_{ck_c} = \frac{1}{6E_F^2}\left[E_F^2 - (m_c - m_m)^2 c^4\right]\left[2E_F^2 + (m_m + m_c)^2 c^4\right]$$

$$\hbar^2 c^2\left(-\frac{k_c^2}{3} + K_m K_c\right) + E_{mk_m} E_{ck_c} = \frac{1}{6E_F^2}\left[E_F^2 - (m_c - m_m)^2 c^4\right]\left[E_F^2 + 2(m_m + m_c)^2 c^4\right]$$

$$\hbar c K_m E_{ck_c} + \hbar c K_c E_{mk_m} = \frac{K_m + K_c}{K_v} \quad \frac{E_F^2 - (m_c - m_m)^2 c^4}{2}$$

$$E_{ck_c} E_{mk_m} = \frac{E_F^4 - (m_c^2 - m_m^2)^2 c^8}{4E_F^2}$$

and also that

$$P_c = \frac{1}{2cE_F}\left[E_F^2 - (m_m + m_c)^2 c^4\right]^{1/2}\left[E_F^2 - (m_m - m_c)^2 c^4\right]^{1/2}$$

$$\frac{dP_c}{dE_F} = \frac{1}{2cE_F^2}\,\frac{\left[E_F^2 + (m_m^2 - m_c^2)c^4\right]\left[E_F^2 - (m_m^2 - m_c^2)c^4\right]}{\left[E_F^2 - (m_m + m_c)^2 c^4\right]^{1/2}\left[E_F^2 - (m_m - m_c)^2 c^4\right]^{1/2}} \qquad (25)$$

Substituting equations (24) and (25) into equation (23), we have the result

$$\frac{1}{\tau_{01}} = \left\{|K_v^{3/2} g_1|^2 \frac{2m_v^2 + (m_m + m_c)^2}{3m_v^2} + 2K_v^3 \mathrm{Re} f_1 g_1^* \frac{m_m + m_c}{m_v} + |K_v^{3/2} f_1|^2 \frac{m_v^2 + 2(m_m + m_c)^2}{3m_v^2}\right\}$$

$$\frac{1}{16\pi}\frac{m_v c^2}{\hbar}\left[1 - \left(\frac{m_m}{m_v} + \frac{m_c}{m_v}\right)^2\right]^{1/2}\left[1 - \left(\frac{m_m}{m_v} - \frac{m_c}{m_v}\right)^2\right]^{3/2}$$

Lifetime for the Capture of the Spin ½ Meson by a Nucleus

The capture of a negative spin ½ meson takes place by one of the three reactions:

$$\text{I} \quad \mu^- \rightarrow \left\{ \begin{matrix} \pi^- + \mu^0 \\ \pi^- + P \end{matrix} \right\} \rightarrow N$$

$$\text{II} \quad P \rightarrow \left\{ \begin{matrix} \pi^+ + N \\ \pi^+ + \mu^- \end{matrix} \right\} \rightarrow \mu^0$$

$$\text{III} \quad P + \mu^- \rightarrow N + \mu^0$$

The lifetime of this capture process is

$$\frac{1}{\tau_1} = \frac{2\pi}{\hbar} \sum_{\text{SPIN}} \int_{\Omega_m} |H_{AF}|^2_{\text{AV.}} \frac{P_m^2}{(2\pi\hbar)^3} \frac{dP_m}{dE_F} \, d\Omega_m V \tag{26}$$

where the averaging is over the spin directions of the initial particles, the sum is over the spin of the neutral particle, and the integration over all directions of the neutral particle. According to the quantum perturbation theory we have

$$H_{AF} = \sum \left[\frac{H_{AI} H_{IF}}{E_A - E_I} + \frac{H_{AII} H_{IIF}}{E_A - E_{II}} \right] + H_{AFIII} \tag{27}$$

To obtain H_{AF} we use plane waves for the emergent neutral meson and we approximate the wave function of the virtual vector π-meson by plane waves. With these wave functions substituted into H_1 we find that the element of H_1 giving (a) H_{AI} is the coefficient of

$$b_c\, b^*_{\nu k_\nu \epsilon}\, a^*_{M k_M s} \qquad \epsilon = 1, 0, -1$$

(b) H_{IF} is the coefficient of

$$b_{\nu k_\nu \epsilon}\, \frac{\tau_x + i\tau_y}{2} \qquad \epsilon = 1, 0, -1$$

(c) H_{AII} is the coefficient of

$$a^*_{\nu k_\nu \epsilon}\, \frac{\tau_x + i\tau_y}{2} \qquad \epsilon = 1, 0, -1$$

(d) H_{IIF} is the coefficient of

$$b_c\, a_{\nu k_\nu \epsilon}\, a^*_{M k_M s} \qquad \epsilon = 1, 0, -1$$

(e) H_{AFIII} is the coefficient of

$$b_c\, a^*_{M k_M s}\, \frac{\tau_x + i\tau_y}{2}$$

These elements are

$$\text{(a)}\ H^\epsilon_{AI} = \frac{\hbar c K_\nu}{V}\frac{1}{\sqrt{2}}\int dV \left\{ \tilde{\psi}_c C\left[g^*_{d_1}(-i)\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{\alpha}\cdot\bar{J}_{\nu k_\nu \epsilon} - f^*_1\left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\,\bar{\gamma}\cdot\bar{J}_{\nu k_\nu \epsilon} - \frac{1}{K_\nu}\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\beta\bar{\sigma}\cdot\bar{J}_{\nu k_\nu \epsilon}\times\bar{k}_\nu\right)\right]\tilde{P}_{M k_M s}\right\} e^{-i(\bar{k}_M+\bar{k}_\nu)\cdot\bar{x}}$$

$$H^0_{AI} = \frac{\hbar c K_\nu}{V}\frac{1}{\sqrt{2}}\int dV \left\{ \tilde{\psi}_c C\left[f^*_{d_1}(-i)\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{\gamma}\cdot\bar{J}_{\nu k_\nu 0} + g^*_{d_1}\left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\,\bar{\alpha}\cdot\bar{J}_{\nu k_\nu 0} - \frac{1}{K_\nu}\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{J}_{\nu k_\nu 0}\cdot\bar{k}_\nu\right)\right]\tilde{P}_{M k_M s}\right\} e^{-i(\bar{k}_M+\bar{k}_\nu)\cdot\bar{x}}$$

$$\text{(b)}\ H^\epsilon_{IF} = \frac{\hbar c K_\nu}{V^{1/2}}\int dV \left\{ \Phi^*_N \frac{\tau_x+i\tau_y}{2}\left[r_\nu i\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{\alpha}\cdot\bar{J}_{\nu k_\nu \epsilon} - S_\nu\left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\,\bar{\gamma}\cdot\bar{J}_{\nu k_\nu \epsilon} - \frac{1}{K_\nu}\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\beta\bar{\sigma}\cdot\bar{J}_{\nu k_\nu \epsilon}\times\bar{k}_\nu\right)\right]\Psi_N \right\} e^{i\bar{k}_\nu\cdot\bar{x}}$$

$$H^0_{IF} = \frac{\hbar c K_\nu}{V^{1/2}}\int dV \left\{ \Phi^*_N \frac{\tau_x+i\tau_y}{2}\left[S_\nu i\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{\gamma}\cdot\bar{J}_{\nu k_\nu 0} + r_\nu\left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\,\bar{\alpha}\cdot\bar{J}_{\nu k_\nu 0} - \frac{1}{K_\nu}\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{J}_{\nu k_\nu 0}\cdot\bar{k}_\nu\right)\right]\Psi_N \right\} e^{i\bar{k}_\nu\cdot\bar{x}}$$

$$\text{(c)}\ H^\epsilon_{AII} = \frac{\hbar c K_\nu}{V^{1/2}}\int dV \left\{ \Phi^*_N \frac{\tau_x+i\tau_y}{2}\left[r_\nu(-i)\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\bar{\alpha}\cdot\bar{J}_{\nu k_\nu \epsilon} - S_\nu\left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\,\bar{\gamma}\cdot\bar{J}_{\nu k_\nu \epsilon} - \frac{1}{K_\nu}\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\,\beta\bar{\sigma}\cdot\bar{J}_{\nu k_\nu \epsilon}\times\bar{k}_\nu\right)\right]\Psi_N \right\} e^{-i\bar{k}_\nu\cdot\bar{x}}$$

$$H^{0}_{A\,II} = \frac{\hbar c K_\nu}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[S_\nu \left(-i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\gamma} \cdot \bar{J}_{\nu k_\nu 0} \right) + \pi_\nu \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\alpha} \cdot \bar{J}_{\nu k_\nu 0} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{J}_{\nu k_\nu 0} \cdot \bar{k}_\nu \right) \right] \Psi_N \right\} e^{-i\bar{k}_\nu \cdot \bar{x}}$$

$$(d)\ H^{\epsilon}_{IIF} = \frac{\hbar c K_\nu}{V} \frac{1}{\sqrt{2}} \int dV \left\{ \tilde{\psi}_c C \left[g_1^* i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\alpha} \cdot \bar{J}_{\nu k_\nu \epsilon} - f_1^* \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\gamma} \cdot \bar{J}_{\nu k_\nu \epsilon} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \beta \bar{\sigma} \cdot \bar{J}_{\nu k_\nu \epsilon} \times \bar{k}_\nu \right) \right] \tilde{\varphi}_{m k_m s} \right\} e^{i(\bar{k}_\nu - \bar{k}_m) \cdot \bar{x}}$$

$$H^{0}_{IIF} = \frac{\hbar c K_\nu}{V} \frac{1}{\sqrt{2}} \int dV \left\{ \tilde{\psi}_c C \left[f_1^* i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\gamma} \cdot \bar{J}_{\nu k_\nu 0} + g_1^* \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\alpha} \cdot \bar{J}_{\nu k_\nu 0} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{J}_{\nu k_\nu 0} \cdot \bar{k}_\nu \right) \right] \tilde{\varphi}_{m k_m s} \right\} e^{i(\bar{k}_\nu - \bar{k}_m) \cdot \bar{x}}$$

$$(e)\ H_{AF\,III} = \frac{\hbar c K_\nu}{V^{1/2}} \frac{1}{\sqrt{2}} \int dV \left\{ S_\nu f_1^* (\tilde{\psi}_c C \bar{\gamma} \tilde{\varphi}_{m k_m s}) \cdot \left(\Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\gamma} \Psi_N\right) + \pi_\nu g_1^* (\tilde{\psi}_c C \tilde{\varphi}_{m k_m s}) \left(\Phi_N^* \frac{\tau_x + i\tau_y}{2} \Psi_N\right) \right\} e^{-i\bar{k}_m \cdot \bar{x}} \tag{28}$$

where the nuclear matrix elements contain implicitly a sum over all protons in the nucleus and $\epsilon = 1, -1$. Now we treat the nucleons non-relativistically. Thus we have

$$\Phi_N^* \frac{\tau_x + i\tau_y}{2} \left\{ \begin{matrix} \bar{\alpha} \\ \bar{\gamma} \end{matrix} \right\} \Psi_N \approx 0 \tag{29}$$

and

$$\Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \bar{\sigma} \Psi_N \approx \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \Psi_N \tag{30}$$

Also we assume that the energy imparted to the nucleons is negligible compared with the rest energy of the vector π-meson. Then

$$E_A - E_I = E_A - E_{II} \approx -E_{\nu k_\nu} \tag{31}$$

Substituting equations (28), (29), (30), and (31) into equation (27), we find

$$H_{AF} = \frac{(\hbar c K_\nu)^2}{V^{3/2}} \frac{1}{\sqrt{2}} \frac{S_\nu}{K_\nu} \sum_{k_\nu} \sum_{\epsilon = 1, -1} \int dV \int dV' \left\{ g_1^* i \frac{\hbar c K_\nu}{E^2_{\nu k_\nu}} (\bar{v}_m \cdot \bar{J}_{\nu k_\nu \epsilon})(\bar{t}'_N \times \bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_\nu) \right.$$

$$\left. - \frac{f_1^*}{K_\nu} \frac{\hbar c K_\nu}{E^2_{\nu k_\nu}} (\bar{\mu}_m \times \bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_\nu)(\bar{t}'_N \times \bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_\nu) \right\} e^{-i\bar{k}_m \cdot \bar{x} + i\bar{k}_\nu \cdot (\bar{x}' - \bar{x})}$$

$$+ \frac{(\hbar c K_\nu)^2}{V^{3/2}} \frac{1}{\sqrt{2}} \frac{\pi_\nu}{K_\nu} \sum_{k_\nu} \int dV \int dV' \Big\{ -f_1^* i \frac{\hbar c K_\nu}{E^2_{\nu k_\nu}} (\bar{\mu}_{o\mu} \cdot \bar{k}_\nu) v'_{oN}$$

$$- \frac{g_1^*}{K_\nu} \frac{\hbar c K_\nu}{E^2_{\nu k_\nu}} v_{o\mu} v'_{oN} k_\nu^2 \Big\} e^{-i\bar{k}_m \cdot \bar{x} + i\bar{k}_\nu \cdot (\bar{x}' - \bar{x})}$$

$$+ \frac{\hbar c K_\nu}{V^{1/2}} \frac{1}{\sqrt{2}} \pi_\nu g_1^* \int dV \, v_{o\mu} v_{oN} e^{-i\bar{k}_M \cdot \bar{x}} \tag{32}$$

where the primed and unprimed quantities refer, respectively, to the variables of V' and V. We now make use of the following identities:

$$\sum_{\epsilon=1,-1} (\bar{v}_\mu \cdot \bar{J}_{\nu k_\nu \epsilon})(\bar{t}'_N \cdot \bar{J}_{\nu k_\nu \epsilon} \times \bar{k}_\nu) = -\bar{v}_\mu \times \bar{t}'_N \cdot \bar{k}_\nu$$

$$\sum_{\epsilon=1,-1} (\bar{\mu}_\mu \times \bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_\nu)(\bar{t}'_N \times \bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_\nu) = (\bar{\mu}_\mu \cdot \bar{t}'_N) k_\nu^2 - (\bar{\mu}_\mu \cdot \bar{k}_\nu)(\bar{t}'_N \cdot \bar{k}_\nu)$$

$$\sum_{k_\nu} \frac{(\bar{v}_\mu \times \bar{t}'_N \cdot \bar{k}_\nu)}{E^2_{\nu k_\nu}} e^{i\bar{k}_\nu \cdot \bar{R}} = -i \frac{V}{4\pi} \frac{1}{\hbar^2 c^2} (\bar{v}_\mu \times \bar{t}'_N \cdot \mathrm{grad}) \frac{e^{-K_\nu R}}{R}$$

$$\sum_{k_\nu} \frac{k_\nu^2}{E^2_{\nu k_\nu}} e^{i\bar{k}_\nu \cdot \bar{R}} = -\frac{V}{4\pi} \frac{1}{\hbar^2 c^2} \nabla^2 \frac{e^{-K_\nu R}}{R}$$

$$\sum_{k_\nu} \frac{(\bar{\mu}_\mu \cdot \bar{k}_\nu)(\bar{t}'_N \cdot \bar{k}_\nu)}{E^2_{\nu k_\nu}} e^{i\bar{k}_\nu \cdot \bar{R}} = -\frac{V}{4\pi} \frac{1}{\hbar^2 c^2} (\bar{\mu}_\mu \cdot \mathrm{grad})(\bar{t}'_N \cdot \mathrm{grad}) \frac{e^{-K_\nu R}}{R}$$

$$\sum_{k_\nu} \frac{\bar{\mu}_{o\mu} \cdot \bar{k}_\nu}{E^2_{\nu k_\nu}} e^{i\bar{k}_\nu \cdot \bar{R}} = -i \frac{V}{4\pi} \frac{1}{\hbar^2 c^2} (\bar{\mu}_{o\mu} \cdot \mathrm{grad}) \frac{e^{-K_\nu R}}{R} \tag{33}$$

where $\bar{R} = \bar{x}' - \bar{x}$ and $R = |\bar{R}|$. We substitute these equations into equation (32). A rough estimate shows immediately that the integrand is appreciable only within a distance of the order $1/K_\nu$ from the nucleus. Since the wave function for the charged spin $\frac{1}{2}$ μ-meson does not vary more than about 10% over this range, we can approximate it by a constant and take this constant to be the value of the wave function at the center of the nucleus. We now perform an integration by parts on the variables of V in order to remove the derivatives in the integrand. The result is

$$H_{AF} = -\frac{(\hbar c K_\nu)^3}{V^{1/2}} \frac{1}{2^{5/2}\pi \hbar^2 c^2} \frac{S_\nu}{K_\nu} \int dV' e^{-i\bar{k}_m \cdot \bar{x}'} \int dV \left\{ -i g_1^* (\bar{v}_\mu(0) \times \bar{t}'_N \cdot \bar{k}_m) + \frac{f_1^*}{K_\nu} \frac{\left[(\bar{u}_\mu(0) \cdot \bar{t}'_N) k_m^2 - (\bar{u}_\mu(0) \cdot \bar{k}_m)(\bar{t}'_N \cdot \bar{k}_m) \right]}{4\pi \hbar^2 c^2} \right\} \frac{e^{-K_\nu R + i\bar{k}_m \cdot \bar{R}}}{R}$$

$$- \frac{(\hbar c K_\nu)^3}{V^{1/2}} \frac{1}{2^{5/2}\pi \hbar^2 c^2} \frac{r_\nu}{K_\nu} \int dV' e^{-i\bar{k}_m \cdot \bar{x}'} \int dV \left\{ -i f_1^* (\bar{u}_{o\mu}(0) \cdot \bar{k}_m) v'_{oN} + \frac{g_1^*}{K_\nu} \frac{v_{o\mu}(0) v_{oN}}{4\pi \hbar^2 c^2} k_m^2 \right\} \frac{e^{-K_\nu R + i\bar{k}_m \cdot \bar{R}}}{R}$$

$$+ \frac{\hbar c K_\nu}{V^{1/2}} \frac{1}{\sqrt{2}} r_\nu g_1^* \int dV \, v_{o\mu}(0) v_{oN} e^{-i\bar{k}_m \cdot \bar{x}} \tag{34}$$

where the argument zero indicates that the quantity is to be evaluated at the center of the nucleus. The integration over the variables of V is now easily performed in the first two integrals and gives

$$H_{AF} = \frac{\hbar c K_\nu}{V^{1/2}} \frac{1}{\sqrt{2}} \frac{K_\nu^2}{k_m^2 + K_\nu^2} \left\{ S_\nu \left[\frac{i g_1^*}{K_\nu} (\bar{k}_m \times \bar{v}_\mu(0)) - \frac{f_1^*}{K_\nu^2} (\bar{u}_\mu(0) k_m^2 - (\bar{u}_\mu(0) \cdot \bar{k}_m) \bar{k}_m) \right] \cdot \int dV \, \bar{t}_N e^{-i\bar{k}_m \cdot \bar{x}} \right.$$

$$\left. + r_\nu \left[i \frac{f_1^*}{K_\nu} (\bar{k}_m \cdot \bar{u}_{o\mu}(0)) - g_1^* v_{o\mu}(0) \right] \int dV \, v_{oN} e^{-i\bar{k}_m \cdot \bar{x}} \right\}$$

$$+ \frac{\hbar c K_\nu}{V^{1/2}} \frac{1}{\sqrt{2}} r_\nu g_1^* \int dV \, v_{o\mu}(0) v_{oN} e^{-i\bar{k}_m \cdot \bar{x}}$$

where the primes have been omitted. From this follows that

$$\int_{\Omega_m} |H_{AF}|^2_{Av.}\, d\Omega_m = \frac{(\hbar c K_\nu)^2}{V} 2\pi \left[\frac{K_\nu^2}{k_m^2+K_\nu^2}\right]^2 \Big\{ |g_1|^2 \Big[|S_\nu|^2 \frac{1}{3}\left(\frac{k_m}{K_\nu}\right)^2 |\bar{v}_\mu(0) \times \int dv\, \bar{t}_N e^{-i\bar{k}_m\cdot\bar{x}}|^2 + |r_\nu|^2 |v_{o\mu}(0)|^2 |\int dv\, v_{oN}\, e^{-i\bar{k}_m\cdot\bar{x}}|^2 \Big]$$

$$+ |f_1|^2 \Big[|S_\nu|^2 \frac{1}{15}\left(\frac{k_m}{K_\nu}\right)^2 \Big(7 |\bar{u}_\mu(0)\cdot \int dv\, \bar{t}_N e^{-i\bar{k}_m\cdot\bar{x}}|^2 + |\bar{u}_\mu(0)|^2 |\int dv\, \bar{t}_N e^{-i\bar{k}_m\cdot\bar{x}}|^2 \Big)$$

$$+ |\Omega_\nu|^2 \frac{1}{3}\left(\frac{k_m}{K_\nu}\right)^2 |u_{o\mu}(0)|^2 |\int dv\, v_{oN}\, e^{-i\bar{k}_m\cdot\bar{x}}|^2 \Big] \Big\} \tag{35}$$

Since we assume that the charged spin $\frac{1}{2}$ μ-meson is captured from the K-orbit of the atom, we may treat it non-relativistically. Thus by summing over the spin directions of the neutral particle, we have in the non-relativistic approximation

$$\sum_{\text{SPIN}} |\bar{v}_\mu(0) \times \int dv\, \bar{t}_N e^{-i\bar{k}_m\cdot\bar{x}}|^2 = \frac{E_{mk_m} + \hbar c K_m}{E_{mk_m}} |\psi_c(0)|^2 |\int dv\, \bar{t}_N e^{-i\bar{k}_m\cdot\bar{x}}|^2$$

$$\sum_{\text{SPIN}} |v_{o\mu}(0)|^2 = \frac{E_{mk_m} - \hbar c K_m}{2E_{mk_m}} |\psi_c(0)|^2$$

$$\sum_{\text{SPIN}} |\bar{u}_\mu(0)\cdot \int dv\, t_N e^{-i\bar{k}_m\cdot\bar{x}}|^2 = \frac{E_{mk_m} - \hbar c K_m}{2E_{mk_m}} |\psi_c(0)|^2 |\int dv\, \bar{t}_N e^{-i\bar{k}_m\cdot\bar{x}}|^2$$

$$\sum_{\text{SPIN}} |\bar{u}_\mu(0)|^2 = 3\, \frac{E_{mk_m} - \hbar c K_m}{2E_{mk_m}} |\psi_c(0)|^2$$

$$\sum_{\text{SPIN}} |\bar{u}_{o\mu}(0)|^2 = 3\, \frac{E_{mk_m} + \hbar c K_m}{2E_{mk_m}} |\psi_c(0)|^2$$

Thus

$$\int_{\Omega_m}\sum_{\text{SPIN}}|H_{AF}|^2_{AV.}\,d\Omega_m=\frac{(\hbar cK_\nu)^2}{V}2\pi\left[\frac{K_\nu^2}{k_m^2+K_\nu^2}\right]^2|\psi_c(0)|^2\Bigg\{|g_1|^2\Bigg[|r_\nu|^2\frac{E_{mk_m}-\hbar ck_m}{2E_{mk_m}}\left|\int dV\,v_{0N}\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2$$

$$+|s_\nu|^2\frac{E_{mk_m}+\hbar ck_m}{2E_{mk_m}}\frac{2}{3}\left(\frac{k_m}{K_\nu}\right)^2\left|\int dV\,\bar{t}_N\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2\Bigg]$$

$$+|f_1|^2\left(\frac{k_m}{K_\nu}\right)^2\Bigg[|r_\nu|^2\frac{E_{mk_m}+\hbar ck_m}{2E_{mk_m}}\left|\int dV\,v_{0N}\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2$$

$$+|s_\nu|^2\frac{E_{mk_m}-\hbar ck_m}{2E_{mk_m}}\frac{2}{3}\left(\frac{k_m}{K_\nu}\right)^2\left|\int dV\,\bar{t}_N\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2\Bigg]\Bigg\} \tag{36}$$

Now one shows easily

$$\frac{p_m^2}{(2\pi\hbar)^3}\frac{dp_m}{dE_F}=\frac{1}{(2\pi\hbar c)^3}\,\hbar ck_m\,E_{mk_m} \tag{37}$$

Hence, substituting equations (36) and (37) into equation (27), and making the summation in the nuclear matrix elements explicit, we obtain the result that

$$\frac{1}{T_1}=\frac{1}{4\pi}\frac{\hbar ck_m}{\hbar}\frac{E_{mk_m}}{\hbar cK_\nu}\left[\frac{K_\nu^2}{k_m^2+K_\nu^2}\right]^2\frac{|\psi_c(0)|^2}{K_\nu^3}\Bigg\{|K_\nu^{3/2}g_1|^2\Bigg[|K_\nu^{3/2}r_\nu|^2\frac{E_{mk_m}-\hbar ck_m}{E_{mk_m}}\left|\sum\int dV\,\Phi_N^*\frac{\tau_x+i\tau_y}{2}\Psi_N\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2$$

$$+|K_\nu^{3/2}s_\nu|^2\frac{E_{mk_m}+\hbar ck_m}{E_{mk_m}}\frac{2}{3}\left(\frac{k_m}{K_\nu}\right)^2\left|\sum\int dV\,\Phi_N^*\frac{\tau_x+i\tau_y}{2}\bar{\sigma}\,\Psi_N\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2\Bigg]$$

$$+|K_\nu^{3/2}f_1|^2\left(\frac{k_m}{K_\nu}\right)^2\Bigg[|K_\nu^{3/2}r_\nu|^2\frac{E_{mk_m}+\hbar ck_m}{E_{mk_m}}\left|\sum\int dV\,\Phi_N^*\frac{\tau_x+i\tau_y}{2}\Psi_N\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2$$

$$+|K_\nu^{3/2}s_\nu|^2\frac{E_{mk_m}-\hbar ck_m}{E_{mk_m}}\frac{2}{3}\left(\frac{k_m}{K_\nu}\right)^2\left|\sum\int dV\,\Phi_N^*\frac{\tau_x+i\tau_y}{2}\bar{\sigma}\,\Psi_N\,e^{-i\bar{k}_m\cdot\bar{x}}\right|^2\Bigg]\Bigg\}$$

APPENDIX 2 SCALAR-SPIN $\frac{1}{2}$ FIELD

Formulation of the Field

The Lagrangian density of the field is

$$L = L_{SCALAR} + L_{CDIRAC} + L_{MDIRAC} + L_2 + L'_{SCALAR} \tag{38}$$

where these terms are defined in equations (1), (4), (5), and (6). The canonical momenta are

$$P(\varphi) = \frac{\partial L}{\partial\left(\frac{\partial \varphi}{\partial t}\right)} = \hbar \rho_0^* = \hbar(\chi_0^* + \eta_s^* v_0^{(N)*})$$

$$P(\psi_{c\alpha}) = \frac{\partial L}{\partial\left(\frac{\partial \psi_{c\alpha}}{\partial t}\right)} = i\hbar \psi_{c\alpha}^*$$

$$P(\psi_{m\alpha}) = \frac{\partial L}{\partial\left(\frac{\partial \psi_{m\alpha}}{\partial t}\right)} = i\hbar \psi_{m\alpha}^* \tag{39}$$

Substituting equations (38) and (39) into

$$H_2 = P(\varphi)\frac{\partial \varphi}{\partial t} + \sum_\alpha P(\psi_{c\alpha})\frac{\partial \psi_{c\alpha}}{\partial t} + \sum_\alpha P(\psi_{m\alpha})\frac{\partial \psi_{m\alpha}}{\partial t} - L$$

we have

$$H_2 = \hbar c \kappa_s\left[\varphi^*\varphi + \rho_0^*\rho_0 + \chi_1^*\chi_1\right] + H_{CDIRAC} + H_{MDIRAC}$$

$$+ \hbar c \kappa_s\left[\eta_s w^{(N)} \varphi^* + c.c.\right]$$

$$+ \hbar c \kappa_s\left[g_1 w^{(M)} \varphi^* + c.c.\right]$$

Thus in vector notation this becomes

$$H_2 = \hbar c \kappa_s \left[|\psi|^2 + |\pi|^2 + \frac{1}{\kappa_s^2} |\operatorname{grad} \psi|^2 \right] + H_{C\,DIRAC} + H_{M\,DIRAC}$$

$$+ \hbar c \kappa_s \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[\Gamma_0 \beta \psi^* \right] \Psi_N + c.c. \right\}$$

$$+ \hbar c \kappa_s \left\{ \tilde{\psi}_M C \left[g_1 \beta \psi^* \right] \psi_C + c.c. \right\} \tag{40}$$

Lifetime of the Scalar Meson

The decay of the scalar π-meson takes place by the reaction

$$\pi^- \rightarrow \mu^- + \mu^0$$

The lifetime of this process is (Appendix 1)

$$\frac{1}{\tau_{02}} = \frac{2\pi}{\hbar} \int_{\Omega_c} \sum_{SPINS} |H_{AF}|^2 \frac{p_c^2}{(2\pi\hbar)^3} \frac{dp_c}{dE_F} \, d\Omega_c \, V \tag{41}$$

The element of H_2 giving rise to the decay process is the coefficient in H_2 of

$$b_{s k_s} \, b^*_{c k_c s} \, a^*_{m k_m r}$$

wherein plane waves are assumed for the wave functions with $\bar{k}_s = 0$ and thus $\bar{k}_c + \bar{k}_n = 0$. This element is

$$H_{AF} = \frac{\hbar c \kappa_s}{2V^{1/2}} \tilde{p}_{m k_m r} C \left[g_2 i \beta \right] \tilde{p}_{c k_c s}$$

By Casimir's spur technique we have

$$\sum_{SPIN} |H_{AF}|^2 = \frac{(\hbar c \kappa_s)^2}{4V} \operatorname{Spur} \left[(-g_2^* i \beta) \left(\frac{\hbar c (\bar{\alpha} \cdot \bar{k}_m + \beta \kappa_m) + E_{m k_m}}{2 E_{m k_m}} \right) (g_2 i \beta) \left(\frac{\hbar c (\bar{\alpha} \cdot \bar{k}_c - \beta \kappa_c) + E_{c k_c}}{2 E_{c k_c}} \right) \right]$$

A calculation of this spur gives

$$\sum_{SPIN} |H_{AF}|^2 = \frac{(\hbar c K_c)^2}{4V} \frac{|g_2|^2}{E_{mk_m} E_{ck_c}} \left[\hbar^2 c^2 (k_c^2 - K_m K_c) + E_{mk_m} E_{ck_c}\right] \tag{42}$$

Using

$$\bar{k}_c + \bar{k}_m = 0$$

and

$$E_{ck_c} + E_{mk_m} = E_F = m_s c^2$$

we find

$$E_{mk_m} E_{ck_c} = \frac{E_F^4 - (m_c^2 - m_m^2)^2 c^8}{4 E_F^2} \tag{43}$$

$$\hbar^2 c^2 (k_c^2 - K_m K_c) + E_{mk_m} E_{ck_c} = \frac{E_F^2 - (m_m + m_c)^2 c^4}{2} \tag{44}$$

and

$$p_c^2 \frac{dp_c}{dE_F} = \frac{1}{8c^3 E_F^4} \left[E_F^2 - (m_m + m_c)^2 c^4\right]^{1/2} \left[E_F^2 - (m_m - m_c)^2 c^4\right]^{1/2} \left[E_F^2 + (m_m^2 - m_c^2) c^4\right] \left[E_F^2 + (m_m^2 - m_c^2) c^4\right] \tag{45}$$

Substituting equations (42), (43), (44), and (45) into equation (41), we have the result

$$\frac{1}{\tau_{02}} = \frac{|K_s^{3/2} g_2|^2}{16\pi} \frac{m_s c^2}{\hbar} \left[1 - \left(\frac{m_m}{m_s} + \frac{m_c}{m_s}\right)^2\right]^{3/2} \left[1 - \left(\frac{m_m}{m_s} - \frac{m_c}{m_s}\right)^2\right]^{1/2}$$

<u>Lifetime for the Capture of the Spin ½ Meson by a Nucleus</u>

The capture of a negative spin ½ meson takes place by one of the two reactions:

$$\text{I} \quad \mu^- \rightarrow \left\{ \begin{matrix} \pi^- + \mu^\circ \\ \pi^- + P \end{matrix} \right\} \rightarrow N$$

$$\text{II} \quad P \rightarrow \left\{ \begin{matrix} \pi^+ + N \\ \pi^+ + \mu^- \end{matrix} \right\} \rightarrow \mu^\circ$$

The lifetime of this capture process is (Appendix 1)

$$\frac{1}{\tau_2} = \frac{2\pi}{\hbar} \sum_{\text{SPIN}} \int_{\Omega_m} |H_{AF}|^2_{AV.} \frac{P_m^2}{(2\pi\hbar)^3} \frac{dP_m}{dE_F} \, d\Omega_m V \tag{46}$$

where

$$H_{AF} = \sum \left[\frac{H_{AI} H_{IF}}{E_A - E_I} + \frac{H_{AII} H_{IIF}}{E_A - E_{II}} \right] \tag{47}$$

To obtain H_{AF} we use plane waves for the emergent neutral meson and we approximate the wave function of the virtual scalar π-meson by plane waves. With these wave functions substituted into H_2 we find that the element of H_2 giving

(a) H_{AI} is the coefficient of

$$b_c \, b^*_{sk_s} \, a^*_{mk_m s}$$

(b) H_{IF} is the coefficient of

$$b_{sk_s} \frac{\tau_x + i\tau_y}{2}$$

(c) H_{AII} is the coefficient of

$$a^*_{sk_s} \frac{\tau_x + i\tau_y}{2}$$

(d) H_{IIF} is the coefficient of

$$a_{sk_s} b_c a^*_{mk_m s}$$

These elements are

$$(a)\ H_{AI} = \frac{\hbar c \kappa_s}{V} \frac{1}{\sqrt{2}} \int dV\, \tilde{\psi}_c C \left[-i g_2^* \beta \sqrt{\frac{\hbar c \kappa_s}{2E_{sk_s}}} \right] \tilde{\rho}_{mk_m s}\, e^{-i(\bar{k}_m + \bar{k}_s)\cdot\bar{x}}$$

$$(b)\ H_{IF} = \frac{\hbar c \kappa_s}{V^{1/2}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[i \eta_s \beta \sqrt{\frac{\hbar c \kappa_s}{2E_{sk_s}}} \right] \Psi_N\, e^{i\bar{k}_s\cdot\bar{x}}$$

$$(c)\ H_{AII} = \frac{\hbar c \kappa_s}{V^{1/2}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[-i \eta_s \beta \sqrt{\frac{\hbar c \kappa_s}{2E_{sk_s}}} \right] \Psi_N\, e^{-i\bar{k}_s\cdot\bar{x}}$$

$$(d)\ H_{IIF} = \frac{\hbar c \kappa_s}{V} \frac{1}{\sqrt{2}} \int dV\, \tilde{\psi}_c C \left[i g_2^* \beta \sqrt{\frac{\hbar c \kappa_s}{2E_{sk_s}}} \right] \tilde{\rho}_{mk_m s}\, e^{i(\bar{k}_s - \bar{k}_m)\cdot\bar{x}}$$

(48)

where the nuclear matrix elements contain implicitly a sum over all protons in the nucleus. We now assume that the energy imparted to the nucleons is negligible compared to the rest energy of the scalar π-meson. Then

$$E_A - E_I = E_A - E_{II} \approx -E_{sk_s} \tag{49}$$

Substituting equations (48) and (49) into equation (47), we find

$$H_{AF} = -\frac{(\hbar c \kappa_s)^3}{V^{3/2}} \frac{g_2^* \eta_s}{2^{3/2}} \sum_{k_s} \int dV \int dV'\, w_\mu w_N' \frac{e^{-i\bar{k}_m\cdot\bar{x}}}{E^2_{sk_s}} \left[e^{i\bar{k}_s\cdot(\bar{x}'-\bar{x})} + e^{-i\bar{k}_s\cdot(\bar{x}'-\bar{x})} \right]$$

where the primed and unprimed quantities refer, respectively, to the variables of V' and V. Making use of the identity

$$\sum_{k_s} \frac{1}{E^2_{sk_s}} \left[e^{i\bar{k}_s\cdot(\bar{x}'-\bar{x})} + e^{-i\bar{k}_s\cdot(\bar{x}'-\bar{x})} \right] = \frac{V}{2\pi} \frac{1}{\hbar^2 c^2} \frac{e^{-\kappa_s R}}{R}$$

where $\bar{R} = \bar{x}' - \bar{x}$, we find

$$H_{AF} = -\frac{(\hbar c K_s)^3}{V^{1/2}} \frac{g_2^* \pi_s}{2^{5/2} \pi \hbar^2 c^2} \int dV' w_N' e^{-i\bar{k}_m \cdot \bar{x}'} \int dV w_\mu \frac{e^{-K_s R + i\bar{k}_m \cdot \bar{R}}}{R}$$

Since the principal contribution of w_μ to this integral occurs in the neighborhood of $R = 0$, we can treat w_μ as a constant which constant we take to be the value of w_μ at the center of the nucleus (Appendix 1). The integration over the variables of V now gives

$$H_{AF} = -\frac{\hbar c K_s}{V^{1/2}} \frac{g_2^* \pi_s}{\sqrt{2}} \left[\frac{K_s^2}{k_m^2 + K_s^2}\right] w_\mu(0) \int dV w_N e^{-i\bar{k}_m \cdot \bar{x}} \tag{50}$$

where the primes have been omitted. Since the charged spin $\frac{1}{2}$ μ-meson may be treated non-relativistically, we have

$$\sum_{SPIN} |w_\mu(0)|^2 = \frac{E_{mk_m} - \hbar c K_m}{2E_{mk_m}} |\psi_c(0)|^2 \tag{51}$$

Substituting equations (50) and (51) into equation (46), making the summation in the nuclear matrix elements explicit, and using the relation

$$\frac{P_m^2}{(2\pi\hbar)^3} \frac{dP_m}{dE_F} = \frac{\hbar c k_m}{(2\pi\hbar c)^3} E_{mk_m}$$

we have the result that

$$\frac{1}{\tau_2} = \frac{|K_s^{3/2} g_2|^2 |K_s^{3/2} \pi_s|^2}{4\pi} \frac{\hbar c k_m}{\hbar} \frac{E_{mk_m}}{\hbar c K_s} \left[\frac{K_s^2}{k_m^2 + K_s^2}\right]^2 \frac{E_{mk_m} - \hbar c K_m}{E_{mk_m}} \frac{|\psi_c(0)|^2}{K_s^3} \left|\sum \int dV \Phi_N^* \frac{\tau_1 + i\tau_2}{2} \Psi_N e^{i\bar{k}_m \cdot \bar{x}}\right|^2$$

APPENDIX 3 PSEUDOSCALAR-SPIN $\frac{1}{2}$ FIELD

Formulation of the Field

The Lagrangian density of the field is

$$L = L_{\text{PSEUDOSCALAR}} + L_{\text{cDIRAC}} + L_{\text{mDIRAC}} + L_3 + L'_{\text{PSEUDOSCALAR}} \tag{52}$$

where these terms are defined in equations (3), (4), (5), and (7). The canonical momenta are

$$p(\chi^{0123}) = \frac{\partial L}{\partial\left(\frac{\partial \chi^{0123}}{\partial t}\right)} = \hbar \varsigma^*_{123} = \hbar \varphi^*_{123}$$

$$p(\psi_{c\alpha}) = \frac{\partial L}{\partial\left(\frac{\partial \psi_{c\alpha}}{\partial t}\right)} = i\hbar \psi^*_{c\alpha}$$

$$p(\psi_{m\alpha}) = \frac{\partial L}{\partial\left(\frac{\partial \psi_{m\alpha}}{\partial t}\right)} = i\hbar \psi^*_{m\alpha} \tag{53}$$

Substituting equations (52) and (53) into

$$H_3 = p(\chi^{0123}) \frac{\partial \chi^{0123}}{\partial t} + \sum_{\alpha} p(\psi_{c\alpha}) \frac{\partial \psi_{c\alpha}}{\partial t} + \sum_{\alpha} p(\psi_{m\alpha}) \frac{\partial \psi_{m\alpha}}{\partial t} - L$$

we have

$$H_3 = \hbar c \kappa_p \left[\bar{\varsigma}_0^* \bar{\varsigma}_0 + \bar{\varphi}_\lambda^* \bar{\varphi}_\lambda + \bar{\chi}^* \bar{\chi} \right] + H_{\text{cDIRAC}} + H_{\text{mDIRAC}}$$

$$+ \hbar c \kappa_p \left[\Omega_p \bar{\varsigma}_N \bar{\chi}^* + \text{c.c.} \right]$$

$$+ \hbar c \kappa_p \left[f_3 \bar{\varsigma}_\mu \bar{\chi}^* + \text{c.c.} \right]$$

where we set for any pseudovector

$$\bar{a}_0 = a_{123} \qquad \bar{a}_l = a_{0ik}$$

and for any pseudoscalar

$$\bar{b} = b_{0123}$$

In vector notation

$$\pi = \bar{\xi}_0 \qquad \psi = -\bar{\chi} \qquad \varphi_i = 1/K_p \operatorname{grad} \psi$$

Thus

$$H_3 = \hbar c K_p\left[|\psi|^2 + |\pi|^2 + 1/K_p^2 |\operatorname{grad}\psi|^2\right] + H_{c\,DIRAC} + H_{\mu\,DIRAC}$$
$$- \hbar c K_p\left[\eta_p \bar{\xi}_N \psi^* + c.c.\right]$$
$$- \hbar c K_p\left[f_3 \bar{\xi}_\mu \psi^* + c.c.\right] \tag{54}$$

Lifetime of the Pseudoscalar Meson

The decay of the pseudoscalar π-meson takes place by the reaction

$$\pi^- \rightarrow \mu^- + \mu^0$$

The lifetime of this process is (Appendix 1)

$$\frac{1}{\tau_{03}} = \frac{2\pi}{\hbar} \int_{\Omega_c} \sum_{SPIN} |H_{AF}|^2 \frac{P_c^2}{(2\pi\hbar)^3} \frac{dP_c}{dE_F}\, d\Omega_c\, V \tag{55}$$

The element of H_3 giving rise to the decay process is the coefficient in H_3 of

$$b_{pk_p} b^*_{ck_cs} a^*_{mk_mr}$$

wherein plane waves are assumed for the wave functions with $\bar{k}_p = 0$ and thus $\bar{k}_c + \bar{k}_n = 0$. This element is

$$H_{AF} = -\frac{\hbar c K_p}{2V} \tilde{\rho}_{mk_mr} C\left[f_3 \beta \gamma^5\right] \tilde{\rho}_{ck_cs}$$

By Casimir's spur technique we have

$$\sum_{SPIN} |H_{AF}|^2 = \frac{(\hbar c K_p)^2}{4V} \operatorname{Spur}\left[\left(-f_3^* \beta \gamma^5\right)\left(\frac{\hbar c(\bar{\alpha}\cdot\bar{k}_m + \beta K_m) + E_{mk_m}}{2E_{mk_m}}\right)\left(f_3 \beta \gamma^5\right)\left(\frac{\hbar c(\bar{\alpha}\cdot\bar{k}_c - \beta K_c) + E_{ck_c}}{2E_{ck_c}}\right)\right]$$

A calculation of this spur gives

$$\sum_{SPIN} |H_{AF}|^2 = \frac{(\hbar c K_p)^2}{4V} \frac{|f_3|^2}{E_{mk_m} E_{ck_c}} \left[\hbar^2 c^2 (k_c^2 + K_m K_c) + E_{mk_m} E_{ck_c}\right] \tag{56}$$

Using

$$\bar{k}_c + \bar{k}_m = 0$$

and

$$E_{ck_c} + E_{mk_m} = E_F = m_p c^2$$

we find

$$\hbar^2 c^2 (k_c^2 + K_m K_c) + E_{mk_m} E_{ck_c} = \frac{E_F^2 - (m_m - m_c)^2 c^4}{2} \tag{57}$$

$$E_{ck_c} E_{mk_m} = \frac{E_F^4 - (m_c^2 - m_m^2)^2 c^8}{4E_F^2} \tag{58}$$

and

$$p_c^2 \frac{dp_c}{dE_F} = \frac{1}{8c^3 E_F^4}\left[E_F^2 - (m_m + m_c)^2 c^4\right]^{1/2}\left[E_F^2 - (m_m - m_c)^2 c^4\right]^{1/2}\left[E_F^2 + (m_m^2 - m_c^2) c^4\right]\left[E_F^2 - (m_m^2 - m_c^2) c^4\right] \tag{59}$$

Substituting equations (56), (57), (58), and (59) into equation (55), we have the result

$$\frac{1}{\tau_{03}} = \frac{|K_p^{3/2} g_3|^2}{16\pi} \frac{m_p c^2}{\hbar}\left[1-\left(\frac{m_m}{m_p}+\frac{m_c}{m_p}\right)^2\right]^{1/2}\left[1-\left(\frac{m_m}{m_p}-\frac{m_c}{m_p}\right)^2\right]^{3/2}$$

Lifetime for the Capture of the Spin $\frac{1}{2}$ Meson by a Nucleus

The capture of a negative spin $\frac{1}{2}$ meson takes place by one of the two reactions:

$$\text{I} \quad \mu^- \rightarrow \left\{\begin{matrix} \pi^- + \mu^0 \\ \pi^- + P \end{matrix}\right\} \rightarrow N$$

$$\text{II} \quad P \rightarrow \left\{\begin{matrix} \pi^+ + N \\ \pi^+ + \mu^- \end{matrix}\right\} \rightarrow \mu^0$$

The lifetime of this capture process is (Appendix 1)

$$\frac{1}{\tau_3} = \frac{2\pi}{\hbar}\sum_{\text{SPIN}}\int_{\Omega_m} |H_{AF}|^2_{\text{AV.}} \frac{P_m^2}{(2\pi\hbar)^3}\frac{dP_m}{dE_F}\, d\Omega_m V \tag{60}$$

where

$$H_{AF} = \sum\left[\frac{H_{AI}H_{IF}}{E_A - E_I} + \frac{H_{AII}H_{IIF}}{E_A - E_{II}}\right] \tag{61}$$

To obtain H_{AF} we use plane waves for the emergent neutral meson and we approximate the wave function of the virtual pseudoscalar π-meson by plane waves. With these wave functions substituted into H_3 we find that the element of H_3 giving

(a) H_{AI} is the coefficient of

$$b_c\, b^*_{pk_p}\, a^*_{mk_{ms}}$$

(b) H_{IF} is the coefficient of

$$b_{pk_p} \frac{\tau_x + i\tau_y}{2}$$

(c) H_{AII} is the coefficient of

$$a^*_{pk_p} \frac{\tau_x + i\tau_y}{2}$$

(d) H_{IIF} is the coefficient of

$$a_{pk_p} b_c a^*_{mk_m s}$$

These elements are

$$\text{(a)}\quad H_{AI} = \frac{\hbar c \kappa_p}{V} \frac{1}{\sqrt{2}} \int dV\, \tilde{\psi}_c C \left[f_3^* \beta \gamma^5 \sqrt{\frac{\hbar c \kappa_p}{2E_{pk_p}}} \right] \tilde{P}_{mk_m s}\, e^{-i(\bar{k}_m + \bar{k}_p)\cdot \bar{x}}$$

$$\text{(b)}\quad H_{IF} = -\frac{\hbar c \kappa_p}{V^{1/2}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[\eta_p \beta \gamma^5 \sqrt{\frac{\hbar c \kappa_p}{2E_{pk_p}}} \right] \Psi_N\, e^{i\bar{k}_p \cdot \bar{x}}$$

$$\text{(c)}\quad H_{AII} = \frac{\hbar c \kappa_p}{V^{1/2}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[\eta_p \beta \gamma^5 \sqrt{\frac{\hbar c \kappa_p}{2E_{pk_p}}} \right] \Psi_N\, e^{-i\bar{k}_p \cdot \bar{x}}$$

$$\text{(d)}\quad H_{IIF} = -\frac{\hbar c \kappa_p}{V} \frac{1}{\sqrt{2}} \int dV\, \tilde{\psi}_c C \left[f_3^* \beta \gamma^5 \sqrt{\frac{\hbar c \kappa_p}{2E_{pk_p}}} \right] \tilde{P}_{mk_m s}\, e^{i(\bar{k}_p - \bar{k}_m)\cdot \bar{x}}$$

where the nuclear matrix elements contain implicitly a sum over all protons in the nucleus. If we compare these expressions with equation (48) of Appendix 2, we see that if we replace $i\beta$ in the latter by $-\beta\gamma^5$ then the scalar-spin $\frac{1}{2}$ field is identical with the pseudoscalar-spin $\frac{1}{2}$ field. Thus by making this replacement in equation (50) of Appendix 2 we find

$$H_{AF} = \frac{\hbar c \kappa_p}{V^{1/2}} \frac{f_3^* \eta_p}{\sqrt{2}} \left[\frac{\kappa_p^2}{k_m^2 + \kappa_p^2} \right] g_r(0) \int dV\, g_N\, e^{-i\bar{k}_m \cdot \bar{x}} \tag{62}$$

Since the charged spin $\frac{1}{2}$ μ-meson may be treated non-relativistically, we have

$$\sum_{\text{SPIN}} |g_\mu(0)|^2 = \frac{E_{mk_m} + \hbar c k_m}{2E_{mk_m}} |\psi_c(0)|^2 \tag{63}$$

Also since the nucleons may be treated non-relativistically, we have from their equation of motion

$$\Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \gamma^5 \Psi_N = -\frac{1}{2iK_N} \operatorname{div}\left(\Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \Psi_N\right)$$

and thus

$$\int dV g_N e^{-i\bar{k}_m \cdot \bar{x}} = -\frac{1}{2K_N} \bar{k}_m \cdot \int dV \,\bar{t}_N e^{-i\bar{k}_m \cdot \bar{x}} \tag{64}$$

Hence, substituting equations (62), (63), and (64) into equation (60), making the nuclear summation explicit, and using the relation

$$\frac{P_m^2}{(2\pi\hbar)^3} \frac{dP_m}{dE_F} = \frac{\hbar c k_m E_{mk_m}}{(2\pi\hbar c)^3}$$

we have the result that

$$\frac{1}{\tau_3} = \frac{|K_p^{3/2} f_3|^2 |K_p^{3/2} r_p|^2}{48\pi} \frac{\hbar c k_m}{\hbar} \frac{E_{mk_m}}{\hbar c K_p} \left(\frac{k_m}{K_N}\right)^2 \left[\frac{K_p^2}{k_m^2 + K_p^2}\right]^2 \frac{E_{mk_m} + \hbar c k_m}{E_{mk_m}} \frac{|\psi_c(0)|^2}{K_p^3} \left|\sum \int dV \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \Psi_N e^{-i\bar{k}_m \cdot \bar{x}}\right|^2$$

APPENDIX 4 VECTOR-SCALAR FIELD

Formulation of the Field

The Lagrangian density of the field is

$$L = L_{VECTOR} + L_{CSCALAR} + L_{MSCALAR} + L_4 + L'_{VECTOR} \tag{65}$$

where these terms are defined in equations (1), (2), (5), and (8). The canonical momenta are

$$p(\varphi_i) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_i}{\partial t}\right)} = \hbar \varrho^*_{oi} = \hbar(\chi^*_{oi} + \varsigma^*_N M^*_{oi})$$

$$p(\varphi_c) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_c}{\partial t}\right)} = \hbar \varrho^*_{co} = \hbar \chi^*_{co}$$

$$p(\varphi_m) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_m}{\partial t}\right)} = \hbar \varrho^*_{mo} = \hbar(\chi^*_{mo} + g_4 \varphi^*_o \varphi_c) \tag{66}$$

where $i = 1, 2, 3$. As in the vector-spin $\frac{1}{2}$ field (Appendix 1) we must eliminate φ_o from the Hamiltonian of the field with the aid of the equation of motion for φ_o, which is

$$\frac{\partial L}{\partial \varphi^*_o} - \sum_\alpha \frac{\partial}{\partial x^\alpha} \frac{\partial L}{\partial\left(\frac{\partial \varphi^*_o}{\partial x^\alpha}\right)} = 0$$

or

$$K_v \eta_o = -\frac{\partial \varrho_{oi}}{\partial x^i} = K_v \varphi_o + K_v \Lambda_v N_o + K_m g_4 \varphi_c \chi_{mo}$$

But now

$$\chi_{mo} = \varrho_{mo} - g^*_4 \varphi^*_c \varphi_o$$

Then

$$K_v \eta_0 = -\frac{\partial S_{0i}}{\partial x^i} = \left[K_v - K_m |g_4|^2 |\varphi_c|^2\right]\varphi_0 + K_v r_v v_0 + K_m g_4 \varphi_c S_{m0}$$

Neglecting the second order term in $|g_4|^2$, we have for the equation of motion of φ_0

$$K_v \eta_0 = -\frac{\partial S_{0i}}{\partial x^i} = K_v \varphi_0 + K_v r_v v_0 + K_m g_4 \varphi_c S_{m0} \tag{67}$$

Substituting equations (65), (66), and (67) into

$$H_4 = \sum_i p(\varphi_i)\frac{\partial \varphi_i}{\partial t} + p(\varphi_c)\frac{\partial \varphi_c}{\partial t} + p(\varphi_m)\frac{\partial \varphi_m}{\partial t} - L$$

we have

$$H_4 = \hbar c K_v\left[\varphi_i^* \varphi_i + \eta_0^* \eta_0 + \tfrac{1}{2}\chi_{ij}^* \chi_{ij} + S_{0i}^* S_{0i}\right] + H_{cSCALAR} + H_{mSCALAR}$$

$$+ \hbar c K_v\left[r_v\left(v_i^{(N)}\varphi_i^* - v_0^{(N)}\eta_0^*\right) + s_v\left(\tfrac{1}{2}M_{ij}^{(N)}\chi_{ij}^* - M_{0i}^{(N)} S_{0i}^*\right) + c.c.\right]$$

$$+ \hbar c K_m\left[g_4^*\left(-\eta_0 \varphi_c^* S_{m0}^* + \varphi_i \varphi_c^* \chi_{mi}^*\right) + c.c.\right]$$

$$+ \hbar c K_m\left[r_v g_4^* \, v_0^{(N)} \varphi_c^* S_{m0}^* + c.c.\right]$$

$$+ \hbar c K_v\left[|r_v|^2 |v_0^{(N)}|^2 + |s_v|^2 |M_{0i}^{(N)}|^2\right]$$

$$+ \hbar c K_m\left[|g_4|^2 |\varphi_c|^2 |S_{m0}|^2\right]$$

$$+ \hbar c K_m\left[|g_4|^2 |\varphi_c|^2 \left|\eta_0 - r_v v_0^{(N)} - \frac{K_m}{K_v} g_4 \varphi_c S_{m0}\right|^2\right]$$

In vector notation this becomes

$$H_4 = \hbar c\kappa_v\left[|\pi|^2 + |\psi|^2 + \tfrac{1}{\kappa_v^2}|\operatorname{curl}\psi|^2 + |\Pi|^2 + |\Psi|^2 + \tfrac{1}{\kappa_v^2}|\operatorname{div}\Psi|^2\right] + H_{e\,SCALAR} + H_{M\,SCALAR}$$

$$+\hbar c\kappa_v\Big[\Phi_N^* \tfrac{\tau_x + i\tau_y}{2}\Big\{\Gamma_v \bar{\alpha}\cdot\psi^* - g_v(\bar{\gamma}\cdot\pi - \tfrac{1}{\kappa_v}\rho\bar{\sigma}\cdot\operatorname{curl}\psi^*)$$

$$+ g_v\bar{\gamma}\cdot\Psi^* + \Gamma_v(\bar{\alpha}\cdot\Pi - \tfrac{1}{\kappa_v}\operatorname{div}\Psi^*)\Big\}\Psi_N + \text{c.c.}\Big]$$

$$+\hbar c\kappa_m\Big[g_4^*\Big\{\tfrac{1}{\kappa_v}\psi_c^*\pi_m \operatorname{div}(\pi^* - \Psi) + \tfrac{1}{\kappa_m}(\psi + \Pi^*)\psi_c^*\operatorname{grad}\psi_m^*\Big\} + \text{c.c.}\Big]$$

$$+\hbar c\kappa_m\Big[\Phi_N^*\tfrac{\tau_x + i\tau_y}{2}\Big\{-\Gamma_v g_4^*\psi_c^*\pi_m\Big\}\Psi_N + \text{c.c.}\Big]$$

$$+\hbar c\kappa_v\left[|\Gamma_v|^2\left|\Phi_N^*\tfrac{\tau_x + i\tau_y}{2}\Psi_N\right|^2 + |g_v|^2\left|\Phi_N^*\tfrac{\tau_x + i\tau_y}{2}\bar{\gamma}\,\Psi_N\right|^2\right]$$

$$+\hbar c\kappa_v\left[\left(\tfrac{\kappa_m}{\kappa_v}\right)^2|g_4|^2|\psi_c|^2|\pi_m|^2\right]$$

$$+\hbar c\kappa_m\left[|g_4|^2|\psi_c|^2\left|\tfrac{1}{\kappa_v}\operatorname{div}(\pi^* - \Psi) - \Gamma_v\Phi_N^*\tfrac{\tau_x + i\tau_y}{2}\Psi_N + \tfrac{\kappa_m}{\kappa_v}g_4\psi_c\pi_m^*\right|^2\right] \tag{68}$$

<u>Lifetime of the Vector Meson</u>

The decay of the vector π-meson takes place by the reaction

$$\pi^- \rightarrow \mu^- + \mu^0$$

The lifetime of this process is (Appendix 1)

$$\frac{1}{T_{04}} = \frac{2\pi}{\hbar}\int_{\Omega_c}|H_{AF}|^2\frac{P_c^2}{(2\pi\hbar)^3}\frac{dP_c}{dE_F}\,d\Omega_c V \tag{69}$$

The element of H_4 giving rise to the decay process is the coefficient in H_4 of

$$b_{\nu k_{\nu} \epsilon}\, b^{*}_{s k_s}\, a^{*}_{m k_m}$$

wherein plane waves are assumed for the wave functions with $\bar{k}_v = 0$ and thus $\bar{k}_c + \bar{k}_n = 0$. This element is

$$H_{AF} = \frac{\hbar c K_\nu}{4V^{1/2}} g_4 \frac{(\bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_c)}{K_\nu} \sqrt{\frac{\hbar c K_m}{E_{m k_m}}} \sqrt{\frac{\hbar c K_c}{E_{c k_c}}}$$

And thus

$$|H_{AF}|^2 = \frac{(\hbar c K_s)^2}{16V} |g_4|^2 \left| \frac{\bar{J}_{\nu k_\nu \epsilon} \cdot \bar{k}_c}{K_\nu} \right|^2 \frac{\hbar c K_m}{E_{m k_m}} \frac{\hbar c K_c}{E_{c k_c}} \tag{70}$$

Using

$$\bar{k}_c + \bar{k}_m = 0$$

and

$$E_{c k_c} + E_{m k_m} = E_F = m_\nu c^2$$

we find

$$E_{m k_m} E_{c k_c} = \frac{E_F^4 - (m_c^2 - m_m^2)^2 c^8}{4E_F^2} \tag{71}$$

and

$$p_c^2 \frac{dp_c}{dE_F} = \frac{1}{8c^3 E_F^4} \left[E_F^2 + (m_m^2 - m_c^2)c^4\right]\left[E_F^2 - (m_m^2 - m_c^2)c^4\right]\left[E_F^2 - (m_m + m_c)^2 c^4\right]^{1/2}\left[E_F^2 - (m_m - m_c)^2 c^4\right]^{1/2} \tag{72}$$

Substituting equations (70), (71), and (72) into equation (69), we have the result

$$\frac{1}{\tau_{04}} = \frac{|K_\nu^{3/2} g_4|^2}{384\pi} \frac{m_\nu c^2}{\hbar} \frac{m_m}{m_\nu} \frac{m_c}{m_\nu} \left[1 - \left(\frac{m_m}{m_\nu} + \frac{m_c}{m_\nu}\right)^2\right]^{3/2} \left[1 - \left(\frac{m_m}{m_\nu} - \frac{m_c}{m_\nu}\right)^2\right]^{3/2}$$

Lifetime for the Capture of the Scalar μ-Meson by a Nucleus

The capture of the negative scalar μ-meson takes place by one of the three reactions:

$$\text{I} \quad \mu^- \rightarrow \left\{ \begin{matrix} \pi^- + \mu^0 \\ \pi^- + P \end{matrix} \right\} \rightarrow N$$

$$\text{II} \quad P \rightarrow \left\{ \begin{matrix} \pi^+ + N \\ \pi^+ + \mu^- \end{matrix} \right\} \rightarrow \mu^0$$

$$\text{III} \quad P + \mu^- \rightarrow N + \mu^0$$

The lifetime of this capture process is (Appendix 1)

$$\frac{1}{\tau_4} = \frac{2\pi}{\hbar} \int_{\Omega_m} |H_{AF}|^2_{AV.} \frac{p_m^2}{(2\pi\hbar)^3} \frac{dp_m}{dE_F} \, d\Omega_m \, V \tag{73}$$

where

$$H_{AF} = \sum \left[\frac{H_{AI} H_{IF}}{E_A - E_I} + \frac{H_{AII} H_{IIF}}{E_A - E_{II}} \right] + H_{AFIII} \tag{74}$$

To obtain H_{AF} we use plane waves for the neutral meson and approximate the wave function of the virtual vector π-meson by plane waves. With these wave functions substituted into H_4 we find that the element of H_4 giving

(a) H_{AI} is the coefficient of

$$b_\epsilon \, b^*_{N k_N \epsilon} \, a^*_{m k_m} \qquad \epsilon = 1, 0, -1$$

(b) H_{IF} is the coefficient of

$$b_{\nu k_\nu \epsilon} \frac{\tau_x + i\tau_y}{2} \qquad\qquad \epsilon = 1, 0, -1$$

(c) H_{AII} is the coefficient of

$$a^*_{\nu k_\nu \epsilon} \frac{\tau_x + i\tau_y}{2} \qquad\qquad \epsilon = 1, 0, -1$$

(d) H_{IIF} is the coefficient of

$$a_{\nu k_\nu \epsilon} b_c a^*_{m k_m} \qquad\qquad \epsilon = 1, 0, -1$$

(e) H_{AFIII} is the coefficient of

$$b_c a^*_{m k_m} \frac{\tau_x + i\tau_y}{2}$$

These elements are

$$\text{(a)}\; H^{\epsilon}_{AI} = \frac{\hbar c K_m}{V} \frac{1}{\sqrt{2}} \int dV \left[-\frac{g_4^*}{K_m} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \sqrt{\frac{\hbar c K_m}{2E_{m k_m}}}\, \bar{\mathcal{J}}_{\nu k_\nu \epsilon} \cdot \operatorname{grad} e^{i\bar{k}_m \cdot \bar{x}} \right] \psi_c^* e^{-i\bar{k}_\nu \cdot \bar{x}}$$

$$H^{\circ}_{AI} = \frac{\hbar c K_m}{V} \frac{1}{\sqrt{2}} \int dV \left\{ g_4^* \left[\frac{i}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \sqrt{\frac{E_{m k_m}}{2\hbar c K_m}}\, e^{-i\bar{k}_m \cdot \bar{x}} \operatorname{div} \bar{\mathcal{J}}_{\nu k_\nu 0} e^{-i\bar{k}_\nu \cdot \bar{x}} - \frac{i}{K_m} \sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} \sqrt{\frac{\hbar c K_m}{2E_{m k_m}}}\, e^{-i\bar{k}_\nu \cdot \bar{x}} \bar{\mathcal{J}}_{\nu k_\nu 0} \cdot \operatorname{grad} e^{i\bar{k}_m \cdot \bar{x}} \right] \psi_c^* \right\}$$

$$\text{(b)}\; H^{\epsilon}_{IF} = \frac{\hbar c K_\nu}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[r_\nu i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\alpha} \cdot \bar{\mathcal{J}}_{\nu k_\nu \epsilon} - s_\nu \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\gamma} \cdot \bar{\mathcal{J}}_{\nu k_\nu \epsilon} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \beta \bar{\sigma} \cdot \bar{\mathcal{J}}_{\nu k_\nu \epsilon} \times \bar{k}_\nu \right) \right] \Psi_N \right\} e^{i\bar{k}_\nu \cdot \bar{x}}$$

$$H^{\circ}_{IF} = \frac{\hbar c K_\nu}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[s_\nu i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\gamma} \cdot \bar{\mathcal{J}}_{\nu k_\nu 0} + r_\nu \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\alpha} \cdot \bar{\mathcal{J}}_{\nu k_\nu 0} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\mathcal{J}}_{\nu k_\nu 0} \cdot \bar{k}_\nu \right) \right] \Psi_N \right\} e^{i\bar{k}_\nu \cdot \bar{x}}$$

$$\text{(c)}\; H^{\epsilon}_{AII} = \frac{\hbar c K_\nu}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[-r_\nu i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\alpha} \cdot \bar{\mathcal{J}}_{\nu k_\nu \epsilon} - s_\nu \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\gamma} \cdot \bar{\mathcal{J}}_{\nu k_\nu \epsilon} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \beta \bar{\sigma} \cdot \bar{\mathcal{J}}_{\nu k_\nu \epsilon} \times k_\nu \right) \right] \Psi_N \right\} e^{-i\bar{k}_\nu \cdot \bar{x}}$$

$$H^{\circ}_{AII} = \frac{\hbar c K_\nu}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[-s_\nu i \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\gamma} \cdot \bar{\mathcal{J}}_{\nu k_\nu 0} + r_\nu \left(\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}}\, \bar{\alpha} \cdot \bar{\mathcal{J}}_{\nu k_\nu 0} - \frac{1}{K_\nu} \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}}\, \bar{\mathcal{J}}_{\nu k_\nu 0} \cdot \bar{k}_\nu \right) \right] \Psi_N \right\} e^{-i\bar{k}_\nu \cdot \bar{x}}$$

$$(d)\; H^{\epsilon}_{IIF} = \frac{\hbar c K_m}{V}\frac{1}{\sqrt{2}}\int dV\left[\frac{g_4^*}{K_m}\sqrt{\frac{\hbar c K_v}{2E_{vk_v}}}\sqrt{\frac{\hbar c K_m}{2E_{mk_m}}}\,\bar{J}_{vk_v\epsilon}\cdot \mathrm{grad}\, e^{-i\bar{k}_m\cdot\bar{x}}\right]\psi_c^* e^{i\bar{k}_v\cdot\bar{x}}$$

$$H^{0}_{IIF} = \frac{\hbar c K_m}{V}\frac{1}{\sqrt{2}}\int dV\left\{g_4^*\left[-\frac{i}{K_v}\sqrt{\frac{\hbar c K_v}{2E_{vk_v}}}\sqrt{\frac{E_{mk_m}}{2\hbar c K_m}}\, e^{-i\bar{k}_m\cdot\bar{x}}\,\mathrm{div}\,\bar{J}_{vk_v0}\, e^{i\bar{k}_v\cdot\bar{x}} - \frac{i}{K_v}\sqrt{\frac{E_{vk_v}}{2\hbar c K_v}}\sqrt{\frac{\hbar c K_m}{2E_{mk_m}}}\, e^{i\bar{k}_v\cdot\bar{x}}\,\bar{J}_{vk_v0}\cdot \mathrm{grad}\, e^{-i\bar{k}_m\cdot\bar{x}}\right]\psi_c^*\right\}$$

$$(e)\; H_{AFIII} = \frac{\hbar c K_m}{V^{1/2}}\frac{1}{\sqrt{2}}\int dV\left\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\left[-\eta_v g_4^*\sqrt{\frac{E_{mk_m}}{2\hbar c K_m}}\,\psi_c^* e^{-i\bar{k}_m\cdot\bar{x}}\right]\Psi_N\right\} \tag{75}$$

where the nuclear matrix elements contain implicitly a sum over all protons in the nucleus and $\epsilon = 1, -1$. Now we treat the nucleons non-relativistically. Thus we have

$$\Phi_N^*\frac{\tau_x+i\tau_y}{2}\begin{Bmatrix}\bar{\alpha}\\ \bar{\gamma}\end{Bmatrix}\Psi_N \approx 0 \tag{76}$$

and

$$\Phi_N^*\frac{\tau_x+i\tau_y}{2}\beta\bar{\sigma}\Psi_N \approx \Phi_N^*\frac{\tau_x+i\tau_y}{2}\bar{\sigma}\Psi_N \tag{77}$$

Also we assume that the energy imparted to the nucleus is negligible compared to the rest energy of the vector π-meson. Then

$$E_A - E_I = E_A - E_{II} \approx -E_{vk_v} \tag{78}$$

Substituting equations (75), (76), (77), and (78) into equation (74), we find

$$H_{AF} = -i\frac{(\hbar c K_c)^3}{V^{3/2}}\frac{s_v g_4^*}{2K_c^2}\sqrt{\frac{\hbar c K_m}{E_{mk_m}}}\sum_{k_v}\sum_{\epsilon=1,-1}\int dV\int dV'\,\frac{(\bar{J}_{vk_v\epsilon}\cdot\bar{k}_m)(\bar{J}_{vk_v\epsilon}\cdot\bar{k}_v\times\bar{t}_N')}{E^2_{vk_v}}\,\psi_c^* e^{-i\bar{k}_m\cdot\bar{x}+i\bar{k}_v\cdot(\bar{x}'-\bar{x})}$$

$$+\frac{(\hbar c K_m)^3}{V^{3/2}}\frac{\eta_v g_4^*}{2K_m^2}\sqrt{\frac{E_{mk_m}}{\hbar c K_m}}\sum_{k_v}\int dV\int dV'\,v_{0N}'\,\psi_c^*\frac{k_v^2}{E^2_{vk_v}}\,e^{-i\bar{k}_m\cdot\bar{x}+i\bar{k}_v\cdot(\bar{x}'-\bar{x})}$$

$$-\frac{\hbar c K_m}{V^{1/2}}\frac{\eta_v g_4^*}{2}\sqrt{\frac{E_{mk_m}}{\hbar c K_m}}\int dV\, v_{0N}\,\psi_c^*\, e^{-i\bar{k}_m\cdot\bar{x}}$$

where the primed and unprimed quantities refer, respectively, to the variables of V' and V. We now make use of the following identities:

$$\sum_{\epsilon=1,-1}(\bar{j}_{\nu k_\nu \epsilon}\cdot\bar{k}_\mu)(\bar{j}_{\nu k_\nu \epsilon}\cdot\bar{k}_\nu\times\bar{t}'_N) = -(\bar{k}_\mu\times\bar{t}'_N\cdot\bar{k}_\nu)$$

$$\sum_{\epsilon=1,-1}\sum_{k_\nu}(\bar{j}_{\nu k_\nu \epsilon}\cdot\bar{k}_\mu)(\bar{j}_{\nu k_\nu \epsilon}\cdot\bar{k}_\nu\times\bar{t}'_N)\frac{e^{-i\bar{k}_\nu\cdot(\bar{x}'-\bar{x})}}{E^2_{\nu k_\nu}} = -i\frac{V}{4\pi}\frac{(\bar{k}_\mu\times\bar{t}'_N)}{\hbar^2c^2}\cdot\operatorname{grad}\frac{e^{-K_\nu R}}{R}$$

$$\sum_{k_\nu}\frac{(\hbar c k_\nu)^2}{E^2_{\nu k_\nu}}e^{-i\bar{k}_\nu\cdot(\bar{x}'-\bar{x})} = \delta(\bar{R}) - \frac{V}{4\pi}K_\nu^2\frac{e^{-K_\nu R}}{R}$$

where $\bar{R} = \bar{x}' - \bar{x}$. Substituting these into H_{AF}, we find

$$H_{AF} = \frac{\rho_\nu g_4^*}{V^{1/2}}\frac{\hbar c K_c}{8\pi}\sqrt{\frac{\hbar c K_\mu}{E_{\mu k_\mu}}}\int dV\int dV'\left[\bar{t}'_N\times\bar{k}_\mu\cdot\operatorname{grad}\frac{e^{-K_\nu R}}{R}\right]\psi_c^* e^{i\bar{k}_\mu\cdot\bar{R}-i\bar{k}_\mu\cdot\bar{x}'}$$

$$-\frac{\eta_\nu g_4^*}{V^{1/2}}\frac{\hbar c K_\mu}{8\pi}K_\nu^2\sqrt{\frac{E_{\mu k_\mu}}{\hbar c K_\mu}}\int dV\int dV'\, v'_{ON}\,\psi_c^*\,\frac{e^{-K_\nu R+i\bar{k}_\mu\cdot\bar{R}-i\bar{k}_\mu\cdot\bar{x}'}}{R}$$

Since the principal contribution of ψ_c^* to this integral occurs in the neighborhood of R = 0, we can treat ψ_c^* as a constant which constant we take to be the value of ψ_c^* at the center of the nucleus (Appendix 1). Carrying out the integration over the variables of V, we find that the first integral in H_{AF} vanishes and we obtain

$$H_{AF} = -\frac{(\hbar c K_\nu)^2}{V^{1/2}}\frac{\eta_\nu g_4^*}{2}\frac{E_{\mu k_\mu}}{(\hbar c k_\mu)^2+(\hbar c K_\nu)^2}\sqrt{\frac{\hbar c K_\mu}{E_{\mu k_\mu}}}\,\psi_c^*(0)\int dV\, v_{ON}\, e^{-i\bar{k}_\mu\cdot\bar{x}} \tag{79}$$

where the primes have been omitted. Substituting equation (79) into equation (73), making the summation in the nuclear matrix element explicit, and using

$$\frac{p_\mu^2}{(2\pi\hbar)^3}\frac{dp_\mu}{dE_F} = \frac{\hbar c k_\mu E_{\mu k_\mu}}{(2\pi\hbar c)^3}$$

we have the result

$$\frac{1}{\tau_4} = \frac{|K_\nu^{3/2} g_4|^2 |K_\nu^{3/2} \Lambda_\nu|^2}{4\pi}\frac{\hbar c k_\mu}{\hbar}\left(\frac{E_{\mu k_\mu}}{\hbar c K_\nu}\right)^2 \frac{K_\mu}{K_\nu}\left[\frac{K_\nu^2}{k_\mu^2 + K_\nu^2}\right]^2 \frac{|\psi_c(0)|^2}{K_\nu^3}\left|\sum\int dV\,\Phi_N^* \frac{\tau_x + i\tau_y}{2}\Psi_N e^{-i\bar{k}_\mu \bar{x}}\right|^2$$

APPENDIX 5 SCALAR-SCALAR FIELD

Formulation of the Field

The Lagrangian density of the field is

$$L = L_{SCALAR} + L_{CSCALAR} + L_{MSCALAR} + L_5 + L'_{SCALAR} \tag{80}$$

where these terms are defined in equations (1), (5), and (6). The canonical momenta are

$$p(\varphi) = \frac{\partial L}{\partial\left(\frac{\partial \varphi}{\partial t}\right)} = \hbar \chi_0^*$$

$$p(\varphi_c) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_c}{\partial t}\right)} = \hbar \chi_{co}^*$$

$$p(\varphi_m) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_m}{\partial t}\right)} = \hbar \chi_{mo}^* \tag{81}$$

Substituting equations (80) and (81) into

$$H_5 = p(\varphi)\frac{\partial \varphi}{\partial t} + p(\varphi_c)\frac{\partial \varphi_c}{\partial t} + p(\varphi_m)\frac{\partial \varphi_m}{\partial t} - L$$

we have

$$H_5 = \hbar c \kappa_s \left[\varphi^*\varphi + \chi_0^*\chi_0 + \chi_i^*\chi_i\right] + H_{CSCALAR} + H_{MSCALAR}$$

$$+ \hbar c \kappa_s \left[\Lambda_s w^{(N)} \varphi^* + c.c.\right]$$

$$+ \hbar c \kappa_s \left[g_5 \varphi_s^* \varphi_c \varphi_m + c.c.\right]$$

In vector notation this becomes

$$H_5 = \hbar c \kappa_s \left[|\psi|^2 + |\pi|^2 + 1/\kappa_s^2 |\mathrm{grad}\,\psi|^2 \right] + H_{cSCALAR} + H_{mSCALAR}$$

$$+ \hbar c \kappa_s \left[\Phi_N^* \frac{\tau_x + i\tau_y}{2} \{ r_s \beta \psi_s^* \} \Psi_N + c.c. \right]$$

$$+ \hbar c \kappa_s \left[g_5 \psi^* \psi_c \psi_m \quad + c.c. \right] \tag{82}$$

Lifetime of the Scalar π-Meson

The decay of the scalar π-meson takes place by the reaction

$$\pi^- \rightarrow \mu^- + \mu^0$$

The lifetime of this process is (Appendix 1)

$$\frac{1}{\tau_{05}} = \frac{2\pi}{\hbar} \int_{\Omega_c} |H_{AF}|^2 \frac{P_c^2}{(2\pi\hbar)^3} \frac{dP_c}{dE_F} d\Omega_c V \tag{83}$$

The element of H_5 giving rise to the decay process is the coefficient in H_5 of

$$b_{sk_s} b_{ck_c}^* a_{mk_m}^*$$

wherein plane waves are assumed for the wave functions with $\bar{k}_c = 0$ and thus $\bar{k}_c + \bar{k}_n = 0$. This element is

$$H_{AF} = \frac{\hbar c \kappa_s}{4V^{1/2}} g_5 \sqrt{\frac{\hbar c \kappa_c}{E_{ck_c}}} \sqrt{\frac{\hbar c \kappa_m}{E_{mk_m}}}$$

And thus

$$|H_{AF}|^2 = \frac{(\hbar c \kappa_s)^2}{16V} |g_5|^2 \frac{\hbar c \kappa_c}{E_{ck_c}} \frac{\hbar c \kappa_m}{E_{mk_m}} \tag{84}$$

Using

$$\bar{k}_c + \bar{k}_m = 0$$

and

$$E_{ck_c} + E_{mk_m} = E_F = m_s c^2$$

we find

$$E_{mk_m} E_{ck_c} = \frac{E_F^4 - (m_c^2 - m_m^2)^2 c^8}{4 E_F^2} \tag{85}$$

and

$$p_c^2 \frac{dp_c}{dE_F} = \frac{1}{8c^3 E_F^4}\left[E_F^2 - (m_m + m_c)^2 c^4\right]^{1/2}\left[E_F^2 - (m_m - m_c)^2 c^4\right]^{1/2}\left[E_F^2 + (m_m^2 - m_c^2)c^4\right]\left[E_F^2 - (m_m^2 - m_c^2)c^4\right] \tag{86}$$

Substituting equations (84), (85), and (86) into equation (83), we have the result

$$\frac{1}{\tau_{0s}} = \frac{|K_s^{3/2} g_s|^2}{32\pi} \frac{m_s c^2}{\hbar} \frac{m_m}{m_s} \frac{m_s}{m_s}\left[1 - \left(\frac{m_m}{m_s} + \frac{m_c}{m_s}\right)^2\right]^{1/2}\left[1 - \left(\frac{m_m}{m_s} - \frac{m_c}{m_s}\right)^2\right]^{1/2}$$

<u>Lifetime for the Capture of the Scalar μ-Meson by a Nucleus</u>

The capture of the negative scalar μ-meson takes place by one of the two reactions:

$$\text{I} \quad \mu^- \rightarrow \left\{\begin{matrix} \pi^- + \mu^0 \\ \pi^- + P \end{matrix}\right\} \rightarrow N$$

$$\text{II} \quad P \rightarrow \left\{\begin{matrix} \pi^+ + N \\ \pi^+ + \mu^- \end{matrix}\right\} \rightarrow \mu^0$$

The lifetime of this capture process is (Appendix 1)

$$\frac{1}{\tau_5} = \frac{2\pi}{\hbar}\int_{\Omega_M} |H_{AF}|^2 \frac{P_M^2}{(2\pi\hbar)^3}\frac{dP_M}{dE_F}\, d\Omega_M V \tag{87}$$

where

$$H_{AF} = \sum\left[\frac{H_{AI}H_{IF}}{E_A - E_I} + \frac{H_{AII}H_{IIF}}{E_A - E_{II}}\right] \tag{88}$$

To obtain H_{AF} we use plane waves for the neutral meson and approximate the wave function of the virtual scalar π-meson by plane waves. With these wave functions substituted into H_2 we find that the element of H_2 giving

(a) H_{AI} is the coefficient of

$$b_c\, b^*_{sk_s}\, a^*_{mk_m}$$

(b) H_{IF} is the coefficient of

$$b_{sk_s}\frac{\tau_x + i\tau_y}{2}$$

(c) H_{AII} is the coefficient of

$$a^*_{sk_s}\frac{\tau_x + i\tau_y}{2}$$

(d) H_{IIF} is the coefficient of

$$a_{sk_s}\, b_c\, a^*_{mk_m}$$

These elements are

(a) $H_{AI} = \frac{\hbar c \kappa_s}{V}\frac{1}{\sqrt{2}}\int dV\left[-g_5^*\sqrt{\frac{\hbar c \kappa_s}{2E_{sk_s}}}\sqrt{\frac{\hbar c \kappa_m}{2E_{mk_m}}}\,\psi_c^*\right] e^{-i(\bar{k}_s + \bar{k}_m)\cdot\bar{x}}$

(b) $H_{IF} = \frac{\hbar c K_s}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[i \Lambda_s \beta \sqrt{\frac{\hbar c K_s}{2 E_{s k_s}}} \right] \Psi_N e^{-i \bar{k}_s \cdot \bar{x}} \right.$

(c) $H_{AII} = \frac{\hbar c K_s}{V^{1/2}} \int dV \left\{ \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[-i \Lambda_s \beta \sqrt{\frac{\hbar c K_s}{2 E_{s k_s}}} \right] \Psi_N e^{-i \bar{k}_s \cdot \bar{x}} \right.$

(d) $H_{IIF} = \frac{\hbar c K_s}{V} \frac{1}{\sqrt{2}} \int dV \left[g_5^* \sqrt{\frac{\hbar c K_s}{2 E_{s k_s}}} \sqrt{\frac{\hbar c K_m}{2 E_{m k_m}}} \psi_c^* \right] e^{i(\bar{k}_s - \bar{k}_m) \cdot \bar{x}}$

(89)

where the nuclear matrix elements contain implicitly a sum over all protons in the nucleus. We now assume that the energy imparted to the nucleons is negligible compared to the rest energy of the scalar π-meson. Then

$$E_A - E_I = E_A - E_{II} \approx -E_{s k_s} \tag{90}$$

Substituting equations (89) and (90) into equation (88), we find

$$H_{AF} = \frac{(\hbar c K_s)^3}{V^{3/2}} \frac{g_5^* \Lambda_s}{2} i \sqrt{\frac{\hbar c K_m}{E_{m k_m}}} \sum_{k_s} \int dV' \int dV \left[\Phi_N'^* \frac{\tau_x + i\tau_y}{2} \Psi_N' \right] \psi_c^* e^{-i \bar{k}_m \cdot \bar{x}} \frac{e^{i \bar{k}_s \cdot (\bar{x}' - \bar{x})}}{E_{s k_s}^2}$$

where the primed and unprimed quantities refer, respectively, to the variables of V' and V. Making use of the identity

$$\sum_{k_s} \frac{e^{i \bar{k}_s \cdot (\bar{x}' - \bar{x})}}{E_{s k_s}^2} = \frac{V}{4\pi} \frac{1}{\hbar^2 c^2} \frac{e^{-K_s R}}{R}$$

where $\bar{R} = \bar{x}' - \bar{x}$, we find

$$H_{AF} = \frac{(\hbar c K_s)^3}{V^{1/2}} \frac{g_5^* \Lambda_s}{8\pi \hbar^2 c^2} i \sqrt{\frac{\hbar c K_m}{E_{m k_m}}} \int dV' \Phi_N'^* \frac{\tau_x + i\tau_y}{2} \Psi_N' e^{-i \bar{k}_m \cdot \bar{x}'} \int dV \, \psi_c^* \frac{e^{-K_s R + i \bar{k}_m \cdot \bar{R}}}{R}$$

Since the principal contribution of ψ_c^* to this integral occurs in the neighborhood of $R = 0$, we can treat ψ_c^* as a constant, which constant we take to be the value of ψ_c^* at the center of the nucleus (Appendix 1). The integration over the variables of V now gives

$$H_{AF} = \frac{\hbar c K_s}{V^{1/2}} \frac{g_5^* \lambda_s}{2} i \sqrt{\frac{\hbar c K_m}{E_{mk_m}}} \left[\frac{K_s^2}{k_m^2 + K_s^2}\right] \psi_c^*(0) \int dV \, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \Phi_N e^{-i\bar{k}_m \cdot \bar{x}} \tag{91}$$

where the primes have been omitted. Substituting equation (91) into equation (87), making the nuclear summation explicit, and using the relation

$$\frac{P_m^2}{(2\pi\hbar)^3} \frac{dP_m}{dE_F} = \frac{\hbar c k_m E_{mk_m}}{(2\pi\hbar c)^3}$$

we have the result that

$$\frac{1}{\tau_5} = \frac{|K_s^{3/2} g_5|^2 |K_s^{3/2} \lambda_s|^2}{4\pi} \frac{\hbar c k_m}{\hbar} \frac{K_m}{K_s} \left[\frac{K_s^2}{k_m^2 + K_s^2}\right]^2 \frac{|\psi_c(0)|^2}{K_s^3} \left| \sum \int dV \, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \Phi_N e^{-i\bar{k}_m \cdot \bar{x}} \right|^2$$

APPENDIX 6 VECTOR-PSEUDOSCALAR FIELD

The Lagrangian density of this field is

$$L = L_{VECTOR} + L_{CPSEUDOSCALAR} + L_{MPSEUDOSCALAR} + L_{G} + L'_{VECTOR}$$

where these terms are defined in equations (2), (3), (5), and (8). If in L we replace

χ_{C0123}	by	$\bar{\varphi}_{C}$
χ_{M0123}	by	$\bar{\varphi}_{M}$
φ_{C0ik}	by	$\bar{\chi}_{C\ell}$
φ_{M0ik}	by	$\bar{\chi}_{M\ell}$.
φ_{C123}	by	$\bar{\chi}_{C0}$
φ_{M123}	by	$\bar{\chi}_{M0}$

then the Lagrangian of this field is identical to that of the vector-scalar field.

APPENDIX 7 SCALAR-PSEUDOSCALAR FIELD

The Lagrangian density of this field is

$$L = L_{SCALAR} + L_{CPSEUDOSCALAR} + L_{MPSEUDOSCALAR} + L_{7} + L'_{SCALAR}$$

where these terms are defined in equations (1), (3), (5), and (6). If in L we replace

χ_{c0123}	by	$\bar{\varphi}_{c}$
χ_{m0123}	by	$\bar{\varphi}_{m}$
φ_{c0ik}	by	$\bar{\chi}_{cl}$
φ_{m0ik}	by	$\bar{\chi}_{ml}$
φ_{c123}	by	$\bar{\chi}_{c0}$
φ_{m123}	by	$\bar{\chi}_{m0}$

then the Lagrangian of this field is identical to that of the scalar-scalar field.

APPENDIX 8 VECTOR-SCALAR-PHOTON FIELD

Formulation of the Field

The Lagrangian density of the field is

$$L = -\hbar c\kappa_v(\varphi_\alpha^*\varphi^\alpha + \tfrac{1}{2}\chi_{\alpha\beta}^*\chi^{\alpha\beta}) - \hbar c\kappa_s(\varphi^*\varphi + \chi_\alpha^*\chi^\alpha)$$

$$-\hbar c\kappa_v(\eta_v v_\alpha^{(N)}\varphi^{\alpha *} + g_v \tfrac{1}{2} u_{\alpha\beta}^{(N)}\chi^{\alpha\beta *} \quad + \text{c.c.})$$

$$-\hbar c\kappa_s(f\varphi_\alpha\chi^{\alpha *} \quad + \text{c.c.})$$

(92)

The canonical momenta are

$$p(\varphi_i) = \frac{\partial L}{\partial\left(\frac{\partial \varphi_i}{\partial t}\right)} = \hbar \xi_{0i}^* = \hbar(\chi_{0i}^* + g_v^* u_{0i}^*)$$

$$p(\varphi) = \frac{\partial L}{\partial\left(\frac{\partial \varphi}{\partial t}\right)} = \hbar \xi_0^* = \hbar(\chi_0^* + f^*\varphi_0^*)$$

(93)

where i = 1, 2, 3. As in the vector-spin $\frac{1}{2}$ field (Appendix 1) we must eliminate φ_0 from the Hamiltonian of the field by means of the equation of motion of φ_0, which is

$$\frac{\partial L}{\partial \varphi_0^*} - \sum_\alpha \frac{\partial}{\partial x^\alpha}\frac{\partial L}{\partial\left(\frac{\partial \varphi_0^*}{\partial x^\alpha}\right)} = 0$$

or

$$\kappa_v \eta_0 = -\frac{\partial \xi_{0i}}{\partial x^i} = \kappa_v(\varphi_0 + \eta_v v_0) + \kappa_s f^*\chi_0$$

But now

$$\chi_0 = \xi_0 - f\varphi_0$$

Then

$$\kappa_v \eta_0 = -\frac{\partial \zeta_{0i}}{\partial x^i} = (\kappa_v - \kappa_s|f|^2)\,\varphi_0 + \kappa_v \lambda_v \nu_0 + \kappa_s f^* \zeta_0$$

But $|f|^2$ is very small. Thus

$$\kappa_v \eta_0 = -\frac{\partial \zeta_{0i}}{\partial x^i} = \kappa_v \varphi_0 + \kappa_v \lambda_v \nu_0 + \kappa_s f^* \zeta_0 \qquad (94)$$

Substituting equations (92), (93), and (94) into

$$H = \sum_i P(\varphi_i)\frac{\partial \varphi_i}{\partial t} + P(\varphi)\frac{\partial \varphi}{\partial t} - L$$

we have

$$H = \hbar c \kappa_v\left[\varphi_i^* \varphi_i + \eta_0^* \eta_0 + \zeta_{0i}^* \zeta_{0i} + \tfrac{1}{2}\chi_{ik}^* \chi_{ik}\right] + \hbar c \kappa_v\left[\varphi^* \varphi + \zeta_0^* \zeta_0 + \chi_i^* \chi_i\right]$$

$$+ \hbar c \kappa_v\left[\lambda_v(\nu_i \varphi_i^* - \nu_0 \eta_0) - \zeta_v(\mu_{0i} \zeta_{0i}^* - \tfrac{1}{2}\mu_{ik} \chi_{ik}^*) + c.c.\right]$$

$$+ \hbar c \kappa_s\left[f(\varphi_i \chi_i^* - \eta_0 \zeta_0^*) + c.c.\right]$$

$$+ \hbar c \kappa_v\left[|\zeta_v|^2 |\mu_{0i}|^2 + |\lambda_v|^2 |\nu_0|^2\right]$$

$$+ \hbar c \kappa_s\left[|f|^2\left(|\eta_0|^2 + \frac{\kappa_s}{\kappa_v}|\zeta_0|^2\right)\right]$$

$$+ \hbar c \kappa_s\left[f \lambda_v \nu_0 \zeta_0^* + c.c.\right]$$

$$- \hbar c \kappa_s\left[|f|^2 \lambda_v \eta_0^* \nu_0 + c.c.\right]$$

$$- \hbar c \kappa_s\left[|f|^2\left(\frac{\kappa_s}{\kappa_v} f^* \zeta_0 \eta_0^* - f \lambda_v \frac{\kappa_s}{\kappa_v} \nu_0 \zeta_0^*\right) + c.c.\right]$$

$$+\hbar c K_s\left[|f|^2|\rho_N|^2|\nu_0|^2\right]$$

$$+\hbar c K_s\left[|f|^4\left(\frac{K_s}{K_N}\right)^2|\xi_0|^2\right]$$

We include the electromagnetic field by the prescription

$$\frac{\partial}{\partial x^\alpha}\rightarrow\frac{\partial}{\partial x^\alpha}-\frac{ie}{\hbar c}A_\alpha$$

and then change to vector notation. The Hamiltonian becomes then

$$H=\hbar c K_v\Big\{\Big[|\psi_N|^2+|\pi_N|^2+1/K_v^2\,|\mathrm{curl}\,\psi_N|^2+|\Psi_N|^2+|\Pi_N|^2+1/K_N^2\,|\mathrm{div}\,\Psi_N|^2\Big]$$

$$-\frac{ie}{\hbar c}\frac{1}{K_v^2}\Big[\bar{A}\cdot(\Psi_N-\pi_N^*)\,\mathrm{div}\,\Psi_N^*+\bar{A}\times(\psi_N+\Pi_N^*)\cdot\mathrm{curl}\,\psi_N^*-\mathrm{c.c.}\Big]$$

$$+\frac{e^2}{(\hbar c K_v)^2}\Big[|\bar{A}\times(\psi_N+\Pi_N^*)|^2+|\bar{A}\cdot(\Psi_N-\pi_N^*)|^2\Big]+\Big[\frac{ie}{\hbar c K_v}A^0(\Psi_N-\pi_v^*)\cdot(\psi_N^*+\Pi_N)+\mathrm{c.c.}\Big]\Big\}$$

$$+\hbar c K_s\Big\{\Big[|\psi_s|^2+|\pi_s|^2+1/K_s^2\,|\mathrm{grad}\,\psi_s|^2\Big]$$

$$+\frac{ie}{\hbar c K_s}\Big[1/K_s\,\psi_s^*\bar{A}\cdot\mathrm{grad}\,\psi_s+A^0\psi_s\pi_s^*-\mathrm{c.c.}\Big]+\frac{e^2}{(\hbar c K_s)^2}|\bar{A}\psi_s|^2\Big\}$$

$$+\hbar c K_v\Big\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\Big[\rho_N\bar{\alpha}\cdot\psi_N^*-\xi_N(\bar{\gamma}\cdot\pi_N-1/K_v\,\beta\bar{\sigma}\cdot\mathrm{curl}\,\psi_N^*)+\xi_N\bar{\gamma}\cdot\Psi_N^*+\rho_N(\bar{\alpha}\cdot\Pi_v+1/K_v\,\mathrm{div}\,\Psi_N^*)$$

$$+\xi_N\frac{ie}{\hbar c K_v}\bar{A}\times(\psi_N^*+\Pi_N)\cdot\beta\bar{\sigma}-\rho_N\frac{ie}{\hbar c K_v}\bar{A}\cdot(\pi_N-\Psi_N^*)\Big]\Psi_N+\mathrm{c.c.}\Big\}$$

$$+\hbar c\kappa_s\left\{f\left[\tfrac{1}{\kappa_s}(\psi_v+\pi_v^*)\cdot(\operatorname{grad}+\tfrac{ie}{\hbar c}\bar{A})\psi_s^*-\tfrac{1}{\kappa_v}\pi_s\left(\operatorname{div}\Psi_v+\tfrac{ie}{\hbar c}\bar{A}\cdot(\pi_v^*-\Psi_v)\right)\right]+c.c.\right\}$$

$$+\hbar c\kappa_v\left[|g_N|^2\left|\Phi_N^*\tfrac{\tau_x+i\tau_y}{2}\gamma\Psi_N\right|^2+|\Omega_N|^2\left|\Phi_N^*\tfrac{\tau_x+i\tau_y}{2}\Psi_N\right|^2\right]$$

$$+\hbar c\kappa_s\left[\frac{|f|^2}{\kappa_v^2}\left|\operatorname{div}\Psi_v^*-\frac{ie}{\hbar c}\bar{A}\cdot(\pi_v-\Psi_v^*)\right|^2+|f|^2\frac{\kappa_s}{\kappa_v}|\pi_s|^2\right]$$

$$-\hbar c\kappa_s\left\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\left[f\Omega_N\pi_s\right]\Psi_N+c.c.\right\}$$

$$-\hbar c\kappa_s\left\{\Phi_N^*\frac{\tau_x+i\tau_y}{2}\left[|f|^2\Omega_N\left(\operatorname{div}\Psi_v^*-\frac{ie}{\hbar c}\bar{A}\cdot(\pi_v-\Psi_v^*)\right)\right]\Psi_N+c.c.\right\}$$

$$+\hbar c\kappa_s\left\{|f|^2\left[\frac{\kappa_s}{\kappa_v}f^*\pi_s^*\left(\operatorname{div}\Psi_v^*-\frac{ie}{\hbar c}\bar{A}\cdot(\pi_v-\Psi_v^*)\right)-\Phi_N^*\frac{\tau_x+i\tau_y}{2}\left(\frac{\kappa_s}{\kappa_v}f\Omega_N\pi_s\right)\Psi_N\right]+c.c.\right\}$$

$$+\hbar c\kappa_s\left[|f|^2|\Omega_N|^2\left|\Phi_N^*\frac{\tau_x+i\tau_y}{2}\Psi_N\right|^2\right]$$

$$+\hbar c\kappa_s\left[|f|^4\left(\frac{\kappa_s}{\kappa_v}\right)^2|\pi_s|^2\right] \tag{95}$$

Lifetime of the Vector Meson

The decay of the vector π-meson takes place by the reaction

$$\pi^-\rightarrow\mu^-+h\nu$$

The lifetime of this process is (Appendix 1)

$$\frac{1}{\tau_0}=\frac{2\pi}{\hbar}\int_{\Omega_\nu}\sum_{\text{Polar.}}|H_{AF}|^2\frac{P_\nu^2}{(2\pi\hbar)^3}\frac{dP_\nu}{dE_F}d\Omega_\nu V \tag{96}$$

where the summation is over the polarization of the photon. The element of H giving rise to the decay process is the coefficient in H of

$$b_{\nu k_\nu e}\, b^*_{s k_s}\, g^*_{\nu k_\nu}$$

wherein plane waves are assumed for the wave functions with $\bar{k}_V = 0$ and thus $\bar{k}_S + \bar{k}_\nu = 0$. This element is

$$H_{AF} = \frac{\hbar c K_s}{\sqrt{V/2}} \frac{f}{2} \frac{ie}{\hbar c}\left[\frac{1}{K_s}\sqrt{\frac{\hbar c K_s}{E_{sk_s}}} - \frac{1}{K_\nu}\sqrt{\frac{E_{sk_s}}{\hbar c K_s}}\right](\bar{e}_k \cdot \bar{j}_{\nu k_\nu e})\sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}}$$

where $\bar{e}_k$ is the direction of polarization of the photon. Now

$$\sum_{\text{POLAR.}} |H_{AF}|^2 = \frac{(\hbar c K_s)^2}{4V} |f|^2 \frac{2\pi e^2}{E_{\nu k_\nu}}\left[\frac{1}{K_s}\sqrt{\frac{\hbar c K_s}{E_{sk_s}}} - \frac{1}{K_\nu}\sqrt{\frac{E_{sk_s}}{\hbar c K_s}}\right]^2 \sin^2\theta \tag{97}$$

where θ is the angle between $\bar{k}_\nu$ and $\bar{j}_{\nu k_\nu e}$. Using

$$\bar{k}_s + \bar{k}_\nu = 0$$

and

$$E_{sk_s} + E_{\nu k_\nu} = E_F = m_\nu c^2$$

we find

$$E_{\nu k_\nu} = \frac{m_\nu c^2}{2}\left[1 - \left(\frac{m_s}{m_\nu}\right)^2\right] \tag{98}$$

$$E_{sk_s} = \frac{m_\nu c^2}{2}\left[1 + \left(\frac{m_s}{m_\nu}\right)^2\right] \tag{99}$$

and

$$P_\nu^2 \frac{dP_\nu}{dE_F} = E_{sk_s} \frac{m_\nu}{4c}\left[1 - \left(\frac{m_s}{m_\nu}\right)^2\right]^2 \tag{100}$$

Substituting equations (97), (98), (99), and (100) into equation (96), we have the result

$$\frac{1}{\tau_0} = \frac{|f|^2}{24} \frac{m_s c^2}{\hbar} \frac{e^2}{\hbar c}\left[1 - \left(\frac{m_s}{m_N}\right)^2\right]^3$$

<u>Lifetime for the Capture of the Scalar Meson by a Nucleus</u>

The capture of a negative scalar meson takes place by one of the three reactions:

$$\text{I} \quad \mu^- \rightarrow \left\{ \begin{matrix} \pi^- + \mu^0 \\ \pi^- + P \end{matrix} \right\} \rightarrow N$$

$$\text{II} \quad P \rightarrow \left\{ \begin{matrix} \pi^+ + N \\ \pi^+ + \mu^- \end{matrix} \right\} \rightarrow \mu^0$$

$$\text{III} \quad \mu^- \rightarrow \left\{ \begin{matrix} \pi^- \\ \pi^- + P \end{matrix} \right\} \rightarrow N + h\nu$$

The lifetime of this process is (Appendix 1)

$$\frac{1}{\tau} = \frac{2\pi}{\hbar} \int_{\Omega_\nu} \sum_{\text{POLAR.}} |H_{AF}|^2_{\text{AV.}} \frac{P_\nu^2}{(2\pi\hbar)^3} \frac{dP_\nu}{dE_F} d\Omega_\nu V \tag{101}$$

where

$$H_{AF} = \sum \left[\frac{H_{AI} H_{IF}}{E_A - E_I} + \frac{H_{AII} H_{IIF}}{E_A - E_{II}} + \frac{H_{AIII} H_{IIIF}}{E_A - E_{III}} \right] \tag{102}$$

To obtain H_{AF} we use plane waves for the photon and approximate the wave function of the virtual vector π-meson. With these wave functions substituted into H and making use of the well-known relation

$$\pi_s = -\frac{i(E_s + e\varphi)}{\hbar c K_s}\psi_s^*$$

for a scalar meson in a Coulomb field φ, we find that the element of H giving

(a) H_{AI} is the coefficient of

$$b_s\, b^*_{\nu k_\nu \epsilon}\, g^*_{\nu k_\nu} \qquad \epsilon = 1, 0, -1$$

(b) H_{IF} is the coefficient of

$$b_{\nu k_\nu \epsilon}\, \frac{\tau_x + i\tau_y}{2} \qquad \epsilon = 1, 0, -1$$

(c) H_{AII} is the coefficient of

$$a^*_{\nu k_\nu \epsilon}\, \frac{\tau_x + i\tau_y}{2} \qquad \epsilon = 1, 0, -1$$

(d) H_{IIF} is the coefficient of

$$a_{\nu k_\nu \epsilon}\, b_s\, g^*_{\nu k_\nu} \qquad \epsilon = 1, 0, -1$$

(e) H_{AIII} is the coefficient of

$$b^*_{\nu k_\nu \epsilon}\, b_s \qquad \epsilon = 1, 0, -1$$

(f) H_{IIIF} is the coefficient of

$$b_{\nu k_\nu}\, g^*_{\nu k_\nu}\, \frac{\tau_x + i\tau_y}{2} \qquad \epsilon = 1, 0, -1$$

Now we treat the nucleons non-relativistically. Thus we have

$$\Phi_N^*\, \frac{\tau_x + i\tau_y}{2} \begin{Bmatrix} \bar{\alpha} \\ \bar{\gamma} \end{Bmatrix} \Psi_N \approx 0$$

$$\Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \bar{\sigma} \Psi_N \approx \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \Psi_N$$

The above elements are then

$$\text{(a)}\ H_{AI}^{\epsilon} = \frac{f}{V} e \sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}} \int dV \left[\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} - \sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} \frac{E_s + e\varphi}{\hbar c K_\nu} \right] (\bar{e}_{k} \cdot \bar{J}_{\nu k_\nu \epsilon}) \psi_s^* e^{-i(\bar{k}_\nu + \bar{k}_\nu)\cdot\bar{x}}$$

$$H_{AI}^{o} = \frac{f}{V} e \sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}} \int dV \left[\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} - \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \frac{E_s + e\varphi}{\hbar c K_\nu} \right] (\bar{e}_{k} \cdot \bar{J}_{\nu k_\nu o}) \psi_s^* e^{-i(\bar{k}_\nu + \bar{k}_\nu)\cdot\bar{x}}$$

$$\text{(b)}\ H_{IF}^{\epsilon} = \frac{s_\nu}{V^{1/2}} i\hbar c \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \cdot \mathrm{curl}(\bar{J}_{\nu k_\nu \epsilon} e^{i\bar{k}_\nu\cdot\bar{x}}) \Psi_N$$

$$H_{IF}^{o} = -\frac{r_\nu}{V^{1/2}} \hbar c \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \mathrm{div}(\bar{J}_{\nu k_\nu o} e^{i\bar{k}_\nu\cdot\bar{x}}) \Psi_N$$

$$\text{(c)}\ H_{AII}^{\epsilon} = -\frac{s_\nu}{V^{1/2}} i\hbar c \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \cdot \mathrm{curl}(\bar{J}_{\nu k_\nu \epsilon} e^{-i\bar{k}_\nu\cdot\bar{x}}) \Psi_N$$

$$H_{AII}^{o} = -\frac{r_\nu}{V^{1/2}} \hbar c \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \mathrm{div}(\bar{J}_{\nu k_\nu o} e^{-i\bar{k}_\nu\cdot\bar{x}}) \Psi_N$$

$$\text{(d)}\ H_{IIF}^{\epsilon} = -\frac{f}{V} e \sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}} \int dV \left[\sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} + \sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} \frac{E_s + e\varphi}{\hbar c K_\nu} \right] (\bar{e}_{k} \cdot \bar{J}_{\nu k_\nu \epsilon}) \psi_s^* e^{i(\bar{k}_\nu - \bar{k}_\nu)\cdot\bar{x}}$$

$$H_{IIF}^{o} = -\frac{f}{V} e \sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}} \int dV \left[\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} + \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \frac{E_s + e\varphi}{\hbar c K_\nu} \right] (\bar{e}_{k} \cdot \bar{J}_{\nu k_\nu o}) \psi_s^* e^{i(\bar{k}_\nu - \bar{k}_\nu)\cdot\bar{x}}$$

$$\text{(e)}\ H_{AIII}^{\epsilon} = -\frac{f}{V^{1/2}} i\hbar c \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \int dV\, (\bar{J}_{\nu k_\nu \epsilon} \cdot \mathrm{grad}\, \psi_s^*) e^{-i\bar{k}_\nu\cdot\bar{x}}$$

$$H_{AIII}^{o} = -\frac{f}{V^{1/2}} i\hbar c \int dV \left[\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} (\bar{J}_{\nu k_\nu o} \cdot \mathrm{grad}\, \psi_s^*) e^{-i\bar{k}_\nu\cdot\bar{x}} + \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} \psi_s^* \mathrm{div}(\bar{J}_{\nu k_\nu o} e^{-i\bar{k}_\nu\cdot\bar{x}}) \frac{E_s + e\varphi}{\hbar c K_\nu} \right]$$

$$\text{(f)}\ H_{IIIF}^{\epsilon} = -\frac{e}{V} \sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}} \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \left[s_\nu \sqrt{\frac{\hbar c K_\nu}{2E_{\nu k_\nu}}} (\bar{\sigma} \cdot \bar{e}_k \times \bar{J}_{\nu k_\nu \epsilon}) + i r_\nu \sqrt{\frac{E_{\nu k_\nu}}{2\hbar c K_\nu}} (\bar{e}_k \cdot \bar{J}_{\nu k_\nu \epsilon}) \right] \Psi_N e^{i(\bar{k}_\nu - \bar{k}_\nu)\cdot\bar{x}}$$

$$H^{\circ}_{\mathrm{III}F} = -\frac{e}{V}\sqrt{\frac{2\pi\hbar^2c^2}{E_{\nu k_\nu}}}\int dV\, \Phi_N^{*}\frac{\tau_x+i\tau_y}{2}\left[S_\nu\sqrt{\frac{E_{\nu k_\nu}}{2\hbar c\kappa_\nu}}(\bar{\sigma}\cdot\bar{e}_{k}\times\bar{J}_{\nu k_\nu 0})+i\Pi_N\sqrt{\frac{\hbar c\kappa_\nu}{2E_{\nu k_\nu}}}(\bar{e}_{k}\cdot\bar{J}_{\nu k_\nu 0})\right]\Psi_N\, e^{i(\bar{k}_N-\bar{k}_\nu)\cdot\bar{x}} \tag{103}$$

where the nuclear matrix elements contain implicitly a sum over the protons in the nucleus and $\epsilon = 1, -1$. Now one shows by an integration by parts that $H^{\epsilon}_{AIII} = 0$, as it must be since spin is conserved. Also one can show that

$$\sum_{k_\nu}(\bar{e}_{k}\cdot\bar{k}_\nu)\int d\bar{R}\; f(\bar{k}_\nu\cdot\bar{R},\bar{k}_\nu\cdot\bar{R},R) = 0 \tag{104}$$

Now we assume that the energy imparted to the nucleons is small compared to the rest energy of the vector π-meson. Then

$$E_A - E_{\mathrm{I}} = E_A - E_{\mathrm{II}} = E_A - E_{\mathrm{III}} \simeq -E_{\nu k_\nu} \tag{105}$$

Making use of these results and substituting equations (103), (104), and (105) into equation (102), we find

$$H_{AF} = -\frac{fS_\nu}{V^{3/2}}e\hbar^2c^2\kappa_\nu\sqrt{\frac{2\pi\hbar^2c^2}{E_{\nu k_\nu}}}\sum_{k_\nu}\sum_{\epsilon=1,-1}\int dV\int dV'\,\frac{(\bar{e}_{k}\cdot\bar{J}_{\nu k_\nu \epsilon})(\bar{t}'_N\cdot\bar{J}_{\nu k_\nu \epsilon}\times\bar{k}_\nu)}{E^2_{\nu k_\nu}}\,\psi_s^{*}\, e^{-i(\bar{k}_\nu)\cdot(\bar{x}'-\bar{x})-i\bar{k}_\nu\cdot\bar{x}}$$

$$+\frac{fS_\nu}{V^{3/2}}\frac{e\hbar^2c^2\kappa_\nu}{2}\sqrt{\frac{2\pi\hbar^2c^2}{E_{\nu k_\nu}}}\sum_{k_\nu}\int dV\int dV'\left(\frac{1}{E_{\nu k_\nu}}-\frac{E_s+e\varphi}{(\hbar c\kappa_\nu)^2}\right)\frac{\bar{t}'_N\cdot\bar{e}_{k}\times\bar{k}_\nu}{E_{\nu k_\nu}}\,\psi_s^{*}\, e^{i\bar{k}_\nu\cdot(\bar{x}'-\bar{x})-i\bar{k}_\nu\cdot\bar{x}'}$$

where the primed and unprimed quantities refer, respectively, to the variables of V' and V. Further, use has been made of the fact that the principal contribution of ψ_s^{*}, φ to these integrals occurs in the neighborhood of $\bar{x}' - \bar{x} = 0$ and thus we may treat ψ_s^{*}, φ as a constant, which constant we take to be the value of ψ_s^{*}, φ at the center of the nucleus (Appendix 1).

Now we have the following relations:

$$\sum_{\epsilon=1,-1} (\bar{e}_k \cdot \bar{J}_{\nu k_\nu \epsilon})(\bar{t}'_N \cdot \bar{k}_\nu \times \bar{J}_{\nu k_\nu \epsilon}) = \bar{e}_k \times \bar{t}'_N \cdot \bar{k}_\nu$$

$$\sum_{k_\nu} \sum_{\epsilon=1,-1} \frac{1}{E^2_{\nu k_\nu}} (\bar{e}_k \cdot \bar{J}_{\nu k_\nu \epsilon})(\bar{t}'_N \cdot \bar{J}_{\nu k_\nu \epsilon} \times \bar{k}_\nu) e^{-i\bar{k}_\nu \cdot (\bar{x}' - \bar{x})} = \frac{iV}{2\pi^2} |\bar{t}'_N| \frac{\sin\alpha \sin\beta \sin\gamma}{R^2} \left(\frac{\partial}{\partial R} - R\frac{\partial^2}{\partial R^2}\right) I_2(R)$$

$$\sum_{k_\nu} \left(\frac{1}{E_{\nu k_\nu}} - \frac{E_s + e\phi(0)}{(\hbar c K_\nu)^2}\right) \frac{\bar{t}'_N \cdot \bar{e}_k \times \bar{k}_\nu}{E_{\nu k_\nu}} e^{i\bar{k}_\nu \cdot (\bar{x}' - \bar{x})} = \frac{iV}{2\pi^2} |\bar{t}'_N| \frac{\sin\alpha \sin\beta \sin\gamma}{R^2} \left(\frac{\partial}{\partial R} - R\frac{\partial^2}{\partial R^2}\right) \left[I_2(R) + \frac{E_s + e\phi(0)}{(\hbar c K_\nu)^2} I_1(R) \right]$$

where $\bar{R} = \bar{x}' - \bar{x}$

$$I_1(R) = \frac{1}{\hbar c}\left[K_0(K_\nu R) + i\pi\,\alpha(R)\right]$$

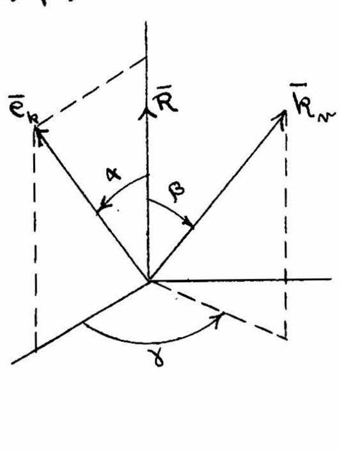
$$I_2(R) = \frac{1}{(\hbar c)^2 K_\nu}\left[\frac{\pi}{2} e^{-K_\nu R} - \pi K_\nu \beta(R)\right]$$

$$\frac{\partial \alpha(R)}{\partial R} = \delta(R) = \text{delta function}$$

$$\frac{\partial \beta(R)}{\partial R} = 0$$

$$\frac{\partial^2 \beta(R)}{\partial R^2} = \delta(R) = \text{delta function}$$

and the angles α, β, γ are shown in the following diagram:



Thus

$$H_{AF}=\frac{fs_{\nu}}{V^{1/2}}\,\frac{ie\hbar^2c^2K_{\nu}}{2\pi^2}\sqrt{\frac{2\pi\hbar^2c^2}{E_{\nu k_\nu}}}\int dV'\,|\bar{t}'_N|\,e^{-i\bar{k}_\nu\cdot\bar{x}'}\int dV\,\psi_s^*(0)\sin\alpha\sin\beta\sin\gamma\left[e^{-i\bar{k}_\nu\cdot\bar{R}}f_1(R)+f_2(R)\right] \tag{106}$$

where

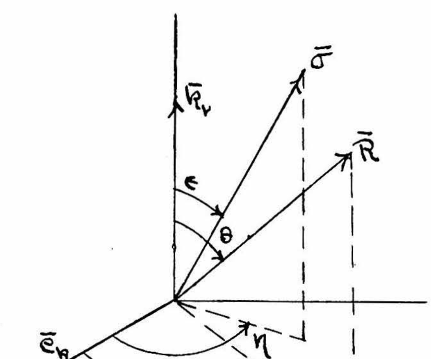
$$f_1(R)=\frac{1}{R^2}\left(\frac{\partial}{\partial R}-R\frac{\partial^2}{\partial R^2}\right)I_2(R)$$

$$f_2(R)=\frac{1}{R^2}\left(\frac{\partial}{\partial R}-R\frac{\partial^2}{\partial R^2}\right)\left[I_2(R)+\frac{E_s+e\varphi(0)}{(\hbar cK_\nu)^2}I_1(R)\right]$$

Now a short analysis shows that

$$\sin\alpha\sin\beta\sin\gamma=\cos\theta\sin\epsilon\sin\eta+\sin\theta\cos\epsilon\sin\varphi \tag{107}$$

where the angles θ, φ, ϵ, η are shown in the following diagram:



Substituting equation (107) into equation (106) and performing the integration over the angles of V, we have

$$H_{AF}=-\frac{fs_{\nu}}{V^{1/2}}\,\frac{2e\hbar^2c^2K_{\nu}}{\pi k_\nu^2}\sqrt{\frac{2\pi\hbar^2c^2}{E_{\nu k_\nu}}}\left[\int dV\,|\bar{t}_N|\,e^{-i\bar{k}_\nu\cdot\bar{x}}\right]\psi_s^*(0)\int dR\,f_1(R)\left[k_\nu R\cos k_\nu R-\sin k_\nu R\right]$$

Since the integrand in R is small except near R = 0, we approximate

$$k_\nu R \cos k_\nu R - \sin k_\nu R \approx -\tfrac{1}{3}(k_\nu R)^3$$

Thus

$$H_{AF} = -\frac{f g_{N}}{V^{1/2}}\, e\, \frac{k_\nu}{K_\nu} \sqrt{\frac{2\pi\hbar^2 c^2}{E_{\nu k_\nu}}}\, \sin\epsilon \sin\eta \left[\psi^*_{s(0)} \int dV\, |\bar{t}_N|\, e^{-i\bar{k}_\nu\cdot\bar{x}}\right] \tag{108}$$

Substituting equation (108) into equation (101), making the nuclear summation explicit, and using the relation

$$\frac{P_\nu^2}{(2\pi\hbar)^3}\frac{dP_\nu}{dE_F} = \frac{P_\nu^2}{(2\pi\hbar)^3 c}$$

we have the result

$$\frac{1}{\tau} = \frac{|f|^2 |K_\nu^{3/2} s_\nu|^2}{3}\, \frac{E_{\nu k_\nu}}{\hbar}\, \frac{4e^2}{\hbar c}\left(\frac{k_\nu}{K_\nu}\right)^2 \frac{|\psi_s(0)|^2}{K_\nu^3} \left|\sum \int dV\, \Phi_N^* \frac{\tau_x + i\tau_y}{2}\, \sigma\, \Psi_N\, e^{-i\bar{k}_\nu\cdot\bar{x}}\right|^2$$

APPENDIX 9 PLANE WAVE EXPANSIONS OF THE FIELD QUANTITIES

$$\psi_s = -\frac{i}{V^{1/2}} \sum_{k_s} \sqrt{\frac{\hbar c k_s}{2E_{sk_s}}} \left[-a_{sk_s} e^{i(\bar{k}_s \cdot \bar{x} - \frac{E_{sk_s}}{\hbar} t)} + b^*_{sk_s} e^{-i(\bar{k}_s \cdot \bar{x} - \frac{E_{sk_s}}{\hbar} t)} \right]$$

$$\pi_s = \frac{1}{V^{1/2}} \sum_{k_s} \sqrt{\frac{E_{sk_s}}{2\hbar c k_s}} \left[a^*_{sk_s} e^{-i(\bar{k}_s \cdot \bar{x} - \frac{E_{sk_s}}{\hbar} t)} + b_{sk_s} e^{i(\bar{k}_s \cdot \bar{x} - \frac{E_{sk_s}}{\hbar} t)} \right]$$

$$\psi_p = -\frac{i}{V^{1/2}} \sum_{k_p} \sqrt{\frac{\hbar c k_p}{2E_{pk_p}}} \left[-a_{pk_p} e^{i(\bar{k}_p \cdot \bar{x} - \frac{E_{pk_p}}{\hbar} t)} + b^*_{pk_p} e^{-i(\bar{k}_p \cdot \bar{x} - \frac{E_{pk_p}}{\hbar} t)} \right]$$

$$\pi_p = \frac{1}{V^{1/2}} \sum_{k_p} \sqrt{\frac{E_{pk_p}}{2\hbar c k_p}} \left[a^*_{pk_p} e^{-i(\bar{k}_p \cdot \bar{x} - \frac{E_{pk_p}}{\hbar} t)} + b_{pk_p} e^{i(\bar{k}_p \cdot \bar{x} - \frac{E_{pk_p}}{\hbar} t)} \right]$$

$$\psi_v = -\frac{i}{V^{1/2}} \sum_{k_v} \sum_{\epsilon = 1,-1} \sqrt{\frac{\hbar c k_v}{2E_{vk_v}}}\, \bar{J}_{vk_v\epsilon} \left[-a_{vk_v\epsilon} e^{i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} + b^*_{vk_v\epsilon} e^{-i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} \right]$$

$$\pi_v = \frac{1}{V^{1/2}} \sum_{k_v} \sum_{\epsilon = 1,-1} \sqrt{\frac{E_{vk_v}}{2\hbar c k_v}}\, \bar{J}_{vk_v\epsilon} \left[a^*_{vk_v\epsilon} e^{-i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} + b_{vk_v\epsilon} e^{i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} \right]$$

$$\Psi_v = -\frac{i}{V^{1/2}} \sum_{k_v} \sqrt{\frac{E_{vk_v}}{2\hbar c k_v}}\, \bar{J}_{vk_v0} \left[-a_{vk_v0} e^{i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} + b^*_{vk_v0} e^{-i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} \right]$$

$$\Pi_v = \frac{1}{V^{1/2}} \sum_{k_v} \sqrt{\frac{E_{vk_v}}{2\hbar c k_v}}\, \bar{J}_{vk_v0} \left[a^*_{vk_v0} e^{-i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} + b_{vk_v0} e^{i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} \right]$$

$$\psi_{1/2c} = \frac{1}{V^{1/2}} \sum_{k_c} \sum_{s=1,2} \left[a_{ck_cs} P_{ck_cs} e^{i(\bar{k}_c \cdot \bar{x} - \frac{E_{ck_c}}{\hbar} t)} + b^*_{ck_cs} \tilde{P}_{ck_cs} e^{-i(\bar{k}_c \cdot \bar{x} - \frac{E_{ck_c}}{\hbar} t)} \right]$$

$$\psi_{1/2m} = \frac{1}{V^{1/2}} \frac{1}{2^{1/2}} \sum_{k_m} \sum_{s=1,2} \left[a_{mk_ms} P_{mk_ms} e^{i(\bar{k}_m \cdot \bar{x} - \frac{E_{mk_m}}{\hbar} t)} + a^*_{mk_m} \tilde{P}_{mk_m} e^{-i(\bar{k}_m \cdot \bar{x} - \frac{E_{mk_m}}{\hbar} t)} \right]$$

$$\bar{A} = \frac{1}{V^{1/2}} \sum_{k_v} \sum_{\text{POLAR.}} \sqrt{\frac{2\pi\hbar^2 c^2}{E_{vk_v}}} \left[\bar{e}_k q_k e^{i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} + \bar{e}_k q^*_k e^{-i(\bar{k}_v \cdot \bar{x} - \frac{E_{vk_v}}{\hbar} t)} \right]$$

APPENDIX 10 LIST OF SYMBOLS

$\hbar$	Planck's constant divided by 2π.
c	Speed of light.
κ	$\equiv \frac{mc}{\hbar}$
m	Mass of a particle.
e	Magnitude of electronic charge.
k	Momentum divided by $\hbar$.
p	Momentum.
E	Energy.
∂_μ	$\equiv \frac{\partial}{\partial x^\mu}$
v	As a subscript denotes vector meson.
s	As a subscript denotes scalar meson.
p	As a subscript denotes pseudoscalar meson.
n	As a subscript denotes neutral particle.
c	As a subscript denotes charged μ-meson.
$\frac{1}{2}$	As a subscript denotes spin $\frac{1}{2}$ meson.
N	As a subscript denotes nucleon.
(μ)	As a superscript denotes light mesons.
(N)	As a superscript denotes nucleon.
φ, χ	Potential and field quantities. Subscripts correspond to components of these quantities.
ψ, π	Potential and field quantities in vector notation.
Φ_N, Ψ_N	Nucleon wave functions.
Φ_v, Ψ_v	Wave functions of longitudinal vector meson.
$\bar{A}$	Vector potential of electromagnetic field.
$\bar{j}_\epsilon$	Direction of polarization of vector meson.

$\bar{e}_k$ Direction of polarization of photon.

f, g Coupling constants between π- and μ-mesons.

r, s Coupling constants between π-meson and nucleons.

p Dirac wave-function matrix

τ_x, τ_y Isotopic spin functions.

$\alpha, \beta, \gamma, \sigma$ Dirac matrices.

$$w_N,\ w^{(N)} = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \Psi_N$$

$$w_\mu,\ w^{(\mu)} = \tilde{\psi}_c C \beta \tilde{\rho}_{mk_m s} \quad , \tilde{\psi}_c C \beta \psi_m$$

$$v_{0N},\ v_0^{(N)} = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \Psi_N \quad , -\Phi_N^* \frac{\tau_x + i\tau_y}{2} \Psi_N$$

$$v_{0\mu},\ v_0^{(\mu)} = \tilde{\psi}_c C \tilde{\rho}_{mk_m s} \quad , -\tilde{\psi}_c C \psi_m$$

$$\bar{v}_N,\ \bar{v}^{(N)} = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\alpha} \Psi_N$$

$$\bar{v}_\mu,\ \bar{v}^{(\mu)} = \tilde{\psi}_c C \bar{\alpha} \tilde{\rho}_{mk_m s} \quad , \tilde{\psi}_c C \bar{\alpha} \psi_m$$

$$\bar{u}_{0N},\ \bar{u}_0^{(N)} = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\gamma} \Psi_N$$

$$u_{0\mu},\ u_0^{(\mu)} = \tilde{\psi}_c C \bar{\gamma} \tilde{\rho}_{mk_m s} \quad , \tilde{\psi}_c C \bar{\gamma} \psi_m$$

$$u_N,\ u^{(N)} = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \bar{\sigma} \Psi_N$$

$$u_\mu,\ u^{(\mu)} = \tilde{\psi}_c C \beta \bar{\sigma} \tilde{\rho}_{mk_m s}, \tilde{\psi}_c C \beta \bar{\sigma} \psi_m$$

$s_N \quad = -i\, \Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \gamma^5 \Psi_N$

$s_\mu \quad = -i\, \tilde{\psi}_c C \beta \gamma^5 \psi_\mu$

$\bar{t}_N,\ \bar{t}^{(N)} \quad = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \bar{\sigma} \Psi_N$

$\bar{t}_\mu,\ \bar{t}^{(\mu)} \quad = \tilde{\psi}_c C \bar{\sigma} \tilde{\rho}_{\mu k_\mu s} \quad , \tilde{\psi}_c C \bar{\sigma} \psi_\mu$

$t_{0N},\ t_0^{(N)} = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \gamma^5 \Psi_N$

$t_{0\mu},\ t_0^{(\mu)} = \tilde{\psi}_c C \gamma^5 \tilde{\rho}_{\mu k_\mu s} \quad , \tilde{\psi}_c C \gamma^5 \psi_\mu$

$q_N \quad = \Phi_N^* \frac{\tau_x + i\tau_y}{2} \beta \gamma^5 \Psi_N$

$q_\mu \quad = \tilde{\psi}_c C \beta \gamma^5 \tilde{\rho}_{\mu k_\mu s}$

*	As a superscript denotes complex conjugate.
†	As a superscript denotes adjoint.
~	Denotes charge conjugate.
c.c.	Complex conjugate.

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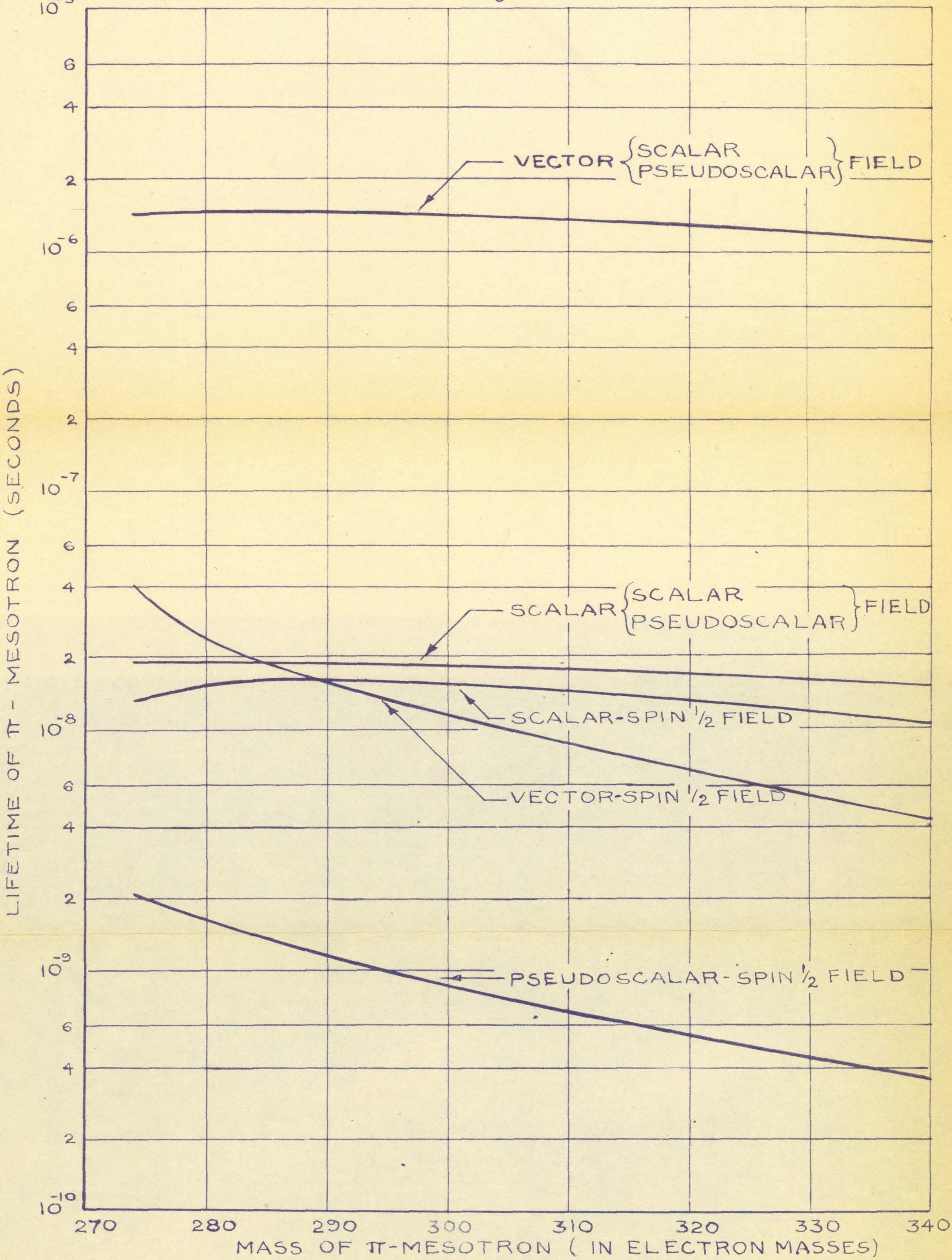
Fig. 1



VECTOR {SCALAR PSEUDOSCALAR} FIELD
SCALAR {SCALAR PSEUDOSCALAR} FIELD
SCALAR-SPIN 1/2 FIELD
VECTOR-SPIN 1/2 FIELD
PSEUDOSCALAR-SPIN 1/2 FIELD
LIFETIME OF π - MESOTRON (SECONDS)
MASS OF π-MESOTRON (IN ELECTRON MASSES)
10^{-5}
6
4
2
10^{-6}
6
4
2
10^{-7}
6
4
2
10^{-8}
6
4
2
10^{-9}
6
4
2
10^{-10}
270
280
290
300
310
320
330
340