AN INVESTIGATION OF THE PERMEABILITY OF IRON AT ULTRA-HIGH FREQUENCIES

Thesis by

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ABSTRACT

This thesis is the report of an investigation of the permeability of iron and its dependence on temperature at ultra high frequencies. The frequency used was 8,900 megacycles, the temperature range covered was from 50°C to 350°C, and two samples of iron were studied. Both samples were approximately $1\frac{1}{2}$ x $1\frac{1}{2}$ x $\frac{1}{4}$ inches and made from standard Armco iron stock. Special heat treatment was used on one sample to remove impurities. The other sample was annealed but not purified.

To measure the permeability the effect of an iron sample was compared with that of copper on the resonant length and the shape of the resonance curve when used as end plates of the resonant cavity. This technique eliminated the effect of the other parts of the cavity on the measurements made.

Two values of permeability are obtained, μ_L and μ_R. The first is obtained from measurements of resonant length while the second is determined from measurements on the shape of the resonance curve. How the two values of permeability are obtained from the measurements made is derived under <u>Theory of Measurement</u>.

For the sample of Armco iron purified by special heat treatment μ_R varied from 80 to 35 in the temperature range 50°C to 300°C. For the annealed iron sample μ_R varied from 67 to 17 in the temperature range 50°C to 360°C. Values of μ_L were obscured by the large probable error that resulted from the type of cavity used in the experiment, but all values were less than ten.

TEXT

I. Preliminary Considerations

No attempt will be made here to discuss the theory of ferromagnetism as developed by Weiss, Heisenberg and others for static magnetic fields.[1] However a few remarks are necessary in order to explain the purpose of the experiment herein described.

The permeability μ is defined by the relation

$$B = \mu H$$

where

$B \equiv$ resultant magnetic flux

$H \equiv$ applied magnetic field

B may also be expressed as

$$B = H + 4\pi I$$

where

$I \equiv$ intensity of magnetization

It may be seen from this that

$$\mu = 1 + 4\pi \frac{I}{H}$$

According to the theory of ferromagnetism[2]

$$I = I_0 \tanh \left[\frac{1}{kT} \left(mH + 2A \frac{I}{I_0} \right) \right]$$

where

$I_0 \equiv$ saturation intensity of magnetization at absolute zero

$k \equiv$ Boltzmann's constant

$T \equiv$ absolute temperature

$A \equiv$ the exchange integral

The Curie point is defined as the temperature at which spontaneous magnetization disappears. Consequently 2A can be replaced by kT_c where T_c is the Curie temperature

$$I = I_o \tanh \left[\frac{1}{kT} \left(mH + kT_c \frac{I}{I_o} \right) \right]$$

It can be seen from this expression for the intensity of magnetization and the relation of permeability to it that no simple expression for μ as a function of T can be written. It can be said however that according to the theory μ decreases steadily and continuously with increasing temperature from a maximum value at absolute zero to one at the Curie point.

It will be noticed that the normal definition of permeability implies that H and I are in the same direction. While this is true for static fields it no longer holds at very high frequencies. The result is that the magnetization of ferromagnetic materials is out of phase with the applied magnetic field and μ must be expressed as a complex quantity. However, because μ is also a function of frequency and because of precedent it is customary in measurements of permeability at high frequencies to continue writing

$$B = \mu H$$

and then measure two different values of μ, μ_R, and μ_L, the former being the value of μ derived from the losses the ferromagnetic material introduces into the Lecher wire system or cavity while the latter is derived from the reactance introduced.(3) The complex expression for μ in terms of μ_R and μ_L is derived in the appendix. The precedence of measuring μ_R and μ_L

has been followed in this experiment for the purpose of comparing results with previous investigations.

The resonant cavity technique is rapidly establishing itself as an accurate method of measuring physical constants. There are, in general, two techniques used in obtaining resonance curves. One is to fix the dimensions of the cavity and sweep through the resonant frequency of the cavity with a variable frequency oscillator. The other is to fix the frequency of the oscillator and vary the length of the cavity through resonance. The former technique is admirably suited to comparing two different cavities. It has the advantage that frequency stabilization is not necessary. It does not, however, yield itself to making accurate measurements of the "Q" of the cavity. The latter method requires frequency stabilization but it can be used to obtain accurate values of "Q" as well as resonant length. For this reason the latter technique was used in this experiment.

II. Description of Apparatus

If the permeability of iron were to be measured by making the entire cavity of iron it would require accurate knowledge of coupling of the cavity to the detecting probe and the high frequency energy source as well as accurate knowledge of the dimensions of the cavity and their variation with temperature. To try to compare a cavity made of iron with one made of copper would involve similar difficulties even though it would eliminate the necessity of knowing the effect of coupling to the energy source and detecting probe. Consequently it was decided to use one cavity made of brass wave guide and place first iron and then copper as the end plate of the cavity and compare the resulting resonance curves. Since the material being investigated is placed only at the end of the cavity, the sample is small in size and easy to prepare. By this method of comparison the entire effects of the walls of the cavity other than the end where the test samples are placed are eliminated. One need know only the conductivity of the iron and copper and the permeability of copper.

The copper sample was prepared by plating a layer of copper about .007 inches thick on a small iron plate the same size and shape as the iron test sample the permeability of which was being investigated. By this means the effect of thermal expansion of the cavity was eliminated since it was the same for the cavity with either sample placed on the end.

The cavity was made of .400 inches by .900 inches rectangular wave guide. The test sample was bolted to one end and a plunger with micrometer adjustment through which the energy from the oscillator was fed formed the other. A schematic diagram of the cavity is shown in plate I. The choice of this type of cavity was unfortunate because the TE_{10} mode of propagation ~~in the cavity~~ requires that current flow across the joint where the test sample contacts the end of the cavity. Consequently minute variations of this contact cause serious variations in the effective resonant length of the cavity. Removing, polishing, and replacing a test sample on the end of the cavity caused a variation of as much as .0005 inches in the resonant length. The "Q" of the cavity is not affected in the same manner because the necessity of the current traversing the joint does not cause any change in losses. The use of the proper mode in a cylindrical cavity would have avoided this source of error. But it was not realized during the construction of the apparatus that the variation resulting from the replacement of a test sample would be so serious.

The energy from the klystron oscillator was fed to the cavity through a small hole in the center of the plunger face that formed the opposite end of the cavity from where the test sample was placed. The plunger was itself a short section of wave guide which fitted snugly into the cavity. It in turn was tightly coupled to the length of guide leading from the oscillator by a length of coaxial cable. Two dielectric attenuators (see plate II) were used to decouple the oscillator from the cavity so that as the cavity length was varied through resonance it would not tend to pull the oscillator off its fixed frequency. The repeller electrode of the klystron was modulated by a 1000 cycle square wave 50 volts in amplitude. This amplitude was sufficient to pulse the klystron so that it oscillated at only one frequency half the time. The purpose of the modulation was to provide some means of detecting the fields in the guide and cavity. A probe was inserted into the cavity .025 inches $1\frac{1}{4}$ wave lengths from the end where the test sample was placed. This probe extracted a very small amount of energy from the cavity which was then demodulated by a germanium crystal. The resulting 1000 cycle signal was then fed to a selective audio amplifier. Since the amplitude of the square wave modulation was held constant the output of the audio amplifier was proportional to the high frequency electric field at the probe in the cavity. To be exact:

$$V = AE^{1.96}$$

where

$V \equiv$ output of audio amplifier

$E \equiv$ electric field at the probe

$A \equiv$ amplification constant

The exponent for E was found experimentally by measuring the field in a shorted section of wave guide where E is known to be a sinusoidal function of distance.

In addition to this primary high frequency circuit beginning with the modulated oscillator the output of which is fed into the cavity through a short section of wave guide and connecting coaxial cable and ending with the audio amplifier and meter which received the signal from the crystal coupled to the detecting probe there was an auxiliary circuit used to monitor the oscillator. This circuit consisted of a branch of wave guide in which was placed a fixed resonant cavity of the cylindrical type. The branch wave guide was connected at right angles to the main section of wave guide. Since the fixed cavity was in series with the guide it acted as a highly selective circuit and allowed energy to reach the demodulating crystal at the end of the branch (see Plate II) only when the oscillator was tuned to the resonant frequency of the fixed or monitoring branch of wave guide so that the klystron could be readjusted if necessary until it was oscillating at the resonant frequency of the fixed cavity. It was determined by measurement that it was possible to hold the frequency of the oscillator to the resonant frequency of the fixed cavity within an error of less than one part in thirty thousand.

To stabilize the output of the klystron and minimize frequency drift a voltage regulator line transformer was used for the square wave generator. The power supply for both the klystron and the square wave generator were voltage regulated. In addition the klystron was placed in a bath of transformer oil contained in a water cooled jacket.

The water flowing through the jacket came from an open reservoir near the ceiling of the room to maintain a constant pressure head. The result was that it was possible to hold the frequency of the oscillation at the resonant frequency of the monitoring cavity without difficulty.

To prevent oxidation of surfaces the entire test cavity assembly was placed in a brass bell jar which was evacuated. The pressure in the bell jar was held to less than one millimeter by a vacuum pump throughout the course of measurement. This pressure was low enough to prevent any noticeable oxidation over a period of many hours even though the temperature of some of the surfaces was more than 400^{o} C. The brass bell jar was necessary because unequal heating resulting from the heating element surrounding the test samples was severe enough to crack a glass jar.

To heat the test samples to the various temperatures at which permeability was measured a small rectangular heating element was made to fit snugly over the sample. This element was made by molding a 1000 watt helix of resistance wire in an aluminum oxide form. It was put in series with several rheostats across a direct current source of 115 volts. The current through the heating coil and hence its temperature was varied by changing the resistance of the rheostats. An alternative method of heating the test samples would have been to pass a very large direct current through the sample itself. But this current would have set up a magnetic field which would have partially magnetized the sample.

While the test sample at the upper end of the test cavity was being heated it was necessary to keep the lower portions of the cavity at least moderately cool. The necessity arose from two different causes. The first is that the detecting probe assembly mounted on the side of the cavity contained a germanium crystal and insulating parts that would have been destroyed by elevated temperatures. The second is that the plunger connected to the micrometer would make poor contact with the lower walls of the cavity if a considerable temperature differential existed. To cool the lower portions of the test cavity a copper tubing was wound around the outside walls of the cavity and soldered to it about $\frac{1}{4}$ inches below the upper end of the cavity which made contact with the heating element. Water circulated through this copper coil at a rate of 15 cubic centimeters per second. Because of thermal expansion small temperature changes of the side walls of the cavity could cause marked changes in the resonant length of the cavity. For this reason this cooling coil was also supplied from an open reservoir near the ceiling of the room to maintain a constant rate of water flow.

To measure the temperature of the test sample two iron-constantin thermocouples were extended through the heating element. The junctions were firmly seated by means of small screws at the bottom of holes which were drilled in the samples. One hole was placed at the center of the sample and the other so that it was just over one wall of the cavity. The holes were drilled almost through the sample so that the thermocouple junctions were 1/64 inches from the surface of the test sample that formed the end of the cavity. The thermocouples were connected to a Leeds-Northrup thermocouple potentiometer calibrated for iron-constantin thermocouples.

To change test samples it was necessary to remove the brass bell jar so that only a temporary seal could be used when placing it over the cavity assembly. A rosin-beezwax mixture makes an ideal temporary seal except that it melts at a relatively low temperature. To avoid melting the seal when the temperature of the heating element was raised another cooling coil was wound around the bell jar and water circulated through it at the rate of 10 cc per second.

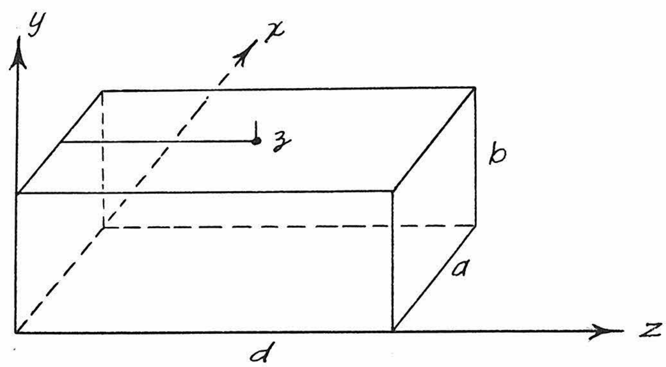
So that the plunger could be moved in or out of the cavity to vary its length by means of a micrometer located outside the evacuated jar a sylphon bellows was used. Inside the cavity the bellows was fastened by means of a rigid guiding plate directly to the lower end of the plunger. Outside the bellows was connected to the micrometer barrel by means of a ball-in-socket joint so that rotation of the micrometer caused pure extension of the bellows. To insure that the position of the plunger was in exact correspondence with the micrometer setting it was necessary that the bellows (to which the plunger is rigidly fastened) always pressed firmly against the end of the micrometer barrel. Since the natural action of the bellows under the influence of the vacuum is to pull away from the barrel strong springs were provided to oppose atmospheric pressure from the outside.

For a clearer understanding of this description of the apparatus reference may be made to the photographs in the appendix where most of the details are clearly illustrated.

III. Theory of Measurement

In this section the effect of the permeability and conductivity of the end wall of a cavity on its resonant length and the shape of the resonance curve is derived in some detail. This is done firstly because the technique of measurement depends on the elimination of effects of other parts of the cavity by comparing measurements made with an iron end with those made with a copper end, and secondly because the shape of the resonance curve which is usually measured by the "Q" of the cavity can be expressed much more usefully for the purposes of this experiment by the slope of a certain linear relationship.

Consider a cavity formed of conducting walls of length d, width a, and height b, the interior of which is evacuated. Let the coordinate axis be oriented as shown in the diagram.



Then in the TE_{10} mode Z is the direction of propagation. At Z = 0 is located the test sample whose physical constants are to be measured. At Z = d is the effective end face of the plunger. In the center of this face is a small hole through which energy is fed into the cavity. At some point $Z = z$ and $X = a/2$ a short probe is inserted into the top of the cavity to detect the electric field present there.

A small amount of energy enters the cavity through the hole at $Z = d$, it excites the TE_{10} mode of propagation, the only mode the dimensions a and b will allow. This field propagates down the cavity with a propagation constant γ_1 and is reflected at $Z = 0$ by the conducting test sample. It is again reflected at $Z = d$ and travels back and forth in the cavity. Thus the electromagnetic field in the cavity can be considered as consisting of an infinite number of waves traveling in both directions. It is assumed that the electromagnetic energy travels back and forth in the cavity in the TE_{10} mode of propagation in a wave guide with perfectly conducting walls with the same dimensions as the cross-section of the cavity. Since the walls of the cavity used in this experiment are brass the assumption is a very good one. At any rate it is the effect of the end of the cavity formed by the test sample at $Z = 0$ on the fields that is of interest, so that if the fields of a TE_{10} mode can satisfy the boundary conditions at the ends it is sufficient to consider only those fields. The fields of a TE_{10} mode are a magnetic field in the Z-direction, H_z , a magnetic field in the X-direction, H_x, and an electric field in the Y-direction, E_y .

Using the practical units these fields for a wave traveling in the negative Z-direction are [4]

$$H_{1z} = A e^{(j\omega t + \gamma_1 z)} \cos \frac{\pi x}{a}$$

$$H_{1x} = -A \frac{\gamma_1 \pi}{a(\gamma_1^2 + K_1^2)} e^{(j\omega t + \gamma_1 z)} \sin \frac{\pi x}{a}$$

$$E_{1y} = -A \frac{j\omega \mu_0 \pi}{a(\gamma_1^2 + K_1^2)} e^{(j\omega t + \gamma_1 z)} \sin \frac{\pi x}{a}$$

The waves traveling in the positive direction are

$$H_{1z}' = C e^{(j\omega t - \gamma_1 z)} \cos \frac{\pi x}{a}$$

$$H_{1x}' = C \frac{\gamma_1 \pi}{a(\gamma_1^2 + K_1^2)} e^{(j\omega t - \gamma_1 z)} \sin \frac{\pi x}{a}$$

$$E_{1y}' = -C \frac{j\omega \mu_0 \pi}{a(\gamma_1^2 + K_1^2)} e^{(j\omega t - \gamma_1 z)}$$

where

A $\equiv$ arbitrary amplitude

C $\equiv$ arbitrary amplitude

x $\equiv$ coordinate across the cavity in meters

a $\equiv$ width of the cavity in meters

$\gamma_1 \equiv$ propagation constant in the Z-direction in meters^{-1}

$\omega \equiv$ frequency of oscillations in radians per second

$\mu_0 \equiv$ permeability of free space in henries per meter

$k^2 = \omega^2 \mu_0 \epsilon_0$

$\epsilon_0 \equiv$ dielectric constant of free space in farads pĕr meter

If the coefficient of reflection of a wave striking an end plate is found so that successively reflected waves can be expressed in terms of the wave initially entering the guide and the coefficient of reflection, and if the infinite series of reflected waves can be summed the problem will be solved since the coefficient of reflection of the end contains the permeability of the material forming the end wall.

The coefficient of reflection may be found by satisfying the boundary conditions at the end. Any field existing in the test sample at $Z=0$ will propagate only in the negative Z-direction since the test sample is conducting and will rapidly attenuate any fields that penetrate it. Consequently the fields in the material forming the end of the cavity are

$$H_{2z} = B e^{(j\omega t + \gamma_2 z)} \cos\frac{\pi x}{a}$$

$$H_{2x} = -B\frac{\gamma_2 \pi}{a(\gamma_2^2 + k_2^2)} \; e^{(j\omega t + \gamma_2 z)} \sin\frac{\pi x}{a}$$

$$E_{2y} = -B\frac{j\omega \mu_2 \pi}{a(\gamma_2^2 + k_2^2)} \; e^{(j\omega t + \gamma_2 z)} \sin\frac{\pi x}{a}$$

where

$k_2^2 \equiv \omega^2 \mu_2 \epsilon_2 \left(1 + \frac{\sigma_2}{j\omega\epsilon_2}\right)$

$\gamma_2 \equiv$ propagation constant in the test sample in meters^{-1}

$\mu_2 \equiv$ permeability of the test sample in henries per meter

$\epsilon_2 \equiv$ dielectric constant in the test sample in farads per meter

$\sigma_2 \equiv$ conductivity of the test sample in mhos per meter

The boundary conditions at $Z=0$ require:

$$H_{1x} + H_{1x}' = H_{2x}$$

$$E_{1y} + E_{1y}' = E_{2y}$$

Substituting the expressions given above for the fields with $Z = 0$ yields

$$\frac{\gamma_1 (C-A)}{(\gamma_1^2 + K_1^2)} = -\frac{\gamma_2 B}{(\gamma_2^2 + K_2^2)}$$

$$\frac{\mu_0 (C+A)}{(\gamma_1^2 + K_1^2)} = \frac{\mu_2 B}{(\gamma_2^2 + K_2^2)}$$

Eliminating B and finding C in terms of A gives the coefficient of reflection

$$C = A\,\frac{\mu_2 \gamma_1 - \mu_0 \gamma_2}{\mu_2 \gamma_1 + \mu_0 \gamma_2}$$

Or the coefficient of reflection, K, is

$$K = \frac{\mu_2 \gamma_1 - \mu_0 \gamma_2}{\mu_2 \gamma_1 - \mu_1 \gamma_2}$$

Let K' be the coefficient of reflection at the plunger ($Z = d$).

Since the detecting probe responds to the electric field it alone needs to be considered. Let the electric field of the wave initially excited by the energy entering the cavity through the hole in the plunger at $Z = d$ have the amplitude A. It may be expressed

$$E_{y1} = A e^{(j\omega t + \gamma_1 z)} \sin \frac{\pi x}{a}$$

Since the detecting probe is located at $x = \frac{a}{2}$ the factor $\sin \frac{\pi x}{2}$ equals one and need no longer be considered. The wave reflected at the test sample is

$$E_{y2} = AK e^{(j\omega t - \gamma_1 z)}$$

The result of the reflection of E_{y2} at Z = d is

$$E_{y3} = AKK'e^{(j\omega t + \gamma_1 z - 2\gamma_1 d)}$$

and so on until there results an infinite series of waves traveling in both directions. The total electric field E_y is

$$E_y = A(e^{\gamma_1 z} + Ke^{-\gamma_1 z})e^{j\omega t}\sum_{n=0}^{\infty}(KK')^n e^{-2n\gamma_1 d}$$

K and K' are in general complex numbers and may be expressed in the form

$$K = e^{-2p}$$

$$K' = e^{-2p'}$$

where p and p' are also complex numbers the values of which are such as to make the above relations true. Thus

$$E_y = A(e^{\gamma_1 z} + e^{-\gamma_1 z - 2p})e^{j\omega t}\sum_{n=0}^{\infty} e^{-2n(\gamma_1 d + p + p')}$$

The infinite series may now be expressed in closed form since

$$1 + \coth x = 2\sum_{n=0}^{\infty} e^{-2nx} \qquad (5)$$

Thus letting $e^{j\omega t}$ be understood

$$E_y = Ae^{-p}\cosh(\gamma_1 z + p)\left[1 + \coth(\gamma_1 d + p_1 + p_2)\right]$$

Obviously Ae^{-p} is a constant independent of the length of the cavity or the value of Z.

The above expression for E_y is an accurate expression for the electric field at a point Z, $x = \frac{a}{2}$ in a cavity of length d with one end at Z = 0 formed a medium causing a coefficient of reflection $K = e^{-2p}$ and the other end at Z = d having a coefficient of reflection $K' = e^{-2p'}$. The only approximation is the assumption that fields of the TE_{10} mode will satisfy the boundary conditions along the sides of the cavity.

To make the expression for E_y

$$E_y = A_0 \cosh(\gamma_1 z + p)\left[1 + \coth(\gamma_1 d + p + p')\right]$$

useful, p and γ_1 must be expressed in terms of the constants of the cavity and its walls.

The propagation constant γ_1 for the TE_{10} mode in a rectangular guide is derived in several texts on the assumption that the walls of the guide are good conductors.(6) So without derivation it may be written

$$\gamma_1 = \alpha_1 + j\beta_1$$

where

$$\beta_1 = \left(\omega^2 \mu_0 \epsilon_0 - \frac{\pi^2}{a^2}\right)^{\frac{1}{2}}$$

$$\alpha_1 = \frac{1}{b}\left(\frac{\omega \epsilon_0}{2\sigma_1}\right)^{\frac{1}{2}} \frac{1 + \frac{2b}{a}\left(\frac{\pi c}{a\omega}\right)^2}{\left[1 - \left(\frac{\pi c}{a\omega}\right)^2\right]^{\frac{1}{2}}}$$

where

$\sigma_1 \equiv$ conductivity of the walls in mhos/meter

$c \equiv$ velocity of light in free space in meters/second

The rest of the nomenclature has been defined previously.

This value of γ_1 is based on the fact that α_1 is very small in relation to β_1 as it will be shown to be. A more rigorous evaluation of γ_1 was made and it was found that it differed from the above expression by a negligible amount for the purposes of this investigation.

To find p in terms of useful constants its definition must be used

$$e^{-2p} = K = \frac{\mu_2 \gamma_1 - \mu_0 \gamma_2}{\mu_2 \gamma_1 + \mu_0 \gamma_2}$$

The propagation constant in the conducting test sample, may be found as follows. A fundamental relationship for the TE_{10} mode is (6)

$$\gamma^2 + K^2 = \frac{\pi^2}{a^2}$$

This must be true in the end of the cavity as well as within it to satisfy the boundary conditions at Z = 0. Applying this relationship to the end conducting wall and substituting the value of K^2

$$\gamma_2^2 = \frac{\pi^2}{a^2} - \omega^2 \mu_2 \epsilon_2 \left(1 + \frac{\sigma_2}{j\omega \epsilon_2}\right)$$

For any metal σ_2 is so large that all terms not containing it are completely negligible and

$$\gamma_2 = (1+j)\sqrt{\frac{\omega \mu_2 \sigma_2}{2}}$$

Before substituting in the values of γ_1 and γ_2 in the expression for e^{-2p} it will be shown that α_1 is negligible in comparison to β_1 and may be neglected.

In this experiment

$a = 2.286 \times 10^{-2}$ meters

$b = 1.016 \times 10^{-2}$ meters

$\omega = 5.58 \times 10^{10}$ radians per second

$\mu_0 = 4 \times 10^{-7}$ henries per meter

$\epsilon_0 = 1/36 \times 10^{-9}$ farads per meter

$c = 3 \times 10^{8}$ meters per second

$\sigma_1 = 1.6 \times 10^{7}$ mhos per meter

σ_2 = same order of magnitude as σ_1

So that

$\beta_1 = 1.252 \times 10^{2}$ meters^{-1}

$\alpha_1 = 4.0 \times 10^{-2}$ meters^{-1}

$|\gamma_2| \cong 8 \times 10^{5}$ meters^{-1}

Thus e^{-2P} is very close to one and an excellent approximation to the true value can be made by neglecting α_1 and letting $\gamma_1 = j\beta_1$

Dividing both numerator and denominator by $\mu_1 \gamma_2$ in the expression for e^{-2P}

$$e^{-2P} = -\frac{1 - \dfrac{j\beta_1\mu_2}{\mu_0(1+j)\sqrt{\omega\mu_2\sigma_2/2}}}{1 + \dfrac{j\beta_1\mu_2}{\mu_0(1+j)\sqrt{\omega\mu_2\sigma_2/2}}}$$

$$e^{-2P} \cong -\left[1 - \frac{2j\beta_1\mu_2}{\mu_0(1+j)\sqrt{\omega\mu_2\sigma_2/2}}\right]$$

This is still an excellent approximation since the last term is very small compared to one, being of the order β_1/γ_2

Now let

$$\Delta = \frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_2}{2\omega\sigma_2}}$$

$$p = \xi + j\eta$$

Expanding e^{-2p} into real and imaginary parts

$$e^{-2\xi}\cos 2\eta - je^{-2\xi}\sin 2\eta = -1 + 2\Delta + j2\Delta$$

or

$$e^{-2\xi}\cos 2\eta = -1 + 2\Delta$$

$$e^{-2\xi}\sin 2\eta = -2\Delta$$

Since Δ is very small it is obvious that η is approximately $\pi/2$ while ξ is approximately zero.

Let

$$\eta = \frac{\pi}{2} + \rho$$

Dividing the equation derived from imaginary parts by that derived from the real parts

$$\tan 2\rho = \frac{2\Delta}{1-2\Delta} \cong 2\Delta$$

or $\qquad \rho \cong \Delta$

Since Δ is of the order of 1×10^{-4} this approximation is good. Substituting $\eta = \frac{\pi}{2} + \Delta$ in $e^{-2\xi}\cos 2\eta = -1 + 2\Delta$ it is easily seen that $\xi = \Delta$.

Thus

$$p = \Delta + j\left(\frac{\pi}{2} + \Delta\right)$$

where

$$\Delta = \frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_2}{2\omega\sigma_2}}$$

γ_1 and p have now been expressed in terms of the constants of the system.

So far nothing specific has been said about $K' = e^{-2p'}$ where K' is the coefficient of reflection at the plunger end of the cavity. While K' cannot be expressed analytically because of the hole in the plunger through which energy enters the cavity it can be safely assumed that the plunger acts as a reflecting plate at the position Z = d with a coefficient or reflection such that

$$p' = \Delta' + j\left(\frac{\pi}{2} + \Delta'\right)$$

where Δ' is left unspecified except that it be regarded as small. This must be true if the cavity is to show the pronounced resonant properties it does. It is very likely that Z = d is not the exact coordinate of the face of the plunger. But whatever the distance between its effective surface and its actual surface, it will remain fixed with any variation of position of the plunger so that it is of no importance in in the analysis.

It has been shown that

$$\gamma_1 = \alpha_1 + j\beta_1$$

$$p = \Delta + j\left(\frac{\pi}{2} + \Delta\right)$$

$$p' = \Delta' + j\left(\frac{\pi}{2} + \Delta'\right)$$

If these values are substituted in the expression

$$E_y = A_0 \cosh(\gamma_1 z + p)\left[1 + \coth(\gamma_1 d + p + p')\right]$$

for the electric field existing in the cavity, there results

$$E_y = A_0 \cosh\left[\alpha_1 z + \Delta + j\left(\beta_1 z + \frac{\pi}{2} + \Delta\right)\right] \times \left[1 + \coth\left(\alpha_1 d + \Delta + \Delta' + j(\beta_1 d + \Delta + \Delta' + \pi)\right)\right]$$

In the cavity used in this investigation the position of the detecting probe is fixed at $Z = 5/4\,\lambda_g$. Likewise the resonance curve investigated was the one occurring around $d = 3/2\,\lambda_g$ where λ_g is the wave length in a wave guide of the same dimensions as the cross-section of the cavity. Thus

$$\beta_1 z = \frac{5}{2}\pi$$

$$\beta_1 d = 3\pi + \beta_1 l$$

where l is a very small deviation of d from $3/2\,\lambda_g$. Substituting these values in E_y

$$E_y = A_0 \cosh\left[\frac{5}{4}\alpha_1 \lambda_g + \Delta + j(3\pi + \Delta)\right] \times \left[1 + \coth\left\{\frac{3}{2}\alpha_1 \lambda_g + \Delta + \Delta' + j(4\pi + \beta_1 l + \Delta + \Delta')\right\}\right]$$

Expanding this relation into real and imaginary parts and using the trigonometric and hyperbolic relations for functions of sums of angles

$$E_y = A_0\left[\cosh\left(\tfrac{5}{4}\alpha_1\lambda_g + \Delta\right)\cos\Delta + j\sinh\left(\tfrac{5}{4}\alpha_1\lambda_g + \Delta\right)\sin\Delta\right]$$

$$\times\left[1 + \frac{\sinh 2\left(\tfrac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right) - j\sin 2(\beta_1 l + \Delta + \Delta')}{\cosh 2\left(\tfrac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right) - j\cos 2(\beta_1 l + \Delta + \Delta')}\right]$$

The first pair of brackets enclose terms that are entirely independent of l. Furthermore, since the arguments of the functions are small its magnitude is very close to unity. Thus the variation of the electric field at the detecting probe located a fixed distance, $5/4\,\lambda_g$, from the end of the cavity as the length of the cavity is varied a small amount, l, about the length $3/2\,\lambda_g$ is given by the terms enclosed in the second pair of brackets. The maximum value that l has in this experiment is about 5×10^{-5} meters so that the arguments of the functions are still small and they may be expanded in a rapidly convergent power series. Although $\sinh 2\left(\tfrac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right)$ and $\sin 2(\beta_1 l + \Delta + \Delta')$ may be approximated to the first order, $\cosh 2\left(\tfrac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right)$ and $\cos 2(\beta_1 l + \Delta + \Delta')$ must both be approximated to the second order to obtain a result other than $E_y = \infty$ since they are both one to a first order approximation. This is to be expected since near resonance the electric field in a cavity becomes very large compared to that of the energizing wave. The detecting probe is not sensitive to the phase of the electric field so only the magnitude of need be considered

$$|E_y|^2 \cong A_0^2 \frac{(\frac{3}{2}\alpha_1\lambda_g+\Delta+\Delta')^2+(\beta_1 l+\Delta+\Delta')^2}{[(\frac{3}{2}\alpha_1\lambda_g+\Delta+\Delta')^2+(\beta_1 l+\Delta+\Delta')^2]^2}$$

$$|E_y|^2 \cong A_0^2\left[(\tfrac{3}{2}\alpha_1\lambda_g+\Delta+\Delta')^2+(\beta_1 l+\Delta+\Delta')^2\right]^{-1}$$

This is an excellent approximation as long as Δ, Δ', l, and α_1 are small and it has been shown that they are.

It is immediately obvious that $|E_y|$ is a maximum when

$$\beta_1 l = -(\Delta+\Delta')$$

and

$$|E_y|^2_{max} = \frac{A_0^2}{(\frac{3}{2}\alpha_1\lambda_g+\Delta+\Delta')^2}$$

Thus the resonant length of the cavity, d_{res}, is

$$d_{res} = \frac{3}{2}\lambda_g - \frac{(\Delta+\Delta')}{\beta_1}$$

The value of l for resonance of the cavity depends on Δ which in turn is a function of μ_2, the permeability of the test sample since

$$\Delta = \frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_2}{2\omega\sigma_2}}$$

To the first order approximation made in this analysis resonant length depends only on reactance phenomena so that the μ_L determined by the measurement of l for resonance of the cavity will be the μ_L discussed by Kittel.[3]

Now let λ be the displacement of the plunger from its resonance position

$$\lambda = l + \frac{\Delta + \Delta'}{\beta_1}$$

Then

$$\frac{|E_y|_\lambda^2}{|E_y|_{max}^2} = \frac{\left[\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right]^2}{\left[\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right]^2 + \beta_1^2\lambda^2}$$

Let

$$\chi^2 = \frac{|E_y|_\lambda^2}{|E_y|_{max}^2}$$

$$\chi^2\left[\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right]^2 + \chi^2\beta_1\lambda^2 = \left[\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right]^2$$

$$\beta_1^2\lambda^2 = \left[\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'\right]^2 \frac{1-\chi^2}{\chi^2}$$

or

$$\sqrt{\frac{1-\chi^2}{\chi^2}} = \frac{\beta_1}{\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'}\,\lambda$$

Hence the slope of the curve $\sqrt{\frac{1-\chi^2}{\chi^2}}$ versus λ gives another determination of Δ. It may be seen that the slope is also a function of α_1 which is to be expected since the steepness of this curve is a measure of width of the resonant curve or of attenuation. Thus the value of permeability, μ_2 determined from the value of Δ given by this slope is the mentioned μ_R by Kittel.(3)

The curve $\sqrt{\frac{1-\chi^2}{\chi^2}}$ versus λ is a straight line since $\alpha_1, \lambda_g, \Delta,$ and Δ' are all constants for the cavity with a given test sample in place. Let the slope of this line be m. Then

$$m = \frac{\beta_1}{\frac{3}{2}\alpha_1\lambda_g + \Delta + \Delta'}$$

From an experimental determination of m, μ_R of the test sample is derived. Likewise from an experimental determination of the value of l for resonance μ_L of the test sample is derived

$$l_{res} = -\frac{\Delta + \Delta'}{\beta_1}$$

It has already been pointed out that Δ' is unknown. In actuality α_1 is also unknown since the detecting probe extracts some energy from the cavity and thereby causes α_1 to have an effectively greater magnitude than that given as far as the ends of the cavity are concerned. Therefore, to obtain an absolute value of Δ for a test sample, Δ' and α_1, must be eliminated. It can be seen that both m and l_{res} are linear functions of Δ, Δ' and α_1 to the approximation made in this derivation and their effects on the measurements made may be eliminated by comparing the values of m and l_{res} obtained with the test sample forming one end of the cavity with the values obtained with a conductor whose permeability and conductivity are known forming the same end. In this experiment copper is used for the known conductor. Let the subscript c denote values for copper as the end of the cavity, and the subscript I denote values with iron as the end.

$$l_{I\,res} = -\frac{\Delta_I + \Delta'}{\beta_1}$$

$$l_{c\,res} = -\frac{\Delta_c + \Delta'}{\beta_1}$$

or

$$l_I - l_c = \frac{\Delta_c - \Delta_I}{\beta_1}$$

$$\frac{\Delta_I}{\beta_1} = \frac{\Delta_c}{\beta_1} + l_c - l_I \quad \text{yields } \mu_L .$$

In a similar manner

$$m_I = \frac{\beta_1}{\frac{3}{2}\alpha_1\lambda_g + \Delta_I + \Delta'}$$

$$m_c = \frac{\beta_1}{\frac{3}{2}\alpha_1\lambda_g + \Delta_c + \Delta'}$$

Or

$$\frac{\Delta_I}{\beta_1} = \frac{\Delta_c}{\beta_1} + \left[\frac{1}{m_I} - \frac{1}{m_c}\right] \quad \text{yields } \mu_R.$$

Using the definition of Δ already given and making use of the knowledge that the permeability of copper is very close to that of free space

$$\sqrt{\frac{\mu_L}{\sigma_I}} = \sqrt{\frac{\mu_o}{\sigma_c}} + \mu_o\sqrt{2\omega}\left[l_c - l_I\right]$$

$$\sqrt{\frac{\mu_R}{\sigma_I}} = \sqrt{\frac{\mu_o}{\sigma_c}} + \mu_o\sqrt{2\omega}\left[\frac{1}{m_I} - \frac{1}{m_c}\right]$$

So that providing the conductivity of copper and iron and the frequency of the electromagnetic waves are known the two values of the permeability of iron, μ_R and μ_L , can be independently measured by finding the difference in resonant length of the cavity with an iron and a copper end plate and by measuring the slopes m_I and m_c of the plot of the function defined above.

Although determinations of μ_R and μ_L are sufficient for the purposes of this investigation it is of interest to find the relationship between μ_R and μ_L and the real and imaginary parts of the permeability. This relationship is derived in the appendix.

TECHNIQUE OF MEASUREMENT

The method of measurement was relatively straightforward and simple. Initially the wave guide and coaxial cable to the test cavity were tuned to give a minumum standing wave ratio at the frequency of the monitoring cavity by adjusting tuning stubs. The adjustment resulted in a standing wave ratio in the guide of 1.05. The audio amplifier was tuned to 1000 cycles, the repetition rate of the square wave modulation on the repeller electrode of the klystron oscillator. From time to time it was necessary to change this adjustment slightly, but the drift was very small. To take a series of measurements on a sample of iron or copper, the inside surface was first gently polished with the finest emery paper available to remove any oxidation or dirt. The sample was then bolted to the top end of the cavity by means of four bolts, one at each corner, the thermocouples fastened under their retaining bolts, the heating element put in place over the test sample and a large block of diatomaceous earth put over this to reduce radiation losses from the surface of the heating element. The bell jar was then placed over the system and evacuated. To get the pressure of the vacuum down to less than one millimeter of mercury required about eight or ten hours and was usually done overnight. The system was then ready to begin a series of measurements. The electronic equipment was turned on, the heating element circuit closed, and the water cooling turned on. The temperature of the test sample as indicated by the thermocouples was first brought to approximately 50°C after which a set of measurements was taken. The temperature was again raised, another set of measurements taken and so on up to the highest temperature that could be obtained without exceeding the wattage rating of the heating element.

As the temperature was raised gases that had occluded to the surface of various parts of the system were driven off so that after each elevation of temperature it often required several hours for the vacuum pressure to be reduced below one millimeter of mercury by the pump. During this time the resistance in series with the heating element was adjusted by small amounts to bring the test sample to the temperature desired.

Since the cross-sectional dimensions of the cavity determine the propagation constant γ_1 , the temperature of the cavity walls had a critical effect on the resonant length of the cavity. Consequently it was necessary to control the rate of flow through the cavity cooling coil as accurately as possible. This was done by measuring the quantity of discharge in a given interval of time with a stop watch and a graduated cylinder. The difficulty encountered in keeping the rate of flow constant was that the heat caused bubbles to form in the cooling coil which restricted flow.

After the entire system had been allowed to come to thermal equilibrium the frequency of the oscillator was tuned to the resonant frequency of the fixed monitoring cavity and the position of the plunger in the test cavity was varied through the resonance curve, the output of the audio amplifier being recorded for each position of the plunger. Before taking a set of readings at any given temperature the gain of the amplifier was adjusted to give about 90 percent the full scale deflection of the output meter at resonance in order to increase accuracy. Readings were then taken at successive points beginning with a position of the plunger enough longer than the resonant length that the output of the audio amplifier was about half maximum, or roughly at the half-power part. Readings were taken every .0002 or .0003 inches, depending on the steepness of rise of the resonance curve, until resonance was approached. About the resonant position of the plunger readings were taken every .0001 inches. Usually the resonance curve was traversed three or four times, the frequency of the oscillator being checked before and after each run and readjusted if necessary. Because there was a slight asymmetry of the resonance curve of the test cavity all readings were taken on the side of the curve where the cavity was slightly longer than the resonant length, except for three or four points taken on the short side of resonance to aid in determining the most probable value of $|E|^2_{max}$.

Usually there resulted from eight to ten points which could be used to calculate the resonant length and the slope of the curve $\sqrt{\frac{1-X^2}{X^2}} = m\lambda$. It can be seen that the actual measurements were simple, but the time necessary for the system to come to equilibrium and to adjust all the possible undesirable variables was quite long.

REDUCTION OF DATA

Since the output of the audio amplifier was proportional to $A|E_y|^{1.96}$ where A is a constant of the amplifier and detecting circuit it was only necessary to raise the amplifier output readings to the 1.04 power to obtain values proportional to $|E_y|^2$. The three or four values of $|E_y|^2$ obtained for each position of the plunger at some particular temperature were then averaged. A value of $|E_y|^2_{max}$ was then obtained by inspection of the resulting set of values for $|E_y|^2_{max}$. Then dividing the average value of $|E_y|^2$ for each position of the plunger by $|E_y|^2_{max}$ a series of values for X^2 were obtained. From these $\sqrt{\frac{1-X^2}{X^2}}$ was calculated. The simple method used for determining does not give the most probable value, but the error is small compared with the mean deviation of the individual values $|E_y|^2$ resulting from repeating the series of readings three or four times.

Since

$$\lambda = d - l$$

where d is the length of the cavity and l is the resonant length of the cavity

$$\sqrt{\frac{1-X^2}{X^2}} = m(d-l)$$

or

$$d = \frac{1}{m}\sqrt{\frac{1-X^2}{X^2}} + l$$

Comparing this linear equation with the usual equation for a straight line it can be seen that l is the intercept of the straight line when $\sqrt{\frac{1-X^2}{X^2}} = 0$ or $X^2 = 1$. Likewise $\frac{1}{m}$ is the slope of d plotted versus $\sqrt{\frac{1-X^2}{X^2}}$.

To find the most probable values of the slope and intercept of a straight line given a series of points which theoretically lie on the line the procedure is as follows.(7)

Let d_i be a particular setting of the plunger and

x_i be the corresponding value of $\sqrt{\frac{1-x^2}{x^2}}$

n be the total number of points

Σ represent the summation over the n points

$$\frac{1}{m} = \frac{\begin{vmatrix} \Sigma d_i & n \\ \Sigma d_i x_i & \Sigma x_i \end{vmatrix}}{\begin{vmatrix} \Sigma x_i & n \\ \Sigma x_i^2 & \Sigma x_i \end{vmatrix}}$$

$$l = \frac{\begin{vmatrix} \Sigma x_i & \Sigma d_i \\ \Sigma x_i^2 & \Sigma d_i x_i \end{vmatrix}}{\begin{vmatrix} \Sigma x_i & n \\ \Sigma x_i^2 & \Sigma x_i \end{vmatrix}}$$

The probable error of the resulting values of $\frac{1}{m}$ and l could also be set down but it was found these errors were so much smaller than the average error resulting from evaluating $\frac{1}{m}$ and l several different times as to be negligible. At most temperatures there were three to five separate values of $\frac{1}{m}$ and l determined, and these values were averaged to give the final values used in calculating μ_L and μ_R.

Of the constants needed in evaluating μ_L and μ_R, ω is easily determined by measuring the wave length in the guide, and μ_0 is known to be $4\pi \times 10^{-7}$ henries per meter. The conductivity of iron and copper is replaced in the formulae for μ_L and μ_R by the reciprocal resistivity. The values as well as the temperature coefficients are taken from the Handbook of Chemistry and Physics[8]. This procedure is valid for the accuracy of this experiment because both the iron and copper used were very nearly pure metals, and because there is no known theoretical reason for the resistivity of metals varying with frequencies. A recent investigation[9] has shown that there is some increase in resistivity at very high frequencies but the increase is not appreciable except at frequencies much higher than those used in this experiment.

The values of resistivity used in this experiment are

$$r_{copper} = .17 \times 10^{-7} [1 + (T-20)(.0042)]$$

$$r_{iron} = 1.0 \times 10^{-7} [1 + (T-20)(.0047 + .000021\,T)]$$

where T is the temperature in degrees centigrade. Finally the values of μ_L and μ_R are given in the results in practical units instead of rationalized MKS units and both l and $\frac{1}{m}$ are measured in units of 10^{-4} inches. The final result of this change of units and the substitution of the values of constants is

$$\mu_L^{1/2} [1+(T-20)(.0047+.000021\,T)]^{1/2} = 3.005\,(l_c - l_I) + \{.17[1+(T-20)(.0042)]\}^{1/2}$$

$$\mu_R^{1/2} [1+(T-20)(.0047+.000021\,T)]^{1/2} = 3.005\left(\frac{1}{m_I} - \frac{1}{m_c}\right) + \{.17[1+(T-20)(.0042)]\}^{1/2}$$

RESULTS

Measurements of μ_R and μ_L were made on two samples of iron. Both were made of $\frac{1}{4}$ inch Armco Iron plate milled into squares approximately $1\frac{1}{2}$ by $1\frac{1}{2}$ inches. One sample was prepared according to a heat treatment described by Cioffi (10) in which the iron was heated to a temperature close to the melting point in an atmosphere of damp hydrogen and held there for eighteen hours to drive out the carbon and oxygen and then lowered to about 835°C and held there for twelve hours for annealing and finally cooled to room temperature at the rate of about 35°C per hour. The second sample was not subjected to the high temperature heat treatment but was annealed in the same manner as the first sample. This heat treatment was done by the Jet Propulsion Laboratory.

In the first sample the high temperature to which it was subjected (1370°C) for the purpose of removing impurities was high enough that the crystals reformed on cooling and resulted in beautifully large crystals, some almost 1/8 inch across. Unfortunately this also resulted in a wrinkling of the surfaces so that the sample had to be repolished. The polishing was done with fine emery and took a very long time. But in spite of this care there is no doubt that surface stresses resulted which had a pronounced effect on the measureable value of the permeability since at the frequencies used in this experiment the permeability is almost entirely a surface phenomenon. In an attempt to reduce the surface strains in the samples resulting from polishing both samples were reannealed in the vacuum already available in the experimental set-up for a period of ten hours. It was still necessary to polish the samples lightly before making a set of measurements to remove surface oxidation. It was found that if a layer of oxidation was allowed to develop on the surface of the iron the measurements varied and were not reproducible. As a consequence it must be remembered that the results given here are for iron, the surface of which has been polished and therefore undoubtedly have surface strains which effect the results. The fine grooves resulting from polishing the surface were in the same direction as the electric field and consequently at right angles to H_x for all three samples, two of iron and one of copper.

Previous measurements of the permeability of iron (see references 3 and 11) at very high frequencies show that in the region from eight to nine thousand megacycles μ_R is much larger than μ_L. On the basis of a low frequency permeability of 100 results show that μ_L is around 2 or 3 while μ_R is around 30 or 40. As is well known values of permeability depend a great deal on the exact composition of the iron investigated and on its history. The results of this investigation seem in good agreement with previous ones.

The results are given in table form below and plotted in graph form in the appendix. Sample one is the sample that was heat treated to remove impurities and annealed, and sample two is the one that was annealed only.

SAMPLE ONE

T	μ_R	μ_L
50°C	75.5 ± 16.5	6.6 ± 25.4
100°C	79.1 ± 11.2	.3 ± 4.7
150°C	77.3 ± 11.5	7.5 ± 18.5
200°C	51.2 ± 7.9	5.0 ± 5.1
250°C	45.9 ± 6.1	5.4 ± 11.3
310°C	36.5 ± 4.6	.5 ± 1.2

SAMPLE TWO

T	μ_R	μ_L
50°C	66.9 ± 6.8	1.7 ± 2.6
115°C	42.8 ± 7.6	0.0 ± .07
160°C	31.6 ± 4.8	5.0 ± 4.5
220°C	29.7 ± 3.5	2.7 ± 5.5
280°C	25.8 ± 4.1	5.8 ± 10.2
360°C	17.3 ± 2.7	- 1.9 ± 6.5

In plotting the values of μ_R for the two samples an attempt is made to draw a curve through the observed points. The effect of surface stresses is probably pronounced and most likely explains why both curves show deviations from a steady decrease of μ_R with increasing temperature. As the sample is heated an annealing effect takes place relieving the surface stresses and increasing the permeability. It seems hard to believe that for sample one any relaxation of stresses could take place at a temperature of 100°C. But it was observed that the resonant length and the slope of the resonance curve did decrease slowly with time with the temperature of the sample held at 100°C. The same effect was noticed around 150°C for sample two.

No attempt is made to plot μ_L because the large probable error obscures the variation with temperature. The observed negative value of μ_L for sample two at 360°C is, or course, an impossible result. But with such a large probable error it is understandable how such a result could be observed.

In the derivation included in the appendix it was found that if

$$\mu = \mu_A + j\mu_B$$

then μ_R and μ_L expressed in terms of the complex components of μ are

$$\mu_R = \sqrt{\mu_A^2 + \mu_B^2} - \mu_B$$

$$\mu_L = \sqrt{\mu_A^2 + \mu_B^2} + \mu_B$$

The results of this experiment, and indeed the results of previous experimental determinations of the permeability of iron (see references 3 and 11 for summaries of previous measurements), show that in the region of 8,000 megacycles is much larger than μ_L. From this it is evident that μ_B is negative and of significant magnitude. Consider, for example, the measured values of μ_L and μ_R for sample two at 50°C. Here the most probable values of μ_R and μ_L were

$$\mu_R = 66.9$$

$$\mu_L = 1.7$$

From these values it may be readily found that

$$\mu_B = 32.6$$

$$\mu_A = 10.7$$

so that with respect to time the magnetic flux lags over 70° behind the applied magnetic field of the electromagnetic wave.

ERRORS

Of all the possible sources of error in this experiment there are two that are much larger than any of the others. One is the variation of contact between the sample and the surface against which it is placed to form the end of the cavity. For the theory to be accurate the joint should be perfectly conducting. But thermal expansion and the slight deviation from a plane surface of the sample and the plate against which it is pressed introduce small cracks and contact differences. Primarily this uncertainty in the contact of the sample with the cavity effects the resonant length since the cracks do not absorb energy themselves. However the walls of the cracks are part brass and part iron so that effects the formulae depend upon being entirely due to iron are partly due to brass.

The other main source of error is the variation of temperature of the walls of the cavity. For a given temperature of the sample the walls of the cavity must be at the same temperature for both the iron and copper sample if the comparison of the two is to eliminate the effects of the cavity walls. Since the brass walls of the cavity form the major part of the conducting area any deviations of its temperature cause a much greater change in the measurements than the same change in temperature of the sample causes. The temperature of the cavity walls depends on two things, (1) the temperature of the heating element, and (2) the cooling of the cavity by the water cooling coil. Cooling is controlled by the rate of water flow through the coil while the temperature of the heating element is controlled by measuring the temperature of the sample. Since the heating element does not make the same contact at all times and the temperature of the water for cooling depends on room temperature it is to be expected that there will be variation in the wall temperature. This error also primarily effects the resonant length and thus the measured value of μ_L since to a first approximation a uniform increase in all cavity dimensions does not effect the "Q".

Other sources of error that undoubtedly affect the results are random fluctuations of the frequency of the oscillator and fluctuations in the amplification of the audio amplifier. These errors however average out over one set of readings in contrast to the previously mentioned errors which vary from one set of readings to the other.

The other error of any importance is due to the contact of the thermocouple junctions with the test sample. The result is that the temperature read on the thermocouple potentiometer may not be the temperature of the test sample. It was found that even slight loosening due to thermal expansion of the bolts that fastened down the thermocouple junctions would cause radical fluctuations in the temperature read. There was also a variation of temperature over the surface of the sample due to the design of the heating element and the thermal conduction of the cavity walls. Consequently there is some uncertainty as to what the effective temperature of the sample should be. However it was found that if the temperature near the cavity walls was held constant while the temperature at the center of the sample was varied there was little effect on the measurements. Consequently the effective temperature was considered to be close to that of the outer temperature.

An example of the measurements made on sample two at the nominal temperature of 50°C will indicate the effect of these various errors.

Let

T_A = temperature at center of sample

T_B = temperature of sample at the cavity wqll.

IRON

<u>First set of readings</u>

$T_A = 48°C$ $\quad \frac{1}{m_I} = (15.65 \pm .05) \times 10^{-4}$

$T_B = 54°C$ $\quad l_I = (2222.61 \pm .04) \times 10^{-4}$

<u>Second set of readings</u>

$T_A = 49°C$ $\quad \frac{1}{m_I} = (15.40 \pm .10) \times 10^{-4}$

$T_B = 53°C$ $\quad l_I = (2222.79 \pm .08) \times 10^{-4}$

<u>Third set of readings</u>

$T_A = 48°C$ $\quad \frac{1}{m_I} = (15.98 \pm .07) \times 10^{-4}$

$T_B = 54°C$ $\quad l_I = (2223.74 \pm .05) \times 10^{-4}$

<u>Average values</u>

$T = 50°C$ $\quad \frac{1}{m_I} = (15.63 \pm .11) \times 10^{-4}$

$l_I = (2222.71 \pm .36) \times 10^{-4}$

COPPER

First set of readings

$T_A = 49^\circ C$ $\quad \frac{1}{m_c} = (12.99 \pm .03)\times 10^{-4}$

$T_B = 53^\circ C$ $\quad l_c = (2222.74 \pm .02)\times 10^{-4}$

Second set of readings

$T_A = 48^\circ C$ $\quad \frac{1}{m_c} = (12.57 \pm .04)\times 10^{-4}$

$T_B = 53^\circ C$ $\quad l_c = (2223.18 \pm .03)\times 10^{-4}$

Third set of readings

$T_A = 48^\circ C$ $\quad \frac{1}{m_c} = (13.12 \pm .06)\times 10^{-4}$

$T_B = 52^\circ C$ $\quad l_c = (2223.15 \pm .05)\times 10^{-4}$

Average values

$T = 50^\circ C$ $\quad \frac{1}{m_c} = (12.85 \pm .11)\times 10^{-4}$

$l_c = (2223.03 \pm .03)\times 10^{-4}$

Result:

$$\sqrt{\mu_R/\sigma_I} = 3.005\ (2.78 \pm .15) + 1/\sqrt{\sigma_c}$$

$$\sqrt{\mu_L/\sigma_I} = 3.005\ (.32 \pm .36) + 1/\sqrt{\sigma_c}$$

μ_R and μ_L are found by substituting into these expressions the values of σ_I and σ_c at 50° C. Although there is a small error in the measurement of temperature it does not compare in effect with the error of the other measurements. It can be seen that the percentage error in determining μ_L is many times larger than that in determining μ_R. It can also be seen that the random error in measuring any one value of $\frac{1}{m}$ or l is negligible compared to the standard deviation resulting from averaging.

CONCLUSIONS

The very small probable error in the values of $\frac{l}{m}$ and l determined by a single set of readings indicate that the cavity resonance technique is an excellent method of measuring the permeability of iron. However the comparison technique used in this investigation would need to be improved to obtain accurate results. As the experiment progressed many improvements in the experimental set-up came to light that seemed so simple it seemed obvious that they should have been used in originally designing the equipment. The most obvious improvement would be to use a cylindrical cavity with the test sample forming one of the plane ends and a plunger forming the other. Not only would contact differences have been greatly reduced since with the proper mode of propagation no current flows across the joint between the cylindrical walls and the plane end but also the thermal problem could have been reduced because the test sample could have been insulated from the rest of the cavity by a ring of quartz or some other refractory dielectric.

The magnitude and temperature dependence of μ_R and μ_L measured in this investigation for the two samples or iron show no remarkable properties. Within experimental errors the values of μ_L and μ_R are in agreement with previous determinations. The temperature dependence of μ_R agrees with the variation of permeability found at low frequencies. The temperature dependence of μ_L is completely obscured by the large experimental error. The one phenomenon of unusual interest observed is the one already measured. That is the apparent inflections in the curve of μ_R as a function of temperature which seem to be due to relaxation of strains on the surface of the samples. Since permeability at very high frequencies is largely a surface phenomenon it is to be expected that such effects should enter into the measured results in a pronounced manner.

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A P P E N D I X

THE RELATIONSHIP BETWEEN μ_R AND μ_L AND THE COMPLEX FORM OF μ

In the theory of measurement the permeability, μ , is treated as a real quantity. The result is that the two independent experimental determinations of the value of that are possible by resonant cavity technique are not consistent. The two different measured values of μ are called μ_L and μ_R and that they are not equal is explained by the fact that μ is actually complex since the intensity of magnetization is not in phase with the applied magnetic field in the material whose permeability is being measured. The relationship between the complex components of μ and μ_R and μ_L is not self evident from the theory of measurement. In fact the approximations made in the theory of this investigation make it difficult to see how a complex permeability can account for the difference in values of μ_R and μ_L .

Suppose one writes

$$\mu_2 = \mu_A + j\mu_B = |\mu_2| e^{i\phi}$$

By following through the theory developed under <u>Theory of Measurement</u> one can see that no difficulty is encountered in substituting the complex form of μ in any of the equations until an attempt is made to solve for ρ . It was found that to an excellent approximation the coefficient of reflection at a conducting end of the cavity is

$$K = -\left[1 - \frac{2j\beta_1\mu_2}{\mu_0(1+j)\sqrt{\omega\mu_2\sigma_2/2}}\right]$$

To express E_y in closed form it was found convenient to write

$$K = e^{-2p}$$

and express E_y in terms of p where p must be evaluated from the expression for K. Thus

$$e^{-2p} = -\left[1 - 2(1+j)\frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_2}{2\omega\sigma_2}}\right]$$

Before, with μ_2 considered as being real,

$$\Delta \equiv \frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_2}{2\omega\sigma_2}}$$

and

$$e^{-2p} = -\left[1 - 2(1+j)\Delta\right]$$

where Δ was very small.

Substituting in the complex form of μ_2

$$\Delta = \frac{\beta_1}{\mu_0}\sqrt{\frac{|\mu_2|}{2\omega\sigma_2}}\, e^{\frac{i\phi}{2}}$$

$$= |\Delta|\, e^{\frac{i\phi}{2}}$$

where $|\Delta|$ is the magnitude of Δ

So that

$$e^{-2p} = -\left[1 - 2(1+j)|\Delta|\, e^{\frac{i\phi}{2}}\right]$$

Now letting $p = \xi + j\eta$ and solving for the approximate values ξ and η in the same fashion as before a different result is obtained

$$e^{-2\xi}\cos 2\eta = -1 + 2|\Delta|\cos\frac{\phi}{2} - 2|\Delta|\sin\frac{\phi}{2}$$

$$e^{-2\xi}\sin 2\eta = +2|\Delta|\cos\frac{\phi}{2} + 2|\Delta|\sin\frac{\phi}{2}$$

$$\tan 2\eta = \frac{2|\Delta|(\cos\frac{\phi}{2} + \sin\frac{\phi}{2})}{1 - 2|\Delta|(\cos\frac{\phi}{2} - \sin\frac{\phi}{2})}$$

$$\cong 2|\Delta|(\cos\frac{\phi}{2} + \sin\frac{\phi}{2})$$

Since $\cos 2\eta$ is negative

$$\eta \cong \frac{\pi}{2} + |\Delta|(\cos\frac{\phi}{2} + \sin\frac{\phi}{2})$$

Since $|\Delta|(\cos\frac{\phi}{2} + \sin\frac{\phi}{2})$ is very small $\eta \cong \frac{\pi}{2}$

$$\cos 2\eta \cong -1$$

and from the value of $e^{-2\xi}\cos 2\eta$

$$\xi \cong |\Delta|(\cos\frac{\phi}{2} - \sin\frac{\phi}{2})$$

The approximation that

$$\eta = \frac{\pi}{2} + \xi$$

no longer holds in general and is valid only if ϕ is very small. This was the implied assumption in treating μ as a real quantity. The result is

$$\rho = |\Delta|(\cos\frac{\phi}{2} - \sin\frac{\phi}{2}) + j\left[\frac{\pi}{2} + |\Delta|(\cos\frac{\phi}{2} + \sin\frac{\phi}{2})\right]$$

This value of ρ along with

$$\gamma_1 = \alpha_1 + j\beta_1$$

$$\rho' = \Delta' + j(\frac{\pi}{2} + \Delta')$$

may be substituted in the expression for E_y

$$E_y = A_0 \cosh[\gamma_1 z + p][1 + \coth(\gamma_1 d + p + p')]$$

and the same process of reduction by expansion and approximation may be followed as was done before in the Theory of Measurement. Without including all the steps in the process since they are exactly analogous to what was done before, the result is obtained that

$$|E_y|^2 \cong A_0^2 \left[\left(\tfrac{3}{2}\alpha_1\lambda_g + \Delta' + |\Delta|\cos\tfrac{\phi}{2} - |\Delta|\sin\tfrac{\phi}{2}\right)^2 + \left(\beta_1 l + \Delta'' + |\Delta|\cos\tfrac{\phi}{2} + |\Delta|\sin\tfrac{\phi}{2}\right)^2\right]^{-1}$$

When μ was treated as a real number

$$|E_y|^2 = \left[\left(\tfrac{3}{2}\alpha_1\lambda_g + \Delta' + \Delta\right)^2 + \left(\beta_1 l + \Delta' + \Delta\right)^2\right]^{-1}$$

μ_L was determined by the resonant length of the cavity, or the value of l for which $\beta_1 l + \Delta'' + \Delta$ equals zero. Hence by comparing the two expressions above for $|E_y|$ it is evident that

$$\frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_L}{2\omega\sigma_2}} = \frac{\beta_1}{\mu_0}\sqrt{\frac{|\mu_2|}{2\omega\sigma_2}}\left(\cos\frac{\phi}{2} + \sin\frac{\phi}{2}\right)$$

$$\sqrt{\mu_L} = \sqrt{|\mu_2|}\left(\cos\frac{\phi}{2} + \sin\frac{\phi}{2}\right)$$

Likewise the Δ which occurred in the slope measurement from which μ_R was determined was the one bracketed with $\frac{3}{2}\alpha_1\lambda_g$ and Δ'. Hence it is also evident that

$$\frac{\beta_1}{\mu_0}\sqrt{\frac{\mu_R}{2\omega\sigma_2}} = \frac{\beta_1}{\mu_0}\sqrt{\frac{|\mu_2|}{2\omega\sigma_2}}\left(\cos\frac{\phi}{2} - \sin\frac{\phi}{2}\right)$$

or

$$\sqrt{\mu_R} = \sqrt{|\mu_2|}\left(\cos\frac{\phi}{2} - \sin\frac{\phi}{2}\right)$$

Consequently μ_R and μ_L expressed in terms of the complex forms for μ_2 are

$$\mu_L = |\mu_2|(1 + \sin\phi)$$

$$\mu_R = |\mu_2|(1 - \sin\phi)$$

where

$$\mu_2 = |\mu_2| e^{i\phi} = \mu_A + j\mu_B$$

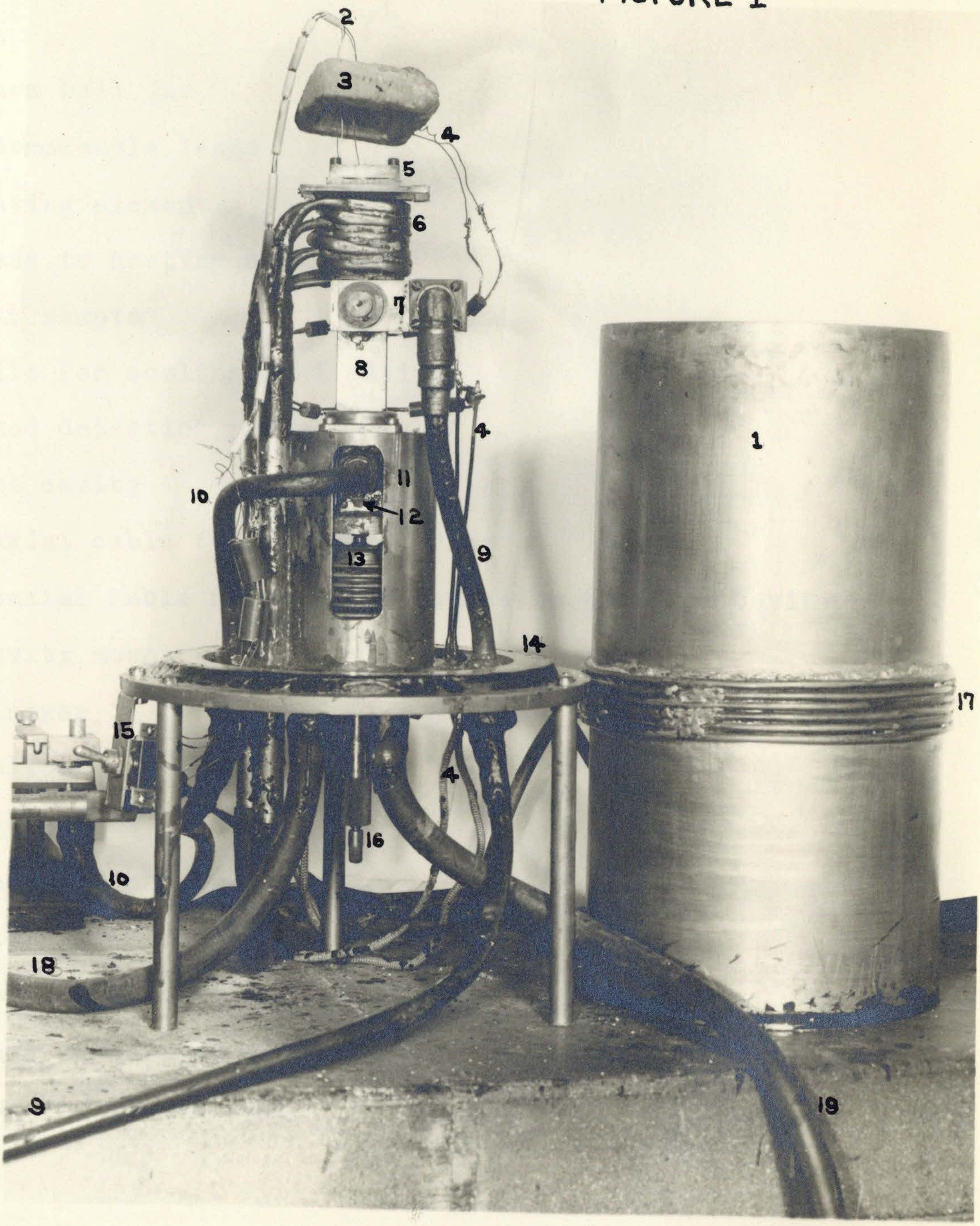
μ_L and μ_R may also be expressed in terms of μ_A and μ_B

$$\mu_L = \sqrt{\mu_A^2 + \mu_B^2} + \mu_B$$

$$\mu_R = \sqrt{\mu_A^2 + \mu_B^2} - \mu_B$$

These are the desired relationships between μ_L and μ_R and the complex value of μ .

PICTURE I

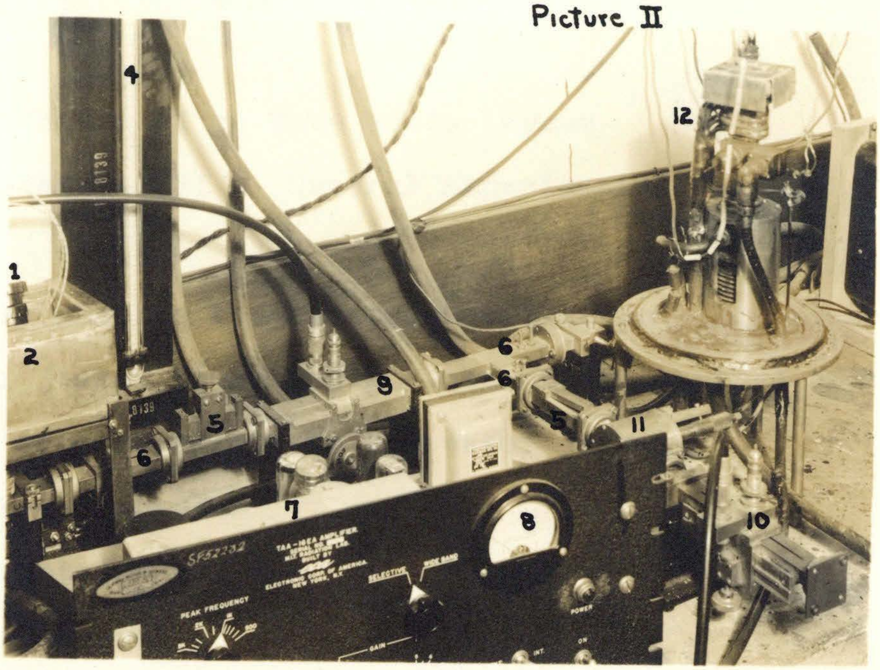


DETAILS OF PICTURE I

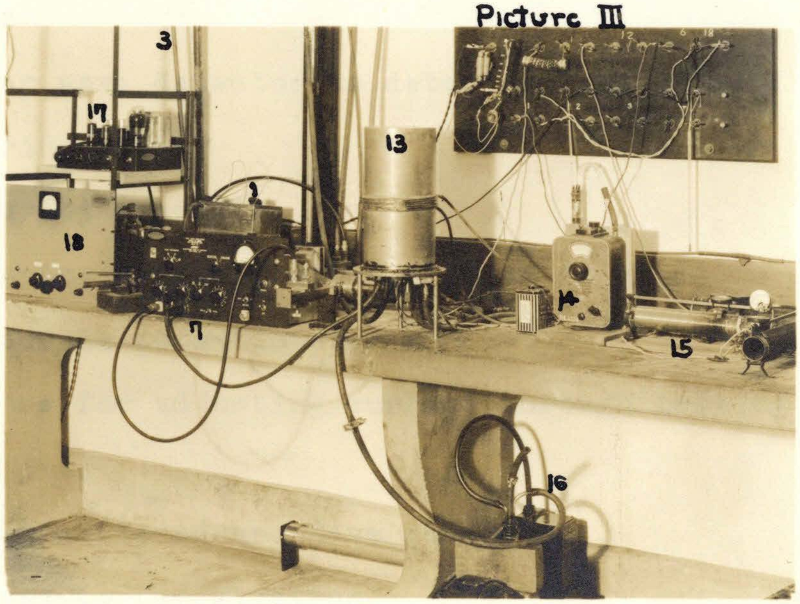
Picture I : Test Cavity Assembly

1. Brass bell jar
2. Thermocouple leads
3. Heating element
4. Leads to heating element
5. Test sample
6. Coils for cooling test cavity
7. Tuned detecting probe
8. Test cavity
9. Coaxial cable to audio amplifier
10. Coaxial cable from energy source to the test cavity
11. Cavity mount
12. Plunger
13. Bellows
14. Base plate for bell jar
15. Thermocouple switch
16. Micrometer for moving the plunger
17. Coils for cooling the bell jar
18. Hose to mercury manometer
19. Hose to vacuum pump

Picture II



Picture III



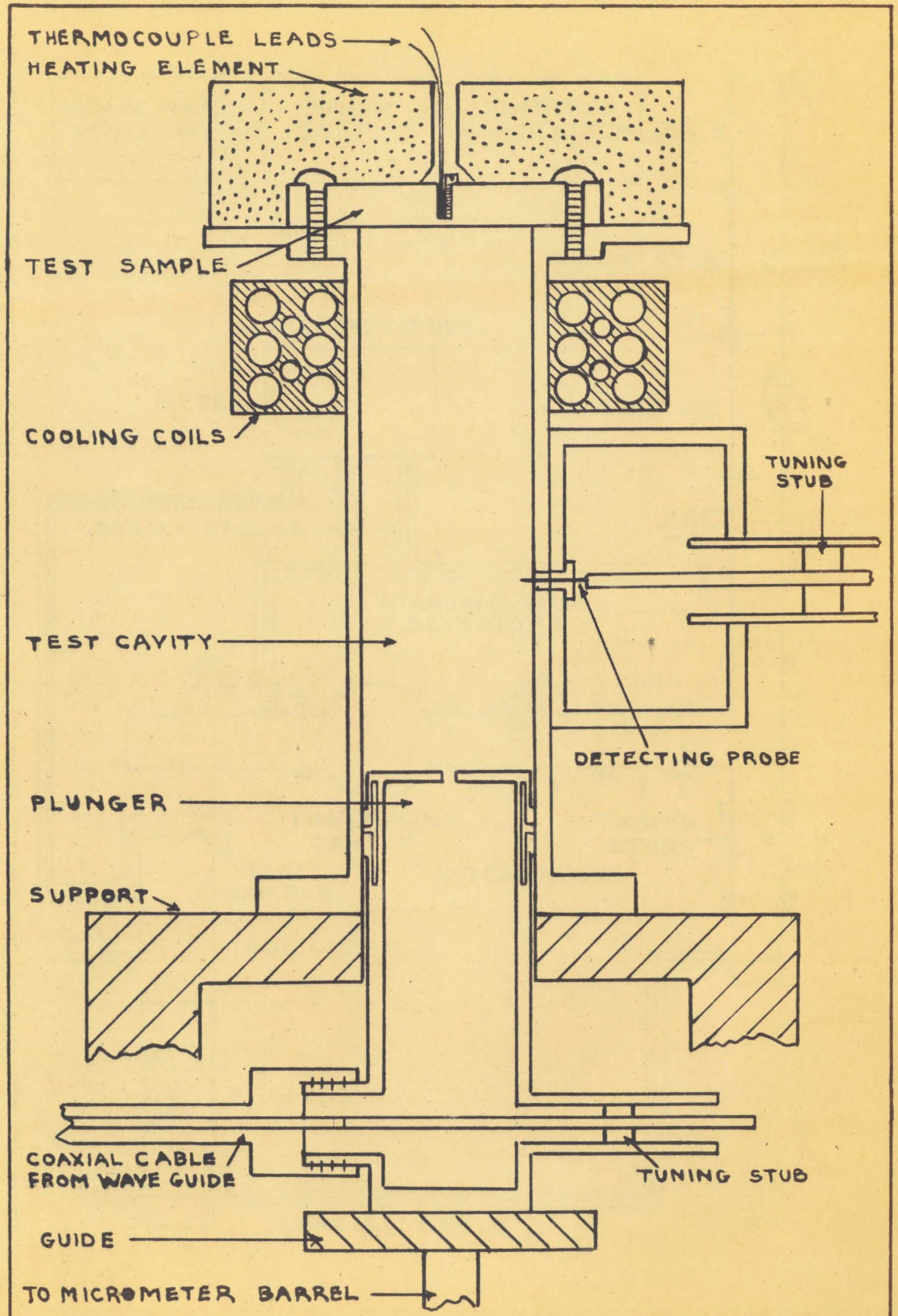
DETAILS OF PICTURES II AND III

Picture II : Wave Guide Layout

Picture III : General Experimental Set-up

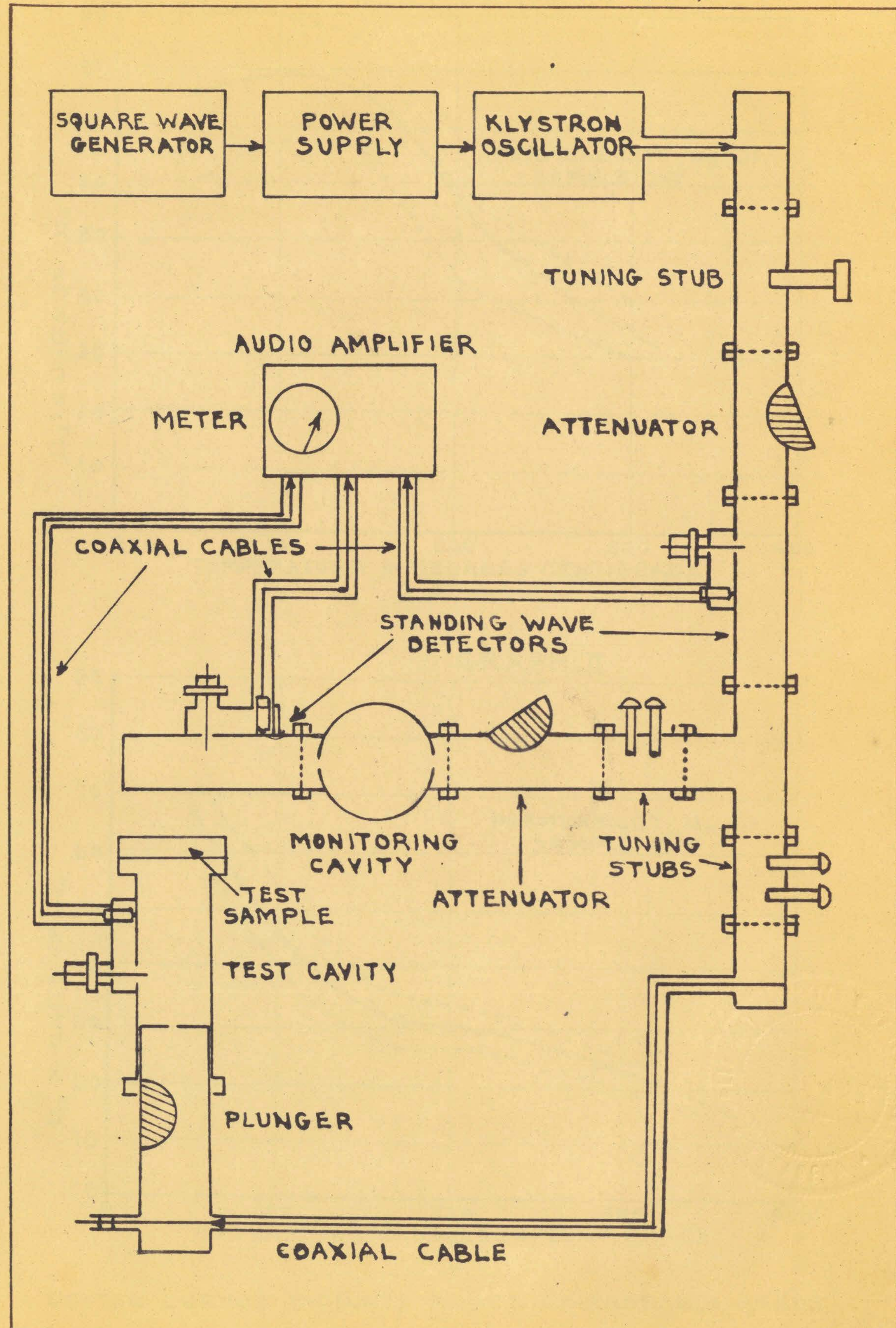
1. Tuning knob on the klystron oscillator
2. Water jacket to maintain constant temperature of the klystron
3. Hose from reservoir to the water jacket
4. Mercury manometer
5. Attenuators
6. Tuning screws
7. Audio amplifier
8. Output meter
9. Standing wave detector to measure wave length and to check the match of the oscillator to the test and monitoring cavities
10. Standing wave detector to detect output of monitoring cavity
11. Monitoring cavity
12. Test cavity assembly
13. Test cavity assembly with bell jar in place
14. Thermocouple potentiometer
15. Resistors for adjusting current through heating element
16. Vacuum pump
17. Thousand cycle square wave generator
18. Power supply for klystron

PLATE I



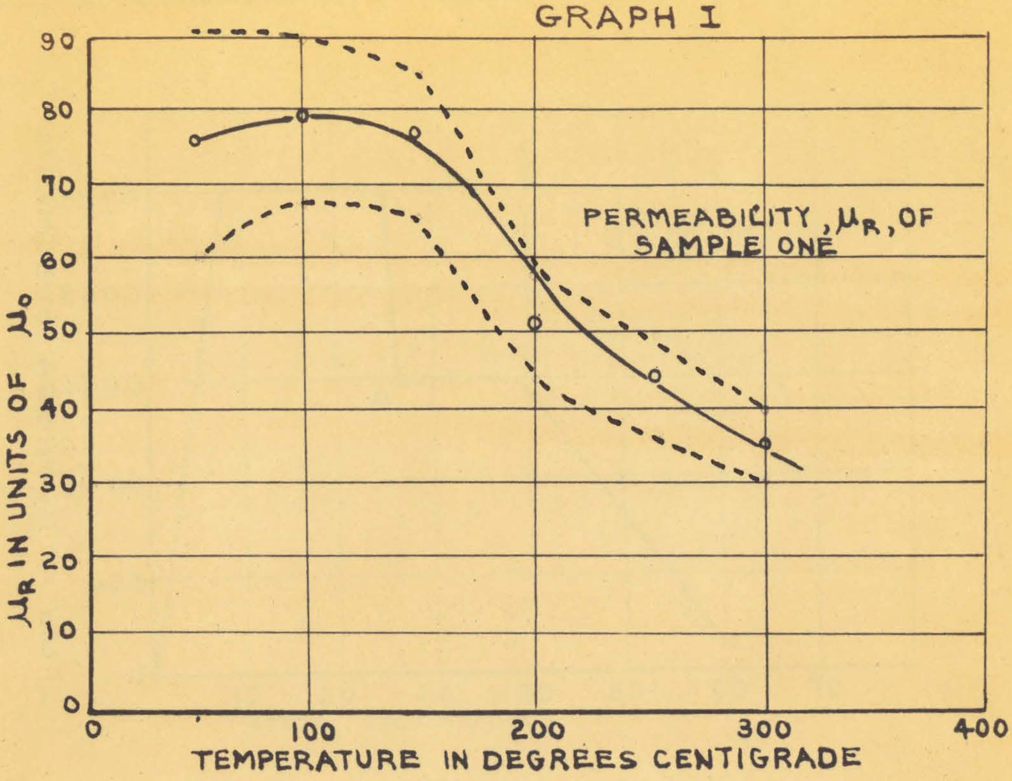
SCHEMATIC CROSS SECTION OF TEST CAVITY

PLATE II

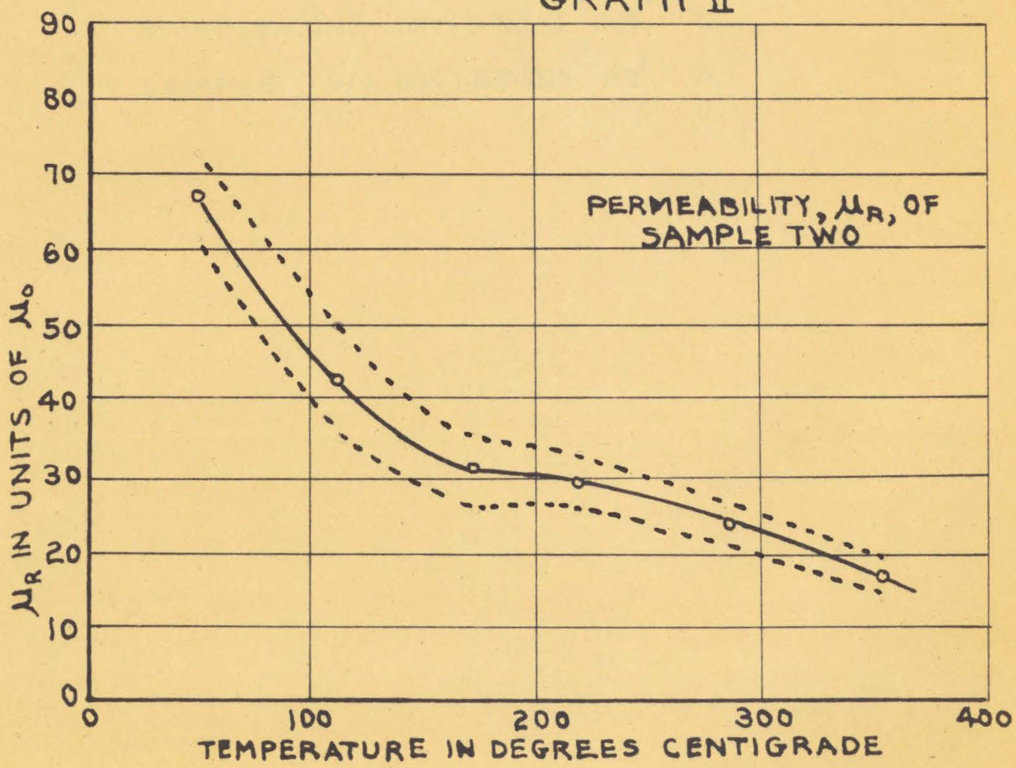


SCHEMATIC DIAGRAM OF ELECTROMAGNETIC CIRCUIT

GRAPH I

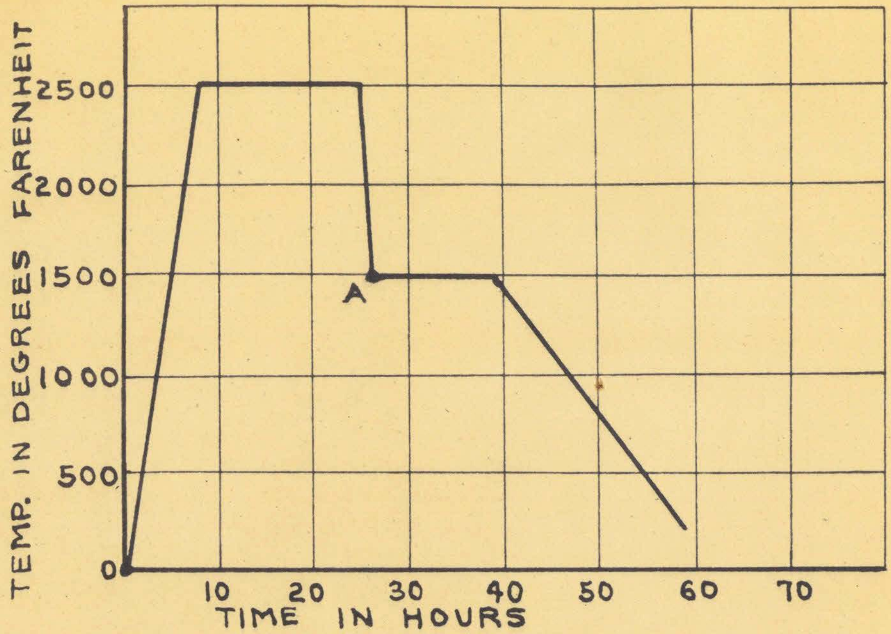


GRAPH II



DOTTED CURVES INDICATE RANGE OF PROBABLE ERROR

GRAPH III



TIME-TEMPERATURE RECORD OF IRON SAMPLES

SAMPLE ONE ENTERED AT 0

SAMPLE TWO ENTERED AT A