LOSSES IN CAVITY RESONATORS

Thesis

by

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ACKNOWLEDGEMENT

I owe thanks to all the faculty members of the Department of Electrical Engineering, especially Dr. S. S. Mackeown and Dr. W. H. Pickering, who both supervised my research, and also to David Hagelbarger.

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NOTATION

		Units
$\bar{E}$	Vector electric field intensity	cgs esu
$\bar{D}$	Vector electric field displacement	cgs esu
K	Dielectric constant	cgs esu
τ	Volume resistivity	cgs esu
x, y, z	Coordinates of length	cgs
$\bar{B}$	Vector magnetic field flux density	cgs emu
$\bar{H}$	Vector magnetic field intensity	cgs emu
μ	Permeability of cavity	cgs emu
c	Velocity of light	cgs
μ'	Permeability of cavity walls	cgs emu
ω , f	Angular frequency, frequency	cgs
$\bar{A}$	Magnetic vector potential	cgs emu
$\bar{i}$, $\bar{j}$, $\bar{k}$, $\bar{u}$	Coordinate vectors	
Δ	Skin thickness $\Delta = \left\{\frac{\tau c^2}{2\pi\mu'\omega}\right\}^{\frac{1}{2}}$	cgs
θ	Phase angle	degrees
t	Time	cgs
λ	Wavelength	cgs
Δ f	Frequency increment between half power points of the resonance curve	cgs
I	Amplitude of alternating current	amperes
V	Amplitude of alternating voltage	volts
R	Resistance	ohms
L	Inductance	henries
C	Capacitance	farads

SUMMARY

The problem investigated was the measurement of the Q, or figure of merit, of a resonant cavity and observation of the variation in Q with changes in the size and shape of the coupling holes. A method of measurement was developed using a spectrum analyzer as a detector and a signal with a known spectrum to calibrate the analyzer for frequency changes. The results indicate that the Q is lowered if the size of the coupling windows is increased. There are two contributions to the losses which may be considered separately; the losses on the inner walls of the resonator, and the losses due to energy radiated out of the coupling hole.

II. THEORETICAL CONSIDERATIONS

Introduction

The striking development of microwave techniques in recent years and the introduction of wave guides and cavity resonators to general usage might give the impression that the ideas and theory are very new. Actually, the theoretical basis for the use of wave guides was laid down a long time ago, in 1897, by Lord Rayleigh.[1] The application of the theory was delayed for a long time after the development of the three element vacuum tube because the wavelengths which could be produced were so large that wave guides and enclosed resonators would have been too large for practical use.

Centimeter wavelengths had been produced late in the nineteenth century by Lecher, but these were only damped waves. It was not until after the first World War that invention of centimeter wavelength generators proceeded far enough to stimulate interest in special transmission systems and resonant elements for this part of the spectrum.

In modern microwave systems resonant cavities perform many vital functions. Cavities are used to measure frequency, to form the oscillating chamber of electron velocity modulated vacuum tubes, and in many special instances.

Since the cavity is almost completely enclosed it does not suffer from radiation losses except through the small openings which are

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1. Phil Mag. V 43 1897 "On the propagation of waves in dielectric cylinders."

required for electromagnetic coupling to the resonator. Possible application of cavity resonators is not limited to microwave frequencies by any electrical properties, but because the size of the cavities cannot be much smaller than half a wavelength, which is too large for use in the radio frequency portion of the spectrum.

The theoretical treatment of completely closed cavity resonators for all simple shapes is well known. Important work in this field has been done by Hansen,[1] Barrow and Chu,[2] Hahn,[3] Hansen and Richtmyer,[4] E. U. Condon, and others.

In 1941 the beginning of World War II interrupted the free publication of work dealing with microwave techniques and theory, so the accumulated discoveries of the war years are just beginning to be available to research workers who were not connected with governmental or military organizations which had access to security classified reports. As a result it is not possible to present a bibliography of theoretical work on cavity resonators with any assurance that it is even partially complete. A great volume of work was done at the Radiation Laboratory of Massachusetts Institute of Technology, where

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1. Hansen, W. W. J. App. Phys., V. 9, Oct. 1938
2. Chu, L. J. and Barrow, W. L., Proc. IRE, V. 26, 1938
3. Southworth, G. C., Proc. IRE, V. 25, 1937
4. Hansen, W. W. and Richtmyer, R. D., J. App Phys.,V. 10, 1939

the principal theoreticians were Hans Bethe, N. H. Frank, Julian Schwinger, and J. C. Slater. Other names will undoubtedly be added to the list when security regulations are relaxed. These men solved some of the problems relating to the effects of windows through conducting barriers in wave guides and resonators. Such a window is often called an iris.

The research described in this thesis was an experimental investigation of the effect of the size and shape of the coupling iris on the Q of a cavity and the development of a suitable method for making the measurement. All the cavities tested were rectangular in shape and were made by closing of the ends of lengths of standard brass wave guide pipe whose cross section is .9 inches wide and .4 inches high. The wavelength was about 3.2 centimeters.

Theory of Q of completely closed rectangular cavities.[1]

When Maxwell's equations are solved for the empty region inside a cavity with perfectly conducting walls, the boundary condition is that the tangential component of the electric field must vanish at the walls. The equations of the field are found by solving the vector form of the wave equation and using the magnetic vector potential $\bar{\mathbf{A}}$,

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1. The basic theory given here is summarized from "Static and Dynamic Electricity," by W. R. Smythe, who follows the methods of E. U. Condon. The notation is the same used in this textbook. Gaussian units are used.

$$\nabla \cdot \bar{D} = 0 \qquad \nabla \times \bar{B} = \frac{4}{c} \frac{\partial \bar{D}}{\partial t} \tag{1}$$

$$\nabla \cdot \bar{B} = 0 \qquad \nabla \times \bar{E} = -\frac{1}{c} \frac{\partial \bar{B}}{\partial t}$$

$$\bar{B} = \nabla \times \bar{A} \qquad \bar{E} = -\frac{1}{c} \frac{\partial \bar{A}}{\partial t}$$

$$\nabla^2 \bar{A} = \frac{4K}{c^2} \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla^2 \bar{B} = \frac{4K}{c^2} \frac{\partial^2 \bar{B}}{\partial t^2}$$

$$\nabla^2 \bar{E} = \frac{4K}{c^2} \frac{\partial^2 \bar{E}}{\partial t^2}$$

Since the divergence of $\bar{E}$ is zero, $\bar{E}$ may be expressed as the curl of a vector and no scalar potential appears in the solution. Using the method described in "Static and Dynamic Electricity," the equation for $\bar{A}$ may thus also be written as the curl of a vector,

$$\bar{A} = \nabla \times \bar{u} W_H + \nabla \times (\bar{u} \times \nabla W_E) \tag{2}$$

where $\bar{u} = \bar{i}, \bar{j}$, or $\bar{k}$

and W is a solution of the scalar wave equation.

The solution for $\bar{E}$ and $\bar{B}$ can be expressed in terms of the W functions as follows,

$$\bar{E} = -\frac{1}{c} \frac{\partial}{\partial t} \left\{ \nabla \times \bar{u} W_H + \nabla \times (\bar{u} \times \nabla W_E) \right\} \tag{3}$$

$$\bar{B} = \nabla \times \bar{A} = -\frac{\omega^2 4K}{c^2} \nabla \times \bar{u} W_E - \nabla \times (\bar{u} \times \nabla W_H)$$

It turns out that this selection of W_E and W_H has physical significance, since for W_E the magnetic field component along $\bar{u}$ is zero, and for W_H the electric field component along $\bar{u}$ is zero. The former is called an E- wave or transverse magnetic (TM) mode, while the latter is called an H- wave or transverse electric (TE) mode.

The modes of oscillation are calculated by finding solutions to the scalar wave equation, which are substituted in the relationships above to give the fields. The different solutions for W are called modes of oscillation.

Once the value of the fields is known, assuming perfect conductivity in the walls, the losses in the walls are calculated using the field solutions and ordinary eddy current theory. This method is justifiable because the fields are not disturbed appreciably by losses which occur when ordinary metals are the walls.

The Q of a cavity is defined as

$$Q = \frac{\omega \times \text{stored energy in the electromagnetic field}}{\text{average power lost}} \qquad (4)$$

In a closed cavity resonator, the magnetic field energy is equal to the electric field energy, and energy is interchanged back and forth from one to the other. Consequently, the stored energy is equal to the peak value of the energy in either field and can be computed directly from the magnetic vector potential by the formula

$$\text{Stored Energy} = W = \frac{\omega^2 K}{8\pi c^2} \int_V |\bar{A}| \cdot |\bar{A}| \, dv \qquad (5)$$

where $|A|$ is the peak amplitude of the magnetic vector potential.

The losses can be computed from the magnetic field by the formula

$$\bar{\bar{P}}_{\text{average}} = \frac{\omega \mu' \Delta}{16\pi \mu^2} \int_S |\bar{B}| \cdot |\bar{B}| \, ds \tag{6}$$

Therefore,

$$Q = \frac{2\omega^2 K \mu^2}{\mu' C^2 \Delta} \frac{\int_V |\bar{A}| \cdot |\bar{A}| \, dv}{\int_S |\bar{B}| \cdot |\bar{B}| \, ds} \tag{7}$$

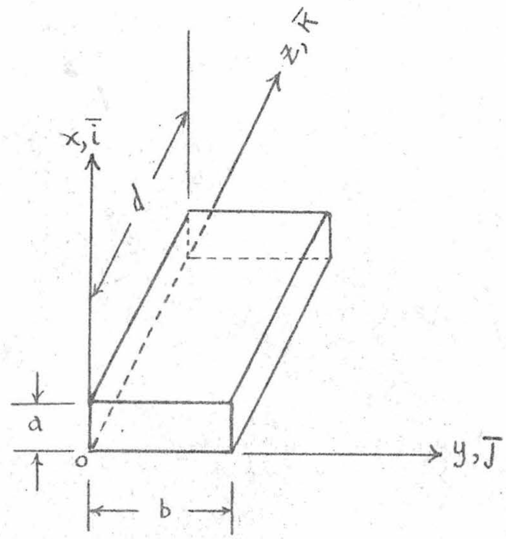

FIGURE 1.

For the cavities used in the experiments, the coordinate system of Figure 1. was employed. The cavity is excited in the mode of oscillation where the electric field is entirely in the x direction, while the magnetic field has components only in the y and z directions.

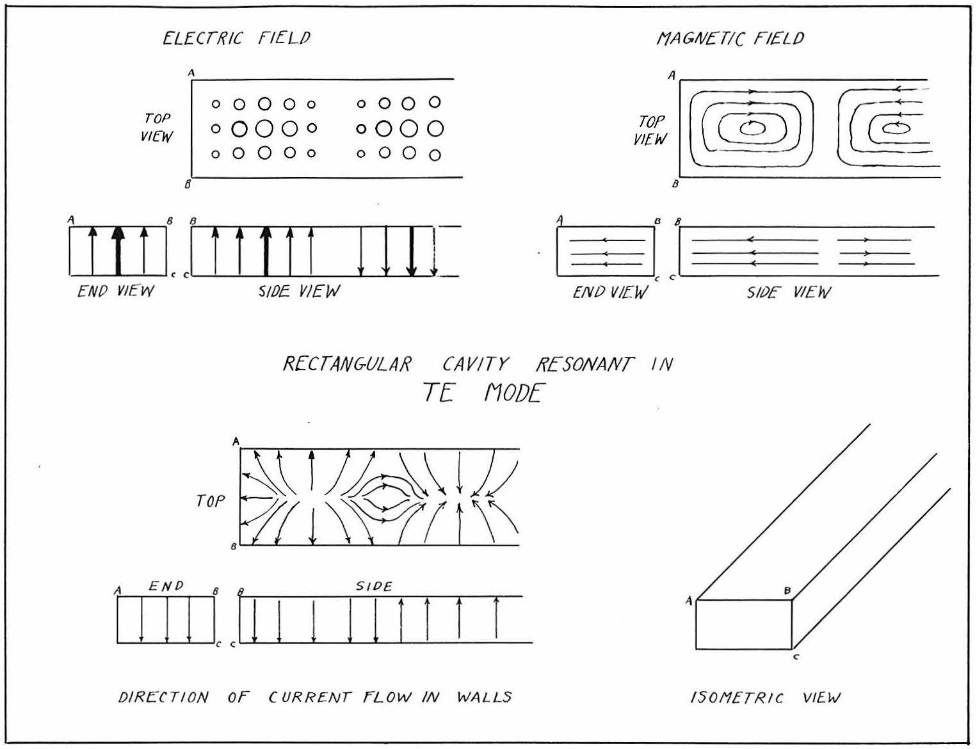

Figure 2

The fields are pictured in the illustration of Figure 2, along with the direction of current flow in the walls. This figure shows one end of a cavity containing a number of wave cells. Cavities of different length may be constructed for a particular frequency by simply adding integral numbers of cells together. The fields in all of these cells will be the same, so it is possible to calculate the Q for a cavity with any number of cells by using the stored energy and losses determined for a single cell.

The mode of oscillation is an H- mode if $\bar{u} = \bar{k}$ in equation 2.

In this case, for a single cell of dimensions a, b, d, the formulas for the vector potential and magnetic may be obtained directly from Smythe

$$\bar{A}_H = -\bar{i}\pi C_H \sin\frac{\pi y}{b} \sin\frac{\pi z}{d} \cos(\omega t + \theta) \tag{8}$$

$$\bar{B}_H = \pi^2 C_H \left\{ -\bar{j}\frac{1}{bd} \sin\frac{\pi y}{b} \cos\frac{\pi z}{d} + \bar{k}\frac{1}{b^2} \cos\frac{\pi y}{b} \sin\frac{\pi z}{d} \right\} \cos(\omega t + \theta)$$

From equations (5) and (8) the stored energy is found to be

$$W = \frac{\omega^2 K}{8\pi c^2} \int_V |\bar{A}|\cdot|\bar{A}|\, dv \tag{9}$$

$$= \frac{\omega^2 K}{8\pi c^2} \frac{\pi^2 C_H^2 d}{4b} = \frac{\omega^2 K \pi d C_H^2}{32 c^2 b}$$

From equations (6) and (8) the losses are found to be

$$\bar{P} = \frac{\omega \mu' \Delta}{16\pi \mu^2} \int_S |\bar{B}|\cdot|\bar{B}|\, ds$$

$$= \frac{\omega \mu' \Delta}{16\pi \mu^2} \left\{ \int_{\text{END WALLS}} |\bar{B}|\cdot|\bar{B}|\, ds + \int_{\substack{\text{TOP AND} \\ \text{BOTTOM}}} |\bar{B}|\cdot|\bar{B}|\, ds + \int_{\text{SIDES}} |\bar{B}|\cdot|\bar{B}|\, ds \right\}$$

$$= \frac{\omega \mu' \Delta}{16\pi \mu^2} \left\{ \frac{\pi^4 C_H^2 a}{b d^2} + \frac{\pi^4 C_H^2}{2} \frac{b^2 + d^2}{b^3 d} + \pi^4 C_H^2 \frac{ad}{b^4} \right\}$$

$$\frac{1}{Q} = \frac{\bar{P}}{\omega W} = \frac{\mu'\Delta/16\pi\mu^2 \left\{ \overbrace{\frac{\pi^4 C_H^2 a}{b d^2}}^{\text{ENDS}} + \overbrace{\frac{\pi^4 C_H^2}{2} \frac{b^2+d^2}{b^3 d}}^{\text{TOP AND BOTTOM}} + \overbrace{\pi^4 C_H^2 \frac{ad}{b^4}}^{\text{SIDES}} \right\}}{\frac{\omega^2 K \pi C_H^2 d}{32 b c^2}} \tag{10}$$

The resonant frequency is found from

$$\omega^2 = \frac{\pi^2 c^2}{4K}\left\{\frac{1}{b^2} + \frac{1}{d^2}\right\} \tag{11}$$

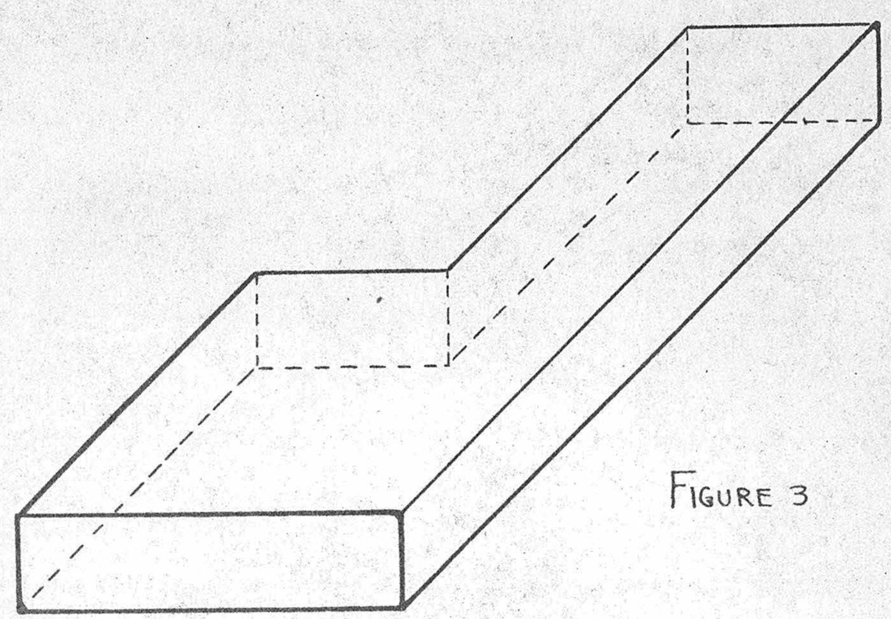

FIGURE 3

Equation 10 is particularly useful since the losses from the various faces are separated. It is possible to use it to compute the Q of any cavity formed by adding wave cells on any face of the resonator. For example, to find the Q of the cavity illustrated in Figure 3, we have four end faces, six tops and bottoms, four side walls, and three cells.

The corresponding terms of equation 10 can simply be multiplied by the appropriate factor.[1] In general, the Q is increased as the ratio of $\frac{\text{volume}}{\text{surface}}$ for the cavity is increased. It is possible to obtain very high Q's by constructing large cubical cavities containing many cells. Odd shapes, such as a cube with many wave cells on a side but with a few individual cells of the cube recessed into its side, could be made to produce high Q with the advantage of suppression of undesired modes of oscillation by judicious placement of the recesses.

The theory for calculation of the Q of a cavity with a coupling iris has only been treated in reports which at present remain classified for purposes of military security.[2] Unfortunately, the theory permits calculation only for the simplest geometrical shapes and for irises whose size is small compared to the wavelength. For a wavelength of 3.2 cm (1.2 inches) a "small" iris would probably be less than 3.2 millimeters (1/8 inch) in its principal dimension. By this standard all the irises tested were large, since the amount of energy that could be extracted from the cavity was so minute with an iris 1/8 inch in diameter that the measurement could not be made with much precision. The principal dimension of the coupling holes varied from 3 millimeters up to 1 centimeter.

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1. This procedure can always be applied to rectangular parallelopiped cavities, but only in special cases to cylindrical or spherical cavities, in which the individual cells are not always of identical shape.
2. Slater, J. C., Radiation Laboratory Report 43-16
   Bethe, H. A., Radiation Laboratory Report 43-22
   Bethe, H. A., Radiation Laboratory Report 43-30
   Bethe, H. A., and Schwinger, J. Radiation Laboratory Report D1-117

It is possible to summarize the effects of the window. It causes a change in the resonant frequency and a power loss by radiation out of the holes.

The shift in the resonant frequency is proportional to the size of the iris and depends on the position on the cavity wall where the hole is located. In general, the resonant frequency may be raised or lowered. For the H- wave mode of this research, an increase in size always lowered the resonant frequency, regardless of the iris shape.[1] A window cut in the wall of the cavity where the magnetic field is large tends to produce an inductive effect, lowering the resonant frequency. Another point of view which explains the behavior is to think of the window as an increase in the volume of the cavity which lowers the frequency of resonance.

The frequency shift due to the window is unaffected by conditions in the system outside. The resonant field inside the cavity is so large that any ordinary reflected wave from outside is negligible by comparison, except in the remote case where the standing wave ratio is of the order of magnitude of the Q of the cavity. Such conditions can be made to occur with the expenditure of some

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1. The susceptance of a given iris for coupling into a cavity is different from the susceptance for the same iris across a wave guide. The cavity fields are different from the wave guide fields in some important features. For instance, in an H_{01} (TE_{01}) mode in a cavity the magnetic field is entirely transverse where the electric field is zero; in a wave guide the magnetic field is entirely transverse where the electric field is maximum. As a result, a "capacitive" wave guide iris may possibly produce an inductive effect when applied to a cavity.

effort (tuning the external circuit to resonance also), but ordinarily the external system simply gives no contribution to the shift of resonant frequency.

The losses due to radiation are also dependent on the size of the iris and its placement. A relatively large hole can be cut off center into the end walls without reducing the Q very much. A much smaller hole cut in the center of the end wall will produce about the same Q. However, the choice of iris size and shape is not always dictated by the Q expected. For example, when using a cavity coupled to a microwave system as a frequency meter, it is not only necessary to get a sharp indication, but also a large enough amplitude of indication. Unfortunately, no striking advantage is to be gained in this respect by proper design of the iris, i.e., the ratio of $\dfrac{\text{transmission into the cavity}}{\text{radiation loss out of the cavity}}$ cannot be increased to any appreciable degree.

The only way to increase the Q and still maintain a given amplitude of fields inside the cavity is to use a cavity with higher intrinsic Q (Q without the iris). The intrinsic Q may be improved by silver plating the walls or by using cavities of large volume as explained at the beginning of this section.

The measurement of Q in this experiment is not performed by direct determination of energy and losses. These quantities are related to the sharpness of the resonance curve and are usually calculated by finding the resonant frequency, f_r, and the frequency increment, Δf, between the points where the amplitude of the power versus frequency curve falls to half the peak amplitude. In terms of voltage or current, the values at the half power points are 70.7% of the peak amplitude.

For an ordinary series resonant circuit, which is a good analog for a cavity in the neighborhood of resonance, the basic definition can be shown to be equivalent to the definition in terms of frequency increments.

By the definition of equation (4), for a series resonant circuit consisting of resistance, R, inductance, L, and capacitance, C, with a sinusoidal current of peak amplitude, I, constant voltage supply, and frequency at resonance, the figure of merit is

$$Q = \frac{\omega_r \times \text{energy stored in the magnetic field}}{\text{average power loss}} \tag{12}$$

$$= \frac{\omega_r \times \frac{LI^2}{2}}{\frac{RI^2}{2}} = \frac{\omega_r L}{R}$$

$$\frac{\text{I at a frequency } \omega}{\text{I at resonance}} = \frac{R}{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}^{\frac{1}{2}}}$$

For the frequencies ω_1 and ω_2 at the half power points this ratio is .707.

$$(.707)^2 = \frac{R^2}{R^2 + \left(\omega_{1,2} L - \frac{1}{\omega_{1,2} C}\right)^2} \tag{13}$$

$$\omega_r^2 = \frac{1}{LC} \tag{14}$$

By combining (13) and (14), and fact that the total reactance is negative below the resonant frequency and positive above it, it can be shown that

$$\left(\omega_{1,2}L - \frac{1}{\omega_{1,2}C}\right)^2 = R^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \qquad \omega_2 L - \frac{1}{\omega_2 C} = +R \qquad \omega_1 < \omega_r < \omega_2$$

$$\omega_1^2 LC - 1 = -R\omega_1 C \qquad \omega_2^2 LC - 1 = R\omega_2 C$$

$$(\omega_2^2 - \omega_1^2)\, LC = RC\,(\omega_2 + \omega_1)$$

$$\frac{\omega_2 + \omega_1}{\omega_2^2 - \omega_1^2} = \frac{L}{R}$$

$$\frac{L}{R} = \frac{1}{\omega_2 - \omega_1}$$

$$\frac{\omega_r L}{R} = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{f_r}{\Delta f}$$

$$\frac{f_r}{\Delta f} = Q \qquad (15)$$

Equation 15 may be applied to resonant circuits in general, including the parallel resonant circuit, which is the dual of the series resonant connection described, and consists of a resistance, a coil, and a capacitor connected in shunt and fed from a constant current source. This representation is commonly used as an analogy for a resonant cavity and was suggested by Hansen.[1]

In either equivalent circuit, the effect of radiation loss out of the coupling holes of the cavity can be represented approximately by a lumped constant. In the parallel tuned circuit an additional resistance is shunted across the two terminals, while in the series circuit a resistance, R_r, is connected in series with R. In both instances the effect of radiation is to lower the Q to a new value.

$$\frac{\text{Q accounting for radiation}}{\text{Q for closed cavity}} = \frac{R}{R+R_r} \qquad (16)$$

To picture the radiation loss as pure resistance is not entirely accurate, because the relative phase angle of the magnetic field components just inside the iris and those just outside the iris is not constant for frequencies near resonance. As a consequence the radiation resistance is a function of frequency near resonance and of increasing magnitude for larger irises.

1. Cited in Publication 23-80 Sperry Gyroscope Company

being adapted to the equipment at hand.

The most important piece of equipment in the apparatus was a spectrum analyzer for use at three centimeter wavelengths.[1] Described in simple terms, this instrument presented on its cathode ray tube screen a picture of the spectrum of the received signal. No frequency calibration was inherent in this picture. The controls permitted a spectrum frequency width of 60 megacycles maximum and a few megacycles minimum. The operation of the analyzer may be understood by reference to the block diagram, Figure 4.

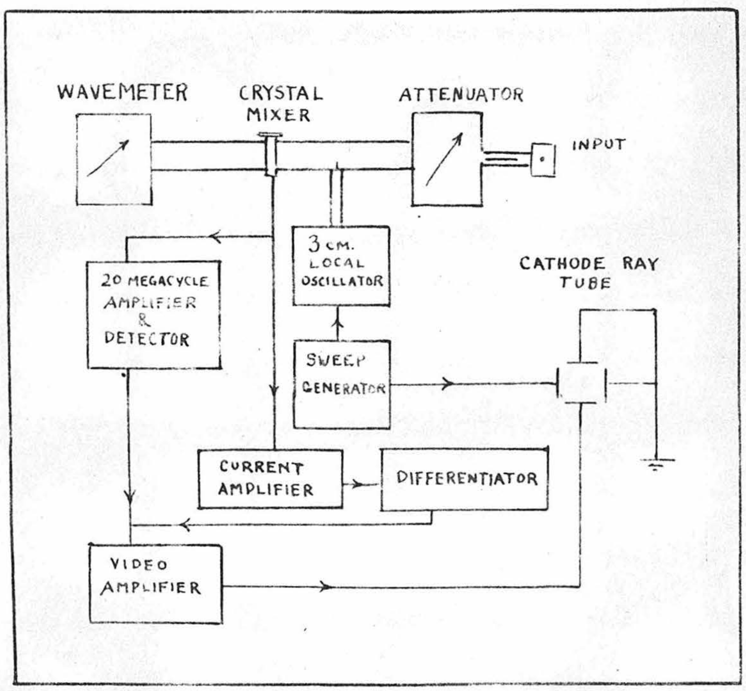

FIGURE 4. BLOCK DIAGRAM OF SPECTRUM ANALYZER

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1. Model TSX -4SE Spectrum Analyzer, Manual PN-2, Sylvania Electric Products, Inc.

The signal input is taken from a coaxial transmission line into a variable attenuator whose attenuation is the result of propagating the signal through a variable length of wave guide operated below cutoff frequency. Then the signal is mixed in the non-linear impedance of a silicon tungsten crystal with a locally generated frequency modulated oscillation. The mixer output is transferred to a high gain tuned radio frequency amplifier which operates at 20 megacycles. Whenever the local oscillator frequency differs from the signal by 20 megacycles, a modulation product is generated at the correct frequency to pass through the amplifier. If the frequency dispersion is very large, the local oscillator may produce two successive signals as its frequency is swept from below the signal frequency to a higher value. The difference frequency voltage is amplified and rectified, so that its envelope appears on the oscilloscope screen. An additional provision is made for tuning an absorption type wavemeter which is coupled to the mixer crystal. When the wavemeter is tuned to a certain frequency it extracts enough energy from the crystal to cause a reduction of the average current in the crystal. The momentary current drop is applied to a differentiating circuit and added into the oscilloscope picture so that the frequency setting of the wavemeter is made to correspond to a small pip on the baseline of the spectrum.

Frequency modulation of the local oscillator is accomplished by applying a sawtooth wave form to the reflecting electrode of a small reflex type velocity modulated tube, the Shepherd-Pierce (723 A) tube. When properly adjusted, this tube gives a sawtooth curve of frequency versus time. In this manner the individual frequencies in the signal spectrum are picked out and shown on the oscilloscope screen at the instant the local oscillator frequency differs by 20 megacycles. Since a sawtooth voltage is a linear function of time, and the local frequency is approximately a linear function of reflector voltage, the spectrum analyzer does actually plot a curve of amplitude versus frequency.

Relative amplitudes of signal components are not automatically reproduced without error. However, with a little caution, it is possible to eliminate this difficulty. The distortion is a result of the fact that the amplitude of the local oscillation is dependent on the reflector voltage. Figure 5 indicates typical modes of oscillation which are produced as the reflector voltage is varied. It is clear

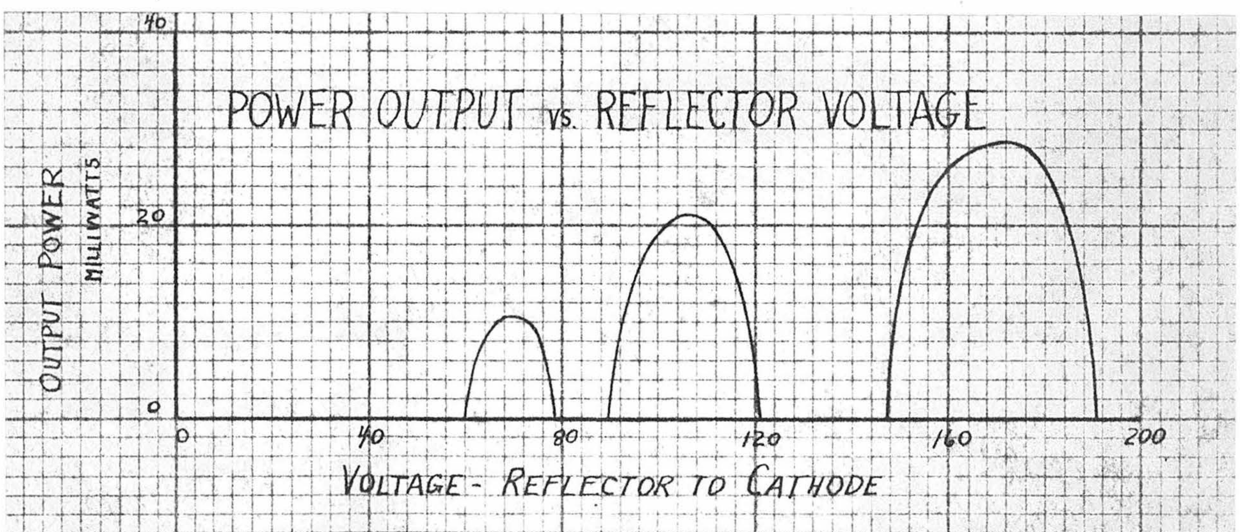

FIGURE 5. OSCILLATING MODES OF SHEPHERD-PIERCE TUBE

that to reduce change of amplitude effects, the dispersion should be made large (i.e. only a small region of the spectrum should be viewed) and the local oscillator should be tuned to the center of the bell-shaped curve for the mode having the highest reflector voltage. In these experiments the reflector voltage change from one of the half power frequencies of the test cavity to the other was ordinarily less than one volt so that the conditions for no amplitude distortion could be met. The mixer type of detection did not cause any amplitude distortion.

The sweep frequency of the modulation of the local oscillation is synchronized with the 60 cycle power supply and can be varied over the range from 10 to 20 cycles per second.

All that was necessary to make the spectrum analyzer capable of measuring high Q values was to provide a method for calibrating the horizontal axis of the sweep in terms of small frequency increments. This was achieved in a convenient and accurate fashion by using an external system for generating a known spectrum. A crystal mixer mounted in a wave guide was excited by another Shepherd-Pierce tube and also by the voltage from a 200 kilocycle quartz crystal oscillator. The quartz crystal oscillation voltage was made relatively large in order to produce large numbers of modulation products. The spectrum resulting from this kind of mixing contained many components of the type $f \pm N \times (200\ \text{KC})$. As many as forty or fifty adjacent spectrum frequencies could be produced easily, although ordinarily only a few of them were used for measurement. When the calibration spectrum with 200 kilocycle frequency separation was fed into the analyzer, the oscilloscope pattern appeared like Figure 6. Since

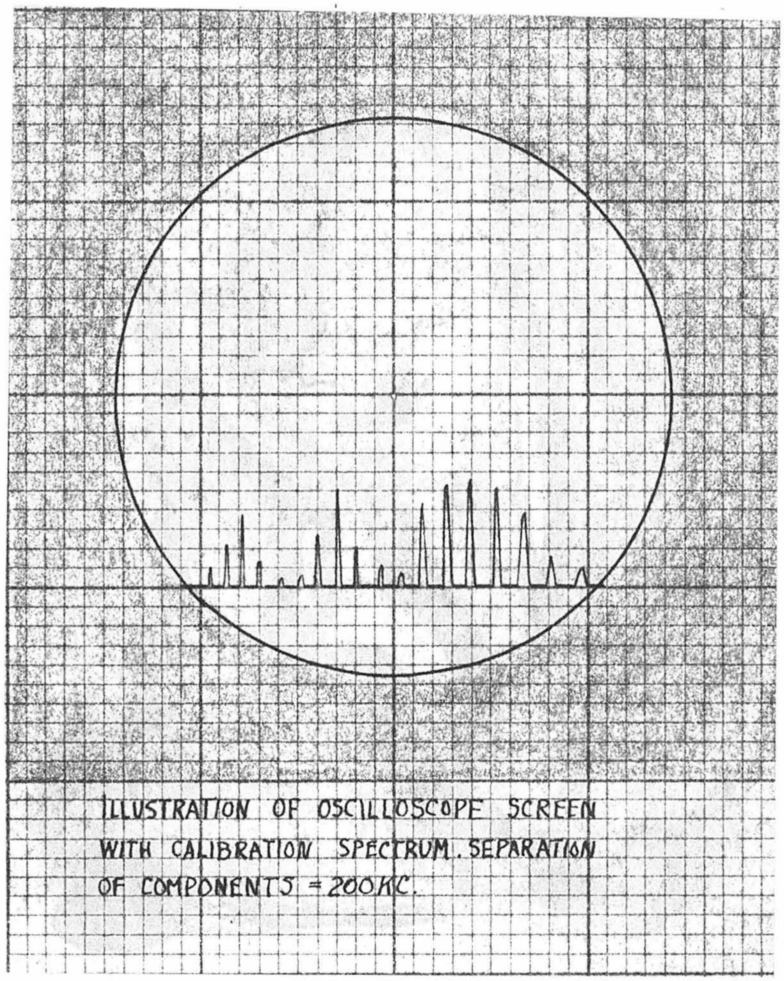

Figure 6

the quartz crystal frequency was accurately known (better than one part in 5000) and was maintained with high stability, the frequency increments on the horizontal axis of the analyzer could be measured very precisely.

Other apparatus

The diagram of Figure 7 illustrates the experimental arrangement and the equipment used.

The source of the signal in the resonant cavity was a 3 centimeter reflex Klystron (Type 419) with a power capability of 250 milliwatts maximum. The tube was immersed in transformer oil which was cooled by tap water circulating in a hollow copper jacket for improved frequency stability.[1] The klystron reflector voltage could be varied and modulating voltages could be superimposed on it. A 1000 cycle square wave generator was the source of modulation.

The reflex Klystron behaves in a fashion very much like the Shepherd-Pierce tube. Its oscillating amplitude and frequency are both dependent on the reflector voltage. In order to tune the Klystron through the resonance curve without amplitude variation it was necessary to observe the precautions mentioned in regard to the type 723 A. However, since the monitor standing wave detector gave readings on alternating voltage, the square wave generator was connected to throw the Klystron in and out of oscillation while making tuning adjustments. By proper adjustment of the tuning knob on the Klystron cavity it was possible to maintain the signal output constant within one per cent as the reflector voltage was changed to tune through resonance. After the initial tuning adjustments

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1. The cooling jacket was built and designed by David Hagelbarger.

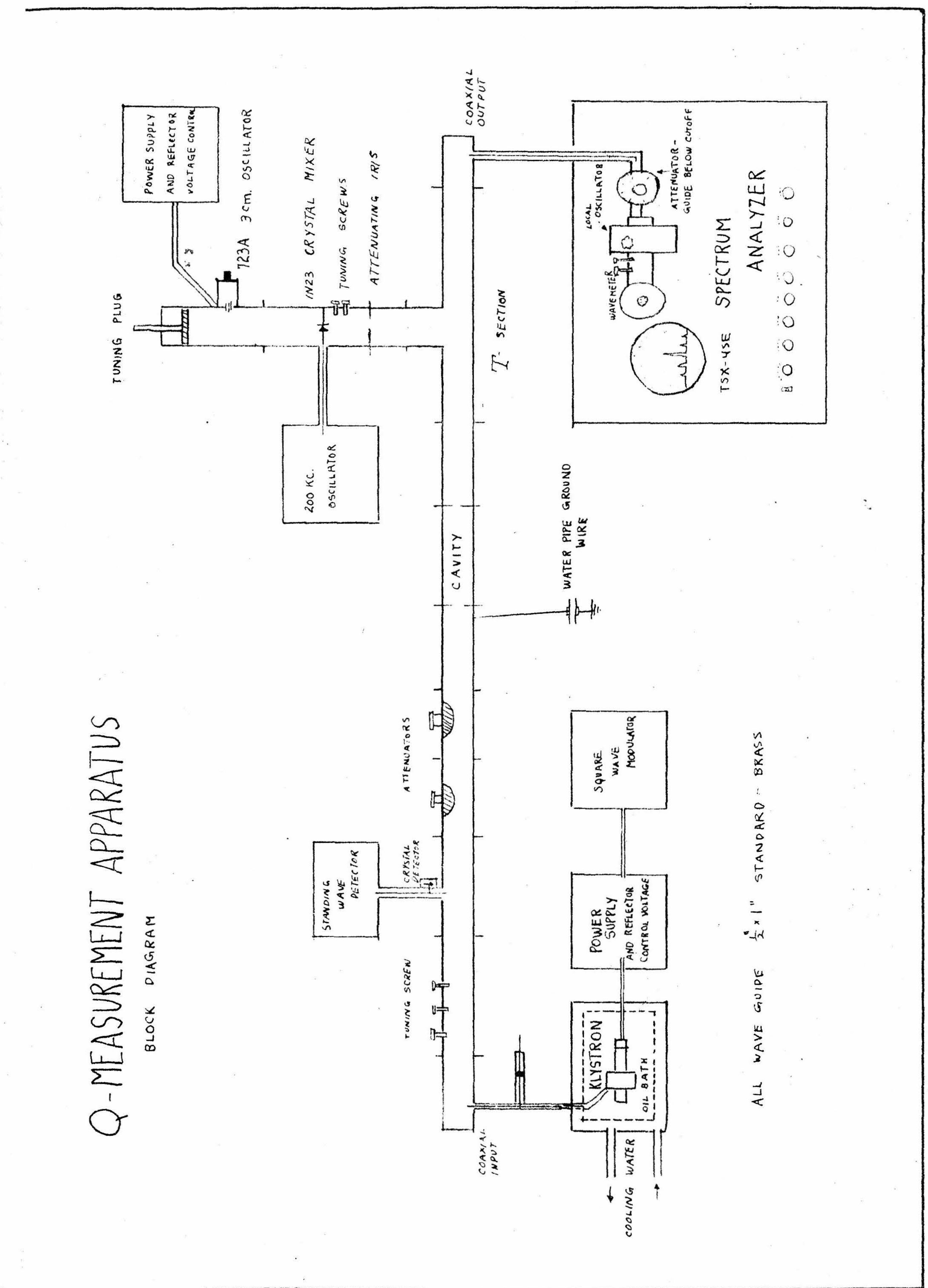
Q-MEASUREMENT APPARATUS
BLOCK DIAGRAM
TUNING PLUG
POWER SUPPLY AND REFLECTOR VOLTAGE CONTROL
723A 3cm. OSCILLATOR
1N23 CRYSTAL MIXER
TUNING SCREWS
ATTENUATING IRIS
COAXIAL OUTPUT
T SECTION
200 KC. OSCILLATOR
LOCAL OSCILLATOR
ATTENUATOR-GUIDE BELOW CUTOFF
WAVEMETER
TSX-4SE SPECTRUM ANALYZER
WATER PIPE GROUND WIRE
CAVITY
ATTENUATORS
STANDING WAVE DETECTOR
CRYSTAL DETECTOR
SQUARE WAVE MODULATOR
POWER SUPPLY AND REFLECTOR CONTROL VOLTAGE
TUNING SCREW
KLYSTRON
OIL BATH
COAXIAL INPUT
COOLING WATER
ALL WAVE GUIDE 1/2" x 1" STANDARD - BRASS

Figure 7

were made, the square wave modulator was shut off and the reflector readjusted to give a steady voltage of the value found to yield constant output amplitude at the chosen frequency.

The signal was fed into standard 1 inch x $\frac{1}{2}$ inch brass wave guide through a short coaxial transmission line with a matching stub. As it proceeded down the guide it came to a section with three adjustable tuning screws, used for impedance matching, on to the probe of the standing wave detector, two attenuator sections and thence to the cavity. The two attenuator sections served to decouple the generator and the cavity so that there was no reaction of the cavity on the frequency-voltage characteristic of the Klystron. As a test that the isolation was adequate, the tuning screw near the cavity was turned until it was completely across the wave guide and then back again. If this produced no variation in the reading of the monitor, the attenuation was considered to be adequate.

From the output iris of the cavity the signal entered a T wave guide joint. The other input arm of the T carried the calibration spectrum. The calibration frequencies were generated by combining the output of a 723 A tube mounted on a wave guide section and the quartz crystal oscillator frequency, introduced by a coaxial connector in series with the silicon-tungsten crystal mixer. Two tuning screws in the guide section served as variable attenuation for the calibration pips and they were aided by a fixed attenuator placed in the leg of the T. The Shepherd-Pierce tube was equipped with a very sensitive adjustment for the reflector voltage, which could be changed as little as a few volts for one nearly complete rotation of a potentiometer knob.

The output leg of the T wave guide junction was terminated in a coaxial connection that led to the spectrum analyzer.

Actual Operating Procedure

The equipment was all turned on and allowed to run for an hour or two until temperature equilibrium was reached and all oscillator frequencies were stable. With low dispersion on the analyzer (a wide range of frequencies) the resonant frequency was located and the analyzer wavemeter was tuned to the two pips on the pattern. The average frequency of these two pips was the resonant frequency of the cavity. Then the analyzer tuning was changed until one of the pips appeared at the center of the bell-shaped curve of the local oscillator output. The analyzer was checked to ascertain that there was no amplitude distortion by varying the reflector control both ways from the desired position and making adjustments until no fluctuation was noted in the height of the pip. The calibration spectrum was brought into view and reduced in size by turning the tuning screws provided to the point where the peaks were about half the height of the resonant signal pip. Then there was no need to change the analyzer cut-off type attenuator setting during the measurement. The main attenuators, tuning screws, and coaxial tuning stub were then set so that the klystron output amplitude was constant over the frequency range and independent of the

IV. RESULTS

Q of a cavity as a function of the number of wave cells.

The first series of measurements was made with a set of cavities and two vertical window irises. The cavities were formed by cutting lengths of wave guide into pieces two, four, six, eight and ten times the length of a single wave cell. Flanges were soldered to the wave guide on both ends and the cavity was formed by placing end plates to close the cavity. The end plates, which contained the coupling holes, were tightly screwed and clamped to the cavity and mounted in the apparatus. In this manner it was possible to test the effect of increasing the number of wave cells. Figure 8 illustrates the shape of the end of one of these cavities (in the upper left hand corner), and the appearance of the end plate used. The inside of the cavity

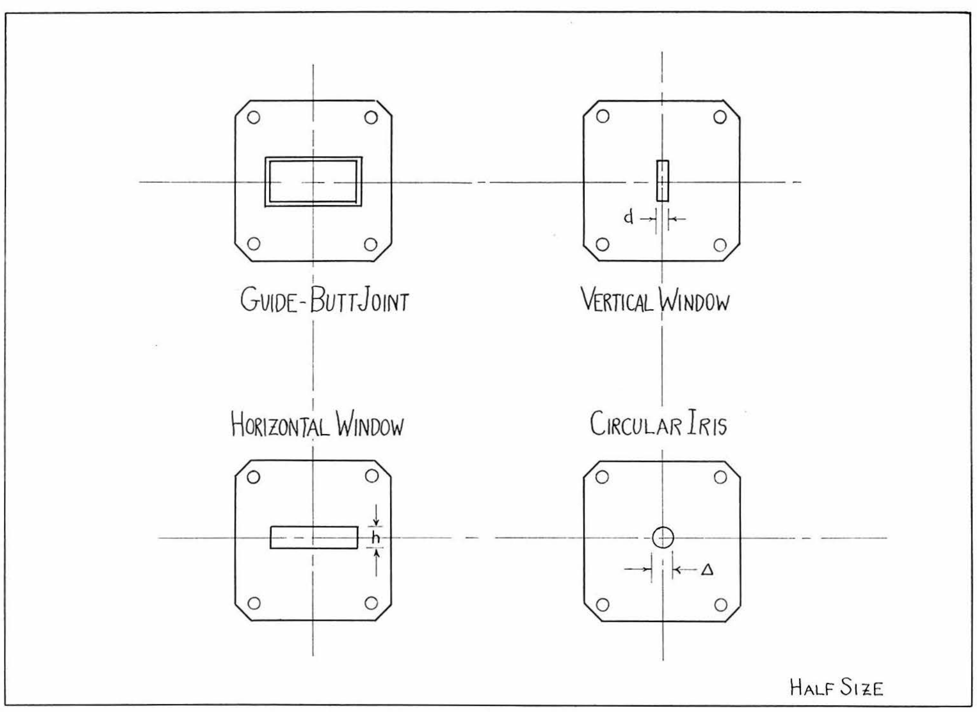

Figure 8

was carefully cleaned with steel wool and the face of the butt joint was turned flat and smooth in a lathe.

From the equation given in the theory section it is found that the Q of a closed cavity of this type with height a, width b, and p wave cells, each of length d_o, should be

$$Q = \frac{a b d_o \; p(d_o^2 + b^2)}{\mu' \Delta \left\{ 2ab^3 + p(d_o b^3 + 2a d_o^3 + b d_o^3) \right\}}$$

When numerical values are substituted, including the value of , 12.7×10^{-5} centimeters for brass, the equation is

$$Q = \frac{9480\, p}{1.73p + .58} \qquad (17)$$

Equation 17 exhibits the properties to be expected. The numerator of the fraction, which should be proportional to the stored energy, contains the factor, p, which is just as it should be, since the stored energy in each wave cell is alike, and the total can be found by taking p times the value for a single cell. The denominator of the fraction contains a constant term, representing the loss in the end faces. If cells are added end to end there are only two such faces regardless of the value of p. The other term in the denominator is multiplied by p, and it is clear that it accounts for the loss in all the walls. Thus the end walls cause a constant loss, while the side walls and top and bottom introduce new loss for each new cell. It is evident that the end losses and the wall losses could be separated for a closed cavity by using a large number of wave cells. End losses would be negligible in that case. The presence of the vertical windows increases the loss but it, too, should not be a function of p.

Experimental data for a series of cavities with vertical window irises are shown in Figure 9. Also plotted are the theoretical values for a closed cavity. Measurements can only be made for integral numbers of wave cells so the graph is plotted with horizontal lines through the ordinate.

The results are in accord with theory, although great difficulty was experienced in obtaining the data. An unexpected effect caused the trouble.

When the first tests were performed, the data were consistent, i.e. with a given setup measurements could be duplicated within a few per cent. However, if the cavity was disassembled and reassembled, the data were sometimes quite different. Careful observation revealed that this apparent inconsistency was due to the fact that the Q was very sensitive to changes in the pressure which was used to hold the end plate to the cavity. Recessed-head screws were tightened to the limit of the experimenter's strength. C-clamps were installed between the screws and tightened on with wrenches. Nor was this sufficient. The results improved in consistency but were never satisfactory. Finally the tests were repeated many times, each time reconstructing the cavity, until enough data were obtained to locate the maximum reading for each cavity, and these are the values given by Figure 9.

Evidently the loss due to current flow across the very slight gap between the butt joint and the iris plate is highly dependent on pressure. It is convenient to make cavities in this manner, but it is

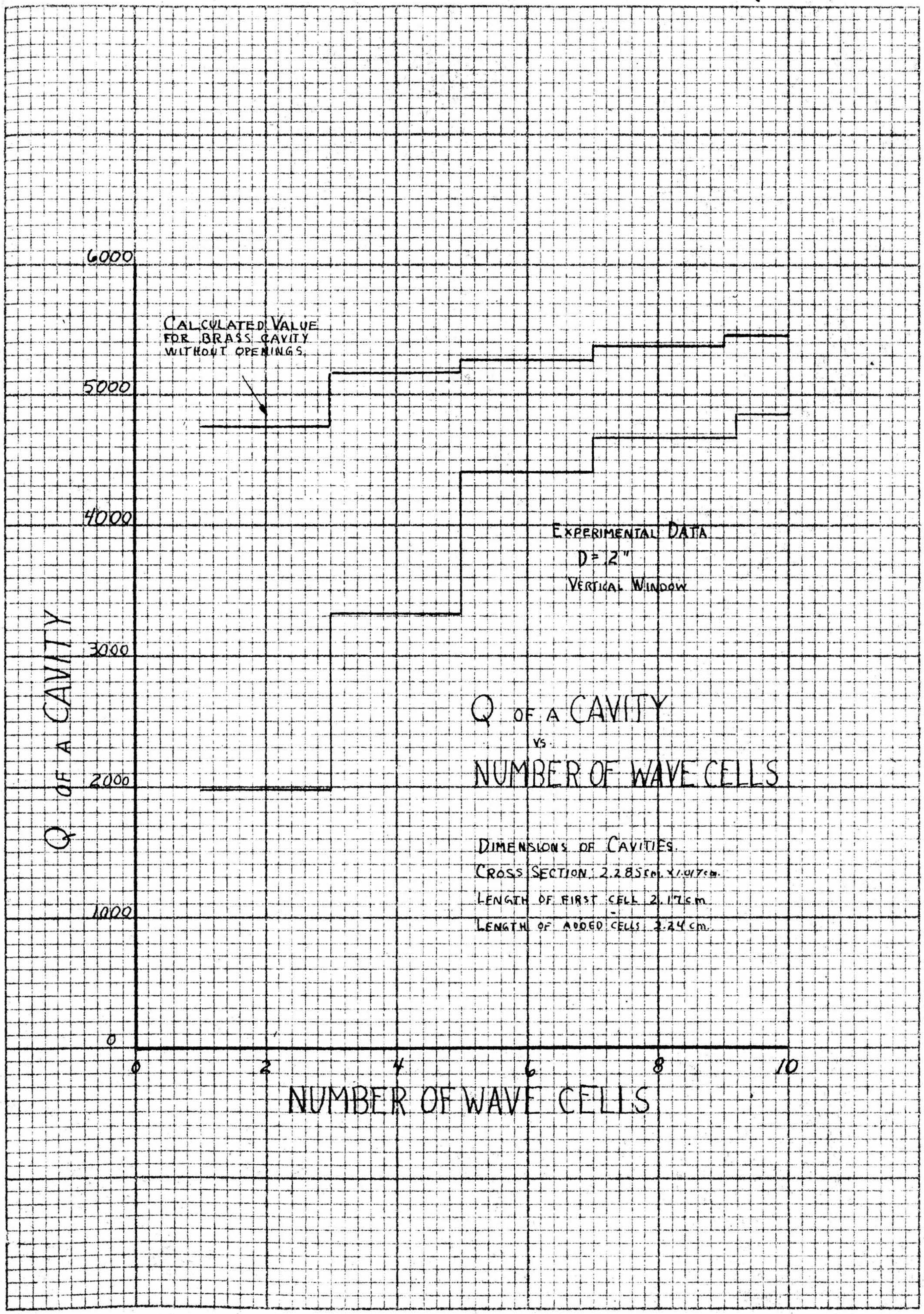
6000
5000
4000
3000
2000
1000
0
0
2
4
6
8
10
CALCULATED VALUE
FOR BRASS CAVITY
WITHOUT OPENINGS
EXPERIMENTAL DATA
D = .2"
VERTICAL WINDOW
Q OF A CAVITY
VS
NUMBER OF WAVE CELLS
DIMENSIONS OF CAVITIES.
CROSS SECTION: 2.285cm. x 1.017cm.
LENGTH OF FIRST CELL 2.17cm.
LENGTH OF ADDED CELLS 2.24 cm.
Q OF A CAVITY
NUMBER OF WAVE CELLS

Figure 9

only advisable when changes in the Q and resonant frequency can be tolerated.

Q versus iris opening and position.

Following this test, a new method of making cavities was adopted. The piece of wave guide was cut to the desired length, cleaned, and the ends milled off square and smooth. It was then placed on the iris plate and held in position with a clamp. A few drops of flux were touched to the joint, which was heated thoroughly in an oxygen gas flame until a drop of solder ran easily under the edge and all around the joint. This kind of assembly was very uniform and no further troubles were experienced with inconsistent results. However, the heat applied to the brass produced a copper coating effect on the inside of the cavity. When the cavities were torn apart, the inside appeared pink, as though it were copper coated. To determine the influence of this coating, a cavity was tested to determine its Q and then it was flushed with concentrated nitric acid, leaving the natural brass finish. It was then tested again. Finally it was copper plated on the inside and tested again. At first it had a value theoretically too high for brass; after flushing with acid its value was below the theoretical value for brass; after copper plating (leaving a few bare spots here and there), the Q returned almost to the original value. This test showed that the wall conductivity was closer to that of copper than that of brass. If the conductivity of copper is used to compute Δ in Equation 17, the values must be increased by a factor

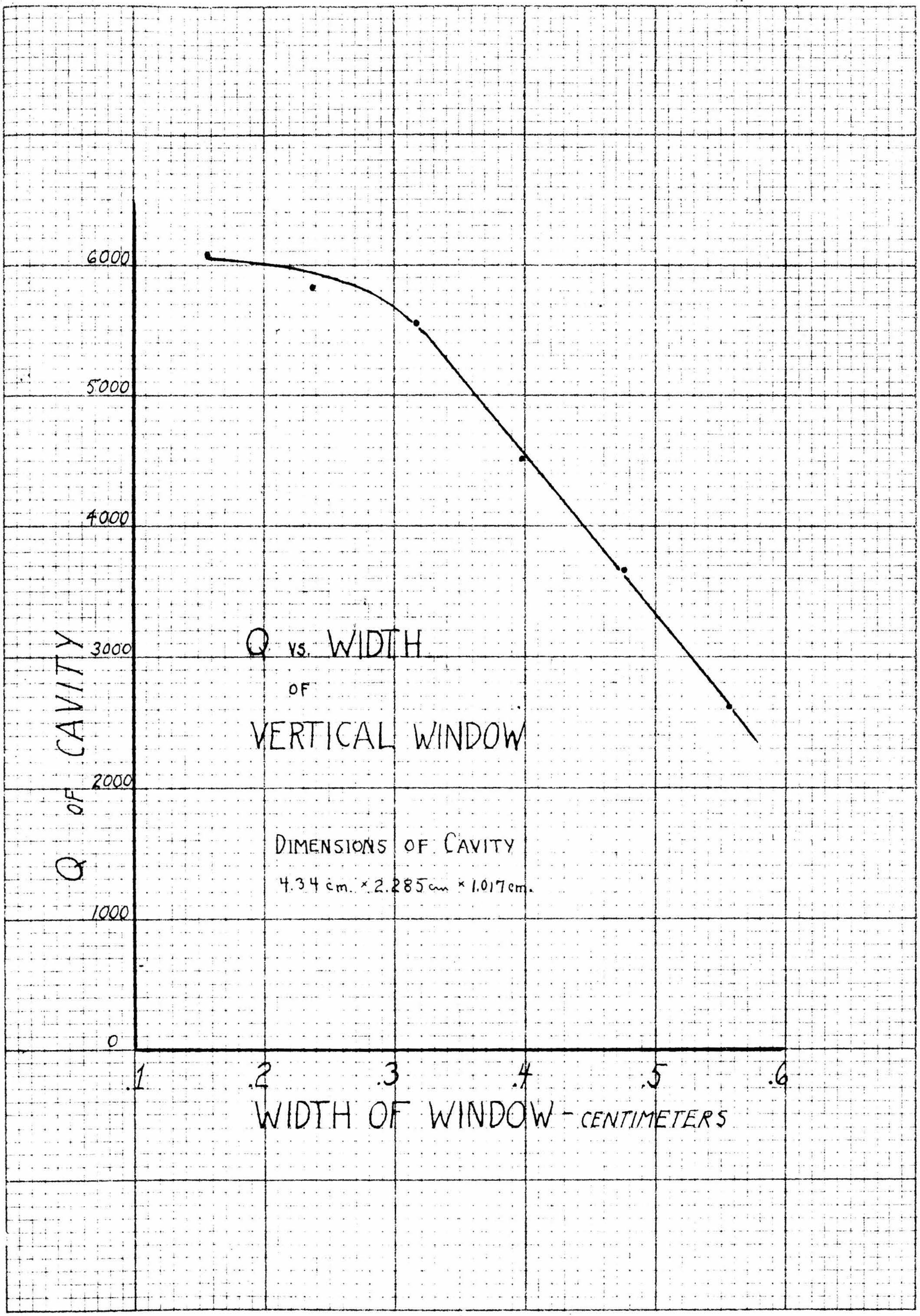
6000
5000
4000
3000
2000
1000
0
Q OF CAVITY
Q VS. WIDTH
OF
VERTICAL WINDOW
DIMENSIONS OF CAVITY
4.34 cm. × 2.285 cm × 1.017 cm.
.1
.2
.3
.4
.5
.6
WIDTH OF WINDOW - CENTIMETERS

Figure 10

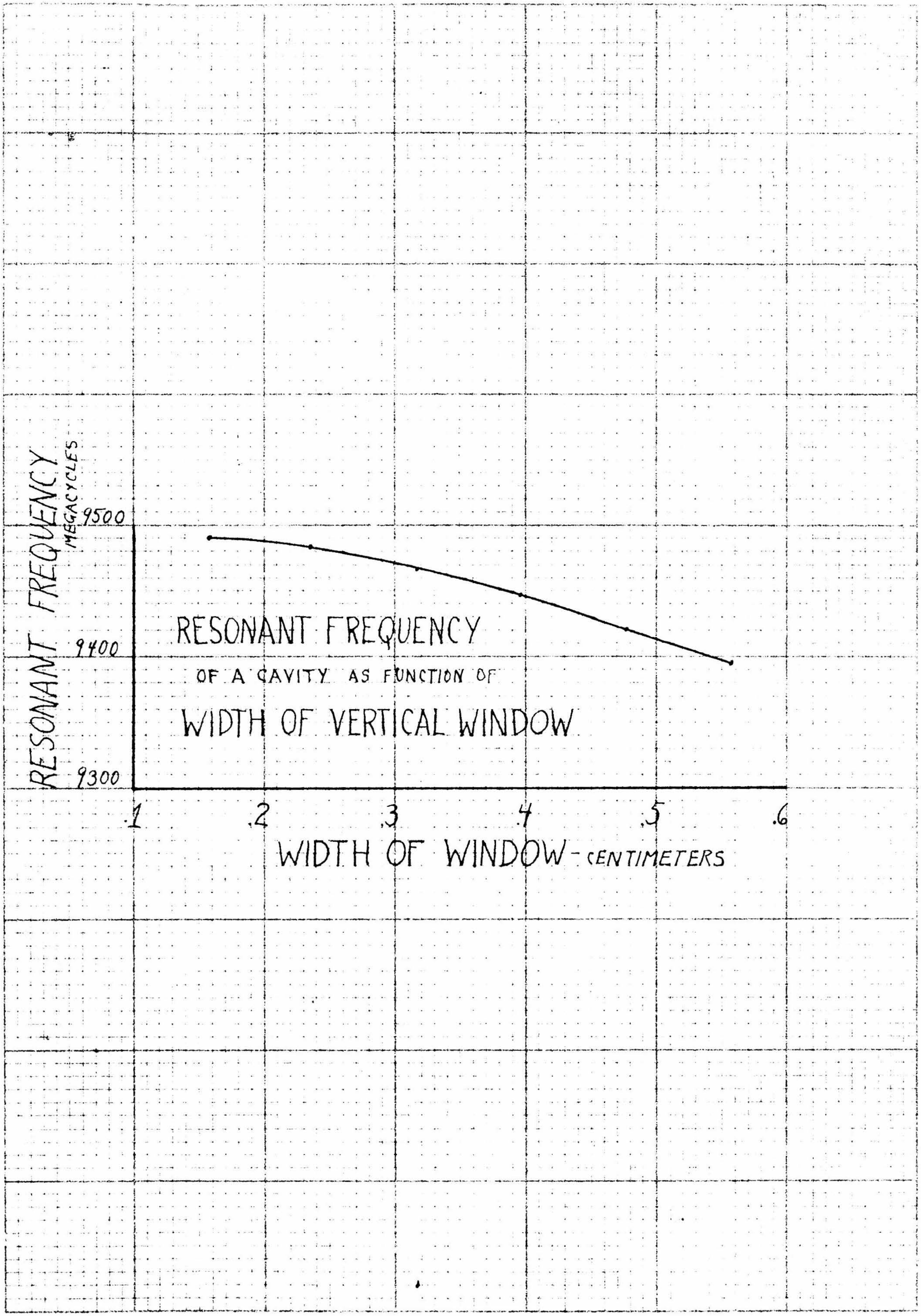
RESONANT FREQUENCY
MEGACYCLES
9500
9400
9300
RESONANT FREQUENCY
OF A CAVITY AS FUNCTION OF
WIDTH OF VERTICAL WINDOW
.1
.2
.3
.4
.5
.6
WIDTH OF WINDOW - CENTIMETERS

Figure 11

inversely proportional to the conductivities, which is 1.96 for these two metals. Consequently, the probable theoretical value of Q for the soldered cavities can be taken as nearly twice that of those with cleaned brass surfaces.

Many cavities prepared in this manner were tested. Figures 10 and 11 summarize the data for a vertical window, .032 inches thick, extending from the bottom of the cavity to the top, and of variable width. Both Q and resonant frequency decrease as the width of the window is increased.

A horizontal window, .032 inches thick, $\frac{3}{32}$ inches high, and of a variable width extending part of the way across the end of the cavity in both directions out from the center, was also tested. This iris had a more severe effect than the vertical window if the results are compared for equal areas of opening. The behavior is logical, inasmuch as the iris begins to behave like a wave guide as it is widened. The greatest width used was 1.1 centimeters, which was close to the value at which the iris would no longer have been operating below cut-off wavelength.

Incidentally, it was interesting to note that this window lowered the frequency of resonance in a manner like the vertical window. It was a clear demonstration of the fact that the properties of an iris in a wave guide as expressed by the wave guide susceptance are not always reliable for predicting the effect on a cavity.

Other irises were placed in the end walls. Figures 13, 14, and 15 are the data for circular irises, .032 inches thick, located at the

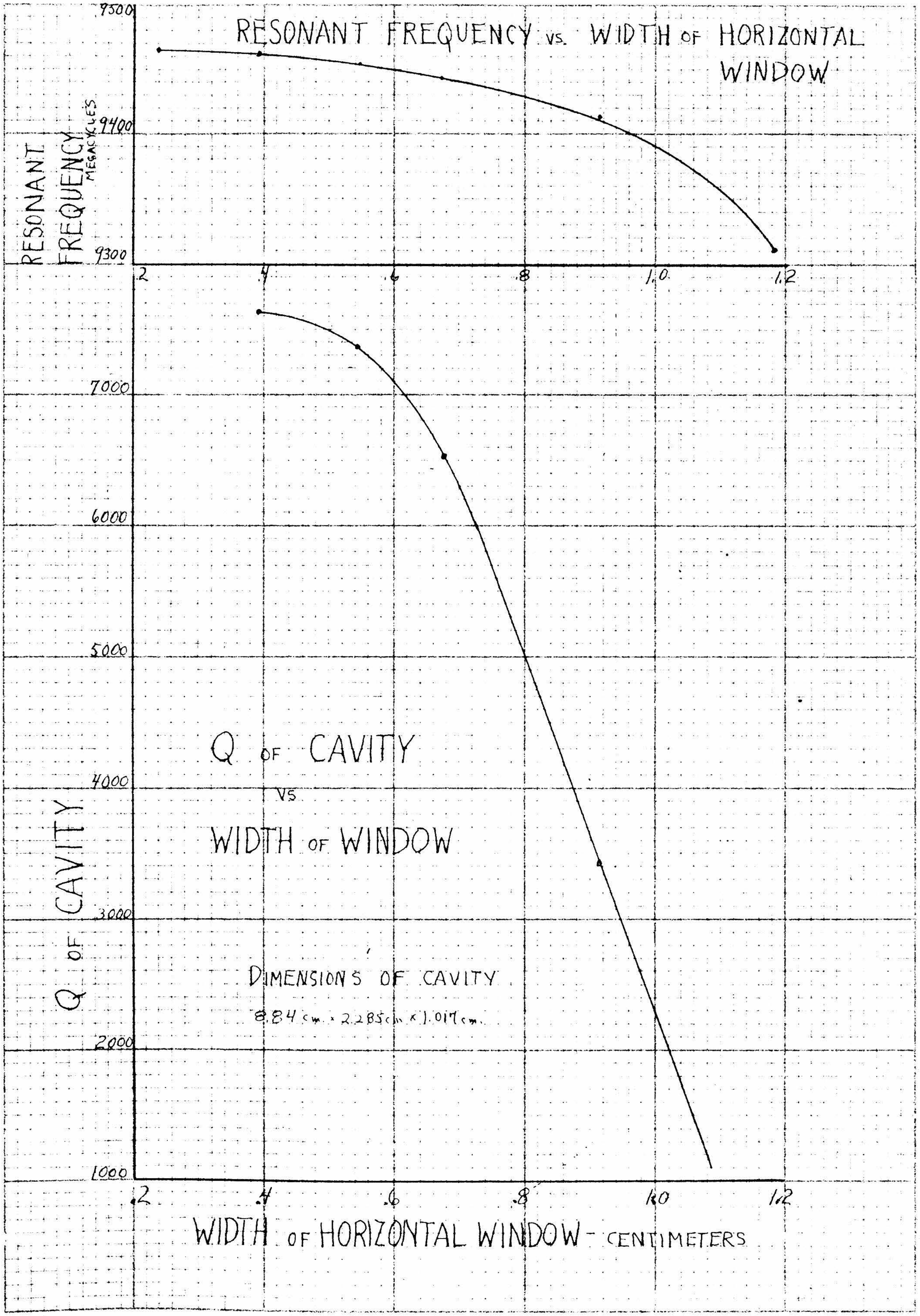
RESONANT FREQUENCY vs. WIDTH OF HORIZONTAL WINDOW
RESONANT FREQUENCY MEGACYCLES
9500
9400
9300
.2
.4
.6
.8
1.0
1.2
7000
6000
5000
4000
3000
2000
1000
Q OF CAVITY
Q OF CAVITY VS WIDTH OF WINDOW
DIMENSIONS OF CAVITY
8.84 cm. × 2.285 cm × 1.017 cm.
WIDTH OF HORIZONTAL WINDOW - CENTIMETERS

Figure 12

center of the end walls. The behavior of Q and resonant frequency is similar to that already described. In these irises, too, Q and resonant frequency decrease when the hole is made larger.

Another experiment, which is not plotted here, used a circular iris, .032 inches thick, placed off center in the end walls about one eighth of an inch in from the side. The coupling of this hole to the resonant mode was so small that the Q measurement could not be made with satisfactory precision. The value for a hole .6 centimeters in diameter was approximately 6500 for a cavity 8.82 centimeters long. This is substantially larger than the value for a centered iris of the same diameter, but the amplitude of the signal through the cavity was very small.

Q versus iris size for coupling holes in the walls.

It is also possible to excite the resonant mode in the cavity by coupling holes in the top and bottom walls. A cavity was constructed using solid end plates and coupling irises in the top and bottom walls midway between the ends. Figure 2 and equation 8 yield the information about the fields at these points. For any cavity with an even number of wave cells, the magnetic field is entirely transverse, while the electric field is zero midway between the ends. Maxwell's equation for the curl of $\bar{\bar{E}}$ signifies that the transverse magnetic field could be excited by establishing an electric field in a perpendicular direction, in the plane of the iris. The electric field was set up in the iris in this experiment by clamping it across the end of a wave guide,

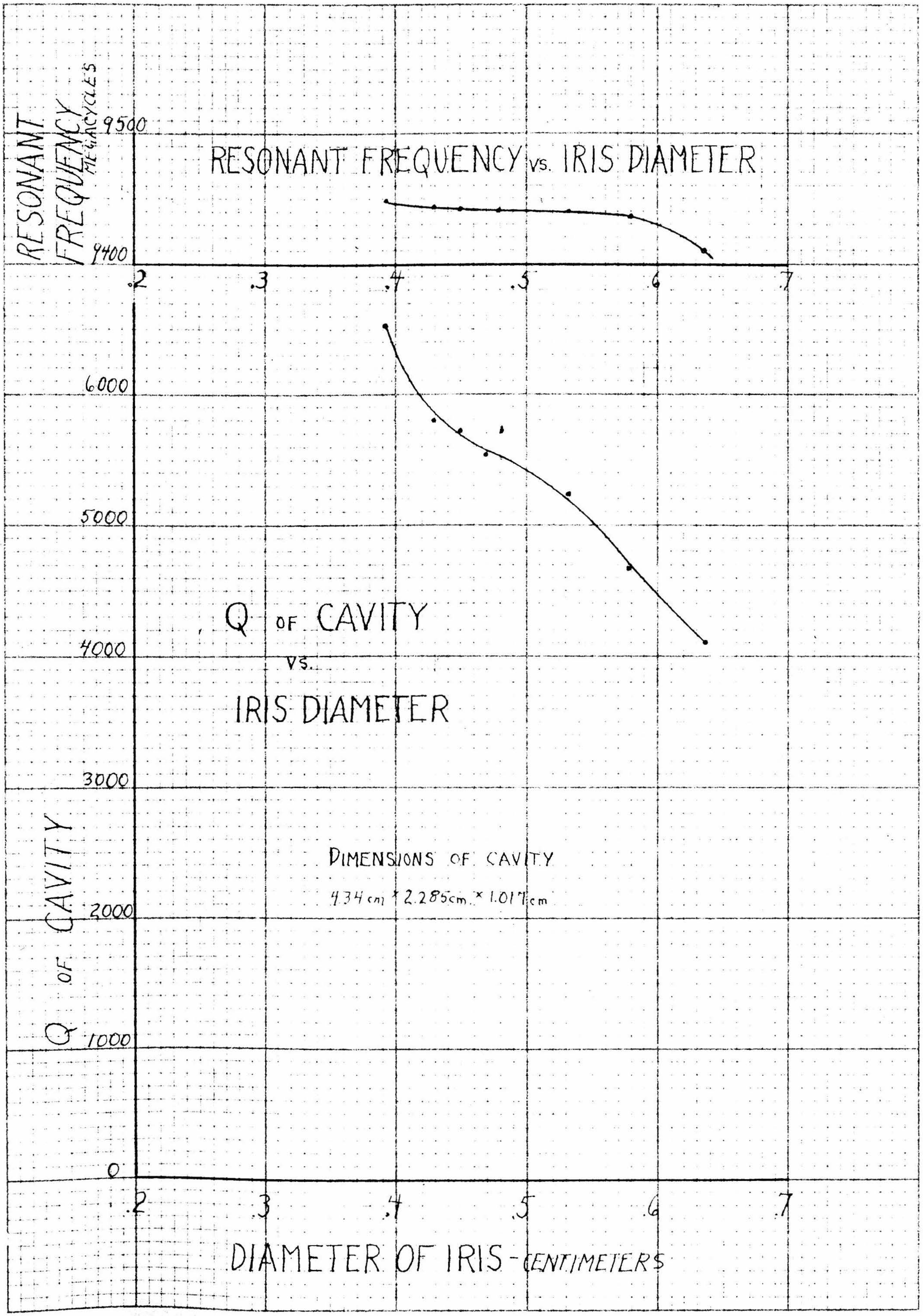
RESONANT FREQUENCY MEGACYCLES
9500
9400
RESONANT FREQUENCY vs. IRIS DIAMETER
.2
.3
.4
.5
.6
.7
6000
5000
4000
3000
2000
1000
0
Q OF CAVITY
Q OF CAVITY
VS.
IRIS DIAMETER
DIMENSIONS OF CAVITY
4.34 cm × 2.285 cm × 1.017 cm
DIAMETER OF IRIS - CENTIMETERS

Figure 13

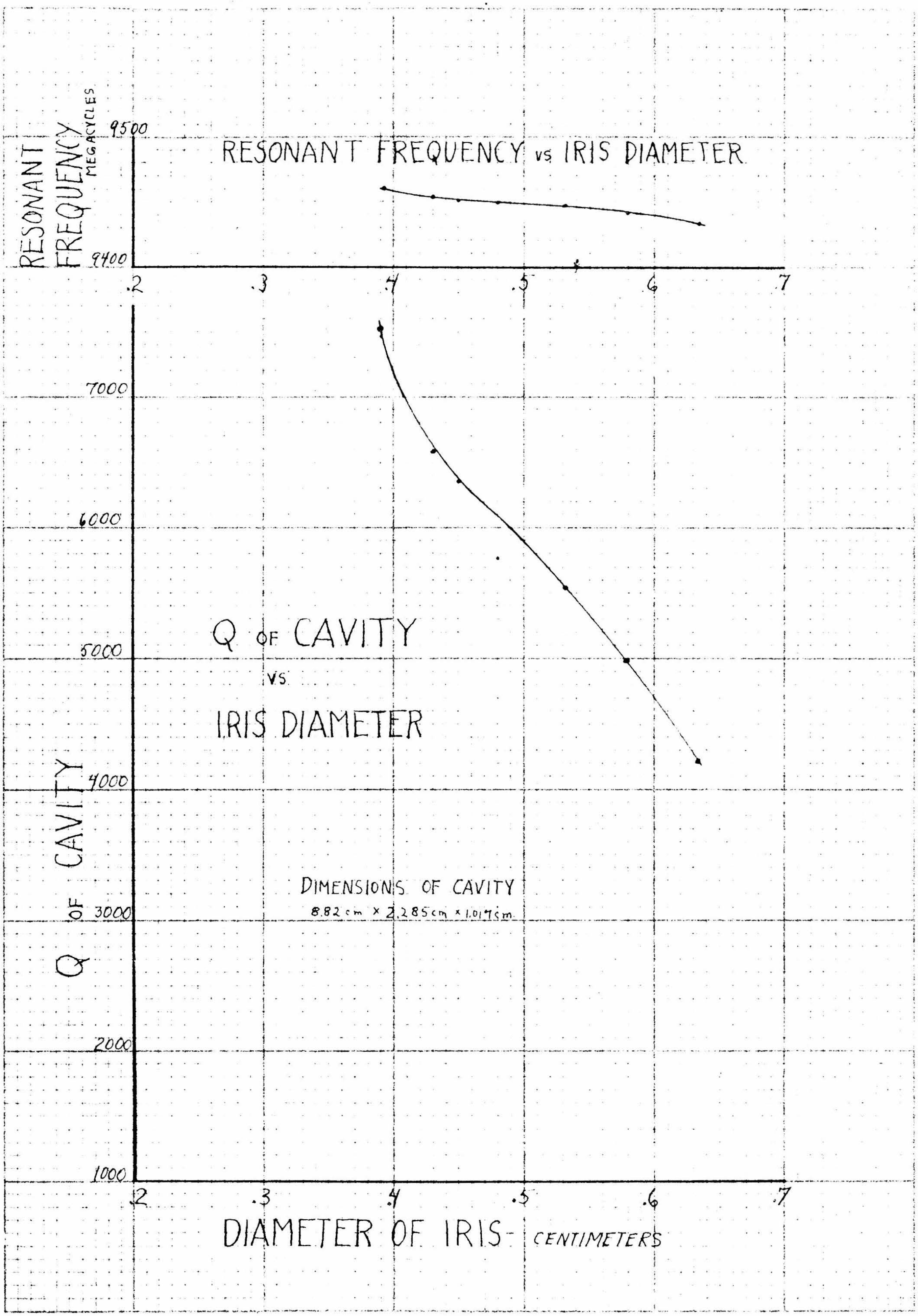
RESONANT FREQUENCY vs IRIS DIAMETER
RESONANT FREQUENCY MEGACYCLES
9500
9400
.2
.3
.4
.5
.6
.7
Q OF CAVITY vs IRIS DIAMETER
Q OF CAVITY
7000
6000
5000
4000
3000
2000
1000
DIMENSIONS OF CAVITY
8.82 cm × 2.285 cm × 1.017 cm
DIAMETER OF IRIS - CENTIMETERS

Figure 14

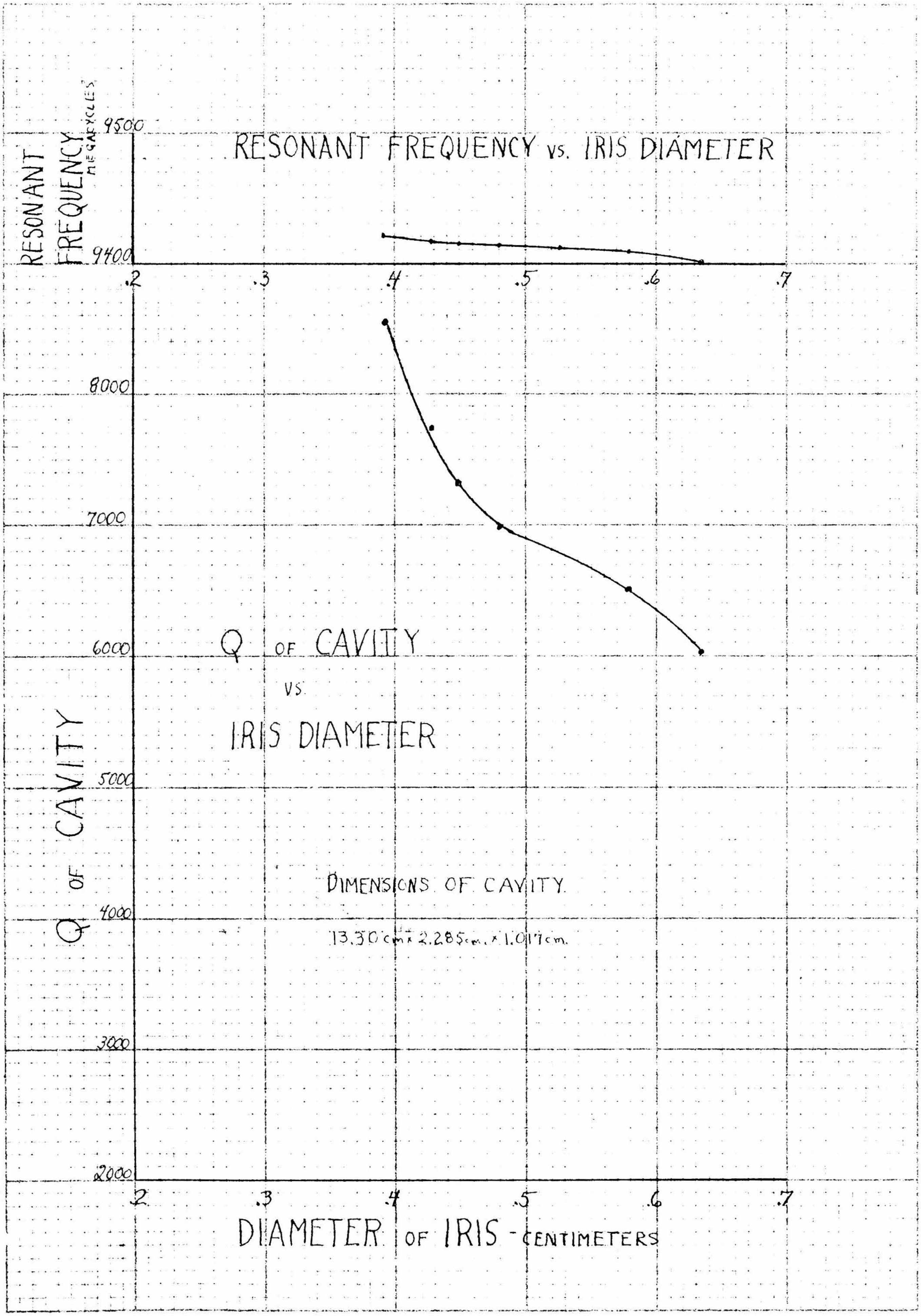
9500
RESONANT FREQUENCY vs. IRIS DIAMETER
RESONANT FREQUENCY MEGACYCLES
9400
.2
.3
.4
.5
.6
.7
8000
7000
Q OF CAVITY
VS.
IRIS DIAMETER
6000
5000
Q OF CAVITY
DIMENSIONS OF CAVITY
13.30 cm × 2.285 cm × 1.017 cm.
4000
3000
2000
DIAMETER OF IRIS - CENTIMETERS

Figure 15

and the output iris was similarly coupled to a guide section. It was found that the resonance could be produced in this manner when the polarization of the exciting field was perpendicular to the magnetic field inside the cavity. When the cavity was turned through a right angle, leaving the exciting field, $\bar{E}$, parallel to the direction of the cavity field, $\bar{H}$, no resonance effect was noted. By increasing the analyzer sensitivity a signal was found coming through the cavity, but it was very small and not resonant in a large range of frequencies near the cavity resonance. With excitation of the wrong polarization only a severely attenuated wave guide pass band was noted, while the correct polarization caused a genuine resonance. The resonant effect disappeared for angles of polarization only 20° or 30° from the correct value.

Figure 16 is the experimental result of varying the size of the coupling iris at this midpoint in the top and bottom walls. The iris was circular, and the wall thickness was .05 inches. The behavior was typical, showing the familiar lowering of the Q and the resonant frequency.

For the sake of curiosity one cavity was constructed with irises in the top and bottom faces one half a cell length away from the mid-point. Inside the cavity near such a point the magnetic field is zero and the electric field maximum, but in a direction perpendicular to the plane of the iris. The field equations show that an exciting electric field in the plane of the iris should not excite any of the fields in the resonant mode. This was verified by test. As in the preceding case, a severely attenuated signal was observed coming through the

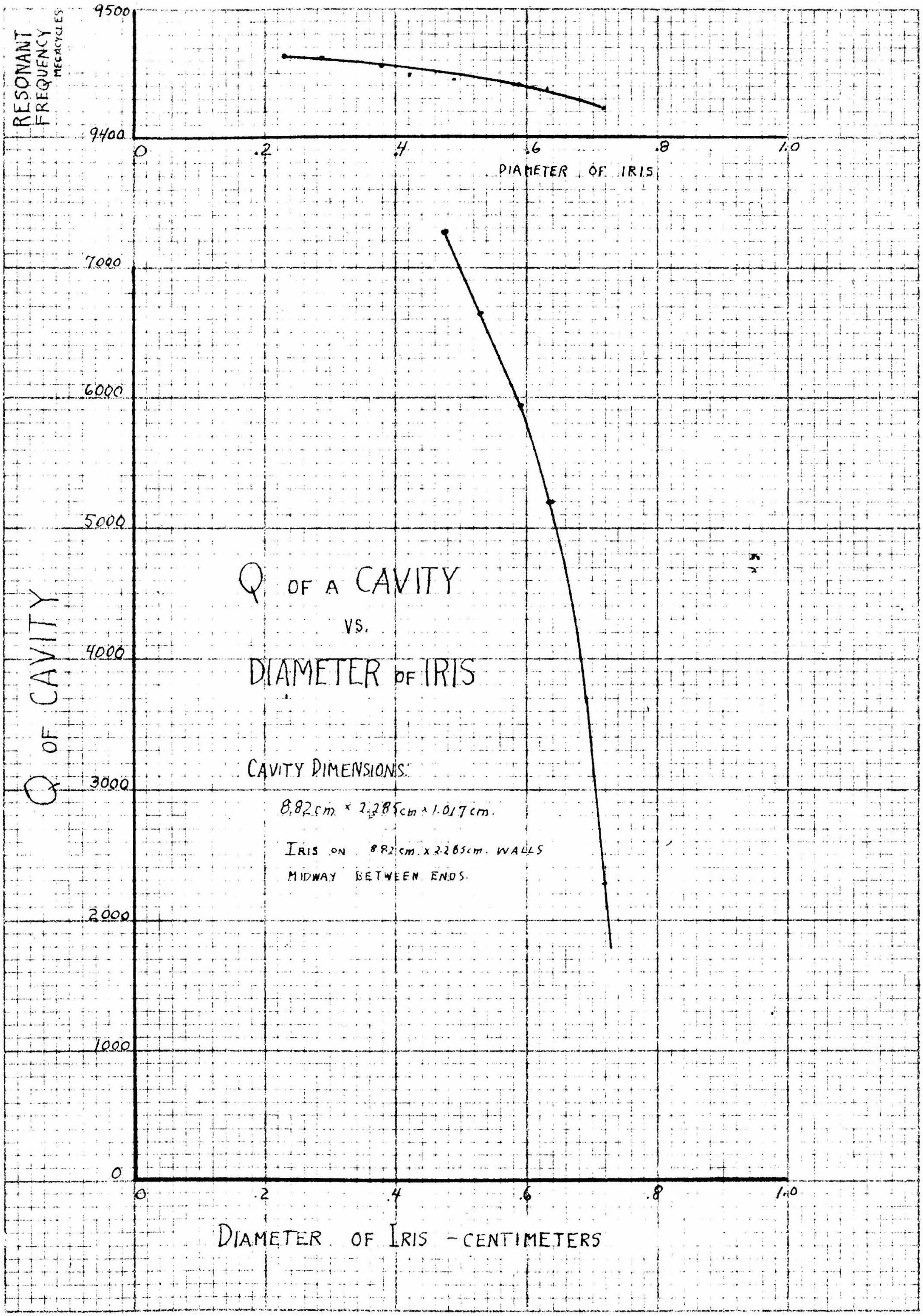
RESONANT FREQUENCY MEGACYCLES
9500
9400
0
.2
.4
.6
.8
1.0
DIAMETER OF IRIS
Q OF CAVITY
7000
6000
5000
4000
3000
2000
1000
0
Q OF A CAVITY
VS.
DIAMETER OF IRIS
CAVITY DIMENSIONS:
8.82cm × 2.285cm × 1.017cm.
IRIS ON 8.82cm. x 2.285cm. WALLS
MIDWAY BETWEEN ENDS.
DIAMETER OF IRIS - CENTIMETERS

Figure 16

cavity, but it showed no signs of being resonant in a wide band of frequencies near the cavity resonance.

Q versus thickness of the iris.

In the theory discussion it was stated that the external conditions in the wave guide should have little effect on the resonant frequency. To test this hypothesis, a cavity was constructed with two circular irises, $\frac{1}{4}$ inch in diameter in the end walls. These irises were made with brass plate $\frac{1}{8}$ inch thick. The thickness of the wall was varied by milling it down. The data show that the theory was correct. Resonant frequency remained unchanged as the thickness was cut down from $\frac{1}{8}$ of an inch to less than $\frac{1}{32}$ of an inch. However, at the same time the Q was decreased. Figure 17 pictures the result.

Q versus material in the end wall

As a last check on the validity of the measurements three cavities were constructed of identical shape with coupling irises $\frac{1}{8}$ inch thick in the end walls. One used copper for the end walls, one used brass, and one steel. The results were as follows:

Q	Material
6750	Copper
6560	Brass
3160	Steel

These values are in accord with the previous results. Using the copper-film value for Δ, the approximate theoretical value for a closed cavity would have been about 9000.

The data for steel end walls is of interest because it shows a larger effect than would be expected using the ordinary values of τ for steel. If the equation (17) is assumed to change only in the constant term of the denominator when steel is used for the end walls, it is possible to compute how much change in this constant is needed to account for the loss in Q. For the data shown, the factor of change in this term is about nine times, and (neglecting the effect of μ') the resistivity of steel would have to be about eighty times the resistivity of copper. In fact, the low frequency resistivity of steel is only 30-40 times the value for copper. The discrepancy may be of some significance,, indicating a change in the electric or magnetic properties of steel at these very high frequencies. Explain

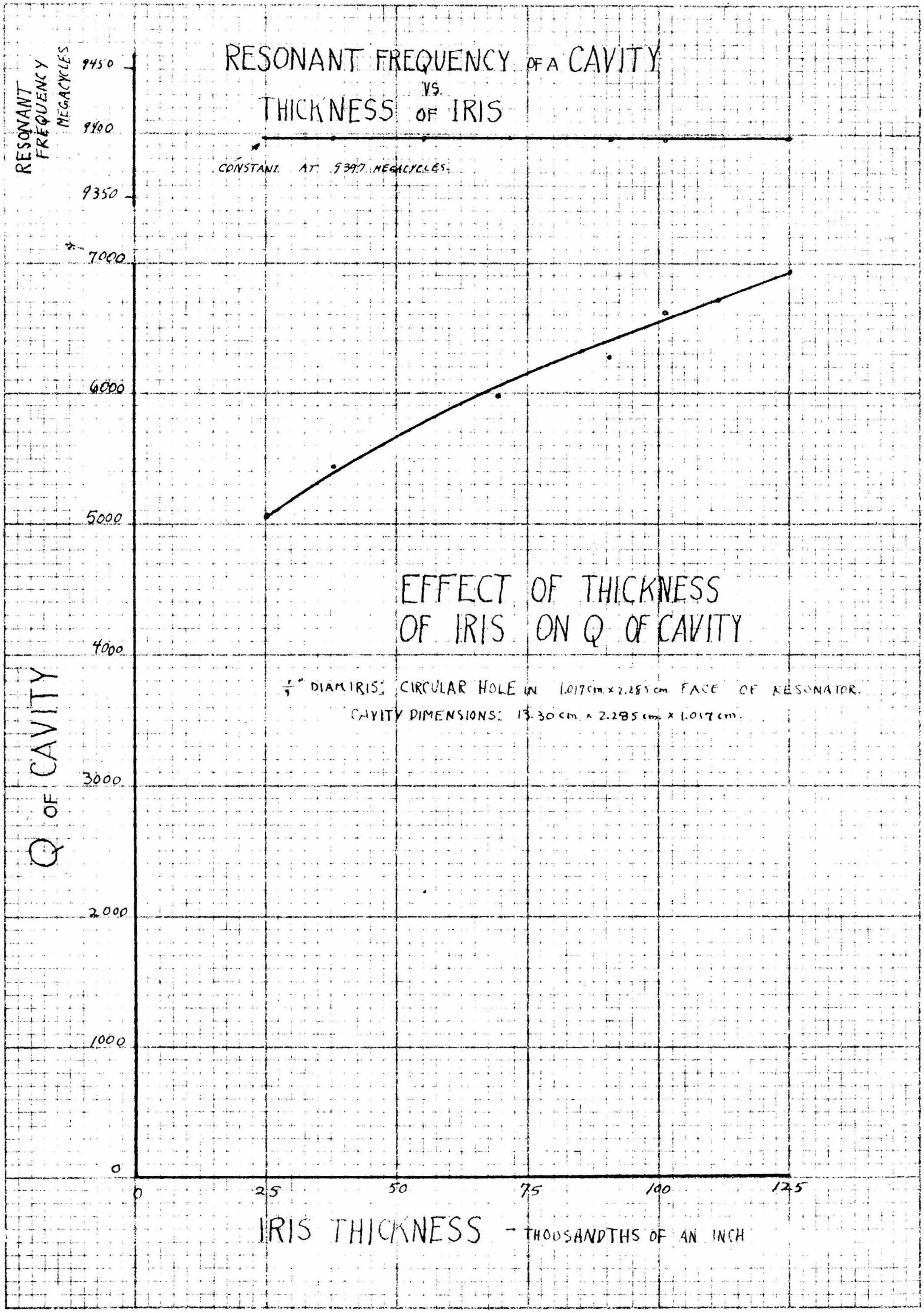
RESONANT FREQUENCY OF A CAVITY
VS.
THICKNESS OF IRIS
RESONANT FREQUENCY MEGACYCLES
9450
9400
9350
CONSTANT AT 9397 MEGACYCLES.
7000
6000
5000
4000
3000
2000
1000
0
EFFECT OF THICKNESS
OF IRIS ON Q OF CAVITY
1/4" DIAM IRIS; CIRCULAR HOLE IN 1.017cm x 2.285cm FACE OF RESONATOR.
CAVITY DIMENSIONS: 13.30 cm x 2.285 cm x 1.017 cm.
Q OF CAVITY
0
25
50
75
100
125
IRIS THICKNESS - THOUSANDTHS OF AN INCH

Figure 17

Discussion of Precision

For all measurements on soldered cavities, the observational error was less than 5%. For the cavities which had to be assembled with clamps, the results varied over a range of 20 - 30%. However, it is believed that the values plotted for the clamped irises are good to about 10%, since the maximum values measured were probably closer to the true value.

Residual errors in the instruments were negligible. The wavemeter on the analyzer had a precision far better than 1%, and the quartz crystal oscillator frequency was known to about .2 %. The greatest observational error arose from difficulty in reading the oscilloscope screen. Estimated precision in measuring in measuring the oscilloscope deflection was about $\frac{1}{3}$ of a division in 10 divisions. Dimensions of the irises were known to a few thousandths of an inch in all cases, which makes their contribution to the error small. Conductivity of the surface is not known precisely, but the relative values of Q were not affected by this factor.

Standing waves in the wave guide system outside the cavity may have contributed a small amount of residual error, although the coupling of the irises is so small that the effect was probably not large. However, due to the fact that the wave guides only propagate certain modes, the results should not be expected to apply to cavities open to a large empty space.