APPLICATION OF THE FLOW NET

TO PERCOLATION

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SYNOPSIS

This paper gives a comprehensive treatment of the fundamental laws of percolation. The problem is attacked from the analytical point of view and the flow-net is used to solve illustrative problems. Darcy's Equation for flow is developed, its applications and limitations are pointed out. The following special cases are taken up: aelotropic soils, variations in soil strata, and two-fluid systems. In addition, problems involving the sand filter, under-drainage, and lateral drainage are worked out. The procedures followed in solving these simpler problems may be extended to the more complicated ones of ground-water movement and the oil-production capacity of wells.

It is hoped that this paper will bring to light some new ideas on the manner in which percolation problems may be analyzed and solved.

CONTENTS

I. Introduction

Percolation is defined as the flow of liquid through a porous medium. In Civil Engineering, it refers to the movement of water through an earth mass. As percolation takes place, the water passes through the interstices between the individual soil particles. These openings, the pore spaces of the soil, may be regarded as a series of more or less tubular passages through which the water flows. Ordinarily it is assumed that the soil particles remain fixed in position. The flow may be either laminar or turbulent depending upon the conditions under which it occurs. The resistance to flow of soil whose particles are smaller than gravel is sufficient to prevent turbulent flow, so that in the usual case of percolation, the flow is laminar.

Permeability is the measure of a soil's capacity to permit percolation. Thus, a permeable soil is one through which water will pass readily, while an impermeable soil retards the flow. The rate of percolation is of importance in many engineering problems. For example, ground-water movement may be forecast if adequate data on geologic sub-strata can be secured. The design of sand-filters, earth-dams, dykes, and other hydraulic structures is influenced by percolation. Permeable soils are preferred for foundations and highway sub-grades where adequate drainage is required. Impermeable soils are utilized in earth-dams and dykes where they prevent the leakage of water.

II. Characteristics of Percolation

A. Darcy's Equation

To get a picture of the flow of water through an earth mass, consider an horizontal tube filled with an isotropic homogeneous soil, and joining two tanks of water of unequal surface elevations as shown in FigureI, page 4. If the particles of soil are relatively small, and if the pressure differential is not excessive, laminar flow will take place through the interstices. The energy equation for hydraulic flow applied between sections (1) and (2) gives:

$$\left(\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1\right) - h_f = \left(\frac{P_2}{w} + \frac{V_2^2}{2g} + z_2\right) \quad *$$

$\frac{P_1}{w} = \frac{P_2}{w} = 0$; since the liquid surfaces are in contact with the atmosphere, the gage pressures are zero.

$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} = 0$; assuming the cross-sectional area of the tanks is relatively large.

Therefore:

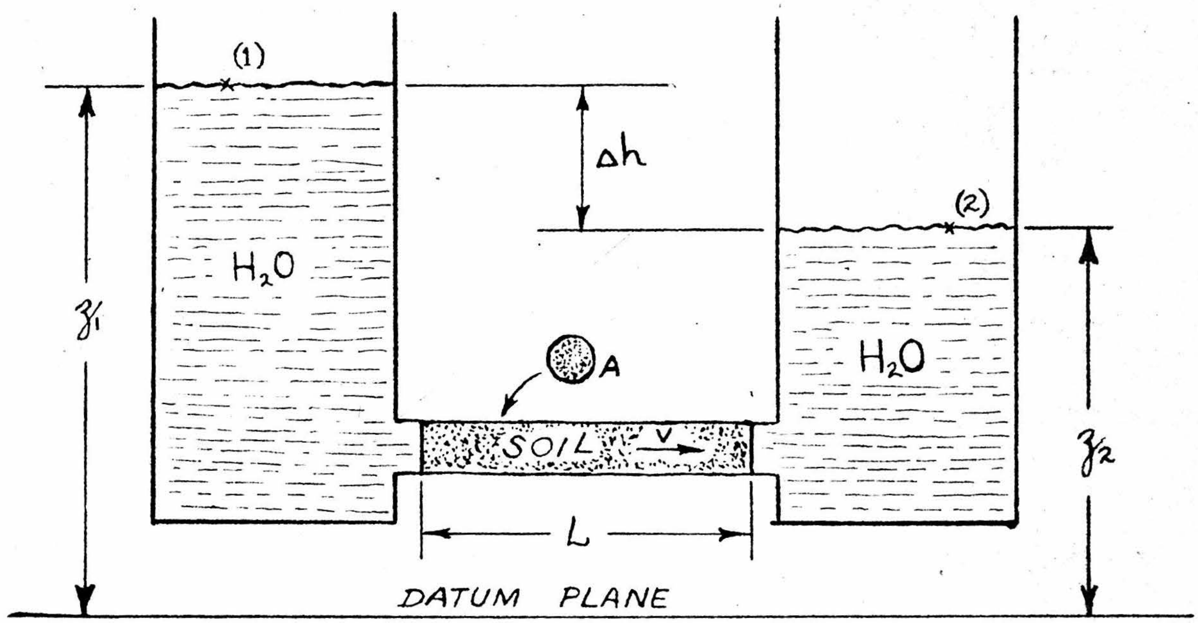
$$z_1 - h_f = z_2$$

Or: $$h_f = (z_1 - z_2) = \Delta h$$

* The letter symbols used in this paper are defined on page 42 . They conform essentially to American Standard Letter Symbols for Hydraulics.

Figure No. I

"Flow of water through soil"



$V = \frac{q}{A}$ (Effective velocity is defined as the rate of discharge q divided by the cross-sectional area of the soil, A.)

Energy Equation:

$$\left(\cancel{\frac{P_1}{w}}^{=0} + \cancel{\frac{V_1^2}{2g}}^{=0} + z_1\right) - h_f = \left(\cancel{\frac{P_2}{w}}^{=0} + \cancel{\frac{V_2^2}{2g}}^{=0} + z_2\right)$$

$$z_1 - h_f = z_2$$

$$h_f = (z_1 - z_2)$$

$$\underline{h_f = \Delta h}$$

The preceding expression indicates that the velocity of flow through the medium will be of such a magnitude that the loss of energy head will be equal to the pressure differential across the soil. This loss of head represents loss of energy due to the resistance to flow offered by the particles of soil. This is similar to pipe friction of ordinary flow in pipe-lines.

From hydraulics, it is known that the energy head loss due to friction $h_f = f \frac{L}{4m} \frac{V^2}{2g}$ * in which "f" is the friction factor, "L" the length of flow, "m" the hydraulic radius, and "V" the average velocity of flow. In the case of laminar flow, the friction factor $f = \frac{c}{R} = \frac{c\mu}{mV\rho}$ where "c" is a constant depending upon the size, distribution, and compaction of the soil grains, μ is the coefficient of absolute viscosity of the percolate, and ρ is the density of the percolate. "R" is Reynold's number, the ratio of the inertia forces to the viscosity forces acting on the water.

* The letter symbols used in this paper are defined on page 42. They conform essentially to American Standard Letter Symbols for Hydraulics.

Therefore for laminar flow:

$$h_f = f\frac{L}{4m}\frac{V^2}{2g} = \frac{c\mu}{mV\rho}\frac{L}{4m}\frac{V^2}{2g} = \Delta h$$

and $\frac{c\mu}{8gm^2\rho} \cdot L \cdot V = \Delta h$

If "k" is defined as: $k = \frac{8gm^2\rho}{c\mu}$

Then, $V = \frac{q}{A} = k\frac{\Delta h}{L}$

Finally; $\underline{\underline{V = ki}}$, where "i" is the hydraulic gradient, that is, the loss in head per length of flow.

This expression is known as the Darcy Equation. It states that the effective velocity of flow "V" of a liquid through a porous medium is equal to the product of "k", the coefficient of permeability of the soil, and "i", the hydraulic gradient.

The effective velocity "V" in the Darcy Equation is much smaller than the actual velocity of movement V_p of the moisture through the soil pores. $V_p = \frac{V}{n}$, where "n" is the porosity of the soil. Since we are interested in the quantity of liquid which passes through the soil per unit of time, we are concerned only with "V", the effective velocity of flow. The rate of discharge through a cross-sectional area of soil "A", is given by: $\underline{q = Aki}$.

B. The Coefficient of Permeability

Before going any further, it should be noted from the Darcy Equation, that since "i", the hydraulic gradient is an abstract number, the coefficient "k" has the dimensions of velocity. Therefore, the coefficient of permeability "k" of a given soil actually represents the effective velocity at which water will pass through the soil under the action of an hydraulic gradient of unity ($i = 1$).

From the analysis in the preceding section it is seen that the coefficient of permeability $k = \frac{8gm^2\rho}{c\mu}$. This indicates that the coefficient of permeability varies inversely with μ, the coefficient of absolute viscosity of the water, and also that it depends upon the characteristics of the soil, such as distribution, shape, size of soil grains, and degree of compaction. Because of the variables involved, an analytical derivation of the functional relationship between "k" and the characteristics of the soil is quite impossible. The particles of two distinct soils for example, may have the same "effective size" but varying

characteristics due to the different "uniformity coefficients", or vice versa. In view of this, the coefficient of permeability of a given soil is usually found by experimental means.
Several devices have been developed to determine "k". Excellent descriptions of these, the constant head permeameter and the variable head permeameter, are given by Krynine [1] in his text: "Soil Mechanics". In order to get good results in determining the coefficient of permeability "k", great care must be used in the field to secure soil samples which are totally undisturbed.

It is of interest to note that various investigators have attempted to establish an empirical relationship between the coefficient of permeability and the soil characteristics. Charles Slichter [2], has perhaps done more work in this field than any other man. He stated permeability as a function of porosity, effective grain size, and temperature of the percolate. His expression does not give as good values for "k" as are secured by experimental means however,

2 - - Charles Slichter; "Motion of Underground Waters" Water Supply Paper, No. 67, U. S. Geological Survey, U. S. Department of Interior.

because of the lack of uniformity present under actual conditions in the field. Some representative values of the coefficient of permeability "k" are given in the following table:

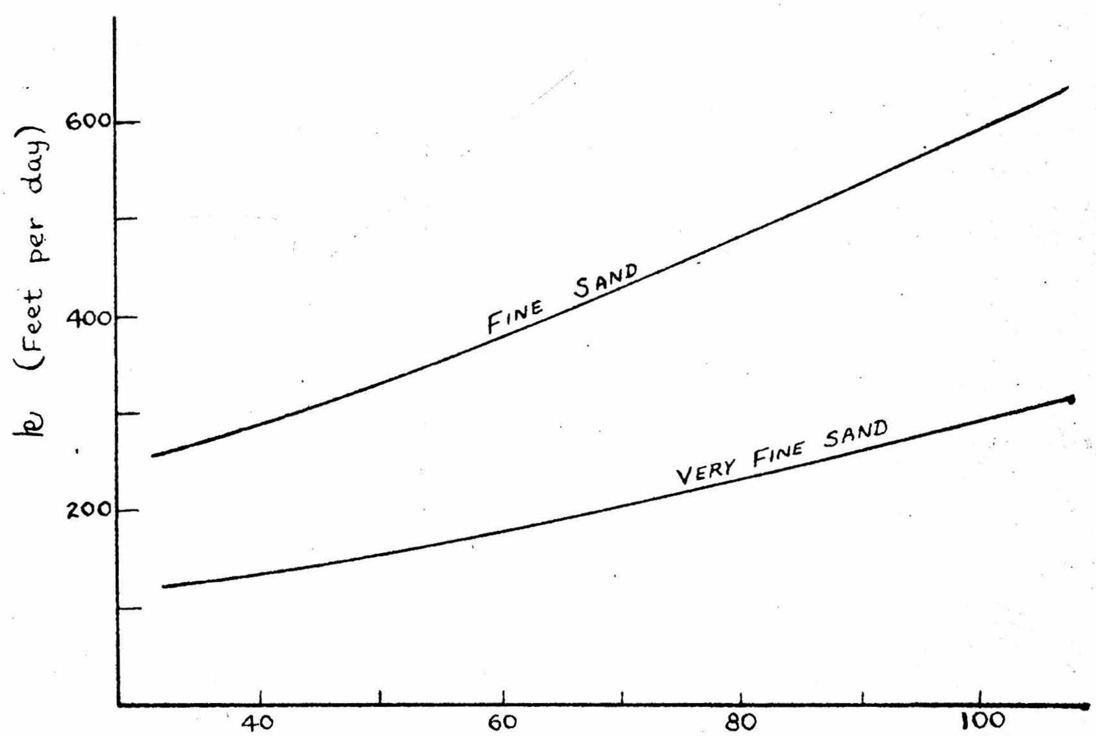
Table I

Kind of Soil	Coefficient of Permeability (Feet per day)
Fine Gravel	4,000
Coarse Sand	600
Fine Sand	50

As already stated, the coefficient "k" varies inversely with μ , the coefficient of absolute viscosity of the liquid, which depends upon the temperature of the percolate. For example, water at 70 degrees Fahrenheit will percolate through soil fifty per cent faster than will water at 40 degrees Fahrenheit. Figure II, page 10 shows the plot of "k" vs. temperature of water. Values of μ , the absolute viscosity coefficient of water, which were used in computing "k", were taken from curves given in Daugherty's "Hydraulics".[3]

Figure No. II

Curve showing variation of the coefficient of permeability with temperature of the percolate.



Temperature; Degrees Fahrenheit

$k = \frac{c}{\mu}$ *

* Data for μ, the coefficient of absolute viscosity of the percolate secured from curves given in Daugherty's "Hydraulics".

C. Limitations

It should be kept in mind that in the foregoing analysis capillary forces, which are of small magnitude, have been neglected. In addition, no consideration has been given to the attached ground water. The moisture which adheres to the walls of the openings is known as attached ground water; its effect on percolation is of minor consequence.

There are definite limits to the applicability of Darcy's Equation. Experimental work in percolation by several investigators indicates that Darcy's Equation gives satisfactory results at extremely low values of hydraulic gradient, but at high values, ($i > 1$), the results are inconsistent. This is accounted for by the fact that when "i" is large, the velocity may become great enough to create turbulent flow. When the flow is turbulent, the Darcy Equation is not valid, for it was derived on the basis of laminar flow. The criterion for the type of flow depends on the value of Reynold's number. In ordinary pipe flow, it will be recalled from hydraulics, that the flow is laminar when R is less than 2000, otherwise the flow is turbulent. A similar condition exists in the case of percolation. Various men have found that the critical Reynold's number for percolation is 10, where R is defined as $R = \frac{d \cdot V \rho}{\mu}$, d being the average grain diameter of the soil.

Previously it has been stated that the loss in head due to flow through pipe-lines or percolation through soil is given by $h_f = f \frac{L}{4m} \frac{V^2}{2g}$. If "f", the friction factor, as defined above, is plotted against R, Reynold's number for various soils we get a series of curves similar to those of Kemler and Piggott shown in Daugherty's "Hydraulics".[3] Figure III, page 13 is a reproduction of "friction-factor" curves for percolation given by Muskat[4] which demonstrate the similarity between percolation and flow in pipe-lines. Data gathered by Tolman[5] shows that

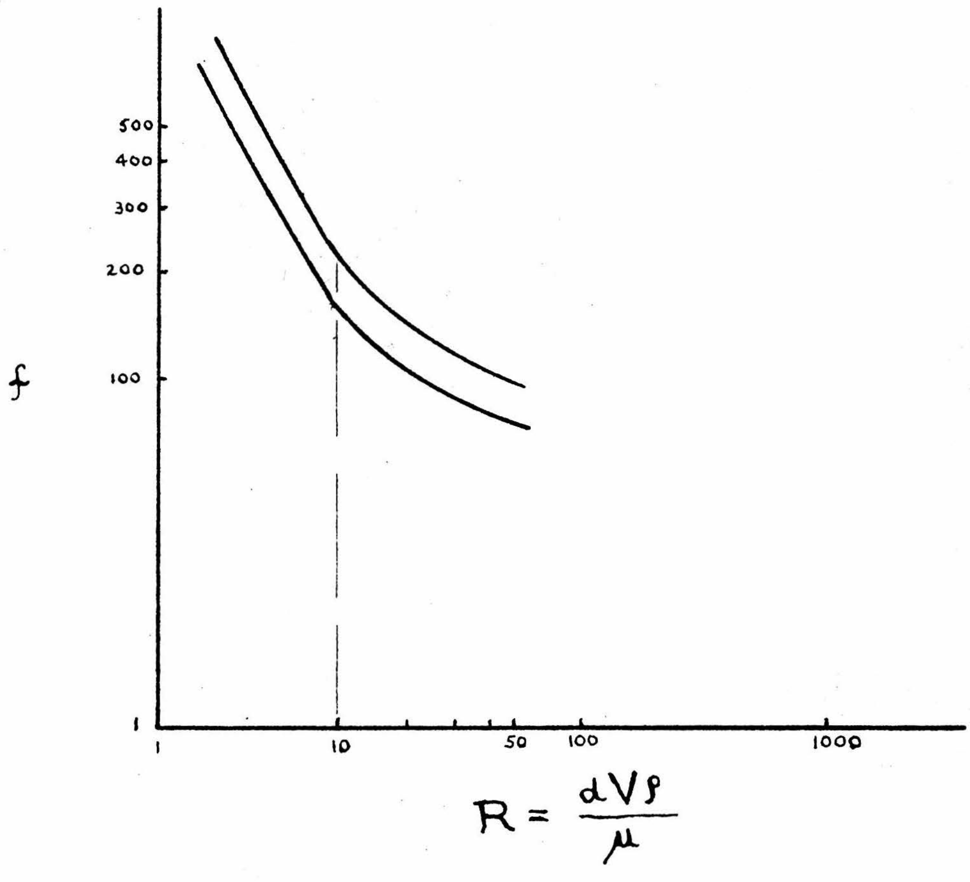
4 - - Muskat: "The Flow of Homogeneous Fluids"

5 - - Tolman: "Ground Water"

Figure No. III

"Friction-Factor Curve"

(Plot of f vs. R)



The break in the straight-line portion of the curve occurs in the vicinity of R = 10. This indicates that if R is greater than 10, the flow is turbulent.

turbulent flow occurs through soils only where the soil particles are of large diameter or where the hydraulic gradient is excessive. The following table taken from Tolman's "Ground Water" gives values of critical velocities in various types of soil.

Table II *

Porosity, per cent	Grain Size in mm. 0.2	1.0	10.0
25	65,400	13,100	1,300
30	50,800	10,200	1,000
35	40,500	8,100	800
40	32,700	6,500	700

* Velocities are in feet per day

If the velocity of flow through soil is less than that shown in the above table, the flow will be laminar and Darcy's Equation will apply.

Our discussion up to this point has been concerned only with homogeneous isotropic soils. As long as the soil particles are of regular shape and uniformily distributed, the permeability factor "k" will be independent of direction. Appreciable stratification however, or the presence of flat grains deposited in a parallel fashion, will result in a lower resistance to flow in one direction than in another. This gives rise to special cases which will be considered later. In nature, we also run across instances where there are layers of different soils of various permeabilities. This problem will also be taken up in a later section of this paper.

III. The Flow Net

The hydraulic flow-net is a graphical representation of the direction of flow and of the potential head available at any point within a flow system. It consists of flow lines and contours of equal potential superimposed on the cross-section through which the flow is taking place. In the case of steady two-dimensional flow, two families of curves may be derived mathematically [6] by means of the continuity equation written as a differential equation. Mathematical solution indicates that these families of curves intersect each other at right angles. There are actually an infinite number of flow lines and equipotential lines in a flow system. It is convenient, however, to choose only a limited number of these lines such that the rate of flow dq between each pair of flow lines is equal, and such that the potential drop dh between successive potential lines is the same. The interval between the flow lines is made equal to the interval between the potential lines thus forming a series of

6 - - Hunter Rouse: "Fluid Mechanics"

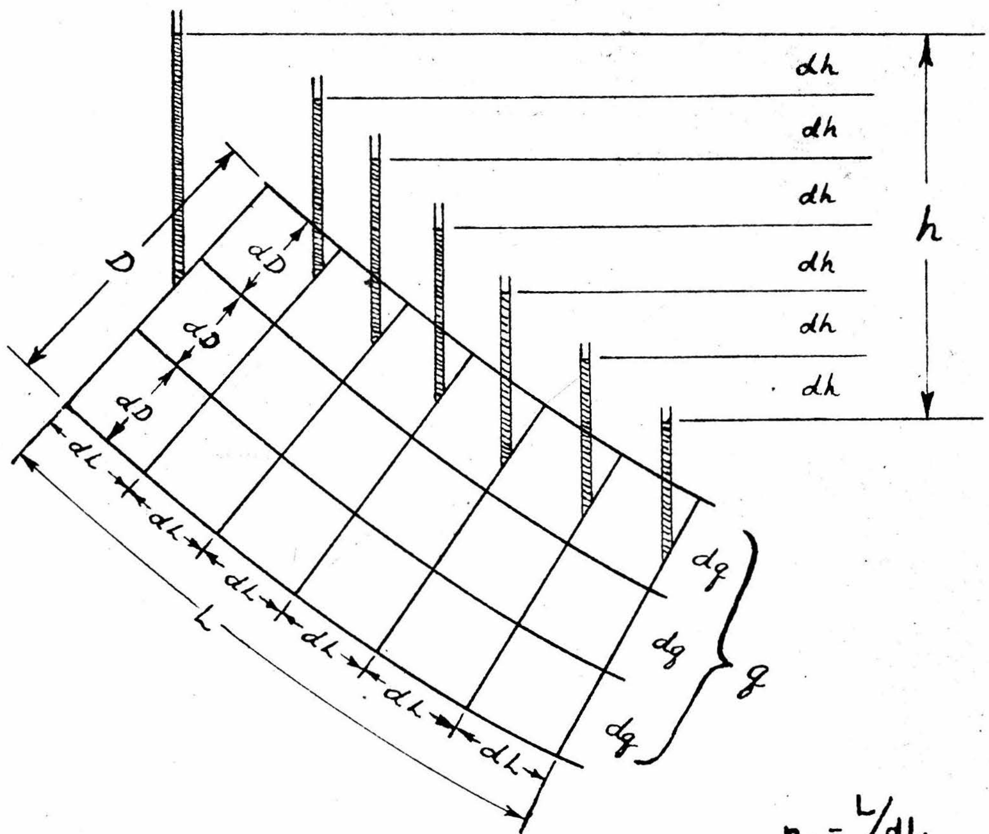
squares. In the vicinity of a bend, the squares will be distorted but they will become more nearly perfect as the number of lines is increased. Mathematically speaking, they approach true squares as they become infinitesimal in size.

The flow-net is usually constructed by free-hand sketching, with gradual adjustment and correction until the flow lines and potential lines intersect at right angles and form curvilinear squares. From the principle of continuity as applied between each pair of flow lines, the velocity must vary inversely with the spacing. Where the flow is in a curved path the position of the flow lines may be adjusted and checked by the relationship that $(\frac{dv}{dy} = \frac{v}{r})$ [6] in which v is the velocity at any point and r is the radius of curvature of the flow line at that point. A graphical method of checking the spacing of the flow-lines is described in a paper by H. Alden Foster [7] in the May 1944 issue of "Proceedings of the American Society of Civil Engineers".

6 - - Hunter Rouse: "Fluid Mechanics"

It will now be shown how the rate of percolation can be computed from the flow-net. Figure IV, page 19 represents a portion of a correctly constructed flow-net for the case of flow through soil. If the distance between the lines of equal potential is dL, the hydraulic gradient is $i = \frac{dh}{dL}$, where dh is the head loss between successive potential lines. For a unit width, the area through which the flow takes place would be 1×dD where dD is the distance between flow-lines. The rate of flow between any two flow-lines as given by the Darcy Equation is $dq = Aki = dD \times k \times \frac{dh}{dL}$, where "k" is the coefficient of permeability. Since the head loss from one potential line to another is equal, the drop dh from one to another is given by $dh = \frac{h}{n_p}$ where h is the total potential drop through the medium under consideration and n_p is the number of increments into which the total potential drop is divided. Substituting this value for dh in the previous equation, the following expression results: $dq = dD \times k \frac{h}{n_p\, dh}$ where dq is the rate of flow between any two successive flow lines. By construction, dD

Figure IV (Flow Net and Discharge)



$n_p = L/dL$

$n_s = D/dD$

$$i = \frac{dh}{dL} = \frac{h}{n_p \times dL} \; ; \quad A = n_s \times dD$$

$$V = ki$$

$$q = AV = Aki$$

$dD = dL$, BY CONSTRUCTION

$$\therefore \; q = kh\,\frac{n_s}{n_p}$$

has been made equal to dL, therefore $dq = \frac{kh}{n_p}$. The total percolation through the medium under consideration would then be $q = n_s\, dq$ where n_s is the number of spaces between the flow lines. The rate of total percolation therefore is given by: $\underline{q = n_s\, dq = kh \frac{n_s}{n_p}}$.

This equation may be used whenever sufficient information is available for the construction of a flow net. The accuracy of the results increases with the number of divisions in the flow net. As long as the flow remains laminar, the same flow-net applies equally will for any rate of discharge, because the pattern of the flow lines depends not upon the velocities, but upon the geometry of the flow boundaries.

It has long been known that percolation through soil is analagous to the propagation of electricity. By virtue of this, percolation problems can be studied by electrical means. In studying percolation by the electric analogy, a model is constructed. The apparatus usually consists of a shallow tray with plate-glass bottom and sides, filled to a depth of one inch or less with an electrolyte. Copper plates under different potentials are used to represent the "source" and "sink" of the flow system. When the current passes from one terminal to the other, the trajectories described are exactly the same as streamlines in the case of percolation. By use of proper electrical equipment, the equipotential lines may be determined and the resulting flow net may then be constructed. The applicability of the electric analogy may be verified by comparing Ohm's law with the Darcy Equation. Ohm's law expresses the fundamental relation for the flow of electric current while Darcy's equation gives the fundamental relation for percolation. Ohm's Law expressed by an equation is $I = \frac{E}{R}$ in which,

I = current in amperes, a measure of quantity of flow

E = pressure in volts, and

R = resistance in ohms.

This equation is in terms of resistance. The Darcy Equation on the other hand, is in terms of conductance. Since electrical conductance, K', is the reciprocal of resistance, Ohm's law may also be written $I = EK'$. The conductance K' varies directly with specific conductivity k', with the area A, and inversely with the length L, then $K' = k' \frac{A}{L}$. Therefore $I = k' \frac{A}{L} E$. If I is the quantity of flow, k' the coefficient of flow, E the potential, L the length of flow, and A the area, this equation is ananagous to the Darcy Equation. That is,

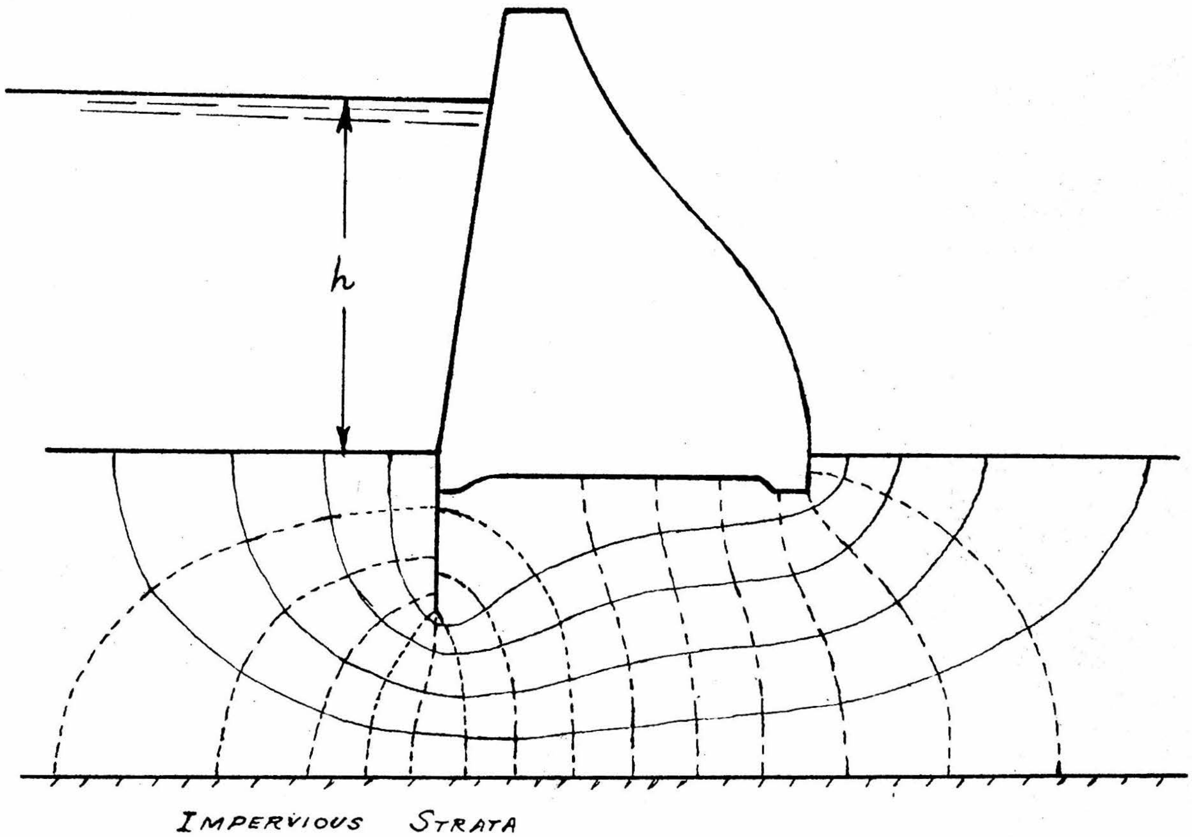
$$q = Aki \qquad \text{(Darcy Equation)}$$

$$I = Ak'\left(\frac{E}{L}\right) \qquad \text{(Ohm's Law)}$$

In addition to the electric analogy technique, models are used to study percolation problems. By this method a glass-walled soil model is constructed of the system to be studied, and the flow-lines are determined by introducing dye at various points in the system. The soil used in the model is usually many times more permeable than the soil of the prototype since in the prototype the velocities are usually extremely low. All discussion up to this point has dealt with two dimensional flow only, but all statements made so far apply equally well to three dimensional flow.

Figure V, page 24 shows the plot of a flow-net for percolation under a dam. Beneath the diagram are shown computations for determining the rate of percolation under the dam by use of the Darcy Equation. The soil sub-layer has been assumed to be homogeneous and isotropic, meaning that the water will flow equally well in all directions.

Figure No. V Flow Net for Percolation Under a Dam with Computations for Rate of Percolation.



$$\left\{\begin{matrix} n_s = 5 \\ n_p = 15 \end{matrix}\right\}$$

SUPPOSE: $k = 200$ FT. PER DAY
$h = 40$ FT.

$$q = kh\frac{n_s}{n_p}$$

$$q = 200(40)\frac{5}{15} = 2{,}667 \text{ CUBIC FEET PER DAY, PER FOOT OF WIDTH}$$

IV. Special Cases

A. Aelotropic Soils

Aelotropic soils are those which have a lower resistance to flow in one direction than in another. Such a condition is found in nature where considerable stratification has occured or where the soil consists of flat grains deposited in a parallel fashion. The permeability of a soil in the horizontal direction k_x is oftentimes five or ten times greater than that in the vertical direction k_z. Conditions of this sort require a modification of the flow net, merely a linear distortion. Samsioe, a Swedish scientist, proposed that the horizontal dimensions be reduced by the factor $\sqrt{\frac{k_z}{k_x}}$. The procedure for constructing such a flow-net is to first plot the flow-net for the reduced structure and then to plot the vertical ordinates of this flow-net under the corresponding points of the natural-shaped, non-reduced structure. On the basis of such a flow-net, the rate of discharge is computed by means of the following expression:

$$q = h \frac{n_s}{n_p} \sqrt{k_x k_z}$$

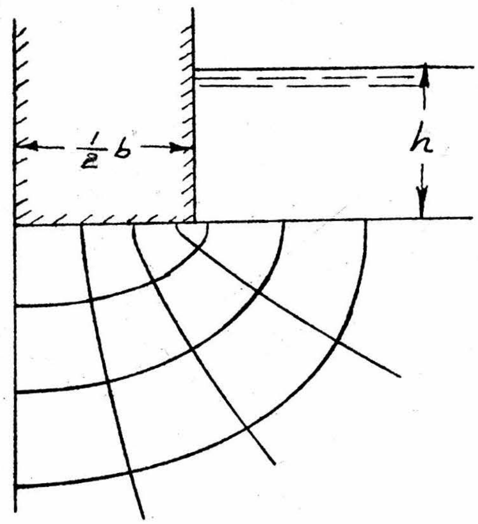
Figure VI, page 26, shows the procedure for such a problem.

<u>Figure No. VI</u> Treatment for Aelotropic Soils

Given: $k_x = 4 k_z$

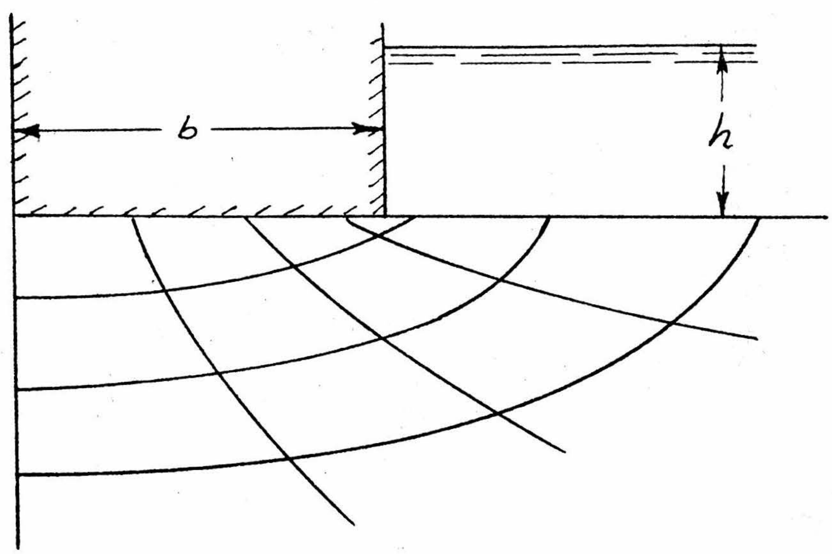
$$\sqrt{k_z / k_x} = 1/2$$

Reduced Scale



$\begin{cases} n_s = 3 \\ n_p = 4 \end{cases}$

Natural Scale



$\begin{cases} n_s = 3 \\ n_p = 4 \end{cases}$

$$q = h \frac{n_s}{n_p} \sqrt{k_x k_z} = h \frac{n_s}{n_p} \sqrt{4 k_z^2}$$

$$q = h \frac{n_s}{n_p} (2 k_z)$$

B. Variations in Soil Strata

There are instances in which a soil mass is composed of successive layers of material which have different coefficients of permeability. Let us consider first the case in which the flow is parallel to the bedding planes. Figure VIIa, page 29 shows a diagram for such a condition. Let k_1, k_2, -----k_n be the coefficients of permeability of the individual layers, and let B_1, B_2, -----B_n be their respective thicknesses. Also let k' represent the average coefficient of permeability parallel to the bedding planes. For such a case, the hydraulic gradient i must be the same everywhere since the same potential drop occurs over identical lengths in each layer. The discharge per unit width through any layer is given by: $q = A(V) = B(ki)$ where B is the thickness of the layer, k its coefficient of permeability, and i the hydraulic gradient common to all layers defined as $(i = \frac{h}{L})$. The total rate of discharge through the mass would be: $q_T = B_1k_1i_1 + B_2k_2i_2 + \text{----}\ B_nk_ni_n$

And the average discharge velocity, is

$$v = \frac{q}{A} = \frac{B_1k_1i_1 + B_2k_2i_2 + \text{----}\ B_nk_ni_n}{B_1 + B_2 + \text{-----}B_n} = k'i$$

Consequently, the average coefficient of permeability parallel to the bedding planes k' is given by:

$$k' = \frac{B_1k_1 + B_2k_2 + \text{----}\ B_nk_n}{B_1 + B_2 + \text{---}B_n}$$

A second case to be considered is one in which the flow is at right angles to the bedding planes. Figure VIIb, page 29 shows a diagram for such a condition. Let k" represent the average coefficient of permeability at right angles to the bedding planes. Let i_1, i_2, -----i_n represent the hydraulic gradients in the different layers and let h_1, h_2, -----h_n represent the potential head losses through the various layers. For this case, the discharge velocity v must be everywhere the same since the flow is continuous. The discharge velocity in any layer is given by: $v = ki$. But as stated above, $v_1 = v_2 = v$. Therefore $k_1 i_1 = k_2 i_2 = \text{-----} k_n i_n$.

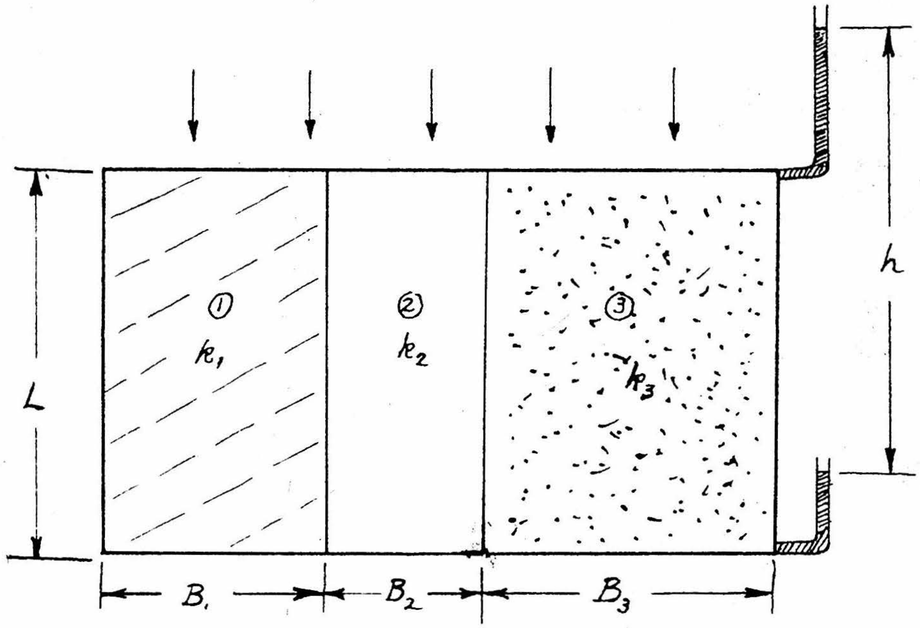
Now $v = k_1 i_1 = k'' \dfrac{h_1 + h_2 + \cdots h_n}{B_1 + B_2 + \cdots B_n}$

And $h = i_1 B_1 + i_2 B_2 + \cdots i_n B_n$

Therefore; $k'' = \dfrac{v(\Sigma B)}{\Sigma h} = \dfrac{ki\,(B_1 + B_2 + \cdots B_n)}{i_1 B_1 + i_2 B_2 + \cdots i_n B_n} = \dfrac{\Sigma B}{B_1/k_1 + B_2/k_2 + \cdots}$

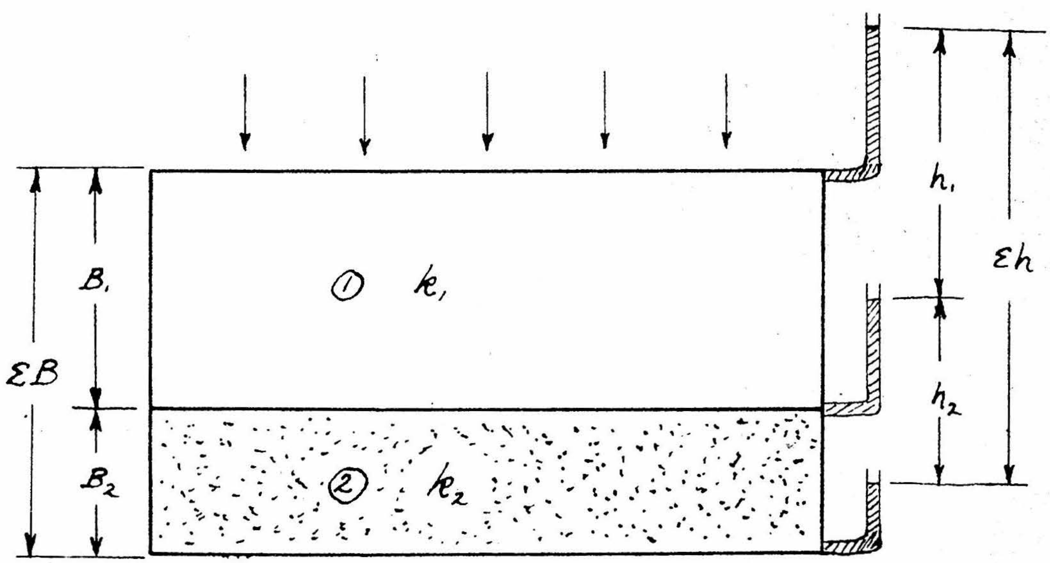
As another illustration, let us consider the case of flow through the earth-mass shown in Figure VIII, page 30. In this case, the flow takes place in a direction that is essentially parallel to the bedding plane between the two layers considered as such. One of these layers is homogeneous, while the other, is composed of two different materials. The construction of the flow-net for this case indicates that if the flow were unrestricted, there would be a slight horizontal drift to the flow.

Figure No. VIIa   Flow Parallel to Bedding Planes



$$k' = \frac{B_1 k_1 + B_2 k_2 + \cdots B_n k_n}{B_1 + B_2 + \cdots B_n}$$

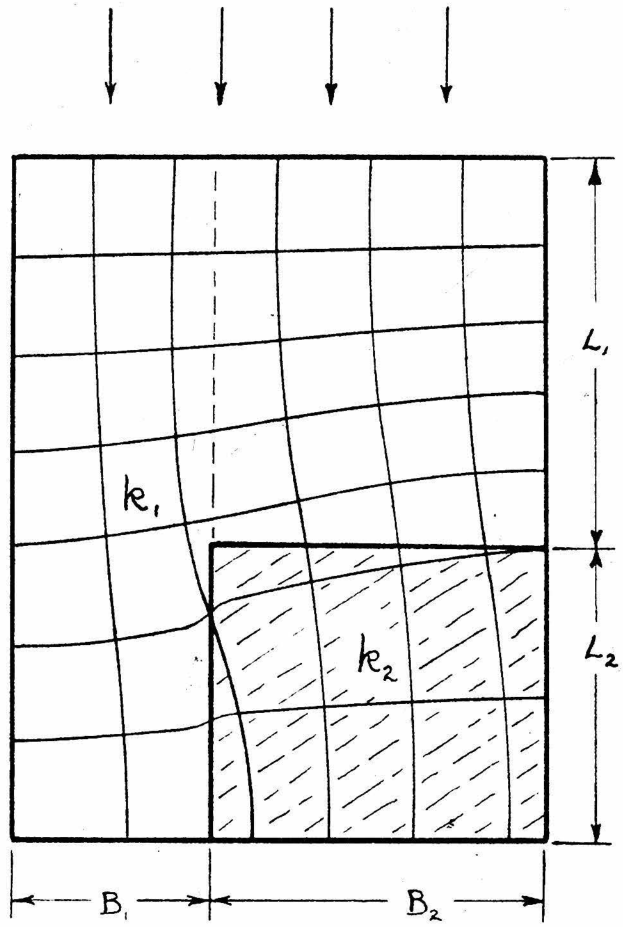
Figure No. VIIb   Flow at Right Angles to Bedding Planes



$$k'' = \frac{B_1 + B_2 + \cdots B_n}{B_1/k_1 + B_2/k_2 + \cdots B_n/k_n}$$

Figure No. VIII   Analysis of Complex System

The superimposed flow-net demonstrates the horizontal drift imposed on the flow.



$$k_1 = k_1$$

$$k'' = \frac{L_1 + L_2}{L_1/k_1 + L_2/k_2}$$

$$k' = \frac{B_1 k_1 + B_2 k''}{B_1 + B_2}$$

$$q = Aki = (B_1 + B_2) \times k' \times \frac{h}{L_1 + L_2}$$

C. Two Fluid Systems

The occurence of ground waters in the vicinity of oil-bearing sands gives rise to a problem which involves two liquids of different viscosities. This problem is of great concern in the field of oil-production technology. Sufficient data and proper analysis will enable one to predict the effect of the movements of ground-waters on the oil-production capacity of nearby wells. In this section a very simple case of "water-encroachment" shall be considered merely to show the general technique for analyzing such a problem. Let us consider a portion of a homogeneous medium through which two liquids of different viscosities are flowing. See Figure IX, page 33. To simplify this illustration let us assume that the potential drop across the soil remains constant as the interface between the two liquids passes through it. The time required for the interface to pass through the sand may be found as follows:

Let k_w represent the coefficient of permeability of the sand in the regions where water is present.

Let k_o represent the coefficient of permeability of sand in the regions where oil is present.

From previous discussion in this paper, (See page 7), it will be recalled that the coefficient of permeability k is inversely proportional to the coefficient of absolute viscosity of the percolate μ; that is, $k = \frac{8gm^2\rho}{c\mu} = \frac{C}{\mu}$ where C is a constant depending on the properties of the soil. Hence $k_w = \frac{C}{\mu_w}$ and $k_o = \frac{C}{\mu_o}$ where μ_w and μ_o are the coefficients of absolute viscosity of the sand with water and with oil respectively. It will be recognized that this condition, the flow of two fluids through an homogeneous soil is analagous to the flow of one fluid through layers of different soils at right angles to the bedding planes. (Discussed on page 28). Consequently the effective coefficient of permeability k''' for this two fluid system is given by: $k''' = \frac{L}{x/k_1 + \frac{(L-x)}{k_2}} = \frac{L}{\frac{x\mu_1}{C} + \frac{(L-x)\mu_2}{C}}$

Refer to Figure IX, page 33.

Now $k''' = \frac{C \cdot L}{x\mu_1 + L\mu_2 - x\mu_2}$

But $v = k''' i = k''' h/L$

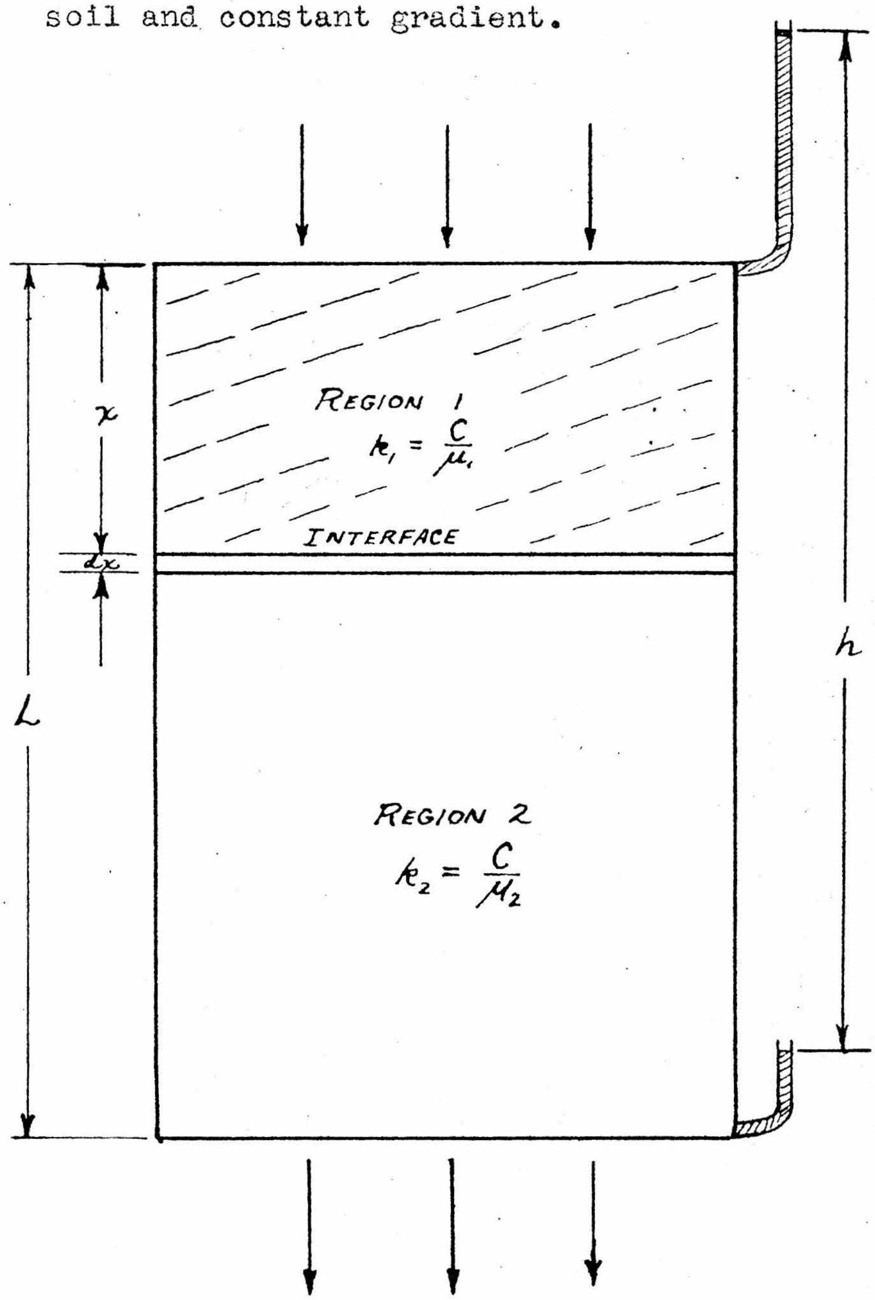
$$dx = v\,dt$$

$$t = \int \frac{dx}{v} = \int_0^L \frac{x\mu_1 + L\mu_2 - x\mu_2}{C \times h}\,dx$$

Finally $t = \frac{L^2}{2Ch}(\mu_1 + \mu_2) = \frac{L(\mu_1 + \mu_2)}{2C\,(h/L)}$

This gives the time required for the interface to pass through the soil mass. Note that the time given by the above expression for an homogeneous soil, $(\mu_1 = \mu_2)$, is: $t = \frac{L\mu}{C(h/L)}$

Figure No. IX Two Fluid System for homogeneous soil and constant gradient.



$$k''' = \frac{C \times L}{x\mu_1 + L\mu_2 - x\mu_2}$$

The time required for the interface to pass through the soil mass is given by: $t = \frac{L(\mu_1 + \mu_2)}{2C(h/L)}$

V. Applications

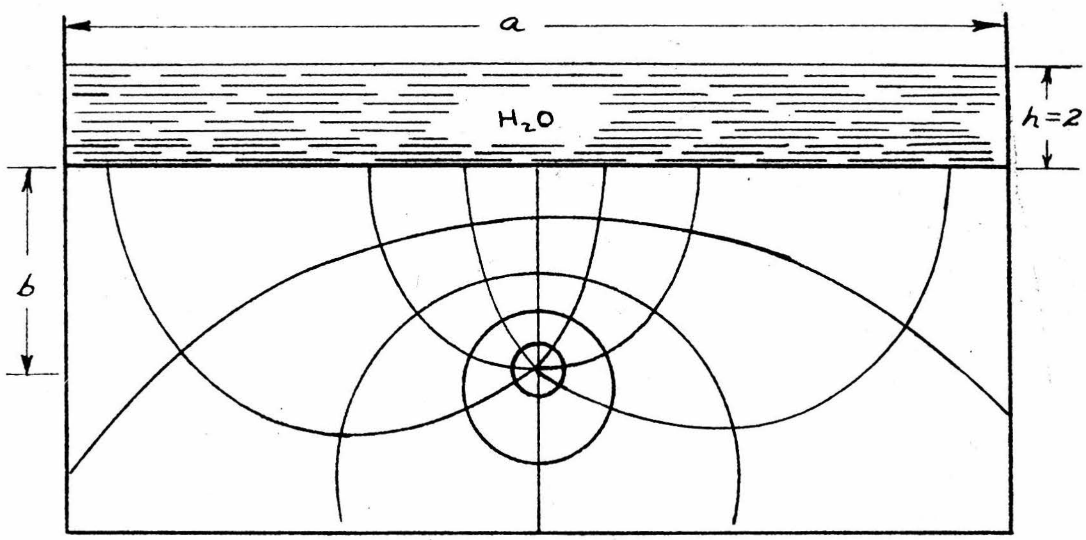
A. Under-Drainage

The flow-net may be applied to various types of percolation problems. In this section, it shall be applied to the underdrain shown in Figure X, page 35. The flow-net for this case is superimposed on the section and computations for the rate of drainage are shown below the diagram. The shape of the flow-net for this case depends on the relationship between a, the width of the basin, and b, the distance the drain is located beneath the soil surface. Figure XI, page 36 shows the effect of varying the ratio of a to b. Where b is small, more drainage tends to take place from the central portions of the basin, while a large value of b tends to induce practically uniform drainage over the basin area.

B. Seepage through Dams

The solution of the very important percolation problem of seepage through earth dams is greatly facilitated by construction of the flow-net. The general procedure is quite similar to that shown in Figure V, page 24 for seepage beneath a dam. When the section of the dam is composed of materials of different permeabilities the flow-net must be adjusted by methods already discussed.

Figure No. X   Flow-Net for the Under-Drain



k = 100 FEET PER DAY

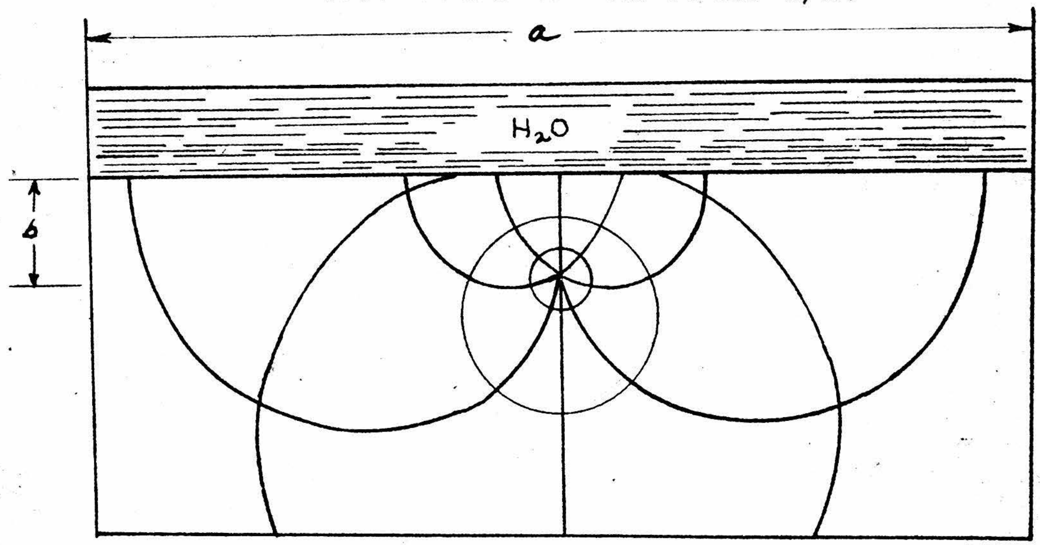
$\begin{cases} n_s = 6 \\ n_p = 5 \end{cases}$

Computations for Rate of Drainage:

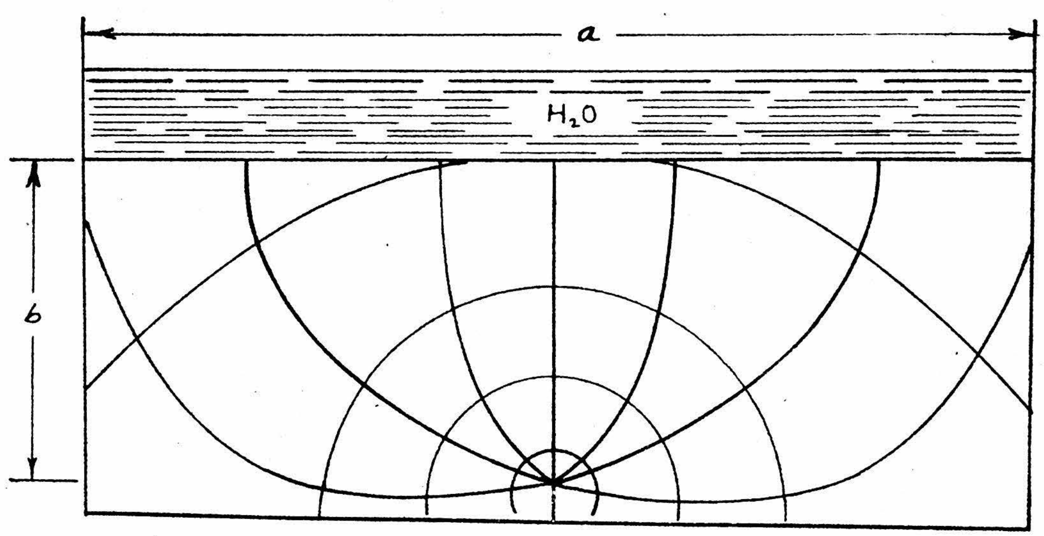
$$q = kh\frac{n_s}{n_p}$$

$$q = 100(2)\frac{6}{5} = 240$$ cubic feet per day per foot.

Figure No. XI   Changes in the Flow-Net induced by variations of the ratio b/a.



Shallow Drain



Deep Drain

C. The Sand Filter

As another interesting illustration of percolation supposing we wish to find the rate of discharge through a sand filter whose dimensions are twenty feet by twenty feet. Let it be composed of three layers of different material as shown in Figure XII, page 38. The thickness B_1 of the first layer is two feet, its coefficient of permeability k_1 is fifty feet per day. The second layer is six feet thick and has a coefficient of permeability k_2 of two hundred feet per day. Let the bottom layer be two feet thick composed of fine gravel with a coefficient of permeability of one thousand feet per day. According to the principles given in section IV B, page 28, the effective permeability k'' for flow at right angles to the planes is:

$$k'' = \frac{B_1 + B_2 + B_3}{\frac{B_1}{K_1} + \frac{B_2}{K_2} + \frac{B_3}{K_3}}$$

$$k'' = \frac{2+6+2}{\frac{2}{50} + \frac{6}{200} + \frac{2}{1000}} = \frac{10}{.072} = 138$$

and $v = k'' i$

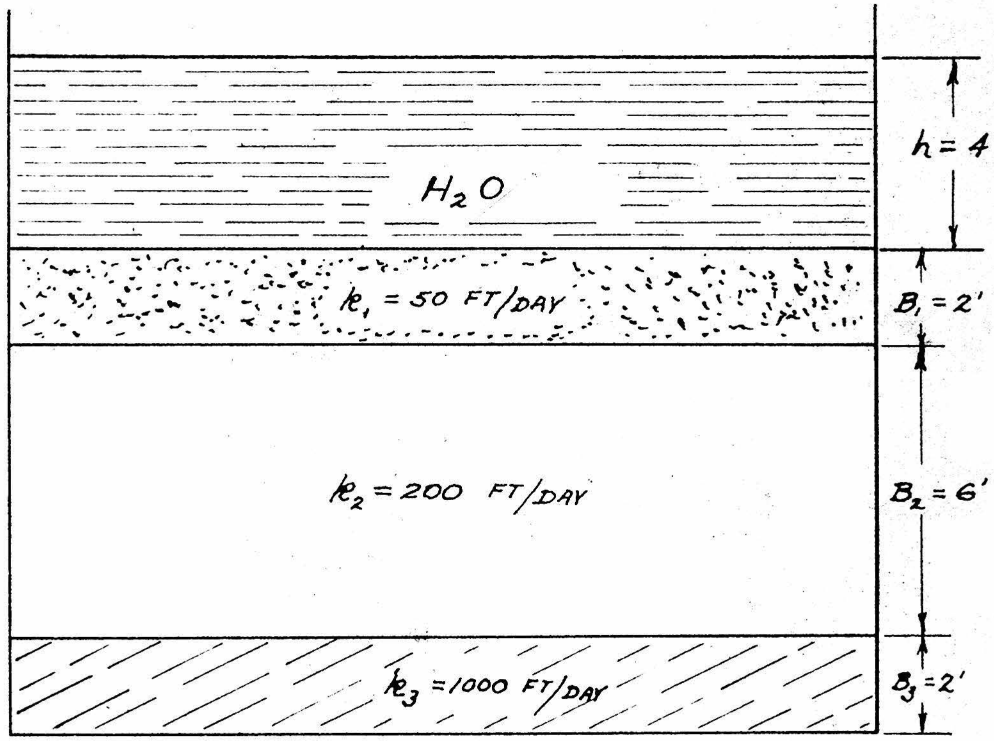
$$v = k'' i = 138\left(\frac{4}{10}\right) = 55.2 \text{ FEET PER DAY}$$

Therefore $q = A(v) = 20(20)(55.2) = 22{,}080$ cubic feet per day.

In instances of this sort where the geometry is extremely simple and symmetrical, the flow-net does not facilitate the computations.

Figure No. 12 The Sand Filter

Refer to Page 37



Depth of Water, h, assumed to be constant.

$$k'' = 138 \text{ FT/DAY}$$

$$i = h/L = 4/10$$

$$v = k'' i = 138\,(4/10) = 55.2 \text{ FT/DAY}$$

D. Lateral Drainage

As a final illustration of the application of the flow-net to percolation let us consider briefly the case of lateral drainage pictured in Figure XIII, page 40. Supposing $k = 200$ feet per day, $z_1 = 40$ feet, $z_2 = 10$ feet, $L = 60$ feet, $w = 100$ feet. The rate at which drainage originally occurs may be readily computed from the flow-net as follows:

$$q = k z_1 \frac{n_s}{n_p} = 200(40)\frac{4}{6} = 5,333 \text{ CUBIC FEET PER DAY}$$

To determine the time required for the water-surface in the reservoir to drop from elevation 40 feet to 10 feet, a simple integration is required as follows:

In any element of time dt, let the level of the reservoir drop the distance dz.

The hydraulic gradient $i = z/L$.

The quantity of water entering the face of the soil bank in any interval of time dt is given by: $dQ = Aki\,dt = z k \frac{z}{L} dt$

But in the time dt the volume of water lost from the reservoir is: $dQ = -w\,dz$

Therefore; $-w\,dz = \frac{k}{L} z^2 \, dt$

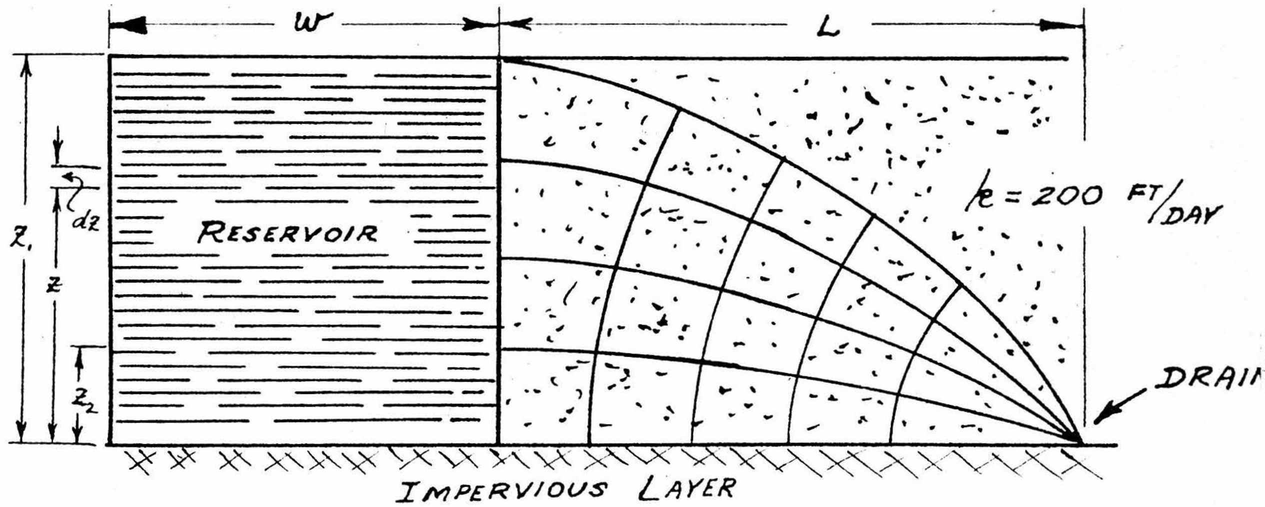
From which $t = -\frac{wL}{k}\int_{z_2}^{z_1} \frac{dz}{z^2}$

Evaluating $t = \frac{wL}{k} \frac{(z_1 - z_2)}{z_1 z_2}$

And finally,

$$t = \frac{100(60)}{200} \frac{30}{40(10)} = \underline{2\tfrac{1}{4} \text{ DAYS}}$$

Figure No. XIII   Lateral Drainage



VI. Conclusions

The preceding discussion demonstrates that the flow-net when properly applied, may greatly facilitate the solving of the more complicated problems of percolation. Several special cases and particular applications have been considered to point out the procedures that must be resorted to in order to solve the various complex problems that are confronted in the field. These may be extended to the more complicated problems of ground-water movement, influent seepage, and the oil-production capacity of wells.

It is hoped that this paper clarifies some of the principles of percolation and that it also presents some new ideas in the methods by which percolation problems may be solved.

NOTATION

The following letter symbols, used in this paper, conform essentially to American Standard Letter Symbols for Hydraulics.

A = cross-sectional area of section through which flow takes place.

B = thickness of layer through which flow occurs.

c = a constant.

f = hydraulic friction-factor.

g = acceleration of gravity.

h = potential head.

h_f = head loss due to frictional resistance.

i = hydraulic gradient, loss in head per unit of length.

k = coefficient of permeability.

L = length of flow path.

m = hydraulic radius, ratio of cross-sectional area to wetted perimeter.

n = porosity of a soil.

n_p = number of spaces between equipotential lines in a given flow net.

n_s = number of spaces between flow lines in a given flow net.

p = hydrostatic pressure.

q = rate of discharge.

R = Reynold's Number, the ratio of inertia forces to viscosity forces acting on a moving liquid.

ρ = density of a liquid.

μ = coefficient of absolute viscosity of a liquid.

v = average effective velocity of flow.

v_p = actual velocity of liquid through the soil pores.

w = weight per unit volume of a liquid.

z = elevation head.

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