List of Figures

2.1 Frequency window $\tilde{U}_{j,\ell}$ (darkened region) defined in the three-dimensional discrete curvelet transform, extracting the frequency content near the wedge with center slope $(1, \alpha_\ell, \beta_\ell)$ (figure based on Ying et al. (2005)) 12

2.2 Example of application of the periodic reconnection algorithm to a set of boundary-intersecting structures obtained from a periodic three-dimensional scalar field. (a) Before periodic reconnection. (b) After periodic reconnection, where the color of each structure indicates the number of pieces involved in the reconnected structure for this particular scenario: blue = 1 (non-intersecting), green = 2, orange = 4, cyan = 8 . . . 14

2.3 2D example of reconnection of a periodic structure spanning across multiple extensions of the periodic domain. The original fragments of the structure are represented in the original domain (central square). The resulting structure, after reconnection, is represented by the thick line. 15

2.4 Transformation from (κ_1, κ_2) to (Υ, Λ) 17

2.5 Range of shape index, Υ, with its most representative associated local shapes (figure based on Koenderink & van Doorn (1992)) 17

2.6 Representative local shape in the combined (κ_1,κ_2) plane 18

2.7 Example of a three-dimensional surface (a) with S and C mapped onto it (bottom and upper halves, respectively) and its corresponding signature (b), for which a projection of the three-dimensional physical structure is shown at the top-right corner, its area-based joint two-dimensional probability density function (in terms of S and C) is presented in the bottom-left area, while the marginal probability density functions of both S and C, are drawn at its top and right sides, respectively. The value of the stretching parameter, λ, is represented below the jpdf by a black bar (in a scale from 0 to 1). Mean and feature centers, as well as upper and lower distances for each variable of the jpdf, are superimposed to the jpdf, as the filled and hollow squares, respectively (refer to §2.3) 20

2.8 Projections of the *visualization space* with the predominantly blob-, tube- and sheet-like regions sketched: three-dimensional perspective projection (left), two-dimensional orthogonal projections (right) of the planes β (formed by the axes $\hat{C}$ and λ) and α (formed by $\hat{S}$ and $\hat{C}$). For example, a glyph consisting of a sphere and four bars along the $\pm\hat{S}$, $\pm\hat{C}$ axes can represent nine parameters of the characterization of the corresponding structure: $\hat{S}, \hat{C}, \lambda$ given by the center of the sphere, upper and lower distances of S and C given by each bar, the surface area A of the associated structure, given by the size of the glyph, and the group to which the structure belongs, given by the color of the glyph 25

3.1 Visualization space with clustering results for the virtual set of modeled structures, with representative examples shown at the sides 28

4.1 Tri-plane cuts of the passive scalar fluctuation field for the original database (top left) and each of the filtered scales resulting from the multi-scale analysis (filtering in curvelet domain) (increasing scale number from left to right and top to bottom) 31

4.2 Plane cuts normal to the x_3-axis at its midpoint of the passive scalar fluctuation field for the original database (top left) and each one of the filtered scales resulting from the multi-scale analysis (filtering in curvelet domain) (increasing scale number from left to right and top to bottom) 32

4.3 Volume-data pdfs of the passive scalar fluctuation field (a) and corresponding spectra (b), associated with the original database (containing all scales) and each of the filtered scales 32

4.4 Iso-contours of the passive scalar fluctuation field for the original database (top left) and each one of the filtered scales resulting from the multi-scale analysis (filtering in the curvelet domain) (increasing scale number from left to right and top to bottom). A contour value equal to the mean plus two times the standard deviation of each resulting scalar field was used 35

4.5 Visualization space with spheres representing the structures educed from the passive scalar fluctuation field at all scales (top), and only at scale 1 (middle) and 2 (bottom). Radii of spheres represents (in normalized logarithmic scale) the surface area of each structure. Color of the spheres represents the scale to which the structure belongs: dark blue (1), light blue (2), green (3), yellow (4), red (5). (Continued on Figure 4.6) 37

4.6 Visualization space with spheres representing the structures educed from the passive scalar fluctuation field at scale 3 (top), 4 (middle), and 5 (bottom). Radii of spheres represents (in normalized logarithmic scale) the surface area of each structure. Color of the spheres represents the scale to which the structure belongs: green (3), yellow (4), red (5). (Continued from Figure 4.5) 38

4.7 Signatures of representative structures (refer to Figure 4.5 (top) for the location of the corresponding points (A–L) in the visualization space) 39

4.8 Result of the clustering algorithm in a visualization space showing spheres representing structures with radii scaled by the value of their silhouette coefficient (renormalized to have only positive values) and colored by the cluster to which they belong. An optimum number of three clusters was automatically found. The seven clustering parameters used to define the feature space are shown for each structure by the center of its representing sphere (of coordinates $\hat{S}, \hat{C}, \lambda$) and the four bars scaled by the value of d_u, d_l of S and C 40

5.1 Volume pdfs in physical domain (left) and spectra in Fourier domain (right) of $\omega_i\omega_i$ and $S_{ij}S_{ij}$ fields for the three grid resolutions (256^3, 512^3, and 1024^3). Note that the volume pdfs use a transformation of the form $\text{sign}(x)\log(1+|x|)$ in the abscissa coordinate, and that curves for $\omega_i\omega_i$ and $S_{ij}S_{ij}$ fields use two different vertical axes (both in the pdfs and the spectra), shifted one decade for a clear view (non-intersecting curves) 46

5.2 Tri-plane cuts of $\omega_i\omega_i$ and its multi-scale component fields for the 512^3 case 46

5.3 Effect of the multi-scale decomposition in the 1024^3 case for $\omega_i\omega_i$ (top) and $S_{ij}S_{ij}$ (bottom) fields on the volume pdfs in physical domain (left) and on the spectra in Fourier domain (right). Note that the volume pdfs have been shifted vertically to accommodate all scales and the original fields in a clearer view. Also, instead of using a log-scale in the abscissa of the pdf plots, since there are negative values for all filtered scales, a transformation of the form $\text{sign}(x)\log(1+|x|)$ is used for each field x 47

5.4 Plane cuts of $\omega_i\omega_i$ (left) and $S_{ij}S_{ij}$ (right) normal to one of the principal directions of the cubic domain at half its side length for the 1024^3 case 48

5.5 Zoomed parts of plane cuts of $\omega_i\omega_i$ (top) and $S_{ij}S_{ij}$ (bottom) in one of the principal directions of the cubic domain at half its side length for the three grid resolutions 256^3 (left), 512^3 (center), and 1024^3 (right). Greyscale has been renormalized to the zoomed region for better clarity 49

5.6 Zoomed parts of plane cuts of component field at scale number 5 for $\omega_i\omega_i$ (top) and $S_{ij}S_{ij}$ (bottom) in one of the principal directions of the cubic domain at half its side length for the three grid resolutions 256^3 (left), 512^3 (center), and 1024^3 (right). Greyscale has been renormalized to the zoomed region for better clarity 49

5.7 3D views of the visualization spaces, with glyphs (spheres) representing educed structures, colored by scale number (all merged in each visualization space) and scaled by the lognormalized area of the corresponding structure, for $\omega_i\omega_i$ (top row) and $S_{ij}S_{ij}$ (bottom row) at 256^3 (left), 512^3 (center), and 1024^3 (right) grid resolutions 51

5.8 Top views ($(\hat{S}, \hat{C})$-plane) of the visualization spaces, with glyphs (spheres) representing educed structures, colored by scale number and scaled by the lognormalized area of the corresponding structure, for $\omega_i\omega_i$ (top row) and $S_{ij}S_{ij}$ (bottom row) at 256^3 (left), 512^3 (center), and 1024^3 (right) grid resolutions 51

5.9 Breakdown, by scale number (increasing top to bottom), of 3D views of the visualization spaces for $\omega_i\omega_i$ at 256^3 (left), 512^3 (center), and 1024^3 (right) grid resolutions 52

5.10 Breakdown, by scale number (increasing top to bottom), of top views ($(\hat{S}, \hat{C})$-plane) of the visualization spaces for $\omega_i\omega_i$ at 256^3 (left), 512^3 (center), and 1024^3 (right) grid resolutions 53

5.11 Breakdown, by scale number (increasing top to bottom), of 3D views of the visualization spaces for $S_{ij}S_{ij}$ at 256^3 (left), 512^3 (center), and 1024^3 (right) grid resolutions 54

5.12 Breakdown, by scale number (increasing top to bottom), of top views ($(\hat{S}, \hat{C})$-plane) of the visualization spaces for $S_{ij}S_{ij}$ at 256^3 (left), 512^3 (center), and 1024^3 (right) grid resolutions 55

5.13 Clustering results in the visualization space—3D view (left) and lateral (center) and top (right) projections—with glyphs (spheres) representing the optimum clusters of structures educed from the stratified random sample with optimum allocation of the sets of $\omega_i\omega_i$ (top) and $S_{ij}S_{ij}$ (bottom) structures. Glyphs are scaled by the normalized value of the silhouette coefficient, which indicates the degree of membership of that element to the assigned cluster 59

5.14 Optimality scores for different number of clusters obtained during the application of the clustering algorithm to the set of structures of $\omega_i\omega_i$ (left) and $S_{ij}S_{ij}$ (right) independently. Optimum number of clusters (square point) of 3 and 2 were automatically determined for $\omega_i\omega_i$ and $S_{ij}S_{ij}$, respectively 59

6.1 Volume pdfs in physical domain (left) and spectra in Fourier domain (right) of Q and $[A_{ij}]_+$ fields for the 1024^3 grid resolution. Note that the volume pdfs use a transformation of the form $\text{sign}(x)\log(1+|x|)$ in the abscissa coordinate, and that curves for Q and $[A_{ij}]_+$ fields use two different vertical axes (both in the pdfs and the spectra), shifted one decade for a clear view (non-intersecting curves) 64

6.2 Plane cuts of Q (left) and $[A_{ij}]_+$ (right) fields normal one of the principal directions of the cubic domain, at half its side length 65

6.3 Iso-contours of Q and $[A_{ij}]_+$ fields extracted at their mean plus 5 and 4 times their standard deviation, respectively 66

6.4 Visualization space—3D view (left) and lateral (center) and top (right) projections—with glyphs (spheres) representing educed structures of Q (blue) and $[A_{ij}]_+$ (red), scaled by the lognormalized area of each corresponding structure 66

6.5 Clustering results in the visualization space—3D view (left) and lateral (center) and top (right) projections—with glyphs (spheres) representing optimum clusters of structures educed from the set union of Q and $[A_{ij}]_+$ structures. Glyphs are scaled by the normalized silhouette coefficient, which indicates the degree of membership of that element to the assigned cluster 67

6.6 Optimality scores for different number of clusters obtained during the application of the clustering algorithm to the set of structures of Q and $[A_{ij}]_+$ together. An optimum number of clusters of 2 was automatically determined (square point) 68

7.1 Left: Plane cut of $S_{ij}S_{ij}$ (red) superimposed over equivalent plane cut of $\omega_i\omega_i$ (blue). Right: Plane cut of $[A_{ij}]_+$ (red) superimposed over equivalent plane cut of Q (blue) . 71

7.2 Schematic of the computation of the conditional array map (CAM) for a given structure a_i (sheet-like) with respect to two close structures c_1 (tube-like) and c_2 (blob-like). Minimum distance maps from c_j $(j = 1, 2)$ to a_i are shown, with a common color scale for both ranging from red to blue, representing smaller to larger minimum distances to a_i, respectively. The proximity value derived from them is mapped on a_i, with a color scale varying from blue (nil proximity) to red (maximum proximity). Those points of a_i tagged during the computation of the minimum distance maps (with proximity values greater than zero) will store also the values $\{g_j, \xi_j, \zeta_j\}$ of the corresponding c_j in the conditional array map 72

7.3 Components of the jpdf+i in terms of $(\hat{S}, \hat{C})$, plus intensity component based on proximity, of structures of $\mathcal{X}(Q) \cup \mathcal{X}([A_{ij}]_+)$ surrounding structures of $\mathcal{X}(Q)$: area-coverage pdf component (top left) using greyscale; intensity component (top right) using blue-cyan-green-yellow-red color-scale; composition of area-coverage pdf and intensity components to obtain the composite plot (bottom left) with bi-dimensional hue-saturation gradient scale (bottom right) corresponding to proximity (hue) and area coverage (saturation) 77

7.4 Representation of the marginal pdf of $[\mathcal{P}(p, g)|_{\mathcal{A} \leftarrow B}$ in terms of the proximity p (thick black line), showing the contribution of each group g by the different colored areas between two black lines (added cumulatively) 78

7.5 Examples of composite structures formed by interaction of tube-like (blue) and sheet-like (red) geometries found in the database. In cases (a) and (b) a tube is mostly embedded by one or more sheets that intersect it and remain at a close distance. Case (c) consists of three tubes and a sheet that follows closely their geometry, intersecting them, and connects them through stretched regions. Cases (d) and (e) show, each, a sheet-like structure wrapping around a tube, without intersecting it but remaining at a close distance and following its curvature. A smaller sheet intersecting the tube is also seen in case (d). Case (f) shows three tubes at moderate distances from each other ($\approx$ 5–10 times their average radius) with a similar orientation and sheets partially surrounding them at close distance or even intersecting them. For clarity, only a subset of all the nearby structures surrounding each tube in every case is shown 80

7.6 Results for structures of Q surrounding themselves: $\left[\vec{\mathcal{PI}}(\hat{S},\hat{C}\,;p)\right|_{\mathcal{A}\leftarrow\mathcal{B}}$ (left) $[\mathcal{P}(p,g)|_{\mathcal{A}\leftarrow\mathcal{B}}$ (right) for the case $\mathcal{A}=\mathcal{B}=\mathcal{X}(Q)$, $g=1$ 81

7.7 Results for structures of $\omega_i\omega_i$ at scale number 3 surrounded by structures of $\omega_i\omega_i$ (top) and by structures of $S_{ij}S_{ij}$ (bottom), at scale numbers $3-6$: $\left[\vec{\mathcal{PI}}(\hat{S},\hat{C}\,;p)\right|_{\mathcal{A}\leftarrow\mathcal{B}}$ (left) $[\mathcal{P}(p,g)|_{\mathcal{A}\leftarrow\mathcal{B}}$ (right) 82

B.1 Fields (left) and corresponding iso-contours (right) for original (top) and filtered scales (below) 97

B.2 Pdfs (a) and spectra (b) associated with the original and each one of the filtered scales 98

C.1 Transformation from (κ_1,κ_2) to (Υ,Λ) 107

C.2 Range of shape index (Υ), with its most representative associated local shapes (figure based on Koenderink & van Doorn (1992)) 108

D.1 $\Psi(\xi)=\tilde{\Psi}(\xi)$ (right) with $\alpha=2$ for a surface M (left) with the local property ξ distributed in patches of constant ξ. Each point of that function (right) represents the area (two-dimensional Hausdorff measure) of the associated patch. Their discrete sum equals the total area of the surface M 113

D.2 $\Psi(\xi)$ (right) with $\alpha = 1$ for a surface M with ξ smoothly distributed throughout M (except one patch of constant $\xi = \xi_{\max}$). In the left diagram, dashed lines represent line contours of constant $\xi = \xi_1, \xi_2$, which have an associated finite value of $\Psi(\xi)$ (since their Hausdorff dimension equals the dimension of the measure used to obtain $\Psi(\xi)$, $\alpha(M^{=}_{\xi_{1,2}}) = 1$), whereas the central patch (filled with oblique lines pattern) of constant $\xi = \xi_{\max}$ has an associated value $\mathcal{H}^1(M^{=}_{\xi_{\max}}) \to \infty$, since its Hausdorff dimension is $\alpha(M^{=}_{\xi_{\max}}) = 2$ 114

D.3 $\tilde{\Psi}(\xi)$ (right) for a surface M with ξ smoothly distributed throughout M (except one patch of constant $\xi = \xi_{\max}$). In the left diagram, dashed lines represent line contours of constant $\xi = \xi_{\min}, \xi_1, \xi_2, \xi_3, \xi_{\max}$, such that they differ in a constant $\Delta\xi$. The central part of the figure represents $\Psi(\xi)$ (top) and $f(\xi)$ (bottom). $f(\xi)$ increases with ξ since the averaged distance between contours (normal to them along n) increases with ξ. On the right, the multiplication of both gives $\tilde{\Psi}(\xi)$ 115

D.4 Example of non-invertible $\xi(u)$ map 117

F.1 Mean ($\bar{x}$) and feature ($\hat{x}$) centers and upper (d_u) and lower (d_l) distances for a sample asymmetric probability density function, $f(x)$ 123

G.1 Generic structure (a) and limiting cases (b) 124

G.2 $\bar{S}$, $\bar{C}$, and λ as a function of ξ for the tube-like and sheet-like limits, evolving from the sphere limit ($\xi = 0$). Note that the abscissa has been rescaled as $\ln(1 + \xi)$ to show more clearly the transition region 126