Abstract

A multi-scale methodology for the study of the non-local geometry of eddy structures in turbulence is developed. Starting from a given three-dimensional field, this consists of three main steps: extraction, characterization, and classification of structures. The extraction step is done in two stages: first, a multi-scale decomposition based on the curvelet transform is applied to the full three-dimensional field, resulting in a finite set of component fields, one per scale; second, by iso-contouring each component field at one or more iso-contour levels, a set of closed iso-surfaces is obtained that represents the structures at that scale. For periodic domains, those structures intersecting boundaries are reconnected with their continuation in the opposite boundaries. The characterization stage is based on the joint probability density function (jpdf), in terms of area coverage on each individual iso-surface, of two differential-geometry properties—the shape index and curvedness—plus the stretching parameter, a dimensionless global invariant of the surface. Taken together, this defines the geometrical signature of the iso-surface. The classification step is based on the construction of a finite set of parameters, obtained from algebraic functions of moments of the jpdf of each structure, that specify its location as a point in a multi-dimensional 'feature space'. At each scale the set of points in feature space represents all structures at that scale, for the specified iso-contour value. This allows the application, to the set, of clustering techniques that search for groups of structures with a common geometry.

Results are presented of a first application of this technique to a passive scalar field obtained from 512^3 direct numerical simulation of scalar mixing by forced, isotropic turbulence ($Re_\lambda = 265$). These show transition, with decreasing scale, from blob-like structures in the larger scales to blob- and tube-like structures with small or moderate stretching in the inertial range of scales, and then

toward tube and predominantly sheet-like structures with high level of stretching in the dissipation range of scales. Implications of these results for the dynamical behavior of passive scalar stirring and mixing by turbulence are discussed.

We apply the same methodology to the enstrophy and kinetic energy dissipation rate instantaneous fields of a second numerical database of incompressible homogeneous isotropic turbulence decaying in time obtained by DNS in a periodic box. Three different resolutions are considered: 256^3, 512^3, and 1024^3 grid points—with $k_{\max}\bar{\eta}$ approximately 1, 2, and 4, respectively, the same initial conditions and $Re_\lambda \approx 77$. This allows a comparison of the geometry of the structures obtained for different resolutions. For the highest resolution, structures of enstrophy and dissipation evolve in a continuous distribution from blob-like and moderately stretched tube-like shapes at the large scales to highly stretched sheet-like structures at the small scales. The intermediate scales show a predominance of tube-like structures for both fields, much more pronounced for the enstrophy field. The dissipation field shows a tendency toward structures with lower curvedness than those of the enstrophy for intermediate and small scales. The 256^3 grid resolution case ($k_{\max}\bar{\eta} \approx 1$) was unable to detect the predominance of highly stretched sheet-like structures at the smaller scales.

The same methodology, but without the multi-scale decomposition, is then applied to two scalar fields used by existing local criteria for the eduction of tube- and sheet-like structures in turbulence, Q and $[A_{ij}]_+$, respectively, obtained from invariants of the velocity gradient tensor and alike in the 1024^3 case. This adds the non-local geometrical characterization and classification to those local criteria, assessing their validity in educing particular geometries.

Finally we introduce a new methodology for the study of proximity issues among different sets of structures, based also on geometrical and non-local analyses. We apply it to four of the fields previously studied. Tube-like structures of Q are mainly surrounded by sheets of $[A_{ij}]_+$, which appear at close distances. For the enstrophy, tube-like structures at an intermediate scale are primarily surrounded by sheets of smaller scales of the enstrophy and structures of dissipation at the same and smaller scales. A secondary contribution results from tubes of enstrophy at smaller scales appearing at farther distances. Different configurations of composite structures are presented.