Chapter 1

Introduction

1.1 The role of geometry

Observation of natural fluid flows indicates the presence of structures with apparent repeating geometries. Vortical structures in multiphase flows are commonly observed. The roll-up of an ocean wave before it breaks (pressure- and gravity-driven flow), the swirling motion of a hurricane around its center (pressure-driven flow affected by Coriolis and friction forces) or that of the stellar gas accretion disk occurring during the formation of galaxies (gravitational-driven flow), and the Kelvin–Helmholtz wave clouds formed between two layers of air of different density and speeds (shear-driven flow) are just a few examples. Turbulent fluid flows are no exception, often adding levels of complexity to the structure geometry owing to the multiple scales that comprise such flows.

Visualization experiments have provided means for the systematic study of geometrical structures in fluid flows and have substantially increased the number of known flows where repeating geometrical patterns are present. Experimental study of the flow past cylinders and spheres led to the discovery of the Kármán vortex street while experiments in turbulent mixing layers resulted in an exhaustive study of 'coherent' vortical structures (see Brown & Roshko, 1974). This work has stimulated theoretical analysis of pattern formation, for example, the description of eddying motions and flow patterns based on critical-point theory (see Perry & Chong, 1987).

Direct numerical simulations (DNS) have also proven to be a valuable tool in the search for geometrical structures in fluid flows. The organized cylindrical elongated vortices (so-called 'worms') found in the intense vorticity of isotropic turbulence (Siggia, 1981; Kerr, 1985; Jiménez et al., 1993) are one remarkable example. The 'worms', however, remain a puzzle; their contribution to the kinetic energy dissipation is almost negligible and their role in turbulence dynamics remains an open question.

Structures in turbulent flows can be considered a consequence of the forces and boundary conditions driving the flow, but also can be seen as themselves producing some intrinsic properties of the turbulence. In the multi-scale ansatz based on self-similarity and the idea of energy cascade (Richardson, 1922; Kolmogorov, 1941*a*,*b*; Onsager, 1945), the external forces and the boundary conditions affect mainly large energy-containing scales, with diminished influence on progressively smaller eddies. The energy-containing scales then depend strongly on the external forces and boundary conditions and are not expected to be universal, while small-scale structures may be related to universal properties of turbulence, thereby exhibiting a generic geometric signature that may be characteristic of efficient cascade dynamics.

A geometrical characterization of those structures could provide improved understanding of cascade mechanics and dissipation-range dynamics, contributing potentially to the development of structure-based models of turbulence fine scales (see Townsend, 1951; Tennekes, 1968; Lundgren, 1982; Pullin & Saffman, 1993), subgrid-scale models for large-eddy simulation LES (see Misra & Pullin, 1997), and simulation methods based on multi-resolution decomposition by means of the wavelet transform (see Farge, 1992; Meneveau, 1991; Farge et al., 1996, 1999). Further, a better understanding of eddy structure at large Reynolds number may provide important insight into possibly singular or near-singular structures in the dynamics of the Euler equations (see Hou & Li, 2006) by elucidating the geometrical characterization of sites within the turbulent field where extreme dissipative or vortical events occur, and which are candidates for singularity formation in the limit of vanishing viscosity.

1.2 Previous identification criteria

Prior work on the identification of structures in turbulence addresses mainly the identification of vortex tube- and sheet-like structures with emphasis on vortex tubes. But the importance of sheet-like structures, where significant turbulent kinetic dissipation may be concentrated owing to their high amplitude of strain rate, and which may produce tubes by roll-up instabilities, has led to renewed interest in sheets. Most identification methods either for tubes, sheets, or both, are based on local measures of scalar fields obtained from the velocity-gradient tensor and/or the pressure field. They rely on physical aspects associated with a particular kind of structure either of turbulent flows or of simpler solutions of the Navier-Stokes equations (e.g., Burgers vortex tubes and sheets), whose phenomenology is extrapolated to turbulence.

Chong et al. (1990) classified regions with complex eigenvalues of the velocity-gradient tensor as vortex tubes (since the local streamlines are then closed or spiral in a reference frame moving with the fluid). The second-order invariant, Q, of the velocity-gradient tensor was used by Hunt et al. (1988), to define a vortex tube as the region with a positive value of Q, and the condition of a pressure lower than the ambient, while Ashurst et al. (1987) based their identification criterion on the sign of the intermediate eigenvalue of the strain-rate tensor, S_{ij}. Tanaka & Kida (1993) extended the identification criterion based on Q for the extraction of both tubes and sheets. Jeong & Hussain (1995) proposed a method based on the second largest eigenvalue, λ_2, of the tensor L_{ij} formed by summing the products of the symmetric, S_{ij}, and antisymmetric, Ω_{ij}, parts of the velocity-gradient tensor with themselves, $L_{ij} = S_{ik}S_{kj} + \Omega_{ik}\Omega_{kj}$. They define a vortex core as the region where λ_2 is negative. Horiuti (2001) combined this methodology with the physical explanations of the alignment of vorticity and the eigenvector associated with the intermediate eigenvalue of S_{ij} (Andreotti, 1997) to develop a new method in which the eigenvalues and eigenvectors of L_{ij} are reordered based on their alignment with the vorticity; then, regions are classified into vortex tubes, and so-called flat vortex sheets and curved vortex sheets depending on the relations of those reordered eigenvalues. Horiuti & Takagi (2005) proposed an improved method for the eduction of vortex sheet structures, based on local values of the largest eigenvalue of the tensor $A_{ij} = S_{ik}\Omega_{kj} + S_{jk}\Omega_{ki}$, once the eigenvalue corresponding to the eigenvector maximally aligned with the vorticity is removed.

Based solely on the pressure field, Miura & Kida (1997) developed a methodology for extracting axes of tubular vortices as the loci of sectionally local minima of the pressure field (obtained by means of the sign of the second largest eigenvalue of the pressure Hessian evaluated at each point; positive values indicate pressure minima).

The majority of existing methods of identification are local, based on pointwise quantities used to discriminate whether each point belongs to one type of structure or another (or none). Regions of points sharing a common identity based on the local criterion applied can then be formed, but often that local analysis is the end of the identification process. Visualization of such regions has proved a helpful tool in its analysis, but here we seek a more automated, systematic approach to structure characterization.

Some non-local methods exist in the fluid mechanics literature. These classify structures considering their spatial extent and can handle a broader range of geometries. While local methods are often based on a priori physical knowledge of the particular geometry to be educed, non-local methods generally draw physical conclusions a posteriori, based on geometrical characteristics obtained from the educed structures. For example, an extended structural and fractal description of turbulence was proposed by Moisy & Jiménez (2004), who applied a box-counting method to sets of points of intense vorticity and strain-rate magnitude (educed by thresholding). They also analyzed geometrically individual structures, defined as a connected set of points satisfying the threshold criterion (thus, considering the spatial extent of such structures), based on their volume and spatial distribution, finding that intense vorticity and dissipation structures are concentrated in clusters of inertial size. Wang & Peters (2006) defined extended dissipation elements as the ensemble of grid cells from which the same pair of extremal points of the scalar field can be reached, and studied their characteristic linear distances.

1.3 Non-local, multi-scale, and clustering features

Our approach is based on a non-local, multi-scale methodology for the extraction, characterization, and classification of structures in turbulence from a geometrical perspective. It is non-local, focusing on the spatial extent of structures. The multi-scale analysis is performed through the curvelet transform, a higher-dimensional generalization of the wavelet transform. Presently, the structures are defined as iso-surfaces extracted, at different scales, from a three-dimensional scalar field obtained from a turbulent flow. The characterization and classification steps are based on measures of the geometry of iso-surfaces. The problem of shape analysis of free-form surfaces has been widely studied in the fields of computer graphics, computer vision and image understanding, (see Campbell & Flynn, 2001; Iyer et al., 2005; Dorai & Jain, 1997; Osada et al., 2001; Zaharia & Prêteux, 2001). Our method characterizes each individual structure in terms of local differential-geometry properties. Structure identification in terms of non-local characterization is done via area-based probability density functions of those geometrical properties. Classification is based on this geometrical characterization of individual structures and is enhanced via clustering techniques. Clustering algorithms allow the eduction of groups of structures without the need for strong a priori assumptions about their properties.

1.4 Choice of applications: passive scalar, enstrophy, and dissipation fields

Presently we apply this methodology, first, to a passive scalar advected and diffused in statistically stationary homogeneous isotropic turbulence with a mean scalar gradient imposed. Second, we study the structures of the enstrophy and dissipation fields obtained from homogeneous isotropic turbulence decaying in time. In all cases the flow is incompressible. The databases under analysis were obtained by DNS.

The dynamics of the passive scalar, c, are governed by the linear advection-diffusion equation:

$$\frac{\partial}{\partial t}c(\boldsymbol{x},t) + u_j(\boldsymbol{x},t)\frac{\partial}{\partial x_j}c(\boldsymbol{x},t) = D\frac{\partial^2}{\partial x_j \partial x_j}c(\boldsymbol{x},t), \tag{1.1}$$

where $\{u_j, j = 1,2,3\}$ are the components of the velocity field, $\boldsymbol{u}$, $\boldsymbol{x}$ is the position vector ($\{x_j, j = 1,2,3\}$ are the spatial coordinates), t is the time variable, and D is the diffusivity. In the presence of a uniform mean scalar gradient of magnitude μ_c in the x_1 direction—which will be preserved by the flow (see Corrsin, 1952)—the passive scalar can be split into its mean component, $\mu_c x_1$, and the passive scalar fluctuation, $c'(\boldsymbol{x},t)$. Thus $c(\boldsymbol{x},t) = \mu_c x_1 + c'(\boldsymbol{x},t)$ and the passive scalar fluctuation is then governed by:

$$\frac{\partial}{\partial t}c'(\boldsymbol{x},t) + u_j(\boldsymbol{x},t)\frac{\partial}{\partial x_j}c'(\boldsymbol{x},t) = D\frac{\partial^2}{\partial x_j \partial x_j}c'(\boldsymbol{x},t) - \mu_c u_1(\boldsymbol{x},t). \tag{1.2}$$

The mean scalar gradient acts as a source term for the scalar fluctuation, and a statistically stationary state can be reached (see Overholt & Pope, 1996). Passive scalars are of paramount importance in turbulent mixing and combustion and a vast effort has been dedicated to their study (see Warhaft, 2000, and the references therein). We choose it as a first case of application of our methodology for being a scalar field itself, governed by a relatively simple equation, before moving to other scalar fields derived from the velocity gradient tensor, with more complicated dynamics, such as the enstrophy and dissipation.

The analysis of the enstrophy and kinetic energy dissipation fields has been recurrent in the study turbulence through experiments (e.g., Zeff et al., 2003), numerical simulations (e.g., Ishihara et al., 2003), and theoretical developments (e.g., Pullin & Saffman, 1997; He et al., 1998; Wu et al., 1999). They are obtained, up to scaling factors, from the double contraction of the rotation- and strain-rate tensor fields. Physically, enstrophy and dissipation correspond to the remaining Galilean-invariant degrees of freedom of fluid particles, rotation and strain, once the dilatation is restricted for incompressible flows. This separation is useful but it does not decouple the equations of fluid motion. On the contrary, both fields appear highly coupled in the equations describing the dynamics of each other (see Appendix A). In fact, the interaction between strain and rotation is intrinsic to the very nature of three-dimensional turbulence; in particular, vortex-stretching occurs when the strain-rate field stretches and amplifies vorticity. A study and comparison of the geometry of structures of both fields, at different scales, might be valuable in our understanding of turbulence. For that reason, we choose them as the second case of application of our methodology.

1.5 Grid resolution effects

Because of its multi-scale nature, a complete study of turbulence requires, both in experiments and in numerical simulations, spatial resolution that resolves the flow up to dissipation scales. A traditional grid resolution criterion used in DNS of homogeneous turbulence in a periodic box, for example, consists in resolving the flow up to scales of the order of the (average) Kolmogorov length scale. But in addition to being multi-scale, turbulence also shows intermittency (Batchelor & Townsend, 1949; Landau & Lifshitz, 1959; Kolmogorov, 1962): fluctuations of flow quantities can reach extreme amplitudes in short intervals of time and spatial distances. Furthermore, fluctuations of different amplitudes tend to cluster. Intermittency increases for higher Reynolds numbers (Okamoto et al., 2007) and also for smaller scales (Brasseur & Wang, 1992).

This suggests that the traditional grid resolution criterion, based on an average dissipation scale, might be inappropriate, since much smaller scales are locally present due to those high fluctuations. Therefore, the resolution required to resolve all scales of turbulent flows increases significantly (see Sreenivasan, 2004). This condition may become even more restrictive when studying the geometry of structures in turbulence, and is explored during the application of our methodology to the enstrophy and dissipation fields by means of databases corresponding to multiple numerical simulations performed at different resolutions but otherwise identical.

1.6 Structure interaction

The dynamics of sheets and tubes are greatly affected by their own interactions. Common examples are the coalescence and reconnection of approaching vortex tubes and the roll-up of vortex sheets to form vortex tubes resulting from the Kelvin-Helmholtz instability. These interactions among sheets and tubes can be seen as the translation of the strain-rotation interaction itself to the structural level of turbulence, and help explain the presence of intermittency and the process of multi-scale energy cascade in turbulence (see Kraichnan, 1974).

An interesting example of the geometrical relations between rotation- and strain-rate fields is the local alignment of the vorticity with the intermediate strain-rate eigendirection, for incompressible homogeneous isotropic turbulence. It was observed first in numerical simulations (Ashurst et al., 1987) and confirmed experimentally (see Tsinober et al., 1992; Tao et al., 2000). Theoretical explanations combine local and non-local arguments (see Jiménez, 1992; Nomura & Post, 1998; Hamlington et al., 2008). But this prevailing alignment between vorticity and the intermediate eigendirection of the strain-rate tensor is observed to switch towards the direction associated with the most negative[1] (compressional) eigenvalue of the strain-rate tensor at the ends of tube-like structures (Nomura & Post, 1998), which is consistent with the compressive straining of the vorticity occurring in those regions.

Other geometrical analyses regarding the proximity of different types of structures, in relation to their shapes, could be useful in further explaining those interactions and also improve structure-based models of the fine scales of turbulence. For that purpose, we have developed a methodology for the study of such proximity issues, from a geometrical viewpoint, among structures of different fields and scales. It takes advantage of many of the features of the methodology for the study of the geometry of structures in turbulence also introduced here.

[1]For incompressible flow the trace of the strain-rate tensor is null, ensuring at least one positive and one negative eigenvalue of that tensor.

1.7 Outline

This thesis is organized as follows: Chapter 2 describes the three main steps of the methodology for the non-local multi-scale study of the geometry of structures—extraction, characterization, and classification—with emphasis on the conceptual basis and on some particular implementation details. In Chapter 3, we present a system test that validates the methodology applied to a virtual world of modeled structures. Chapter 4 shows results of its first application to extended passive scalar structures educed from a DNS database of incompressible homogeneous isotropic turbulence stationary in time. We apply, in Chapter 5, the same methodology to structures of the enstrophy and dissipation fields, comparing the results of both fields, from another DNS database of incompressible homogeneous isotropic turbulence decaying in time. This database includes three different grid resolutions, allowing us to study how this parameter affects the geometry of educed structures and the validity of the traditional grid resolution criterion in DNS from a geometrical standpoint. In Chapter 6, we combine our non-local methodology with two local criteria of identification of vortex tubes and sheets in turbulence (Horiuti & Takagi, 2005) that are based on scalar fields obtained from the velocity gradient tensor. An assessment of the geometries expected from those local criteria is done. Chapter 7 introduces a new methodology for the study of the proximity of multiple sets of structures, also in terms of geometry and based on non-local measures through area-coverage quantification. We apply this methodology to the pairs of two scalar fields used by the local identification criteria in Chapter 6 and also to the enstrophy and dissipation fields, considering the multi-scale decomposition performed in Chapter 5. Chapter 8 summarizes the conclusions of this work and comments on its possible future directions. The contents of Chapters 2, 3, and 4, along with the corresponding conclusions included in Chapter 8, will appear in Bermejo-Moreno & Pullin (2008).

We emphasize that the tools developed here—the multiresolution analysis, geometric characterization, spectral projection, clustering algorithms, and proximity analysis—can be applied to many scalar and tensor fields in turbulence, and in fields beyond fluid dynamics.