Appendix F

Definition of feature center and upper and lower distances of a probability density function.

Consider a real-valued random variable X with probability density function $f(x)$, $x \in \mathbb{R}$. We define the *feature center* $\hat{x}$ as

$$\hat{x} \equiv \begin{cases} \bar{x} - d_l\sqrt{1-(d_l/d_u)^2} & \text{if } d_l < d_u \\ \bar{x} + d_u\sqrt{1-(d_u/d_l)^2} & \text{if } d_l > d_u \end{cases} \tag{F.1}$$

where $\bar{x}$ is the *mean* or *expected value* of X, $\bar{x} \equiv \int x f \mathrm{d}x$. The *lower* and *upper distances* are defined by

$$d_l \equiv \sqrt{\frac{\int_{x\leq\bar{x}} (\bar{x}-x)^2 f \mathrm{d}x}{\int_{x\leq\bar{x}} f \mathrm{d}x}}, \qquad d_u \equiv \sqrt{\frac{\int_{x\geq\bar{x}} (\bar{x}-x)^2 f \mathrm{d}x}{\int_{x\geq\bar{x}} f \mathrm{d}x}}. \tag{F.2}$$

The feature center can be interpreted as a correction to the mean that accounts for the asymmetry (skewness) of the density function $f(x)$ with respect to its mean, defining a new point closer to the region of higher density. When the probability density function $f(x)$ is symmetric, the feature center and mean coincide ($\hat{x} = \bar{x}$). The upper and lower distances, d_u and d_l, can be regarded as the r.m.s. of the part of the pdf above and below its mean value, respectively. A graphical example is shown in Figure F.1, for a probability density function $f(x) = x^2 \exp\left(-\sqrt{x}\right) / \int_0^\infty \xi^2 \exp\left(-\sqrt{\xi}\right) \mathrm{d}\xi$ that shows a long tail in one direction. The mean, $\bar{x}$, feature center, $\hat{x}$, and lower and upper distances, d_l

and d_u, are superimposed on the probability density function. These definitions can be immediately extended to higher-dimensional probability density functions.

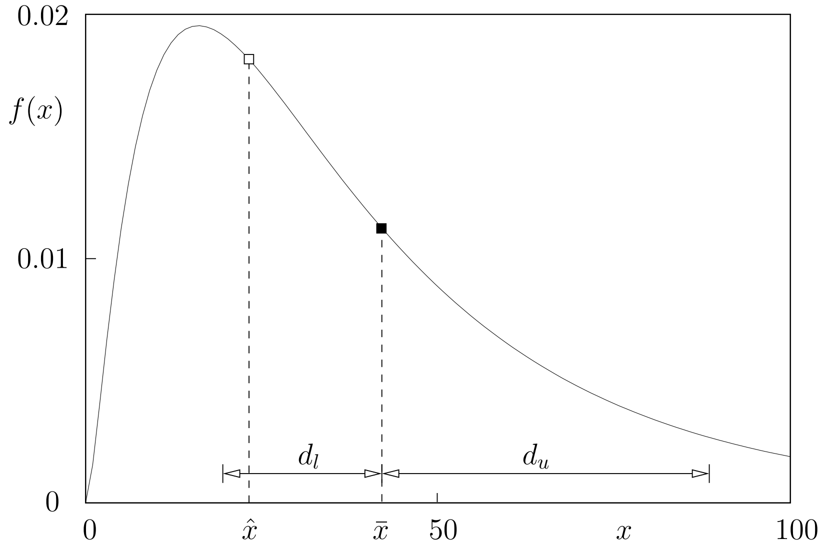

Figure F.1: Mean ($\bar{x}$) and feature ($\hat{x}$) centers and upper (d_u) and lower (d_l) distances for a sample asymmetric probability density function, $f(x)$