Appendix B

Interpretation of extracted structures

Here we give an interpretation of the physical meaning of the educed structures at different scales resulting from the extraction step of the present methodology. For simplicity and clarity, a two-dimensional scalar field is used. A 1024×1024 greyscale image (with values in the range [0–255]) (see top left of Figure B.1) obtained from a particular realization of a Julia set has been chosen as the scalar field. This set is of interest since it contains self-similar structures at different scales. The outer region has been faded to white so that all boundaries have the same value.

The extraction procedure described in the main text is applied to the two-dimensional image. The result of the multi-scale decomposition provided by the curvelet transform can be seen on the left images of Figure B.1. The effect in Fourier space is shown by the spectra on Figure B.2(b), while Figure B.2(a) shows the bin pdfs of the original and filtered fields, in physical space. Note that, in this case, the low-pass filter used for the coarsest scale is (in logarithmic scale) wider than the others. It can be thought of as two scales merged into one (the coarsest scale, in this case), and could be done also for other groups of scales.

Each filtered field (image) corresponding to each scale is then iso-contoured at a value equal to its mean plus 3/2 times its standard deviation: see right plots on Figure B.1. The original field has also been iso-contoured (top right) for comparison.

From the way in which the decomposition is done, as observed in the spectra, the structures educed for each filtered scale have a correspondence with the different ‘energetic’ bands of the

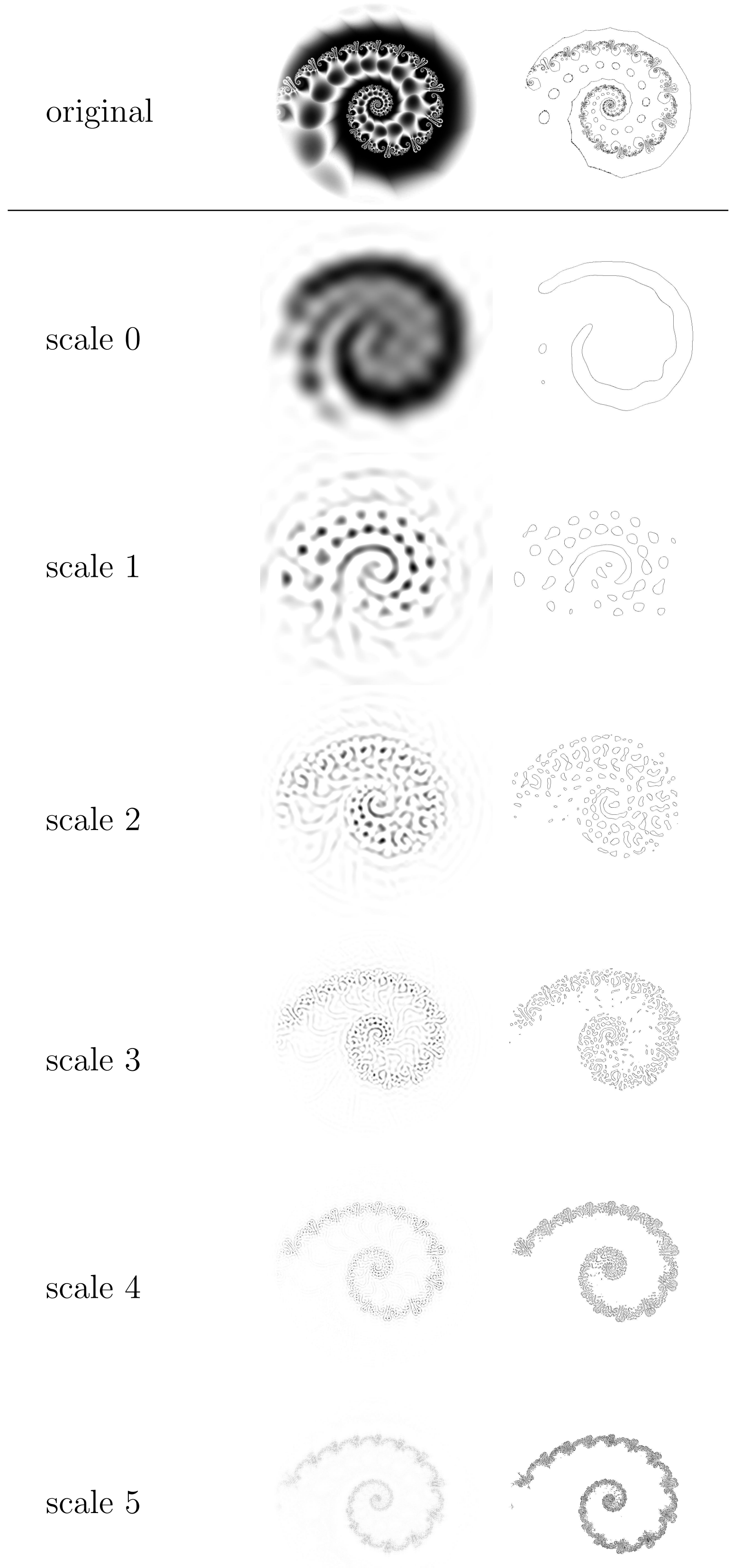

Figure B.1: Fields (left) and corresponding iso-contours (right) for original (top) and filtered scales (below)

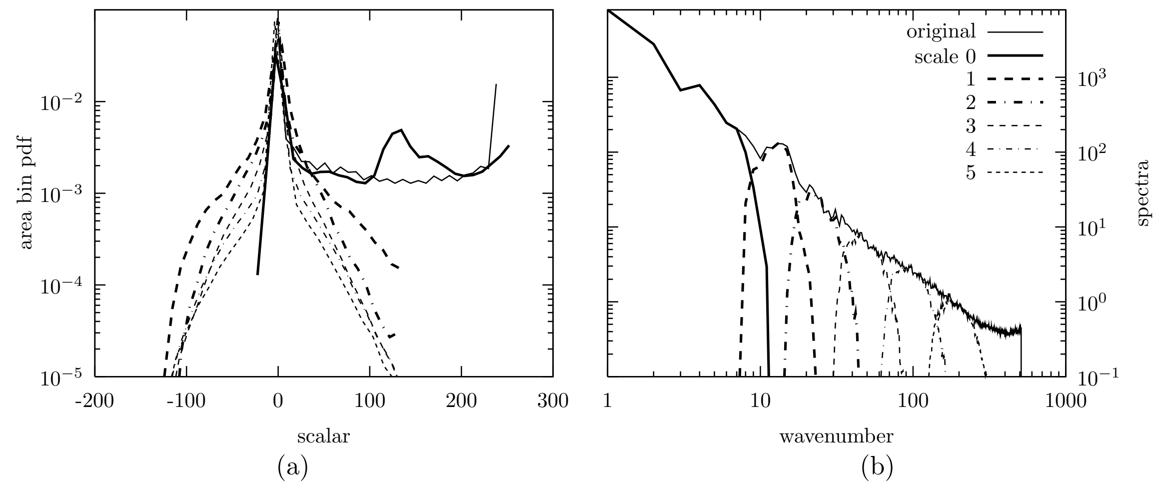

Figure B.2: Pdfs (a) and spectra (b) associated with the original and each one of the filtered scales

field, as defined by frequency corona in Fourier space. But furthermore, they also have a direct correspondence in physical space with the structures of the original field (features of the image). First, we notice that the spatial localization of the features (structures) educed for each scale is retained, with respect to the original image. As expected, they vary in relative sizes (scales), from one filtered scale to the next. Some features of the original image that span across different scales are split as a result of the decomposition. See, for example, the dark continuous arm of the spiral: scale 0 captures its largest portion, but the remainder can be seen also in the rest of the scales. The geometry of each part resembles that of the structure from which it was derived. Shape is preserved and thus a geometrical analysis of the educed structures is meaningful in this context. The iso-contour obtained from the original field (top right of Figure B.1) contains a large individual structure, rich in features, and a few simpler structures, but is missing many other features of the original image. In contrast, contours of the filtered fields tend to contain many more (simpler) structures that capture the essential features of the original field at that scale. The fact that the spatial localization is kept can be used for the study of relative positioning, clustering, and other organizational aspects of the sets of structures.

The extension of this reasoning to three dimensions is immediate. The complexity of the structures that can be found increases. For example, structures that appear as circular in two dimensions could become either blob- or tube-like, while elongated structures in two dimensions could become either tube- or sheet-like structures. We note that an alternative to the multi-scale decomposition of the scalar field applied here is to perform a multi-resolution analysis applied to the iso-contours extracted from the original database. Since there is a loss of information by iso-contouring, we choose to perform the multi-scale decomposition first over the entire field and then iso-contour each one of the filtered scales.

As an analogy, consider the decomposition of a tree into its trunk, branches, leaves, etc. The outer surface of the tree, containing all those elements, would correspond to the iso-surface of the original field. It is generally too rich and complex to study as a whole. By applying a multi-scale decomposition before iso-contouring, we can separate the tree into its individual components, ranged by the scale. Then, iso-contouring extracts structures at those different levels, whose properties can be studied individually. This is the philosophy applied in our methodology for the study of structures in turbulence. In the same manner that the geometry of the elements of a tree has a relation to their physical functionality, perhaps that is also the case for those structures present in turbulent flows. A multi-scale decomposition followed by surface identification (by iso-contouring based on global contour values or other means) seems an appropriate framework for this study. Its current form can be considered a starting point, but there is much room for refinement: for example, use of additional multi-resolution capabilities (as outlined in the body of the thesis), such as multi-orientation decomposition, and selection of locally adapted contour levels for optimal feature extraction are two possible paths for improvement.