On the non-local geometry of turbulence

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To my family

and to the memory of Pawel Buraczewski

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Abstract

A multi-scale methodology for the study of the non-local geometry of eddy structures in turbulence is developed. Starting from a given three-dimensional field, this consists of three main steps: extraction, characterization, and classification of structures. The extraction step is done in two stages: first, a multi-scale decomposition based on the curvelet transform is applied to the full three-dimensional field, resulting in a finite set of component fields, one per scale; second, by iso-contouring each component field at one or more iso-contour levels, a set of closed iso-surfaces is obtained that represents the structures at that scale. For periodic domains, those structures intersecting boundaries are reconnected with their continuation in the opposite boundaries. The characterization stage is based on the joint probability density function (jpdf), in terms of area coverage on each individual iso-surface, of two differential-geometry properties—the shape index and curvedness—plus the stretching parameter, a dimensionless global invariant of the surface. Taken together, this defines the geometrical signature of the iso-surface. The classification step is based on the construction of a finite set of parameters, obtained from algebraic functions of moments of the jpdf of each structure, that specify its location as a point in a multi-dimensional 'feature space'. At each scale the set of points in feature space represents all structures at that scale, for the specified iso-contour value. This allows the application, to the set, of clustering techniques that search for groups of structures with a common geometry.

Results are presented of a first application of this technique to a passive scalar field obtained from 512^3 direct numerical simulation of scalar mixing by forced, isotropic turbulence ($Re_\lambda = 265$). These show transition, with decreasing scale, from blob-like structures in the larger scales to blob- and tube-like structures with small or moderate stretching in the inertial range of scales, and then

toward tube and predominantly sheet-like structures with high level of stretching in the dissipation range of scales. Implications of these results for the dynamical behavior of passive scalar stirring and mixing by turbulence are discussed.

We apply the same methodology to the enstrophy and kinetic energy dissipation rate instantaneous fields of a second numerical database of incompressible homogeneous isotropic turbulence decaying in time obtained by DNS in a periodic box. Three different resolutions are considered: 256^3, 512^3, and 1024^3 grid points—with $k_{\max}\bar{\eta}$ approximately 1, 2, and 4, respectively, the same initial conditions and $Re_\lambda \approx 77$. This allows a comparison of the geometry of the structures obtained for different resolutions. For the highest resolution, structures of enstrophy and dissipation evolve in a continuous distribution from blob-like and moderately stretched tube-like shapes at the large scales to highly stretched sheet-like structures at the small scales. The intermediate scales show a predominance of tube-like structures for both fields, much more pronounced for the enstrophy field. The dissipation field shows a tendency toward structures with lower curvedness than those of the enstrophy for intermediate and small scales. The 256^3 grid resolution case ($k_{\max}\bar{\eta} \approx 1$) was unable to detect the predominance of highly stretched sheet-like structures at the smaller scales.

The same methodology, but without the multi-scale decomposition, is then applied to two scalar fields used by existing local criteria for the eduction of tube- and sheet-like structures in turbulence, Q and $[A_{ij}]_+$, respectively, obtained from invariants of the velocity gradient tensor and alike in the 1024^3 case. This adds the non-local geometrical characterization and classification to those local criteria, assessing their validity in educing particular geometries.

Finally we introduce a new methodology for the study of proximity issues among different sets of structures, based also on geometrical and non-local analyses. We apply it to four of the fields previously studied. Tube-like structures of Q are mainly surrounded by sheets of $[A_{ij}]_+$, which appear at close distances. For the enstrophy, tube-like structures at an intermediate scale are primarily surrounded by sheets of smaller scales of the enstrophy and structures of dissipation at the same and smaller scales. A secondary contribution results from tubes of enstrophy at smaller scales appearing at farther distances. Different configurations of composite structures are presented.

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Nomenclature

Greek letters

α_ℓ, β_ℓ	Angles defining the center slope of the frequency wedge (curvelets)	12
γ_k	Array of parameters stored in the conditional array map (CAM)	71
ϵ	Local dissipation	45
ϵ_{ijk}	Levi-Civita symbol	45
$\bar{\eta}$	Average Kolmogorov length scale	30
ϑ	Eigenvalues of A_{ij}	63
κ_1	Maximum curvature	15
κ_2	Minimum curvature	15
κ_n	Normal curvature	16
Λ	Curvedness	15
λ	Stretching parameter	19
μ	Characteristic length scale of a closed surface	18
μ_c	Mean passive scalar gradient magnitude in x_1 direction	6
ν	Kinematic viscosity	44
ξ, ζ	Local properties mapped onto a surface	72
ρ	Density	63
ϱ	Radial polar coordinate in the plane of principal curvatures	16
σ_l	Local scaling parameter	22
Υ	Shape index	15

ϕ	Azimuthal polar coordinate in the plane of principal curvatures	16
$\varphi^D_{j,\ell,k}$	Curvelets in physical space	11
$\hat{\varphi}^D_{j,\ell,k}$	Curvelets in Fourier space	11
Ω_{ij}	Rotation-rate tensor	45
ω	Wavenumber (Fourier domain)	11
ω_i	Vorticity field ($i = 1, 2, 3$)	45
$\omega_i\omega_i$	Local enstrophy	45

Roman letters

$\hat{\boldsymbol{A}}$	Locally scaled affinity matrix	22
A	Area of a surface	18
$\boldsymbol{a}$	Vector contained on the tangent plane at a point P of a surface	16
a_i	Average distance from an element to other elements of its same cluster (in the clustering algorithm)	26
a_i	Element i of the set $\mathcal{A}$ (in the structure interaction analysis)	71
A_{ij}	Symmetric second-order tensor $S_{ik}\Omega_{kj} + S_{jk}\Omega_{ki}$	3
$[A_{ij}]_+$	Largest remaining eigenvalue of A_{ij} after removing $[A_{ij}]_\omega$	63
$[A_{ij}]_-$	Smallest remaining eigenvalue of A_{ij} after removing $[A_{ij}]_\omega$	63
$[A_{ij}]_\omega$	Eigenvalue of A_{ij} associated with the eigenvector most aligned with the vorticity field, ω_i	63
b_i	Average distance from an element to the elements in the closest cluster	26
C	Dimensionless curvedness	18
$\hat{C}$	Feature dimensionless curvedness center	24
c	Passive scalar	6
c'	Passive scalar fluctuation	6
c_j	Element j of the set $\mathcal{C}$ (in the structure interaction analysis)	71

$c^D(j,l,k)$	Curvelet coefficients	11
$\boldsymbol{D}$	Diagonalizing matrix	22
D	Diffusivity	6
d	Distance	72
D_{a_i}	Non-dimensionalizing length scale of the structure a_i	72
d_{ij}	Distance matrix	21
d_l	Lower distance of a probability density function	24
d_u	Upper distance of a probability density function	24
E	Set of elements, e_i, to cluster, $i = 1, ..., N$	21
$E(k)$	Energy spectrum of original field (containing all scales)	33
$E_i(k)$	Energy spectrum associated with component field at scale number i	33
F	Distance function in the space of parameters	22
f_k	Cumulative marginal probability density function of proximity for group indices g from 1 to k	78
g	Group index	72
G_g	Groups contained in set $\mathcal{B}$	72
j	Scale number (curvelets)	11
j_0	Minimum scale number (curvelets)	11
j_e	Maximum scale number (curvelets)	11
k	Spatial location index, $\{k_i, i = 1, 2, 3\}$ (curvelets)	11
$k_{\max}$	Largest dynamically significant wavenumber	30
$\boldsymbol{L}$	Normalized locally scaled affinity matrix	22
ℓ	Orientation index (curvelets)	11
L_i, L'_i	Integral length scales of component field at scale number i	33
$\boldsymbol{N}$	Normal vector to the tangent plane at a point P of a surface	16
N	Number of elements to cluster	21
n	Grid size of side cubic domain	11

N_g	Number of G_g groups contained in set $\mathcal{B}$	72
N_P	Number of parameters defining the feature space	21
p	Pressure field (in the equations of fluid mechanics)	63
p	Proximity (in the structure interaction analysis)	72
$\mathcal{P}$	Area-based joint probability density function	19
$\mathcal{P}_\mathcal{C}$	Area-based (marginal) probability density function of C	19
$\mathcal{P}_\mathcal{S}$	Area-based (marginal) probability density function of S	19
$\vec{\mathcal{P}\mathcal{I}}(\xi,\zeta;p)$	Area-based joint probability density function in terms of the local properties (ξ,ζ) with averaged intensity component in terms of the local property p	73
$p[k]$	Parameters of feature space	21
R	Radius of curvature	16
r	Number of closest neighbors for local scaling (in the clustering algorithm)	22
Re	Reynolds number	42
Re_λ	Taylor Reynolds number	30
S	Absolute value of the shape index	16
$\hat{S}$	Feature absolute value of the shape index center	24
SC	Silhouette coefficient	26
Sc	Schmidt number	30
S_{ij}	Strain-rate tensor	45
$S_{ij}S_{ij}$	Local dissipation renormalized by $(2\,\nu)^{-1}$	45
t	Characteristic thickness of a sheet-like structure	76
t	Time variable (in the equations of fluid mechanics)	6
$\boldsymbol{u}$	Velocity vector field (with components $u_j, j=1,2,3$)	6
$\overline{u^2}$	Characteristic squared integral velocity of original field	33

$\overline{u_i^2}$	Characteristic squared integral velocity of component field at scale number i	33
$\tilde{U}_{j,\ell}$	Frequency window (curvelets)	11
V	Volume inside a closed surface	18
$\tilde{V}_{j,\ell}$	Angular frequency window (curvelets)	11
$\tilde{W}_j$	Radial frequency window (curvelets)	11
$\boldsymbol{X}$	Matrix of eigenvectors	22
$\boldsymbol{x}$	Position vector (spatial coordinates $x_j, j = 1, 2, 3$)	6
$\mathcal{X}(\alpha)$	Set of extracted structures from a three-dimensional scalar field α	75
$\boldsymbol{Y}$	Renormalized matrix of eigenvectors	23

Acronyms

BIC	Bayesian information criterion	26
CAM	Conditional array map	71
DNS	Direct numerical simulation(s)	1
HSB	Hue-saturation-brilliance color space	76
jpdf	Joint probability density function	19
jpdf+i	Joint probability density function with intensity component	73
LES	Large eddy simulation(s)	2
MDM	Minimum distance map	71
pdf	Probability density function	13