THE MEASUREMENT OF ALTITUDE AND INCLINATION OF AIRCRAFT BY THE ECHO METHOD

Thesis by

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SUMMARY

During the past twenty years the Echo Method of measuring distances has been successfully employed by surface ships both for charting and navigating. The present investigation was undertaken in order to determine the advantages and limitations of the parallel application of measuring the altitude of modern aircraft.

Although similar in principle these two applications involve different problems. Some of these arise from the different media in which the sounds are transmitted, some from the vastly different velocities involved and others from the different disturbing influences in the two cases. The investigation has been divided into four parts.

The first section of the paper is devoted to a theoretical examination of the sound waves transmitted from and received on a moving ship. Expressions are derived for the Doppler Effect as a function of the inclination of the reflecting surface and the ratio of ship velocity to that of sound.

The second section is devoted to the corrections introduced by the speed of the ship, its direction relative to the reflecting surface and the temperature. It is shown that, for practical purposes, these corrections can be made from observations in the ship.

A third section is devoted to the measurement of the sound spectra of typical ships in flight and to the attenuation of sound in the atmosphere.

A fourth section describes an experimental altimeter together with the results of field tests in the Goodyear Airship "Volunteer". Altitudes as great as 700 feet and as low as 4 feet were measured. Subsequent work indicates that in order to obtain a maximum range of 1000 feet under unfavorable atmospheric conditions a source radiating 500 watts of acoustical energy is required.

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The Measurement of Altitude and Inclination of Aircraft by the Echo Method*

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THE time of transit of a sound pulse from a moving ship to a reflecting surface and back to the ship has been used by several investigators to measure altitude.[1] The first part of the present paper calls attention to the use of the Doppler effect to measure also the angle of inclination of the ship relative to the reflecting surface. The second part deals with the corrections introduced into the measurement of altitude by the velocity of the ship, its inclination with the reflecting surface and the temperature. The third considers some of the practical difficulties encountered in the application of the above theory, in particular with the measurement of sound spectra of typical aircraft in flight and the selection of a source of sound that can be filtered through the ship's noise upon returning as an echo from a reflecting surface. The fourth reports one way of overcoming these difficulties and gives the details of an experimental altimeter, together with the results of some preliminary field tests.

THE DOPPLER EFFECT APPLIED TO THE MEASUREMENT OF THE INCLINATION OF AIRCRAFT

The time of transit of a sound pulse from a moving ship to a reflecting surface and back as an echo may be used to measure the distance to the reflecting surface. A comparison of the pitch of the echo and that of the emitted sound taken together with the air speed of the ship makes it possible to calculate also the angle of inclination of the ship with the reflecting surface. The complete treatment of the problem of sound emitted on a moving ship and received again after reflection from an absorptive surface, presents many theoretical difficulties. An approximate solution of the problem may, however, be obtained by assuming specular reflection and neglecting the ship's size.

Take as an example a ship moving through still air with a velocity v and making an angle α with the reflecting surface. Fig. 1 shows the geometry of this situation. If the velocity of sound in air is C we may write for the ratio

$$C/v = p. \tag{1}$$

If a short pulse of sound of frequency n_s is transmitted from the ship at a particular instant and received at the ship one second later we may

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[1] Behm, Sci. Am. **136**, 418–419 (1927).

C. W. Rice, Aeronautical Engineering **4**, (2), 61–77.

P. Léglise L'eronautique, No. 169, 128–133 (June, 1933); No. 170, 160–170 (July, 1933).

write for the distance the plane travels $d=v$ and for the total distance the sound travels $s=C$. The component of the velocity of the ship in the direction AO would be

$$v_1=v\cos\theta. \tag{2}$$

The wave-length of the sound proceeding in this direction would be given by

$$\lambda_1=(C-v_1)/n_s=(C-v\cos\theta)/n_s. \tag{3}$$

The frequency received by an observer at O would then be

$$n_0=C/\lambda_1=n_sC/(C-v\cos\theta). \tag{4}$$

Upon arrival at the point B the ship would be moving relative to the point O with a velocity component given by

$$v_2=-v\cos\phi. \tag{5}$$

The number of waves received per second would be given by

$$n_R=n_0(C+v_2)/C=n_0(C-v\cos\phi)/C. \tag{6}$$

The net result of these two effects is given by

$$n_R=n_s(p-\cos\phi)/(p-\cos\theta). \tag{7}$$

We may now obtain n_R in terms of p and α by

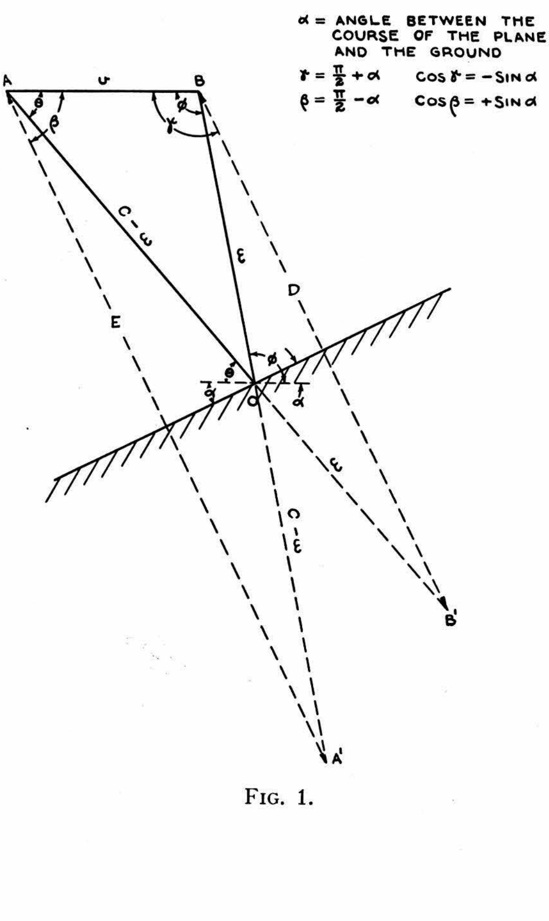

FIG. 1.

considering the geometry of a typical reflection.

From Fig. 1 we obtain, by applying the cosine law twice,

$$C^2=D^2+v^2-2Dv\cos\gamma, \tag{8}$$

$$D^2=C^2+v^2-2Cv\cos\theta. \tag{9}$$

Eliminating D between Eqs. (8) and (9), and substituting $\cos\gamma=-\sin\alpha$ gives

$$v^2-2Cv\cos\theta+v^2 \\ +2v\sin\alpha(C^2+v^2-2Cv\cos\theta)^{\frac{1}{2}}=0.$$

Dividing by C^2 and substituting $p=C/v$ results in

$$\cos^2\theta-(2/p)(1-\sin^2\alpha)\cos\theta \\ +(1/p^2)(1-\sin^2\alpha)-\sin^2\alpha=0. \tag{10}$$

The solution of Eq. (10) for $\cos\theta$ is then

$$\cos\theta=(\cos^2\alpha\pm\sin\alpha(p^2-\cos^2\alpha)^{\frac{1}{2}})/p. \tag{11}$$

Similarly

$$\cos\phi=(\cos^2\alpha\pm\sin\alpha(p^2-\cos^2\alpha)^{\frac{1}{2}})/p. \tag{12}$$

If now we substitute Eqs. (11) and (12) in (7) we obtain

$$n_R=n_s\frac{p^2-\cos^2\alpha+\sin\alpha(p^2-\cos^2\alpha)^{\frac{1}{2}}}{p^2-\cos^2\alpha-\sin\alpha(p^2-\cos^2\alpha)^{\frac{1}{2}}}. \tag{13}$$

The $+$ sign in the numerator preceding $\sin\alpha$ and the $-$ sign in the denominator preceding $\sin\alpha$ are used since for $+$ values of α, $n_R>n_s$.

If now we set $n=n_R/n_s$ we may reduce Eq. (13) to the polar equation

$$n^2(2n(p^2-\cos 2\alpha))/(p^2-1)+1=0. \tag{14}$$

Eq. (14) agrees well with the following polar equation for a circle for large values of p, namely

$$n^2-4np\sin\alpha/(p^2-1)-1=0.$$

A polar graph for this equation for $p=7.5$, $v=100$ mi./hr. is shown in Fig. 2. The center is given by $a=2p/(p^2-1)$ and the radius is given by $R=(p^2+1)/(p^2-1)$. Plotting $n^{\frac{1}{2}}=\beta$ instead of n gives[2]

$$\beta^4-2\beta^2(p^2-1+2\sin^2\alpha)/(p^2-1)+1=0,$$

which reduces to

$$\beta^2-\frac{2\beta\sin\alpha}{(p^2-1)^{\frac{1}{2}}}-1=0.$$

[2] This procedure was suggested by W. M. Whyburn of the Department of Mathematics at the University of California at Los Angeles.

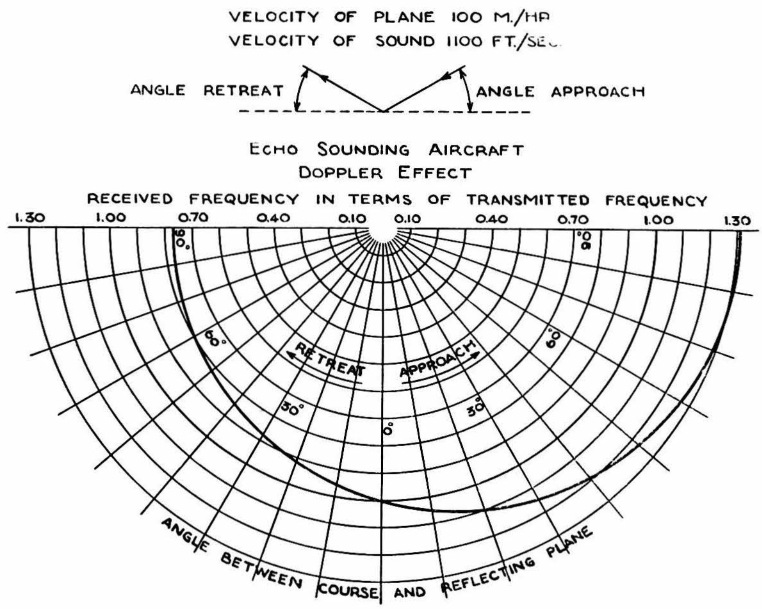

FIG. 2.

The polar equation for a circle with its center a distance a from the origin and having a radius r is

$$\beta^2-2\beta a \sin\alpha = r^2-a^2.$$

Since in this case for $\alpha=0$, $n=1$, $r^2-a^2=1$ so we may write

$$\beta^2-2\beta a\sin\alpha-1=0.$$

We now identify the shift of the center as

$$a=1/(p^2-1)^{\frac{1}{2}}$$

and the radius as

$$r^2-1/(p^2-1)=1 \quad \text{or} \quad r=p/(p^2-1)^{\frac{1}{2}}.$$

This is true for any value of p and provides a rapid graphical method of obtaining the change in pitch in terms of p and α.

An experimental check on this effect was obtained in the Goodyear Airship Volunteer. The source of sound was provided by a standard horn loudspeaker driven through a two-stage power amplifier from an oscillator. The echo was picked up with a dynamic microphone and amplified by a three-stage amplifier. A switching device made it possible to record on an oscillograph a sample of the outgoing sound and after a short time interval a sample of the echo. An electrically driven fork in the oscillograph traced a 50~ time line on the film. The ratio of the received to the transmitted frequencies was found by counting the number of waves due to the echo in a given time interval and comparing this number with that obtained from the source in an equal time interval. Two sets of observations were made, one when the ship was being driven 55 mi./hr. downward at an angle of 10° into a slight wind and second when the ship was headed upward 10° into the same wind at the same speed. The ship was slightly light. The experimental and theoretical values obtained are given in Table I.

TABLE I.

α	n (Calc.)	n (Exp.)	
+10°	1.025	1.028	Av. 1.025
+10°		1.025	
+10°		1.022	
−10°	0.976	0.963	Av. 0.965
−10°		.968	
−10°		.965	

The good agreement on approaching the ground seems to indicate that the top wind compensated for the slight excess buoyancy and that the ship was actually travelling a downward course of 10°. The lack of agreement on receding from the ground can be similarly explained. In this case the ship was headed up at an angle of 10° but because of the buoyant effect and the bottom wind, was actually climbing at a somewhat

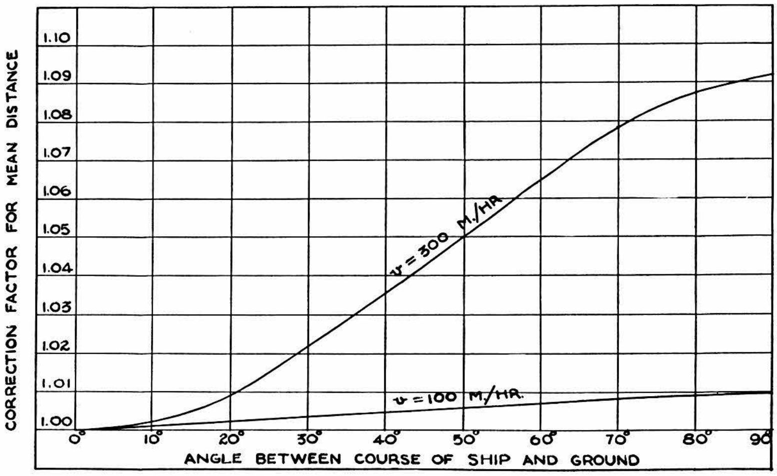

FIG. 3.

steeper angle. If we assume the theory to be correct the difference would be accounted for if the ship headed upward at an angle of 14°. This seems entirely reasonable. Although the experiments were confined to angles of 10° the results show that the effect can be used to indicate to a pilot the attitude of the ship relative to the ground.

ALTITUDE CORRECTION FOR THE SPEED OF A SHIP, THE ANGLE IT MAKES WITH A REFLECTING SURFACE, AND THE TEMPERATURE

The distance of the ship from a reflecting surface may be obtained from Fig. 1 where it was shown that

$$D^2 = C^2 + v^2 - 2Cv \cos\theta \tag{9}$$

and

$$\cos\theta = (\cos^2\alpha \pm \sin\alpha(p^2 - \cos^2\alpha)^{\frac{1}{2}})/p. \tag{11}$$

Eliminating $\cos\theta$ between these two equations gives

$$D = \pm v\{p^2 + 1 - 2\cos^2\alpha \pm 2\sin\alpha(p^2 - \cos^2\alpha)^{\frac{1}{2}}\}^{\frac{1}{2}}.$$

If now the meter is calibrated to read correctly when $\alpha = 0$, and calling this value of D, D_0 we have $D_0 = \pm v(p^2 - 1)^{\frac{1}{2}}$. We may now calculate a correction factor for D_0 to obtain D so that $D_0 K = D$, giving

$$K = \left\{ \frac{p^2 + 1 - 2\cos^2\alpha \pm 2\sin\alpha(p^2 - \cos^2\alpha)^{\frac{1}{2}}}{p^2 - 1} \right\}^{\frac{1}{2}}. \tag{15}$$

The value of K with the $-$ sign in (15) gives the correction for the meter reading in order to obtain D; with the $+$ sign it gives the correction factor in order to obtain C. The mean distance to the reflecting surface during the transit of the sound pulse is $(D+C)/2$. The average of these two values of K then gives the correction factor by which to multiply the meter reading in order to obtain the mean distance during the transit of a sound pulse.

This correction factor has been plotted for air speeds of 100 and 300 miles an hour in Fig. 3. From these considerations we see that for even the highest cruising speed used at present, an acoustic altimeter will read very nearly the mean distance a ship is from the reflecting surface during the transit of a sound pulse, although for large values of v and α the distance when the pulse is transmitted or received may be quite different.

The correction for air speed and temperature for some constant value of α, say $\alpha = 0$, can be obtained from Eq. (15), since both of these factors affect $p = C/v$. The net correction for the air speed and temperature has been plotted in Fig. 4. It is assumed that the altimeter reads correctly at $\alpha = 0$, $v = 0$ (air speed $= 0$) and temperature $= 20°$C. In practice a meter would be calibrated at the cruising speed of the ship. The corrections for variable air speeds and temperature above and below the calibrated values are, in general, small but can be made easily by changing the speed of the timing device.

It must be pointed out, in concluding this section, that for a given course of a ship, the Doppler effect determines only the angle the course makes with the reflecting surface and the time of transit of a sound pulse determines only the distance to that surface. The surface may be tangent anywhere around an ellipsoid of revolution having a major axis along the ship's course equal to the distance the sound pulse travels, and an eccentricity ϵ given by $\epsilon = v/C$.

A change in the course of a ship, however, should show, upon turning away from the reflecting surface, a decrease in the pitch of the echo and an increase in the distance; or, upon turning toward the reflecting surface, the reverse, an increase in the pitch of the echo and a decrease in the distance. Thus a decrease in the altimeter reading and a rise in pitch of the echo both serve as warnings of approaching danger.

Practical Difficulties in the Application of the Theory

It is at once apparent that there are many practical difficulties in the application of the foregoing theory. These may be classified as follows: (*a*) the detection of a relatively weak echo in the presence of the noise of the ship itself; (*b*) the absorption of sound by the air (including the effect of turbulence); (*c*) the absorption of sound at the reflecting surface. At the beginning of the experimental work, it was obvious that in order to be able to detect a weak echo through the background of a ship's noise, it would be necessary to investigate in considerable detail the sound emitted by typical aircraft in flight. In order to accomplish this purpose, it became necessary to design a portable analyzer that could be used under these conditions. This instrument operates essentially as follows.

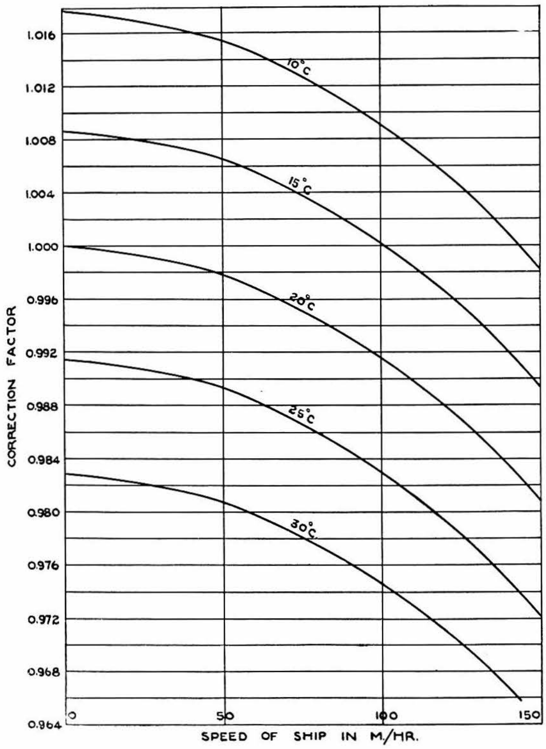

Fig. 4.

The complex sound produced by a ship is converted into potential fluctuations by a microphone and high quality amplifier. The output of the amplifier is applied to a modified quadrant electrometer, the needle of which forms part of a sharply resonant mechanical system. The natural period of this system is made continuously variable by a tri-filar suspension. A micrometer makes it possible to tune the mechanical system to each of the components of the complex sound. The amplitude of the components is observed on a transparent scale by the spreading of the image of a straight filament lamp, while the frequencies of the components are obtained from the micrometer readings. Further mechanical and electrical details may be obtained from the original paper.[3] The intensity of the sound components was obtained by a subsequent calibration in the laboratory. The method used to accomplish this is a modification of a method due to Gerlach[4] in which the variable pressure developed by the sound wave is opposed by a variable electrodynamic force of the same frequency and opposite phase. The details of the apparatus are shown in Fig. 5. The sound wave falls on a light circular metal disk, *D* in Fig. 5, which is attached to the coil of a dynamic loudspeaker and suspended in the face of a heavy guard ring by three fine steel wires. Attached to the back of the disk is a carbon microphone unit. This button is connected

[3] L. P. Delsasso, *A New Acoustic Analyzer*, J. Acous. Soc. Am. **3**, 167–178 (1931).

[4] Wiss. Veröfft. d. Siemens-Konzern, p. 139 (1923).

through an amplifier to a detector and serves to indicate the adjustment at which the disk is motionless. In practice, the analyzer itself is used as a vibration electrometer to indicate zero motion of the disk. The sound that is to be measured is impressed on one side of the disk, while current of the same frequency as the sound is sent through the coil attached to the disk. The amplitude of the current through the coil can be varied by the resistance R_3 and the phase of the current, relative to that of the sound, can be varied by the combination of capacitance and resistance C and R_2. In order to measure the intensity of a sound these two factors are adjusted until the disk stands motionless under the joint action of the force due to the sound and the force due to the current. The constant of the instrument was obtained by passing direct currents through the coil and measuring the mechanical force required to restore the disk to its equilibrium position. If this value of force is f dynes, the corresponding current I_0, in amperes and A the area of the disk, the constant of the instrument is given by $k=f/AI_0$ dynes/cm^2 amp. The effective sound pressure under the conditions of balance is then given by $P=kI$, where I is the effective alternating current in amperes required to produce the balance. The device thus permits the measurement of sound intensity by using the relation $J=P^2/\rho C$, where ρ is the density of the medium and C the velocity of sound.

Observations were made in typical aircraft while in flight with the analyzer. The instrument was suspended by long rubber shock cords from a convenient support in the cabin of the ship and the microphone similarly suspended in the sound field to be analyzed. The amplitude of vibration of the beam of light and the micrometer reading for observed frequency components were recorded. In most cases it was found necessary to repeat the observation many times in order to obtain a good average since the speed of a ship's motor is rarely absolutely constant.

Upon returning to the laboratory, the analyzer-microphone was sealed in a hole of the guard plate as shown in Fig. 5 and the loudspeaker cone clamped over it. The frequencies of the sound components were next obtained from the frequency calibration curve of the analyzer.[3] The intensity of each component was obtained in two ways. First, current of the frequency of a particular component was sent through the loudspeaker and adjusted until the amplitude of vibration of the analyzer was the same as that found in the ship. Since other factors remained constant the intensity of the sound striking the microphone was assumed equal to the intensity of that component of the ship's sound. Without altering the input to the loudspeaker it was then transferred to the second position on the guard plate shown in Fig. 5 and the sound intensity measured as previously described by the null method. A second, although not an absolute method of measuring this intensity, was also

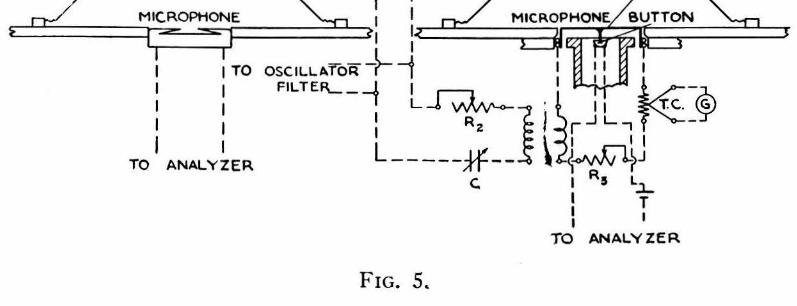

FIG. 5.

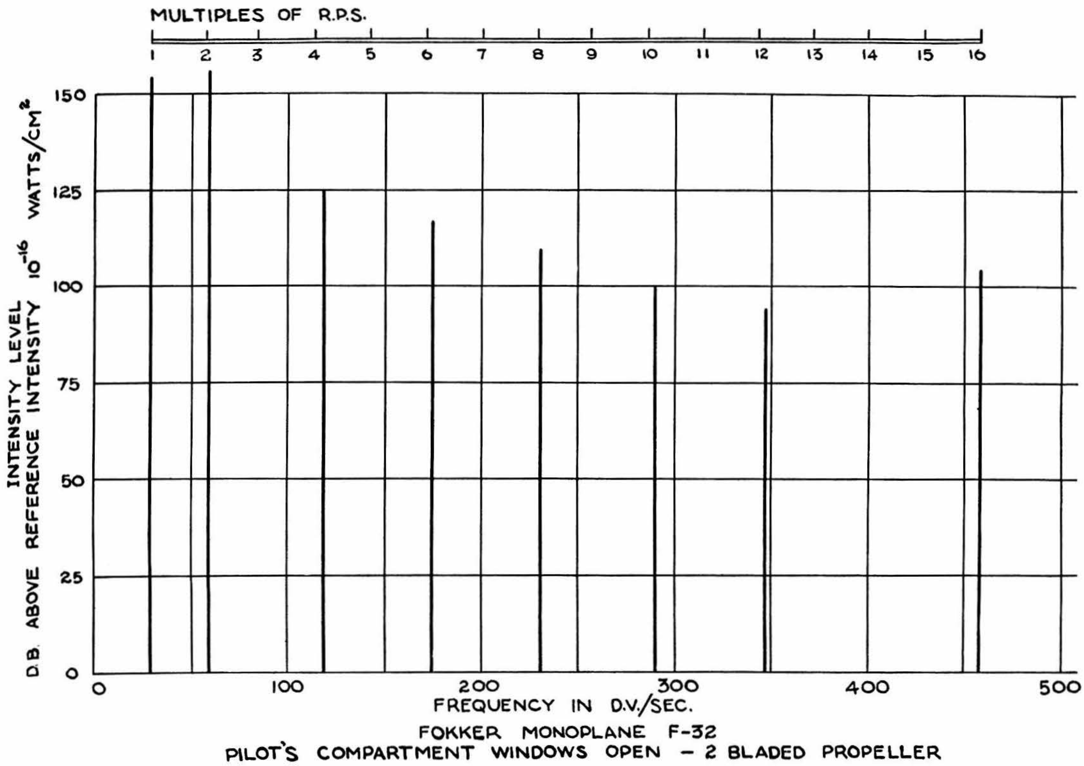

FIG. 6.

used. The microphone in the first position was removed and the ear of an observer substituted in its place, care being taken to obtain a tight seal about the ear. The current through the loudspeaker was then reduced to the threshold of hearing. A linear relation between the current through the loudspeaker and the sound pressure was assumed and the intensity of the sound component above the threshold of hearing at that particular frequency was calculated in the usual manner from the expression

$$\text{Intensity difference in db} = 20 \log_{10} i/i_0,$$

where i is the original current through the speaker and i_0 the current that produced the threshold intensity. By the use of Fletcher's threshold data the intensity level above the reference intensity (10^{-16} watt/cm^2) was obtained. A typical sound spectrum is shown in Fig. 6. The most intense component is seen to have a frequency just equal to the number of blades of the propeller passing a particular point per second. In addition, many harmonics are observed. With one exception these may be classified by the following expression $n_s = lNS$, where l is the number of blades in the propeller, N is the speed of the propeller in revolutions per second, S is the order of the harmonic (one for the fundamental, etc.).

In addition to the above harmonics, a component having a frequency just equal to N is always observed when measurements are made in the ship itself. This is believed to be due to the vibration of the ship as a whole because of motor and propeller unbalance. With this one exception, the frequency of the components observed in a ship in flight agrees with observations made by Obata and Yoshida[5] on the ground of sound produced by planes passing overhead, after correction for the Doppler effect had been made. The intensity distribution is naturally different. Fig. 6 may be taken as representative of the sound spectra of ships in flight. The intensity of the components is seen to decrease for the higher frequencies in the range observed. Although a complete analysis has not been made above 500 cycles a second, other experiments indicate that with the exception of high pitched

[5] Report of the Aeronautical Research Institute, Tokyo Imperial University, Vol. 6, No. 59 (March, 1930).

sounds produced by vibrating wires or the aeolian sounds from small structural parts, the energy continues to decrease as we go to higher frequencies. It is at once apparent, from the results of the above survey, that the selection of a suitable pitch for the source of an acoustic altimeter must be restricted to frequencies above the intense component of the engine and propeller noise, if a relatively weak echo is to be filtered out.

Another factor of great importance in the selection of a suitable pitch, is the attenuation of the sound as it passes through the atmosphere. The experimental work of Knudsen[6] has shown the absorption of sound in the audible range to be several times that predicted by the classical theory involving only the viscosity and heat conduction. The absorption coefficient increases rapidly with the frequency. This effect by itself would indicate that the frequency should be chosen as low as possible in order to reduce to a minimum the medium absorption. In addition to the above effects, the intensity of the echo is greatly influenced by the turbulence of the atmosphere. Little experimental work has as yet been done on this effect but such data as are available indicate it to be of considerable magnitude.

An attempt was made on the campus of the University to measure the variation of absorption with weather conditions. A loudspeaker with an exponential horn was located in one window on the third floor of a building facing the flat brick wall of another 350 feet away. A parabolic horn and an electrodynamic microphone were located in an adjacent window. The same oscillator and amplifier arrangement as was used in the Doppler effect experiment made it possible to send out a short pulse of sound and record the echo on an oscillograph. Observations were made only late at night or early in the morning when the air was quite still. The intensity of the outgoing sound was maintained constant and records taken at frequencies ranging from 100 cycles per second to 4000 cycles per second. It was at once apparent, after a very few readings had been taken, that turbulence, even in relatively still air, plays an important part in the scattering of sound. One echo was found to differ from another by as much as 50 percent even when they were taken only a few seconds apart. The wind velocity during these preliminary experiments was well below half a mile an hour. The average results of observations taken at different relative humidities agree reasonably well with those of Knudsen in the range 20 to 100 percent relative humidity. The absorption decreased as the humidity increased. A series of readings taken in a very heavy rain showed the absorption to be no greater than for 20 percent relative humidity for any frequency.

A second series of readings was taken with the loudspeaker horn located at one corner of the building on the roof. The parabolic horn was mounted on top of an automobile and arranged so that it could be directed at the horn on the roof. The recording equipment was placed inside the automobile. Observations were made only on the maximum intensity of the sound over a period of time, for a fixed intensity of the source and for distances ranging from 100 meters to 370 meters. A typical set of data follows. For a temperature of $23\frac{1}{2}$°C and 18 percent relative humidity the value of m is, from Knudsen's data, 0.00015 cm^{-1}. Using this value of m and assuming a parallel beam of sound, we can calculate the ratio of the sound pressure at different distances. For two distances, 109 meters and 214 meters, this pressure ratio turns out to be 2.197. The pressure ratio actually observed by experiment was 2.297. Again, for distances of 109 meters and 377 meters, the calculated pressure ratio was 7.482 and the experimentally observed ratio, 8.882. A large part of the difference between the theory and the experiments is undoubtedly due to the fact that the sound in the experiments was not confined to a parallel beam; but a part of the discrepancy is also due to turbulence in the atmosphere, as evidenced by the large variations in the received intensity.

The results of these preliminary experiments seem to show that an altimeter designed to work near the maximum absorption range, 15 to 20 percent relative humidity for ordinary frequencies, should prove entirely reliable in heavy fog and even rain. The experiments are to be repeated under controlled conditions in a large,

[6] V. O. Knudsen, *The Absorption of Sound in Air, in Oxygen, and in Nitrogen*, J. Acous. Soc. Am. **5**, 112–122 (1933).

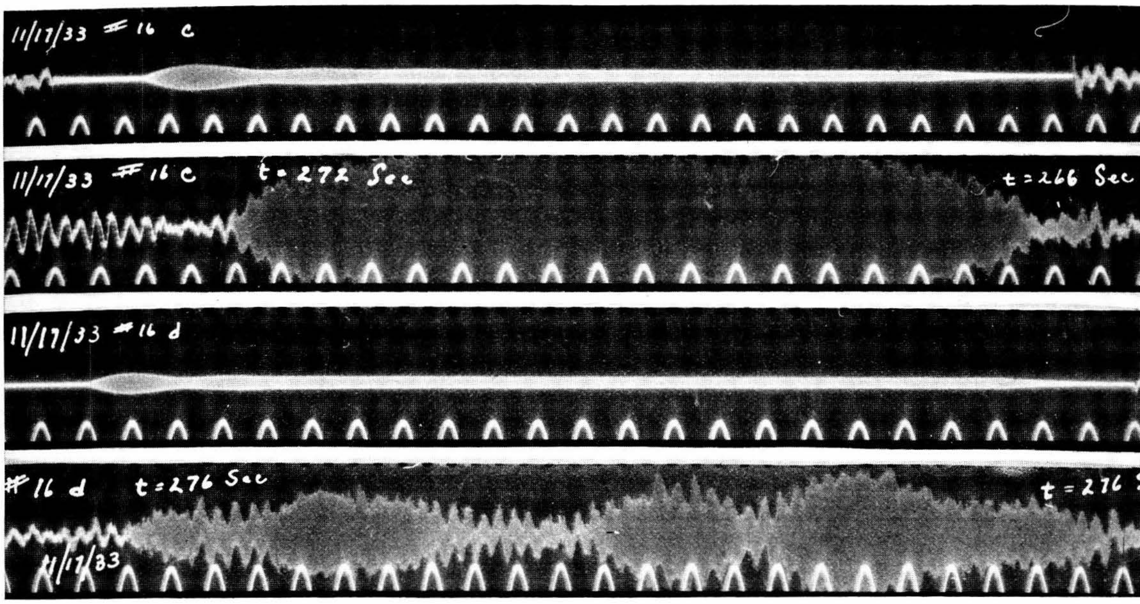

FIG. 7.

well-padded room where the effect of introducing artificial turbulence can be studied.

The intensity of an echo will be further influenced by the absorption coefficient of the reflecting surface and by its shape. A typical example of the variation of the intensity of an echo is shown in Fig. 7. The first record is a sample of a 2000 cycle sound pulse sent out from an airship standing practically still at an altitude of 1530 feet, over plowed ground. The second is a record of an echo from this pulse 2.72 seconds later. The third is a record of a second pulse sent out only a few seconds after the first echo was received, and the fourth is the echo from this second pulse at an altitude of 1550 feet. The change in the pattern of the second echo indicates the wide variation in the character of echoes taken under practically identical conditions. The final selection of a suitable frequency to use in acoustic altimeters will require many such observations over a wide variety of reflective surfaces, at various altitudes, and under different weather conditions. Some work of this nature has been reported by Eisner and Krüger.[7] They used a whistle of 2900 cycles per second, attached to one side of the gondola of an airship and picked up the echo by a tuned microphone located at the other side. Taking the reflection coefficient of water as unity they obtained for thin ice 1.07 and for meadow land 0.49. In order to obtain reflection coefficients from observations made at various altitudes, the correction for absorption in the medium must be introduced. Such observations as have been made up to the present time indicate that a suitable frequency for the source of an echo altimeter should lie between 2000 and 3000 cycles per second. The final selection of the most suitable pitch must await the completion of many more experiments similar to those described above.

THE DESIGN OF AN EXPERIMENTAL ALTIMETER

From the data presented it is apparent that the main problem to be considered in the design of an acoustic altimeter is the one of selecting a source of sound of a pitch that can be successfully filtered through the background of ship's noise as an echo. Since the attenuation of sound increases rapidly with frequency the most efficient frequency would appear to be that which is just high enough to be filtered through the particular ship's noise. This will, of course, vary with the selectivity of the device and the sound spectra of

[7] Eisner and Krüger, Hochfrequenztechn. Electroakustik **24**, 64–67 (1933).

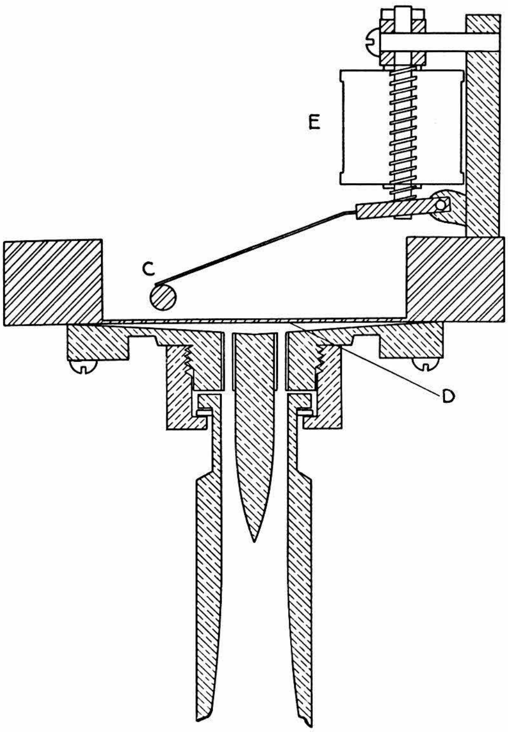

FIG. 8.

the ship. From these considerations a frequency of 2000 cycles per second was chosen for use in an experimental altimeter.

In order to measure low altitudes a sound pulse of very short duration is required. The details of a source of sound of moderate intensity and short duration are shown in Fig. 8. *D* is a steel diaphragm cut from a solid steel ring and having a fundamental frequency of 2000 cycles per second. A clapper *C* is arranged to strike the diaphragm a sharp blow whenever current to the electromagnet *E* is broken. An efficient transfer of the energy of the diaphragm to the air is accomplished by the coupling used in the Bostwick high frequency loudspeaker. An oscillograph of the sound pulse produced by this source is shown in the upper half of Fig. 9. The duration is approximately 0.02 second.

The filtering of the sound pulse through the ship's noise is accomplished in the microphone itself. The microphone is a modification of Bragg's amplitude meter. The details are shown in Fig. 10. *D* is a 0.001-inch duralumin diaphragm, tuned to the pitch of the source (2000 cycles per second) and has attached to its center a small platinum button. Pressing against this button is a second platinum contact attached to a light platinum spring *S*. The spring has a natural frequency of 95 cycles per second in this case. A small drying tube *R* serves to keep the contacts dry. For sounds of even great intensity and low frequency the spring follows the diaphragm, while a weak sound of 2000 cycles per second causes the contact to break once each vibration of the diaphragm. The filtering action is brought out more fully in the following: If the amplitude

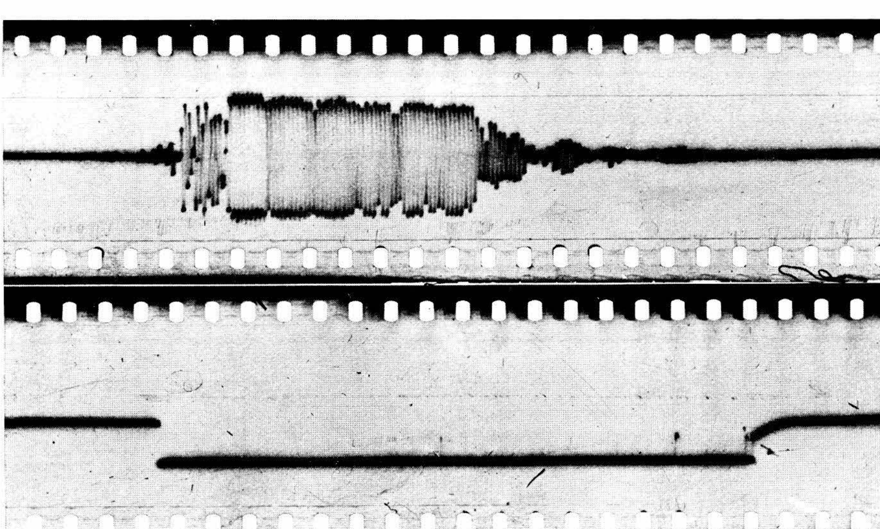

FIG. 9.

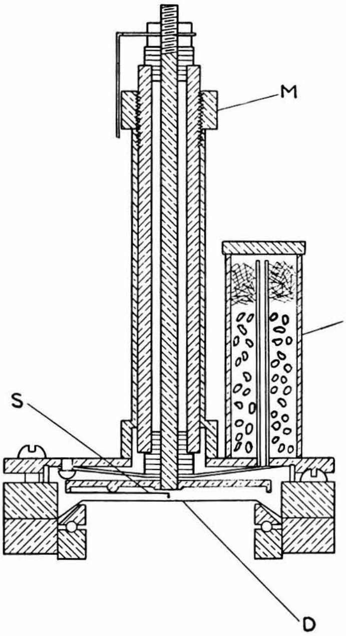

FIG. 10.

of vibration of the diaphragm is r_D and the frequency at which the diaphragm is vibrating f_D, the maximum acceleration for sinusoidal motion is given by $a_D=4\pi^2 r_D f_D^2$. If the spring is given an initial displacement r_S then at the extreme upward displacement of the diaphragm the acceleration of the spring will be given by $a_S=4\pi^2(r_S+r_D)f_S^2$. Breaking of the contact will occur when a_D slightly exceeds a_S, or we may set for this critical condition $(r_S+r_D)f_S^2=r_D f_D^2$, which, if we solve for r_D, we obtain $r_D=r_S f_S^2/(f_D^2-f_S^2)$. This expression indicates that for a source of sound equal to the natural frequency of the spring f_S an infinite amplitude would be required to break the contact. In practice we can, of course, only approach this condition.

If we take as a measure of the selectivity of the microphone the ratio of the amplitude required to break the contact at a frequency f_D to that of the resonant frequency, 2000 cycles per second, we may write

$$s=r_D/r_{D_{2000}}=(\overline{2000}^2-f_S^2)/(f_D^2-f_S^2).$$

If this ratio is expressed in db above the resonant amplitude we obtain the curve shown in Fig. 11. Three curves have been calculated for three natural frequencies of the spring: $f_S=20$, 50, and 95 cycles per second. This simple theory does not

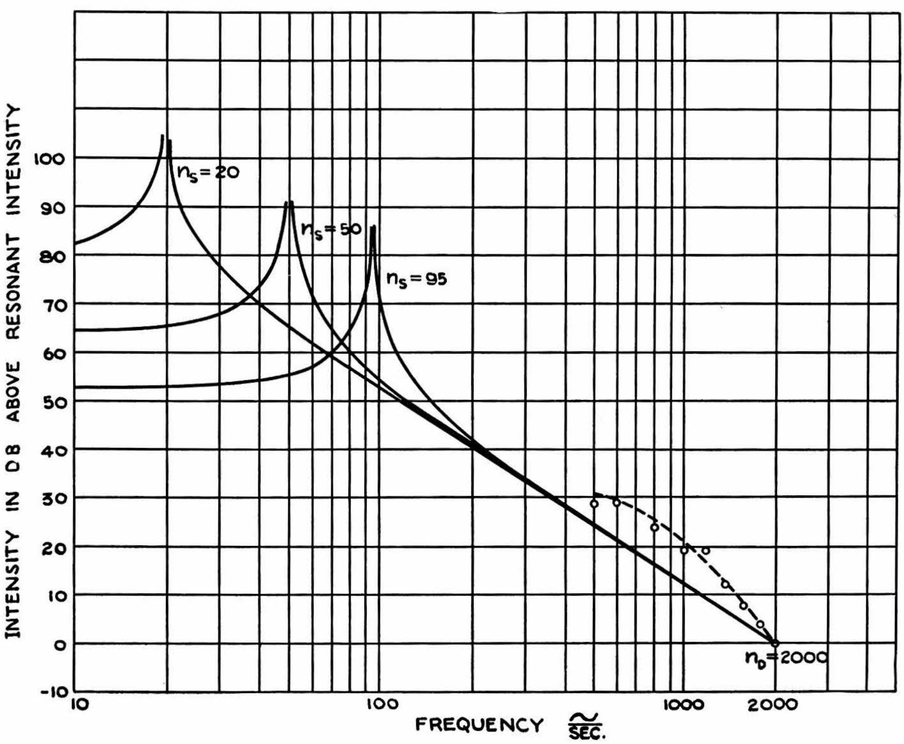

FIG. 11.

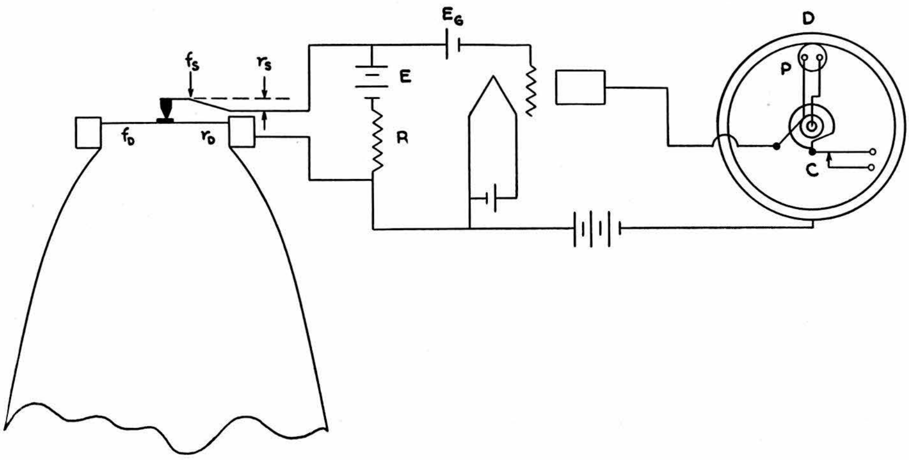

FIG. 12.

take into account the change in amplitude due to the resonant character of the diaphragm itself. This effect will increase the selectivity. An experimental determination of the selectivity over a small frequency range was obtained by driving the diaphragm by an electrostatic grill. These data have been plotted as a broken line in Fig. 11.

The electrical connections of the microphone and the indicating mechanism are shown in Fig. 12. A single three-element vacuum tube has connected in its plate circuit a neon lamp P. With the microphone contact closed, the small battery E and the resistance R are short-circuited and the grid potential now is maintained at a negative value E_G. The value of E_G is chosen so that the neon tube in the plate circuit is below its striking potential. When the microphone contacts are broken, the potential on the grid is $(E-E_G)$, a sufficiently positive potential to cause the neon lamp to glow. When a sound of sufficient intensity strikes the microphone diaphragm it causes the contact to break. For the duration of the sound the contact is made only momentarily once each cycle, resulting in substantially a direct current through the neon tube. An oscillogram, taken on a cathode-ray oscillograph, of the potential across the neon tube is shown in the lower half of Fig. 9. The potential across the lamp drops below the striking potential for only a very short interval of time each cycle. The drop is so rapid that only a few irregular contacts show on the oscillogram. The neon lamp is mounted on a shaft which is driven at a constant speed by a small electric motor. Attached to this shaft is a cam which makes and breaks a contact once each revolution. This contact breaks a local circuit through the electromagnet E (Fig. 8) and causes the clapper C to strike the diaphragm once each revolution of the neon lamp. At each contact a short pulse of sound is sent out. The neon lamp is adjusted so that it is exactly behind the zero of a circular scale at this instant. The return of the sound pulse as an echo is indicated by a flash of the lamp, of short duration. The position of the flash on the circular scale depends on the time of transit of the sound pulse and on the speed of rotation of the neon lamp. The experimental altimeter is calibrated for a temperature of 20°C and for zero air speed. Corrections for other ships' speeds and temperatures were summarized in Fig. 3 and Fig. 4. These corrections can be applied by adjusting the speed of rotation of the lamp.

A check on the mechanical operation and calibration of the altimeter was obtained by measuring distances in a large gymnasium. The altimeter was set up on a movable table and moved until an observer read a given distance on the instrument. The actual distance was then measured. A typical set of readings taken by two different observers are given in Table II. In order to determine the overall selectivity of the device, a loudspeaker was located in approximately the same relative position from the microphone as the engines would be in an actual ship. A phonograph record of the noise of a plane

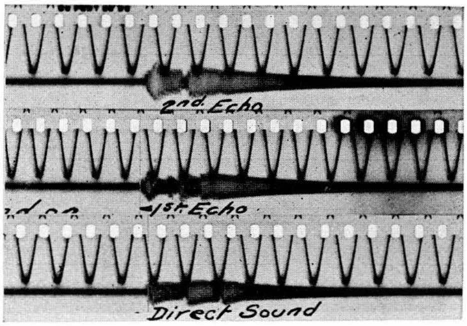

FIG. 13.

TABLE II.

Distance by altimeter	Measured Distance L. P. D.		J. H. M.		Average
10′	10′ 0″	10′ 3″	10′ 0″	10′ 5″	10.16′
50′	48′ 0″	50′ 1″	52′ 1″	52′ 0″	50.5′
120′	122′ 2″	122′ 9″	121′ 2″	122′ 5″	122.08′

was then introduced into the loudspeaker and adjusted so that the intensity was equal to that of an airplane. Oscillograms of the direct sound from the impulsive source, together with the first and second echoes obtained from reflections between two buildings 350 feet apart, are shown in Fig. 13. The absence of any disturbance, other than that due to the impulsive source, shows clearly the filtering action of the microphone.

FIELD TESTS OF THE ALTIMETER

Field tests of the altimeter were made from the Goodyear Airship Volunteer. The general arrangement of the impulsive source and selective microphone is shown in Fig. 14. The microphone

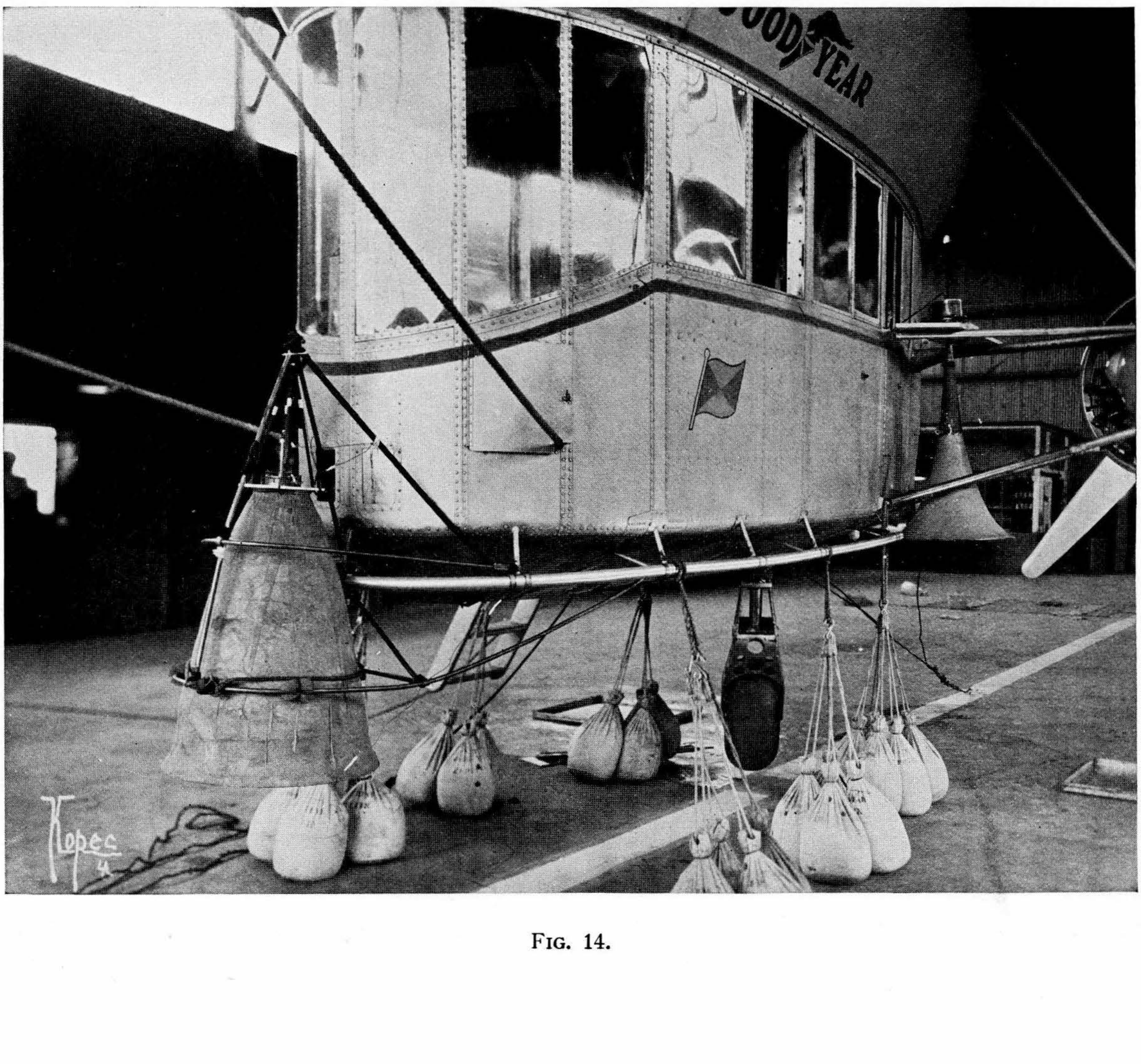

FIG. 14.

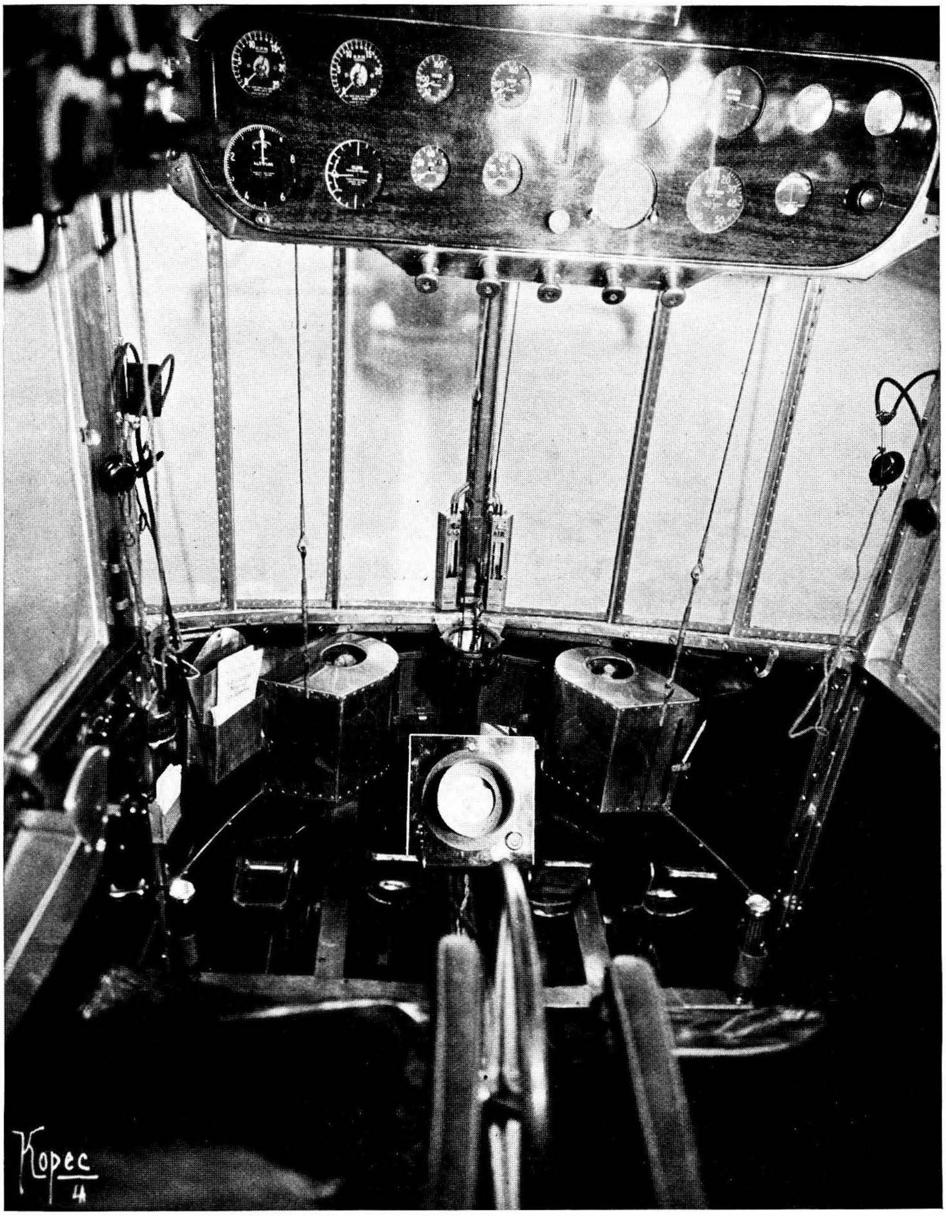

FIG. 15.

and parabolic reflector were suspended by rubber shock cords and held clear of the supporting frame by sponge rubber in order to minimize mechanical vibrations due to the ship. Fig. 15 shows the temporary location of the indicator in the cabin of the ship. At the beginning of a flight the contact spring S (Fig. 10) was adjusted so that the neon lamp just failed to flicker with the ship's noise alone. The indicator was then started. Observations were made over a variety of reflecting surfaces. It was found possible to measure altitudes as high as 350 feet and as low as 4 feet with this source of sound and as high as 700 feet with a somewhat stronger source.

A typical set of readings on the acoustic altimeter with the corresponding barometric altimeter readings is shown in Fig. 16. Since over short periods of time the barometric altimeter may be assumed constant, these two sets of readings make it possible to plot a cross section of

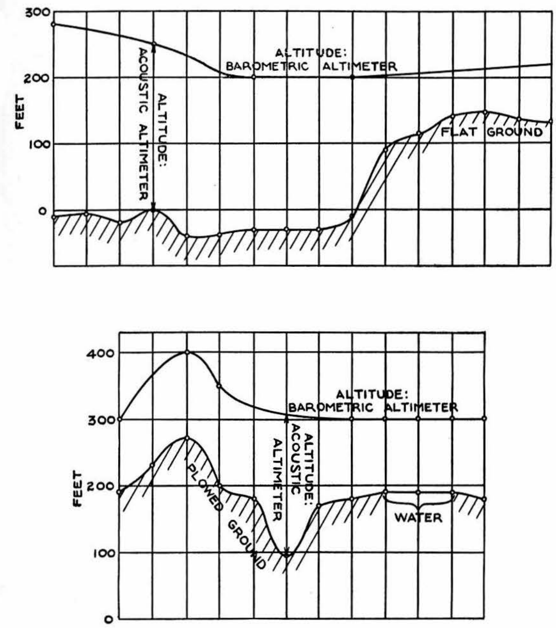

FIG. 16.

the surface beneath a ship. This may prove useful in a heavy fog, as an example, to determine when a pass has been successfully negotiated. Under abnormal storm conditions the barometric altimeter may be in error by as much as 700 feet.

It should be pointed out that although some progress has been made in the use of sound as an aid to avigation, several problems of both a physical and strictly engineering nature remain to be considered. Important among the purely physical problems is the study of the attenuation of sound in a turbulent atmosphere, while among those of an engineering nature is the problem of reducing the weight of the altimeter.

In concluding this paper I wish to express my gratitude to the Guggenheim Fund for the Promotion of Aeronautics for the financial assistance, to the Bureau of Aeronautics of the Navy Department, to the Army Air Corps, and to many commercial companies, for the use of equipment. The investigation was made particularly pleasant by the kindly interest and helpfulness of Dr. V. O. Knudsen. Mr. John Hammond Munier assisted in much of the experimental work.

A NEW ACOUSTIC ANALYZER—DETERMINATION OF THE SOUND SPECTRA PRODUCED BY AIRCRAFT IN FLIGHT[1]

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ABSTRACT

An Electro-Mechanical Frequency Analyzer has been adapted to the determination of the sound spectra of aircraft in flight. The complex source of sound is converted into potential fluctuations by a microphone and high quality amplifier. The output of the amplifier is applied to a modified quadrant electrometer, the needle of which forms part of a sharply resonant mechanical circuit. The natural period of the mechanical circuit is made continuously variable by a tri-filar suspension and can be determined from a single micrometer reading. An analysis of a complex source is made by observing the amplitude of vibration and natural period of the mechanical circuit for each component. By a previous calibration of the instrument these observations are sufficient to determine the amplitude and frequency components of the complex sound. The instrument has been used to measure the sound spectra of typical passenger and military planes in flight.

During the summer of 1927 some preliminary work was undertaken to determine the practicability of using acoustic methods for measuring the altitude of aircraft. An attempt was first made to determine, with the aid of a portable oscillograph and amplifier, the sound spectra produced by typical planes while in flight. Records were obtained which, while they made possible an estimate of the frequency range to be expected, required an inordinate amount of labor for their interpretation under all operating conditions. It became obvious after several such tests that an instrument capable of making an analysis while in flight was needed. The size, weight, or fragility of the available electrical analyzers rendered them impractical for this special work. It is the purpose of this paper to report the theory, design, and calibration of an analyzer which has been found well suited for use in aircraft, and which it is believed may find other applications in the laboratory.

Theory

In common with other acoustic analyzers this instrument consists principally of a sharply resonant system which is capable of being tuned to the various components of the complex sound to be analyzed. It differs from similar instruments in that the resonant system is me-

[1] This work has been made possible by the financial assistance of the Daniel Guggenheim Fund for the Promotion of Aeronautics Inc. The use of the Goodyear Blimp, *Volunteer*, was kindly donated by the Goodyear Zeppelin Company for this preliminary experimental work.

chanical rather than electrical. The details of the resonant system are as follows: A small aluminum needle and concave mirror (Fig. 1) is supported by three fine tungsten wires (a_1) (a_2) (a_3) (Fig. 2). Tension on the system is regulated by a micrometer head through an elliptic spring (Fig. 2). The angular separation of the wires (a_2) and (a_3) can be ad-

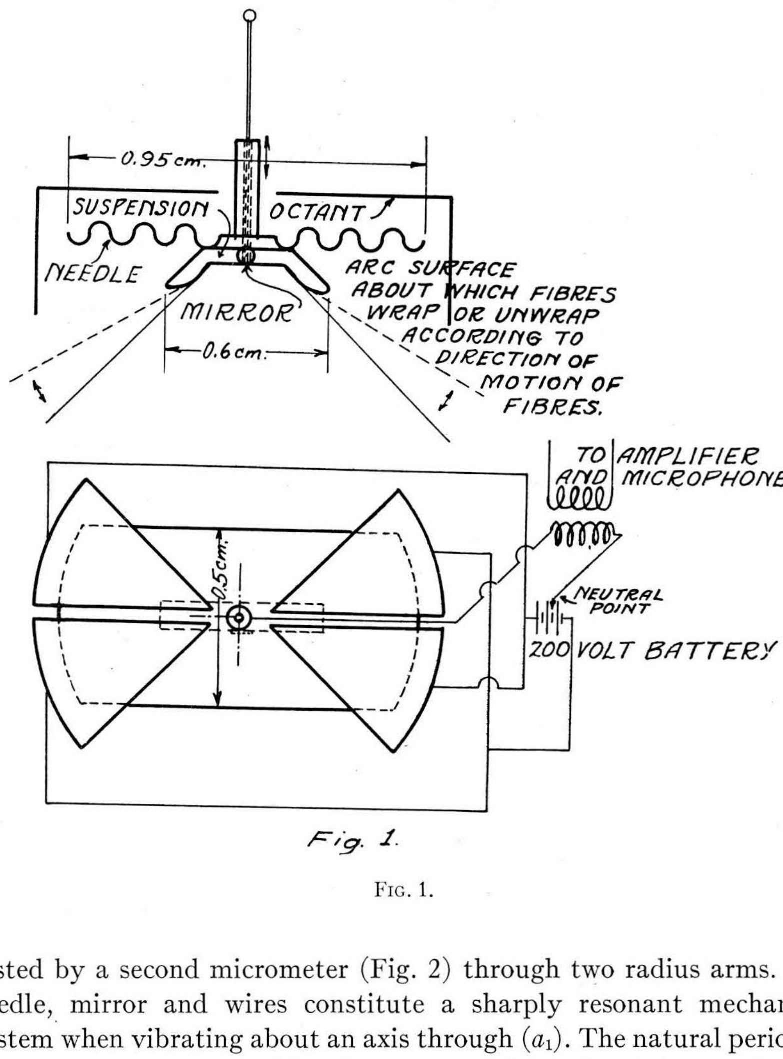

FIG. 1.

justed by a second micrometer (Fig. 2) through two radius arms. The needle, mirror and wires constitute a sharply resonant mechanical system when vibrating about an axis through (a_1). The natural period of the system is determined by the moment of inertia of the needle and mirror and by the restoring torque supplied by the suspensions. The restoring torque is due partly to the twisting of the suspensions but principally to the bifilar action of the two lower ones (a_2) (a_3). The manner in which the natural period of the system varies with the angular separation of (a_2) and (a_3) can be seen from the following considerations.

In Fig. 3 let the points A and B represent the points on the needle to which the suspensions (a_2) and (a_3) are attached.

$2b$ = the distance from A to B

a = the length of the suspensions (a_2) and (a_3)

2ϕ = the angular separation of the suspensions (a_2) and (a_3)

G_0 = torque per radian supplied by the suspensions to the needle

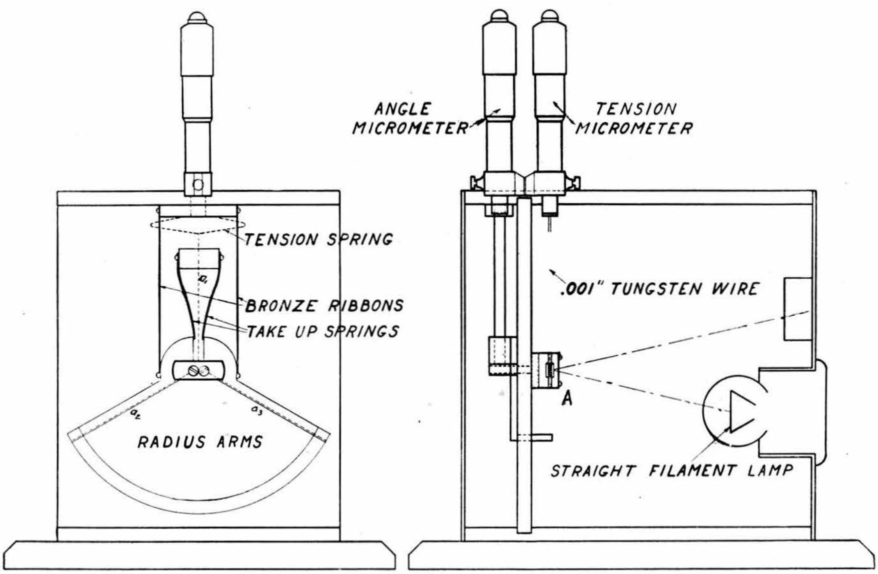

FIG. 2.

If we let A' and B' represent the points A and B after the element has been turned through a small angle θ, and assume that the wires do not stretch, a slight motion (h_1) of the element parallel to (a_1) will result. The force (f) in suspension (a_1) will remain practically constant for small displacements of the needle. Applying these conditions it is possible to obtain an approximate expression for the restoring moment and natural frequency of the system. In Fig. 3 imagine a plane passed through the point B perpendicular to the line CD meeting it at E.

From the geometry of the system we obtain the following relations:

$$x = b\cos\theta \quad y = c - b\cos\theta$$

$$z^2 = a^2 - y^2$$

$$s^2 = z^2 + x^2$$

$$h_\theta^2 = s^2 - b^2$$

$$h_\theta^2 = a^2 - y^2 + x^2 - b^2$$

$$h_0 = a \cos \phi \qquad h_1 = h_0 - h_\theta .$$

Substituting for x and y

$$h_1 = a \cos \phi - \sqrt{a^2 - b^2 - c^2 + 2cb \cos \theta}. \tag{1}$$

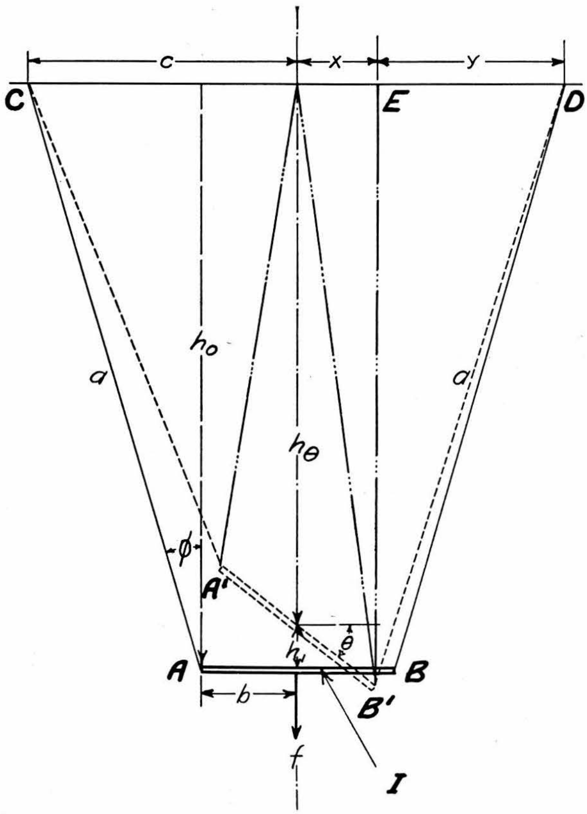

FIG. 3.

If G represents the couple to hold the rod at an angle θ then the work to turn it through an angle $d\theta$ will be $-Gd\theta$ and the work to move it through a distance dh_1 will be fdh_1 giving

$$- G\, d\theta + f\, dh_1 = 0 \tag{2}$$

$$\frac{dh_1}{d\theta} = \frac{G}{f} \cdot$$

Since

$$h_1 = a \cos \phi - \sqrt{a^2 - b^2 - c^2 + 2cb \cos \theta}.$$

and

$$\frac{dh_1}{d\theta} = + \frac{cb \sin \theta}{\sqrt{a^2 - b^2 - c^2 + 2cb \cos \theta}}$$

$$G = f\frac{dh_1}{d\theta} = \frac{fcb \sin \theta}{\sqrt{a^2 - b^2 - c^2 + 2cb \cos \theta}} \cdot$$

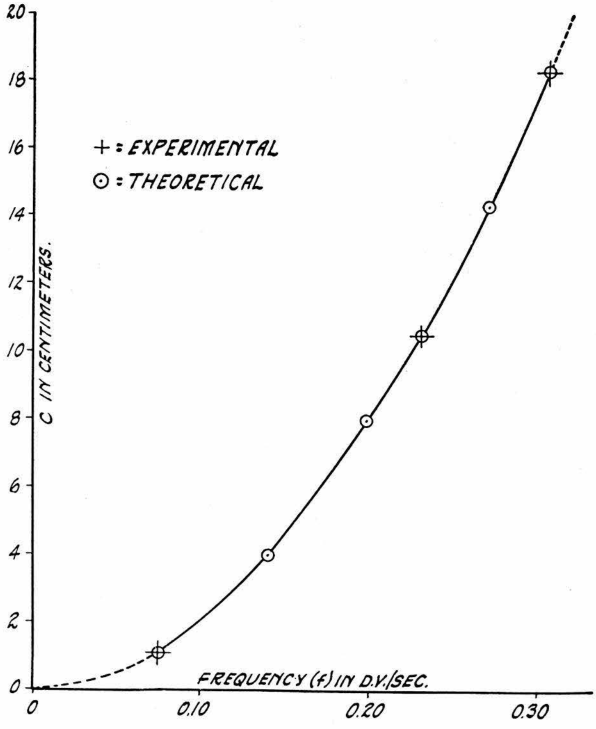

FIG. 4.

So that G is given by

$$G = \frac{fcb \sin \theta}{\sqrt{a^2 - b^2 - c^2 + 2cb\left(1 - 2\sin^2 \frac{\theta}{2}\right)}} \cdot$$

Now since θ is assumed small we may set $\sin \theta = \theta$ and neglect $\sin^2 \theta/2$ giving as the torque per radian

$$G_0 = \frac{G}{\theta} = \frac{fcb}{\sqrt{a^2 - b^2 - c^2 + 2cb}} = \frac{fcb}{\sqrt{a^2 - (c-b)^2}} \cdot$$

Now substituting the value of G_0 in the expression for the frequency (n)

$$n = \frac{1}{2\pi}\sqrt{\frac{G_0}{I}}$$

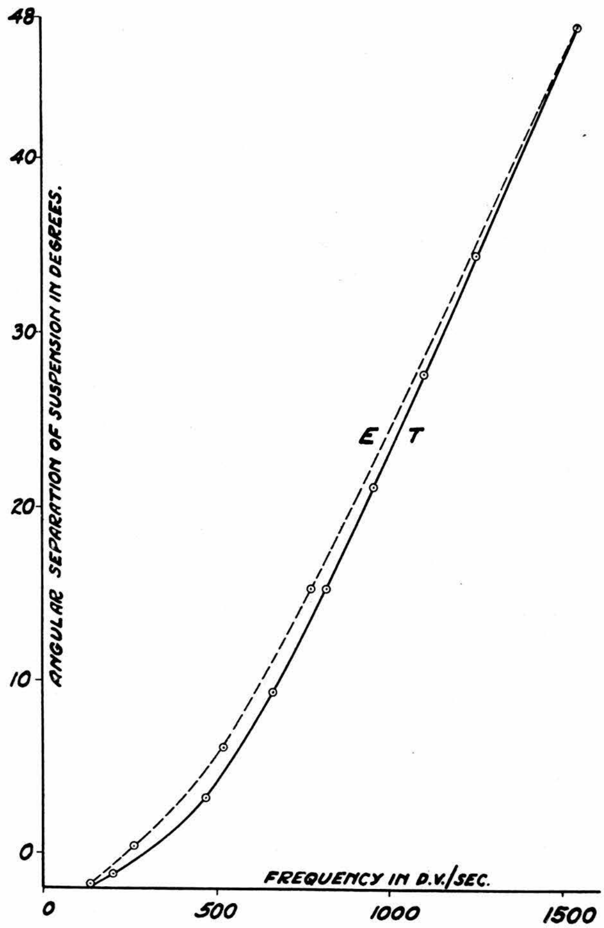

FIG. 5.

we have

$$n = \frac{1}{2\pi}\left[\frac{fcb}{I\left[a^2 - (c - b)^2\right]^{1/2}}\right]^{1/2}$$

where I is the moment of inertia of the needle. It may be noted that this expression reduces to the familiar one for a parallel bifilar suspension when we set $c = b$, and that the natural frequency (n) increases rapidly with increasing angular suspension of (a_2) and (a_3).

The resonant system consisting of the needle and mirror is stimulated electrostatically from the source to be analyzed by surrounding it with the four quadrants of an electrometer. The electrical connections to this part of the analyzer are shown in Fig. 1. For the analysis of weak

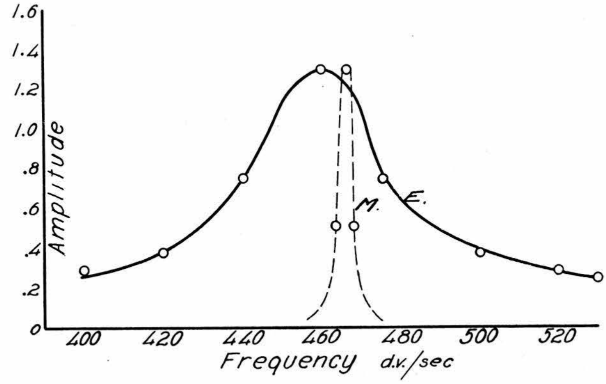

Fig. 6.

sounds a condenser transmitter and a high quality amplifier are used to supply the fluctuating potential to the needle. A potential difference of 200 volts is maintained between the quadrants to increase the sensitivity of the instrument.

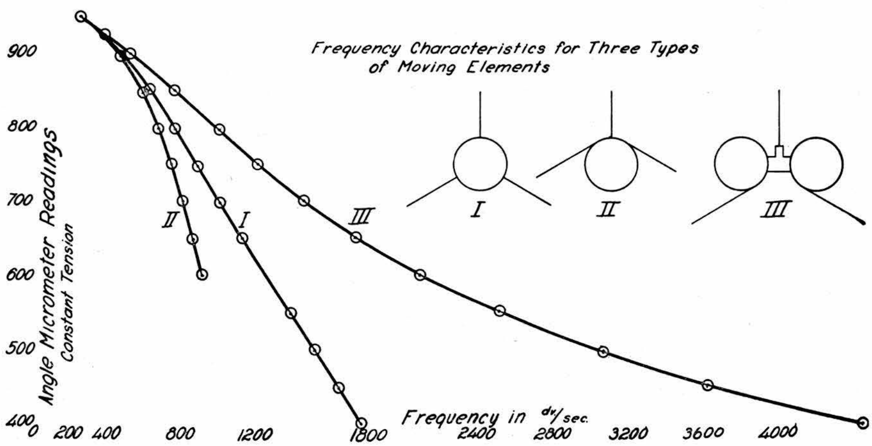

Fig. 7.

The agreement between the theoretical curve obtained by assuming perfectly flexible suspensions and an experimental calibration of a system suspended by strings is shown in Fig. 4. The agreement between the theoretical curve and a calibration of the analyzer in which the effect of torsion of the suspensions is present is shown in Fig. 5. The "b" distance in this instrument was fixed. A comparison of the resonance curves for the mechanical system of this analyzer and a typical electrical system in the same frequency range is shown in Fig. 6. The sharp resonance of the mechanical system permits adjacent frequency components to be readily resolved. The frequency range of the present

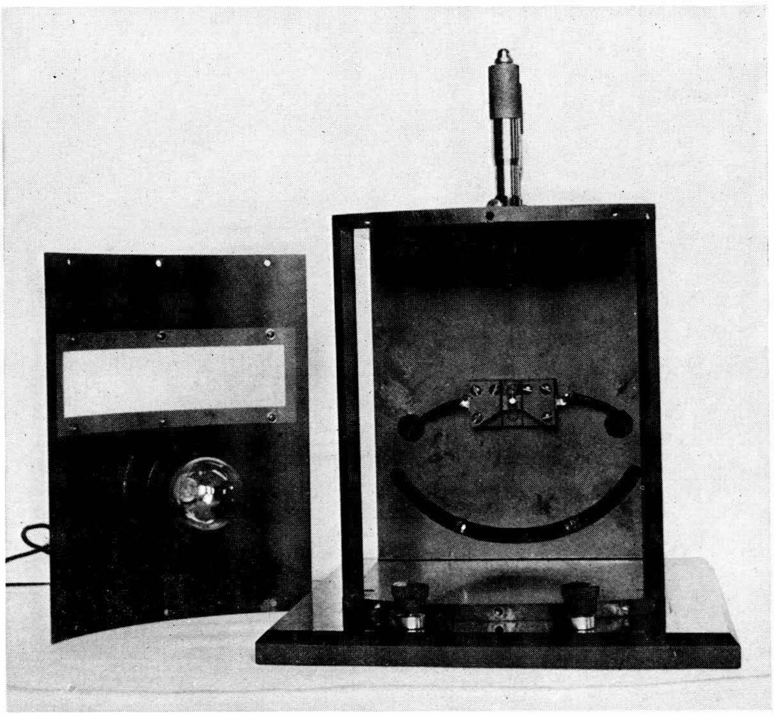

FIG. 8.

analyzer is from 20 to 550 d.v./sec. It is apparent from inspecting the expression for the natural frequency that this range can be extended upward by decreasing the moment of inertia of the needle, by using shorter suspensions or by increasing the "b" distance. The tension may also be increased to this end.

DESIGN

The incorporation of the foregoing theory into a rugged portable instrument was naturally attended with some difficulties. It was found that the simple construction of the element shown as I of Fig. 7 was not practical because of the constant bending of the suspensions at the points of attachment. The construction shown as II of Fig. 7 was discarded because of the limited frequency range that could be obtained. Finally the construction shown as III in Fig. 7 was adopted since it is free from these defects. The "b" distance of the formula in this case is not constant but increases with the angular separation of the suspensions. The calibrations in this figure have been reduced to a common frequency at the lower end in order to compare the three methods of attaching the suspensions to the element.

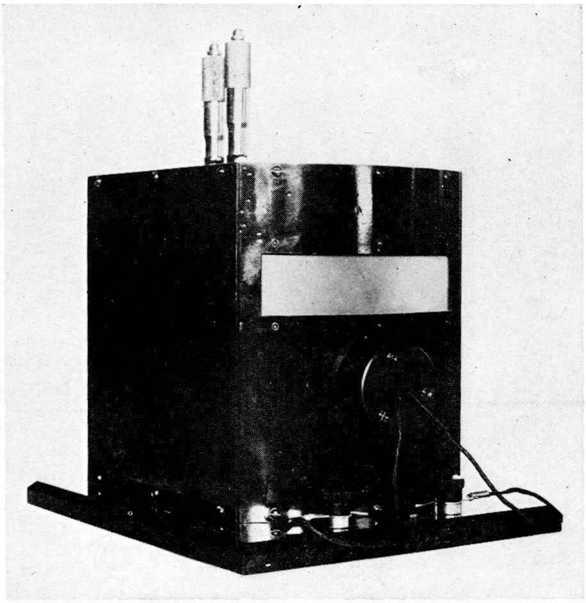

FIG. 9.

A small concave mirror attached to the needle reflects an image of a straight filament lamp onto a frosted scale. The amplitude of vibration of the needle is measured by the spreading of this image. The resonant setting for any particular component is readily made by turning the micrometer head until a maximum spreading of the filament image results. Fig. 8 shows a broken, and Fig. 9 an assembled, view of the analyzer. A double shielded box contains the two stage amplifier and batteries necessary to operate the instrument. The shielded box serves also as a base upon which to mount it.

When used in aircraft, or under other conditions where mechanical vibration is encountered, the complete unit is supended by elastic cords. The microphone is also elastically suspended. Fig. 10 is a complete view of the instrument as used under these conditions.

CALIBRATION AND USE

The calibration of the instrument for frequency was made by two methods. In the first the sounds from standard turning forks were

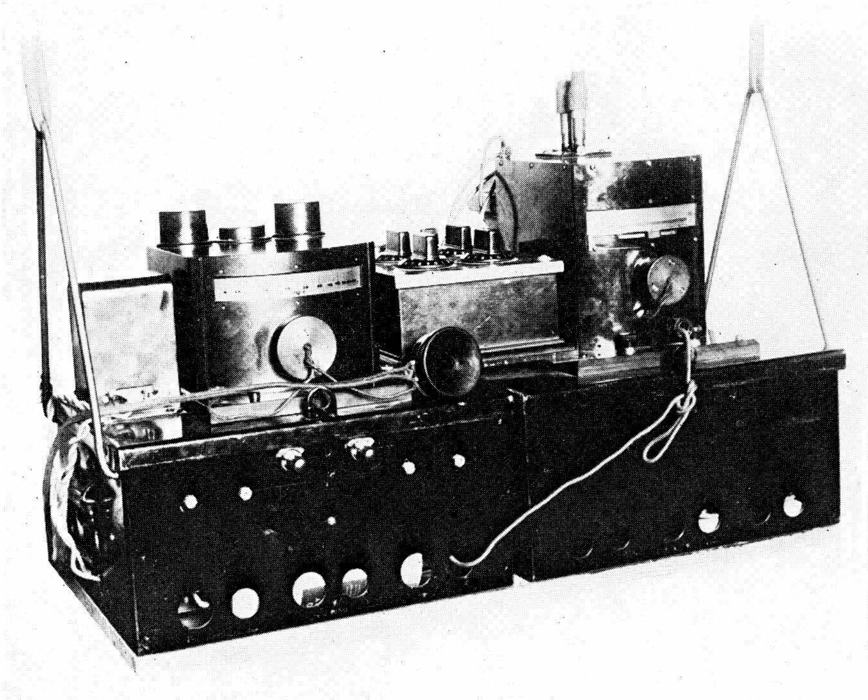

FIG. 10.

amplified and the setting of the angle micrometer noted which gave a maximum amplitude of vibration of the filament image in each case. In the second method the harmonics obtained by overloading the amplifier were used. The results of these two methods of calibration are plotted in Fig. 11.

Calibrations of the microphones, amplifier and analyzer for intensity were made by a modification of the method due to Gerlach in which the sound pressure is opposed by electrodynamic forces.

The analysis of a steady sound is made by turning the micrometer head slowly through its entire range observing the readings at which the component frequencies give maximum spreading of the filament

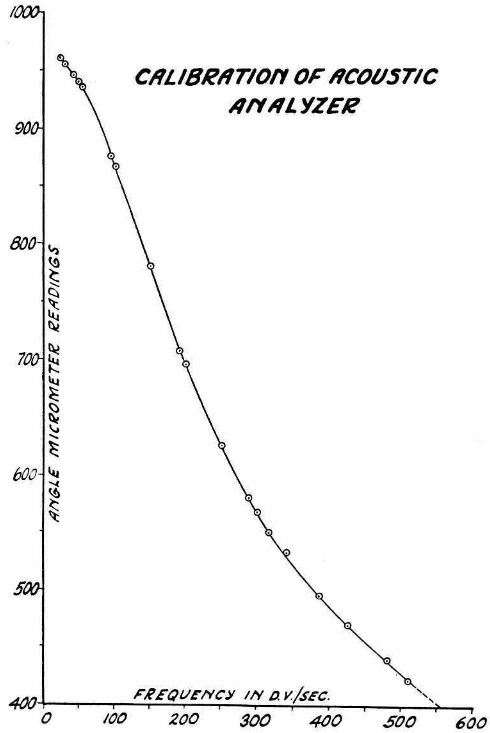

FIG. 11.

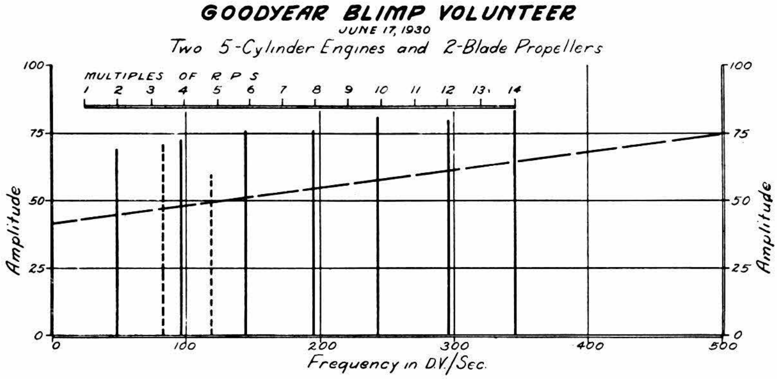

FIG. 12.

image. The spread of the image in each case is a measure of the amplitude of the particular component, and the micrometer reading a measure of the frequency. A typical sound spectrum, that of the Goodyear Blimp, *Volunteer*, is shown in Fig. 12. This record was obtained at an average altitude of 1000 feet with the microphone in the center of the passenger compartment and the windows open. Frequency components which are even multiples of the angular velocity of the propellers are very prominent in this case.

The analyzer has been used up to the present principally for the analysis of intense sounds such as those produced by aircraft in flight and has demonstrated its ruggedness and constancy of calibration in this severe service. It is believed that because of these features the analyzer may find other uses in the laboratory.