CONSTRUCTION OF A MODEL NOMOGRAM TO SHOW THE VARIATION OF INSOLATION WITH SEASON AND LATITUDE

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In constructing this Model Nomogram, the main objective was to give a graphical three dimensional picture of relative amounts of insolation received over the earth's surface at all seasons. In this way, not only the amounts of change could be shown, but also the different degrees of the rates of change, which play a very important part in the location of the Arctic, Polar, and Sub-Polar fronts.[3]

The amount of Solar radiation reaching a unit area of any part of the earth's surface in one day depends upon:

a. The Solar constant
b. The transparency of the atmosphere
c. The latitude
d. The time of year

Since this problem is mainly concerned with the variation of the sun's radiation with season and latitude, certain assumptions must be made. In order to make the variation presentable, the earth's atmosphere is assumed to be transparent. By doing this, a uniform non-variation of the sun's radiation field is obtained over the whole earth. It is of interest to note at this point that about 43 per cent of the total heat received at the outer layers of the atmosphere is lost, due to scattering and reflection of the rays.*

Another term that must be considered is that of the solar intensity, or solar constant. The value of the solar constant is derived from a series of observations over a period from 1905 to 1926, taken over various parts of the world. The value varied very little in the series of observations, and is therefore taken as a constant of 1.94 gram calories per cm^2 per minute, at the outer limit of the earth's atmosphere with the sun at the zenith, and reduced to its value at mean solar distance--which is the distance equal to one-half of the major axis of the earth's orbit.

The actual variations in the amount of heat energy received from the sun thus conveniently becomes a function of latitude and season. The season is dependent on the position of the earth in its orbit. It will be noted that figures for the Southern Hemisphere do not correspond with the figures for the Northern Hemisphere shifted through six months. This is due to variation

* According to Abbot, Fowle and Aldrich in Ann. Astrophys. Obs. Smithsonian Instituion, 4; 381, 1922.

in the distance from the earth to the sun across the major axis of the earth's orbit. The totals for the whole earth show a maximum in December for the Southern Hemisphere, and a maximum in June for the Northern Hemisphere. The maximum of December being greater than the maximum of June, corresponds to the times of perihelion and aphelion, or the times when the earth is closest to and farthest from the sun respectively. At aphelion, the distance of the earth from the sun is about 1.034 that at perihelion, thus by the law of inverse squares the intensity of heat energy must be at perihelion, 1.069 that at aphelion.[1.] It must be noted here, however, that in the course of a year, each hemisphere receives equal amounts of heat energy because although at perihelion the earth is closer to the sun, the path of the orbit is shorter than at aphelion. Thus what is lost or made up in distance is also lost or made up in time, as the case may be.

In the consideration of the variation of the latitude, it is shown that a variation of the intensity of the sun's radiation on a sloping surface is a function of the angle that this sloping surface makes with a surface perpendicular to the sun's radiation. Or that the sun's radiation is directly proportional to the sine of the angle or solar altitude, or to the cosine of the sun's zenith distance. These angles can be expressed in observable quantities as a function of latitude, of the time of day, and solar declination, and may be expressed by the following formula:

1. $$Q = \frac{2I}{\omega}(\sin\varphi \sin\delta H + \cos\varphi \cos\delta \sin H)$$

in which Q= amount of solar energy delivered in a given time, δ is the sun's declination, φ is the latitude of the point in question, H is the sun's hour angle, ω is angular velocity of earth's rotation, and I is the solar constant.

In order to represent the degree of insolation received at various latitudes and seasons, the following unit is calculated. On the day of the vernal equinox at a station located on the cellestial equator, it follows that $\delta = 0$ and the hour angle is $\frac{\pi}{2}$ and the latitude is 0°. Substitution in equation 1. yields:

$$Q = \frac{2I}{W}$$

Where I is the solar constant and equal to 1.94, and $W = \frac{2\pi}{1436}$ radians per minute, then:

$$Q = \frac{2(1.94)}{\frac{2\pi}{1436}} = 889 \text{ Calories/cm}^2$$

The values applied to the model nomogram are expressed in terms of the unit calculated above, namely, the insolation received at the equator on the day of the vernal equinox.

The Model Nomogram is constructed in three dimensions with Latitude and Seasons (by months) on a horizontal plane at right angles to each other, and units of insolation on the vertical scale. The horizontal base is made of plywood, 24 x 24 x 3/8 inches. The background is painted white with lines of latitude and months ruled in black ink. The month lines are 2 inches apart, the latitude lines are 1¼ inches apart. The vertical curves are drawn on a celluloid sheet 10 inches high by 23 inches long, and represent the variation of insolation with latitude for each individual month. There are thirteen such curves, with December being the first and last curve in consecutive order. The curves are mounted upright on the plywood base.

The nomogram is made with a vertical scale equal to 4 such units for each inch, hence for any latitude and/or season the degree of insolation may be rapidly scaled to a small degree of error.

It is well to take each monthly curve as representative for the 15th day of that month; therefore, for the 15th day of October at latitude 60°N there would be 9/30 times 889 calories per cm^2 received during that day. Likewise for the 15th day of April at latitude 20°N there would be 30.7/30 times 889 calories per cm^2 received during that day. A curve running parallel to the latitude can be extrapolated between the 15th day of consecutive months. It would, therefore, follow that the 1st day would be midway between each monthly curve. Using this method it would have to be assumed that each month contained 30 days, which is a satisfactory assumption.

If it is further desired to reduce the degree of insolation with respect to time, reference is made to a Thesis by Capt. J. Vern Hales, in which he has divided the day into hourly variation of total insolation incident since sunrise for the Northern Hemisphere.[4]

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